The importance of the embodied question revisited*

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Abstract

In order to assess the importance of embodiment, we build up an endogenous growth model in which learning by doing is the engine of both embodied and disembodied technological progress. In sharp contrast to Phelps (1962), we show that a change in the composition of technical change affects the growth rate in the long run. We also provide an alternative explanation for the *productivity slowdown*: an increase in the fraction of embodied technical progress, through an improvement in the learning efficiency of the capital goods sector, permanently lowers the growth rate of technological progress, by increasing the obsolescence costs of investment. The productivity slowdown occurs together with a rise in the rate of decline of investment goods prices. Finally, we show that an increase in the embodied fraction of technical change reduces the gap between the optimal and the decentralized growth rates.

Keywords: Embodied technical progress, Obsolescence, Learning by doing, Productivity slowdown, Optimal growth.

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1 Introduction

Considered roughly unimportant by Denison (1964) and irrelevant in the long run by Phelps (1962) three decades ago, the embodied nature of a substantial fraction of technological progress has been repeatedly invoked in a number of recent theoretical and empirical contributions to growth and business fluctuations research areas. In particular, Greenwood, Hercowitz and Krusell (1997) have found that around 60% of US productivity growth can be attributed to embodied technological change. Among other interesting features, the embodied characteristic of technological progress has been shown to be relevant in explaining a number of puzzling stylized facts of the US economy, such as the productivity slowdown, the decline in the relative price of investment goods and the persistent rise in the equipment to output ratio.[1] It is now widely admitted that embodiment is crucial in the understanding of these phenomena and several other related issues.

On a purely theoretical ground, some important advances have been already accomplished regarding the way embodiment can affect growth and business fluctuations through the corollary machine replacement and technology adoption problems. A key aspect of the analysis turns out to be *learning*. Indeed, since technological progress is partly investment specific and innovations occur continuously, the effects of embodiment rely heavily on the ability of the economy to learn quickly how to use efficiently the new capital goods. In most of the recent contributions, learning is modeled in a purely mechanical way: Assuming that workers are initially unable to use efficiently the new capital goods, *ad-hoc* learning rules are specified often *via* human capital accumulation.[2] Alternatively, and in line with Arrow (1962)'s learning by doing theory, this paper investigates a simpler and much more traditional approach to learning, in a context where both embodied and disembodied technological progress are present. Greenwood and Jovanovic (1998) have briefly mentioned that Arrow's learning by doing has a number of interesting implications, in particular regarding the relative price of capital. This line of research is comprehensively explored in this paper. As in the exogenous growth model proposed by Greenwood, Hercowitz and Krusell (1997), the state of technology is defined by both total factor productivity in the production of consumptions goods and the marginal productivity of new in-

[1] The contributions of Cooley, Greenwood and Yorukoglu (1997), Greenwood, Hercowitz and Krusell (1997), Greenwood and Yorukoglu (1997), Hornstein and Krusell (1996), and Krusell (1998) are some appealing examples of this recent trend in the macroeconomic literature. More theoretical contributions in this field using explicit vintage capital settings can be found in Benhabib and Rustichini (1991), Boucekkine, Germain and Licandro (1997) and Boucekkine, del Río and Licandro (1999).

[2] Recent examples of this kind of model are Parente (1994), Greenwood and Yorukoglu (1997), and Greenwood and Jovanovic (1998).

vestment goods. However in our model, the engine of growth is learning by doing (LBD), the technological variables being isoelastic functions of cumulative efficient investment. LBD in the production of consumption goods is consistent with Romer (1986) whereas LBD in the production of investment goods is close to Greenwood and Jovanovic (1998), a slightly modified version of Arrow (1962). The LBD process in the former (Resp. latter) sector represents *disembodied* (Resp. *embodied*) technological change.

This simple theoretical framework allows us to bring out a number of meaningful results on embodiment and learning. Indeed, our contribution is twofold. The first implication of our framework is that a change in the composition of technological progress, through an increase in the efficiency of LBD in the capital goods sector, affects the long run growth rate of the economy. This result is very appealing from a theoretical point of view. Indeed, since the theoretical contribution of Phelps (1962), a view emerged as embodied technical progress only could matter in the short run. Phelps considered an exogenous growth model in which the rates of both embodied and disembodied technical change were constant. He found that while the former speeds up convergence to the steady state growth path, the long run growth rate only depends on the sum of these two exogenous rates. Hence, according to Phelps, embodied technological progress is unimportant in the long run. This paper contradicts this claim. We show that once the two forms of technological progress are endogenized using learning by doing, Phelps prediction does not hold. The key economic mechanism behind this result is related to the obsolescence cost specific to embodied technological progress: Since a drop in the fraction of embodied technical change has a direct effect on the obsolescence costs, it implies a change in the user cost of capital, which typically determines the growth rates in endogenous growth settings.

On the other hand, our model provides an alternative explanation for the so-called *productivity slowdown*, consistently with the most relevant results reported in the recent empirical and theoretical literature on embodiment and learning. Indeed, some authors led by Hornstein and Krusell (1996) argue that the productivity slowdown that has occurred in most developed countries since the first oil shock, may well have been caused by an acceleration in embodied technological progress. In effect, the decline of the relative price of equipment, which can be seen as a proxy of the importance of the latter form of technical change, has experienced a significant acceleration after 1974 in the US economy, passing from 3.3% per year to 4%. Greenwood and Yorukoglu (1997) succeed at replicating these stylized facts in a computable general equilibrium framework once adoption costs are incorporated, mainly in the form of direct learning costs. In this framework, an unexpected and permanent rise in the rate of embodied technological change will cause labor productivity to slowdown during a certain number of periods, the exact duration of the slowdown depending on the size of the learning costs.[3] The productivity slowdown arises because adoption through learning is costly: With an increase in the rate of adoption more resources are devoted to new technologies where experience is low. As a result, labor productivity and total factor productivity growth fall temporarily. Our model puts forward an alternative mechanism through which an increase in the fraction of embodied technological progress allows to replicate all the elements of the productivity slowdown, as observed in the US economy. Despite accumulating knowledge has no direct cost under learning by doing, embodied technical change implies an indirect cost due to the obsolescence of the existing equipment. An acceleration in embodied technical change increases the user cost of capital, by rising obsolescence costs, and then reduces the endogenous growth rate of output via a decrease in the total rate of technological progress.

So in explaining the productivity slowdown, our model puts forward a kind of technological composition or *reassignment* effect: Since the first oil shock, technological progress has mainly consisted in a continuous increase in the efficiency of the capital goods sector while the gains in efficiency of the consumption goods sector, if any, have been much lower. The subsequent natural question is: Does the increase in productivity growth in the capital goods sector characterize conveniently the post first oil shock period? We do think this is indeed one of the important features of this period. A substantial empirical evidence supports this view (see for example Baily, Barstelsman and Haltiwanger, 1994). As Kortum (1997) noticed, "two industries display much more rapid productivity growth after 1974 than before, industrial machinery (which include computing equipment) and electrical equipment. All the rest either had roughly constant productivity growth or slower productivity growth after 1974". This change in the composition of technological progress may well be the major characteristic of the post first oil shock period, ingeniously referred to as the Third Industrial Revolution by Greenwood and Yorukoglu (1997).

We organize the rest of the paper as follows. In Section 2 we present our general modeling strategy, compute the long run growth rate and show some convergence properties. Indeed, our model specification is so general that it can be shown to hold in quite different frameworks. For example, our general model is completely consistent either with vintage capital set-ups or with the two sector models as built up in Greenwood *et al.* (1997, 1998). In section 3 we relate our findings to Phelps theory (1962) on the irrelevance

[3]These numerical findings show clearly how important is learning in Greenwood and Yorukoglu's model. In the absence of adoption costs, as in Boucekkine *et al.* (1998), the productivity slowdown does not last more than a few years.

of embodiment in the long run. Section 4 analyzes the interaction between learning efficiency in the capital goods sector and the productivity slowdown. We also investigate in this section whether an increase in the fraction of embodied technical change reduces the gap between the optimal and the decentralized growth rates. Section 5 concludes.

2 The model

As mentioned just above, we present our arguments on a quite general theoretical framework that fits several important set-ups proposed in the literature of this field. Indeed, the equilibria of many growth models can be described by the following four equations under a Cobb-Douglas technology and assuming preferences with constant intertemporal elasticity of substitution:

$$y_t = z_t k_t^{1-\alpha} = c_t + i_t, \tag{1}$$

$$\dot{k}_t = q_t i_t - (\delta + n)\, k_t, \tag{2}$$

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}\left((1-\alpha)\, z_t q_t k_t^{-\alpha} - \delta - \frac{\dot{q}_t}{q_t} - \rho \right), \tag{3}$$

$$\lim_{t\to\infty} \frac{k_t}{q_t} e^{-(\rho-n)t} c_t^{-\sigma} = 0, \tag{4}$$

where y_t, k_t, c_t and i_t are production, capital, consumption and gross investment at time t, respectively. All variables are in per-capita terms. Equation (1) is the usual resource constraint with $0 < \alpha < 1$, the labor share. Equation (2) gives the law of motion of efficient capital per-capita, with $0 < \delta < 1$ the depreciation rate, and $n > 0$ the population growth rate. The third equation is the standard Euler equation yielded by growth models, with $\sigma > 0$ the inverse of the intertemporal elasticity of substitution, and $\rho > n > 0$ the rate of time preference. Finally, equation (4) is the transversality condition.

The variables z_t and q_t represent the state of knowledge at time t. An increase in z_t rises the marginal productivity of all the capital stock, independently of its age structure. Hence, z_t represents *disembodied* technological progress. In sharp contrast, q_t only affects new equipment by equation (2), and represents *embodied* technological progress. There is a much more crucial difference between the two forms of technical change: Embodiment implies obsolescence, and this is reflected in our model through the term $-\frac{\dot{q}_t}{q_t}$ appearing in the Euler equation (3). Since technological progress is partly embodied, the user cost of capital includes the loss of value due to future technological improvements, which will only affect future capital goods. This feature is the major departure with respect to the standard optimal growth model. To keep the presentation as simple as possible, we report in the Appendix a rather sophisticated vintage capital foundation of the system (1)-(4). As the latter system may as well characterize the equilibrium of simple two-sector economies as in Greenwood *et al.* (1997, 1998), we decided to adopt this simpler and more general approach.

Growth models essentially differ in their assumptions concerning z_t and q_t. In the neoclassical growth theory (see Solow, 1956), q_t is kept constant and z_t is allowed to grow exponentially at a constant rate. In a subsequent work, Solow (1960) proposes the inverse specification based on the observation that "...many if no most innovations need to be embodied in new kinds of durable equipment before they can be made effective". The capital stock is thus seen as a sequence of vintages, the new vintages being more productive. If accordingly z_t is kept constant and q_t is allowed to grow exponentially at a constant rate, a steady state growth path is shown to arise with a long run growth rate proportional to the growth rate of q_t. Phelps contribution (1962) is a worthwhile complement to the two previous approaches, as it puts together both embodied and disembodied technological progress in the same exogenous growth framework. We will come back to this seminal paper in the next section.

In this paper, we consider that technological progress is essentially endogenous and to this end, we assume *learning by doing* (LBD) as in Arrow (1962). By proceeding in this way, we do not only endogenize technological progress, we place a concept of learning at the heart of our analysis. We set $z_t = z\, k_t^\gamma$ and $q_t = q\, k_t^\lambda$, with z, q, γ and λ four strictly positive real numbers. Additionally we assume that: i) social returns to capital are constant, namely $\gamma + \lambda + 1 - \alpha = 1$ or $\gamma + \lambda = \alpha$, and ii) the effects of capital accumulation on technical progress are not internalized by firms. As usual, condition i) is needed for a balanced growth path to exist, and condition ii) is consistent with the existence of a competitive equilibrium. With these assumptions, the system (1)-(4), describing the decentralized equilibrium of the considered economy, can be rewritten as a differential equation system on k_t and c_t, $\forall t \geq 0$,

$$\dot{k}_t = (zq - \delta - n)\, k_t - q\, k_t^\lambda\, c_t, \tag{5}$$

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}\left((1-\alpha) zq - \delta - \rho - \lambda \frac{\dot{k}_t}{k_t}\right), \tag{6}$$

$$\lim_{t \to \infty} \frac{k_t^{1-\lambda}}{q} e^{-(\rho-n)t} c_t^{-\sigma} = 0, \tag{7}$$

given the initial condition $k_0 > 0$, under the additional assumption $i_t \geq 0$, $\forall t \geq 0$. As it is standard in LBD models, the aggregate technology is linear, where zq is equivalent to the A term in an AK technology. Since individual firms do not internalize the LBD externality, the individual marginal productivity of capital is a fraction $(1-\alpha)$ of the aggregate marginal productivity zq.

Our formalization of technological progress departs from Arrow's specifications in several respects. Apart from the fact that we are clearly distinguishing between the two forms of technological progress,[4] we do not consider Leontief technologies as in the original Arrow's contribution. Furthermore, the technological variables q_t and z_t are taken as functions of the cumulative *efficient* investment *per capita* while in Arrow, the efficiency of capital goods is measured by cumulative investment.[5] Our model can also be closely related to two other major references in the field, Frankel (1962) and Romer (1986). In the former, the limit case $\lambda = 0$ is analyzed, i.e., technological progress is totally disembodied. Under constant social returns to capital, namely if $\gamma = \alpha$, Frankel shows that a steady state growth path with an endogenous growth rate does arise. Frankel's model is at the basis of Romer's paper, as noticed by Aghion and Howitt (1998), since it can be perfectly seen as a limit case of Romer's analysis in which social returns to capital are assumed increasing.

The steady state growth rate can be very easily computed from (5) and (6):

$$g = \frac{1}{\hat{\sigma}}\left((1-\alpha) zq - \delta - \rho\right), \tag{8}$$

where $\hat{\sigma} = \sigma + \frac{\lambda}{1-\lambda} \geq \sigma$, with the growth rate of capital being $g_k = \frac{g}{1-\lambda} > g$. The key difference with the standard results comes from the obsolescence term, $\frac{\lambda}{1-\lambda}$, which alters the corresponding term of the inverse of the intertemporal elasticity of substitution. To guarantee that the growth rate is positive at the balanced growth path and that utility at the decentralized equilibrium is bounded, we impose the following assumption:

[4] Indeed, Arrow's specifications correspond basically to our special case $\gamma = 0$.

[5] In Greenwood and Jovanovic (1998), the efficiency of capital goods is captured by an isoelastic function of the cumulative efficient gross investment. We define our technological variables in per capita terms, in order to remove scale effects.

Assumption 1

$$(1-\alpha)\, zq > \delta + \rho > (1-\sigma)\, g + \delta + n.$$

The first part of this condition implies $g > 0$. The second part ensures that the transversality condition (7) holds and that equilibrium utility is bounded.

It remains to see whether consumption is positive along a balanced growth path with the growth rate given by (8). Indeed, although Assumption 1 guarantees the positivity of consumption's growth rate, it does not ensure at first glance the positivity of consumption as computed from the resource constraint (5). The following proposition provides a condition for this property to hold.

Proposition 1 *Under Assumption 1, a steady state equilibrium with a positive consumption to output share exists if and only if the following condition is fulfilled*

Assumption 2

$$\frac{g}{1-\lambda} + \delta + n < zq.$$

Proof: Let us define the consumption to output share as $\chi_t = \frac{c_t}{z{k_t}^{1-\lambda}}$ and rewrite the resource constraint (5) as follows

$$\frac{\dot{k}_t}{k_t} = zq - \delta - n - zq\chi_t. \tag{9}$$

Since $\frac{\dot{c}_t}{c_t} = \frac{\dot{\chi}_t}{\chi_t} + (1-\lambda)\, \frac{\dot{k}_t}{k_t}$, the Euler equation (6) can be rewritten after some algebra as

$$\frac{\dot{\chi}_t}{\chi_t} = \eta + \zeta\chi_t, \tag{10}$$

where

$$\eta = \frac{(1-\alpha)\, zq - \delta - \rho}{\sigma} - \frac{(1-\lambda)\sigma + \lambda}{\sigma}\, (zq - \delta - n)\,,$$

and

$$\zeta = \frac{(1-\lambda)\sigma + \lambda}{\sigma} zq.$$

The BGP solution is $\chi = -\frac{\eta}{\zeta}$. Since ζ is strictly positive, we need Assumption 2 to ensure the strict positivity of χ. □

It should be noted that Assumption 2 implies an upper bound for the long run growth rate of capital $g_k = \frac{g}{1-\lambda} < zq - \delta - n$. Our interpretation of Assumption 2 follows : For a fixed λ, if g_k is "too" high, the obsolescence cost as measured by $\frac{\dot{q}_t}{q_t} = \lambda g_k$, will be so high that the rise in investment required to sustain this capital growth rate will induce negative consumption. Indeed, we can prove a much stronger result under Assumptions 1 and 2: The previous steady state growth path is the unique solution path of the dynamic system (5)-(7). That is to say our model behaves like the standard AK model, in particular, it does not display any transitional dynamics. This property is established in the following proposition.

Proposition 2 *Under Assumption 1 and Assumption 2, for any $k_0 > 0$, the dynamic system described by (5)-(7) yields a unique solution path, in which consumption per capita and the capital stock per capita grow at the constant rates g and g_k, respectively.*

Proof: Denote by χ the (positive) steady state value of variable χ_t, which dynamics is described by equation (10). Observe that by (7) the transversality condition does not hold if

$$\rho < (1-\lambda)\frac{\dot{k}_t}{k_t} - \sigma\left[\frac{\dot{\chi}_t}{\chi_t} + (1-\lambda)\frac{\dot{k}_t}{k_t}\right] + n \tag{11}$$

when t goes to infinity. We can prove now that $\chi_t = \chi$. We prove this by contradiction. If $\chi_t < \chi$ then $\frac{\dot{\chi}_t}{\chi_t}$ goes to $\eta < 0$ and χ_t goes to zero when $t \to \infty$. From (9) $\frac{\dot{k}_t}{k_t}$ goes to $zq - \delta - n$. Substituting $\frac{\dot{k}_t}{k_t}$ and $\frac{\dot{\chi}_t}{\chi_t}$ by their limit values, it can be shown after some trivial algebra that the induced right hand side of (11) is higher than ρ. Thus, the transversality condition fails to hold, and this path cannot be an equilibrium.

If $\chi_t > \chi$, then $\frac{\dot{\chi}_t}{\chi_t} > 0 \; \forall t \geq 0$ and χ_t goes to infinity. Thus, there exists a date $\bar{t}$ such that the resource constraint is violated (from (9) $\frac{\dot{k}_t}{k_t}$ goes to minus infinity, and then from (2) investment should become negative, which is excluded by assumption) and, therefore, this path can not be an equilibrium.

Let us now show that, given $k_0 > 0$, c_t and k_t grow at constant rates for all $t \geq 0$. By definition of χ, $c_0 = \chi \; z \; k_0$. Moreover, from (9) the growth rate of k_t is constant $\forall t \geq 0$, which implies, by constancy of χ that the grow rate of c_t is also constant $\forall t \geq 0$. □

With the latter analytical characterization of the equilibrium dynamics, we are now ready to rigorously analyze Phelps prediction within our set-up.

3 Revisiting Phelps irrelevance prediction

As mentioned in the introduction, Phelps assumes that both z_t and $q_t^{1-\alpha}$ grow exponentially at the exogenous constant rates g_d and g_m, respectively, and he finds that the nature of technological progress is unimportant in the long run, since the stationary growth rate is proportional to $g_d + g_m$. This property can be easily checked in the system (1) to (4).

For comparison purposes, we will investigate if in our model the composition of technological progress matters in the long run. Let us define the rate of technological progress as in Phelps (1962)

$$\frac{\dot{a}_t}{a_t} \equiv \frac{\dot{z}_t}{z_t} + (1-\alpha)\,\frac{\dot{q}_t}{q_t} = (\gamma + (1-\alpha)\lambda)\,g_k = \alpha\,(1-\lambda)\,g_k. \tag{12}$$

The last equality comes from the assumption of constant social returns to capital. One can notice that a fraction $\frac{(1-\alpha)\lambda}{\alpha(1-\lambda)}$ of the technological progress is embodied and a fraction $\frac{\alpha-\lambda}{\alpha(1-\lambda)}$ is disembodied. For a given α, an increase in λ rises the former fraction and reduces the latter. Consequently, the effect on growth of an increase in the fraction of embodied technological progress, at given α, can be computed by differentiating g with respect to λ in equation (8). It can be easily shown that an increase in the proportion of embodied technical progress reduces the growth rate, by increasing the obsolescence costs. The growth rate reaches its maximum at $\lambda = 0$. So in sharp contrast to Phelps finding, a composition effect does arise: The long run growth rate does depend on the composition of technological progress.

The previous results can be interpreted taking into account the obsolescence effect due to any shift in λ, and having in mind that in our exercise *à la* Phelps, an increase in λ should be compensated by a decrease in γ to hold α constant. Due to the production function specification (1), a decrease in γ has a direct and negative effect on the marginal productivity of capital. However, an increase in λ improves the efficiency of new equipment due to the rise in the learning ability in the capital goods sector, and this tends to increase the marginal productivity of capital. Because social returns to capital are assumed to be constant, this positive effect completely compensates the negative effect of the decrease in γ.[6] However, the obsolescence effect does remain effective: An increase in λ rises the obsolescence cost through

[6] A quick look at the Euler equation (6) is sufficient to conclude for this. Indeed, the

the term $\frac{\dot{q}_t}{q_t} = \lambda g_k = \frac{\lambda}{1-\lambda} g$. From (6), it follows that an increase in the obsolescence cost lowers the equilibrium interest rate and so tends to decrease the growth rate of consumption, giving rise to the typical intertemporal substitution effect in optimal growth models. An increase in the fraction of embodied technological progress is bad for growth since it only affects the new capital goods, which in turn rises the velocity at which the old equipment becomes obsolete. In contrast, an increase in the weight of disembodied technological progress is good for growth since it affects all the capital goods, independently on their vintage.

It is important to notice that the negative effect on output growth of an increase in the weight of embodied technical progress can be associated with either an increase or a decrease in the growth rate of capital. Remember that

$$g_k = \frac{g}{1-\lambda} = \frac{(1-\alpha)\, zq - \delta - \rho}{\lambda\,(1-\sigma) + \sigma},$$

implying that the effect of an increase in λ on the growth rate of the capital stock depends on σ. When the intertemporal elasticity of substitution is high ($\sigma < 1$), an increase in the obsolescence costs has an important (negative) effect on growth, generating a decrease in the growth rate of capital. On the other hand, when the intertemporal elasticity of substitution is low ($\sigma > 1$), an increase in the obsolescence costs has a less important effect on growth inducing an increase in the growth rate of capital. Note that when $\sigma = 1$, i.e., under logarithmic preferences, an increase in λ has no effect on the growth rate of the capital stock, but Phelps prediction is still ruled out.

The analysis above makes clear why Phelps prediction does not hold in our setting and why it should not hold generally in endogenous growth models. Indeed, while the steady state growth rates are determined exogenously by the rate of technological progress in exogenous growth models *à la* Phelps, they strongly depend on the incentives to accumulate capital in our framework and generally in endogenous growth settings. Since these incentives depend on the marginal return to capital, and since the two forms of technological progress have very different effects on the latter, the equilibrium growth rates should be altered by a change in the composition of technological progress. In our framework, the obsolescence cost is specific to embodied technical change. This is enough to generate the composition effects analyzed above.

marginal productivity of capital at the decentralized equilibrium is given by $(1-\alpha)\, zq$. A shift in λ has not effect on this expression.

4 The economic consequences of learning by doing

As mentioned in the introduction section, our very simple model has several other appealing implications both at an empirical and a theoretical level. Hereafter, we display its most salient features.

4.1 The productivity slowdown

Recall that the productivity slowdown is characterized by a sharp reduction in both the growth rate of per capita output and the growth rate of total factor productivity. Moreover, the productivity slowdown is contemporaneously associated with a rise in the decline rate of investment goods prices, as reported by Greenwood and Yorukoglu (1997), and a reduction in the rate of disembodied technological progress, as reported by Hornstein and Krusell (1996).

In our model, the increase in the decline rate of investment goods prices is quite a direct consequence of an increase in λ. Indeed, if we denote by π the decline rate of the relative price of investment goods, and since it is by construction the opposite of the growth rate of the efficiency variable q_t, we have

$$\pi = \frac{\dot{q}_t}{q_t} = \frac{\lambda}{1-\lambda} g, \tag{13}$$

which can be rewritten, using (8), as

$$\pi = \frac{(1-\alpha)\, zq - \delta - \rho}{\frac{1-\lambda}{\lambda}\sigma + 1}. \tag{14}$$

As desired, the decline rate of the relative price of capital is an increasing function of λ.

It remains to prove that the rate of disembodied technical progress is a decreasing function of λ. Since $z_t = zk_t^{\gamma}$, the rate of disembodied technical progress is $\frac{\dot{z}_t}{z_t} = \gamma g_k$. Using equation (8) together with $g_k = \frac{g}{1-\lambda}$ and the assumption of constant social returns to capital, we can find an expression for $\theta \equiv \frac{\dot{z}_t}{z_t}$ involving λ:

$$\theta = \frac{\alpha - \lambda}{1-\lambda}\, g = \frac{\alpha - \lambda}{(1-\lambda)\,\sigma + \lambda} \left((1-\alpha)\, zq - \delta - \rho \right). \tag{15}$$

As desired, θ is a decreasing function of λ.

It is therefore clear from equations (8), (14) and (15) that an increase in λ can account for the empirical puzzle described in the introduction. If the post-1974 period implied a higher learning efficiency in the sector of capital goods, captured here by an increase in λ, and a subsequent decrease in the learning efficiency in the consumption goods sector, as captured by a decrease in γ, our model forecasts the productivity slowdown and the increase in the decline rate of investment goods prices, as observed in the US data.[7] The economic growth rate must also go down for the same technological reassignment effect.

An important consequence of our model is that the productivity slowdown is a permanent phenomenon and not just the result of a very long adjustment process, as in Greenwood and Jovanovic (1998, section 4.4), depending on learning effects and diffusion lags.

A quantitative exercise

In order to evaluate the empirical relevance of our model, we perform the following quantitative exercise, trying to be as close as possible to Greenwood, Hercowitz and Krusell (1997). Their production technology employs two different types of capital goods, namely equipments and structures. For the US economy, they calibrate the equipment and structures factor shares to 0.17 and 0.13, respectively, with the corresponding depreciation rates 0.056 for structures and 0.124 for equipments. The average depreciation rate, consistent with their Cobb-Douglas technology is around 0.095. Consequently, we set $\alpha = 0.7$ and $\delta = 0.095$. We take $\rho = 0.05$ from the same authors. Finally, they assume that only equipments profit from embodied technical change, implying that their rate of embodied technological progress for aggregate capital is around 1.81% (3.2% for equipments and zero for structures). They set the average growth rate of per capita output to 1.24%. We have calibrated the growth rate of per capita output to 1.4% and 1%, and the growth rate of embodied technical change to 1.7% and 2%, before and after 1973, which correspond to the same average rates than in Greenwood, Hercowitz and Krusell.

Given the growth rate of per capita output and the decline rate of investment goods prices, before and after 1973, from equation (13) we can deduce the values of λ consistent with our model, for the referred sample periods. We find that λ should have taken the following values: 0.55 and 0.67, before and after 1973. Given α and λ, we can compute the corresponding fractions of embodied technical change: 0.52 and 0.86, before and after 1973 (the corresponding average is 0.66, slightly higher than the 0.6 found by Greenwood,

[7]The decrease in γ comes exclusively from the fact that we assume α constant according to US data, as reported by Blanchard (1997).

Hercowitz and Krusell). In order to satisfy equation (8), the intertemporal elasticity of substitution should be 1.33 and zq should be equal to 0.58.

Consequently, our model can reproduce the salient features of the productivity slowdown of the US economy for plausible parameter values. Note also that the quantitative implications for the fraction of embodied technological progress are rather consistent with the empirical studies recently devoted to this topic. Obviously, a more complete story is needed here to explain why and how technological progress has mainly consisted in continuous efficiency gains in the capital goods sector since the first oil shock, whereas these gains, if any, have been much lower in the consumption goods sector. This drop in the composition of technological progress is in our opinion completely in line with the view that the first oil shock has notably raised the incentives to produce new, more efficient and less energy demanding capital goods. In other words, the first oil shock has redirected the R&D activity towards the capital goods sector, which explains for example the boom of the non-electrical machinery sector as observed on US data (see Baily and Gordon, 1988). Note also that our interpretation of the productivity slowdown is rather consistent with the early explanations of this puzzle. For example, Baily (1981) has argued that the productivity slowdown results from to the obsolescence of a significant part of the capital stock due to the dramatic increase in energy costs. In our view, this a good argument to start with and the obsolescence concept should be placed at the heart of any comprehensive analysis of the productivity slowdown. However, in contrast to the early literature in this field, we don't think that the productivity slowdown is merely a story of intensive scrapping of out-dated machines. It is just one of the most salient features of the technological reassignment effect described above.

4.2 Optimal versus decentralized growth

In our model, the decentralized equilibrium is not Pareto optimal, because firms do not internalize the effects of capital accumulation on technical progress. If we understand that a third industrial revolution has taken place since 1974, and if we characterize this revolution by the technological reassignment effect described above, one would like to know to which extent the markets perform better in terms of output growth in this new situation . To this end, we first compute the growth rate corresponding to the social optimum. A planner maximizes the infinitely-lived representative household's utility subject to the resource constraint (5). Typically, the Hamiltonian of the social planner's optimization problem looks like:

$$H_t\left(c_t, k_t, \mu_t\right) = \frac{c_t^{1-\sigma}}{1-\sigma} e^{-(\rho-n)t} + \mu_t\left(\left(zq-\delta-n\right)k_t - qk_t^{\lambda}c_t\right).$$

The corresponding optimality conditions for an interior maximum of H can be written as, $\forall t \geq 0$,

$$\dot{k}_t = (zq - \delta - n)\, k_t - q\, k_t^{\lambda}\, c_t, \tag{16}$$

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}\left(zq - \delta - \rho - \lambda q k_t^{\lambda-1} c_t - \lambda \frac{\dot{k}_t}{k_t}\right). \tag{17}$$

$$\lim_{t\to\infty} \frac{k_t^{1-\lambda}}{q} e^{-(\rho-n)t} c_t^{-\sigma} = 0, \tag{18}$$

given $k_0 > 0$. There are two important differences between (17) and (6). First, the central planner evaluates the marginal productivity of capital taking into account that social returns to capital are constant. Secondly, by increasing the capital stock, investment goods become more productive, which rises the value of consumption in the resource constraint (16). This negative externality is also considered by the planner and it corresponds to the term $-\lambda\, q\, k_t^{\lambda-1}\, c_t$.

Using the resource constraint (16) in order to remove the term $k_t^{\lambda-1}\, c_t$, from (17), we can compute the optimal growth rate

$$g^o = \frac{1}{\sigma}\left((zq - \delta - \rho) - \underbrace{\lambda\,(zq - \delta - n)}_{embodimentcosts}\right), \tag{19}$$

which is time independent.[8] Without embodied technical progress, the term on λ vanishes and we have the standard optimal growth rate in LBD growth models. However, when the embodied fraction of technological progress increases, the optimal growth rate clearly decreases. Remember that, from the first inequality in Assumption 1 and $\rho > n$, $zq - \delta - n > 0$. Embodiment costs come from both obsolescence of capital goods and the rise in consumption value. Since the planner takes into account the cost associated with the increase in consumption value, the optimal growth rate g^o does depend on the population growth rate n.

The proposition below summarizes our findings regarding the gap between the optimal growth rate g^o and the decentralized growth rate g. Among other findings, it is shown that the gap between the two growth rates is a decreasing function of λ, the learning ability in the capital goods sector.

[8] If we substitute g by g^o in Assumptions 1 and 2, then we can easily prove existence and uniqueness of the balanced growth path, and instantaneous convergence, as in Propositions 1 and 2.

Proposition 3 *The optimal growth rate g^o checks the following three properties: i) $g^0 > g$, ii) $\frac{\partial g^o}{\partial \lambda} < 0$, and iii) $\frac{\partial (g^o - g)}{\partial \lambda} < 0$.*

Proof: Properties i) and ii) come quite immediately from the expressions of g^o, g and b under Assumption 1 and Assumption 2. Property iii) is a little bit more complicated and we demonstrate it just below. From (8) and (19), we get

$$\begin{aligned}\frac{\partial (g^o - g)}{\partial \lambda} &= \frac{1}{\sigma}\left(-\left(zq-\delta\right)+n\right)+\frac{(1-\alpha)\,zq-\delta-\rho}{\left(\sigma\left(1-\lambda\right)+\lambda\right)^2} = \\ &= \frac{1}{1-\lambda}\left(-g^o-\frac{\rho-n}{\sigma}+\frac{g}{\left(\sigma\left(1-\lambda\right)+\lambda\right)}\right).\end{aligned}$$

If $\sigma < 1$ then $\frac{1}{(\sigma(1-\lambda)+\lambda)} \leq \frac{1}{\sigma}$ and therefore

$$\frac{\partial (g^o - g)}{\partial \lambda} \leq \frac{1}{1-\lambda}\left(-g^o-\frac{\rho-n}{\sigma}+\frac{g}{\sigma}\right).$$

Under Assumption 1, $g > \frac{g-\rho+n}{\sigma}$. Hence

$$\frac{\partial (g^o - g)}{\partial \lambda} < \frac{1}{1-\lambda}\left(-g^o+g\right) < 0.$$

If $\sigma \geq 1$ then $\frac{1}{(\sigma(1-\lambda)+\lambda)} \leq 1$, and therefore

$$\frac{\partial (g^o - g)}{\partial \lambda} \leq \frac{1}{1-\lambda}\left(-g^o-\frac{\rho-n}{\sigma}+g\right) < 0.$$

It follows that under Assumption 1, $\frac{\partial (g^o - g)}{\partial \lambda} < 0$ for all σ. □

As explained previously, the planner internalize two externalities. The first one corresponds to the difference between the aggregate and the individual marginal productivity of capital. It is constant and positive, and does not depend on the learning efficiency in the investment goods sector. The second externality is negative and, as explained before, it corresponds to the rise in the consumption value that follows an increase in the capital stock. Its magnitude is an increasing function of the learning efficiency in the investment goods sector. Property i) states that the positive externality has always a greater effect on the optimal growth rate than the negative externality. Since a rise in the learning efficiency in the investment goods sector only increases the magnitude of the negative externality, it reduces the optimal growth rate (property ii). Finally, the difference between the optimal and the decentralized growth rates falls down when the learning efficiency in the investment goods sector goes up because the planner only takes into account the consumption value externality in this case (property iii).

5 Conclusion

We have developed a very simple endogenous growth model in which learning by doing is the engine of both embodied and disembodied technical progress, in line with Arrow (1962), Frankel (1962) and Romer (1986). In this endogenous growth set-up, we have shown that the nature of technological progress does matter in the determination of the long run growth rate, in sharp contrast to the findings of Phelps (1962) under exogenous growth. The key mechanism in our model is related to obsolescence costs, which are specific to embodied technological change. Since the growth rate is endogenously determined by the marginal return to capital, and since the latter is not insensitive to the nature of technological progress because of the obsolescence cost, Phelps prediction cannot hold.

We also suggest that if the post-1974 period implied a higher learning capacity in the sector of capital goods, our model accounts for the productivity slowdown as observed after the first oil shock. A higher learning elasticity in the capital goods sector implies a fall in the growth rate, an increase in the decline rate of the relative price of capital goods and a reduction in the rate of disembodied technical progress. In contrast to the usual learning explanation, as reported in Hornstein and Krusell (1996) and Greenwood and Yorukoglu (1997), ours relies only on a technological reassignment effect and not on any loss of resources due to costly adoption. In our framework, the Third Industrial Revolution is at first characterized by an increase in the efficiency of learning in the capital goods sector, while the gains in learning efficiency in the consumption goods sector, if any, have been much lower. If we believe that the first oil shock has raised the incentives to produce new, more efficient and less energy demanding capital goods, the following step in this research project should be to build a R&D model with the fraction of embodied technological change depending on energy costs. Also note that, if after 1974 the economic growth rate is lower because of a higher fraction of embodied technological progress, our model also predicts that the gap with respect to the optimal growth rate should have been lowered. This is a strong result that deserves a more comprehensive appraisal in alternative settings.

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Appendix: A vintage capital foundation of the model

In this appendix we will provide a vintage capital foundation of the system (1)-(4) (it founds also the system (5)-(7), under $z_t = z\ k_t^{\gamma}$ and $q_s = q\ k_t^{\lambda}$).

In our economy, we assume that the population grows at a constant rate $n > 0$. The infinitely-lived representative household's optimization problem is

$$U = \max \int_0^{\infty} \frac{{c_t}^{1-\sigma}}{1-\sigma} e^{-(\rho-n)t} dt,$$

subject to

$$\dot{a}_t = w_t L_t - c_t + (r_t - n)\, a_t,$$

where a_t is the value of per capita assets at time t, w_t the wage rate and r_t the interest rate. The Euler and transversality conditions of this very standard dynamic optimization problem are:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma} (r_t - \rho)\,, \tag{20}$$

$$\lim_{t\to\infty} a_t {c_t}^{-\sigma} e^{-(\rho-n)t} = 0. \tag{21}$$

On the production side of the economy, let $E_{s,t}$ represent the number of machines or capital units produced at time s (i.e., the vintage s) and still in use at time $t \geq s$. The quantity $I_s = E_{s,s}$ stands for gross investment, i.e., capital goods production at time s. We assume that the physical depreciation rate, δ, is constant so

$$E_{s,t} = I_s e^{-\delta(t-s)}.$$

At time $t \geq s$, the vintage s is operated by a certain amount of labor, say $L_{s,t}$. Let $Y_{s,t}$ be the output produced at time t with vintage s, under the following Cobb-Douglas technology

$$Y_{s,t} = z_t \left(q_s E_{s,t}\right)^{1-\alpha} L_{s,t}^{\alpha}. \tag{22}$$

Total output at time t, say Y_t, is the sum of outputs produced with all vintages,

$$Y_t = z_t \int_{-\infty}^{t} \left(q_s E_{s,t}\right)^{1-\alpha} L_{s,t}^{\alpha} ds. \tag{23}$$

The discounted profits of investing I_t, in vintage t, are given by:

$$\Phi(t) = \int_t^{\infty} [Y_{t,s} - w_s L_{t,s}] e^{-\int_t^s r_\tau d\tau} ds - I_t, \tag{24}$$

where the exponential term is the discounted factor at time s, r_τ is the interest rate at time τ and w_s is the wage at time s. The representative firm chooses investment and the labor allocation across vintages in order to maximize its discounted profits taking prices as given and subject to its technological constraint. We assume that each firm is small enough to neglect its own contribution into the aggregate efficient capital stock and therefore the representative firm treats z and q as given. The first order conditions characterizing an interior maximum for $\Phi(t)$ are

$$(1-\alpha) q_t^{(1-\alpha)} I_t^{-\alpha} \int_t^{\infty} e^{-\int_t^s r_\tau d\tau - \delta(1-\alpha)(s-t)} z_s L_{t,s}^{\alpha} ds = 1, \tag{25}$$

and $\forall s \geq t$,

$$\alpha z_s (q_t E_{t,s})^{1-\alpha} L_{t,s}^{\alpha-1} = w_s. \tag{26}$$

Equation (25) determines investment at time t by equalizing marginal returns to marginal costs. Equation (26) determines the labor allocation at time s to vintage t. Using (26), the aggregate employment at time t is:

$$\begin{aligned} L_t &= \int_{-\infty}^{t} L_{s,t} ds = \left(\frac{w(t)}{z_t \alpha}\right)^{\frac{1}{\alpha-1}} \int_{-\infty}^{t} q_s E_{s,t} ds = \\ &= \left(\frac{w(t)}{z_t \alpha}\right)^{\frac{1}{\alpha-1}} K_t, \end{aligned} \tag{27}$$

where K_t is total efficient capital, by definition. Note that the latter is the sum of the "surviving" investments at time t weighted by their respective productivity. Indeed, as $E_{s,t} = I_s e^{-\delta(t-s)}$, one obtains:

$$K_t = \int_{-\infty}^{t} q_s I_s e^{-\delta(t-s)} ds. \tag{28}$$

If we differentiate (28), we obtain the following equation for the effective capital accumulation, $\dot{K}_t = q_t I_t - \delta K_t$, which in per-capita terms becomes

$$\dot{k}_t = q_t i_t - (\delta + n) k_t. \tag{29}$$

If we solve (26) for $L_{s,t}$ and substitute it in the definition of the aggregate production, (23), we get

$$Y_t = z_t \left(\frac{w_t}{z_t\alpha}\right)^{\frac{\alpha}{\alpha-1}} \int_{-\infty}^{t} q_s E_{s,t} ds = z_t \left(\frac{w_t}{z_t\alpha}\right)^{\frac{\alpha}{\alpha-1}} K_t.$$

Using (27), it follows

$$Y_t = z_t K_t^{1-\alpha} L_t^{\alpha}, \tag{30}$$

which in per-capita terms is

$$y_t = z_t k_t^{1-\alpha}.$$

The equilibrium condition in the goods market can be written as

$$y_t = z_t k_t^{1-\alpha} = c_t + i_t. \tag{31}$$

The first order conditions of the representative firm program, (25) and (26) can be rewritten in terms of the aggregate variables. Solving (27) for w_t, we obtain

$$w_t = \alpha z_t k_t^{1-\alpha}, \tag{32}$$

which establishes that the wage rate is equal to the marginal productivity of labor. Solving (26) for $L_{t,s}$ and substituting in (25), it yields

$$(1-\alpha)\, q_t \int_t^{\infty} e^{-\int_t^s r_\tau d\tau - \delta(s-t)} z_s^{\frac{1}{1-\alpha}} \left(\frac{w_s}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} q_t ds = 1,$$

then using (32), the equation just above can be rewritten as

$$(1-\alpha)\, q_t \int_t^{\infty} e^{-\int_t^s r_\tau d\tau - \delta(s-t)} z_s k_s^{-\alpha} q_t ds = 1.$$

Differentiating the latter equation, we get:

$$(1-\alpha)\, z_t q_t k_t^{-\alpha} = r_t + \delta + \frac{\dot{q}_t}{q_t}, \tag{33}$$

which establishes that the marginal productivity of effective capital is equal to its user cost. If now (33) is substituted into (20), we obtain the optimal consumption rule

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}\left((1-\alpha)\, z_t q_t k_t^{-\alpha} - \delta - \frac{\dot{q}_t}{q_t} - \rho\right). \tag{34}$$

Finally, as $a_t = \frac{1}{q_t} k_t$, the transversality condition (21) can be rewritten as

$$\lim_{t\to\infty} \frac{1}{q_t} k_t {c_t}^{-\sigma} e^{-(\rho-n)t} = 0. \tag{35}$$

Equations (29), (31), (34) and (35) characterize an equilibrium of the vintage capital model. The same equilibrium characterization is adopted in the main text.