Bargaining with Externalities: Licensing of an Innovation

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Abstract

The objective of this paper is to analyse the relationship between bargaining organizational forms and licensing of cost-reducing innovations, in order to assess the patent holder's optimal policies as well as the welfare properties. Up to now two trading mechanisms for selling an innovation have dominated the literature: a license auction game and a fixed fee licensing game. But a duopolistic industry is a small market for an innovation. The case where two licenses are sold is just a bargaining problem, and predictions of auction theory are quite sensitive to the *bargaining* model used. Only, when there are strictly less licenses than firms, auction theory is insentive to the bargaining theory used as its foundations. Also, the terms of trade between any two agents are mostly determined by negotiation, the course of which is influenced by the agents' opportunities for matching and trading with other partners. Therefore, alternative modes of licensing, where license trade is carried out through matching and bargaining, may deserve some interests.

In a two-period Cournot duopoly, we consider the licensing of a cost-reducing innovation by means of a take-it-or-leave-it, an alternating bids, and a simultaneous bids mechanism. All these bargaining mechanisms incorporates voluntary matching. Our major finding is that the patent holder prefers the take-it-or-leave-it licensing mechanism to the fixed fee, alternating bids or simultaneous bids mechanisms. The simultaneous bids licensing mechanism is always worse for him. However, from a social point of view (an agency maximizing the domestic welfare where the patentee is a foreign laboratory), it is better licensing through the simultaneous bids mechanisms.

1 Introduction

MOTIVATION. What's new?

Auction theory $\rightarrow$ its limits for small markets:
the problem of the reserve price.

The objective of this paper is to analyse the relationship between bargaining organizational forms and licensing of cost-reducing innovations, in order to assess the patent holder's optimal policies as well as the welfare properties. Up to now two trading mechanisms for selling an innovation have dominated the literature: a license auction game (Kamien [4], Katz and Shapiro [7])and a fixed fee licensing game (Kamien [4], Kamien and Tauman [5]).

But our market for an innovation is small. The case where two licenses are sold is just a bargaining problem, and predictions of auction theory are quite sensitive to the *bargaining* model used. That is, in Kamien's [4] auction game, the seller's ability or commitment to set any particular reserve price and stick to it becomes questionable. Indeed, if at the end of the auction game the seller has to choose between selling the licenses at the highest bids or withdrawing the licenses from the auction, then fixing a reserve price different from the continuation value is not a credible commitment. Only, when there are strictly less licenses than firms, auction theory is insentive to the bargaining theory used as its foundations. Also, the terms of trade between any two agents are mostly determined by negotiation, the course of which is influenced by the agents' opportunities for matching and trading with other partners. Therefore, alternative modes of licensing, where license trade is carried out through matching and bargaining, may deserve some interests.

Answer: bargaining models incorporating voluntary matching.
To study these alternative modes of licensing from a point of
view of the patentee's preferences and the social welfare
(domestic or world welfare).

EXAMPLES. Franchise + Innovation.

RELATED LITERATURE.

1) Innovation : Kamien [4], Kamien and Tauman [5], Kamien *et al.* [6], Katz and Shapiro [7], Reinganum [11], Sempere [12];

2) Bargaining + externalities: Jéhiel and Moldovanu [8],[9];

3) Bargaining + matching: Hendon and Tranæs [2] analyzes a matching and bargaining model in a market with one seller and two buyers, differing only in their reservation price

within an infinite horizon framework (see also Osborne and Rubinstein [10], Hendon *et al.* [3]).

MODEL + RESULTS. In a two-period Cournot duopoly, we have considered the licensing of a cost-reducing innovation by means of a take-it-or-leave-it, an alternating bids, and a simultaneous bids mechanism. All these bargaining mechanisms incorporates voluntary matching. Our major finding is that the patent holder prefers the take-it-or-leave-it licensing mechanism to the fixed fee, alternating bids or simultaneous bids mechanisms. The simultaneous bids licensing mechanism is always worse for him. However, from a social point of view (an agency maximizing the domestic welfare where the patentee is a foreign laboratory), it is better licensing through the simultaneous bids mechanisms.

STRUCTURE. The paper is organized as follows. In Section 2 we give a description of the market for the cost-reducing innovation and we consider two classic modes of licensing: a license auction game and a fixed fee licensing game. Section 3 is devoted to alternative modes of licensing, where license trade is carried out through matching and bargaining. Section 4 undertakes a social welfare appraisal. Section 5 concludes.

2 Description of The Market

We posit an duopolistic industry consisting of two identical firms. A patentee with a cost-reducing innovation seeks to license the patent to both firms, one or none so as to maximize his profits. We assume that the patentee is an independent research laboratory and cannot enter the market of the final good directly.

Time Period One	Stage One : patentee chooses k - or - a fee
	Stage Two : trading stage
	Stage Three : Cournot competition
Time Period Two	Stage Four : trading stage
	Stage Five : Cournot competition

Table 1: Two-Period / Five-Stage Licensing Games

Our noncooperative game between the patentee and the duopolists consists of two periods (see Figure 1). Period One is subdivided in three stages (Stage One - Stage Three), while Period Two is subdivided in two stages (Stage Four - Stage Five). In this noncooperative game, the patentee moves first. In Stage One, either he chooses the number of licenses to sell, k, belonging to $\{0, 1, 2\} \equiv K$, or he settles a fixed fee. In Stage Two and Stage Four the license(s) is (are) traded following the trading rules in force. In Stage Three (Period One) and Stage Five (Period Two) Cournot competition takes place.

Without the new technology, the duopolists are producing the same good with a linear cost function $f(q_i) = c\ q_i$, where q_i is the quantity produced by firm i ($i = 1, 2$), and $c > 0$ is the constant marginal cost of production. Both firms are facing an homogenous linear inverse demand for the good given by $P(Q) = a - Q$, where $a > c$ and $Q = q_1 + q_2$ is the aggregate quantity demanded and produced in each period. Firm i's production profits in each period is given by $\Pi_i(q_1, q_2) = (a - q_1 - q_2)\, q_i - cq_i$. The patentee owns a cost-reducing innovation that reduces the marginal cost of production from c to $c - \varepsilon$, $\varepsilon > 0$. Both firms' technologies are common knowledge when the duopolists are choosing their quantities to produce. Let Π_i ($\overline{\underline{\Pi}}_i$) be firm i's Cournot-Nash equilibrium production profits when both firms produce with the old (new) technology. Let $\overline{\Pi}_i$ ($\underline{\Pi}_i$) be firm i's Cournot-Nash equilibrium production profits when firm i produces with the new (old) technology while firm j produces with the old (new) technology; $j \neq i$. Analytically, we have that

$$\begin{aligned}
\Pi_i &= \tfrac{1}{9}(a-c)^2 \\
\overline{\Pi}_i &= \tfrac{1}{9}(a-c+2\varepsilon)^2 \\
\underline{\Pi}_i &= \tfrac{1}{9}(a-c-\varepsilon)^2 \\
\overline{\underline{\Pi}}_i &= \tfrac{1}{9}(a-c+\varepsilon)^2
\end{aligned}$$

where it is true that $\overline{\Pi}_i > \overline{\underline{\Pi}}_i > \Pi_i > \underline{\Pi}_i$. It should be noted that, if only one firm owns the innovation then the other one is worse off since there exits a negative externality due to the market interdependence between the duopolists.

Assumption 1 *All innovations are nondrastic ones. That is,* $\varepsilon \leq a - c$.

Note that $\varepsilon \leq a - c$ is the same condition for the nonpurchasing firm produces a positive quantity in the case where only one licence is sold. We restrict our analysis to this nondrastic case. Time is costly. All agents have the same common discount factor $\delta \in [0, 1]$. The patentee's objective function is to maximize its discounted revenue from licensing its innovation. Also, we assume that the only available mode of patent licensing is lump-sum fees, F_i (we do not exclude a-priori $F_i \neq F_j$), which are independent of the quantity produced. Both firms seek to maximize their discounted production profits less their discounted licensing costs.

Two trading mechanisms for selling an innovation have mainly been studied in the literature: a license auction game and a fixed fee licensing game. These mechanisms can be adapted to fit our framework.

2.1 The License Auction Game

The license auction game is close to the game developed by Katz and Shapiro [7] and Kamien [4] except that in our version we have a duopolistic industry and a multi-period model. Our license auction game is a five-stage noncooperative game (see Figure 1). In Stage One the patentee decides how many licenses $k \in \{0, 1, 2\} \equiv K$ to auction. Let $v = 0$ be the monetary value of the innovation to the patentee. We focus on the case where the patentee cannot choose a reservation price different from v under which he will not sale a license. Stage Two is the sealed-bid first price auction. Both firms decide independently and simultaneously how much to bid for a license. Licenses are sold to the highest bidders at their bid price and in the event of a tie, licensees are chosen arbitrarily. If the k licenses have been sold in the first period, then Stage Four is empty; otherwise, the unsold licenses are again auctioned in Stage Four. Stage Three and Stage Five are the production stages wherein each firm, licensed or unlicensed, competes on the good market and chooses its Cournot profit-maximizing level of output. We denote by $A(K)$ our license auction game without reservation price. The subgame-perfect equilibrium (SPE) in pure strategies is the solution concept employed. Thus, the license auction game $A(K)$ is solved backwards from its last stage to its first. Let $A(k)$ be the auction game where the patentee offers k licenses in Stage One; $A(1)$ is the license auction game with exclusive licensing.

At the SPE of the license auction game $A(1)$, both firms make a bid equal to

$$F_i^* [A(1)] = \left[\overline{\Pi}_i - \underline{\Pi}_i\right](1+\delta) = \frac{1}{9}\left[6(a-c)+3\varepsilon\right](1+\delta)\,\varepsilon, \qquad i = 1, 2,$$

and the licensee is chosen arbitrarily. But at the SPE of the license auction game $A(2)$, both firms make a bid equal to $F_i^* [A(2)] = 0$, $i = 1, 2$, and obtain the license. Therefore, at the SPE of the license auction game $A(K)$, the patentee auction only one license. Table 2 gives us the agents' (patentee / firm i / firm j) SPE payoffs of $A(K)$.

$\Phi[A(K)]$	$=$	$\frac{1}{9}\left[6(a-c)+3\varepsilon\right](1+\delta)\,\varepsilon$
$\Psi_i[A(K)]$	$=$	$\frac{1}{9}\left[a-c-\varepsilon\right]^2(1+\delta)$
$\Psi_j[A(K)]$	$=$	$\frac{1}{9}\left[a-c-\varepsilon\right]^2(1+\delta)$

Table 2: Agents' SPE payoffs of A(K)

This result is due to the absence of reserve prices. Reserve prices are prices below which the seller refuses to sell. They can increase the seller's revenue, and their effect is to make the auction more like a regular fixed-price market.

Let $A^*(K)$ be the license auction game where the patentee also chooses a reserve price in Stage One. At the SPE of the license auction game $A^*(1)$, both firms makes a bid equal to $F_i^* [A(1)]$, $i = 1, 2$, and the licensee is chosen arbitrarily. The reserve price matters only for $A^*(2)$. Indeed, at the SPE of the license auction game $A^*(2)$, both firms make a bid equal to $\left(\underline{\Pi}_i - \Pi_i\right)(1+\delta) = \frac{4}{9}(a-c)(1+\delta)\,\varepsilon$ and obtain the license. To sustain such a SPE the patentee must, along with his announcement that two licenses will be auctioned, state a reserve price slightly below the benefit to a firm if all are licensed, below which he will not sell a license. The use of the license reserve price prevents a firm from offering nothing for a license because it knows it will get one anyway, what occurs in $A(2)$. Therefore, at the SPE of the license auction game $A^*(K)$ the patentee will auction $k^* = 1$ if $\varepsilon \in \left(\frac{2}{3}(a-c), a-c\right]$, and $k^* = 2$ if $\varepsilon \in \left(0, \frac{2}{3}(a-c)\right)$. That is, if the innovation affords only a modest reduction in unit production costs, then it is optimal for the patentee to license all the firms in the industry. Table 3 gives us the agents' (patentee / firm i / firm j) SPE payoffs of $A^*(K)$.

$$
\begin{aligned}
\Phi[A^*(K)] &= \begin{cases} \frac{1}{9}[6(a-c)+3\varepsilon](1+\delta)\varepsilon & \text{if } \varepsilon \in \left(\frac{2}{3}(a-c), a-c\right] \\ \frac{8}{9}(a-c)(1+\delta)\varepsilon & \text{if } \varepsilon \in \left(0, \frac{2}{3}(a-c)\right) \end{cases} \\
\Psi_i[A(K)] &= \tfrac{1}{9}[a-c-\varepsilon]^2(1+\delta) \\
\Psi_j[A(K)] &= \tfrac{1}{9}[a-c-\varepsilon]^2(1+\delta)
\end{aligned}
$$

Table 3: Agents' SPE payoffs of $A^*(K)$

But the seller's ability or commitment to set any particular reserve price and stick to it may be questionable. If at the end of the auction game the seller has to choose between selling the licenses at the highest bids or withdrawing the licenses from the auction, then fixing a reserve price different from the continuation value is not a credible commitment. Indeed, the case $k = 2$ is just a bargaining problem, and predictions of auction theory are quite sensitive to the bargaining model used. Actually, when there are strictly less licenses than firms, auction theory is insentive to the bargaining theory used as its foundations.

2.2 The Fixed Fee Licensing Game

The fixed fee licensing game has been introduced by Kamien and Tauman [5] (see also Kamien [4]). This game is similar to the license auction game except that now in Stage One the patentee also sets a price at which any firm wishing to can buy a license. The license price is independent of the number of units produced with the new technology and therefore is a fixed cost to the firm just as in the auction case. In Stage Two, both firms choose simultaneously whether or not to buy a license at the settled price. In Stage Four, if licenses are still available, then unlicensed firms choose simultaneously whther or not buy a license at the price settled in Stage One. Ties are broken randomly. The production stages do not change. We denote by FF this fixed fee licensing game and we solve it backwards. For all nondrastic innovations, the SPE licensing policy of the patentee is to set $k^* = 2$ and a fee equal to

$$F_i^*[FF] = \frac{4}{9}(a-c)(1+\delta)\varepsilon, \qquad i = 1, 2.$$

Both firms buy the license at this settled price. Whatever the extent of the cost-reducing innovation the fixed fee licensing game has a unique SPE in pure strategies in which two licenses are sold if the innovation is not drastic. Table 4 gives us the agents' (patentee / firm i / firm j) SPE payoffs of FF.

$\Phi[FF]$	$=$	$\frac{8}{9}(a-c)(1+\delta)\varepsilon$
$\Psi_i[FF]$	$=$	$\frac{1}{9}[a-c-\varepsilon]^2(1+\delta)$
$\Psi_j[FF]$	$=$	$\frac{1}{9}[a-c-\varepsilon]^2(1+\delta)$

Table 4: Agents' SPE payoffs of FF

From Tables 3 and 4, we recover Kamien's [4] result that the patentee's licensing profits are, in general, lower under fixed fee licensing than under license auctioning. This result is no more valid once we exclude the use of reserve prices: $\Phi[A(K)] > \Phi[FF]$ if $\varepsilon \in \left(\frac{2}{3}(a-c), a-c\right]$, and $\Phi[A(K)] < \Phi[FF]$ if $\varepsilon \in \left(0, \frac{2}{3}(a-c)\right)$. Nonetheless, fixed fee licensing is better for consumers because the market price declines with the number of licensees.

Our market for an innovation is small. Indeed, the case where two licenses are sold is just a bargaining problem. Also, the terms of trade between any two agents are mostly determined by negotiation, the course of which is influenced by the agents' opportunities for matching and trading with other partners. Therefore, alternative modes of licensing, where license trade is carried out through matching and bargaining, may deserve some interests.

3 Licensing Games with Matching and Bargaining

All licensing games with matching and bargaining we consider are two-period noncooperative games (see Table 5). The first period is subdivised in three stages. In Stage One the patentee always decides how many licenses $k \in K$ to sell. In Stage Two the agents are matched and negotiate the terms of trade. In Stage Three, each firm, licensed or unlicensed competes on the good market and chooses its Cournot profit-maximizing level of output.

The second period is subdivised in two stages. If no total agreement has been reached in Stage Two, then the negotiation proceeds in Stage Four. Stage Five is similar to Stage Three. These licensing games will only differ in respect with the matching & bargaining stages. All these games are solved backwards.

Time Period 1	Stage One : the patentee chooses k
	Stage Two : matching & bargaining
	Stage Three : Cournot competition
Time Period 2	Stage Four : matching & bargaining
	Stage Five : Cournot competition

Table 5: Two-Period / Five-Stage Licensing Games

3.1 The Take-it-or-Leave-it Licensing Game

We first consider the licensing game where the patentee can offer a take-it-or-leave-it contract to both firms at each period. We denote by $T(K)$ our take-it-or-leave-it licensing game; $T(k)$ is the take-it-or-leave-it licensing game where the patentee offers k licenses in stage one.

Take-it-or-leave-it licensing game $T(1)$. In Stage Two, the patentee voluntary matches with one of the firms and makes a take-it-or-leave-it offer. The matched firm either accepts or rejects the offer. If it accepts, then the negotiation ends and it starts to produce in Stage Three with the new technology. If it rejects, then the negotiation will proceed in Stage Four. In Stage Four, if no agreement has been reached previously, then the patentee voluntary matches with one of the firms (not necessarily the same firm as in Stage Two) and again makes a take-it-or-leave-it offer which is accepted or rejected.

Take-it-or-leave-it licensing game $T(2)$. In Stage Two, the patentee makes a take-it-or-leave-it offer (F_1, F_2). Both firms, simultaneously, accept or reject the offer. If both firms accept the offer, then the negotiation ends and both firms start producing with the new technology. If at least one firm rejects the offer, then the negotiation will proceed in Stage Four. In Stage Four, the patentee makes again a take-it-or-leave-it offer to the firms which haven't yet bought the new technology. Then, the offer(s) are (simultaneously) accepted or rejected.

First we consider the take-it-or-leave-it licensing game $T(1)$. The computation of the SPE is given in Appendix ??. At the SPE, the patentee makes a take-it-or-leave-it offer

$$F_i^*[T(1)] = \frac{1}{9}\left[2(2+3\delta)(a-c) + (4+3\delta)\varepsilon\right]\varepsilon.$$

This take-it-or-leave-it offer is accepted by the matched firm i in the first period. Table 6 gives us the agents' (patentee / firm i / firm j) SPE payoffs of $T(1)$.

$\Phi[T(1)]$	$=$	$\frac{1}{9}[2(2+3\delta)(a-c)+(4+3\delta)\varepsilon]\varepsilon$
$\Psi_i[T(1)]$	$=$	$\frac{1}{9}[a-c+2\varepsilon]^2(1+\delta)-\frac{1}{9}[2(2+3\delta)(a-c)+(4+3\delta)\varepsilon]\varepsilon$
$\Psi_j[T(1)]$	$=$	$\frac{1}{9}[a-c-\varepsilon]^2(1+\delta)$

Table 6: Agents' SPE payoffs of T(1)

The main difference between $T(1)$ and FF is that, in the first period of $T(1)$, the patentee can credibly threaten firm i to match next period with firm $j \neq i$ if firm i rejects the patentee's current offer. Using this credible threat the patentee will obtain a higher revenue in $T(1)$. Next we consider the take-it-or-leave-it licensing game $T(2)$. The computation of the SPE is given in Appendix ??. At the SPE, the patentee makes a take-it-or-leave-it offer

$$F_i^*[T(2)] = \frac{4}{9}(a-c)(1+\delta)\varepsilon, \qquad i = 1,2.$$

This take-it-or-leave-it offer is accepted by both firms in the first period. Table 7 gives us the agents' (patentee / firm i / firm j) SPE payoffs of $T(2)$.

$\Phi[T(2)]$	$=$	$\frac{8}{9}(a-c)(1+\delta)\varepsilon$
$\Psi_i[T(2)]$	$=$	$\frac{1}{9}[a-c-\varepsilon]^2(1+\delta)$
$\Psi_j[T(2)]$	$=$	$\frac{1}{9}[a-c-\varepsilon]^2(1+\delta)$

Table 7: Agents' SPE payoffs of T(2)

Comparing $\Phi[T(1)]$ with $\Phi[T(2)]$, we can state the following proposition.

Proposition 1 *Consider the take-it-or-leave-it licensing game $T(K)$. Then, the SPE licensing policy of the patentee is to sell*

- $k = 1$ *if* $\varepsilon \in \left(\frac{4+2\delta}{4+3\delta}(a-c), a-c\right]$,
- $k = 2$ *if* $\varepsilon \in \left(0, \frac{4+2\delta}{4+3\delta}(a-c)\right)$.

Which modes of licensing does the patentee prefer? The answer is given by Figure 1.

The left-hand figure compares the take-it-or-leave-it game $T(K)$ with the license auction game $A(K)$: $\Phi[A(K)] > \Phi[T(K)]$ if $\varepsilon \in \left(\frac{2}{3}(a-c), a-c\right]$ (non-shaded area), and $\Phi[A(K)] < \Phi[T(K)]$ if $\varepsilon \in \left(0, \frac{2}{3}(a-c)\right)$ (shaded area). Remark that $\Phi[A^*(K)] = \Phi[T(K)]$ if $\varepsilon \in \left(0, \frac{2}{3}(a-c)\right)$. Indeed, the license auction game with a reserve price is a *take-it-or-leave-it auction* game whenever two licenses are sold. The right-hand figure compares the take-it-or-leave-it game $T(K)$ with the fixed fee licensing game FF:

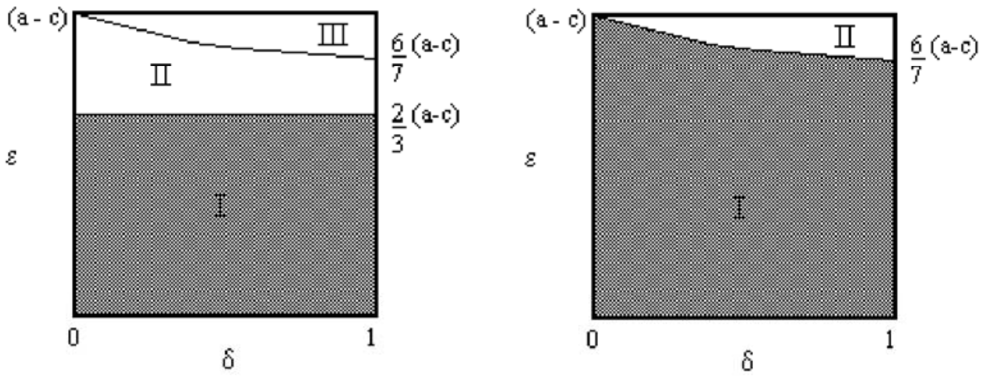

Figure 1: Left-hand figure: $T(K)$ vs $A(K)$, Right-hand figure: $T(K)$ vs FF.

$\Phi[T(K)] > \Phi[FF]$ if $\varepsilon > \frac{4+2\delta}{4+3\delta}(a-c)$ (non-shaded area), and $\Phi[T(K)] = \Phi[FF]$ if $\varepsilon \leq \frac{4+2\delta}{4+3\delta}(a-c)$ (shaded area). Remark that the credible threat the patentee possesses in $T(1)$ disappears in $T(2)$. Therefore, it is not surprising that the patentee is indifferent between the take-it-or-leave-it mechanism and the fixed fee whenever it is optimal for him to sell two licenses by means of both mechanisms.

3.2 The Alternating Bids Licensing Game

The alternating bids licensing game is a game where the patentee and the potential buyers negotiate the license price or fee. In each period, these agents possibly alternate in making offers. We denote by $AB(K)$ our alternating bids licensing game; $AB(k)$ is the game where the patentee offers k licences in Stage One.

The alternating bids licensing game $AB(1)$. In Stage Two, the patentee voluntary matches with one of the firms and offers a fee. The matched firm either accepts or rejects the offer. If it accepts, then the bargaining ends and it starts to produce in Stage Three with the new technology. If it rejects, then the negotiation will proceed in Stage Four. In Stage Four, if no agreement was reached previously, then the patentee voluntary matches with one of the firms (not necessary the same firm as in Stage Two). The proposer is chosen randomly between the patentee and the selected firm (with equal probability), and the proposer offers a fee which is accepted or rejected.

The alternating bids licensing game $AB(2)$. In Stage Two, the patentee offers (F_i, F_j). Both firms, simultaneously, accept or reject the offer. If both firms accept the offer, then the bargaining ends and both firms start producing with the new technology. If one of them rejects, then the negotiation will proceed in Stage Four. In Stage Four, the proposer(s) is(are) chosen randomly, with equal probability, between the patentee and the firms which are still bargaining. Then, the offer(s) are, simultaneously, accepted or rejected.

First we consider the alternating bids licensing game $AB(1)$. The computation of the SPE is given in Appendix A.3. At the SPE, the patentee offers a contract where

$$F_i^*[AB(1)] = \frac{1}{9}[2(2+3\delta)(a-c)+(4+3\delta)\varepsilon]\varepsilon$$

This contract is accepted by the matched firm i in the first period. Table 8 gives us the agents' (patentee / firm i / firm j) SPE payoffs of $AB(1)$.

$\Phi[AB(1)]$	$=$	$\frac{1}{9}[2(2+3\delta)(a-c)+(4+3\delta)\varepsilon]\varepsilon$
$\Psi_i[AB(1)]$	$=$	$\frac{1}{9}[a-c+2\varepsilon]^2(1+\delta)-\frac{1}{9}[2(2+3\delta)(a-c)+(4+3\delta)\varepsilon]\varepsilon$
$\Psi_j[AB(1)]$	$=$	$\frac{1}{9}[a-c-\varepsilon]^2(1+\delta)$

Table 8: Agents' SPE payoffs of AB(1)

Next we consider the alternating bids licensing game $AB(2)$. The computation of the SPE is given in Appendix A.4. At the SPE, the patentee offers two contracts where

$$F_i^*[AB(2)] = \left(1+\frac{\delta}{2}\right)\frac{4}{9}(a-c)\varepsilon,\ i=1,2$$

These contracts are accepted by both firms in the first period. Table 9 gives us the agents' (patentee / firm i / firm j) SPE payoffs of $AB(2)$.

$\Phi[AB(2)]$	$=$	$(2+\delta)\frac{4}{9}(a-c)\varepsilon$
$\Psi_i[AB(2)]$	$=$	$\frac{1}{9}[a-c+\varepsilon]^2(1+\delta)-\left(1+\frac{\delta}{2}\right)\frac{4}{9}(a-c)\varepsilon$
$\Psi_j[AB(2)]$	$=$	$\frac{1}{9}[a-c+\varepsilon]^2(1+\delta)-\left(1+\frac{\delta}{2}\right)\frac{4}{9}(a-c)\varepsilon$

Table 9: Agents' SPE payoffs of AB(2)

Comparing $\Phi[AB(1)]$ with $\Phi[AB(2)]$, we can state the following proposition.

Proposition 2 *Consider the alternating bids liccensing game $AB(K)$. Then, the SPE licensing policy of the patentee is to sell*

- $k=1$ *if* $\varepsilon \in \left(\frac{4-2\delta}{4+3\delta}(a-c), a-c\right]$,
- $k=2$ *if* $\varepsilon \in \left(0, \frac{4-2\delta}{4+3\delta}(a-c)\right)$.

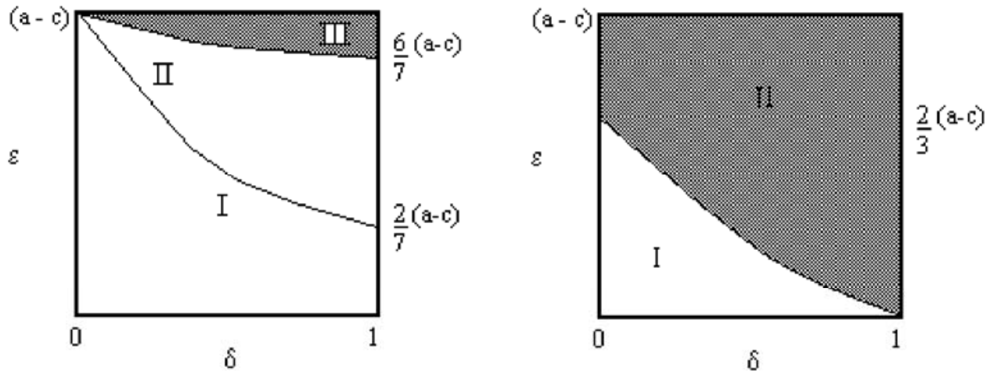

Figure 2: Left-hand figure: $AB(K)$ vs $T(K)$ and $AB(K)$ vs FF; Rigt-hand figure: $AB(K)$ vs $A(K)$.

Which modes of licensing does the patentee prefer? The answer is given by Figure 2.

The left-hand figure compares the alternating bids game $AB(K)$ with the take-it-or-leave-it game $T(K)$: $\Phi[AB(K)] = \Phi[T(K)]$ if $\varepsilon > \frac{4+2\delta}{4+3\delta}(a-c)$ (shaded), and $\Phi[AB(K)] < \Phi[T(K)]$ if $\varepsilon < \frac{4+2\delta}{4+3\delta}(a-c)$ (non-shaded). The only difference between $AB(1)$ and $T(1)$ is the proposer is chosen randomly in the Stage Four of $AB(1)$, while the patentee makes all offers in $T(1)$. But this difference does not matter at equilibrium. Both games yield the same outcome $F_i^*[AB(1)] = F_i^*[T(1)]$. Indeed, in both cases, the patentee can credibly threaten firm i to match next period with firm $j \neq i$ if firm i rejects the patentee's first-period offer. And, the unique SPE of $AB(1)$ and $T(1)$ are supported by this credible threat. This credible threat vanishes in $AB(2)$ and $T(2)$. Therefore, the patentee's SPE revenue is higher in $T(2)$ compared to $AB(2)$ since the buyers have more *bargaining power* (the proposer is chosen randomly in the second period) in $AB(2)$. The left-hand figure could also be used to compare the alternating bids game $AB(K)$ with the fixed fee licensing game FF: $\Phi[AB(K)] > \Phi[FF]$ if $\varepsilon > \frac{4+2\delta}{4+3\delta}(a-c)$ (shaded), and $\Phi[AB(K)] < \Phi[FF]$ if $\varepsilon < \frac{4+2\delta}{4+3\delta}(a-c)$ (non-shaded). Finally, the right-hand figure compare the alternating bids game with the license auction game $A(K)$: $\Phi[A(K)] > \Phi[AB(K)]$ if $\varepsilon > \frac{2}{3}\frac{1-\delta}{1+\delta}(a-c)$ (shaded), and $\Phi[A(K)] < \Phi[AB(K)]$ if $\varepsilon < \frac{2}{3}\frac{1-\delta}{1+\delta}(a-c)$ (non-shaded).

3.3 The Simultaneous Bids Licensing Game

Now, we consider a bargaining game with iterated demands to model the negotiation of the fee. Let $SB(K)$ be our simultaneous bids licensing game; $SB(k)$ is the game where the seller offers k licences in stage one.

The simultaneous bids licensing game $SB(1)$. In Stage Two, the patentee voluntary matches with one of the firms. The patentee and the matched firm (i) simultaneously announce bids: $\left(\overline{F}_{pi},\ \overline{F}_i\right)$, where $\bar{F}_{pi}$ and $\bar{F}_i$ are respectively the bids of the patentee and firm i. If $\bar{F}_{pi} \leq \bar{F}_i$, then the agreement is reached on a fee $F_i = \bar{F}_{pi}$ and the bargaining ends. Then, licensee i starts to produce in Stage Three (or first-period) with the new technology. If the bids are incompatible $(\bar{F}_{pi} > \bar{F}_i)$, then the negotiation will proceed in Stage Four (or second-period). In Stage Four, the patentee voluntary matches with one of the firms (not necessarily the same firm as in Stage Two). Again, the patentee and the matched firm (i) simultaneously announce bids: $(\bar{F}_{pi},\ \bar{F}_i)$. If $\bar{F}_{pi} \leq \bar{F}_i$, then the agreement is reached on a fee $F_i = \bar{F}_{pi}$ and the bargaining ends. Otherwise, the bargaining ends with disagreement.

The simultaneous bids licensing game $SB(2)$. In Stage Two, the patentee and both firms simultaneously announce bids $(\bar{F}_{pi},\ \bar{F}_i)$ and $(\bar{F}_{pj},\ \bar{F}_j)$. If $\bar{F}_{pi} \leq \bar{F}_i$ and $\bar{F}_{pj} \leq \bar{F}_j$, then $F_i = \bar{F}_{pi}$ and $F_j = \bar{F}_{pj}$, the bargaining ends and both firms start producing with the new technology. When at least one of the bids is not compatible, the negotiation will proceed in Stage Four (or next period). In Stage Four, the patentee matches with the potential buyer(s) and they simultaneously announce bids. Bids which are not compatible lead to the disagreement event.

Both Stage Two and Stage Four of the simultaneous bids licensing game $SB(k)$ possess many Nash equilibria. To overcome such multiplicity of Nash equilibria and the equilibrium selection problem, we introduce some vanishing uncertainty[1] at these stages of the game $SB(k)$. Then we characterize which outcomes of $SB(k)$ can be supported as a subgame perfect equilibrium (SPE) in which agreement is reached with positive probability.

First, we consider the simultaneous bids licensing game $SB(1)$. The computation of the SPE in which agreement is reached with positive probability is given in Appendix A.5. Then, the SPE fee is

$$F_i^*\left[SB(1)\right] = \frac{(4+8\delta)(a-c)\varepsilon + (4+5\delta)\varepsilon^2}{18}$$

Table 10 gives us the agents' (patentee / firm i / firm j) SPE payoffs of $SB(1)$.

[1] Note that both Stage Two and Stage Four of $SB(k)$ are Nash demand ones. To overcome the multiplicity of Nash equilibria in the Nash demand game, Osborne and Rubistein [10] have considered perturbations in the Nash demand game. By introducing some uncertainty in the neighbourhood of the boundary of the set of posible agreements, they have shown that when this uncertainty is sufficiently small, all Nash equilibria (in which agreement is reached with positive probability) of the perturbed Nash demand game approximate the Nash bargaining solution (see Binmore et al. [1]) to the bargaining problem considered.

$\Phi[SB(1)]$	$=$	$\frac{1}{18}[(4+8\delta)(a-c)+(4+5\delta)\varepsilon]\varepsilon$
$\Psi_i[SB(1)]$	$=$	$\frac{1}{9}[a-c+2\varepsilon]^2(1+\delta)-\frac{1}{18}[(4+8\delta)(a-c)+(4+5\delta)\varepsilon]\varepsilon$
$\Psi_j[SB(1)]$	$=$	$\frac{1}{9}[a-c-\varepsilon]^2(1+\delta)$

Table 10: Agents' SPE payoffs of SB(1)

Next we consider the simultaneous bids licensing game $SB(2)$. The computation of the SPE in which agreement is reached with positive probability is given in Appendix A.6. Then, the SPE fees are,

$$F_i^*[SB(2)] = \frac{2}{9}(a-c)(1+\delta)\varepsilon, \qquad i=1,2.$$

Table 11 gives us the agents' (patentee / firm i / firm j) SPE payoffs of $SB(2)$.

$\Phi[SB(2)]$	$=$	$\frac{4}{9}(1+\delta)(a-c)\varepsilon$
$\Psi_i[SB(2)]$	$=$	$\frac{1}{9}(1+\delta)\left[(a-c)^2+\varepsilon^2\right]$
$\Psi_j[SB(2)]$	$=$	$\frac{1}{9}(1+\delta)\left[(a-c)^2+\varepsilon^2\right]$

Table 11: Agents' SPE payoffs of SB(2)

Comparing $\Phi[SB(1)]$ with $\Phi[SB(2)]$, we can state the following proposition.

Proposition 3 *Consider the simultaneous bids licensing game $SB(K)$. Then, the SPE licensing policy of the patentee is to sell*

- $k=1$ *if* $\varepsilon \in \left(\frac{4}{4+5\delta}(a-c), a-c\right]$,
- $k=2$ *if* $\varepsilon \in \left(0, \frac{4}{4+5\delta}(a-c)\right)$.

Which modes of licensing does the patentee prefer? The simultaneous bids mechanism gives more bargaining power to the buyers than any other modes of licensing considered. Therefore, next result is obvious: the patentee strictly prefers the fixed fee, the auction (with or without reserve price), the take-it-or-leave-it, and the alternating bids to the simultaneous bids licensing mechanism.

The left-hand Figure 3 summarizes the patentee's preferences at equilibrium. We have four areas which are delimited as follows.

Area I : for all $\delta \in [0,1]$, if $\varepsilon \in \left(0, \frac{2}{3}\frac{1-\delta}{1+\delta}(a-c)\right)$ then $\Phi[A^*(K)] = \Phi[T(K)] = \Phi[FF] > \Phi[AB(K)] > \Phi[A(K)] > \Phi[SB(K)]$;

Area II : for all $\delta \in [0,1]$, if $\varepsilon \in \left(\frac{2}{3}\frac{1-\delta}{1+\delta}(a-c), \frac{2}{3}(a-c)\right]$ then $\Phi[A^*(K)] = \Phi[T(K)] = \Phi[FF] > \Phi[A(K)] > \Phi[AB(K)] > \Phi[SB(K)]$;

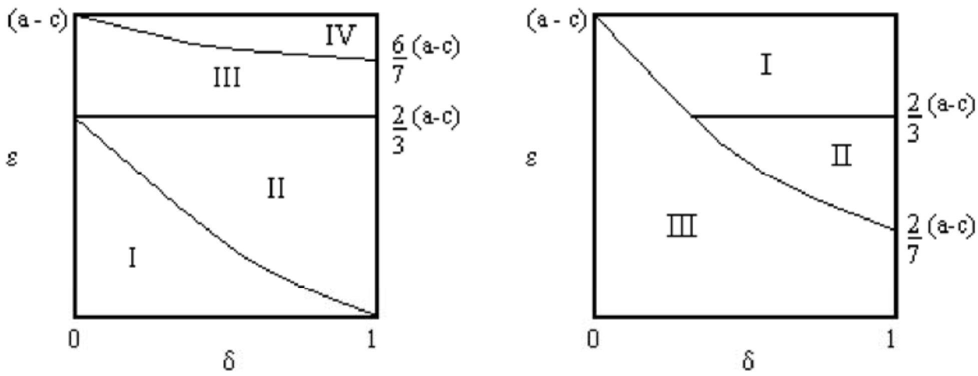

Figure 3: Left-hand figure: patentee's preferences; Right-hand figure: the social (domestic) welfare.

Area III : for all $\delta \in [0,1]$, if $\varepsilon \in \left(\frac{2}{3}(a-c), \frac{4+2\delta}{4+3\delta}(a-c)\right]$ then $\Phi[A^*(K)] = \Phi[A(K)] > \Phi[T(K)] = \Phi[FF] > \Phi[AB(K)] > \Phi[SB(K)]$;

Area IV : for all $\delta \in [0,1]$, if $\varepsilon \in \left[\frac{4+2\delta}{4+3\delta}(a-c), (a-c)\right]$ then $\Phi[A^*(K)] = \Phi[A(K)] > \Phi[T(K)] = \Phi[AB(K)] \geq \Phi[FF] > \Phi[SB(K)]$.

Proposition 4 *At the SPE, the patent holder prefers the take-it-or-leave-it licensing mechanism to the fixed fee, alternating bids or simultaneous bids mechanisms. The simultaneous bids licensing mechanism is always worse for him.*

4 Welfare Considerations

We proceed in this section to consider the desirability of licensing modes when a public agency cares either for *domestic* welfare (DW) or for *world* welfare (SW). Indeed, the patentee could be a foreign independent laboratory. Expressions for DW and SW are given below:

$$\begin{aligned} DW &\equiv CS + \Psi_i + \Psi_j; \\ SW &\equiv CS + \Psi_i + \Psi_j + \Phi. \end{aligned}$$

where CS is the consumer surplus, Ψ_i is firm i's discounted profits, and Φ is the patentee's discounted revenue from licensing. Expressions for the consumer surplus are given below; $CS[k]$ denotes the consumer surplus when k licenses have been sold.

$$
\begin{aligned}
CS[1] &= \frac{1}{18}\left[4(a-c)^2+4(a-c)\varepsilon+\varepsilon^2\right](1+\delta) \\
CS[2] &= \frac{1}{9}\left[4(a-c)^2+4(a-c)\varepsilon+2\varepsilon^2\right](1+\delta)
\end{aligned}
$$

It is obvious that $CS[2] > CS[1]$. In Appendix A.7 we give the expressions of DW and SW, at equilibrium, for the different modes of licensing. The right-hand Figure 3 concerns the domestic welfare (DW) at equilibrium. We have three areas which are delimited as follows.

Area I : for all $\delta \in [0,1]$, if $\varepsilon \in \left[\max\left\{\frac{2}{3}, \frac{4-2\delta}{4+3\delta}\right\}(a-c), (a-c)\right]$ then $DW[SB(K)] > DW[FF] \geq DW[T(K)] \geq DW[AB(K)] > DW[A(K)]$;

Area II : for all $\delta \in \left[\frac{1}{3}, 1\right]$, if $\varepsilon \in \left[\frac{4-2\delta}{4+3\delta}(a-c), \frac{2}{3}(a-c)\right]$ then $DW[SB(K)] > DW[FF] = DW[T(K)] > DW[AB(K)] > DW[A(K)]$;

Area III : for all $\delta \in [0,1]$, if $\varepsilon \in \left[0, \frac{4-2\delta}{4+3\delta}(a-c)\right]$ then $DW[SB(K)] > DW[AB(K)] > DW[FF] = DW[T(K)] > DW[A(K)]$.

For world welfare (or total welfare), that is, adding profits earned by the patentee to consumer surplus and firms' profits, it is always better that two licenses are sold. Therefore, an agency maximizing the world welfare will recommend the use of the fixed fee mechanism which guarantees that two licenses are sold whatever the externality. The next proposition summarizes our welfare recommendations.

Proposition 5 *A public agency maximizing the domestic welfare will recommend: (i) It is better licensing through the simultaneous bids mechanisms; (ii) It is worse licensing through the auction without reserve price; (iii) Licensing through the fixed fee is better than licensing through the take-it-or-leave-it or both auction mechanisms; (iv) Licensing through the the take-it-or-leave-it is better than licensing through both auction mechanisms. A public agency maximizing the world welfare will recommend: It is better licensing through fixed fees; indeed,*

$$SW[FF] \geq SW[T(K)] \geq SW[SB(K)] \geq SW[AB(K)] \geq SW[A(K)].$$

5 Concluding Comments

The objective of this paper has been to analyse the relationship between bargaining organizational forms and licensing of cost-reducing innovations, in order to assess the patent holder's optimal policies as well as the welfare properties. Up to now two trading mechanisms for selling an innovation have dominated the literature: a license auction game and a fixed fee licensing game. But a duopolistic industry is a small market for an innovation. The case where two licenses are sold is just a bargaining problem, and predictions of auction theory are quite sensitive to the *bargaining* model used. Only, when there are strictly less licenses than firms, auction theory is insentive to the bargaining theory used as its foundations. Also, the terms of trade between any two agents are mostly determined by negotiation, the course of which is influenced by the agents' opportunities for matching and trading with other partners. Therefore, alternative modes of licensing, where license trade is carried out through matching and bargaining, may deserve some interests.

In a two-period Cournot duopoly, we have considered the licensing of a cost-reducing innovation by means of a take-it-or-leave-it, an alternating bids, and a simultaneous bids mechanism. All these bargaining mechanisms incorporates voluntary matching. Our major finding is that the patent holder prefers the take-it-or-leave-it licensing mechanism to the fixed fee, alternating bids or simultaneous bids mechanisms. The simultaneous bids licensing mechanism is always worse for him. However, from a social point of view (an agency maximizing the domestic welfare where the patentee is a foreign laboratory), it is better licensing through the simultaneous bids mechanisms.

A Appendix

A.1 The Take-it-or-Leave-it Licensing Game $T(1)$

We characterize the SPE for the take-it-or-leave-it licensing game $T(1)$. Take the fourth-stage subgame where the patentee voluntary matches with firm i. It is optimal for the patentee to make the take-it-or-leave-it offer $F_i = \overline{\Pi}_i - \Pi_i$; offer which is accepted by firm i. Then, the agents' (patentee / firm i / firm j) second-period discounted payoffs are, respectively, $\delta\left(\overline{\Pi}_i - \Pi_i\right)$, $\delta\Pi_i$, and $\delta\underline{\Pi}_j$. Now, take the second-stage subgame where the patentee voluntary matches with firm i and makes the take-it-or-leave-it offer F_i such that firm i is indifferent between accepting and rejecting. If firm i rejects the offer, then firm i will obtain in the second-period the payoff of the unmatched buyer, i.e. $\delta\underline{\Pi}_i$. Indeed, the patentee can credibly threaten firm i to match next period with firm j if firm i rejects the patentee's current offer. Therefore, in stage two, the SPE take-it-or-leave-it offer is $F_i = (1+\delta)\overline{\Pi}_i - \Pi_i - \delta\underline{\Pi}_i$. Then, the agents' (patentee / firm i / firm j) SPE payoffs of

$T\,(1)$ are, respectively,

$$\begin{aligned}
\Phi\,[T\,(1)] &= (1+\delta)\ \overline{\Pi}_i - \Pi_i - \delta\ \underline{\Pi}_i \\
&= \tfrac{1}{9}\,[2\,(2+3\delta)\,(a-c) + (4+3\delta)\,\varepsilon]\,\varepsilon \\
\Psi_i\,[T\,(1)] &= \Pi_i + \delta\ \underline{\Pi}_i \\
&= \tfrac{1}{9}\,[a-c+2\varepsilon]^2\,(1+\delta) - \tfrac{1}{9}\,[2\,(2+3\delta)\,(a-c) + (4+3\delta)\,\varepsilon]\,\varepsilon \\
\Psi_j\,[T\,(1)] &= (1+\delta)\ \underline{\Pi}_j \\
&= \tfrac{1}{9}\,[a-c-\varepsilon]^2\,(1+\delta)
\end{aligned}$$

Result 1 *Consider the take-it-or-leave-it licensing game $T\,(1)$. At the SPE, the patentee makes a take-it-or-leave-it offer*

$$F_i^*\,[T\,(1)] = \frac{1}{9}\,[2\,(2+3\delta)\,(a-c) + (4+3\delta)\,\varepsilon]\,\varepsilon.$$

This take-it-or-leave-it offer is accepted by the matched firm i in the first period.

A.2 The Take-it-or-Leave-it Licensing Game $T\,(2)$

We characterize the SPE for the take-it-or-leave-it licensing game $T\,(2)$. Take the fourth-stage subgame when no agreement was reached in stage two. The patentee offers two take-it-or-leave-it contracts. Both firms, simultaneously, either accept it (Buy) or reject it (Don't buy). Table 12 gives us the matrix second-period undiscounted payoffs for both firms.

i \ j	Buy	Don't buy
Buy	$\overline{\underline{\Pi}}_i - F_i$; $\overline{\underline{\Pi}}j - F_j$	$\overline{\Pi}_i - F_i$; $\underline{\Pi}_j$
Don't buy	$\underline{\Pi}_i$; $\overline{\Pi}_j - F_j$	Π_i ; Π_j

Table 12: Take-it-or-leave-it: fourth-stage subgame

We compute a pair (F_i, F_j) that maximizes the patentee's second-period revenue. Two Nash equilibria of Table 12 are analysed: (Buy, Buy) and (Buy, Don't buy). The following conditions on $F_i,\ F_j$, are necessary and sufficient for (buy, buy) to be a Nash equilibrium: $\overline{\underline{\Pi}}_i - F_i \geq \underline{\Pi}_i$ and $\overline{\underline{\Pi}}_j - F_j \geq \underline{\Pi}_j$. That is,

$$F_i \leq \frac{4\,(a-c)\,\varepsilon}{9} \text{ and } F_j \leq \frac{4\,(a-c)\,\varepsilon}{9}$$

Therefore, when (Buy,Buy) is the Nash equilibrium outcome the patentee's undiscounted revenue is $\frac{8}{9}\,(a-c)\,\varepsilon$. The following conditions on $F_i,\ F_j$, are necessary and sufficient for

(Buy, Don't buy) to be the unique Nash equilibrium: $\overline{\Pi}_i - F_i \geq \Pi_i$ and $\Pi_j \geq \overline{\Pi}_j - F_j$. That is,

$$F_i \leq \frac{4\,(a-c+\varepsilon)\,\varepsilon}{9} \text{ and } F_j > \frac{4\,(a-c+\varepsilon)\,\varepsilon}{9}$$

Therefore, when (Buy, Don't buy) is the unique Nash equilibrium the patentee's undiscounted revenue is $\frac{4}{9}\,(a-c+\varepsilon)\,\varepsilon$. Comparing the patentee's revenue for (Buy, Buy) and (Buy, Don't buy), the patentee's optimal decision is to offer the license at the price $\frac{4}{9}\,(a-c)\,\varepsilon$ to both firms.

i j	Buy	Don't buy
Buy	$(1+\delta)\overline{\underline{\Pi}}_i - F_i$ $(1+\delta)\,\overline{\underline{\Pi}}j - F_j$	$\overline{\Pi}_i - F_i + \delta\,\overline{\underline{\Pi}}_i$ $(1+\delta)\underline{\Pi}_j$
Don't buy	$(1+\delta)\underline{\Pi}_i$ $\overline{\Pi}_j - F_j + \delta\,\overline{\underline{\Pi}}_j$	$\Pi_i + \delta\,\underline{\Pi}_i$ $\Pi_j + \delta\,\underline{\Pi}_j$

Table 13: Take-it-or-leave-it: second-stage subgame

Now, take the second-stage subgame. Table 13 gives us the matrix payoffs for both firms. In order to have (Buy, Buy) as SPE outcome the following conditions must hold simultaneously: $(1+\delta)\left[\overline{\underline{\Pi}}_i - \underline{\Pi}_i\right] \geq F_i$ and $(1+\delta)\left[\overline{\underline{\Pi}}_j - \underline{\Pi}_j\right] \geq F_j$. That is,

$$F_i \leq \frac{4}{9}\,(a-c)\,(1+\delta)\,\varepsilon \text{ and } F_j \leq \frac{4}{9}\,(a-c)\,(1+\delta)\,\varepsilon$$

Therefore, when (Buy, Buy) is the SPE outcome the patentee's discounted revenue is $\frac{8}{9}\,(a-c)\,(1+\delta)\,\varepsilon$. The following conditions on F_i, F_j, are necessary and sufficient for (Buy, Don't buy) to be the SPE: $\overline{\Pi}_i - \Pi_i + \delta\,\left[\overline{\underline{\Pi}}_i - \underline{\Pi}_i\right] \geq F_i$ and $\overline{\Pi}_j - \Pi_j + \delta\,\left[\overline{\underline{\Pi}}_j - \underline{\Pi}_j\right] < F_j$. That is,

$$F_i \leq \frac{4}{9}\,(a-c)\,(1+\delta)\,\varepsilon + \frac{4}{9}\varepsilon^2 \text{ and } F_j > \frac{4}{9}\,(a-c)\,(1+\delta)\,\varepsilon + \frac{4}{9}\varepsilon^2$$

Comparing the patentee's revenue, the patentee's SPE decision is to offer the license at the price $\frac{4}{9}\,(a-c)\,(1+\delta)\,\varepsilon$ to both firms. Then, the agents' (patentee / firm i / firm j) SPE payoffs of $T\,(2)$ are, respectively,

$$\begin{aligned}
\Phi\,[T\,(2)] &= 2\,(1+\delta)\left[\overline{\underline{\Pi}}_i - \underline{\Pi}_i\right] \\
&= \tfrac{8}{9}\,(a-c)\,(1+\delta)\,\varepsilon \\
\Psi_i\,[T\,(2)] &= (1+\delta)\;\underline{\Pi}_i \\
&= \tfrac{1}{9}\,[a-c-\varepsilon]^2\,(1+\delta) \\
\Psi_j\,[T\,(2)] &= (1+\delta)\;\underline{\Pi}_j \\
&= \tfrac{1}{9}\,[a-c-\varepsilon]^2\,(1+\delta)
\end{aligned}$$

Result 2 *Consider the take-it-or-leave-it licensing game $T(2)$. At the SPE, the patentee makes a take-it-or-leave-it offer*

$$F_i^* [T(2)] = \frac{4}{9}(a-c)(1+\delta)\varepsilon, \qquad i = 1, 2.$$

This take-it-or-leave-it offer is accepted by both firms in the first period.

A.3 The Alternating Bids Licensing Game $AB(1)$

We characterize the SPE for the alternating bids licensing game $AB(1)$. Take the fourth-stage subgame where the patentee voluntary matches with firm i. A random event chooses whether the patentee or firm i will make the offer. If firm i makes the offer, then it makes a take-it-or-leave offer $F_i = 0$ which is accepted by the patentee and leads to the following second-period discounted payoffs at equilibrium: 0 (for the patentee), $\delta\overline{\Pi}_i$ (for firm i), and $\delta\underline{\Pi}_j$ (for firm j). If the patentee makes the offer, then he makes the highest offer, $F_i = \overline{\Pi}_i - \Pi_i$, which is accepted by firm i and leads to the following second-period discounted payoffs at equilibrium: $\delta\left[\overline{\Pi}_i - \Pi_i\right]$ (for the patentee), $\delta\Pi_i$ (for firm i), and $\delta\underline{\Pi}_j$ (for firm j). Therefore, at the beginning of stage four, but after the patentee has voluntary matched with firm i, the expected discounted payoffs are: $\frac{1}{2}\delta\left[\overline{\Pi}_i - \Pi_i\right]$ (for the patentee), $\frac{1}{2}\delta\left[\overline{\Pi}_i + \Pi_i\right]$ (for firm i), and $\delta\underline{\Pi}_j$ (for firm j).

Now, Take the second-stage subgame where the patentee voluntary matches with firm i and offers F_i such that firm i is indifferent between accepting and rejecting. If firm i rejects the patentee's offer, then firm i will obtain the second-period expected payoff of the unmatched buyer, i.e. $\delta\underline{\Pi}_i$. Indeed, the patentee can credibly threaten firm i to match next period with firm j whenever firm i rejects the patentee's current offer. Therefore, in stage two, the patentee SPE offer is $F_i = (1+\delta)\overline{\Pi}_i - \Pi_i - \delta\underline{\Pi}_i$. Then, the agents' (patentee / firm i / firm j) SPE payoffs of $AB(1)$ are, respectively,

$$\begin{aligned}
\Phi[AB(1)] &= (1+\delta)\ \overline{\Pi}_i - \Pi_i - \delta\ \underline{\Pi}_i \\
&= \tfrac{1}{9}\left[2(2+3\delta)(a-c) + (4+3\delta)\varepsilon\right]\varepsilon \\
\Psi_i[AB(1)] &= \Pi_i + \delta\ \underline{\Pi}_i \\
&= \tfrac{1}{9}\left[a-c+2\varepsilon\right]^2(1+\delta) - \tfrac{1}{9}\left[2(2+3\delta)(a-c) + (4+3\delta)\varepsilon\right]\varepsilon \\
\Psi_j[AB(1)] &= (1+\delta)\ \underline{\Pi}_j \\
&= \tfrac{1}{9}\left[a-c-\varepsilon\right]^2(1+\delta)
\end{aligned}$$

Result 3 *Consider the alternating bids licensing game $AB(1)$. At the SPE, the patentee offers a contract where*

$$F_i^* [AB(1)] = \frac{1}{9}\left[2(2+3\delta)(a-c) + (4+3\delta)\varepsilon\right]\varepsilon$$

This contract is accepted by the matched firm i in the first period.

A.4 The Alternating Bids Licensing Game $AB\,(2)$

We characterize the SPE for the alternating bids licensing game $AB\,(2)$. Take the fourth-stage subgame where no agreement was reached in stage two (or period one). A random event chooses the proposer(s). If the patentee is chosen, then it is straightforward that the patentee offers $F_i = \overline{\Pi}_i - \underline{\Pi}_i = \frac{4}{9}\,(a-c)\,\varepsilon$, $i = 1, 2$. If the firms are chosen, then both firms offer $F_i = 0$, $i = 1, 2$. Therefore, at the beginning of this subgame, the second-period expected payoffs (at equilibrium) are: $\delta\left[\overline{\Pi}_i - \underline{\Pi}_i\right]$ (for the patentee), $\frac{1}{2}\delta\left[\overline{\Pi}_i + \underline{\Pi}_i\right]$ (for both firms).

Now, take the fourth-stage subgame where only one agreement was reached in the stage two (or period one). This subgame is structurally identical to one fourth-stage subgame of $AB\,(1)$ but taking into account that the rival j has already bought and adopted the new technology. At the beginning of this subgame, the second-period expected payoffs (at equilibrium) are: $\delta\frac{1}{2}\left[\overline{\Pi}_i - \underline{\Pi}_i\right]$ (for the patentee), $\delta\frac{1}{2}\left[\overline{\Pi}_i + \underline{\Pi}_i\right]$ (for firm i), and $\delta\overline{\Pi}_j$ (for firm j).

Finally, at the beginning of the fourth-stage subgame where both firms have bought the technology in stage two (or period one), the second-period expected payoffs (at equilibrium) are: 0 (for the patentee), $\delta\overline{\Pi}_i$ (for both firms).

i \ j	Buy	Don't buy
Buy	$(1+\delta)\overline{\Pi}_i - F_i$ $(1+\delta)\,\overline{\Pi}j - F_j$	$\overline{\Pi}_i - F_i + \delta\,\overline{\Pi}_i$ $\left(1+\frac{\delta}{2}\right)\underline{\Pi}_j + \frac{\delta}{2}\overline{\Pi}_j$
Don't buy	$\left(1+\frac{\delta}{2}\right)\underline{\Pi}_i + \frac{\delta}{2}\overline{\Pi}_i$ $\overline{\Pi}_j - F_j + \delta\,\overline{\Pi}_j$	$\Pi_i + \frac{\delta}{2}\left[\overline{\Pi}_i + \underline{\Pi}_i\right]$ $\Pi_j + \frac{\delta}{2}\left[\overline{\Pi}_j + \underline{\Pi}_j\right]$

Table 14: Alternating bids licensing game: second-stage subgame

Now, take the second-stage subgame. Table 14 gives us the matrix payoffs for both firms. In order to have (buy, buy) as SPE the following conditions must hold simultaneously: $F_i \leq \left(1+\frac{\delta}{2}\right)\left[\overline{\Pi}_i - \underline{\Pi}_i\right]$ and $F_j \leq \left(1+\frac{\delta}{2}\right)\left[\overline{\Pi}_j - \underline{\Pi}_j\right]$. If only one of these conditions is not satisfied, then either (buy, don't buy) or (don't buy, buy) will be the SPE. If none of them are satisfied, then (don't buy, don't buy) is the unique SPE. It is straightforward to check which licensing policy yields the highest income for the patentee. First, (don't buy, don't buy) in the second-stage implies both firms accepting in the fourth-stage (or period two) and the patentee having as profits, $\delta\left[\overline{\Pi}_i - \underline{\Pi}_i\right]$. If firm i accepts in the second-stage while firm j accepts in the fourth-stage, then the patentee obtains the following profits: $\delta\left[\overline{\Pi}_i - \underline{\Pi}_i\right] + \overline{\Pi}_j - \Pi_j$. Finally, both firms accepting the second-stage (or period one) offer implies the next patentee profits: $(2+\delta)\left[\overline{\Pi}_i - \underline{\Pi}_i\right]$. Therefore, in stage two, the patentee SPE offer is $F_i = \left(1+\frac{\delta}{2}\right)\left[\overline{\Pi}_i - \underline{\Pi}_i\right]$, $i = 1, 2$. Then, the agents' (patentee / firm i / firm j) SPE payoffs of $AB\,(2)$ are, respectively,

$$\begin{aligned}
\Phi\,[AB\,(2)] &= (2+\delta)\left[\overline{\Pi}_i - \underline{\Pi}_i\right] \\
&= (2+\delta)\tfrac{4}{9}(a-c)\,\varepsilon \\
\Psi_i\,[AB\,(2)] &= \underline{\Pi}_i + \delta\tfrac{1}{2}\left[\overline{\Pi}_i + \underline{\Pi}_i\right] \\
&= \tfrac{1}{9}[a-c+\varepsilon]^2(1+\delta) - \left(1+\tfrac{\delta}{2}\right)\tfrac{4}{9}(a-c)\,\varepsilon \\
\Psi_j\,[AB\,(2)] &= \underline{\Pi}_j + \delta\tfrac{1}{2}\left[\overline{\Pi}_j + \underline{\Pi}_j\right] \\
&= \tfrac{1}{9}[a-c+\varepsilon]^2(1+\delta) - \left(1+\tfrac{\delta}{2}\right)\tfrac{4}{9}(a-c)\,\varepsilon
\end{aligned}$$

Result 4 *Consider the alternating bids licensing game $AB\,(2)$. At the SPE, the patentee offers two contracts where*

$$F_i^*\,[AB\,(2)] = \left(1+\frac{\delta}{2}\right)\frac{4}{9}(a-c)\,\varepsilon,\ i = 1, 2$$

These contracts are accepted by both firms in the first period.

A.5 The Simultaneous Bids Licensing Game $SB\,(1)$

We compute the SPE of $SB\,(1)$ in which agreement is reached with positive probability. Take the fourth-stage subgame where the patentee voluntary matches with firm i. The outcome will be the Nash bargaining solution (see Binmore et al. [1]) obtained from

$$F_i = \underset{F_i\in\mathbb{R}_+}{\text{ArgMax}}\left[(F_i)\left(\overline{\Pi}_i - F_i - \Pi_i\right)\right] \ .$$

Then, the agents (patentee / firm i / firm j) second-period payoffs are, respectively, $\delta\frac{1}{2}\left[\overline{\Pi}_i - \Pi_i\right]$, $\delta\frac{1}{2}\left[\overline{\Pi}_i + \Pi_i\right]$, and $\delta\underline{\Pi}_j$. The outcome of the second-stage subgame where the patentee voluntary matches firm i will be the Nash bargaining solution, with disagreement payoffs equal to $\delta\frac{1}{2}\left[\overline{\Pi}_i - \Pi_i\right]$ and $\Pi_i + \delta\underline{\Pi}_i$, respectively for the patentee and firm i. Indeed, it is still a credible threat for the patentee to match with firm j in case of no agreement in stage two (or period one). Therefore, the Nash bargaining solution is obtained from

$$F_i^*\,[SB\,(1)] = \underset{F_i\in\mathbb{R}_+}{\text{ArgMax}}\left[\left(F_i - \delta\frac{1}{2}\left[\overline{\Pi}_i - \Pi_i\right]\right)\left((1+\delta)\overline{\Pi}_i - F_i - [\Pi_i + \delta\underline{\Pi}_i]\right)\right] \ .$$

Then, the agents (patentee / firm i / firm j) SPE payoffs of $SB\,(1)$ are, respectively,

$$\begin{aligned}
\Phi\,[SB\,(1)] &= \tfrac{1}{4}\left[(2+3\delta)\,\overline{\Pi}_i - 2\delta\underline{\Pi}_i - (2+\delta)\,\Pi_i\right] \\
&= \tfrac{1}{18}[(4+8\delta)(a-c) + (4+5\delta)\,\varepsilon]\,\varepsilon \\
\Psi_i\,[SB\,(1)] &= \tfrac{1}{4}\left[(2+\delta)\,\overline{\Pi}_i + 2\delta\underline{\Pi}_i + (2+\delta)\,\Pi_i\right] \\
&= \tfrac{1}{9}[a-c+2\varepsilon]^2(1+\delta) - \tfrac{1}{18}[(4+8\delta)(a-c) + (4+5\delta)\,\varepsilon]\,\varepsilon \\
\Psi_j\,[SB\,(1)] &= (1+\delta)\ \underline{\Pi}_j \\
&= \tfrac{1}{9}[a-c-\varepsilon]^2(1+\delta)
\end{aligned}$$

Result 5 *Consider the simultaneous bids licensing game $SB(1)$. Then, the SPE fee is*

$$F_i^*[SB(1)] = \frac{(4+8\delta)(a-c)\varepsilon + (4+5\delta)\varepsilon^2}{18}$$

A.6 The Simultaneous Bids Licensing Game $SB(2)$

We compute the SPE of $SB(2)$ in which agreement is reached with positive probability. Take the fourth-stage subgame where no agreement was reached in stage two (or period one). The fourth-stage outcome will be the Nash bargaining solution (see Binmore et al. [1]) obtained from

$$F_i = \underset{F_i \in \mathbb{R}_+}{\text{ArgMax}} \left[(F_i) \left(\overline{\Pi}_i - F_i - \underline{\Pi}_i \right) \right]; \qquad i = 1, 2.$$

Then, the agents (patentee / firm i / firm j) second-period payoffs are, respectively, $\delta\left(\overline{\Pi}_i - \underline{\Pi}_i\right)$, $\delta\frac{1}{2}\left(\overline{\Pi}_i + \underline{\Pi}_i\right)$, and $\delta\frac{1}{2}\left(\overline{\Pi}_j + \underline{\Pi}_j\right)$. The SPE outcome of $SB(2)$ will be the Nash bargaining solution with the agents' (patentee / firm i / firm j) disagreement points equal, respectively, to

$$\delta\,\frac{1}{2}\left(\overline{\Pi}_i - \underline{\Pi}_i\right),\ \underline{\Pi}_i + \frac{1}{2}\delta\left(\overline{\Pi}_i + \underline{\Pi}_i\right),\ \text{and}\ \underline{\Pi}_j + \frac{1}{2}\delta\left(\overline{\Pi}_j + \underline{\Pi}_j\right)\ .$$

Therefore, the Nash bargaining solution $\left(F_i^*[SB(2)], F_j^*[SB(2)]\right)$is obtained from

$$\begin{cases} F_i^*[SB(2)] = \text{ArgMax}_{F_i \in \mathbb{R}_+} \left[\left(F_i + F_j^* - \left[\delta\,\frac{1}{2}\left(\overline{\Pi}_i - \underline{\Pi}_i\right) + F_j^* \right] \right) \left(\left(1 + \frac{\delta}{2}\right) \left[\overline{\Pi}_i - \underline{\Pi}_i \right] - F_i \right) \right] \\ F_j^*[SB(2)] = \text{ArgMax}_{F_j \in \mathbb{R}_+} \left[\left(F_j + F_i^* - \left[\delta\,\frac{1}{2}\left(\overline{\Pi}_j - \underline{\Pi}_j\right) + F_i^* \right] \right) \left(\left(1 + \frac{\delta}{2}\right) \left[\overline{\Pi}_j - \underline{\Pi}_j \right] - F_j \right) \right] \end{cases}$$

Then, the agents (patentee / firm i / firm j) SPE payoffs of $SB(2)$ are, respectively,

$$\begin{aligned} \Phi[SB(2)] &= \tfrac{4}{9}(1+\delta)(a-c)\varepsilon \\ \Psi_i[SB(2)] &= \tfrac{1}{9}(1+\delta)\left[(a-c)^2 + \varepsilon^2\right] \\ \Psi_j[SB(2)] &= \tfrac{1}{9}(1+\delta)\left[(a-c)^2 + \varepsilon^2\right] \end{aligned}$$

Result 6 *Consider the simultaneous bids liccensing game $SB(2)$. Then, the SPE fees are,*

$$F_i^*[SB(2)] = \frac{2}{9}(a-c)(1+\delta)\varepsilon, \qquad i = 1, 2.$$

A.7 The Social Wefare

In this appendix, we give the expressions of DW and SW, at equilibrium, for the different modes of licensing.

$$
\begin{aligned}
DW[A(1)] &= \frac{1}{18}\left[8(a-c)^2 - 4(a-c)\varepsilon + 5\varepsilon^2\right](1+\delta) = DW[A^*(1)] \\
DW[A(2)] &= \frac{4}{9}\left[(a-c)^2 + \varepsilon^2\right](1+\delta) = DW[A^*(2)] = DW[FF] = DW[T(2)] \\
DW[T(1)] &= \frac{1}{18}\left[8(a-c)^2(1+\delta) - 4\delta(a-c)\varepsilon + (3+5\delta)\varepsilon^2\right] = DW[AB(1)] \\
DW[AB(2)] &= \frac{4}{9}\left[(a-c)^2(1+\delta) + \delta(a-c)\varepsilon + (1+\delta)\varepsilon^2\right] \\
DW[SB(1)] &= \frac{1}{18}\left[8(a-c)^2(1+\delta) - 4(a-c)\varepsilon + (7+6\delta)\varepsilon^2\right] \\
DW[SB(2)] &= \frac{4}{9}\left[(a-c)^2 + (a-c)\varepsilon + \varepsilon^2\right](1+\delta)
\end{aligned}
$$

$$
\begin{aligned}
SW[1] &= \frac{1}{18}\left[8(a-c)^2 + 8(a-c)\varepsilon + 11\varepsilon^2\right](1+\delta) \\
SW[2] &= \frac{4}{9}\left[(a-c)^2 + 2(a-c)\varepsilon + \varepsilon^2\right](1+\delta)
\end{aligned}
$$

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