Export Restraints and Horizontal Product Differentiation[†]

N. Boccard[*] & X. Wauthy[**]

First draft : 01/97, revised : 09/97

Abstract

We consider the effects of export restraints on price competition in the Hotelling model of horizontal product differentiation. We characterise the Nash equilibrium for all possible values of the quota and compare our results with those of Krishna [89]. We show that a foreign producer would choose a Voluntary Export Restraint in the vicinity of the Free Trade Equilibrium. In order to maximise domestic welfare, a government would not necessarily choose complete protectionism nor free trade.

JEL Classification : D43, F13, L13
Keywords : Hotelling, optimal quota, price competition

[†] The authors thank Nicolas Schmitt and Philippe Monfort for useful comments. The usual disclaimer applies.

[*] CORE, Université Catholique de Louvain. Research supported by a "Marie Curie" fellowship of the European Community.

[**] FNRS and IRES, UCL. Research supported by a grant "Actions de Recherche Concertées" 93/98-162 of the Ministry of Scientific Research Of the Belgian French Speaking Community.

1) Introduction

In a seminal paper, Krishna [89] shows that the presence of a quota on the foreign producer's sales has drastically different implications depending on the nature of the strategic variable chosen by the firms. The intuition for her result is that a quota is formally equivalent to the presence of a capacity constraint for the foreign producer. It is therefore easily understood that their implications are different depending on whether firms use quantities or prices as their strategic variables. It is indeed well-known that the presence of capacity constraints may destroy the existence of pure strategy equilibrium in price-setting models whereas such a problem does not arise in quantity-setting models.

Krishna [89] shows that the presence of Voluntary Export Restraints[1] (VER) tends to yield more collusive outcomes, thereby explaining why export restraints can be voluntary. To this end, she develops a price setting model in which two firms sell symmetrically differentiated products. Moreover, she is interested by the effects of a quota set in the vicinity of the Free Trade Equilibrium (FTE hereafter), and in particular slightly above this level.

The present paper studies a similar problem within the horizontal product differentiation model of Hotelling [29]. As noted by Krishna [89], it offers a natural application of her analysis (cf. page 260) and has also been used in the recent literature on international trade (see for instance Schmitt [90], [96]). Indeed, the Hotelling model neatly captures the idea that intra-industry trade occurs because of the variety in consumers' tastes and products' characteristics while allowing for strategic interactions between firms. We consider a game similar to that of Krishna [89] : a domestic and a foreign producer compete in prices on the domestic market. The foreign producer is facing a quota when price competition takes place. Products are imperfect substitutes and we model product differentiation using the address- model of Hotelling [29]. This will allow us to characterise explicitly Nash equilibrium in prices and therefore address the question of the optimal level of the quota.

Our main findings are the following.

---

[1] We use the terms "quota" and "export restraints" indifferently in order to capture the idea that there is a bound to the quantity of the good that the foreign producer is allowed to export to the domestic market.

i) Quotas in the vicinity of the FTE level tend to destroy the existence of a pure strategy equilibrium. In this respect our results mimic those of Krishna. However, we identify the precise range of quota levels where this phenomenon occurs. The relationship between this range and the degree of product differentiation is then studied.

ii) A pure strategy equilibrium with large prices still exist if the quota is very restrictive and the consumers are very sensitive to product differentiation, thereby making products relatively poor substitutes. This equilibrium illustrates the effect of quantitative restrictions in the Hotelling model : the presence of a quota allows both firms to benefit from a local monopolist structure.

iii) Contemplating the issue of Voluntary Export Restraints, we show that the foreign producer would always choose a quota in the vicinity of Free Trade.

iv) For most values of the parameters, the domestic government would choose complete protectionism in order to maximise domestic welfare. However, when the valuation of the product by the consumers does not differ too much from costs levels, a domestic monopolist would not cover the market in equilibrium. In this case, limited competition through restrictive, but positive, quota is desirable because it allows for market coverage, without leading to a too large profits diversion.

The paper is organized as follows. In section 2, the implications of a quota in a price-setting industry are briefly recalled. In section 3, we consider the effects of a quota under horizontal differentiation whereas section 4 is devoted to the analysis of the Welfare implications of the quota. Section 5 concludes.

2) Export Restraints and Product Differentiation

The implications of an export restraint in models of price competition is probably best understood by noting that export restraints are formally equivalent to the presence of a capacity constraint on the foreign producer. Price competition in the presence of capacity constraints has been extensively studied since the pioneering work of Edgeworth [25], although most often in the context of homogeneous product. In the present context, the problem amounts to consider a price game with product differentiation, in which one of the two firms only is capacity constrained (see Levitan & Shubik [72] for a similar analysis with homogeneous product).

In order to capture the intuition underlying our analysis, consider two firms selling substitute products and competing in prices in a differentiated industry. Let us denote (p_1^*, p_2^*), the unique Nash equilibrium which prevails when firms do not face any form of quantitative restriction. Suppose now that firm 2 is facing a quota at the FTE level, i.e. $q \equiv D_2(p_1^*, p_2^*)$. Is (p_1^*, p_2^*) still an equilibrium ? Presumably not. Indeed, if firm 1 raises p_1, against p_2^*, her demand should decrease whereas the demand addressed to firm 2 should increase. However, firm 2 cannot meet this demand since it exceeds the quota. Accordingly, rationing appears in the market. It is then sufficient that some rationed consumers turn back to firm 1 in order to make the deviation profitable, thereby destroying our equilibrium candidate. Now, the question is to find out the nature of the new equilibrium.

As noted in the preceding argument, the presence of the quota typically implies that the payoffs of the domestic producer are not quasi-concave, so that the existence of an equilibrium can be problematic. Yet, contrarily to models of homogeneous goods, payoffs are continuous under product differentiation, therefore it is only the existence of an equilibrium in pure strategies which is problematic.

Summing up, we note that the presence of quantitative restrictions generates incentives for the domestic producer to name high prices in order to create rationing at the foreign firm and benefit from those rationed consumers who turn back to her. In other words, the presence of the quota allows the domestic firm to act as a monopolist along a residual demand. The level of this residual demand depends on the level of the quota and the extent to which rationed consumers are willing to buy the domestic product instead of refraining from consuming. Other things being equal, the larger the residual demand, the greater the incentive to quote high prices. Therefore, the extent to which the domestic producer recovers rationed consumers is of crucial importance for the analysis. Product differentiation should play an important role in this respect. The other crucial element is the form of the rationing rule which determines *who* are the rationed consumers (they have unit demand for the products). Moreover, in the Hotelling model, each consumer is characterised by a particular reservation price for each product, therefore the identity of the rationed consumers is directly linked to their willingness to report their purchase to the domestic firm in case of rationing.

In the present paper, we consider the efficient rationing rule so that *rationed consumers are characterised by the lowest reservation prices for the foreign product*. Although efficient rationing is not necessarily the most intuitive rationing rule, it has been widely used in the literature (see for instance Kreps & Scheinkman [83]). Moreover, it easily compares with the implicit rationing rule considered by Krishna[2].

3) Price Equilibrium in the Presence of a Quota

In order to capture horizontal differentiation, we consider the most simple version of the Hotelling model with fixed locations (this point shall never change).

3.1) The Free Trade benchmark

An indivisible homogeneous good is sold by a domestic firm at a price p_d and by a foreign firm at a price p_f. They are respectively located at the left end and the right end of the [0;1] segment. Consumers are uniformly distributed over this space of characteristics and identified by their address **x**. A consumer buy at most one unit of the good, bears a linear transportation cost and has a reservation price **S** for the good. Hence, the utility derived by a consumer located at **x** in the interval [0,1] when buying the domestic product is $S - tx - p_d$ while he gets $S - t(1-x) - p_f$ if he buys from the foreign producer. Refraining from consuming any of the two products yields a nil level of utility[3]. Since we can normalise prices, we set the transportation cost **t** at 1$; thus, a large S either means that consumers like the good very much or that the two firms are poorly differentiated.

Although being a fairly standard result, we first characterise the Hotelling equilibrium in full length. Indeed, this will provide a useful benchmark for the analysis to follow.

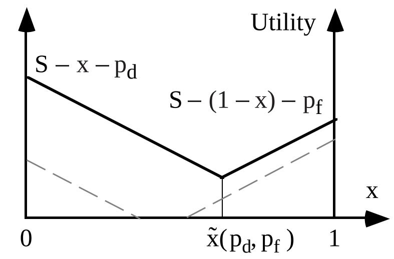
[2] We will discuss later on the robustness of our results to the introduction of other rationing rules.

[3] In Hotelling's original model, this possibility is not considered, formally, this correspond to an infinite S.

Lemma 1 (Hotelling)

If S > 3/2 and firms face no quantitative constraints, the only Nash equilibrium of the pricing game is (1,1) and the market is covered.

<u>Proof</u> As one can see with the plain lines of figure 1, if prices are not too large, all agents buy the good at one of the shops : agents living in the segment $\left[0;\tilde{x}(p_d,p_f)\right]$ will buy at the domestic firm and the rest buy at the foreign firm.



<u>Figure 1</u>

Since the consumers are uniformly distributed on [0;1], the demands are respectively $\tilde{x}(p_d,p_f)$ and $1-\tilde{x}(p_d,p_f)$. We compute $\tilde{x}(p_d,p_f)$ by solving $S - tx - p_d = S - t(1-x) - p_f$ and we get $D_i = \frac{1-p_i+p_j}{2}$ for i = d,f. It is also clear that if prices are too large the market is not covered (cf. dashed lines of figure 1) and the demand addressed to firm **i** is $S - p_i$; this happens if $S - \tilde{x}(p_i,p_j) - p_i < 0 \Leftrightarrow p_i > 2S-1-p_j$. In conclusion, the demand function of firm **i** is

$$D_i(p_i,p_j) = \begin{cases} \frac{1-p_i+p_j}{2} & \text{if } p_i \le 2S-1-p_j \\ S-p_i & \text{if } p_i > 2S-1-p_j \end{cases}$$

The associated profit functions are second degree polynomials (parabola) with maximum at $H(p_j) \equiv \frac{1+p_j}{2}$ and $\frac{S}{2}$. The best reply of firm **i** will depend on the relative ordering of $H(p_j)$, S/2 and $2S-1-p_j$; we analyse this in the following table.

Price p_j		$S-1$	$\frac{3S}{2}-1$
Ordering	$H(p_j) < S/2 < 2S-1-p_j$	$S/2 < H(p_j) < 2S-1-p_j$	$2S-1-p_j < S/2 < H(p_j)$
Best reply	$H(p_j)$	$H(p_j)$	S/2

For $p_j < S - 1$, $H(p_j)$ is in the right domain while S/2 is not,thus, at $2S-1-p_j$, both parabola are decreasing and $p_i = H(p_j)$ is the overall maximiser of the profit function. The case $p_j \in \left[S-1; \frac{3S}{2}-1\right]$ has an identical conclusion while for $p_j > \frac{3S}{2} - 1$, it is reversed.

Figure 2 displays the best reply functions (note that $S > 3/2 \Rightarrow \frac{3S}{2} - 1 > 1$). Since the demand $D_i(p_i,p_j)$ is linear in p_j, the profit function is concave so that the best reply to a mixed strategy is the best reply to its expectation which is a pure strategy. Therefore, the unique Nash equilibrium of this pricing game is pure ; as seen on the figure, the best reply lines intersect at the unit price for both firms. ♦

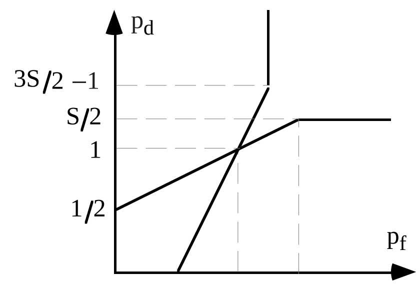


Figure 2

Note that the equilibrium prices do not depend on S and are "too low" in the sense that all consumers enjoy a strictly positive surplus. Clearly, firms could benefit from the presence of the quota to relax the price competition.

3.2) Export Restraints

The analysis of the pricing game with the quota proceeds as follows. Against a foreigner's price, the domestic producer contemplates two options : by naming a high price, she will make the quota binding, thereby generating rationing and spillovers. By naming a low price, she fights for market shares, exactly as under free trade. First, we characterise the shape of demands, corresponding to these two strategic options for all possible price constellations. Second, we compute the firms' best replies : the domestic one is discontinuous, reflecting the two strategic options mentioned above whereas the foreign one is kinked. With these best replies in hand, we characterise the Nash equilibrium in prices for all possible constellations of **q** and **S**. These equilibria may involve either pure or mixed strategies.

We will assume that the efficient rationing rule is at work in the market, as in Kreps & Scheinkman [83] : rationed consumers are those exhibiting the lowest reservation price for the rationed good. In other words, among the set of potential consumers of the foreign product, they are the most inclined to turn to the domestic firm. Consider the example depicted on Figure 3 below : some consumers willing to buy at the foreign firm are rationed. Under efficient rationing, they are located in the interval $\left[\tilde{x}(p_d, p_f); q\right]$ and thus are precisely the most inclined to switch to the domestic firm. Despite the latter has a potentially low demand (p_d is large relative to p_f), the fact that the foreign firm is constrained by the quota, could give the domestic firm an effective demand of $1 - q$.

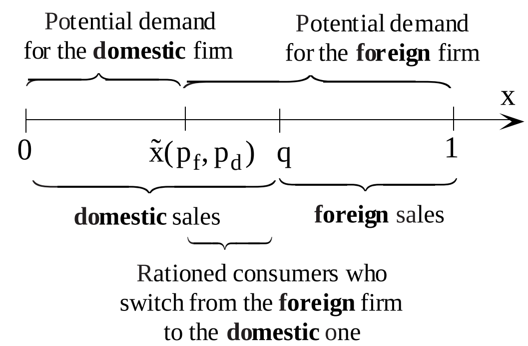


Figure 3

More precisely, as long as p_d is less than $S - (1 - q)$ which measure the net utility of the consumer located in **q**, the effective demand of the domestic firm is $1 - q$. This feature of the market allocation rule also lowers domestic firm's incentives to enter a price competition "à la Bertrand" since her demand is locally independent of her own price p_d. Note thus that within our framework, efficient rationing defines the largest residual demand for the domestic firm, so that, contrarily to Kreps & Scheinkman, the incentives to use rationing strategically are maximised.

3.3) The quota-constrained Equilibrium

The formal derivation of the demand functions can be found in our paper (Boccard & Wauthy [97b]) on capacity pre-commitment in the Hotelling model which is more general on this point. Figure 4 below will help to understand how the quota affects the game. Lets start from region A where the market is covered and the foreign firm is not constrained by the quota.

If p_f increases, we leave area A to get into area D as in the free trade analysis as the marginal consumer $\tilde{x}(p_d, p_f)$ ceases to buy the good. The complex part is when p_d increases because the domestic firm expects to benefit from spillovers. Indeed, in area B, the domestic firm recovers all rationed consumers while for larger prices some consumers cease to buy.

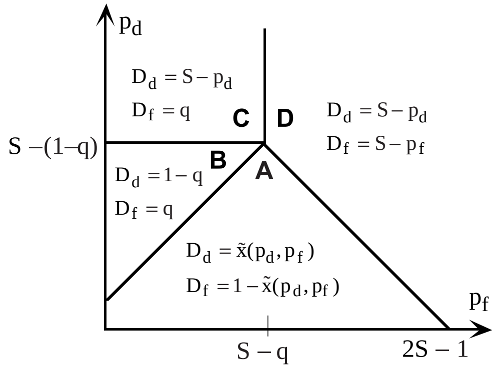


Figure 4

This is area C where the foreign producer is still constrained by the quota. The equation of the frontier A/B is the solution of $1 - \tilde{x}(p_d, p_f) = q$ and gives $p_d = p_f + 1 - 2q$, the equation of the frontier A/D is $p_d = 2S - 1 - p_f$. We can now derive the best reply functions by considering in turn the optimal responses in the four areas.

Intuitively, against a low p_d, the foreign firm responds in an aggressive manner to gain market shares. As p_d increases, her sales increases and finally reach the quota ; for domestic prices above that threshold, the foreign producer can only sale the quota at a maximum price (see the kinked bold line on figure 5 below). Analytically, the free trade analysis of lemma 1 applies in area A, the best reply is $\psi_f(p_d) = \frac{1 + p_d}{2}$ whenever it belongs to area A. Indeed, the demand is $D_f = 1 - \tilde{x}(p_d, \psi_f(p_d)) = \frac{1 + p_d}{4}$, it reaches the quota at $\hat{p}_d \equiv 4q - 1$. Against a larger domestic price, the foreign firm sticks to the quota.

In areas B and C where the quota is binding, the foreign demand is constant, thus the optimal price is the largest possible one which leads us to the frontiers with Areas A and D. Note then that the optimal price in B∪C is dominated by that of A∪D. The only (technical) problem is the domain of monopoly demand D.

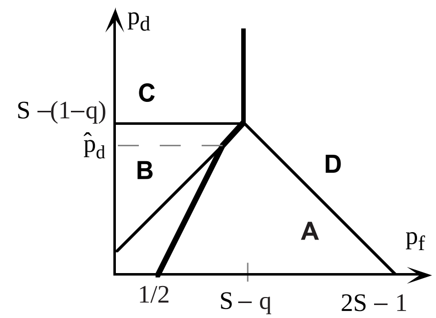


Figure 5

If, as displayed on figure 5, the monopolistic price S/2 is less than S – q, the monopoly profit function is decreasing everywhere in D and the optimal choice is S – q. The remaining possibility, S/2 > S – q is irrelevant[4]. In conclusion, the best reply of the foreign firm is the continuous function :

$$\psi_f(p_d) = \begin{cases} (1 + p_d)/2 & \text{if} \quad p_d \leq \hat{p}_d \\ p_d + (1 - 2q) & \text{if} \quad \hat{p}_d < p_d < S - 1 + q \\ S - q & \text{if} \quad S - 1 + q \leq p_d \end{cases}$$

We now turn to the domestic best reply. Area C∪D is straightforward to analyse since there is no competition as the market is left uncovered. As the monopolistic price S/2 is the overall maximiser of the profit, if it lies above S – 1 + q, then it is the dominant strategy played by the domestic firm in equilibrium. This happens when q < 1 – S/2 i.e., S must be small and the quota very restrictive (less than 1/4 as S > 3/2). Otherwise, the optimal price is S – 1 + q.

The crucial point that drives all our results is the behaviour of the domestic firm in a competitive context i.e., area A∪B where the market is covered. She can act in a classical fashion by fighting for market share with a low price or she can take advantage of the quota with a high price in order to create some rationing at the foreign firm and recover the rationed consumers. Intuitively, if the foreign firm is aggressive, the price

[4] Indeed, q must be greater than 3/4 as S > 3/2 ; in that case, the best reply of the domestic firm to S/2 would create competition i.e., we are driven back in area A. Thus, the foreigner monopoly strategy S/2 won't appear in equilibrium and this is why omit it.

competition generated by the first option drives the profits to zero, it is therefore better to hide behind the quota in order to be able to act as a monopolist on a residual demand. However, if the foreign firm becomes less aggressive then it is optimal to revert to an aggressive pricing. The optimal behaviour of the domestic firm can drastically change, depending on her perception of the foreign firm pricing.

The first option corresponds to area A where the free trade analysis applies, the optimal price is $(1+p_f)/2$. The second option is area B where the demand is always $1-q$, the largest price $S-1+q$ is therefore optimal. The associated profits are respectively $\frac{(1+p_f)^2}{8}$ and $[S-1+q](1-q)$, they are equal at $\hat{p}_f \equiv \sqrt{8[S-1+q](1-q)}-1$. We obtain the discontinuous best reply function of the domestic firm :

- if $q < 1-S/2$, $\psi_d(p_f) = S/2$

- if $q \geq 1-S/2$, $\psi_d(p_f) = \begin{cases} S-1+q & \text{if} \quad p_f \leq \hat{p}_f \\ (1+p_f)/2 & \text{if} \quad \hat{p}_f < p_f < 3S/2-1 \\ S/2 & \text{if} \quad 3S/2-1 \leq p_f \end{cases}$

The fact that the best reply function of the domestic firm is discontinuous at $\hat{p}_f$ can preclude the existence of pure strategy equilibria but contrarily to the case studied by Kreps & Scheinkman [83] and Osborne & Pitchik [86], there is no density of prices in equilibrium. This is a property of product differentiation.

Proposition 2

The unique equilibrium of the pricing game is as follows :

i) *if $q < 1-S/2$, the domestic firm acts as a pure monopolist and the foreign one sell her quota at a maximum price, the market is uncovered.*

ii) *if $1-S/2 < q < 1-S/3$, the domestic firm covers the market but do not enter a price competition with the foreign firm.*

iii) *if $1-S/3 < q < \bar{q}$, an Edgeworth cycle appears where the domestic firm mixes between aggressive pricing and hiding behind the quota while the foreign firm plays a pure strategy.*

iv) *if $q > \bar{q}$, firms play the Hotelling unit price.*

<u>Proof</u> Observed from figure 3 above that, for any p_d, $D_f(p_d,.)$ is a weakly decreasing function, so that the profit function $\Pi_f(p_d,.)$ is concave for any p_d. Thus, whatever mixed strategy F_d the domestic firm might play, $\Pi_f(F_d,.) \equiv \int \Pi_f(p_d,.)dF_d(p_d)$ is again concave and has a unique maximiser which means that in a Nash equilibrium, the foreign firm plays a pure strategy.

When the quota is very loose (case **iv**), the "classical" Hotelling equilibrium (1,1) remains an equilibrium because the residual demand is too small. The analytical conditions derived from the best reply functions are $\hat{p}_f < 1$ and $1 < \hat{p}_d$. From the first, we get $[S-1+q](1-q) < 1/2 \Rightarrow q > \bar{q} \equiv 1 - \frac{S-\sqrt{S^2-2}}{2}$ (one root is negative). From the second, we obtain $q > 1/2$ which is satisfied by $\bar{q}$ as $S > 3/2$.

At the other extreme where $q < 1-S/2$ (case **i**), the domestic firm plays the dominant strategy $S/2$ and the best reply of the foreign firm is then $S-q$; those prices form the unique Nash equilibrium which features an uncovered market because $D_d = S/2 < 1-q$ and $D_f = q$. If the quota is only slightly larger (case **ii**), this kind of equilibrium where firms do not compete, stills prevails. The only difference is that the domestic plays $p_d = S-1+q$ against $S-q$ and the market is exactly covered. This behaviour is optimal for the domestic firm if $S-q < \hat{p}_f$ which leads to $q < 1-S/3$.

When $q \in [1-S/3;\bar{q}]$ (case **iii**), we have $1 < \hat{p}_f < S-q$ as on figure 6 below which depicts a typical configuration of the best reply functions (in bold).

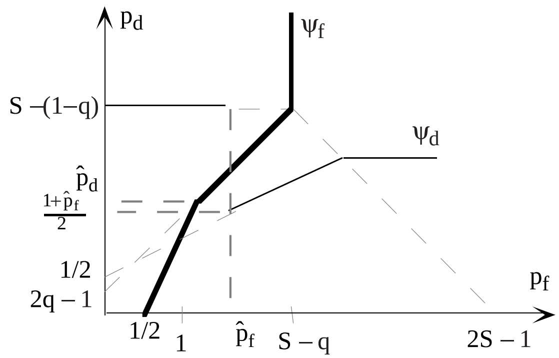


Figure 6

The curves do not intersect, hence there exists no pure strategy equilibrium. Still, the foreign firm plays a pure strategy in equilibrium, it must be $\hat{p}_f$ because it is the only one that enables the domestic firm to mix between S – 1 + q and $\frac{1+\hat{p}_f}{2}$. The weight μ put by the domestic firm on S – 1 + q is chosen to make $\hat{p}_f$ a best reply for the foreign firm against that mixture. More precisely, the foreign profit against the mixed strategy F_d is

$$\Pi_f(F_d, p_f) = p_f\left[(1-\mu)q + \mu\tilde{x}\left(\tfrac{1+\hat{p}_f}{2}, p_f\right)\right]$$

By solving $\frac{\partial \Pi_f(F_d, p_f)}{\partial p_f} = 0$, we get the argmax as a function P(.) of μ, we then solve P(μ) = $\hat{p}_f$ to get the desired weight $\bar{\mu} = \frac{4q}{4q - 3 + 3\hat{p}_f}$. ♦

The following comments are in order. Note first that by considering the complete range of possible values of the quota, we give a precise content to the idea of a quota "in the vicinity of the FTE" considered by Krishna [89]. More precisely, a mixed strategy equilibrium is found to exist only for a range of intermediate values of the quota (case **iii**). Second, it is easy to relate the size of this interval $[1 - S/3; \bar{q}]$ to S, the fundamental parameter of the model. As $\bar{q}$ increases with S, the larger S, the larger the interval which supports a mixed strategy equilibrium. The reasons for this are quite intuitive : when S is large, the profit levels at the Hotelling equilibrium are well below the monopoly profit levels[5]. Since the security strategy basically allows the domestic producer to reach her monopoly profit curve, she has a great incentive to do so.

A third observation is that a pure strategy equilibrium can exist under highly restrictive quota levels (case **ii**). This possibility was not considered by Krishna [89] and is indeed not relevant in her setting. This result is very specific to the Hotelling model and relies on the idea of localised competition which is embodied into the Hotelling framework : the quota weakens the incentives to compete in prices by allowing both firms to play along their respective local monopolists' curves. Stated differently, a quota under horizontal differentiation essentially allows both firms to benefit from their local market advantages. In the limit, the domestic firm enjoys her full monopoly profits if both q and S are low (case **i**).

4) Quotas and Welfare

In this section we address two questions. First, to what extent could the quota be voluntary ? Second, what is the optimal level of the quota from a domestic welfare viewpoint ? The first question is prompted by the comparison of this model with that of Krishna [89]. She concentrates on quota in the vicinity of the FTE. We will show that in our model the foreign firm would indeed choose the level of the VER in this region in order to maximise profits. Depending on the value of the parameters, this voluntary export restraint will be chosen above or below the FTE level. On the other hand, a government aiming at maximising domestic welfare will have to account of three factors when choosing the level of the quota. First, there is the profit diversion effect, i.e. the part of the total welfare which is captured by the foreign producer in the presence of the quota. Second, total welfare is maximised when prices are equal. In this case indeed, utility losses reflecting the fact that consumers are not able to buy their ideal product are minimised. Therefore, domestic welfare will be lower when prices are not equal since the marginal consumer will not be located in the middle of the market. Third, a quota affects welfare negatively if it prevents full market coverage.

[5] Either because the reservation price is large or because the transportation cost is low i.e., products are poorly differentiated and the price competition is fierce.

We concentrate first on the implications of the quota on the foreign producer's payoffs. We summarise hereafter the payoffs of the firms in the four kind of equilibria.

$q \in$	Π_f	Π_d
$\left[0;1-\frac{S}{2}\right[$	$q(S-q)$	$S/4$
$\left[1-\frac{S}{2};1-\frac{S}{3}\right[$	$q(S-q)$	$(1-q)(S-1+q)$
$\left[1-\frac{S}{3};\bar{q}\right[$	$\frac{2q(\hat{p}_f)^2}{4q-3+3\hat{p}_f}$	$(1-q)(S-1+q)$
$\left[\bar{q};1\right]$	$1/2$	$1/2$

We assumed $S \geq \frac{3}{2} \Rightarrow S > \frac{6}{5} \Rightarrow \frac{S}{2} > 1 - \frac{S}{3} \Rightarrow \forall q \in \left[0;1-\frac{S}{3}\right], S - 2q > 0$, this means that Π_f is increasing with the quota over $\left[0;1-\frac{S}{3}\right]$. The formula of Π_f over $\left[1-\frac{S}{3};\bar{q}\right[$ involves an intricate polynomial expression which cannot be studied analytically in q. However, it can be shown that on this domain the function is smooth, so that we can rely on numerical computations. These computations indicate that the foreign producer's payoff is strictly concave in the domain of the mixed strategy equilibrium. Since profit is constant over $\left[\bar{q};1\right]$, we may thus conclude that Π_f reaches a maximum for a quota interior to $\left[1-\frac{S}{3};\bar{q}\right[$. Moreover, a numerical maximisation indicate that this optimal quota is slightly increasing in S and lie in the vicinity of Free Trade demand level (i.e., 1/2). We summarise our findings in the following proposition.

Proposition 3

The foreign producer would choose a VER in the vicinity of the FTE.

On figure 7, we provide a plot of the foreign profit surface in the (q,S) space ; the various levels of grey correspond to the four different kind of pricing equilibria.

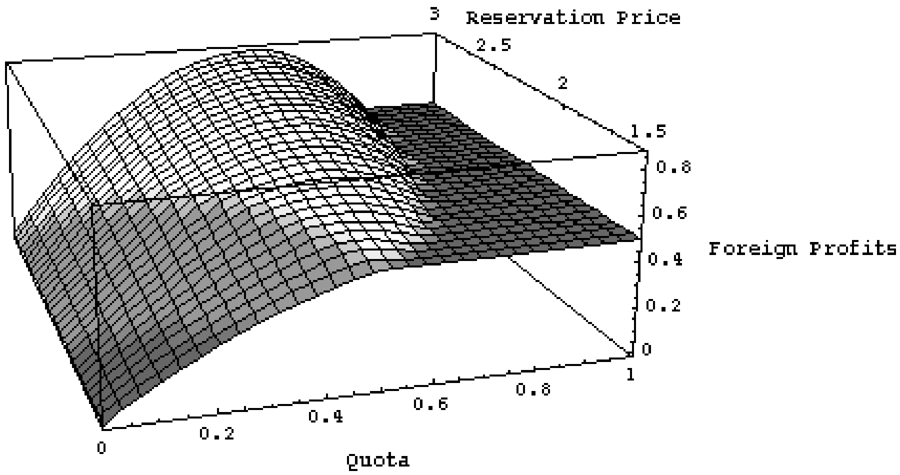


Figure 7

This result of proposition 3 is intuitive. Choosing a VER in the vicinity of Free Trade allows the foreign producer to take advantage of the price effect associated with the quota without penalising her too much in terms of potential sales. Note however that the profitability of the quota does not strictly depend on the equilibrium being in mixed strategies. One can see on figure 7 that for some combinations of q and S, the pure strategy equilibrium where the quota is binding (case **ii**) pays more than the free trade one because the lower sales are more than compensated by higher prices.

We turn now to the domestic welfare issue. The domestic welfare W_d is obtained from the total welfare by subtracting the foreign profit. In the Hotelling model, the total welfare is easily derived because, as long as the market is covered, it does not depend on the level of prices but only on the position of the marginal consumers (which depends in turn on price differentials only). Total welfare is :

$$W(p_d, p_f) = \int_0^{\tilde{x}} (S-x)dx + \int_{\tilde{x}}^1 (S-1+x)dx = S - \frac{2\tilde{x}(p_d,p_f)^2 - 2\tilde{x}(p_d,p_f)+1}{2} \qquad \text{(E1)}$$

It is immediate to see that total welfare is maximised when $\tilde{x}(p_d, p_f) = 1/2$ i.e., for identical prices. The quota will therefore affect domestic welfare in two obvious ways. First is the impact of foreign profits and second is the impact of price differentials.

Proposition 4

The domestic government would choose protectionism unless the domestic firm does not cover the market by itself in which case the foreign firm is allowed to sell the complement.

Proof When the market is uncovered (case **i**), the domestic firm is a pure monopolist and the foreign firm is constrained by the quota, thus we have :

$$\forall q \in \left[0;1-\tfrac{S}{2}\right[,\ W^{\mathbf{i}} = \int_0^{\frac{S}{2}}(S-x)dx + \int_{1-q}^{1}(S-1+x)dx = \frac{3S^2}{8} + q\left(S-\frac{q}{2}\right)$$

As $\Pi_f = q(S-q)$ on this domain, the domestic welfare is $W_d^{\mathbf{i}} = \dfrac{3S^2}{8} + \dfrac{q^2}{2}$ which is increasing in the quota. From this observation, we conclude that for low values of S, the domestic government will not implement a full protectionism policy, he will issue a quota that enables the foreign firm to serve the part of the market left uncovered by the domestic firm (who act as a monopolist on this range of quotas).

In case **ii** where $q \in \left[1-\frac{S}{2};1-\frac{S}{3}\right[$, the marginal consumer is located at **q** and (E1) becomes $W^{\mathbf{ii}} = S - \dfrac{2(1-q)^2 - 2(1-q)+1}{2} = S - \dfrac{2q^2-2q+1}{2}$. As $\Pi_f = q(S-q)$, we get $W_d^{\mathbf{ii}} = q(1-S) + S - \dfrac{1}{2}$ which is decreasing with the quota. Hence, over $\left[0;1-\frac{S}{3}\right[$, the optimal quota is $1-\frac{S}{2}$ which we call the "market-complement".

Case **iii** is the most complex because the domestic firm mixes between S − 1 + q and $\frac{1+\hat{p}_f}{2}$ with probabilities $\overline{\mu}$ and $1-\overline{\mu}$, the foreign firm sticks to $\hat{p}_f$. Total welfare is thus the average formula $W^{\mathbf{iii}} = \overline{\mu}W(S-1+q,\hat{p}_f) + (1-\overline{\mu})W(\frac{1+\hat{p}_f}{2},\hat{p}_f)$. This function cannot be studied analytically. However, it is smooth in the relevant domain and numerical computations indicate that domestic welfare is strictly convex, therefore it cannot be optimal for the government to set a quota in this region. Intuitively, this could have been expected since foreign profits tends to be higher in this region so that profit

diversion affects domestic welfare negatively, moreover prices are not equal so that total welfare must also be lower.

Lastly, when the Free Trade equilibrium prevails (case **iv**), total welfare is maximum because the marginal consumer is in the middle of the market. Since neither the foreign profit nor the total welfare depend on the quota, the domestic welfare is constant on $[\overline{q};1]$ and equal to S – 3/4.

To find the overall optimal quota for the government, we have to compare the free trade solution (q = 1) to the "market-complement" one ($q = 1-\frac{S}{2}$). When S > 2, this latter option is irrelevant and complete protectionism is optimal. However, when S < 2, $W_d^{\mathbf{i}} = \dfrac{3S^2}{8} + \dfrac{q^2}{2}$ evaluated at $q = 1-\frac{S}{2}$ is greater than the free trade welfare S – 3/4, thus it is optimal to let the foreign firm cover the part of the market not served by the domestic monopolist. Note furthermore that this particular choice of quota is not followed by price competition as case **i** or **ii** applies in the equilibrium of the game. ♦

Figure 8 provides a plot of the domestic welfare surface in the (q,S) space.

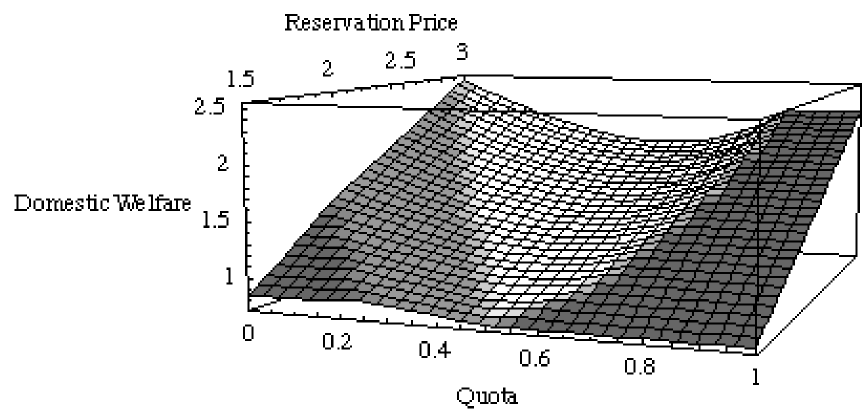


Figure 8

5) Final remarks

In this note, we have studied by mean of an example the implication of Export Restraints in an address-model of horizontal differentiation. Our findings are clearly specific to the particular case we have studied, however several generalisations can be considered. First, our results have been derived using an efficient rationing rule. In this respect, it must be noted that this form of rationing is in fact the most favourable for the domestic producer. In this sense, any other rationing rule would make deviations less profitable, thereby reducing the domain in which the quota will lead to a mixed strategy equilibrium. On the other hand, under other rationing rules, it could happen that some of the rationed consumers do not consume at an equilibrium. This clearly has a negative impact on Welfare.

Second, one could argue that the parameter constellations in which a pure strategy equilibrium is compatible with a restrictive quota (cases **i** and **ii**) are not the most likely to be observed as we must have $S \in \left[\frac{3}{2};3\right[$ in order that the quota interval $\left[0;1-\frac{S}{3}\right[$ is non void. In this respect, it is important to recall that the analysis has been performed under zero production cost. It is easy to see that under symmetric constant marginal cost **c**, the relevant constellation is $S - c \in \left[\frac{3}{2};3\right[$. Thus, the case for a restrictive quota depends in fact on the difference between the valuation of the product on the consumers' side in the domestic market and the production cost. Note also that the presence of a cost differential would not affect qualitatively our results. If the domestic producer faces a cost disadvantage, this reinforces the case for protectionism, other things being equal.

Clearly, assuming that products are located at both ends of the interval is restrictive. This assumption has been made in order to preserve the tractability of the problem. However, as long as efficient rationing prevails, the implications of inside locations are easy to trace. The only possible effect is that the residual demand addressed to the domestic producer becomes smaller. This would be the case if rationed consumers are located close to the right end of the interval. Thus, the incentive to hide behind the quota are weaker. An interesting extension of the present paper would consist in endogeneizing the choice of products' characteristics in the presence of the quota. However, this task is clearly beyond the scope of the present paper.

Note finally that our analysis has been confined to the case of horizontal differentiation. In this respect, we have shown that the quota relaxes price competition by allowing firms to exploit the local monopoly structure which is inherent to the Hotelling model. Therefore, one should not expect to observe the same kind of result in a vertical differentiation model. In another paper (Boccard & Wauthy [97a]) we study this issue. Relying on our findings in this paper and the present one, we are quite confident on the fact that the effects of a quota dramatically depend on the nature of product differentiation

References

Boccard N. & X. Wauthy (1997a) : Export Restraints and Vertical Differentiation, Mimeo, UCL.

Boccard N. & X. Wauthy (1997b) : Capacity pre-commitment in Hotelling's model, Mimeo, UCL.

Edgeworth F. (1925), The theory of pure monopoly, in *Papers relating to political economy*, vol. 1, MacMillan, London

Hotelling H. (1929), Stability in competition, *Economic Journal*, 39, 41-57

Kreps D. M. and J. Scheinkman (1983), Quantity precommitment and Bertrand competition yields Cournot outcomes, *Bell Journal of Economics*, 14, 326-337

Krishna K. (1989): Trade Restrictions as Facilitating Practices, *Journal of International Economics*, 26, p251-270.

Levitan R. & M. Shubik (1972) : Price Duopoly and Capacity Constraints, International *Economic Review* , 13, p111-122.

Osborne M. and C. Pitchick (1986), Price competition in a capacity-constrained Duopoly, *Journal of Economic Theory*, 38, p238-260.

Osborne M. and C. Pitchick (1986), Equilibrium in Hotelling's model of spatial competition, *Econometrica*, 55, p911-922.

Schmitt N. (1990) : Two country Trade Liberalisation in an Address Model of Product Differentiation, *Canadian Journal of Economics* , 23, p654-675.

Schmitt N. (1995) : Product Imitation, Product Differentiation and International Trade, *International Economic Review*, 36, p583-608.