Efficient Wage Subsidies in Private Firms and Deadweight Spending

PICARD P.M
I.R.E.S. U.C.L. (Belgium)

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Abstract

This paper links the old literature on employment subsidies with the current theories of contract and regulation. One important source of inefficiency of employment subsidies is deadweight spending. This refers to the cases in which private firms receive a subsidy for employment creation that would have been reached without this financial support. We identify the asymmetry of information between the regulator and the private firm as the source of this deadweight spending. The private firm knows its hiring capability while the government does not. We then derive the optimal incentive contracts for the regulator. In these contracts deadweight spending is reduced to the information rent to private firms. We derive conditions for which this deadweight spending is zero.

Résumé

Dans cet article, nous entreprenons de relier la littérature sur les subsides à l’emploi aux théories de la régulation et des contrats. Une importante source d’inefficience lors de l’octroi des subsides à l’emploi est l’effet d’aubaine. Cet effet apparaît dès que des firmes privées reçoivent un subside pour des emplois qui auraient été créés sans ce subside. Nous identifions la source de cet effet dans l’asymétrie d’information entre le gouvernement et la firme. Une firme privée est capable d’évaluer sa capacité d’embauche alors que le gouvernement n’a pas accès à cette information. En construisant les subsides à l’emploi comme des contrats incitatifs, nous réduisons l’effet d’aubaine au niveau d’une rente d’information laissée aux firmes privées par le gouvernement. Nous dérivons enfin les conditions pour lesquelles cette rente peut être éliminée.

Picard Pierre
IRES UCL,
3, pl. Montesquieu,
1348 Louvain-La-Neuve,
Belgium
email : picard@ires.ucl.ac.be

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1 Introduction

The persistence of unemployment in modern economies leads governments to intervene in the labour market. A significant part of these interventions is made on the demand side of the labour market in the form of employment subsidies to private firms. Actually, this subsidy seems increasingly justified since, as Phelps (1994) and Snower (1994) explain, it is a means to intervene on the direct cause of unemployment while avoiding the harmful effect of the welfare system.

A first discussion on employment subsidies is proposed by Kaldor (1976) who analyses the macroeconomic implications of a general employment subsidy. This type of subsidy reduces the labour cost of each worker in each firm. It appears to be very costly since it is paid for jobs that already exist in firms. To reduce this effect, Layard & Nickell (1980) propose a marginal employment subsidy (MES). This subsidy is paid only for new job creations. As it applies only to a small fraction of the overall work force, the MES leads to a substantial saving in public outlay.

The Layard & Nickell’s approach does not consider the long run and dynamic aspects. However, Luskin (1986) shows that the long run impact of the MES is weaker than what Layard & Nickell present. Indeed as the private firms make no profit in the long run, their average costs equal their product price. Since the MES has a significant impact on the marginal cost but not on the average cost, its effect on output and employment remains very weak in the long run. Steinberr (1985) & Michel (1990) analyze the dynamic implications of the MES. They show how the private firms will increase the job turnover in order to benefit more from the MES. However the benefit from this type of subsidy is still substantial when compared to the general subsidy introduced by Kaldor.

Beside these criticisms, the issue of deadweight spending is questioned by Layard and Nickell. As they conclude in their paper, "the performance of the marginal employment subsidy depends on the size of the deadweight spending to jobs that would have been provided anyway" (p.67). Deadweight spending is also called windfall gain or non-additional spending (H.M. Treasury (1988, p.26)). In fact the deadweight spending is measured as the part of subsidy that is not strictly required to make the firm hire a new employee: this employee would have been hired with a lower subsidy. The complement of deadweight spending is the transfer with no deadweight or minimal transfer. This is the smallest transfer with which the regulator is able to attain a desired level of employment: this could not have been achieved without the subsidy. From another point of view, the transfer with no deadweight corresponds to the contract that makes the firm indifferent to the "null contract" in which no transfer is provided and no employment increase is obtained. Any unit of transfer beyond the minimal transfer is a deadweight transfer.

The problem for the government and its agency is to avoid the deadweight spending. But is this administratively possible? “Unfortunately not”, Layard and Nickell reply. “For it requires that the government can get firms to report truthfully what their employment would have been in the absence of the subsidy” (p.52). The source of the deadweight spending must be found in the asymmetry of information between the private firm. The government agency is not able to select the private firms and the government since the latter does not know the true hiring capabilities of private firms.

Economic researches on subsidies usually focus on the full information setting. For instance, Holden & Swales (1993) assess the efficiency of this relationship in a full information setting. They relate the cost-per-job with the characteristics of the technology and the demand for the produced good. In this full information setting there is no deadweight spending. However, in the real regulatory situation, the government or its agency has neither the legitimacy nor the time to audit each private firm in order to assess accurately its true hiring capability.

The government must set up institutions or devices that induce the private firms to reveal their true hiring capabilities. In this paper we construct employment subsidies as incentive contracts that maximize the government’s socio-economic objective under the constraint that the firms reveal their true hiring capabilities. Such incentive contracts enable the government to reduce the deadweight spending. In some particular situations it will even be eliminated. Incentive contracts are often
used in the theory of regulation. In this sense, the current paper links the old literature on employment subsidies to the current theories of regulation and contract.

In the present paper we mainly extend the general employment subsidy proposed by Kaldor to the context of asymmetric information. The government never observes the level of employment before the date of application of the subsidy. He is then unable to infer the increase in employment due to the subsidy. He must pay the firm on the basis of total employment as in the general employment subsidy. We devote however small extension on the MES introduced by Layard & Nickell. The government can then observe ex-ante the level of employment in the firm. He bases its incentive contracts on increase in employment. We however focus on the first extension for two reasons: firstly, we believe in Luskin’s argument that MES scheme is not efficient in the long run and secondly we find that ex-ante observation introduces strategic behaviors on the firm’s part that are out of the scope of this work.

Whereas the present paper focuses on the problem of deadweight spending, three other effects must be assessed to measure the actual impact of employment subsidies. There are the substitution effect that can take place between subsidized and non-subsidized workers inside firms (Van der Linden (1995)) and the activity displacement effect that occurs between subsidized and non-subsidized firms operating in the same industry (Holden & Swales (1995)). Finally there is the effect of non-additional employment from which deadweight spending should be distinguished. Non-additional employment concerns the number of new jobs that would have been created without the subsidy. Deadweight spending relates to the part of subsidy that is not strictly required to make the firm hire new employees.

In the static (one shot) analysis developed here, the issue of non-additional employment is not relevant. Any job creation is indeed additional provided that the private firm is at equilibrium. If a job creation were non-additional, the firm could have created the job without any transfer from the government. However, why didn’t the firm create the job while it was optimal to do so at equilibrium? This is a contradiction. In fact, employment non-additionality becomes relevant only in a dynamic or strategic framework. The paper will focus only on deadweight spending.

Deadweight spending and employment non-additionality are important effects. Three studies by Hamermesh (1978), Foley (1992) and Van der Linden (1995) show the very large proportion of non-additional jobs against the total jobs created by various subsidies - 50 to 80 percent - during the last decade. Deadweight spending is in the same proportion. Being aware of the importance of deadweight spending we can now explain the model and the results of the paper.

In this paper we consider an economy characterized by involuntary unemployment due to labour market imperfections. Formally we allow for these imperfections by introducing a wedge between the market wage and the socially optimal shadow wage. The government and its agency is modeled as a regulator that is in charge to promote employment in private firms. He takes maximizes the expected social welfare that takes into account the consumer and producer surpluses, as well as the social cost of public funding and the welfare valuation of employment through the wage wedge (Drèze & Stern (1987)).

In this partial equilibrium analysis, the private firm adjusts its level of employment as to maximize its profit. All other factors of production are exogenously fixed to the firm. The regulator is ignorant about the productivity parameter of the firms applying to the subsidy. This introduces an asymmetry of information between the private firm and the regulator as in the models of Fudenberg & Tirole (1993). A main difference is that the reservation profits - i.e. the minimum profits for the firms to participate into the subsidy scheme - vary with the productivity parameter of the firm.

We show how the problem of deadweight spending is related to the concept of asymmetric information. When the regulator introduces incentive contracts to promote employment, the deadweight spending is minimized and equals the information rent, i.e. the part of transfer granted to the firm to reveal its true information. In some situations this rent may even be zero. We characterize one class of contracts where the deadweight spending is equal to the expected information rent and
two other classes where no information rents are required so that there is no deadweight spending.

In our analysis, the firms with the highest hiring capabilities benefit from the information rents and are the cause of deadweight spending. The firms with the lowest capability are disregarded by the regulator. The latter is not willing to promote the employment in these firms as he prefers to pay for the firm with the highest hiring capability.

In the hypotheses of our model, the firms with the highest hiring capability are the most competitive firms in their product markets. When the regulator applies the prescribed incentive contracts, he indirectly generates some pick-the-winner strategy by overpayment for these market stars. Incentive contracts are, however, intrinsically different from this type of strategy. In the pick-the-winner strategy, the regulator has complete information about each firm’s characteristics and deliberately chooses the market stars. Within our model, it is the existence of asymmetric information that makes him reward the winner with an extra-profit - the information rent.

Note that this paper shows that in the context of asymmetric information deadweight spending is not synonymous to inefficiency. Indeed, the cases in which the regulator pays an information rent and therefore a deadweight, is constrained efficient given the information at the disposal of the regulator.

The paper is structured as follows: Section 2 sets the assumptions of the model. Section 3 describes to the full information case. Section 4 proceeds with the construction of incentive contracts in the asymmetric information context. It discusses cases for which the full information contract can still be attained and pinpoints the information rent as the source of the deadweight transfer. The results are collected in three classes of contracts and an example is provided. The Section 4 considers the case of an ex-ante observation by the regulator and the last section suggests the possible extensions to our model.

2 The Model

2.1 The Private Firm

In the real world, the information about the private firm’s hiring capability is hidden to the regulator. The private firm has usually better knowledge about its parameters of production than the regulator, it is more able to assess its hiring capability. When it is the case we say that there is an asymmetry of information between the firm and the regulator.

In this text we will make four assumptions on this asymmetric information. First, it can be summarized in one dimension, that we call the firm’s type $\theta$. Secondly, there are two realizations of this type, $(\theta_l, \theta_h)$ where $\theta_l$ and $\theta_h$ are the lowest and highest types: $\theta_l < \theta_h$. Thirdly, the type $\theta$ will be restricted to describe some parameter of the production technology so that the production function depends on the type $\theta$ and the level of employment $L$: $Q = Q(\theta, L)$. The function $Q(\theta, L)$ is increasing and strictly concave in $L$. We assume finally that the type $\theta$ ranks the production schedule positively:

$$\frac{\partial}{\partial \theta} Q(\theta, L) > 0 \quad (1)$$

The current paper focuses on the distortion introduced by the labour market. To focus on this distortion, we assume that the private firm is perfectly competitive and sells its product $Q$ at a price $P$ fixed by the product market. The revenue from sales is then $P Q(\theta, L)$.

The firm has a homogeneous work force (same skills, same weekly work hours per employee) and the wage $w$ is exogenously fixed in the labour market. The private firm’s profit has the following form:
\[ \Pi = P Q(\theta, L) - wL + t \]  

(2)

where the right hand side terms denote the revenue \( P Q(\theta, L) \), the cost of labour \( wL \) and the transfer \( t \) granted by the regulator. By the hypothesis on \( Q(\theta, L) \), the revenue is concave in \( L \).\(^1\)

Finally we make use of the Single Crossing Property for the firm’s profit: \( \frac{\partial}{\partial \theta} \frac{\Pi}{\Pi_t} > 0 \). As the profit is linear in \( t \) and the wage \( w \) is fixed, this yields:

\[ \frac{\partial^2}{\partial L \partial \theta} PQ(\theta, L) > 0 \forall L \]  

(3)

With this hypothesis\(^2\), the parameter \( \theta \) must be chosen such that it monotonically ranks the values of each firm’s marginal revenues of labour (or returns to scale of labour). The higher the type, the higher the marginal revenue. Examples of such kinds of asymmetric information are the marginal productivity and the staff performance.

When the firm is free from regulation, it receives no transfer and maximizes its profit. The following first order condition yields the level of employment \( L^F \):

\[ PQ_L(\theta, L^F) = w \]

By the strict concavity of the revenue, this yields a unique maximum at the level of employment \( L^F(\theta) \), and the profit, \( \Pi^F(\theta) \) that depend on the firm’s type \( \theta \). For conciseness we will denote these values as the free level of employment and the free profit.

By the hypotheses on production function, the free levels of employment and the free profits are ranked according to the firm’s types: \( L^F(\theta) \) and \( \Pi^F(\theta) \) increase with \( \theta \). So, with two types, \( L^F_l < L^F_h \) and \( \Pi^F_l < \Pi^F_h \). This property is important as the free profit levels will be used to determine the reservation value of the firm when it makes its decision to participate to the subsidy scheme.

2.2 The Regulator

The regulator operates in an economy with involuntary unemployment caused by distortions in the labour market. To model this we assume that the market wage \( w \) is larger than some shadow wage \( w_s \). This justifies the existence of an excess supply of labour. The intensity of this distortion is denoted by \( \frac{w}{w_s} \geq 1 \).

Suppose for the time being that the regulator can distinguish each type of firm. This will be relax at the end of this section. His objective is then defined as:

\[ W = PQ(\theta, L) - w_s L - \lambda t \]  

(4)

where the \( PQ(\theta, L) \) denotes the gross consumer surplus provided by the production \( Q(\theta, L) \) in each firm. The group of terms \( PQ(\theta, L) - w_s L(\theta) \) is what we will call the social surplus. It consists of the social surplus that would be reached if the price of labour was set at the shadow wage, \( w_s \). The term \(-\lambda t \) is the welfare loss incurred by a transfer \( t \) from the tax payer to the firm. The social welfare effect of this transfer is negative because of the use of distortionary taxes on

\(^1\)In the following text, we will often discuss the properties of the revenue \( P Q(\theta, L) \) instead of those of the production function \( Q(\theta, L) \). The reason is that our results on competitive firms can be easily extended to monopolistic firms if we use the revenue function.

\(^2\)The following weaker condition is sufficient in this paper: \( (PQ(\theta_h, L_h) - PQ(\theta_h, L_l)) - (PQ(\theta_l, L_h) - PQ(\theta_l, L_l)) > 0 \)
income, capital and consumption to finance the transfer. The parameter $\lambda$ reflects this distortion and is called the shadow cost of public fund.\(^3\) We assume that $\lambda > 0$.

A basic element of this model is the shadow wage $w_s$. It captures the marginal social costs associated with unemployment. For instance it includes the social cost of the workers’ loss of leisure time, the social cost of the distortion in the labour market or the social cost of the unemployment benefit financing.\(^4\) The rigorous estimate of $w_s$ should be addressed with the methodology of Cost and Benefit Analysis (See Drèze & Stern (1987))

We note that, by hypothesis, the consumer surplus $PQ(\theta, L)$ and the social surplus, $PQ(\theta, L) - w_sL(\theta)$, are concave in $L$.

Finally given that the regulator has only some belief ($p, 1-p$) on the possible realization of the firm’s type, its objective is then the expectation of the social welfare.

3 The Full Information Contracts

When he has full information about the type of firm he faces, the regulator is able to propose a distinct optimal contract $(L^*(\theta), t^*(\theta))$ to each type. In this section we characterize this full information contract and we relate it to the issues of deadweight transfers and non constant reservation profits.

For each type, the regulator maximizes its objective given the participation constraint $(PC)$ that imposes a profit above the free profit $\Pi^F$. This gives the following program where the reference to the type $\theta$ can be temporarily discarded:

$$\max_{\{L,t\}} PQ(L) - w_sL - \lambda t$$

s. t. \hspace{1em} $(PC): PQ(L) - wL + t \geq \Pi^F$

Since the transfer $t$ is costly to the regulator and appears linearly in the program, it will be minimized by making the voluntary participation constraint $(PC)$ binding. (Any transfer that does not make this constraint binding can be lowered without modifying the required level of employment. It is a gain for the regulator since the transfer can be lowered while the employment level and thus the social surplus remain constant.) At the full information level of employment $L^*$, the transfer is

$$t^* = \Pi^F - (PQ(L^*) - wL^*) \hspace{2em} (5)$$

It consists of the compensation for the firm’s loss associated with the increased work force, that is the difference between the free profit and the (out of transfer) profit achieved under the contract. The program simplifies to

$$\max_{L} [PQ(L) - w_sL - \lambda (\Pi^F - (PQ(L) - wL))]$$

that yields the following first order condition:

$$PQL(L^*) - w_s = \lambda(w - PQL(L^*)) \hspace{2em} (6)$$

\(^3\)See Atkinson & Stiglitz (1980).
\(^4\)The latter equals $-b\frac{(L-L^*)}{L}$ where $b$ is the unemployment benefit and where $\frac{(L-L^*)}{L}$ is the proportion of unemployed workers $(L-L)$ in the active population $L$. The marginal social cost is therefore captured by $\frac{\lambda b}{L}$
By this equality, the marginal social surplus should equate the marginal cost of the transfer that makes the private firm voluntarily participate. In order to make the firm participate, the regulator is required to compensate for the productivity loss caused by the higher level of employment.$^5$

We can rearrange this equation and reintroduce the type $\theta$ of the firm. So,

$$PQ_L(\theta, L^*) = \frac{w_s}{\lambda} + \frac{\lambda}{1+\lambda} < w$$

This equality shows that, by concavity of $Q(L)$, the full information level of employment $L^*$ is higher than the free level $L^F$. The economic intuition goes as follows. The regulator tries to correct the firm’s perceived wage to the level of the shadow wage $w_s$. It does not however reach that level due to the shadow cost $\lambda$ of financing the transfer $t^*$. $^6$

From this last identity and the previous results, we can draw the following proposition:

**Proposition 1.** The full information contract $(L^*, t^*)$ is the solution of equations (7) and (5). The full information level of employment is $L^* = L^F(w, w_s, \lambda; \theta)$. It decreases with lower $w_s$, with higher $\lambda$ and with lower $\theta$. The transfer involves no deadweight. $^6$

**Proof:** In equation (7), the concavity of $Q(\theta, L)$ implies that any decrease in $w_s$ and $\theta$ should increase $L^*$. The effect of $\lambda$ is opposite. The transfer includes no deadweight since the firm’s profit is always reduced to participation level. The firm is indifferent between participating or not.

Q.E.D

The full information level of employment $L^*$ decreases with higher shadow cost of public fund $\lambda$. A less expensive public financing context decreases the social cost of transfer as compared to the social benefits from employment creation. The regulator’s trade-off will be to increase $L^*$ and to raise slightly this cost. Besides, the level of employment $L^*$ increases with higher intensity of the distortion in the labour market $\frac{w_s}{w}$, higher distortions requiring stronger interventions from the regulator. Finally a higher production parameter $\theta$ will yield higher marginal revenues and thus more job creation in the firm.

The full information allocation can be presented in the following figure. The transfers to the firms, $t$, are represented on the vertical axis, and the levels of employment, $L$, on the horizontal axis. The upward sloping curve, $\Pi^F$, is the iso-profit curve when the firm is free from regulation. As the transfers enter linearly in the profit, the iso-profit curves are invariant to vertical translation. The higher the profit the higher the iso-profit curve.

The regulator is characterized by concave indifference curves such as $W$. The higher the regulator’s objective the lower the curve. The full information level of employment appears at the tangency point $T$ of the two types of curves.

In this full information context, we must notice that the absence of deadweight transfer does not imply social efficiency. As the figure shows, there are two contracts with no deadweight: $(L, t_a)$ and $(L^*, t^*)$. The firm is always indifferent between these contracts and no contract. However, only the full information contract $(L^*, t^*)$ is efficient since it leads to the efficient level of employment.

Unfortunately the full information contract can not be implemented by the regulator. Due to the existence of hidden information in the private firm, he is not able to characterize completely the behavior of the firm. The regulator must build incentive contracts in order to maximize his objective given this informational restriction. This is the purpose of the next section.

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$^5$Since by hypothesis $PQ(\theta, L) - w_sL(\theta)$ and $P Q(\theta, L)$ are concave in $L$, one can readily show that the second order condition is met and that $(w - PQ(\theta, L)) \geq 0$ at $L = L^*$ and thus $L^F < L^*$. 

$^6$This full information allocation is second best. A first best allocation would be achieved when there regulator would place no value on the losses in the private firm. That would mean that $\lambda \rightarrow 0$. So, $PQ(\theta, L^{fb}) = w_s$ and $L^{fb} > L^*$ for any $\lambda > 0$. 


4 Incentive Contracts under asymmetric Information

Suppose now that the regulator is no longer informed about the firm’s type $\theta$. What contracts can he give to the firm in order to maximize his objective? How important will be the induced deadweight transfers? The regulator can propose the following game to the firm:

In this one shot game, the regulator proposes a menu $\{C_i\}$ of two contracts $C_L = (L_l, t_l)$ and $C_H = (L_h, t_h)$ without knowing the true state of nature. Then nature moves by selecting $\theta_l$ or $\theta_h$ with the probabilities $(p, 1 - p)$. The private firm is informed about the selected type and moves in last place; it either selects one of the two contracts or chooses not to participate i.e. it chooses the free profit $\Pi^F$. Finally the contracted levels of employment and transfers are realized.

The regulator maximizes the expected welfare gains subject to the participation constraints ($PC'$) and the incentive compatibility constraints ($IC'$). The latter reflects that one type of firm is enticed not to mimic the other. Consequently it reveals its true type through the choice of the contract. This is an adverse selection problem with the unusual specificity that it includes non constant reservation values $\Pi^F(\theta)$. To our knowledge this specificity has never been discussed in a two types adverse selection model.\textsuperscript{7} The problem is that it may modify the binding incentive compatible constraints. This will be checked in the next sections. The program can be written in the following way:

\textsuperscript{7}See Fudenberg & Tirole (1993) p.263 for more detail and refer to Champsaur & Rochet (1989) for the same specificity with a continuum of types.
Let us define the contract. This will be the class (a).

In this section we will derive the condition that allows the regulator to offer the full information contract. This will be the class (a).

Class a  Let us define the pooling level of employment $L^p$ such that the minimal transfers that makes the firms indifferent to participate are equal for both types. At a level of employment $L$, the minimal transfer to any firm is given by the following expression:

$$t^F_i = \Pi^F_i - (PQ(\theta_i, L_i) - wL_i) \quad i = l, h$$

Equating the minimal transfers $t^F_i = t^F_h$ at $L^p$ yields the following implicit expression:

$$[PQ(\theta_h, L^p) - \Pi^F_h] = [PQ(\theta_i, L^p) - \Pi^F_i] \quad (8)$$

The pooling level of employment is positively correlated to the $\theta_i$ and $\theta_h$. It depends also on $w$ through $\Pi^F_i$ and $\Pi^F_h$. So $L^p = L^p(w; \theta_i, \theta_h)$. 8

We are now able to express the condition for a class (a) contract in the following proposition:

**Proposition 2.** Consider the full information contract $\{(L^*_l, t^*_l), (L^*_h, t^*_h)\}$. This contract solves the program $P^{IC}$ if and only if $L^*_l \leq L^p$. In this situation, there is no deadweight transfer.

**Proof:**

At the full information allocation, the constraint $IC_h$ requires that

$$PQ(\theta_h, L^*_h) - wL^*_h + t^*_h \geq PQ(\theta_h, L^*_l) - wL^*_l + t^*_l$$

Using the expression (5) for the transfers $t^*_l$ and $t^*_h$, we get

$$\Pi^F_h \geq \Pi^F_l + PQ(\theta_i, L^*_l) - PQ(\theta_i, L^*_l)$$

By the definition of $L^p$ this can be transformed in

$$0 \geq (PQ(\theta_h, L^*_l) - PQ(\theta_i, L^*_l)) - (PQ(\theta_h, L^p) - PQ(\theta_i, L^p))$$

By the Single Crossing Property this is satisfied if and only if $L^*_l \leq L^p$.

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8The reader can check that, by the Single Crossing Property, $L^p$ increases as the parameter of productivity $\theta_i$ and $\theta_h$ increase.
At the full information allocation, the constraint $IC_l$ is

$$PQ(\theta_l, L^*_l) - wL_l + t^*_l \geq PQ(\theta_l, L^*_h) - wL^*_h + t^*_h$$

Using the same reasoning, we can reduce it to

$$0 \geq (PQ(\theta_l, L^*_h) - PQ(\theta_h, L^*_h)) - (PQ(\theta_l, L^*_l) - PQ(\theta_h, L^*_l))$$

Again, by the Single Crossing Property this is satisfied only and only if $L^* = L^*_h$. The reader can check that, by construction, $L^*$ takes value in the range $[L^*_f, L^*_h]$. Therefore, $L^* \leq L^*_h$. Finally, since $L^*_h < L^*_h$, we have $L^* \leq L^*_h$ and the constraint is always satisfied.

Q.E.D

This proposition defines the menu of contracts $\{(a_l), (a_h)\}$ that we depict in the following figure. The profit are represented by the iso-profit curves $\Pi^F$ and $\Pi^F_h$. Larger profits will yield higher iso-profit curves. By (1) the iso-profit curve of the lowest type is drawn at low levels of employment $L$ (to the left), while the highest type’s is drawn at high levels (to the right). By definition, the pooling level of employment $L^*$ corresponds to the horizontal projection of the intersection of the iso-profit curves $\Pi^F$ and $\Pi^F_h$.

The free profit iso-curves $\Pi^F_h$ and $\Pi^F_l$ represent also the participation constraints $PC$. They depict the minimal transfers that make the firms indifferent between participating or not to the contracts. As before, the full information allocation is attained at the tangency point of the iso-profit curve $\Pi^F$ and the regulator’s indifference curve $W$.

The proposition tells that if the full information level of employment in the least hiring capable firm, $L^*$, is sufficiently close to its free level $L^*_f$ (see $(a_l)$ in the figure), then the full information contract will be incentive compatible and will respect the participation constraints. This remains true as long as $L^* \leq L^*_f$.

**Corollary 1.** The likelihood of a class (a) economic situation with attainable full information contracts increases as the distortion in the labour market decreases, as the shadow cost of public funding increases and as the decreases in the lowest type’s returns to scale of labour is more important.

**Proof:**
The occurrence of class (a) depends on whether the condition $L_l^*(w, w_s, \lambda; \theta_l) \leq L_P(w; \theta_l, \theta_h)$ is satisfied. On the one hand, we remarked in section 3 that $L_l^*$ decreased as $\lambda$ increased and $w_s$ decreased. Therefore the chance to a satisfied condition is greater. On the other hand, larger decreases in the lowest type’s returns to scale of labour implies lower marginal revenues to labour and then lower levels of employment $L_l^*$ at full information. In the figure 3, this means that the lowest type has a more convex curve. The likelihood of $L_l^* < L_P^*$ increases too.

The class (a) of incentive contracts allows to promote the socially efficient levels of employment in all types of firms in the economy. Moreover these include no deadweight transfers. When the condition for such a class of contracts is not satisfied, some inefficiency and some deadweight can occur. The next section will demonstrate that, when optimal contracts are proposed by the regulator, the source of deadweight lies in the information rent that the private firm holds about its hiring capability.

4.2 Information Rent as a Source of Deadweight

When the condition for the full information contracts is not satisfied, the problem becomes a basic adverse selection problem. As in the basic models, a menu of contracts $\{(L_l, t_l), (L_h, t_h)\}$ is proposed to the private firm by the regulator. The highest type is characterized by an efficient level of employment and gets a positive or null information rent while the lowest type is induced to an inefficient level and gets only its reservation profit. We will characterize such a menu of contracts in the current and following sections.

When the condition to obtain a full information allocation is not satisfied, we have the following proposition:

**Proposition 3.** If $L_l^*(w, w_s, \lambda; \theta_l) > L_P^*(w; \theta_l, \theta_h)$ then (a) levels of employment are monotonically ranked: $L_l \leq L_h$, (b) $PC_l$ is binding, (c) $IC_h$ is binding and (d) $IC_l$ is slack (e) $PC_h$ may be slack or binding.

The proof is presented in the Appendix A. The specificity of this proof lies in the existence of non constant reservation profits $\Pi^F_l$ and $\Pi^F_h$ in the participation constraints. These require that $L_l^* > L_P^*$ in order to get the binding constraints as specified here, otherwise the full information contract is achievable.

By this proposition we can restrict our attention to the problem where only the constraints ($IC_h$) and ($PC_l$) are binding and where the constraint ($IC_l$) is slack. The constraint ($PC_h$) may be binding or slack. The regulator has the following reduced program:

$$
\begin{align*}
\text{P}^{IC} & \max_{\{L_l, L_h, t_l, t_h\}} p[PQ(\theta_l, L_l) - w_s L_l - \lambda t_l] + (1 - p)[PQ(\theta_h, L_h) - w_s L_h - \lambda t_h] \\
\text{s. t.} & \quad PC_l : PQ(\theta_l, L_l) - w L_l + t_l = \Pi^F_l \\
& \quad IC_h : PQ(\theta_h, L_h) - w L_h + t_h = PQ(\theta_h, L_l) - w L_l + t_l \\
& \quad PC_h : PQ(\theta_h, L_h) - w L_h + t_h \geq \Pi^F_h (\mu)
\end{align*}
$$

where $\mu$ is the Kuhn-Tucker multiplier of ($PC_h$).

The Superscript $^{**}$ will denote the optimal values of this program. By rearranging the two last constraints we get the expressions of profits and transfers as functions of the free profits:

$$
\Pi^{**}(\theta_l) = \Pi^F_l
$$
\[ \Pi^{**}(\theta_h) = \Pi^F_h + \Phi(L^*_h) \]  

(10)

\[ t^*_h = \Pi^F_h - [PQ(\theta_h, L^*_h) - wL^*_h] \]  

(11)

\[ t^*_h = \Pi^F_h - [PQ(\theta_h, L^*_h) - wL^*_h] + \Phi(L^*_h) \]  

(12)

The term \( \Phi(L^*_h) \) is the information rent that the regulator must give to the firm such that the highest type does not mimic the lowest type. The information rent is expressed as:

\[ \Phi(L^*_l) = (PQ(\theta_h, L_l) - wL_l) - \Pi^F_h - (PQ(\theta_l, L_l) - wL_l) - \Pi^F_l \]  

or, after simplification,

\[ \Phi(L^*_l) = (PQ(\theta_h, L_l) - PQ(\theta_l, L_l)) - (\Pi^F_h - \Pi^F_l) \]  

(13)

As the first expression shows, the information rent corresponds to the difference in losses of profit, \( (PQ(\theta, L_l) - wL_l) - \Pi^F \), between the two types of firms if the regulator imposed a level of employment \( L_l \). The information rent increases with the level of employment in the firm \( \theta_l \). Indeed differentiating by it by \( L_l \) and using the Single Crossing Property we get this relationship:

\[ \frac{d\Phi(L_l)}{dL} > 0 \]  

(14)

The intuition here is that, as the high type has larger returns to scale of labour, its extra-profit increases faster than the low type. So, does the information rent.

Note finally that at the pooling level of employment \( L^P \), \( \Phi(L^*_l) = 0 \); at any \( L_l > L^P \), \( \Phi(L_l) > 0 \).

From the equations (9)-(12) we observe that the lowest type gets the free profit as in the regulation free situation. It receives a transfer that exactly compensates the loss in profit due to the contracted level of employment \( L^*_l \), as it was the case with full information. It is indifferent between participating or not to the contract: there is thus no deadweight transfer. On the other side, the highest type has a profit level above its free profits: it receives an information rent. This rent makes it prefer the contract \( (L_h, t_h) \) to the contract \( (L_l, t_l) \) and thus the rent induce the firm to reveal its true type. Since the highest type always prefers the contract \( (L_h, t_h) \) to no contract at all, there will be a deadweight transfer that corresponds to the information rent.

4.3 The Condition for Deadweight Spending

The previous section has presented that the information rent can be the source of deadweight transfer. It did not tell that this rent was always positive. The current section shows that it can be null in particular economic situations. To demonstrate this, we develop the formal solution of the program \( PIC \).

Using (12) the participation constraint \( PC_h \) can be rewritten as follows:

\[ PC_h : \Phi(L_l) \geq 0 \ (\mu) \]

The Lagrangian of the problem is written in the following form (where we removed the Superscript ** for readability):

\[ 9It induces the lowest (highest) type to choose the contract \((L_l, t_l)\) \((L_h, t_h)\). By observing the chosen contract, the regulator can infer the type of the firm. In this sense, those are separating contracts.
\[ L = p[PQ(\theta_l, L_l) - w_s L_l] + (1-p)[PQ(\theta_h, L_h) - w_s L_h] \]
\[ + p\lambda[\Pi^F_l - (PQ(\theta_l, L_l) - w L_l)] + (1-p)\lambda[\Pi^F_h - (PQ(\theta_h, L_h) - w L_h)] \]
\[ - (1-p)\lambda\Phi(L_l) \]
\[ - \mu\Phi(L_l) \]

The First Order conditions are obtained by differentiating \( L \) by \( L_l \) and \( L_h \). The optimal levels of employment \( L_l^{**} \) and \( L_h^{**} \) are given by

\[ PQ_l(\theta_l, L_l^{**}) - w_s = \lambda [w - PQ_l(\theta_h, L_h^{**})] \] (15)

\[ PQ_h(\theta_h, L_h^{**}) - w_s = \lambda [w - PQ_h(\theta_l, L_l^{**})] \]
\[ + \frac{(1-p) + \mu}{p} \lambda d\Phi(L_i^{**}) \] (16)

We have now two classes of contracts depending on whether the Kuhn-Tucker multiplier \( \mu \) is positive or null.

**Class (b)** In this class of contracts \( \mu > 0 \) and \( PC_h \) is binding. Thus, by (10),

\[ \Pi^{**}(\theta_h) = \Pi^F_h + \Phi(L_l^{**}) = \Pi^F_h \iff \Phi(L_l^{**}) = 0 \]

Therefore, by definition (8) of the pooling level of employment, \( L_l^{**} = L^F(w; \theta_l, \theta_h) \). This expresses that there is no point for the regulator to diminish \( L_l^{**} \) beyond the pooling level of employment \( L^F \).

The first order condition (15) holds and is identical to equation (6) in the full information setting. It expresses that the expected marginal social surplus yielded by the highest type of firm should equate the expected marginal transfer to make this firm participate voluntarily. The optimal level of employment is then \( L_h^{**} = L_h^P(w, w_s, \lambda; \theta_l) \).

As both participation constraints bind, no firm earns more than its free profit \( \Pi^F \), there is no information rent and no deadweight transfer. The transfer for the most capable hiring firm is the full information transfer \( t_h^* \) and, by (11), the transfer for the least capable firm is \( t^P = \Pi^F_l - [PQ(\theta_l, L_l^P) - w L^F] \).

All these developments lead us to the following proposition:

**Proposition 4.** In class (b) incentive contracts, the level of employment in the most hiring capable firm is the optimal level in full information: \( L_h^{**} = L_h^P(w, w_s, \lambda; \theta_l) \). The level of employment in the least capable firm is distorted downward and equal to the pooling level of employment: \( L_l^{**} = L^P(w; \theta_l, \theta_h) \). The transfers do not include any deadweight.

**Class (c)** In this class of contracts \( \mu = 0 \) and \( PC_h \) is slack. By (10) that means that \( \Phi(L_l^{**}) > 0 \). Since \( \Phi(L) \) increases in \( L \) and \( \Phi(L^F) = 0 \), then

\[ L_l^{**} > L^P \] (17)
The optimal level of employment is always larger that the pooling level of employment in the class (c) contracts.

The first order condition (15) for the highest type of firm does not depends on $\mu$. The optimal level of employment $L_{h}^{**}$ for the highest type is the same as in class (b).

The first order condition (16) for the lowest type does however matter as it now determines the level of employment for the lowest type. It has the following form:

$$PQ_{L}(\theta_{l},L_{l}^{**}) - w_{s} = \lambda [w - PQ_{L}(\theta_{l},L_{l}^{**})]$$

$$+ \frac{(1-p)}{p} \lambda \frac{d}{dL_{l}} \Phi(L_{l}^{**})$$  \hspace{1cm} (18)

This equation expresses that, at the margin, the regulator must make the trade-off between the cost of information rent $\Phi(L_{l}^{**})$ - the cost of deadweight transfer - and the cost in social surplus through the decrease in $L_{l}^{**}$. The optimal level of employment in the lowest type of firm is $L_{l}^{**} = L_{l}^{ic}$ where $L_{l}^{ic}(w,w_{s},\lambda;\theta_{l},\theta_{h},p)$ is the solution of this equation.

The following proposition collects these results for class (c) and describes how the levels of employment vary with the parameter of the economy.

**Proposition 5.** In class (a) incentive contracts, the most hiring capable firm achieves the full information level of employment $L_{h}^{**} = L_{h}^{*}$, while the least capable one is proposed a level of employment lower than the full information level: $L_{l}^{ic}(w,w_{s},\lambda;\theta_{l},\theta_{h},p) < L_{l}^{*}(w,w_{s},\lambda;\theta_{l})$. It decreases with lower $w_{s}$, with higher $\lambda$, with higher $\theta_{h}$, and with lower $p$.

**Proof:** Indeed, the equality (6) can be multiplied by $p$,

$$pPQ_{L}(\theta,L) - w_{s} = p\lambda(w - PQ_{L}(\theta,L))$$

at $L = L^{*}$. We compare it to (16). By (14), $\frac{d}{dL_{l}} \Phi(L_{l}^{**}) > 0$. We find that $L_{l}^{**} < L_{l}^{*}$.

The variations $L_{l}^{ic}(w,w_{s},\lambda;\theta_{l},\theta_{h},p)$ according to the parameter $w_{s}$, $\lambda$, $\theta_{h}$ and $p$ are obtained by differentiating (18). We do not provide the computation in this text.

Q.E.D

The intuition behind the last statement in the proposition is the following one. Lower distortion $w_{s}$ implies smaller intervention from the regulator. The employment in the least hiring capable firm is therefore less supported. Larger cost of public fund means that the regulator will be also less incline to support this employment. Higher parameter ($\theta_{h}$) in the production function of the most hiring capable firm will make this firm more attractive for the regulator. The regulator will desire to reduce further the employment in the least hiring capable firm in order to diminish the information rent further. Finally, the level of employment $L_{l}^{ic}$ decreases when the probability $p$ of the least hiring capable firm ($\theta_{l}$) decreases. The smaller the probability of this firm, the smaller weight in the regulator puts on it and less incline is the regulator to loose some employment - and consequently some welfare - in this type of firm.

The transfers are given by (11) and (12).

$$t_{l}^{ic} = \Pi_{l}^{F} - [PQ(\theta_{l},L_{l}^{ic}) - wL_{l}^{ic}]$$

$$t_{h}^{ic} = \Pi_{h}^{F} - [PQ(\theta_{h},L_{h}^{*}) - wL_{h}^{*}] + \Phi(L_{l}^{ic}) = t_{h}^{*} + \Phi(L_{l}^{ic})$$

13
The last equality expresses that the transfer to the most hiring capable firm includes some deadweight.

The next section will summarize and explain the three classes of incentive contracts we have just obtained.

### 4.4 Three Classes of Incentive Contracts

We can now summarize the three classes of incentive contracts in the following table. Two first columns give the states of the participation and incentive constraints of the highest hiring capable firm. The next columns present the forms of incentive contracts and the last column shows the expected value of the deadweight spending $EDWS$ (or information rent) to the private firm.

<table>
<thead>
<tr>
<th>Class</th>
<th>$PC_h$</th>
<th>$IC_h$</th>
<th>${L_{i*}^{<strong>}, t_i^{</strong>}}$</th>
<th>${L_h^{<strong>}, t_h^{</strong>}}$</th>
<th>EDWS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>binding</td>
<td>slack</td>
<td>${L_i^l, t_i^l}$</td>
<td>${L_h^l, t_h^l}$</td>
<td>0</td>
</tr>
<tr>
<td>(b)</td>
<td>binding</td>
<td>binding</td>
<td>${L_P^l, t_P^l}$</td>
<td>${L_h^l, t_h^l}$</td>
<td>0</td>
</tr>
<tr>
<td>(c)</td>
<td>slack</td>
<td>binding</td>
<td>${L_{i*}^{ic}, t_{i*}^{ic}}$</td>
<td>${L_h^{ic}, t_h^{ic} + \Phi(L_{i*}^{ic})}$</td>
<td>$(1-p)\Phi(L_{i*}^{ic})$</td>
</tr>
</tbody>
</table>

Table 1: Three Classes of Incentive Contracts

This table shows that only class (c) introduces some deadweight spending in the form of information rent $(1-p)\Phi(L_{i*}^{ic})$ to the most hiring capable firm. The transfers are otherwise transfers with no deadweight since this firm’s participation constraints $(PC_l)$ and $(PC_h)$ are binding. The firm is then indifferent between participating to the contract or not.

The level of employment in most capable firm is always the optimal level with full information: there is no distortion introduced by the asymmetry of information over the firm’s hiring capability. In the least capable firm, the levels of employment are distorted downwards for classes (b) and (c). This is the consequence of the regulator’s desire to reduce the information rent to the most capable firm at the cost of some employment reduction in the least capable one.

The contracts corresponding to these three classes are displayed in figure 4. The full information levels $\{(a_l),(a_h)\}$ are reached where the slopes of the iso-profit curves (II) are small. At this level, by tangency, the slope of the iso-welfare (W) are small, too. (This is depicted only for $(a_h)$.) This indicates a weak marginal social surplus from employment, which characterizes a weak distortion in the labour market or a large shadow cost of public fund.

The incentive contracts $\{(b_l),(b_h)\}$ and $\{(c_l),(c_h)\}$ are reached for higher slopes of iso-profit curve. The difference between the two classes of contract lies in the weight the regulator puts on the least hiring capable firm. In class (c) the regulator makes a trade-off between reducing the information rent to the most hiring capable firm and reducing the employment level of the least capable firm. In class (b), the regulator attaches a smaller weight on the least capable firm because it is unfrequent. The regulator is then willing to reduce much employment in this unfrequent firm in order to decrease the cost of highly probable information rent to other firm. He decreases this employment down to the level $L_P^l$. Any decrease beyond this level is inefficient to him since the information rent can no more be reduced.

We may now question in which economic situations the various classes of incentive contracts occur. In the previous text, we have defined three important levels of employment for the least hiring capable firm: the level of employment $L_l^*(w, w_s, \lambda, \theta_l)$ in the full information contract, the pooling level $L_P^l(w; \theta_l, \theta_h)$ for the incentive contract with no deadweight and the level $L_{i*}^l(w, w_s, \lambda; \theta_l, \theta_h, p)$ for the incentive contract with deadweight (solution of (18)). All these levels depends on the (different) sets of parameters of the economy: $w, w_s, \lambda; \theta_l, \theta, p$. We may now make the following...
proposition that collects the results of the current and the previous sections.

Proposition 6.

- If \( L^*_l(w, w_s, \lambda; \theta_l) \leq L^P(w; \theta_l, \theta_h) \), the regulator can implement Class (a) incentive contracts. These contracts include no deadweight spending.

- If \( L^*_l(w, w_s, \lambda; \theta_l) > L^P(w; \theta_l, \theta_h) \) and \( L^P(w; \theta_l, \theta_h) \geq L^c_L(w, w_s, \lambda; \theta_l, \theta_h, p) \), the regulator can implement Class (b) incentive contracts. These include also no deadweight transfer.

- Otherwise, the regulator must implement Class (c) incentive contracts which include some deadweight transfer to the private firm. The forms of the corresponding incentive contracts are given in table 1.

The first condition in the proposition is demonstrated through the section 4.1. The second condition is elaborated in the discussion of \( \mu \) in section 4.3.10

The current results are imprecise about the effect that each parameter of the economy have on the deadweight spending. As we have obtained no general results we turn to an example to highlight these effects.

4.5 Example

We can characterize the welfare value of these contracts as function of the market distortion in the labour market \( \frac{w}{w_s} \), the belief over the types of firms \( p \) and \( (1 - p) \), and the spread in types

---

10A final discussion of this section carries over the impact of the private firm’s reservation profits. The classic literature in regulation proposes adverse selection models where the participation constraints include constant reservation values. In our particular application, the private firms’ reservation values differ according to the firms’s types. The most hiring capable firm has a larger reservation value since its profit when it is free from regulation is higher than this of the least capable firm. The impact of these varying reservation values is analyzed in the Appendix A. We can briefly show this by analyzing the form of the information rent. We write it here again:

\[ \Phi(L_1) = [PQ(\theta_h, L_1) - PQ(\theta_l, L_1)] - [\Pi^F_h - \Pi^F_l] \]

The difference between our model and the classic models with constant reservation values lies in the presence of the difference in free profits \([\Pi^F_h - \Pi^F_l]\). As this grows, the information rent decreases. The impact of asymmetric information is therefore diminished.
\[ s \equiv \frac{\theta_h - \theta_l}{\theta_l}. \]  Let us fix the spread and exploit the example of a price taker firm with productivity \( Q = \theta \sqrt{L} \) where \( \theta \) is a productivity parameter. The price of the product is set to \( P = 1 \).

The full information class (a) occurs when \( LP > L_l^c \). After computation this leads to the following inequality:

\[
\frac{\theta_h + \theta_l}{4w} > \frac{\theta_l}{2w} \frac{1 + \lambda}{w_s + \lambda w}
\]

This means that the distortion in the labour market, \( \frac{w}{w_s} \), should be lower than this particular value:

\[
\left( \frac{w}{w_s} \right) = \frac{1 + \frac{1}{2}s}{1 - \frac{1}{2}\lambda s} > 1
\]

The class (b) occurs when \( LP > L_l^c \). This condition is transformed to the following expression:

\[
\tilde{p} \equiv \frac{1}{1 - \frac{w_s}{w} + \frac{\lambda s}{2 + \frac{1}{2}s}}
\]

The probability \( \tilde{p} \) is the regulator’s maximum belief on the least hiring capable firm that allows him to propose an incentive contract with no deadweight. This probability increases with larger \( \lambda \) and \( s \) but with lower \( \frac{w}{w_s} \). A larger cost of public fund \( \lambda \) or a larger spread will imply a larger difference between the optimal levels of employment in the two types of firm. The regulator will be more enticed to reduce the employment in the least capable firm down to \( LP \). This will lead to a class (b) contract. Finally a lower distortion in the labour market, \( \frac{w}{w_s} \), means that the regulator desires to promote less employment, in particular within the least hiring capable firm. The optimal level of employment in this firm may become sufficiently low so that the most capable firm is not induced to mimic the least capable one. There will be no information rent and then no deadweight spending. This is again the class (b).

In the other case, we achieve an incentive contract with non null information rent (class (c)).

The following graph depicts the three possible classes for various believes and intensities of distortion in the labour market. It is remarkable that the regulator can implement contracts with transfer with no deadweight in the classes (a) and (b).

In this particular example, we can note that when \( s \to 0 \), our model with a discrete distribution of types is transformed into a model with a continuous distribution of types and the boundary \( \left( \frac{w}{w_s} \right) \to 0 \). The class (a) disappears and the full information contracts are no more possible. We expect therefore to loose this property in the continuous type model.

In this example, the intersect \( X \) between the two limit curves drifts to the North as the spread \( s \), increases. This means that the class (c) shrinks as the firms are more and more differentiated by their types. Therefore the economic situations in which transfers with no deadweight are possible, become more probable. The intuition is that, within this particular example, a larger spread will involve a larger difference in the free profits and will augment the likelihood of contracts with no deadweight transfer.

The current results focus on the optimal incentive contracts of private firms with different hiring capabilities. The regulator lacked information on the firm’s productivity parameter and could not observe the firm before offering the contracts. We now turn to the situation where the regulator \textit{ex-ante} observes the level of employment in the firm.
In the previous section, we made the hypothesis that the regulator could not observe ex-ante the level of employment in each private firm. We assumed that the regulator "blindly" offered incentive contracts because we wanted to make clear the link between deadweight spending, efficiency and information rent. We will now modify the model such that the regulator observes each firm’s level of employment before the incentive contract is signed.

The problem becomes one of selecting the private firm with the largest hiring capability among those having the same free level of employment $L_F$. This model can be viewed as an adverse selection problem based on the increase in the firm’s employment $\Delta L = L - L_F$ instead of the absolute level of employment $L$ as in the previous section. The regulator can now check the difference between the achieved level of employment $L$ and the observed free level $L_F$. This ex-ante observation provides some information about the various parameters unknown to the regulator. In fact, it will help to determinate one of these parameters but still some other parameters will have to be revealed through the incentive contract.

The role of ex-ante observation becomes clear when we assume two kinds of information: $\theta$ and $\gamma$. The firm’s revenue becomes $PQ(\theta, \gamma, L)$ while its profit function is transformed to $\Pi = PQ(\theta, \gamma, L) - wL + t$. When the firm is free from regulation, the transfer $t$ is null and the firm chooses the free level of employment $L_F$ such that $PQ_L(\theta, \gamma, L_F) = w$. This equality gives a relation between $\theta$, $\gamma$, and $L_F$ that we express as $\gamma = \gamma(L_F, \theta)$. After the observation of $L_F$ the regulator can infer the firm’s profit: $\Pi = PQ(\theta, \gamma(L_F, \theta), L) - wL + t$. This is a function of $\theta$ and $L$ and the problem is identical to this of the previous sections.$^{11}$

The contracts with ex-ante observation can be explained in the figure 6. As this figure shows, the nature of the asymmetric information relates now to the concavity of the iso-profit curve and thus to the importance of decreasing returns to scale of labour. (There is no longer a translation effect between the curves, as in the previous section.) When they are free from regulation both firms’ iso-profit curves intersect at the same level of employment $L_F (= L_h = L_l)$ and the firm with the largest (lower) hiring capability is characterized by the flatter (steepest) iso-profit curve.

In this particular model we get the following proposition:

**Proposition 7.** If the regulator ex-ante observes the free level of employment, $L_F$,

- the full information contracts (class a) can never be reached.
- If the probability $p$ of the lowest type of firm is low, a menu of incentive contracts with no

$^{11}$A minor difference lies in the violation of the Single Crossing Property (3): $Q_L(\theta, L) = 0$ at $L = L_F$. However, as long as this property holds for other levels of employment $L \neq L_F$ our model still remain valid.
information rent (class b) can be achieved.

• Otherwise a menu of incentive contracts with non null information rent must be implemented (class c).

• Non additional transfers are only available in (class b) and correspond to contracts that disregard the lowest type.

Proof:

We content ourselves to the informal proof since this results can easily be obtained from the previous sections and figure 6. Remark that the pooling level of employment \(L_P\) is pushed down to \(L_P = L_F\). Note then that the lowest type’s full information level of employment is such that \(L_P \leq L_l^c\). Indeed, the hypothesis on the social surplus implies that the regulator will always gain from increasing \(L_l\) above \(L_F\). Therefore, the occurrence of class (a) is impossible.

At \(L_P = L_F\), the transfer to the lowest type is \(t_l^* = O\) and the lowest type is at equilibrium. By application of the previous proposition, the class (b) corresponds to the menu of contracts \(\{(L_F, 0), (L_l^*, t_l^*)\}\). On one side, it would decrease the social surplus. On the other side, any reduction of \(L_l\) below the level \(L_P\) would necessitate an increase in the transfer and thus an increase in the social cost. There is thus no gain for the regulator to decrease \(L_l\) below \(L_P\). The class (c) is has the same characteristics.

Q.E.D

Note that the Class (b) contracts discards the lowest type since the latter always receives a null transfer and never participates.

To conclude, we note that the results for incentive contracts with ex-ante observation of the free level of employment are similar to those of the previous model. However, on one hand ex-ante observation withdraws the possibility of full information outcome (class (a)) so that transfers with no deadweight are less probable to occur. On the other hand this ex-ante observation provides more information on the firms’ characteristics, it would also reduce the size of information rent and thus the importance the deadweight transfers in class (c).

Finally, we note an inconsistency in this model. The observation of the free level of employment means that the private firms exist before the contracts are proposed. This breaks the hypothesis of
one-shot (or static) game. The introduction of dynamics in the model will imply strategic behaviors of the firms on their levels of employment but also on the information revelation strategies.

6 Extensions

In this paper we applied strong hypotheses about the private firm. How robust is this model to relaxing these? A first step is to relax the assumption on the firm behavior in the product market. When the firm is a monopoly, the same model can easily be adapted by considering the new revenue function $P(\theta, L)Q(\theta, L)$ instead of $PQ(\theta, L)$. The same conclusions will be derived. The additional specificity will be the regulator’s intervention against the monopoly’s power in the product market. This element has extensively been discussed in the literature and is not developed here.

We can also relax the hypothesis on the nature of the type $\theta$. We assumed that it described a production parameter. The introduction of a parameter about the demand function is trivial and the results will be similar. However, we note that this parameter should monotonically rank the firm’s revenue, which might be an unrealistic property.

Other relevant extensions must be reported. As presented in the previous section, the private firm’s strategic behavior and the regulator’s commitment are important. The firm can strategically modify any ex-ante characteristic in order to increase its profit from the contracts. Also, in a repeated relationship, the regulator might not be able to commit in the long run. A ratchet effect will appear such that the firm will avoid to reveal all its information in the first period as required in the current model. We can mention the example of a highly capital intensive firm which contracts with the regulator for a specified increase in employment and a specified annual subsidy. Within our model, the regulator is able to know all the firm’s private information after the first period. He has then the incentive to break his commitment and to grant minimal transfers in the next periods. Will the firm still make the investment linked to the specified increase in employment?

Finally, a major but difficult extension of this model would be the introduction of peer firms. When several private firms are (imperfectly) competing in the product market, a subsidy to one firm affects the others. The following questions arise: what are the characteristics of optimal contracts when we monitor employment in a specific industry? Can the information obtained from one firm be used to infer the type of the other firm when the types of the firms are correlated?

References


\(^{12}\)We mention here the ratchet effect, and the renegotiation possibilities.


APPENDIX A:

Rename the revenue $PQ(\theta, L)$ by $R(\theta, L)$. By the definition of $L^P$ the inequality $L_l > L^P$ is equivalent to $[PQ(\theta_h, L^P) - \Pi_h^E] - [PQ(\theta_l, L^P) - \Pi_l^E] > 0$ that can be summarized as $\Phi(L_l) > 0$. Call $(L_l^{**}, L_h^{**}, t_i^{**}, t_h^{**})$ the solution of program $P_{IC}^*$. The proposition to prove becomes then:

**Proposition.** Assume $\theta_0 > \theta_1$ and $R_{L,0} > 0$. Levels of employment are monotonic: (a) $L_l^{**} \leq L_h^{**}$.

If $\Phi(L_l) > 0$ then (b) $PC_l$ binds, (c) $IC_h$ binds and (d) $IC_l$ does not bind. Only the constraint $PC_h$ is undetermined.

**Proof:**

(a) $L_l^{**} \leq L_h^{**}$:

We prove that $L_l^{**} > L_h^{**}$ is a contradiction.

Suppose $L_l^{**} > L_h^{**}$.

If $IC_l$ and $IC_h$ could be satisfied, then the sum of $IC_l$ and $IC_h$ could be satisfied, too:

$$(R(\theta_h, L_l^{**}) - R(\theta_h, L_h^{**})) - (R(\theta_l, L_l^{**}) - R(\theta_l, L_h^{**})) \leq 0$$

Which contradicts the single crossing property $R_{L,0} > 0$.

(b) If $\Phi(L_l^{**}) > 0$, then $PC_l$ binds:

We prove here that, if $\Phi(L_l^{**}) \geq 0$ and if $PC_l$ is not binding then $PC_h$ is not binding and the transfers $t_i^{**}$ and $t_h^{**}$ are not optimal for the regulator.

Suppose $PC_l$ is not binding at the optimum, i.e. $R(\theta_l, L_l^{**}) - wL_l^{**} + t_i^{**} > \Pi_l^E$. Add the inequality $\Phi(L_l^{**}) > 0$ from hypothesis and get $R(\theta_h, L_l^{**}) - wL_l^{**} + t_i^{**} > \Pi_l^E$. Using $PC_h$, $R(\theta_h, L_h^{**}) - wL_l^{**} + t_h^{**} > R(\theta_h, L_l^{**}) - wL_l^{**} + t_i^{**} > \Pi_h^E$. The first and last term of this chain of inequalities correspond to the constraint $PC_h$. So, none of the participation constraints are binding. It is possible to gain some welfare by reducing $t_i^{**}$ and $t_h^{**}$ without modifying the levels of employment $L_l^{**}$ and $L_h^{**}$.

(c) If $\Phi(L_h^{**}) > 0$, then $IC_h$ binds:

We prove that, if $\Phi(L_h^{**}) > 0$ then, if $IC_h$ does not bind then $PC_h$ does not bind and $t_h$ is not an optimal transfer.

If $IC_h$ is not binding, then $R(\theta_h, L_h^{**}) - wL_h^{**} + t_h^{**} > R(\theta_h, L_l^{**}) - wL_l^{**} + t_i^{**}$. Add the inequality $\Phi(L_h^{**}) > 0$, then $R(\theta_h, L_l^{**}) - wL_h^{**} + t_h^{**} + \Phi(L_h^{**}) > R(\theta_h, L_l^{**}) - wL_l^{**} + t_i^{**}$. This yields $R(\theta_h, L_h^{**}) - wL_h^{**} + t_h^{**} > \Pi_l^E + (R(\theta_h, L_l^{**}) - wL_l^{**} + t_i^{**} - \Pi_l^E) > \Pi_h^E$, as the parenthesis in the middle term corresponds to the constraint $PC_l$ and is positive or null. So, we have that $PC_h$ and $IC_h$ are not binding. Ceteris paribus, the regulator can therefore reduce $t_h^{**}$ and gain some welfare: the transfer $t_h^{**}$ is not optimal.

(d) If $\Phi(L_l^{**}) > 0$, then $IC_l$ does not bind:

We prove here that, if $IC_l$ binds, then by the single crossing property $R_{L,0} > 0$, $IC_h$ cannot bind, which contradicts the previous paragraph.

Suppose $IC_l$ is binding, then $R(\theta_l, L_l^{**}) - wL_l^{**} + t_i^{**} = R(\theta_l, L_h^{**}) - wL_h^{**} + t_i^{**}$. By $R_{L,0} > 0$,

$R(\theta_h, L_l^{**}) - R(\theta_l, L_l^{**}) > R(\theta_h, L_h^{**}) - R(\theta_l, L_h^{**})$. Add both inequalities and get $R(\theta_h, L_l^{**}) - wL_l^{**} + t_i^{**} > R(\theta_h, L_h^{**}) - wL_h^{**} + t_i^{**}$, that is $IC_l$ is not satisfied, which is a contradiction.
The reader may wonder what happens when $\Phi(L_{l}^{**}) < 0$. In this situation, the constraints $PC_l$ and $PC_h$ are binding, but none of the constraints $IC_h$ and $IC_l$ are binding. The level of employment $L_{l}^{**}$ is not optimal since the regulator can increase $L_{l}^{**}$ without modifying the states of the constraints. He can increase it up to the level $L^P$ where $\Phi(L^P) = 0$. In fact as will be discussed later, there is no point to decrease $L_{l}^{**}$ below $L^P$.

Q.E.D