

# Asymmetries of information in centralized order-driven markets\*

N. Boccard<sup>†</sup> and R. Calcagno<sup>‡</sup>

June 1999

## Abstract

We study the efficiency of the equilibrium price in a centralized, order-driven market where many asymmetrically informed traders are active for many periods. We show that asymmetries of information can lead to sub-optimal information revelation with respect to the symmetric case. In particular, we assess that the more precise the information the higher the incentive to reveal it, and that the value of private information is related to the volume of exogenous trade present on the market. Moreover, we prove that any informed trader, whatever his information, reveals its private signal during an active phase of the market, concluding that long pre-opening phases are not effective as an information discovering device in the presence of strategic players.

Keywords: asymmetric information, pre-opening, insider trading.

JEL Classification: D, G.

---

\*We would like to thank Ronald Anderson, Olivier Gossner, Jean-Francois Mertens for their valuable suggestions and comments.

<sup>†</sup>CORE, UCL and Université de Liège.

<sup>‡</sup>CORE and IRES, UCL. Financial support from CORE and IRES in the PAI program of the Université Catholique de Louvain. E-mail : calcagno@ires.ucl.ac.be

# 1 Introduction

Professional traders often collect accurate information about the future value of an asset, and it is therefore likely that they will use it strategically. Doing this, they should be aware of two risks: to be identified as insiders, since insider trading is illegal, and avoid the imitation effect by the other operators. If efficiency of markets is a goal of a market designer, one should look for a mechanism guaranteeing the fastest information disclosure in the presence of strategic insiders. This task requires the analysis of the incentives for the insiders to reveal their private information. In dealing with this problem, we notice that private information is a peculiar good, intrinsically non homogeneous, since even the same confidential news could be interpreted differently by economic agents differing in experience or education. For these reasons we think that private information originating from diverse sources (confidential or technical reports) is spread asymmetrically among (potentially many) individuals. The non-homogeneity of information is then an intrinsic characteristic of this good. In order to formalize this, we assume that insiders receive private signals having different precisions. Contrary to the model studied by Kyle & Wang (1997) we will assume as in the traditional rational expectation paradigm that these different precisions are commonly known by the agents; our model is then in a sense a particular case of Kyle & Wang (1997), but it introduces a dynamic dimension in the choice of information revelation. Our purpose is in fact to understand if asymmetrically informed traders reveal their inside information as soon as they get it.

Microstructure literature has been concerned with a limited form of information asymmetry, i.e. situations of information monopoly. Here we focus on oligopoly of information. Intuitively, the competition between insiders with different information should be strategically different from the (perfect) competition between equally informed traders. The implications in terms of efficiency of prices could be completely different in the two cases. In this paper we assess the incentive for the better informed trader to reveal his information in the beginning of the game. On the other hand, we show that there exist equilibria in which the trader who receives the less precise signal waits until the last stage to reveal his information, since at that stage he is effectively the only agent possessing an informational advantage. We find as in Kyle & Wang that the best informed trader (in their contest the overconfident one) trade more aggressively on his information, since it has a higher precision.

The information revelation is often studied in markets characterized by distinct phases of activity. In many existing stock markets (e.g., NYSE, Paris, Madrid) the working day is preceded by a phase called pre-opening where orders come to the market and equilibrium prices are quoted, but no exchanges occur until the *fixing*. The existence of these pre-opening phases is often motivated for purposes of information disclosure. Biais, Hillion & Spatt (1997)

have studied empirically the information disclosure process during the pre-opening period in the Paris “Bourse”. They show that informed traders take care not to reveal their information *before* their opponents in such periods where no trades occur. Insiders may be reluctant to disclose their information in order to use it optimally in later trades, but they also have an incentive to place informative orders (at least at the end of the pre-opening stage) because of the risk of a communication breakdown in the last minutes, in order to take advantage of the large liquidity trade present on the market at 10:00, and also for priority rules in execution of equal orders that exist in many exchange markets (for ex. the Paris “Bourse”). Hence, a long pre-opening phase seems to be unuseful for efficiency purposes, since all the “serious” orders are submitted at the very end of that phase. In our paper we show why insiders wait until the last minutes to submit true orders, verifying that it is not optimal to give to their opponents an information advantage for an active market phase.

Existing works on insider trading literature start with the contribution of Kyle (1985), who develops a model in which a single privately informed trader optimally exploits his monopoly power over several periods of trade. The main result is that the optimal behavior of the informed trader is to gradually incorporate his information into the price in order to keep the market depth constant.

Holden & Subrahmanyam (1992) extend the Kyle model by adding many equally informed traders. This generates Bertrand competition between traders and enables those authors to obtain a rapid revelation of information in contrast to Kyle’s result even in the limiting case of two insiders. Indeed, as all insiders share the *same* private information, any revelation by one of them removes the informational advantage of the others. Hence, all insiders have an incentive to use (and thus reveal) immediately their common private information. As we have said before, a problem closely related our model is presented by Kyle & Wang (1997) in a static model with differences in beliefs.

The pre-opening stage existing in many stock exchanges gives us the opportunity to study theoretically whether it plays efficiently the role of a price discovery device in the presence of informed traders. Vives (1995) supports this view claiming that it permits the formation of an efficient price with a process of successive offers by the investors similar to the classical Walrasian “tâtonnement” process. He shows that the more the tâtonnement process lasts, the more prices convey information. Hence he can conclude that “this information tâtonnement proves effective in resolving quickly the uncertainty about the fundamental value of the asset”. This theoretical result is not supported by empirical studies (as already mentioned) and lacks game theoretical foundation. The fact that traders wait until the very last minute to submit revealing orders (Biais & al. (1997)) seems totally in contrast with the assumption of competitive tâtonnement made by Vives (1995), where furthermore no strategic consideration is made.

We consider a three stages game describing a centralized and order-driven market with two insiders having asymmetric private information. We enquire about the incentives to reveal the private information in the first two stages, since we know by simple backward induction argument that it is always optimal to reveal it in the last one (in fact, once the information is publicly revealed, its value is zero, hence the insider has to exploit it before the public announcement). On the one hand, using private information to trade with uninformed agents is beneficial but an insider gives an advantage to its competitor for the last stage. We show that the amount of liquidity trade present on the market can be considered as a “cake” to be divided between the two insiders. If the size of this “cake” increases in the last stage, only the best informed trader reveals before this last stage. In the case this “cake” is very large, nobody reveals before the last stage.

At each stage the informed traders and the liquidity traders simultaneously choose the quantities they trade. The insiders’ information consists of their observation of two signals with different precision correlated with the liquidation value of the asset, of the past history of prices and volumes, and they also observe, as in Kyle (1985), the amount of liquidity trade present into the market. In stage zero, orders are notional, in the sense that they are not concluded: here we do not address the problem of the *length* of the pre-opening and the strategic considerations inherited, but we only study if any information revelation can occur in a stage of fictitious trade. Stages one and two are usual periods of market activity. Market makers set a price and trade the quantity which clears the market. The competition between them makes the equilibrium price equal to the expected value of the asset given the observable order flows; market makers do not observe the individual demands but only the aggregated order flow. Consequently, the equilibrium price changes only if the successive order flows contain new information.

We characterize the set of equilibria in linear strategies for any couple of exogenous liquidity trade volume respectively in stage one and two. For a constant volume across periods, the two players reveal their signal in the first stage if their information is almost equally precise (the result of Holden & Subrahmanyam). However, if asymmetries are large, the trader with better information reveals and the opponent conceals in the first period. The better informed trader will trade more aggressively on his information given that market makers cannot detect perfectly his move. Hence, the more informed player acts as a leader in the “information game” and must use his advantage immediately.

The less informed trader, however, may conceal his type until the last stage during which he can exploit an informational advantage that was not relevant in the previous stages for the presence of a better informed opponent. The competition between asymmetric agents is not of Bertrand-type because the information released by the traders is non-homogeneous.

As the volume of exogenous trade in the first stage reduces with respect to the second

period volume, traders tend to conceal their information in order to exploit it at a more profitable stage. However, the asymmetric equilibrium described before exists until the volume of the first period is one-tenth of that of the second one.

With as slightly different model, we show that revelation during the pre-opening is not rational, and that the equilibrium set of the game does not change adding a notional trade stage before the game. To reveal before the opening makes the opponent stronger since he can rely on a two-signals information and act even more aggressively once the market is open. As a trade-off, the revelation before the opening allow the trader to get advantage in case of equal orders (for priority reasons considered for example in Paris) or to protect himself in case of a communication breakdown. We quantify the expected loss due to earlier revelation; this permits to determine when information will be released as a function of the probability of a tie in orders or a communication breakdown.

The paper is organized as follows: in section 2 we present the three-stage model with simultaneous moves that generalizes Kyle (1985) to the case of asymmetric insiders. We solve the market makers equilibrium in section 3; then we tackle the informed traders problem in the various stages. The results are gathered in section 5. In section 6, we develop a sequential version of the game and finally section 7 collects conclusions. The appendix contains a brief description of the Paris Bourse trading organization that has motivated our setup (section 8.1) and the set of calculations not included in the body of the text (sections 8.2 to 8.5).

## 2 The model

### 2.1 Players and timing

We study a market for a risky asset where the exchanges occur between three kinds of agents: informed agents, market makers and liquidity traders. The risky asset has a random liquidation value  $\tilde{v}$  normally distributed with Normal law  $\mathcal{N}(0, 1)$ . Trader  $i^1$  observes a private signal  $s_i = \tilde{v} + \varepsilon_i$  where the error term  $\varepsilon_i$  has law  $\mathcal{N}(0, \tau_i^{-1})$  and  $\tau_i$  is interpreted as the ex-ante precision with which the trader can guess the true value of  $\tilde{v}$ . All the three random variables  $\tilde{v}$ ,  $\varepsilon_i$ , and  $\varepsilon_j$  are assumed to be independent.

We model the trading day in three stages. Stage zero represents the pre-opening phase<sup>2</sup> which is purely notional in the sense that no trade takes place during this initial stage. Yet, orders are collected and a theoretical equilibrium price is computed. This price is observed by all the traders admitted on the market.

---

<sup>1</sup>*Notation* Whenever a formula or definition is given for  $i$  only, the  $j$  formula is obtained by interverting symbols  $i$  and  $j$ , otherwise both the formulae are given.

<sup>2</sup>We will describe in the appendix the organization of the Paris Bourse, in order to motivate our assumptions on the timing of the trading day.

Stages one and two represents normal periods of market activity. Possible way of thinking if this is to refer to the rules in the Paris Bourse. There, although the pre-opening last officially from 8:30 until 10:00, we interpret the last minutes as stage one, where for the time constraint, it is not possible to cancel a notional order anymore. All orders submitted in that stage are then entered into the electronic order book and become effective; the fixing price is then our equilibrium price for stage one.

Each of stages 1 and 2 is structured as follows: the informed players choose the quantities they trade,  $q_{i,t}$  and  $q_{j,t}$  knowing their signals while liquidity traders submit an aggregated order  $\tilde{u} \rightsquigarrow \mathcal{N}(\bar{u}, \tau)$ . We normalize their average trade to  $\bar{u} = 0$ . Insiders observe the realization of  $\tilde{u}$ , but market makers do not. Given the realized order flow  $\omega = q_i + q_j + \tilde{u}$ , market makers formulate an equilibrium price  $p_t$  and trade the quantities that clear the market. The ex-post profit is  $q_{i,t}(\tilde{v} - p_t)$  while the ex-interim expectation conditional on trader  $i$ 's private information  $\mathcal{H}_i$  is  $\Pi_{i,t} = E[q_{i,t}(\tilde{v} - p_t) / \mathcal{H}_{i,t}]$ . We will consider also the ex-ante profit obtained integrating the latter with respect to the joint measure of private signals and liquidity trade.

## 2.2 Behavioral assumptions

- As in Kyle (1985, 89), we assume for tractability that market makers price the asset in all 3 periods according to a linear rule  $p_t(\omega_t) = \mu_t + \lambda_t \omega_t$  for  $t = 0, 1, 2$ . Perfect competition between market makers also guarantees a weak form of efficiency: the price is equal to the expectation of  $\tilde{v}$  conditional on the observed order flows of the current and previous stages ( $\mu_2$  will depend on the previously observed order flow  $\omega_1$ ).
- Trader  $i$  plays a strategy linear with respect to his private information i.e.,  $q_i(\mathcal{H}_i) = \alpha_i + \beta_i E[\tilde{v} / \mathcal{H}_i]$ . This can be interpreted as an ex-ante commitment to a particular revelation rule (concealing its information amounts to choose  $\beta_i = 0$ ). We are thus excluding partially revealing strategies again for analytic tractability of the model. Moreover, we exclude that players can explicitly manipulate their signal:  $q_i$  is a function of the true  $\mathcal{H}_i$ .
- As in Kyle (1985) and Admati & Pfleiderer (1987), we suppose that informed traders behave strategically one against the other **but** competitively against the market makers i.e., they do not manipulate the market makers pricing rule. It is then natural to assume that profit maximization occurs independently in stages 1 and 2<sup>3</sup>.

---

<sup>3</sup>As the reader can realize, the strategy space is then considerably small; our purpose is in fact to stress the dynamic information revelation strategies, that would be impossible from an analytic point of view if we enlarge the strategy space.

### 3 Market Makers behavior

Our first task is to assess how the revelation of private information in the first stage affects the pricing function used by the market makers in the second stage. This behavior is at the heart of our model. Because market makers use the fixing price and its underlying information to name prices later in the day, insiders may not want to reveal an information that will bring the price too close to their private expectation of the underlying value of the asset.

We require markets to be efficient (at least in a weak sense) in any stage, that means that prices  $p_1(\omega_1)$  and  $p_2(\omega_1, \omega_2)$  convey all the information incorporated in the two respective order flows. Formally, this implies that

$$p_1(\omega_1) = E[\tilde{v} / \omega_1] \quad (1)$$

$$p_2(\omega_1, \omega_2) = E[\tilde{v} / \omega_1, \omega_2] \quad (2)$$

These two conditions have a remarkable interpretation obtained by the application of iterated conditional expectation. In efficient markets, prices are martingales with respect to the public information i.e., the order flows.

$$E[p_2 / \omega_1] = E[E[\tilde{v} / \omega_1, \omega_2] / \omega_1] = E[\tilde{v} / \omega_1] = p_1 \quad (3)$$

A prerequisite to the derivation of prices is to be able to compute the functional form of the order flows  $\omega_1$  and  $\omega_2$ . We thus assume a linear pricing rule in both periods.

$$p_1(\omega_1) = \mu_1 + \lambda_1 \omega_1 \quad (4)$$

$$p_2(\omega_1, \omega_2) = \mu_2 + \lambda_2 \omega_2 \quad (5)$$

This will imply that in the game played by the informed traders, the order flows take the following "simple" functional forms.

$$\omega_1 = \frac{k_i + k_j}{\lambda_1} \tilde{v} + \frac{k_i \varepsilon_i}{\lambda_1} + \frac{k_j \varepsilon_j}{\lambda_1} + \eta_1 \tilde{u}_1 \quad (6)$$

$$\omega_2 = \frac{m_i + m_j}{\lambda_2} \tilde{v} + \frac{m_i \varepsilon_i}{\lambda_2} + \frac{m_j \varepsilon_j}{\lambda_2} + \eta_2 \tilde{u}_2 \quad (7)$$

Since  $\tilde{v}$ ,  $\varepsilon_i$ ,  $\varepsilon_j$ ,  $\tilde{u}_1$  and  $\tilde{u}_2$  have been previously assumed to have zero mean, it is also the case for  $\omega_1$  and  $\omega_2$ : notice that  $\tilde{u}_1$  and  $\tilde{u}_2$  have no informational role. A useful property of normal variables is that the conditional expectation is a linear function of the observations, so that we can write, in our case<sup>4</sup>

$$E[\tilde{v} / \omega_1, \omega_2] = a_1 \omega_1 + a_2 \omega_2 \quad (8)$$

---

<sup>4</sup>There is no constant because all variables have zero mean.

By the law of iterated expectations, we have

$$E[\tilde{v}/\omega_1] = E[E[\tilde{v}/\omega_1, \omega_2]/\omega_1] = a_1\omega_1 + a_2E[\omega_2/\omega_1] \quad (9)$$

$$E[\tilde{v}/\omega_2] = E[E[\tilde{v}/\omega_1, \omega_2]/\omega_2] = a_2\omega_2 + a_1E[\omega_1/\omega_2] \quad (10)$$

Notice that those formulae are independent of the timing of the game at stake, they only reflect properties of the underlying random variables. It is the derivation of the market depths  $\lambda_1$  and  $\lambda_2$  that will depend on the timing of the game.

We will check that if  $\omega_1$  is non informative (traders play independently of their private signal in the first stage) then  $a_1 = 0$  while  $a_2 > 0$ . This is perfectly consistent with (9) and (10) because the first one becomes  $E[\omega_2/\omega_1] = 0$  which is true in the non informative case as  $E[\omega_2/\omega_1] = E[\omega_2]$  while the second equation becomes (8) for a single conditioning variable.

Considering the functional form of  $\omega_1$  given by (6), we have <sup>5</sup>

$$E[\tilde{v}/\omega_1] = \frac{(k_i + k_j)\omega_1}{\lambda_1 Var(\omega_1)} \quad (11)$$

with  $Var(\omega_1) = \left(\frac{k_i + k_j}{\lambda_1}\right)^2 + \frac{k_i^2}{\lambda_1^2 \tau_i} + \frac{k_j^2}{\lambda_1^2 \tau_j} + \frac{\eta_1^2}{\tau_1}$ . Remember that we assume that market makers do not know the realization of  $\tilde{u}$ .

Given (1) and (4) and combining it with (11), we obtain  $\mu_1 + \lambda_1\omega_1 = \frac{(k_i + k_j)\omega_1}{\lambda_1 Var(\omega_1)}$ . Identifying  $\mu_1$  and  $\lambda_1$  in this linear equation, we get  $\mu_1 = 0$  and  $\lambda_1^2 Var(\omega_1) = k_i + k_j$ . Developing the variance, we solve for the market depth of the first period, obtaining

$$\lambda_1 = \frac{\sqrt{\tau_1}}{\eta_1} \sqrt{k_i + k_j - (k_i + k_j)^2 - \frac{k_i^2}{\tau_i} - \frac{k_j^2}{\tau_j}} \quad (12)$$

The ex-ante variance of exogenous trade positively influence the depth of the market as in Kyle (1985).

To find  $\lambda_2$  and  $\mu_2$ , we use similar techniques in appendix 8.2. First, we rewrite (9) using formulae (6) and (7) to expand  $E[\omega_2/\omega_1]$ . With a completely symmetric rewriting of (10), we are able to solve for  $a_1$  and  $a_2$  in the variables  $\lambda_1$  and  $\lambda_2$ . Combining the two expressions for the price (5) and (8), we can write

$$\mu_2 + \lambda_2\omega_2 = a_1\omega_1 + a_2\omega_2$$

Identifying the coefficients of this linear equation, we obtain  $\mu_2 = a_1\omega_1$  and  $\lambda_2 = a_2$ . The last equality enables to solve for  $\lambda_2$  in a fashion similar to that used for  $\lambda_1$  (see the Appendix 4.2, (40)-(41)) and another complex expression for  $\mu_2$  (see (42)). Both depend on the first period choices  $k_i, k_j, \eta_1$ , the second period ones  $m_i, m_j, \eta_2$  and the fundamentals of the model  $\tau_i, \tau_j, \tau_1$  and  $\tau_2$ .

---

<sup>5</sup>According to the standard projection of normal variables : if  $Y = X + Z$  where  $Z$  is independent of  $X$  and all three variables have zero mean, then  $E[X/Y] = \frac{Var(X)Y}{Var(X) + Var(Z)}$



## 4 Informed traders behavior

### 4.1 Information revelation

The trades of stage 1 might reveal some private information. Indeed, traders observe the order flow  $\omega_1 = q_i + q_j + \tilde{u}_1$  and they know the amount of liquidity trade  $\tilde{u}_1$ . Trader  $i$  can then recognize the order of trader  $j$  and if the latter has played according a revealing strategy  $q_j(s_j)$ , trader  $i$  can deduce the private signal  $s_j$  and thus increase his private information.

We denote  $\mathcal{H}_t$  the information revealed to market makers at the beginning of stage  $t$  and  $\mathcal{H}_{t,i}$  the private information of trader  $i$  in that stage. Since we consider only full revelation of information or full concealing behavior, the second stage can start with four possible information structures.

$$\begin{array}{ll}
 i \text{ reveals} & \mathcal{H}_{2,i} = \{s_i\}, \mathcal{H}_{2,j} = \{s_i, s_j\}, \mathcal{H}_2 = \{s_i\} \\
 j \text{ reveals} & \mathcal{H}_{2,i} = \{s_i, s_j\}, \mathcal{H}_{2,j} = \{s_j\}, \mathcal{H}_2 = \{s_j\} \\
 \text{Nobody reveals} & \mathcal{H}_{2,i} = \{s_i\}, \mathcal{H}_{2,j} = \{s_j\}, \mathcal{H}_2 = \{\emptyset\} \\
 \text{Both reveal} & \mathcal{H}_{2,i} = \{s_i, s_j\}, \mathcal{H}_{2,j} = \{s_i, s_j\}, \mathcal{H}_2 = \{s_i, s_j\}
 \end{array}$$

Observe that  $\mathcal{H}_{2,i}$  is always finer than the observation of the total order flow  $\omega_1$  which can therefore be safely ignored in the second stage calculations. In the symmetric revelation case, the second period is a Cournot game with complete information i.e., each pair  $\{s_i, s_j\}$  defines a proper subgame. The game is solved for any couple of given private signals in the space of linear strategies. When only trader  $i$  reveals his information in stage one, trader  $j$  behaves as in the previous case, since he knows  $\{s_i, s_j\}$ . In equilibrium, trader  $i$ , despite the fact that he does not know the signal  $s_j$ , anticipates the rule  $q_j(s_j, s_i)$  used by his opponent. Hence, we deal with a form of Stackelberg game. When no revelation has occurred, traders play a game with incomplete information on both sides. Each trader has to optimize against a rule and not against a single order.

An obvious consequence of the finite number of stages in the game is that informed traders have an incentive to use their private information in stage 2, thus  $\omega_2$  conveys a lot of information about the underlying liquidation value of the asset. Yet, if one or both traders have revealed information during the first stage, then the order flow  $\omega_1$  is also an informative statistic for the market makers.

According to our behavioral assumptions, we analyze a  $2 \times 2$  matrix game. We study trader  $i$ 's behavior according to  $j$ 's one. We obtain the following 4 configurations:

- CC* Both traders conceal in stage 1 so that public information in stage 2 is empty
- RC* Trader  $i$  reveals in stage 1, so that trader  $j$  has an added information in stage 2
- CR* Reverse of the preceding case
- RR* Both trader reveal in stage 1 and have a symmetric information in stage 2

For any pair of strategies  $h \in \{CC, RC, CR, RR\}$ , the ex-ante global profit is the sum of profits obtained in each active trading stage  $\Pi_i^h = \Pi_{1,i}^h + \Pi_{2,i}^h$ .

We first study the best reply of trader  $i$  when trader  $j$  conceals his information in stage 1 to show that he ought to reveal even though he is giving an advantage to  $j$  for the second stage i.e.,  $\Pi_i^{RC}$  is greater than  $\Pi_i^{CC}$ . Thanks to this result, there is (almost) always revelation in equilibrium. It is then sufficient to solve the trade-off between concealing and revealing for a trader who face a (rational) revealing player i.e., determine when  $\Pi_i^{CR}$  is greater than  $\Pi_i^{RR}$ .

In solving the game by backward induction we shall develop the minimal amount of calculation in the body of the text; complex computations are deferred to the appendices.

## 4.2 Both players conceal their information (CC case)

### 4.2.1 Second period analysis

Trader  $j$  places an order  $q_j$  independent of his private signal. As trader  $i$  chooses to conceal his own information, he places a market order  $q_i$  constant across  $s_i$ . The first stage order flow  $\omega_1 = q_i + q_j + \tilde{u}_1$  contains no information on the underlying value of the asset, thus  $E[\tilde{v}/\omega_1] = E[\tilde{v}/p_0] = p_0$ , the closing price of the previous day which is normalized to zero (as it is assumed to be efficient). The zero profit condition for market makers leads to  $0 = p_1(\omega)$  so that any  $q_i$  is indeed optimal. The commitment by informed traders to conceal their information drives their first period profits to zero on average. In terms of sensitivity to the signals, we have  $k_i = k_j = 0$  and  $\eta_1 = \lambda_1 = 1$ .

The second stage starts with an empty public information set  $\mathcal{H}_2 = \{\emptyset\}$  and private information set  $\mathcal{H}_{2,i} = \{s_i\}$ . The ex-post profit for trader  $i$  of a trade  $q_i$  is  $(\tilde{v} - p_2)q_i$ . As it is always optimal to use its information in the last stage of the game, the ex-interim profit is

$$\Pi_{2,i}(\mathcal{H}_i) = q_i(s_i)E[\tilde{v} - p_2 / \mathcal{H}_i] \quad (13)$$

Substituting for the market makers strategy  $p_2(\omega_2) = \mu_2 + \lambda_2\omega_2$  and using the order flow decomposition  $\omega_2 = q_i + q_j + \tilde{u}_2$ , where now  $\tilde{u}_2$  is known to player  $i$ , the expected profit is

$$\begin{aligned}
\Pi_{2,i}(\mathcal{H}_i) &= q_i E [\tilde{v} - \mu_2 - \lambda_2(q_i + q_j + \tilde{u}_2) / \mathcal{H}_i] \\
&= q_i (E [\tilde{v} / \mathcal{H}_i] - \mu_2 - \lambda_2 q_i - \lambda_2 E [q_j / \mathcal{H}_i] - \lambda_2 \tilde{u}_2)
\end{aligned}$$

Using the specific form of trader  $j$ 's linear strategy  $q_j(\mathcal{H}_j) = \alpha_j + \beta_j E [\tilde{v} / \mathcal{H}_j]$ , the first order condition (FOC) for trader  $i$  reads

$$\begin{aligned}
2\lambda_2 q_i &= E [\tilde{v} / \mathcal{H}_i] - \mu_2 - \lambda_2 \tilde{u}_2 - \lambda_2 E [q_j / \mathcal{H}_i] \\
&= E [\tilde{v} / \mathcal{H}_i] - \mu_2 - \lambda_2 \tilde{u}_2 - \lambda_2 \alpha_j - \lambda_2 \beta_j E [E [\tilde{v} / \mathcal{H}_j] / \mathcal{H}_i]
\end{aligned} \tag{14}$$

The projection result for normal variables leads to  $E [\tilde{v} / \mathcal{H}_i] = \frac{s_i \tau_i}{\tau_i + 1}$ . Using  $E [E [\tilde{v} / \mathcal{H}_j] / \mathcal{H}_i] = \frac{\tau_j}{\tau_j + 1} E [\tilde{v} / \mathcal{H}_i]$ , we can identify the intercept and the slope of the linear strategy to obtain

$$2\lambda_2 \alpha_i = -\mu_2 - \lambda_2 \alpha_j - \lambda_2 \tilde{u}_2 \quad \text{and} \quad 2\lambda_2 \beta_i = 1 - \lambda_2 \beta_j \frac{\tau_j}{\tau_j + 1}$$

Putting together the symmetric equations for  $j$ , we solve the system to get

$$\alpha_i = -\frac{\tilde{u}_2}{3} - \frac{\mu_2}{3\lambda_2} \quad \text{and} \quad \beta_i = \frac{(\tau_j + 2)(\tau_i + 1)}{\lambda_2(3\tau_i \tau_j + 4\tau_j + 4\tau_i + 4)} \tag{15}$$

We now use our previous computation regarding the market makers pricing rule. Since there is no revelation in stage one,  $\mu_2$  must be zero. Substituting the traders optimal strategies in the order flow definition leads to

$$\begin{aligned}
\omega_2 &= -\frac{2\mu_2}{3\lambda_2} + \frac{\beta_i \tau_i}{\tau_i + 1} (\tilde{v} + \varepsilon_i) + \frac{\beta_j \tau_j}{\tau_j + 1} (\tilde{v} + \varepsilon_j) + \frac{\tilde{u}_2}{3} \\
&= \frac{m_i + m_j}{\lambda_2} \tilde{v} + \frac{m_i \varepsilon_i + m_j \varepsilon_j}{\lambda_2} + \frac{\tilde{u}_2}{3}
\end{aligned} \tag{16}$$

with  $m_i = \frac{(\tau_j + 2)\tau_i}{3\tau_i \tau_j + 4\tau_j + 4\tau_i + 4}$  and  $\eta_2 = \frac{1}{3}$ .

The analysis of the first period has shown that  $p_1$  is always nil, hence the market depth is infinite i.e.,  $\lambda_1 = 0$ . Using the notation developed in the previous sub-section,  $\omega_1 = q_i + q_j + \tilde{u}_1$  implies that  $k_i = k_j = 0$ . We finally obtain  $\mu_2^{CC} = 0$  and

$$\lambda_2^{CC} = \frac{3\sqrt{\tau_2 Z(\tau_i, \tau_j)}}{3\tau_i \tau_j + 4\tau_j + 4\tau_i + 4} \tag{17}$$

with  $Z(\tau_i, \tau_j) = 4\tau_j + 4\tau_i + 4\tau_i^2 + 8\tau_i \tau_j + 5\tau_i^2 \tau_j + 5\tau_i \tau_j^2 + 2\tau_i^2 \tau_j^2 + 4\tau_j^2$ .

### 4.2.2 Ex-ante profits in the CC case

>From (15), we derive the optimal orders

$$\begin{aligned} q_i(s_i) &= \alpha_i + \beta_i \frac{\tau_i s_i}{\tau_i + 1} \\ &= \frac{1}{3\sqrt{\tau_2}} \left( \frac{(\tau_j + 2)\tau_i}{\sqrt{Z(\tau_i, \tau_j)}} s_i - \sqrt{\tau_2} \tilde{u}_2 \right) \end{aligned} \quad (18)$$

and the equilibrium price is

$$\begin{aligned} p_2(s_j, s_i) &= \lambda_2^{CC} (q_i(s_i) + q_j(s_j) + \tilde{u}_2) + \mu_2^{CC} \\ &= \lambda_2^{CC} \frac{\tilde{u}_2}{3} + \frac{(\tau_j + 2)\tau_i s_i + (\tau_i + 2)\tau_j s_j}{3\tau_i \tau_j + 4\tau_j + 4\tau_i + 4} \end{aligned}$$

Notice that the equilibrium price function pools all the information as it depends on  $s_j$  and  $s_i$ . The ex-interim profit of trader  $i$  is the expected profit conditional on its private information  $s_i$ , thus we have to average  $p_2$  with respect to  $s_j$  using  $E[s_j / s_i] = E[\tilde{v} + \varepsilon_j / s_i] = E[\tilde{v} / s_i] = \frac{\tau_i s_i}{\tau_i + 1}$  (the error terms are independent random variables). Furthermore, straightforward algebraic manipulations show that  $E[\tilde{v} - p_2 / s_i] = \lambda_2 q_i(s_i)$ . Hence, substituting into (13) and using (18), we get the traditional Cournot formula for the expected profit conditional on the private signal in the second stage (ex-interim profit).

$$\begin{aligned} \Pi_{2,i}^{CC}(s_i) &= \lambda_2^{CC} q_i(s_i)^2 \\ &= \frac{\sqrt{Z(\tau_i, \tau_j)}}{3\sqrt{\tau_2}(3\tau_i \tau_j + 4\tau_j + 4\tau_i + 4)} \left( \frac{(\tau_j + 2)\tau_i}{\sqrt{Z(\tau_i, \tau_j)}} s_i - \sqrt{\tau_2} \tilde{u}_2 \right)^2 \end{aligned} \quad (19)$$

We use  $E[s_i^2] = \frac{1+\tau_i}{\tau_i}$ ,  $E[\tilde{u}_2^2] = \tau_2^{-1}$  and the independence of  $s_i$  and  $\tilde{u}_2$  to compute the ex-ante expectation of  $\Pi_{2,i}^{CC}(s_i)$  (before knowing the realization of  $s_i$  and  $\tilde{u}_2$ ) to obtain an increasing and concave function of both precisions  $\tau_i$  and  $\tau_j$ .

$$\begin{aligned} \Pi_i^{CC} &= E[\Pi_{2,i}^{CC}(s_i)] \\ &= \frac{\sqrt{Z(\tau_i, \tau_j)}}{3\sqrt{\tau_2}(3\tau_i \tau_j + 4\tau_j + 4\tau_i + 4)} \left( \frac{(\tau_j + 2)^2(1 + \tau_i)\tau_i}{Z(\tau_i, \tau_j)} + 1 \right) \end{aligned} \quad (20)$$

### 4.3 Trader $i$ reveals while $j$ conceal (RC case)

#### 4.3.1 Second period analysis

We solve this case by backward induction since in the first stage, the optimal strategy for the trader who reveals depends on the second stage behaviors.. At the beginning of stage 2, the public information in the present context is  $\mathcal{H}_2 = \{s_i\}$ , thus  $\mathcal{H}_{2,i} = \{s_i\}$  and  $\mathcal{H}_{2,j} = \{s_j, s_i\}$  so that

$$\begin{aligned} E[E[\tilde{v} / \mathcal{H}_j] / \mathcal{H}_i] &= E[\tilde{v} / \mathcal{H}_i] = \frac{s_i \tau_i}{\tau_i + 1} \\ \text{and } E[E[\tilde{v} / \mathcal{H}_i] / \mathcal{H}_j] &= E[\tilde{v} / \mathcal{H}_j] = \frac{s_i \tau_i + s_j \tau_j}{\tau_j + \tau_i + 1} \end{aligned}$$

The FOC for trader  $i$  is still

$$2\lambda_2 q_i(\mathcal{H}_i) = E[\tilde{v} / \mathcal{H}_i] - \mu_2 - \lambda_2 \tilde{u}_2 - \lambda_2 E[q_j / \mathcal{H}_i] \quad (21)$$

Because firm  $j$  knows  $\mathcal{H}_i$ , its FOC reads

$$2\lambda_2 q_j(\mathcal{H}_j) = E[\tilde{v} / \mathcal{H}_j] - \mu_2 - \lambda_2 \tilde{u}_2 - \lambda_2 q_i(\mathcal{H}_i) \quad (22)$$

and solving, we obtain

$$q_i(s_i) = \frac{s_i \tau_i}{3\lambda_2(\tau_i + 1)} - \frac{\tilde{u}_2}{3} - \frac{\mu_2}{3\lambda_2} \quad (23)$$

We can now solve for the more informed trader  $j$  substituting (23) in (22)

$$q_j(s_i, s_j) = \frac{s_i \tau_i + s_j \tau_j}{2\lambda_2(\tau_j + \tau_i + 1)} - \frac{s_i \tau_i}{6\lambda_2(\tau_i + 1)} - \frac{\tilde{u}_2}{3} - \frac{\mu_2}{3\lambda_2} \quad (24)$$

Observe from (23) that the slope term for trader  $i$  is  $\beta_i = \frac{1}{3\lambda_2}$  like in the previous complete information case while trader  $j$  has a more complex behavior since he puts more weight on his own information.

The optimal market orders are used to recompute the order flow.

$$\omega_2 = -\frac{2\mu_2}{3\lambda_2} + \frac{m_i + m_j}{\lambda_2} \tilde{v} + \frac{m_i}{\lambda_2} \varepsilon_i + \frac{m_j}{\lambda_2} \varepsilon_j + \frac{\tilde{u}_2}{3} \quad (25)$$

where  $m_i = \frac{1}{2} \frac{\tau_i}{\tau_i + \tau_j + 1} + \frac{1}{6} \frac{\tau_i}{\tau_i + 1}$ ,  $m_j = \frac{1}{2} \frac{\tau_j}{\tau_i + \tau_j + 1}$  and  $\eta_2 = \frac{1}{3}$ . We can now compute the price

$$p_2(s_i, s_j) = \mu_2 + \lambda_2 \omega_2 = \frac{\mu_2}{3} + \lambda_2 \frac{\tilde{u}_2}{3} + \frac{s_i \tau_i + s_j \tau_j}{2(\tau_i + \tau_j + 1)} + \frac{s_i \tau_i}{6(\tau_i + 1)} \quad (26)$$

Notice that  $E[p_2 / s_j, s_i] = p_2(s_i, s_j)$  and  $E[\tilde{v} - p_2 / s_i] = \lambda_2 q_i(s_i)$ , thus the ex-interim profits in stage two are

$$\Pi_{2,i}^{RC}(s_i) = \lambda_2 q_i(s_i)^2 \quad (27)$$

$$\Pi_{2,j}^{RC}(s_j, s_i) = \lambda_2 q_j(s_i, s_j)^2 \quad (28)$$

### 4.3.2 First period analysis in the RC case

We have previously assumed that insiders are not manipulating the market makers pricing rule of the current period, thus it is natural to assume that they would not influence  $\lambda_2$  with their first stage strategies through a strategic manipulation of  $\omega_1$ . To comply with our previous notations, we write the linear pricing rule of the market makers as  $p_1(\omega_1) = \mu_1 + \lambda_1(\omega_1 - E[\omega_1])$  since in the present case the constant demand of trader  $j$  adds a non informative constant term in the first period order flow. The first stage profit for trader  $i$  is

$$\Pi_1^{RC}(\mathcal{H}_i) = q_i(\mathcal{H}_i)E[\tilde{v} - \mu_1 - \lambda_1(q_i + q_j + \tilde{u}_1 - E[\omega_1]) / \mathcal{H}_i]$$

Solving the two maximization problems (with the same procedure used before) gives

$$q_i = -\frac{\mu_1}{3\lambda_1} - \frac{\tilde{u}_1}{3} + \frac{s_i\tau_i}{2\lambda_1(\tau_i + 1)} \quad (29)$$

$$q_j = -\frac{\mu_1}{3\lambda_1} - \frac{\tilde{u}_1}{3} \quad (30)$$

Notice that  $q_j$  is independent of  $\tilde{v}$  since player  $j$  voluntarily ignores his private information at stage 1. As trader  $i$  is revealing its private information, the order flow observed by market makers conveys some of it.

$$\omega_1 = \frac{k_i}{\lambda_1}(\tilde{v} + \varepsilon_i) + \frac{\tilde{u}_1}{3} + \rho_1 \quad (31)$$

where  $k_i = \frac{\tau_i}{2(1+\tau_i)}$ ,  $k_j = 0$ ,  $\eta_1 = \frac{1}{3}$  and  $\rho_1 = -\frac{2\mu_1}{3\lambda_1}$ .

Recalling that  $\mu_1 = 0$  and the expression for  $\lambda_1$  given in (12) we get the following result

$$\lambda_1^{RC} = \frac{3}{2}\sqrt{\tau_1}\sqrt{\frac{\tau_i}{\tau_i + 1}} \quad (32)$$

As expected,  $\lambda_1^{RC}$  is increasing in  $\tau_i$  because the more precise the signal, the more aggressive the trader and the more aggressive the market makers response that reduces the depth of the market.

### 4.3.3 Ex-ante profits in the RC case

Integrating (32) into optimal demands (23,24) and price (26), we get the ex-interim profit

$$\Pi_{1,i}^{RC}(s_i) = \lambda_1^{RC} \left( \frac{s_i\tau_i}{2\lambda_1^{RC}(1 + \tau_i)} - \frac{\tilde{u}_1}{3} \right)^2 \quad (33)$$

Taking into account  $E[s_i^2] = \frac{1+\tau_i}{\tau_i}$ ,  $E[\tilde{u}_2^2] = \tau_2^{-1}$  and the independence of  $s_i$  and  $\tilde{u}_2$ , the ex-ante expectation is

$$\Pi_{1,i}^{RC} = \frac{1}{3\sqrt{\tau_1}} \sqrt{\frac{\tau_i}{\tau_i + 1}} \quad (34)$$

As intuition would suggest, there is a first stage advantage to reveal since  $\Pi_{1,i}^{RC} > 0$  while we saw that  $\Pi_{1,i}^{CC} = 0$ . Having solved for the first stage price equilibrium, we can find out the updating performed by the market makers in the second period. We compute the complex form of  $\lambda_2^{RC}$  which is positive like  $\lambda_1^{RC}$  i.e., the market makers are spreading their reactivity towards adverse selection over the two periods of trade. Plugging  $\lambda_2^{RC}$  and  $\mu_2^{RC}$  back into (27), we obtain a formula for  $\Pi_{2,i}^{RC}$  that is similar to that of  $\Pi_i^{CC}$  (cf. 20) albeit more complex. The global ex-ante profit,  $\Pi_i^{RC} = \Pi_{1,i}^{RC} + \Pi_{2,i}^{RC}$  appears to be an increasing and concave function of  $\tau_i$  and  $\tau_j$  (cf. appendix 8.3).

If trader  $i$  uses his private information in stage 1, he makes profits but he starts stage 2 with a handicap having made the second period price come closer to its own estimation of the underlying asset value. Whatever the ranking between  $\tau_i$  and  $\tau_j$ , trader  $j$  starts stage 2 with a better information since it is the aggregate of  $\mathcal{H}_{1,i}$  and  $\mathcal{H}_{1,j}$ . Noticing that hedging and liquidity orders are presumably spread over the day in an equal manner, we are now in position to derive a first result.

**Lemma 1** *If liquidity trade is constant across stages 1 and 2, it is optimal to reveal its information when the opponent conceals it.*

*Proof* We have normalized the precision of the underlying asset  $\tilde{v}$  to unity, thus the relevant range for the precision parameters  $\tau_i$  and  $\tau_j$  is  $[a, b]$  with  $a < 1 < b$ . Both  $\Pi_i^{RC}$  and  $\Pi_i^{CC}$  are polynomial fractions of  $\tau_i$  and  $\tau_j$  whose coefficients are positive, thus denominators are never zero on the relevant range so that we can rely on a graphical representation. Assuming an ex-ante identical volume of trade in stages 1 and 2 amounts to set  $\tau_1 = \tau_2$ . For that specification of the parameters, the graph of  $\Pi_i^{RC} - \Pi_i^{CC}$  is displayed on figure 1 below ( $\tau_i$  on the right axis,  $\tau_j$  on the left axis), it is indeed a positive function. More precisely, because  $\Pi_{2,i}^{RC}$  is only slightly lesser than  $\Pi_{2,i}^{CC}$ , we can say that the benefit of revealing is the first period profit generated by the use of private information. *Q.E.D.*

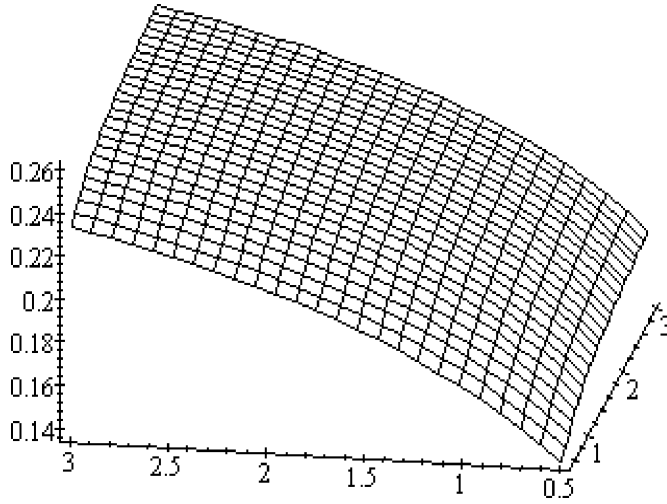


Figure 1

#### 4.4 Trader $i$ conceals while $j$ reveals (CR case)

The preceding lemma implies that the equilibrium is found by looking at a trader's best reply when the other trader reveals his private information. In this CR case we find the best non-informative reply for  $i$  to the optimal revealing strategy of  $j$ . This is different from the RC case before, the difference being that trader  $i$  is now concealing optimally his information while in the RC case analyzed before, trader  $j$  was playing any concealing strategy. The full analysis performed in appendix 8.4 leads to a payoff function  $\Pi_i^{CR}$  that is again an increasing and concave function of  $\tau_i$  and  $\tau_j$ .

#### 4.5 Traders $i$ and $j$ reveal (RR case)

In this final case, traders play in the first period as if they were in the second period without any previously revealed information. Referring to the second stage analysis of the (CC) case, we have

$$\begin{aligned} q_i(s_i) &= \alpha_i + \beta_i \frac{\tau_i s_i}{\tau_i + 1} \\ &= \frac{1}{3\sqrt{\tau_1}} \left( \frac{(\tau_j + 2)\tau_i}{\sqrt{Z(\tau_i, \tau_j)}} s_i - \sqrt{\tau_1} \tilde{u}_1 \right) \end{aligned}$$

Plugging those optimal strategies into the order flow lead to the following decomposition:

$$\omega_1 = \frac{k_i + k_j}{\lambda_1} \tilde{v} + \frac{k_i \varepsilon_i + k_j \varepsilon_j}{\lambda_1} + \frac{\tilde{u}_1}{3}$$

with  $k_i = \frac{(\tau_j + 2)\tau_i}{3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4}$ , and  $\eta_1 = \frac{1}{3}$ . Hence, from (12) we obtain a positive depth in the market with

$$\lambda_1^{RR} = \frac{3\sqrt{\tau_1 Z(\tau_i, \tau_j)}}{3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4} \quad (35)$$



and no history effect with  $\mu_1 = 0$ . The expectation of the ex-interim profit reads

$$\Pi_{1,i}^{RR} = \frac{\sqrt{Z(\tau_i, \tau_j)}}{3\sqrt{\tau_1}(3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4)} \left( \frac{(\tau_j + 2)^2(1 + \tau_i)\tau_i}{Z(\tau_i, \tau_j)} + 1 \right) \quad (36)$$

As intuition suggest, there is a first period advantage to reveal its information. Furthermore, since trader  $j$  is always revealing, the better informed he is, the better it is for trader  $i$  to use that public information for himself ; hence  $\Pi_{1,i}^{RR} - \Pi_{1,i}^{CR}$  is a positive and is bigger for large values of  $\tau_j$ .

In the second stage, the public information in this case is  $\mathcal{H}_2 = \{s_j, s_i\} = \mathcal{H}_{2,i} = \mathcal{H}_{2,j}$ , thus  $E[\tilde{v} / \mathcal{H}_{2,i}] = \frac{s_i\tau_i + s_j\tau_j}{\tau_j + \tau_i + 1}$ . The usual methods of decomposition for the order flow give coefficient  $m_i = \frac{\tau_i}{3(\tau_j + \tau_i + 1)}$  and  $\eta_2 = \frac{1}{3}$  leading to a complex formula of  $\lambda_2$  and an even more complex one for the ex-ante profit  $\Pi_{2,i}^{RR}$  (cf. appendix 8.5). The overall expected payoff  $\Pi_i^{RR} = \Pi_{1,i}^{RR} + \Pi_{2,i}^{RR}$  is "as usual" an increasing and concave function of  $\tau_i$  and  $\tau_j$ .

## 5 The equilibrium of the game with simultaneous moves

The previous analysis allows to characterize the equilibrium of the simple  $(2 \times 2)$  matrix game with simultaneous moves described in section 4. The equilibrium set crucially depends on the relative size of the liquidity trade in the two periods, because this ratio influence the depth parameter: the amount of liquidity trade appears to be a "cake" to be divided between the two informed traders. More precisely, the game can be considered as a constant-sum game between  $i$  and  $j$  once the dimension of the exogenous trade is fixed. As intuition suggests, the bigger the volume of trade present in the market for hedging or other liquidity reasons, the higher the profit insiders can get. Hence, if stage 2 displays significantly more liquidity than stage 1, traders have the incentive to hide their information in order to exploit it successfully in the last stage. For instance, interpreting stage 2 as the period lasting from the fixing until the closing time (17:00) is equivalent to set  $\tau_2 = \frac{\tau_1}{100}$  because the fixing is about 1/100 of the day's trade ( $\tau$  is an inverse variance). In that case the conclusion of Lemma 2 is reversed.

**Lemma 2** *If liquidity trade grows much bigger from stage 1 to stage 2, it is optimal to conceal its information when the opponent conceals it.*

**Proof** As seen on figure 2 below ( $\tau_i$  on the right axis,  $\tau_j$  on the left axis), the difference  $\Pi_i^{RC} - \Pi_i^{CC}$  is now negative. *Q.E.D.*

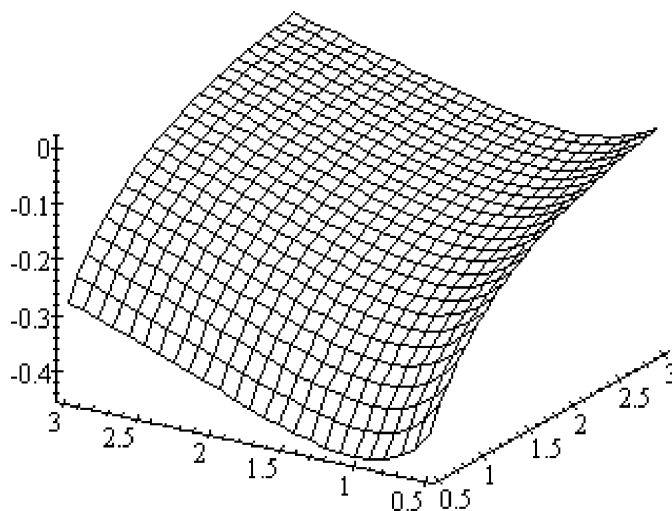


Figure 2

In the literature (Holden & Subrahmanyam (1992), Kyle (1989), Foster & Viswanathan (1993)) it is well known that increasing the competition between informed trader leads towards more efficient prices, but this result is obtained keeping the average liquidity trade volume constant. We find that this result is robust with the possibility of informational asymmetries between traders. In fact, assuming is constant liquidity trade across stages 1 and 2 (i.e.,  $\tau_1 = \tau_2 = 1$ ) we have the following result:

**Proposition 3** *If  $\tau_1 = \tau_2$ , the set of Nash equilibria of the game is  $(R, R)$  for precisions  $(\tau_i, \tau_j)$  not too dissimilar, otherwise the equilibrium is asymmetric and thus inefficient in terms of information revelation: in particular, if  $\tau_i \gg \tau_j$  the unique Nash equilibrium of the game is  $(R, C)$ .*

**Proof** By lemma 2, if a player conceals then the best reply for the other trader is to reveal. Therefore in equilibrium at least one trader ( $j$  w.l.o.g.) reveals which leads us to compare  $\Pi_i^{RR}$  and  $\Pi_i^{CR}$ . As seen on figure 3 below ( $\tau_i$  on the right axis,  $\tau_j$  on the left axis),  $\Pi_i^{CR} - \Pi_i^{RR} < 0$  when  $\tau_i$  is larger than  $\tau_j$  which means that trader  $i$  will optimally reveal, thus the equilibrium is  $(R, R)$ . On the other hand, if  $\tau_j$  is large and  $\tau_i$  quite low, then trader  $i$  prefers to conceal so that the equilibrium is asymmetric and *inefficient*. Observe that the benefit of revealing for trader  $i$  increases with his own quality of information (the surface decreases). *Q.E.D.*

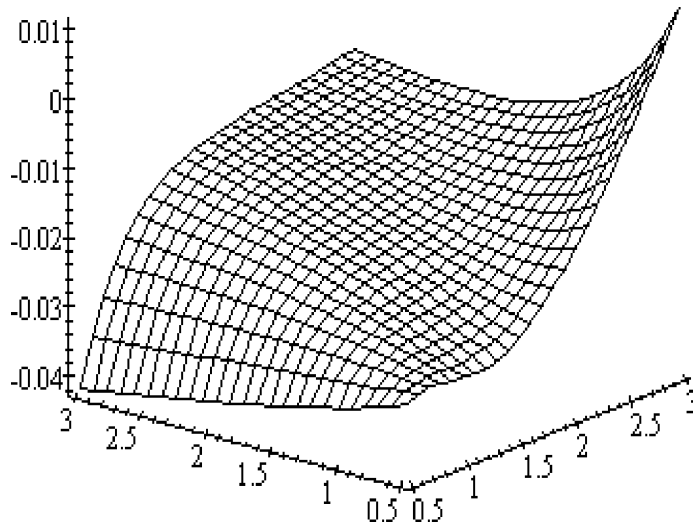


Figure 3

The better informed trader (as the overconfident one in Kyle & Wang) trades more aggressively because market depth is high, since market makers are willing to provide more depth than they otherwise would since they cannot distinguish between informed and uninformed orders. Badly informed player trades a little since he gets higher profit waiting to be the unique "informed" on the market in the second period.

The size of liquidity trade can totally change this result. When the second stage is 10 times more active than the first one, then the variance is squared and  $\tau_2 = \frac{\tau_1}{100}$ .

**Proposition 4** *If  $\tau_2 = \frac{\tau_1}{100}$ , both traders conceal their information in equilibrium.*

**Proof** As seen on figure 4 below, we always have  $\Pi_i^{CR} - \Pi_i^{RR} > 0$ . Hence at least one trader ( $j$  w.l.o.g.) conceal his information. By lemma 4, if a player conceals then the best reply for the other trader is to conceal. Thus the equilibrium is  $CC$  for any pair  $(\tau_i, \tau_j)$ . *Q.E.D.*

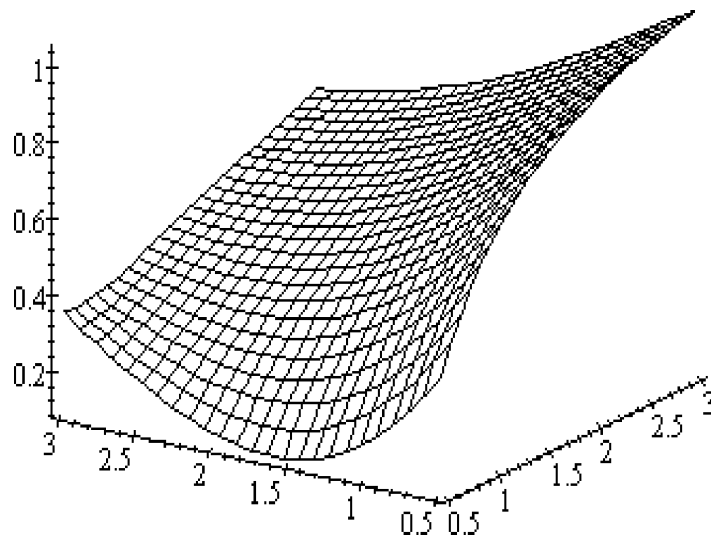


Figure 4

Because traders have the possibility to gain on a substantially bigger liquidity trade during the rest of the day, they both prefer to wait to use their private information. The more the exogenous trade the higher, *ceteris paribus*, the depth of the market, the higher profit possibilities for insiders.

Clearly, given the structure of the game that uses simultaneous moves, the correct comparison must be between the trading volume at the fixing and the trading volume at another precise instant of the day: this permits to consider  $\tau_1$  (the precision of the volume at the fixing) relatively high with respect to  $\tau_2$ , and our analysis shows that for those value of  $(\tau_1, \tau_2)$  the fixing price is presumably efficient at least if information asymmetries are not too important: in this sense, a model including the analysis of these asymmetries is more general than a symmetric one. However, relevant cases of inefficiency could arise, in particular if during the day other moments of high activity are expected (as the opening of American markets for European stock exchanges).

## 6 Sequential revelation of information

In the previous section we have characterized the equilibrium set in linear pure strategies of the two-stage game with simultaneous moves. Changing the timing of the moves for informed players, allows us to analyze the problem of information revelation in the pre-opening period.

Empirically, Biais & al. (1997) show that, in the case of Paris Bourse, the greatest activity occurs in the very last minutes of the pre-opening, typically, between 9:50 and 10:00. This phenomenon suggests that traders are careful not to reveal their information before the market opens. On the ground of this empirical evidence, we have interpreted stage 0 as the time period between 8:30 and 9:50 where traders submit only notional orders and stage 1 the remaining time to 10:00, where they simultaneously submit “real” orders.

We will now motivate this ad-hoc setting by showing that no trader in equilibrium submit a revealing order before the market opens. In practice, this guarantees that traders wait until the very last minutes of the pre-opening before using their inside, and a long-lasting pre-opening phase is completely ineffective as a information disclosure device<sup>6</sup>.

Consider the case where trader  $i$  can submit an order before trader  $j$  moves in stage one: in other words, he has the choice to reveal before  $j$  or wait until stage one, and in this case  $i$  and  $j$  will play (simultaneously) the game as in stage one described before. Revealing before  $j$  could give to player  $i$  an advantage in the sense of Stackelberg leadership, and moreover, it permits to  $i$  to get priority for his orders in the fixing but it reduces the impact of his private information that will be known to  $j$  at the moment to make his own choice.

---

<sup>6</sup>Yet we have shown that the pre-opening phase is profitable for informed traders who can estimate the exogenous volume of trade.

Notice that to reveal his own information before the opponents does not simply mean to submit a notional order before them. The revealing trader must commit to implement the desired belief revision. Indeed, if he changes this order by a new one before 10:00, the opponents and the market makers will also revise their belief on the ground of this new order.

The modified game is now composed by two phases: one occurs during the pre)opening, and the second one will be denoted as the continuation game. In the first phase, trader  $i$  chooses the optimal order to enter the book, and this order has to be considered as a notional order, since it is set before 10:00 am. He can then choose to reveal his type, or simply he can not exploit this opportunity, leaving the game as before. For simplicity, we suppose now that the continuation game is a one-shot, simultaneous moves game equal to that presented before with only the final stage. At this stage, both informed players reveal their information for sure, and we can then refer to the correspondent payoffs previously determined. If trader  $i$  does not reveal before the simultaneous game, he will be as in the case  $(C, C)$  obtaining an expected profit equal to  $\Pi_{2,i}^{CC}$ . If trader  $i$  reveals before  $j$  moves, the latter will be in a situation equivalent to the case  $(R, C)$ . Then, the profit for player  $i$  is given by  $\Pi_{2,i}^{RC}$ . A trade-off arises for player  $i$  when he submits a revealing order before stage one. Revealing his type gives to his opponent a strategic advantage for the real game, but gives to  $i$  the right of priority in execution for his order at the fixing. We can easily quantify the loss for  $i$  due to revealing behavior before the fixing with a plot of  $\Pi_{2,i}^{CC} - \Pi_{2,i}^{RC}$  shown on figure 5 below ( $\tau_i$  on the right axis,  $\tau_j$  on the left axis).

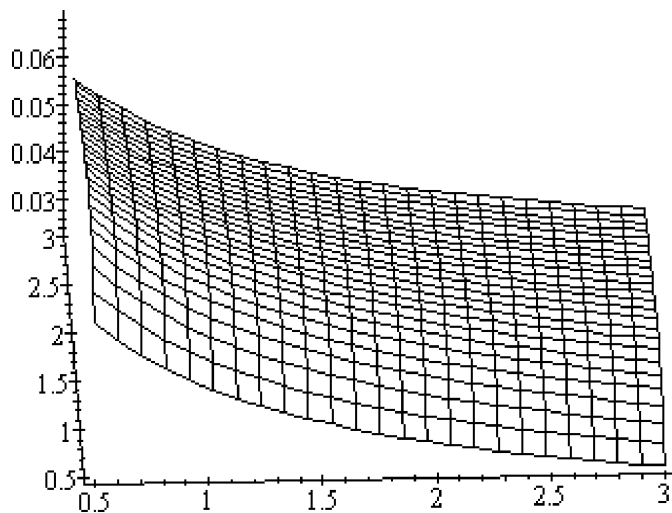


Figure 5

The gain for  $i$  due to priority execution in case of ties in the order book is linked to the specific matching rule used in many stock exchanges. In our model we suppose traders use only market orders, hence the problem of ties does not exist. We can hence state the following:

**Proposition 5** *During the pre-opening phase no information is revealed.*

**Proof** See figure 5 above.

Quantitatively in our model the equilibrium expected profits for the final period belong to the interval  $[1.5 - 2.5]$  for any vector  $(\tau_i, \tau_j)$ . Hence, to reveal information in a phase of notional orders the expected gain should be (see figure 5) in  $(0.03 - 0.06)$  that is almost 3% of the equilibrium profit. We can conclude that players will begin to reveal information when they estimate that the risk of communication breakdown or the expected percentage gain in obtaining a priority in execution sums at least to 3%. This happens typically at the very last minutes of the pre-opening, and then this behavior, observed by Biais & al. (1997), seems to be rational, at least in the sloppy sense described here.

To understand more formally the role of pre-opening, we are studying a one stage model where both insiders and market makers cannot observe the exogenous trade present in the market. Thus it seems informed traders have an interest to estimate it during the pre-opening. But it could be that both players want to bias the estimate of their opponent in order to mislead his perception of the signal revealed at that stage. A formal analysis of this case will be presented in the following version of the paper.

## 7 Conclusions

In the studies that address the problem of aggregation of information by equilibrium prices the role of asymmetries between informed traders has been neglected. The results of Kyle (1985), Holden & Subrahmanyam (1992) consider the equilibrium in a centralized, order-driven continuous financial market with a monopolist of information or more competitors with the same information. They point out that even when the insider is relatively small with respect to the dimension of the market, he can strategically influence the prices, and it is only the competition among informed that matters.

We show that the role of asymmetries is crucial in order to define the efficiency properties of prices. We characterize the linear, pure strategies equilibrium set as a function of the precision of private signals and the volume of liquidity trade present on the market. Keeping aside the influence of the latter, we show that the more precise the signal, the higher the incentive to reveal it at the first stage, but the optimal response of a less-informed trader can be to hide its own information. Asymmetric equilibria hence arise if the insides have considerably different precision. In the symmetric case, we find the result of Holden & Subrahmanyam as a special case. This result goes in the direction of Kyle & Wang (1997) where they assess in a static model the more aggressiveness of an overconfident player.

The competition between asymmetric agents has a really different nature from the one between equally informed insiders.

The model presented here provides a meaningful extension of the literature on insider trading since the information typically differs across individuals (even the same signal could be interpreted in different ways between two different economic agents). Our result could be extended to a multi-stage game but the analytical solution would be practically impossible to assess.

Finally, we have been able to explain the puzzle described by Biais & al. (1997) on the role of the pre-opening phase in information disclosure. Contrary to the theoretical prediction of Vives (1995), Biais & al. notice that in the Paris “Bourse” the disclosure of private information happens from 9:45 to 10:00 while some gaming activity is observed before.

With a slight by different timing in our model we easily prove that informed traders in equilibrium never reveal information before the fixing. It is only in the very last minutes of the pre-opening stage that informed traders reveal some information because the risk of communication breakdown becomes serious and the importance of priority in order execution is relevant. We can hence conclude that a long-lasting phase of pre-opening plays no role in the information revelation.

## 8 Appendix

### 8.1 The Paris Bourse<sup>7</sup>

Paris bourse works as a centralized, order-driven market where a computerised system collects the orders and quotes the equilibrium price maximizing trades, breaking ties by selecting the price nearest to the last quotation at 17:00, the preceding day. This algorithm is a good proxy of the theoretical clearing price. There are about 60 firms who have the right to place orders on the market ; they get the most complete information in real time (to be described later on) through a network. We call a trader any employee of those firms, while a large investor designates any mutual fund, bank or financial institution and a small investor is any private household.

The market opens at 8:30 but until 10:00, all orders are notional, thus the price computed by the system is also notional (“cours théorique d’ouverture”). This stage is known as the pre-opening while the rest of the day from 10:00 until 17:00 is known as the opening-active period.

Any trade that has not been cancelled before 10:00 will be executed at 10:00 or later, depending on the liquidity of the market and on the size and the type of the order. The first quoted price of the day at 10:00 is call the *fixing*. An important feature is that orders submitted before the fixing are filled in priority to those placed later on.

---

<sup>7</sup>All these informations were provided to us by the Bourse de Paris (SBF).

The pre-opening is useful for large investors who want to trade the asset for portfolio hedging reasons. A large order placed later than 10:00 faces the risk of being sliced and executed at prices arising in a less liquid market than in the fixing.

Another source of liquidity trade comes from private households who place orders to their banks in the evening so that many small orders are transmitted to the traders early in the morning (before 9:00). They constitute a significant part of the notional orders ( $\approx 10\%$ ) but a lesser part of the global amount ( $\approx 5\%$ ).

The fixing collects 10% of the whole day trade, it is therefore one of the most “liquid” moment of the day. This characteristic is very important given our results on the link between liquidity volume and efficiency of prices.

In Paris, the only accepted orders are those set by the specialist allowed to operate in the market. Moreover, all specialists can observe the orders placed by the others. Under this observable structure, we have conjectured that for any player is optimal to mislead the estimate the opponents try to obtain of liquidity trade with fake orders. We now turn to a more rigorous argument in defense of this conjecture.

## 8.2 Market Makers Updating Rule

We derive the second stage rule used by market makers. We recall some formulae.

$$\omega_1 = \frac{k_i + k_j}{\lambda_1} \tilde{v} + \frac{k_i \varepsilon_i}{\lambda_1} + \frac{k_j \varepsilon_j}{\lambda_1} + \eta_1 \tilde{u}_1 \quad (6)$$

$$\omega_2 = \frac{m_i + m_j}{\lambda_2} \tilde{v} + \frac{m_i \varepsilon_i}{\lambda_2} + \frac{m_j \varepsilon_j}{\lambda_2} + \eta_2 \tilde{u}_2 \quad (7)$$

$$E[\tilde{v} / \omega_1, \omega_2] = a_1 \omega_1 + a_2 \omega_2 \quad (8)$$

$$E[\tilde{v} / \omega_1] = a_1 \omega_1 + a_2 E[\omega_2 / \omega_1] \quad (9)$$

$$E[\tilde{v} / \omega_2] = a_2 \omega_2 + a_1 E[\omega_1 / \omega_2] \quad (10)$$

$$E[\tilde{v} / \omega_1] = \frac{(k_i + k_j) \omega_1}{\lambda_1 \text{Var}(\omega_1)} \quad (11)$$

To solve (9), we use the order flow formulae (6) and (7) to perform the following decomposition

$$\begin{aligned} E[\omega_2 / \omega_1] &= E\left[\frac{m_i + m_j}{\lambda_2} \tilde{v} / \omega_1\right] + E\left[\frac{m_i \varepsilon_i}{\lambda_2} / \omega_1\right] + E\left[\frac{m_j \varepsilon_j}{\lambda_2} / \omega_1\right] + E[\eta_2 \tilde{u}_2 / \omega_1] \\ &= \frac{m_i + m_j}{\lambda_2} \frac{\lambda_1}{k_i + k_j} E\left[\frac{k_i + k_j}{\lambda_1} \tilde{v} / \omega_1\right] + \frac{m_i}{\lambda_2} \frac{\lambda_1}{k_i} E\left[\frac{k_i}{\lambda_1} \varepsilon_i / \omega_1\right] + \frac{m_j}{\lambda_2} \frac{\lambda_1}{k_j} E\left[\frac{k_j}{\lambda_1} \varepsilon_j / \omega_1\right] \\ &= \left( \frac{\lambda_1 (m_i + m_j)}{\lambda_2 (k_i + k_j)} \left( \frac{k_i + k_j}{\lambda_1} \right)^2 + \frac{\lambda_1 m_i}{\lambda_2 k_i} \left( \frac{k_i}{\lambda_1} \right)^2 + \frac{\lambda_1 m_j}{\lambda_2 k_j} \left( \frac{k_j}{\lambda_1} \right)^2 \right) \frac{\omega_1}{\text{Var}(\omega_1)} \\ &= \frac{Q_z \omega_1}{\lambda_1 \lambda_2 \text{Var}(\omega_1)} \end{aligned} \quad (37)$$

where  $Q_z = (m_i + m_j)(k_i + k_j) + k_i m_i + k_j m_j$ .



Combining (11) and (37), equation (9) becomes

$$a_1\omega_1 + a_2\omega_1 \frac{Q_z}{\lambda_1\lambda_2 Var(\omega_1)} = \frac{k_i + k_j}{\lambda_1} \frac{\omega_1}{Var(\omega_1)}$$

$$\Rightarrow a_2 = \frac{\lambda_2(k_i + k_j - a_1\lambda_1 Var(\omega_1))}{Q_z} \quad (38)$$

Symmetrically, (10) leads to

$$a_1 = \frac{\lambda_1(m_i + m_j - a_2\lambda_2 Var(\omega_2))}{Q_z} \quad (39)$$

with  $Var(\omega_2) = \left(\frac{m_i+m_j}{\lambda_2}\right)^2 + \frac{m_i^2}{\lambda_2^2\tau_i} + \frac{m_j^2}{\lambda_2^2\tau_j} + \frac{\eta_2^2}{\tau_2}$ . Solving this system gives

$$a_1 = \lambda_1 \frac{Q_z(m_i + m_j) - (k_i + k_j)\lambda_2^2 Var(\omega_2)}{Q_z^2 - (k_i + k_j)\lambda_2^2 Var(\omega_2)} \quad (40)$$

$$a_2 = \lambda_2 \frac{(k_i + k_j)(Q_z - (m_i + m_j))}{Q_z^2 - (k_i + k_j)\lambda_2^2 Var(\omega_2)} \quad (41)$$

Now we can easily obtain the linear pricing rule for the market makers for the second stage. Combining  $p_2(\omega_1, \omega_2) = \mu_2 + \lambda_2\omega_2$  and (8) we obtain  $\mu_2 + \lambda_2\omega_2 = a_1\omega_1 + a_2\omega_2$ . Identifying the coefficients of this linear equation, we obtain  $\mu_2 = a_1\omega_1$  and  $\lambda_2 = a_2$  leading through (41) to the equation that will enable to solve for  $\lambda_2$ .

$$\lambda_2 = \lambda_2 \frac{(k_i + k_j)(Q_z - (m_i + m_j))}{Q_z^2 - (k_i + k_j)\lambda_2^2 Var(\omega_2)} \quad (42)$$

and finally

$$\lambda_2 = \frac{\sqrt{\tau_2}}{\eta_2} \sqrt{\frac{Q_z(Q_z - k_i - k_j)}{k_i + k_j} + (m_i + m_j) - (m_i + m_j)^2 - \frac{m_i^2}{\tau_i} - \frac{m_j^2}{\tau_j}} \quad (43)$$

Notice that when  $k_i$  and  $k_j$  tend to zero,  $\frac{Q_z(Q_z - k_i - k_j)}{k_i + k_j}$  tends also to zero, thus (43) is the exact counterpart to (12) the market depth in the first stage.

Observe from (42) that  $\lambda_2^2 Var(\omega_2) = \frac{Q_z(Q_z - k_i - k_j)}{k_i + k_j} + m_i + m_j$ , thus we can also simplify  $a_1$  with to obtain

$$a_1 = \lambda_1 \frac{Q_z(m_i + m_j) - Q_z(Q_z - k_i - k_j) - (m_i + m_j)(k_i + k_j)}{Q_z^2 - Q_z(Q_z - k_i - k_j) - (m_i + m_j)(k_i + k_j)}$$

Lastly, using  $\lambda_1\omega_1 = k_i s_i + k_j s_j + \lambda_1\eta_1\tilde{u}_1$ , we get

$$\mu_2 = \frac{Q_z(m_i+m_j) - Q_z(Q_z - k_i - k_j) - (m_i+m_j)(k_i+k_j)}{Q_z^2 - Q_z(Q_z - k_i - k_j) - (m_i+m_j)(k_i+k_j)} (k_i s_i + k_j s_j + \lambda_1\eta_1\tilde{u}_1) \quad (44)$$

### 8.3 Trader $i$ reveals while $j$ conceal : RC case

In the body of the text, we obtain for the first period  $k_i = \frac{\tau_i}{2(1+\tau_i)}$ ,  $k_j = 0$ ,  $\eta_1 = \frac{1}{3}$  and  $\rho_1 = -\frac{2\mu_1}{3\lambda_1}$  so that  $\mu_1 = 0$  and  $\lambda_1 = \frac{3\sqrt{\tau_1}}{2}\sqrt{\frac{\tau_i}{\tau_i+1}}$  and  $\Pi_{1,i}^i = \frac{1}{3\sqrt{\tau_1}}\sqrt{\frac{\tau_i}{\tau_i+1}}$ . In the second period, we had  $m_i = \frac{1}{2}\frac{\tau_i}{\tau_i+\tau_j+1} + \frac{1}{6}\frac{\tau_i}{\tau_i+1}$ ,  $m_j = \frac{1}{2}\frac{\tau_j}{\tau_i+\tau_j+1}$  and  $\eta_2 = \frac{1}{3}$  so that

$$\lambda_2^{RC} = \frac{\sqrt{\tau_2 Y(\tau_i, \tau_j)}}{2\sqrt{2}(\tau_i + \tau_j + 1)(1 + \tau_i)^{3/2}} \quad (45)$$

where  $Y(\tau_i, \tau_j) = \left( \begin{array}{l} 18\tau_j+16\tau_i+68\tau_i\tau_j+16\tau_i^2+18\tau_j^2+43\tau_i\tau_j^2+32\tau_i^2\tau_j^2 \\ +84\tau_i^2\tau_j+32\tau_i^5+34\tau_i^4\tau_j+11\tau_i^3\tau_j^2+68\tau_i^3\tau_j+48\tau_i^4+16\tau_i^3 \end{array} \right)$  and

$$\mu_2^{RC} = \frac{4\tau_i + \tau_i\tau_j + 3\tau_j + 6 - 2\tau_i^2}{12(\tau_i + \tau_j + 1)(\tau_i + 1)^2} \left( \tau_i s_i + \sqrt{(\tau_i + 1)\tau_i\tau_1}\tilde{u}_1 \right) \quad (46)$$

Finally, we are able to compute the ex-interim profits (27) and (28)

$$\begin{aligned} \Pi_{2,i}^{RC}(s_i) &= \frac{1}{9\lambda_2} \left( \frac{\tau_i s_i}{\tau_i + 1} - \lambda_2^{RC} \tilde{u}_2 - \mu_2^{RC} \right)^2 \\ \Pi_{2,j}^{RC}(s_j, s_i) &= \frac{1}{\lambda_2^{RC}} \left( \frac{s_i\tau_i + s_j\tau_j}{2(\tau_i + \tau_j + 1)} - \frac{\tau_i s_i}{6(\tau_i + 1)} - \lambda_2^{RC} \frac{\tilde{u}_2}{3} - \frac{\mu_2^{RC}}{3} \right)^2 \end{aligned}$$

Since  $s_i$  and  $\mu_2$  are correlated, we need to factorize  $s_i$  and  $\tilde{u}_2$  before proceeding to the ex-ante profits. We obtain

$$\frac{s_i\tau_i}{\tau_i + 1} - \mu_2^{RC} = A\tau_i s_i - B\sqrt{\tau_i\tau_1}\tilde{u}_1$$

with  $A = \frac{14\tau_i^2+20\tau_i+11\tau_i\tau_j+9\tau_j+6}{12(\tau_i+\tau_j+1)(\tau_i+1)^2}$  and  $B = \frac{4\tau_i+\tau_i\tau_j+3\tau_j+6-2\tau_i^2}{12(\tau_i+\tau_j+1)(\tau_i+1)^{3/2}}$ . Using  $E[s_i^2] = \frac{1+\tau_i}{\tau_i}$ ,  $E[\tilde{u}_1^2] = \tau_1^{-1}$  and  $E[\tilde{u}_2^2] = \tau_2^{-1}$  and the expectation of  $\Pi_{2,i}^{RC}(s_i)$  is

$$\Pi_{2,i}^{RC}(\tau_i, \tau_j, \tau_1, \tau_2) = \frac{A^2\tau_i(1 + \tau_i) + B^2\tau_i + \tau_2^{-1}(\lambda_2^{RC})^2}{9\lambda_2^{RC}} \quad (47)$$

The global ex-ante profit is

$$\Pi_i^{RC}(\tau_i, \tau_j, \tau_1, \tau_2) = \frac{1}{3\sqrt{\tau_1}}\sqrt{\frac{\tau_i}{\tau_i+1}} + \Pi_{2,i}^{RC}(\tau_i, \tau_j, \tau_1, \tau_2)$$

## 8.4 Trader $i$ conceals while $j$ reveals : CR case

This case is quite symmetric with the RC one. We obtain for the first period,  $k_j = \frac{\tau_j}{2(\tau_j+1)}$ ,  $k_i = 0$  and  $\eta_1 = \frac{1}{3}$ , we get  $\mu_1^{CR} = 0$  and  $\lambda_1^{CR} = \frac{3}{2}\sqrt{\frac{\tau_1\tau_j}{\tau_j+1}}$ . Now, trader  $i$  is placing a constant order  $q_i = -\frac{\tilde{u}_1}{3}$  but in a *strategic manner*, there is more competition on the market and therefore the reaction of the market makers is stronger:  $\omega_1 = \frac{s_j\sqrt{\tau_j}}{3\sqrt{\tau_1(\tau_j+1)}} + \frac{\tilde{u}_1}{3}$ . The price thus obtained is

$$p_1(\omega_1) = \mu_1^{CR} + \lambda_1^{CR}\omega_1 = \frac{s_j\tau_j}{2(\tau_j+1)} + \lambda_1\frac{\tilde{u}_1}{3}$$

The first period profit is

$$q_i E[\tilde{v} - p_1] = \frac{\tilde{u}_1}{3} \left( \lambda_1 \frac{\tilde{u}_1}{3} - \frac{E[\tilde{v} / s_j]}{2} \right)$$

Because trader  $i$  does not use his information (i.e.,  $E[E[\tilde{v} / s_j]] = 0$ ), the ex-interim expectation is  $\frac{\tilde{u}_1^2}{6}\sqrt{\frac{\tau_1\tau_j}{\tau_j+1}}$  and using  $E[\tilde{u}_1^2] = \tau_1^{-1}$ , we obtain the ex-ante first period profit as

$$\Pi_{1,i}^{CR} = \frac{1}{6}\sqrt{\frac{\tau_j}{\tau_1(\tau_j+1)}} \quad (48)$$

In the second stage,  $m_j = \frac{1}{2}\frac{\tau_j}{\tau_i+\tau_j+1} + \frac{1}{6}\frac{\tau_j}{\tau_i+1}$ ,  $m_i = \frac{1}{2}\frac{\tau_i}{\tau_i+\tau_j+1}$  and  $\eta_2 = \frac{1}{3}$ , thus

$$\lambda_2^{CR} = \frac{1}{\tau_i^2 + 2\tau_i + \tau_i\tau_j + \tau_j + 1} \sqrt{\frac{\tau_2 X(\tau_i, \tau_j)}{8(\tau_j + 1)}} \quad (49)$$

where (notice the asymmetry between  $X$  and  $Y$ )

$$X(\tau_i, \tau_j) = \begin{pmatrix} 18\tau_i + 54\tau_i^2 + 10\tau_j^3 + 12\tau_j^4 + 2\tau_j^5 + 18\tau_i^4 + 54\tau_i^3\tau_j + 54\tau_i^2\tau_j^2 + 20\tau_i^2\tau_j^2 \\ + 16\tau_j - 18\tau_i\tau_j^2 - 20\tau_j^2 + 9\tau_i^4\tau_j + 18\tau_i^3\tau_j^2 + 44\tau_i\tau_j^3 + 32\tau_i^2\tau_j^3 + 16\tau_i\tau_j^4 + 68\tau_i\tau_j \end{pmatrix}$$

Similarly

$$\mu_2^{CR} = (9\tau_i + 3\tau_i^2 - 2\tau_j - 2\tau_i\tau_j - 2\tau_j^2 + 6) \frac{\tau_j s_j + \sqrt{\tau_1\tau_j(\tau_j+1)}\tilde{u}_1}{12(\tau_i + \tau_j + 1)(1 + \tau_i)(\tau_j + 1)} \quad (50)$$

Using symmetry with care, the ex-interim profit formula of the concealing agent is the  $\Pi_{2,j}^{RC}$  presented in the preceding case but with indices  $i$  and  $j$  inverted i.e., we obtain

$$\Pi_{2,i}^j(s_j, s_i) = \frac{1}{\lambda_2^{CR}} \left( \frac{s_i\tau_i + s_j\tau_j}{2(\tau_j + \tau_i + 1)} - \frac{s_j\tau_j}{6(\tau_i + 1)} - \lambda_2^{CR} \frac{\tilde{u}_2}{3} - \frac{\mu_2^{CR}}{3} \right)^2$$

Working out the correlations between the random variables involved, we are led to the following constants

$$\begin{aligned} s_j &\rightarrow A = \tau_j \frac{-8\tau_j - 6 - 14\tau_i\tau_j - 3\tau_i + 4\tau_j^2 + 3\tau_i^2}{-36(\tau_i + \tau_j + 1)(1 + \tau_i)(\tau_j + 1)} \\ s_i &\rightarrow B = \frac{\tau_i}{2(\tau_j + \tau_i + 1)} \\ \sqrt{\tau_2}\tilde{u}_2 &\rightarrow C = \frac{1}{3(\tau_i^2 + 2\tau_i + \tau_i\tau_j + \tau_j + 1)} \sqrt{\frac{X(\tau_i, \tau_j)}{8(\tau_j + 1)}} \\ \sqrt{\tau_1}\tilde{u}_1 &\rightarrow D = \frac{(9\tau_i + 3\tau_i^2 - 2\tau_j - 2\tau_i\tau_j - 2\tau_j^2 + 6)\sqrt{\tau_j(\tau_j + 1)}}{-36(\tau_i + \tau_j + 1)(1 + \tau_i)(\tau_j + 1)} \end{aligned}$$

in order to get the ex-ante second period profit

$$\Pi_{2,i}^{CR}(\tau_i, \tau_j, \tau_1, \tau_2) = \frac{1}{\lambda_2^{CR}} \left( A^2 \frac{1 + \tau_j}{\tau_j} + B^2 \frac{1 + \tau_i}{\tau_i} + C^2 + D^2 \right) \quad (51)$$

The global ex-ante profit is the sum of (48) and (51)

$$\Pi_i^{CR}(\tau_i, \tau_j, \tau_1, \tau_2) = \frac{1}{6} \sqrt{\frac{\tau_j}{\tau_1(\tau_j + 1)}} + \Pi_{2,i}^{CR}(\tau_i, \tau_j, \tau_1, \tau_2)$$

## 8.5 Traders $i$ and $j$ reveal : RR case

In the first period, the players choices are  $k_i = \frac{(\tau_j + 2)\tau_i}{3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4}$ ,  $k_j = \frac{(\tau_i + 2)\tau_j}{3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4}$  and  $\eta_1 = \frac{1}{3}$ , thus  $\mu_1 = 0$  and

$$\lambda_1^{RR} = \frac{3\sqrt{\tau_1 Z(\tau_i, \tau_j)}}{3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4}$$

We derive

$$\Pi_{1,i}^{RR} = \frac{\sqrt{Z(\tau_i, \tau_j)}}{3\sqrt{\tau_1}(3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4)} \left( \frac{(\tau_j + 2)^2(1 + \tau_i)\tau_i}{Z(\tau_i, \tau_j)} + 1 \right) \quad (52)$$

For the second period, we get  $m_i = \frac{\tau_i}{3(\tau_j + \tau_i + 1)}$ ,  $m_j = \frac{\tau_j}{3(\tau_j + \tau_i + 1)}$ ,  $\eta_2 = \frac{1}{3}$  leads to is true

$$\lambda_2^{RR} = \frac{\sqrt{\tau_2 K(\tau_i, \tau_j)}}{(\tau_i + \tau_j + 1) \sqrt{2L(\tau_i, \tau_j)}} \quad (53)$$

with  $K(\tau_i, \tau_j) = \left( \begin{aligned} &16\tau_i^2 + 8\tau_j^3 + 8\tau_j^4 + 8\tau_i^4 + 26\tau_i^3\tau_j + 8\tau_i^3 + 64\tau_i^2\tau_j + 76\tau_i^2\tau_j^2 + 64\tau_i\tau_j^2 + 16\tau_j^2 \\ &+ 10\tau_i^4\tau_j + 24\tau_i^3\tau_j^2 + 26\tau_i\tau_j^3 + 24\tau_i^2\tau_j^3 + 10\tau_i\tau_j^4 + 32\tau_i\tau_j + 3\tau_i^4\tau_j^2 + 6\tau_i^3\tau_j^3 + 3\tau_i^2\tau_j^4 \end{aligned} \right)$  and  
 $L(\tau_i, \tau_j) = 3\tau_i^2\tau_j^2 + 7\tau_i^2\tau_j + 7\tau_i\tau_j^2 + 12\tau_i\tau_j + 4\tau_j^2 + 4\tau_i^2 + 4\tau_i + 4\tau_j$

Finally we get

$$\mu_2^{RR} = \frac{(s_i\tau_i\tau_j + 2s_i\tau_i + \tau_j s_j\tau_i + 2\tau_j s_j + \sqrt{\tau_1 W(\tau_i, \tau_j)} u_1) (3\tau_i^2\tau_j + 2\tau_i^2 + 14\tau_i\tau_j + 3\tau_i\tau_j^2 + 2\tau_j^2 + 6\tau_i + 6\tau_j)}{6(3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4)(\tau_i\tau_j + \tau_i + \tau_j)(\tau_i + \tau_j + 1)} \quad (54)$$

with  $W(\tau_i, \tau_j) = 4\tau_j + 4\tau_i + 4\tau_i^2 + 8\tau_i\tau_j + 5\tau_i^2\tau_j + 5\tau_i\tau_j^2 + 2\tau_i^2\tau_j^2 + 4\tau_j^2$ .

As usual, to compute the ex-ante profit we take into account that  $s_i$  and  $s_j$  are correlated with  $\mu_2$ , thus from

$$\Pi_{2,i}^{RR}(s_i, s_j) = \frac{1}{9\lambda_2^{RR}} \left( \frac{s_i\tau_i + s_j\tau_j}{\tau_j + \tau_i + 1} - \lambda_2^{RR}\tilde{u}_2 - \mu_2^{RR} \right)^2$$

we obtain

$$\Pi_{2,i}^{RR}(\tau_i, \tau_j, \tau_1, \tau_2) = \frac{A^2 \frac{1 + \tau_j}{\tau_j} + B^2 \frac{1 + \tau_i}{\tau_i} + C^2 + D^2}{9\lambda_2^{RR}} \quad (55)$$

where a constant is computed for each of the 4 independent random variables.

$$\begin{aligned} s_j \quad A &= \tau_j \frac{15\tau_i^2\tau_j^2 + 22\tau_i^2\tau_j + 34\tau_i\tau_j^2 + 38\tau_i\tau_j + 20\tau_j^2 + 14\tau_i^2 + 12\tau_i + 12\tau_j - 3\tau_i^3\tau_j - 2\tau_i^3}{6(3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4)(\tau_i\tau_j + \tau_i + \tau_j)(\tau_i + \tau_j + 1)} \\ s_i \quad B &= -\tau_i \frac{-15\tau_i^2\tau_j^2 - 34\tau_i^2\tau_j - 22\tau_i\tau_j^2 - 38\tau_i\tau_j - 14\tau_j^2 - 20\tau_i^2 - 12\tau_i - 12\tau_j + 3\tau_i\tau_j^3 + 2\tau_j^3}{6(3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4)(\tau_i\tau_j + \tau_i + \tau_j)(\tau_i + \tau_j + 1)} \\ \sqrt{\tau_2}\tilde{u}_2 \quad C &= -\frac{\sqrt{K(\tau_i, \tau_j)}}{(\tau_i + \tau_j + 1) \sqrt{2L(\tau_i, \tau_j)}} \\ \sqrt{\tau_1}\tilde{u}_1 \quad D &= -\sqrt{W(\tau_i, \tau_j)} \frac{3\tau_i^2\tau_j + 2\tau_i^2 + 14\tau_i\tau_j + 3\tau_i\tau_j^2 + 2\tau_j^2 + 6\tau_i + 6\tau_j}{6(3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4)(\tau_i\tau_j + \tau_i + \tau_j)(\tau_i + \tau_j + 1)} \end{aligned}$$

To conclude, the global ex-ante profit is the sum of (52) and (55).

$$\Pi_i^{RR}(\tau_i, \tau_j, \tau_1, \tau_2) = \Pi_{1,i}^{RR}(\tau_i, \tau_j, \tau_1) + \Pi_{2,i}^{RR}(\tau_i, \tau_j, \tau_1, \tau_2) \quad (56)$$

## References

- [1] Admati, A., & P. Pfleiderer (1991). “Sunshine Trading and Financial Market Equilibrium”. *Review of Financial Studies* **1**, Spring, 3-40.
- [2] Amihud, Y., & H. Mendelson (1987). “Trading Mechanism and Stock Returns: An Empirical Investigation”. *Journal of Financial Economics*, **8** : 31-53.
- [3] Bhattacharya, U. & M. Spiegel (1991). “Insiders, Outsiders and Market Breakdowns”. *Review of Financial Studies*, **4** : 255-282.
- [4] Biais, B., P. Hillion & C. Spatt (1997). “Price Discovery and Learning during the Pre-opening Period in the Paris Bourse”, Mimeo University of Toulouse.
- [5] Foster, F. D. & S. Viswanathan (1993). “A Theory of the Intraday Variations in Volume, Variance and Trading Costs in Securities Markets”. *Review of Financial Studies*, **3** : 593-624.
- [6] Holden, C. W. & A. Subrahmanyam (1992). “Long-Lived Private Information and Imperfect Competition”. *The Journal of Finance*, **47** : 247-270.

- [7] Kyle, A. S. (1985). "Continuous Auctions and Insider Trading". *Econometrica*, **53** : 1315-1335.
- [8] Kyle, A. S. & F. A. Wang (1997). "Speculation Duopoly with Agreement to Disagree: Can Overconfident Survive the Market Test?". *The Journal of Finance*, **52** : 2073-2090.
- [9] Kyle, A. S. (1989). "Informed Speculation with Imperfect Competition". *Review of Economic Studies*, **56** : 317-356.
- [10] Madhavan, A. (1992). "Trading Mechanisms in Securities Markets". *The Journal of Finance*, **47** : 607-642.
- [11] O' Hara, M. (1995). *Market Microstructure Theory*. Blackwell Publishers, Oxford.
- [12] Spiegel, M. & A. Subrahmanyam (1992). "Informed Speculation and Hedging in a Non-competitive Securities market". *Review of Financial Studies*, **2** : 307-329.
- [13] Vives, X. (1995). "The Speed of Information Revelation in a Financial Market Mechanism". *Journal of Economic Theory*, **67** : 178-204.