Is the allocation of voting power among EU states fair ?*

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Abstract

It has often been claimed that the current voting process within the EU Council of Ministers is not fair. In this paper we verify this assertion by carrying out an evaluation of the distribution of power among the member states. The results show that the current distribution of votes for the qualified majority does not lead to a fair distribution of power whatever definition of the EU is considered. It can not be claimed however that the current voting process has a systematic bias in favor of certain states. We also present a simple method to derive voting weights which lead to a fair allocation of power.

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1 Introduction

This paper deals with the fairness of national representation in the European Union (EU). An often heard argument is that in the current decision-making the largest countries are under-represented while the reverse holds for the small ones. During the European Free Trade Area (EFTA) countries' accession talks there was some debate regarding the small versus the large countries' influence in the Union, which is easy to understand as the EFTA countries' entry increased the relative proportion of small countries in the Union. As the Union is now taking the first steps of enlargement to the Central and Eastern Europe, where the potential candidates are in terms of population again smaller than the Union average, and as the Union has decided to reform its institutions before their entry, the question is once again very topical.

The voting process in the EU involves mainly three institutions: the Parliament, the Commission and the Council of Ministers. Since this paper is devoted to the distribution of power among the member states, we focus on the Council of Ministers, which represents the national interests. Indeed in the Parliament representatives' voting patterns are likely to be based on ideology rather than nationality and the Commission represents, at least theoretically, the supranational view.

The current distribution of votes is far from reflecting the relative sizes of the member states. At the extremes, Germany for example has 8 million electors per vote whereas Luxembourg has less than 200,000. The general feature of the current distribution of votes is that small countries are relatively over-endowed and their large counterparts have less than proportional weight. How fair are these arrangements can thus be questioned.

The purpose of this paper is to find out whether the current decision-making process can make sense in terms of fairness, and if not, to propose a redistribution of the votes among the member states. The approach followed here is to model the voting process in the EU Council of Ministers by a simple superadditive game, which allows us to measure the distribution of power by the Banzhaf index. The current indices within the Council of Ministers are computed and compared with indices considered to be fair. In order to derive such fair indices, the issue of what is meant by fairness is first clarified in the European context. Fairness criterion in this case depends on how the EU is defined . As the nature of the EU is not quite clear, several possible definitions are considered here: association of states, federal state or single state. The results crucially depend on which definition of the EU is taken into account.

We show that the current distribution of votes cannot give rise to a fair allocation of power regardless of the EU definition we choose: the votes should be redistributed. We propose a method to derive the distribution of votes which leads to a given distribution of voting power. So with regards to the definition of the EU taken into account, it is possible to reach the fair distribution of voting power.

The rest of the paper is organized as follows. Section 2 models voting processes as simple games and defines the Banzhaf index of power. Section 3 presents the particular features of the EU and proposes the simple-game representation of the voting processes within the Council of Ministers. Section 4 derives the fair distributions of power for different definitions of the EU. Section 5 computes the current distributions of power and deals with the question of fairness. Section 6 searches for distributions of votes that ensures fair allocation of power. Finally Section 7 concludes.

2 Voting games and power indices

A (0-1)-game is defined as a pair (N, v) where $N = \{1, .., n\}$ denotes the set of players and v a real-valued function defined on the subsets of N to $\{0, 1\}$. Any non-empty subset of N is called a coalition. Let us denote an arbitrary coalition by S. Without a loss of generality we can refer to coalitions with the property $v(S) = 1$ as winning while those with $v(S) = 0$ as losing. Function v defines a simple game when it is not identically 0 and is monotonic, i.e. $v(S) \leq v(T)$ whenever $S \subseteq T$. In this context, a game is superadditive if the complement of a winning coalition is always losing: $v(S) + v(N \setminus S) \leq 1$ for all $S \subset T$[1].

A voting process can be modelled as a (0-1)-game where the winning coalitions are defined as those which can make a decision without the vote of the remaining players. The coalitional form of a voting process can be described as follows:

1. Unanimity of players leads to the passage of a proposal: $v(N) = 1$.
2. A subset of a losing coalition is always losing: $v(T) = 0 \Rightarrow v(S) = 0$ for all $S \subset T$
3. Two nonintersecting coalitions are not winning at the same time: if $v(S) = 1$ then $v(T) = 0$ for all T such that $S \cap T = \emptyset$.

Properties 1 and 2 define a simple game. As the game is monotonic, property 3 is equivalent to saying that the game is superadditive. Thus a voting process can be modelled as a simple superadditive game.

There are direct and indirect voting processes. In indirect cases individual players are partitioned into groups and each group selects a representative to vote for an outcome of

[1] In the context of simple games, this condition is equivalent to the general definition of superadditivity [See Dubey and Shapley, 1979]:

$$v(S) + v(T) \leq v(S \cup T) \text{ for all } S \text{ and } T \text{ such that } S \cap T = \emptyset$$

the game. Let $M = \{1, ..., m\}$ be a set of players divided into n nonintersecting groups of players $(m_1 + m_2 + ... + m_n = m)$:

$$M = \left\{ \underbrace{1(1), ..., m_1(1)}_{M_1}, \underbrace{1(2), ..., m_2(2)}_{M_2}, ..., \underbrace{1(n), ..., m_n(n)}_{M_n} \right\}$$

In the first stage each group $M_j = \{1, ..., m_j\}$ elects its representative, i.e. the games (M_j, y_j) are played within groups j $(j = 1, ..., n)$. In the second stage, the representatives $N = \{1, ..., n\}$ vote for the outcome, i.e. the game (N, v) is played among n representatives. The indirect voting process among the individual players is called a compound game (M, u) having a coalitional form:

$$u(S) = v\left(\{j \mid y_j(S \cap M_j) = 1\}\right)$$

The coalitional form of a compound voting game is denoted $u = v(y_1, ..., y_n)$ to underline that u is made of a game v of the sub-games y_j.

A player is said to exerts power in a coalition if she allows the coalition to make a decision. In other words a player has power in a coalition if the outcome depends on her vote. Formally, player i exerts power in $S \Longleftrightarrow v(S) = 1$ and $v(S \setminus \{i\}) = 0$. A player's a priori power is defined as the probability that she exerts power in a coalition. Thus power depends on the probability of coalition formation. In the literature, there are two standard assumptions on the coalitions formation. Under the first assumption, all coalitions have an equal probability of formation. Under the second assumption all the sizes of coalition are equiprobable and all the coalitions of a given size have the same probability of forming. The first assumption leads to the non-normalized Banzhaf index while the second one leads to the Shapley-Shubik index. [See Weber,1988]. In the absence of information on these probabilities, we give an equal probability for all coalitions[2]. As there are 2^{n-1} coalitions including a given player, player i's power can be written as follows:

$$\widetilde{\beta}_i(v) = \frac{1}{2^{n-1}} \cdot \sum_{\substack{S \subset N \\ (S \ni i)}} [v(S) - v(S \setminus \{i\})] \tag{1}$$

which is referred to as the non-normalized Banzhaf index. The following normalization is often used:[3]

$$\beta_i(v) = \frac{\widetilde{\beta}_i(v)}{\sum_{k \in N} \widetilde{\beta_k}(v)} \tag{2}$$

[2]Straffin (1988) has shown that the non normalized Banzhaf index may be obtained by assuming that each voter's probability of voting "yes" on a bill is chosen from the uniform distribution on [0,1]. As this paper deals with an abstract design of a voting body, this behavioral background seems the most reasonable.

[3]However Straffin's probabilistic interpretation holds no longer.

Owen (1975) has shown that, for a compound game (M, u), the Banzhaf index can be defined as a product of indices of the stages and thus the following holds:

$$\widetilde{\beta_{i(j)}}(u) = \widetilde{\beta_j}(v) \;.\; \widetilde{\beta_i}(y_j) \text{ for all } i \in M_j,\ j = 1, ..., n. \tag{3}$$

3 Voting within the EU as a simple game

3.1 Special features of the EU

There are several ways of making a decision in the Council of Ministers depending on the issue which is voted upon. These include: (1) unanimity, (2) simple majority of member states, (3) qualified majority with voting weights and (4) qualified majority with voting weights and a minimum number of states. Whenever a qualified majority is used, the national voting weights are the following: 10 votes each for Germany, the United Kingdom, France and Italy; 8 votes for Spain; 5 votes each for the Netherlands, Greece, Belgium and Portugal; 4 votes each for Sweden and Austria; 3 votes each for Denmark, Finland and Ireland; and 2 votes for Luxembourg. A qualified majority can be reached by 62 votes out of 87. For some qualified majorities a minimum of 10 members out of 15 is required.

In order to evaluate the fairness of the power distribution among member states, one has to define a criterion for fairness within the EU. This gives rise to a fair distribution of power which can then be compared to the current distribution. Since the definition of the EU determines the fairness criterion among the states, the nature of the EU must be taken into account. If the EU were a single state, all citizens should be treated equally. The "One Man, One Vote" principle should apply. If the EU were an association of states, no state should be treated differently from another. Thus the "One State, One Vote" should be applied. But the EU is neither a single state nor an association of states; the basic nature of the Union lies somewhere between these extremes. Hence in some sense the EU can be defined as a federal state, which means that both principles must be taken into account. The degree of federalism should determine the importance accorded to the "One Man, One Vote" principle relative to the "One State, One Vote" principle. To take into account both principles, the vote within the EU Council must be approach from two points of view, the point of view of the member states and the point of view of the EU citizens.

3.2 Voting games among member states

Let $N = \{1, ..., 15\}$ be the set of member states. The four different voting rules in the Council of Ministers can be defined formally as follows:

1) Unanimity (N, v_u)

$$v_u(S) = \begin{cases} 1 & \text{if } S = N \\ 0 & \text{otherwise} \end{cases}$$

2) Simple Majority (N, v_{sm})

$$v_{sm}(S) = \begin{cases} 1 & \text{if } s \geq 8, \text{ where } s \text{ denotes the number of voters in } S \\ 0 & \text{otherwise} \end{cases}$$

3) Qualified Majority (N, v_{qm})

$$v_{qm}(S) = \begin{cases} 1 & \text{if } \sum_{j \in N} w_j \geq 62, \text{ where } w_j \text{ denotes state } j\text{'s number of votes} \\ 0 & \text{otherwise} \end{cases}$$

4) Qualified Majority with a minimum number of member states (N, v_{qm+})

$$v_{qm+}(S) = \begin{cases} 1 & \text{if } \sum_{j \in N} w_j \geq 62 \text{ and } s \geq 10 \\ 0 & \text{otherwise} \end{cases}$$

3.3 Voting games among citizens

Let $M = \{1, ..., m\}$ be the set of the EU citizens. The citizens do not vote directly in the Council of Ministers. They are only represented via their national governments. Hence from the EU citizens' point of view the voting processes in the Council are indirect. We model them as compound games. The set of the EU citizens may be divided into 15 separate subgroups: $M = M_1 \cup ... \cup M_{15}$. In the first stage, the citizens elect their national government, i.e. the games (M_j, y_j) are played within states j $(j = 1, ..., 15)$. We assume that the government election games are simple majority games:

$$y_j(S) = \begin{cases} 1 & \text{if } s > m_j/2 \\ 0 & \text{otherwise} \end{cases}$$

In the second stage the member states representatives vote in the Council of Ministers, i.e. the game (N, v) is played among the member states. Then the coalitional forms of the compound games among the EU citizens are respectively as follows $u_u = v_u(y_1, ..., y_{15})$, $u_{sm} = v_{sm}(y_1, ..., y_{15})$, $u_{qm} = v_{qm}(y_1, ..., y_{15})$ and $u_{qm+} = v_{qm+}(y_1, ..., y_{15})$.

4 On fairness within the EU

The way fairness is defined depends on the underlying conception of the EU. As it is not quite clear, several possible definitions of the EU are considered here. The following notational conventions are adopted: (N, v_α), $\widetilde{\alpha}$, α denote a fair game played among states in the Council, the fair non-normalized Banzhaf index and the fair normalized index respectively.

4.1 The case of an association of states

If the EU is an association of totally independent states, each state must have the same treatment. Thus in a fair game in the Council of Ministers (N, v_α) it means that each state must have the same probability of being crucial: $\widetilde{\alpha_1}(v_\alpha) = ... = \widetilde{\alpha_{15}}(v_\alpha)$ whatever this probability is.[4] It is equivalent to require that each player has the same normalized power index. Hence

$$\alpha_j(v_\alpha) = 1/15 \text{ for all } j \tag{4}$$

4.2 The case of a single state

If the EU is a single state, each individual citizen must be treated equally. It means that no citizen may exert more power than any other, regardless of her nationality, in EU decisions.

Let (M, u_α) be a fair compound game of the EU decision making as outlined above. So we have $M = M_1 \cup ... \cup M_{15}$ and $u_\alpha = v_\alpha(y_1, ..., y_{15})$ where (M_j, y_j) $(j = 1, ..., 15)$ are the government election games in the member states and (N, v_α) is the voting game in the Council of Ministers that guarantees fairness in the compound game.

In the case of a single state, requiring the equal treatment of the EU citizens leads to the following property

$$\widetilde{\alpha}_{1(1)}\ (u_\alpha) = ... = \widetilde{\alpha}_{m_1(1)}\ (u_\alpha) = ... = \widetilde{\alpha}_{1(n)}\ (u_\alpha) = ... = \widetilde{\alpha}_{m_n(n)}\ (u_\alpha) \tag{5}$$

Since we are dealing with the Council of Ministers we reduce the voting game of individual citizens to a voting game among the member states. Using (3), we get

$$\widetilde{\alpha}_j\,(v_\alpha) = \frac{\widetilde{\alpha}_{i(j)}\,(u_\alpha)}{\widetilde{\beta}_i(y_j)} \tag{6}$$

Then, as m is sufficiently large, we obtain (see the appendix for a proof):

$$\alpha_j(v_\alpha) = \frac{\sqrt{m_j}}{\sum\limits_{k=1}^{n} \sqrt{m_k}} \tag{7}$$

[4]Requiring a certain probability would imply a choice concerning the risk of the status quo. The non-normalized Banzhaf index gives some information on the difficulty of making a decision : the higher the non-normalized Banzhaf index is, the easier it is to make a decision [see Widgrén (1994a)].

4.3 The case of a federal state

If the EU is a federal state, each state must be treated in accordance to a weighted average between the two extreme principles ("One Man, One Vote" and "One State, One Vote"). In the corresponding fair game in the Council of Ministers (N, v_α), it means that each state must have the following index of power:

$$\alpha_j(v_\alpha) = x \,.\, \frac{\sqrt{m_j}}{\sum_{k=1}^{n} \sqrt{m_k}} + (1-x) \,.\, \frac{1}{15} \tag{8}$$

where $x \in (0,1)$ represents the weight given to the "One Man, One Vote" principle. Figure 1 gives the fair distributions of power for different values of x.

[Insert Figure 1]

Three general aspects may be underlined. First, as could be expected, the fair share increase with the weight given to the "One Man, One Vote" principle for the large states (from Germany to Spain), while the reverse holds for medium-sized countries (from the Netherlands to Portugal) and small states (from Sweden to Luxembourg). Second, the fair share of power varies widely with x for the four largest states (from Germany to Italy) or for the two smallest states (Ireland and Luxembourg). For medium-sized countries the sensitivity to x is not that significant. In particular the Netherlands' fair share remains nearly constant irrespective of how the EU is defined. Third, when the "One Man, One Vote" principle is taken into account ($x > 0$), the United Kingdom, France, and Italy all have similar fair shares of power. This also holds for Greece, Belgium and Portugal or Denmark, and Finland.

5 Power distribution within the EU Council of Ministers

5.1 Current distribution of power

Table 1 shows the current non-normalized Banzhaf indices for each way of making a decision.

[Insert Table 1]

A striking feature of this table is the difference between the unanimity and the simple majority voting rules. In the former case the probability of being crucial is very small for any member while it is quite high in the simple majority. This may be explained by the phenomenon that when unanimity is required, decision making is far from being easy and therefore the status quo is very likely to be the outcome of such a vote (see Widgrén 1994a). Table 1 also shows that the requirement of at least 10 members in a qualified majority increases small states' power. The reverse holds for large states. The explanation of this phenomenon is the following. Because of this "minimum number" requirement, coalitions of 7-9 countries are no longer winning. Since these coalitions are mainly formed by large countries, small countries gain power from the minimum requirement, while large states lose.

In order to compare the distribution of power between members, the normalization of these indices seems more appropriate. Table 2 gives the current normalized Banzhaf indices within the Council of Ministers.

[Insert Table 2]

It can be seen that the distribution of power is the same when either unanimity or a simple majority is required. Both rules are symmetric and thus treat all countries equally. In the qualified majorities, however, the power share increases with the state's population. Large states receive a bigger share in the qualified majorities than when symmetric rules are used. The opposite holds for medium-sized and small countries.

5.2 Are the current allocations of power fair?

The "One Man, One Vote" principle is obviously fulfilled when a simple majority or unanimity is used.[5] The underlying definition of the EU is thus clearly an association of states. Therefore this analysis will now focus on the qualified majorities. Table 3 reports the difference between the fair power indices and the current indices in the two following cases:

1. the EU is a single state: application of the "One Man, One Vote" principle ($x = 1$).
2. the EU is a federal state: the average between the two extreme principles ($x = 1/2$).

[Insert Table 3]

5.2.1 The case of a single state

The following remarks could be made on the basis of the calculations. If a qualified majority is required, the current index gives excessive power to most medium-sized countries and to all small states (from Greece to Luxembourg). Large states and the Netherlands have less power than they should be entitled to. Luxembourg's power, as well as that of

[5]As far as fairness is concerned, the simple majority of member states and the unanimity rules are equivalent since they are symmetric. The difference between the two procedures lies in the probability of making a decision which is higher in the simple majority case, as mentioned above. That is why a simple majority contradicts with the usual idea of an association of states where member countries have a right to veto.

most medium-sized countries (Greece, Belgium and Portugal), is totally excessive compared with their population. On the contrary Germany is notably under-represented: in particular Germany's number of votes has not been adjusted to the new population figures after German unification. For Denmark and Finland the current index seems to fit quite well. Table 3 also shows that the distortions significantly increase when there is a requirement concerning the minimal size of a winning coalition, except for the Netherlands, whose current index is then close to being fair.

5.2.2 The case of a federal state

As Table 3 shows, the current distribution gives an unfairly great amount of power to large states while small states suffer from a lack of power. Medium-sized countries have more or less a fair share of power, except the Netherlands, which has less than the fair share. If a minimum number of members is required, the distortions are smaller with the exception of Greece, Belgium and Portugal; in which case Germany receives less power than it should.

Figure 2a (for the qualified majority) and 2b (for the qualified majority requiring a minimum number of members) present the results for the different degrees of federalistic definitions of the EU (x, the weight given to the "One Man, One Vote" principle varying on a range between 0 and 1).

[Insert Figure 2a and Figure 2b]

On the basis of these figures, we may argue that the current distribution of votes is not fair when the two versions of qualified majorities are used and regardless of the definition of the EU is considered. The current distribution of votes does not, however, systematically give excessive power to some small states. The favored and disfavored states depend on the chosen definition of the EU. Nevertheless the Netherlands are always under-represented (in a qualified majority).

5.3 To which definition of the EU do the current indices correspond the best?

Unanimity and simple majority processes fit perfectly with the definition of the EU as an association of states. On the other hand, qualified majorities do not correspond to a particular conception of the EU. This means that the weight that the current distribution of power gives to the "One Man, One Vote" principle varies according to the state, which is illustrated in Figure 3.

[Insert figure3]

It is also shown that for large and small states more importance is currently accorded to the "One Man, One Vote" principle, while the reverse holds for most medium-sized countries (Greece,Belgium and Portugal). As previously mentioned the Netherlands' power index does not correspond to any of the weighted averages of the two extreme principles. Thus in a strict sense there exists no consistent definition of the EU that would give rise to the current distribution of power.[6]

It may be interesting, however, to determine which definition of the EU would be best reflected by the current qualified majorities processes. This can be done by estimating simple regressions where the current index is the dependent variable and the square root share of the population is the independent variable. Then the regression coefficient shows directly the share of the "One State, One Vote" principle. For the ordinary qualified majority rule we obtain the following equation[7]

$$\beta_j = \underset{(0.037)}{0.79} \cdot \frac{\sqrt{m_j}}{\sum_{k \in N} \sqrt{m_k}} + \underset{(0.003)}{0.014} \tag{9}$$

$$R^2 = 0.97$$

and for qualified majority with the minimum requirement of member states we obtain

$$\beta_j = \underset{(0.029)}{0.60} \cdot \frac{\sqrt{m_j}}{\sum_{k \in N} \sqrt{m_k}} + \underset{(0.002)}{0.026} \tag{10}$$

$$R^2 = 0.97$$

Thus the respective estimated weights for the "One Man One Vote" principle are 0.79 for the qualified majority and 0.60 for the qualified majority with a number of members requirement. Table 4 presents the difference between the current index and the fair index for these weights, i.e. the residuals of the regressions in (9) and (10).

[Insert Table 4]

Given the high values of R^2 we could expect small residuals and not any systematics in their behavior. The absolute values are, indeed, quite small and one cannot find systematic

[6]It is worth noting that the Netherlands has not always been underrepresented [see Laruelle (1994)]. This under-representation can be explained by the phenomenon that her population has quite significantly increased during her membership in the EU.

[7]The standard errors of the regression coefficients are shown in parentheses.

fluctuation in their values.[8] It can be seen, however, that the relative differences are far from being negligible. Compared to the fair distribution of voting power, Germany is still largely under-represented, which is also the case in relative terms for Denmark and Finland, while most medium-sized states (Greece, Belgium and Portugal) have too much voting power. This still emphasizes that the current distribution of votes under the qualified majorities is inappropriate to ensure a fair distribution of power among the member states.

6 Searching for fair allocation of power

The previous sections have shown that the current distribution of votes is not fair, in the sense that it does not ensure a fair distribution of power. Then the next question to pose is whether there exists a fair distribution of votes. In other words, given there exists a distribution of power considered to be fair, is there a distribution of votes that gives rise to this distribution of power? In this section we try to give an empirical answer to this question. We propose an iterative process to derive voting weights which lead to a fair allocation of power.

Let β_{fair} be the fair distribution of power. We start by computing the Banzhaf index obtained when the fair distribution of power is used as the distribution of votes: we compute $\beta(\omega_{(0)})$, the Banzhaf index where $\omega_{(0)} = \beta_{fair}$. The derivative power index fits better to the fair distribution of power than the current index. Then we derive the power coefficient, defined as the ratio between the corresponding Banzhaf index and the fair distribution of power.

$$r_0 = \frac{\beta_{(\omega_{(0)})}}{\beta_{fair}} = \frac{\beta_{(\omega_{(0)})}}{\omega_{(0)}} \tag{11}$$

The comparison of this coefficient with the unit vector indicates which countries are over-endowed ($r_0 > 1$) and which countries are under-endowed ($r_0 < 1$). We use this information to derive a new distribution of votes, which is the fair distribution of power divided by the power coefficient.

$$\omega_{(1)} = \frac{\omega_{(0)}}{r_0} = \frac{\omega_{(0)}}{\beta_{(\omega_{(0)})}/\omega_{(0)}} = \frac{(\omega_{(0)})^2}{\beta(\omega_{(0)})} \tag{12}$$

The corresponding Banzhaf index ($\beta(\omega_{(1)})$) is still closer to the fair distribution of power. In other words, the corresponding power coefficient (r_1) is closer to 1. The distribution of

[8] So in statistical sense we may argue that the current distribution of vote could make sense when a weight of 0.79 is given to the "One Man, One Vote" principle.

votes has thus been improved by the power coefficient.

$$r_1 = \frac{\beta_{(\omega_{(1)})}}{\beta_{fair}} = \frac{\beta_{(\omega_{(1)})}}{\omega_{(0)}} \tag{13}$$

This suggests an iterative process in order to achieve the fair distribution of power where the second step is:

$$\omega_{(2)} = \frac{\omega_{(1)}}{r_1} = \frac{\omega_{(1)}}{\beta(\omega_{(1)})/\omega_{(0)}} = \frac{(\omega_{(0)})^3}{\beta(\omega_{(0)})\beta(\omega_{(1)})} \tag{14}$$

Generally the kth step can be written

$$\omega_{(k)} = \frac{\omega_{(k-1)}}{r_{k-1}} = \frac{(\omega_{(0)})^{k+1}}{\prod_{i=0}^{k-1}\beta(\omega_{(i)})} \tag{15}$$

$$r_k = \frac{\beta_{(\omega_{(k)})}}{\beta_{fair}} \tag{16}$$

Basically the aim of equations (11)-(16) is that the kth vector of lagged power coefficient converges to the unit vector.

The special case is a symmetric game where the fair distribution of votes and power are equal. In this case the zero order lagged power coefficient is a unit vector. So if the EU is defined as an association of states, an equal distribution of votes ensures an equal distribution of power. For the other possible definitions of the EU the above-described iterative process must be applied.

In this section we still focus on three possible definitions:
1) the EU is a state: $x = 1$
2) the EU is a federal state: $x = 1/2$
3) the EU is a modified federal state: $x = 0.79$
Table 5 reports the calculation of the iteration towards the fair distribution of power in the case of a state and compare this with countries' population shares.

[Insert Table 5]

From Table 5 we see that r_0 varies on a range from 0.872 to 1.095 . Thus the differences are quite moderate in the EU Council game. It can also be seen that the convergence towards the fair allocation of power is fast and that only two adjustments are required when we start from the given fair allocation of votes. The comparison between the population share and the fair distribution of votes shows that large countries should have a weight less than proportional to their population while the reverse holds for medium-sized and small countries. This result reflects the well-known fact that in weighted voting power tends to exceed voting weight for large voters as the reverse holds for small ones. Table 6 gives the fair distribution of votes depending on the definition taken into account for the EU (in the sense that they lead to the fair distribution of power).

[Insert Table 6]

The results shows that the fair distribution of votes depends on the definition of the EU and that it cannot be asserted that small countries have an excessive share in the current distribution of votes. It can be mentioned, however, that the Netherlands has too small a share.

7 Conclusion

This paper has provided a game theoretic framework to analyze fairness within the EU Council of Ministers. To assess fairness within a decision-making process, power indices are more appropriate measures for voters influence than voting weights. It is commonly observed that voters with higher shares of votes tend to have more power relative to their voting share than small ones. Thus the argument of the large states under-representation, based on the distribution of votes, is ill-founded.

The quantitative results of this paper are the following: what can be considered as a fair distribution of power among states depends on how the EU is defined. The unanimity and simple majority processes can be evaluated being fair if the EU is defined as an association of states. The qualified majorities do not lead to a fair distribution of power whether the EU is defined as a single state, or as an association of states or as a federal state (whatever the degree of federalism is). Moreover the Netherlands is always under-represented when the qualified majority is used. Thus a redistribution of the votes is necessary. It cannot be argued, however, that the current voting process has a systematic bias in favor of some states, in that the definition of EU cannot be made precise.

This analysis suggests the following two policy recommendations. First, with regard to the current distribution of votes there are no reasonable fairness arguments to concentrate the allocation of votes towards the largest member states. Second, the necessary redistribution of the votes among the states requires first an agreement concerning the definition of the EU that should be reflected in the distribution of power. Seemingly such an agreement would be very difficult to reach, but it is the only way to guarantee a fair distribution of power among the states.

The paper has also provided a method by which fair power allocations can be obtained. The results show that the fair distribution is rather easy to reach with an iterative process by taking the fair allocation as the base in terms of the distribution of votes. Detailed analysis of this method and the study between the weights and the voting power are left for future research.

A Appendix

Fair Banzhaf index

Let C denotes a individual citizen's probability of being crucial. Hence

$$C = \widetilde{\alpha}_{1(1)}(u_\alpha) = ... = \widetilde{\alpha}_{m_n\ (n)}(u_\alpha). \tag{17}$$

The computation of $\widetilde{\beta}_i(y_j)$ will allow us to derive $\widetilde{\alpha_j}(v_\alpha)$ from (6), which can be rewritten as

$$\widetilde{\alpha_j}(v_\alpha) = \frac{C}{\widetilde{\beta}_i(y_j)} \tag{18}$$

Banzhaf index in the government election games

The games (M_j, y_j) $(j = 1, ..., n)$ are symmetric since they have been defined as simple majority games. Moreover a player is only crucial in coalitions whose size is such that $s > m_j/2$ and $s - 1 \leq m_j/2$. Hence $s = \frac{m_j+1}{2}$ if m_j is odd while $s = \frac{m_j}{2}$ if m_j is even.

- if m_j is odd:

By definition of the non-normalized Banzhaf index, we obtain

$$\begin{aligned}
\widetilde{\beta}_i(y_j) &= \frac{1}{2^{m_j-1}} \begin{pmatrix} \frac{m_j-1}{2} \\ \\ m_j-1 \end{pmatrix} \\
&= \frac{1}{2^{m_j-1}} . \frac{(m_j-1)!}{\left(\frac{m_j-1}{2}\right)! \left(\frac{m_j-1}{2}\right)!}
\end{aligned}$$

By Stirling's approximation[9], we get

$$\widetilde{\beta}_i(y_j) = \frac{\sqrt{2.\pi.(m_j-1)}\ (m_j-1)^{m_j-1}\ e^{\frac{m_j-1}{2}}\ e^{\frac{m_j-1}{2}}}{2^{m_j-1}\ e^{m_j-1}\ \sqrt{2.\pi.\left(\frac{m_j-1}{2}\right)}\sqrt{2.\pi.\left(\frac{m_j-1}{2}\right)}\ \left(\frac{m_j-1}{2}\right)^{\frac{m_j-1}{2}}\ \left(\frac{m_j-1}{2}\right)^{\frac{m_j-1}{2}}}$$

[9]if n is sufficiently large, the following holds:

$$n! \simeq \sqrt{2.\pi.n}\, n^n\ e^{-n}$$

Since m_j is sufficiently large, this expression may be reduced to

$$\widetilde{\beta}_i(y_j) = \frac{\sqrt{2}}{\sqrt{\pi}\sqrt{m_j}} \tag{19}$$

- if m_j is even

By definition of the non-normalized Banzhaf index, we obtain

$$\begin{aligned}
\widetilde{\beta}_i(y_j) &= \frac{1}{2^{m_j-1}} \begin{pmatrix} \frac{m_j}{2} \\ \\ m_j-1 \end{pmatrix} \\
&= \frac{1}{2^{m_j-1}} . \frac{(m_j-1)!}{\left(\frac{m_j}{2}\right)! \left(\frac{m_j-2}{2}\right)!}
\end{aligned}$$

By Stirling's approximation, we get

$$\begin{aligned}
\widetilde{\beta}_i(y_j) &= \frac{\sqrt{2.\pi.(m_j-1)}\ (m_j-1)^{m_j-1}\ e^{\frac{m_j}{2}}\ e^{\frac{m_j-2}{2}}}{2^{m_j-1}\ e^{m_j-1}\ \sqrt{2.\pi.\left(\frac{m_j-2}{2}\right)}\sqrt{2.\pi.\left(\frac{m_j}{2}\right)}\ \left(\frac{m_j-2}{2}\right)^{\frac{m_j-2}{2}}\ \left(\frac{m_j}{2}\right)^{\frac{m_j}{2}}} \\
&= \frac{\sqrt{2}}{\sqrt{\pi}} . \frac{\sqrt{m_j-1}}{\sqrt{m_j-2}\sqrt{m_j}} . \frac{(m_j-1)^{m_j-1}}{\left(\frac{m_j-2}{2}\right)^{\frac{m_j-2}{2}}\ \left(\frac{m_j}{2}\right)^{\frac{m_j}{2}}}
\end{aligned}$$

Since m_j is sufficiently large, this expression may be approximated by

$$\widetilde{\beta}_i(y_j) = \frac{\sqrt{2}}{\sqrt{\pi}\sqrt{m_j}} \tag{20}$$

Fair Banzhaf index in the Council of Ministers

Using (19) or (20) in (18), we obtain

$$\widetilde{\alpha_j}(v_\alpha) = \frac{C.\sqrt{\pi}\sqrt{m_j}}{\sqrt{2}}$$

Thus

$$\alpha_j\left(v_\alpha\right) = \frac{\sqrt{m_j}}{\sum_{k\ \in\ N}\sqrt{m_k}} \tag{7}$$

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	U	SM	QM	QM+
Germany	0.000061	0.209	0.113	0.098
United Kingdom	0.000061	0.209	0.113	0.098
France	0.000061	0.209	0.113	0.098
Italy	0.000061	0.209	0.113	0.098
Spain	0.000061	0.209	0.093	0.083
Netherlands	0.000061	0.209	0.059	0.059
Greece	0.000061	0.209	0.059	0.059
Belgium	0.000061	0.209	0.059	0.059
Portugal	0.000061	0.209	0.059	0.059
Sweden	0.000061	0.209	0.048	0.051
Austria	0.000061	0.209	0.048	0.051
Denmark	0.000061	0.209	0.036	0.042
Finland	0.000061	0.209	0.036	0.042
Ireland	0.000061	0.209	0.036	0.042
Luxembourg	0.000061	0.209	0.023	0.030

Table 1: Current non-normalized Banzhaf indices for unanimity (U), a simple majority(SM), a qualified majority (QM) and a qualified majority with at least ten countries (QM+).

	U	SM	QM	QM+
Germany	0.067	0.067	0.112	0.101
United Kingdom	0.067	0.067	0.112	0.101
France	0.067	0.067	0.112	0.101
Italy	0.067	0.067	0.112	0.101
Spain	0.067	0.067	0.092	0.085
Netherlands	0.067	0.067	0.059	0.061
Greece	0.067	0.067	0.059	0.061
Belgium	0.067	0.067	0.059	0.061
portugal	0.067	0.067	0.059	0.061
Sweden	0.067	0.067	0.048	0.052
Austria	0.067	0.067	0.048	0.052
Denmark	0.067	0.067	0.036	0.043
Finland	0.067	0.067	0.036	0.043
Ireland	0.067	0.067	0.036	0.043
Luxembourg	0.067	0.067	0.023	0.031

Table 2: Current normalized Banzhaf indices for unanimity (U), a simple majority(SM), a qualified majority (QM) and a qualified majority with at least ten countries (QM+).

	QM (x=1)	QM+(x=1)	QM (x=.5)	QM+ (x=.5)
Germany	-0.028	-0.039	+0.009	-0.002
United Kingdom	-0.007	-0.018	+0.019	+0.008
France	-0.006	-0.017	+0.020	+0.009
Italy	-0.006	-0.017	+0.020	+0.009
Spain	-0.005	-0.012	+0.010	+0.003
Netherlands	-0.002	+0.0001	-0.005	-0.003
Greece	+0.009	+0.011	+0.0003	+0.003
Belgium	+0.009	+0.012	+0.0007	+0.003
Portugal	+0.010	+0.012	+0.0009	+0.003
Sweden	+0.002	+0.006	-0.008	-0.004
Austria	+0.004	+0.008	-0.007	-0.003
Denmark	+0.0005	+0.008	-0.015	-0.008
Finland	+0.0009	+0.008	-0.015	-0.008
Ireland	+0.007	+0.014	-0.012	-0.005
Luxembourg	+0.013	+0.021	-0.016	-0.007

Table 3: The difference between the current index and the fair one when the EU is defined as a state ($x = 1$) or as a federal state ($x = 0.5$) for a qualified majority (QM) and a qualified majority with at least ten countries (QM+).

	QM	%	QM+	%
Germany	-0.013	-10	-0.010	-8.7
United Kingdom	+0.004	+3.8	+0.003	+3.3
France	+0.004	+4.1	+0.003	+3.5
Italy	+0.005	+4.5	+0.004	+3.9
Spain	+0.001	+1.6	+0.0004	+0.4
Netherlands	-0.003	-5.4	-0.002	-3.4
Greece	+0.005	+9.8	+0.004	+7.6
Belgium	+0.006	+11	+0.005	+8.4
Portugal	+0.006	+12	+0.005	+9
Sweden	-0.002	-4.6	-0.002	-3.6
Austria	-0.0007	-1.5	-0.0007	-1.4
Denmark	-0.006	-14	-0.005	-9.8
Finland	-0.006	-13	-0.004	-9.4
Ireland	-0.001	-3	-0.001	-2.4
Luxembourg	+0.001	+5.3	-0.001	-4.5

Table 4: The absolute and relative error terms for equations (9) and (10).

	% *pop*	$w_{(0)}$	$\beta(w_{(0)})$	r_0	$w_{(2)}$	$\beta(w_{(2)})$	r_2	$\beta(w_{(2)})/w_{(2)}$
Germany	0.219	0.140	0.153	1.095	0.129	0.140	0.999	1.079
United Kingdom	0.157	0.119	0.124	1.049	0.113	0.119	1.000	1.045
France	0.156	0.118	0.124	1.048	0.113	0.118	1.000	1.044
Italy	0.155	0.118	0.123	1.047	0.113	0.118	1.000	1.043
Spain	0.106	0.097	0.098	1.008	0.096	0.097	1.000	1.011
Netherlands	0.041	0.061	0.058	0.947	0.064	0.061	1.000	0.955
Greece	0.028	0.050	0.047	0.931	0.053	0.050	1.000	0.938
Belgium	0.027	0.049	0.046	0.930	0.053	0.049	1.000	0.937
Portugal	0.027	0.049	0.045	0.929	0.052	0.049	1.000	0.936
Sweden	0.024	0.046	0.042	0.924	0.049	0.046	1.000	0.932
Austria	0.022	0.044	0.041	0.921	0.047	0.044	1.000	0.929
Denmark	0.014	0.035	0.032	0.909	0.039	0.035	1.000	0.915
Finland	0.014	0.035	0.032	0.908	0.038	0.035	1.000	0.915
Ireland	0.010	0.029	0.026	0.900	0.032	0.029	1.000	0.906
Luxembourg	0.001	0.010	0.009	0.872	0.011	0.010	1.000	0.875

Table 5: The iteration towards the fair allocation of power when the EU is defined as a state.

	current	x=0.5	x=0.79	x=1
Germany	0.115	0.098	0.117	0.129
United Kingdom	0.115	0.090	0.103	0.113
France	0.115	0.089	0.103	0.113
Italy	0.115	0.089	0.103	0.113
Spain	0.092	0.081	0.090	0.096
Netherlands	0.057	0.065	0.064	0.064
Greece	0.057	0.060	0.056	0.053
Belgium	0.057	0.059	0.055	0.053
Portugal	0.057	0.059	0.055	0.052
Sweden	0.046	0.058	0.053	0.049
Austria	0.046	0.057	0.051	0.047
Denmark	0.034	0.053	0.045	0.039
Finland	0.034	0.053	0.044	0.038
Ireland	0.034	0.050	0.040	0.032
Luxembourg	0.023	0.041	0.024	0.011

Table 6: The current distribution of votes and the shares of votes that lead to the fair distribution of power for three possible definitions of the EU.

Figure 1 : Fair power indices (depending on the weight accorded to the "One Man, One Vote" principle)

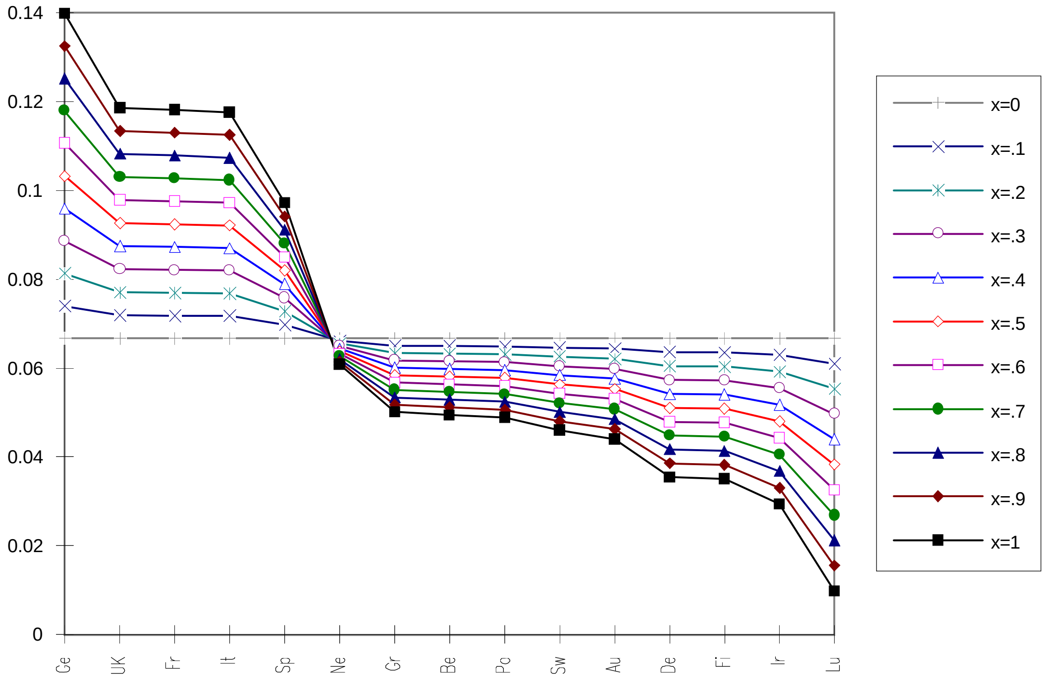

Figure 2a : Difference between the current index and the fair index (depending on the weight accorded to the "One Man, One Vote" principle) for the qualified majority

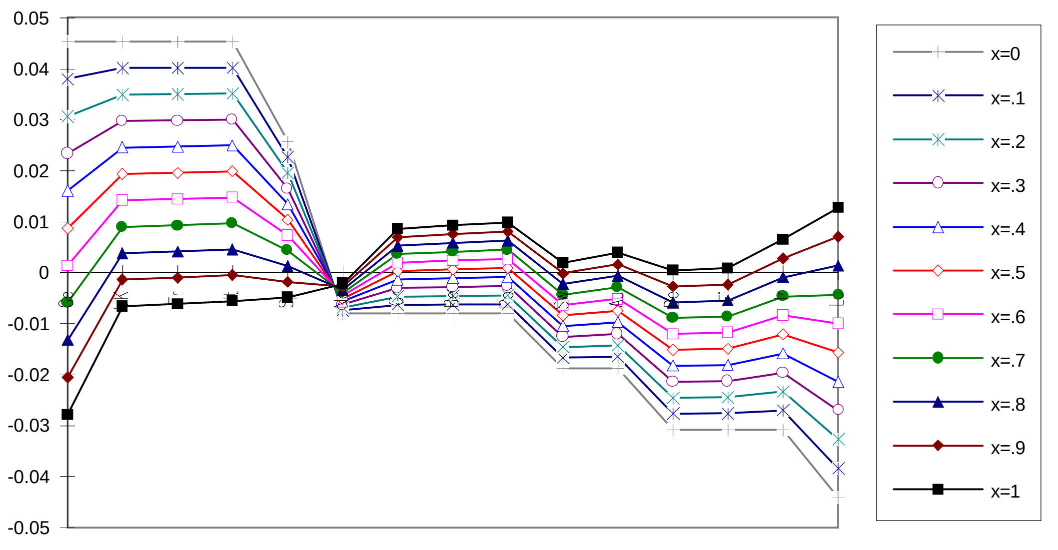

Figure 2b : Difference between the current index and the fair index (depending on the weight accorded to the "One Man, One Vote" principle) for the qualified majority +

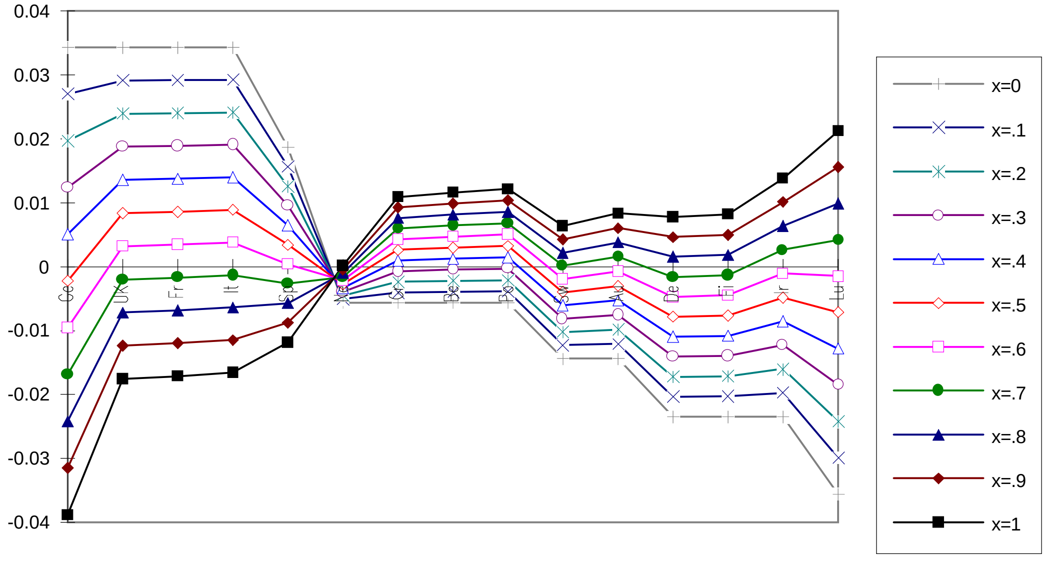

Figure 3 : Weight that the current indices give to the "One Man, One Vote" principle for each state (in the qualified majorities)