Stochastic Nominal Wage Contracts in a Cash–in–Advance Model

Fabrice Collard Guy Ertz*

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Abstract

We build a simple cash–in–advance model for the German economy, in which we introduce stochastic nominal wage contracts. This allows to weaken the negative effect of the inflation tax such that monetary shocks exert a positive effect on output dynamics. The nominal wage contract model is able to mimic the correlation of inflation and real balances with output. It also lowers the standard deviation of inflation relative to that of output. Further, the variance decomposition analysis indicates that in this setting, monetary shocks explain between 30% and 45% of output volatility in the first quarter. Moreover, it indicates that this model generates a long lasting effect of monetary shocks on output dynamics.

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*Respectively, IRES Department of Economics, Université catholique de Louvain (UCL), MAD Université Paris-I and IRES, UCL. The financial support of the European HCM network ("Imperfect competition in intertemporal general equilibrium macro models") is gratefully acknowledged. This text presents research results of the Belgian programme on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the authors. We are thankful to G. Ascari, P. Malgrange, F. Portier, H. Sneessens and all the participant to the T2M conference in Louvain–la–Neuve (May 1997) for their comments on earlier drafts, and to P. Brandner and K. Neusser for providing German data.
Correspondence to : Guy Ertz, IRES, Place Montesquieu 3, B–1348 Louvain la Neuve, Belgium. E-mail: `Ertz@ires.ucl.ac.be`

Introduction

Many economists agree that the evidence supports the existence of a positive and lasting effect of monetary shocks on output in the short run. The basic cash–in–advance business cycle model (Cooley and Hansen [1989]) does not mimic all these stylized facts. In particular, it predicts an instantaneous fall in output after a positive money growth shock. Indeed, after a money injection, the purchasing power of all nominal balances is reduced. This induces households to increase leisure. This is the so–called inflation tax effect. Recently a class of limited participation or sluggish capital models[1] has focused on liquidity effects that push interest rates down after a positive money growth shock and enhance a positive response of output. But, as noticed by [?], *"these models have one major empirical shortcoming: the inability to account for long–lasting effects of monetary stimulus"*. By allowing for real effects of monetary shocks within the business cycle, the literature on nominal contracts — and more specifically staggered contracts — seems to provide an appropriate way to circumvent this shortcoming[2]. The aim of this paper is to propose a model with stochastic nominal wage contracts able to assess for a positive and long–lasting effect of monetary stimuli.

The introduction of wage contracts has already been studied by RBC theorists. These contracts were initially introduced in the literature by Fisher [1977] and Taylor [1980]. Their aim was to show that the rational expectation hypothesis is not inconsistent with a real effect of monetary policy. The introduction of nominal wage contracts in a RBC framework aimed to account for the propagation of nominal shocks. Thus, Cho [1990], Cho and Cooley [1995] assume that period t nominal wage contracts are set $t-j$ periods in advance on the basis of the rational expectation of the wage that clears the labor market. We will not use this type of contracts in our model Economy. We assume that in each period a worker has a non zero probability to be laid off[3]. The hiring period and thus the length of the contract is random. This structure of contracts is based on Calvo [1983][4]. The choice of this modeling is motivated by the fact that it reinforces the internal persistence mechanism of the model. The duration of nominal wage contracts being stochastic, a monetary shock may alter the behavior of the real economy for a higher number of periods, depending on the mean duration of contracts. Ambler, Cardia and Phaneuf [1991] introduced this type of contracts in a general equilibrium framework. They assume that the economy is characterized by monopolistic competition and endogenous growth. Nevertheless they do not explicitly describe the underlying optimizing behavior of agents. Further the main objective of these papers is to provide a framework that

[1] See Lucas [1990], Christiano [1991], Fuerst [1992] and [?].

[2] See for example Cho and Cooley [1995], Chari, Kehoe and McGrattan [1996].

[3] It shall be clear to the reader that we will not model the hiring–firing process.

[4] Calvo built a model with a continuum of price-setters and in which the probability of a given price-setter changing his price is constant at any point in time

allows to explain the dynamics of employment and wages in the labor market.

As previously noted, we depart from these studies in the sense that we are interested in explaining the long lasting effect of money injections. The introduction of these stochastic nominal wage contracts leads to the abandon of the traditional assumption that the labor supply behavior is determined by intertemporal substitution motives. This allows to weaken the negative effect of the inflation tax such that monetary shocks have a positive effect on output dynamics. The variance decomposition analysis suggests that monetary shocks explain up to 45% of the total variance of output and have a long lasting effect. Further, the model also mimics the correlation between output and inflation and real balances. We also propose an evaluation of the effects of variations in the mean duration of contracts on these indicators.

The remaining of the paper proceeds as follows. The first section describes the competitive cash–in–advance Model. In section 2, we define the structure of the stochastic nominal wage contracts. Section 3 is devoted to the validation of the theoretical model. A last section concludes.

1 The Competitive Cash–in–Advance Model

This model relies on previous work by Cooley and Hansen [1989] and Hairault and Portier [1995]. This section is devoted to the presentation of the set of hypothesis concerning the behavior of households, firms and the monetary authorities.

1.1 The Representative Household

The economy is populated by many identical infinitely lived agents. Each household has preferences on consumption and leisure represented by the following intertemporal utility function:

$$\mathcal{U} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, \ell_t) \right\} \tag{1.1.1}$$

where E_0 denotes the conditional expectation operator[5] at time $t = 0$. β is the discount factor, C_t and ℓ_t denote respectively consumption and leisure. Finally, $U(.,.)$ is the instantaneous utility function, satisfying the traditional Inada conditions.

The household has access to complete financial markets on which he can sell or purchase contingent claims. The household enters period t with some nominal balances, M_t, that corresponds to its money demand at the end of period $t-1$, and an asset portfolio of contingent claims with real value B_t purchased in period $t-1$. The household offers his work on the labor market at the real wage rate W_t/P_t. The household can buy a

[5] Hereafter we will use the notation: $E_t(X_{t+1}) = \int_{\Omega} X(\omega_{t+1}) d\mathcal{F}(\omega_{t+1}|\omega^t)$.

consumption good or a portfolio of assets, $B_{t+1}(\omega_{t+1},\omega^t)$, defining its intertemporal behavior. Contingent claims are purchased in period t for period $t+1$ at intertemporal price $\frac{\rho(\omega_{t+1},\omega^t)}{\rho_t}$. $\rho(\omega_{t+1},\omega^t)$ will thus be interpreted as the discount factor of the firm, as it issues contingent claims to finance investment. $\omega_t \in \Omega$ denotes a particular realization of stochastic shocks, that will be defined later. These two shocks will be defined later on. ω^t denotes the past realizations of ω_t : $\omega^t = \{\omega_1,\ldots,\omega_t\}$. Then the value of household portfolio is given by $\int_\Omega \frac{\rho(\omega_{t+1},\omega^t)}{\rho_t} B_{t+1}(\omega_{t+1},\omega^t)d\omega_{t+1}$.

Then the intertemporal budget constraint of the household is given by:

$$C_t + \frac{M_{t+1}}{P_t} + \int_\Omega \frac{\rho(\omega_{t+1},\omega^t)}{\rho_t} B_{t+1}(\omega_{t+1},\omega^t)d\omega_{t+1} \leq B_t + \frac{W_t}{P_t}h_t + \frac{M_t}{P_t} + \frac{N_t}{P_t} \tag{1.1.2}$$

where P_t denotes the nominal price level, and N_t denotes a lump–sum money transfer from the monetary authorities.

The household also faces a cash–in–advance constraint:

$$C_t \leq \frac{M_t}{P_t} \tag{1.1.3}$$

Finally, the household has a unit time endowment that he allocates between leisure, ℓ_t and working time h_t:

$$\ell_t + h_t = 1 \tag{1.1.4}$$

Then, the household maximizes (1.1.1) subject to (1.1.2)–(1.1.4). The household's optimal behavior is then given by the set of first order conditions and the transversality conditions:

$$U_C(t) = x_t^1 + x_t^2 \tag{1.1.5}$$

$$U_\ell(t) = x_t^1 \frac{W_t}{P_t} \tag{1.1.6}$$

$$\frac{\rho(\omega_{t+1},\omega^t)}{\rho_t} = \beta \frac{x_{t+1}^1}{x_t^1} \frac{d\mathcal{F}(\omega_{t+1}|\omega^t)}{d\omega_{t+1}} \tag{1.1.7}$$

$$\frac{x_t^1}{P_t} = \beta E_t\left(\frac{x_{t+1}^1 + x_{t+1}^2}{P_{t+1}}\right) \tag{1.1.8}$$

$$\lim_{i\to\infty} \beta^{t+i} x_{t+i}^1 B_{t+1+i}(\omega_{t+1+i},\omega^{t+i}) = 0 \tag{1.1.9}$$

$$\lim_{i\to\infty} E_t\left\{\beta^{t+i} x_{t+i}^2 M_{t+1+i}\right\} = 0 \tag{1.1.10}$$

x_t^1 and x_t^2 are the lagrangian multipliers associated respectively to the intertemporal budget constraint and the cash–in–advance constraint. Equation (1.1.5) and (1.1.6) define Frishian demand functions for consumption and leisure. Equation (1.1.7) provides the

Lucas [1978] formula for pricing kernel: the price of a contingent claim in state ω_{t+1} is given by the discounted intertemporal rate of substitution in state ω_{t+1} times its density. (1.1.8) defines the intertemporal behavior of demand for money. Finally, (1.1.9) and (1.1.10) provide terminal conditions to the evolution of assets and money.

1.2 The Representative Firm

The homogeneous good, accumulated and consumed, is produced according to the technology represented by the following production function:

$$Y_t = F(K_t, h_t; A_t, X_t) \tag{1.2.1}$$

where K_t, h_t denote respectively private capital and hours used in the production process. X_t denotes exogenous Harrod neutral technical progress evolving according to :

$$X_{t+1} = \gamma X_t, \ \gamma > 1$$

$F(.)$ is increasing concave with respect to each argument and satisfies Inada conditions.

A_t is an exogenous technological shock affecting total factor productivity. $\log(A_t)$ is assumed to follow a first order autoregressive stationary process:

$$\log(A_t) = \rho_a \log(A_{t-1}) + (1-\rho_a)\log(\overline{\mathrm{A}}) + \varepsilon_{a,t} \tag{1.2.2}$$

with $-1 < \rho_a < 1$, and $E(\varepsilon_{a,t}) = 0$ and $E(\varepsilon_{a,t}^2) = \sigma_a^2$. $\log(\overline{\mathrm{A}})$ denotes the unconditional mean of the process.

The firm invests I_t to form capital according to the following law of motion:

$$K_{t+1} = I_t + (1-\delta)K_t \tag{1.2.3}$$

m where $0 < \delta < 1$ denotes the depreciation rate of capital. Let $\Pi_t = Y_t - \frac{W_t}{P_t}h_t - I_t$ denotes the instantaneous profit of the representative firm, then the firm chooses investment and hours such that it maximizes the discounted intertemporal flow of profits:

$$h_t, I_t \in \text{argmax} \sum_{t=0}^{\infty} \int_{\Omega} \rho_t(\omega_t, \omega^{t-1}) \Pi_t d\omega_t \tag{1.2.4}$$

subject to (1.2.3) and transversality conditions. The optimal choice for the firm is defined by the first order optimality conditions, capital law of motion:

$$F_h(t) = \frac{W_t}{P_t} \tag{1.2.5}$$

$$q_t = 1 \tag{1.2.6}$$

$$q_t = \int_{\Omega} \frac{\rho(\omega_{t+1}, \omega^t)}{\rho_t} \left(F_K(t+1) + (1-\delta)q_{t+1}\right) d\omega_{t+1} \tag{1.2.7}$$

$$\lim_{i\to\infty}\int_{\Omega}\frac{\rho(\omega_{t+i},\omega^{t+i-1})}{\rho_{t+i-1}}q_{t+i}K_{t+1+i}d\omega_{t+i}=0 \tag{1.2.8}$$

Condition (1.2.5) defines the demand for hours, while (1.2.6) and (1.2.7) correspond to the capital accumulation behavior. (1.2.8) furnishes a terminal condition to the evolution of the marginal Tobin's Q.

1.3 Money Supply

Money is assumed to grow at a rate $(g_t - 1)$:

$$M_{t+1} = g_t M_t \tag{1.3.1}$$

We assume that g_t follows an exogenous stochastic process of the following form:

$$\log(g_t) = \rho_g \log(g_{t-1}) + (1-\rho_g)\log(\bar{g}) + \varepsilon_{g,t} \tag{1.3.2}$$

where $\varepsilon_{g,t}$ is a gaussian white noise with mean zero and variance σ_g^2. Finally, $\log(g_t)$ is a second order stationary process, $|\rho_g| < 1$.

We finally assume that the money created in period t is entirely distributed to households:

$$(g_t - 1)M_t = N_t \tag{1.3.3}$$

1.4 Competitive Equilibrium

In a competitive equilibrium, the resources constraint imposes:

$$Y_t = C_t + I_t$$

Equilibrium on the financial market imposes that the value of contingent claims corresponds to the value of the firm.

Since, the economy grows at an exogenous rate γ, we divide each variable that grows by X_t[6]. Further, nominal variables are deflated by P_{t-1}[7]. We finally define, for convenience, $\lambda_t = q_t x_t^1 X_t^{\varphi}$ and $\mu_t = q_t x_t^2 X_t^{\varphi}$, where $\varphi = 1 - \nu\,(1-\sigma)$ is the intertemporal elasticity of substitution with respect to consumption. Thus λ_t can be interpreted as the stationary marginal value of capital in terms of utility.

Definition 1 *The competitive equilibrium of the economy is a set of policy rules:*

$$z_t = \mathcal{Z}(k_t, m_t, a_t, g_t),\;\; z \in \{c, h, i, y, k_{+1}, m_{+1}\}$$

[6]Let Z_t be a real growing variable, then we define $z_t = Z_t/X_t$

[7]Let Z_t be a nominal growing variable, then we define $z_t = Z_t/(X_t P_{t-1})$. $f_t = \frac{P_t}{P_{t-1}}$ then denotes the inflation factor.

such that:

$$u_c(t) = \lambda_t + \mu_t \tag{1.4.1}$$

$$u_\ell(t) = \lambda_t f_h(t) \tag{1.4.2}$$

$$\lambda_t = \frac{\beta}{\gamma^\varphi} E_t \left\{ \lambda_{t+1} \left(f_k(t+1) + 1 - \delta \right) \right\} \tag{1.4.3}$$

$$\lambda_t = \frac{\beta}{\gamma^\varphi} E_t \left\{ \frac{\lambda_{t+1} + \mu_{t+1}}{f_{t+1}} \right\} \tag{1.4.4}$$

$$f(k_f, h_t, A_t) = c_t + i_t \tag{1.4.5}$$

$$c_t \leq \frac{m_t}{f_t} \tag{1.4.6}$$

$$\gamma k_{t+1} = i_t + (1-\delta) k_t \tag{1.4.7}$$

$$\gamma m_{t+1} = \frac{g_t}{f_t} m_t \tag{1.4.8}$$

$$\lim_{i \to \infty} E_t \left\{ \beta^{t+i} \lambda_{t+i} k_{t+1+i} \right\} = 0 \tag{1.4.9}$$

$$\lim_{i \to \infty} E_t \left\{ \beta^{t+i} \mu_{t+i} m_{t+1+i} \right\} = 0 \tag{1.4.10}$$

2 The Nominal Wage Contract Cash–in–Advance Model

In this section, we depart from the canonical Cooley and Hansen [1989] case presented in the previous section, and we introduce nominal wage contracts. We focus on differences arising in this framework, rather than presenting the full model.

2.1 Nominal wage contracts

The introduction of nominal wage contracts has been widely studied in the RBC literature[8]. In the nominal wage contracts framework, nominal wages are set some period in advance : A contract set at period $t-j$ will prevail in period t in the case of a j periods contract. As soon as the nominal wages are determined by a contract, the household gives the firm the right to manage hours: the households commits itself to provide the labor the firm needs to achieve its production level.

In some studies, the contracts can be staggered, meaning that in each period only a given proportion of contracts (θ_k) are renegotiated. The wage is then defined by the following equation:

$$\log(W_t^c) = \sum_{k=0}^{j} \theta_k E_{t-k} \log(W_t)$$

where W_t^c denotes the contractual wage. In that version, the length of the contracts is given.

[8] See for example Cho [1990] or Fairise [1995].

In this paper, we will consider a more general class of contracts in which the duration of contracts is stochastic. This class of contracts was first introduced by Calvo [1983]. The contract is defined relative to a wage target $W_t^\star$. In this setting, the labor market can be viewed as being composed by many cohorts of households which all sign a contract — χ_t — with the firm. Each contracts is fixed relative to the expected path of a target wage — $W_t^\star$ (equation (2.1.1)), whose meaning will be defined later by reference to a walrasian equilibrium wage. The length of each contract is stochastic. $\Xi(i)$ then represents the probability that a contract applies at least i periods:

$$\log(\chi_t) = \sum_{i=0}^{\infty} \Xi(i) E_t \log\left(W_{t+i}^\star\right) \tag{2.1.1}$$

We assume that contracts end according to a Poisson's process, with extinction probability π. Thus:

$$\begin{aligned} \Xi(i) &= \pi\left(1 - \sum_{j=1}^{i} \xi(j)\right) \\ \xi(j) &= \pi(1-\pi)^j,\ j \geq 0 \end{aligned}$$

Since we are interested in the average agent, we have to aggregate these contracts. This implies that we will only focus on the average nominal wage. This later is given by the sum of non–extinguished contracts :

$$\log(W_t^c) = \sum_{i=0}^{\infty} \Xi(i) \log(\chi_{t-i}) \tag{2.1.2}$$

Using properties of lag operators, we can show that (2.1.1) and (2.1.2) admit the following recursive form:

$$\left\{ \begin{aligned} \log(\chi_t) &= \pi \log(W_t^\star) + (1-\pi) E_t \log(\chi_{t+1}) \\ \log(W_t^c) &= \pi \log(\chi_t) + (1-\pi) \log(W_{t-1}^c) \end{aligned} \right.$$

This system describes the law of motion for nominal wages in a contractual framework. The existence of nominal wage contracts affects the behavior of the household. Following Cho [1990], we assume that the firm has the right to manage employment[9], so that the household has to provide the amount of hours that the firm demands:

$$F_h(K_t, h_t^d X_t; A_t) = \frac{W_t^c}{P_t}$$

[9]We assume that the firm has the right to manage employment so as to avoid time inconsistency problems. Assume for example that a shock occurs in the economy and that the nominal wage contract and employment are bargained between the two agents, this could lead one of the agent to break the contract. In order to avoid this, once the contracts are signed, the firm has the right to freely adjust employment in order to smooth the effects of shocks.

Then, the household is submitted to an additional constraint:

$$h_t^s = h_t^d \tag{2.1.3}$$

Otherwise stated actual hours worked must equal labor demand whatever the state of Nature. In this framework, the optimal labor supply behavior for the household reduces to:

$$U_\ell(t) - x_t^1 \frac{W_t^c}{P_t} = x_t^3$$

where x_t^3 is the lagrangian multiplier associated to the constraint (2.1.3). It can then be interpreted as the cost, expressed in term of utility, of being constrained on the labor market. We assume that the contract is fixed by reference to the nominal wage that would minimize the cost of being constrained on the labor market, that is $x_t^3 = 0$. This defines the wage target in (2.1.1). This amounts to set the nominal wage target by reference to the nominal wage that would prevail in a walrasian framework, as in Cho [1990] and Cho and Cooley [1995]. Thus, $W_t^\star$ is given by:

$$W_t^\star = \frac{P_t U_\ell(t)}{x_t^1}$$

2.2 Equilibrium in the Nominal Wage Contracts Economy

Equilibrium in the nominal wage contract economy differs from the competitive equilibrium, since now hours are demand determined and we have to take into account the two equations defining the law of motion of nominal wages.

Definition 2 *The equilibrium in the nominal wage contracts economy is a set of policy rules:*

$$z_t = \mathcal{Z}(k_t, m_t, a_t, g_t), \;\; z \in \{c, h, i, y, k_{+1}, m_{+1}\}$$

such that:

$$u_c(t) = \lambda_t + \mu_t \tag{2.2.1}$$

$$f_h(t) = \frac{w_t^c}{f_t} \tag{2.2.2}$$

$$\lambda_t = \frac{\beta}{\gamma^\varphi} E_t \left\{\lambda_{t+1} \left(f_k(t+1) + 1 - \delta\right)\right\} \tag{2.2.3}$$

$$\lambda_t = \frac{\beta}{\gamma^\varphi} E_t \left\{\frac{\lambda_{t+1} + \mu_{t+1}}{f_{t+1}}\right\} \tag{2.2.4}$$

$$f(k_f, h_t, A_t) = c_t + i_t \tag{2.2.5}$$

$$c_t \leq \frac{m_t}{f_t} \tag{2.2.6}$$

$$\gamma k_{t+1} = i_t + (1-\delta) k_t \tag{2.2.7}$$

$$\gamma m_{t+1} = \frac{g_t}{f_t} m_t \tag{2.2.8}$$

$$\log(x_t) = \pi \log\left(\frac{u_\ell(t) f_t}{\lambda_t}\right) + (1-\pi) E_t \log(\gamma x_{t+1} f_t) \tag{2.2.9}$$

$$\log(w_t) = \pi \log(x_t) + (1-\pi) \log\left(\frac{w_{t-1}}{\gamma f_{t-1}}\right) \tag{2.2.10}$$

$$\lim_{i\to\infty} E_t \left\{\beta^{t+i} \lambda_{t+i} k_{t+1+i}\right\} = 0 \tag{2.2.11}$$

$$\lim_{i\to\infty} E_t \left\{\beta^{t+i} \mu_{t+i} m_{t+1+i}\right\} = 0 \tag{2.2.12}$$

This latter system, as the one defining the competitive equilibrium, admits no analytical solution. So we log–linearize the system around the deterministic steady–state and solve the obtained linear system. It is then possible to assess for the validity of the model and to characterize its qualitative properties.

3 Validation of the Model

This section is devoted to the analysis of the dynamic properties of the model in both model economies. We then assess for the ability of the model to mimic the main features of the German business cycle. But, since the model admits no analytical solution, we use a numerical approach that first necessitates the calibration of structural parameters.

3.1 Calibration

We calibrate the model for the German economy, from 1960:1 to 1989:4[10]. We use seasonally adjusted quarterly data. Most of the series are taken from the Deutsche Institut für Wirtschaftsforschung (DIW) just as in Brandner and Neusser [1992]. Details on the data used can be found in appendix A.

To calibrate the model we followed the method initiated by Kydland and Prescott [1982] and developed by Cooley and Prescott [1995]. This calibration exercise consists in choosing parameter values such that the balanced growth path (steady state) of our model economy matches certain long-term features of the data.

The household We consider the general parametric class of preferences of the form:

$$U(C_t, \ell_t) = \frac{\left(C_t^\nu \ell_t^{1-\nu}\right)^{1-\sigma} - 1}{1-\sigma}$$

For β and σ we choose the same value as Hairault and Portier [1995] used for France. β was estimated by Laffargue, Malgrange and Pujol [1990] to be 0.99, while σ is taken from Eichenbaum, Hansen and Singleton [1988] to be 4/3.

[10]This choice is related to the break in 1990 due the unification.

We normalize the total time available to work to one and calibrate the parameter ν such that the steady state number of hours is 0.4 which corresponds to the historical share of total disposable time endowment. This lead to a value for ν equal 0.44.

Table 1: Household's Structural Parameters

β	σ	ν
0.99	4/3	0.44

The firm The firm's production possibilities are summarized by the following Cobb-Douglas production function:

$$Y_t \leq A_t K_t^{\alpha}(X_t H_t)^{1-\alpha}$$

This functional form is suggested by the basic observation that capital and labor shares of output have been approximately constant over time. The parameter $1-\alpha$ is referred to as the labor share in output in a competitive environment:$1-\alpha = 0.66$, the historical share of labor income in real GNP.

We chose a benchmark value for capital depreciation rate ($\delta = 0.025$) and constructed a capital series using the law of motion for capital (equation (1.2.3)). Then, we can compute a series for the Solow residual. Since, under perfect competition and constant returns to scale, the growth rate of the Solow residual represents a measure of the growth rate of technical progress augmenting the global productivity of factors, A_t is measured by:

$$\log(A_t) = \log(Y_t) - \alpha \log(K_t) - (1-\alpha)\log(H_t)$$

ρ_a and σ_a are then obtained by estimating a first-order autoregressive equation, AR(1), on RS_t. We obtain a persistence for the technological shock of 0.96 and a standard deviation of 0.01.

Table 2: Firm's Structural Parameters

γ	δ	α	ρ_a	ϵ_a
1.0067	0.025	0.34	0.96	0.01

Money Supply and nominal wage contracts In order to calibrate the parameters of the money supply process, we estimate an AR(1) on the growth rate of the money aggregate M1. We use a seasonally adjusted (Census X-11) series for $M1$ taken from the

Bundesbank (Seasonally adjusted business statistics). The persistence of the monetary shock is estimated to be 0.495 with a standard deviation of 0.011.

Since we have no insight on the value of π, the extinction probability , we will try several values.

Table 3: Money Supply Structural Parameters

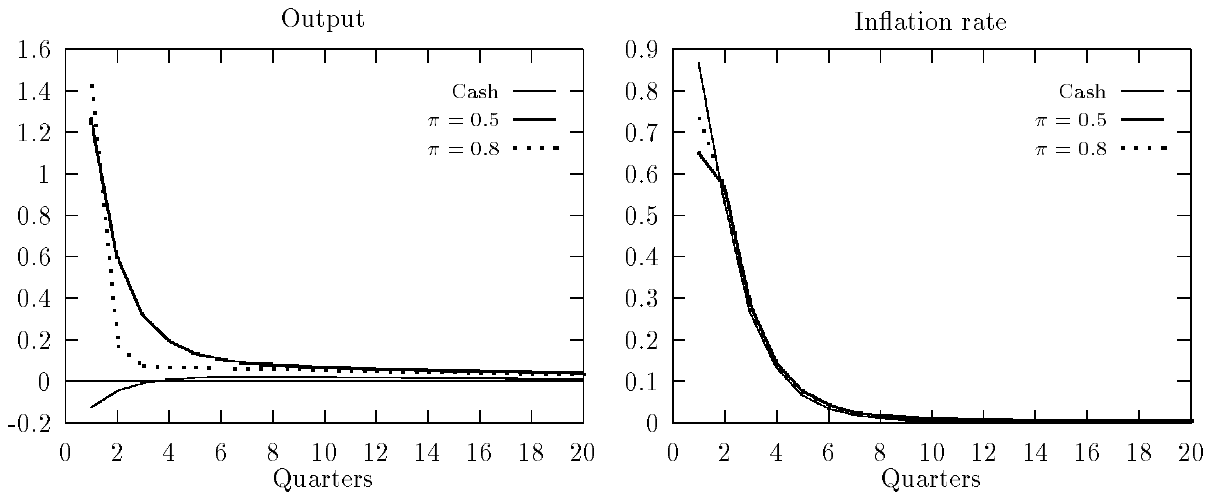
$\bar{g}$	ρ_g	ϵ_g
1.018	0.495	0.011

3.2 Impulse Response Function Analysis

Impulse response functions (IRF) provide information on the response of the system to a stochastic shock in period t. It then allows one to feature the qualitative dynamic properties of the model economy with respect to an exogenous macroeconomic event.

In this model, the shape of IRF of the economy to a unit technological shock does not differ from the benchmark RBC model. Thus, we only discuss the response of the system to a monetary shock. This response is reported in figure (1).

Figure 1: Impulse response functions to a monetary shock



Instantaneously, inflation rises. Thus the purchasing power of the transferred monetary balances decreases, and so does consumption via the cash–in–advance constraint. This effect is common to all cash–in–advance models and on the so–called inflation tax phenomenon. In the simple cash–in–advance model, the household will report its consumption willingness to goods that do not bear the inflation tax, namely leisure and assets. So he will transfer consumption to future periods, in which inflation tax will be lower. Saving increases, and so does investment. Further, the household will increase his demand for leisure so that hours will fall in equilibrium. The decrease in hours worked implies that output will instantaneously be below its steady state level, since capital is predetermined. We thus find the traditional negative effect of money in the simple cash–in–advance model.

In the nominal wage contract economy, the first effect that transits through investment remains. It is further reinforced by the fact that the household cannot freely adjust labor supply: labor is set by the firm. So the household can only respond to the inflation tax by transferring consumption to future periods. So, at general equilibrium, the response of investment is reinforced.

Further, the nominal wage rate does not react instantaneously. Thus, the rise in inflation leads to a lowering in the real wage rate. *Ceteris Paribus*, the demand for labor increases, and so do hours worked[11]. Given the predetermined capital stock, output rises above its steady state level. So in this model, because of nominal wage contracts, money injection exerts a positive effect.

As π increases, the mean duration of a contract diminishes[12]. Thus, the length of the period during which firms can benefit from wage stickiness is shorter. So they will respond more strongly to a monetary shock, and will increase more sharply their demand for hours. This explains the higher magnitude and lower persistence of the response of output.

3.3 Quantitative validation

In this section, we simulate 2 types of models, to assess for their ability to mimic the German business cycle:

- The monetary BC model without wage contracts;
- The monetary BC model with wage contracts for different values of π

The data generated by the theoretical model economies are logged and detrended using the Hodrick-Prescott's filter. We compute a set of second order moments characterizing the business cycle. These statistics are reported in table (4).

The main features of the German data are alike those of the U.S. economy. Investment is more volatile than output while consumption exhibits less variability. Hours are less volatile than output. Inflation is far less volatile than output, while the volatility of the

[11] Just recall that the firm has the right to manage employment.
[12] The mean duration of a contract is given by $1/\pi$.

Table 4: Cyclical properties of the model

	Data	Cash	$\pi = 0.3$	$\pi = 0.5$	$\pi = 0.8$	$\pi = 0.95$
σ_y	1.62	1.70	2.53	2.47	2.47	2.52
σ_c/σ_y	0.92	0.72	0.44	0.46	0.47	0.47
σ_i/σ_y	2.82	2.93	3.91	4.10	4.23	4.27
σ_h/σ_y	0.81	0.38	1.03	1.01	1.05	1.07
σ_f/σ_y	0.23	0.67	0.38	0.42	0.43	0.42
$\sigma_{m/p}/\sigma_y$	1.39	0.46	0.38	0.36	0.33	0.33
ρ_f	0.24	0.38	0.37	0.38	0.37	0.36
$\rho(c,y)$	0.70	0.69	0.28	0.14	0.06	0.04
$\rho(i,y)$	0.83	0.85	0.94	0.94	0.94	0.94
$\rho(h,y)$	0.69	0.91	0.87	0.86	0.86	0.86
$\rho(f,y)$	0.03	-0.32	0.19	0.17	0.14	0.15
$\rho(m/p,y)$	0.30	0.62	0.51	0.37	0.27	0.26

real balances is higher than that of output. All aggregates are procyclical. But, the inflation rate is almost acyclical, meaning that demand and supply shocks both matter.

For the calibration we used, the simple cash–in–advance model mimics in a quite good way the volatility of output. As soon as we introduce nominal rigidities, the volatility of output increases. This is due to the abandon of the intertemporal substitution in labor supply behavior, that traditionally leads to a smoothing in hours worked. Nominal wage contracts reinforce volatility of hours, and thus that of output.

Whatever the model we consider, the general pattern of relative volatilities is reproduced. But, the different models underestimate the relative volatility of consumption. This is essentially due to the permanent income hypothesis that implies a too high consumption smoothing. Further, investment's relative volatility is too high in the model. It appears that as π increases — *i.e.* as the mean duration of contracts diminishes — the volatility of investment increases. This is so because hours react more strongly. Since nominal wages are fixed, the wealth of an individual increases instantaneously. This additional wealth will not be consumed because of the inflation tax, and will be invested.

The relative volatility of inflation is too high whatever the model we consider. In the simple cash cash–in–advance model the relative volatility is 3 times the one observed in the data. As soon as we introduce nominal wage contracts, it decreases considerably. As seen from figure (1), the magnitude of the response of inflation decreases with the length of the contract. So decreasing π allows to lower, slightly, the volatility of inflation. Nevertheless, the volatility of real balances is badly replicated by all models, including those with contracts, since historical real balances are always more volatile than output.

In terms of correlation of aggregates with output, all models predict procyclical aggregates. We focus on correlations between inflation, real balances and output. The simple cash–in advance model fails to mimic $\rho(f, y)$ and $\rho(m/p, y)$. The correlation between inflation and output is negative. Indeed, as the variance decomposition exercise shows, productivity shocks dominate. Further, after a monetary shock inflation goes up while output falls, as the IRF analysis shows. The introduction of contracts breaks this negative link. After a monetary shock, output and inflation raise. So we obtain a weak positive correlation between output and inflation, which is consistent with german data.

Concerning the real balances, the cash–in–advance model strongly overestimates the correlation with output. Introducing nominal wage contracts allows to reduce sharply this correlation, so that the model is able to mimic the data for high values of π — *i.e.* for short contracts.

We now turn to the analysis of the variance decomposition for output and inflation rate. This allows us to precise the weight of monetary shock in the dynamics of these aggregates. As can be seen from table (5), monetary shocks do not explain the dynamics of output in the simple cash–in–advance model. Since prices freely adjust instantaneously after an increase in money supply, monetary shocks only affect marginally the dynamics of output.

Table 5: Variance Decomposition

	Cash		$\pi = 0.3$		$\pi = 0.5$		$\pi = 0.8$		$\pi = 0.95$	
	Y	f	Y	f	Y	f	Y	f	Y	f
1	1.1	77.8	31.4	52.3	36.7	60.8	41.2	67.9	42.5	69.8
4	0.34	83.7	28.3	71.0	22.2	75.5	22.6	78.5	23.6	79.2
8	0.2	83.7	21.2	71.2	15.1	75.6	15.1	78.5	15.8	79.2
20	0.12	83.7	13.6	71.3	9.2	75.6	9.0	78.5	9.4	79.2
40	0.09	83.5	10.9	71.1	7.2	75.5	7.1	78.4	7.4	79.1

Note: Each column gives the percentage of variance explained by the monetary shock

In the nominal wage contracts economy, monetary shocks explain between 30% and 45% of output. As seen from the IRF analysis, the raise in money supply leads to an increase in hours worked, and thus output. The magnitude of this real effect is higher than the one obtained in the cash–in–advance model as the elasticity of labor supply is infinite in the nominal wage contract case. Concerning the inflation rate, the introduction of contracts raises the weight of technological shocks. Since households cannot adjust hours worked, consumption becomes more reactive to a technological shock. So, the response of inflation is reinforced *via* the cash–in–advance constraint.

An even more striking feature, appearing through this variance decomposition, is that monetary shocks exert a long lasting effect on output dynamics. After four quarters, monetary shocks still explain between 20% and 30% of output volatility in the nominal wage contracts model, whereas they only explain less than 1% in the cash–in–advance model. After 20 quarters, the share of output volatility explained by monetary shocks still lies between 9% and 14% in the nominal wage contracts model, whatever the length of the contracts.

4 Concluding remarks

We have built a simple cash–in–advance model, in which we introduced stochastic nominal wage contracts. At each period, a given contract has a positive probability to end. We showed that the introduction of these contracts allows to weaken the negative effect of the inflation tax such that monetary shocks exert a positive effect on output dynamics.

Contrary to the basic cash–in–advance model, this model is able to mimic the correlation of inflation and real balances with output. It also lowers the standard deviation of inflation relative to that of output. But, the relative standard deviation of real balances remains far too low. Further, the variance decomposition analysis indicates that, in this setting, monetary shocks explain between 30 and 45 % of the variance of output, compared to 1% for the cash–in–advance model. Moreover, it also indicates that monetary shocks have a long lasting effect on output dynamics, as suggested by several studies (See Chari et al. [1996].)

A Data

If not indicated otherwise, data are from the Deutsche Institut für Wirtschaftsforschung, Berlin (DIW). As in Brandner and Neusser (1990) annual values for total population (Source: Austrian Institute of Economic Research (WIFO) database) have been interpolated to get quarterly series for per capita calculations. All series have been seasonally adjusted (Census X-11).

Output: Real gross national product (GNP), at constant prices 1980, per capita.

Consumption: Real private consumption, at constant prices 1980, per capita.

Fixed investment: Real gross investment, at constant prices 1980, per capita.

Hours: Total hours worked, per capita.

Real wage: Compensation of employees, divided by the deflator of private consumption, per employee.

Price level: Consumer price index (CPI), IMF statistics.

Inflation: Δ LN (CPI).

Money supply: M1, Seasonally adjusted (Census X-11) business statistics.

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