Learning and Agglomeration in Industrial Networks

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June 1998

Abstract

The main aim of this study is to evaluate the nature and the dynamics of agglomeration forces inside specific network structures like industrial districts. Firms seem to be attracted in these locations by the opportunity of exploiting the advantages of an inter-firm coordinating structure. This structure enables them to share the cumulated know-how available in a district. This is the real and inner agglomerating force. An original feature rises in correspondence of it: it is not so evident that a district, which has an initial advantage (in knowledge) over the others, will keep that advantage and be always the most attractive one.

JEL Classification: L22, R12, R3

Key-words: Agglomeration, Industrial Districts, Firm Location.

1 Introduction

The resurgence of *regional economies* and the *territorial specialisation* are some of the major phenomena which need explanations in economic geography. The term localisation stands for something more than an accidental

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concentration in one place of production processes, which have been attracted there by already-existing localising factors (Pyke et alii.1990).

We may admit that there exists two kinds of forces that induce the agglomeration of firms. These forces can be divided into two groups: internal and external. By *external forces*, we are referring to the classical centripetal forces that the new models of economic geography adopt, while by *internal forces* we consider some cohesive components that make firms locate in a specific area. If the first kind of forces is useful for explaining the geographical location choices, i.e. explaining the reasons that make a region an attractive industrial pole (a so-called 'core'), the second ones should detect the sectors in which firms decide more easily to produce in an agglomerated area.

The idea of *centripetal forces,* for a geographical delocalisation, finds its roots in the Krugman' analysis in 1991. He stated that there are three principal criteria which let a region be selected as an attractive pole: transport costs, local level of revenue and increasing returns to scale. This last factor is the explicitly trait-d'union between the two types of forces. The exploitation of *increasing returns to scale* is the principal requirement which firms look for when they decide to install in a limited area.

The concentration of firms is not so surprising when one remarks that *clustering* is particularly strong for firms in *high-technology* and *information-intensive* sectors, given the recent development in the new information technologies (Lawson 1997). For this reason the attention is more and more focused on the peculiarities of linkages that might exist among firms. This setting generally concerns industrial networks but, above all, some their specific forms as *industrial districts.* Districts consist in locales that accommodate a large number of small firms which produce similar goods and benefit from the accumulation of know-how associated with workers residing there (Souberyran -Thisse 1997). Here, Smith's notion of *division of labour* is combined with a form of *evolutionary theory* for which a firm's survival is taken to depend on an increased differentiation and co-ordination (Lawson 1997). Altogether, the main aim of this study is not to focus primarily on the economic behaviour or the performances of districts themselves. Indeed we are interested in analysing which internal centripetal forces may produce an agglomeration inside them.

Empirical findings recognize that the real competitive advantages of districts are in the *cumulated know-how* embodied in *workers* (which makes them more productive) and in all the *formal* (or *informal*) *inter-firm struc-*

tures that coordinate economic activities. In particular, our last goal will be to state the way in which an inter-firm structure and the cumulated experience in production (which represents one of the basic forces for attracting firms inside a specific location) could be real aggregating forces. In districts capital is complementary to human skillness and not a direct substitute of it. The proper specialisation of workers inside a district forms the real competitive advantage of its firms. The skillness embodied in workers (as in Soubeyran-Thisse (1997)) is modelled through the assumption of a decreasing cost function against the total cumulated output of a district. In districts, the technical co-operation is realized through the social co-operation that is based on the relation of trust and reciprocity between firms, rather than hierarchical relations. As a consequence, any economy of scale is external to any particular (small) firm, but internal to the productive system as a whole. This idea implies that the *knowledge is embodied in workers* that live in a bordered geographical area and interact together. By this interaction we derive a spread *informational process* among firms, in a network, according to the natural formal or informal interactions across members. Besides, we suppose that firms are price-takers and decide to locate in a specific area for maximizing their current profit. It is evident that firms do not make any expectations about their future and they do not assume any strategical behaviour like hit-and-run. This hypothesis is not so unrealistic if it is applied to district systems. Participating to a district, means also to come into an *informal system of reciprocal trust* at least as far as the behaviours among members is concerned. As well presented in Pyke at alii (1990), in an industrial district still exists a competitive mechanism across firms, but this specific structure implies that all partecipants respect the rules established in that community. The efficiency of the information system is also very important since it opens a deep debate on the selective mechanisms across districts and on their temporal evolution.

In general, firms and their distribution affect the production history of each district. An equilibrium state can be destroyed by firms that have an incentive to move away from a specific location. Indeed firms may be profitably attracted by places in which the *stock of know-how* is large, but also in which there is a better organisation for *spreading and sharing of the common knowledge*. Consequently, by hypothesis, the presence of an inter-firm structure which fosters exchanges of information (represented by the parameter b_i) makes small firms can combine the advantages of flexibility with the support of stability that comes from larger networks (Pyke et alii. 1990). As it is reported in Onida et alii (1992), in industrial districts, it is the presence of autonomous institutions that manage different kinds of services (addressed to firms) the real source of competitiveness for district firms. In general its tasks cover a quite wide range. First of all it picks up all possible data on the market conditions (even foreign markets) and it continously updates the firms that belong to its districts. In addition it organises special meeting for training the local labour force and it often finances (at very advantageous conditions) the modernisation of the capital stock at firm level. Sometimes it may also have other tasks as being financial intermediary in granting special or extraordinary loans. At the same time, its duties include the task to certificate the standard of the quality of goods that are produced in a district, beside supervising the marketing behaviour of each firm for avoiding situations of unfair competition.

In this model, our basic hypotheses imply that the more a firm produces, the more it gains experience and the more know-how is accumulated inside the district it belongs to. If we assume a simple direct relation among output, know-how and productiveness, it will be evident that this cumulated process may continue to the infinity. On the contrary, it is quite evident (by the stylized facts on the cumulation of knowledge) that there is a sort of superior bound to the most profitable exploitation of cumulated knowledge.

This specification gives us the possibility to extend dynamically our analysis. If we considered simply the direct proportion between cumulated experience and productivity, it would be evident that the oldest districts would always have a competitive advantage over the new ones. This formulation cannot explain the birth of new agglomerations, while older ones still exists, and it is also unhelpful for understanding the changes in dimensional ranking among several districts. For instance, a static setting would not be able to explain why (in Italy) there has been a recent large (and profitable) spreading of industrial districts in the shoe-sector in Montegranaro (Marche), while the oldest concentration in Vigevano still exists (Lombardia)(Onida et alii 1992).[1]

[1] This is only an example. We can account also some other examples. In the wood and furniture sector we have the oldest activities around Cantu' (Lombardia), while the most recent activities are settled in Triveneto (Veneto and Friuli Venezia Giulia). The same can be observed even for the gold and jewellery sectors with the districts around Valenza Po (Piemonte) and around Bari (Puglia).

It should be clear that the initial endowment in human capital and organisational structure is not a sufficient condition for assuring an economic advantage for several periods. So, it will be interesting to detect the conditions under which an initial hierarchy is respected or catching-up processes develop. For reaching this object we will produce some static comparisons among the main outcomes of a mathematical framework. Our features are not plugged in a general geographic model since we want just to focus on a peculiar aspect. Therefore it might be possible to think of this model as a device to extend to a geographic model.

The remainder of this paper is organised as follows: section 2 introduces a model for defining the general behaviour of firms in a district, while section 3 contains the analysis of the effects of agglomeration forces on the district structure and section 4 concludes.

2 The model

The general features of the model follow the principal ideas proposed by Soubeyran and Thisse (1997).

We consider a generic industrial district i, in which there is a number $n_i(t)$ of firms and a fixed population of workers Li. Each worker supplies one unit of labour and has a zero reservation wage. Each firm demands two inputs: labour and capital. We have the complete depreciation of the capital stock in each period[2], while the knowledge, derived from the productive process, is directly embodied in workers.

Firms are supposed to be price-takers and so they sell their (homogeneous) good at price $p_i(t)$, while they sustain costs for wages and for buying the capital stock[3]. We assume also that the capital requirement is the same everywhere.

We take into account the spatial delimitation of industrial district, by assuming that firms hire workers only in a fixed area and we prevent workers from migration.

Altogether the real advantage of belonging to a district is the real sharing

[2]It has to be replaced at the end of each period. This means that the intertemporal profit maximisation is equivalent to the profit maximisation in each istant t.

[3]At this level, we suppose that the interest rate payed by the firms is r and it is constant across time.

process of the cumulated knowledge. Indeed, workers better their productiveness by a learning process, filled by the total previous quantity of good produced at district level. This is the central point.

The collective advantage can be exploited by a firm if and only if it belongs to a district. This advantage consists of increasing its efficiency (at productive level), through a better productivity of workers which depends directly on the cumulated know-how.

The cumulated know-how, at time t, is supposed to be dependent on the total quantity of output produced by all firms up to t. In addition a district differentiates itself from another one by the efficiency and the coordination in sharing information. This condition implies the assumption to deal with an inter-firm organisation.This detail will be represented by a positive parameter b_i and the greater b_i is, the more efficient the internal organisation is.

In order to take into account the relationship between produced output and cumulated know-how[4], we represent the accumulation of know-how by a diffusion process.

Definition 1:

An industrial district is a local agglomeration of firms. It is made of a fixed number of workers and a variable number of firms which rely on an inter-firm system of coordination.

We consider that *firms* (inside a district) are all *symmetric*, and so the output is the same for each of them. In particular we consider that firms *produce uniformly* their goods along the time.

Knowing that each firm takes into account the total experience cumulated up to that period (and without any kind of expectation on future), in each instant t, each district firm maximises its profit function:

$$\Pi_i(q_i, w_i, S_i) = p_i(t)\, q_i(t) - C_i(q_i, w_i, S_i), \tag{1}$$

where

$$C_i(q_i, w_i, S_i) = w_i(t)\, l\,[S_i]\, q_i(t) + rK(q_i(t)). \tag{2}$$

At time t, the output of a firm is denoted by $q_i(t)$ and $p_i(t)$ is the selling price of goods. The cost function (2)consists of two components. The former

[4]There is a non-linear dependence between produced output and cumulated know-how along time.

is related to the labour cost [5]. It is expressed as labour cost per-output. The variable $w_i\,(t)$ represents the level of wages in a district. Given the *immobility* of workers, this variable has different values in different districts. The total cost of the work force is not always the same across districts. It depends on the labour coefficient (considered as workers per-quantity) $l(S_i\,(t))$, which is a decreasing function of the cumulative knowledge, denoted by $S_i\,(t)$. The latter (the second term of expression (2)) represents the cost of the capital (which is the same across the districts).

By maximizing equation (1), we obtain the following f.o.c.:

$$\frac{\delta \Pi_i}{\delta q_i} = p_i\,(t) - w_i\,(t)\,l\,[S_i\,(t)] - rK'(q_i\,(t)) = 0 \tag{3}$$

from which we derive the optimal quantity q_i^* (that maximizes firm profits) as :

$$q_i^* = \{K'\left(\frac{p_i\,(t) - w_i\,(t)\,l\,[S_i\,(t)]}{r}\right)\}^{-1}. \tag{4}$$

Knowing that all firms are identical, this equation states the optimal level of production for each firm.

This result is also important for determining the total number of firms that are present in a district in a specific moment.

We supposed that all workers are hired in firms of the same district.

This means that :

$$L_i = n_i\,(t)\,l\,[S_i\,(t)]\,q_i^*\,(t) \tag{5}$$

and so the optimal total number of firms n_i in i is :

$$n_i\,(t) = \frac{L_i}{l\,[S_i\,(t)]\,q_i^*\,(t)}.$$

At the same time, always by the f.o.c., we determine that :

$$p(t) = w_i(t)l(S_i(t)) + rK'(q_i^*(t)) \tag{6}$$

and so by the f.o.c. we derive also that:

[5]The capital requirement of a firm is increasing and strictly convex in its output.

$$w_i(t) = \frac{[p_i(t) - rK'(q_i^*(t))]}{l[S_i(t)]}. \tag{7}$$

This last condition specifies that the level of wages is inversely proportional to the total amount of know-how cumulated in a district.

In general a district is active if the firms which belong to it produce strictly positive outputs, i.e. pay positive wages. By applying this condition to the equation (7) and considering both equation (5) and the convexity of $K(q_i)$, we deduce that positive wages implies the presence (in the district) of an actual number of firms greater than a minimum size (that is fixed in correspondence of the zero level of wages)[6].

3 The Agglomeration Effect

3.1 General Features

Generally, firms try to distribute themselves in locales in order to maximize their profits.

As widely described in sections 1 and 2, *joining to an industrial district* means to be associated into a specific structure in which each activity, at firm level, is strictly *interrelated* to the activities of the other firms. Each firm experience becomes common knowledge in the district and it is included in the general know-how which is shared by all members. By this situation, firms reach higher level of efficiency in production and they increase their profitability by production. Onida et alii (1992) report that the real capacity of exploiting the learning effect is a discriminatory component, which allows firms to select a location to another one. In this framework, we assume that the cumulative knowledge of a district is a function of the total quantity of goods (produced by all the firms of the district) up to a fixed moment. It is indirectly assumed that the experience derives from a specific form of learning: the learning by doing. At time t, we know that the total quantity of good produced by a district is equal to the quantity of good produced by each firm times the total number of firms present (at that moment) in the district. This means that the total quantity Q at time t is equal to :

[6] In a general model of economic geography, imposing the positive-wage condition would prevent a district from loosing all its firms, when a migration mouvement appeared.

$$Q_i(t) = n_i(t)\, q_t(t) = \frac{L_i}{l\,[S_i(t)]} \tag{8}$$

We suppose that the learning process is not a simple direct function of the cumulated past production. Indeed it is not true that the more a district has produced in the past, the more it attracts firms and keeps its advantages over competitors.

Onida et alii (1992) show that it is possible to have a growing or a declining process, in the sense that a district may be more attractive for fixed periods, while it may loose its attractiveness in others up to the point to disappear out of the market. There is a sort of dynamic turnover among districts as agglomerating poles. In our idea the cumulation of knowledge, produced by a learning by doing process, depends on two factors: the previous production and the inter-firm organisation.

In a district, taking into account the impact of learning (considered as learning by doing) across time, we suppose that the cumulated knowledge (defined as S_i) depends on the cumulated quantity by a logistic function.

As stated before, each firm produces uniformly over time. In equation (4), we stated the intensity of the production at an instant t. This means that the total quantity that is produced by a firm in period $[a, b]$ is:

$$q_i(a, b) = \int_a^b q_i(s)\, ds \tag{9}$$

so that, defining (Q_i)as the total quantity of good that is produced by the whole district, over the period (a, b), we get :

$$Q_i(a, b) = n_i(a, b)\, q_i(a, b) = \int_a^b Q_i(s)\, ds \tag{10}$$

Besides, given this expression, we may affirm that the function of cumulated quantity of good that is produced by the district up to time T can be easily expressed by:

$$\overline{Q_i}(T) = \int_0^T Q_i(s)\, ds$$

and given the definition of Q_i, the cumulated quantity is a continuous function.

To explain the central role of the inter-industrial structure for creating know how [7], we consider a diffusion model. We define a functional form of cumulated know-how, through the equation proposed by Mansfield (1961) to describe the diffusion process of technological innovations in the industrial fields[8].

In order to fit that idea to our problem, we introduce some further specifications. We know that the labour coefficient is a decreasing function of the stock of knowledge. In a district we know (by the previous hypothesis) that in the initial period we have a positive stock of knowledge ($S_i(0)$). So, we assume that the initial level *of* disposable knowledge in a district at ($t = 0$) is exactly equal to the level of quantity (Q_0). To this stock we may associate a specific value of l, $l_i(0)$,which is the highest value of l (which means the lowest level of efficiency) over a generic period $(0, T)$. In the same way, we admit that there is a lowest value of l, l^*, that is reached with the largest stock of know-how S_i^* (to which each district tries to converge).

In general, we manage the following differential equation for stating the level of cumulated know-how. In it, the variable b_i represents the inter-firms structure[9]:

$$\frac{dS_i(t)}{dQ_i(t)} = \frac{dS_i(t)/dt}{dQ_i(t)/dt} = b_i S_i(t) \frac{[S_i^* - S_i(t)]}{S_i^*}. \tag{11}$$

where $S_i(t)$ is the level of knowledge at time t (that depends directly on the cumulated level of produced goods) while S_i^* corresponds to the optimal level of knowledge to which it is associated *the* most efficient labour coefficient l^*.

By this differential equation, associating at time $t = 0$ (in correspondence of $l_i(0)$) the level of knowledge $S_i(0)$ and the level of production $Q_i(0)$, we derive (for a generic district i) the analytical formula for the cumulated know-how (see appendix A) :

$$S_i(t) = \frac{S_i^*}{1 + \frac{\left[S_i^* - \frac{L}{l_i(0)}\right]}{\frac{L}{l_i(0)}} e^{-b_i\left[\overline{Q_i}(t)\right]}}. \tag{12}$$

[7]In this way we are able to distinguish districts from a simple collection of firms.

[8]This equation is often spreadly applied in marketing models as De Palma et alii. (1991) reports, and it is suitable to our final object.

[9]In general b_i is positive and the greater it is, the more efficient the structure is.

Assumption 1:

In a district, the cumulation of knowledge follows a logistic representation over time. It directly depends on the total experience in production, embodied by the level of cumulated production of local firms.

The cumulation of knowledge does not seem to suffer of firm-congestion problems. Indeed looking at equation (12) the number of firms influences directly the total production of a district. By that equation it may be easily checked that the more firms there are in a district, the more knowledge is cumulated in that district and this process makes that district more and more attractive.[10]

3.2 Principal Properties

The analysis of the cumulated know-how (12) is useful for understanding the dynamics inside a district.

It is easily verified that :

$$\frac{dS_i(t)}{db_i} > 0$$

$$\frac{dS_i(t)}{dL_i} > 0$$

or better, by equation (12), we are able to state that the cumulated knowledge is directly correlated to the number of workers in the district and to the level of the efficiency of the inter-firm organisation.

We reproduce useful graphs that correspond to these results[11].

In Figure 1, the effect of an increase in the parameter b_i is reported. With a fixed number of agents, a more efficient organisation makes it possible to achieve a better efficiency and so reaching the highest level of know-how more quickly.

[10] This effect is quite closed to the 'backward linkage' effect of economic geographic models.

[11] They are obtained by representing the analytical formula, for specific (and coherent) values of the parameters and supposing that the initial level of knowledge and production is always positive.

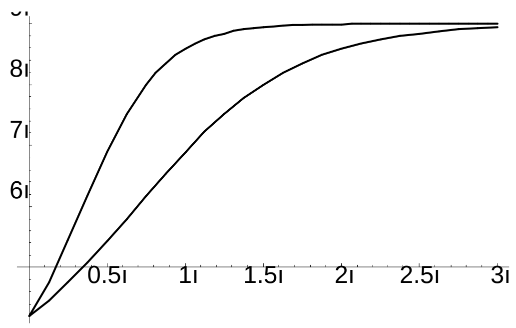

Figure 1: The effect of an increase of the parameter b_i

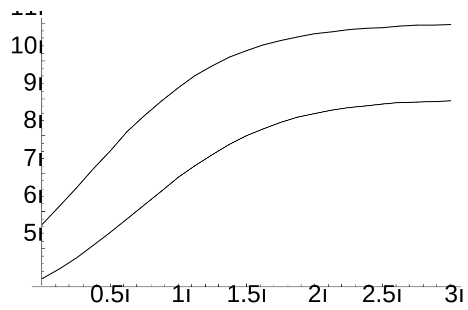

Figure 2: The effect of an increase of the parameter L

On the contrary, in Figure 2, we present the effect of an increase of the number of workers (that might mean an indirect increase of the number of firms). We find that this last event shifts the S-curve up. This means the projection of the system toward a potential higher final stock of know-how (which implies a final higer level of labour efficiency), keeping the same type of inter-firm organisation.

Observing the equation (11), we realise that the degree of the cumulation of the knowledge is dependent on the level of interpersonal communication and on the width of the gap for reaching the maximum efficiency.

Observation 1:

The cumulation of know-how depends directly on the activity of the information transmission among members of a same location. It is as much higher as much efficient is the inter-firm organisation structure.

A direct consequence of equations (11) and (12) is the following proposition:

Proposition 1:

In a district, at the generic time t, the presence of an inter-firm structure may make the cumulated knowledge proportionally greater than the correspondent level of total production.

Proof. Equation (11) states the dynamics of the cumulation of knowledge. At the istant t *knowledge* is directly dependent on the level of production but it is not strictly equal to it. This is principally due to the existence of the inter-firm organisation structure (b_i). If our hypothesis is true, this means that there exists some conditions such that:

$$S_i\left(t\right) > \overline{Q_i}\left(t\right)$$

By equations (8), (11) and (12) the previous condition can be expressed as :

$$\frac{S_i^*}{1+\frac{\left[S_i^*-\frac{L}{l_i(0)}\right]}{\frac{L}{l_i(0)}}e^{-b_i\left[\overline{Q_i}(t)\right]}} > \overline{Q_i}\left(t\right)$$

and so

$$\frac{S_i^* - \overline{Q_i}(t)}{\overline{Q_i}(t)} > \left[\frac{S_i^* - \frac{L}{l_i(0)}}{\frac{L}{l_i(0)}}\right] e^{-b_i\left[\overline{Q_i}(t)\right]}$$

Supposing that both the LHS and RHS are positive, i.e. respecting the following conditions:

$$\overline{Q_i}(0) < \overline{Q_i}(t) \text{ and } 1 < \frac{S_i^*}{\overline{Q_i}(t)} < \frac{S_i^*}{\overline{Q_i}(0)}$$

we may apply the logaritm function to the equation and so we obtain:

$$\ln\left[\frac{S_i^*}{\overline{Q_i}(t)} - 1\right] > \ln\left[\frac{S_i^* - \frac{L}{l_i(0)}}{\frac{L}{l_i(0)}}\right] - b_i\left[\overline{Q_i}(t)\right]$$

and through the condition (A.1) in appendices we derive that:

$$b_i\left[\overline{Q_i}(t)\right] > \ln\left[\frac{S_i^*}{\overline{Q_i}(0)} - 1\right] - \ln\left[\frac{S_i^*}{\overline{Q_i}(t)} - 1\right]$$

$$b_i\left[\overline{Q_i}(t)\right] > \ln\left[\frac{\frac{S_i^*}{\overline{Q_i}(0)} - 1}{\frac{S_i^*}{\overline{Q_i}(t)} - 1}\right] \tag{13}$$

This condition is well fulfilled the greater b_i is and the smaller the gap between the two level of knowledge (the starting and the final ones) is. This dynamics shows the importance of the coordinating structure across firms in a district, for offering them proportional increasing benefits in the industrial activity.

3.3 The stock of knowledge in two districts

We produce, now, a comparison on the cumulation of the knowledge across two districts for deriving further suggestions about the agglomeration forces. Let us suppose that districts 1 and 2, have the same number of workers, the same organisation and the same best level of efficiency of the labour force (l^*) that implies the same higher level of knowledge [12]. If we have

[12] We consider also equal the per-firm level of production.

$$S_1(0) < S_2(0) \qquad \textit{this means} \;\; l_1(0) > l_2(0)$$

i.e. a lower initial level of knowledge is due only to less efficient workers.

Considering the same hypothesis as before, what could be the relation between the stock of knowledge of the districts ? Given the functional form of the cumulation of knowledge, we may observe a catching-up (or even a leapfrogging) process between districts in terms of stockage of know-how.

The condition for which at generic time t,

$$S_2(t) \leq S_1(t)$$

starting form

$$S_1(0) < S_2(0)$$

is

$$\frac{S_1^*}{1 + \frac{\left[S_1^* - \frac{L_1}{l_1(0)}\right]}{\frac{L_1}{l_1(0)}} e^{-b_1\left[\overline{Q_1}(t)\right]}} \geq \frac{S_2^*}{1 + \frac{\left[S_{2i}^* - \frac{L_2}{l_2(0)}\right]}{\frac{L_2}{l_2(0)}} e^{-b_2\left[\overline{Q_2}(t)\right]}}$$

For describing all the potential dynamics (independently of the level of production) we take the assumption to have the same level of cumulated output in the two districts $\left(\overline{Q_1}(t) = \overline{Q_2}(t) = \overline{Q_i}(t)\right)$ and we keep fixed and equal all these parameters[$S_1^* = S_2^*$ and $L_1 = L_2$]. Now, the previous condition becomes

$$\frac{\left[S_1^* - \frac{L_1}{l_1(0)}\right]}{\frac{L_1}{l_1(0)}} e^{-b_1\left[\overline{Q_i}(t)\right]} \leq \frac{\left[S_{2i}^* - \frac{L_2}{l_2(0)}\right]}{\frac{L_2}{l_2(0)}} e^{-b_2\left[\overline{Q_i}(t)\right]}$$

Imposing that $\frac{1}{a} = \frac{S_i^*}{L_i}$, we reduce the previous equation to the followig form:

$$\left(\frac{l_1(0) - a}{l_2(0) - a}\right) e^{-(b_1 - b_2)\overline{Q_i}(t)} \leq 1 \qquad (14)$$

In equation (14) we specify different parameter $b_1 \neq b_2$ respectively for each district. In this equation there is clearly a trade-off between the initial conditions and the organisational systems. Starting from a disadvantageous initial condition, it is more probable that district 1 can have cumulated (at time *t)* more knowledge than district 2 if it is endowed with a better inter-firm organisation system (i.e. $b^1 > b^2$). We may deal also with some further hypothetical scenarios:

a) with $b^1 = b^2$ and $l_0^1 > l_0^2$, the catching up process can never be realised. The district, that has an initial advantage keeps it

b) with $b^1 > b^2$ and $l_0^1 > l_0^2$, there is both a catching up and a leapfrogging process by district 1 over district 2. Indeed, district 1 begins with a disadvantage, but it has a better structure that allows it to cover this gap

c) with $b^1 < b^2$ and $l_0^1 > l_0^2$ the catching up never happens.

With the same criteria, we may describe the dynamics considering an equal level of initial labour coefficient :

d) with $b^1 = b^2$ and $l_0^1 = l_0^2$ everything is fixed and stable.

e) with $b^1 > b^2$ and $l_0^1 = l_0^2$ district 1 cumulates knowledge faster than district 2

f) with $b^1 < b^2$ and $l_0^1 = l_0^2$ district 2 cumulates knowledge faster than district 1

The final conclusion that we can derive from this first comparative analysis is the turnover dynamics that characterizes the evolutionary path of districts.

Indeed, the centripetal (agglomerating) force of a district lies principally on its efficiency that represents the real comparative advantage for production at firm level.

Observation 2:

In a district, the stock of knowledge is fed by a coordinating structure among firms. This structure may also be able to fill up the competitivity lag of a district in comparison to another one.

Supposing, now, that $\left(\overline{Q_1}(t) \neq \overline{Q_2}(t) \neq \overline{Q_i}(t)\right)$. Equation (14) becomes:

$$\left(\frac{l_1(0) - a}{l_2(0) - a}\right) e^{-\left(b_1\overline{Q_1}(t) - b_2\overline{Q_2}(t)\right)} \leq 1 \tag{15}$$

so that the analysis has to take into account another parameter, making the comparison more complex and uncertain.

In general, if we admit that

$$b_1 > b_2 \text{ and } \overline{Q_1}(t) > \overline{Q_2}(t)$$

and

$$b_1 < b_2 \text{ and } \overline{Q_1}(t) < \overline{Q_2}(t)$$

we obtain the same outcomes as we detected in the previous six situations. On the contrary, supposing that

$$b_1 > b_2 \text{ and } \overline{Q_1}(t) < \overline{Q_2}(t)$$

and

$$b_1 < b_2 \text{ and } \overline{Q_1}(t) > \overline{Q_2}(t)$$

the relative outcomes for the same six previous situations are uncertain, since they depend on the reciprocal interactions of the values of all parameters (even if it could not be so reasonable to think that the less efficient district displays higher level of production).

Knowing these results we will pass now to detect the conditions for which a district becomes more attractive than another one.

3.4 The agglomeration forces of districts

In equation (5) we stated the relation between the number of firms, of workers and their productivity in a district. By the same equation, fixing the total number of workers and the level of per-firm production, we derived that the number of firms in a district is inversely proportional to the productivity of workers.

Analytically, we express this situation for districts 1 and 2 at time t as follows:

$$\frac{n_2(t)}{n_1(t)} = \frac{l_1[S_1(t)]}{l_2[S_2(t)]} \tag{16}$$

As proved in the previous subsection, the initial advantage in terms of labour productivity is not sufficient for assuring a constant advantage of a district. There is a multiplicity of factors and situations that make the analysis quite complex.

We affirm that (coherently) firms decide to migrate in the more profitable district.

This means that a progressive concentration of firms in district 1 arrives when :

$$\frac{n_2(t)}{n_1(t)} > \frac{n_2(t+1)}{n_1(t+1)}, \text{ which implies } \frac{l_1[S_1(t-1)]}{l_2[S_2(t-1)]} > \frac{l_1[S_1(t)]}{l_2[S_2(t)]} \tag{17}$$

We know that l is a decreasing function of S, and if we make the further assumption that there is a *linear dependence* between l *and* S, the necessary and sufficient condition for firms to agglomerate in district 1 becomes:

$$\frac{S_1(t-1)}{S_2(t-1)} < \frac{S_1(t)}{S_2(t)}$$

We replace this last expression with the proper functions, to obtain

$$\left(\frac{1+\frac{\left[S_1^*-\frac{L}{l_1(0)}\right]}{\frac{L}{l_1(0)}}e^{-b_1\left[\overline{Q_1}(t-1)\right]}}{1+\frac{\left[S_1^*-\frac{L}{l_1(0)}\right]}{\frac{L}{l_1(0)}}e^{-b_1\left[\overline{Q_1}(t)\right]}}\right) > \left(\frac{1+\frac{\left[S_2^*-\frac{L}{l_2(0)}\right]}{\frac{L}{l_2(0)}}e^{-b_2\left[\overline{Q_2}(t-1)\right]}}{1+\frac{\left[S_{21}^*-\frac{L}{l_2(0)}\right]}{\frac{L}{l_2(0)}}e^{-b_2\left[\overline{Q_2}(t)\right]}}\right) \tag{18}$$

After some mathematical manipulations (reported in appendix B), supposing that for both the districts $\overline{Q_1}(t-1) = \overline{Q_2}(t-1) = \overline{Q_i}(t-1)$ and $Q_1(t-1,t) = Q_2(t-1,t) = Q_i(t-1,t)$ and we reduce expression (17) to the following form :

$$e^{(b_1-b_2)\overline{Q}_i(t-1,t)}\left[\frac{\frac{a}{(l_1(0)-a)e^{-b_1\overline{Q_i}(t-1)}}+1}{\frac{a}{(l_2(0)-a)e^{-b_2\overline{Q_i}(t-1)}}+1}\right] > 1. \tag{19}$$

In this equation, the general trade-off between initial conditions and organisational structure is present.

We have different hypothesis to consider for evaluating a possible concentration of firms in district 1:

a) with $b_1 = b_2$ and $l_0^1 = l_0^2$ there is no agglomeration process. Firms are equally distributed across the two districts;

b) with $b_1 > b_2$ and $l_0^1 = l_0^2$ there exists a concentration process of firms in district 1. Firms are indeed attracted by the most efficient district;

c) with $b_1 > b_2$ and $l_0^1 > l_0^2$ there still exists a concentration process in district 1. In fact, this is the case that has been described in the previous subsection as a 'leapfrogging' case. District 1 has an initial disadvantage in comparison with district 2, but it is better organized and it attracts firms more than the other one;

d) with $b_1 < b_2$ and $l_0^1 = l_0^2$ there is an agglomeration in district 2, because of the less efficient organisation structure of district 1 (at the same initial conditions);

e) with $b_1 < b_2$ and $l_0^1 < l_0^2$ there is always concentration in district 2 (with the same reasons as in point d);

f) with $b_1 < b_2$ and $l_0^1 > l_0^2$ we are in an apparent uncertain case. We are in a situation in which both the terms of equation (18) are less than 1. This means that we are in a situation of agglomeration in district 2. Economically it depends on the initial advantage of this district and, at the same time, on the quality of its internal organisation;

g) with $b_1 = b_2$ and $l_0^1 > l_0^2$ there is an original outcome. The inter-firm organisation is the same in the two districts, but district 1 has a lower labour efficiency in the initial period. In this case, equation (18) states that we have a convergence of firms in district 2. Why ? From equation (7) we state that a lower labour coefficient *makes* firms pay lower wages. But in industrial districts, firms are more interested in the quality (i.e. skillness expressed by efficiency) of the labour force. So, setting in locations with more efficient labour force (at initial level) is more attractive.

Proposition 2:

A district attracts firms if:

- *its coordination system is more efficient than another one, even with a disadvantageous initial level of labour coefficient.*

- *the coordinating system is equivalent in the two districts, but one of them has a more skilled initial labour force.*

These results have been obtained considering the same productive capacity at firm level. It is a simplification, but it does not discredit results. Supposing a difference between them, we may derive that some conditions, for a concentration in one of the two districts are reinforced (for example when the most efficient firms belong to the most efficient structure), while the outcomes in other cases become less evident. However, at qualitative level, the general features are unchanged. The same kind of objection should be also applied to the variable that represents the per-firm level of quantity. For analyzing the agglomeration forces we have supposed that it is equal in both the district. It should be possible integrate our results considering a different level of total quantities at time t. One option might be to include the opportunity that the most efficient district provides higher per-firm quantities at time t, developing a specific functional form between the level of knowledge S and the labour coefficient l.

4 Conclusions

The focal results of this analysis stand around the conditions for which we may have agglomeration in a district. The initial situation is not so essential for imprinting the evolutionary path of a district. The most important component is the inter-firm organisation structure; it is the real centripetal force toward an agglomeration in a specific location. This kind of results is quite coherent with stylized facts.

The coordination component offers useful suggestions for defining the conditions of the rise and the fall in dimensions (and also importance) of different industrial districts. Our last outcomes give us the possibility to sketch some hints at this purpose.

The inner centripetal forces of a district find their principal strenght in the ability of co-ordinating activities among firms. The real dynamism of a district has to be detected by its capacity of adapting that advantage along all the paths of its evolution. Indeed the 'economic' decadence of some districts is signed by a default in co-ordination. For this purpose, more hints might be also obtained evaluating b_i as a direct function of the number of firms and so making endogenous the co-ordination structure of the model. Useful results could also be derived by relaxing the fixity of the number of workers, introducing a migration process.

At this point, the next (and probably natural) stage in the evolution of this kind of analysis would be mergering together the inner and external centripetal forces as sources of the concentration of activities. Through this specific formalisation we might obtain a joint description of all possible determinants that influence location decisions and agglomeration rise.

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5 Appendices

A) CUMULATED KNOW-HOW.

Giving equation (11), for deriving the cumulated level of know-how over a period $(t_0 = 0, T)$ we procede by the following device. At time T , equation (8) states that the cumulated quantity of good produced in the district is equal to $\overline{Q_i}(T)$ and so :

$$dS_i(t) - \left(b_i S_i(t) - \frac{b_i [S_i(t)]^2}{S_i^*}\right) dQ_i(t) = 0$$

$$\int_0^T \frac{S_i^*}{S_i^* S_i(t) b_i - b_i [S_i(t)]^2} dS_i(t) = \int_0^T dQ_i(t) = \overline{Q_i}(T)$$

We solve the LHS of this last equation applying a standard device of the integral solutions :

$$\frac{S_i^*}{b_i} \int_0^T \frac{dS_i(t)}{S_i(t)[S_i^* - S_i(t)]} = \frac{S_i^*}{b_i} \int_0^T \frac{1}{S_i^*} \left[\frac{1}{S_i(t)} + \frac{1}{S_i^* - S_i(t)}\right] dS_i(t) = \overline{Q_i}(T)$$

$$[\ln(S_i(t)) - \ln(S_i^* - S_i(t))]_0^T = b_i \overline{Q_i}(T)$$

$$\ln \frac{\frac{S_i(t)}{S_i^* - S_i(t)}}{\frac{S_i(0)}{S_i^* - S_i(0)}} = b_i \overline{Q_i}(T)$$

$$S_i(t) = \frac{S_i^*}{1 + \left(\frac{S_i^* - S_i(0)}{S_i(0)}\right) e^{-b_i \left[\overline{Q_i}(T)\right]}}$$

We know that the lowest level of knowledge is correlated with the initial labour coefficient and it influences the level of production. By these conditions and by equation (8), assuming that :

$$S_i\,(0) = Q_i\,(0) = \frac{L}{l_i\,(0)} \tag{A.1}$$

and substituing this expression in the previous equation we obtain equation (12), i.e.

$$S_i\,(t) = \frac{S_i^*}{1 + \frac{\left[S_i^* - \frac{L}{l_i(0)}\right]}{\frac{L}{l_i(0)}} e^{-b_i\left[\overline{Q_i}(t)\right]}}$$

B) AGGLOMERATION CONDITIONS

Through mathematical computations we transform equation (17) in the following one (always considering that $a = \frac{L}{S^*}$)

$$\frac{1 + \left(\frac{l_1(0)}{a} - 1\right) e^{-b_1 Q_i(t-1)}}{1 + \left(\frac{l_1(0)}{a} - 1\right) e^{-b_1 Q_i(t)}} \geq \frac{1 + \left(\frac{l_2(0)}{a} - 1\right) e^{-b_2 Q_i(t-1)}}{1 + \left(\frac{l_2(0)}{a} - 1\right) e^{-b_2 Q_i(t)}} \tag{B.1}$$

By equations (9) and (10) and assuming that (in both the districts):

$$Q\,(t) = Q\,(t-1) + \int_{t-1}^{t} Q\,(s)\,ds = Q\,(t-1) + Q\,(t-1, t)$$

we derive from equation (B.1) the following condition:

$$\frac{e^{-b_2 Q_i(t-1,t)}}{\left(\frac{l_1(0)}{a} - 1\right) e^{-b_1 Q_i(t-1)}} + \frac{1}{\left(\frac{l_2(0)}{a} - 1\right) e^{-b_2 Q_i(t-1)}} + e^{-b_2 Q_i(t-1,t)} \geq$$

$$\frac{e^{-b_1 Q_i(t-1,t)}}{\left(\frac{l_2(0)}{a} - 1\right) e^{-b_2 Q_i(t-1)}} + \frac{1}{\left(\frac{l_1(0)}{a} - 1\right) e^{-b_1 Q_i(t-1)}} + e^{-b_1 Qi(t-1,t)}$$

Simplifying the previous expression, we transform it in the following condition:

$$e^{(b_1 - b_2)\overline{Q_i}(t-1,t)} \left[\frac{\frac{a}{(l_1(0)-a)e^{-b_1\overline{Q_i}(t-1)}} + 1}{\frac{a}{(l_2(0)-a)e^{-b_2\overline{Q}_i(t-1)}} + 1}\right] > 1.$$