# A Real Business Cycle Model of the Phillips Curve.

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**Abstract**

The aim of this article is to show that RBC models can account for the so-called Phillips curve. We propose an efficiency wage model in which money is introduced via a cash-in-advance constraint. Households choose how much effort to devote by comparing present real and nominal wages with past ones. This special intertemporal effort function implies wage sluggishness and a higher volatility of employment compared to standard RBC models. It also reduces a negative contemporaneous correlation between inflation and output, one of the more difficult moments to match for a cash-in-advance model. The model allows to match labor market moments and also to build up a transmission mechanism that affects employment through nominal wage growth. The model generates a Phillips curve that is able to mimic US data.

JEL classification: E24, E31, E32.

Keywords: efficiency wage, RBC, wage sluggishness, cash-in-advance, Phillips curve.

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# 1 Introduction

Fisher (1926) reported a significant positive correlation between employment and prices in the US Economy. The rationale is that as prices increase, firms receive more income. As their costs are fixed, they increase production and so employment, although costs will adjust after some periods and profits will return to their initial level. Phillips (1958) studied the statistical relationship between unemployment and the rate of change of money wage rates and found evidence for a negative relationship between these two aggregates. This is the so-called Phillips curve. The higher employment is compared to the "natural" rate of employment, the lower the nominal wage growth is, and vice-versa. This equation establishes a connection between the real and monetary sides of the Keynesian theory. Macroeconomists viewed this equation as a rationale to the inflation-employment trade off.

This conclusion was criticized by Friedman (1975). He pointed out the importance of anticipated real wages related to nominal ones. If prices are higher than the anticipated ones, real wages decrease and firms decide to hire more workers. Friedman admitted that there could exist a short run Phillips curve due to asymmetric information on prices between workers and firms, stressing the role of expectations. However, the Phillips curve would not hold in the long run, which is only determined by the fundamentals of the economy.

Nevertheless, actual data exhibits a positive correlation between employment and nominal wage growth that none of the alternative models suggested after Friedman's have been able to explain. This article aims to show that we can find theoretical arguments that support the idea of a Phillips curve in RBC models.

One issue is related to the way money is introduced in the model. Some previous models, especially those of Cooley and Hansen (1989) or Hairault and Portier (1995), have introduced money in a real business cycle model through a cash-in-advance constraint. In these models, money is needed to consume but has to be held one period in advance. Inflation acts as a tax on consumption and induces a substitution effect towards leisure and investment, both which are not taxed. However, these models fail to account for several moments on nominal variables, especially for the contemporaneous inflation-output correlation.

In order to improve the results of these cash-in-advance models, we build a stochastic dynamic general equilibrium model with efficiency wages and money. The latter is introduced through a cash-in-advance constraint. Money follows a stochastic process which allows to include a monetary shock as well as a technological one. It also allows us to check how well our model fits monetary moments, especially the output-inflation correlation in order to analyze the slope of the Phillips curve.

In order to account for the standard deviation of labor, most RBC models consider the "labor indivisibility" hypothesis of Hansen (1985). Workers have to take decisions on lotteries rather than hours. However, some alternative hypothesis, such as the efficiency wage theory by Danthine and Donaldson (1990), have been proposed. In their article households supply labor inelastically but choose how much effort to devote. Danthine and Donaldson prove that an effort function which compares current wages to an alternative wage cannot account for wage rigidities. More recently, De la Croix and Collard (1997) have used an effort function which compares present wages to a norm of past wages. They have managed to reproduce the higher variability of employment and lower variability of wages with respect to output. This scheme implies a wage sluggishness and a high variation of employment related to the business cycle.

Following Akerlof (1982), we consider labor contracts as a partial gift exchange: the firm pays wages higher than those of the Walrasian equilibrium in exchange for a higher effort by workers. In its original presentation, workers devote effort comparing their wages to alternative ones. Now, following De la Croix and Collard (1997) in an intertemporal setting, the effort function depends on past wage considerations. Thus, efficiency of labor will be a function of current, nominal and real wage, alternative wage and past wages. Workers choose an effort level comparing their present nominal and real wages to past ones of their peers and the alternative wage they can earn in some other firm.

The rationality for the special effort function comes from Bewley (1997) who analyzed the wage rigidity through a Survey of 300 business people, labor leaders, business consultants and counselors of unemployed people in the US. The survey rejects most of the alternative theories of wage rigidity except the morale model due to Solow (1979) and Akerlof (1982). According to this theory, the firms pay special attention to the morale, which depends either on the level of wages or changes in levels. However, the survey neglects the importance of pay levels and reinforces the idea that workers react to changes in wages. It also shows that the actual wage policies of firms comprise elements of both real and nominal wages. On the one hand, comparisons in terms of real wages affect the standard of living of households. On the other hand, comparisons in terms of nominal wages imply a "motivation- insult" effect. The household perceives increases in nominal wages as an approbation and a reward. The firm promotes in this way high productivity, good morale and a good company reputation which helps future recruiting and reduces turnover costs. On the contrary, workers feel insulted by a nominal pay cut, even if prices are falling.

Section 2 presents the model. Section 3 calibrates it and explains the impulse-response functions. Section 4 deals with the business cycle features and the Phillips curve. The model is able to replicate the correlation between unemployment and nominal wage growth, the so-called Phillips curve. It also aims to match some other labor moments from the US business cycle. Finally, section 5, concludes.

## 2 Description of the model

The model has to be solved in a decentralized way as the equilibrium is not Pareto efficient. This is due to the cash-in-advance constraint and the determination of wages. Instead of arising from a market equilibrium condition the firm chooses the wage. As a social capital case is considered, the firm takes previous wages as socially settled. This implies that the firm does not internalize in its wage decision the influence of individual wages on social ones. The wage chosen by the firm also affects the economy in form of an externality through the alternative wage. Both these externalities mean that unemployment may not be an optimal solution for a central planner whereas it is for the individual firm.

## 2.1 The Household

The economy is populated by many identical infinitely lived agents, uniformly distributed over $[0,1]$. Each household has to define a consumption–savings plan that maximizes its discounted expected utility, subject to the intertemporal budget constraint and a cash-in-advance constraint. This implies choosing a portfolio of assets and money in each period. The household has two different type of assets. Bonds, which pay a rate of return contingent on the realized state of nature and money, which is a dominated asset. Money has a negative rate of return because of inflation. Even if the return to money is negative, it is still held, as is necessary in order to carry out consumption activities.

The utility function depends positively on consumption and negatively on effort. It is assumed to be separable in consumption and effort. For simplicity, consumption enters through a CES function. This implies a constant intertemporal elasticity of substitution, given by $1/\sigma$. $d_t$, is a dummy variable which takes the value 1 when the agent is employed and zero otherwise. If the agent is unemployed, he has no utility or disutility from effort. The level of effort is a dynamic function increasing in the comparison of current, nominal and real wages with past ones. It also depends on the comparison of present real wages with a current alternative wage the household could earn in some other firm. This special characterization results from the way in which wages are determined, as explained by Bewley (1997). Households maximize[1]:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - d_t \left( \begin{array}{c} e_t - \phi - \gamma \log\left(\frac{w_t}{w_t^a}\right) - \psi \log\left(\frac{w_t}{w_{t-1}}\right) - \\ -\varphi \left[ \log\left(\frac{w_t}{w_{t-1}}\pi_t\right) - \log \pi \right] \end{array} \right)^2 \right\} \quad (1)$$

$$\text{s.t.} \qquad c_t + \int_S \frac{q_{t+1}(s_{t+1})}{q_t(s_t)} b_{t+1} ds_{t+1} + m_{t+1} \leq b_t + w_t(1-u_t) + \frac{m_t + n_t}{\pi_t}$$

$$c_t \leq \frac{m_t}{\pi_t}$$

The variables $c_t$ and $e_t$ denote consumption and effort, $w_t$ and $w_t^a$ are present and alternative wages. $\phi, \gamma, \psi, \varphi$ are positive parameters of the effort function. Clearly, nominal wages matter, or similarly, firms consider inflation as affecting

[1] Hereafter, only $q_t$, prices of assets, will have an explicit reference to $s_t$, the state of nature although all variables are functions of $s_t$.
Note that $E_t(X_{t+1}) = \int_S X_{t+1}(s_{t+1}) f(s_{t+1}|s_t)\, ds_{t+1}$

the worker's effort. This is expressed through the parameter $\varphi$. $\pi$ denotes the steady state rate of inflation. The term $\log\left(\frac{w_t}{w_{t-1}}\pi_t\right)$ denotes the comparison in terms of real wages.

We specify the alternative wage as the current wage in the economy times the employment rate, i.e. a kind of probability of finding a new job outside the firm

$$w_t^a = l_t w_t.$$

Capital letters denote nominal variables, whilst lower-case letters denote real ones. $p_t$ is the nominal price of one unit of consumption at period t. It refers to the quantity of money, $M_t$, you need to purchase one unit of the real consumption good. Real balances are defined as $m_t = M_t/p_{t-1}$. Inflation in the steady state is the gross rate of growth of price levels, $\pi_t = p_t/p_{t-1}$. In a steady state this rate of growth, $\pi$, is equal to the rate of growth of money. Money injections in real terms are $n_t = N_t/p_{t-1}$ per period.

The household can either consume or save, the last through contingent claims, one of which is money. We assume that it has access to a complete system of markets on which it can trade contingent claims. These are purchased in period $t$ at price $q_t(s_t)$ and sold in period $t+1$ at price $q_{t+1}(s_{t+1})$. $s_t \in S$ denotes a particular realization of the stochastic productivity shock which will be defined later. $\{q_t(s_t)\}_{t=0}^{\infty}$ is the Arrow-Debreu price system such that $1/(1+r_{t+1}(s_{t+1})) = q_{t+1}(s_{t+1})/q_t(s_t)$. The price system determines the real rate of interest or discount rate of the economy. Money does not yield any return but depreciate due to inflation.

Households are ex-ante homogeneous. However, the possibility of unemployment makes them heterogeneous ex-post. Those who are employed are randomly chosen from the total labor supply. To keep the model simple, there is a perfect insurance system for the unemployed. It is paid for by employees, who receive only a proportion of their wage $w(1-u_t)$ while, $wu_t$ is paid to unemployed households.

The household selects present consumption $c_t$,effort devoted $e_t$, the demand for bonds $b_{t+1}, \forall s_{t+1}$ and the demand for money $m_{t+1.}$ Labor supply $l_{t,}$ is inelastic and always equal to one.

The set of first order conditions defining optimal consumption, savings and the effort plan of the household can be stated as follows:

$$c_t^{-\sigma} = \lambda_t + \mu_t \tag{2}$$

$$e_t = \phi + \gamma \log\left(\frac{w_t}{w_t^a}\right) + \psi \log\left(\frac{w_t}{w_{t-1}}\right) + \varphi \log\left(\frac{w_t}{w_{t-1}}\pi_t\right) \tag{3}$$

$$\frac{q_{t+1}(s_{t+1})}{q_t(s_t)} = \beta\,\frac{\lambda_{t+1}}{\lambda_t}\,f(s_{t+1}|s_t) \tag{4}$$

$$\lambda_t = \beta E_t\left(\frac{\lambda_{t+1}+\mu_{t+1}}{\pi_{t+1}}\right) \tag{5}$$

where $\lambda_t$ is the multiplier associated to the intertemporal budget constraint and $\mu_t$ is the one associated to the cash-in-advance constraint. We add the following limiting conditions to avoid "Ponzi-games":

$$\lim_{j\to\infty} \beta^{t+j}\lambda_{t+j}b_{t+1+j}(s_{t+1+j}) = 0 \quad (6)$$

$$\lim_{j\to\infty} \beta^{t+j}\mu_{t+j}m_{t+1+j}(s_{t+1+j}) = 0 \quad (7)$$

Equation (2) yields the demand for consumption as a function of its shadows prices. Equation (2.3) defines the effort function, which is taken into account in the firms' plan.

The relation (4) corresponds to the traditional asset pricing formula given by Lucas (1978): the price of contingent claims in state $s_{t+1}$ is determined by the discounted rate of intertemporal substitution, weighted by the occurrence probability of $s_{t+1}$. Relation (5) relates the shadow price of money today, $\lambda_t$, to the shadow price of holding money tomorrow. The latter yields a higher financial endowment in the budget constraint, $\lambda_{t+1}$,and more consumption in the cash-in-advance constraint, $\mu_{t+1}$. However, as the supply of money grows, real balances decrease by an amount equal to inflation. The parameter $\beta$ discounts next period's utility. Finally, equation (6) and (7) furnish terminal conditions to the evolution of $k_t$ and $m_t$.

## 2.2 Money Supply

Following Cooley and Hansen (1989), money supply is given by

$$m_{t+1} = g_t \frac{m_t}{\pi_t}$$

while money injection is given each period by

$$n_t = (g_t - 1)m_t$$

$g_t$ is the rate of growth of real money in the economy. The rate of growth is assumed to be random, and follows a stationary exogenous AR(1) process

$$log(g_t) = (1-\rho_m)log(\bar{g}) + \rho_m log(g_{t-1}) + \xi_t.$$

Parameter $|\rho_m| < 1$ represents the autocorrelation parameter and $\xi_t$ the unknown shock, distributed as Gaussian white noise with $E(\xi_t) = 0$ and $E(\xi_t^2) = \sigma_m^2$.

This stochastic process introduces uncertainty in the economy. The household does not know how much money will be available in the next period and, therefore, does not know the price level.

## 2.3 The Firm

The firm produces a homogeneous good which can be either consumed or accumulated, either in the form of bonds or money. The technology is described by the following production function

$$y_t = f(k_t, e_t l_t; a_t) = a_t k_t^{\alpha} (l_t e_t)^{1-\alpha} \quad (8)$$

where $k_t$ denotes the firm's capital stock, $l_t$ the level of employment, $e_t$ the level of effort devoted by its workers and $a_t$ a technological shock. As there are different effort levels, productivity of workers will differ. The firm is hence interested in effective units of labor $l_t e_t$, and not only in the amount of labor hours. $k_t$ evolves over time according to the usual law of accumulation:

$$k_{t+1} = i_t + (1-\delta) k_t \quad (9)$$

where $i_t$ denotes net investment in period $t$ and $\delta \in [0,1]$ is the rate at which capital depreciates.

The technological shock, $a_t$, is assumed to be stationary and known by all firms at the beginning of period t. It follows a stationary exogenous $AR(1)$ process

$$\log(a_t) = \rho \log(a_{t-1}) + \varepsilon_t \quad (10)$$

with $|\rho| < 1$. $\varepsilon_t$ is Gaussian white noise with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t^2) = \sigma^2$.

It is assumed that the firm has no control over "social past wages" with which its workers compare their actual wage. This implies that past wages are treated as an externality. The wage setting behavior of the firm is therefore static. It maximizes over real wages.

The "personal norm" case, an alternative wage setting behavior, was also considered. In this scheme the firm only considers the previous wages it selected in the previous period. This induces the firm to consider the influence of current wage setting in next periods. However, the results are very different and have been omitted.

The firm seeks to maximize the sum of its expected discounted profit flows

$$\max \sum_{t=0}^{\infty} \int_S q_t(s_t) \{y_t - w_t l_t - i_t\} \ ds_t$$

subject to (8) and (9). The Lagrangian multiplier associated to the law of motion of capital is denoted by $\tau_t$. It is the shadow price of capital in period t in terms of period t units of the physical good. The firm maximizes over $i_t$, $l_t$, $w_t$ and $k_{t+1}$. A special feature of efficiency wages models is that firms choose the wage they pay instead of behaving as a price-taker. The technological shock takes place whilst the firm chooses, investment, labor and wages.

The first order conditions are given by

$$\tau_t = 1 \tag{11}$$

$$w_t = (1-\alpha)\frac{y_t}{l_t} \tag{12}$$

$$l_t = (1-\alpha)\frac{y_t}{e_t}\left(\frac{\gamma+\psi+\varphi}{w_t}\right) \tag{13}$$

$$\tau_t = \int_S \frac{q_{t+1}(s_{t+1})}{q_t(s_t)}\left(\alpha\frac{y_{t+1}}{k_{t+1}} + (1-\delta)\tau_{t+1}\right) ds_{t+1} \tag{14}$$

and the transversality condition:

$$\lim_{j\to\infty}\int_S \frac{q_{t+1+j}(s_{t+1+j})}{q_{t+j}(s_{t+j})}\tau_{t+j}k_{t+1+j}ds_{t+1+j} = 0 \tag{15}$$

Equation (11) states that the price of capital in period t is equal to one, the number of consumption units one unit of capital can be exchanged for. Equations (11) and (14) show that the firm has a static demand for capital: the rate of return on one unit of capital net of depreciation is equal to the return the bond provides in the financial market.

Equation (12) represents the labor demand of the firm. As profit flows are maximized, hiring will occur up to the point where the marginal productivity of labor equals the real wage. The only difference between this case and the traditional approach is that the real wage is not set according to the Walrasian equilibrium motive, but is settled by the firm. This can be seen in equation (13), which corresponds to the wage setting behavior of the firm. It states that the firm will increase wages until its marginal cost equals the marginal return in terms of effort. By combining (12) and (13) it is possible to obtain the so-called Solow condition

$$\frac{\partial e(w_t, w_t^a, w_{t-1}, \pi_t)}{\partial w_t}\frac{w_t}{e(w_t, w_t^a, w_{t-1}, \pi_t)} = 1$$

which states that the firm considers a wage such that the marginal productivity of effort equals average productivity. Hence, in this model the firm chooses the real wage in such a way that effort is constant over the business cycle:

$$e_t = \gamma + \psi + \varphi$$

However, the firm considers the influence of nominal wages (or inflation) on effort through the parameter $\varphi$. It adjusts real wages to smooth effort over the business cycle under the influence of current alternative wages $w_t^a$, past social wages $w_{t-1}$, and inflation $\pi_t$. Given the incentive mechanism available to the firm regarding the effort of workers, it chooses an optimal level of effort, which is kept constant through the adjustment of the real wage rate.

## 2.4 Equilibrium

We define an equilibrium as a sequence of prices $\{w_t, q_t(s), \pi_t\}_{t=0}^{\infty}$ and quantities $\{c_t, i_t, e_t, y_t, n_t, k_t, m_t\}_{t=0}^{\infty}$ such that,

1) for a given sequence $\{w_t, q_t(s), \pi_t\}_{t=0}^{\infty}$ , $\{c_t, i_t, e_t, m_t\}_{t=0}^{\infty}$ maximize household's utility,

2) for a given sequence $\{w_t, q_t(s), \pi_t\}_{t=0}^{\infty}$ , $\{e_t, n_t, k_t\}_{t=0}^{\infty}$ maximize firm's profit,

3) for a given sequence $\{q_t(s), \pi_t\}_{t=0}^{\infty}$ , $\{c_t, i_t, e_t, y_t, n_t, k_t, m_t\}_{t=0}^{\infty}$ clear the financial and goods markets,

4) for a given sequence $\{c_t, i_t, e_t, y_t, n_t, k_t, m_t\}_{t=0}^{\infty}$ , $w_t$ is determined by an efficiency wage mechanism[2].

# 3 Implications of the Model

## 3.1 calibration

The model does not allow for any obvious analytical solution. The dynamic system is thus log-linearized around the deterministic steady state and solved numerically using Farmer's method. We have to assign values to key parameters. These values correspond to values for the US economy in quarterly data.

The parameter for the comparison with alternative wages, $\gamma$ is set to 0.9, following Danthine and Donaldson (1990) who used a similar effort function. $\delta$, the depreciation rate of capital is 0.025, $\alpha$, the share of labor in the production function is equal to 0.36 and $\beta$, the personal discount rate, is 0.99. Futhermore, the autoregressive parameters are $\rho = 0.95$ and $\rho_m = 0.48$. The standard deviation of the monetary shock, $\sigma_m^2$, is set to 0.009. Both, the autocorrelation term and the standard deviation of the monetary shock are taken from Cooley and Hansen (1989) who estimate them from US time series of money. The standard deviation of the technological shock is such that the standard deviation of output matches its actual value of 1.72. This implies a value of $\sigma^2 = 0.0081$, close to the value observed in the related literature.

We choose the values for parameters $\phi, \psi$ and $\varphi$ such as to match different actual moments. We choose them such that they are not relevant in analyzing the Phillips curve. In this way the results can be compared with the actual data for the Phillips curve. The unemployment rate is set at 10% in the steady state. Correlation between hours and output is fixed at 0.86. Correlation between real wages and output is chosen to be 0.68. To match these moments, the parameters have to take the following values: $\psi = 2.21$, $\varphi = 0.34$ and $\phi = \psi + 0.986\varphi + 0.8052$.

[2] Appendix I shows the equilibrium conditions.

## 3.2 Impulse-Response Functions

In this section we analyze the response of several variables to a 1% technological and monetary shock. The analysis is done in terms of deviation from the steady state, which is the initial value for all variables. The shocks are not known in advance.

Agents have several transmission mechanisms to transfer resources in and between periods. As a CES utility function is assumed, the intertemporal elasticity of substitution of consumption between periods is constant for any consumption level. As the future matters, households want to save part of their current increase in wealth, implying an intertemporal shift of resources. Of course this model does not show any substitution effect for the labor supply which is assumed to be inelastic. However, it affects workers' effort and hence effective labor. This last variable is the one relevant to the firm in order to produce. As a consequence of it, changes in the unemployment rate result from the substitution effect of labor demand. The firms demands more labor in those periods in which productivity is relatively larger.

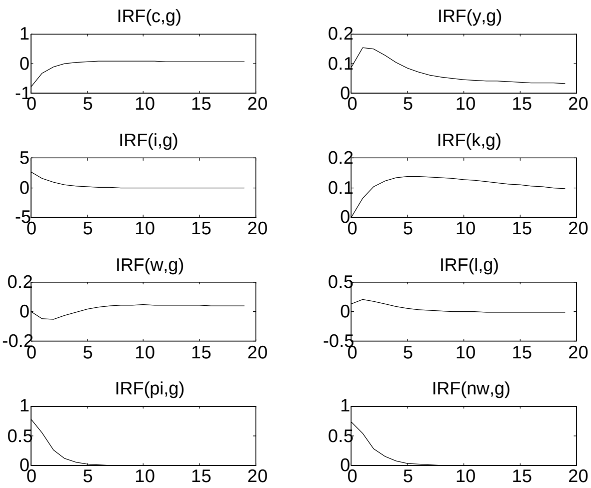
### 3.2.1 A technological shock

A positive unexpected technological shock of 1% occurs at period t. It implies a 1% gain of productivity at period t, which evolves as equation (10) states. The shock increases present productivity and firms are able to produce more output given the same inputs. This will have only a temporary effect because the shock is not a permanent one. It also induces a substitution effect from future toward present labor in order to take advantage of the higher current productivity which reinforces the rise in output. Thus the firm hires more workers today, which increases present employment. Real wages, which are related to productivity, also increase. As the productivity shock is highly autocorrelated ($\rho = 0.95$), real wages converge quite slowly to the steady state.

In the shock period, productivity increases the supply of goods which, given a fixed amount of money holdings, reduces prices and inflation. The technological shock being temporary, inflation falls only for that period. As the inflationary tax is reduced, households increase consumption and demand for money in order to also take advantage of it in the next period. As households want to smooth consumption over the cycle, part of the increase in output is saved to maintain future consumption. The only way to transfer wealth to the future is through bonds or money. Bonds are invested by firms which results in a short but high growth of investment at period t.

### 3.2.2 A monetary shock

The figures below plot the response of several variables to a positive unexpected monetary shock of 1% occurring at period t. In the first period money increases for a given supply of goods, which implies that prices increase. As inflation rises and households need money to buy consumption goods, it is now more expensive to consume due to the inflation tax. This makes households reduce their present consumption (which is relatively expensive) and substitute it for cheaper (future) consumption. Hence households' savings, which are invested in firms, increases. As the monetary shock is not very autocorrelated, consumption quickly returns to its original level. Due to the rise in inflation, nominal wages are higher too, inducing workers to devote more effort. However, firms prefer to maintain the level of effort constant over the business cycle. As real wages are selected by the firm, they will be reduced to compensate for the higher nominal wages in order to keep effort constant. In subsequent periods the firm increases real wages as nominal wages slowly decrease. The cost of hiring new workers decreases and so the firm chooses to increase employment. But, as the monetary shock is not very autocorrelated, the effect is a short term one.




Impulse-Response Functions to a Monetary Shock

The model generates two different effects. One is similar to the one found in Cooley and Hansen (1989) and Hairault and Portier (1995), where consumption is substituted by investment, which increases the next period capital stock and hence output. Futhermore though, this model has a different effect compared to the other two models. In both there exist a substitution effect from consumption toward leisure as this is not taxed by inflation. In our model that effect does not exist since the labor supply is inelastic, however, employment is affected by effort which determines effective labor. In this sense, inflation increases present nominal wages. The workers' norm increases as they compare present nominal to past nominal wages. This induces a rise in the effort devoted by workers, which is seen in the impulse-response function of output. It shows a short term, positive relation to a monetary shock which is quite similar to what is observed in actual data.

Although this positive relation exists, the inflation-output correlation is negative. This is explained by the fact that, given the particular calibration of the model, most of the disturbance is caused by the technological shock. A positive correlation between the technological shock, employment and output exists. As money is given and output increases, prices fall. This determines the negative inflation-output correlation.

# 4 Can the model match the Business cycle?

The theoretical moments are summarized in table 1[3].

Table 1: Basic Moments

| | US. data | | Current Model | | Cooley-Hansen | | Hairault-Portier | |
|---|---|---|---|---|---|---|---|---|
| | (a) | (b) | (a) | (b) | (a) | (b) | (a) | (b) |
| c | 0.74 | 0.83 | 0.51 | 0.38 | 0.36 | 0.72 | 0.51 | 0.52 |
| i | 4.79 | 0.91 | 3.60 | 0.93 | 3.29 | 0.97 | 2.97 | 0.93 |
| l | 0.92 | 0.86 | 0.75 | 0.86 | 0.77 | 0.98 | 0.75 | 0.98 |
| w | 0.44 | 0.68 | 0.52 | 0.68 | 0.29 | 0.87 | 0.30 | 0.87 |
| $\pi$ | 0.33 | 0.34 | 0.49 | -0.18 | — | — | 0.45 | -0.28 |

(a): *Standard deviations relative to output.*
(b): *Contemporaneous correlations with output*

We decided to replicate actual contemporaneous correlations of employment and labor with output in our model. Although this implies that additional moments need to be matched, we are more confidence of the results. Especially as this allows to fix values for parameters $\varphi$ and $\psi$ in the effort function which have not been previously tested.

The introduction of a monetary shock has the same effect for investment and consumption. This new shock affects both directly through a substitution effect of consumption towards investment. On the one hand, it implies a higher standard deviation for both variables. On the other hand, this new source of influence reduces the correlation with output, for both variables. It should be noticed that consumption deviations are mainly driven by the monetary shock. Around 87% of the deviation of consumption in the first period is due to the monetary shock. This points to the importance of this kind of shock in explaining the results.

In the labor market, we decided to calibrate the correlations of output with wages and labor. Our model matches the standard deviation of labor better than the other models. However, it is only two thirds of the actual statistic. Recall that our models relies on the theory of efficiency wages. This mechanism is completely different to the one of indivisible labor used in the other two models. In both cases a infinitely elastic intertemporal labor supply is assumed so that labor is totally determined by the firm.

The theory of efficiency wages relies on the assumption that the wage is the only instrument of the firm and is the incentive of effort of workers. As the firm wants to maintain a constant level of effort over the business cycle, wages deviate

[3]Theorical moments are obtained using the spectral approach advocated by Uhlig (1995).

with any kind of shock. However, any shock has a long run adjustment which implies a long deviation of wages to keep effort constant. This slow adjustment can be seen as a wage sluggishness, which implies a higher standard deviation of wages. The latter is observed no matter whether it is compared to the other models or actual data.

The model of Cooley and Hansen does not report any nominal moment. This means that all our comparisons are done with respect to the model of Hairault and Portier. The negative inflation-output correlation is verified in the supply side of the economy: when a positive technological shock occurs, the firm takes advantage of it by intertemporal labor substitution. It increases output which, given the same cash balances, reduces prices and inflation. This holds for all cash-in-advance models. What is new in our model is that positive monetary shocks increase inflation and nominal wages, which induces workers to supply more effort. To keep effort constant, the firm has to lower real wages, reducing labor costs. As a consequence, the firm recruits more workers and hence increases output. However, the positive inflation-output correlation due to the monetary shock explains only around 1% of the total correlation. The negative correlation is then explained by the pre-eminent technological shock.

## 4.1 Analysis of parameter $\varphi$

We analyze how the model reacts to a variation of $\varphi$ holding $\psi$, $\gamma$ and $\phi$ constant. By doing so we check the influence of nominal wage comparisons on several variables and results. The main statistics can be found in Table 2.

Table 2: Moments for different values of $\varphi$

| | $\varphi = 0.34$ | | $\varphi = 0$ | | $\varphi = 0.7$ | | $\varphi = 1$ | | $\varphi = 3$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | (a) | (b) | (a) | (b) | (a) | (b) | (a) | (b) | (a) | (b) |
| c | 0.51 | 0.38 | 0.51 | 0.42 | 0.49 | 0.35 | 0.48 | 0.33 | 0.42 | 0.29 |
| i | 3.60 | 0.93 | 3.54 | 0.92 | 3.66 | 0.93 | 3.69 | 0.93 | 3.73 | 0.95 |
| l | 0.75 | 0.86 | 0.73 | 0.85 | 0.78 | 0.86 | 0.80 | 0.87 | 0.97 | 0.89 |
| w | 0.52 | 0.68 | 0.54 | 0.70 | 0.51 | 0.64 | 0.50 | 0.60 | 0.47 | 0.30 |
| $\pi$ | 0.49 | -0.18 | 0.51 | -0.24 | 0.47 | -0.13 | 0.46 | -0.01 | 0.38 | 0.02 |

(a): *Standard deviations relative to output.*
(b): *Contemporaneous correlations with output.*

We analyze those moments which are greatly affected when $\varphi$ rises. One of them is the wage-output correlation, which decreases. When $\varphi = 0$, nominal wage comparison has no effect on the effort function and only real and alternative wages matter. This can be seen in the recursive equation for wages we obtain from the effort function:

$$\log(w_t) = a_0 + \lambda \log(y_t) + (1-\lambda)\log(w_{t-1}) - \frac{\varphi}{\gamma+\psi+\varphi}\log(\pi_t), where \lambda = \frac{\gamma}{\gamma+\psi+\varphi} \tag{16}$$

As $\varphi$ increases, nominal wages are more important in determining effort. However, nominal wages are real wages times inflation. The latter is determined in the money market and depends directly on money growth, which follows an exogenous stochastic process. Obviously, this makes effort depend less on output and more on inflation, which is shown by a lower correlation of effort with output. This can be seen in the fact that corr$(w_t, g_t)$ becomes more important and corr$(w_t, y_t)$ decreases.

We now explain the mechanism through which the standard deviation of labor increases when $\varphi$ rises. Again, we begin by analyzing the case when $\varphi = 0$. In this case the equation for labor becomes:

$$\log(l_t) = b_0 + \frac{\psi+\varphi}{\gamma+\psi+\varphi}[\log(y_t) - \log(w_{t-1})] + \frac{\varphi}{\gamma+\psi+\varphi}\log(\pi_t). \tag{17}$$

If $\varphi = 0$, labor is fully determined by previous real wages and output and hence is only affected by the technological shock. As $\varphi$ increases, nominal wages (or inflation) become more important in the determination of labor. This implies that the monetary shock begins to affect labor too. However, the monetary shock explains about 5% of labor variance while only 1% of the output variance. This means that the deviation of labor is more affected by the monetary shock than the deviation of output. The ratio of both deviations increases, becoming one when $\varphi = 3$.

Corr$(\pi_t, y_t)$ goes from negative values to positive ones for values of $\varphi$ close to 3. The main transmission mechanism operates through the technological shock. When a positive productivity shock occurs, the firm increases labor to take advantage of it. This implies a rise in output which, given cash balances, produces a reduction in prices and inflation.

A second monetary mechanism leads to a different relation. Effort depends on nominal wages, which are formed by real wages and inflation. If inflation increases as a result of a money injection, workers devote more effort given the real wage. However, the firm wants to maintain effort constant over the cycle to be sure of the effectiveness of its labor. This leads the firm to decrease real wages, which makes it more profitable to hire workers since labor costs have decreased. As the firm has to pay less for the same effort, it can hire more workers and increase output. We hence find the positive inflation-output correlation.

In our model the technological shock affects the economy more than the monetary shock does. However, when $\varphi$ rises, nominal wages become more important in determining effort. In this case, the larger is $\varphi$, the larger is the response of effort to inflation, as seen in (3). In fact, for values of $\varphi$ greater than 3, the increase of output, via a monetary shock, more than compensates the decrease due to the technological shock.

It remains to be explained why the standard deviation of inflation relative to output falls as $\varphi$ increases, as the parameter $\varphi$ has no influence on the standard deviation of inflation. However, due to nominal wages, inflation influences the effort devoted by workers. To keep effort constant the firm reduces real wages and hires more workers, which increases output. As a consequence, the larger is $\varphi$, the more dependent output is from the monetary shock. This implies that the standard deviation of output is larger and hence, the ratio of the standard deviation of inflation relative to output decreases.

## 4.2 On the Phillips curve

To deal with the analysis we consider the original Phillip's curve in terms of nominal wage growth and the unemployment rate. However, none of the previous articles report this data which were obtained from the Bureau of Labor Statistics. To compute the growth of nominal wages we use quarterly data. As nominal wage series considered are the "average hourly earnings of production workers", which are reported monthly, we transform them into quarterly data by taking the average over three months. The hour series considered are the "indexes of aggregate weekly hours", which are reported monthly. Again, the series have been transformed into quarterly data. As we are interested in the unemployment rate, we use the negative of work hours. Both series, nominal wage growth and the unemployment rate, are detrended by the Hodrick-Prescott filter. Both series are used to compute past, present and future correlations between them. Results are presented in table 3.

Table 3: $Corr(nw_t, U_{t+j})$

| Period | t-4 | t-3 | t-2 | t-1 | t | t+1 | t+2 | t+3 | t+4 |
|---|---|---|---|---|---|---|---|---|---|
| Data | -0.09 | -0.10 | -0.08 | -0.09 | -0.18 | -0.18 | -0.10 | -0.05 | -0.04 |
| Model | 0.02 | -0.04 | -0.13 | -0.24 | -0.34 | -0.28 | -0.08 | 0.03 | 0.07 |

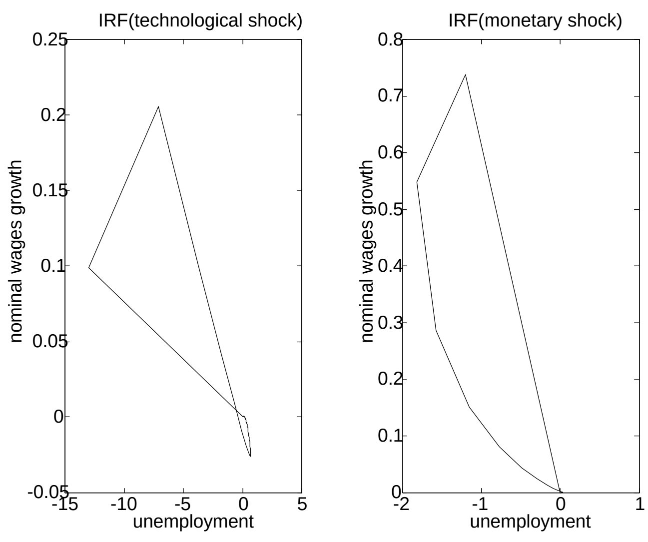
The current model displays correlations at period $t$ and $t+1$ which are close to those obtained by actual data. For these periods the correlation is higher both for the model and the actual data. In both periods correlations are clearly negative as are those of the model. For periods $t-1$ and $t-2$ correlations of the model are larger compared to ones of the actual data. However, this can be explained by the wage sluggishness due to the special intertemporal effort function considered.

The correlation between both variables is close to zero after and before two periods, hence it can be interpreted as if it were only a short run Phillips curve. In the long run employment is not affected by nominal variables and is fully determined by real variables. Even in the short run the cycle is mainly driven by technology rather than monetary shocks.

Nominal wage growth is able to reduce the unemployment rate through its influence on the effort function. As workers devote more effort which the firm wants to keep constant, the latter reduces the real wage and, as a consequence, hires more labor. As households supply labor inelastically and the demand of labor increases, unemployment rate decreases.

The Phillips curve in terms of deviation from the steady state can be seen in the figure below. The deviation from the steady state due to a positive technological shock is also depicted. The technological shock has a higher effect on the rate of unemployment, with almost no nominal wage growth. This is explained by the direct effect of the technological shock on employment. However, nominal wage growth hardly increases as inflation decreases and compensates the increase in real wages. After one period employment begins to adjust to its steady state level. Nominal wage growth first increases before returning to its steady state level.

Compared to a technological shock, the effect of a positive monetary shock has a larger effect on nominal wage growth than on the unemployment rate. In the first period nominal wage growth increases due to inflation. As workers compare their wage with the one in the previous period, they offer more effort. This implies that the firm hires more workers to take advantage of it. There is no influence of nominal wage growth on employment after several periods. As the monetary shock tends to zero, nominal wage growth returns to its steady state level. Employment also falls to its steady state as it follows nominal wage growth. The model is able to replicate the right direction of response of the nominal wage growth and unemployment rate to both kind of shocks.




Phillips Curve

# 5 conclusion

The negative correlation between unemployment rate and nominal wage growth was a puzzle unsolved for a standard RBC model. This model is able to replicate a Phillips curve for the US economy, i.e., the negative correlation between nominal wage growth and the rate of unemployment. It is also able to match past and future correlations for the same variables, which implies a short run influence between them.

The model includes efficiency wage characteristics of the "gift exchange" type. Each worker decides how much effort to supply through an effort function, which compares present real and nominal wages to past ones. This specific effort function of the model implies a kind of sluggishness in wages and a higher volatility of employment compared to classical RBC models. It also reduces the negative contemporaneous correlation between inflation and output, one of the more difficult moments to match for a cash-in-advance model. The efficiency wage theory is able to solve the labor market puzzle. This is done due to the introduction of a dynamic effort function instead of a static one.

The model is also able to match moments of the US economy, especially the volatility of hours and wages, and to improve the results related to inflation. The moments related to inflation differ from actual ones for any of the previous models that have been considered. All of them display a negative inflation-output correlation, while actual data show a clear positive one. This is due to the way money is introduced in the model. Households are constrained and money injections act as a tax, reducing consumption, which seems to be counterfactual. This problem is partially solved in this model because money injections also increase effort and have a direct effect on the supply of goods, which compensates the inflation tax effect.

The efficiency wage theory seems to be a promising research line in the future as it has proven to be capable of reproducing a Phillips curve. This theory also solves the business cycle puzzle of labor volatility and wages sluggishness and improves the results for the nominal variables.

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## 5.2 Appendix I

We show here the equilibrium conditions of the model. Using the equilibrium condition of the financial market and gathering together the equations obtained from the households' and the firm's maximization programs we have the following set of equilibrium conditions:

$$
\begin{aligned}
c_t^{-\sigma} &= \lambda_t + \mu_t \\
e_t &= \phi + \gamma \log\left(\frac{w_t}{w_t^a}\right) + \psi \log\left(\frac{w_t}{w_{t-1}}\right) + \varphi \log\left(\frac{w_t}{w_{t-1}}\pi_t\right) \\
w_t &= (1-\alpha)\frac{y_t}{l_t} \\
\lambda_t &= \beta E_t\left[\lambda_{t+1}\left(\alpha\frac{y_{t+1}}{k_{t+1}} + 1 - \delta\right)\right] \\
y_t &= a_t k_t^{\alpha}(e_t l_t)^{1-\alpha} \\
y_t &= c_t + i_t \\
k_{t+1} &= i_t + (1-\delta)k_t \\
w_t^a &= l_t w_t \\
\lambda_t &= \beta E_t\left[\frac{\lambda_{t+1} + \mu_{t+1}}{\pi_{t+1}}\right] \\
m_{t+1} &= g_t\frac{m_t}{\pi_t} \\
c_t &= \frac{m_t}{\pi_t} \\
\log(a_t) &= \rho \log(a_{t-1}) + \varepsilon_t \\
\log(g_t) &= (1-\rho_m)\log(\bar{g}) + \rho_m \log(g_{t-1}) + \xi_t \\
e_t &= \gamma + \psi + \varphi
\end{aligned}
$$