MULTIDIMENSIONAL POVERTY MEASUREMENT WITHOUT MISSING POOR BIAS

Roberto Mauricio Sanchez-Torres

LIDAM Discussion Paper IRES
2026 / 19



UCLouvain

IRES | LIDAM
Louvain Institute of Data Analysis and Modeling in economics and statistics

Multidimensional poverty measurement without missing poor bias

Roberto Mauricio Sánchez-Torres*,†

Abstract

Lifespan is strongly shaped by standard of living. When premature mortality disproportionately affects the poor, conventional poverty measurements are likely to be underestimated, since they only account for the living, among whom poor groups are systematically underrepresented. The *missing poor* refers to individuals who died prematurely and who, had they survived, would have been counted among the poor. This research revisits the missing poor framework within the context of multidimensional poverty measurement, a particularly important extension given that mortality inequality has gradients beyond income alone. The objective of this study is to identify and assess the missing poor bias and its relationship with poverty as measured multidimensionally. It is shown that the missing poor bias is driven by differences in the survival probabilities of population subgroups, as well as by the weights assigned to each dimension in the poverty index. Empirical evidence from Cambodia, El Salvador and Zambia illustrates these theoretical findings, demonstrating that mortality inequality plays a crucial role in multidimensional poverty measurement, and that the missing poor bias is substantially larger in countries where mortality rates and their inequality are higher.

keywords: mortality paradox, multidimensional poverty, poverty measurement, premature mortality, selection biases.

JEL: I14, I32

Funding: This research has been funded by the FNRS WellCHANGE Program through the project "The Missing Poor: Measurement and Policy" (Ref.40020237).
Acknowledgments: I am especially grateful to Professor Gregory Ponthiere for his continuous support, insightful suggestions, and careful and numerous readings of this manuscript. I would also like to thank Jean-Marie Baland, Catherine Guirkinger, Rigas Oikonomou, William Parienté, Pierre Pestieau, Josselin Thuilliez, and the participants of research seminars at IRES UCLouvain and Université de Rennes for their valuable comments and discussions.

*IRES/LIDAM, Université Catholique de Louvain.
†Escuela de Economía, Universidad Nacional de Colombia.

1 Introduction

Poverty is broadly understood as the failure to achieve a basic standard of living within a given society at a particular point in time. The criteria for classifying a household as poor typically involve determining whether it meets a minimum level of economic welfare, as defined by the social context in which it is assessed. The measurement of poverty has conventionally addressed two core aspects: first, the identification of those considered poor; and second, their aggregation into a statistical or synthetic indicator that quantifies the extent of the problem (Sen, 1976; Ravallion, 2012).

The most conventional approach to measuring poverty is monetary, classifying individuals or households according to their income or consumption relative to a poverty line[1] (Ravallion, 2012). The simplest aggregation indicator is the headcount ratio (HR), representing the proportion of the population classified as poor[2]. However, income plays an instrumental role in poverty analysis and does not necessarily capture access to essential dimensions of basic well-being -such as housing, education and health- either due to consumption choices or institutional exclusion (Sen, 1999). Given this limitation, multidimensional poverty measurement emerges as an alternative framework that more comprehensively captures the aspects of individuals' and households' lives that influence well-being. Households are identified as multidimensionally poor when their deprivations across defined dimensions[3] exceed a given threshold[4], and the simplest aggregation indicator remains analogous to the HR.

The measurement of poverty has conventionally overlooked a crucial stylized fact: survival probabilities are substantially reduced for households at the bottom of the income and wealth distribution and with more deprivations in basic needs (Fogel, 2004; Van den Berg et al., 2006). When mortality disproportionately affects population groups with greater deprivations, reflecting Malthus' "positive population check", whereby adverse living conditions constrain population growth through higher mortality, poverty measurement is subject to a "mortality paradox": measured poverty appears lower precisely because mortality is higher among the poor (Lefebvre et al., 2013; Kanbur & Mukherjee, 2007). Consequently, poverty estimates at older ages are systematically biased downward, as those who died prematurely due to poor living conditions are excluded from measurement. These individuals have been termed the *missing poor*, following Sen's concept of *missing women* (Sen, 1990).

The objective of this research is to revisit the missing poor framework within the context of multidimensional poverty measurement, emphasizing that mortality has multiple gradients beyond income. The central question addressed is: how does differential mortality across population groups with heterogeneous living standards affect the measurement of multidimensional poverty? The analysis departs from the observation that differences in

[1] The threshold can be absolute or relative, and is regularly adjusted to the social and economic context.

[2] HR is a particular case of the FGT indices (Foster et al., 1984), which aggregate poor individuals using weights based on the distance between their income and the poverty line. HR assigns equal weight to every person below the threshold.

[3] A widely used approach was developed by the Oxford Poverty and Human Development Initiative [OPHI], whose Global Multidimensional Poverty Index considers access to health, education, and housing (Alkire et al., 2022).

[4] The *union* criterion classifies a household as poor if it is deprived in at least one dimension, whereas the *intersection* criterion requires deprivation in all dimensions; intermediate thresholds exist between these two extremes.

survival probabilities extend beyond income distribution (Cutler et al., 2006; Elo & Preston, 1996), and that other mortality gradients are essential to explaining premature death and, consequently, the bias in old-age poverty measurement. For instance, two households with equivalent income levels (at purchasing power parity) may differ substantially in survival probabilities if they face unequal access to healthcare, schooling, sanitation, and childcare[5]. Such disparities arise because prices are not always comparable across contexts, may reflect externalities, and are shaped by heterogeneous welfare state arrangements and institutional settings; moreover, market imperfections further limit the comparability of income across households[6] (Bourguignon & Chakravarty, 2003; Sen, 1997).

Multidimensional poverty and the missing poor are two closely interrelated topics. On the one hand, income is not the sole determinant of differential mortality across population groups; relative to the monetary approach, a multidimensional perspective may reveal mortality inequalities that remain concealed when income is not the primary mortality gradient. Therefore, a multidimensional poverty approach makes visible missing poor that remain hidden under the monetary perspective. For instance, when the access to services that are essential determinants of mortality, such as access to healthcare services, does not directly depend on income[7], inequalities that drive mortality differences, and thus part of the missing poor, remain undetected under a purely monetary framework.

On the other hand, accounting for the missing poor in multidimensional poverty measurement corrects the bias introduced by differential premature mortality between poor and non-poor groups. When poverty measurement is subject to the mortality paradox, a recorded decline in poverty may be erroneously attributed to improved living conditions, when it instead reflects the deaths of the most deprived. This is particularly relevant when comparing multidimensional poverty across countries or regions and over time[8]. Given that income is not always a reliable basis for comparing living standards[9] and that mortality inequality operates along multiple gradients[10], a multidimensional approach provides a more comprehensive understanding of the relationship between mortality and poverty, while also improving the

[5]Sen (1998, 1999) shows that life expectancy among African-American men is lower than among rural populations in India, despite considerably higher income levels in the former group.

[6]For instance, price discrimination in essential housing services, such as electricity tariffs, means that the same nominal income can translate into different levels of access, rendering direct comparisons across households unreliable.

[7]This may occur where hospitals, health facilities, or medical personnel are absent in certain areas, or where individuals are excluded from the healthcare system due to migrant legal status or informal employment.

[8]Multidimensional poverty measurement may be biased even when its monetary counterpart is not, specifically when income-mortality differentials are negligible yet other mortality gradients more adequately explain lifespan differences between deprived and non-deprived populations across the dimensions included in the poverty measure.

[9]Externalities, imperfect markets, and heterogeneous welfare state arrangements, among other institutional factors, render income an inaccurate variable for comparing living standards and poverty.

[10]Nutrition (Fogel, 2004), medical treatment and vaccination (Cutler, 2004), public health infrastructure (Cutler & Miller, 2005), social and ecological environment (Van den Berg et al., 2006), education (Lleras-Muney, 2005; Pamuk et al., 1998), and lifestyle (Cutler et al., 2006) have been identified as essential determinants of mortality differentials. In sum, "individuals with low income, low health, low education or low social status often die younger than those who are better off" (Cutler et al., 2006), indicating that mortality differences extend well beyond income distribution.

comparability of quality of life analyses.

To illustrate the usefulness of the multidimensional analysis, Figure **1** presents a hypothetical scenario in which survival probabilities are shown for groups deprived and non-deprived across two dimensions (income and education) in countries A and B. It depicts the relevance of the multidimensional approach both for the identification and counting of the missing poor, and for a more precise measurement of multidimensional poverty. Country A exhibits a substantial inequality in survival probability between income-deprived and non-deprived populations (i.e., those below and above the poverty line), whereas in Country B survival probability is roughly uniform regardless of monetary poverty status. Consequently, it may be concluded that Country B has no missing poor. Nevertheless, the opposite would hold if education, rather than income, were the dimension considered in the poverty measure. Therefore, measuring poverty beyond income may reveal missing poor and survival differences that would otherwise remain hidden under a purely monetary approach.

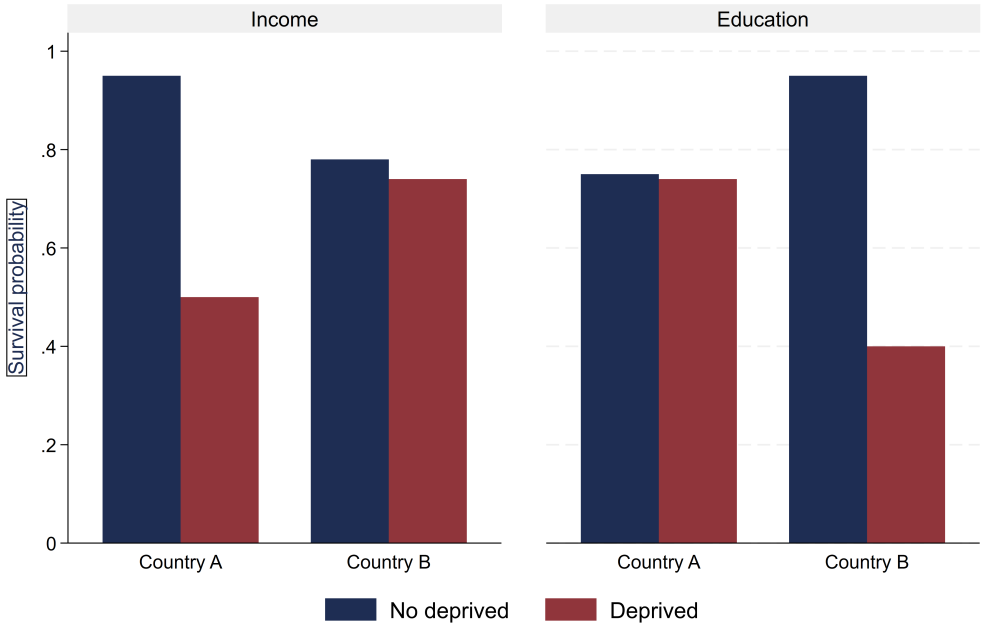
The missing poor analysis not only exposes biases in poverty measurement but also enhances the comparability of poverty indicators across countries. Relying on a single dimension to explain the missing poor problem is inherently limited, given the multiple gradients of mortality; as noted earlier, no single variable such as income can fully capture mortality differences across population groups. Classifying the population into groups based on the number and type of deprivations, and accounting for the resulting heterogeneities in survival probabilities, yields more comparable poverty indicators. As illustrated in Figure **1**, when poverty is measured along both income and education dimensions, and the identification criterion requires at least one deprivation, the average survival probabilities of poor and non-poor groups would be similar in both countries[11], while if it is considered only monetary poverty survival of poor group would be lower in country A, and for non-poor it would be lower en country B. Therefore, correcting for the missing poor bias produces a more comparable measure than one based on income alone. This underscores the central argument of the analysis: mortality inequality is driven by multiple gradients that a unidimensional approach fails to capture.

This research focuses on the role of multidimensional poverty in the identification and measurement of the missing poor. It includes a theoretical model explaining the mechanism to identify, count, and measure poverty without the missing poor bias; and an empirical analysis that demonstrates the correction of the missing poor bias in the measurement of multidimensional poverty.

Regarding the theoretical approach, a two-period model in which individuals are young or old is set up. The population is classified based on deprivation across the dimensions considered in the measurement of poverty (Alkire & Foster, 2011). Each group has a different survival probability, and old-age population is made up of survivors from young to old age (Lefebvre et al., 2013); additionally, individuals can change population group given their multidimensional mobility. The key feature of the analysis is survival differences across population groups, therefore, poverty is underestimated if survival rates are lower for groups classified as poor. To identify missing population and correct the poverty indicator, a hypo-

[11]Incorporating additional dimensions into poverty measurement and broadening comparability across countries, regions, and over time would introduce greater heterogeneity, underscoring the importance of adjusting for the missing poor bias.

Figure 1. Survival probabilities for deprived and no deprived groups in two dimensions of well-being



thetical transition matrix is proposed for those who did not survive, and a baseline scenario is considered in the adjustment of the poverty measurement. The missing poor bias [MPB] is the result of comparing the observed and adjusted poverty measures.

In relation to the empirical evidence, it shows the lack of comparability in conventional measurement of multidimensional poverty and underscore the importance of considering different gradients of mortality inequalities. The analysis of the case study estimates multidimensional poverty, classifies population based on deprivations, and computes life tables for each population group. The results reveal the importance of the weighting setting and differences in survival probabilities to adjust the measurement of poverty and compute the MPB.

The main results of this research focus on the MPB in the context of multidimensional poverty. It is shown theoretically and empirically that the MPB is a function of weights and, therefore, of the dimensions included in the analysis and their assigned relevance. Furthermore, the marginal effect of survival on the MPB depends on the conditions of dimensional mobility; particularly, it is not only important if the prematurely deceased individual was poor before dying, but also what the probability of being poor in old age would have been if she had survived. Additionally, the empirical evidence suggests that multidimensional poverty measurement is systematically underestimated due to mortality inequality.

The mortality paradox and the MPB have been analyzed previously under a unidimensional/monetary framework (Kanbur & Mukherjee, 2007; Lefebvre et al. 2013, 2019), and considering certain scenarios of hypothetical income mobility for prematurely dead individuals (Baland et al. 2021; Lefebvre et al. 2013, 2019). Baland et al. (2021, 2022) include premature mortality as an intrinsic deprivation and assume that everyone who dies prematurely is considered a missing poor, whereas similar assumption is adopted by Lefebvre et

al. (2013), based on a welfare-neutral fictitious income which makes individuals indifferent between being alive with that income and death. Baland et al. (2021) propose that people are poor if they have a shorter lifespan, which can also be interpreted as including longevity as a dimension/characteristic in the measurement of poverty; hence, the missing population is equal to the missing poor. This is a criterion subject to the normative position related to the comparable value of death. Other alternatives exist, such as assuming that those who died would have had the same level of income as before dying, regardless if they were poor or not (Kanbur & Mukherjee, 2007), or assuming that the missing population would have had the same income mobility as the population group to which they belonged when alive (Lefebvre et al., 2019).

This research contributes to this literature by analizing a multidimensional measurement of poverty, given that income is a limited variable for comparing poverty in countries with different provision of public goods and the role of the welfare state. It also proposes an adjustment in the measurement of poverty and the size of the MPB based on the hypothetical dimensional mobility of prematurely dead population, namely, assuming that they would have had the same mobility of the population group they belonged.

The paper is organized as follows. Section 2 presents the basic framework of the proposed two-period model for analyzing population dynamics and introduces the multidimensional poverty setting. Section 3 develops the identification and measurement of the missing poor, presents the suggested correction in the measurement of multidimensional poverty, and concludes with propositions regarding the MPB. Section 4 analyzes the implications for comparability in multidimensional poverty measurement when differences in survival probabilities exist and when dimensions have different levels of importance in explaining mortality inequalities. The empirical analysis is presented in section 5, while conclusions and final remarks are provided in section 6.

2 Model

2.1 Basic framework

The model considers two periods: when individuals are young and when they are old[12]. Some individuals die prematurely and not achieve old age, being deprived of a long life. Consequently, the size of each population group can differ between periods. Vectors N^Y and N^O (1) indicate the size of population groups for young-age population and old-age population, respectively. Each element of these vectors represents the number of young (N_i^Y) or old (N_i^O) individuals[13] that belong to population group i.

[12]It is therefore the population dynamics of one generation in two periods.

[13]Although poverty is identified at the household level, the size of each group is given by the number of individuals. A common practice in the measurement of poverty is to identify poverty at the household level, while aggregating at the individual level, given the potential underestimation of poverty when —as is frequently the case— poor households are larger.

$$N^Y = \begin{bmatrix} N_1^Y \\ N_2^Y \\ \vdots \\ N_{2^J}^Y \end{bmatrix}, \quad N^O = \begin{bmatrix} N_1^O \\ N_2^O \\ \vdots \\ N_{2^J}^O \end{bmatrix} \tag{1}$$

The standard of living of the population is measured based on **J** dimensions, which represent a normative set that, following Sen (1985, 1999), allows individuals to develop their capabilities to achieve better functionings in society[14]. In particular, in this research, poverty is defined as deprivation in those dimensions (see Section 2.2). Individuals can be deprived[15] or not in each dimension; hence, each of them can be dichotomized if there is (=1) or not (=0) deprivation. To formally characterize the multidimensional deprivation structure of the population, it is useful to define the deprivation profile associated with each population group. This profile summarizes the presence or absence of deprivation across the J dimensions included in the analysis.

Definition 1 (Deprivation profile) *The deprivation profile associated with population group i is a binary vector*

$$d_i = (g_{i1}, g_{i2}, \ldots, g_{iJ}) \in \{0,1\}^J$$

where $g_{ij} = 1$ indicates that population group i is deprived in dimension j, and $g_{ij} = 0$ otherwise, $j \in \{1, ..., J\}$.

The population is classified into different groups (for each period) based on the number and type of deprivations. A population group i is the set of individuals sharing the same *deprivation profile* d_i. Each deprivation profile represents a unique combination of deprivations across the J dimensions included in the analysis. Since each dimension can take only two values (deprived or non-deprived), the total number of possible deprivation profiles is 2^J, which determines the total number of population groups in the model ($i \in \{1, ..., 2^J\}$). The deprivation matrix $\mathbf{G} \in \{0,1\}^{2^J \times J}$ (2) summarizes how deprivation and non-deprivation across J dimensions are distributed in the population.

$$G = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_{2^J} \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1J} \\ g_{21} & g_{22} & \cdots & g_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ g_{2^J 1} & g_{2^J 2} & \cdots & g_{2^J J} \end{bmatrix} \tag{2}$$

Each row in the matrix represents the deprivation profile of a population group. The *uniqueness* of each group refers to the fact that no other group shares the same set of deprivations. This criterion for dividing the population into groups holds regardless of time. Population groups are identified based on the deprivation profile, and it does not change

[14] In this paper, the discussion about which dimensions must be included is not strictly relevant; rather, the theoretical focus relates to the fact that some of those dimensions are simultaneously gradients of mortality.

[15] That is, they do not achieve a minimum level of access to a given dimension.

across the time. However, individuals/households can change their number and type of deprivations, and thus the deprivation profile and the population group that they belong. Formally, an individual initially in group i experiences mobility if she belongs to group j $(i \neq j)$ when old.

As mentioned above, some individuals die prematurely and do not achieve old age. Given the focus of this research, attention is directed toward the differences in survival probabilities across population groups and, therefore, their link with multidimensional deprivation. For each population group i, $\pi_i \in [0,1]$ represents the probability that an individual belonging to group i when young survives when old. The vector $\Pi = (\pi_1, \pi_2, ..., \pi_{2^J})' \in [0,1]^{2^J}$, contains the survival probabilities of population groups, it captures the heterogeneity in mortality across groups with different deprivation profile[16].

$$\Pi = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_{2^J} \end{bmatrix} \tag{3}$$

Old-age population

The distribution of the young-age population across different groups is determined by the presence or absence of deprivation across the **J** dimensions. A young individual may or may not survive; therefore, in this framework, the old-age population may be smaller when some individuals die between periods. The focus of this research is on the measurement of poverty among the old-age population and the role of mortality inequality in the missing poor bias; hence, a two-period analysis is the most straightforward way to understand the issue at stake[17].

The model assumes the following sequence between two periods. First, individuals belong to a population group determined by their deprivation profile in when they are young, and multidimensional poverty is identified according to the deprivation structure observed in that period. Second, individuals survive from young to old age with group-specific probability π_i. Third, conditional on survival, individuals can transition between population groups according to the probabilities of transition (δ_{ij}). Finally, multidimensional poverty status of old-age population is determined by the deprivation profile associated with the destination group.

In addition to mortality inequality, the distribution of population groups for old population also depends on multidimensional mobility between periods. Any household/individual can change group (dimensional mobility) from young to old age if the number or type of dimensions in which they have deprivation changes. The change in the aggregate deprivation profile of survivors, or how population moves from one group to another between periods is

[16] π_i can be interpreted as the proportion of individuals in population group i who are expected to survive between periods.

[17] Accordingly, this basic two-period model does not consider new generations, fertility differentials, population growth, or migration.

summarized by the transition probability.

Definition 2 (Dimensional transition probability of survivors) *The parameter* $\boldsymbol{\delta_{ij}}$ *represents the conditional probability that an individual belonging to group* i *when young belongs to group* j *when old, conditional on survival. Additionally,*

$$\sum_{j=1}^{2^J} \delta_{ij} = 1 \quad \forall\, i \in \{1, ..., 2^J\}.$$

The size of each old-age group N_i^O is equal to:

$$N_i^O = \pi_1 N_1^Y \delta_{1i} + \pi_2 N_2^Y \delta_{2i} + ... + \pi_{2^J} N_{2^J}^Y \delta_{2^J i} \tag{4}$$

Where π_i is the survival probability, δ_{ij} is the conditional probability that an individual belonging to group i when young belongs to group j when old, conditional on survival, and N_i^Y the size of group i of young population. N_i^O is basically the result of survivors who stayed in the group i plus those who moved from any other population group to i when old. The total old-age population is equal to the total surviving young-age population: $\sum_{i=1}^{2^J} N_i^O = \sum_{i=1}^{2^J} \pi_i N_i^Y$. If dimensional mobility is lower, the survival probability of each group is more relevant to explain their size in the period 2. Indeed, in the absence of mobility ($\delta_{ii} = 1$ and $\delta_{ij} = 0, \forall i \neq j$), the role of mortality is independent of mobility, and the size of each old-age group depends only on the survival probability: $N_i^O = \pi_i N_i^Y$.

Vector N^O indicates the size of population groups in old age, and it is equal to:

$$N^O = \Psi' \cdot N^Y = \begin{bmatrix} \pi_1\delta_{11} & \pi_2\delta_{21} \dots & \pi_{2^J}\delta_{2^J 1} \\ \pi_1\delta_{12} & \pi_2\delta_{22} \dots & \pi_{2^J}\delta_{2^J 2} \\ \vdots & \vdots & \vdots \\ \pi_1\delta_{12^J} & \pi_2\delta_{22^J} \dots & \pi_{2^J}\delta_{2^J 2^J} \end{bmatrix} \cdot \begin{bmatrix} N_1^Y \\ N_2^Y \\ \vdots \\ N_J^Y \end{bmatrix} \tag{5}$$

The transition matrix $\Psi \in \mathbb{R}^{2^J \times 2^J}$ describes the survival-weighted transitions of individuals across population groups between periods. The vectors $N^Y, N^O \in \mathbb{R}^{2^J}$ are column vectors containing the size of each population group for young and old-age population, respectively.

2.2 Multidimensional poverty measurement

The multidimensional measurement of poverty classifies households according to the presence or absence of deprivation across dimensions considered essential for a basic standard of living, and then compares the degree —number and type— of deprivation with a multidimensional threshold to classify the population as poor or non-poor[18].

As explained above, in this model the population is classified precisely according to the presence and absence of deprivation across **J** dimensions. Following the most standard approach to the measurement of multidimensional poverty, the Multidimensional Poverty Index

[18]Monetary poverty measurement is a particular case, in which only one dimension is weighted to classify the population and the "multidimensional" threshold is equal to 1 (completely deprived in the single dimension included).

(MPI) will be adopted, which is essentially a weighted sum of deprivations[19]. To compute the MPI, it is necessary to know the number and type of deprivations of households/individuals and the weight assigned to each dimension in the measurement. Matrix $\mathbf{G}$ captures the first element, while the assignment of weights is a normative matter.

While in applied research dimensions are conventionally equally weighted (Atkinson et al., 2002), the relative importance of different dimensions in measuring poverty is not necessarily the same, and therefore some criterion is needed to weight them differently[20]. For instance, Battiston et al. (2013) proposed a weighting scheme based on the subjective valuations of the poor regarding the dimensions they would prioritize for improving their standard of living, and this approach was further formalized by Decanq et al. (2019). In this theoretical analysis, the weighting of dimensions determines the degree of poverty given the type of deprivations as well as the identification of poor and non-poor households/ individuals. However, this research is not strictly about the measurement of poverty, but rather about how it is biased by mortality inequalities; hence, the findings are much more general and the correction applies regardless of the criterion adopted concerning dimensions and their weighting in the measurement[21].

Nonetheless, any normative criterion for assigning weights in the measurement of multidimensional poverty is feasible within the framework of the model[22]. Consider that dimension j has weight w_j, $w_j \in [0,1]$; and $\sum_{i=1}^{J} w_i = 1$. The vector $\mathbf{W}$ (6), $W = (w_1, w_2, ..., w_J)' \in [0,1]^J$, includes the weights for each dimension.

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_J \end{bmatrix} \tag{6}$$

The MPI is then the resulting vector of the dot product of G and W (7). mpi_i is the computed multidimensional poverty index for the group i, and it is equal to: $mpi_i =$

[19]The conventional multidimensional poverty measure defines the MPI as the headcount ratio of the multidimensionally poor adjusted by their average intensity of deprivation. This research, however, focuses on the counting problem and the resulting misestimation of the poverty headcount. Further research could examine the implications of mortality-related mismeasurement for adjusted measures that also incorporate the depth and severity of deprivation, such as those proposed by Alkire & Foster (2011).

[20]As in the conventional measurement of monetary poverty, where the same poverty line is applied to younger and older populations, the weights assigned to the dimensions of multidimensional poverty are assumed to be independent of the age composition of households. This assumption is maintained throughout the analysis. Nevertheless, the subjective and objective relevance of certain dimensions, such as education and health, may differ substantially across age groups. Examining how age-specific weights could be incorporated into multidimensional poverty measurement is left for future research.

[21]As highlighted in the following sections, two basic conditions must be satisfied. First, the same dimensions and weights must be used for comparability; otherwise, any comparison is meaningless and the bias correction does not address this issue. Second, the empirical analysis should, as noted, consider ordinal robustness; consequently, to draw stronger conclusions, different dimensions and weights should be used to guarantee a valid time-space comparison.

[22]The role of the weighting structure in the measurement of poverty and the MPB within a multidimensional framework is formalized later (proposition 2).

$g_{i1}w_1 + g_{i2}w_2 + ... + g_{iJ}w_J$.

$$MPI = G \cdot W = \begin{bmatrix} mpi_1 \\ mpi_2 \\ \vdots \\ mpi_{2^J} \end{bmatrix} \tag{7}$$

$mpi_i \in [0, 1]$ is the MPI of the population group i whose size is N_i. This structure includes the simplest case of uni-dimensional measurement of poverty (monetary approach, $w_1 = 1$) when there are only two groups (and two values for the MPI) those who belong to the group of people whose income (the unique dimension) is lower than the poverty line (poor group, $MPI = 1$) and those who have income higher than the threshold (non-poor group, $MPI = 0$).

A population group has the same MPI in each period because it preserves the same deprivation profile, but its size change between periods given the mortality and multidimensional mobility (5). Vectors N^Y and N^O of size 2^J include the young and old population, respectively, distributed in the different values of the MPI, then each cell N_i^Y and N_i^O is equal to the number of individuals with the same deprivation profile and the same mpi_i, in the corresponding period.

The identification of the multidimensionally poor population is determined by a multidimensional threshold $mpi_T \in (0, 1]$, the precise value is absolutely subjected to discussion[23] but the model is flexible enough to accept any value, affecting solely the identification of poor people given their level of deprivation[24] (Alkire & Foster, 2011). Indeed, it is frequent to measure multidimensional poverty considering several multidimensional thresholds to show ordinal robustness (Alkire & Santos, 2014). The identification of poor population is given by the multidimensional poverty indicator function:

$$p_i = 1(mpi_i >= mpi_T) \tag{8}$$

The population group, whose deprivation profile is d_i, is poor ($p_i = 1$) if its MPI is higher or equal than the multidimensional threshold, it means, if their synthetic measurement of multidimensional deprivation is higher o equal than the normative minimum to be considered poor. Consider the vector P whose elements are the identifying poverty value of each population group, then each element p_i of the vector P is given by the function (8), $P = (p_1, p_2, ..., p_{2^J})' \in \{0, 1\}^{2^J}$. Therefore, the multidimensional poor young-age population is $P'N^Y$ and the multidimensional young-age poverty headcount ratio is equal to:

$$H^Y = \frac{P'N^Y}{I'N^Y} = \frac{\sum_{i=1}^{2^J} p_i N_i^Y}{\sum_{i=1}^{2^J} N_i^Y} \tag{9}$$

[23]It has conventionally been a value between 0.3 and 0.5.

[24]To clarify, "solely" means that whatever the multidimensional poverty threshold is, the correction by missing poor bias will adjust accordingly, and the resulting measurement of the model can be easily adjusted to the new identification. However, the size of the adjusted measurement of poverty and the MPB would be different when the threshold change.

Being I the all-ones vector, $I = (1, 1, ..., 1)' \in \mathbb{R}^{2^J}$. H^Y is essentially the ratio of individuals whose MPI is greater than or equal to the multidimensional threshold to the total population. The population dynamics between periods are summarized in (5). There is upward (downward) multidimensional mobility between periods, if a household moves from population group i to population group j such that $mpi_i > mpi_j$ ($mpi_i < mpi_j$). Given the multidimensional mobility and the differences in survival probabilities, the number and proportion of poor individuals at young age and old age are expected to differ.

The following definition introduces an important feature to analyze the composition of old-age poverty in the model.

Definition 3 (Old-age poverty risk) *For each young population group $i \in \{1, \ldots, 2^J\}$, define*

$$\tau_i \equiv \sum_{j=1}^{2^J} p_j \delta_{ij}, \tag{10}$$

The quantity $\tau_i \in [0, 1]$ therefore represents the conditional probability that a survival individual belonging to population group i when young is poor when old.

Considering observed data (deterministic), τ_i is the proportion of young people from the group i that ends up multidimensionally poor when old.

The old-age headcount ratio of multidimensional poverty is analogous to (9), but accounts for survival probabilities and multidimensional mobility. The old-age multidimensional poverty headcount ratio is equal to:

$$H^O = \frac{P'N^O}{I'N^O} = \frac{P'\Psi'N^Y}{I'\Psi'N^Y} = \frac{\sum_{j=1}^{2^J}\sum_{i=1}^{2^J} p_j \pi_i \delta_{ij} N_i^Y}{\sum_{i=1}^{2^J} \pi_i N_i^Y} = \frac{\sum_{i=1}^{2^J} \tau_i \pi_i N_i^Y}{\sum_{i=1}^{2^J} \pi_i N_i^Y} \tag{11}$$

The numerator in (11) is the sum of every surviving individual who, independently of the population group to which they belonged when young, ends up when old in a group whose MPI is greater than or equal to the multidimensional threshold. The denominator is simply the total old-age surviving population. H^O depends on survival probabilities if there is, at least, one population group i whose π_i differs from that of another group ($\exists i \wedge j \mid \pi_i \neq \pi_j$). Since the central objective of this research is to analyze how mortality inequalities across population groups affect the measurement of multidimensional poverty, it is useful to consider the particular case in which survival probabilities are homogeneous across groups.

Lemma 1 (Old-age multidimensional poverty and homogeneous survival probabilities) *If survival probabilities are homogeneous across population groups, such that*

$$\pi_i = \pi^* \quad \forall i \in \{1, \ldots, 2^J\},$$

then the old-age multidimensional poverty headcount ratio becomes

$$H^O = \frac{\sum_{i=1}^{2^J} \tau_i N_i^Y}{\sum_{i=1}^{2^J} N_i^Y}. \tag{12}$$

Hence, old-age multidimensional poverty is independent of differential mortality across population groups.

Beyond this benchmark, the fact that H^O (11) actually depends on survival probabilities is a crucial feature because it explains how this measurement is subjected to the "mortality paradox". The *lemma 2* formalizes this:

Lemma 2 (Mortality paradox and old-age multidimensional poverty measurement) *Consider the old-age multidimensional poverty headcount ratio given by equation (11). The marginal effect of π_i on aggregate old-age poverty is:*

$$\frac{\partial H^O}{\partial \pi_i} > 0 \qquad \textit{if and only if} \qquad \tau_i > H^O,$$
$$\frac{\partial H^O}{\partial \pi_i} < 0 \qquad \textit{if and only if} \qquad \tau_i < H^O.$$

Proof: *See Appendix 1: Proof of Lemma 2.*

The lemma 2 shows that a marginal increase in the survival probability of group i raises aggregate old-age poverty whenever the group's probability of becoming poor in old age exceeds the aggregate poverty rate ($\tau_i > H^O$), or analogously, poverty indicator declines when mortality (survival) is higher (lower) for population groups that are more likely to be poor when old. Conversely, increasing the survival probability of a group with below-average old-age poverty risk decreases aggregate poverty ($\tau_i < H^O$). Hence, mortality inequality affects aggregate poverty through heterogeneity in intertemporal poverty risk across population groups.

Corollary 1 (No mobility across population groups) *Suppose there is no dimensional mobility across population groups, so that $\delta_{ii} = 1$, $\delta_{ij} = 0$ for all $i \neq j$. The marginal effect of π_i on H^O is:*

$$\frac{\partial H^O}{\partial \pi_i} > 0 \qquad \textit{if and only if} \qquad p_i = 1,$$
$$\frac{\partial H^O}{\partial \pi_i} < 0 \qquad \textit{if and only if} \qquad p_i = 0.$$

Corollary 1 illustrates a particular case in which there is no dimensional mobility. In this scenario, the mortality paradox becomes evident: old-age multidimensional poverty decreases as the probability of dying rises for the poor population (individuals who are poor when young and, given the absence of mobility between periods, would have remained poor when old). In sum, Lemma 2 and Corollary 1 show that the conventional measurement of old-age multidimensional poverty is subject to the mortality paradox.

3 Missing poor and adjusted multidimensional measurement of old-age poverty

The adjustment of a multidimensional poverty measure must account not only for the missing population, but also for potential changes in living standards over the life course. Figure **2** summarizes the problem under consideration. Multidimensional poverty is measured across three dimensions, giving rise to eight possible deprivation profiles. With a multidimensional poverty threshold equal to 2/3, four profiles are identified as multidimensionally poor, namely, those deprived in at least two dimensions. The first panel shows the size of each population group at young age, while the second shows the size of the same groups at old age. Differences in group size between the two periods may arise for two reasons: some individuals do not survive to old age; survivors may also move between deprivation profiles as their shortfalls change over time. The third panel shows the missing population associated with each initial profile, corresponding to individuals who died prematurely. The last panel shows the net balance of dimensional mobility among survivors: a positive value indicates a net inflow into the group, while a negative value indicates a net outflow.

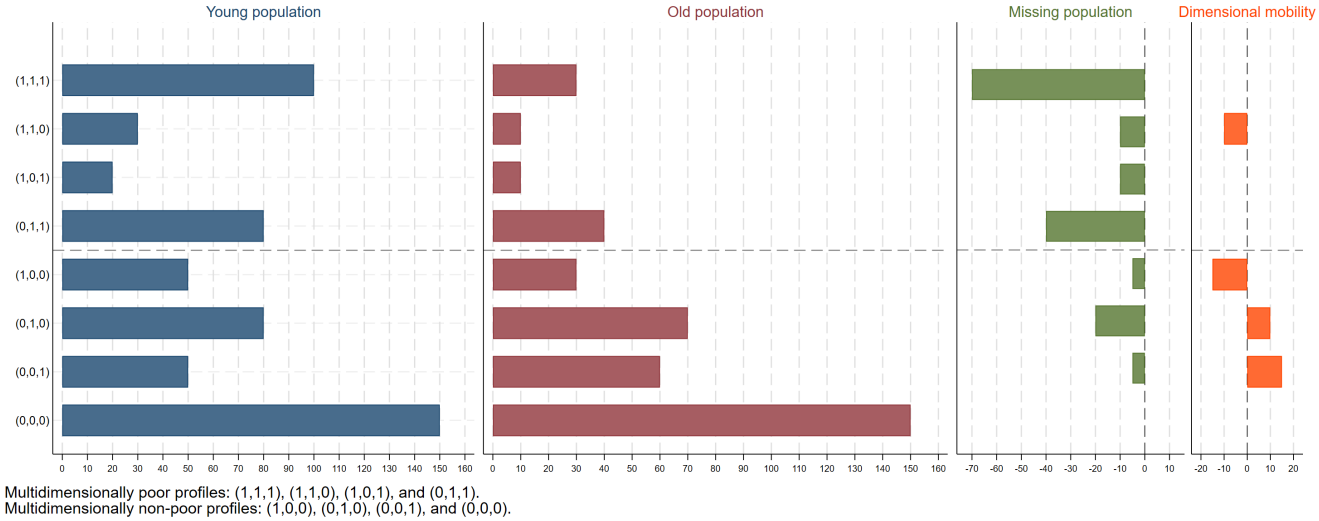
Conventional measures of old-age poverty consider only individuals who are alive at old age and therefore implicitly ignore mortality inequalities. In the illustration, mortality affects multidimensionally poor groups disproportionately. For example, the large reduction in the size of the most deprived profile, (1,1,1), is entirely explained by premature mortality: more than half of its young population does not survive to old age, while there is no net dimensional mobility into or out of this group. At the opposite extreme, the size of the completely non-deprived group, (0,0,0), remains unchanged because there is neither premature mortality nor net mobility. The comparison between profiles (1,1,0) and (0,1,0) illustrates the additional role of dimensional mobility. These groups differ only in deprivation in Dimension 1. For profile (1,1,0), the decline in population size results from both premature mortality and a net outflow toward other profiles. By contrast, profile (0,1,0) also loses population through premature mortality, but this loss is partially offset by a positive net inflow of survivors. Thus, observed differences in old-age deprivation profiles reflect the joint effects of mortality selection and dimensional mobility.

One additional element remains unobserved: the counterfactual mobility of individuals who died prematurely. The figure shows that premature mortality is more concentrated among multidimensionally poor profiles, but it is impossible to observe how these individuals would have evolved in terms of deprivation had they survived. Some might have remained in the same deprivation profile, while others might have experienced improvements or deteriorations in their living standards and consequently moved to a different profile. The adjustment developed in this section addresses this missing counterfactual population.

3.1 Missing poor

Missing poor are individuals who died prematurely and therefore are absent from the observed population, but who would likely have been identified as multidimensionally poor had they survived to the period in which poverty is measured. They represent the portion of poverty that remains statistically invisible because mortality is not randomly distributed

Figure 2. Deprivation profiles, missing population and mobility ($mpi_T = 2/3$).



across population groups, but is associated with higher levels of deprivation and disadvantage.

Assume a benchmark survival probability equal to π^*, which is the highest observed survival probability[25] $(\pi^*|\pi^* \geq \pi_i, \forall i \in \{1, 2, ..., 2^J\})$. The missing problem arises as long as there are survival inequalities, it means, if there is at least one population group whose survival is lower than π^*. However, it does not necessarily mean that missing population should have been counted as poor. In order to identify the missing poor is necessary to know mortality/survival inequalities (vector Π) and the trajectory of them if they had survived. Elements of vector Π can be observed or estimated. Nevertheless, the mobility across population groups of premature dead population is a matter of a normative or theoretical analysis because it requires to assumed what could have happened with people who are not observed anymore.

Premature dead population in group i is equal to $(\pi^* - \pi_i)N_i^Y$. Similarly to the mobility of survivors (matrix Ψ), the best case scenario would be to know exactly the trajectory of people who died prematurely. The matrix Γ (eq. 13) represents the hipothetical transition of this population, each cell $(\pi^* - \pi_i)\gamma_{ij}$ captures the proportion of premature dead members of group i, who could have been in the group j when old if they had survived $(\sum_{j=1}^{2^J} \gamma_{ij} = 1, \forall i \in \{1, 2, ..., 2^J\})$. Then, (13) captures the hypothetical distribution of deprivation profiles of premature dead population if they had survived from young to old age.

$$\Gamma = \begin{bmatrix} (\pi^* - \pi_1)\gamma_{11} & (\pi^* - \pi_1)\gamma_{12} \dots & (\pi^* - \pi_1)\gamma_{12^J} \\ (\pi^* - \pi_2)\gamma_{21} & (\pi^* - \pi_2)\gamma_{22} \dots & (\pi^* - \pi_2)\gamma_{22^J} \\ \vdots & \vdots & \vdots \\ (\pi^* - \pi_{2^J})\gamma_{2^J 1} & (\pi^* - \pi_{2^J})\gamma_{2^J 2} \dots & (\pi^* - \pi_{2^J})\gamma_{2^J 2^J} \end{bmatrix} \tag{13}$$

[25] It could potentially be the survival of the group with the lowest MPI ($mpi^* = 0$).

In order to formalize the adjustment for mortality inequality, it is defined the concept of missing population, namely, the individuals who are absent from the observed population because they died prematurely relative to a benchmark survival probability.

Definition 4 (Missing Population) *Let N_i^Y denote the young population initially belonging to group i, π_i the survival probability of group i, and $\pi^* = \max\{\pi_i\}$ the benchmark survival probability. Then, the missing population is defined as individuals who would have survived if every population group had experienced the benchmark survival probability:*

$$N^M = (\Pi^* - \Pi)' N^Y = \sum_{i=1}^{2^J} (\pi^* - \pi_i) N_i^Y. \tag{14}$$

$(\pi^* - \pi_i)N_i^Y$ represents the prematurely deceased population in group i associated with mortality inequality.

Nevertheless, the identification of the *missing poor* is much less obvious since it depends on the hipothetical dimensional mobility of people who died, we should therefore consider information in Γ. Nonetheless, it is not feasible to know the exact multidimensional mobility presented in matrix Γ. There are many different possible scenarios based on different assumptions, in the context of monetary poverty, Baland et al. (2021) and Lefebvre et al. (2013) assumed that everyone who died prematurely should have included as missing poor; Lefebvre et al. (2013) also consider the case when premature dead individuals would not have had mobility; and Lefebvre et al. (2019) analyzed the scenario when premature dead individuals would have had the same mobility than survivors[26].

In this research it would be considered that multidimensional mobility of people who died prematurely is the same as the mobility of the population group that they belonged when they were young. In the appendix 5 are presented the same results when premature dead individuals have no mobility[27]. Under the chosen scenarion, $\gamma_{ij} = \delta_{ij}\ \forall\ i, j \in \{1, 2, \ldots, 2^J\}$. The multidimensional mobility is exactly the same regardless survival, then the matrix of multidimensional mobility of prematurely dead population is equal to: $\Gamma^\bullet = \Gamma_{\{\gamma_{ij}=\delta_{ij}\}}$.

This scenario of multidimensional mobility is justified from a theoretical and empirical view. In theoretical terms this scenario lets to find an adjusted poverty measure independent on mortality or survival probabilities, hence, the mortality paradox can be ruled out (proposition 1), and this is precisely the interest in the correction of the missing poor bias. Furthermore, the tractability of the theoretical results of this paper, and especially in section 3.3, are unambiguous and straighforward because under this condition about mobility the missing poor bias can be characterized exclusively by the survival composition of the poor population or the correlation between survival and poverty risk.

[26]Every scenario is, of course, subjeted to discussion because it is based on assumptions about unobserved data. Moreover, given the lack of awareness about the situation of population who died prematurely, it is certainly a much better option to think about how these people would have moved in case of survival, than simply assume that every population group would have had exactly the same survival probability regardless their level of deprivation, which is in fact empirically a wrong assumption.

[27]Under no multidimensional mobility for both poor and non-poor people who died prematurely, the matrix Γ is equal to a diagonal matrix $\Gamma^\diamond$ whose cells are the proportion of premature dead population for each group ($(\pi^* - \pi_i)$ in the row i). Then, $\gamma_{ii} = 1\ \forall i \in \{1, 2, \ldots, 2^J\}$ ($\gamma_{ij} = 0$, if $i \neq j$).

From an empirical point of view, as will be shown in section 5, this assumpion about mobility is the most practial to estimate the adjusted poverty and the the size of the missing poor bias. With cross-sectional data, there is information on the current standard of living of households and retrospective information on whether there was any deceased member in the recent period, usually within the last one to three years. This source does not provide information about the set of deprivations of the household at the time of the death of the deceased member. Life tables and their corresponding estimates of survival probabilities are based on this reference period for deaths (last one to three years). Therefore, the most precise assumption applicable in this scenario is that the mobility of those who died would have been the same as the mobility of survivors ($\gamma_{ij} = \delta_{ij}$). Moreover, since only households that experienced a recent member's death are considered in the estimations of life tables, it is reasonable to assume that the dimensional mobility of both the deceased and the corresponding household would have been low; consequently, we may assume that $\gamma_{ii} = \delta_{ii} \to 1, \quad \sum_{\substack{j=1 \\ j \neq i}}^{2^J} \delta_{ij} \to 0$. As a result, in the empirical analysis the considered baseline scenario and the scenario with no mobility for missing population will tend to converge under the indicated data structure.

Definition 5 indicates the identification of the size of missing poor when the entire information about dimensional mobility is known, and when alive and premature dead population have the same dimensional transition probability[28].

Definition 5 (Missing Poor) *The missing poor are defined as the subset of the missing population that would have been identified as multidimensionally poor under the hypothetical survival trajectory encoded in* Γ*:*

$$N^{MP} = P'\Gamma' N^Y = \sum_{j=1}^{2^J} \sum_{i=1}^{2^J} p_j(\pi^* - \pi_i)\gamma_{ij} N_i^Y. \tag{15}$$

In particular, when $\gamma_{ij} = \delta_{ij} \; \forall \; i, j \in \{1, 2, \ldots, 2^J\}$*. The missing poor are defined as the prematurely deceased individuals who would have been identified as multidimensionally poor under the same mobility structure observed among survivors:*

$$N^{MP} = P'(\Gamma^\bullet)' N^Y = \sum_{j=1}^{2^J} \sum_{i=1}^{2^J} p_j(\pi^* - \pi_i)\delta_{ij} N_i^Y = \sum_{i=1}^{2^J} (\pi^* - \pi_i)\tau_i N_i^Y. \tag{16}$$

3.2 Adjusted measurement of multidimensional poverty

The change in the measurement of poverty to include mortality differences across population groups depends on the multidimensional mobility of premature dead population, namely, the matrix Γ (13). Specifically, if we had complete information of vector Π and matrices Ψ and Γ, the adjusted indicator would be simply the counting of missing population and their classification between poor and non poor. The resulting adjusted old-age multidimensional poverty headcount ratio is shown in equation 17, which is equivalent to the equation 11 adding

[28] Appendix 5 shows the identification of missing poor when $\gamma_{ii} = 1 \; \forall \; i \in \{1, 2, \ldots, 2^J\}$, as well as, other results developed in this section, considering that type of mobility.

missing population (eq.14) in the denominator and missing poor (eq.15) in the numerator.

$$\widehat{H}^O = \frac{P'\Psi' N^Y + \mathrm{P}'\Gamma' N^Y}{I'\Psi' N^Y + [\Pi^* - \Pi]' N^Y} = \frac{\sum_{i=1}^{2^J} \tau_i \pi_i N_i^Y + \sum_{j=1}^{2^J}\sum_{i=1}^{2^J} p_j(\pi^* - \pi_i)\gamma_{ij} N_i^Y}{\sum_{i=1}^{2^J} \pi_i N_i^Y + \sum_{i=1}^{2^J} (\pi^* - \pi_i) N_i^Y} \tag{17}$$

Equation 17 can be express as 18. Total population (denominator of eq.18) in period 2, including those who died prematurely is equal to population that should have survived if every group had the highest survival probability. While old-age poor people, including those who would have been poor in case of survival depends on the distribution of young population, the vector of survival probabilities and the multidimensional mobility for both survivors and non-survivors.

$$\widehat{H}^O = \frac{\sum_{j=1}^{2^J}\sum_{i=1}^{2^J} p_j N_i^Y \left[\pi^* \delta_{ij} + (\pi^* - \pi_i)(\gamma_{ij} - \delta_{ij})\right]}{\pi^* \sum_{i=1}^{2^J} N_i^Y} \tag{18}$$

To estimate the old-age multidimensional poverty headcount ratio would be simple if we had information of the young-age population structure, the survival probabilities for each group and information about the multidimensional mobility for each population group and for survivors and non-survivors. The latest is the tricky issue because there is not information about the trajectory of people that did not survive. Following the previous section, it is consider the plausible case[29] when $\delta_{ij} = \gamma_{ij}\ \forall\ i, j \in \{1, 2, \ldots, 2^J\}$. Taking equation (18), it is straightforward to see that the right-hand side inside the paranthesis cancels out and the adjusted headcount of multidimensional poverty is equal to[30]:

$$\widehat{H}_A^O = \frac{\sum_{i=1}^{2^J} \tau_i N_i^Y}{\sum_{i=1}^{2^J} N_i^Y} \tag{19}$$

Since changes in the population composition for poor groups are the same for alive and missing people, it is irrelevant if there are differences in survival probabilities and the counting of poor depends solely on the size of the poor when young, and their poverty risk[31]. The adjusted poverty measures proposed in equations (18) and (19) explicitly incorporate missing population generated by mortality inequality and, therefore, correct the compositional

[29] For the alternative case when $\delta_{ii} = 1, \forall i \in \{1, 2, ..., 2^J\}$ see the Appendix 5.

[30] This adjusted measure (19) is equivalent to the case analyzed in Lemma 1 (12).

[31] As it will be underscored in the next section, the MPB is the result of comparing the observed and the adjusted indicator, therefore, the MPB is affected by differencial mortality among population groups, while here, the adjusted measure is independent of survival probabilities.

bias that comes when survival probabilities differ across population groups. The following proposition formalizes the relationship between the adjusted measures and the mortality paradox[32].

Proposition 1 (Adjusted Poverty Measures and the Mortality Paradox) *Let $\widehat{H}^O$, and $\widehat{H}^O_A$ denote the adjusted poverty measures defined in equations (18) and (19), respectively. The adjusted poverty measures are not subject to the mortality paradox. In particular:*

(i) ***General adjusted measure ($\widehat{H}^O$).*** *Equation 18 internalizes mortality differentials through the explicit incorporation of missing population and their hypothetical multidimensional mobility, hence:*

$$\frac{\partial \widehat{H}^O}{\partial \pi_i} > 0 \iff \sum_{j=1}^{2^J} p_j \gamma_{ij} < \tau_i,$$

and

$$\frac{\partial \widehat{H}^O}{\partial \pi_i} < 0 \iff \sum_{j=1}^{2^J} p_j \gamma_{ij} > \tau_i.$$

(ii) ***Adjusted measure with equal mobility of survivors and non-survivors ($\widehat{H}^O_A$).*** *Differentiating equation (19) with respect to π_i yields:*

$$\frac{\partial \widehat{H}^O_A}{\partial \pi_i} = 0, \qquad \forall i.$$

Proof: *See Appendix 2: Proof of Proposition 1*

The proposition establishes that the adjusted multidimensional poverty measures proposed in equations (18) and (19) are robust to the mortality paradox because they explicitly account for missing population generated by survival inequalities. In the general case of equation 18, changes in poverty generated by mortality depend exclusively on differences between the multidimensional mobility of survivors and prematurely deceased individuals, rather than on a compositional effect generated by selective survival. Indeed, the result shows that the measure only aligns with the mortality paradox in the unfeasible case that the probability of being poor is higher for survivors. Likewise, when equal mobility of survivors and non-survivors is considered, the proposition states a straightforward result: the adjusted poverty measure is completely independent of survival probabilities and therefore fully eliminates the mortality paradox.

3.3 Missing poor bias and multidimensional poverty measurement

The adjusted measures of old-age multidimensional poverty proposed in the previous section do not convey anything about the magnitude bias that comes from the mortality selection differential. Indeed, in equation 19 survival probabilities are not even needed to estimate

[32]See the Appendix 5 to show that also when there is no mobility for prematurely dead population the adjusted measure is not subjeted to the mortality paradox.

multidimensional poverty. Consequently, it is essential to quantify the difference between the adjusted and the conventional multidimensional poverty measure to highlight the magnitude of the bias, which is defined as follows:

Definition 6 (Missing poor bias (MPB)) ***The missing poor bias*** *in the measurement of old-age multidimensional headcount poverty ratio is the difference between the adjusted and the observed HR as a proportion of the observed.*

$$MPB \equiv \frac{\widehat{H}^O - H^O}{H^O} \tag{20}$$

Based on this definition, the remainder of this section will show how the MPB depends on the weighting structure, how it responds to changes in survival probability, and how it varies with mortality inequality. To measure the MPB and the theorethical results developed in this section, it will be considered the scenario in which mobility of survivors and non survivors is equal, therefore, the MPB would be equal to $\frac{\widehat{H}^O_A - H^O}{H^O}$.

The construction of any multidimensional poverty measure is inherently shaped by the choice of dimensions and the weights assigned to reflect their relative importance in classifying households as poor (eq. 6). When dimensions carry unequal weights and simultaneously exhibit differential roles in driving mortality inequality, the magnitude of the correction for the missing poor bias will itself change accordingly. The MPB depends on the multidimensional weighting structure, this relationship operates through a specific channel: changes in the weights alter the identification of individuals expected to be poor in old age and, therefore, the average survival probability of that group. The following proposition formalizes it:

Proposition 2 (Missing Poor Bias and the weighting structure) *Let multidimensional poverty identification depends on the weighting structure W (6), so that the average survival probability of expected old-age poor population $\bar{\pi}_{poor}(W)$ also depends on that vector of weights. Then the Missing Poor Bias, under $\delta_{ij} = \gamma_{ij}$, satisfies*

$$MPB_A(W) = \frac{\bar{\pi}}{\bar{\pi}_{poor}(W)} - 1.$$

Moreover, the marginal change in the weighing structure results:

$$\frac{\partial MPB_A(W)}{\partial w_k} > 0 \iff \frac{\partial \bar{\pi}_{poor}(W)}{\partial w_k} < 0.$$

Proof: *See Appendix 3: Proof of Proposition 2.*

The proposition shows that, under $\delta_{ij} = \gamma_{ij}$, the weighting structure affects the Missing Poor Bias only through its effect on the survival composition of those expected to be poor in old age[33]. Changes in W increase the bias whenever they lower $\bar{\pi}_{poor}(W)$, and reduce it whenever they raise $\bar{\pi}_{poor}(W)$. Thus, the relevant issue is not merely whether poverty is

[33]A similar proposition cannot be derived as cleanly for the general case or when no mobility among

higher or lower under a given weighting scheme, but whether the selected weights identify as poor groups with systematically different survival probabilities[34]. Therefore, the magnitude of the MPB changes according to the dimensions emphasized in the poverty measure. If dimensions strongly associated with premature mortality receive greater weight, the population identified as poor will exhibit lower average survival probabilities, increasing the MPB. Conversely, dimensions weakly related to mortality may generate substantially smaller biases.

In contrast to conventional measures, old-age multidimensional poverty HR proposed in this paper do not mechanically decline when mortality is disproportionately concentrated among poor population groups. While this property is normatively important, it is less informative about the extent to which mortality inequalities bias the conventional observed measure of poverty. For this reason, the MPB becomes a crucial concept for understanding how mortality dynamics affect the underestimation of old-age multidimensional poverty. The remainder of this section therefore introduces in-depth analysis addressing how changes in survival probabilities affect the MPB, and how variations in mortality inequality influence its magnitude.

Proposition 3 (Marginal effect of survival on the MPB) *The marginal effect of a change in survival probability π_i, on the MPB_A is given by:*

$$\frac{\partial MPB_A}{\partial \pi_i} \begin{cases} < 0 & \text{if } \tau_i > H^O, \\ > 0 & \text{if } \tau_i < H^O. \end{cases}$$

Proof: *See Appendix 4: Proof of Proposition 3*

This proposition reveals that, although survival probabilities do not affect the adjusted poverty measure (under $\delta_{ij} = \gamma_{ij}$), they do affect the magnitude of the MPB through their impact on observed poverty[35]. Since observed old-age poverty is subject to the mortality paradox, changes in survival among specific population groups modify the extent to which poverty is underestimated due to selective mortality. In particular, increasing mortality among groups with above-average probability of being poor in old age reduces the observed poverty headcount ratio, thereby increasing the relative correction associated with the missing poor. Hence, the MPB is not driven by mortality differentials per se, but by mortality differentials among groups with different exposures to future poverty.

premature dead people is assumed because, unlike the analyzed case, the bias is no longer determined exclusively by the survival composition of the poor population. In the analyzed case with same mobility for survivors and non-survivors, the adjusted poverty measure is independent of survival probabilities, which allows the MPB to collapse into a simple expression based on the ratio between average survival in the population and average survival among the poor. By contrast, in the other scenarios, the adjusted poverty measure depends simultaneously on several interacting components: initial poverty identification, survival probabilities, transition dynamics, and assumptions regarding the counterfactual mobility of prematurely deceased individuals. As a consequence, the effect of increasing the weight of a mortality-related dimension on the MPB becomes theoretically ambiguous.

[34] As shown in Appendix 3: Proof of Proposition 2, $\bar{\pi}_{poor}$ is weighted by the population share of poor. Therefore, increasing the poverty HR, or share of poor, implies lower gap with $\bar{\pi}$. Indeed, section 5 illustrates clearly this result.

[35] An equivalent proposition when it is considered no mobility for premature dead population is developed in Appendix 5.

More specifically, the proposition shows that increasing the survival probability of groups whose probability of being poor in old age exceeds the observed old-age poverty rate reduces the bias. Intuitively, when more individuals from high-poverty-risk groups survive into old age, observed poverty becomes more representative of the true underlying poverty distribution, reducing the extent of downward bias generated by selective mortality. This result is particularly relevant in the multidimensional setting because both the probability of being poor in old age (τ_i) and the survival probability (π_i) depend on the specific deprivation profile characterizing each group.

The following proposition shows that the MPB can be represented entirely as a function of the covariance between survival probabilities and old-age poverty risk.

Proposition 4 (Covariance representation of the MPB) *Denote the population share of deprivation profile i for young population as $s_i = \frac{N_i^Y}{\sum_{k=1}^{2^J} N_k^Y}$, then the average survival probability can be express as $\bar{\pi} = \sum_{i=1}^{2^J} s_i \pi_i$ and the adjusted poverty measure as $\widehat{H}_A^O = \sum_{i=1}^{2^J} s_i \tau_i$.*
Define the weighted covariance between survival probabilities and old-age poverty risk as:

$$\mathrm{Cov}(\pi, \tau) = \sum_{i=1}^{2^J} s_i (\pi_i - \bar{\pi})(\tau_i - \widehat{H}_A^O).$$

Then the observed old-age poverty measure satisfies:

$$H^O = \widehat{H}_A^O + \frac{\mathrm{Cov}(\pi, \tau)}{\bar{\pi}},$$

and the MPB_A can be expressed as:

$$MPB_A = -\frac{\mathrm{Cov}(\pi, \tau)}{\bar{\pi} H^O}.$$

Proof: *See Appendix 6: Proof of Proposition 4*

Corollary 2 (Absence of mortality inequality) *If survival probabilities are identical across all deprivation profiles, namely, $\pi_i = \bar{\pi}, \quad \forall i$, then: $MPB = 0$.*

Corollary 3 (Absence of mortality selection) *If survival probabilities and old-age poverty risk are uncorrelated across deprivation profiles, namely, $Cov(\pi, \tau) = 0$, then: $MPB = 0$.*

The MPB is entirely determined by the covariance between survival probabilities and old-age poverty risk. This representation highlights the central role of mortality inequality in shaping observed poverty outcomes because the MPB increases precisely when low-survival groups are also those with higher probabilities of being poor in old age. In this sense, the observed old-age poverty measure becomes systematically downward biased whenever future poor individuals are disproportionately removed from the population through premature mortality. This result is especially pertinent in a multidimensional setting, as heterogeneous deprivation profiles entail differential survival probabilities and distinct likelihoods of persistent poverty in old age. The covariance representation captures precisely this mechanism,

showing how mortality inequalities across multidimensional deprivation profiles translate into an underestimation of old-age poverty.

The following extension analyzes how changes in the dispersion of survival probabilities across groups affect the magnitude of the bias. In particular, it will be shown that, under a stable mortality-poverty gradient, greater inequality in survival probabilities increases the MPB because high-poverty-risk groups become increasingly underrepresented in the observed old-age population.

Proposition 5 (Survival inequality and the MPB) *Assume that:*

1. *The correlation between survival probabilities and old-age poverty risk is negative and constant,* $\text{Corr}(\pi, \tau) = \rho < 0$*;*
2. *The variance of old-age poverty risk is fixed,* $\text{Var}(\tau) = \sigma_\tau^2$*;*
3. *Local changes in the variance of survival probabilities do not affect the observed old-age poverty rate*[36]*,* $\frac{\partial H^O}{\partial \text{Var}(\pi)} \approx 0$*.*

Then the MPB_A *can be expressed as:*

$$MPB_A = -\frac{\rho \sigma_\tau \sqrt{\text{Var}(\pi)}}{\bar{\pi} H^O},$$

and its marginal effect with respect to the variance of survival probabilities is positive because $\rho < 0$*:*

$$\frac{\partial MPB_A}{\partial \text{Var}(\pi)} = -\frac{\rho \sigma_\tau}{2\bar{\pi} H^O \sqrt{\text{Var}(\pi)}} > 0$$

Proof: *See Appendix 7: Proof of Proposition 5*

The assumptions of the proposition hold constant the multidimensional poverty structure and the negative association between survival and future poverty risk, while focusing on the effect of increasing mortality inequality across groups. Under these conditions, greater dispersion in survival probabilities amplifies the covariance between mortality and future poverty risk. The result is straightforward and unambiguous. The negative ρ guarantees that even when the variance of π changes as a result of shifts in survival probabilities exclusively among poor (non-poor) population groups, the MPB will increase for two reasons: first, the level of observed poverty is assumed to be fixed and unaffected by changes in the variance of π, so the marginal change in the dispersion of survival does not alter the structure of poverty; and second, higher mortality inequality, holding constant the dispersion of poverty-risk ($Var(\tau)$), implies a higher number of uncounted individuals with higher poverty risk (or higher over-representation of those with lower old-age poverty risk).

The multidimensional approach is crucial because both survival probabilities and future poverty risk depend on the specific deprivation profile characterizing each group. Different combinations of deprivations may therefore generate distinct mortality gradients and different probabilities of remaining poor in old age. For example, profiles combining poor health

[36]This is a needed assumption, otherwise the "pure selection effect" generated by mortality inequality cannot be isolated, since $Var(\pi)$ changes both the covariance $Cov(\pi, \tau)$ and the observed poverty mechanically. The idea behind this assumption is that small redistributions of survival probabilities around their mean primarily affect the covariance structure, while the induced change in aggregate observed poverty is of second-order importance.

and inadequate housing may exhibit both low survival and high poverty persistence, whereas profiles associated mainly with temporary income deprivation may imply weaker mortality disadvantages. The MPB therefore depends on how multidimensional deprivation structures jointly shape survival and future poverty.

The proposition also provides an intuitive framework for comparing societies. Two countries may exhibit close levels of observed old-age poverty rates and average survival while differing substantially in the extent to which poverty is underestimated given their mortality inequality. For a given relationship between survival and poverty, greater inequality in mortality leads to a larger number of missing poor among the elderly. The proposition therefore suggests that societies characterized by stronger mortality inequalities across multidimensional deprivation profiles are also those in which conventional old-age poverty measures are more likely to underestimate the true extent of deprivation.

4 Comparability of old-age multidimensional poverty measurement

This section examines how the MPB is influenced by two key features discussed theoretically in the preceding section: weighting structures and inequalities in survival. Adopting a more applied perspective, it first analyses how the MPB within a given society changes as the weights assigned to different dimensions vary. It then considers how both the mean and the dispersion of survival probabilities affect the comparability of multidimensional poverty measures and their corresponding MPBs.

4.1 Changes in dimensions and weights to measure poverty

The proposition 2 established that the Missing Poor Bias depends on the weighting structure through the average survival probability of the population identified as poor. Because alternative weights may change which deprivation profiles satisfy the multidimensional poverty threshold, they may also alter the survival composition of the poor population, even when all demographic and mobility characteristics of the society remain fixed. The following corollary extends this result to any finite set of alternative weighting structures and provides a general ordering: weighting schemes that identify poor populations with lower average survival probabilities necessarily generate larger MPBs within the same society.

Corollary 4 (Comparison of the MPB across weighting structures) *Consider a society in which multidimensional poverty is measured based on different weighting structures, and the MPB is computed accordingly. Let $W^1, W^2, \ldots, W^M$ be alternative weighting structures. From proposition 2, if*

$$\bar{\pi}_{poor}(W^1) \leq \bar{\pi}_{poor}(W^2) \leq \cdots \leq \bar{\pi}_{poor}(W^M),$$

then

$$MPB(W^1) \geq MPB(W^2) \geq \cdots \geq MPB(W^M).$$

In short, corollary 4 underscores that a weighting structure produces a larger MPB whenever the poverty identification it generates yields a lower average survival probability among people expected to be poor when old. Therefore, regardless the change in the size of the multidimensional poverty, what matters is how the weighting structure is linked to the mortality gradients, and therefore, how the change in the identification of poor and non-poor groups reshape the average survival probability of poor population. A weighting structure may identify a larger or smaller share of the population as poor, but the decisive factor is the mortality composition of the groups whose poverty status changes. When greater weight is assigned to dimensions that are more strongly associated with premature mortality, the newly identified poor are more likely to belong to low-survival groups, thereby reducing the average survival probability among the poor and increasing the MPB. Conversely, emphasizing dimensions with weak or negligible mortality gradients may have a smaller effect on the bias, even if measured poverty changes substantially. Thus, the weighting structure has both a normative role in defining poverty and a demographic implication, since it determines how mortality inequality enters the measurement of old-age multidimensional poverty.

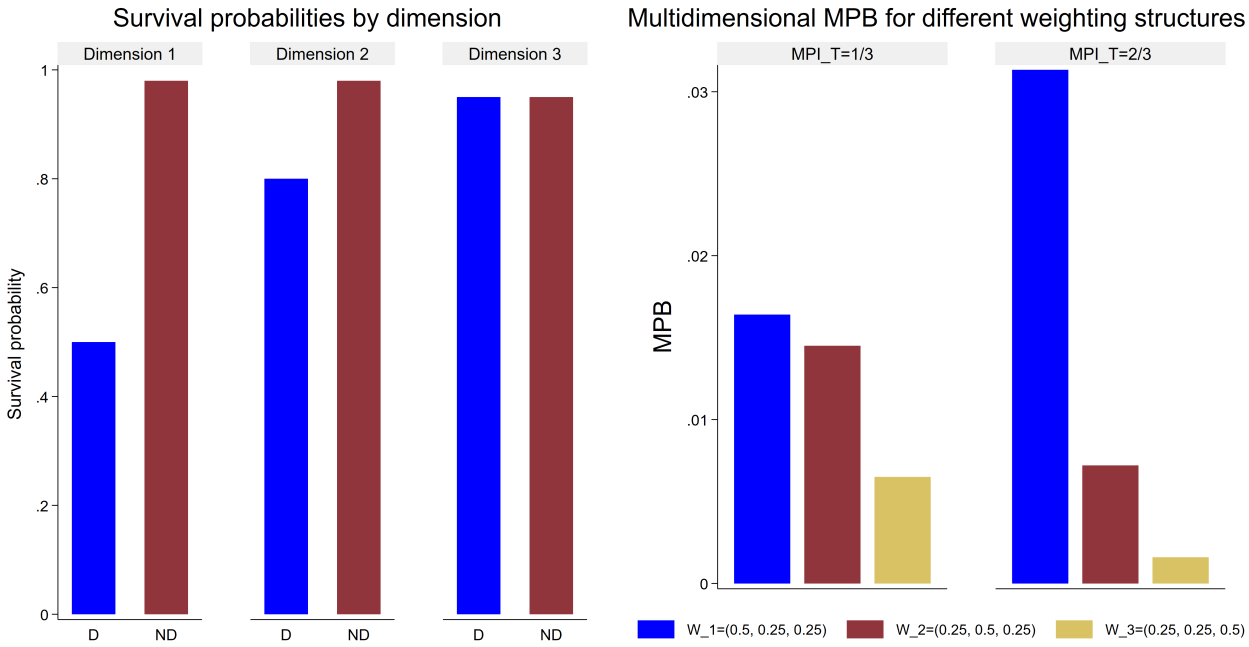
To illustrate Proposition 2 and Corollary 4, consider a single country in which multidimensional poverty is measured across three dimensions and one-half of the population is deprived in each dimension. Figure 3 reports survival probabilities by deprivation status and the resulting MPB under three alternative weighting structures and two multidimensional poverty thresholds. Dimension 1 exhibits the strongest mortality gradient, since the survival probability of the deprived population is approximately one-half that of the non-deprived population. Dimension 2 presents a smaller survival differential, whereas Dimension 3 displays virtually no difference in survival between deprived and non-deprived groups.

The resulting ordering of the MPB is consistent with Corollary 4. The weighting structure that assigns greater importance to Dimension 1 generates the highest MPB because the poverty identification it produces yields a poor population with a lower average survival probability. By contrast, assigning greater weight to Dimension 3 produces the lowest MPB, since deprivation in that dimension is weakly related to survival. The largest bias arises when $MPI_T = 2/3$ and $w_1 = 0.5$, as the groups identified as poor under this configuration are those deprived in the dimension with the strongest mortality gradient. Thus, as established by Proposition 2, the relevant mechanism is not the change in poverty incidence itself, but the effect of the weighting structure on $\bar{\pi}_{poor}(W)$: the lower the average survival probability among the identified poor, the larger the MPB.

4.2 Differences in survival probabilities

The same old-age multidimensional poverty headcount ratio (H^O) for two countries (A and B) are not comparable if they have different levels of mortality inequality. Consider that in country A poor people die at higher proportion and a lot of them do not achieve "old age", while survival probability from young to old age in country B is quite similar for poor and non-poor groups. Consequently, in country A the mortality paradox plays a role in the selection bias (missing poor) and therefore as a cause of poverty decline. Under this scenario the same reduction in poverty cannot be compared for countries A and B, because in the former the selection process explains that decline, while in country B, whose mortality inequality is low, it can be the result of an effective policy against poverty.

Figure 3. Survival probabilities and multidimensional MPB



Proposition 5 showed the role of survival/mortality inequality in the size of the MPB. It could be inferred that countries with higher inequality in mortality could underestimate poverty more and therefore the MPB would be higher. Nevertheless, it is not always the case if there are differences in mobility, the spread of mortality in the entire population, and the size of initial poverty. The following proposition formalizes the result, showing that two societies with the same observed poverty levels may display different adjusted poverty rates because mortality is more strongly concentrated among high-poverty-risk groups, or because average survival is lower.

Proposition 6 (Mortality-selection dominance and the MPB) *Consider two societies, indexed by* $s \in \{A, B\}$. $\bar{\pi}^S$ *is the average survival probability,* $H^{O,S}$ *the observed old-age multidimensional poverty headcount ratio,* MPB^S *the missing poor bias, and* $C^S \equiv -Cov^S(\pi, \tau)$ *the negative of the covariance between survival probabilities and poverty risks* ($C^S > 0$). *Then, when comparing their old-age multidimensional poverty measures and the corresponding missing poor bias, the following results hold.*

1. *If two societies have the same average survival probability and the same observed old-age poverty level,*

$$\bar{\pi}^A = \bar{\pi}^B, \qquad H^{O,A} = H^{O,B},$$

then

$$C^A > C^B \quad \Rightarrow \quad MPB^A > MPB^B.$$

2. *If two societies have the same observed old-age poverty level and the same mortality-selection intensity,*

$$H^{O,A} = H^{O,B}, \qquad C^A = C^B,$$

then

$$\bar{\pi}^A < \bar{\pi}^B \quad \Rightarrow \quad MPB^A > MPB^B.$$

Proof: *See Appendix 8: Proof of Proposition 6.*

The proposition indicates that observed poverty comparisons across societies can be misleading when mortality is unequally distributed across poverty-risk groups. The result is useful because it separates two sources of incomparability: the degree to which mortality is concentrated among high-poverty-risk groups, captured by C^s, and the overall level of survival, captured by $\bar{\pi}^s$. Adjusting for missing poor therefore makes poverty comparisons more meaningful, since it compares societies after accounting for the population that is absent because of premature mortality.

Let's consider two societies with the same population structure, three ($J = 3$) dimensions to classify population in 8 different groups ($2^J = 8$). The vector N^Y is the same[37]. For simplicity, consider that both societies have exactly the same transition mobility, and in particular, mobility is the same for survivors and non-survivors. The only difference between societies is the vector of survival probabilities (Π), therefore, the old-age population size in each group would be different.

Figure 4 illustrates the implications of Proposition 6 by comparing the mentioned countries, under three alternative survival structures. The first case provides the benchmark: survival probabilities differ across countries, but within each country survival is constant across population groups. Hence, there is no mortality selection because survival is unrelated to multidimensional deprivation, implying $C^S = \text{Cov}^S(\pi, \tau) = 0, \quad s \in \{A, B\}$ and therefore no missing poor bias. It cannot be included as a particular case of the second scenario of proposition 6 because results hold when there is a negative relationship between survival probability and poverty risk ($C^S > 0$). On the contrary, this is indeed the illustration of corollaries 2 and 3. It shows that mortality by itself is not the source of the bias; what matters is mortality inequality across groups with different multidimensional poverty risks.

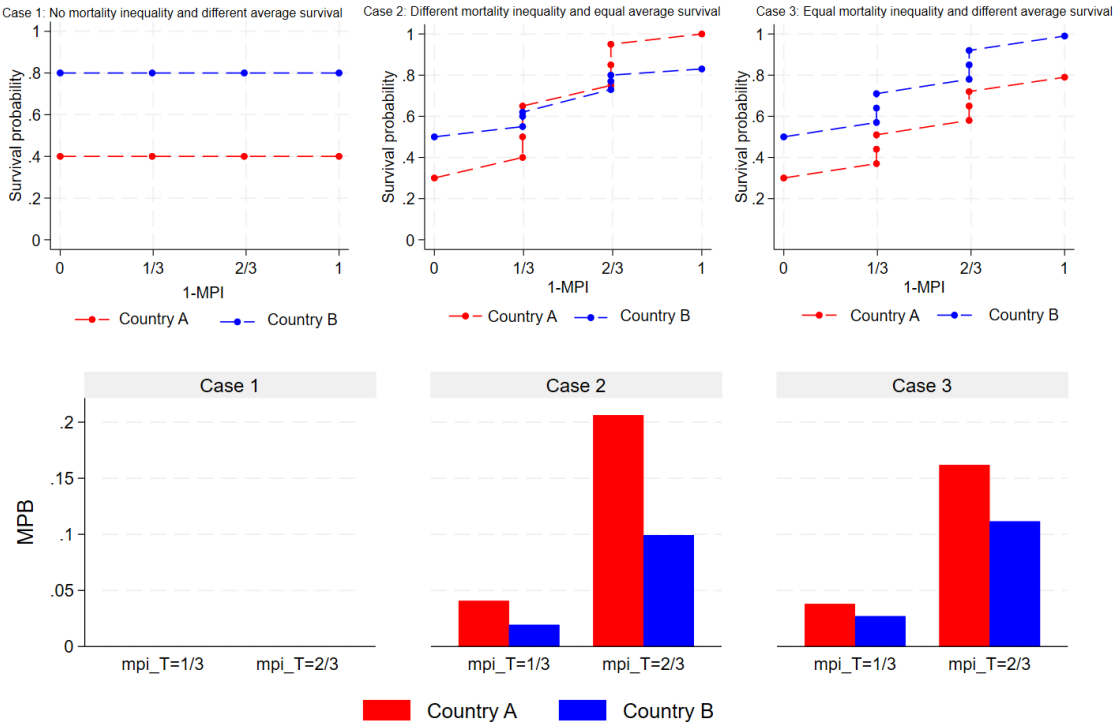
In contrast, the second case holds average survival constant across countries but allows survival to vary across deprivation groups. Country A displays a steeper survival gradient, meaning that groups with higher poverty risk have lower survival probabilities. As predicted by the proposition, stronger mortality-selection intensity, $C^A > C^B$, produces a larger missing poor bias in country A. Although the observed old-age poverty is the same in both countries, in country A is operating the selection process of missing poor much more than in country B, where poorer groups are less likely to survive and are consequently underrepresented in the observed old-age population. Poverty measures are not directly comparable.

The third case isolates the role of average survival. The two countries have a similar pattern of mortality inequality (variance) across deprivation groups, represented by approximately parallel survival profiles, but country A has lower average survival. Given positive mortality selection, the proposition implies that lower $\bar{\pi}$ increases the relative missing poor bias. Thus, even when the association between deprivation and survival is similar across countries, the country with lower average survival can display a larger bias in observed old-age poverty.

The figure also shows why the multidimensional structure is central to the argument. Since the mobility of survivors and prematurely deceased individuals is assumed to be the

[37]It can be also the same vector by a scalar, given the invariance to the scale of the headcount ratio of poverty.

Figure 4: **Comparison of MPB under different mortality regimes**



Notes: The upper panels compare survival probabilities across multidimensional deprivation profiles, while the lower panels report the implied missing poor bias. Case 1 shows that differences in average survival do not generate a bias when survival is unrelated to deprivation. Case 2 shows that, holding average survival fixed, stronger mortality selection among high-poverty-risk groups increases the missing poor bias. Case 3 shows that, given positive mortality selection, lower average survival also increases the bias.

same, the MPB is driven by the association between survival probabilities and poverty-relevant deprivation profiles[38]. Hence, the relevant object is not only the survival gap between poor and non-poor groups, but the survival gradient across the full distribution of multidimensional deprivation states.

The next section presents an empirical application that illustrates the theoretical results developed so far. In particular, it applies the proposed adjustment to show how selective mortality across multidimensional deprivation profiles affects observed poverty measures and the size of the missing poor bias in practice.

5 Empirical evidence

In this section, the empirical analysis compares multidimensional poverty measurement in Cambodia, El Salvador, and Zambia. Specifically, it illustrates unidimensional and multidi-

[38]Remember that when poverty is measured under the monetary approach there are only two deprivation profiles, and even when survival probabilities were computed for different points of the income distribution, there could still some concealed mortality inequalities that the multidimensional perspective can reveal.

mensional poverty levels, survival inequalities across population groups within each country, the sensitivity of the MPB to changes in the weighting structure, and the magnitude of the MPB in each country given the observed survival inequalities. The section is structured in three parts. First, the sources of information and the rationale for the selection of the three countries are presented. Second, the empirical procedure is described. Finally, the results are presented and analyzed in light of the theoretical contributions developed in the previous sections.

5.1 International comparability and source of information: Cambodia, El Salvador and Zambia

Considering the theoretical model, the empirical analysis requires countries with information on mortality, together with household and individual-level data that allow the multidimensionally poor population to be identified. Conventionally, this information is available in population censuses, which allow for relatively detailed analyses at different geographical levels and provide a sufficiently large sample of deceased individuals to estimate survival probabilities for different population groups. Nevertheless, an important limitation is the restricted set of variables that can be used to measure multidimensional poverty.

For this empirical illustration, three developing countries were selected: Cambodia, El Salvador, and Zambia. Four main reasons motivated their selection. First, they have relatively small population sizes and broadly similar age structures (see Table 1). Second, they represent different regions of the world, which helps illustrate the relevance of adjusting for the missing poor across substantially different institutional settings where multidimensional poverty remains widespread. Third, their population censuses were conducted in relatively similar years, and their census databases are harmonized by IPUMS International, allowing the use of a set of sufficiently comparable variables to construct the multidimensional poverty measure. Fourth, mortality information collected through the population censuses is available for all three countries, which is essential for estimating life tables and survival probabilities.

The empirical analysis uses harmonized census microdata from El Salvador (2007), Cambodia (2008), and Zambia (2010), obtained from IPUMS International (Ruggles et al., 2025), together with the corresponding mortality records on recently deceased household members. The original data were collected by the Dirección General de Estadística y Censos of El Salvador, the National Institute of Statistics of Cambodia, and the Central Statistical Office of Zambia, respectively. The analysis considers only variables that had already been harmonized by IPUMS. To estimate the life tables, and specifically the probability of survival to age 60, mortality information was used, namely, records of household members who died during the year preceding the census. For all three countries, IPUMS provides a 10% sample of the census records; therefore, population weights were applied to recover estimates representative of the total population.

Table 1 summarizes selected demographic and socioeconomic characteristics of Cambodia, El Salvador, and Zambia around the corresponding census years. In demographic terms, the three countries have relatively small population sizes and broadly similar age structures, which facilitates the comparison of mortality patterns across population groups. At the same

time, the indicators of living standards and monetary poverty show that socioeconomic deprivation remains relevant in all three countries, although its magnitude differs across them. These similarities provide a useful common basis for the empirical exercise, while the differences in income levels, poverty incidence, and institutional settings introduce meaningful heterogeneity. Therefore, the purpose is not to assume that the three countries are fully comparable in their socioeconomic conditions, but rather to analyze the proposed adjustment in different contexts using harmonized census information and a common multidimensional poverty measurement framework.

Table 1: Selected demographic and socioeconomic characteristics

	Cambodia (2008)	**El Salvador (2007)**	**Zambia (2010)**
Panel A. Demographic characteristics			
Population (millions)	13.39	5.74	13.22
Women (%)	51.4	52.7	50.8
Rural population (%)	80.5	37.3	60.5
Population aged 0–5 (%)	12.5	11.8	20.4
Population aged 0–12 (%)	28.4	29.1	40.4
Population aged 13–24 (%)	27.5	23.6	25.7
Population aged 60+ (%)	5.8	8.7	3.6
Panel B. Economic and poverty indicators			
GDP per capita (current US$)	866	2,786	1,451
Extreme monetary poverty (%)	40.9	5.9	64.8
Multidimensional poverty (%)	72.0	43.0	72.0

Notes: Population and demographic characteristics correspond to the respective census years. Extreme monetary poverty refers to the share of the population living below US$2.15 per person per day at 2017 PPP.
Sources: IPUMS International; World Bank, World Development Indicators; World Bank, Poverty and Inequality Platform.

5.2 Empirical procedure

The empirical procedure consists of four steps. First, the dimensions and indicators of multidimensional poverty are selected according to data availability and comparability across countries. Second, individuals are classified into unique deprivation profiles based on their deprivation status in each dimension, and the observed old-age multidimensional poverty headcount ratio is computed using the poverty status and population size of each profile under the specified weighting structure, with equal weights ($w_i = 1/3$) in the baseline specification. Third, life tables and profile-specific survival probabilities are estimated for each country and population group and used to infer the missing population associated with premature mortality. This missing population is then incorporated into the observed population to obtain the adjusted old-age multidimensional poverty headcount ratio. Finally, the missing

poor bias (MPB) is calculated as the difference between the adjusted and observed poverty measures relative to the observed measure. The detailed description of the procedure in each step is the following:

1. **Variables and dimensions to measure multidimensional poverty:** The measurement considers three dimensions and 10 indicators. The chosen variables were selected based on three main criteria: availability and comparability among studied countries; similarity with previous measurement of multidimensional poverty based on censuses (Stankiewicz Serra et al., 2021; Statistics South Africa, 2014; Uganda Bureau of Statistics, 2026) and particularly, as long as possible, similarity with the Global MPI (Alkire et al., 2022); and third, since the analysis asummes that there was not significat changes in households' well-being as a result of a recent premature death, hence, variables that have a mechanical link to deaths were not considered, such as overcrowding, shool attendance or household economic dependency[39]. The following are the criteria to identify dimensional deprivation in each of the three dimensions considered, namely, basic services, education, and economic vulnerability.

   Basic services[40]*:* Four variables were considered: connection to a sewage system, access to piped water, electricity in the dwelling, and type of cooking fuel. For each indicator, a household was classified as experiencing a shortfall if it lacked access to the corresponding service. Specifically, indicator-level shortfall was defined as follows: no connection to a sewage system; no access to piped water; no electricity in the dwelling; and the use of dung, wood, charcoal, or coal as the primary cooking fuel. A household was classified as deprived in the *basic services* dimension if it experienced shortfalls in at least two of these four indicators.

   Education[41]*:* Three indicators were considered in the education dimension: adult literacy, educational attainment, and years of schooling. A household was classified as experiencing a shortfall in each indicator under the following conditions: at least one adult member aged 18 or older was illiterate; no household member had completed secondary education; or the average number of years of schooling among adult household members (18+) was below six. A household was classified as deprived in the *education* dimension if it experienced shortfalls in at least two of the three indicators.

   Economic vulnerability: Three indicators were used: asset ownership, dwelling ownership, and work vulnerability. The asset-ownership indicator included four types of assets: a television, a mobile phone, a computer, and a motor vehicle. A household was classified as experiencing a shortfall if it owned fewer than two of these

[39]Mortality variables were also excluded, since the underlying idea in this research is that premature mortality is associated with pre-existing poverty, rather than poverty being defined as a consequence of mortality.

[40]In the Global MPI, these four variables are included in the "living standards" dimension, together with housing materials and asset ownership. Since the four variables considered here are strictly linked to the provision of basic services, "Basic services" was chosen as the name of the dimension. Asset ownership, in turn, is included in the economic vulnerability dimension, while housing materials were excluded due to data limitations.

[41]School attendance was not included because it has a mechanical link with a member's death.

assets. Dwelling-ownership deprivation was defined according to whether the household owned the dwelling in which its members lived. Work vulnerability was assessed using age-specific criteria. Prime-age adults aged 25–69 were classified as vulnerable if they were unemployed. Children aged 12–16 were classified as vulnerable if they were working, while young people aged 17–24 were classified as vulnerable if they were not in education, employment, or training (NEET). Older household members (70+) were classified as vulnerable if they were working or unemployed. A household was considered as experiencing a shortfall in this indicator if at least one member met the relevant age-specific criterion. Finally, a household was classified as deprived in the *living standards* and economic vulnerability dimension if it experienced deprivation in at least two of the three indicators: asset ownership, dwelling ownership, and work vulnerability.

2. **Deprivation profile and aggregation of old-age multidimensional poverty:** Once each household has been classified as deprived or non-deprived in each of the three dimensions, the population is grouped according to its specific pattern of dimensional deprivation. With three binary dimensions, there are eight possible deprivation profiles in each country ($2^3 = 8$). Households are then identified as multidimensionally poor according to the selected poverty cutoff, with two thresholds considered: $mpi_T = 1/3$ and $mpi_T = 2/3$. The old-age multidimensional poverty headcount ratio (HR) is defined as the proportion of individuals aged 60 or older who live in multidimensionally poor households relative to the total old-age population. Equation 21 presents the corresponding multidimensional poverty measure. The baseline specification assigns equal weights to the three dimensions. To examine more closely the relationship between dimension-specific mortality inequality and poverty measurement, the analysis subsequently considers the full range of feasible continuous weighting structures and evaluates the resulting changes in the average survival probability among the poor and in the missing poor bias.

$$MPI = G \cdot W = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2/3 \\ 2/3 \\ 2/3 \\ 1/3 \\ 1/3 \\ 1/3 \\ 0 \end{bmatrix} \tag{21}$$

3. **Life tables and survival probabilities:** Using the available mortality data, life tables were estimated separately for each deprivation profile. The population was divided into ten-year age intervals, with three exceptions: infants younger than one year, children aged one to four years, and children aged five to nine years, they were treated as separate groups to account for the distinctive patterns of infant and early-childhood mortality. The remaining population was classified into ten-year age groups, with an open-ended final interval for individuals aged 100 or older. This classification yields 13 age groups and, combined with the eight deprivation profiles, 104 population groups in

each country. Life-table estimation followed the standard demographic methodology described by Preston, Heuveline, and Guillot (2001). For the infant and early-childhood age intervals, the Coale–Demeny model-based method was applied (Coale, Demeny, & Vaughan, 1983; Romero-Prieto, Verhulst, & Guillot, 2024).

The population census makes it possible to identify each household's deprivation profile and whether the household experienced the death of a member during the preceding year. To assign a deprivation profile to individuals who died prematurely, the empirical analysis adopts the central assumption introduced in the theoretical model: multidimensional mobility is the same for survivors and non-survivors, such that $\gamma_{ij} = \delta_{ij}$. Under this assumption, had a deceased individual survived, that person would have been classified in the same deprivation profile as the surviving members of the household. Equivalently, the household is assumed not to have changed its multidimensional poverty status as a direct consequence of the recent death.

This assumption is plausible in the present empirical setting for two reasons. First, the indicators used to measure multidimensional poverty were selected to minimize mechanical changes in household deprivation status following the death of a member.[42] Second, the analysis considers only deaths occurring during the previous year. Structural characteristics such as access to water and electricity, educational attainment, and dwelling ownership are unlikely to change substantially within such a short period solely because of a household member's death.

The assumption is also consistent with the interpretation of the missing poor bias. The mechanism of interest is that poorer population groups face higher mortality and are therefore underrepresented in observed old-age poverty measures. In this sense, the missing poor died prematurely partly because poverty exposed them to disproportionately high mortality risks, rather than becoming poor as a consequence of the death itself. This assumption would be less plausible for monetary poverty, since the death of an income-earning household member may immediately affect household income and poverty status.

Life tables were estimated separately for each deprivation profile within each country. An individual was considered to have died prematurely if death occurred before age 60. Accordingly, the probability of surviving to age 60 was used to estimate the population missing because of premature mortality and to adjust the observed population size. For illustration, Appendix 10 presents the life tables for Cambodia and for deprivation profile 1, comprising individuals deprived in all three dimensions ($mpi_1 = 1$), and profile 8, comprising individuals deprived in none of the dimensions ($mpi_8 = 0$). The probability of surviving to age 60 is defined as the probability of surviving all age intervals from birth to exact age 60 and is calculated as the product of the conditional survival probabilities over those intervals. As shown in Equation 22, $1 - q_{ix}$ denotes the conditional probability of surviving age interval x, given survival to the beginning of that interval.

[42] Indicators such as overcrowding, school attendance, and economic dependency were excluded because they may change mechanically following the death of a household member.

$$\pi_i^{60} = \prod_{x<60} (1 - q_{ix}) \tag{22}$$

4. **Adjusted multidimensional poverty and the missing poor bias:** Missing old-age people in group i is equal to:

$$M_i^{60+} = \frac{\pi^{*60} - \pi_i^{60}}{\pi_i^{60}} N_i^{60+} \tag{23}$$

Where π^{*60} is the highest observed survival probability at age 60 considering population groups within each country, and N_i^{60+} is the population size of group i for people aged 60 or older[43]. Adjusted old-age population size of group i, including missing people is:

$$\hat{N}_i^{60+} = \frac{\pi^{*60}}{\pi_i^{60}} N_i^{60+} \tag{24}$$

The adjusted old-age multidimensional poverty HR $\hat{H}^{60+}$, is basically the ratio between the sum of adjusted population within groups identified as poor, over the total expanded population including missing people. As it has been considered, two thresholds are used, $mpi_T = 1/3$ and $mpi_T = 2/3$, it means in the first case -and given an equal weighting structure- every group that has at least one deprivation is considered poor, therefore there are seven poor groups and one non-poor group; similarly, in the latter case there are 4 poor groups and 4 non-poor groups. Finally, the Missing Poor Bias is estimated accordingly and based on the definition in equation 20.

5.3 Results

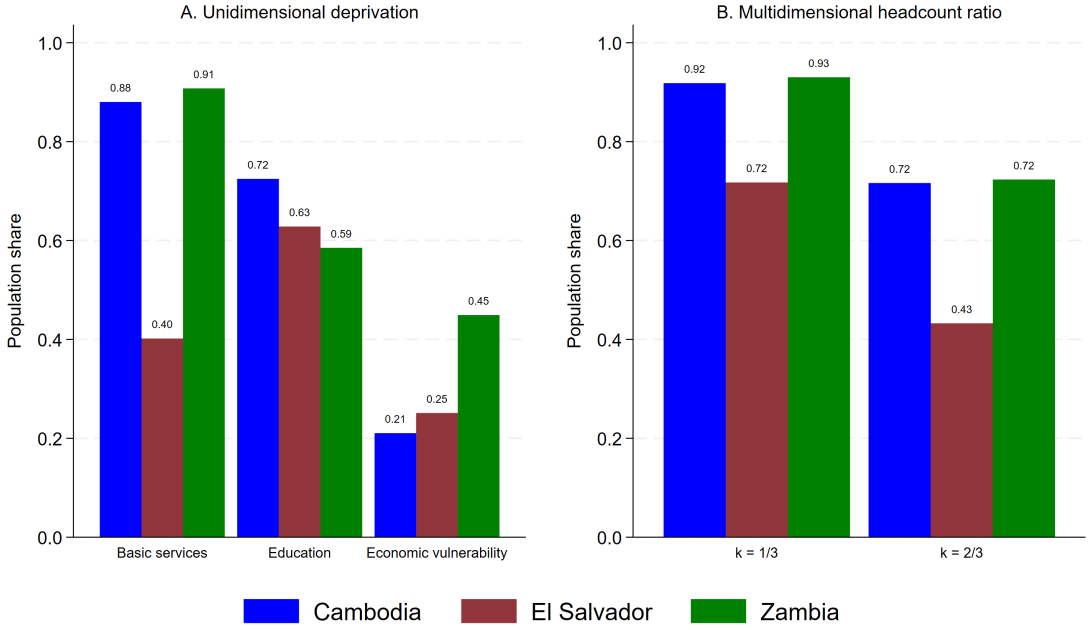
Figure 5 shows the levels of unidimensional and multidimensional poverty in Cambodia, El Salvador, and Zambia. Poverty is widespread in the three countries. In Cambodia and Zambia, more than 90% of the population is deprived in at least one dimension, while in El Salvador the corresponding figure exceeds 70%. Likewise, Cambodia and Zambia display similarly high multidimensional poverty headcount ratios. However, the composition of deprivation differs considerably across countries. Zambia has the lowest level of deprivation in education but the highest level of economic vulnerability. Cambodia, in contrast, exhibits widespread deprivation in basic services and education, while only about one-fifth of its population is economically vulnerable. This relatively low level of economic vulnerability, compared with deprivation in the other dimensions, is partly explained by the small share

[43]The reason to consider the size of old-age population in the expansion of the missing people is that if we considered the size of young population there is a wrong identification of missing poor. For instance, a higher proportion of deprived people in education at old ages is related to a dimensional progressive transition from low to middle levels of education in the population, consequently we could infer mistakenly that a population group with higher survival probability is integrated by more missing poor population. It is basically explained due to data structure, we do not really have the same population in two periods but we have two generations in the same period of time.

of Cambodian households that do not own their dwelling, which reduces deprivation in one of the indicators included in this dimension.

Two observations are relevant before analyzing survival probabilities. First, the widespread incidence of poverty could suggest that mortality inequalities have limited relevance in explaining potential biases in poverty measurement and, consequently, that the MPB may be negligible when poverty is already very high. The empirical analysis below examines whether this intuition holds. Second, the figure highlights the importance of analyzing poverty from a multidimensional perspective, since relying on a single dimension could lead to misleading comparisons across countries with different standards of living. For instance, if education were the only dimension used to identify poverty, Zambia would appear to have the lowest poverty level, even though this conclusion does not hold when the other dimensions or the multidimensional measure are considered.

Figure 5: **Unidimensional deprivation and multidimensional poverty for old population**



Source: Author's calculations using IPUMS International census for each country (Ruggles et al., 2025).

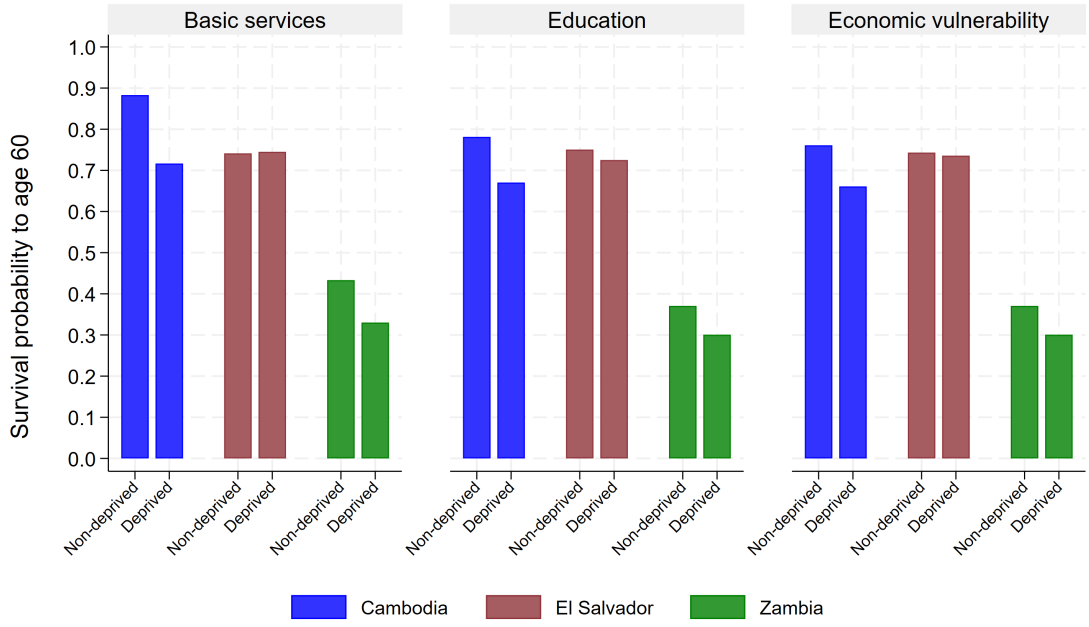
The probability of surviving to age 60 is substantially lower in Zambia, while it is relatively similar in Cambodia and El Salvador.[44] Figure 6 shows that deprivation is clearly associated with differences in survival, although the role of each dimension varies and its influence is heterogeneous across countries. Despite the similar overall survival probabilities observed in El Salvador and Cambodia, mortality inequalities between deprived and non-deprived groups are more pronounced in Cambodia. In El Salvador, by contrast, survival differences are generally smaller, with only a slight gap between deprived and non-deprived individuals in the education dimension.

[44]Lower survival and higher mortality in Zambia are observed across all age groups, including infancy. Consequently, life expectancy is systematically lower in this country from birth through old age.

Taken together, Figures 5 and 6 reveal the relevance of comparing these three countries. On the one hand, Cambodia and Zambia exhibit very similar levels of poverty. Estimating the missing poor bias therefore allows us to assess whether these apparently comparable poverty levels remain so once premature deaths associated with lower standards of living are incorporated into the analysis. On the other hand, Cambodia and El Salvador have similar probabilities of surviving to old age, but markedly different mortality gradients between deprived and non-deprived groups. The multidimensional analysis thus makes it possible to examine the role of these gradients and, ultimately, how they affect poverty measurement.

Nevertheless, the joint mortality and poverty structure of the three countries does not allow the first case of Proposition 6 to be applied directly. Establishing the ordering of the MPB under that case requires two countries with the same poverty level and the same average survival probability. In the present comparison, Cambodia and Zambia have approximately the same poverty level, whereas Cambodia and El Salvador have similar average survival probabilities. The second case of Proposition 6, however, provides a closer approximation to the comparison between Cambodia and Zambia. They exhibit very similar poverty levels and, as shown in Figure 7, similar mortality-inequality gradients, but the average survival probability is substantially lower in Zambia. Accordingly, the theoretical results predict a higher MPB in Zambia.

Figure 6: **Survival probabilities to age 60 by dimensional deprivation**



Source: Author's calculations using IPUMS International census and mortality records for each country (Ruggles et al., 2025).

The left-hand side of Figure 7 shows survival probabilities to age 60 for the eight population groups in each country. Survival is substantially lower in Zambia: even the group with the highest survival probability remains well below the lowest-survival groups in Cambodia and El Salvador. The relationship between multidimensional poverty and survival to old age is clear in Cambodia and Zambia, but much weaker in El Salvador. For instance, in El

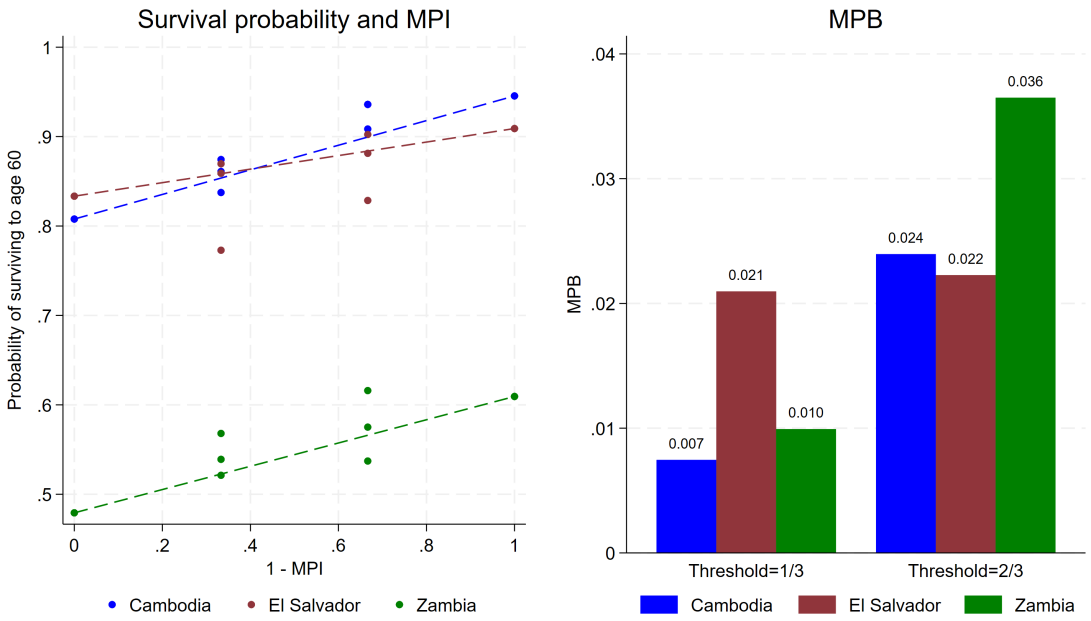
Salvador the most deprived group ($1 - MPI = 0$) has a higher survival probability than some groups with lower poverty levels. Figures 6 and 7 therefore suggest that the dimensions included in the poverty measure generate only weak mortality gradients in El Salvador, and the association between survival probabilities and poverty risk is limited. In contrast, survival is clearly related to multidimensional poverty in Cambodia and Zambia. Moreover, the covariance between survival and poverty risk is relatively similar in these two countries, despite the large difference in their average survival probabilities.

The empirical estimates in the right-hand side of Figure 7 further illustrate why the conditions of proposition 2 and 6 matter. At MPI_T=1/3, the almost universal incidence of poverty in Cambodia and Zambia makes the average survival probability among the poor close to the population average, thereby limiting the MPB. As shown in Proposition 2, its magnitude depends on the gap between the average survival probability of the overall population and that of the poor population. This result holds despite the stronger mortality gradients observed in these two countries. The initially unexpected higher MPB in El Salvador can be explained by the fact that, although the dimensions used to measure poverty are less strongly associated with mortality, poverty is less widespread. This can lead to a larger difference between the average survival of the poor and that of the total population, resulting in a higher MPB, as established by Proposition 2. When the threshold increases to k=2/3, poverty identification becomes more concentrated among the most deprived groups, for whom survival is particularly low in Cambodia and Zambia. Under this more selective poverty structure, Zambia exhibits the highest MPB, consistent with the theoretical role of lower average survival in amplifying mortality selection.

Figure 8 illustrates how alternative weighting structures affect the composition of the population identified as multidimensionally poor and, through this channel, the MPB when the poverty threshold is $k = 2/3$. In each triangle, the vertices correspond to assigning all the weight to basic services (w_1), education (w_2), or economic vulnerability (w_3), while interior points represent combinations of these weights. The left-hand panels show the average survival probability at age 60 among the population identified as poor, where warmer colors (yellow, orange, and red) indicate higher survival probabilities and blue tones indicate lower survival; the right-hand panels show the corresponding MPB, with blue indicating a smaller bias and yellow, orange, and red indicating progressively larger values.

A clear inverse association emerges between both sides of the figure, consistently with Proposition 2: weighting structures that identify poor populations with lower average survival probabilities tend to generate a larger MPB. In Cambodia, survival among the poor remains comparatively high under most weighting structures, and the MPB is correspondingly relatively low, although increasing the weight of economic vulnerability moves the identified poor population toward somewhat lower survival and higher MPB. A similar pattern is observed in El Salvador, where weighting basic services more heavily is associated with relatively high survival among the poor and a low MPB, whereas shifting weight toward economic vulnerability reduces survival and increases the bias. The relationship is particularly pronounced in Zambia: survival probabilities among the poor are substantially lower than in the other two countries, and weighting structures that place greater importance on education, and to a lesser extent economic vulnerability, generate some of the highest MPB values in the figure. Thus, the results emphasize that weights are not merely a normative component of multidimensional poverty measurement; by changing which deprivation pro-

Figure 7: **Survival probabilities, multidimensional poverty and MPB**



Note: The left-hand side panel illustrates the survival probability for each population group given their MPI. The x-axis measures 1-MPI, therefore, the poorest group is located when the axis is zero. *Source:* Author's calculations using IPUMS International census and mortality records for each country (Ruggles et al., 2025).

files are classified as poor, they also change the mortality composition of the poor population and therefore the magnitude of the mortality-selection bias identified in Proposition 2.

6 Conclusions

This research revisits the missing poor problem within a multidimensional framework. Conventional poverty measures are computed over the population that is alive at the time of measurement and therefore implicitly condition on survival. When premature mortality is disproportionately concentrated among population groups with a higher probability of being poor, these groups become underrepresented among survivors and observed poverty is consequently underestimated. The main contribution of this paper is to show that this selection problem extends beyond monetary poverty and depends on the multidimensional structure through which poverty is identified. By classifying the population according to deprivation profiles, assigning group-specific survival probabilities, and allowing for multidimensional mobility between periods, the proposed framework identifies the population missing because of mortality inequality and the subset that would have been poor had they survived. The resulting adjustment provides a measure of old-age multidimensional poverty that corrects for this selection process, while the Missing Poor Bias (MPB) quantifies the extent to which the conventional measure differs from the adjusted one.

The theoretical results show that the magnitude of the MPB is not mechanically determined by mortality differences alone. What matters is the interaction between survival and

Figure 8: **Mortality selection and multidimensional poverty weights**



Note: Each point in the ternary plots represents a weighting structure satisfying $(w_1 + w_2 + w_3 = 1)$, where (w_1), (w_2), and (w_3) correspond to basic services, education, and economic vulnerability, respectively. The left panels report the population-weighted survival probability at age 60 among the identified poor, while the right panels report the corresponding Missing Poor Bias (MPB); the diamond indicates equal weights ((1/3,1/3,1/3)).
Source: Author's calculations using IPUMS International census and mortality records for each country (Ruggles et al., 2025).

poverty risk. When population groups with a higher probability of being poor in old age also exhibit lower survival probabilities, selective mortality reduces their representation in the observed old-age population and increases the MPB. Under the baseline assumption that survivors and prematurely deceased individuals would experience the same multidimensional mobility, the adjusted poverty measure is independent of survival probabilities and therefore eliminates the mortality paradox. Moreover, the MPB can be represented through the negative covariance between survival probabilities and old-age poverty risk. This provides a direct interpretation of mortality selection: mortality inequality generates a downward bias in observed poverty insofar as low survival is systematically associated with high future poverty risk.

A particularly relevant result arises from the multidimensional identification of poverty. Both the probability of being poor in old age and the probability of survival depend on the specific deprivation profile characterizing each population group. Consequently, the dimen-

sions included in the poverty measure and their associated weights determine not only who is classified as poor, but also the survival composition of that population. A weighting structure generates a larger MPB when it identifies as poor groups with lower average survival probabilities. In this respect, multidimensional identification goes beyond the conventional monetary missing-poor approach. Income represents only one possible mortality gradient, while disadvantages associated with education, basic services, economic vulnerability, health, or other dimensions may be related to survival independently of monetary poverty. A monetary measure may therefore fail to identify missing poor when survival inequalities operate primarily through non-monetary dimensions.

The empirical application to Cambodia, El Salvador, and Zambia illustrates these mechanisms. The three countries exhibit substantially different survival levels and mortality gradients across deprivation profiles despite some similarities in observed multidimensional poverty. At the lower poverty threshold, the almost universal incidence of poverty in Cambodia and Zambia makes average survival among the poor relatively close to average population survival, limiting the MPB despite substantial mortality inequalities. At the higher threshold, poverty identification becomes concentrated among more deprived and lower-survival groups, and the bias increases, with Zambia displaying the largest MPB. The analysis of alternative weighting structures also confirms the theoretical mechanism: weights that shift poverty identification toward deprivation profiles with lower survival probabilities generate larger biases. Thus, neither poverty incidence nor mortality inequality considered separately is sufficient to predict the magnitude of the missing poor problem.

These findings also have implications for international poverty comparisons. Two societies displaying similar observed poverty levels need not have comparable underlying poverty when mortality selection differs between them, while similar average survival probabilities do not guarantee comparability when mortality is distributed differently across deprivation profiles. Correcting for the missing poor could therefore contribute to improving worldwide comparability in poverty measurement by separating differences in observed living standards from differences generated by selective survival. This adjustment does not replace the conventional requirements for comparability—common dimensions, deprivation cutoffs, weights, reference populations, and sufficiently comparable data—but addresses an additional source of incomparability: the fact that the population available to be counted may itself be differently selected by mortality across societies.

Several limitations qualify these results and point toward future research. The most important concerns the counterfactual multidimensional trajectory of individuals who died prematurely. The baseline assumption that their transition probabilities would have been the same as those observed for survivors makes the adjustment tractable and is plausible when deaths are recent and relatively structural deprivation indicators are used, but it cannot be directly tested. Future work could therefore develop bounds and sensitivity analyses allowing the counterfactual transition matrix to vary. The empirical analysis is also constrained by census data: although censuses provide sufficiently large samples to estimate mortality by deprivation profile, they contain a restricted set of comparable indicators, deaths may be imperfectly reported, and their cross-sectional structure does not allow the trajectories of the same individuals to be observed. The association between deprivation and mortality should therefore not be interpreted as a causal effect of particular deprivations on survival. A promising empirical agenda would combine censuses with mortality registers, longitudinal

surveys, and administrative information, using the former to characterize population structure and the latter to estimate survival and multidimensional transitions more precisely. Longitudinal information would also allow multidimensional mobility to be studied directly and the assumption concerning the mobility of the missing population to be relaxed.

Finally, a crucial limitation of the present framework is that it provides a one-shot representation of a process that is intrinsically cumulative. The two-period model isolates the mechanism through which mortality between young and old age affects the population observed at the end of the period, but the missing poor problem is generated repeatedly across cohorts and generations. Populations observed today are already the outcome of previous mortality selection, while premature deaths in one generation may affect the deprivation profiles, resources, and opportunities of subsequent generations. Lefebvre et al. (2023) analyzed this cumulative nature of the missing poor from a unidimensional perspective, applying their approach to pre-industrial societies that had not yet undergone the demographic transition. A natural theoretical extension is therefore to develop a dynamic or overlapping-generations version of the model in which mortality selection and multidimensional mobility operate recursively over time. Such a framework would make it possible to distinguish the contemporaneous MPB from the historical accumulation of missing poor and to assess whether conventional cross-sectional poverty measures conceal a larger cumulative effect of mortality inequality. More generally, the results of this paper emphasize that changes in measured poverty cannot always be interpreted independently of changes in the population that survives to be observed. Incorporating multidimensional deprivation into the missing poor framework makes this selection visible along several dimensions of well-being and provides a basis for poverty comparisons that account more explicitly for inequalities in the opportunity to survive.

References

[1] Alkire, S., & Foster, J. (2011). Counting and multidimensional poverty measurement. Journal of public economics, 95(7-8), 476-487.

[2] Alkire, S., & Santos, M. (2014). Measuring acute poverty in the developing world: Robustness and scope of the multidimensional poverty index. World Development, 59, 251-274.

[3] Alkire, S., Roche, J. M., Ballon, P., Foster, J., Santos, M. E., & Seth, S. (2015). Multidimensional poverty measurement and analysis. Oxford University Press, USA.

[4] Alkire, S., Kanagaratnam, U., Nogales, R., & Suppa, N. (2022). Revising the global multidimensional poverty index: Empirical insights and robustness. Review of Income and Wealth, 68, S347-S384.

[5] Association of Southeast Asian Nations (ASEAN). (2023). Old age poverty and active ageing in ASEAN. The ASEAN Secretariat

[6] Atkinson, A., Cantillon, B., Marlier, E. & Nolan, B. (2002). Social Indicators - The EU and Social Inclusion. Oxford University Press, Oxford.

[7] Baland, J. M., Cassan, G., & Decerf, B. (2021). "Too Young to Die": Deprivation Measures Combining Poverty and Premature Mortality. American Economic Journal: Applied Economics, 13(4), 226-257.

[8] Battiston, D., Cruces, G., Lopez-Calva, L. F., Lugo, M. A., & Santos, M. E. (2013). Income and beyond: Multidimensional poverty in six Latin American countries. Social indicators research, 112(2), 291-314.

[9] Beccaria, L., & Fernández, A. L. (2020). Measuring multidimensional poverty using households surveys. Problemas del desarrollo, 51(200), 129-156.

[10] Bourguignon, F., & Chakravarty, S. R. (2003). The measurement of multidimensional poverty. The Journal of Economic Inequality, 1, 25-49.

[11] Burchi, F., De Muro, P., & Kollar, E. (2018). Constructing well-being and poverty dimensions on political grounds. Social Indicators Research, 137(2), 441-462.

[12] Coale, A. J., Demeny, P., & Vaughan, B. (1983). Regional model life tables and stable populations (2nd ed.). Academic Press.

[13] Cutler, D. M. (2004). Your money or your life: Strong medicine for America's health care system. Oxford University Press.

[14] Cutler, D., & Miller, G. (2005). The role of public health improvements in health advances: the twentieth-century United States. Demography, 42(1), 1-22.

[15] Cutler, D., Deaton, A., & Lleras-Muney, A. (2006). The determinants of mortality. Journal of economic perspectives, 20(3), 97-120.

[16] Decancq, K., Fleurbaey, M., & Maniquet, F. (2019). Multidimensional poverty measurement with individual preferences. The Journal of Economic Inequality, 17(1), 29-49.

[17] Elo, I. T., & Preston, S. H. (1996). Educational differentials in mortality: United States, 1979–1985. Social science & medicine, 42(1), 47-57.

[18] Evans, B., & Palacios, R. (2015). Who is poorer?: poverty by age in the developing World (Vol. 18). Washington, DC, USA: World Bank.

[19] Fogel, R. W. (2004). The escape from hunger and premature death, 1700-2100.

[20] Foster, J., Greer, J., & Thorbecke, E. (1984). A class of decomposable poverty measures. Econometrica: journal of the econometric society, 761-766.

[21] Kanbur, R., & Mukherjee, D. (2007). Premature mortality and poverty measurement. Bulletin of economic Research, 59(4), 339-359.

[22] Lefebvre, M., Pestieau, P., & Ponthière, G. (2013). Measuring poverty without the mortality paradox. Social Choice and Welfare, 40, 285-316.

[23] Lefebvre, M., Pestieau, P., & Ponthière, G. (2018). FGT Old-Age Poverty Measures and the Mortality Paradox: Theory and Evidence. Review of Income and Wealth, 64(2), 428-458.

[24] Lefebvre, M., Pestieau, P., & Ponthiere, G. (2019). Missing poor and income mobility. Journal of Comparative Economics, 47(2), 330-366.

[25] Lefebvre, M., Pestieau, P., & Ponthière, G. (2018). Counting the missing poor in pre-industrial societies. Cliometrica, 17, 155-183.

[26] Lleras-Muney, A. (2005). The relationship between education and adult mortality in the United States. The Review of economic studies, 72(1), 189-221.

[27] Nussbaum, M. (2011). Creating capabilities. The human development approach. Cambridge, MA: Hardvard University Press.

[28] Nussbaum, M., & Sen, A. (Eds.). (1993). The quality of life. Clarendon press.

[29] Organization for Economic Co-operation and Development. (2025). Pensions at a Glance 2025. OECD and G20 Indicators. OECD Publishing, Paris.

[30] Oxford Human Development Initiative. (2024). Global Multidimensional Poverty Index 2024: Poverty amid conflict.

[31] Pamuk, E. (Ed.). (1998). Health in the United States 1998: with socioeconomic status and health chart book. DIANE Publishing.

[32] Preston, S. H., Heuveline, P., & Guillot, M. (2001). Demography: Measuring and modeling population processes. Blackwell Publishers.

[33] Ravallion, M. (2012). Poverty lines across the world (pp. 76-104). Oxford: Oxford University Press.

[34] Romero-Prieto, J., Verhulst, A., & Guillot, M. (2024). Estimating ${}_1a_0$ and ${}_4a_1$ in a life table: A model approach based on newly collected data. Demography, 61(3), 643–664.

[35] Ruggles, S., Cleveland, L. L., Lovatón Dávila, R., Sarkar, S., Sobek, M., Burk, D., Ehrlich, D. E., Lee, J., & Merrill, N. (2025). IPUMS International: Version 7.7 [Data set]. IPUMS.

[36] Sen, A. (1976). Poverty: an ordinal approach to measurement. Econometrica: Journal of the Econometric Society, 219-231.

[37] Sen, A. (1979). Personal utilities and public judgements: or what's wrong with welfare economics. The economic journal, 89(355), 537-558.

[38] Sen, A. (1983). Poor, relatively speaking. Oxford economic papers, 35(2), 153-169.

[39] Sen, A. (1985). Well-Being, Agency and Freedom: The Dewey Lectures 1984. The Journal of Philosophy, 82(4): 169–221.

[40] Sen, A. (1990). More than 100 million women are missing. New York Review of Books 37(20).

[41] Sen, A. (1997). From income inequality to economic inequality. Southern Economic Journal, 64(2), 384-401.

[42] Sen, A. (1998). Mortality as an indicator of economic success and failure. The economic journal, 108(446), 1-25.

[43] Sen, Amartya. (1999). Development as Freedom. NewYork: Alfred Knopf.

[44] Stankiewicz Serra, A., Yalonetzky, G. I., & Maia, A. G.. (2021). Multidimensional poverty in Brazil in the early 21st century: Evidence from the demographic census. Social Indicators Research, 154(1), 79–114.

[45] Statistics South Africa. (2014). The South African MPI: Creating a multidimensional poverty index using census data (Report No. 03-10-08).

[46] Uganda Bureau of Statistics. (2026). National Population and Housing Census 2024 monograph: Multidimensional poverty index.

[47] United Nations. (2024). World population prospects. Departement of economic and social affairs.

[48] Van den Berg, G. J., Lindeboom, M., & Portrait, F. (2006). Economic conditions early in life and individual mortality. American Economic Review, 96(1), 290-302.

[49] World Bank. (2026). World Development Indicators [Data set]. World Bank.

Appendices

Appendix 1: Proof of Lemma 2

Proof. Starting from

$$H^O = \frac{\sum_{i=1}^{2^J} \tau_i \pi_i N_i^Y}{\sum_{i=1}^{2^J} \pi_i N_i^Y}.$$

Define

$$\alpha \equiv \sum_{i=1}^{2^J} \tau_i \pi_i N_i^Y, \qquad \beta \equiv \sum_{i=1}^{2^J} \pi_i N_i^Y,$$

so that

$$H^O = \frac{\alpha}{\beta}.$$

Applying the quotient rule:

$$\frac{\partial H^O}{\partial \pi_i} = \frac{\left(\frac{\partial \alpha}{\partial \pi_i}\right)\beta - \alpha\left(\frac{\partial \beta}{\partial \pi_i}\right)}{\beta^2}.$$

Since

$$\frac{\partial \alpha}{\partial \pi_i} = \tau_i N_i^Y, \qquad \frac{\partial \beta}{\partial \pi_i} = N_i^Y,$$

it follows that

$$\frac{\partial H^O}{\partial \pi_i} = \frac{\tau_i N_i^Y \beta - \alpha N_i^Y}{\beta^2}.$$

Factoring out N_i^Y:

$$\frac{\partial H^O}{\partial \pi_i} = \frac{N_i^Y(\tau_i \beta - \alpha)}{\beta^2}.$$

Using the identity $\alpha = H^O \beta$:

$$\frac{\partial H^O}{\partial \pi_i} = \frac{N_i^Y(\tau_i \beta - H^O \beta)}{\beta^2}.$$

Simplifying:

$$\frac{\partial H^O}{\partial \pi_i} = \frac{N_i^Y(\tau_i - H^O)}{\beta}.$$

Substituting back for β gives

$$\frac{\partial H^O}{\partial \pi_i} = \frac{N_i^Y(\tau_i - H^O)}{\sum_{k=1}^{2^J} \pi_k N_k^Y}.$$

Since $N_i^Y > 0$ and $\sum_{k=1}^{2^J} \pi_k N_k^Y > 0$, the sign of the derivative is entirely determined by the sign of $(\tau_i - H^O)$. Therefore:

$$\frac{\partial H^O}{\partial \pi_i} > 0 \quad \Leftrightarrow \quad \tau_i > H^O,$$

and

$$\frac{\partial H^O}{\partial \pi_i} < 0 \quad \Leftrightarrow \quad \tau_i < H^O.$$

■

Appendix 2: Proof of Proposition 1

. The mortality paradox arises when higher mortality among disadvantaged population groups mechanically reduces observed poverty because poor individuals disappear from the surviving population. Therefore, a poverty measure is not subject to the mortality paradox if selective mortality cannot artificially improve poverty outcomes through a compositional effect.

Throughout the proof, let

$$D = \pi^* \sum_{k=1}^{2^J} N_k^Y > 0,$$

where $\pi^* = \max\{\pi_i\}$ is treated as fixed.

(i) General adjusted measure ($\widehat{H}^O$). Consider the adjusted multidimensional poverty headcount ratio:

$$\widehat{H}^O = \frac{\sum_{j=1}^{2^J} \sum_{i=1}^{2^J} p_j N_i^Y \left[\pi^* \delta_{ij} + (\pi^* - \pi_i)(\gamma_{ij} - \delta_{ij})\right]}{D}.$$

Define the numerator as

$$A = \sum_{j=1}^{2^J} \sum_{i=1}^{2^J} p_j N_i^Y \left[\pi^* \delta_{ij} + (\pi^* - \pi_i)(\gamma_{ij} - \delta_{ij})\right].$$

Hence,

$$\widehat{H}^O = \frac{A}{D}.$$

Since the denominator is constant with respect to π_i,

$$\frac{\partial \widehat{H}^O}{\partial \pi_i} = \frac{1}{D} \frac{\partial A}{\partial \pi_i}.$$

Only terms involving π_i vary. Therefore,

$$\frac{\partial A}{\partial \pi_i} = \sum_{j=1}^{2^J} p_j N_i^Y \frac{\partial}{\partial \pi_i} \left[(\pi^* - \pi_i)(\gamma_{ij} - \delta_{ij})\right].$$

Using

$$\frac{\partial}{\partial \pi_i}(\pi^* - \pi_i) = -1,$$

it follows that

$$\frac{\partial A}{\partial \pi_i} = -N_i^Y \sum_{j=1}^{2^J} p_j (\gamma_{ij} - \delta_{ij}).$$

Substituting back,

$$\frac{\partial \widehat{H}^O}{\partial \pi_i} = -\frac{N_i^Y}{\pi^* \sum_{k=1}^{2^J} N_k^Y} \sum_{j=1}^{2^J} p_j (\gamma_{ij} - \delta_{ij}).$$

Since

$$N_i^Y > 0, \qquad \pi^* \sum_{k=1}^{2^J} N_k^Y > 0,$$

the sign of the derivative is entirely determined by

$$\sum_{j=1}^{2^J} p_j (\gamma_{ij} - \delta_{ij}).$$

Rearranging terms,

$$\sum_{j=1}^{2^J} p_j(\gamma_{ij} - \delta_{ij}) = \sum_{j=1}^{2^J} p_j\gamma_{ij} - \sum_{j=1}^{2^J} p_j\delta_{ij}.$$

Using the definition

$$\tau_i = \sum_{j=1}^{2^J} p_j\delta_{ij},$$

we obtain

$$\frac{\partial \widehat{H}^O}{\partial \pi_i} > 0 \iff \sum_{j=1}^{2^J} p_j\gamma_{ij} < \tau_i,$$

and

$$\frac{\partial \widehat{H}^O}{\partial \pi_i} < 0 \iff \sum_{j=1}^{2^J} p_j\gamma_{ij} > \tau_i.$$

Therefore, the effect of mortality depends on the difference between the multidimensional mobility of survivors and the hypothetical mobility of prematurely deceased individuals. Mortality no longer changes poverty mechanically through selective survival, since missing individuals are explicitly reincorporated into the poverty measure through γ_{ij}.

(ii) Adjusted measure with equal mobility of survivors and non-survivors ($\widehat{H}_A^O$).
Consider

$$\widehat{H}_A^O = \frac{\sum_{i=1}^{2^J} \tau_i N_i^Y}{\sum_{i=1}^{2^J} N_i^Y}.$$

Observe that neither the numerator nor the denominator depends on π_i. Therefore,

$$\frac{\partial \widehat{H}_A^O}{\partial \pi_i} = 0, \qquad \forall i.$$

Hence, the adjusted poverty measure is completely invariant to mortality differentials across groups. Selective survival cannot alter measured poverty because the measure is constructed as if all groups experienced the same benchmark survival probability and mobility were independent of survival status.

Consequently, equation 19 fully eliminates the mortality paradox.

■

Appendix 3: Proof of Proposition 2

. Consider that $s_i = \frac{N_i^Y}{\sum_{k=1}^{2^J} N_k^Y}$ denotes population shares, $\pi_i \in (0,1]$ denotes survival probabilities, and average survival in the population is given by $\bar{\pi} = \sum_{i=1}^{2^J} s_i \pi_i$. The old-age poverty risk, $\tau_i(W)$, is defined as in (10), but it is written explicitly as a function of W because the weighting structure determines who is identified as multidimensionally poor in both periods through

$$p_i(W) = \mathbf{1}(mpi_i(W) \geq mpi_T).$$

Accordingly, the average survival probability among individuals expected to be poor in old age is

$$\bar{\pi}_{poor}(W) = \frac{\sum_{i=1}^{2^J} s_i \pi_i \tau_i(W)}{\sum_{i=1}^{2^J} s_i \tau_i(W)}.$$

Under equal mobility for survivors and premature dead population, the adjusted poverty measure is

$$\widehat{H}_A^O(W) = \sum_{i=1}^{2^J} s_i \tau_i(W),$$

while observed poverty is

$$H^O(W) = \frac{\sum_{i=1}^{2^J} s_i \pi_i \tau_i(W)}{\sum_{i=1}^{2^J} s_i \pi_i} = \frac{\sum_{i=1}^{2^J} s_i \pi_i \tau_i(W)}{\bar{\pi}}.$$

Using the definition of $\bar{\pi}_{poor}(W)$,

$$\sum_{i=1}^{2^J} s_i \pi_i \tau_i(W) = \bar{\pi}_{poor}(W) \sum_{i=1}^{2^J} s_i \tau_i(W)$$

Therefore,

$$H^O(W) = \frac{\bar{\pi}_{poor}(W)}{\bar{\pi}} \widehat{H}_A^O(W).$$

Rearranging,

$$\widehat{H}_A^O(W) = \frac{\bar{\pi}}{\bar{\pi}_{poor}(W)} H^O(W).$$

Substituting into the definition of the Missing Poor Bias,

$$MPB_A(W) = \frac{\widehat{H}_A^O(W) - H^O(W)}{H^O(W)},$$

yields

$$MPB_A(W) = \frac{\bar{\pi}}{\bar{\pi}_{poor}(W)} - 1.$$

Differentiating with respect to w_k,

$$\frac{\partial MPB_A(W)}{\partial w_k} = -\frac{\bar{\pi}}{\bar{\pi}_{poor}(W)^2}\frac{\partial \bar{\pi}_{poor}(W)}{\partial w_k}.$$

Since $\bar{\pi} > 0$ and $\bar{\pi}_{poor}(W)^2 > 0$, the sign of the derivative is entirely determined by the sign of

$$\frac{\partial \bar{\pi}_{poor}(W)}{\partial w_k}.$$

Hence,

$$\frac{\partial MPB_A(W)}{\partial w_k} > 0 \iff \frac{\partial \bar{\pi}_{poor}(W)}{\partial w_k} < 0.$$

■

Appendix 4: Proof of Proposition 3

. Under de baseline scenario, $\delta_{ij} = \gamma_{ij}$, the adjusted old-age multidimensional poverty measure is given by

$$\widehat{H}_A^O = \frac{\sum_{i=1}^{2^J} \tau_i N_i^Y}{\sum_{i=1}^{2^J} N_i^Y}$$

which is independent of survival probabilities π_i. Therefore, $\frac{\partial \widehat{H}_A^O}{\partial \pi_i} = 0$.

By definition,

$$MPB_A = \frac{\widehat{H}_A^O - H^O}{H^O} = \frac{\widehat{H}_A^O}{H^O} - 1.$$

Differentiating with respect to π_i yields

$$\frac{\partial MPB_A}{\partial \pi_i} = \frac{\partial}{\partial \pi_i}\left(\frac{\widehat{H}_A^O}{H^O} - 1\right).$$

Since $\widehat{H}_A^O$ does not depend on π_i, and applying the quotient rule,

$$\frac{\partial MPB_A}{\partial \pi_i} = -\frac{\widehat{H}_A^O}{(H^O)^2}\frac{\partial H^O}{\partial \pi_i}.$$

From Lemma 2 (appendix 1),

$$\frac{\partial H^O}{\partial \pi_i} = \frac{N_i^Y}{\sum_k N_k^Y \pi_k}\left(\tau_i - H^O\right).$$

Substituting into the previous expression gives

$$\frac{\partial MPB_A}{\partial \pi_i} = -\frac{\widehat{H}_A^O}{(H^O)^2}\frac{N_i^Y}{\sum_k N_k^Y \pi_k}\left(\tau_i - H^O\right),$$

Since

$$\widehat{H}_A^O > 0, \qquad (H^O)^2 > 0, \qquad \frac{N_i^Y}{\sum_k N_k^Y \pi_k} > 0,$$

the sign is determined entirely by the term $-(\tau_i - H^O)$. Hence,

$$\frac{\partial MPB_A}{\partial \pi_i} < 0 \quad \text{if} \quad \tau_i > H^O,$$

and

$$\frac{\partial MPB_A}{\partial \pi_i} > 0 \quad \text{if} \quad \tau_i < H^O.$$

■

Appendix 5: Multidimensional poverty measurement and the missing poor bias when there is no mobility for premature dead population $\gamma_{ii} = 1,\ \forall i \in \{1, 2, \ldots, 2^J\}$

The baseline setting of this model assumes that the mobility of premature dead is the same that the mobility of alive population, both groups would have had exactly the same transition between populations groups from young to old age. Nevertheless, as Lefebvre et al. (2013, 2018) analyzed in the monetary approach, it is also plausible that missing people would have remained in the same group provided they survived, while the household they belonged could face a different trajectory. In this appendix the basic theoretical setting will be summarized consider this escenario of multidimensional mobility.

This scenario of no mobility means that in the matrix given in equation 13, $\gamma_{ii} = 1\ \forall\ i \in \{1, 2, \ldots, 2^J\}$. Following definition 5 and equation 14, the missing poor are defined as the prematurely deceased individuals who belonged to population groups identified as poor at the initial period:

$$N^{MP} = P'\Gamma^{\diamond}N^Y = \sum_{i=1}^{2^J} p_i(\pi^* - \pi_i)N_i^Y$$

Therefore, the adjusted multidimensional poverty headcount ratio woud be:

$$\widehat{H}_B^O = \frac{\sum_{i=1}^{2^J} N_i^Y\left[p_i(\pi^* - \pi_i) + \tau_i\pi_i\right)]}{\pi^*\sum_{i=1}^{2^J} N_i^Y}$$

In the numerator we have people who did not survive and were poor when young, plus

the survivors who ended up in a poor group when old. Poor population is adjusted considering in the counting of poor, the fraction of non-survivors in poor population groups ($p_i = 1$), while adding to the total population the amount of people that were supposed to survive if the benchmark survival probability had been observed for every population group. It is importat to underline that under this scenario the adjustment still depends on π_i because the prematurely dead are kept in their initial poverty status (p_i), while survivors are classified according to their old-age poverty risk. The numerator in $\widehat{H}_B^O$ can be written as $\sum_{i=1}^{2^J} N_i^Y[\pi^* p_i - \pi_i(p_i - \tau_i)]$, the first component sums all people initially poor under the mortality benchmark, while the second subtracts those survivors who were initially poor but did not end up as poor. The adjusted poverty measure still depends on group-specific survival probabilities because only surviving individuals experience poverty transitions, whereas the prematurely dead are assumed to retain their initial deprivation status. Consequently, the survival probability π_i weights the difference between the initial poverty status and the expected old-age poverty status of each population group, preventing π_i from cancelling out in the adjustment.

Proposition 1 showed that the adjusted poverty measures introduced in this article are not subject to the mortality paradox, in particular, assuming equal mobility of survivors and non-survivors the measure of poverty is fully immune to that paradox. It is also the case when we consider no multidimensional mobility, extending proposition 1 to this case, we have that, differentiating $\widehat{H}_B^O$ with respect to π_i yields:

$$\frac{\partial \widehat{H}_B^O}{\partial \pi_i} = \frac{N_i^Y}{\pi^* \sum_{k=1}^{2^J} N_k^Y}(\tau_i - p_i).$$

Hence,

$$\frac{\partial \widehat{H}_B^O}{\partial \pi_i} > 0 \iff \tau_i > p_i,$$

and

$$\frac{\partial \widehat{H}_B^O}{\partial \pi_i} < 0 \iff \tau_i < p_i.$$

In particular, if group i is initially poor ($p_i = 1$), then

$$\frac{\partial \widehat{H}_B^O}{\partial \pi_i} \leq 0,$$

since $0 \leq \tau_i \leq 1$. Therefore, increasing mortality among poor groups cannot mechanically reduce adjusted poverty.

Proposition 3 states that, assuming that $\delta_{ij} = \gamma_{ij}$, a reduction in the survival probability (increase in mortality) of a population group that has a high poverty risk increases the Missing Poor Bias through its effect in the observed poverty measure. Here it is confirmed that under no mobility for premature dead population, the same conclusion is found. The marginal effect of a change in survival probability π_i under $\gamma_{ii} = 1,\ \forall i$ is:

$$\frac{\partial \text{MPB}_B}{\partial \pi_i} = \frac{N_i^Y}{H^O}\left[\frac{1}{\pi^* \sum_{k=1}^{2^J} N_k^Y}(\tau_i - p_i) - \frac{\widehat{H}_B^O}{H^O \sum_{k=1}^{2^J} N_k^Y \pi_k}(\tau_i - H^O)\right].$$

The sign of the marginal effect depends jointly on:

1. the initial poverty status of group i, through $(\tau_i - p_i)$;
2. the relative old-age poverty risk of group i, through $(\tau_i - H^O)$.

In particular:

$$\frac{\partial \text{MPB}_B}{\partial \pi_i} \begin{cases} < 0 & \text{if } p_i = 1 \text{ and } \tau_i \geq H^O, \\ \gtrless 0 & \text{if } p_i = 1 \text{ and } \tau_i < H^O, \\ > 0 & \text{if } p_i = 0 \text{ and } \tau_i \leq H^O, \\ \gtrless 0 & \text{if } p_i = 0 \text{ and } \tau_i > H^O. \end{cases} \tag{25}$$

The effect of survival probabilities on the MPB is more complex than the baseline because survival affects both the observed and the adjusted poverty measures. The sign of the effect depends jointly on the initial poverty status of a group and on its probability of being poor in old age relative to the observed poverty rate. Empirically, the most plausible cases are those in which initially poor groups ($p_i = 1$) exhibit above-average old-age poverty risk ($\tau_i \geq H^O$), while initially non-poor groups ($p_i = 0$) exhibit below-average poverty risk ($\tau_i \leq H^O$). In these cases, higher survival among initially poor groups reduces the MPB, whereas higher survival among initially non-poor groups increases it.

Appendix 6: Proof of Proposition 4

. If the transition mobility from young to old age is the same for survivors and non-survivors, the adjusted old-age poverty measure is $\widehat{H}_A^O = \sum_{i=1}^{2^J} s_i \tau_i$. The observed old-age poverty measure is:

$$H^O = \frac{\sum_{i=1}^{2^J} s_i \pi_i \tau_i}{\sum_{i=1}^{2^J} s_i \pi_i} = \frac{\sum_{i=1}^{2^J} s_i \pi_i \tau_i}{\bar{\pi}}.$$

Using the standard covariance decomposition,

$$\sum_{i=1}^{2^J} s_i \pi_i \tau_i = \bar{\pi} \widehat{H}_A^O + \text{Cov}(\pi, \tau),$$

it follows that:

$$H^O = \frac{\bar{\pi} \widehat{H}_A^O + \text{Cov}(\pi, \tau)}{\bar{\pi}} = \widehat{H}_A^O + \frac{\text{Cov}(\pi, \tau)}{\bar{\pi}},$$

By definition,

$$MPB_A = \frac{\widehat{H}_A^O - H^O}{H^O}.$$

Substituting the previuos result,

$$MPB_A = \frac{\widehat{H}_A^O - \left(\widehat{H}_A^O + \frac{\text{Cov}(\pi,\tau)}{\bar{\pi}}\right)}{H^O},$$

which simplifies to:

$$MPB_A = -\frac{\text{Cov}(\pi,\tau)}{\bar{\pi}H^O}.$$

Therefore, the Missing Poor Bias depends exclusively on the association between survival probabilities and old-age poverty risk across deprivation profiles. ■

Appendix 7: Proof of Proposition 5

. From Proposition 4,

$$MPB_A = -\frac{\text{Cov}(\pi,\tau)}{\bar{\pi}H^O}.$$

Using the standard covariance decomposition,

$$\text{Cov}(\pi,\tau) = \text{Corr}(\pi,\tau)\sqrt{\text{Var}(\pi)}\sqrt{\text{Var}(\tau)},$$

and assumptions (1) and (2) of Proposition 5,

$$\text{Cov}(\pi,\tau) = \rho\sigma_\tau\sqrt{\text{Var}(\pi)}.$$

Substituting into the covariance representation of the MPB yields:

$$MPB_A = -\frac{\rho\sigma_\tau\sqrt{\text{Var}(\pi)}}{\bar{\pi}H^O},$$

Differentiating with respect to

$$\text{Var}(\pi),$$

and using assumption (3),

$$\frac{\partial H^O}{\partial \text{Var}(\pi)} \approx 0,$$

gives:

$$\frac{\partial MPB_A}{\partial \text{Var}(\pi)} = -\frac{\rho\sigma_\tau}{2\bar{\pi}H^O\sqrt{\text{Var}(\pi)}},$$

Finally, since:

$$\bar{\pi} > 0, \qquad H^O > 0, \qquad \sigma_\tau > 0, \qquad \sqrt{\text{Var}(\pi)} > 0,$$

the sign of the derivative depends exclusively on $-\rho$. Therefore, if $\rho < 0$, then:

$$\frac{\partial MPB_A}{\partial \text{Var}(\pi)} > 0.$$

■

Appendix 8: Proof of Proposition 6

. From Proposition 4,

$$MPB = -\frac{\text{Cov}(\pi, \tau)}{\bar{\pi} H^O}.$$

Define $C \equiv -\text{Cov}(\pi, \tau)$, then $C > 0$. $s \in \{A, B\}$ indexes two societies. The proposition follows directly from the monotonicity properties of

$$MPB(C, \bar{\pi}, H^O) = \frac{C}{\bar{\pi} H^O}.$$

For the first result, suppose that

$$\bar{\pi}^A = \bar{\pi}^B, \qquad H^{O,A} = H^{O,B}.$$

Then,

$$MPB^A = \frac{C^A}{\bar{\pi}^A H^{O,A}}, \qquad MPB^B = \frac{C^B}{\bar{\pi}^B H^{O,B}}.$$

Since the denominators are identical and $C^A > C^B$, it immediately follows that

$$MPB^A > MPB^B.$$

For the second result, suppose that

$$H^{O,A} = H^{O,B}, \qquad C^A = C^B.$$

Then,

$$MPB^A = \frac{C^A}{\bar{\pi}^A H^{O,A}}, \qquad MPB^B = \frac{C^B}{\bar{\pi}^B H^{O,B}}.$$

Since the numerators are identical and $\bar{\pi}^A < \bar{\pi}^B$, the denominator of MPB^A is smaller, implying

$$MPB^A > MPB^B.$$

■

Appendix 9: Life tables for groups 1 and 8 in Cambodia

Age interval	N_x	D_x	${}_nM_x$	${}_nq_x$	l_x	${}_nd_x$	${}_nL_x$	T_x
Panel A. Group 1: deprived in all dimensions $(1,1,1)$								
0	28,690	1,390	0.0484	0.0480	100,000	4,802	99,107	7,906,548
1–4	102,200	1,270	0.0124	0.0463	95,198	4,408	354,755	7,807,442
5–9	147,040	790	0.0054	0.0265	90,790	2,407	436,182	7,452,686
10–19	433,800	370	0.0009	0.0085	88,383	751	880,080	7,016,505
20–29	213,000	330	0.0015	0.0154	87,633	1,347	869,591	6,136,424
30–39	121,310	270	0.0022	0.0220	86,285	1,899	853,358	5,266,834
40–49	150,660	240	0.0016	0.0158	84,386	1,334	837,193	4,413,476
50–59	79,390	220	0.0028	0.0273	83,052	2,270	819,175	3,576,282
60–69	45,850	150	0.0033	0.0322	**80,782**	2,600	794,823	2,757,108
70–79	88,070	230	0.0026	0.0258	78,182	2,015	771,744	1,962,284
80–89	16,750	190	0.0113	0.1073	76,167	8,176	720,787	1,190,540
90+	1,520	220	0.1447	1.0000	67,991	67,991	469,753	469,754
Panel B. Group 8: non-deprived in all dimensions $(0,0,0)$								
0	18,820	180	0.0096	0.0096	100,000	956	99,926	9,127,083
1–4	73,560	140	0.0019	0.0075	99,044	745	391,517	9,027,157
5–9	93,470	160	0.0017	0.0085	98,299	838	484,947	8,635,640
10–19	268,170	80	0.0003	0.0030	97,461	290	973,162	8,150,692
20–29	343,770	60	0.0002	0.0017	97,171	169	970,864	7,177,530
30–39	168,180	140	0.0008	0.0083	97,002	804	965,996	6,206,666
40–49	143,290	150	0.0010	0.0104	96,198	1,002	956,966	5,240,671
50–59	103,320	70	0.0007	0.0068	95,196	643	948,743	4,283,705
60–69	45,140	130	0.0029	0.0284	**94,553**	2,684	932,107	3,334,962
70–79	17,510	70	0.0040	0.0392	91,869	3,601	900,682	2,402,854
80–89	5,540	80	0.0144	0.1347	88,268	11,888	823,239	1,502,172
90+	800	90	0.1125	1.0000	76,380	76,380	678,933	678,933

Notes: N_x denotes the population exposed to mortality in age interval x; D_x is the number of deaths; ${}_nM_x$ is the age-specific mortality rate; ${}_nq_x$ is the conditional probability of dying within the interval; l_x is the number of survivors at the beginning of the interval from a radix of 100,000; ${}_nd_x$ is the number of life-table deaths; ${}_nL_x$ denotes person-years lived within the interval; and T_x is the total number of person-years remaining above age x.

The final age interval is open-ended and therefore ${}_nq_x = 1$ by construction.

Since $l_0 = 100{,}000$, the probability of survival from birth to exact age 60 is $S(60) = l_{60}/100{,}000$. Hence, the estimated probabilities are $S(60) = 0.8078$ for Group 1 and $S(60) = 0.9455$ for Group 8.

Source: Author's calculations using IPUMS International census and mortality records.

INSTITUT DE RECHERCHE
ÉCONOMIQUES ET SOCIALES

Place Montesquieu 3
1348 Louvain-la-Neuve

ISSN 1379-244X D/2026/3082/19

UCLouvain

IRES | LIDAM
Louvain Institute of Data Analysis and Modeling in economics and statistics