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\HideDisplacementBoxes
\usepackage{arapa}

\newenvironment{Proof}{\noindent {\bf Proof.}}{ \hfill{\sc q.e.d}\smallskip}
\newtheorem{Prop}{Proposition}
\newtheorem{Lem}{Lemma}


\title{Labour Supply when Consumption Takes Time}
\author{Pascal Berten\\IRES, Department of Economics\\ Universtité Catholique de
Louvain\\Louvain-la-Neuve, Belgium.\thanks{This research has been supported by a
grant ``Actions de Recherche Concertées" n$^\circ$ 93/98-162 of the Ministery of
Scientific Research of the Belgian French Speaking Community.} \thanks{I would like
thanks Michel DeVroey for his support to the present work, and to Pierre Dehez for mo
helpful discussions and comments.}}
\date{Avril 1997}


\begin{document}
\nocite{AtkinsonStern79}
\nocite{Owen69}
\setcounter{page}{0}
\maketitle
\begin{abstract}
When dealing with labour supply, the classical microeconomic theory does not investi
cautiously the way consumer-workers allocate time. The non-working time (more ofte
``leisure") is assumed to be a normal good on which the individual preferences are defi
This paper is based on the works of \cite{Becker65} and aims at strenghtening the th
foundation of the individual labour supply function by assuming that consumption take
The model so developed is then compared to the classical formulation of the individua
behaviour on the labour market. It is shown that, in such a framework, individuals may
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not to work {\it whatever the wage level}.
\end{abstract}

\clearpage

\section{Introduction}

The purpose of this paper is to analyse the behaviour of economic agents on the labou
market when consumption takes time. The microeconomic theory has always dealed
with the simple trade-off between work and leisure, assuming disutility of work. No
description is made of the non-working use of time. In such a framework, it seems th
leisure is just a part of time in which nothing is done. Using Becker's
words, the analysis of the non-working use of time should be more sophisticated:

\begin{quote}
``{\it It is perhaps surprising that economists have not been equally sophisticated abc
other non-working uses of time. For exemple, the cost of a service like the theatre or
good like meat is generally simply said to equal their market prices, yet everyone wo
agree that the theatre and even dining takes time, (...), time that often could have beer
used productively. If so, the full costs of these activities would equal the sum of mar
prices and the foregone value of time used up}".\footnote{\cite{Becker65}.}
\end{quote}

This lack of sophistication in the description of the behaviour of economic agents in
economic theory has been criticized by several authors. In the context of consumption
theory, Lancaster's critics is summarized in the following quotation:

\begin{quote}
``{\it Johnson's comment, on reflection, would seem to be almost the best summary th
be given of the current state of the theory of consumer behaviour. All intrinsic prope
of particular goods, those properties that make a diamond quite obviously something
different from a loaf of bread, have been omitted from the theory, so that a consumer
consumes diamonds alone is as rational as a consumer who consumes bread alone, but
who sometimes consumes bread, sometimes diamonds, is irrational. Thus the only prc
which the theory can build on is the property shared by all goods, which is simply tha
are goods}."\footnote{\cite{Lancaster66}.}
\end{quote}

Though this comment applies to consumption theory, one could think that such incons
would also exist in the framework of labour supply analysis. The inability of economi
theory to reproduce some well-known observed facts on the labour market (as involur
unemployment) indicate that one should go deeper into the analysis of time allocation
\vspace{3mm}

In the models developped below we assume that individuals do not suffer any {\it dire

disutility of work. However, since the part of time spent at work reduces the time
available for consumption, an individual will suffer an indirect disutility of work. It
be assumed that non-working time is used, either for consumption, or for the producti
domestic goods (like cooking)\footnote{See \cite{Becker65} and \cite{AtkinsonStern7
will be shown that such sophistication will not fundamentally change the results obta
the classical analysis (i.e. the shape of the individual labour supply curve).
\vspace{3mm}

In the second section of this paper the classical model of individual optimization witl
labour supply will be formulated. The purpose of this section is to let the reader have
mind the fundamental results that one can derive from such a model. The third sectior
makes precise the analysis proposed by Becker in his 1965 paper. The third section
compares the results obtained in the first two sections.

%%
%%

\section{A classical model of individual optimization with labour supply}

There are n goods indexed $i=1,...,n$. A consumer owns T units of time that he can
devote either to work (t) or to leisure (l). A rational consumer with preferences
represented by a utility function $U({\bf x},l)$ (which is assumed to be well behaved)
chooses the commodity bundle ${\bf x}=(x_1,...,x_n)$ and the time spent at work ($T-l
maximizes this function under his budget constraint:

\vspace{3mm}
 $\displaystyle \mbox{Max }U(x,l)$
\vspace{3mm}

subject to

\vspace{3mm}
$\left\{
\begin{tabular}{ll}
 $\displaystyle \sum p_i x_i\leq w(T-l)+R$ &\\
 $0\leq l\leq T$, $x_i\geq 0 \hspace{8mm} \forall i$ &
\end{tabular}
\right.$
\vspace{3mm}

\noindent where w is the wage and p_i is the unit price of good i.
\vspace{3mm}

The first order conditions of this simple maximization problem, for an {\it interior} s
are given by\footnote{The monotonicity of preferences assumption implies that the bu

```
constraint holds with equality.}:

\vspace{3mm}
  $\displaystyle \frac{u_i}{u_j}=\frac{p_i}{p_j}\hspace{10mm}\forall i,j$
\vspace{3mm}

  $\displaystyle \frac{u_l}{u_i}=\frac{w}{p_i}\hspace{10mm}\forall i$
\vspace{3mm}

 $\displaystyle \sum p_i x_i+lw=I,$\
\hfill (1)
\vspace{3mm}

\noindent where $u_i=\partial U/\partial x_i$ is the marginal utility for commodity $
$u_l=\partial U/\partial l$ is the marginal utility of leisure and $I=R+wT$ is total inc
\vspace{3mm}

Treating $I$ parametrically, the solution to this problem can be written
${\bf x}=\tilde{\xi}({\bf p},w,I)$ and $l=\tilde l({\bf p},w,I)$. The demand functions fc
commodities and leisure are then defined by:

\vspace{3mm}
$\displaystyle \xi({\bf p},w,R)=:\tilde{\xi}({\bf p},w,R+wT)$
\vspace{3mm}

$\displaystyle l({\bf p},w,R)=:\tilde l({\bf p},w,R+wT).$
\vspace{3mm}

\noindent Using the Slutsky equations, one can write

\vspace{3mm}
  $\displaystyle \frac{\partial \xi_i}{\partial w}=s_{il}+(T-l)\frac{\partial
\tilde{\xi}}{\partial I}
$ \
\hfill (2)

\vspace{3mm}
  $\displaystyle \frac{\partial l}{\partial w}=s_{ll}+(T-l)\frac{\partial \tilde l}{\partia
\hfill (3)
\vspace{3mm}

\noindent where $s_{il}=\partial \tilde{\xi_i}/\partial w\mid_{\bar u}$ and $s_{ll}=
\tilde l/\partial w\mid_{\bar u}$ are the substitution effects derived when $I$ is tre
parmatrically.
\vspace{3mm}
```

In terms of labour supply, we have $\tilde t({\bf p},w,I)=T-\tilde l({\bf p},w,I)$, and t
supply function is then defined by $t({\bf p},w,R)=\tilde t({\bf p},w,R+wT)$.

\vspace{3mm}
 $\displaystyle \frac{\partial t}{\partial w}=-s_{ll}+t \frac{\partial \tilde t}
{\partial I},$\ \hfill (4)
\vspace{3mm}

The sign of the partial derivative (4) is, {\it a priori}, not defined. Indeed, assuming tl
leisure is a normal good, the substitution effect (s_{ll}) and the income effect ($t
\tilde t/\partial I$) are both negative. This result implies that the labour supply cur\
have many different shapes depending on the preferences. In the tradition of the econ
literature it is, most of the time, assumed that the substitution effect dominates for
wages (i.e. equation (4) is positive) and that the income effect dominates for high wa
(i.e. equation (4) is negative). It is important to know that there is large class of
preferences with standard properties (i.e. strict quasi-concavity, non satiation,...) lea
to labour supply functions that do not match these assumptions. Thus the restriction
particular type of function can be justified only through the observation of {\it valid}
economic data.

%%
%%

\section{Becker's theory of the allocation of time}

In the light of his comments (reproduced in the introduction) Becker proposes a
refinement of the time allocation analysis in order to take into account the indirect c
of consumption (in terms of time). He develops a model in which the consumer-worker
does not derive direct utility of the market goods, but uses them (in combination with
time) in order to produce final goods on which preferences are defined.

\subsection{The model}

There are m market commodities x_i, $i=1,...,n$. Households own T units of tim
they can devote either to produce goods (z_j, $j=1,...,m$) or to work at a wage rate
Preferences are defined by a utility function $U\equiv U(z_1,...,z_m)$
\vspace{3mm}

The households' technology is described by the following production function:

\vspace{5mm}
 $\displaystyle z_j=f_j({\bf x_j},T_j),$\ \hfill (5)
\vspace{5mm}

\noindent where ${\bf x_j}$ is a vector of market commodities and T_j the time
input\footnote{Becker actually assumes that T_i is a vector allowing to distinguish
for exemple, between hours used during the day or weekdays and hours used during nig
week-ends}. For simplicity, we will assume that ${\bf x_j}$ and thus ${\bf p_j}$ are
and that $n=m$\footnote{Thus indices i and j will no longer be distinguished in th
following (for the purpose, i will be used). The qualitative results are not affected
simplification.}.
\vspace{3mm}

The households maximize their utility function subject to

\vspace{5mm}
$\displaystyle \sum_i p_i x_i\leq R+w t$\ \hfill (6)

\vspace{3mm}
$\displaystyle \sum_i T_i\leq T$\ \hfill (7)
\vspace{5mm}

\noindent where t is the time spent at work. Since there is no disutility of work an
preferences are monotone, one can write $t:=T-\sum_i^n T_i$ and
constraint (7) can be replaced by:

\vspace{3mm}
$\displaystyle \sum_i T_i+t= T$\ \hfill (7')
\vspace{5mm}

Assuming a linear activity model, the technology can be described as
follows:
\vspace{5mm}

$\left\{
\begin{tabular}{rcr}
T_i & $\equiv$ & α_iz_i\\
x_i & $\equiv$ & b_iz_i
\end{tabular}
\right.$
\ \hfill (8)
\vspace{5mm}

\noindent where α_i is the time input per unit of z_i and b_i is the comm
input per unit of z_i. This specification is very restrictive but leads to some
interesting simplifications in the computation of the individual labour supply curve.
\vspace{3mm}

Thus, substituting for t in (6) its equivalent in (7') gives,

```
\vspace{5mm}
 $\displaystyle \sum p_ix_i + \sum w T_i\leq R+wT.$\ \hfill (9)
\vspace{5mm}

 Using (8), (9) can be written as

\vspace{5mm}
 $\displaystyle \sum (p_ib_i+\alpha_iw)z_i\leq R+wT.$\ \hfill (10)
\vspace{5mm}

Letting

\vspace{5mm}

$
\left\{
\begin{tabular}{rcr}
$\pi_i$ & $\equiv$ & $p_ib_i+\alpha_iw$\\
$I$ & $\equiv$ & $R+wT$,
\end{tabular}
\right.\ \hfill (11)
$
\vspace{5mm}

\noindent the maximization problem becomes

\vspace{5mm}
 $\displaystyle \mbox{Max }U(z_1,...,z_n)$
\vspace{3mm}

 subject to

\vspace{3mm}
 $\displaystyle \sum \pi_i z_i\leq I$

\vspace{3mm}
 $\displaystyle \sum \alpha_i z_i\leq T,$
\vspace{5mm}

\noindent where $\pi_i$ can be interpreted as the shadow price of commodity $i$;
$p_ib_i$ is the cost associated with the purchase of $x_i$, and $\alpha_i w$ the cost
associated with the use of time in the production of $z_i$.  Thus, this model makes th
cost of non-working time enter the prices of domestic goods.
\vspace{3mm}
```

```
\subsection{Results and comments}

The first order conditions of this maximization problem are (assuming interior soluti
i.e.
$z_i>0$, $\forall i$),

\vspace{5mm}
 $\displaystyle u_i(z)-\lambda\pi-\mu \alpha_i=0\mbox{\hspace{10mm}i=1,...,n}$\ \
(12)
\vspace{3mm}

 $\lambda\geq 0$, $\mu\geq 0$
\vspace{3mm}

 $\displaystyle \sum\pi_i z_i\leq I$ \hspace{11mm} with equality if $\lambda>0$\
(13)
\vspace{3mm}

 $\displaystyle \sum \alpha_i z_i\leq T,$ \hspace{10mm} with equality if $\mu>0$\
(14)
\vspace{5mm}

\noindent where $u_i(z)=\partial U/\partial z_i$ is the marginal utility of the final g
$z_i$.
\vspace{3mm}

A rational individual will chose a bundle such that the marginal rate of substitution b
any pair of domestic goods equals, either the price ratio, or the ``time-input" ratio (i.e
$\alpha_i/\alpha_j$). Treating $I$ parametrically, the solution to this problem is uniq
can be written ${\bf z}={\bf \tilde z}({\bf \pi},I)$. We then define $z(p,w,I)=:{\bf \tild
\pi},R+wT)$.
\vspace{3mm}

Surprisingly, in his 1965 paper, Becker implicitly assumes that $\mu=0$ and that
$\lambda>0$ while this is not necessarily the case. Obviously, $\lambda$ and $\mu$ c
be simultaneously nil and three cases must be considered:

\vspace{5mm}
\begin{tabular}{llll}
1) & $\lambda>0$ &and& $\displaystyle T-\sum \alpha_i z_i> 0$\\
2) & $\displaystyle I-\sum\pi_i z_i>0$ &and& $\mu>0.$\\
3) & $\lambda>0$ &and& $\mu>0.$
\end{tabular}
\vspace{5mm}
```

```
Even if the first case is the most interesting one, the other two should not be neglect
Indeed, case 2 and 3 are straightforward to analyse in a context of individual behavio
(we will see that $t=0$ in both cases).
\vspace{5mm}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

{\bf \underline{Case 1}}. In the first case, $\sum \alpha_i z_i<T$ and $\sum\pi_i z_i=
the time constraint is not binding while the budget constraint is binding. The figures
describes the five possible subcases compatible with  $\lambda>0$ and $\displaystyl
\alpha_i z_i> 0$. Such a case arises when the non labour income $R$ is sufficiently lo
subcases compatible with these assumptions are depicted in figure 1.

\begin{figure}[p]
\centering
\begin{tabular}{cc}
\BoxedEPSF{TAfig1a.EPSF scaled 600}
&
\BoxedEPSF{TAfig1b.EPSF scaled 600}
\ \
\BoxedEPSF{TAfig1c.EPSF scaled 600}
&
\BoxedEPSF{TAfig1d.EPSF scaled 600}
\ \
\multicolumn{2}{c}{\BoxedEPSF{TAfig1e.EPSF scaled 600}}
\end{tabular}
\caption{$\lambda>0$ and $\displaystyle T-\sum \alpha_i z_i> 0$}
\end{figure}
\clearpage

Each subcase is restricted by the following condition\footnote{It is interesting to no
that for intermediate levels of $R$ the three possible situations may arise depending
the utility function.}:

\begin{itemize}
\item[(a)] $\displaystyle \frac{T b_1 p_1}{\alpha_1}<R<\frac{T b_2 p_2}{\alpha_2}.$
\item[(b)] $\displaystyle \frac{T b_1 p_1}{\alpha_1}>R>\frac{T b_2 p_2}{\alpha_2}.$
\item[(c)] $\displaystyle \frac{T b_1 p_1}{\alpha_1}>R,\, \frac{T b_2 p_2}{\alpha_2}>
\mbox{ and }\, \frac{\alpha_1}{\alpha_2}<\frac{b_1 p_1}{b_2 p_2}.$
\item[(d)] $\displaystyle \frac{T b_1 p_1}{\alpha_1}>R,\,  \frac{T b_2 p_2}{\alpha_2}
\mbox{ and }\, \frac{\alpha_1}{\alpha_2}>\frac{b_1 p_1}{b_2 p_2}.$
\item[(e)] $\displaystyle \frac{T b_1 p_1}{\alpha_1}>R,\,  \frac{T b_2 p_2}{\alpha_2}
\mbox{ and }\, \frac{\alpha_1}{\alpha_2}=\frac{b_1 p_1}{b_2 p_2}.$
\end{itemize}
```

The maximization problem becomes of the standard form and thus, using the Slutsky e

\vspace{3mm}
$\displaystyle \frac{\partial z_i}{\partial \pi_j}=s_{ij}-z_j \frac{\partial \tilde z_i}{\partial I},$\ \hfill (15)
\vspace{3mm}

\noindent or in matrix form:

\vspace{3mm}
$\displaystyle {\bf Z_{\pi}}={\bf S}-{\bf z' z_I},$\ \hfill (15')
\vspace{3mm}

\noindent where ${\bf S}$ is the Slutsky matrix of this maximization problem, ${\bf z^T}=(z_1,...,z_n)^T$ and ${\bf z_I}=(\partial \tilde z_1/\partial I,...,\partial \tilde z_n/\partial I)$.
\vspace{3mm}

Defining $t({\bf p},w,R)=T-\sum_i \alpha_i z_i({\bf p},w,R)$ and differentiating equat
(keping in mind that $T_i\equiv \alpha_i z_i$), one can write:

\vspace{5mm}
$\displaystyle \frac{\partial t}{\partial w}=-\sum_i \alpha_i \frac{\partial z_i}{\p
w}.$\ \hfill (16)
\vspace{5mm}

\noindent Moreover,

\vspace{5mm}
$\displaystyle \frac{\partial z_i}{\partial w}=\sum_j\frac{\partial \tilde z_i}{\par
\pi_j} \frac{\partial \pi_j}{\partial w}+\frac{\partial \tilde z_i}{\partial I}\frac{\pa
{\partial w}.$\ \hfill (17)
\vspace{5mm}

\noindent Since $\partial \pi_j/\partial w=\alpha_j$ and $\partial I/\partial w=T$ e
(17) can be rewritten as

\vspace{5mm}
$\displaystyle \frac{\partial z_i}{\partial w}=\sum_j\frac{\partial \tilde z_i}{\par
\pi_j} \alpha_j +\frac{\partial \tilde z_i}{\partial I} T.$\ \hfill (18)
\vspace{5mm}

```
\noindent Substituting (15) into (18) one gets

\vspace{5mm}
 $\displaystyle \frac{\partial z_i}{\partial \pi_j}=\sum_j[s_{ij}-z_j\frac{\partial
\tilde z_i}{\partial I}] \alpha_j +\frac{\partial \tilde z_i}{\partial I} T.$\ \hfill (18')
\vspace{5mm}

\noindent Substituting (18') into (16) yields

\vspace{5mm}
 $\displaystyle \frac{\partial t}{\partial w}=-\sum_i \sum_j \alpha_i s_{ij}
\alpha_j+\sum_i\sum_j \alpha_i z_j\frac{\partial \tilde z_i}{\partial I} \alpha_j -\
\alpha_i \frac{\partial \tilde z_i}{\partial I} T.$\ \hfill (16')
\vspace{5mm}

\noindent Rearranging (16') one gets

\vspace{5mm}
 $\displaystyle \frac{\partial t}{\partial w}=-\sum_i \sum_j \alpha_i s_{ij}
\alpha_j+\sum_i \alpha_i\frac{\partial \tilde z_i}{\partial I}[\sum_j \alpha_j z_j-T
\vspace{5mm}

\noindent Knowing that $\sum_i \alpha_i z_i-T=t$, we get (in matrix form),

\vspace{5mm}
 $\displaystyle \frac{\partial t}{\partial w}=-{\bf \alpha S \alpha'}-t {\bf \alpha z_I
\hfill (19)
\vspace{5mm}

\noindent where ${\bf {\bf \alpha}}=(\alpha_1,...,\alpha_n)$. Since ${\bf S}$ is negati
${\bf {\bf \alpha} S {\bf \alpha'}}<0$. Moreover, $t {\bf {\bf \alpha} z_I}$ is positive. T
sign of equation (19) is not defined.
\vspace{5mm}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

{\bf \underline{Case 2}}. In this case one has $\sum \alpha_i z_i=T$ which implies th
$t=0$. Individuals do not have enough time to consume what they could buy ($R$ is
relatively high). In fact this is the case {\it independently of} $w$. To prove this, we
remark that several cases must be considered (see figure 2):

\begin{itemize}
\item[(a)] $\displaystyle \frac{T b_1 p_1}{\alpha_1}<R<\frac{T b_2 p_2}{\alpha_2}.$
\item[(b)] $\displaystyle \frac{T b_1 p_1}{\alpha_1}>R>\frac{T b_2 p_2}{\alpha_2}.$
\item[(c)] $\displaystyle \frac{T b_1 p_1}{\alpha_1}<R,\, \frac{T b_2 p_2}{\alpha_2}<F
```

```
\mbox{ and }\, \frac{\alpha_1}{\alpha_2}>\frac{b_1 p_1}{b_2 p_2}.$
\item[(d)] $\displaystyle \frac{T b_1 p_1}{\alpha_1}<R,\,  \frac{T b_2 p_2}{\alpha_2}<
\mbox{ and }\, \frac{\alpha_1}{\alpha_2}<\frac{b_1 p_1}{b_2 p_2}.$
\item[(e)] $\displaystyle \frac{T b_1 p_1}{\alpha_1}<R,\,  \frac{T b_2 p_2}{\alpha_2}<
\mbox{ and }\, \frac{\alpha_1}{\alpha_2}=\frac{b_1 p_1}{b_2 p_2}.$
\end{itemize}

\noindent It is obvious that, whatever the case, for small variations of $w$, the solut
does not change. Indeed, since $T=\sum \alpha_iz_i$ one has $\sum \alpha_i \partial
z_i/\partial w=0$ which implies that $\partial t/\partial w=0$.

\begin{figure}[p]
\centering
\begin{tabular}{cc}
\BoxedEPSF{TAfig2a.EPSF scaled 600}
&
\BoxedEPSF{TAfig2b.EPSF scaled 600}
\ \
\BoxedEPSF{TAfig2c.EPSF scaled 600}
&
\BoxedEPSF{TAfig2d.EPSF scaled 600}
\ \
\multicolumn{2}{c}{\BoxedEPSF{TAfig2e.EPSF scaled 600}}
\end{tabular}
\caption{$\displaystyle I-\sum\pi_i z_i>0$ and $\mu>0.$}
\end{figure}
\clearpage

In order to show that this is also the case for larger variations in $w$ we will procee
steps\footnote{All the reasonning is made considering the case (a) of figure 1 (i.e. the
slope of the time constraint is higher than the slope of the budget constaint). The
reasoning can be easily transposed to symmetric cases.}.
\vspace{3mm}

The first step is summarized by the following proposition:

\begin{Prop}
Any two budget constraints (defined for two different wages, all other parameters be
have an intersection that does not depend on the wage level. Moreover, this intersecti
hyperplane always belongs to the time constraint (i.e. it is also the intersection betwe
budget constraint and the time constraint).
\end{Prop}

\begin{Proof}
Define two such budget constraints using two different wage levels (say $w'$ and $w'
```

```
\vspace{5mm}
  $\left\{
\begin{tabular}{lll}
$\displaystyle \sum_i z_i (p_i b_i+\alpha_i w')$ & $=$ & $T w'+R$\\
$\displaystyle \sum_i z_i (p_i b_i+\alpha_i w'')$ & $=$ & $T w''+R$
\end{tabular}
\right.$
\ \hfill (20)
\vspace{5mm}

\noindent Replacing the second equation of this system by their difference, one can
rewrite the system as

\vspace{5mm}
  $\left\{
\begin{tabular}{lll}
$\displaystyle \sum_i z_i (p_i b_i+\alpha_i w')$ & $=$ & $T w'+R$\\
$\displaystyle \sum_i \alpha_i z_i $ & $=$ & $T$
\end{tabular}
\right.$
\ \hfill (21)
\vspace{5mm}

\noindent Hence solving the system (20) for $z$ is equivalent to search for the
intersection between any one of the two budget constraints and the time constraint (i
the hyperplane that solves the system belongs to the time constraint).
\vspace{3mm}

Finally, (21) can be rewritten as (replacing the first equation of the system by their
difference):

\vspace{5mm}
  $\left\{
\begin{tabular}{lll}
$\displaystyle \sum_i p_i b_i z_i $ & $=$ & $R$\\
$\displaystyle \sum_i \alpha_i z_i $ & $=$ & $T$
\end{tabular}
\right.$
\ \hfill (22)
\vspace{5mm}

\noindent The system of equations (22) defines an hyperplane of dimension $n-1$ whic
not depend on
$w$.
```

\end{Proof}
\vspace{3mm}

Secondly it is easy to see that as w goes to infinity, the slope (in any direction) of
budget constraint {\em tends} to the slope of the time constraint. Indeed, the slope of
budget constraint (in a particular direction) is given by π_i/π_j, and the slope o
time constraint (in the same direction) by α_i/α_j. Since,

\vspace{5mm}
$\displaystyle \lim_{w\rightarrow \infty}\frac{b_i p_i+\alpha_i w}{b_j p_j+\alpha
w}=\frac{\alpha_i}{\alpha_j}.$\
\vspace{5mm}

\noindent the result follows.
\vspace{3mm}

This last result combined with the previous one tells us that one can never switch fro
case to another (for exemple, from the case described in figure 2(a) to the case of fig
Thus an individual in a situation where he decides not to work will keep his decision
w.

%%

\vspace{5mm}
{\bf \underline{Case 3}}. Since both constraints are binding, the third case also implie
$t=0$. As in the second case, a small increase (or decrease) in w does not affect $t
The same argument prevails for larger variations in w.
\vspace{5mm}

These last two cases should not be neglected. If the results are uninteresting in the
framework of individual behaviour analysis, this may obviously not be the case in the
aggregate. The result obtained in these two cases is somewhat intriguing. they tell us
that even with an extremely high wage level (commodity prices being given) some
individuals will find optimal not to work. However, in a general equilibrium framework
wage increase would induce price variations. Such price variations would push the bu
constraint upward or downward and thus lead some individuals to switch either from
2 or from 3 to case 1.

\begin{figure}[h]
\centering
\begin{tabular}{cc}
\BoxedEPSF{TAfig3a.EPSF scaled 600}
&
\BoxedEPSF{TAfig3b.EPSF scaled 600}
\end{tabular}

```
\caption{$\lambda>0$ and $\mu>0.$}
\end{figure}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

\section{Conclusion}

It is interesting to notice the similarity between equation (4) and equation (19). Both
equations have an ambiguous sign and the two models allow for a wide class of indivi
labour supply curves. The main difference between the classical model and Becker's o
in the possible existence of individuals who do not supply any labour {\it whatever} $\
Though in the classical model it can be the case that an individual decides not to work
given wage level), this will never be the case ``whatever the wage level". This is not
unrealistic. For instance, think of extremely wealthy persons who live on their
annuities.\vspace{3mm}

Even more surprising\footnote{In a static frame work where individuals do not
derive any direct utility of income.} is the fact that individuals for whom the budget
constraint is not binding do not spend all their income. If we suppose that the number
commodities is high, it would be realistic to assume the existence of commodities w
sufficiently high prices (relatively to the non-wage income). Such an assumption coul
the possibility of ``case 2 and case 3 individuals".
\vspace{3mm}

Since economists should be more concerned by aggregate behaviour, the next step shou
the construction of the aggregate labour supply function. The point is then to see unde
conditions this function has the standard ``textbook" shape, and to test the empirical
of this conditions.
\vspace{3mm}

Consumer-workers can be differentiated in many ways, in particular in terms of
preferences and non-wage income, the former being however unobservable. Labour sup
aggregation could be studied along two lines as exemplified by the works on the aggre
of demand by \cite{Hildenbrand94} and \cite{Grandmont92}.
\clearpage
\bibliographystyle{arapa}
\bibliography{Doc}

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