Debt Valuation and Marketability Risk*

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Abstract

This paper studies the valuation of corporate debt contracts in an intertemporal setting under uncertainty taking into account the possibility that the bondholder will be unable to sell his asset. The model considers a coupon paying debt contract with default risk in a binomial setting. Randomly matched investors who place different values upon the firm in bankruptcy bargain for the price of the asset in a secondary market. With this framework we are able to isolate the influence of liquidity risk in the pricing of risky debt contracts. This influence is shown to be function of the heterogeneity of investors' valuations and the range of uncertainty concerning potential bankruptcy costs. In particular, even though mean bankruptcy costs may be relatively low, uncertainty about them can generate relatively large spreads. Furthermore this model is capable of generating a large variety of shapes for the term structure of yield spreads. Finally, the model captures the fact that early after the issue, a bond is relatively liquid and later becomes relatively illiquid depending on the underlying asset value.

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1 Introduction

In an early paper, Fisher (1959) already suggested that risk premia on corporate bonds depend on the "marketability" of the bond. Much academic interest in the pricing of corporate bonds has however only centered on determining the influence of interest rate and default risks[1]. The risk that the bondholder will be unable to sell the instrument (marketability risk) has surprisingly received little attention in the literature. In this paper, we attempt to address this important issue i.e. the incorporation of marketability risk in the pricing of contingent claims.

Jones, Mason and Rosenfeld (1984) show that classical Merton's types models are unable to generate the yield spreads observed in the market for sufficiently realistic values of the underlying parameters such as the volatility of the value of the firm. Kim, Ramaswamy and Sundaresan (1993) include bankruptcy costs and a stochastic term structure of interest rates but are also unable to obtain realisitic spreads except for unreasonable values of the bankruptcy cost. More recently, Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997 forthcoming) have introduced game theoretic bargaining among debt and equity-holders in standard valuation models[2] and show that concessions extracted by equity-holders from debt-holders go some way toward explaining observed yield spreads. These models do not take into account illiquidity and are for instance unable to explain the flat term structure of spreads observed in high-grade corporate bonds [see Duffee (1996)].

There are however strong reasons to believe that corporate bond markets are relatively illiquid. A standard text book states

> "Markets for Treasury securities are extremely liquid because, relative to these securities, matching buyers and sellers of corporate bonds is more difficult. Therefore corporate bond markets are much less liquid, and so investors require an additional premium in their yields" [Hubbard (1995)].

Altman (1991) claims that the lack of liquidity and marketability is an important determinant of the value of distressed securities. Shulman, Bayless and Price (1993) provide some empirical evidence that investors require an additional premium for marketability risk as measured by trading frequency. Despite this evidence, illiquidity has not been incorporated into models of corporate bond pricing.

[1] Recent papers include Kim et. al. (1993), Longstaff and Schwartz (1995), Leland and Toft (1996) and Mella-Barral and Tychon (1996).

[2] See also Lambrecht and Perraudin (1996) for a model where creditors will race to grab assets in a continuous-time framework and Anderson, Sundaresan and Tychon (1996) who present a way to include a strategic dicrete-time modelling of the bankruptcy process into a continuous-time pricing model.

Empirical evidence also shows that bankruptcy is costly. However several authors have reported very different magnitudes for the bankruptcy cost when one takes into account direct and indirect costs[3]. Indirect costs of financial distress come from the firm's inability to conduct business as usual. These indirect costs are particularly difficult to assess. A recent study by Alderson and Betker (1996) gives evidence on the difficulty to explain the variations in liquidation costs by classical accounting variables used to proxy these costs.

Classical models do not include bankruptcy costs [Merton (1974), Black and Cox (1976)]. More recent models include a bankruptcy cost but assume it is fixed and known [Leland (1994), Anderson and Sundaresan (1996) among others]. Still other papers assume that the probabilities of default and recovery rates follow an exogenous stochastic process, on which firm value can have no impact [Duffie and Singleton (1995) among others].

In this paper, we propose a model of contingent claims pricing which explicitly take into consideration (i) the determinants of liquidity of the secondary market for claims and (ii) the fact that bankruptcy costs are unknown. In particular we assume there exists different types of investors in the market who have different bankruptcy costs. Furthermore we consider a secondary market for corporate bonds in which there are matching problems. It is shown that the frictions generated by the secondary market can account for a substantial portion of observed premia.

The model considers a coupon paying debt contract with default risk in a dynamic setting. For simplicity the underlying asset value is assumed to follow a binomial process. Potential investors in the market are assumed to place different values on the value of the firm in the event of default. Each period the current bondholder is matched in the secondary market with an investor chosen at random from the population of market participants. Depending on valuation about collateral value a secondary market transaction may or may not result. With this framework we are able to isolate the influence of liquidity risk in the pricing of risky debt contracts. This influence is shown to be function of the heterogeneity of potential investors in the market and the range of uncertainty concerning potential bankruptcy costs. In particular, even though mean bankruptcy costs may be relatively low, uncertainty about them can generate relatively large spreads. Furthermore this model is capable of generating a large variety of shapes for the term structure of yield spreads. Finally, the model captures the fact that early after the issue, a bond is relatively liquid and later becomes relatively illiquid depending on the underlying asset value.

To our knowledge there exists no model that incorporates "marketability" or liquidity

[3] Warner (1977) found direct bankruptcy costs to be about 5.3% of firm's market value while Alderson and Betker (1995) reported mean total liquidation costs of 36.5% (ranging from 12.8% to 61.8%).

risk in the valuation of debt contracts. Grinblatt (1994) models liquidity as an exogenous state variable that follows an Ornstein-Uhlenbeck process in order to find a closed form solution for interest rate swap spreads. Reduced form models [for instance Duffie and Singleton (1995)] tend to replicate observed yield spreads by an exogenous stochastic process which also may include potential liquidity or credit risks. Note that we are able to isolate the influence of these two components without assuming any exogenous stochastic process which is proved to be very difficult to justify. Boudhouck and Whitelaw (1993) study the issue of liquidity from Japanese government bond markets by considering a model where heteregoneous investors in their endowments with short-sales restrictions limiting their trading strategies explain price differentials. A closer related article is Williams (1995) which studies the valuation of housing properties in a model consisting of a costly search process followed by a Nash bargaining. Williams does not explicitly study liquidity issues, and is not concerned with the valuation of corporate bonds. Also, matching problems in our model are induced by heterogeneous valuations of the event of default.

The paper is organized as follows : in the second section, we present the model and its general solution as well as as a simple three periods-three types of investors example. In the third section, we present different numerical solutions of the implied term structure of credit spreads for different distributions of the heterogeneous bankruptcies' valuations and for different trading frequencies and we show their influence on the term structure of risky spreads as well as on the liquidity premium itself. Section four concludes.

2 The model

2.1 Economic environment

A firm wants to finance a project by issuing a debt contract calling for the payment of a coupon c each period t until the principal repayment P at the maturity date T. Let r be the risk-free interest rate. Once issued, the bond will trade on a secondary market consisting of many potential risk-neutral investors or traders.

The present value V_t of current and future cash-flows of the project, at period t, is represented as a stochastic process which intends to capture all the uncertainty affecting the project if it remains active. We assume that the present value V_t follows a simple binomial process with probabilities, q, which are time and state invariant; q is the probability of an "up move" while $1-q$ is the probability of a "down move".

The firm is assumed to go bankrupt as soon as V_t falls below a time-independent constant $\underline{V}$. This refers for instance to a debt protected by a net-worth covenant where bankruptcy is triggered as soon as V_t falls below the value of the principal P; i.e. $\underline{V}= P$. In this case, the bankruptcy point, $\underline{V}$, is an absorbing state. We choose this bankruptcy scenario for simplicity in order to concentrate on modelling the liquidity of secondary trading. However, we note that other default triggers are possible : bankruptcy can be triggered by a strict liquidity constraint such as in Kim *et al.* (1993) where bankruptcy occurs as soon as firm's cash-flows are not sufficient to cover coupon payments or at the equity holders' optimal abandonment point such as in Leland (1994). Our model could also be easily adapted for the case where there would be a regime change as soon as the underlying variable crosses some specific point. One may think for instance to a reorganization regime where heterogeneous investors would place different values upon the reorganization process [see Franks and Torous (1989) and (1994) and Anderson and Sundaresan (1996)].

In case of bankruptcy, debt-holders take possession of the firm only at a proportional cost α. We assume that potential investors place different values upon this bankruptcy cost. This might be interpreted as a rough proxy for different information about bankruptcy costs; however note that we do not incorporate the complications of asymmetric information into our framework. Heterogeneous valuations can also be justified by the existence of different rating agencies which even do not always agree on a rating indicating that they are not able to perfectly measure crediworthiness. More simply, the fact that investors for corporate bonds are of different types (private individuals and institutional investors including insurance companies, pension funds, investment trusts, investment banks and corporations) gives scope for different valuation methodologies. Different investors may have different abilities of extracting value from bankrupt firms. A recent empirical study by Bernardo and Cornell (1997) shows clearly that sophisticated investors are heterogeneous in the value they place on fixed income securities.

A finite number I of types or bankruptcy costs is assumed. Let α_i be the expected bankruptcy cost of an investor of type i, $i = 1, ..., I$. These types are classified in a way such that $\alpha_1 > \alpha_2 > ... > \alpha_I$. We denote by $\gamma(i)$ the probability of being of type i. Let $\#(i)$ be the number of investors of type i among the global population of investors.

2.2 The game

At period 0 the firm is matched randomly with one of the potential investors and makes a take-it-or-leave-it offer. This can be justified by search costs which may prevent the firm to obtain the best deal at issuance or by the need for immediacy. Also, the underwriting business is often dominated by major investment banking firms which generally practice

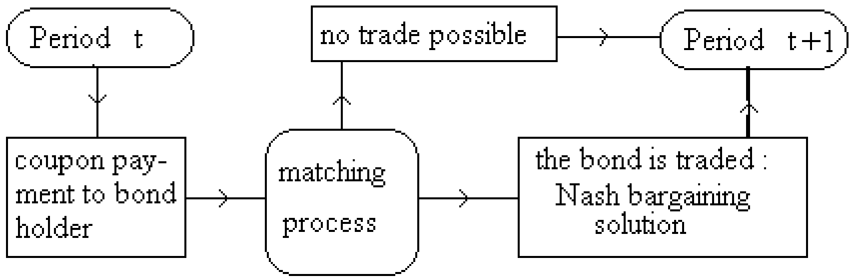

Figure 1: The Subgame at period t

some degree of underpricing[4]. This kind of simplification does not preclude the case where the bond is issued by mean of first-price auctions where only a subset of potential investors are bidding. The probability of being matched with an investor of type i is given by $\gamma(i)$. For simplicity we assume a large population . This simply means that the probabilities $\gamma(i)$ do not change during the life of the contract, i.e.

$$\gamma(i) = \frac{\#(i)}{\sum_{j=1}^{I} \#(j)} \simeq \frac{\#(i) - 1}{\left(\sum_{j=1}^{I} \#(j)\right) - 1}$$

Note that this assumption could be relaxed in order to study small markets; however the algebra would be slightly more complicated.

Thereafter, at each subsequent period t or nodes the debt contract can be traded on the market according to the following stages (see Figure 1).

Step 1: the bond holder receives the coupon payment of period t.

Step 2: the bond holder randomly matches a potential investor. Random matching means that the role of the matchmaker is left unmodelled. Random matching may be interpreted as a "black box" describing the activity of brokerage. It can also be seen as a proxy for information problems. We consider two different matching processes representing two potential extreme forms of the organization of the market in order to characterize a liquidity premium which will be the difference between these two benchmark processes.

"illliquid" matching process: In the "illiquid" matching process, the bondholder is matched with all the population of investors. Therefore, the bond holder and a

[4] For evidence on underpricing of corporate straight debt, see Datta et. al. (1997).

potential investor are matched without avoiding a possible match where no gain from trade is possible which results in no transaction. That is, at each period $\gamma(i)$ is the probability for the bond holder of matching an investor of type i, $i = 1, ..., I$.

"liquid" matching process: In the "liquid" matching process, the bonholder is matched with the subset of investor's population which always results in a transaction. That is, the bond holder and a potential investor are matched in a way such that gain from trade is always possible. At each period

$$\frac{\gamma(i)}{\sum\limits_{j=k}^{I} \gamma(j)}$$

is the probability for a bond holder of type k of matching an investor of type $i \geq k$, while the probability of matching an investor of type $i < k$ is equal to zero, $i = 1, ..., I$.

Step 3: the bond holder and the matched investor bargain over the selling price of the debt contract whenever gain from trade is possible. We denote a given node of the value tree by (t, m) where t is the time period while m is the number of "up moves" needed for reaching this node. Let $\Pi_i(t, m)$ be the bond holder's value from selling the debt contract at node (t, m); $\Pi_i^0(t, m)$ is the bond holder's value from not selling the debt contract at node (t, m); $U_j(t, m)$ is the investor's value from buying the debt contract at the negotiated price at node (t, m); and $U_j^0(t, m)$ is the investor's statu-quo value. At each node (t, m), a bondholder of type i and an investor of type j will agree to trade if and only if gain from trade is possible:

$$\Pi_i(t, m) \geq \Pi_i^0(t, m) \text{ and } U_j(t, m) \geq U_j^0(t, m) \tag{1}$$

with

$$\begin{aligned}
\Pi_i(t, m) &= p_{t,m}^{i,j} \\
\Pi_i^0(t, m) &= q\left(\frac{1}{1+r}\right)\left[c + B_{t+1,m+1}^i\right] + (1-q)\left(\frac{1}{1+r}\right)\left[c + B_{t+1,m}^i\right] \\
U_j(t, m) &= q\left(\frac{1}{1+r}\right)\left[c + B_{t+1,m+1}^j\right] + (1-q)\left(\frac{1}{1+r}\right)\left[c + B_{t+1,m}^j\right] - p_{t,m}^{i,j} \\
U_j^0(t, m) &= 0
\end{aligned}$$

where $p_{t,m}^{i,j}$ is the selling price and $B_{t+1,m+1}^i$ is the expected price of a bond held by an investor of type i at node $(t+1, m+1)$.

The bargaining solution concept we use is the asymmetric Nash Bargaining Solution [see Binmore *et al.* (1986) or Osborne and Rubinstein (1990)] in which the bargaining power of the bond holder facing an investor is $\lambda \in (0,1)$. This allows us to capture the fact that the current asset holder may have greater or less power than the potential investor. For example, the ability of investors to continue searching for alternative investors in the future may give him relatively greater bargaining power. Also, previous bargain experience might give the current bondholder the edge.

Therefore, at each node (t,m) of the binomial tree, the matching will result either in a transaction at price

$$p_{t,m}^{i,j} = \arg\max \left[\Pi_i(t,m) - \Pi_i^0(t,m)\right]^{\lambda} \left[U_j(t,m) - U_j^0(t,m)\right]^{1-\lambda} \qquad \text{if } j \geq i \quad (2)$$

or in disagreement if $j < i$, in which case the bond is not traded. Equation (2) is the asymmetric Nash solution. This asymmetric Nash solution may be also viewed as the limiting subgame perfect equilibrium to an alternating-offer bargaining model with complete information [Osborne and Rubinstein (1990)][5]. Finally, note that take-it-or-leave-it solutions are limit solutions ($\lambda = 1$ or $\lambda = 0$) of our bargaining model.

2.3 The General Solution

At each node (t,m), the (expected) price of the bond when the market is "illiquid" is denoted by $p_{t,m}^{i,j}$. Then, for each $j \geq i$ where $\alpha_i \geq \alpha_j$, gain from trade between a bondholder of type i and and an investor of type j is possible. Therefore, the selling price $p_{t,m}^{i,j}$ is the Asymmetric Nash bargaining solution to the bargaining problem:

$$\begin{aligned} p_{t,m}^{i,j} \quad = \quad & (1-\lambda) * (q/(1+r) * (c + B_{t+1,m+1}^{i}) + (1-q)/(1+r) * (c + B_{t+1,m}^{i})) \\ & + \lambda * (q/(1+r) * (c + B_{t+1,m+1}^{j}) + (1-q)/(1+r) * (c + B_{t+1,m}^{j})) \end{aligned} \quad (3)$$

Meanwhile, for each $j < i$ where $\alpha_i > \alpha_j$, no gain from trade is possible. Therefore, the bondholder of type i will keep the bond and the price of the bond will be equal to the bondholder's reservation price:

$$p_{t,m}^{i,j} = p_{t,m}^{i,i} = ((q/(1+r) * (c + B_{t+1,m+1}^{i}) + (1-q)/(1+r) * (c + B_{t+1,m}^{i}) \quad (4)$$

When there is always a match resulting in a transaction the market is described as "liquid" and the price of the bond is denoted by $pl_{t,m}^{i,j}$. Then, for each $j \geq i$ where $\alpha_i \geq \alpha_j$, gain from trade between a bondholder of type i and and an investor of type j is

[5] For example, in the case of bargaining with a risk of breakdown of negotiations, λ is derived from the parties' beliefs concerning the likelihood of a breakdown.

possible. Therefore, the selling price $pl_{t,m}^{i,j}$ is the Asymmetric Nash bargaining solution to the bargaining problem and is equal to :

$$\begin{aligned} pl_{t,m}^{i,j} &= (1-\lambda)*(q/(1+r)*(c+BL_{t+1,m+1}^{i})+(1-q)/(1+r)*(c+BL_{t+1,m}^{i})) \\ &\quad +\lambda*(q/(1+r)*(c+BL_{t+1,m+1}^{j})+(1-q)/(1+r)*(c+BL_{t+1,m}^{j})); \end{aligned} \tag{5}$$

But, when bankruptcy is imminent, i.e. when the value of the firm has the probability $(1-q)$ to fall below the bankruptcy point $\underline{V}$, the selling price $p_{t,m}^{i,j}$ resulting from the bargaining process is:

$$\begin{aligned} p_{t,m}^{i,j} &= (1-\lambda)*(q/(1+r)*(c+B_{t+1,m+1}^{i})+(1-q)/(1+r)*(1-\alpha_i)*\underline{V})) \\ &\quad +\lambda*(q/(1+r)*(c+B_{t+1,m+1}^{j})+(1-q)/(1+r)*(1-\alpha_j)*\underline{V})) \end{aligned} \tag{6}$$

for each $j \geq i$ where $\alpha_i \geq \alpha_j$, where $((1-\alpha_i)*\underline{V})$ is the expected residual value an investor of type i places upon the firm in default. Meanwhile, for each $j < i$ where $\alpha_i > \alpha_j$, no gain from trade is possible. Therefore, the price of the bond will be equal to the bondholder's reservation price:

$$p_{t,m}^{i,j} = p_{t,m}^{i,i} = q/(1+r)*(c+B_{t+1,m+1}^{i})+(1-q)/(1+r)*(1-\alpha_i)*\underline{V} \tag{7}$$

When there is always a match resulting in a transaction, the selling price $pl_{t,m}^{i,j}$ is given by:

$$\begin{aligned} pl_{t,m}^{i,j} &= (1-\lambda)*(q/(1+r)*(c+BL_{t+1,m+1}^{i})+(1-q)/(1+r)*((1-\alpha_i)*\underline{V})) \\ &\quad +\lambda*(q/(1+r)*(c+BL_{t+1,m+1}^{j})+(1-q)/(1+r)*(1-\alpha_j)*\underline{V})) \end{aligned} \tag{8}$$

To value the "illiquid" bond $B_{t,m}^{i}$ for the bondholder of type i at node (t,m), we simply have to weight the different transaction prices by the probabilities associated with the different types of investors :

$$B_{t,m}^{i} = \sum_{j} p_{t,m}^{i,j} * \gamma(j) \tag{9}$$

where $\gamma(j)$ is the probability of matching an investor of type j which is common knowledge and $\sum_{j=1}^{I} \gamma(j) = 1$.

Similarly, in order to value the "liquid" bond $BL_{t,m}^{i}$ for the bondholder of type i at node (t,m), we need to weight the effective transaction prices by their respective probabilities :

$$BL^i_{t,m} = \frac{\sum\limits_{j=i}^{I} pl^{i,j}_{t,m} * \gamma(j)}{\sum\limits_{j=i}^{I} \gamma(j)} \tag{10}$$

To find the expected values of the "illiquid" bond $B_{t,m}$ and the "liquid" bond $BL_{t,m}$ at node (t, m), we simply have to weight the different bond values by the probabilities associated with the different types of investors :

$$B_{t,m} = \sum_j B^j_{t,m} * \gamma(j) \tag{11}$$

and,

$$BL_{t,m} = \sum_j BL^j_{t,m} * \gamma(j) \tag{12}$$

Hence, we define the marketability risk as follows:

Marketability risk: the marketability risk is defined as the risk that the bond holder will be unable to sell the instrument. Hence the difference between the value of the bond in a "liquid" market and the value of the same debt contract in a market where one can face matching problems characterises the marketability risk.

Therefore, our definition of marketability risk induces the following definition of the liquidity premium, $LP_{t,m}$.

$$LP_{t,m} \equiv BL_{t,m} - B_{t,m}$$

Therefore at each node of the tree we need to calculate different expected values for each type of investor. While this may appear computer time-consuming, we have developped a procedure that can considerably speed up calculations. In Appendix A.1. we show how to simplify the calculations by only looking at the nodes where bargaining takes place. The values of the bond at the other nodes are known : for sufficiently high values of the underlying variable, the value of the bond is equal to its riskless value. This also shows that gains from trade will be possible only for sufficiently low values of the expected variable, i.e. when market participants anticipate the event of default[6]. The model therefore captures the fact that early after the issue, a bond is relatively liquid and later may become relatively illiquid.

[6] Shulman, Bayless and Price (1993) empirically show that bond market's anticipation of default results in greater trading frequency.

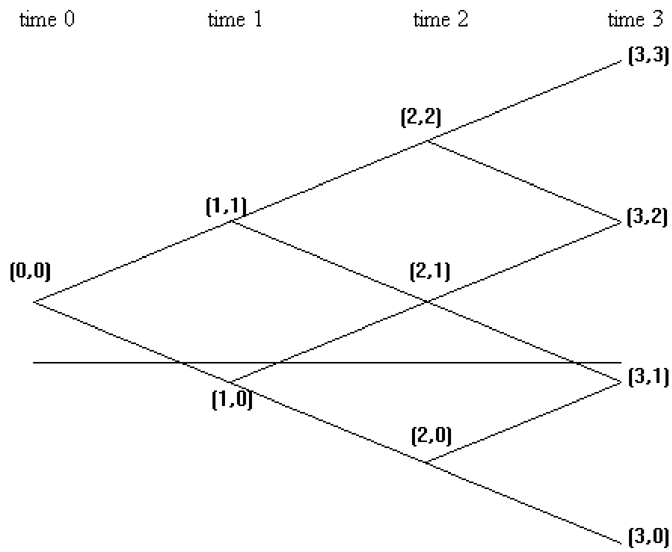

Figure 2: Binomial Tree

2.4 An Example : 3 Periods and 3 Types

We consider now a simple version of our pricing model with three periods. Figure 2 gives us the binomial tree. The market consists of three types of players or potential investors. These investors place heterogeneous values upon the bankruptcy cost: $\alpha_i \in [0,1]$ is the bankruptcy cost of an investor of type i, $\alpha_1 > \alpha_2 > \alpha_3$. Without too much loss of generality, let $(\gamma_1, \gamma_2, \gamma_3)$ be the probability distribution of types $(\alpha_1, \alpha_2, \alpha_3)$, and $\sum_{j=1}^{3} \gamma_j = 1$. A high γ_3 corresponds therefore to a large proportion of investors with low bankruptcy costs. Remember that $B_{0,0}$ and $BL_{0,0}$ are the expected prices of the bond at period 0 with the "illiquid" matching process and with the "liquid" matching process, respectively. These prices are solved backwards by beginning at the end of the binomial tree (see Appendix A.2 where we derive in detail the results for this simple version).

Then, the expected price of the bond with "illiquid" matching process is given by

$$\begin{aligned} B_{0,0} = \; & q\,(2+r)\,c\,(1+r)^{-2} + (2-q)\,q^2\,(c+P)\,(1+r)^{-3} \\ & + \left[(1-q)\,(1+r)^{-1} + q\,(1-q)^2\,(1+r)^{-3}\right] \left[\gamma_1\,(1-\alpha_1) + \gamma_2\,(1-\alpha_2) + \gamma_3\,(1-\alpha_3)\right] \underline{V} \\ & + q\,(1-q)^2\,(1+r)^{-3}\left[\gamma_1\gamma_2\lambda\,(2-\lambda\,(1-\gamma_1))\,(\alpha_1-\alpha_2) + \right. \\ & \left. \gamma_1\gamma_3\lambda\,(2-\lambda\,(1-\gamma_1))\,(\alpha_1-\alpha_3) + \gamma_2\gamma_3\lambda\,(\gamma_1\lambda - \gamma_3\lambda + 2)\,(\alpha_2-\alpha_3)\right] \underline{V} \end{aligned}$$

And, the expected price of the bond with "liquid" matching process is given by

$$
\begin{aligned}
BL_{0,0} = \; & q\,(2+r)\,c\,(1+r)^{-2} + (2-q)\,q^2\,(c+P)\,(1+r)^{-3} \\
& + \left[(1-q)\,(1+r)^{-1} + q(1-q)^2\,(1+r)^{-3}\right]\left[\gamma_1\,(1-\alpha_1) + \gamma_2\,(1-\alpha_2) + \gamma_3\,(1-\alpha_3)\right]\underline{V} \\
& + q(1-q^2)\,(1+r)^{-3}\,\underline{V}\gamma_1\lambda\,(2-\lambda\,(1-\gamma_1))\,(\gamma_2\,[\alpha_1-\alpha_2] + \gamma_3[\alpha_1-\alpha_3]) \\
& + q\,(1-q)^2\,(1+r)^{-3}\,(1-\gamma_1)^{-1}\,\gamma_2\gamma_3\lambda\left[\lambda\gamma_1 + (1-\gamma_1)^{-1}\,(\gamma_2 + (1-\gamma_1) + \gamma_3 - \gamma_3\lambda)\right] \\
& (\alpha_2-\alpha_3)\,\underline{V}
\end{aligned}
$$

Then,

$$
\begin{aligned}
LP_{0,0} \;=\; & q\,(1-q)^2\,(1+r)^{-3}\,(\gamma_2+\gamma_3)^{-2}\,\gamma_2\gamma_3\lambda[(1-\gamma_2-\gamma_3)\lambda(\gamma_2+\gamma_3) + 2(\gamma_2+\gamma_3) - \gamma_3\lambda + \\
& (-(1-\gamma_2-\gamma_3)\lambda - 2 + \gamma_3\lambda)(\gamma_2+\gamma_3)^2]\,(\alpha_2-\alpha_3)\,\underline{V}
\end{aligned}
$$

Because $\sum_{j=1}^{3}\gamma_j = 1$ and $\lambda \in (0,1)$ we have that $LP_{0,0} \geq 0$. Comparative static results give $\partial LP_{0,0} \,/\, \partial\gamma_2 > 0$, $\partial LP_{0,0} \,/\, \partial\,[\alpha_2-\alpha_3] > 0$ and $\partial LP_{0,0} \,/\, \partial\lambda > 0$. Next proposition summarizes our main results.

Proposition 1 *Consider the debt pricing game with 3 periods and 3 types where* $(\gamma_1, \gamma_2, \gamma_3)$ *is the probability distribution over types* $(\alpha_1, \alpha_2, \alpha_3)$ *and* $\sum_{j=1}^{3}\gamma_j = 1$*. Then,*
(i) The stronger the bond holder (λ) *the higher the value of the debt contract* $(B_{0,0})$ *;*
(ii) As the bargaining power (λ) *of the bond holder approaches zero the liquidity premium* $(LP_{0,0})$ *vanishes;*
(iii) The stronger the bond holder (λ) *the higher the liquidity premium* $(LP_{0,0})$*;*
(iv) The higher γ_2 *the higher the liquity premium* $(LP_{0,0})$*;*
(v) The greater $[\alpha_2-\alpha_3]$ *the higher the liquidity premium* $(LP_{0,0})$*.*

The intuition behind the results shown in Proposition 1 is the following : (i) The higher the bargaining power of the bondholder, the higher the selling price he will obtain from a transaction and therefore the higher the expected value of the debt contract. (ii) The lower the bargaining power of the bondholder, the lower the difference between the value of the bond in a "liquid" and "illiquid" market and therefore the lower the liquidity premium. When the bargaining power of the bondholder vanishes, the bondholder will obtain at most his reservation value whatever the next matches. Therefore, the value of the bond in the liquid and illiquid markets will converge towards the same value. (iii) Conversely, the higher the bargaining power of the seller, the higher the liquidity premium. (iv) The greater the proportion of investors with high bankruptcy costs, the higher the chance to have a match which will not result in a transaction and thus the higher the liquidity premium. (v) The greater the uncertainty about bankruptcy costs, the higher the liquidity premium. Intuitively, if all investors had the same bankruptcy costs, the liquidity premium would be equal to zero as all investors would agree on the same prices. When they have heterogeneous bankruptcy costs, they know that the expected values of the different investors will be even more different the more heterogeneous are the bankruptcy costs. This predicts that even though mean bankruptcy costs may be relatively low, the uncertainty about them can generate relatively large spreads.

Note also that our model captures the fact that the bond may be relatively liquid early after the issue and then becomes relatively illiquid. In particular, in the bargaining zone [points (0,0), (1,1), (2,1) in Figure 2] where the value of the asset is affected by the risk of default, the probability to have a transaction decreases with the time to maturity. Consider the three types cases with $(\gamma_1, \gamma_2, \gamma_3)$ the probability distribution of types $(\alpha_1, \alpha_2, \alpha_3)$. In the first period, the probability to have a transaction at node (1,1) is equal to $\gamma_1 + \gamma_2 * [\gamma_2 + \gamma_3] + (\gamma_3)^2$; the next period, at node (2,1), the probability to have a transaction is decreasing to $(\gamma_1)^2 + \gamma_2 * [1 + 2 * \gamma_1] * [\gamma_2 + \gamma_3] + (\gamma_3)^2 [1 + \gamma_1 + \gamma_2]$. As the the time to maturity increases, the probability to have a transaction converges towards γ_3, the probability to be of the type with the higher value in default.

Proposition 2 *Given our debt pricing game, the probability to have a transaction is decreasing with the time to maturity of the debt contract.*

When investors anticipate the event of default, as a transaction occurs only if gain from trade is possible, then as time passes, the bond will be in the hands of an investor with a lower bankruptcy cost who will find it more and more difficult to sell his asset.

3 Term Structure of risky spreads

Before presenting numerical simulations on the term structure of risky spreads, note that our framework is rich enough to accomodate for different descriptions of the distribution of investor's types. In the following simulations, we will consider four simple different investors' distributions:

1. Investors with different valuations of the bankruptcy costs $\alpha_l \in [0.5, 0]$ are equally distributed, i.e, $\gamma\ (l) = (I)^{-1}$.
2. Investors with different valuations of the bankruptcy costs $\alpha_l \in [0.5, 0]$ are "pessimists", i.e, the higher the investor's bankruptcy cost, the higher the probability to meet him :

   $\gamma\ (i) = \varepsilon^i$ with $i \in [1, I]$ such that $\varepsilon \in [0, 1]$ and $\sum_{i=1}^{I} \gamma(i) = 1$.

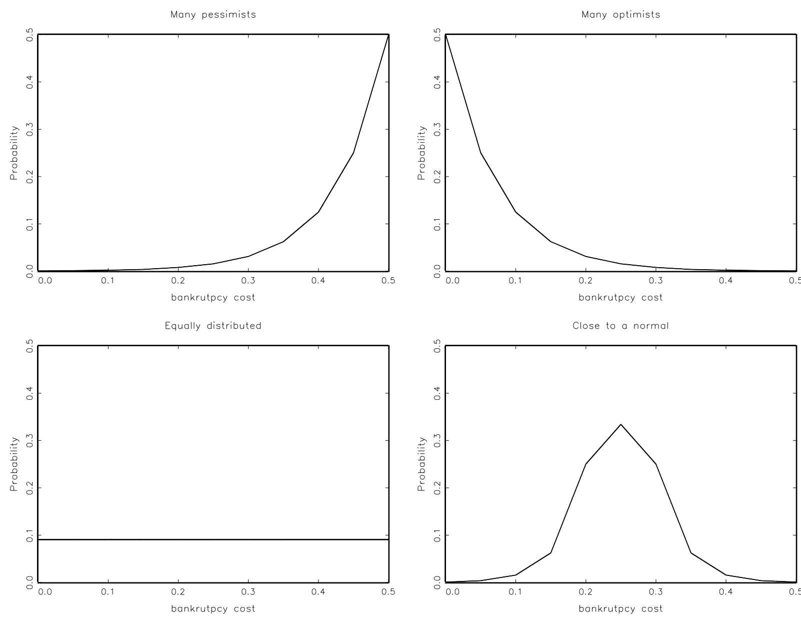

Figure 3: Distributions of investors' bankruptcy costs.

3. Investors with different valuations of the bankruptcy costs $\alpha_i \in [0.5, 0]$ are "optimists", i.e, the lower the investor's bankruptcy cost, the higher the probability to meet him :
$\gamma\ (i) = \varepsilon^{I-i+1}$ with $i \in [1, I]$ such that $\varepsilon \in [0,1]$ and $\sum_{i=1}^{I} \gamma(i) = 1$.
4. Investors with different valuations of the bankruptcy costs $\alpha_i \in [0.5, 0]$ are "close to a Normal" , i.e, the investors' bankruptcy costs are distributed around the mean bankruptcy cost of the range of possible bankruptcy costs:
$\gamma\ (i) = \varepsilon^{(I-1)/2-i+1}$ with $i \in [1, I]$ such that $\varepsilon \in [0,1]$ and $\sum_{i=1}^{I} \gamma(i) = 1$.

These different distributions can be interpreted as a proxy for market conditions or market participants. These distributions are plotted in Figure 3.

In Figures 5, 6 and 7 we present the term structure of credit spreads, i.e. the risk premium of debt function of the maturity under a variety of assumptions about the distribution of the investors' types and the bargaining power of the seller for the "illiquid" and "liquid" cases. Throughout our numerical simulations, we have adopted the following values for the other parameters : $V_0 = 100$, $P = 50$, $\sigma = 0.15$, $\alpha \in [0, 0.5]$, $c = 0.066$, $r = 0.06$ and the number of types of investors $I = 10$. Given some empirical evidence, we have chosen a time step of $1/52$ representing one transaction a week[7].

In Figure 5, we plot the term structures of credit spreads for the four different cases considered whith $\lambda = 0.5$.

Observe that our framework can generate yield spreads close to those observed in the market. Kim, Ramaswamy and Sundaresan (1993) report 77 basis points as the average spread for investment-grade corporate bonds, and Litterman and Iben (1991) report historical ranges for par spreads between 20 and 130 basis points. Duffee (1996) computes mean yield spreads on Baa bonds between 115 and 198 basis points.

The difference between the two curves can be seen as a proxy for liquidity risk. Observe that the magnitude as well as the shape is substantially different depending on the set of assumptions describing the market environment. This could prove very useful if we want to fit observed yield spreads in the markets. The term structure of credit spreads is monotone increasing for both the "equally distributed" and the "optimists" cases; it is hump-shaped for the cases where the market is pessimist and in the case where investors' bankruptcy costs are distributed around an average value. The hump-shaped curve can be explained by the fact that the longer the time to maturity, the higher the chance to find a match, thus the lower the discount for liquidity risk. This effect tends thus to lower the influence of default risk which increases with the time to maturity.

Not surprisingly, the magnitude of risk premia is higher in a market where many investors place a low value upon the firm in default. This is consistent whith one may observe in reality : when agents are pessimists, in a period of recession for instance, they tend to ask a higher risk premium. This higher risk premium is not only due to a more pessimist anticipation of the event of default but also to a less liquid market due to an anticipation of problems of matching buyers and sellers.

Figures 6, and 7 present the term structures of credit spreads for two different values of the bargaining power of the seller. Observe that both the magnitude and the shape of the term structure depend on λ . There is thus a clear influence of the relative power of the seller on the term structure of risk premia. The risk premium is higher for small values of λ , i.e. when the bondholder has less bargaining power, the resulting price from the transaction tends to be lower which induces a higher risk premium. Observe also that in the "equally distributed" case, the shape of the term structure can also be different according to λ . It is decreasing for $\lambda = 0.1$ and increasing for $\lambda = 0.9$.

Figure 8 presents risk premia function of the bargaining power λ for $T = 2$ and $T = 10$ for the assumed different distributions of beliefs. Risk premia are sharply decreasing with

[7] Shulman, Bayless and Price (1993) give some statistics on bond trading frequency distributions and show that 32% of the bonds in their sample traded less than 25% of the time.

the bargaining power of the seller. This is consistent with the intuition : if the seller has all the bargaining power, the resulting price from the bargaining will be higher and this will induce a lower risk premium. Observe also that the risk premia in the "illiquid" and "liquid" markets tend to converge to the same value when λ approaches 1. The convergence is less rapid in a "pessimist" market as in this case, the probability to get a match will be more difficult.

When we look at the premium generated by the lack of liquidity (see Figure 9), we observe that the liquidity premium reaches a maximum at an intermediate value of λ. This is due to two effects which occur as λ increases (at $\lambda = 0$ the liquidity premium is equal to zero). There is a direct effect which pushes up the price the seller will obtain. Also, there is an indirect effect due to the continuation value of the bondholder. Indeed, when the market is "liquid", the bondholder will be able to sell the bond whatever the next matches while he will face matching problems in the "illiquid" case. It implies that the bond price will be increasing less rapidly in function of λ in the "illiquid" case. Finally, not surprisingly as λ approaches 1, one converges to the take-it-or-leave it solution whatever the liquidity of the market. Nevertheless, the convergence is faster in the "liquid" case. Indeed, if the buyer has less bargaining power, then he is willing to concede more rapidly in the "liquid" case than in the "illiquid" case where he anticipates problems of matching.

Figure 10 presents the yield spreads in the "illiquid" market as a function of time to maturity for three smaller supports of the distribution of bankruptcy costs keeping the lower bound constant ($\alpha \in [0, 0.4]$, $\alpha \in [0, 0.3]$, $\alpha \in [0, 0.2]$) and for two different distributions of investors' types ("Equally distributed" and "Close to a Normal"). In each case, the larger the support of the distribution, the larger are the risk premia. Figure 11 presents the term structure of credit spreads in the "liquid" and "illiquid" markets for different supports of the distribution each with an average bankruptcy cost of 25% ($\alpha \in [0, 0.5]$, $\alpha \in [0.1, 0.4]$, $\alpha \in [0.2, 0.3]$) for the "Equally distributed" and "Close to a Normal" cases. Observe than in both cases, the higher the lower bound of the support of the distribution, the higher are the risk premia in the "illiquid" market. For instance, in the equally distributed case, risk premia are above 200 basis points when $\alpha \in [0.2, 0.3]$, while they are about 40 basis points when $\alpha \in [0, 0.5]$. Hence, while the average bankruptcy cost is in both cases equal to 25%, the resulting risk premia are substantially different and are actually higher when the support of the distribution is smaller. When the support of the distribution is smaller, the lower bound is larger and this results in a higher risk premia. In the "Close to a Normal" case, we also observe that the liquidity premium, hence the difference between the spreads in the "liquid" and "illiquid" markets is increasing with the support of the distribution. Therefore, the higher the lower bound of the support of the distribution the higher are the risk premia while the larger the support, the larger the premium due to the marketability risk. This confirms the results obtained in the simple three periods-three types case. Notice also that our model is able to generate a relatively flat term structure of yield spreads.

It is also interesting to look at the influence of trading frequency on the term structure of credit spreads. In Figure 12 we plot yield spreads function of the maturity of the debt contract when there are 3 different types of investors and where trades are supposed to occur monthly, weekly and daily. Not surprisingly, risk premia are decreasing with trading frequency, furthermore the premium due to marketability risk decreases with trading frequency and observe in the example considered that this premium vanishes when trades occur daily. Observe also that the shape of the term structure is dependent on trading frequency. While the term structure is monotone increasing with weekly and daily trades, it is hump shaped in the monthly case. The hump-shaped term structure can be explained by the tradeoff between marketability risk and default risk. While default risk is increasing with the maturity of the contract, marketability risk is decreasing with the number of trades which is function of the maturity of the debt contract.

4 Concluding Comments

This article has presented a simple framework for valuing corporate risky debt contracts which incorporates the risk that the bondholder will be unable to sell his asset. We have developed a pricing framework that includes potential frictions in a secondary market where different investors are supposed to place heterogeneous values upon the the event of default. These heterogeneities induce problems of matching buyers and sellers which result in an additionnal premium due to the risk that investors will encounter difficulties to sell their assets. This premium is shown to be function of the heterogeneity of potential investors in the market and the range of uncertainty concerning potential bankruptcy costs. We have also presented different characterisations of the distribution of probabilities of investors' types and we have shown that these distributions as well as the bargaining power of the debt-holder are important determinants of the risk premium attached with corporate bonds. Even though mean bankruptcy costs may be relatively low, uncertainty about them can therefore generate relatively large spreads. Furthermore this model is capable of generating a large variety of shapes for the term structure of yield spreads as well as spreads consistent with empirical evidence. Finally, the model captures the fact that early after the issue, a bond is relatively liquid and later becomes relatively illiquid depending on the underlying asset value.

We view this article as a first attempt to incorporate marketability risk in the pricing of corporate bonds. Beyond this we believe our framework suggests a general approach incorporating market microstructure considerations in a valuation setting which can be usefully explored in future work. In our analysis different market structures are represented in part by the parameter λ, the bargaining power of the seller. One can imagine extending the analysis to model market structures in more detail. For example, one could conceive of bargaining with asymmetric information [see Gül, Sonnenschein and Wilson (1986) and Gül and Sonnenschein (1988)]. In such a context privileged information for one agent would likely enhance her bargaining power. In a sequential model of the secondary market similar to ours reporting of past transactions would tend to bestow this advantage on current buyers.

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A Appendix

A.1 Simplified Computational Procedure

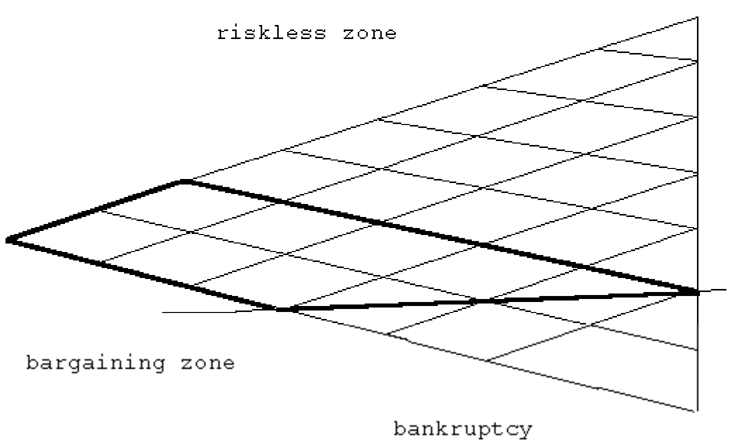

Figure 4: Simplified computational procedure

As in Cox and Rubinstein (1985) let a stand for the minimum number of upward moves that the value of the firm must make over the next m periods for the value of the bond to finish riskless. Thus a will be the smallest nonnegative integer such that $u^a d^{m-a} V > P$. The problem is therefore reduced to the calculation of the expected values in the bargaining zone which is of dimension $[a * m]$. The value of the corporate bond in the riskless zone is simply the value of a riskless claim on a coupon and a principal repayment : $\sum_{j=1}^{m-j+1} \frac{c}{(1+r)^j} + \frac{P}{(1+r)^{m-j+1}}$

A.2 3-Types 3-Periods Case

$$\begin{aligned}
p_{2,1}^{12} &= \frac{1}{1+r}\left[q\left(c+P\right)+\left(1-q\right)\underline{V}\left[1-\alpha_1\left(1-\lambda\right)-\alpha_2\lambda\right]\right] \\
p_{2,1}^{13} &= \frac{1}{1+r}\left[q\left(c+P\right)+\left(1-q\right)\underline{V}\left[1-\alpha_1\left(1-\lambda\right)-\alpha_3\lambda\right]\right] \\
p_{2,1}^{23} &= \frac{1}{1+r}\left[q\left(c+P\right)+\left(1-q\right)\underline{V}\left[1-\alpha_2\left(1-\lambda\right)-\alpha_3\lambda\right]\right] \\
p_{2,1}^{22} &= \frac{1}{1+r}\left[q\left(c+P\right)+\left(1-q\right)\underline{V}\left(1-\alpha_2\right)\right] = p_{2,1}^{21} \\
p_{2,1}^{11} &= \frac{1}{1+r}\left[q\left(c+P\right)+\left(1-q\right)\underline{V}\left(1-\alpha_1\right)\right] \\
p_{2,1}^{33} &= \frac{1}{1+r}\left[q\left(c+P\right)+\left(1-q\right)\underline{V}\left(1-\alpha_3\right)\right] = p_{2,1}^{31} = p_{2,1}^{32}
\end{aligned}$$

$$\begin{aligned}
B_{2,1}^{1} &= \frac{1}{1+r}\left[q\left(c+P\right)+\left(1-q\right)\underline{V}\left[\left(1-\alpha_1\right)+\lambda\gamma_2\left(\alpha_1-\alpha_2\right)+\lambda\gamma_3\left(\alpha_1-\alpha_3\right)\right]\right] \\
B_{2,1}^{2} &= \frac{1}{1+r}\left[q\left(c+P\right)+\left(1-q\right)\underline{V}\left(1-\alpha_2\right)+\left(1-q\right)\underline{V}\lambda\gamma_3\left(\alpha_2-\alpha_3\right)\right] \\
B_{2,1}^{3} &= \frac{1}{1+r}\left[q\left(c+P\right)+\left(1-q\right)\underline{V}\left(1-\alpha_3\right)\right]
\end{aligned}$$

$$p_{2,2}^{ij} = \frac{1}{1+r}\left[c+P\right] = B_{2,2}^{i}$$

$$\begin{aligned}
p_{1,1}^{12} &= q\left[\frac{c+B_{2,2}^{i}}{1+r}\right]+\left(1-q\right)\frac{c}{1+r}+\frac{1-q}{1+r}\left[\left(1-\lambda\right)B_{2,1}^{1}+\lambda B_{2,1}^{2}\right] \\
p_{1,1}^{13} &= q\left[\frac{c+B_{2,2}^{i}}{1+r}\right]+\left(1-q\right)\frac{c}{1+r}+\frac{1-q}{1+r}\left[\left(1-\lambda\right)B_{2,1}^{1}+\lambda B_{2,1}^{3}\right] \\
p_{1,1}^{11} &= q\left[\frac{c+B_{2,2}^{i}}{1+r}\right]+\left(1-q\right)\left[\frac{c+B_{2,1}^{1}}{1+r}\right] \\
p_{1,1}^{22} &= q\left[\frac{c+B_{2,2}^{i}}{1+r}\right]+\left(1-q\right)\left[\frac{c+B_{2,1}^{2}}{1+r}\right] = p_{1,1}^{21}
\end{aligned}$$

$$
\begin{aligned}
p^{23}_{1,1} &= q\left[\frac{c+B^i_{2,2}}{1+r}\right]+(1-q)\frac{c}{1+r}+\frac{1-q}{1+r}\left[(1-\lambda)B^2_{2,1}+\lambda B^3_{2,1}\right] \\
p^{33}_{1,1} &= q\left[\frac{c+B^i_{2,2}}{1+r}\right]+(1-q)\left[\frac{c+B^3_{2,1}}{1+r}\right]=p^{32}_{1,1}=p^{31}_{1,1}
\end{aligned}
$$

$$
\begin{aligned}
B^1_{\mathbf{0,0}} &= \frac{q}{1+r}\left[c+\gamma_1 p^{11}_{1,1}+\gamma_2 p^{12}_{1,1}+\gamma_3 p^{13}_{1,1}\right]+\frac{1-q}{1+r}(1-\alpha_1)\underline{V} \\
B^2_{\mathbf{0,0}} &= \frac{q}{1+r}\left[c+(1-\gamma_3)p^{22}_{1,1}+\gamma_3 p^{23}_{1,1}\right]+\frac{1-q}{1+r}(1-\alpha_2)\underline{V} \\
B^3_{\mathbf{0,0}} &= \frac{q}{1+r}\left[c+p^{33}_{1,1}\right]+\frac{1-q}{1+r}(1-\alpha_3)\underline{V}
\end{aligned}
$$

$$
B_{\mathbf{0,0}}=\gamma_1 B^1_{\mathbf{0,0}}+\gamma_2 B^2_{\mathbf{0,0}}+\gamma_3 B^3_{\mathbf{0,0}}
$$

Now we turn to the case where at each node there will be transactions.

$$
\begin{aligned}
BL^1_{2,1} &= \frac{1}{1+r}\left[q(c+P)+(1-q)\underline{V}\left[(1-\alpha_1)+\lambda\gamma_2(\alpha_1-\alpha_2)+\lambda\gamma_3(\alpha_1-\alpha_3)\right]\right] \\
BL^2_{2,1} &= \frac{1}{1+r}\left[q(c+P)+(1-q)\underline{V}(1-\alpha_2)+\frac{\gamma_3}{1-\gamma_1}(1-q)\underline{V}\lambda(\alpha_2-\alpha_3)\right] \\
BL^3_{2,1} &= \frac{1}{1+r}\left[q(c+P)+(1-q)\underline{V}(1-\alpha_3)\right]
\end{aligned}
$$

$$
BL^i_{2,2}=\frac{1}{1+r}\left[c+P\right]
$$

$$
\begin{aligned}
p^{12}_{1,1} &= q\left[\frac{c+BL^i_{2,2}}{1+r}\right]+(1-q)\frac{c}{1+r}+\frac{1-q}{1+r}\left[(1-\lambda)BL^1_{2,1}+\lambda BL^2_{2,1}\right] \\
p^{13}_{1,1} &= q\left[\frac{c+BL^i_{2,2}}{1+r}\right]+(1-q)\frac{c}{1+r}+\frac{1-q}{1+r}\left[(1-\lambda)BL^1_{2,1}+\lambda BL^3_{2,1}\right] \\
p^{11}_{1,1} &= q\left[\frac{c+BL^i_{2,2}}{1+r}\right]+(1-q)\left[\frac{c+BL^1_{2,1}}{1+r}\right] \\
p^{22}_{1,1} &= q\left[\frac{c+BL^i_{2,2}}{1+r}\right]+(1-q)\left[\frac{c+BL^2_{2,1}}{1+r}\right]=p^{21}_{1,1} \\
p^{23}_{1,1} &= q\left[\frac{c+BL^i_{2,2}}{1+r}\right]+(1-q)\frac{c}{1+r}+\frac{1-q}{1+r}\left[(1-\lambda)BL^2_{2,1}+\lambda BL^3_{2,1}\right] \\
p^{33}_{1,1} &= q\left[\frac{c+BL^i_{2,2}}{1+r}\right]+(1-q)\left[\frac{c+BL^3_{2,1}}{1+r}\right]=p^{32}_{1,1}=p^{31}_{1,1}
\end{aligned}
$$

$$
\begin{aligned}
BL_{0,0}^{1} &= \frac{q}{1+r}\left[c+\gamma_1 p_{1,1}^{11}+\gamma_2 p_{1,1}^{12}+\gamma_3 p_{1,1}^{13}\right]+\frac{1-q}{1+r}\left(1-\alpha_1\right)\underline{V} \\
BL_{0,0}^{2} &= \frac{q}{1+r}\left[c+\frac{\gamma_2}{1-\gamma_1} p_{1,1}^{22}+\frac{\gamma_3}{1-\gamma_1} p_{1,1}^{23}\right]+\frac{1-q}{1+r}\left(1-\alpha_2\right)\underline{V} \\
BL_{0,0}^{3} &= \frac{q}{1+r}\left[c+p_{1,1}^{33}\right]+\frac{1-q}{1+r}\left(1-\alpha_3\right)\underline{V}
\end{aligned}
$$

Let BL the price of the liquid bond at time zero. Then,

$$
BL_{0,0}=\gamma_1 BL_{0,0}^{1}+\gamma_2 BL_{0,0}^{2}+\gamma_3 BL_{0,0}^{3}
$$

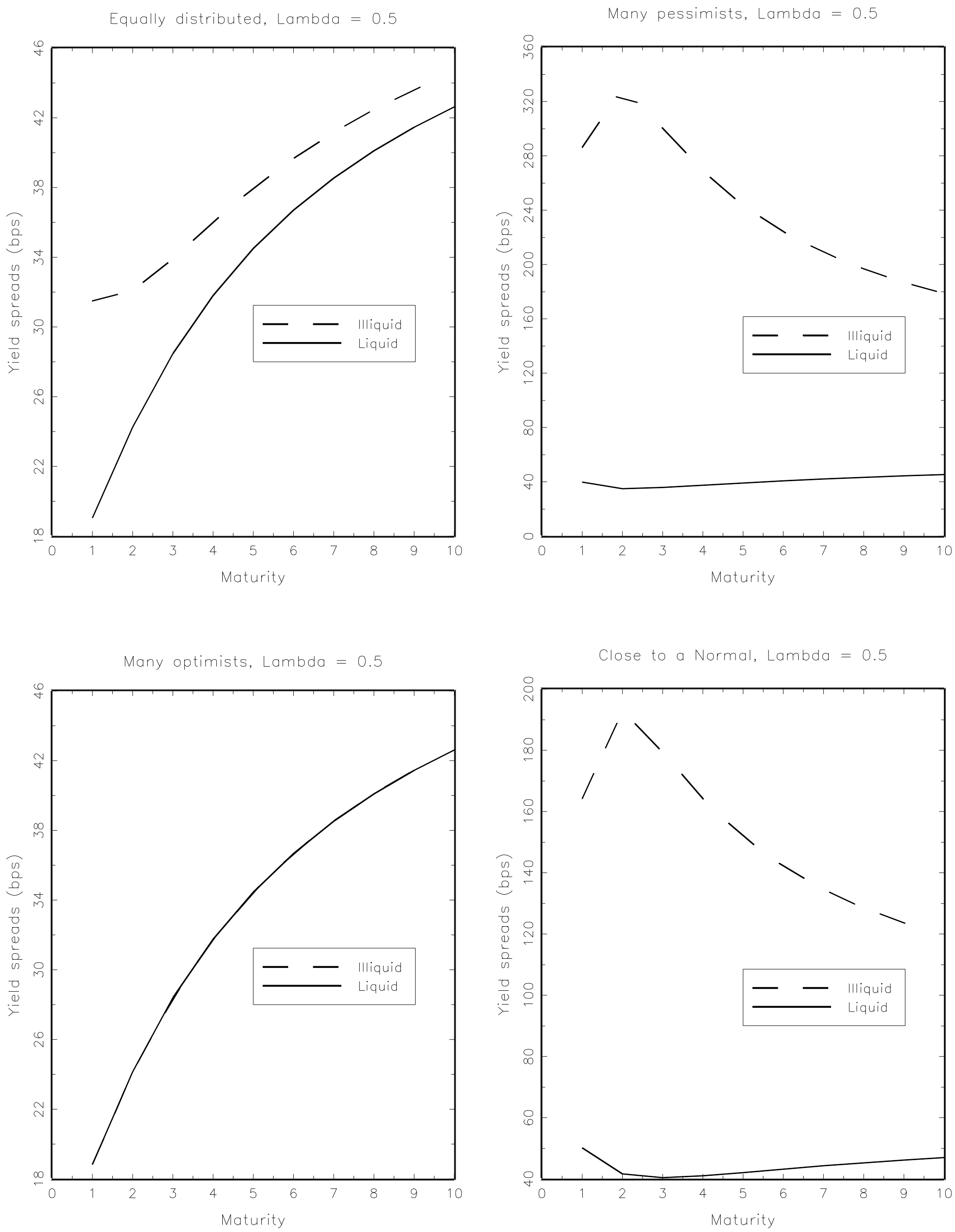

Figure 5: Term structure of credit spreads for different distributions of investors' types and for $\lambda = 0.5$.The other parameters are those of the base-case example.

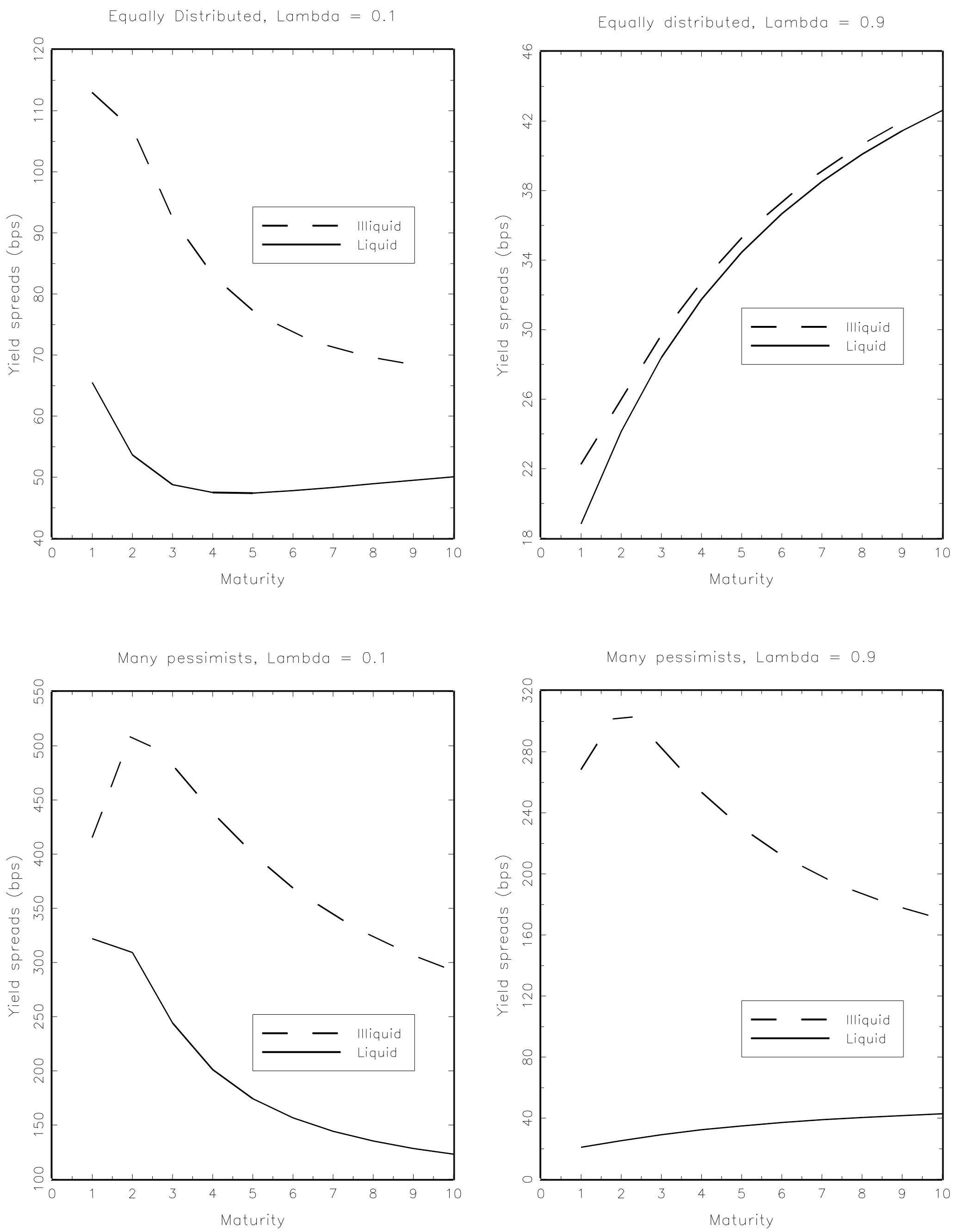

Figure 6: Term structure of credit spreads for different distributions of investors' types and for different values of λ.The other parameters are those of the base-case example.

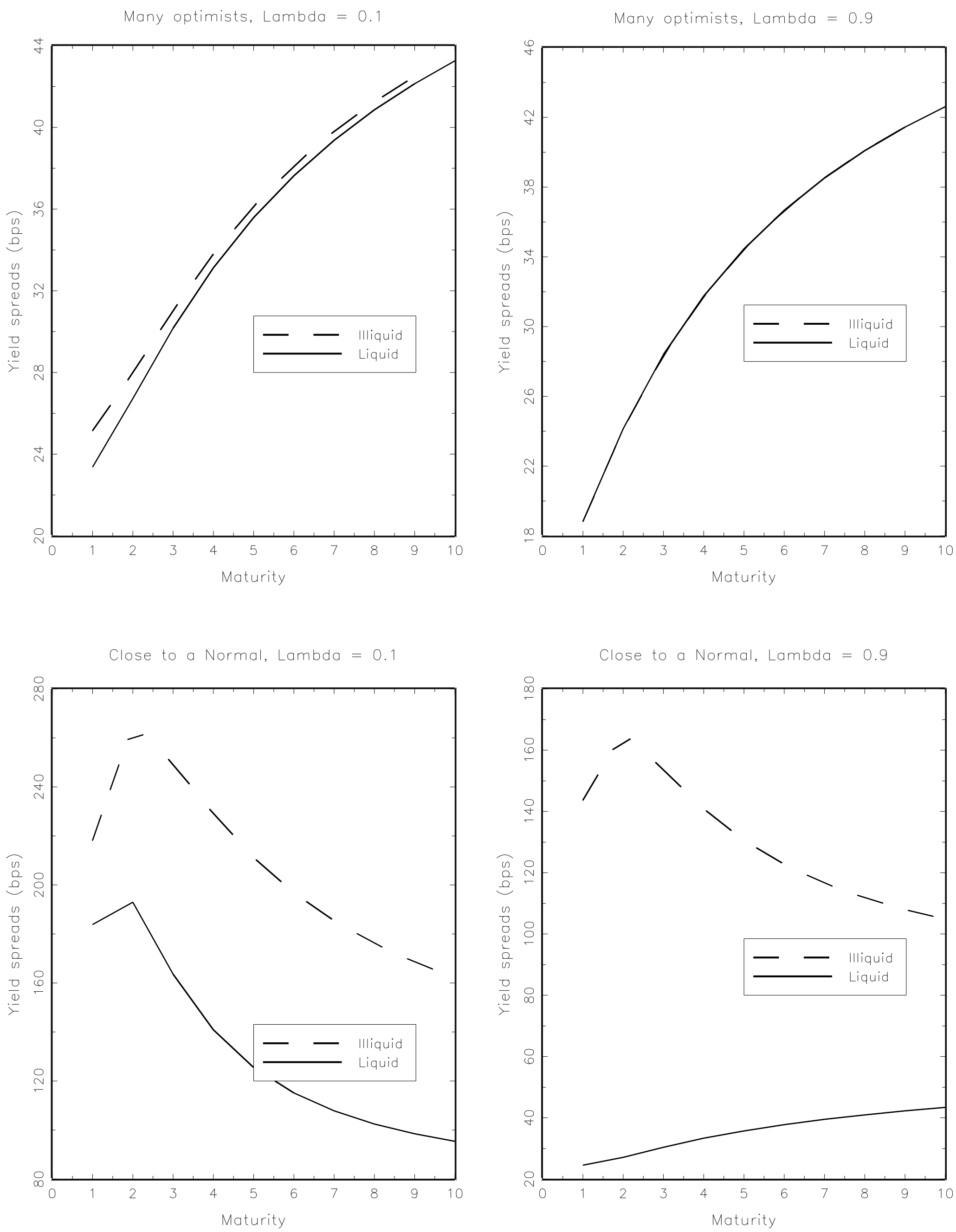

Figure 7: Term structure of credit spreads for different distributions of investors' types and for different values of λ.The other parameters are those of the base-case example.

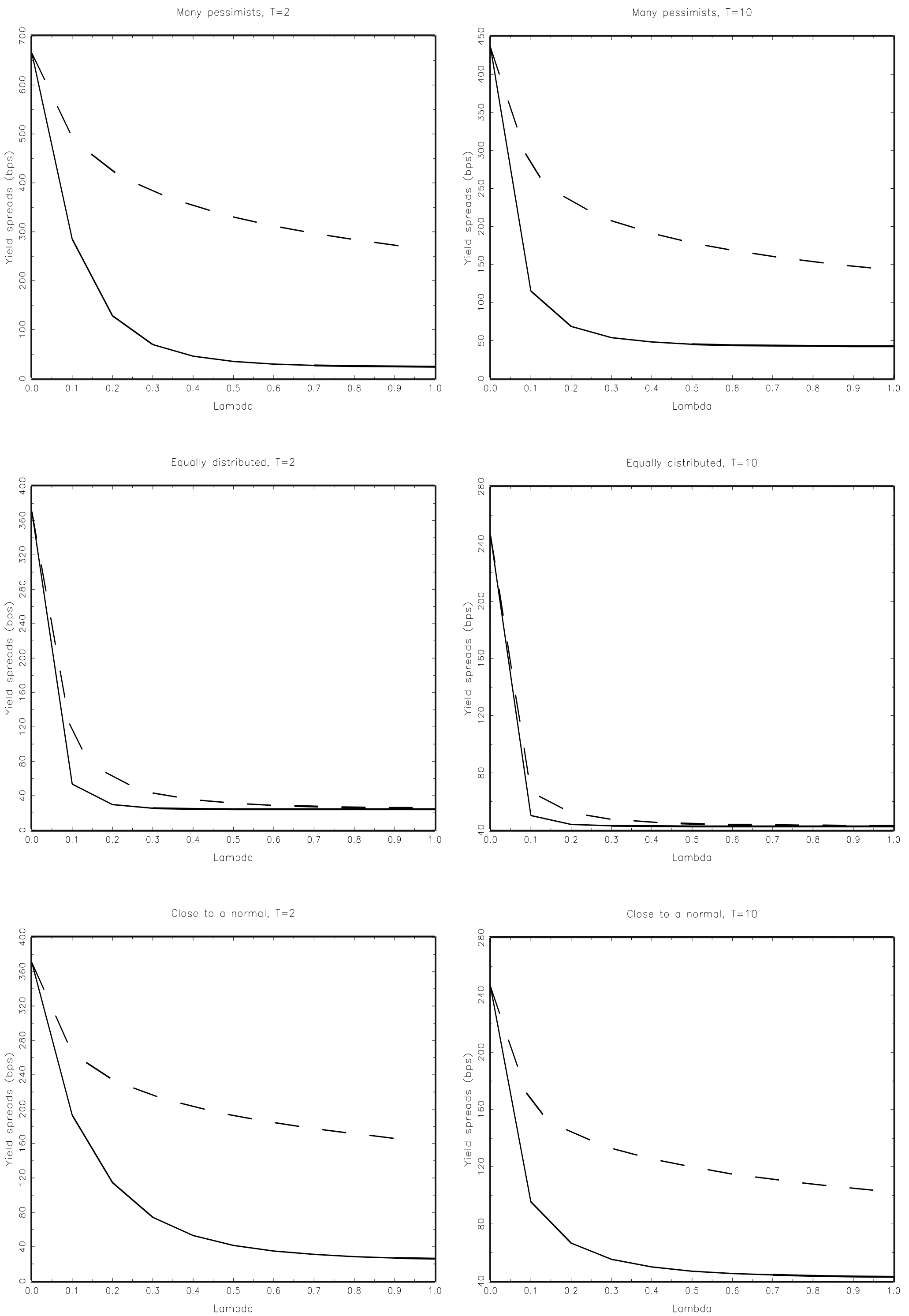

Figure 8: Effect of λ on the risk premium for the "illiquid" and "liquid" cases.

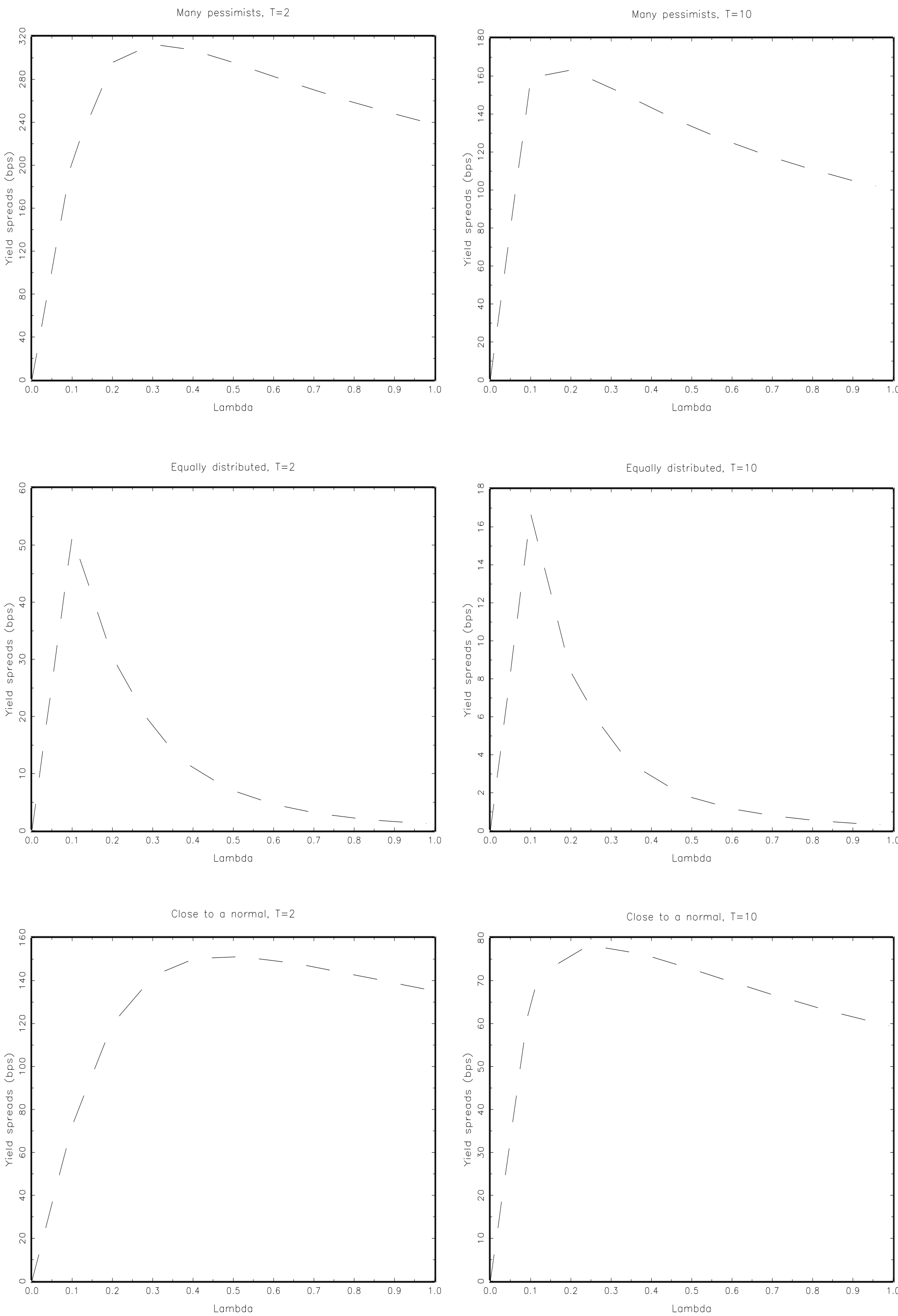

Figure 9: Liquidity Premium function of λ

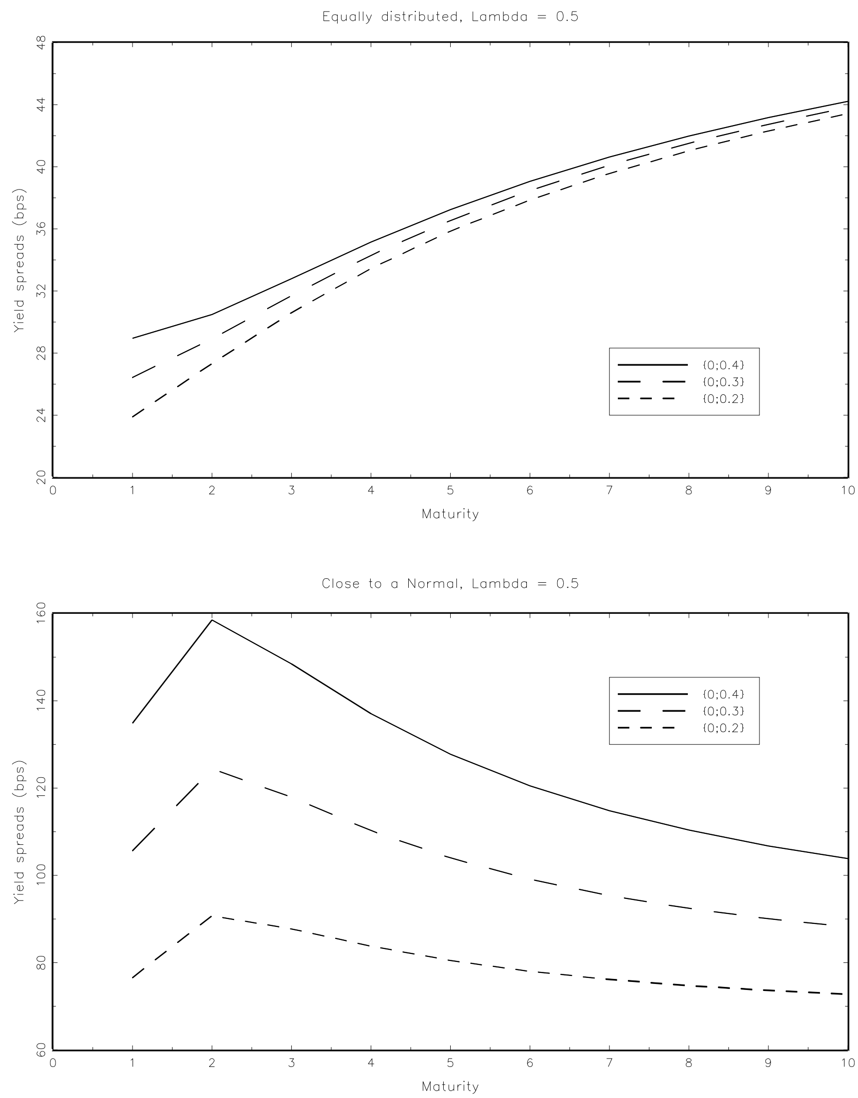

Figure 10: Term structure of credit spreads for different supports of the distribution of bankruptcy costs.

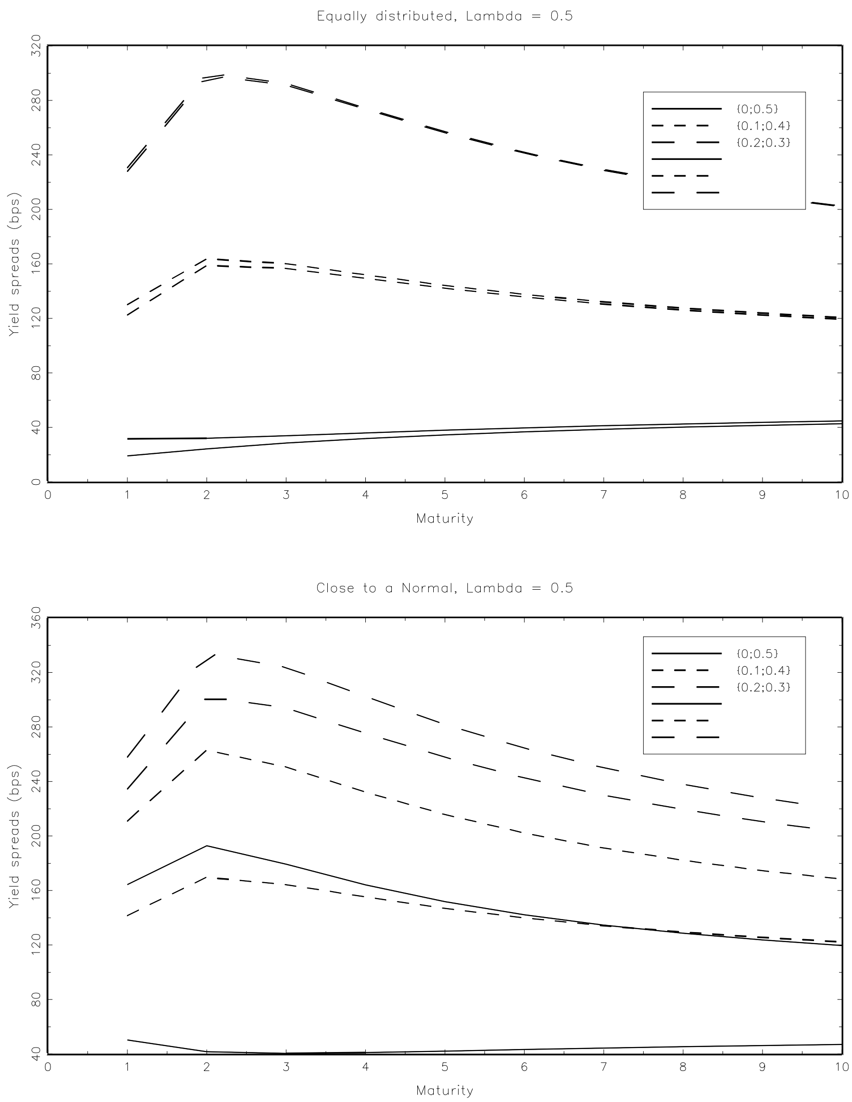

Figure 11: Term structure of credit spreads for different supports of the distribution keeping the average bankruptcy cost constant.

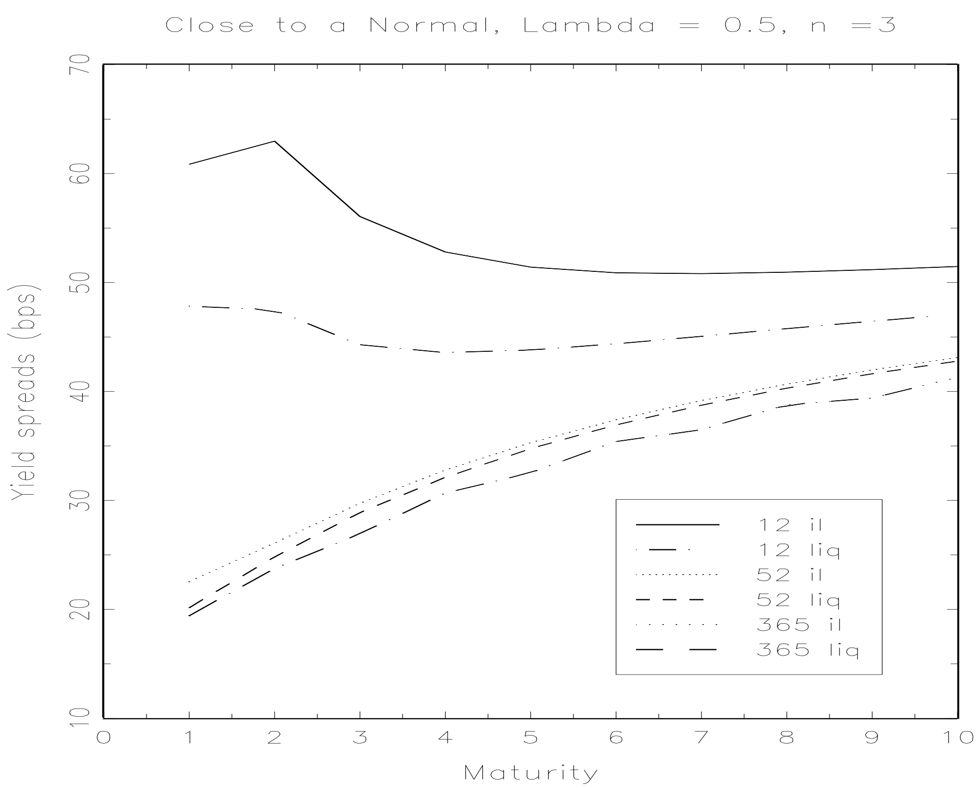

Figure 12: Term structure of credit spreads for different trading frequencies (monthly : 12 trades a year, weekly : 52 trades a year and daily : 365 trades a year) when there are 3 different types of investors in the "close to a normal" case.