Pensions and voting equilibria in an overlapping generation model with heterogeneous agents*

Géraldine Mahieu† and Stéphane Rottier‡

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Abstract

We model how a Beveridgean pay-as-you-go pension system may be supported by a majority of heterogeneous voters in a general equilibrium OLG model. The introduction of heterogeneity creates intragenerational transfers among workers which may lead to different optimal taxation rates within young individuals and to a positive taxation rate as outcome of the political choice. We underline the general equilibrium effects of a PAYG pension system on the interest rate, on future wages and therefore on the future level of pensions. We obtain an equilibrium tax rate and pension level that do not depend on population growth rate and on the capital stock.

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†IRES, Department of Economics, Université Catholique de Louvain. Financed by a scholarship from FNRS. E-mail: mahieu@ires.ucl.ac.be

‡IRES, Department of Economics, Université Catholique de Louvain. E-mail: rottier@ires.ucl.ac.be

1 INTRODUCTION

Two types of pension systems exist all over the world : Fully-funded and Pay-as-you-go pension systems. The Pay-as-you-go (PAYG) system is widespread in Europe and America and implies an intergenerational redistribution[1]. However, the future of this system often seems problematic. Pension expenditures increase because people have a longer life-span; the ratio of contributors to beneficiaries is reduced because of the decreasing growth rate of population; high public debt often prohibits debt financing[2]. Nevertheless, in these democratic countries, the system persists.

'Why do we not choose a fully-funded system?' How to explain the political support of a PAYG system?

Several arguments have been put forward to show that a majority of voters supports this system. It seems clear that old voters always prefer a Pay-as-you-go-system. They do not have to contribute anymore to its financing and receive a pension paid by the young. But why do the young vote for such a system? The arguments of altruism (Tabellini (1990)), social contract (Sjoblom (1985), Verbon (1987)), or 'efficiency reasons' (growth rate of population higher than the rate of return on savings, as in Boldrin and Rustichini (1999)) have been put forward. We show that, even in a economy with selfish agents and a low growth rate of population (as today), *heterogeneity* within generations may explain the political support of this system by young voters. The difference in skillness among agents divides young voters in terms of preferences: the low-skilled agents may prefer a Pay-as-you-go system because of the intragenerational transfer it may create.

Romer (1975), Roberts (1977) and Meltzer and Richard (1981) have looked at intragenerational transfers through linear income taxes in static voting models. Except from Tabellini (1990), who considered heterogeneous dynasties of altruistic families, the heterogeneity within generations had not been considered in a dynamic framework. Recently Conesa and Krueger (1998) considered a *simulation* model with agents facing idiosyncratic income uncertainty. Three very recent papers propose concurrently and independently of our paper *partial* equilibrium models with heterogeneous agents. Casamata and Al.(1999) and De Donder and Hindricks (1999) study the case where voters vote for a pension that may be partly distributive and partly not. Galasso and Conde Ruiz (1999) introduce in addition to pensions a lump-sum transfer within the young generation.

We will analyze the political choice of the pension level in a Beveridgean system, a system that moreover prevails in many Western countries such as

[1] See Browning (1975), Boadway and Wildasin (1989), Esteban and Sakovics (1993), Cooley and Soares (1999)

[2] As in Tabellini (1991)

Canada, the United States, Belgium, New-Zealand,... In a Beveridgean PAYG system, every old receives the same pension. Conversely, in a Bismarckian PAYG system, pension received depends exactly on the amount that has been paid when working, so that there does not exist any redistribution within generations. We model the tax rate as the result of a political equilibrium : it is the outcome of a majority vote involving both workers and retirees. The voted tax rate will depend on the optimal tax rate of the different voters and on their relative importance. Each agent votes *sincerely* (abstracting from any strategic consideration and according to his true preference) for his optimal tax rate. He supposes that the tax rate voted today will be valid when he will be retired, considering it as a *vested interest* [3]. Once the tax rate is voted, individuals choose their optimal consumption plan given the voted tax rate and pension. We do not restrict the analysis to a steady state economy, contrary to many authors[4]. Steady State may be far away or may never happen. Moreover, voters only care about what happens during their life and not about the far future.

We obtain two important results. The higher the heterogeneity level (namely the difference between the various skills) and the more numerous the low-skilled, the closer will be the optimal tax rate of the (relatively) poor working class to that of the old and the higher the voted taxation level. In addition, thanks to the general equilibrium approach, we investigate many effects of the tax rate neglected in a partial equilibrium approach. It influences savings, but also interest rate, future wages and so future level of pension. This leads to a remarkable result : our optimal tax rate does not depend on capital level nor on population growth.

The structure of the rest of the paper will be the following. First, we will analyze the economic equilibrium : we take an OLG model with two generations. In the next section, we look at the political choice of the tax rate. We determine the choice of the median voter of a continuity of agents. Eventually, we examine some comparative statics.

2 THE ECONOMIC ENVIRONMENT

To show that individuals may have different preferences regarding the choice of the tax rate, we develop a dynamic general equilibrium model with overlapping generations of *heterogeneous* individuals, who live for two periods. Time t is discrete and goes from 0 to ∞.

[3] Different arguments have been developed in the literature to support this hypothesis. We analyze them in section 3.1.

[4] See Casamata and al.(1999), Boldrin and Rustichini(1999).

2.1 The Consumer Side

Individuals are heterogeneous in terms of their innate economic ability or skills in production e. The skill level e (expressed in efficiency units of labor) is a continuous variable, distributed on the support (e_L, e_H) according to the probability density function $f(e)$, which is constant over time. We denote the average skill level as $\bar{e} = \int_{e_L}^{e_H} f(e)e \; de$.

The population is assumed to grow at a constant rate n:

$$N_{t+1} = N_t(1+n) \tag{1}$$

Each individual lives for two periods, only works in the first period and consumes in both periods. Young individuals who are endowed with one unit of labor in the first period of life supply it *inelastically*. By this assumption, we gain in simplicity, but we rule out the potential negative effect of taxation on labor supply and so on actual pensions. Consequently, labor supply in efficiency units is equal to $N_t\bar{e}$.

Each individual is assumed to have the following homothetic lifetime utility function:

$$V_{et} = \ln C_{et} + \beta \ln D_{et+1} \tag{2}$$

with

$$C_{et} + S_{et} = ew_t(1-\tau_t) \tag{3}$$

$$D_{et+1} = R_{t+1}S_{et} + P_{t+1} \tag{4}$$

where $\beta < 1$ is the subjective discount factor. With this specification, the intertemporal elasticity of substitution is equal to one. In the first period of life ('when young'), individual's income is equal to the after-tax wage $(ew_t(1-\tau_t))$, with w_t the wage per efficiency unit of labor. He allocates it between current consumption C_{et} and savings S_{et}. In the second period of life t+1 ('when old'), each individual is retired and consumes D_{et+1}. His income in this period comes from two sources: the pension received in t+1 (P_{t+1}) assumed to be identical for all individuals and the return on savings made at time t.

Individuals have perfect foresight. Therefore, the maximization problem of each individual under his budget constraints can be expressed as follows:

$$MAX_{S_{et}} \ln \, (ew_t(1-\tau_t) - S_{et}) + \beta \ln(R_{t+1}S_{et} + P_{t+1}) \tag{5}$$

with τ_t the proportional labor income tax rate at time t and $R_{t+1}$the rate of return on savings held from period t to period t+1.

Using the First Order Condition, it can easily be verified that the optimal savings of the individual e is given by :

$$S^*_{et} = \frac{1}{1+\beta}\left[\beta\ ew_t(1-\tau_t) - \frac{P_{t+1}}{R_{t+1}}\right] \tag{6}$$

The optimal savings of the individual depend positively on his wage (after taxes) and on the interest rate which represents the return on savings made from t to t+1. Note that since utility is log-linear (therefore the intertemporal elasticity of substitution is equal to one), the marginal propensity to save out of income does not depend on the interest rate. Optimal savings are negatively related to the pension received in t+1.

Aggregate savings will be:

$$S(\bar{e}w_t, \tau_t, P_{t+1}, R_{t+1}) = N_t\frac{1}{1+\beta}\left[\beta\ \bar{e}w_t(1-\tau_t) - \frac{P_{t+1}}{R_{t+1}}\right] \tag{7}$$

2.2 The Production Side

Firms in economy produce one good according to the following Cobb-Douglas production function, assumed to be the same in each period:

$$F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha} \tag{8}$$

with K_t the quantity of physical capital used at time t and
$L_t = N_t\bar{e}$, the total labor input in efficiency unit at time t.

The productive actor equates the marginal productivity of factors with their market price:

$$w_t = (1-\alpha)K_t^{\alpha} L_t^{-\alpha} \tag{9}$$

$$1 + r_t = R_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha} \tag{10}$$

where full capital depreciation is assumed.

2.3 Government

In this model, the role of the government is very limited. It only intervenes by running a social security system. For this purpose, the government levies labor taxes on young workers and redistributes these via old-age pensions. There is no tax distortion since labor supply is inelastic. We constrain the tax rate to be positive ($\tau_t \geq 0$) so that the system implies a transfer from young to old individuals. Moreover, we assume that budget deficits are not permitted[5]. Hence, total pensions paid at time t to the old born in t-1 must entirely be financed by the taxes levied on the young of period t. The budget constraint of the government is given by:

$$P_t = \tau_t \bar{e} w_t (1+n) \tag{11}$$

This social security is therefore of the '*Pay-as-you-go*' type. Moreover, this system is redistributive since pension received is independent of contributions, a characteristic of the so-called 'Beveridgean system'. In case where $\tau_t = 0$ individuals only finance their retirement by their own savings when young, which is the characteristic of a 'fully-funded' social security system.

2.4 The economy equilibrium and the dynamics of capital accumulation

To define the equilibrium path of the economy, we have to account for the market clearing conditions for labor and capital:

$$L_t = N_t \bar{e} \tag{12}$$

and

$$K_{t+1} = S_t(\bar{e}w_t, \tau_t, w_{t+1}, n, R_{t+1}) \tag{13}$$

where we have substituted P_{t+1} from (11) in the aggregate saving function (7). Together with conditions (9) and (10), they characterize the time path of $\{K_t, R_{t,} w_t\}$.

From these four conditions some computation allows us to determine the following equation determining the dynamics of capital accumulation :

[5] Intergenerational transfers via debt as in Tabellini (1991) are not possible.

$$K_{t+1} = \left[\frac{\beta(1-\alpha)(1-\tau_t)}{(\bar{e})^{\alpha-1}(1+\beta+\frac{1-\alpha}{\alpha}\tau_{t+1})} \right] K_t^{\alpha} N_t^{1-\alpha} \tag{14}$$

The social security system influences the accumulation of capital of the economy in two ways. On the one hand, it increases pensions received in t+1 and so reduces savings. This effect accounts for the term $\frac{1-\alpha}{\alpha}\tau_{t+1}$ in the denominator. On the other hand, increasing pensions requires an increase in the tax rate in order to respect the government budget constraint (11). This higher taxation reduces the disposable income in period t and in turn savings and capital in period t+1 as reflected in the term $(1-\tau_t)$ in the denominator[6].

3 THE POLITICAL SYSTEM

This section analyzes the political equilibrium, namely the tax rate (and thus social security system) that comes out of the voting procedure. The tax rate is decided by majority voting. The voting process involves all individuals, both young and old. We assume that every individual votes '*sincerely*', that is according to his preferences and abstracting from any strategic consideration.

The analysis of the tax rate resulting from the voting procedure requires first to determine the *optimal* tax rate of each individual, namely the one that maximizes his utility. Next, the *equilibrium tax rate* will be computed, taking into account the distribution of voters and their optimal choices.

3.1 The optimal tax rate

Every agent has an optimal tax rate. We suppose that each agent expects, when voting, that the tax rate he chooses for today will also be the one that will prevail in the next period ($\tau_t = \tau_{t+1} = \tau$). This simplifying assumption can be justified in several ways. First, it is acceptable if we suppose that the vote does not happen frequently (at least not before the death of the currently young workers), which seems to be the case for important changes in social security system. Second, we may suppose that individuals are convinced to have *vested interests* : their past contributions to social security are considered by them as a guarantee of receiving a pension later on, so that they feel entitled to a given level of pension. The third and most important argument relies on the fact that, as will be shown later on, with homothetic preferences and a Cobb-Douglas production function the optimal tax rate does not depend on the

[6] This can directly be seen by looking at equation (13). Substituting in (13), the equation (6), (9) and (10), we get $\frac{1+\beta}{N_t}K_{t+1} = \beta\hat{e}w_t(1-\tau_t) - \tau_{t+1}\frac{1-\alpha}{\alpha}K_{t+1}N_t^{-1}$. This expression highlights the two effects of taxation on the capital accumulation. We then express K_{t+1} as a function of the parameters τ_t and τ_{t+1} and obtain (14).

capital stock. It only depends on the distribution of skills across individuals, on a psychological discount factor and on technology.

We must distinguish the behavior of old and young individuals. Since individuals only work when young and are taxed on their labor income only, we can consider old individuals as an homogeneous group of agents as far as voting behavior is concerned. The voting behavior of young workers will however change with their relative skill level.

3.1.1 The optimal tax choice of the old individuals

The old at time t choose a τ_o^* that maximizes their utility over their remaining lifetime. Their utility is strictly increasing in τ_o^*. Therefore, they choose the maximum tax rate $\tau_o^* = 1$. In a model with endogenous labor supply, their optimal tax rate would be determined by the Laffer curve.

3.1.2 The optimal tax choice of young individuals

The young individuals care about the present and the future. Young individual e will vote for a τ_{ye}^* that maximizes their indirect lifetime utility V_{et}. He chooses a $\tau_{ye}^* \geq 0$ given by:

$$\tau_{ye}^* = \arg\max_{\tau} V_{et} = \arg\max_{\tau} \ \ln C_{et} + \beta \ln D_{et+1} \tag{15}$$

s.t. (3), (4), (6), (9),(10), (11), (14).

By the envelop theorem and (6), the first order condition for young agent e is given by :

$$\left[-\ ew_t + S_{et} \frac{\frac{dR_{t+1}}{d\tau}}{R_{t+1}} + \frac{\frac{dP_{t+1}}{d\tau}}{R_{t+1}} \right] \frac{1}{C_{et}} = 0 \tag{16}$$

with

$$\frac{dR_{t+1}}{d\tau} = -(1-\alpha)\ R_{t+1} \frac{1}{\tau} \epsilon_{K_{t+1},\tau} \tag{17}$$

$$\frac{dP_{t+1}}{d\tau} = P_{t+1}(1 + \alpha \epsilon_{K_{t+1},\tau}) \frac{1}{\tau} \tag{18}$$

and $\epsilon_{K_{t+1},\tau}$, the elasticity of capital with respect to the tax rate τ, given by:

$$\epsilon_{K_{t+1},\tau} = \frac{dK_{t+1}}{d\tau}\frac{\tau}{K_{t+1}} = -\left[\frac{(1+\beta+\frac{1-\alpha}{\alpha})}{(1+\beta+\frac{1-\alpha}{\alpha}\tau)}\right]\frac{\tau}{1-\tau} < 0 \qquad (19)$$

Note that $\frac{dR_{t+1}}{d\tau} > 0$ (since $\epsilon_{K_{t+1},\tau} < 0$) and $\frac{dP_{t+1}}{d\tau} > 0$ if $\epsilon_{K_{t+1},\tau} > -\frac{1}{\alpha}$.

When computing his optimal tax rate, each young individual takes three effects into account:

First, an *income effect* : the tax rate reduces his current labor income by $-ew_t$. The higher the wage of the agent (the higher his skills in production), the more costly for him the social security system.

Second, an *interest rate effect*: an increase in the tax rate reduces the capital available in the next period by affecting the current optimal savings of the individuals. The interest rate R_{t+1} is thus *increased*, which augments the return on savings made in t. This effect will be higher, the higher the savings level of the individual.

Third, a *pension effect* : increasing the tax rate has two opposite effects on the future pension of young individuals. On one hand, a rise in the tax rate augments pension by increasing the contributions of the agents to social security system. On the other hand, the tax rate has a negative effect on capital accumulated in t+1 and through this on the wage and pension received in t+1. Therefore, the sign of the effect of an increase in τ on pension $(\frac{dP_{t+1}}{d\tau})$ is a-priori *ambiguous*. As can be seen from (18), it will be positive if the negative effect on the accumulation of capital is not too large. And the higher is α, the lower will be this effect ($d(\frac{dP_{t+1}}{d\tau})/d\alpha < 0$).

Note that the choice of the tax rate at time t does not affect capital of the current period (K_t)[7], which was determined by the savings realized during the previous period and so does not depend on τ_{t+1}. Therefore, current wages (ew_t) will remain unchanged.

The optimal tax rate τ^*_{ye} for the group of young individuals with skills e is thus the result of the combination of these three different effects. Using (3), (4), (6), (9), (10),(11), (14) and after some computations, we find that the optimal tax rate of young individuals is given by the following implicit function :

[7] Remember K_t may be outside steady state.

$$
\begin{aligned}
J &= \\
&-(1+\beta)(\frac{1-\alpha}{\alpha})^2(1+\beta\frac{\bar{e}}{e})\tau_{ye}^{*^2} \\
&+(\frac{1-\alpha}{\alpha})\begin{bmatrix} -2(1+\beta)^2+\beta(1-\alpha)(1+\beta+\frac{1-\alpha}{\alpha})-\beta\frac{\bar{e}}{e}(1+\beta)^2 \\ +\beta\frac{1-\alpha}{\alpha}\frac{\bar{e}}{e}(1+\beta)-\beta\frac{\bar{e}}{e}(1+\beta+\frac{1-\alpha}{\alpha})(1+\alpha\beta) \end{bmatrix}\tau_{ye}^{*} \\
&+(1+\beta)\left[-(1+\beta)^2+\beta(1-\alpha)(1+\beta+\frac{1-\alpha}{\alpha})+\beta(\frac{1-\alpha}{\alpha})\frac{\bar{e}}{e}(1+\beta)\right] \\
&= 0 \qquad (20)
\end{aligned}
$$

This expression is a second order equation in τ_{ye}^{*} and can be summarized by the following expression:

$$J \equiv A\tau_{ye}^{*^2} + B\tau_{ye}^{*} + C = 0 \qquad (21)$$

with A, B, C depending on parameters (α, β) and on the skill level of the individual e relatively to the weighted skill level average ($\frac{\bar{e}}{e}$).

The First Order Condition of the maximization problem of young individuals given by (16) will be sufficient to establish that τ_{ye}^{*} is the optimal choice of individual e, if the objective function V_{et}(15) is *single peaked* in $\tau_{ye} \geqslant 0$. Proposition 1 establishes the conditions under which this is true.

Proposition 1 *The objective function of young agents (V_{et}) will be single peaked iff $\alpha \geq \alpha_{\min}$, where $\alpha_{\min} \leq 1/3$*[8].

Proof : see appendix 1.

From formula (20) we can deduce some properties of the optimal tax rate of young individuals:

1) The optimal tax rate τ_{ye}^{*} does *not* depend *on the level of capital* (at steady state or not) in our dynamic general equilibrium framework. It only depends on the parameters of the model $(\alpha, \beta, e, \bar{e})$, as can directly be seen from the observation of the function (20). This remarkable characteristic supports our assumption of a vote forever.

[8] The higher α, the lower the effect of the tax rate on the accumulation of capital ($\frac{d\epsilon_{K_{t+1},\tau}}{d\alpha} > 0$, with $\epsilon_{K_{t+1},\tau} < 0$) and the lower will be the interest rate effect. In addition, the higher the α, the smaller the pension effect. Therefore, for α sufficiently high, the (positive) interest rate effect is dominated by the pension and income effects and the objective function is single peaked.

2) The optimal tax rate τ^*_{ye} does not depend on the growth rate of population n. This happens because individuals consider pension as fixed in their saving decision so that n only intervenes in the third term of (16) (namely in $\frac{\frac{dP_{t+1}}{d\tau}}{R_{t+1}}$). Since n affects both the numerator and the denominator of this term, it disappears from the equation.

As it appears in (20), the optimal tax rate τ^*_{ye} of young individuals depends on the value of the ratio $\frac{\bar{e}}{e}$. Therefore, the relatively rich (skilled, e close to e_H) and relatively pour (unskilled, e close to e_L) individuals, having a different ratio ($\frac{\bar{e}}{e}$), will have a different optimal tax rate. It reflects the fact that earning a different wage, they contribute in various proportions to the social security system and have different levels of savings. The relation between the relative skill level of a young agent $\frac{\bar{e}}{e}$ and his optimal tax rate, τ^*_{ye} , is given in the next proposition.

Proposition 2

*a) The **higher** the skill level e for a given distribution of skills (or equivalently the lower the ratio $\frac{\bar{e}}{e}$), the **lower** the optimal tax rate τ^*_{ye}, as long as $\alpha > \alpha_1$[9] , with $\alpha_1 \leq 0.25$, $\forall\beta$ (details and proof given in appendix 2). Therefore: if $\alpha > \alpha_1$: $e_i > e_j \Rightarrow \tau^*_{yi} < \tau^*_{yj}$, for all e_i,$e_j \in [e_L, e_H]$.*

b) Moreover, there exists a relative skill level (e_0) above which the optimal tax rate is negative. In this case, the constraint $\tau \geq 0$ is binding and τ^* is set to 0: if $\frac{e}{\bar{e}} > e_0$, $\tau^* = 0$. This is proved in appendix 3.

Remark 3 *One could argue that by restricting our analysis to a Beveridgean PAYG pension system, we implicitly suppose a social contract, transferring resources from less-skilled to high-skilled. However, in our framework, voters can always refuse this redistributive scheme by voting for a zero tax rate. Consequently, a fully-funded system is not excluded ex ante. However, our analytical results show that the equilibrium tax rate would imply a mixed equilibrium (positive savings and positive tax rate) if one takes standard simulation values of f(e), α and β. This result is driven by the heterogeneity : without it, the optimal tax rate of the median voter would be zero*[10].

[9] This insures that the (negative) income effect dominates the (positive) interest rate effect.

[10] The optimal tax rate in a Bismarkian framework (where pension is defined as $P_t = ew_t\tau(1+n)$) is given by the following implicit function:

$$\begin{aligned} & -(1+\beta)(\frac{1-\alpha}{\alpha})^2((\frac{e}{\bar{e}})^2+\beta){\tau^*_{ye}}^2 \\ & +(\frac{1-\alpha}{\alpha})\left[\begin{array}{c} -2\frac{e}{\bar{e}}(1+\beta)^2+\beta(1-\alpha)(1+\beta+\frac{1-\alpha}{\alpha}\frac{e}{\bar{e}})-\beta(1+\beta)^2 \\ +\beta\frac{1-\alpha}{\alpha}(1+\beta)-\beta(1+\beta+\frac{1-\alpha}{\alpha}\frac{e}{\bar{e}})(1+\alpha\beta) \end{array}\right]\tau^*_{ye} \\ & +(1+\beta)\left[-(1+\beta)^2+\beta(1-\alpha)(1+\beta+\frac{1-\alpha}{\alpha}\frac{e}{\bar{e}})+\beta(\frac{1-\alpha}{\alpha})(1+\beta)\right] \\ = & \ 0 \end{aligned}$$

3.2 The political equilibrium and the equilibrium tax rate

Depending on his relative skillness, each individual wishes (and so votes for) a different tax rate. Since the tax rate is the result of the vote of all individuals, the equilibrium tax rate will be such that there does not exist another one that is preferred by a majority of voters.

Given the single peakedness of the preferences regarding to a positive optimal tax rate (established by proposition 1), the equilibrium tax rate τ^* will be the tax rate preferred by the *median voter of total population* (young and old) with skill level e_m.

Retirees, which represent a fraction $\frac{1}{(2+n)}$ of citizens, are in favor of the highest possible tax rate, independently of their skill level. On the other hand, the preferences of young individuals are function of their relative skill level : the lower their skill e, the higher their preferred tax rate (proposition 2). Therefore, the decisive voter (and the only Condorcet winner) will be a *young* individual (except if $n \leq 0$) with a skill level lower than the median of young individuals. And, the equilibrium tax rate will always be *higher* than the one wished by the median of young individuals.

This underlines the effect of the political participation of old individuals. The votes of retirees (and almost retirees), who do not bear the cost of the pension system, push the tax rate higher than the one that would be implemented, were the decision power in the sole hands of young workers. Consequently, in the majority coalition, one finds *old* individuals (the retirees and old individuals close to retirement who have the same reasoning) and workers with relatively *low* wages.

Remark 4 *The political outcome can be considered as inefficient since it is not identical to the one that could be implemented by a benevolent government maximizing the utility of the median young individual.*

4 Comparative statics

4.1 The effects of the population growth rate

The equilibrium tax rate is a *decreasing* function of the growth rate of the population for $\alpha > \alpha_1$. Since the growth rate of population (n) does not

Without heterogeneity ($e = \bar{e}$), the Bismarkian optimal tax rate is the same as the homogeneous Beveridgean optimal tax rate. It hits the non-negativity constraint with standard simulation value ($\alpha = 0.36$; $\beta = 0.4$).

influence the optimal tax rate of workers (τ^*_{ye}), a change in n will affect τ^* only by changing the weight of the old.

The lower is n, the higher the relative weight of old individuals in the vote ($\frac{N_{t-1}}{N_t} = \frac{1}{1+n}$), the lower is the part of young people necessary to get a majority and the higher will be the equilibrium tax rate. Note that, because of our simplifying assumptions (no disutility of labor and a two-periods OLG model), when $n \leq 0$ old individuals constitute a majority and the tax rate will be close to one.

4.2 The effects of the distribution of the skills on the equilibrium tax rate.

Equation (20) shows that the equilibrium tax rate (τ^*) only depends on $\frac{e_m}{\bar{e}}$ in our model. A change in the distribution of skills will affect the numerator of this ratio (by changing the *identity* of the median voter) and the denominator (by affecting the *average* skill level of the population). If the resulting ratio $\frac{e_m}{\bar{e}}$ is **higher** (lower), the equilibrium tax rate will be **lower** (higher).

Proposition 5 *In case of a **uniform distribution of skills** , a reduction of the lower bound of the skill distribution (which reduces the average skill level and increases the dispersion of skill level) increases the equilibrium tax rate:* $\frac{d\tau^*}{de_L} < 0$ *for* $\tau^* > 0$.

Proof. In case of a uniform distribution ($e \backsim U[e_L, e_H]$):
- the probability density function is constant: $f(e_H) = f(e_L) = f(e_m) = \frac{1}{e_H - e_L}$.
- the median voter of total population is : $e_m = e_L + (e_H - e_L)\nu$,
with $\nu = \frac{1}{2} - \frac{1}{(2+n)} = \frac{n}{2}\frac{1}{2+n}$.
- the average skill level is equal to: $\int_{e_L}^{e_H} f(e)ede = \frac{e_H + e_L}{2}$.

Consequently: $\cdot \frac{de_m}{de_L} = 1 - \nu > 0$

$\cdot \frac{d\bar{e}}{de_L} = \frac{1}{2} > 0$

$\cdot d(\frac{\bar{e}}{e_m})/de_L = \frac{(\nu - \frac{1}{2})e_H}{e_m^2} < 0$, since $(\nu - \frac{1}{2}) = -\frac{1}{(2+n)} < 0$

and by proposition 2, $\frac{d\tau^*}{de_L} < 0$ for $\tau^* > 0$. ■

5 CONCLUSION

In this paper, we have computed a majority voting equilibrium level of a Beveridgean PAYG social security system in an overlapping generations model with heterogeneous individuals differing by their innate ability in production. The introduction of *heterogeneity* in a political OLG model allows for intragenerational transfers among workers and leads to different optimal tax rates within young individuals. Unless the young rich individuals constitute the majority in the voting population, we may have a *positive* tax rate as outcome of the political negotiations. In the majority coalition, one finds old individuals and relatively poor workers for whom the optimal tax rate is relatively high[11]. The *lower* the growth rate of population (which determines the weight of retirees) and the *lower* the *relative* skill level of the median voter, the *higher* the equilibrium tax rate and the higher will be the support for a Pay-as-you-go social security system.

Unlike the majority of contributions in this field, we have tackled this question in a *general equilibrium model*. This framework puts into evidence other and less noticed effects of social security system. In addition to the well-known intergenerational transfers that a social security system implies, we underline the effects of a PAYG pension system on the interest rate, on the future wages and so on the future level of pensions. We have obtained a remarkable result: the independence of the optimal tax rate on the population growth rate and on the capital level. The optimal tax rate depends only on technology, preferences and on the distribution of skills.

Higher heterogeneity and the aging of population, a phenomenon expected to rise in the next years, will even increase the support to a PAYG pension system, although it becomes in these cases even more problematic.

[11] With a positive growth rate of population, the median voter will be a relative low-skilled young voter.

APPENDIX

Appendix 1: Proof of proposition 1 : Condition to guarantee the single peakedness of the objective function V_{et} , $\forall \tau \geq 0$

1. First derivative

The first derivative of the objective function V_{et} of young agents with respect to the tax rate (τ) can be expressed as the product of two terms:

$$\frac{dV_{et}}{d\tau} = (A\tau^2 + B\tau + C)\left(\frac{ew_t}{C_t(_{1+\beta})}\right) = (A\tau^2 + B\tau + C)\left(\frac{ew_t}{ew_t + \tau(\frac{1-\alpha}{\alpha}K_{t+1}N_t^{-1} - ew_t)}\right)$$

with the value of A, B, C given by (20).

For simplicity, we denominate this expression as :
$\frac{dV_{et}}{d\tau} = J * \xi$

with $J = (A\tau^2 + B\tau + C) > 0$ for $\tau < \tau^*_{ye}$
< 0 for $\tau > \tau^*_{ye}$,
since $J(\tau^*_{ye}) = 0$ (from (21)) and $J(0) > 0$ (see appendix 3),

and
$\xi = \left(\frac{ew_t}{ew_t + \tau(\frac{1-\alpha}{\alpha}K_{t+1}N_t^{-1} - ew_t)}\right) > 0$, $\forall \tau > 0$,
since $w_t > 0$ and $e > 0$.

Consequently, the first derivative of the objective function is **positive** before τ^*_{ye} and **negative** after τ^*_{ye} (where it reaches a maximum):

$$\frac{dV_{et}}{d\tau} = J * \xi \qquad \begin{array}{l} > 0 \ \forall \tau < \tau^*_{ye} \\ < 0 \ \forall \tau > \tau^*_{ye}. \end{array}$$

2. Second derivative

To show that the function V_{et} admits only one unique maximum in τ^*_{ye} , we compute the second derivative of the objective function. This can be expressed as the following: $\frac{d^2V_{et}}{d\tau^2} = \frac{dJ}{d\tau}\xi + \frac{d\xi}{d\tau}J$. Each term can be computed separately.

2.1. First term of the second derivative ($\frac{dJ}{d\tau}$):

$\frac{dJ}{d\tau} = 2A\tau + B < 0$ iff $\alpha \geq \alpha_{\min}$.

Indeed, after some computation, the expression $2A\tau + B$ can be reduced to:

$$\begin{aligned}
&\bar{e}[(\frac{1-\alpha}{\alpha}2\tau+2)\beta+\frac{\beta^2}{\alpha}((3-2\tau)\alpha+2\tau-1)+(1+\alpha)\beta^3] \\
&+(2+2\tau\frac{1-\alpha}{\alpha})+\frac{\beta}{\alpha}[(5-2\tau)\alpha+2\tau-1]+\beta^2(1+\alpha) \qquad (22)
\end{aligned}$$

Sufficient condition for $\frac{dJ}{d\tau} < 0$:

Since the first and third term between the brackets in the first line are always positive, and since it is the same for the first and third term in the second line, a sufficient condition for this expression to be positive is that the second terms in the first and second lines are positive. This is the case iff:

$$\frac{\beta^2}{\alpha}((3-2\tau)\alpha+2\tau-1) \geq 0 \text{ iff } \alpha \geq 1/3$$

and

$$\frac{\beta}{\alpha}[(5-2\tau)\alpha+2\tau-1] \geq 0 \quad \text{iff } \alpha \geq 1/5$$

Consequently,

$$\alpha \geq 1/3$$

is a **sufficient but not necessary condition** for $\frac{dJ}{d\tau} < 0$ to be assured.

Necessary condition for $\frac{dJ}{d\tau} < 0$:

The necessary condition implies a **minimal value on** τ . But this condition will not be binding if the minimal value imposed on τ is negative, which is the case iff :

$$\alpha \geq \alpha_{\min} \qquad (23)$$

Indeed, the necessary condition for $\frac{dJ}{d\tau} < 0$ is

$$\begin{aligned}
\tau \quad \geq \quad & [(1+\beta\bar{e})2(1+\beta)\frac{1-\alpha}{\alpha}]^{-1}[-\frac{\bar{e}}{e}\frac{\beta}{\alpha}(2\alpha+(3\alpha-1)\beta+\alpha(1+\alpha)\beta^2) \\
& -\frac{1}{\alpha}(2\alpha+(5\alpha-1)\beta+(\alpha+\alpha^2)\beta^2)]
\end{aligned}$$

The denominator (the first term) is positive. Therefore, this condition will always be satisfied if the numerator (the second term) is negative. The numerator can be expressed as a second order equation in α:

$$[-\frac{\bar{e}}{e}\frac{\beta}{\alpha}(2\alpha+(3\alpha-1)\beta+\alpha(1+\alpha)\beta^2)-\frac{1}{\alpha}(2\alpha+(5\alpha-1)\beta+(\alpha+\alpha^2)\beta^2)]$$

$$=-[\frac{\bar{e}}{e}\beta^3+\beta^2]\alpha^2-[\frac{\bar{e}}{e}(2\beta+3\beta^2+\beta^3)+(2+5\beta+\beta^2)]\alpha+[\frac{\bar{e}}{e}\beta^2+\beta]$$

This expression will be negative iff :

$$\alpha > \alpha_{\min}$$

or iff

$$\alpha < \alpha_{\max}$$

Since $\alpha_{\max}$ is negative, only the first condition remains on α .

Hence,

$$\frac{dJ}{d\tau} < 0$$

iff

$$\alpha > \alpha_{\min} < 1/3 \tag{24}$$

with

$$\alpha_{\min} = [-\frac{\bar{e}}{e}(2\beta+3\beta^2+\beta^3)-(2+5\beta+\beta^2)+\sqrt{\Delta}][2(\frac{\bar{e}}{e}\beta^3+\beta^2)]^{-1} \tag{25}$$

and with

$$\begin{aligned}\Delta \;=\; & (\frac{\bar{e}}{e})^2[(2\beta+3\beta^2+\beta^3)^2+4\beta^5] \\ & +[2(2\beta+3\beta^2+\beta^3)(2+5\beta+\beta^2)+8\beta^4]\frac{\bar{e}}{e} \\ & +(2+5\beta+\beta^2)^2+4\beta^3\end{aligned}$$

Since the boundary imposed on the share of capital α (namely $\alpha > \alpha_{\min}$) is always strictly inferior to 1/3 ($\alpha_{\min} < 1/3$), the constraint on α is not binding for the value suggested by the empirical studies (around 0.36).

2.2. Second term of the second derivative ($\frac{d\xi}{d\tau}$) :

$\frac{d\xi}{d\tau} < 0 \quad \forall \alpha, \beta, \tau.$

Indeed, $\frac{d\xi}{d\tau} < 0$

iff $\quad \frac{1-\alpha}{\alpha} K_{t+1} N_t^{-1} - ew_t > 0$

or equivalently

iff $\quad \frac{K_{t+1}}{K_t^{\alpha}} > \frac{e}{\bar{e}^{\alpha}} \alpha \frac{N_t}{N_t^{\alpha}}.$

By using (14) and after some computations, one can show that this condition is satisfied since: $\frac{\beta(1-\tau)(\frac{1-\alpha}{\alpha})\frac{\bar{e}}{e} N_t^{-1}}{1+\beta+\tau(\frac{1-\alpha}{\alpha})} > 0$

2.3. Sign of the second derivative

Since $\frac{d^2 V_{et}}{d\tau^2} = \frac{dJ}{d\tau}\xi + \frac{d\xi}{d\tau} J$

with $\frac{dJ}{d\tau} < 0$ iff $\alpha > \alpha_{\min}$
$\xi > 0$
$\frac{d\xi}{d\tau} < 0$
$J > 0 \quad \forall \tau < \tau_{ye}^*$ and $J < 0 \quad \forall \tau > \tau_{ye}^*$

$\frac{d^2 V_{et}}{d\tau^2}$ **will be negative iff**:

$\alpha > \alpha_{\min} < 1/3$

and

$\forall \tau < \tau_{ye}^*.$

For $\forall \tau > \tau_{ye}^*$, the sign of $\frac{d^2 V_{et}}{d\tau^2}$ cannot be determined a-priori.

3. Single-peakedness

The conditions that :

$$\frac{dV_{et}}{d\tau} > 0 \quad \forall \tau < \tau_{ye}^*$$
$$< 0 \quad \forall \tau > \tau_{ye}^*$$

and

$$\frac{d^2 V_{et}}{d\tau^2} < 0 \quad \forall \tau < \tau_{ye}^* \text{ (iff } \alpha > \alpha_{\min})$$

guarantee that the objective function V_{et} is single peaked.

Appendix 2: Proof of proposition 2.a : the effect of the skill level on the optimal tax rate

The optimal tax rate of young individuals is given by the implicit function:

$$J \equiv A\tau_{ye}^{*^2} + B\tau_{ye}^{*} + C = 0$$

with the value of A, B, C given by (20). The optimal tax rate of young agents is the value of τ_{ye}^{*} that sets this expression to zero.

This first order condition can be rewritten as follows to isolate the term e:

$$\left\{ \begin{array}{c} [-(1+\beta)+\beta(1-\alpha)\frac{(1+\beta+\frac{1-\alpha}{\alpha})}{(1+\beta+\frac{1-\alpha}{\alpha}\tau)}]e+\beta\frac{1-\alpha}{\alpha}(\bar{e})^{\alpha} \\ [(1+\beta)(1+\beta+\frac{1-\alpha}{\alpha}\tau)(1-\tau)-(1+\beta+\frac{1-\alpha}{\alpha})\tau(1+\alpha\beta)] \end{array} \right\} = 0 \qquad (26)$$

As it can be seen from (26), the optimal tax rates of two individuals belonging to the same population (namely with the same $\bar{e}$) but having a different skill level e, will only differ to the extent of the effect of the first term on the τ_{ye}^{*}. Namely of :

$$[-(1+\beta)+\beta(1-\alpha)\frac{(1+\beta+\frac{1-\alpha}{\alpha})}{(1+\beta+\frac{1-\alpha}{\alpha}\tau)}]$$

Hence, the sign of this first term will be decisive in determining the effect of the skill level on the optimal tax rate. The first bracket $(-(1+\beta))$ represents the (negative) income effect, while the remaining terms represent the (positive) interest rate effect, which is a decreasing function of α. Consequently, there is a limit value on α above which the income effect will dominate the interest rate effect.

The function J is concave $(A<0)$ and decreasing for all $\tau>0$ $(2A\tau+B<0$ $if\ \alpha>\alpha\min)$. Therefore, if this first term is strictly negative, the function J will be *lower* for a young individual with a *higher* e ($e_2>e_1$). And since the optimal tax rate is defined as the τ_{ye}^{*} that cancels the function (namely the τ_{ye}^{*} at the intersection of the function J with the horizontal axe), the optimal tax rate of this individual will be *lower* $(\tau_{y2}^{*}<\tau_{y1}^{*})$.

The first term will be negative iff:

$$\tau>\tau_{\min}=[(1-\alpha)^2\beta-(1+\alpha\beta)\alpha(1+\beta)][(1+\beta)(1-\alpha)]^{-1} \qquad (27)$$

And this condition will always be satisfied if $\tau_{\min}<0$.

This condition on τ implies a minimal value of α: $\tau_{\min}<0$ iff :

$$\alpha>\alpha_1 \qquad (28)$$

with α_1 given by:

$$\alpha_1 = \frac{-3\beta - 1 + \sqrt{4\beta^3 + 9\beta^2 + 6\beta + 1}}{2\beta^2} > 0 \tag{29}$$

This expression gives α_1 as a function of β. This function is always positive, since the numerator is always positive, namely: $\sqrt{4\beta^3 + 9\beta^2 + 6\beta + 1} > 3\beta + 1$. Moreover, the values given by this function remain small for any β (for example, $\alpha_1 = 0.057$ if $\beta = 0.4$) and this function reaches a maximum value of 0.25 at $\beta = 2$. Therefore, given the numerical estimation of α, this constraint ($\alpha > \alpha_1$) can be considered as not binding.

Hence, if

$$\alpha > \alpha_1 \Rightarrow \tau > \tau_{\min}, \forall \beta$$

$$\Rightarrow$$

$$e_i > e_j \ \Rightarrow \tau^*_{yi} < \tau^*_{yj} \ , \forall \ e_i, e_j \in [e_L, e_H]$$

This proves the proposition 2: the *higher* the skill level e for a given distribution of skills, the *lowe*r the optimal tax rate τ^*_{ye} , as long as $\alpha > \alpha_1$.

Appendix 3: Proof of proposition 2.b: Minimal value of $\frac{\bar{e}}{e}$

$$\tau_{ye}^* = \frac{-B - \sqrt{B^2 - 4AC}}{2A} = \frac{B + \sqrt{B^2 - 4AC}}{-2A}$$

with $A < 0$.

Hence, $\tau_{ye}^* > 0$ iff $B + \sqrt{B^2 - 4AC} > 0$

$\Leftrightarrow$

iff $4AC < 0$

$\Leftrightarrow$

iff $C > 0$

with

$$C = (1+\beta)$$
$$\left[-(1+\beta)^2 + \beta(1-\alpha)(1+\beta+\frac{1-\alpha}{\alpha}) + \beta(\frac{1-\alpha}{\alpha})\frac{\bar{e}}{e}(1+\beta) \right]$$

After some simple computations, one can prove that :

$\tau_{ye}^* > 0$ iff $C > 0$

$\Leftrightarrow$

iff

$$\frac{\bar{e}}{e} > \frac{1}{e_0} = \frac{\alpha(1+\beta)}{(1-\alpha)\beta} - \alpha - \frac{1-\alpha}{1+\beta} \tag{30}$$

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