PRODUCTIVITY LABOUR ADJUSTMENT COSTS. HOW DO NEW HIRES AND LEAVERS (INCL. RETIREES) COMPARE?

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How do new hires and leavers (incl. retirees) compare?

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Abstract

Labour turnover is a crucial element of contemporary economic life. It can improve productivity if more productive workers replace less productive ones. However, in the short run, it generates sizeable labour adjustment costs (LACs), including productivity losses. This paper sheds new light on the turnover-productivity relationship focusing on productivity LACs. We use firm-level 2014-2022 Belgian data with information on stayers, new hires and leavers: those who are fired, those leaving voluntarily, and those who are about to retire. We use the Hellerstein-Neumark (HN) framework to quantify the productivity of these different labour types, using stayers as a benchmark. We posit that evidence of significant productivity handicaps is a good indicator of productivity LACs. Results suggest no productivity LACs for new hires. By contrast, for leavers, they point to significant ones. What is more, findings for prospective (early) retirees indicate a very sizeable drop in productivity during their last year of employment.

Keywords: Labour Turnover, Labour Adjustment Costs, Labour Productivity Differences, Prospective Retirees, Short Horizon

JEL Codes: J24, J63, J26

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1 Introduction

Labour turnover measures the rate employees leave and are replaced within a company. In the U.S. and Europe, annual turnover rates generally range between 10% and 20%, though they vary by industry and country. For example, Belgium often experiences lower turnover rates with its robust labour protections. Nonetheless, in both the US and Europe, the retirement of Baby Boomers is a significant factor contributing to the increase in turnover rates.

Turnover has mixed implications for firms. It can enhance productivity over the medium to long term by introducing new skills and perspectives, replacing underperformers, or improving job-role alignment. However, in the short term, it may cause Labour Adjustment Costs (LACs), particularly productivity losses ("productivity LACs"). These differ from "financial LACs", which include expenses like recruitment, training, and severance. Despite the importance of productivity LACs, existing studies primarily focus on financial LACs due to challenges in measurement.[1]

The key idea underpinning productivity LACs is that firms incur drops in productive performance when changing the composition (and, potentially, the overall level) of their labour force, whatever the intrinsic/long-term productivity of the people involved. Although, by definition, these are short-term/transitory costs, they matter economically (Hamermesh, 1995). For example, in the presence of LACs, Dixit (1997) predicts that a firm cannot adjust its labour demand costlessly and has the incentive to minimize labour turnover, something called the labour demand "stickyness" or labour "hoarding".

Productivity LACs for new hires could stem from a lack of firm/job-specific experience or firm-related human capital. Exiting employees may also generate productivity LAC before leaving the firm, as stressed by the literature on short horizons and how these negatively affect productivity (Hall and Lazear, 1984; Akerlof, 2005. One of the points we make in this

[1]While the LACs hypothesis has been mobilized in many important works on labour demand, conducting a formal test of this hypothesis has been challenging due to the inherent difficulty of identifying accurate proxies or directly measuring LACs, and even more so for their different components, i.e. financial vs productivity LACs (Golden et al., 2020). The existing literature has relied chiefly on proxies of LACs and has generally focused on financial LACs. For example, Banker et al. (2013) consider country-level employment protection legislation (EPL) as a proxy for LACs associated with the legal impediments to firing workers in an international context. Nguyen (2022) use level of labour skills as a proxy for LACs costs incurred to search, hire, train, retain, and fire employees. Another example is Dierynck et al. (2012) considering the Belgian legal requirements that result in higher costs related to adjusting the number of white-collar employees compared to the costs of adjusting the number of blue-collar employees. But, in these three studies, the proxies only capture financial LACs and ignore productivity LACs.

paper is that assessing the productivity of prospective leavers should be possible if exits are gradual and anticipated. In the Belgian context[2], notice period rules *de facto* mean that most separations take the form of transition work spells, sometimes lasting several months for the workers with the most seniority. Regulations about notice intend to minimize disruption to business operations by mandating a minimal transition period. However, in practice, employees who have resigned or been given notice may mentally disengage from their current role as they focus on their next career move. Both employers and employees may feel less motivated to put in extra effort because they will not benefit from long-term outcomes. The same might apply to employees approaching retirement.

In short, this empirical paper relates to the economic literature on labour turnover and productivity. It focuses on the short-term productivity costs/losses associated with labour turnover. It specifically differentiates between new hires and leavers, as well as among various categories of leavers, including those retiring. Its contribution is essentially fourfold:

- First, to go beyond the existing literature on turnover and labour productivity that focuses on the overall level of turnover and its (primarily long-term) impact on average productivity (De Winne et al., 2018).[3] This paper suggests that the overall impact conflates plusses and minuses and structural vs. more transitory effects, i.e. those corresponding to productivity LACs. The paper also shows that these transitory effects vary in intensity depending on the type of worker considered: i.e. not all leavers display the same level of productivity LACs.

- Second, more on the methodological side, we argue that it is possible to quantify these short-term productivity LACs using methods developed by economists to estimate a private-economy production function where labour is heterogeneous. This implies using firm-level data containing detailed information on *i)* the standard determinants of labour productivity (capital intensity, total labour, educational attainment, industry, year of observation...) and *ii)* annual information on the stayers, new hires, and leavers[4] To retrieve estimates of productivity LACs for each, we mobilise the Hellerstein-Neumark (HN) framework. It has been developed and extensively used to quantify the (relative) labour productivity of workers who differ in terms of gender, ethnicity, age or education (Hellerstein et al., 1999; Vandenberghe et al., 2013). The intuition

[2] Which is fairly representative of what happens in Continental Europe

[3] Results generally show a non-linear relationship such that organizations' labour productivity increases at low levels of turnover, reaches a peak and decreases afterwards in a negatively attenuated fashion.

[4] In our data, we can distinguish those who were fired, those who left voluntarily, and those who retired upon becoming eligible for old-age (early) pension benefits).

underpinning HN is that workers forming a firm's workforce may be modelled as substitutes who vary in quality (i.e. productivity). Using the coefficients associated with each type of worker in the production function, their quality/productivity can be estimated relative to a benchmark (i.e. the reference type). Three key ideas/assumptions underpin the results presented in this paper. First, new hires and different groups of leavers are labour types in their own right and can be considered in the same way as types defined by age, gender or educational attainment. Using the HN framework, their marginal productivity can be estimated against a benchmark (the stayers). Second, by definition, these individuals have either just joined or will soon be leaving the firms. As measured using HN, their productivity is likely influenced by a transitory productivity handicap conducive to LACs. Third, evidence of significant productivity handicaps (i.e., productivity lower than that of the stayers) is a strong indicator of transitory productivity LACs, provided there are no significant unaccounted intrinsic productivity differences between the new hires/leavers and the benchmark group

- Third, the paper examines and quantifies productivity LACs characterizing different categories of leavers, particularly those who are about to `retire`. As far as we know, the LACs literature has never examined this group, although it fully qualifies as a potential source of productivity LACs. This void in the literature exists even though retirement-related exits have started to rise dramatically as a reflection of workforce ageing in Europe[5] and are bound to keep rising over the 2 to 3 coming decades. Moreover, the broader economic literature has long emphasized the risk of a (negative) impact of short work horizons on labour productivity. For instance, Hall and Lazear (1984) highlights the challenges of incentivizing short-term employees, who may have diminished motivation to perform well as their tenure ends. Also Akerlof (2005) discusses how labour contracts often involve a reciprocal relationship where workers provide effort beyond the minimum required in exchange for fair treatment and future rewards. When future rewards are no longer relevant due to a short work horizon, this reciprocal relationship decreases effort and productivity. The point is that prospective retirees[6] are quintessentially the group facing a short work horizon. They tend to focus on planning their post-retirement life rather than continuing to engage deeply with their work. The prospect of (definitely) leaving the workforce can reduce the drive (also in the eyes of employers) to achieve long-term goals or take on new challenges. In other words, retirees in their final year of work are particularly likely to display productivity LACs.

[5] For a review of issues related to ageing workforces, see Skirbekk (2004) or Vandenberghe (2022).

[6] Particularly in Europe, where quasi-mandatory retirement age rules still prevail.

- Fourth, this paper assesses the relative importance of gross vs. net productivity LACs. The literature distinguishes *i)* gross LACs incurred when a worker leaves or joins the firm and are independent of the change in the overall level of employment: these can be linked to the status of the individual joining or about to leave a job, and *ii)* net LACs strictly related to changing the firm's overall number of workers/hours. We do that by re-estimating our key results for firms that have not experienced a significant change in their overall labour force over the years present in our panel, and comparing these to the estimates delivered by the entire sample of firms.

The rest of this paper is structured as follows. The HN method and why it can generate productivity LACs are presented in Section 2. Section 3 discusses the endogeneity problem inherent to most production function estimation using firm-level data and how we cope with it. The data and variables entered into the estimated models are described in Section 4. Section 5 presents the results, and Section 6 concludes.

2 The Hellerstein-Neumark Framework

As said above, we propose using the framework developed by HN to quantify productivity LACs attached to each type of worker involved in labour turnover: new hires, leavers.... Moreover, we propose implementing this distinguishing different categories of leavers, including prospective retirees.

At its core, the HN method aims to retrieve (relative) labour productivity directly from estimating a value-added production function using firm-level data. It owes a lot to the seminal work of economists like Griliches (1979). These authors pioneered using firm-level data to study productivity, mainly focusing on the roles of research and development (R&D) and capital investment. Other authors, such as Mairesse and Dormont (1985), considered this approach promising for evaluating the influence of worker characteristics on productivity, distinct from other firm characteristics like capital intensity. HN extended Mairesse's seminal work by showing that estimates of (relative) marginal labour productivity of different types of workers can be retrieved from the estimation of a "labour-augmented" production function. In such a production function, the contribution of labour to output is modelled by the weighted sum of different types of labourers, where the weights reflect their (relative) marginal productivity.

HN propose estimating (relative) marginal labour productivity for different types of workers using between- and within-firm variations of their share in total employment and how they relate to (firm-level) average labour productivity. Their model rests on the seminal Cobb-Douglas production function and is traditionally used to assess gender/age/education-related productivity differences (Ours and Stoeldraijer, 2011;Dostie, 2011; Vandenberghe, 2011; Vandenberghe et al., 2013; Vandenberghe, 2013; Lebedinski and Vandenberghe, 2014). We assume below that it can be applied to situations where labour heterogeneity consists of stayers vs. new hires vs. (different categories of) leavers. The HN model is quite flexible and can be augmented with many controls. It also accommodates multiple simultaneous decompositions of the labour force (e.g., in terms of gender or education, alongside the decomposition between stayers/leavers/new hires, which we are centred upon here). A significant restriction is that the HN model is log additive[7] in the different labour types. So, by assumption, complementarities or spillover effects between types of workers are excluded: we cannot assess whether new hires and leavers affect each other's productivity or that of the stayers.

HN consider an augmented Cobb-Douglas value-added production function :

$$Y_i = AQL_i^{\alpha} K_i^{\beta} \tag{1}$$

where Y_i is output/value-added in firm i, K_i is capital, and labour heterogeneity → [quality of] labour index QL_i. For the simplicity of exposure hereafter, we drop the time subscript t.

We have that L_{ij} is the number of type j workers (e.g. young/prime-age/old; men/women; stayers/new hires/leavers) in firm i. Types are perfectly substitutable with different weights/marginal products μ_j (assumed identical across firms, meaning that i can be dropped from μ). Each type j is an input of the quality of labour aggregate (QL, with one type (type 0 below) being the reference or benchmark:

$$QL_i = \sum_j \mu_j L_{ij} = \mu_0 L_{i0} + \mu_j L_{i1} + ...\mu_n L_{in} \tag{2}$$

with $j = 0, \ldots, n$ and μ_0 the marginal productivity of (benchmark) type 0 workers.

[7]In other words, labour types are perfect substitutes.

$$
\begin{aligned}
Y_i &= A\left(\mu_0 L_{i0} + \mu_1 L_{i1} + \ldots \mu_n L_{in}\right)^{\alpha} K_i^{\beta} \\
Y_i &= A(\sum_j \mu_j L_i)^{\alpha} K_i^{\beta}
\end{aligned}
\tag{3}
$$

Note that the marginal labour productivity (MLP) of labour types is

$$
\begin{aligned}
MLP_0 \equiv \frac{\partial Y}{\partial L_0} &= A\alpha[\mu_0 L_0 + \mu_1 L_1 + \cdots + \mu_n L_n]^{\alpha-1}\mu_0 K^{\beta} \\
MLP_j \equiv \frac{\partial Y}{\partial L_j} &= A\alpha[\mu_0 L_0 + \mu_j L_j + \cdots + \mu_n L_n]^{\alpha-1}\mu_1 K^{\beta}
\end{aligned}
\tag{4}
$$

thus the relative $MLP_{j;0} = \mu_j/\mu_0$, $j > 0$.

From there, it is easy to show (see Appendix 8.1 for the full development) that the production function becomes:

$$
lnY_i = lnA + \alpha\left(ln\mu_0 + lnL_i + \sum_{j>0}(\lambda_j - 1)S_{ij}\right) + \beta lnK_i \tag{5}
$$

or, equivalently, as a fully linearized expression of the production function that is easy to estimate econometrically

$$
y_i = B + \alpha l_i + \sum_{j>0} \eta_j S_{ij} + \beta k_i \tag{6}
$$

where $y_i \equiv lnY_i, l_i \equiv lnL_i, k_i \equiv lnK_i; B \equiv lnA + \alpha ln\mu_0$ and in particular $\eta_j = \alpha(\lambda_j - 1), j > 0$ with the candidate for type j's MLP equal to $\widehat{\lambda_j} = 1 + \frac{\widehat{\eta_j}}{\widehat{\alpha}}$

As an alternative, one can consider the modified (but strictly equivalent) version of that function – i.e. the one we estimate in this paper – which is the average labour productivity function

$$
ay_i = B + \tilde{\alpha} l_i + \sum_{j>0} \eta_j S_{ij} + \beta k_i \tag{7}
$$

where the right-hand side term is the (log of) average firm-level productivity $ay_i \equiv ln(Y_i/L_i)$ and where $\tilde{\alpha} = \alpha - 1$ and where the candidate MLP of type j is now equal to $\lambda_j = 1 + \frac{\widehat{\eta_j}}{1+\widehat{\tilde{\alpha}}}$.

Finally, turning to the econometric version of the model we estimate hereafter, the above HN equation easily accommodates the presence of controls (X: year, region and sector/industry fixed effects in our case). It can also include the breakdown of the labour force along several distinct/independent dimensions: gender (here female vs (ref.) male workers); educational attainment (here tertiary educated workers vs (ref.) less educated workers) and stayers vs non-stayers).[8] Hellerstein et al. (1999) shows that conditional on the equiproportionality (e.g. same proportion of new hires. . . across education and gender cat.. . .) and equality of relative marginal productivities (e.g same productivity of new hires. . . across education and gender cat.) assumptions, we can confidently estimate

$$ay_i = B + \tilde{\alpha} l_i + \eta_{fem} S_{fem} + \eta_{teduc} S_{teduc} + \sum_{j>0} \eta_j S_j + \beta k + \rho X_i + \epsilon_i \tag{8}$$

and use $\widehat{\eta_j}$ to compute the marginal productivities of type j workers, i.e. new hires and different types of leavers, while also gauging the productivity of women and tertiary-educated workers. As always with HN, these will be relative marginal productivities, i.e. in comparison with a benchmark, which in this case are the (male, non-tertiary educated) stayers. What is more, we will posit that the productivity handicap that we compute as $\widehat{\lambda}_j - 1 = \frac{\widehat{\eta_j}}{1+\widehat{\tilde{\alpha}}}$ are a good proxy of the transitory/short-term productivity LACs. Strictly speaking, this will be the case if there are no significant intrinsic unaccounted[9] labour productivity differences between the new hires/leavers and the benchmark.

3 Coping with simultaneity-reverse causation bias

Estimating a unique (common to all firms/industries) HN production function, as we do in this paper with eq.(8)), is complicated by between-firm heterogeneity and endogeneity problems. The HN approach retained here and applied to labour turnover data is no exception. We explain what we do regarding firm heterogeneity in Section 4.2; namely the inclusion as controls of year, region and sector 4-digit fixed effects. Regarding endogeneity, we have decided to focus on the sort most likely to affect our estimates: the simultaneity-reverse causation bias. This problem ultimately stems from unobserved (non-random) productivity differences at the firm level. For instance, the simultaneity of a (non-observed) negative productivity shock[10], and dismissal/early retirement scheme, or a recruitment freeze is a source

[8] A fully-fledged breakdown along these three dimensions combined would amount to estimating eq. (7).
[9] Via the inclusion of female and tertiary-educated shares.
[10] E.g. due to the loss of a major contract?

of reverse causality: it goes from productivity developments to labour shares (including those forming turnover), which is precisely the reverse of the one we would like to capture with our data. More formally, we have that the OLS sample-error term of the HN production function is extremely likely to consist of *i)* a purely random shock ϵ_{it}; but also *ii)* a productivity term χ_{it}, unobserved by the econometrician but whose level and dynamic is (partially) anticipated by the firm. Hence, it should (partially) drive labour inputs and turnover. The term χ_{it} generates a spurious correlation between the share of new hires/leavers and firms' productivity, even when resorting to identification using solely within NACE 4-digit variation, as we do here.

Over the past decades, economists have implemented different methods to cope with this problem. One approach involves using panel data with firm-level fixed effects (FE), in other words, by assuming that $\chi_{it} = \chi_i$. This also assumes that all simultaneity/reverse causation problems are related to a time-invariant term χ_i. Unfortunately, FE has not worked well in practice (Griliches and Mairesse, 1999). This is due to problems with the assumption: it is indeed unrealistic to assume that decisions about labour inputs are solely driven by something that is time-invariant. We also need a way to control for relatively short-term, unobserved productivity developments. The other well-documented problem with FE[11] is that it exacerbates measurement errors, a problem known for causing estimates that are downwards biased. Another, more recent set of techniques follows the dynamic panel literature (Arellano and Bond, 1991; Vandenberghe, 2011), which consists of using lagged values of labour inputs as instrumental variables (IV-GMM) for contemporary input. The control function method is a third set of somewhat more structural techniques. In this paper, we adopt such a method, and more precisely, the version advocated by Levinsohn-Petrin and Ackerberg et al. (Levinsohn and Petrin, 2003; Ackerberg et al., 2015) (LP-ACF hereafter).

The idea consists of using intermediate inputs to control/proxy short-term unobserved productivity χ_{it} that may partially drive labour turnover and is the source of reverse causation we want to address. The LP-ACF control function approach capitalises on (and improves) the methods developed by Olley and Pakes (1996) (OP). The essence of OP is that, under certain theoretical and statistical assumptions, one can invert optimal input decisions to allow an econometrician to account for the unobserved productivity term χ_{it}. More precisely, OP identified conditions under which firm-level investment (conditional on capital stock k_{it}) is a strictly increasing function of firm-level unobserved productivity term χ_{it}. This strict monotonicity implies that the investment demand function can be inverted and

[11] That amounts to mean-centering or first-differencing all variables.

deliver a proxy of the unobserved productivity term, i.e. $\chi_{it} = f^{-1}(inv_{it}, k_{it})$. In practice, OP suggest representing $f^{-1}(.)$ as a n^{th} degree polynomial in capital stock and investment. Then, the OLS regression of production[12] on that polynomial plus labour inputs, achieves the goal of "controlling for" the unobserved productivity χ_{it} and directly delivering estimates for the labour terms.[13]

The problem with the OP approach is that most firms do not invest over the typical duration of a panel, implying a lot of zeros/missing observations. Levinsohn and Petrin (2003) (LP) overcomes this problem using intermediate inputs (consumption of raw materials, electricity, . . .) instead of investment. The rationale remains that the demand for intermediate inputs is related to the firm's unobserved productivity term χ_{it}. Identification also assumes that firms can relatively easily (and quickly) adjust their use of intermediate inputs in response to productivity developments χ_{it}. Following OP, LP assume the intermediate demand function can be inverted and represented as a n^{th} degree polynomial in capital stock and intermediate goods (instead of investment). Then the OLS regression of firm's output[14] on that polynomial and labour terms directly identifies the causal contribution of labour.

ACF fundamentally adheres to the LP idea. But they convincingly argue that labour[15], akin to capital, is a determinant of the demand for intermediates. It is thus an argument[16] of the inverted demand function. Hence, conditioning on that inverted function (now represented by a polynomial function of intermediate goods, capital **and labour**) leaves no room to directly identify the contribution of labour via OLS. This results in the need to fully mobilize the OP-LP two-stage procedure and resort to GMM at the end of stage two to identify the coefficients for the labour inputs. The LP-ACF estimator's full exposure and details (including the LP/ACF differences) are available in Appendix 8.2.

4 Data and key variables

Our firm-level panel data includes 7,069 firms established in Belgium, tracked from 2014 to 2022, resulting in 49,529 firm-year observations. This data is sourced from the Bel-first database, compiled by the Belgian National Bank and published by the Van Dijk Bureau.[17]

[12] Or average productivity, see eq.(7).

[13] And by extension delivers the HN candidates for relative marginal productivity, see eq.(6), eq.(7).

[14] Or average labour productivity.

[15] And by extension labour turnover in the context of this paper.

[16] In the mathematical sense.

[17] Now a Moody's Analytics Company.

The core Bel-first dataset is comprehensive, encompassing bookkeeping data for all private-economy firms established in Belgium to be submitted to the National Bank (a legal obligation). However, our study uses only a subset of this data, centred on the so-called Social Balance Sheet module that contains information on annual labour turnover. This module is not mandatory and is predominantly completed by large firms with higher than average capital intensity (see Table 1). In Section 5.2, we provide evidence that there is no significant risk of bias in our econometric results stemming from the non-representativeness of our sample in terms of firm size.

Table 1: Sample representativeness in terms of firm size (number of FTE workers) and capital intensity: Bel-first population vs our sample

	Bel-first Population			Our Sample		
	mean	min	max	mean	min	max
Firm size (number of workers)	89.2	1.0	395,530.0	182.7	1.0	33,272.0
Capital per worker (th. EUR)	2,928.1	0.0	6,956,521.0	2,853.8	2.6	3,375,049.0

Source: Bel-first

Additionally, to address the representativeness of our sample, particularly concerning (early) retirees, we present the results of two basic OLS regressions in Table 5. These show that, overall, exit via retirement has risen in Belgium between 2014 and 2022; in line with what we said in the introduction about the rising importance of retirement due to workforce ageing. Also, and this has to do with institutional reforms implemented since the mid-2000s in Belgium, Table 4, column 2 shows the gradual disappearance of early retirement. This reflects the dismantling of this exit route from the labour market in Belgium as part of a policy aimed at lifting the old employment rate (Vandenberghe, 2010).

4.1 Turnover data

The Social Balance Sheet enables us to calculate, for each firm i, the number of (full-time-equivalent) employees at the end of the year (L_{it}^{eoy}), but also the number of new hires (L_{it}^{hires}) and the number of people who left the company (for a variety of motives k) **during** the year ($L_{ikt}^{leavers}$). Assuming the actual moment of departure is uniformly distributed over the year, the total number of workers that worked during that year can be expressed as the sum of stayers ($\tilde{L}_{it}^{stayers}$), 50% of the new hires ($\tilde{L}_{it}^{hires}$) and 50% of the different categories of leavers ($\sum_k \tilde{L}_{ikt}^{leavers}$). The key descriptive statistics about the turnover labour share forming the core of our HN estimated model are reported in Table 2.

$$\begin{aligned}
\tilde{L}_{it} &= \tilde{L}_{it}^{stayers} + \tilde{L}_{it}^{hires} + \sum_k \tilde{L}_{ikt}^{leavers} \\
\text{with } \tilde{L}_{it}^{stayers} &\equiv L_{it}^{eof} - L_{it}^{hires} - \sum_k L_{ikt}^{leavers} \\
\tilde{L}_{it}^{hires} &\equiv .5 \; L_{it}^{hires} \\
\tilde{L}_{ikt}^{leavers} &\equiv .5 \; L_{ikt}^{leavers} \\
k &= \text{retired, early-retired, dismissed, other}
\end{aligned} \tag{9}$$

4.2 Control variables and estimated models

In what follows, the HN technology is assumed to be the same for all firms and corresponds to (average) labour productivity eq.(7),(8). However, we condition on a large set of dimensions, synonymous with heterogeneity, in the econometric version of that model.[18] First, our vector of control comprises the share of female and tertiary-educated workers (see bottom of Table 2) to control for two key determinants of labour productivity and account for the intrinsic productivity difference between stayers and leavers/joiners. Second, it comprises year, region (Brussels, Flanders and Wallonia) (see Table 3), and NACE-4 digits sector/industry fixed effects.[19] Table 4 gives an idea of the type of private-economy sectors/industries in our sample. Note the importance of the manufacturing sector, which is probably not unrelated to the over-representation of large and capital-intensive firms identified above (Table 1).

De facto, including NACE-4 (and also year) fixed effects, reinforces the plausibility of the HN model's unique technology assumption. This controls for economy-wide productivity developments and sector-specific productivity levels. Sectoral fixed effects reflect labour quality and intensity of efficiency wages heterogeneity across sectors and other sources of systematic productivity differentials (Hellerstein et al., 1999). More importantly, since our dataset does not contain sector price deflators, introducing the NACE 4-digit is a way to control the price of firms' outputs at the sector level. More in econometric terms, using these fixed effects means that our identification of labour productivities entirely rests on "within" NACE 4-digit variation. The potentially important "between" sector/industry differences play no role, and nothing can be said about their role in explaining, e.g. the estimated importance of the productivity handicap of (early) retirees.

[18] Refer to Section 3 for a discussion of how we cope with endogeneity/simultaneity: a distinct problem that also stems from unobserved differences at the firm level.

[19] No to confound with fully-fledged (here firm-level) fixed effect (FE) analysis commonly used in short-panel econometrics and discussed in Section 3 as inadequate for HN identification.

Table 2: Descriptive statistics

	mean	sd	min	max
Per worker net value added th. EUR (log)	11.713	0.818	2.471	19.092
Capital th. EUR (log)	10.194	1.408	4.419	18.966
Purch. int. goods & serv. th. EUR (log)	10.249	1.221	1.386	18.029
Labour (log)	4.108	1.406	0.000	10.412
Share new hires	0.137	0.126	0.000	1.000
Share leavers	0.123	0.112	0.000	1.000
Share retirees	0.006	0.018	0.000	1.000
Share early retirees	0.001	0.007	0.000	0.500
Share dismissed	0.022	0.043	0.000	1.000
Share oth. leavers	0.100	0.104	0.000	1.000
Share female	0.297	0.231	0.000	1.000
Share tertiary-educated	0.334	0.312	0.000	1.000
N		49,529		

Source: Bel-first
Reported total labour and labour shares correspond to $\tilde{L}$'s defined in eq. (9).

Table 3: Descriptive statistics: regions[a]

	Percent
Region	
Brussels	23.15
Flanders	54.49
Wallonia	22.37
Total	100.00

Source: Bel-first
[a]: note that the percentage breakdowns reported here are based on FTE employment count (and not the number of firms count)

Table 4: Descriptive statistics: sector/industry (NACE 1-digit)[a]

NACE 1-digit	Percent
0. A AGRICULTURE, FORESTRY AND FISHING	0.10
1. B MINING AND QUARRYING	0.29
2. C MANUFACTURING	36.29
3. D ELECTRICITY, GAS, STEAM AND AIR CONDITIONING SUPPLY	2.15
4. E WATER SUPPLY; SEWERAGE, WASTE MANAGEMENT AND REMEDIATION ACT.	2.35
5. F CONSTRUCTION	7.75
6. G WHOLESALE AND RETAIL TRADE	19.46
7. H TRANSPORTATION AND STORAGE	15.01
8. I ACCOMMODATION AND FOOD SERVICE ACT.	1.10
9. J PUBLISHING, BROADCASTING, AND CONTENT PRODUCTION AND DISTRI. ACT.	5.98
10. J TELECOMMUNICATION, COMPUTER PROGRAMMING, CONSULT;, COMPUT. INFR.;...	0.65
11. K FINANCIAL AND INSURANCE ACTIVITIES	1.30
12. L REAL ESTATE ACTIVITIES	0.83
13. M PROFESSIONAL, SCIENTIFIC AND TECHNICAL ACT.	6.74
Total	100.00

[a]: note that the percentage breakdowns reported here are based on FTE employment count (and not the number of firms count)
Source: Bel-first

Table 5: Share of leaving retirees and early retirees: regional differences and time trends

	(1) Share retirees	(2) Share early retirees
Flanders (ref. Brussels)	-0.000474	0.000393***
	(-1.88)	(3.99)
Wallonia	0.000503	0.000413***
	(1.68)	(3.52)
2015 (ref. 2014)	0.000241	-0.00106***
	(0.71)	(-8.00)
2016	-0.0000657	-0.00206***
	(-0.19)	(-15.47)
2017	0.00107**	-0.00202***
	(3.16)	(-15.18)
2018	0.00155***	-0.00215***
	(4.58)	(-16.21)
2019	0.00130***	-0.00265***
	(3.85)	(-20.10)
2020	0.00179***	-0.00242***
	(5.36)	(-18.55)
2021	0.00293***	-0.00248***
	(8.74)	(-18.98)
2022	0.00338***	-0.00278***
	(9.22)	(-19.42)
_cons	0.00524***	0.00303***
	(16.09)	(23.83)
N	49,529	49,529

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Source: Bel-first

5 Results

Below, in subsection 5.1, (Table 6) we first report the OLS econometric results (col. 1) corresponding to the estimation of eq. (8). We then present our preferred results (col. 2, col. 3), i.e. those delivered by the LP-ACF method that addresses the risk of simultaneity-reverse causation bias. Further down, we report (and synthesise visually) the labour productivity implied by these econometric results for the new hires and the different categories of leavers. Results of robustness tests aimed at assessing the risk of sample bias are reported in section 5.2 while those addressing the question of the relative importance of so-called gross vs. net productivity LACs.

5.1 Econometric results

In the upper part of Table 6, we report the OLS, LP and ACF estimates for parameters η in eq. (8). The bottom of Table 6 reports the implied marginal productivity $\lambda = 1 + \frac{\widehat{\eta}}{1+\widehat{\widetilde{\alpha}}}$.

The most interesting findings relate to leavers, who display significant productivity disadvantages compared to our benchmark category (the stayers). Assuming no major intrinsic productivity differences, this suggests the presence of productivity LACs. For instance, prospective retirees display ACF η's of -.53 (statistically significantly different from zero), which means a relative productivity of .04 (i.e. a 96% productivity handicap compared to stayers). For early retirees, we even have evidence of negative λ's; in other words, value destruction. Even if the assumption of no intrinsic productivity difference with stayers does not perfectly hold, such handicaps undoubtedly support productivity LACs. We will say more about the possible causes of these drops in productivity during the final months of work in the concluding Section 6.

For the dismissed workers, ACF-estimated λ is .62, which suggests a productivity handicap of 38% points that also hints at sizeable LACs; although of a lesser magnitude than those for (early)retirees. For the other (presumably voluntary) leavers, the ACF estimated productivity handicap is 33% points, which is compatible with significant LACs.

The interpretation of the results is less straightforward for new hires than for the other groups. The ACF-estimated η is .075 (statistically different from zero), corresponding to a $\lambda = 1.13$; a value pointing at productivity gains relative to the male, non-tertiary educated stayers. Note that LP results does not support such a gain. We interpret this as evidence of no or limited LACs or, said differently, that new hires' LACs are not large enough to make them appear less productive during their first year of employment. Remember that *stricto sensu* HN delivers relative productivity estimates. For new hires in particular, whose intrinsic productivity is more likely to deviate from that of stayers, the HN estimates could still conflate *i)* unaccounted intrinsic/long-term productivity differences vis-à-vis the benchmark stayers and *ii)* the transitory LACs we are interested in.

In Fig. 1, we display the relative productivity (λ's) implied by the econometric results. The reported distance to the magenta vertical line expresses labour productivity as percentage-point gaps vis-à-vis the reference group (i.e. the stayers). We also translate these values into Euros in Fig. 2). For that, we multiply our estimates of (marginal) labour productivity advantages/handicaps (λ-1) by the year 2021 average labour productivity of stayers underpinning our Bel-first data that we estimate to be equal to 87,300 euros. In Fig. 2, referring to our preferred ACF results, we verify what was already visible in the previous tables. New hires represent a productivity gain of max. 22,000 euros. By contrast, all leavers are synonymous with (relative) productivity losses. They range from -25,000 (other leavers), -32,000 (dismissed leavers), -54,000 (retirees), to -178,000 euros (early retirees).

Table 6: Estimates of relative labour productivity: OLS vs LP vs ACF (Standard errors)

	1.OLS	2.LP	3.ACF
Capital	0.372***	0.385***	0.377***
	(0.004)	(0.015)	(0.022)
Labour ($\tilde{\alpha}$)	-0.428***	-0.432***	-0.450***
	(0.004)	(0.008)	(0.007)
Share female	0.014	-0.009	-0.022
	(0.022)	(0.030)	(0.034)
Share tertiary-educated	0.242***	0.233***	0.256***
	(0.013)	(0.003)	(0.008)
Share retirees (η)	-0.669***	-0.573***	-0.529***
	(0.223)	(0.012)	(0.077)
Share early retirees	-0.601**	-0.575***	-0.739***
	(0.301)	(0.026)	(0.165)
Share dismissed	-0.180**	-0.182***	-0.208***
	(0.090)	(0.007)	(0.026)
Share oth. leavers	-0.140***	-0.142***	-0.181***
	(0.042)	(0.012)	(0.018)
Share new hires	-0.019	-0.020*	0.075***
	(0.037)	(0.011)	(0.012)
Controls		Year, Region, NACE4	
N	49,701	40,903	40,903

* p<0.10, ** p<0.05, *** p<0.01

Standard errors in parentheses, estimated allowing for intrafirm correlation.

Implied marginal productivity $\lambda = 1 + \frac{\widehat{\eta}}{1+\widehat{\tilde{\alpha}}}$, [p-value H0: $\lambda = 1$]			
Female	1.024(0.535)	0.985(0.770)	0.959(0.508)
Tertiary Educated	1.423(0.000)	1.409(0.000)	1.467(0.000)
Retirees	-0.170(0.003)	-0.009(0.000)	0.037(0.000)
E.retirees	-0.052(0.046)	-0.012(0.000)	-0.344(0.000)
Dismissed	0.685(0.044)	0.679(0.000)	0.622(0.000)
Oth. leavers	0.756(0.001)	0.749(0.000)	0.670(0.000)
New hires	0.967(0.610)	0.965(0.066)	1.136(0.000)

Source: Bel-first

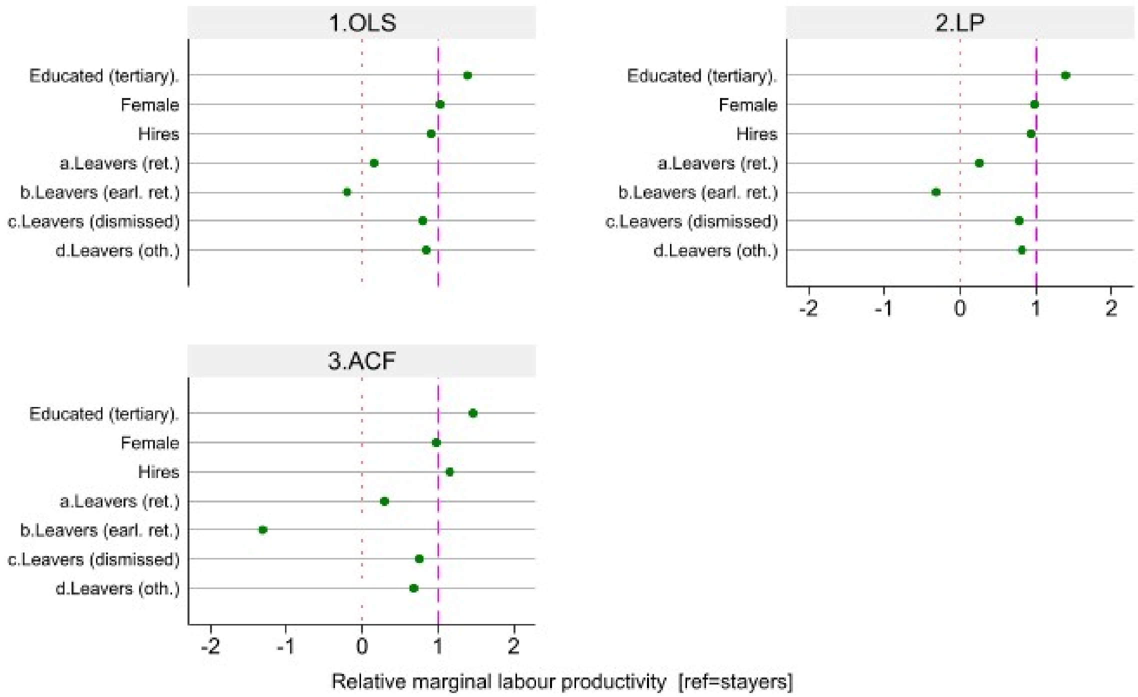

Figure 1: Productivity (ref. male, non-tertiary educated stayers' productivity)

Reported values correspond to λ.

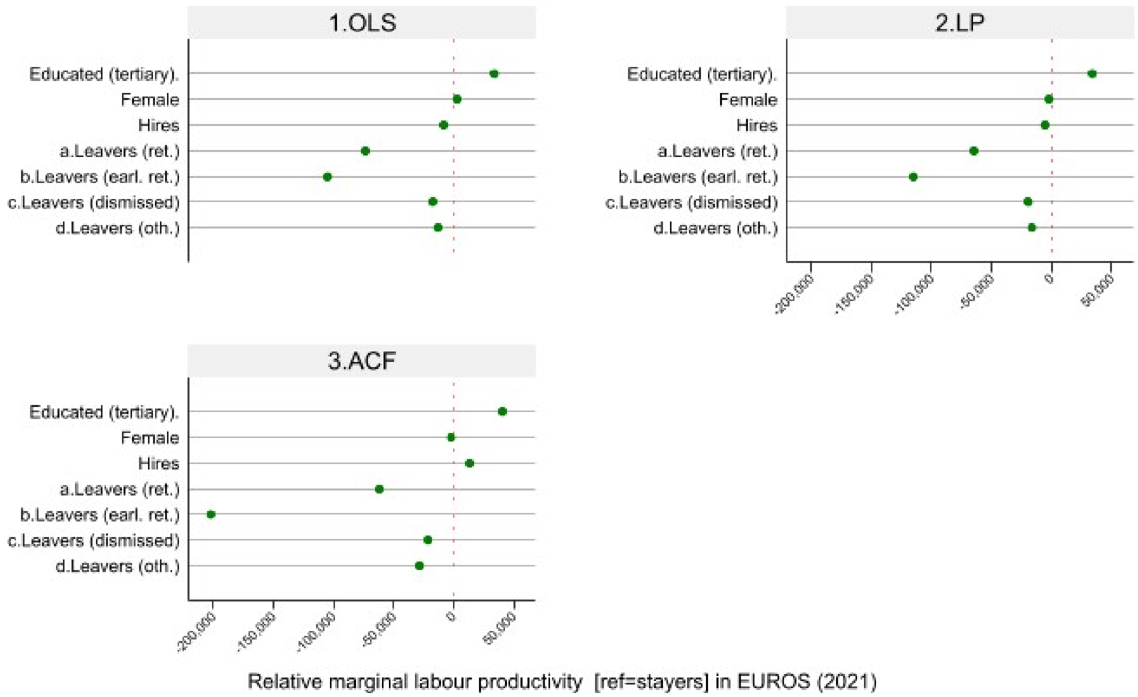

Figure 2: Productivity in EUROs (ref. male, non-tertiary educated stayers' productivity)

Reported values are obtained by multiplying the estimates of (marginal) labour productivity losses (λ) by the year 2021 average labour productivity of stayers i.e. 87,300 euros.

5.2 Robustness test: do the results hold for (underrepresented) smaller firms?

We have seen in the data section (Table 1) that our sample consists of larger firms using a more capital-intensive technology. To assess the bias this may represent, we re-estimate our key results using the smaller firms of our sample only.

Results are reported in Table 7. We note the absence of significant deviation with the results reported in Table 6. Our most interesting results for leavers are confirmed. For instance, prospective retirees display ACF λ's of -29 (i.e. a 71% productivity handicap compared to stayers). For early retirees, we still have evidence of negative λ's synonymous with value destruction. For the dismissed workers, ACF λ is .75, which suggests a productivity handicap of 25% points. For the other, the ACF estimated productivity handicap is 25% points. All these results tend to point to sizeable LACs. Finally, we still find little/no evidence supporting LACs for new hires, as they appear more productive than the stayers.

Table 7: OLS-LP-ACF estimates of relative labour productivity and labour cost (Standard errors)- Assessing the role of size: <=50 percentile of the employment level distribution

	1.OLS	2.LP	3.ACF
Capital	0.433***	0.457***	0.434***
	(0.007)	(0.026)	(0.034)
Labour ($\tilde{\alpha}$)	-0.507***	-0.508***	-0.541***
	(0.008)	(0.007)	(0.007)
Share female	0.016	-0.011	-0.011
	(0.029)	(0.020)	(0.025)
Share tertiary-educated	0.192***	0.191***	0.213***
	(0.017)	(0.005)	(0.009)
Share retirees (η)	-0.416*	-0.365***	-0.323***
	(0.230)	(0.013)	(0.056)
Share early retirees	-0.591	-0.648***	-1.063***
	(0.377)	(0.041)	(0.303)
Share dismissed	-0.099	-0.109***	-0.112***
	(0.103)	(0.010)	(0.032)
Share oth. leavers	-0.074	-0.092***	-0.148***
	(0.048)	(0.009)	(0.021)
Share new hires	-0.045	-0.033***	0.071***
	(0.044)	(0.011)	(0.014)
Controls		Year, Region, NACE4	
N	24,887	19,839	19,839

* p<0.10, ** p<0.05, *** p<0.01

Standard errors in parentheses, estimated allowing for intrafirm correlation.

	Implied marginal productivity $\lambda = 1 + \frac{\widehat{\eta}}{1+\widehat{\alpha}}$, [p-value H0: $\lambda = 1$]		
Female	1.033(0.581)	0.977(0.571)	0.976(0.662)
Tertiary Educated	1.388(0.000)	1.388(0.000)	1.464(0.000)
Retirees	0.157(0.071)	0.258(0.000)	0.296(0.000)
E.retirees	-0.199(0.117)	-0.318(0.000)	-1.315(0.000)
Dismissed	0.800(0.336)	0.779(0.000)	0.757(0.000)
Oth. leavers	0.849(0.124)	0.813(0.000)	0.677(0.000)
New hires	0.908(0.308)	0.934(0.003)	1.154(0.000)

Source: Bel-first. Reported results are obtained with the subsample of firms displaying a lower level of change in their overall labour force; thus, those where, presumably, so-called net LACs (strictly related to changes in the overall level of employment/hours) are minimal.

5.3 Gross vs net adjustment costs

So far, we have implicitly assumed that we were assessing the role of what the literature calls "gross" LACs: those strictly linked to the status of the individual joining or about to leave a job. However, it could be that our results also capture what the literature refers to as "net" adjustment costs, i.e. those stemming from changes in the overall level of employment underpinning labour turnover.

To assess the potential role of those, we implement a simple robustness strategy: we characterize the firms forming our sample in terms of change in the overall level of employment and re-estimate all the above results using only firms with low changes in the overall labour force size. How do we define low vs. high change? For each firm, we compute the standard deviation of the level of employment over the panel years (2014-2022). To account for the inflationary effect of being observed for more years, we regress the standard deviation over a constant and the number of spells characterizing the firm. We retain the residuals as an adjusted measure of change of employment.[20] The final step is to compute the deciles of that adjusted change measure and re-estimate our results for firms belonging to the 5 first deciles, i.e. the low-employment-change ones.

Results are reported in Table 8. Due to the absence of significant deviation with the results reported in Table 6, including for new hires, we conclude that what we have estimated so far corresponds to gross LACs. Note that this result aligns with Hamermesh (1995) and other works on LACs: the largest part of LACs firms are confronted with correspond to gross LACs.

[20]A measure of employment change netted out of the impact of the flow of time.

Table 8: OLS-LP-ACF estimates of relative labour productivity and labour cost (Standard errors)- Assessing the role of size: <=50 percentile of the employment level distribution

	1.OLS	2.LP	3.ACF
Capital	0.403***	0.417***	0.403***
	(0.006)	(0.032)	(0.033)
Labour ($\tilde{\alpha}$)	-0.468***	-0.449***	-0.491***
	(0.006)	(0.008)	(0.017)
Share female	0.034	0.019	-0.003
	(0.030)	(0.021)	(0.037)
Share tertiary-educated	0.240***	0.226***	0.244***
	(0.017)	(0.006)	(0.010)
Share retirees (η)	-0.474**	-0.387***	-0.399***
	(0.219)	(0.012)	(0.089)
Share early retirees	-0.462	-0.490***	-0.773***
	(0.462)	(0.026)	(0.144)
Share dismissed	-0.167	-0.132***	-0.125***
	(0.116)	(0.006)	(0.041)
Share oth. leavers	-0.059	-0.060***	-0.104***
	(0.057)	(0.008)	(0.027)
Share new hires	0.008	-0.010	0.120***
	(0.054)	(0.008)	(0.020)
Controls		Year, Region, NACE4	
N	24,850	21,262	21,262

* p<0.10, ** p<0.05, *** p<0.01

Standard errors in parentheses, estimated allowing for intrafirm correlation.

	Implied marginal productivity $\lambda = 1 + \frac{\widehat{\eta}}{1+\widehat{\tilde{\alpha}}}$, [p-value H0: $\lambda = 1$]		
Female	1.063(0.256)	1.034(0.381)	0.994(0.934)
Tertiary Educated	1.452(0.000)	1.410(0.000)	1.480(0.000)
Retirees	0.108(0.031)	0.297(0.000)	0.216(0.000)
E.retirees	0.131(0.317)	0.110(0.000)	-0.519(0.000)
Dismissed	0.685(0.149)	0.760(0.000)	0.754(0.003)
Oth. leavers	0.889(0.305)	0.892(0.000)	0.795(0.000)
New hires	1.014(0.888)	0.982(0.237)	1.235(0.000)

Source: Bel-first. Reported results are obtained with the subsample of firms displaying a lower level of change in their overall labour force; thus, those where, presumably, so-called net LACs (strictly related to changes in the overall employment/hours) are minimal.

6 Concluding remarks

Labour turnover impacts firms in contrasting ways. It potentially improves productivity through the changing mix of skills and experiences of new hires versus those who leave. This paper shows that turnover also generates important (transitory/short-term) labour adjustment costs (LACs), synonymous with productivity losses. These have been somehow overlooked by the LACs literature. For that, it uses the Hellerstein-Neumark (HN) framework commonly used in the production function literature to estimate the relative productivity of different types of labour as defined by gender, age or educational attainment) and applies it to firm-level data containing information about worker flows.

This paper shows that departing employees display sizeable productivity handicaps relative to stayers, supporting the existence of productivity LACs. This is even more evident for (early) retirees, who exhibit negative or nearly zero productivity compared to stayers, highlighting a significant productivity drop in their final year of work. Another (less robust) result is that no evidence of sizeable productivity LACs for new hires was found. For them, better data would be needed to disentangle the intrinsic versus transitory productivity differences they display vis-à-vis stayers.

The findings related to (early)retirees also suggest a need for additional research. Works on the relationship between productivity and retirement have been rare in the pension or labour literature.[21] A starting point could be to investigate the role of short-horizon effects and how they negatively affect labour productivity during the final years or months of presence in the labour market. One avenue for future investigation could involve re-examining the early work of Hall and Lazear (1984) and Akerlof (2005) on how short-horizon negatively affects labour productivity. One needs to understand better how (in particular mandatory) retirement contributes to the emergence of negative short-horizon effects. Such a research agenda could inform policies and practices to sustain productivity levels and minimise the negative economic impact of the important retirement-related workforce turnover most advanced economies will undergo for the foreseeable future.

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[21] One possible exception is Ippolito (1998).

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8 Appendix

8.1 The HN model: development

Consider the following Cobb-Douglas-augmented HN production function in logs

$$lnY_i = lnA + \alpha lnQL_i + \beta lnK_i \tag{10}$$

and then transform the labour quality index as

$$\begin{aligned} QL_i &= \mu_0 L_{i0} + \mu_1 L_{i1} + \dots \mu_n L_{in} \\ &= \mu_0 L_{i0} + \mu_1 L_{i1} + \dots \mu_n L_{in} + \mu_0 L_i - \mu_0 L_i \\ &= \mu_0 L_i + \mu_0 L_{i0} + \mu_1 L_{i1} + \dots \mu_n L_{in} - \mu_0 L_{i0} - \mu_0 L_{i1} - \dots \mu_0 L_{in} \\ &= \mu_0 L_i + (\mu_1 - \mu_0) L_{i1} + \dots (\mu_n - \mu_0) L_{in} \\ &= \mu_0 L_i + \sum_{j>0} (\mu_j - \mu_0) L_{ij} \end{aligned} \tag{11}$$

Multiplying/dividing 2nd right-hand-side term by $\mu_0 L_i$

$$\begin{gathered} QL_i = \mu_0 L_i + \mu_0 L_i \sum_{j>0} (\frac{\mu_j}{\mu_0} - 1) \frac{L_{ij}}{L_i} \\ \mu_0 L_i (1 + \sum_{j>0} (\lambda_j - 1) S_{ij}) \end{gathered} \tag{12}$$

where $S_{ij} \equiv L_{ij}/L_i$ and $\lambda_j \equiv \mu_j/\mu_0$.

Taking the log

$$lnQL_{it} = ln\mu_0 + lnL_i + ln(1 + \sum_{j>0} (\lambda_j - 1) S_{ij}) \tag{13}$$

Since $ln(1+x) \approx x$ for small values of x[22], in eq. (14) $\sum_{j>0} (\lambda_j - 1) S_{ij}$ becomes

[22] Mathematical reminder:

- Linear approx. (Taylor expansion) rule for $f(x)$ in $x = a$
  $f(x) \approx f(a) + f'(a)[x - a]$ if $x - a$ is small

$$lnQL_{it} \approx ln\mu_0 + lnL_i + \sum_{j>0} (\lambda_j - 1) S_{ij} \tag{14}$$

The production function becomes:

$$lnY_i = lnA + \alpha \left(ln\mu_0 + lnL_i + \sum_{j>0} (\lambda_j - 1) S_{ij} \right) + \beta lnK_i \tag{15}$$

or, equivalently, as a fully linearized expression (that can be estimated with OLS)

$$y_i = B + \alpha l_i + \sum_{j>0} \eta_j S_{ij} + \beta k_i \tag{16}$$

where $y_i \equiv lnY_i, l_i \equiv lnL_i, k_i \equiv lnK_i$
$B \equiv lnA + \alpha ln\mu_0; \eta_1 = \alpha(\lambda_1 - 1) \ldots \eta_n = \alpha(\lambda_n - 1)$

8.2 The LP-ACF estimation method

Turning to the `econometric estimation` of HN using firm-level panel data, the OLS sample-error term is extremely likely to consist of:

- a purely random shock ϵ_{it}
- but also productivity term χ_{it}, unobserved by the econometrician but whose level and dynamic are (partially) anticipated by the firm. Hence, it should (partially) drive labour inputs, including labour turnover $l_{it}; S_{ijt}$

- Application to $f(z) = ln(z)$ in $z = a = 1$ with $z \equiv 1 + x$
  $ln(z) \approx ln(a) + ln(a)'[z - a]$
  $ln(1 + x) \approx ln(1) + 1/1[1 + x - 1]$
  $\rightarrow ln(1 + x) \approx x$ if x is small
  NB: $ln(z)' = 1/z = 1$ in $z = 1$

In other words, the econometric version of eq. 6 becomes

$$\begin{aligned} y_{it} &= B + \alpha l_{it} + \sum_{j>0} \eta_j S_{ijt} + \beta k_{it} + \nu_{it} \\ \nu_{it} &= \chi_{it} + \epsilon_{it} \end{aligned} \tag{17}$$

where $cov(\chi_{it}, l_{it}) \neq 0, cov(\chi_{it}, S_{ijt}) \neq 0, E(\epsilon_{it}) = 0$

How to cope with χ_{it}? Our preferred approach consists of proxying it (Olley and Pakes, 1996) using demand for intermediate inputs as recommended by Levinsohn and Petrin (2003) (LP), and Ackerberg et al. (2015) (LP). The LP version writes

$$int_{it} = f(\chi_{it}, k_{it}) \tag{18}$$

while the ACF version is

$$\begin{aligned} int_{it} &= f(\chi_{it}, k_{it}, ql_{it}) \\ \text{where } ql_{it} &\equiv \alpha l_{it} + \sum_{j>0} \eta_j S_{itj} \end{aligned} \tag{19}$$

Indeed, ACF convincingly argue that labour[23], akin to capital, is a determinant of the demand for intermediates. Hence, it is an argument of the inverted demand function.

Assuming $f(.)$ can be inverted, the LP version of the production function becomes

$$y_{it} = B + ql_{it} + \beta k_{it} + \overbrace{f^{-1}(int_{it}, k_{it},)}^{\chi_{it}} + \epsilon_{it} \tag{20}$$

LP propose representing f^{-1} as a 3^{rd} degree polynomial in capital stock and intermediate goods. Then the OLS regression of firm's output[24] on that polynomial and labour terms directly identifies the causal contribution of labour term ql_{it}.

ACF convincingly argue that labour[25], akin to capital, is a determinant of the demand for intermediates. It is thus an argument[26] of the inverted demand function.

[23] And by extension labour turnover in the context of this paper.
[24] Or average labour productivity.
[25] And by extension labour turnover in the context of this paper.
[26] In the mathematical sense.

Thus, the ACF version writes

$$y_{it} = B + ql_{it} + \beta k_{it} + \overbrace{f^{-1}(int_{it}, k_{it}, ql_{it})}^{\chi_{it}} + \epsilon_{it} \tag{21}$$

and conditioning on that augmented inverted function (now represented by a polynomial function of intermediate goods, capital **and labour**) leaves no room to identify the contribution of labour via OLS directly. ACF propose a two-stage procedure that uses OLS at stage 1 and GMM at stage 2

Stage 1: OLS regress y_{it} on a composite term Ψ_{it} that comprises a constant, a 3rd order polynomial expansion in $int_{it}, k_{it}, ql_{it}$.

$$y_{it} = \Psi(const, int_{it}, k_{it}, ql_{it}) + \epsilon_{it} \tag{22}$$

Note that Ψ encompasses $\chi_{it} = f^{-1}(int_{it}, k_{it}, ql_{it})$ and that both capital coefficient β and labour coefficients η_j, α are not identified yet

Stage 2: generate implied values for χ_{it} using stage 1 estimates $\widehat{\Psi}$ & candidate values[27] for the coefficients β, η_j, α

$$\chi_{it} = \widehat{\Psi} - B^c - ql_{it}(\alpha^c, \eta_j^c) - \beta^c k_{it} \tag{23}$$

ACF assume further that the evolution of $\chi_{i,t-1}$ follows a first-order Markov process

$$\chi_{it} = E[\chi_{it} \mid \chi_{it-1}] + \xi_{it} \tag{24}$$

where χ_{it} decomposes into its conditional expectation a time $t-1$ i.e. $g(\chi_{it-1})$ (known by the firm) and a time t innovation term ξ_{it}.

$$\chi_{it} = g(\chi_{it-1}) + \xi_{it} \tag{25}$$

ACF assume that $g(\chi_{it-1})$ can be approximated by a nth-degree polynomial in $\chi_{it-1}, \chi_{it-2}, \chi_{it-3} \ldots$. A regression of χ_{it} on that polynomial delivers, as residuals, the im-

[27] For example OLS estimates

plied values for ξ_{it} that can be used to define moment conditions. This is where the firms' timing of decisions assumed by LP/ACF plays an important role. LP/ACF assume capital in period t was determined at period $t-1$ (or earlier) (i.e., it may take a full period for new capital to be ordered and put to use). Thus, it must be uncorrelated with the implied innovation terms ξ_{it} delivered by the inverted demand function, and hence the first-moment condition to estimate the capital coefficient

$$E[\xi_{it} \mid k_{it}] = 0 \tag{26}$$

ACF assume that labour inputs observed in t are probably chosen sometime before, although after capital – say in $t-b$, with $0 < b < 1$. As a consequence, l_{it}, ql_{it} will be correlated with at least part of the productivity innovation term ξ_{it}. But lagged labour inputs in $t-1, t-2, \ldots$ should be uncorrelated with ξ_{it}. Logically, ACF moments conditions identifying the coefficients of labour inputs write

$$E[\xi_{it} \mid ql_{it-1}, ql_{it-2} \ldots] = 0 \tag{27}$$

However, given the Belgian context and the fact that we are dealing with retirement, we assume that labour and turnover decisions were made before the year t. Thus, the moment condition identifying the (relative) marginal productivity of labour inputs, including that of leavers/new hires, writes

$$E[\xi_{it} \mid ql_{it}] = 0 \tag{28}$$

This also means that the relative productivities we capture are contemporary and are thus likely to reflect the transitory LACs we focus on in this paper. Not using lagged values of labour input at stage two of the OP/LP/ACF procedure means that we assume that – as standardly done to identify for capital (see supra) – decisions about labour are made somewhat before firms learn about the (innovative) part of the productivity term χ_{it}. This is particularly likely to be the case for retirement or early retirement. Referring to Ackerberg et al. (2015), this is also plausible for the other components of ql_{it} in situations (like in Belgium) where there are significant hiring or firing costs or labour market rigidities, possibly due to government- or social-partner-enforced regulations.

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