Does Modern Econometrics Replicate The Phillips Curve ?*

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Abstract

This paper reexamines the existence of a long-run relationship between wages and unemployment in the U.K., with data over the period 1860-1913 used by A.W. Phillips to derive the well-known Phillips Curve. Using Johansen's maximum likelihood method of testing for cointegration, a long-run inverse relationship is indeed depicted between the rate of inflation and the unemployment rate. However, the main impact of deviations from this long-run equilibrium is on the unemployment rate rather than the rate of inflation.

Keywords: Phillips Curve, long-run equilibrium, cointegration.

JEL classification: C32, C52, E24.

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1 Introduction

It is probably fair to say that the analysis of the wage-price mechanism is one of the areas of applied macroeconomics that has received most attention. This is of course especially true ever since the publication of A.W. Phillips' article in 1958. The "Phillips Curve" became, at first an essential, and then a controversial ingredient of macro-models of the economy. Over the years, numerous studies have been undertaken in different countries, to estimate and test many variants of the Phillips curve (Nickell (1990) and Bean (1994) provide excellent surveys of the applied research in this area over the last two decades). Although there are some results with which most economists seem to agree, one is nevertheless left with the impression that the empirical evidence for or against the existence of a stable long-term relationship between the rate of change of wages (or of prices) and the rate of unemployment remains contradictory.

The last decade has seen much development in dynamic econometric modelling of economic time series. These developments, which have gone mainly ignored in most empirical studies of the wage-price mechanism, concern issues related to the exogeneity of variables, cointegration or the existence of a long-run relation between 'integrated' economic variables, and finally single equation versus system modelling. Much of the empirical evidence reviewed in Nickell (1990) and in Bean (1994) is based on the estimation of conditional wage and price equations. If the regressor variables can legitimately be assumed to be weakly exogenous for the parameters of interest in the conditional model, then efficient estimation and testing may be conducted by analysing only the conditional model. On the other hand, ignoring the exogeneity status of a variable such as the unemployment rate, can lead to invalid inference (Engle *et al.* (1983)). Another common practice in the applied literature is to invert estimated wage and price equations to derive the so-called 'natural' rate of unemployment or the $NAIRU$. But such an inversion of conditional models, need not at all give estimated coefficients which are close to the parameters of the uninverted conditional model for the unemployment rate (Ericsson (1992), Shadman-Mehta (1996)).

A further dimension needs to be borne in mind when dealing with integrated series which are expected to be cointegrated, such as wages, prices and productivity. Even the unemployment rate itself behaves at times as a unit root process. Granger (1981,1986) and Engle and Granger (1987) have established the isomorphism between cointegration and error correction models. It can therefore generally be expected that the parameters of interest in a conditional wage equation are linked with the parameters of the marginal distribution of the regressor variables, through the common cointegration vector(s), which would violate weak exogeneity.

These results underline further the importance of employing a directed research strategy of modelling from the general to the specific. Apart from ensuring that the analysis begins from a congruent model of the data and avoiding the need to correct obvious shortcomings, such a strategy naturally widens the concept to define the optimal strategy as one that comprises an appropriate set of variables which ought to be modelled jointly. A system approach is preferable to single-equation modelling until weak exogeneity is ascertained (Banerjee *et al.* (1993)).

The aim of this paper is to use these important developments in econometric methodology to reevaluate the relationship between the unemployment rate and the rate of change of wages in the U.K. over the period 1860-1913. It seems a befitting tribute to A. W. Phillips' contribution to empirical economics, to apply econometric methods which were unavailable to him, to the same data set used in his study, to determine whether similar conclusions may be drawn[1].

The plan of the paper is as follows. Section 2 briefly describes the Phillips data set. Section 3 investigates the existence of long-run equilibrium relations between the basic variables in this data set, using the maximum likelihood method developed by Johansen (1988) and Johansen and Juselius (1990). A question of interest is whether the Phillips Curve, or a variant of it, is a long-run equilibrium relation in this approach which calculates LR tests obtained in a vector autoregressive framework, with a given lag structure. Section 4 models the unemployment rate equation. Section 5 concludes.

2 Data Description

The basic variables used in this study are W, the index of average full-time weekly wage rates, P, the index of retail prices, U, the percentage unemployed of the working population, and Q, which is the measure of average labour productivity. Phillips did not publish the data he used to estimate his famous curve, no doubt because at the time of publication, he did not expect the impact that his work was going to have. Lipsey (1960), who attempted soon after to interpret the relationship as a structural one, also failed to publish his data. The only available empirical evidence are the scatter diagrams and the "crosses" provided in the various figures in Phillips' article.[2] Other researchers have attempted to reconstruct the series, by using Phillips' data sources (see *inter alia* Gilbert (1976) and Wulwick(1989)). The basic data set for $\dot{W}$, $\dot{P}$, U and $\dot{U}$ used in this study,[3] were reconstructed by A. Sleeman of Western Washington University, [4] for the years 1860 to 1957, and completed to 1979 by J.J.

[1]A separate paper (Shadman-Mehta (1996), chapter 4) extends the analysis to the period 1860-1990.

[2]The fact that Phillips' original data are now available, were brought to my attention, after the completion of this study, by R. Leeson .

[3]A dot over a variable signifies the first central difference of that variable, e.g. defined as: $\dot{W}_t = 50 * \left[\frac{W_{t+1}-W_{t-1}}{W_t}\right]$

[4]Sleeman (1981). I am indebted to J.J. Thomas at the L.S.E. for giving me this data set.

Thomas (see Thomas(1984)). Using this data set, levels of W, and P were calculated on the basis of $\dot{W}$, $\dot{P}$ and by reference to actual values available for later years. For observations between 1980 and 1990, actual values of W, P and U are used. Finally, the productivity variable Q, defined as output per worker, was also added to this basic data set. W, P and Q are expressed as indices, with 1985 as the base. [5]

The first step was to apply Phillips' own procedure to the data set, to ascertain that his results can be reproduced. This can be confirmed for the period 1861-1913 (there are some small discrepancies for the period 1920-1939 and 1947-1957). Figure 6, at the end of the paper, shows the scatter diagram of U and $\dot{W}$ for 1861-1913.[6]

Figure 1 (a,b,d and c clockwise) shows some of the features of these series during the period 1860-1913. Figure 1a graphs the logarithm of annual observations on W, the index for average full-time weekly wage rate (1985=1), and P, the index of retail prices (1985 =1) in the U.K.. The evolution of the logarithm of their ratio, namely the index of real wages is also graphed for comparison. Let us denote these series by w_t, p_t and $(w-p)_t$ (The means and ranges of the variables have been adjusted to show maximum correlation). For much of this period, prices were actually falling, beginning to rise from about 1896. Nominal wages on the other hand, rose almost continually, with a sharp increase in the early 1870's which was also accompanied by a rise in prices. But real wages nevertheless rose during this period.

Figure 1b graphs the first differences of these series. Compared to the levels, they are much more erratic, but as far as prices or wages are concerned, the growth rates still appear rather autocorrelated. Wages have had positive growth over most of the sample. The rate of inflation on the other hand, fluctuated around zero much more frequently, starting an upward drift from the mid-1890's. The growth rate of real wages appears to be stationary, especially towards the end of the period. However, visual inspection alone does not establish the stationarity of a series. Formal tests are required to help clarify the issue.

Figure 1c graphs the evolution of the logarithm of real wages $(w-p)$, the unemployment rate U and the inflation rate Δp (means and ranges have once again been adjusted). The unemployment rate series appears stationary (with fluctuations around a positive value for U). The visual inspection of figure 1c shows no pairwise correlation between U_t and $(w-p)_t$. The inflation rate however moves closely with U and a negative correlation can be detected between them.

Finally, figure 1d shows a cross-plot of real wages against productivity. A regression line is also fitted to the sample. It shows clearly the co-movement of these series and points to the possibility of co-integration between them with a unitary coefficient.

[5]Full details on the various variables and their sources are given in Shadman-Mehta (1996).

[6]A comparison with Phillips' original data (Leeson (1995)) reveals that the reconstruction by Sleeman corresponds exactly to Phillips' for 1861-1913. Differences do exist however for later years.

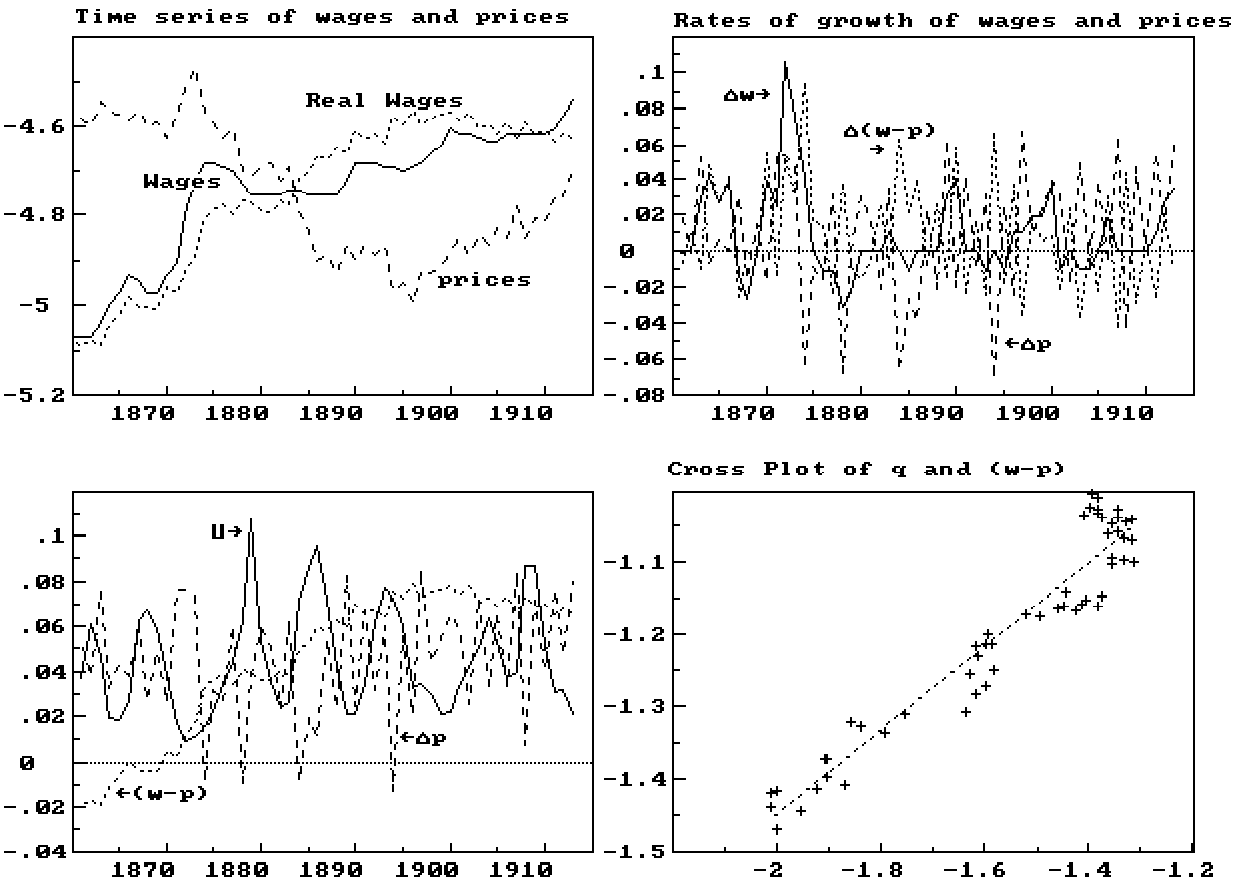

Figure 1 (a-d): Features of the Phillips data set (means and ranges adjusted).

3 Cointegration and Long-run relations: 1860-1913

The most commonly used approach to the wage-setting equation in the literature today, can be summarised by the following equation:

$$\Delta w = (p^e - p_{-1}) - \beta_1 U - \beta_2 (w-p)_{-1} + Z_w \Gamma + \epsilon_w \tag{1}$$

where Z_w is a host of variables believed to influence the mark-up over the reservation wage (unemployment benefits, real interest rates, skill mismatch, productivity, tax wedge, ...). Written in this form, this equation allows the comparison of the case when $\beta_2 = 0$, which is interpreted as the traditional Phillips Curve relating the rate of change of wages to the unemployment rate, with the more general error correction representation, first introduced by Sargan (1964), which allows the level of real wages to be related to the unemployment rate. As already mentioned, it is also generally the practice to derive the so-called 'natural' unemployment rate from the estimated wage equation as $Z_w \Gamma / \beta_1$ in this case.

In this paper, the econometric analysis of the relation between the variables appearing in the above equation follows a general to specific modelling strategy, that is beginning with the joint density of the observations. To investigate the existence of a long-run relation between the variables in the Phillips data set, use is made of the concept of cointegration which formalises such a property in statistical terms. A variable is integrated of order 1 ($I(1)$), if it requires differencing to make it stationary. A set of $I(1)$ time series is cointegrated if some linear combination of such (non-stationary) series is stationary. Thus, if the joint density of a vector process $\underset{\sim}{x}_t$, with n variables, takes the form of a p-th order vector autoregression (VAR), we have:

$$\underset{\sim}{x}_t = \sum_{i=1}^{p} \pi_i \underset{\sim}{x}_{t-i} + \underset{\sim}{\epsilon}_t \qquad \underset{\sim}{\epsilon}_t \sim \mathcal{IN}(\underset{\sim}{0}, \Omega) \tag{2}$$

A constant term or dummies may also be added to (2). A simple reparameterisation of (2) will lead to:

$$\Delta \underset{\sim}{x}_t = \pi \underset{\sim}{x}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \underset{\sim}{x}_{t-i} + \varepsilon_t \tag{3}$$

with $\Gamma_i = -(\pi_{i+1} + \ldots + \pi_p)$ and $\pi \equiv (\sum_{i=1}^{p} \pi_i) - I$. As shown by Engle and Granger (1987), π may be of reduced rank r, where $0 < r < n$. In this case the elements of $\underset{\sim}{x}_t$ are $I(1)$, but there are r linear combinations of $\underset{\sim}{x}_t$ which are stationary. The components of $\underset{\sim}{x}_t$ are then said to be cointegrated and π can be written as the product of two full column rank $(n \times r)$ matrices $\underset{\sim}{\alpha}$ and $\underset{\sim}{\beta}$, i.e.:

$$\pi = \underset{\sim}{\alpha} \underset{\sim}{\beta}^{'} \tag{4}$$

where $\beta^{'}$ is the matrix of cointegrating vectors, and α is the matrix of 'weighting elements' or speeds of adjustment. Using Granger's representation theorem (Granger(1983)) as well as (4), equation (3) can be written in its error correction form:

$$\Delta \underset{\sim}{x}_t = \alpha\beta' \underset{\sim t-1}{x} + \sum_{i=1}^{p-1} \Gamma_i \Delta \underset{\sim t-i}{x} + \varepsilon_t \tag{5}$$

This study investigates cointegration in the VAR, involving the variables $(w_t - p_t, \Delta p_t, q_t, U_t)$, where lower case letters denote logarithms of the corresponding variables. This formulation imposes long-run price homogeneity, acceptable from the point of view of economic theory, but also allows the analysis of the role of inflation as a proxy for agents' price expectations. It is clear that a number of other variables are also likely to play a significant role in the determination of wages and prices. But this exercise is aimed at investigating the conclusions reached when applying new techniques to the same basic data used by Phillips. It would indeed be difficult to obtain reliable data for most other variables of interest, stretching back into the last century. The only additional variable used is productivity q, which was discussed by Phillips in his article.[7]

Since the degree of integration of a series is not an inherent property, and may change over different sample periods, it is important to base the analysis on a model which is $I(0)$ congruent and invariant, and not dependent on assumptions such as constancy of the order of integration. The starting point therefore, will be the analysis of the system of the four stochastic variables $(w_t - p_t, \Delta p_t, q_t, U_t)$, with the aim of first arriving at such a model. Testing for cointegration will follow this initial stage.

A constant and a trend are included in the system. The inclusion of deterministic variables in the model calls for special attention. The constant cannot *a priori* be restricted to lie in the cointegration space, since we expect real wages and productivity to have an autonomous rate of growth, even though the unemployment rate should have no long-run autonomous growth, and therefore no separate intercept. The trend term, on the other hand, should be restricted to lie in the cointegration space only. If it was allowed to lie outside, it would create a quadratic trend in the levels of the variables. There is no evidence to suggest this might be a realistic representation.

3.1 The general system.

The first step was an analysis of the lag structure of the VAR, starting with a maximal lag of 4. All selection criteria, as well as the F-tests of the validity of reducing the lag length, pointed to the choice of 2 as the appropriate lag length. In what follows, 2 is the maximum lag in the series, although higher order lags were tried without the results changing significantly.

[7] One other important varibale which is expected to be cointegrated with the rate of inflation is the interest rate (see the study of the rate of inflation in the U.K. over a century in Hendry and Doornik (1995). It is hoped that a further extension of the model will include this variable.

Table 1 reports some of the statistics that help evaluate the system. * and ** refer to significance at the 5% and 1% levels respectively. The standard deviations of the residuals provide a useful measure of the goodness of fit, because they are either in the same units as their corresponding dependent variable (e.g. U), or are a proportion in the case of log models. They are also invariant under linear transformations of the variables. If the misspecification tests allow us to safely assume that the system errors are white noise, these standard deviations can act as the baseline innovation standard errors. The correlations between the residuals help guide the direction of modelling. In this case we observe a large negative correlation between the residuals of $w - p$ and Δp as well as correlations between residuals of q and $w - p$, and U and Δp of the order of .45.

	$w-p$	Δp	q	U
Δp	-0.84	.	.	.
q	0.42	-0.37	.	.
U	0.26	-0.45	0.23	.

(a): Residual Correlations

	$w-p$	Δp	q	U				
$F_{s=1}(4,38)$	18.60^{**}	5.54^{**}	3.31^{*}	7.91^{**}				
$F_{s=2}(4,38)$	2.87^{*}	0.72	1.84	5.77^{**}				
$\lvert\lambda_{(\pi(1)-I)}\rvert$	0.98	0.75	0.59	0.03				
$\lvert\lambda_{Comp}\rvert$	0.75	0.75	0.75	0.47	0.47	0.17	0.17	0.96

(b): Dynamics

Statistic	$w-p$	Δp	q	U	VAR
$\hat{\sigma}$	0.032	0.033	0.017	0.015	
$F_{ar}(2,39)$	0.34	3.44^{*}	0.69	0.69	
$F_{arch}(1,39)$	0.10	0.48	0.33	0.02	
$F_{het}(18,22)$	0.78	0.85	0.90	0.55	
$\chi^2_{nd}(2)$	0.47	1.21	1.40	7.46^{*}	
$F^v_{ar}(32,112)$					1.21
$F^v_{het}(180,135)$					0.71
$\chi^{2v}_{nd}(8)$					12.27

(c): Evaluation

Table 1: Goodness of fit and misspecification tests, 1863-1913

Next we examine the reduction of the system to an appropriate lag length, as well as analyse its dynamics. The statistic $F_{s=i}$ tests the hypothesis of an i-period lag. As is clear from table 1(b), except for the second lag on Δp and q, lags 1 and 2 of all variables are significantly different from 0. $|\lambda_{(\pi(1)-I)}|$'s are the moduli of the eigenvalues of the long-run matrix $\hat{P}_o = \hat{\pi}(1) - I$, and the $|\lambda_{Comp}|'s$ those of its companion matrix. From the values of the $|\lambda_{(\pi(1)-I)}|$'s, it appears that the rank of $\hat{P}_o$ is less than four, as at least one of them is quite small, but the rank is also greater than 0, with one eigenvalue having a modulus of 0.98. If so, then there is cointegration between the variables. As for the eigenvalues of the companion matrix, none is greater than one, which would imply an explosive system, and the number of roots close to one is less than 4, thus confirming that the system is indeed $I(1)$.

Other reported tests are tests of misspecification. A satisfactory model should have constant parameters (see Figure 3) and residuals that are homoscedastic innovations. These tests can be performed both at the single equation level, and at the system level. $F_{ar}()$ is the Lagrange-Multiplier test for autocorrelated residuals (here of the second-order). $F_{arch}()$ is the $ARCH$ test for autoregressive conditional heteroscedasticity, or autocorrelated squared residuals (here of order 1) (Engle(1982)). $F_{het}()$ is the test of the null hypothesis of unconditional homoscedasticity, testing the significance of the regressors as well as their squares in the squares of the estimated residuals (White(1980)). $\chi^2_{nd}(2)$ is a chi-square test for normality. The corresponding tests applied to the system are denoted by ^{v} (see Doornik J.A. and Hendry, D.F. (1994)).

As suggested by table 1(c), most outcomes are satisfactory, except for some remaining autocorrelation in the errors of the inflation equation, and possibly non-normal errors in the unemployment equation, although the latter is probably not so important when testing for cointegration, given that the analysis by Cheung and Lai (1993) shows that Johansen's trace test is quite robust to both skewness and excess kurtosis in innovations. Note that the tests of heteroscedasticity are insignificant both at the individual and the general model level. This indicates that although the representation chosen here is linear in U, unlike Phillips', there is no evidence of misspecification due to non-linearity.

Figures 2, 3, and 4 summarise more of the information about the estimated system. Figure 2 shows the fitted and actual values of the four variables, their cross-plots and their scaled residuals. One can detect clearly that there is a greater scatter for the equations relating to Δp and to U. Figure 3 shows graphically, diagnostic checking of parameter constancy, through recursive estimation of the system. One can notice from the one-step residuals $\pm 2\hat{\sigma}$, that all four equations can reasonably be assumed to have constant parameters, with one exceptional outlier for U. The individual equation break-point Chow(1960) F-tests are never larger than the corresponding 5% critical value, and the system break-point test values also remain insignificant throughout. None of these results changed substantially, when a lag larger than two was adopted. Finally, figure 4 presents informal graphical representation of some of the tests reported in table 1(c). The correlogram shows no obvious dependence between successive residuals, and the histograms with non-parametric densities, and the cumulative distribution show no substantial departure from normality.

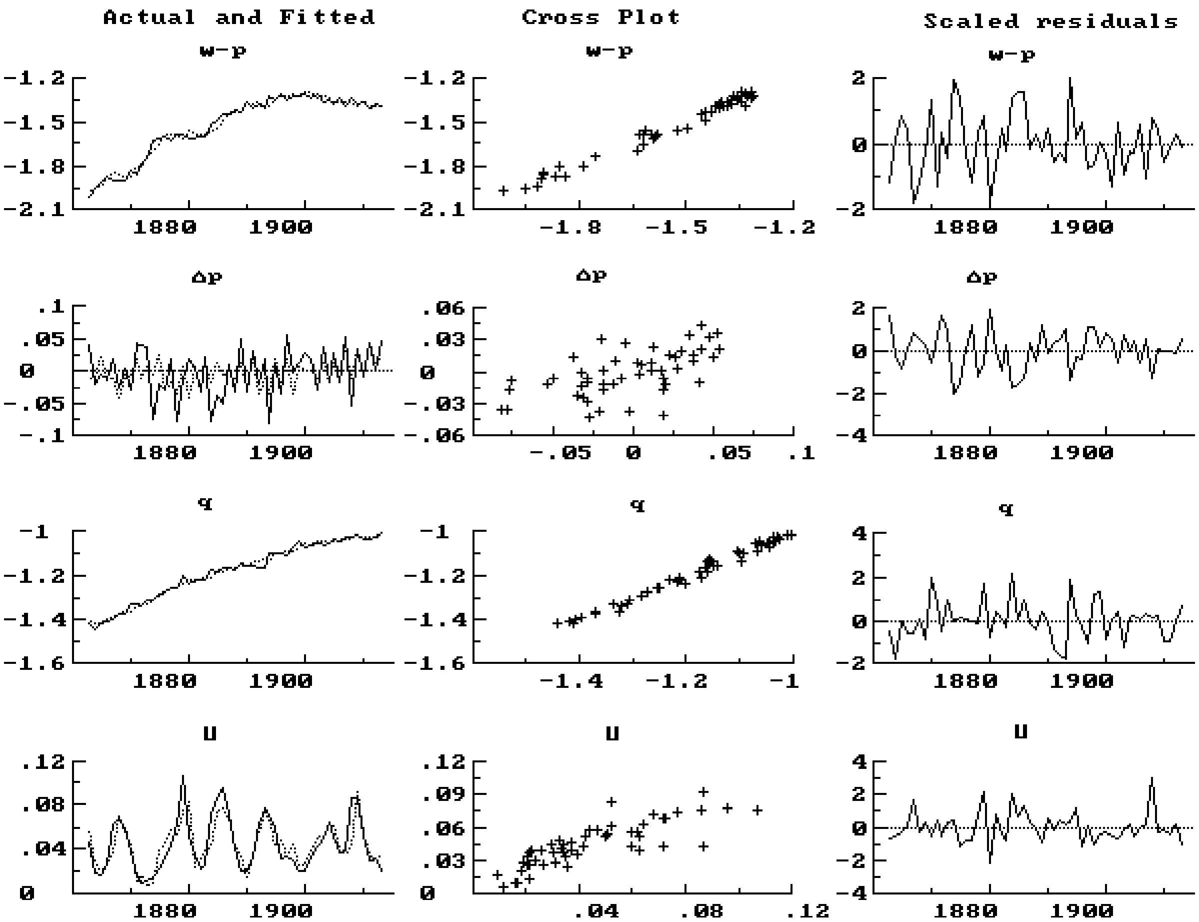

Figure 2: Actual and Fitted values and scaled residuals, 1863-1913.

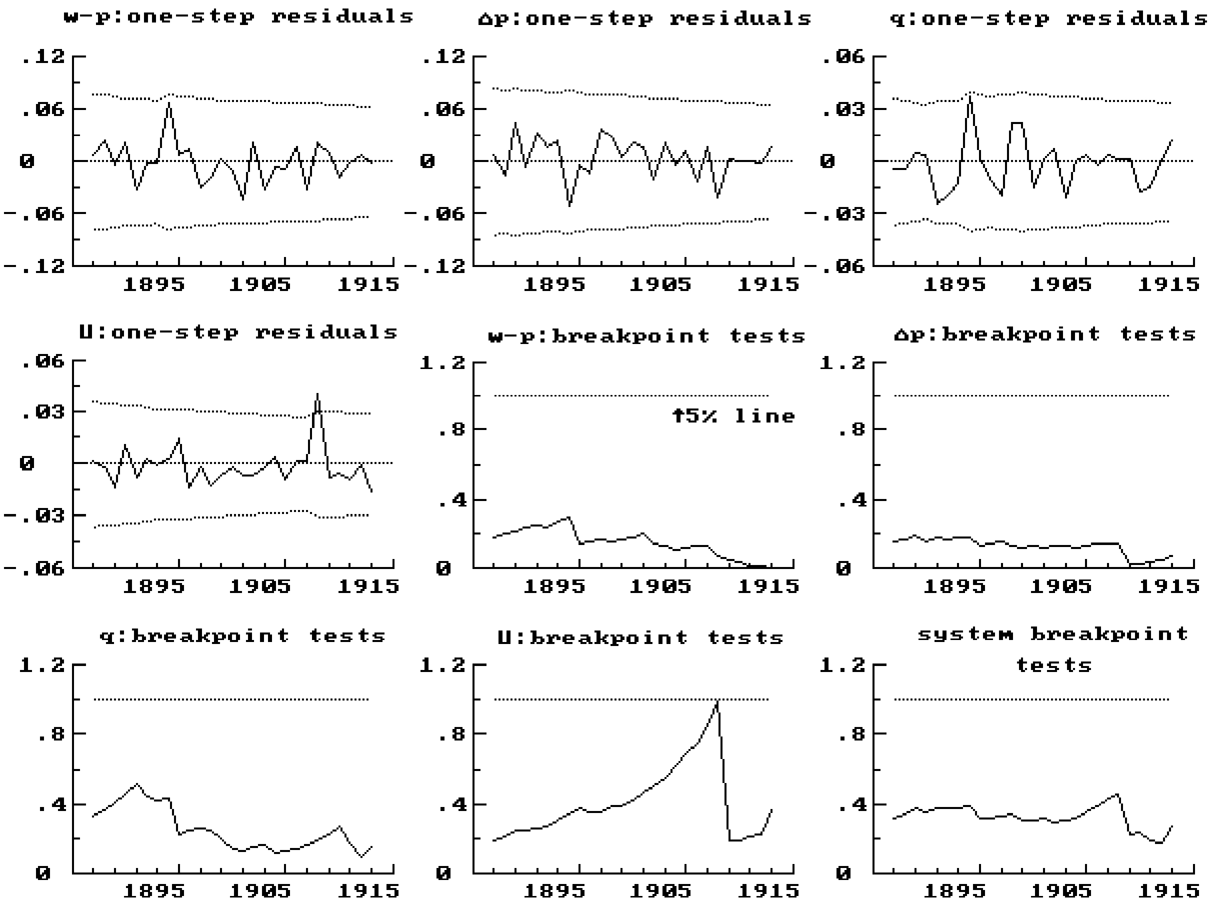

Figure 3: Recursive evaluation of the system, 1863-1913.

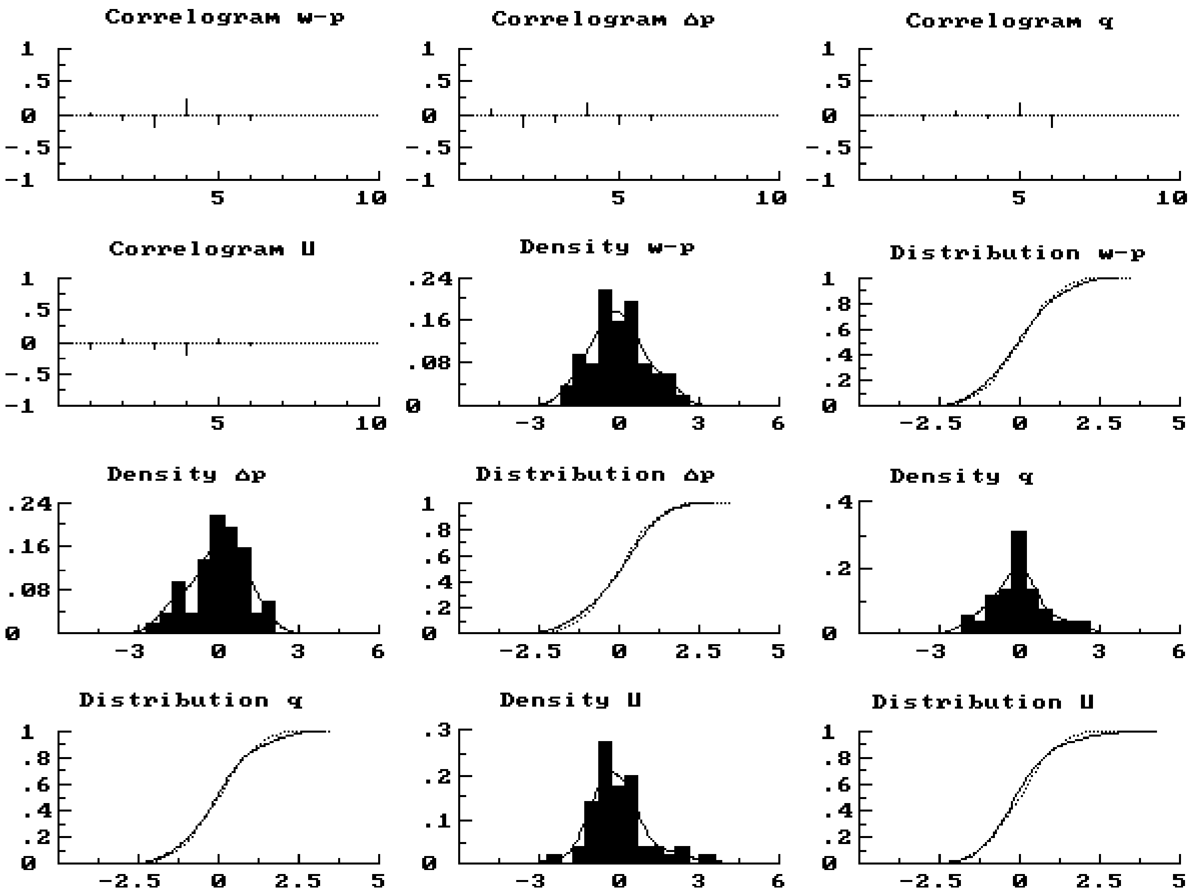

Figure 4: Graphical Diagnostics, 1863-1913.

3.2 Cointegration analysis.

The next step is to test for cointegration in the system. The trend is entered restricted to lie in the cointegration space, but the intercept is unrestricted. Table 2 gives the eigenvalues (μ), the associated maximal eigenvalue statistic (Max), as well as the trace statistic (Tr). These are adjusted for degrees of freedom, by multiplying by $(T - nk)/T$, where T is the sample size, n is the number of variables in the VAR, and k is the lag length (see Reimers (1992)). The critical values are from Osterwald-Lenum (1992). At the 5% significance level, we can clearly accept the hypothesis that there is one cointegrating vector or one stationary combination of the basic variables.

r	1	2	3	4
μ	0.52	0.34	0.21	0.12
Max	31.64*	18.22	10.4	5.5
Tr	65.76*	34.12	15.9	5.5

Table 2: Cointegration analysis, 1863-1913

Table 3 reports all eigenvectors of the system, the first row being the stationary component. The variables in the system have also been rearranged in the following order $(U, \Delta p, w - p, q)$, given that in fact it is more meaningful to normalise this vector by U. The loading factors α are also reported.

Vector\Variable	U	Δp	$w - p$	q	t
$\beta_1^{'}$	1.	0.82	0.05	-0.09	-0.00004
$\nu_2^{'}$	0.04	1.	0.17	-0.33	0.0003
$\nu_3^{'}$	0.80	-0.64	1.	-3.95	0.02
$\nu_4^{'}$	0.20	1.14	0.34	1.	-0.004

$\hat{\alpha}$	1	2	3	4
U	-0.79	0.27	0.001	-0.002
Δp	-0.02	-1.10	0.035	0.033
$w - p$	0.05	0.60	-0.071	-0.061
q	-0.10	0.15	0.107	-0.025

Table 3: Eigenvectors of the Π matrix and their loading factors.

Thus the cointegrating relation over this sample period suggests a definite negative effect from inflation onto the unemployment rate, with the real wage and productivity having very little effect. The estimated long-run relationship between inflation and unemployment over the period 1863-1913[8] is: $U = -.82\Delta p - 0.05(w-p) + 0.09q + .00004t$. The vector of adjustment coefficients with the rank of the Π matrix set to one shows that the main effect of this cointegrating vector is on U rather than on $\Delta(w-p)$ or $\Delta^2 p$. Figure 5 shows the estimated deviations from equilibrium for the cointegration vector, as well as the other components. It also portrays the behaviour of the eigenvalues, when estimated recursively. They all remain relatively stable over this period.

One important advantage of modelling a system is that it is possible to test the stationarity of individual series, having taken due account of the dynamics. Another important advantage of the Johansen maximum likelihood method is that it allows testing hypothesis about the long-run parameters, thus allowing them to be identified in a form that is interpretable by economic theory, as well as testing the weak exogeneity of various variables, at least for the long-run parameters. At this point therefore, we can test a number of interesting hypotheses. Starting with stationarity of the individual series, below are the list of hypotheses tested and their outcomes:

$H_0^1 : (0,0,1,0,a) \in Sp(\beta) \equiv$ real wages stationary	$\chi^2(3) = 30.78\,[0.00]^{**}$
$H_0^2 : (0,1,0,0,a) \in Sp(\beta) \equiv$ inflation rate stationary	$\chi^2(3) = 20.11\,[0.00]^{**}$
$H_0^3 : (0,0,0,1,a) \in Sp(\beta) \equiv$ productivity stationary	$\chi^2(3) = 30.68\,[0.00]^{**}$
$H_0^4 : (0,0,1,-1,a) \in Sp(\beta) \equiv$wage share stationary	$\chi^2(3) = 30.59\,[0.00]^{**}$
$H_0^5 : (1,0,0,0,a) \in Sp(\beta) \equiv$ unempl. rate stationary	$\chi^2(3) = 9.92\,[0.02]^{*}$
$H_0^6 : (1,0,0,0,0) \in Sp(\beta) \equiv$ unempl. rate stationary (no trend)	$\chi^2(4) = 9.95\,[0.04]^{*}$

and as for cointegration, we get:

$$
\begin{array}{ll}
H_0^7 : (1,a,0,0,0) \in Sp(\beta) & \chi^2(3) = 1.56\,[0.6680] \\
H_0^8 : (1,a,0,0,0) \in Sp(\beta) & \chi^2(6) = 3.00\,[0.8088] \\
\text{and } \alpha = 0 \text{ for } w-p,\ \Delta p,\ q. &
\end{array}
$$

To summarise, the likelihood ratio tests indicate that none of the series: real wages $w-p$, rate of inflation Δp, productivity q, or the wage share $w-p-q$ are stationary over the period 1862-1913. For the unemployment rate, on the other hand, the hypothesis that it is stationary, even without a trend, is rejected at 5% but cannot be rejected at the 1% significance level. One might be tempted to conclude that U is a stationary variable. Finally, the hypothesis H_0^7, where the cointegrating vector simplifies to a relation between the rate of inflation and the unemployment rate, cannot be rejected. Given that Δp is not stationary (H_0^2), the latter result provides further evidence against the stationarity of U, since stationarity of U together with H_0^7 or H_0^8 implies stationarity of Δp.

[8]Phillips' estimated equation was: $log(\dot{W} + 0.9) = 0.984 - 1.394 logU$

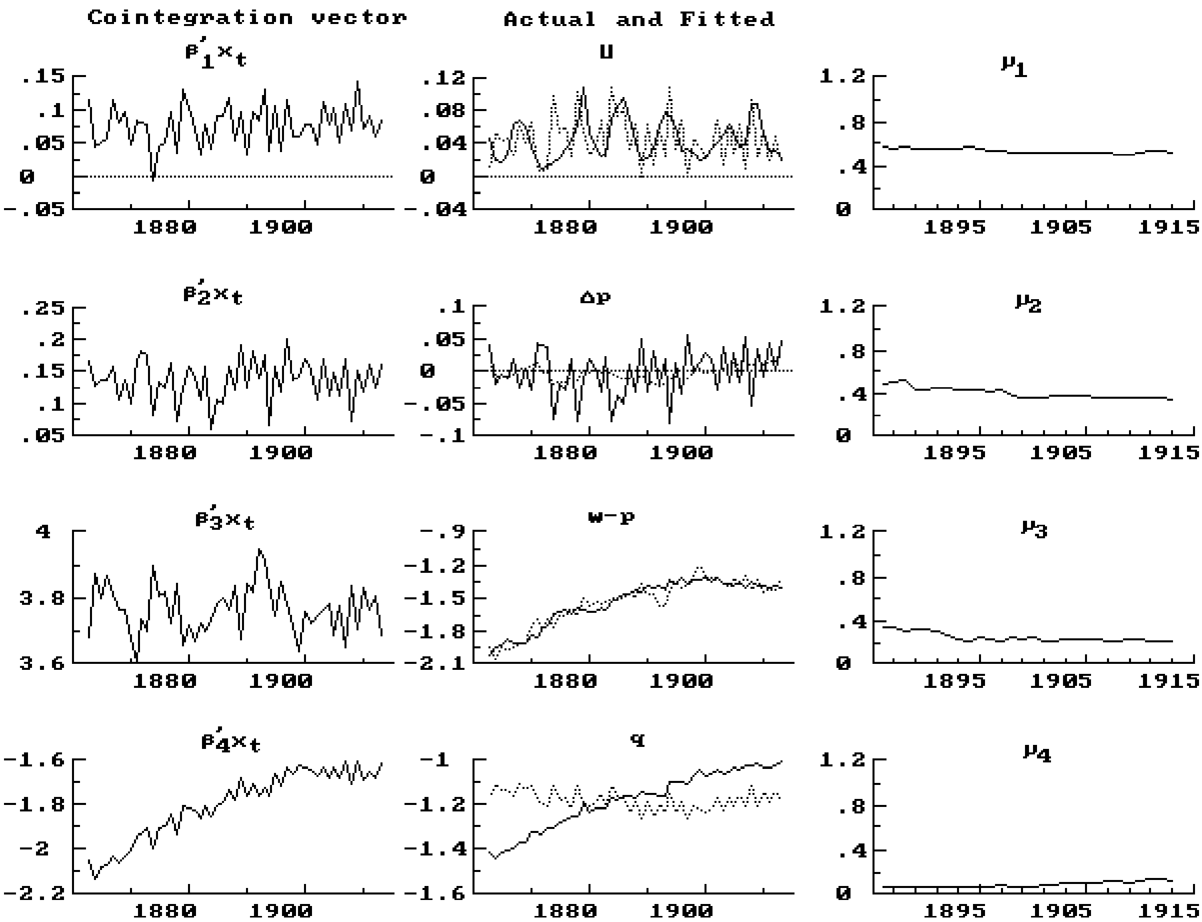

Figure 5: Cointegration vectors and recursive eigenvalues, 1863-1913

The graphs in the centre refer to, for example in the case of U, a comparison between the actual value of U and that obtained from the first cointegration vector, i.e. $-.82\Delta p - .05(w-p) + .09q + .00004t$.

Although the Johansen trace test indicates that in this period there is only one cointegration relation, the results obtained for the period 1868-1990 (Shadman-Mehta (1996)), showed the presence of a second cointegration relation as well. A closer look at table 3 also suggests that the second vector has a substantial impact, especially on the rate of inflation. Setting the cointegration rank to 2, and imposing overidentification restrictions on the cointegration vectors yields the following result:

$$H_0^9 \quad : \quad \beta^{'} = \begin{pmatrix} 1 & \beta_{21} & 0 & 0 & 0 \\ 0 & 1 & 0 & \beta_{42} & -.009\beta_{42} \end{pmatrix} \in Sp(\beta) \quad \chi^2(4) = 4.095[0.39]$$

Note that the coefficient of trend in the second vector is restricted such that the significant variable in this relation is the deviations of productivity from its long-run trend (the average value of Δq over this period is about 0.009, very close to average rate of productivity growth between 1860 and 1990). Finally, the coefficient of q was restricted to the value of -0.1063, obtained in the large sample, with the following outcome:

$$H_0^{10} \quad : \quad \beta^{'} = \begin{pmatrix} 1 & \beta_{21} & 0 & 0 & 0 \\ 0 & 1 & 0 & -0.1063 & +.009 * 0.1063 \end{pmatrix} \in Sp(\beta) \quad \chi^2(5) = 7.748[0.17]$$

The weak exogeneity of each variable may be considered next. This can be achieved by testing whether the adjustment coefficient or the α corresponding to each cointegration vector is zero in each equation. If no cointegration vector is present in the marginal distribution of a particular variable, this indicates that the variable may be treated as weakly exogenous, as far as the long-run parameters are concerned. In this case, sequential setting of various adjustment coefficients to zero leads to the conclusion that the hypothesis that the first vector appears only in the unemployment equation, whereas the second vector appears only in the inflation and the real wage equations cannot be rejected, with a test statistic $\chi^2(10) = 9.591[0.48]$.

4 Modelling the unemployment rate, 1860-1913.

Given this outcome, the data can be mapped to $I(0)$ space by defining the error correction mechanisms obtained under hypothesis H_0^{10}, that is:

$$\begin{aligned} c_{1t} &= U_t + 0.554\Delta p_t \qquad (6) \\ c_{2t} &= \Delta p_t - 0.1063(q_t - 0.0088t) \end{aligned}$$

The mapped data will then define a new system with six variables (ΔU_t,$\Delta^2 p_t$,$\Delta(w-p)_t$,Δq_t, c_{1t}, c_{2t}) where both c_{1t} and c_{2t} are identities and the maximal lag is 1. The hypotheses tests in the previous section led to the conclusion that the variables ($\Delta^2 p_t$,$\Delta(w-p)_t$,Δq_t) may be treated as weakly exogenous for the unemployment rate. This follows from the observation that the first cointegration vector which involves U is insignificant in the equations relating to the other three variables, and the second cointegration vector which is significant in the equations relating to the inflation and the real wage rates does not involve U. Therefore, as the necessary condition for the weak exogeneity of the last three variables of the system is satisfied, one could at this stage proceed with estimating the conditional model for U, without losing information which could jeopardise inference. The following results are nevertheless based on continuing with the complete system.

The initial step is to reestimate the new $I(0)$ system and verify its stationarity with a cointegration analysis. The rank of the system is indeed confirmed as 4. Similarly, the error correction term c_{1t-1} is significant only in the equation for U, and c_{2t-1} is significant in the equations for $\Delta^2 p_t$ and $\Delta(w-p)_t$. Removing them, as well as all the other insignificant variables, leads to the model reported in the table below [9].

$$
\begin{aligned}
\Delta U_t &= \underset{(0.100)}{0.556}\,\Delta U_{t-1} - \underset{(0.065)}{0.073}\,\Delta^2 p_t - \underset{(0.010)}{0.232}\,\Delta(w-p)_{t-1} - \underset{(0.103)}{0.327}\,\Delta q_{t-1} \\
&\quad - \underset{(0.104)}{0.717}\,c^a_{1t-1} + \underset{(0.002)}{0.005} \\
\Delta^2 p_t &= -\underset{(0.117)}{0.421}\,\Delta^2 p_{t-1} - \underset{(0.137)}{0.351}\,\Delta U_{t-1} - \underset{(0.189)}{0.853}\,c^a_{2t-1} \\
\Delta(w-p)_t &= \underset{(0.119)}{0.456}\,\Delta(w-p)_{t-1} + \underset{(0.122)}{0.407}\,\Delta^2 p_{t-1} + \underset{(0.144)}{0.271}\,\Delta q_{t-1} + \underset{(0.189)}{0.511}\,c^a_{2t-1} \\
\Delta q_t &= -\underset{(0.128)}{0.342}\,\Delta q_{t-1} + \underset{(0.003)}{0.011}
\end{aligned}
$$

Table 4: Model estimates (constrained $FIML$), 1863-1913

Table 5 summarises tests of model evaluation and misspecification. The hypotheses of interest include autocorrelation F_{ar}, autoregressive conditional heteroscedasticity ($ARCH$) F_{arch}, the normality of the distribution of the residuals χ^2_{nd}, heteroscedasticity F_{het}, and functional form misspecification F^{mod}_{func}. The $p-values$ for the model statistics are given inside brackets. The final statistic reported for the model is $\chi^2_{ov.ident}$ which tests the validity of the overidentifying restrictions. The model misspecification tests are all insignificant (except for normality which is insignificant only at 1%) thus confirming that the estimated model is a congruent model. Although real wages and productivity had no role to play in the long-run relationship, they both have a significant short-run effect on the unemployment rate over the period 1863-1913. The contemporaneous effect of the acceleration in the rate of inflation is not very significant, but the inclusion of this variable reduces residual correlations. As for

[9] c^a_{1t-1} and c^a_{2t-1} are the two cointegration vectors adjusted for their sample means.

the other variables, both the acceleration in the rate of inflation and the rate of growth

Statistic	$\Delta(w-p)$	$\Delta^2 p$	Δq	ΔU	*model*
$\hat{\sigma}$	0.032	0.033	0.019	0.013	
$F_{ar}(2,42)$	0.73	6.27^{**}	3.24^{*}	0.83	$F_{ar}^{mod}(32,134) = 1.02[0.44]$
$F_{arch}(1,42)$	0.01	0.60	0.50	0.04	
$F_{het}(12,31)$	1.04	0.84	1.30	0.68	$F_{het}^{mod}(120,208) = 1.04[0.40]$
$\chi^2_{nd}(2)$	1.43	3.22	1.85	7.76^{*}	$\chi^2_{nd}(8) = 18.64[0.02]^{*}$
					$F_{func}^{mod} = 0.72[0.98]$
					$\chi^2_{ov.ident}(11) = 6.35[0.85]$

Table 5: Model Evaluation Statistics, 1863-1913.

of productivity exhibit negative autocorrelation. Similar results are obtained for the acceleration in inflation both in the full sample (1868-1990) estimates (Shadman-Mehta (1996)) and in other studies (Hendry and Doornik (1994)). There is no autonomous growth either in the acceleration in the inflation rate or in the rate of growth of real wages over this period.

Residual	*correlations*			
	$\Delta(w-p)_t$	$\Delta^2 p_t$	Δq_t	ΔU_t
$\Delta(w-p)_t$	0.032	-0.84	0.42	0.26
$\Delta^2 p_t$	-0.83	0.033	-0.37	-0.45
Δq_t	0.42	-0.41	0.019	0.23
ΔU_t	0.10	-0.27	0.17	0.013

Table 6: Residual Correlations, 1863-1913.

Table 6 gives the matrix of residual correlations for this system. The diagonal terms give residual standard deviations. The error covariances below the diagonal are those between the structural residuals, and those above the diagonal are the reduced form correlations. The remaining high correlation of -0.83 between the residuals of the two equations for $\Delta^2 p_t$ and $\Delta(w-p)_t$, suggest that other important variables affecting both these variables over this period are missing from the analysis. Nevertheless, the model in table 4 does offer an explanation of the data features.

Although doubts about the potential non-constancy of the parameters of econometric equations under changed states of nature have a long history, it is the critique voiced by Lucas (1976) about the use of econometric models in general, and of the Phillips Curve in particular, for policy analysis that seems to have marked the literature. But the critique should be viewed as a potential denial of the invariance of the parameters of interest to a particular set of interventions. Only then can its applicability be tested meaningfully, since refuting it in one instance cannot rule out the possibility that it might be confirmed in other instances. Engle and Hendry (1993) propose tests of superexogeneity based on checking whether the parameters of a conditional model are invariant to changes in the parameters of the marginal processes. If the determinants of the non-constancies of those processes are statistically insignificant when added to the conditional model, then superexogeneity cannot be refuted for that particular instance. Practically, this can be achieved by the inclusion of dummies. Here, the inclusion of an impulse dummy in the system for the year 1874 improves the general fit of the model in the sense that it removes all the remaining problems of autocorrelation in the residuals of $\Delta^2 p$ and Δq as well as lack of normality. This dummy variable turns out to be very significant in both the real wage equation and the equation for the acceleration in the rate of inflation. But it is insignificant in the unemployment rate equation, and this fact together with the weak exogeneity of these variables confirms their super exogeneity for the unemployment rate equation over this particular sample. Thus, if $\Delta^2 p$ had been a proxy for expectations, the parameters of the unemployment rate equation should have manifested changes when the expectations process changed during the sample. The evidence therefore favours agents using data-based expectations which do not require further modelling.

5 Conclusions.

Allowing for the fact that the relation derived here is expressed slightly differently, in terms of U rather than $logU$ as Phillips had done, the results are remarkably close to his (see figures 6 and 7). In other words, if Phillips was conducting his analysis of the sample period 1862-1913, with the current developments in econometric theory, his overall conclusions would have been much the same. There existed indeed an apparent inverse relationship between the rate of inflation and the level of the unemployment rate.

However, the following remarks also result from such an analysis: Firstly, the unemployment rate in the U.K. in that period, was not an autonomous causal factor, but an endogenous variable of the economy. Thus Phillips' views regarding the intervention of government through demand management in order to stabilise the inflation rate would probably have been different.

Secondly, the cointegration relation depicted in figure 7 does not equate equilibrium unemployment with zero inflation, as suggested in much of the discussions around the Phillips Curve. Equilibrium, in the sense that there is no tendency for the unemployment rate to change, is concievable with both inflation and deflation. Any point on the line drawn in figure 7 can be an equilibrium unemployment rate. The points to the right of the line represent

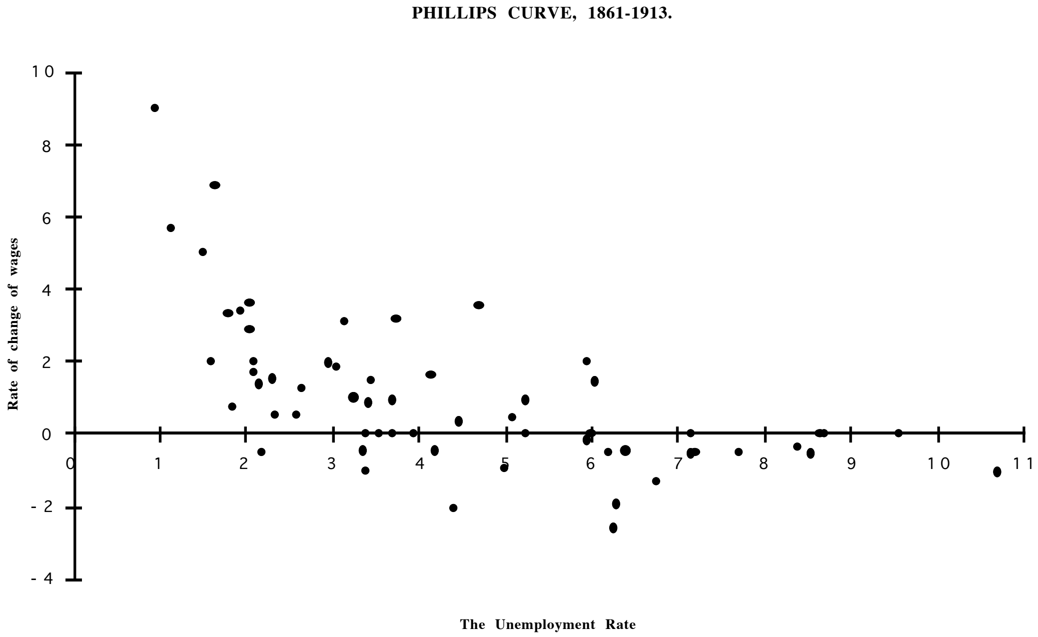

Figure 6: The variable on the vertical axis is $\dot{W}$. The encircled points are Phillips' six average points.

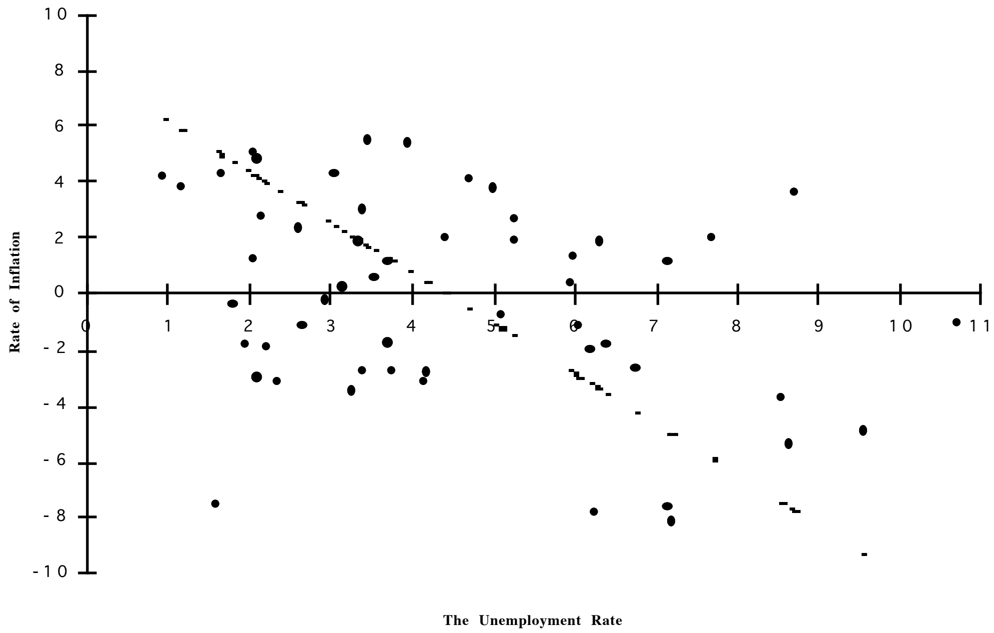

Figure 7: The cointegration relationship between the rate of inflation Δp and the unemployment rate in the U.K., 1863-1913.

situations where unemployment was too high for the existing level of inflation (or deflation). Similarly, the points to the left of the line represent cases where unemployment was too low for the given level of inflation (or deflation).

Thirdly, as far as inflation or the level of real wages are concerned, the level of the unemployment rate played no role in determining whether they remained stable or not. Their stability depended on discrepancies between the inflation rate and deviations of productivity from long-run trend. There is therefore no evidence to suggest that there was any long-term effect from the level of the unemployment rate to the level of real wages or the level of price inflation.

Further results can be summarised as follows. There existed a constant econometric equation for this sample period, relating the change in the unemployment rate to the acceleration in inflation, the rate of growth of real wages, and that of productivity, even though some of these determinants themselves were subject to shifts. Although inflation, real wages and productivity are themselves endogenous variables, their marginal density contains no information of relevance to the long-run parameters of the unemployment rate equation. They can therefore be treated as weakly exogenous. Moreover, the irrelevance of the deterministic step dummy in this equation, despite its importance in both the equations for the rate of growth of real wages and the change in inflation, confirms superexogeneity of these variables. This result provides evidence against the applicability of the Lucas critique as far as the parameters of interest of the unemployment rate equation for this period are concerned, and highlights the importance of actually testing those aspects of the critique that are testable.

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