Money Injections in a Neoclassical Growth Model

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Abstract

This paper analyzes the effects and transmission mechanism related to the alternative injection channels - i.e to households versus a financial intermediary - in a neoclassical growth model with reserve requirements and money multiplier effects. The money injected directly to a financial intermediaries is not subject to reserve requirements while deposits are. As suggested in Fuerst [1994], we show that it does matter what injection channel is used as long as reserve requirements on saving deposits are nonzero. However, it matters only for a scale factor and that the transmission mechanism of money are identical. There are no additional tax avoidance effects that would stimulate intermediation when money is injected directly to the financial intermediary.

The model allows for the definition of a set of monetary aggregates, from the most narrow (nonborrowed reserves) to the largest (M1). There is therefore a potential room to understand why different aggregates display different cyclical pattern.

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Introduction

Recently, a number of articles have emphasized the failures of existing monetary business cycle models to account for statistical properties of nominal variables[1]. The main remaining puzzles refer to a broader set of nominal stylized facts than considered in the pioneering articles like Stockman [1981] or Cooley and Hansen [1989]. To analyze the effects of monetary actions, Eichenbaum [1992], Strongin [1995] and Chari, Christiano and Eichenbaum [1995] discuss the importance of considering different monetary aggregates. Only narrow aggregates - i.e. non-borrowed reserves are dominated by exogenous monetary policy shocks while movements in broader aggregates - i.e. $M1$ - are dominated by endogenous response to nonpolicy shocks.

To be able to analyze a broader set of nominal stylized facts and account for both exogenous and endogenous movements in money aggregates, one need to introduce financial intermediation. Before building a complex model, it is worth studying the consequences of introducing a simple financial structure with reserve requirements and money multiplier effects in the basic cash–in–advance model. We use the financial intermediation framework proposed by Fuerst [1994] but put it into a standard neoclassical growth model[2].

The central issue in this paper is the analysis of the effects and transmission mechanism related to the alternative injection channels in a neoclassical growth model with reserve requirements and money multiplier effects. The money injected to banks is not subject to reserve requirements while both savings and checking accounts are. As suggested in Fuerst [1994], it does matter what injection channel is used as long as reserve requirements on saving deposits are nonzero. We show that it matters only for a scale factor and that the transmission mechanisms of money are identical. There are no additional tax avoidance effects that would stimulate intermediation when money is injected directly to the financial intermediary.

Therefore, introducing a financial intermediary and reserve requirements in such a model matters only for monetary national accounting: it allows for the definition of a set of monetary aggregates, from the most narrow (nonborrowed reserves) to the largest (M1).

The paper is organized as follows: the first section presents the basic model and the competitive equilibrium. The analysis of the effects and transmission mechanism of the monetary injections to bank versus households is presented in section 2. Finally, a few conclusive remarks are collected in section 3.

[1] For example King and Watson [1996] and Christiano, Eichenbaum and Evans [1997].

[2] Fuerst [1994] presents a model that departs somewhat from the neoclassical growth framework but uses this particular financial framework.

1 A Neoclassical Growth Model with Financial Intermediation

There are three representative agents in this economy: a households, a firm and a bank, each one acting as an atomistic competitor. Given this assumption, we will present each agent in turn. The model's notations are kept as close as possible to Fuerst [1994].

1.1 Representative Household

The representative household maximizes the following utility criteria:

$$U_0 \quad = \quad E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (\log(C_t) + v(\ell_t)) \right\} \tag{1.1.1}$$

where $\beta_t \in]0,1[$, C_t is commodity consumption, ℓ_t is leisure and v is a strictly increasing concave function. The household enters the period with a predetermined level of money balances M_t and saving account N_t. The interest paid on this account between period $t-1$ and t is denoted i_t^s. The household is constrained on its consumption goods expenditures by the existing money balances at the beginning of the period:

$$C_t \leq \frac{M_t}{P_t} \tag{1.1.2}$$

During the period, the household consumes $P_t C_t$, receives labor income $W_t H_t$, savings N_t with interest payments $i_t^s N_t$, profits from firms and banks Π_t^f and Π_t^b. In addition, these balances are augmented with a lump sum transfer equal to Ω_t (money injection to households). At the end of the period, the household chooses how much resources to transfer via cash (M_{t+1}) or via the saving account (N_{t+1}). The representative household's budget constraint at period t is

$$M_{t+1} + N_{t+1} \leq M_t - P_t C_t + (1 + i_t^s) N_t + W_t H_t + \Pi_t^f + \Pi_t^b + \Omega_t \tag{1.1.3}$$

Finally, the household has a unit of time endowment that it allocates between leisure, ℓ_t and working time H_t:

$$\ell_t + H_t = 1 \tag{1.1.4}$$

The household maximizes (1.1.1) subject to (1.1.2), (1.1.3) and (1.1.4). The optimal behavior of households is given by a set of optimality conditions that is presented in the appendix.

1.2 Representative Firm

The homogeneous good, accumulated and consumed, is produced according to the following production function:

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}, \qquad \alpha \in]0,1[\tag{1.2.1}$$

where K_t, H_t denote respectively private capital and hours used in the production process. A_t represents total factor productivity. $\log(A_t)$ is supposed to follow a first order autoregressive stationary process :

$$\log(A_t) = \rho_a \log(A_{t-1}) + (1-\rho_a)\log(\overline{\mathrm{A}}) + \varepsilon_{a,t} \tag{1.2.2}$$

with $-1 < \rho_a < 1$, and $E(\varepsilon_{a,t}) = 0$ and $E(\varepsilon_{a,t}^2) = \sigma_a^2$. $\log(\overline{\mathrm{A}})$ denotes the unconditional mean of the process.

The representative firm maximizes the discounted flow of expected profits:

$$E_0\left\{\sum_{t=0}^{\infty}(\rho_0\cdots\rho_t)\Pi_t^f\right\} \tag{1.2.3}$$

under the linear capital accumulation rule:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{1.2.4}$$

where $0 < \delta < 1$ denotes the depreciation rate of capital and ρ_t the firm discount factor between period $t-1$ and t. We assume that ρ_t is such that the firm maximizes the intertemporal utility of its shareholder (the representative household)[3]

The firm owns its capital, and we assume that it must finance its investment purchases by contracting a debt at the bank. No self-financing is allowed - i.e. all profits must be distributed to households. The debt contract is written as follows. During period t, the firm subscribes a one period debt D_{t+1}, at interest R_{t+1} between t and $t+1$. This borrowed amount is put on check account at the bank, for a level $d_{t+1}^c = D_{t+1}$, that bears an interest i_{t+1}^c. The firm's net cost of funds is $(R_{t+1} - i_{t+1}^c)$. The firm uses its check account to buy investment[4]:

$$P_t I_t = d_{t+1}^c \tag{1.2.5}$$

It is assumed that all the checks signed in period t will be paid by the bank at the beginning of the period $t+1$[5]. The money cash flow of the firm is therefore limited to goods that were sold for consumption purposes, and that have been paid with money. This cash flow is used to pay wages, to reimburse the debt the firm has contracted in $t-1$ an to distribute profits.

In period $t+1$, the firm will receive the money from the check that it has (including an interest payment) and will reimburse its debt to the bank.

[3]One will then have at the equilibrium $\rho_t = \frac{1}{(1+i_{t+1}^s)}$

[4]Englund and Svensson [1988] and Hartley [1988] also consider a cash–in–advance economy with checking accounts but these are used by households to buy a subset of their consumption goods. We focus on the intermediation service of banks.

[5]There is an apparent asymmetry between selling one unit of good to a consumer or to an investor, since the former pays in cash and the latter in checks that will be paid one period later. At the equilibrium, prices and interest rates will be such that firms are indifferent in selling one unit of good to a consumer or to an investor.

The representative firm's profit is given by:

$$\Pi_t^f = P_tC_t + P_tI_t + d_{t+1}^c - (W_tH_t + (R_t - i_t^c)D_t + P_tI_t + D_{t+1}) \qquad (1.2.6)$$

As $D_{t+1} = d_t^c = P_tI_t$, one gets

$$\Pi_t^f = P_tC_t - (R_t - i_t^c)D_t - W_tH_t \qquad (1.2.7)$$

The firm maximizes (1.2.3) subject to (1.2.4), (1.2.5) and (1.2.6). The optimal behavior of firm is then given by a set of optimality conditions that is presented in the appendix.

1.3 Representative Bank

During period t, the bank collects savings accounts (N_{t+1}) and checking accounts (d_{t+1}^c) and uses those to provide loans to firms. The monetary authorities lend money, X_t, to the bank by purchasing a one-period bond, B_t issued by the bank at zero interest rate[6]. Along with determining the monetary growth rate, the monetary authority regulates the bank, as it requires it to hold currency reserves in proportion to its deposits. We will denote by ρ^c and ρ^s the fractional reserve requirements on checking and saving accounts, respectively.

During period t, the bank's actions are the following: *(i)* Begin period with $\rho^cD_t+\rho^sN_t$, the reserves corresponding to the credit of period $t-1$; *(ii)* Receive $(R_t - i_t^c)D_t$ from the firm; *(iii)* Pay back $(1 + i_t^s)N_t$ to the household; *(iv)* Pay back the monetary injection of the past period $X_{t-1} = B_{t-1}$; *(v)* Distribute π_t^b to the household; *(vi)* Get X_t, the money injection from the central bank and N_{t+1} from the household; *(vii)* Lend D_{t+1} to the firm and put it on the check account, d_{t+1}^c; *(viii)* Keep $\rho^cD_{t+1} + \rho^sN_{t+1}$ as reserves corresponding to the new credit.

The bank's profit is defined in the following way:

$$\Pi_t^b = (1 + R_t)X_{t-1} - B_{t-1} + (1 - \rho^c)(1 + R_t)d_t^c + \rho^cd_t^c + (1 - \rho^s)(1 + R_t)N_t$$
$$+\rho^sN_t - (1 + i_t^c)d_t^c - N_t(1 + i_t^s)$$

Rearranging the terms we get

$$\Pi_t^b = [(1 - \rho^c)R_t - i_t^c]d_t^c + [(1 - \rho^s)R_t - i_t^s]N_t + R_tX_{t-1} \qquad (1.3.1)$$

The bank's balance sheet at period t can be written as follows:

[6]This assumption is made for simplicity but we could assume that this rate is positive.

Assets	Liabilities
D_{t+1}	N_{t+1}
	d^c_{t+1}
$\rho^c d^c_{t+1} + \rho^s N_{t+1}$	B_t

Let us notice here that given the linearity of bank profits, the following no-arbitrage relations will hold at the equilibrium:

$$\begin{aligned} [(1-\rho^c)R_t - i^c_t] &= 0 \\ [(1-\rho^s)R_t - i^s_t] &= 0 \end{aligned}$$

1.4 Money Multiplier

The bank has reserve requirements on its saving and check deposits, respectively, ρ^s and ρ^c. So for every dollar that is deposited on the saving account, it can lend $\rho^s N_{t+1}$ to firms (the same is true for the check deposits). There are no reserve requirements for the outside money, X_t, that is for the money injected by the monetary authorities to the financial intermediary. If the bank would only get X_t, it would lend $D_{t+1} = X_t$ to the firm and the firm would put it on the check account. So the bank could again lend $(1-\rho^c)\ d^c_t$ to the firm etc... The total amount the bank would be permitted to lend given X_t would be given by $X_t + (1-\rho^c)X_t + ((1-\rho^c))^2 X_t + ... = \frac{X_t}{\rho^c}$ The same is true for household's deposits, N_{t+1}, so we have the following general expression:

$$D_{t+1} = \frac{X_t}{\rho^c} + \frac{(1-\rho^s)N_{t+1}}{\rho^c}$$

1.5 Money Supply and Monetary Aggregates

We assume the monetary authorities have two possible injection channels: (i) one through the households (Ω_t) and (ii) one through the financial intermediary (X_t).

The money injected to the financial intermediary is assumed grow at a the rate $1-\phi_t$:

$$X_{t+1} = \phi_t\ X_t \tag{1.5.1}$$

ϕ_t is supposed to follow an exogenous stochastic process of the form

$$\log\ \phi_t = \rho_\phi \log\ \phi_{t-1} + (1-\rho_\phi)\log\bar{\phi} + \epsilon_{\phi,t} \tag{1.5.2}$$

The money injected directly to the households is assumed to grow at a the rate $1-\omega_t$

$$\Omega_{t+1} = \omega_t\ \Omega_t \tag{1.5.3}$$

ω_t is supposed to follow an exogenous stochastic process of the form:

$$\log \omega_t = \rho_\omega \log \omega_{t-1} + (1-\rho_\omega)\log \bar{\omega} + \epsilon_{\omega,t} \tag{1.5.4}$$

where both monetary shocks $\epsilon_{\phi,t}$ and $\epsilon_{\omega,t}$ are gaussian white noise with zero mean and variance σ_ϕ^2 and σ_ω^2. Finally, $\log(\phi)$ and $\log(\omega)$ are stationary process, $\mid \rho_\phi \mid < 1$ and $\mid \rho_\omega \mid < 1$

The following monetary aggregates can be defined in the model: nonborrowed reserves (X_t), total reserves ($N_{t+1}+X_t$), monetary base ($M_{t+1}+N_{t+1}+X_t$), M1 ($M_{t+1}+N_{t+1}+d^c_{t+1}$). Total reserves can be divided into borrowed and nonborrowed reserves. Following Strongin [1995], nonborrowed reserves are the *"policy-induced supply innovation"*, while borrowed reserves are due to accommodation of innovations in the demand for reserves. In our model, the *"policy-induced supply innovation"* would be X_t. The borrowed reserves would then be represented by N_{t+1}, the saving deposits[7].

1.6 Competitive Equilibrium and Steady State

The competitive equilibrium of the economy is defined by the first-order conditions of the household, firm and bank programs, and by market equilibrium equations for the good, credit, labor and money market $f_t = \frac{P_t}{P_{t-1}}$ will denote the inflation factor.

We assume that the steady state growth rate of the two money injections are equal ($\phi = \omega$), so that a balanced growth path exists for nominal variables. The steady state inflation rate is thus $f = \phi = \omega$. The inflation tax arises from the assumption that currency pays a zero nominal return and therefore the real return being minus the inflation rate. Higher inflation will lower this real return and thus discourage activities that require cash - i.e. consumption. In our model, as in Fuerst (1994), higher inflation will also affect firm's net cost of borrowing and discourage intermediation. This is true because the competitive bank system will pass that inflation tax onto the firm and the household in the form of a positive spread between the loan rate (R) and the rate paid on checking (i^c) and savings (i^s) deposits:

$$(R - i^c) = \frac{\rho^c}{(1-\rho^s)}\left(\frac{f-\beta}{\beta}\right)$$

$$(R - i^s) = \frac{\rho^s}{(1-\rho^s)}\left(\frac{f-\beta}{\beta}\right)$$

Note that if $\rho^c = 0$, the money multiplier is infinite and inputs are a pure credit good. The checking deposits spread, and thus this inflation tax disappears. ρ^c will not affect the spread on saving deposits. Eliminating the reserve requirements on savings will eliminate

[7] Chari et al. [1995] assume that the growth rate of their money base consist of two components: one purely exogenous (that follows the same stochastic process as in our model) and one that is a function of time t innovations to the economy (past and current productivity shocks).

the tax on intermediation but the tax on transaction remains. Higher money growth rates will increase both spreads by increasing the steady state inflation rate. As in Cooley and Hansen [1989], higher steady state inflation rates will imply a lower level of steady state capital and output. Higher reserve requirements will increase the check account spread and thus imply a lower steady state level of capital and output.

1.7 Calibration

No closed form solution of the model can be computed, and we will use a log-linear approximation around its steady-state to solve it. To get some quantitative results, the model is calibrated according to the monetary business cycle literature for the U.S., on a quarterly basis (see table 1)

Table 1: Calibration

α	δ	β	ω	ϕ	H
.42	.0125	.988	1.014	1.014	.2
ρ_c	ρ_s	ρ_a	ρ_ϕ	ρ_ω	
.2	0 or .2	.95	.377	.377	

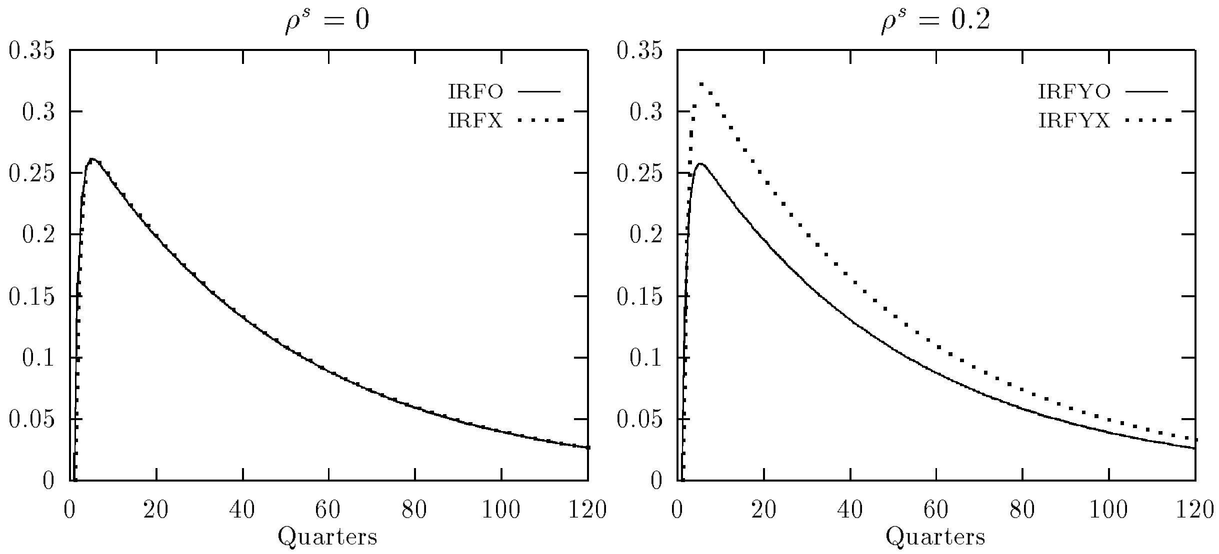
We consider two cases for the disutility of labor function v: an inelastic supply case ($v(1-H) = 0\ \forall H$) and an elastic case ($v(\cdot) = \log(\cdot)$).

2 Does the Way Money is Injected Matter?

In this section, we show that no specific propagation mechanism is added by the introduction of financial intermediaries and reserve requirement in the way that is done in this paper, as an extension of the Fuerst [1994] setup. For that purpose, we focus on the response of output to a money supply shock, injected to banks or to households. We have normalized the steady state ratio $\frac{X}{(M+N)}$ to 1%. Therefore, the "one dollar injection" that we will consider corresponds to a 1% deviation of the bank injection growth rate and to a .01% of the household injection growth rate from their respective steady state levels. We are not interested with the absolute levels of the responses but in their relative size, whether a given dollar is injected though households or banks. Let us first consider the model with inelastic labor supply.

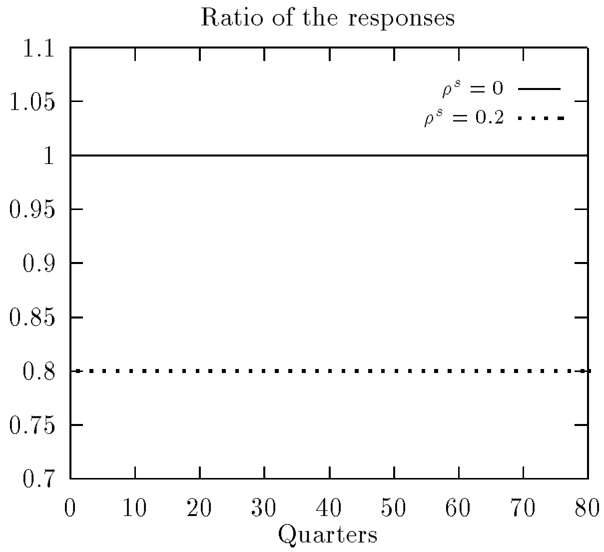
As indicated by Fuerst [1994], the stimulus that the monetary injection has on real activity depends on the reserve requirement that the intermediary face: as soon as ρ^s is positive, a one dollar injection to the bank will have a larger effect that a one dollar injection to households. Does this mean that with reserve requirement, a model with financial intermediaries gives a new propagation channel to a non-borrowed reserve shock?

Figure 1: Response of Output with Inelastic Labor Supply, one dollar injection



The answer is clearly no. This effect is a pure scale effect related to the money multiplier. Given the equation of the money multiplier, ‘ it is easy to see that a one dollar in injection to the bank becomes $\frac{x_t}{\rho^c}$, whereas the same injection to the household becomes $\frac{(1-\rho^s)n_{t+1}}{\rho^c}$. Thus, there will be a scale effect as soon as, $\rho^s > 0$ (see figure 3.1).

Figure 2: Ratio of the two Responses, Inelastic Labor supply Case



To make it fully clear that it is only this scale effect that differs during all the transition, we plot on figure 2 the ratio of response to a monetary shock to households and to banks with different reserve requirements. The responses to a bank or household injection are just homothetic, the ratio bank/household being greater that one as soon as ρ^s is positive.

With elastic labor supply (figure 3), we are back to the Cooley and Hansen [1989] model, and the same scale effect explains the differences in the responses of output to a monetary injection to households or banks, as soon as ρ^s is different from zero.

Figure 3: Response of Output with Elastic Labor Supply, one dollar injection

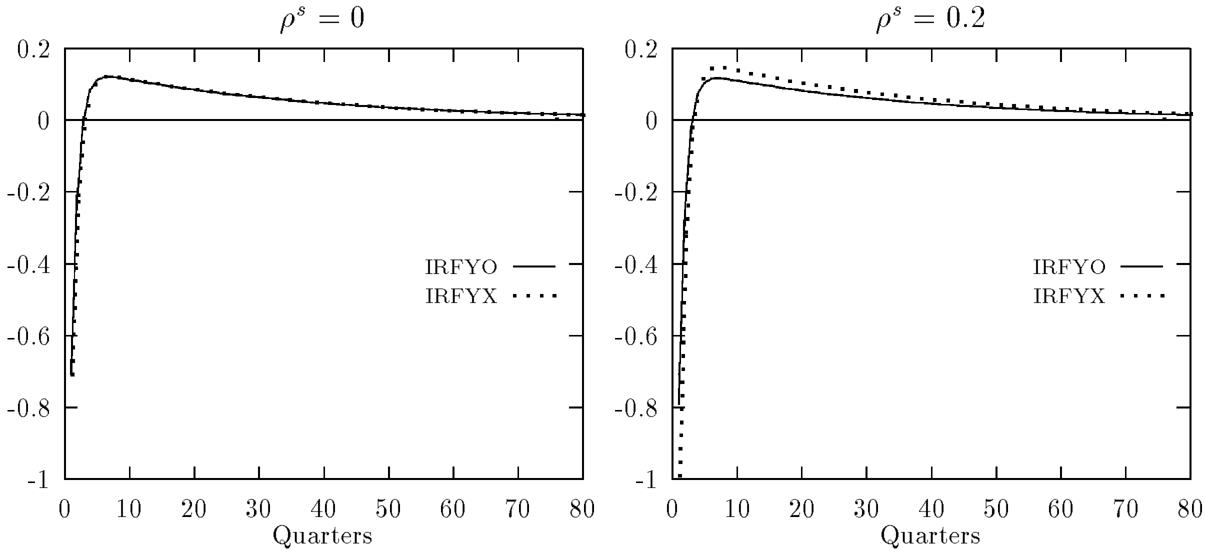
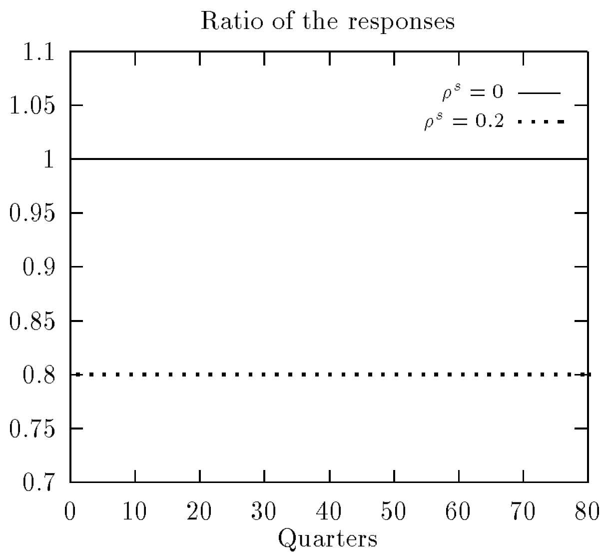


Figure 4: Ratio of the two Responses, Elastic Labor supply Case



The positive response of output that we found in the previous model, is purely related to the assumption of the absence of intertemporal labor substitution, as already documented in Hairault and Portier [1995]. Thus, in the neoclassical growth model with the most simple Fuerst [1994] type of financial environment, the response of real output to a monetary shock is negative, whatever the injection channel used. The model display interesting features such as endogeneity of M1, following a technological shock, but no specific propagation mechanisms are added.

3 Concluding remarks

In this paper we have built a most simple neoclassical growth model with financial intermediation and reserve requirements. We used this framework to study whether the way money injections make their way into the economy matters, when injections to the bank are not subject to reserve requirements. As suggested by Fuerst [1994], we show that the stimulus that the monetary injection has on real activity depends on the reserve requirement that the intermediary face. But this effect is a pure scale effect related to the money multiplier and the ratio of the response of output for the two type of injection remains constant. It does matter how money is injected in the economy, but only for the amplitude of the effect (via a very simple multiplier effect) and not for the transmission mechanism itself. There are no additional tax avoidance effects that would stimulate intermediation when money is injected directly to the financial intermediary.

The potential interest of such models, as already illustrated by Chari et al. [1995], is to allow for the definition of different monetary aggregate, whose cyclical behavior was shown to be different, for example with respect to the short-term interest rates. What our analysis suggest is that one can easily extend the benchmark cash-in-advance model in order to model a broad set of monetary aggregates, but that we need to think more deeply about what a financial intermediary does to get any new transmission mechanisms of monetary supply shocks.

4 Appendix

4.1 Household

This section presents the first-order conditions of the household problem. λ_t and μ_t are the Lagrangian multipliers associated to the intertemporal budget constraint and the cash-in-advance constraint respectively.

$$U_C(t) = (\lambda_t + \mu_t)\ P_t \tag{4.1.1}$$

$$U_\ell(t) = \lambda_t \frac{-W_t}{P_t} \tag{4.1.2}$$

$$\lambda_t = \beta E_t\left[(\lambda_{t+1} + \mu_{t+1})\right] \tag{4.1.3}$$

$$\lambda_t = \beta E_t[\lambda_{t+1}(1 + i^s_{t+1})] \tag{4.1.4}$$

$$\lim_{i\to\infty} E_t\left\{\beta^{t+i}\lambda_{t+i}N_{t+1+i}\right\} = 0 \tag{4.1.5}$$

$$\lim_{i\to\infty} E_t\left\{\beta^{t+i}\mu_{t+i}M_{t+1+i}\right\} = 0 \tag{4.1.6}$$

Equation (4.1.1) and (4.1.2) imply the equalization of the marginal utility of consumption of the good and of leisure respectively to their anticipated and discounted opportunity costs in terms of utility.

Equation (4.1.4) defines the anticipated discounted value of revenue when it is transferred via savings. It depends on tomorrow return on the saving account and also on the value of one unit of revenue (or savings) the period thereafter.

Equation (4.1.3) defines that the anticipated discounted value of revenue (shadow price) when it is transferred via cash. Money does not give any direct return in the next period but it reduces the cost of bearing a cash constraint (μ_t) in period t+1. Taking equation (4.1.3) and (4.1.4) together indicates that the household is indifferent at the optimum, about the way it transfers revenue into the future.

4.2 Firm

This section presents the first-order conditions of the firm problem. We will denote Λ_t, the multiplier associated to the capital accumulation constraint.

$$F_h(t) = \frac{W_t}{P_t} \tag{4.2.1}$$

$$\rho_t E_t[\frac{P_{t+1}}{P_t}(\alpha\frac{Y_{t+1}}{K_{t+1}} + \frac{\Lambda_{t+1}}{P_{t+1}}(1-\delta))] = \frac{\Lambda_t}{P_t} \tag{4.2.2}$$

$$1 + \rho_t E_t[(R_{t+1} - i^c_{t+1})] = \frac{\Lambda_t}{P_t} \tag{4.2.3}$$

$$\lim_{i\to\infty} E_t \left\{\beta^{t+i}\Lambda_{t+i}K_{t+1+i}\right\} = 0 \tag{4.2.4}$$

Equation (4.2.1) is the standard condition for hiring labor. Equation (4.2.2) indicates that the anticipated discounted value of capital tomorrow depends on tomorrow's productivity of capital (net of depreciation) and also on the value of capital the period thereafter. Note that the equation is in real terms and that Λ represents the nominal value of capital. Inflation appears on the left side of the equation as an opportunity gain. The intuition is that if the firm does not invest one more unit and transfers it as profits distributed to households, this value will be reduced by the inflation tax just as for the transfer via cash for the households (recall that dividends can only be distributed the next period). This equation simply states that at the optimum, the firm that maximizes the discounted expected flow of profits is indifferent between distributing one more unit of profit or invest one more unit. The equation also states that the value of one unit of capital today equal the expected discounted marginal revenue of investing today. Equation (4.2.3) states that at equilibrium, the value of one unit of capital today equals the expected discounted marginal cost of investing today. The marginal cost is, in this case, the net cost of borrowing to the bank. To invest one unit today you need to repay $1 + (R_t - i_t^c)$ tomorrow. Taking equation (4.2.2) and (4.2.3) together, indicates that it is optimal to invest until the expected discounted marginal revenue equals the marginal cost of investing one unit today. Finally, equation 4.2.4 provide the terminal condition for the evolution of capital.

4.3 Competitive Equilibrium

In the following, since monetary aggregates grow, nominal variables will deflated by P_{t-1}[8]. We also define $\Lambda_t^\star = \frac{\Lambda_t}{P_t}$.

$$Y_t = C_t + I_t \tag{4.3.1}$$

$$[(1-\rho^c)R_t - i_t^c] = 0 \tag{4.3.2}$$

$$[(1-\rho^s)R_t - i_t^s] = 0 \tag{4.3.3}$$

$$U_C(t) = (\lambda_t + \mu_t)\ P_t \tag{4.3.4}$$

[8] Let Z_t be a nominal growing variable, then we define $z_t = Z_t/(X_t P_{t-1})$.

$$U_\ell(t) = \lambda_t \frac{-W_t}{P_t} \tag{4.3.5}$$

$$\lambda_t = \beta E_t \left[(\lambda_{t+1} + \mu_{t+1}) \right] \tag{4.3.6}$$

$$\lambda_t = \beta E_t[\lambda_{t+1}(1 + i^s_{t+1})] \tag{4.3.7}$$

$$F_h(t) = \frac{W_t}{P_t} \tag{4.3.8}$$

$$\rho_t E_t[f_{t+1}\ (\alpha \frac{Y_{t+1}}{K_{t+1}} + \Lambda^\star_{t+1}(1-\delta))] = \Lambda^\star_t \tag{4.3.9}$$

$$1 + \rho_t E_t[(R_{t+1} - i^c_{t+1})] = \Lambda^\star_t \tag{4.3.10}$$

$$f(k_f, h_t, A_t) = C_t + I_t \tag{4.3.11}$$

$$C_t \leq \frac{m_t}{f_t} \tag{4.3.12}$$

$$K_{t+1} = I_t + (1-\delta)K_t \tag{4.3.13}$$

$$(m_{t+1} + n_{t+1}) = \omega_t\ (m_t + n_t)\ \frac{1}{f_t} \tag{4.3.14}$$

$$D_{t+1} = \frac{X_t}{\rho^c} + \frac{(1-\rho^s)N_{t+1}}{\rho^c} \tag{4.3.15}$$

$$\rho_t = \frac{1}{(1 + i^s_{t+1})} \tag{4.3.16}$$

$$x_t = \phi_t\ \frac{x_{t-1}}{f_t} \tag{4.3.17}$$

$$[(1-\rho^c)R_t - i^c_t] = 0 \tag{4.3.18}$$

$$[(1-\rho^s)R_t - i^s_t] = 0 \tag{4.3.19}$$

$$\lim_{i\to\infty} E_t \left\{ \beta^{t+i} \lambda_{t+i} N_{t+1+i} \right\} = 0 \tag{4.3.20}$$

$$\lim_{i\to\infty} E_t \left\{ \beta^{t+i} \mu_{t+i} M_{t+1+i} \right\} = 0 \tag{4.3.21}$$

$$\lim_{i\to\infty} E_t \left\{ \beta^{t+i} \Lambda_{t+i} K_{t+1+i} \right\} = 0 \tag{4.3.22}$$

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