Efficiency wages, labor heterogeneity and the financing of the training cost

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Abstract

We consider a dual labor market with a continuum of heterogeneous workers differentiated by their ability of acquiring a specific skill. In the primary sector, jobs require firm-specific training and firms set efficiency wages. In the secondary sector, wages are competitive and no training is required. Given workers heterogeneity, firms in the primary sector face an elastic labor supply, so that they can be labor constrained at the efficiency wage. When this is the case, we show that firms may optimally choose to bear all the training cost in order to relax the labor supply constraint.

Keywords: dual labor markets, efficiency wages, training costs, workers heterogeneity, labor supply constraint.

J.E.L. Classification: J31, J41

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1 Introduction

E¢ciency wage models explain why rms do not want to cut wages when involuntary unemployment prevails. They provide a theoretical foundation for the existence of a dual labor market (Doeringer and Piore, 1971). A dual labor market consists of two sectors. In the primary sector, jobs are stable and are well paid. In the secondary one, these features are absent. Jobs tend to be short-term and low wage. The main di/erence between the two sectors is the type of jobs o/ered. Assuming that primary jobs are more complex than secondary jobs so that it is more di¢cult to monitor workers performance, it is optimal for rms to pay an e¢ciency wage to deter shirking. The wage differences between sectors re ect the presence of an e¢ciency wage (Akerlof and Yellen, 1986) in the primary sector which is above the market clearing wage or the minimum wage in the secondary sector (Bulow and Summers, 1987, Krueger and Summers, 1988, Klundert, 1989, Albrecht and Vroman, 1992, Saint-Paul, 1996, Smith and Zenou, 1997 among others).[1] Thus, it is the di/erence in wage setting behavior that explains why, in equilibrium, identical workers earn di/erent wages.

In the present paper, we also consider a dual labor market with e¢ciency wages in the primary sector. Relying on the idea that jobs are more complex in this sector, we assume that a speci c quali cation level must be acquired by workers which involves a training cost (see Taubman and Wachter, 1986, for other justi cations of the fact that rm-speci c training is important in the primary sector). Our key assumption is that workers are totally heterogeneous in their ability of acquiring the required skill It follows that, if workers bear a part of the training cost, they are not equally willing to enter into the primary sector so that this sector faces a nitely elastic labor supply In the secondary sector, no training is required so that workers can be viewed as homogeneous with respect to the secondary sector.

Observe that the idea of introducing labor heterogeneity in a dual labor market is not new. In particular, workers can di/er in terms of the value placed on leisure (Albrecht and Vroman, 1992), their exogenous turnover probability (Jones 1987), their productivity (McCormick, 1990), Gottfries and McCormick, 1995).

In our model, rms set an e¢ciency wage (Solow, 1979) in the primary sector since labor productivity depends on workers e/ort: rms induce optimal e/ort through (e¢ciency) wages. In this context, they set optimally their labor demand but can however face labor supply constraint at this wage. We show that two market outcomes may result at the Solow e¢ciency wage.

[1] Another way of di/erentiating the two sectors is to consider that primary sectors are more unionized than secondary ones. In this case, it is not the type of jobs that makes the two sectors di/erent but the fact that institutions di/er between sectors.

First, a standard dual labor market may be observed. Labor rationing can arise in equilibrium when some workers are forced to work in the secondary sector even though they want to work in the primary one at the prevailing wage. If the rm bears a part of the training cost, these workers will be the ones that have the average level of ability of the distribution. Why ? Because workers with very low ability will never apply for a primary job since their training costs are too large and very high ability workers will always be hired in the primary sector since rms bear a part of the training cost. Thus workers with middle ability level will be very sensitive to demand shocks and to the way training costs are shared between the workers and the rm. At the prevailing wage, workers bene t from di/erent net wages in the primary sector since they di/er in training costs. For very low ability workers, the net wage is higher ir the secondary sector, so that they choose to work there. More able workers will nd it pro table to apply in the primary sector but not all of them will be hired. Typically, workers with intermediate abilities will be rationed and forc to work in the secondary sector.

Second, and more importantly, the presence of the training cost may result in a situation where the primary rm is rationed by the labor supply at the e¢ciency wage. In this case, setting the standard e¢ciency wage is not anymore an optimal strategy. There is a trade o/ for primary rms between inducing the optimal e/ort through e¢ciency wages and having enough workers. Since the rm has to solve two problems with only one instrument (the wage), it is obvious that it could bene t from using a second instrument. Therefore, the real question for the rm is: who nance the training cost ? In our setting, two policies are possible. In the rst one, the rm does not internalize the traini cost and sets a wage greater than the Solow e¢ciency wage along the labor supply. When this happens, there is no rationing in the labor market so that labor dualism does not prevail. In the other one, the rm bears all the training cost. In this case, labor supply is in nitely elastic so that we observe ration at the equilibrium wages, i.e. labor dualism prevails in our model. Thus suggests that in the presence of an heterogeneous workforce, e¢ciency considerations may explain why rms would be willing to bear the training costs.

The remainder of the paper is organized as follows. Section 2 presents the general model when the share of the training cost is exogenously set. We show that three market outcomes may result from e¢ciency wage considerations. In section 3, we show what is the optimal policy for the primary rm when it chooses wages, the employment level and its part of the training cost. We derive some numerical simulations in section 4 to illustrate our main results. Finall section 5 concludes.

2 The model when the share of the training cost is exogenous

There are two sectors in the economy characterized by a large number of identical firms in each of them. Without loss of generality, we consider one representative firm in each sector.

In the primary sector, jobs are complex and require a training period in order to meet the productivity level required by the (representative) primary firm. This means that when a worker is hired a contract is signed stipulating that the worker must be perfectly matched with the firm and thus produces the productivity level required. The qualification level required by the primary firm is equal to Q which is normalized to 1 for analytical simplicity. We assume that each worker is endowed with an indivisible unit of labor. We also assume that, *ex ante*, there is a continuum of workers ranked by their decreasing ability of learning a specific training and distributed uniformly in the interval $[0, \bar{x}]$. Firms are able to identify ex ante workers' type. For simplicity, the density of workers in each point of the interval is taken to be unity so that the total workers in the economy is equal to $N = \bar{x}$. Each individual is characterized by a parameter $x \in [0, \bar{x}]$ which is defined so that the training cost is x, increasing in x. We assume here that the training cost is exogenously shared between the worker and the firm, $0 \le \alpha < 1$ being the part borne by the firm.[2] In this context, a worker of type x must bear a training cost equal to $(1-\alpha)x$ in order to work in the primary firm. Therefore, after the training period (*ex post*) all workers are identical and provide the same level of effort e within the firm.

In this sector, all workers who are *ex post* identical have a utility function equal to $V(w, e)$, where w is the wage in the primary sector. We assume that $V_w > 0, V_{ee} < 0, V_{ew} > 0, V_{eww} \le 0$ and $V_{eew} \le 0$. We will see later the point of the last two hypotheses. Each worker solves the following program:

$$\max_e V(w, e) \tag{1}$$

and obtains:

$$V_e(w, e) = 0 \tag{2}$$

Equation (2) leads to an effort function: $e = e(w)$, with $e(0) > 0$. By totally differentiating (2), we have:

$$e'(w) = \frac{de}{dw} = -\frac{V_{ew}}{V_{ee}} > 0$$

and

$$e''(w) = \frac{d^2e}{dw^2} = \frac{-[V_{eww}V_{ee} - V_{ew}V_{eew}]}{(V_{ee})^2} < 0$$

since $V_{eww} \le 0$ and $V_{eew} \le 0$. We assume that the effort e is unobservable so that the employer has to rely on his wage w to provide the motivation (for a discussion

[2]We will relax this assumption in the next section.

of this type of effort function see Layard, Nickell and Jackman, 1991, ch.3). The employer knows that, given w, the worker solves (1) $e(w)$. Thus given , the firm selects a wage w^* (the efficiency wage) that maximizes its profit (Solow, 1979). At this efficiency wage, all workers provide the corresponding effort $e(w^*)$ (see Figure 1 for an illustration of the effort function). Moreover, we assume that the net equilibrium utility level for a worker of type x in the primary sector is equal to $V(w^*; e^*) - (1 - x)$. Thus $V(w^*; e^*)$ and the training cost problem are separable from the worker's viewpoint.

To summarize there are two different but independent problems, one *ex ante*, one *ex post*. *Ex ante*, workers are heterogeneous and must be trained in order to provide . *Ex post*, all workers are identical from the firm's viewpoint and the optimal policy is to set an efficiency wage that maximize its profits. Since the efficiency wage does not depend on the initial ability of workers and since the net equilibrium utility for a type x worker is $V(w^*; e^*) - (1 - x)$, the two problems are totally separable. We have therefore collapsed the two period model into a one period one. An alternative approach would have been to set a wage for each worker and therefore not to impose a perfect match between workers and the firm. In equilibrium, there would be a distribution of wages and all workers would still be heterogenous *ex post*. We have privileged the former approach since our focus is on dual labor market where in the primary sector all workers earn the same wage and are rationed. In the latter approach there will obviously not be any rationing.

[Figure 1]

In the secondary sector, jobs are less complex so that no training is required. We also assume the effort $\overline{e}$ in the secondary sector is perfectly observable and is negligible. Perfect competition prevails in this market so that in equilibrium the competitive wage is such that workers in the secondary sector are indifferent between working in there or being unemployed. For the simplicity of exposition let us denote by $\overline{w} = V(\overline{e}; w)$ the reservation utility in the secondary sector.

In order to decide whether to work or not in the primary sector, a worker of type x trades off $V(w^*; e^*) - (1 - x)$ and $\overline{w}$. According to our previous assumptions, the reservation wage is positively related to workers' ability. When $V(w^*; e^*) - (1 - x) \geq \overline{w}$, the worker decides to apply to the primary firm. The marginal worker $\tilde{x}$ who is indifferent between working in the primary firm and in the secondary one is defined by: $V(w^*; e^*) - (1 - \tilde{x}) = \overline{w}$. We have therefore:

$$\tilde{x} = \frac{V(w^*; e^*) - \overline{w}}{(1 - \ )} \tag{3}$$

Equation (3) expresses the labor supply of the primary sector as a function of the utility difference between the two sectors and the share of training cost. This means that the number of applicants to the primary sector is endogenous and is equal to $\tilde{x}$. By differentiating (3), we obtain:

$$\frac{\partial \tilde{x}}{\partial V(w^*; e^*)} > 0 \quad ; \quad \frac{\partial \tilde{x}}{\partial} > 0 \quad ; \quad \frac{\partial \tilde{x}}{\partial \overline{w}} < 0 \tag{4}$$

The larger the di/erence between the net primary and the secondary sector utilities or the greater the part of the training cost nanced by the rm, the higher x . Thus, a worker of a lower ability is more and more ready to work in the primary sector as soon as net utilities increase or training cost are lowere

We have now to determine the level of wage and employment set by the primary rm. Observe that in the secondary sector, rms pay the market clearing wage and at this wage they can hire all workers they want $\overline{w}$ Obviously   can be interpreted as the level of unemployment bene t since in equilibrium workers will be indi/erent between being unemployed and working in the secondary sector (there is no rationing in this sector). The pro t function of the primary rm writes:

$$= pF(e(w)L) \quad wL \quad \int_0^L xdx \tag{5}$$

where p is the output price, $e(w)$, the (aggregate) e/ort function of all workers, L, the level of employment and $F(e(w)L)$, the production function. We assume that $F(0) = 0$, $F^0(0) = 1$, $F^0(1+) = 0$ (Inada conditions), $F^0(:) > 0$ and $F^{00}(:) < 0$ Observe that what matters to rms is not the level of employment but the e¢ciency level of it, $L_e = e(w)L$: We can rewrite (5) as:

$$= pF(L_e) \quad \frac{w}{e(w)} L_e \quad \int_0^L xdx \tag{6}$$

where $w=e(w)$ is the labor cost per unit of e/ort. Inspection of (6) suggests that the wage determination is independent of the employment one. In a rst stage, the rm solves the following program:

$$\min_w \frac{w}{e(w)} \tag{7}$$

It is easy to show that we obtain:

$$_{e;w} = \frac{w\ e^0(w)}{e(w)} = 1 \tag{8}$$

where w is the e¢ciency wage and $_{e;w}$, the elasticity of e/ort with respect to wage. This is the so-called Solow condition (see Solow, 1979) stating that the e¢ciency wage is such that the mean e/ort is equal to the marginal e/ort (see Figure 1). Observe that the e¢ciency wage (8) does not depend on the production function but only on the parameters of the e/ort function. Observe also that it does not depend on , the training cost, since once employed all workers must contribute to the same level of output whatever their initial ability For this (e¢ciency) wage, the rm obtain an optimal e/ort level equal to:

$$e = e(w) \tag{9}$$

Note that there will be rents for workers in the rm since there is a distribution of net utilities associated with the e¢ciency wage (a worker of type x earns a net utility level equal to $V(w, e)\ (1 \quad x)$. The efficiency wage is such that (ex post) trained workers are all motivated and provide the effort level e. So the higher the initial ability, the greater the net utility level (see Figure

[Figure 2]

We can now determine the level of efficient employment in the firm. The latter solves the following program:[3]

$$\max_{L_e} \quad = pF(L_e) \quad \frac{w}{e} L_e \quad \frac{}{2} \left[\frac{L_e}{e} \right]^2 \tag{10}$$

We obtain:

$$\frac{w}{e} + \frac{L_e}{e^2} = pF'(L_e) \tag{11}$$

Since $L = L_e / e$; the labor demand is defined by:

$$\frac{w + L}{e} = pF'(e\,L) \tag{12}$$

Thus the optimal level of labor demand is obtained by equating the marginal productivity and the general cost of labor per unit of effort. Three types of labor market outcomes can obtain when the firm adopts this strategy.

Outcome 1: $L < xe$ (see Figure 3)

In this case, only workers of type $x \quad L$ are hired by the primary firm.[4] This means that there is an endogenous rationing due to the fact that at the efficiency wage the labor demand is lower than the indifferent worker xe. Even if workers of type $L < x \quad xe$ propose to work at a lower wage than the efficiency wage w (any wage that guarantee them to have a net wage greater than the secondary wage), the primary firm refuses to hire them because they will not provide the optimal effort level e. In other words, for the primary firm, a wage cut will decrease its profit because of the reduction of the marginal productivity. These workers are typically rationed since for all of them the utility level in the primary sector $V(w, e)\ (1 \quad x \quad x2(L, xe])$ is strictly greater than the utility level in the secondary sector, i.e., $\overline{w}$. Thus, we can denote the workers who are involuntary unemployed (or employed in the secondary sector)[5] by U_I and they are equal to:

$$U_I = xe \quad L > 0 \tag{13}$$

with

$$\frac{\partial U_I}{\partial} > 0 \quad ; \quad \frac{\partial U_I}{\partial V(w, e)} > 0 \quad ; \quad \frac{\partial U_I}{\partial \overline{w}} < 0 \quad ; \quad \frac{\partial U_I}{\partial p} < 0 \tag{14}$$

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[3] It is readily verified that the second order condition is always satisfied.

[4] Note that we assume here that the firm is able to distinguish workers type, so that it hires first the most able workers in order to minimize training costs.

[5] Since the secondary sector is a waiting sector, workers are indifferent between working there or being unemployed. Throughout the text, we will use indifferently voluntary or involuntary employed in the secondary sector and voluntary or involuntary unemployed.

Clearly, outcome 1 exhibits the main feature of a dual labor market under efficiency wage since there is rationing in equilibrium. The only difference, which results from workers heterogeneity is that only part of the secondary sector workers are frustrated from being there. As it can be seen from (13), these static comparative results on U_I stem from two effects: the marginal worker and the labor demand ones. The first result in (14) is due to the fact that when rises, the position of the marginal worker x increases and the labor demand L decreases. The increase of the net utility level $V(w;e)$ has a similar interpretation. Concerning the secondary wage, $\overline{w}$, its effect is the following: when it rises, it does not affect L but reduces the position of x which switches to the right and thus cuts U_I. Last, when there is a negative demand shock, i.e., the output price decreases, profit maximizing firms hire less workers and U_I is increased. What about $U_V = \overline{N} - x$, the voluntary part of unemployment? Since these workers would all have a lower utility level in the primary sector than in the secondary sector ($V(w;e)(1-x) < \overline{w}$), they prefer to work in the secondary sector and they are therefore not rationed. We have:

$$\frac{\partial U_V}{\partial} < 0 \quad ; \quad \frac{\partial U_V}{\partial V(w;e)} < 0 \quad ; \quad \frac{\partial U_V}{\partial \overline{w}} > 0 \quad ; \quad \frac{\partial U_V}{\partial p} = 0 \qquad (15)$$

We obtain the opposite results than (14) since an increase in , $\overline{w}$ or affects only the marginal worker x. Of course, the latter is not affected by an output price cut.

More generally, this equilibrium shows that, by introducing workers heterogeneity, the view of dual labor market becomes different. The main part of the secondary labor force is composed by workers who have a lower level of ability than primary workers. Just a small part of the secondary sector can be employed in the primary sector and that depends on the labor demand and thus on the state of nature. Thus, the size of secondary sector is largely contingent on the occurrence on demand shocks but also on the matching technology (through). The higher the degree of specialization of firms, the greater the firm-specific training and the larger the involuntary unemployment. Observe also that in equilibrium, heterogeneous workers earn the same wage and enjoy the same utility level in the secondary sector whereas in the primary sector, they have the same wage but there is a distribution of utility levels, the more talented workers enjoying the higher utility level. In some sense, the secondary sector does not recognize workers heterogeneity whereas the primary sector does. Let us now define our second type of equilibrium.

[Figure 3]

Outcome 2: $L = x$ (see Figure 4)

The conjuncture is better and the primary sector can hire all workers that have a utility level greater than $\overline{w}$. Nobody is rationed and according to his initial ability each worker is pleased with his situation. We have:

$$U_I = 0 \quad \text{and} \quad U_V = \overline{N} - x \qquad (16)$$

Thus with no rationing dual labor market still exists. It is the heterogeneity of workers that creates labor dualism. In equilibrium, we have the same results as before in terms of wages and utility levels in the two sectors.

[Figure 4]

Outcome 3: $L > \tilde{x}$ (see Figure 5)

All workers of type $x \in [\tilde{x}; L]$ refuse to work in the primary sector because their utility level is lower than the one in the secondary sector. Thus secondary employment would coexist with labor demand rationing in the primary sector.

[Figure 5]

This suggests that Outcome 3 is not an equilibrium for the firm. Indeed, it could contemplate to raise the wage in order to relax the labor supply constraint. This illustrates the fact that efficiency wages are optimal only under excess supply. However, when workers are not equally willing to enter the primary sector, more generally when labor supply in the primary sector is not perfectly elastic, we have no particular reason to think that excess supply will prevail at efficiency wages. Note also that increasing would have the same effect. Indeed, we may argue that part of the problem comes from the fact that the firm has only one instrument in order to solve two different problems: ensure efficiency in the firm and attract workers. In the next section we will tackle this issue. The following proposition summarizes our result.

Proposition 1 When is exogenous, three outcomes may emerge at the Solow efficiency wage.

- If the primary firm is not rationed with $L < \tilde{x}$, the employment level is determined by profit maximization. Some workers are involuntary unemployed.
- If the primary firm is not rationed with $L = \tilde{x}$, it does the same policy but nobody is involuntary unemployed.
- If the primary firm is rationed, i.e. $L > \tilde{x}$, the standard efficiency wage policy is not an equilibrium.

3 The endogenous choice of the share of the training cost

In the previous section, we have assumed that the training cost was shared by the firm and the workers. One could easily argue however that the primary firm could be tempted to transfer as much as possible of that cost onto workers. This is particularly clear when the primary firm benefits from monopsonistic power. Outcome 3 however has shown that the presence of positive training costs on the workers' side can prevent the firm from implementing strict efficiency wage policy. Obviously, one possible strategy for the primary firm in case of excess demand at the efficiency wage would be to bear more training cost in order to relax the constraint while preserving efficiency of labor in the firm. In order to address this problem, we consider now that the firm can choose optimally w, L and . Therefore, the firm's problem is stated as follows:

$$\max_{w;L;} \left[= pF(e(w)L) - wL - \frac{L^2}{2} \right]$$

$$s:t: e = x(V(w;e) - \bar{w}) + 1 \quad L \tag{17}$$

$$0 \quad 1$$

The Lagrangian associated with (17) is given by (is the Lagrange multiplier):

$$\mathcal{L} = pF(eL) - wL - \frac{L^2}{2} + \lambda\left(V(w;e) - \bar{w} - L + L\right) \tag{18}$$

Proposition 2 It is optimal for the firm to set either $= 1$ or $= 0$.

Proof. By differentiating $\mathcal{L}$ with respect to , one obtains:

$$\frac{\partial \mathcal{L}}{\partial} = -\frac{L^2}{2} \quad L \lessgtr 0$$

In other words, the Lagrangian is linear in and its slope can either be negative or positive so that the optimal value of can be either 0 or 1.

The following comments are in order. First, since the Lagrangian is linear in , there are other solutions in which could take any value in $[0;1]$. However, we will focus only on corner solutions. Second, if the firm were not rationed ($= 0$ at the efficiency wage solution), it would have chosen always $= 0$. Thus the possibility of $= 1$ arises because, in this case, the firm is no more rationed ($e = + 1$) and can hire as many workers as it wants. We have to study now the two cases separately ($= 0$ and $= 1$) and then compute the firm's profit in each case. In some sense, it is a two stage decision in which the firm first maximizes its profit by choosing the optimal w and L for each , and then decides which it chooses. Let us study first the case $= 0$.

The Lagrangian (18) writes now:

$$\mathcal{L} = pF(e(w)L) - wL + (V(w;e) - \bar{w} - L) \tag{19}$$

First order conditions are given by:

$$\frac{\partial \mathcal{L}}{\partial w} = pF'(:)e'(:)L - L + V_w = 0 \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial L} = pF'(:)e(:) - w - L = 0 \tag{21}$$

$$\frac{\partial L}{\partial} = V(w;e) \quad \bar{w} \quad L \quad ; \qquad 0 \quad ; \quad \frac{\partial L}{\partial} = 0 \tag{22}$$

In order to solve this problem two different cases must be contemplated:

CASE 1:[6] The firm sets $= 0$ and is not rationed: $V(w_1; e_1) \quad \bar{w} > L_1$

We consider an equilibrium candidate (w_1, L_1) such that $\frac{\partial L}{\partial} > 0$: By using (22), this implies that $= 0$. By combining (20) and (21), we obtain the efficiency wage as defined by the Solow condition:

$$\frac{e'(w_1)}{e(w_1)} = \frac{1}{w_1} \tag{23}$$

and the optimal level of employment is given by:

$$pF'(e(w_1)L_1)\, e(w_1) = w_1 \tag{24}$$

In this case, its (optimal) profit function writes:

$$_1 = pF(e(w_1)L_1) \quad w_1 L_1 \tag{25}$$

Proposition 3 If the firm chooses optimally $= 0$ and if $V(w_1; e(w_1)) \quad \bar{w} > L_1$; then the firm sets the efficiency wage (23) and the employment level (24).

This proposition is quite intuitive. Since the firm is not rationed, it can hire as many workers as it wants. In this case, it is obvious that $= 0$ is the optimal policy since from the firm s viewpoint all workers are identical and we are back to the standard efficiency wage model with homogeneous workers. In other words, transferring all training costs onto workers is compatible with pur efficiency wage considerations. It is therefore the optimum . We now turn to the case in which $= 0$ leads to a labor supply shortage at the efficiency wage.

CASE 2:[7] The firm sets $= 0$ and is rationed: $V(w_2; e(w_2)) \quad \bar{w} = L_2$

We consider an equilibrium candidate (w_2, L_2) such that $\frac{\partial L}{\partial} = 0$: According to (22), two cases arise: > 0 and $= 0$. If $= 0$, we are back to the previous case in which $V(w_1; e(w_1)) \quad \bar{w} = L_1$: When > 0, we must solve (20) et (21) for w_2 and under the constraint that $V(w_2; e(w_2)) \quad \bar{w} = L_2$. First, it its easily checked by combining (20) et (21) that:

$$= \frac{L_2 \left[e(:) \quad w_2 e'(:) \right]}{e'(:)(L_2 \quad ^2) + e(:) V_{w_2}}$$

Since > 0, it must be that $e'(w_2) < e(w_2)/w_2$. This second condition implies that the firm never sets the Solow efficiency wage w_1 (in which case $e'(w_1) = e(w_1)/w_1$) but always a greater one, i.e. $w_2 > w_1$. We can calculate this new efficiency wage. By using (20) and (21), it is readily verified that this efficiency wage is equal to:

$$w_2 = pF'(:)\, e(:) + \frac{(L_2 \quad)^2}{V_{w_2}} \left(pF'(:)\, e'(:) \quad 1 \right) \tag{26}$$

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[6] We use the subscript 1 to refer to this case.

[7] We use the subscript 2 for this case.

and the associated optimal employment level is given by:

$$L_2 = V(w_2; e(w_2))\,\overline{w} \tag{27}$$

where w_2 is defined by (26). Observe that $L_2 < L_1$ because the labor demand defined by (26) is below the one define in case 1 (see Figure 6). We can write now the optimal profit level as:

$$_2 = pF(e(w_2)V(w_2 \quad e(w_2)\,)\,\overline{w}\,))\, w_2 V\,[w_2; e(w_2)\,(\overline{w})\,] \tag{28}$$

Proposition 4 Let the firm choose optimally $= 0$. If $V(w_1; e(w_1))\,\overline{w} = L_1$; then the firm sets the Solow e¢ciency wage (23). If $V(w_1; e(w_1))\,\overline{w} < L_1$; then the firm pays the e¢ciency wage (26) $w_2 > w_1$ and employs $L_2 = V(w_2; e(w_2))\,\overline{w} < L_1$.

The following comments are in order. First, the firm does not internalize the training costs and thus labor heterogeneity. In this context, there is a clear hierarchy in terms of net utility levels in the primary sector, the more able workers obtaining the highest ones. Second, when the firm is constrained in his labor supply and even if $= 0$, it will never be optimal to set an e¢ciency wage defined by the Solow condition. The firm is induced to increase the Solow e¢ciency wage up to the point where the marginal gain of employing an extra worker is equal to the marginal loss in terms of global e/ort e¢ciency. Contrary to the Solow case, this e¢ciency wage depends now on the shape of the production function and thus on the firm s technology. Last, the only possible unemployment here is voluntary because some workers of type x with middle level abilities prefer $\overline{w}$ than $V(w; e)$ x. Figure 6 illustrates Proposition 4 by comparing cases 1 and 2. If the firm is not rationed, it chooses the Solow solution $(w_1; L_1)$. However, the labor supply constraint $L^S = V(w; e(w))\,\overline{w}$ is violated at $w = w_1$ since at this wage the labor demand is L_1 and the labor supply is L^S_1, with $L_1 > L^S_1$. According to (21), the labor demand when > 0 is strictly below the one corresponding to the Solow case and therefore the firm s optimal policy is to increase the wage along the labor supply curve L^S. The equilibrium is thus characterized by a higher wage and a lower level of employment than in the Solow case.

[Figure 6]

Case 3[8]: The firm sets $= 1$

In this case, all workers are willing to work in the primary firm as soon as $V(w_3; e(w_3))$ is greater than $\overline{w}$. There is no rationing and thus no constraint. The firm solves now the following program:

$$\max_{w;L} \; _3 = pF(e(w_3)L_3) - w_3 L_3 - \frac{(L_3)^2}{2}$$

[8] Subscript 3 refers to this case.

First order conditions are given by:

$$\frac{\partial \Pi_3}{\partial w_3} = pF'(.)e'(.)L_3 - L_3 = 0 \tag{29}$$

$$\frac{\partial \Pi_3}{\partial L_3} = pF'(.)e(.) - w_3 - \gamma L_3 = 0 \tag{30}$$

By combining (29) and (30), we obtain the following efficiency wage:

$$\frac{e'(w_3)}{e(w_3)} = \frac{1}{w_3 + \gamma L_3} \tag{31}$$

which is defined by a modified Solow condition in which the training cost is strictly positive. Then by using (30), L_3 is the solution of the following equation:

$$w_3 + \gamma L_3 = pF'(e(w_3)L_3)e(w_3) \tag{32}$$

Observe that this efficiency wage is always greater than the one given by the standard Solow condition[9] (23). We have to check that there always exists a unique labor market equilibrium. By differentiating (31), we have:

$$\frac{dw_3}{dL_3} = -\frac{e'(.)\gamma}{e''(.)(w_3 + \gamma L_3)} > 0$$

and by differentiating (32), we obtain:

$$\frac{dw_3}{dL_3} = -\frac{pF''(.)e(.)^2 - \gamma}{pF''(.)e'(.)L_3 e(.) + pF'(.)e'(.) - 1} \tag{33}$$

which is negative if $\eta < 1 = pF'(.)e'(.)$, where $\eta = -F''(.)e(.)L/F'(.)$ is the elasticity of the marginal product with respect to the efficient units of labor. Thus, there exists a unique labor market equilibrium in the plane (L_3, w_3). The optimal profit function is equal to:

$$\Pi_3 = pF(e(w_3)L_3) - w_3 L_3 - \gamma\frac{(L_3)^2}{2} \tag{34}$$

Proposition 5 *If the firm chooses optimally $\alpha = 1$, then it sets the modified Solow efficiency wage (31) $w_3 > w_1$ and employs $L_3 < L_1$ (defined by (32)) workers.*

Observe that here contrary to cases 1 and 2, the primary firm internalizes labor heterogeneity so that all workers in the primary sector obtain the same utility level $V(w_3; e_3) > \overline{w}$. In this context, ex ante heterogeneous workers obtain ex post two utility levels $V(w_3; e_3)$ and $\overline{w}$ depending on their initial ability. Observe also that the only type of unemployment is involuntary since

[9] Let $F_1(w) = e'(w)w - e(w)$ and $F_3(w) = e'(w)(w + \gamma L_3) - e(w)$. Then it is easily checked that $F_1'(w) < 0$ and $F_3'(w) < 0$. Then it must be that $w_3 > w_1$ since $F_1(w_3) < F_3(w_3)$ (i.e. $F_1^{-1}(0) < F_3^{-1}(0)$).

all workers will apply in the primary sector. Figure 7 gives some intuition to Proposition 5 by comparing it with case 1 (Solow). In case 3, we have an increasing relation between wage and employment (31) which we called the modified Solow relation (when $L_3 = 0$, we obtain the Solow efficiency wage). Indeed, when the firm employs an extra worker it pays a higher training cost since the marginal training cost increases with L_3. It must therefore rise the efficiency wage in order to motivate this marginal worker. We have shown that in this case the labor demand is strictly below the one corresponding to case 1 because of positive training cost. Consequently, as in case 2, equilibrium involves a higher wage and a lower level of employment than in the Solow's case.

[Figure7]

We have now to analyze the optimal policy of the primary sector firm. It is obvious that the firm will always choose $= 0$, if this is compatible with the Solow efficiency wage w_1 (23) and the corresponding employment level L_1 (24), i.e. if the firm faces excess supply at this wage. This is the first best choice. When it is rationed, it has two possibilities. If it chooses $= 0$, the firm bears no training cost but it cannot set anymore the Solow efficiency wage since there is a labor supply constraint. It will set a wage above the Solow efficiency wage. It is important to notice that in this case all workers exhibit the same level of efficiency $F(\cdot)$ within the firm since there are all identical from the firm's viewpoint. Thus when the firm increases w_2 it employs more workers (since $L_2 = V(w_2; e(w_2))\ \overline{w}$) but the labor costs increases due to the fact that the wage is above the efficiency level. If it chooses $= 1$, there is no more a labor supply constraint but a training cost constraint. This training cost increases with the employment level since at the margin the firm employs workers that have to be more and more trained (the training cost rises with the type x). In this case, the firm sets the modified Solow wage. Here contrary to the previous case the trade-off is between $F(e(w_3)L_3)$ $w_3 L_3$ and $+L_3^2 = 2$. In this context, each worker can be viewed as exhibiting different efficiency levels, reflecting their training cost. The problem is not anymore how many workers to hire but how many will be efficient enough to cover their training cost. Note that in this case, very able workers "subsidize" the training costs of lower ability ones.

The firm faces thus a trade-off between what is lost in terms of each individual's efficiency, due to the higher wage (recall that the efficiency unit labor cost increases for wages above w_1) and what is gained by employing more workers. Choosing $= 1$ allows the firm to get rid of the labor supply constraint and to use the wage only to induce effort but it imposes an extra cost (the training cost). Choosing $= 0$ implies that the wage will be used only to increase the labor supply while imposing efficiency losses for all workers. It is then obvious that the shape of the effort function will play a crucial role in determining which of the two policies will be chosen. Indeed, the efficiency loss associated with a given increase in the wage will be more important if the effort function is very concave. In this case, it is more likely that the firm will prefer to bea the training cost in order to accommodate for a lower wage increase, and thus a lower efficiency loss at the individual level. On the other hand, the higher the reservation utility in the secondary sector, the larger the excess demand at th Solow wage, the greater the incentive to relax totally this constraint, i.e. t greater the incentive to bear the training cost. Finally, we should not expect that the production function will play a very important role in the analysis sinc it does not really discriminates between the two policies. In the next section w study a numerical example which will help us to clarify these intuitions.

4 A Numerical example

We assume that the utility function for a worker is:

$$V(w;e) = e\left(-a + w^b\right) - \frac{1}{2}e^2 \tag{35}$$

with $a > 0$ and $0 < b < 1$. It is easily checked that:

$$V_w = ebw^{b-1} > 0 \quad ; \quad V_{ee} = -1 < 0 \quad ; \quad V_{ew} = bw^{b-1} > 0$$

$$V_{eww} = b(b-1)w^{b-2} < 0 \quad ; \quad V_{eww} = 0$$

By choosing e that maximizes (35), each worker obtains the following effort function (see Figure 1 for an illustration):

$$e(w) = -a + w^b$$

with

$$e'(w) = bw^{b-1} > 0 \quad ; \quad e''(w) = b(b-1)w^{b-2} < 0$$

We assume that the production function writes:

$$F(e(w)L) = [e(w)L]^c \tag{36}$$

where $0 < c < 1$. In this context, the Solow efficiency wage corresponding to case 1 writes now:

$$w_1 = \left(\frac{a}{1-b}\right)^{1/b} \tag{37}$$

and the optimal effort function is:

$$e_1 = e(w_1) = \frac{ab}{1-b} \tag{38}$$

The corresponding labor demand is given by:

$$L_1 = (pc)^{\frac{1}{1-c}} \left(\frac{a}{1-b}\right)^{\frac{b-c}{(1-c)bc}} b^{\frac{1}{(1-c)c}} \tag{39}$$

and the optimal profit is:

$$\pi_1 = (pd)^{\frac{1}{1-c}} \left(\frac{a}{1-b}\right)^{\frac{b-c}{(1-c)b}+c} \left[b^{\frac{1}{(1-c)}+c} - c^{\frac{1}{(1-c)}+c} \right] \left(\frac{ab}{1-b}\right)^{\frac{1}{c}-c} \tag{40}$$

Last, the optimal utility function is equal to:

$$V(w_1; e_1) = \frac{1}{2}\left(\frac{ab}{1-b}\right)^2$$

For the two other cases (2 and 3), we cannot determine explicitly wage, employment and profit levels. However, by using numerical simulations we can compute their values in all cases and compare them. In particular, we will see the impact of a variation of $\bar{w}$, of the shape of the effort function through b, and of the shape of the production function through c on the different equilibria (Solow, case 2 and case 3). We start with the following benchmark numerical values:

$$a = 0.5 \; ; \quad b = 0.5 \; ; \quad c = 0.5 \; ; \quad p = 2 \; ; \quad \bar{w} = 0 \tag{41}$$

With (41), we obtain[10]:

Table 1: The benchmark numerical simulation model

	wage: w	Labor Demand: L^D	Labor Supply: L^S	Profit: π
Solow	1	0.5	0.125	0.5
Case 2	1.58	-	0.287	0.479
Case 3	1.44	0.245	-	0.445

It can easily seen that the best solution (in terms of profit) for the firm is to set the Solow efficiency wage. However it is not feasible since at this wage and because workers bear the training costs, the firm is constrained in its labor demand $L^S < L^D$. In this context, the optimal solution is to choose $= 0$ (Case 2) and the efficiency wage (26), here equal to 1.58, since it yields the highest profit. Let us study the impact of a variation of $\bar{w}$ on this equilibrium. We take exactly the same numerical values as in (41), and we give different values to $\bar{w}$:

Table 2: Variation of $\bar{w}$

	w	L^D	L^S	
Solow with $\bar{w} = 0.2$	1	0.5	0	0.5
Case 2 with $\bar{w} = 0.2$	2.008	-	0.220	0.457
Case 3 with $\bar{w} = 0.2$	1.44	0.245	-	0.446
Solow with $\bar{w} = 0.3$	1	0.5	0	0.5
Case 2 with $\bar{w} = 0.3$	2.23	-	0.195	0.445
Case 3 with $\bar{w} = 0.3$	1.44	0.245	-	0.446

[10] In all tables, a numerical value with a star as a superscript indicates the best policy.

When $\overline{w}$ varies, it affects x and thus the labor supply. In this context, it has no influence on cases 1 and 3 in which there is no labor constraint: it thus affects only case 2. Indeed, when $\overline{w}$ is very high, less and less individuals are willing to work in the primary sector and thus the labor supply constraint is greater. When $\overline{w}$ increases, x decreases and the firm must set higher wages to attract more workers; this reduces its profit. Table 2 shows that when $\overline{w}$ varies from 0 to 0.2, the firm has to increase its wage (from 1.58 to 2.008) and to reduce its profit but $= 0$ is still the best policy (Case 2). However, when $\overline{w} = 0.3$, this is not anymore true because the increase in wage yields a too large efficiency loss and the firm prefers to bear all the training costs ($= 1$), thereby allowing the firm to hire as many workers as it wants, at a lower wage.

Let us focus now on the case when the shape of the effort function varies. This is captured by a variation of b: when b is close to 1, the effort function is nearly linear whereas when it close to 0 the effort function is very concave. Once again we start with the numerical values of (41) and we change only the value of b.

Table 3a: Increasing b

	w	L^D	L^S	
Solow with $b = 0.6$	1.45	0.356	0.281	0.517
Case 2 with $b = 0.6$	1.56	-	0.327	0.516
Case 3 with $b = 0.6$	1.95	0.212	-	0.482
Solow with $b = 0.7$	2.07	0.271	0.681	0.562
Case 2 with $b = 0.7$	-	-	-	-
Case 3 with $b = 0.7$	-	-	-	-

When b increases slightly (from 0.5 to 0.6), Case 2 is still the optimal solution whereas when $b = 0.7$, the Solow solution becomes feasible since the firm is not anymore constraint ($L^S = 0.681 > L^D = 0.271$) and this is obviously the first best solution in which $= 0$ and the efficiency wage is defined by (23), here equal to 1.45. This result is quite intuitive since when $b = 0.7$, the effort function becomes more linear and workers are more paid (they need to be induced more): the efficiency wage switches from 1 to 2.07. In this context, the rise of the efficiency wage and thus of $V(\overline{w}_1; e_1)$ and x is sufficiently large to release the firm's labor constraint.

Table 3b: Decreasing b

	w	L^D	L^S	
Solow with $b = 0.4$	0.634	0.829	0.055	0.525
Case 2 with $b = 0.4$	1.599	-	0.249	0.441
Case 3 with $b = 0.4$	1.052	0.289	-	0.430
Solow with $b = 0.3$	0.326	2.019	0.022	0.657
Case 2 with $b = 0.3$	1.61	-	0.214	0.403
Case 3 with $b = 0.3$	0.717	0.353	-	0.441

When b decreases, the effort function becomes more and more concave. The standard efficiency wage is decreasing and the associated labor demand is increasing. In this case, increasing the wage becomes more and more costly in terms of efficiency loss for each worker. Therefore, the primary firm is more likely to support all the training costs. Table 3b gives us a good illustration this point. When switches from 0.4 to 0.5, the firm is constrained and = 0 is still the best policy. However, as soon as = 0.3, it is optimal for the firm to bear all the training cost and to set the modified efficiency wage defined by (31), here equal to 0.717. The last effect that we want to study is the modification of the production function through a variation of . We start again with the numerical values of (41) and we vary .

Table 4: Variation of

	w	L^D	L^S	
Solow with = 0.7	1	0.609	0.125	0.261
Case 2 with = 0.7	1.63	-	0.302	0.232
Case 3 with = 0.7	1.436	0.238	-	0.199
Solow with = 0.6	1	0.557	0.125	0.371
Case 2 with = 0.6	1.62	-	0.299	0.346
Case 3 with = 0.6	1.45	0.247	-	0.312
Solow with = 0.4	1	0.434	0.125	0.651
Case 2 with = 0.4	1.52	-	0.268	0.636
Case 3 with = 0.4	1.42	0.230	-	0.606
Solow with = 0.3	1	0.358	0.125	0.836
Case 2 with = 0.3	1.42	-	0.242	0.826
Case 3 with = 0.3	1.386	0.209	-	0.801

The following comments are in order. First, when varies, the Solow efficiency wage is not affected (it is always equal to 1) but the labor demand is. This is a standard result in which this wage does not depend on the parameters of the production function. Second, whatever the value of and thus of the shape of the production function, the best policy seems to be = 0 (Case 2). This is due to the fact that the shape of the production function affects both policies = 0 and = 1 whereas a variation of (the effort function) affects only the policy = 1. Note that for a sufficiently low value of , the Solow efficiency wage will become an equilibrium, since labor demand is positively related to .

5 Final Remarks

In this paper, we have considered a labor market where labor dualism is likely to prevail due to efficiency wage setting in the primary sector. However, when access to the primary sector is conditional on training cost, it is not obviou that the primary sector will face excess supply at the efficiency wage. In our framework, workers are heterogeneous through training costs, so that the labor supply in the primary sector is finitely elastic. We have shown that when the wage alone is not able to ensure both individual efficiency and a sufficient labor supply, firms choose among two strategies: either they cope with the labor supply constraint and let workers bear all the training cost, or they relax the constraint by bearing all the training costs. In the first case, we will not obser labor dualism since workers of the secondary sector do not apply in the primary one. In the second one, a standard dual labor market structure prevails, i.e. there is excess supply in the primary sector. When the firm is constrained in its labor supply at the standard efficiency wage, it has to increase wages so that all workers are less efficient. However, by bearing the training cost, the firm can afford a lower wage increase. Therefore, it prefers this policy whenever either the supply constraint is very important or when the efficiency loss associated with a wage increase is very large.

Our model can be extended in two different directions. First, firms optimally choose between bearing all the training cost or none of it. This is due to the fact that the objective function is linear in the share of the training cost supported the firms. It is obvious that more general settings will lead to interior solutio in which firms a bear part of the training cost. However, the flavor of our results will not be affected since the more costly in terms of individual efficiency to increase the wage above the Solow wage, the greater the share of the training cost for the firm.

Second, our results depend crucially on the fact that firms are assumed to observe perfectly workers types since firms are allowed to hire only the most able workers when it internalizes training costs. Although restrictive, this sumption may not be too demanding since firms may find objective signals of applicants through the quality of their general education levels and past working experience. Alternatively, we may also assume that firms are able to screen applicants during the hiring procedure. However, if workers type are not perfectl observable, the training cost supported by the workers can act as a self-selectio device, which may dispense the firm to screen them.

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