The consequences of a shorter working time: Some lessons from a general equilibrium analysis

Pierre CAHUC
MAD, Université de Paris I
Pierre GRANIER(•)
GREQM, Université d'Aix-Marseille II

Introduction

A reduction in working time is often seen as a way of sharing the available numbers of jobs and increasing employment. If unemployment is keynesian, reducing working time induces work sharing and increases employment, because global demand for hours is given. But, the persistence of unemployment in Europe suggests that unemployment is to a large extent of classical nature. In this perspective, neoclassical labour demand theory can lend some support to a reduction in working time. With fixed hourly wages, a reduction in hours increases employment provided that the elasticity of labour's marginal product with respect to hours is smaller than unity (Andersen [1987], Houpis [1993]). But it is generally recognised that hourly wages may increase after a reduction in working time, because workers are generally reluctant to accept a decrease in their revenue. Such a reaction should question the efficiency of work sharing policies. Therefore, it is worth coping with the relationship between wages and working time in order to examine the consequences of policies that aim at work sharing.

Some contributions analyse such an issue in a partial equilibrium framework. Hoel [1984], Calmfors [1985] and Booth and Schiantarelli [1987] study the reaction of the hourly wage to reductions in hours in monopoly unions models, or in wage bargaining models. They conclude that a reduction in working time has either a negative, or, at best, an ambiguous effect on employment. Hoel and Valle [1986], and Layard,

(•)We wish to thank Xavier Fairisse, Louis-André Gérard-Varet, Hubert Kempf, Alan Manning, André Zylberberg, an anonymous referee of the Recherches Economiques de Louvain, and participants at seminars in Paris (MAD), Louvain la Neuve (IRES) and Aix en Provence (GREQM) for comments. All remaining errors are ours.
Nickell and Jackman [1991] reach the same result in general equilibrium models with negotiated wages, or efficiency wages.

However, these results are questionable. They rely upon a framework that neglects many phenomenon that can influence the relationship between working time and employment. The general equilibrium models of Hoel and Valle [1986] and Layard, Nickell and Jackman [1991] take into account neither the utility of leisure, nor the influence of working time on labour force participation, and moreover, they consider hours as exogenous. Nevertheless, the relationship between the legal and the actual working time, negotiated or chosen by the firm, is not obvious. Hart [1984, 1987] and Calmfors and Hoel [1988] showed that a reduction in the legal working time can have ambiguous effects on the actual working time, at the firm level, with exogenous hourly wages. This underscores that it is worth analysing the influences of a reduction in the legal working time, both at partial and general equilibrium levels, when wages and hours are endogenous. Moreover, a more recent paper by Houpis [1993], who introduces the utility of leisure in a general equilibrium model with endogenous wages, claims that reductions in hours of work are not likely to lead to an increase in the hourly wage and will reduce unemployment.

The aim of our paper is to analyse, both at partial and general equilibrium, the consequences of work sharing policies, when the wages, the length of the working time and the participation rates are endogenous, in an economy where workers get utility from leisure. The behaviour of the individuals and of the firms are presented in the first section. Section two copes with the determination of employment, hours and wages when collective bargaining on wages only is compulsory. In such a framework, we explain why firms are incited to precommit on hours, before the wage bargaining. This result induces us to study the effects of work sharing policies when firms are precommitted on hours. We evaluate the consequences of different work sharing policies. Namely, we consider the effects of variations in the legal working time, in overtime premium, in the maximal working time and the consequences of the introduction of compulsory negotiations on hours. The consequences of these policies are studied at partial equilibrium in a first step (section 3). Then, we cope with general equilibrium at stationary state. We consider a model in which the natural rate of unemployment is determined in the same way as Layard, Nickell and Jackman [1991], but with endogenous labour force and number of firms (section 4). This allows us to show that the consequences of work sharing policies are very different at partial and at general equilibrium. The limits of the model and some possible extensions are discussed in the conclusion.
1 The model

We study the steady state of a continuous time economy with two goods: labour, that can be offered by individuals living forever, and a good produced by many infinitely lived identical competitive firms.

1.1 Individuals

The economy is composed of a continuum of heterogeneous individuals. The size of this continuum is normalised to one. Each individual has the choice of staying out of the labour force and enjoying a real present discounted utility $V_0$, or entering the labour force to look for a job, for a real present discounted expected utility $V_u$. The return when out of the labour force, $V_0$ differs across individuals. We denote by $G(V_0) : [0, +\infty] \rightarrow [0, 1]$, the repartition function of individuals. An individual will participate if $V_0 \leq V_u$. Therefore, the size of the labour force is equal to: $G(V_u)$. These assumptions describe a discouraged-worker effect. A worker is more reluctant to participate when the return from participation is low. It could be possible to take into account an added-worker effect, that appears when one considers the participation decision within households. But empirical studies found that the discouraged worker effect dominates the added-worker effect (Pissarides [1990], chap 6). Accordingly, we do not consider this last effect.

Each member of the labour force (henceforth a worker) can be either employed or unemployed. An employed worker is paid a revenue $\Omega$ and works $h$ hours during a small interval of time $dt$. An unemployed worker has no revenue. The instantaneous utility of a worker, paid a revenue $\Omega$ and working $h$ hours during a small interval of time $dt$, is defined by:

$$U(\Omega, h) = \Omega^\gamma g(h) \quad ; \quad \gamma > 0$$

(1)

with $g(0) > 0$ ; $g'(h) \leq 0$; $g''(h) \leq 0$.

Such an utility function implies that the optimal number of hours desired by a worker does not depend on the hourly wage $w$, if $\Omega = wh$ (i.e. the income effect exactly compensates the substitution effect). Empirical studies on this issue yield questionable results: they estimate the observed hours, which do not correspond to desired hours, because there are many constraints on working time (such as demand side constraints or institutional constraints). Moreover, empirical studies found small, either positive or negative elasticities of the hours with respect to the hourly wage (Fallon and Very [1988], Pencavel [1986], Killingsworth and Heckman [1986]). Thus, it seems relevant to assume, as a first approximation, that the desired hours do not depend on hourly wages.

Workers move between employment and unemployment. The flow into unemployment results from exogenous job separation that follows
a Poisson process with rate $s > 0$. In a steady state, if we denote by $u$ the unemployment rate, the probability, denoted by $q$, of leaving unemployment during a small interval of time $dt$ is:

$$q = \frac{s(1-u)}{u}.$$  \hfill (2)

Let $V$ be the present discounted expected utility of a worker employed in a given firm, and $V_u$ the present discounted expected utility of an unemployed worker. If the workers have a discount rate $r$, $V$ and $V_u$ satisfy:

$$rV = \Omega^\gamma g(h) + s(V_u - V)$$  \hfill (3)

$$rV_u = q(\overline{V} - V_u)$$  \hfill (4)

where $\overline{V}$ represents the present discounted expected utility of being employed in any firm. For the sake of simplicity, we assume that the unemployed workers do not receive unemployment benefits. The consequences of unemployment benefits are discussed in appendix.

1.2 Firms

During a small interval of time $dt$, each firm can produce an output $Y$ with the following technology:

$$Y = F(e(h)N) ; \quad F'(.) > 0, F''(.) < 0, e'(.) \geq 0$$  \hfill (5)

where $N$ is the employment level. $e(h)$ is a twice differentiable function, that measures the efficiency of an individual as a function of the length of the working time. The multiplicative separability between individual efficiency and employment in the production function means that the number of efficient labour units produced by each worker does not depend on the employment level within the firm. This assumption does not seem to be very restrictive in order to analyse the consequences of variations in the length of working time. This is why it is widely used in the literature that focus on such an issue (see, e.g., Ehrenberg, [1971], Hoel, [1986], Hart, [1987], Fitzroy and Hart, [1985], Calmfors and Hoel, [1988]).

We assume that $e'(h) \geq 0$. The individual efficiency function $e(h)$ plays a crucial role in the analysis of the consequences of variations in hours. It will appear that the elasticity of work efficiency, defined as $\varepsilon_e = (he'(h)/e(h))$, is a key variable. This is not surprising. If $\varepsilon_e$ is larger than one, an increase in working hours increases the efficiency of each hour of work. But, if $\varepsilon_e$ is smaller than one, an increase in working hours decreases the efficiency of each hour of work.
It is worth noting that a reduction in working time for a given hourly wage increases employment if the elasticity of the marginal product of labour with respect to hours (i.e. $F_N h/F_N$) is lower than one (Andersen [1987], Houpis [1993]). Hence, our specification of the technology allows us to consider cases such as a reduction in working time has either a positive or a negative effect on employment when the hourly wage is taken as given. Moreover, it allows us to take into account different degrees of substitutability between jobs and hours. If $e(h) = ah$, $a > 0$, there is a perfect substitutability between jobs and hours. But if $e(h) = ah^\sigma; \sigma \neq 1$; jobs and hours are imperfectly substitutable.

Empirical works that have been undertaken on this issue have given mixed findings. Feldstein [1967] and Craine [1973] find that the elasticity of work efficiency with respect to hours is larger than one, implying imperfect substitutability, but Leslie and Wise [1980] find an elasticity very close to unity, implying perfect substitutability. Hence, in this paper, we do not make a priori restrictions on the form of function $e(h)$.

We assume that there are no hiring cost. Hence, the labour cost per worker depends only on the hourly wage, on the length of working time and the overtime premium. Actually, in most of the OECD countries, when the actual working time is higher than the legal working time, denoted by $h_L$, firms pay exogenous overtime premium, denoted by $x \geq 0$. Then the instantaneous cost per worker is:

$$\Omega = w (h + x \max (h - h_L, 0))$$  \hspace{1cm} (6)

where $w$ denotes the hourly wage. Moreover, firms cannot go beyond a maximum working time denoted by $\bar{h}$.

Let $\Pi$ be the present-discounted value of profit in a firm. $\Pi$ satisfies:

$$r\Pi = F (e(h)N) - Nw (h + x \max (h - h_L, 0)).$$  \hspace{1cm} (7)

2 Wages, employment and working time determination: why firms should precommit on hours?

In this economy, wages are determined by collective bargaining at the firm level. We take as a benchmark a situation where the firms keep the right to manage employment and hours. Each firm is unionised and bargains the wage with one union. The union is only concerned with the $M$ given proportion of workers who belong to the employment pool of the firm, and wishes to maximise the present value of the expected utility of these workers. Since workers are identical, we can assume
that layoffs are by random assignment. Then, the objective of a union is:

$$V = \frac{N}{M} V + \left( 1 - \frac{N}{M} \right) V_u ; \quad N \leq M. \quad (8)$$

The outcome of the bargaining is given by the generalised Nash bargaining solution. We assume that, in case of strike, the firm makes no profit and the strikers can get the same expected utility as the unemployed. Therefore, the statu quo payoffs of the firm and the union are respectively $\bar{H} = 0$ and $\bar{V} = V_u$. Thus, the outcome of the wage bargaining is the solution to the maximisation of the following objective:

$$\Phi = N^{\beta} (V - V_u)^{\beta} (\Pi)^{1-\beta} ; \quad 0 \leq \beta \leq 1. \quad (9)$$

These assumptions are not sufficient to describe precisely the determination of wage, hours and employment, because the timing of the choice of variables must be specified. The firm can decide to be either precommitted or not precommitted on hours and/or employment. Usually, most of the literature assumes that firms are not precommitted on employment: the so called "right to manage model" (Nickell and Andrews, [1983]) rests on the assumption that firms adjust employment once the wage has been negotiated. We will keep this assumption throughout the paper. But there is no consensus about the strategy of the firm concerning hours. Earle and Pencavel [1990] stressed that industrial relations do not provide clear empirical evidence on this problem. Booth and Schiantarelli [1988] and Houpif [1993] assume that both hours of work and employment are set simultaneously, after the wage has been determined.

Indeed, as it is shown by proposition 1, such an assumption is questionable, because the firm prefers to make a precommitment on hours, before the wage bargaining.

**Proposition 1** The firm gets higher profits if it is precommitted on hours than if it adjusts hours once the wage has been determined.

**Proof**
The firm has two different strategies. It can be either precommitted on hours or not precommitted (these strategies are respectively denoted by $P_h$ and $A_h$).
If the firm chooses \( A_h \), the profit maximisation program yields the following working time and labour demand.

\[
    h^* = \text{Min} \left( \hat{h}, \bar{h} \right) \quad \text{with} \quad \begin{cases} 
    \frac{e'(\hat{h})}{e(\hat{h})} = 1 & \text{if } \hat{h} \in [0, h_L] \\
    1 \leq \frac{e'(\hat{h})}{e(\hat{h})} \leq 1 + x & \text{if } \hat{h} = h_L \\
    \frac{e'(\hat{h})}{e(\hat{h})} = \frac{1 + x}{1 + x (1 - h_L / h)} & \text{if } \hat{h} \in [h_L, \bar{h}] 
\end{cases}
\]

(10)

\[
    N^*(\Omega, h) = \frac{1}{e(h)} F^{\gamma - 1} \left( \frac{\Omega}{e(h)} \right) ; \quad \Omega \equiv w(h + x \max(h - h_L, 0)) ; \quad h = h^*. \quad (11)
\]

The wage negotiated by the union and the firm is the solution to the following program:

\[
    \text{Max}_w \Omega^\gamma g(h) - rV_u \beta N^\beta (F(e(h)N) - \Omega N)^{1-\beta} \quad (12)
\]

subject to:

\[
    N = N^*(\Omega, h) ; \quad \Omega = w(h + x \max(h - h_L, 0)) ; \quad h = h^*.
\]

Denoting respectively by \( \varepsilon_\pi < 0 \) and \( \varepsilon_N < 0 \) the elasticity of profit and employment with respect to \( \Omega \), we get a remuneration \( \Omega^*(h) ; h = h^* \) that depends on the working time that will be chosen by the firm:

\[
    \Omega^*(h) = \left( \frac{rV_u}{g(h)} \left( \frac{(1 - \beta)\varepsilon_\pi + \beta \varepsilon_N}{(1 - \beta)\varepsilon_\pi + \beta \varepsilon_N + \gamma \beta} \right) \right)^{1/\gamma} ; \quad h = h^* \quad (13)
\]

assuming: \( (1 - \beta)\varepsilon_\pi + \beta \varepsilon_N + \gamma \beta < 0 \).

Therefore, the optimal instantaneous profit of the firm can be written as:

\[
    \pi^A = \text{Max}_w F(e(h)N^*(\Omega, h)) - \Omega N^*(\Omega, h) \quad (14)
\]

subject to:

\[
    \Omega = w(h + x \max(h - h_L, 0)) ; \quad w = \frac{\Omega^*(h^*)}{h^* + x \max(h^* - h_L, 0)}.
\]

If the firm chooses strategy \( P_h \), the negotiation program is:

\[
    \text{Max}_w \Omega^\gamma g(h) - rV_u \beta N^\beta (F(e(h)N) - \Omega N)^{1-\beta} \quad (15)
\]

subject to:

\[
    N = N^*(\Omega, h) ; \quad \Omega = w(h + x \max(h - h_L, 0)).
\]
In programs (12) and (15) the length of the working time does not depend on the hourly wage. Therefore, the solution to program (15) is again the remuneration function $\Omega^*(h)$ defined in equation (13). But when the firm chooses the hours before the wage bargaining, the optimal instantaneous profit can be written as follows:

$$\pi^P = \operatorname{Max}_h F(e(h)N^*(\Omega^*(h), h)) - \Omega^*(h)N^*(\Omega^*(h), h).$$ (16)

Then, program (14) and (16) imply that: $\pi^P \geq \pi^A$. □

Proposition 1 explains why a firm prefers to sign contracts that specify the length of the working time when the firm keeps the possibility to adjust employment at each point of time. To precommit on the length of working time is a strategic choice, which allows the firm to get a lower wage. Actually, the very long-term of labour relations gives to the firms important incentives to respect their precommitments, in order to benefit from a good reputation (Bull [1987]). Thus, we shall assume that firms can make credible precommitments on hours. According to proposition 1, this assumption implies that firms are precommitted on hours in our model.

It is important to understand that this result does not mean that employment is less variable than hours, or that firms adjust more easily employment than hours. In the case of uncertainty, firms and workers should sign contingent contracts, that stipulate hours and wage as a function of the realisation of the states of nature. Thus, hours that maximise profit (equation (16)), would be variable, with, possibly, overtime working hours in some states of nature.

It can be easily checked that proposition 1 holds with a quasi-concave instantaneous utility function $U(\Omega, h)$. Hence, this result does not rely upon very restrictive assumptions. A sufficient condition is the multiplicative separability of individual efficiency and employment in the production function.

The next sections show that precommitments on hours have important consequences to examine the effects of policies that aim at reducing the length of working time.

3 Work sharing policies: partial equilibrium analysis

In this section, we examine the consequences of three different policies at the firm level. First, we study the effects of a shorter legal working time and of a higher overtime premium. Second, we analyse
the consequences of a shorter maximum working time. Actually, there
is generally a link between the legal working time and the maximum
working time. For instance, in France, the difference between the maxi-
mum and the legal working time amounts to 130 hours per year and per
worker. Therefore, a modification in the legal working time can induce
a modification in the maximum working time. But, here we study the
implications of these two variables separately. Such a distinction allows
us to show that the effects of variations in legal and maximum working
time are very different, when hours are endogenously determined by
firms. Eventually, we focus on the consequences of compulsory bargain-
ing on hours.

3.1 Legal working time and overtime premium

It is often considered that a decrease in the length of the legal
working time induces work sharing by decreasing the actual length of
working time. Indeed, such a measure would induce firms to cut back
on overtime hours and hire more workers. If hourly wages are taken as
given, labour demand theory lends some support to such a claim. But
if wages are endogenous, variations in the length of the legal working
time or in the overtime premium have rather different consequences.

Lewis [1969] argued that overtime pay regulation should not modi-
ify the working conditions if the hourly wages can be fully adjusted in
a bargaining process. The basic idea is that a job is simply a package
of compensation and working conditions. Accordingly, modifying over-
time premium or legal working time will have no real effects since, by
adjusting the hourly wage, the firm can offer the same package of com-
penations and hours that was acceptable to the worker initially. The
following proposition shows that the bargaining process should end up
in such a wage adjustment.

**Proposition 2** The employment, the remuneration per worker, and the
actual length of working time are independent from the legal length of
working time and from the overtime premium.

**Proof**
Let us describe the choice of employment, hours and hourly wages when
the firm pays an overtime premium \( x \), and when the legal working time
is \( h_L \). The instantaneous cost per worker is defined in equation (6).
The firm chooses the hours, then the hourly wage is negotiated, and
finally, employment is adjusted. Let us solve this game recursively. The
labour demand is a function \( N^*(\Omega, h) \), defined in equation (11), with
\( \Omega = (h + x \max(h - h_L, 0)) \). Thus, the remuneration \( \Omega \) determined
by the bargaining process is still the solution to program (15), that
yields a function $\Omega^*(h)$ defined in equation (13). Since $h_L$ and $x$ are not arguments of this function, the optimal length of working time, that maximises program (16), is independent from those two variables. Accordingly, equation (11) implies that employment is also independent from $h_L$ and $x$.

When the wage is endogenous, the modifications in overtime pay regulation are neutral. This result holds whatever the specification of the utility function and of the technology as long as the wage can be adjusted. This can be the case if a minimum wage level is not binding. Empirical evidence, given by Trejo [1991], suggests that wages do mitigate the effects of overtime pay regulation, even if the wages adjustment does not seem to completely neutralise such policies. Actually, minimum legal wages are often binding in OECD countries. The analysis of the relationship between employment and legal length of working time with an exogenous hourly wage has been undertaken by Calmfors and Hoel [1988]. They show that this relation is ambiguous and they conclude that, in such a case, "a reduction in normal working time does not seem to be the way to increase employment". Thus it is worth examining other ways to increase employment.

3.2 The maximum length of working time

If the maximum length of working time is binding, hours are exogenous, at level $\bar{h}$. In that case, the effect of a shorter working time critically depends on the level of $\bar{h}$.

**Proposition 3** The length of working time $h^N$ that maximises employment is shorter than the length of working time $h^\pi$ chosen by the firm.

**Proof**
The optimal working time chosen by the firm is defined by the following program:

$$
\begin{align*}
h^\pi &= \arg\max_h \pi^*(h) \\
&= \arg\max_h F(e(h)N(\Omega^*(h), h)) - N(\Omega^*(h), h)\Omega^*(h)
\end{align*}
$$

(17)

where $\Omega^*(h)$ is defined in equation (13).

Let us define the function $\Lambda(h) \equiv d\pi^*(h)/dh$, then first order condition can be written as follows, for an interior solution $h^\pi \in [0, \bar{h}]$:

$$
\Lambda(h^\pi) = 0
$$

(18)
assuming that the profit function is concave with respect to hours, \( \Lambda'(h) < 0 \), implies that the second order condition is satisfied.

The value \( h^N \in [0, h] \) that maximises employment can be obtained by differentiating equation (11) and looking for the value of \( h \) such that \( dN^*(\Omega^*(h), h)/dh = 0 \), one gets, assuming that the second order condition is fulfilled:

\[
\Lambda (h^N) = -e' (h^N) e (h^N) (N^*)^2 F'' (e(h^N)N^*) > 0 \tag{19}
\]

since the concavity of the profit function entails \( \Lambda'(h) < 0 \), equations (18) and (19) imply: \( h^\pi > h^N \).

Proposition 3 shows that, given the outcome of the wage bargaining, the firm’s precommitment on hours entails a relatively low employment level. It is easy to check that this result holds with a quasi-concave instantaneous utility function \( U(\Omega, h) \).

This proposition also shows that it is possible to increase employment by reducing working time if the maximal length of working time is higher than \( h^N \). A slight decrease in working time can increase employment, but a large decrease can have a negative effect. Actually, the firm takes into account the reaction of the wage to the variations in hours that is determined by the elasticity of the disutility of hours, denoted by \( \epsilon_g \equiv hg'(h)/g(h) \) (see equation (13)). Indeed, if one assumes that \( eNF'/F \) is a constant, denoted by \( \alpha \), it can be easily shown, from equation (18), that a decrease in working time increases employment if and only if: \( |\epsilon_g| > \alpha \gamma \epsilon_e \). If workers have a strong preference for a shorter working time, a decrease in hours will be consistent with a strong decrease in the labour cost \( \Omega \), because worker will agree to work a shorter time with a lower remuneration. But, if it is not the case, workers will be reluctant to accept a reduction in working hours without compensation (i.e. an increase in the hourly wage), because their main objective is to keep a high revenue.

It is worth noting that in the case where the wage is considered as exogenous, the consequences of a decrease in the maximal length of working time \( h^\pi \) is different. It depends on the elasticity of the work efficiency with respect to hours. In such a case, assuming that \( eNF'/F \) is a constant, denoted by \( \alpha \), a reduction in working time increases employment if the elasticity of work efficiency with respect to hours, denoted by \( \epsilon_e \) is lower than the inverse of the elasticity of the production with respect to the service of labour, that is to say: \( \epsilon_e < 1/\alpha \) (Andersen [1987], Houpis [1993]).

Hence, we have shown that, at the firm level, there exists a way of increasing employment by substituting jobs to hours. This suggests that compulsory bargaining on hours could increase employment.
3.3 Compulsory bargaining on hours

It is often argued that negotiations on hours are a means to induce work sharing. Indeed, if unions have some bargaining power on hours, they can offer to “exchange” hours against employment to the firm. This idea is supported by our model in the case where the elasticity of production with respect to the service of labour is constant:

**Proposition 4** Assuming that the elasticity of production with respect to the service of labour, $eNF'/F$, is constant, working time is reduced and employment is increased when compulsory bargaining on hours is introduced.

**Proof**
Without loss of generality, we look for interior solutions such that $h \in ]0, \bar{h}[,$ to the maximisation programs that we consider: Maximisation of program (15) with respect to $w$ yields the remuneration function $\Omega^*(h)$, defined in equation (13), and optimal hours $h^B$ can then be easily obtained by maximising (15) with respect to $h$, with $\Omega = \Omega^*(h)$ (defined in equation (13)) and $N = N^*(\Omega^*(h), h)$ (defined in equation (11)). Using equations (11), (13) and (17), and assuming that $\alpha = eNF'/F$ is a constant, this amounts to maximise: $(1 - \beta) \log(\pi^*(h)) + \beta \log(N^*(\Omega^*(h), h))$. Thus, $h^B$ is implicitly defined by:

$$\Lambda(h^B) = \frac{-\beta \pi^*(h^B)}{(1 - \beta)} \times \frac{dN^*(\Omega^*(h^B), h^B)}{dh}$$

(20)

where $\Lambda(h), \Lambda'(h) < 0$, is defined in equation (18).

We know, from equation (18), that $\Lambda(h^\pi) = 0$, and that

$$\frac{dN^*(\Omega^*(h), h)}{dh} < 0,$$

if and only if $h > h^N$, since $N(\Omega^*(h), h)$ is assumed to be a concave function of $h$. Suppose that $h^B > h^\pi$, from proposition 3 we know that $h^\pi > h^N$, thus equations (18) implies that $\Lambda(h^B) < 0$ (remember that $\Lambda'(h) < 0$), which is impossible from equation (20), since $dN^*(\Omega^*(h), h)/dh < 0$. Suppose that $h^B < h^N$, then equation (18) implies that $\Lambda(h^B) > 0$, which is impossible from equation (20) since in that case $dN^*(\Omega^*(h), h)/dh > 0$. Thus we have necessarily: $h^B \in [h^N, h^\pi]$. □

This proposition relies upon the following mechanism: bargaining on hours does not modify the relationship between the remuneration and the hours. The remuneration is still defined by the same function $\Omega^*(h)$ as previously. Since the union uses its bargaining power to exchange
hours against employment, the union will not try to get less hours than \( h^N \), the number of hours that maximises the employment level, for the remuneration function \( \Omega^*(h) \). Accordingly, the stronger the union's bargaining power, the higher is the gain in employment that results from compulsory bargaining on hours. Hence, to bargain on hours increases employment and induces, at the firm level, work sharing.

As a counterpart, profits are reduced by compulsory bargaining on hours. The firm loses some power, because it cannot unilaterally precommit itself on working time: it must give up some right to manage to the union. This may explain the opposition between firms and unions on this issue: firms are generally reluctant to bargain on hours, but unions claim bargaining power on hours.

Let us remark that proposition 4 relies upon the homogeneity of the production function with respect to efficient labour \( e(h)N \). If the production is not homogeneous, the elasticity of labour demand with respect to the labour cost depends on hours, and the so-called wage pressure (which is defined as the ratio of the utility of an insider to the utility of an unemployed worker in the wage equation (13)), also depends on hours. Hence, the union can have interest in decreasing hours below \( h^N \) in order to increase the wage pressure. But such an effect, that relies upon the manipulation of the labour demand elasticity by the union, should be negligible.

The results obtained up to now suggest that a reduction in working time can increase employment. Among the policies that we have studied, only a reduction in maximal working time and the introduction of compulsory negotiations on hours can increase employment. But the analysis has been limited to a partial equilibrium framework. We shall see, in the next section, that a general equilibrium analysis yields very different results.

4 Work sharing policies: general equilibrium analysis

This section allows us to underscore that a reduction in working time may have very different effects at general and partial equilibrium levels. Actually, work sharing policies that can raise employment at the firm level can have an opposite effect at aggregate level. We consider a stationary state, with endogenous number of firms and labour force. The number of firms is determined by a zero profit condition, and the size of the labour force depends on the behaviour of individuals, who can choose either to stay out of the labour force, or to look for a job. In such a framework, we can prove the following proposition.
Proposition 5 Assuming that the elasticity of production with respect to
the service of labour, $eNF'/F$, is constant, if a binding maximal working
time is reduced, or if compulsory bargaining on hours are introduced,
aggregate employment decreases.

Proof
We begin to compute the equilibrium unemployment rate (i). Then we
show that the expected utility of an unemployed worker and the size of
the labour force decrease with respect to $h$ if $h < h^\pi$, where $h^\pi$
is the length of working time that maximises profits (ii).

(i) From equations (3) and (4), the present discounted expected utility
of an unemployed worker is

$$rV_u = \frac{q\Omega^\gamma g(h)}{r + s + q}. \quad (21)$$

The remunerations are always determined by a bargaining process,
which implies (see equation (13): $\Omega = \Omega^*(h)$. Then, equation (21)
together with equation (13) yields the equilibrium quit rate from
unemployment:

$$q = \frac{(r + s)(\alpha + \beta(1 - \alpha)(1 - \gamma))}{\beta \gamma (1 - \alpha)} \quad (22)$$

with $\alpha = eNF'/F$.

Equation (22) together with (2) defines the equilibrium unemploy-
ment rate, which is independent from the length of working time.

(ii) At symmetric general equilibrium, equations (21) and (22) are al-
ways fulfilled. Let us assume that a firm can be created with a fixed
cost, denoted by $C > 0$. From equation (17), the entry equilibrium
condition for firms is:

$$\pi^*(h) = \left( \frac{e(h)}{\Omega^*(h)} \right)^{\alpha/(1 - \alpha)} \quad K = rC \quad ; \quad K = (1 - \alpha)\alpha^{\alpha/(1 - \alpha)}. \quad (23)$$

This condition defines the equilibrium remuneration as a function
of hours. (21), (22), and (23), allow us to define the expected dis-
counted utility of an unemployed worker as follows:

$$rV_u = (e(h))^\gamma g(h)\frac{\beta(1 - \alpha)(1 - \gamma) + \alpha}{\beta(1 - \alpha) + \alpha} \left( \frac{K}{rC} \right)^{[\gamma/(1 - \alpha)]/\alpha}. \quad (24)$$

An individual enters the labour force if the discounted expected
utility of an unemployed worker is larger than his own discounted utility
outside the labour force, i.e. if: \( V_0 \leq V_c \). Since \( G(V_0) \) is the repartition function of \( V_0 \), the size of the labour force is equal to \( G(V_u) \); \( G'(.) \geq 0 \). It can be easily checked, from equations (13), (18) and (23) that \((e(h))^{\gamma} g(h)\) has a maximum at \( h = h^* \). Therefore, equation (24) implies that the equilibrium value of \( G(V_u) \) has a maximum at the working time that maximises profits: \( h = h^* \). Assuming that \((e(h))^{\gamma} g(h)\) is concave, this entails that a decrease in \( \bar{h} \), when \( \bar{h} \) is binding, or the introduction of compulsory bargaining on hours (see equation (20)) decreases the size of the labour force. Since the unemployment rate \( u \) is independent from hours, a decrease in the size of the labour force entails a decrease in aggregate employment, equal to \( G(V_u)(1-u) \).

Let us describe the mechanism that entails a negative relationship between aggregate employment and working time.

Firstly, the unemployment rate of the members of the labour force is independent from hours. Here, we find the same result as Layard, Nickell and Jackman (1991), but in a more general framework, since we take into account the utility of leisure and we do not assume a perfect substitutability between jobs and hours. The equilibrium unemployment rate does not depend on hours because it is determined by the wage pressure, that depends on the workers' wage bargaining power and the elasticity of the labour demand with respect to employment. Namely, a reduction in working time, for \( h > h^N \) increases employment in each firm. This raises the expected utility of the unemployed workers and then the wage negotiated in each firm (equation (13)). At the end of this process, for a given size of the labour force, the increase in employment at the firm level is completely supplanted by the wage increase. Accordingly, a reduction in working time does not modify the unemployment rate (the generality of this result is discussed in appendix).

Secondly, a reduction in hours decreases profits. Hence, in the long-run, the number of firms is increasing with respect to hours, for \( h < h^* \). Since the unemployment rate \( u \) is always given by the wage pressure, the firms destruction, that follows a reduction in hours, entails a decrease in wages, in the expected utility of an unemployed worker, and thus in the size of the labour force.

This result is logical, a decrease in hours, by a binding maximal working time or by compulsory bargaining, entails necessarily a decrease in profits, and thus, in the long-run in the equilibrium wage and in the size of the labour force. This result is always satisfied if the wage pressure does not depend on hours, because in that case, the entry equilibrium condition implies that a raise in profits increases the workers' remuneration. Actually, a reduction in working time has the
same effects as a negative productivity shock, that does not modify the unemployment rate in an equilibrium unemployment rate model (Layard, Nickell and Jackman [1991], Manning [1993]), but that reduces the workers' welfare.

Proposition 5 shows that the previous results, concerning partial equilibrium properties of the model, are not fulfilled at aggregate level. This suggests that a partial equilibrium analysis could be misleading, when studying the consequences of work sharing. We have seen that a decrease in the maximal working time can increase employment at the firm level. Compulsory bargaining on hours also increases employment in a firm. Hence, each decentralised union should claim such policies in order to increase employment in the labour pool of its firm. But, the counterpart of such policies is a decrease in profits. At aggregate level, once the firms creation process and the workers participation decision have been taken into account, the decrease in working time decreases aggregate employment.

It is worth noting that a decrease in working time decreases the unemployed level. But this relies upon the decrease in the size of the labour force, and the following corollary underscores that such a decrease is not desirable.

**Corollary**

*A decrease in the length of working time decreases welfare.*

**Proof**

We have shown, in proposition 5, that a decrease in working time decreases the discounted expected utility of an unemployed worker. From equations (4) and (23), the ratio of $V_u$ to $V$ does not depend on hours, therefore a reduction in working time decreases the expected utility of an insider.

A decrease in working time decreases welfare because, in our model, the firm's choice of hours maximises welfare. Therefore, aggregate production decreases, and the welfare of each member of the labour force is reduced when working time is decreased, either by compulsory negotiations on hours or by a reduction in the maximal working time. Hence, at aggregate level, a reduction in working time has mainly negative consequences.

**Conclusion**

In this paper, we have analysed the microeconomics of wage, employment and hours determination. Such an analysis has allowed us
to show that the effects of a shorter working time are very different at the firm level and at the aggregate level. It has been shown that a (slight) decrease in working-time is likely to increase employment at the firm level, but has an opposite effect on the steady state aggregate employment level and worker’s welfare. This last result is particularly important, since a reduction in working time is often considered as irreversible. Hence, the main conclusion of this paper is that a reduction in working time is likely to decrease employment and welfare in the long-run.

It is worth stressing some limits of our analysis.

First, we studied the consequences of work sharing policies under the assumption that firms are precommitted on hours and not on employment. We just showed that firms are incitated to precommit on hours if they are not precommitted on employment. But we analysed neither the issue of precommitment on employment, nor the reputation argument that we put forward to justify that the firms do make credible precommitment. This rests to be done.

Second, the equilibrium unemployment rate does not depend on work sharing policies in our framework, as in Layard, Nickell and Jackman [1991] model. We underscored that such a property can be considered as relevant in a steady state. But it could be interesting to analyse how the unemployment rate is influenced, in the short run, by work sharing policies, and to study the dynamics of the unemployment rate. This also rests to be done.
APPENDIX

The relationship between unemployment rate and working time

We claimed that the steady state unemployment rate does not depend on hours. This result relies upon two particular assumptions that we discuss in this appendix.

1. If the production function is not homogeneous, the labor demand and the profit elasticities with respect to the labor cost (respectively denoted by $\varepsilon_N$ and $\varepsilon_\pi$) depend on hours, and thus the wage pressure also depends on hours. In this case we get from equation (13) a new expression for equation (22):

\[
q = \frac{(r + s)((1 - \beta)\varepsilon_\pi + \beta \varepsilon_N + \beta \gamma)}{-\beta \gamma}.
\]

Therefore, the unemployment rate depends on hours because elasticities $\varepsilon_N, \varepsilon_\pi$ are functions of hours. But the effects of a variation in hours on those elasticities is a priori ambiguous and should be weak.

2. If we suppose that an unemployed worker receives unemployment benefits denoted by $B$, the expected utility of an unemployed worker is defined by

\[
rV_u = \frac{q\Omega^\gamma g(h) + (r + s)B^\gamma g(0)}{r + s + q}.
\]

This equation, together with equations (13) and (23) determines the unemployment quit rate:

\[
q = \frac{(r + s)\beta(1 - \alpha) + \alpha}{\beta\gamma(1 - \alpha)} \left( \frac{\beta(1 - \alpha)(1 - \gamma) + \alpha}{\beta(1 - \alpha) + \alpha} - \frac{B^\gamma g(0)}{\Omega^\gamma g(h)} \right),
\]

with

\[
\Omega = e(h) \left( \frac{K}{rC} \right)^{(\alpha - 1)/\alpha}.
\]

Equation (A2) defines a relationship between $q$ and $h$. If $B = b\Omega, 0 < b < 1$, it can be easily verified that $dq/dh < 0$, and thus, $du/dh < 0$. But if $B$ is exogenous, (A2) implies, together with equations (18), (19) and (20), that $dq/dh > 0$ and then $du/dh > 0$. Accordingly, the sign of the relationship between $u$ and $h$ is a priori ambiguous. It depends on the determination of the unemployment benefits.
REFERENCES

ANDERSEN, T.M. [1987], Short and long run consequences of shorter working hours, in P.J. Pedersen and R. Lund (eds), Unemployment Theory, Policy and Structure, Amsterdam, De Gruyte, pp. 147–165.


EHRENBERG, R.G. [1971], Heterogeneous labor, the internal labor market and the dynamics of the employment-hours decision, Journal of Economic Theory, 3(1), pp. 85–105.


HOEL, M. [1986], Employment and allocation effects of reducing the length of the workday, Economica, 53(209), pp. 75-85.


LEWIS, H.G. [1969], Employer interests in employee hours of work, Mimeo, University of Chicago.


