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PABLO GARCIA SANCHEZ, LUCA MARCHIORI, AND OLIVIER PIERRARD

Abstract. We propose a two-period overlapping generation economy that incorporates health investment in preventive measures during youth. These preventive measures contribute to increased longevity and reduced frailty, which influence old-age dependency and pension costs. As these costs are partly funded through pay-as-you-go social security contributions, investment in prevention creates externalities for the next generation. We analytically determine the optimal level of prevention and characterize the optimal health policy that a government should implement to achieve it. Our findings reveal that the optimal subsidy to long-term care exceeds the optimal subsidy to preventive measures. Furthermore, both subsidies are inversely related to the generosity of the public pension scheme. We explore the robustness of our results through various extensions and demonstrate their consistency with several patterns observed in cross-country OECD data.

JEL Codes: H23, I18, O41.

Keywords: Health, Prevention, Optimal Ramsey policy, Overlapping generations.

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Old age is like everything else. To make a success of it, you've got to start young.

— Theodore Roosevelt

1. Introduction

Dependency costs surge in old age and are expected to increase further in the future (Frank, 2012; French et al., 2016; EC, 2024). However, young adults may opt to invest in preventive healthcare (such as medical screening, immunization, healthy diet, but also rehabilitation and treatments), as early investments in prevention may slow down the deteriorating health conditions associated with aging (Sirven et al., 2017). In turn, this reduction in frailty has been shown to help lower health care costs in old age (Sirven and Rapp, 2017). Nonetheless, preventive measures may also contribute to a longer life expectancy and therefore may potentially raise total health expenditures (see for instance Breyer and Lorenz, 2021, for a discussion). Given these two transmission channels (frailty and longevity), we analytically determine the optimal level of prevention and whether it can be obtained through a private allocation of resources. As the answer turns out to be negative, we characterize the optimal health policy needed to attain the efficient equilibrium and we demonstrate how it interacts with other characteristics of the economy.

To this end, we construct a general equilibrium overlapping generation (OLG) economy in which agents live for two periods. For simplicity, we introduce quasi-linear preferences and assume the absence of capital. Therefore, income in the second period is limited to benefits from a pay-as-you-go pension system. We allow for health investment in prevention during youth, which boosts longevity (or, equivalently, the likelihood of surviving the first period) and reduces frailty (or, equivalently, the likelihood of dependency in old age, given survival). We show that the total number of dependent individuals is the combined outcome of longevity and frailty. As a result, the effect of health investment on dependency is a priori ambiguous.[1] Importantly, being dependent in old age entails costs (dependency or long-term care costs).

The choice of health investment by the current generation influences the cost of social security (pension and dependency) by the next generation. Specifically, the longevity channel raises pension and dependency costs. Because this is not internalized by the current generation, it creates an incentive to over-invest. On the other hand, the frailty channel decreases dependency costs and generates an incentive to under-invest. In other words, our model

[1]With a continuum of identical agents, the law of large numbers ensures that the number of agents who survive the first period and become dependent in the second period equals the corresponding ex-ante probabilities for a single individual. As a result, we use the two concepts interchangeably.

includes a pension externality due to the longevity channel (over-investment), and a dependency externality due to both the longevity and frailty channels (over- or under-investment depending on their relative strengths). As a result, the steady state of the decentralized equilibrium is suboptimal. To address the two externalities, the government considers two health policy instruments: the relative price of health investment (via subsidies to prevention) and the relative price of dependency costs (via subsidies to long-term care). A subsidy to prevention obviously encourages preventive measures and reduces consumption, while a subsidy to long-term care decreases the incentive for prevention and yields uncertain effects on consumption. We show that when the frailty channel is sufficiently strong, an interior optimal Ramsey steady state exists and is stable. Hence, a Ramsey equilibrium cannot exist in a model with only the longevity channel (that is with no frailty channel). To achieve this Ramsey equilibrium, we demonstrate that the subsidy to long-term care must be higher than the subsidy to prevention, and that the gap between them is positively linked to the strength of the longevity channel. Put differently, in the absence of the longevity channel, the two subsidy rates would need to be equal. The intuition is that a robust longevity channel leads to excessive health investment, requiring a higher subsidy for long-term care (relative to prevention) to counterbalance it. Moreover, both subsidies are inversely related to the generosity of the pension system. Therefore, in our stylized model, a more generous pension scheme should be accompanied by less state support for the health system.

To explore the robustness of our results, we impose concavity on utility functions of young and old, consider a fully-funded pension scheme, and introduce the notions of incompressible longevity and incompressible frailty. We also consider informal care, where relatives reduce their market labor supply to provide free long-term care, and demonstrate that this is a special case of our model. We also illustrate our analytical results through a parametrization exercise. Finally, we gather cross-country data on prevention, public health, and pension policies, to show that our results align with several observed patterns. These patterns include a positive correlation between the two health subsidies, a positive correlation between the prevention subsidy and health investment, a negative correlation between the long-term care subsidy and health investment, and a negative correlation between the subsidy to health investment and the pension replacement rate.

Our paper contributes to three strands of the theoretical literature. First, it relates to studies examining the economic implications of prevention through longevity and/or frailty channels. In his classic model of the demand for health, Grossman (1972, 2000) models health as a capital stock, yielding an output of healthy time and allowing for increased longevity. Health naturally depreciates over time but can be enhanced through investment in prevention. Courbage and Rey (2012) propose a simple two-period model where prevention in the first period reduces the probability of disease in the second period. Both prevention and disease affect wealth, and they determine the optimal level of prevention based on the properties of the utility function. Menegatti (2014) extends their model by adding another choice variable: if disease occurs despite prevention, the agent can attempt to cure it. Dalgaard and Strulik (2014) develop a different life-cycle model where health deficits accumulate through aging, but health investment reduces the speed of this accumulation, thus increasing life expectancy. They show that their model reproduces the Preston curve, which demonstrates a positive link between longevity and income. Schünemann et al. (2022) follow a similar approach but add a frailty channel: prevention also reduces the probability of being dependent in old age. They show that the effect of better health on long-term care (LTC) expenditures is ambiguous, as the longevity and frailty channels have opposite impacts. They examine the quantitative importance of each channel by projecting the future evolution of income and medical technology. All these models do not include externalities, so optimal prevention (from an individual perspective) is also socially optimal.

This leads us to the second strand of literature, which examines socially optimal health investment within models that include externalities. Kuhn et al. (2011) build a life-cycle model with a longevity channel, where health decisions may affect others positively (e.g., vaccination) or negatively (e.g., congestion). Depending on the type of externality, a subsidy or a tax on prevention is required. Leroux et al. (2011) propose a two-period overlapping generations (OLG) model with endogenous longevity. Public intervention is justified because agents underestimate the effects of prevention, which generates positive externalities on others' survival. Cremer et al. (2012) also develop a two-period OLG model where myopic individuals do not realize that consuming sin goods in the first period negatively affects their health in the second period. The central planner can tax sin goods in the first period and/or subsidize health expenditures in the second. Atolia et al. (2021) construct a growth model where health investment enhances labor productivity. However, health acts as an externality since it is a non-remunerated input in the production function, leading to under-investment in health and the need for a positive subsidy. While we acknowledge the relevance of these externalities, our focus is on another one inherent to the OLG setup: a generation does not internalize that their choices may affect other generations through a pay-as-you-go social security system. This type of externality is explored by Marchiori and Pierrard (2023). In their model, without the longevity channel, prevention in the first period unequivocally reduces dependency expenditures in the second period, which are fully subsidized. Our model generalizes their framework by incorporating a longevity channel and partially private expenditures on long-term care. We demonstrate that including longevity and private expenditures mitigates the under-investment in health identified by Marchiori and Pierrard (2023), thereby reducing the optimal prevention subsidy.

Third, our research closely relates to studies on optimal health insurance. For instance, Ellis and Manning (2007) introduce private insurance in a utility-based framework to determine the optimal coinsurance rate for prevention and treatment. The private optimum does not consider the effect of prevention on the insurance premium, while the social optimum does. In our paper, the two subsidy rates mirror their two coinsurance rates, the tax rate to maintain social security equilibrium corresponds to their premium, and the decentralized equilibrium does not internalize the effects of prevention on the tax, similar to their model's treatment of premiums. Ellis and Manning (2007) also show that an increased demand for health care treatment makes it attractive to cover preventive care more generously, an outcome we also observe. Cremer et al. (2016) specifically examine long-term care insurance within a one-period model, excluding prevention measures. They identify the typical tradeoff between risk-sharing and ex post moral hazard (where healthcare insurance increases individuals' demand) as noted by Blomqvist (1997). Additionally, they find that optimal LTC insurance coverage should decrease with the significance of informal care. In our model, LTC demand is price inelastic, and we do not encounter direct ex post moral hazard. However, there is indirect ex post moral hazard because LTC coverage reduces prevention efforts, thereby increasing LTC demand. Our numerical simulations further demonstrate that informal care reduces prevention in both decentralized and social equilibria, necessitating a higher optimal prevention subsidy.

Lastly, while the model developed here is very simple, the insights we gain hold promise for considerable generalization. Hence, it can serve as a useful guide to exploring larger and more quantitative models in the future. The remainder of the paper is organized as follows. Section 2 presents the dynamic equilibrium and Section 3 discusses the resulting steady state. Section 4 presents the Ramsey equilibrium and proves its stability. Section 5 discusses several extensions. Section 6 provides a parametric illustration and presents selected cross-country data. Section 7 concludes.

2. The model

In our model, time is discrete and runs to infinity, but individuals' lives span at most two periods. For simplicity, the size of each new generation is constant and normalized to 1. In the first period, individuals supply one unit of labor inelastically, earning the wage rate w_t. Their after-tax income is divided between consumption c_t and health investment x_t. The probability of surviving the first period is $\epsilon(x_t)$, where $\epsilon(x_t)$ is increasing and concave in x_t. Conditional on surviving the first period, individuals retire with a pension of γw_t, face long-term care (LTC) costs with probability $\pi(x_t)$, which is decreasing and convex in x_t, and consume their residual income d_{t+1}.[2] The government uses a labor income tax τ_t to subsidize a fraction ϕ of health investment and a fraction θ of LTC costs. For simplicity, we do not introduce capital but discuss this extension in Section 5.

2.1. **Households.** An individual of generation t chooses consumption levels $\{c_t, d_{t+1}\}$ for the two periods of life and health investment x_t, to maximize

$$\log c_t + \beta\epsilon(x_t)\,\mathbb{E}_t\,\bar{v}d_{t+1}\,, \tag{1}$$

subject to the first period budget constraint

$$c_t + (1-\phi)x_t = (1-\tau_t)w_t\,, \tag{2}$$

as well as to the second period budget constraint

$$d_{t+1} + (1-\theta)\mu w_{t+1} = \gamma w_t\,, \quad \text{with probability } \pi(x_t)\,, \tag{3a}$$

$$d_{t+1} = \gamma w_t\,, \quad \text{with probability } 1-\pi(x_t)\,, \tag{3b}$$

where $\beta \in (0,1)$ is the discount factor and $\bar{v} > 0$ governs the marginal utility of consumption at $t+1$. We assume a quasi-linear lifetime utility function, which enables us to solve both the decentralized equilibrium and the Ramsey allocation analytically (see for instance Koskela et al., 2002, for a similar quasi-linear utility function in a 2-period OLG model). In Section 5, we also introduce concavity in the second period to explore the robustness of our results. In the first period, τ_t is the labor income tax and the net wage is used to consume or invest in health. In the second period, $\gamma \in (0,1)$ is the replacement rate of the pension scheme. Without frailty (as shown in equation (3b)), income is entirely consumed. With frailty (as shown in equation (3a)), income is split between consumption and LTC costs. In this case, $\mu > 0$ is the number of LTC units needed and w_{t+1} represents the cost per LTC unit.[3] Public policy parameters ϕ and $\theta \in (-1,1)$ either tax (if negative) or subsidize (if positive) health investment and LTC costs, respectively. We allow for negative values because there is no theoretical reason to exclude ex ante that the optimal policy will be a tax. Every period, the labor income tax is adjusted to satisfy the government budget constraint (see below). Importantly, we consider here formal – or market – LTC care. However, LTC is sometimes supplied informally by relatives, mainly family members and predominantly women (OECD,

[2]The unconditional dependency probability is therefore $\epsilon(x_t)\,\pi(x_t)$, which also equals the number of dependent individuals.

[3]Appendix A provides micro-foundations. Briefly, we assume a 2-sector economy (goods and LTC services) and we show that if the LTC sector is labor intensive and operates a linear technology, then the price of LTC services is equal to the wage in this sector. Moreover, when the labor supply is perfectly substitutable between the two sectors, wages are equal.

2023). We show in Section 5 that informal care is a special case of our model, and we discuss optimal prevention in this case.

Inserting (2), (3a) and (3b) into (1) results in an unconstrained maximization program in x_t.[4] The resulting optimality condition can be written as

$$\frac{1-\phi}{c_t} = \beta\bar{v}\left[\epsilon'(x_t)d_{t+1} - \epsilon(x_t)(1-\theta)\mu w_{t+1}\pi'(x_t)\right]. \tag{4}$$

This Euler equation balances the marginal cost of investing an extra unit in health today (left-hand side) with its marginal benefit tomorrow (right-hand side). As mentioned earlier, the latter has two components: higher longevity and lower frailty.

2.2. **Firms.** The production side of the economy is standard, with a set of competitive firms operating a linear technology with labor as the only input. At equilibrium, the wage rate is therefore

$$w_t = A, \tag{5}$$

where $A > 0$ captures labor productivity.

2.3. **The Government.** The government subsidizes (or taxes) a share ϕ of health investment ('prevention subsidy') and a share θ of LTC costs ('care subsidy'). In addition, it funds the pension scheme with replacement rate γ.[5] In our model, one period represents half a lifetime. Hence, we assume the government balances its budget every period, covering its expenditures through the income tax τ_t

$$\phi x_t + \Lambda(x_{t-1})\theta\mu w_t + \epsilon(x_{t-1})\gamma w_{t-1} = \tau_t w_t. \tag{6}$$

Here we define $\Lambda(x_t) = \epsilon(x_t)\pi(x_t)$. As already explained, $\Lambda(x_t)$ is the unconditional probability of becoming dependent or, equivalently, the number of individuals facing LTC costs. We discuss theses functions below.

2.4. **Frailty, Longevity and Dependent Individuals.** Functions $\epsilon(x)$ and $\pi(x)$ are crucial in our analysis, because they determine the effects of health investment. We assume the following.

Assumption 1. *The longevity function* $\epsilon(x)$*:* $\mathbb{R}^+ \to [0,1)$ *has* $\epsilon'(x) > 0$*,* $\epsilon''(x) \leq 0$*,* $\epsilon(0) = 0$*,* $\epsilon(\infty) = 1$*, and* $\epsilon'(0) = a$ *with* $a > 0$*. The frailty function* $\pi(x)$*:* $\mathbb{R}^+ \to [0,1)$ *has* $\pi'(x) < 0$*,* $\pi''(x) \geq 0$*,* $\pi(0) = 1$*,* $\pi(\infty) = 0$*,* $\pi'(0) = -b$ *with* $b > 0$*.*

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[4] Because of the linear utility in the second period, equations (3a) and (3b) aggregate into $d_{t+1} + \pi(x_t)(1-\theta)\mu w_{t+1} = \gamma w_t$, which can be directly inserted in (1).

[5] We show later we have two externalities (namely the pension and the dependency externalities) and we therefore need two policy instruments in the Ramsey problem. We choose the two subsidy rates (θ and ϕ) as instruments. Obviously, the optimal settings of these instruments will depend on the value of the pension replacement rate γ. Any other instrument choice (e.g. γ and ϕ with θ given) would produce a similar analysis.

These assumptions are standard in the literature. Assumption 1 implies that health investment boosts the probability of surviving the first period, and lowers the conditional (on surviving) probability of incurring LTC costs in the second period. In addition, the effect of each extra unit of x on these probabilities decreases in x, ensuring the maximization problem is well defined.

As noted earlier, the product $\Lambda(x)$ of the functions $\epsilon(x)$ and $\pi(x)$ determines the number of dependent individuals in old age; that is, those facing LTC costs (or dependency costs) in the second period. Assumption 1 immediately implies the following properties.

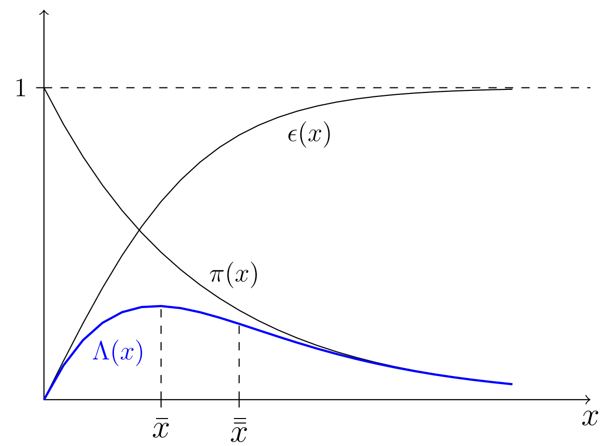
- $\epsilon'(\infty) = \epsilon''(\infty) = 0$;
- $\pi'(\infty) = \pi''(\infty) = 0$;
- $\Lambda(x)$: $\mathbb{R}^+ \to [0, 1]$;
- $\Lambda(0) = \Lambda(\infty) = 0$;
- $\Lambda'(0) = a$ and $\Lambda'(\infty) = 0$;
- $\Lambda(x) \leq \epsilon(x)$, $\Lambda(x_t) \leq \pi(x)$, $\Lambda'(x) \leq \epsilon'(x)$ and $\Lambda'(x) \geq \pi'(x)$;
- $\Lambda''(0) = \epsilon''(0) - 2ab < \epsilon''(0) \leq 0$ and $\Lambda''(\infty) = 0$;
- $\exists!\ \bar{x} \in (0, \infty)$ such that $\Lambda'(\bar{x}) = 0$;
- $\exists!\ \bar{\bar{x}} \in (0, \infty)$ such that $\Lambda''(\bar{\bar{x}}) = 0$;
- $\bar{\bar{x}} > \bar{x}$.

Since these are not particularly enlightening, Figure 1 provides a graphical analysis. As probabilities, $\epsilon(x)$ and $\pi(x)$ range between 0 and 1. The former is increasing and concave, whereas the latter is decreasing and convex. This makes sense: though x raises (lowers) the probability of survival (facing LTC costs), there are decreasing returns to scale. That is, the return of each extra unit of x declines as x increases.

Given these properties, the number of dependent individuals, $\Lambda(x)$, also ranges between 0 and 1, forming an inverted U-shaped function of x. Indeed, raising the health investment of generation t from very low levels results in a non-monotonic impact on the number of dependent individuals. Up to the threshold $\bar{x}$, health investment increases the number of individuals incurring LTC costs. However, once that threshold is crossed, health investment has the opposite effect. The logic is straightforward: higher health investment boosts individuals' lifespans, expanding the pool of potentially dependent individuals. At the same time, higher health investment lowers the incidence of LTC costs. The balance between these competing effects determines how the share of dependent individuals reacts to higher health investment.

For future reference, Figure 1 also highlights the inflection point $\bar{\bar{x}}$, where $\Lambda(x)$ transitions from being concave to convex. By construction, $\bar{\bar{x}}$ always lies to the right of $\bar{x}$. This inflection point $\bar{\bar{x}}$ will play an important role in the analysis of the decentralized and Ramsey equilibria.

FIGURE 1. Graphical Analysis of $\epsilon(x)$, $\pi(x)$, and $\Lambda(x)$



Notes. $\epsilon(x)$ and $\pi(x)$ are the longevity and frailty functions, respectively, whose properties are given in Assumption 1. $\Lambda(x) = \epsilon(x)\pi(x)$ is the resulting dependency function. $\bar{x}$ and $\bar{\bar{x}}$ are such that $\Lambda'(\bar{x}) = 0$ and $\Lambda''(\bar{\bar{x}}) = 0$, respectively.

Indeed, Section 4 will show that the optimal steady state equilibrium obtained by solving the Ramsey problem can only exist to the right of $\bar{\bar{x}}$, and that conditional on its existence, this equilibrium is locally stable. As a consequence, in Section 3, we introduce restrictions on parameters such that the decentralized steady state equilibrium also locates to the right of $\bar{\bar{x}}$. Given the appropriate policies, the decentralized equilibrium thus converges to the Ramsey equilibrium.

Finally, in Section 5, we modify the $\epsilon(x)$ and $\pi(x)$ functions to take into account the notions of incompressible longevity and incompressible frailty. We show how these affect the $\Lambda(x)$ function and hence the Ramsey equilibrium.

2.5. **Decentralized Equilibrium and Externalities.** Combining (2) and (6) to eliminate τ_t, inserting (3a) and (3b) into (4) to eliminate d_{t+1}, and using (5) to eliminate w_t results in the following dynamic system of equations

$$\frac{1-\phi}{c_t} = \beta \bar{v} A \left[\epsilon'(x_t)(\gamma - \pi(x_t)(1-\theta)\mu) - \epsilon(x_t)(1-\theta)\mu\pi'(x_t)\right], \tag{7}$$

$$c_t + x_t = A\left[1 - \epsilon(x_{t-1})(\theta\mu\pi(x_{t-1}) + \gamma)\right], \tag{8}$$

in the unknowns c_t and x_t. Hence, given the predetermined variable x_{t-1}, equation (8) determines the total level of resources available to the young generation, and equation (7) determines how the young generation splits these total resources between consumption and health investment.

Importantly, the decentralized equilibrium provided by (7)-(8) is not optimal. Indeed, two externalities violate the necessary conditions of the Fundamental Welfare Theorems. The first one is the *pension externality*. Higher health investment of generation t results in a larger population of elderly individuals at $t+1$. The ensuing pensions must be financed by taxing the next generation of workers. This higher tax burden for individuals born at $t+1$ is not internalized by generation t when choosing its health investment. This externality, which is due to the longevity channel only, leads to an over-investment in health and is magnified by the replacement rate γ.

The second one is the *dependency externality*. This reflects the need to finance the LTC subsidy and therefore only exists when θ is different from zero. In this case, the logic is as before: generation t does not internalize that its health investment at t determines the public LTC bill that will be financed at $t+1$ by the next generation of workers. This second externality is due to both the longevity and the frailty channels. The longevity channel produces over-investment while the frailty channel produces under-investment (assuming a positive θ). The sign of the net effect is therefore ambiguous and depends on the relative strength of the two channels. In any case, the parameter μ (size of required LTC services) amplifies the net effect.

3. Decentralized Steady State Equilibrium

As explained in the introduction, we focus on the steady state, where all variables are constant. In Section 3.1, we first examine the first order equation (7) which will be rewritten as $c = 1/f(x)$. In Section 3.2, we then study the budget equation (8) which will be rewritten as $c = g(x)$. In Section 3.3, we finally characterize the steady state decentralized equilibrium (x^*, c^*) as the intersection of these two curves.

3.1. **First Order Condition.**

Recall that this first equilibrium condition balances the marginal utility of consumption of each new generation with the marginal return of health investment. At the steady state, this equation is

$$\frac{1}{c} = f(x)\,,$$

where

$$f(x) = \frac{\tilde{v}A\,(\gamma\epsilon'(x) - (1-\theta)\mu\Lambda'(x))}{1-\phi}\,. \tag{9}$$

Here $\tilde{v} = \beta\bar{v}$ is the discounted marginal utility of consumption in the second period conditional on surviving from the first period. In addition, the term $A\gamma\epsilon'(x)$ represents the extra income earned from increasing longevity. In turn, the term $(1-\theta)A\mu\Lambda'(x)$ captures the extra cost incurred from raising dependency. Lastly, $1-\phi$ is the relative price of health investment. We assume the following.

Assumption 2. *$\gamma > 2\mu$.*

In words, income in the second period (γA) must be at least twice the level of LTC costs (μA). This assumption provides a sufficient condition ensuring that $f(x)$, and hence consumption, is always positive. Under Assumption 1, we also have $f(\infty) = 0$. However, we cannot infer the slope of $f(.)$, which can be positive or negative.

3.2. **Budget Constraint.** The second equilibrium condition ensures that total income in the first period equals total expenditures. At the steady state, this implies

$$c = g(x)\,,$$

where

$$g(x) = A\left(1 - \gamma\epsilon(x) - \theta\mu\Lambda(x)\right) - x\,. \tag{10}$$

Hence, consumption in the first period can be expressed as income net of taxes (to fund the pension scheme and the long-term care subsidy) minus health investment. Under Assumption 1, we have $g(0) > 0$, $g(\infty) = -\infty$. Once again, however, we cannot determine the slope of $g(.)$, which can be positive or negative.

3.3. **Equilibrium.** The preceding subsections analyzed each equilibrium curve separately and their intersection represents the decentralized steady state equilibrium (x^*, c^*). However, since both curves may be upward or downward sloping, we cannot guarantee the existence and uniqueness of an equilibrium. Moreover, we are interested in a decentralized equilibrium $x^* > \bar{\bar{x}}$. Indeed, Section 4 shows that the Ramsey equilibrium, if it exists, is to the right of $\bar{\bar{x}}$ and is locally stable. Having the decentralized equilibrium in its neighborhood would imply its convergence to the Ramsey equilibrium under the optimal policy. Here, we therefore impose parameter restrictions such that one and only one decentralized equilibrium exists to the right of $\bar{\bar{x}}$, for all values of ϕ and $\theta \in (-1, 1)$.

More precisely, it is straightforward to show that $1/f(x)$ is positive and increasing on $(\bar{\bar{x}}, \infty)$. The following assumption ensures that $g(x)$ is decreasing on $(\bar{\bar{x}}, \infty)$.

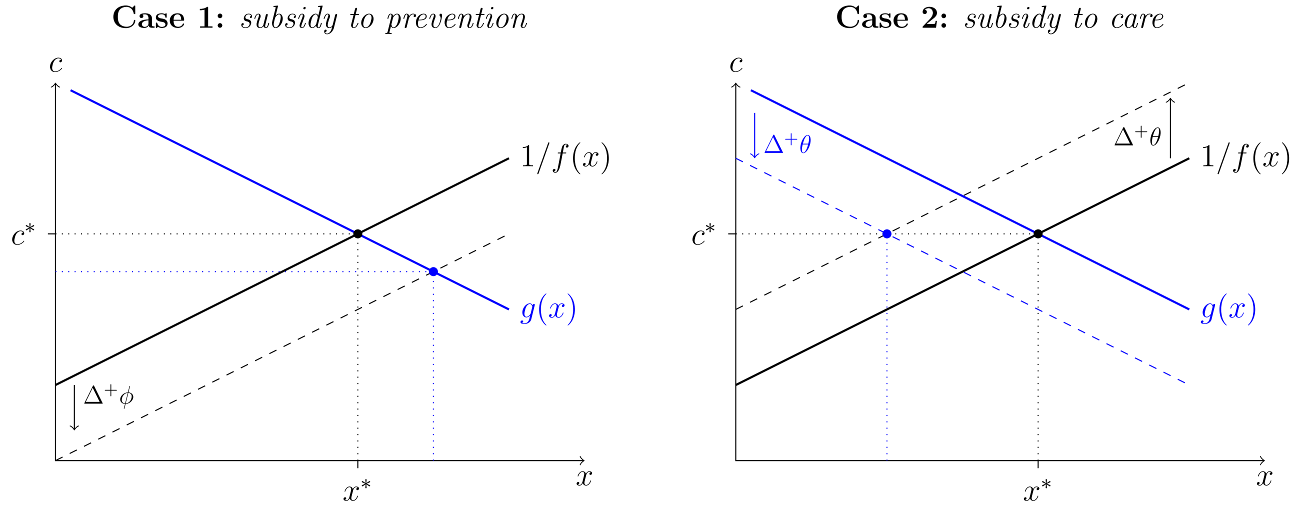
Assumption 3. $-A(\gamma\epsilon'(\tilde{x}) + \mu\Lambda'(\tilde{x})) < 1$, *with* $\tilde{x} = \underset{x \in (\bar{\bar{x}}, \infty)}{\arg\min}\ \gamma\epsilon'(x) + \mu\Lambda'(x)$.

The left-hand side of the inequality represents the tax reduction due to an increase in health investment and the inequality means that the tax reduction must be below one for all $x \in (\bar{\bar{x}}, \infty)$. In this case, an increase in x can never be fully offset by lower taxes and consumption must decrease. The next assumption ensures that $1/f(\bar{\bar{x}}) < g(\bar{\bar{x}})$.

Assumption 4. $\frac{2}{\bar{v}A\gamma\epsilon'(\bar{\bar{x}})} < A(1 - \gamma\epsilon(\bar{\bar{x}}) - \mu\Lambda(\bar{\bar{x}})) - \bar{\bar{x}}$.

This assumption requires that the right-hand side is positive and that productivity (or gross output) A is sufficiently high. Recall that $g(\infty) = -\infty$ and that the decentralized equilibrium is the intersection of curves $1/f(x)$ and $g(x)$. Then we have the following proposition.

FIGURE 2. Decentralized Equilibrium when $x^* > \bar{\bar{x}}$



Notes. The intersection of the curves $1/f(x)$ and $g(x)$ is the decentralized equilibrium (x^*, c^*). Case 1 shows how an increase in ϕ (subsidy to prevention) moves the two curves and hence the decentralized equilibrium. Case 2 shows how an increase in θ (subsidy to care) moves the two curves and hence the decentralized equilibrium. In Case 2, we draw the curves such that $\Delta c^* = 0$ but we could also get a higher c^* or a lower c^*.

Proposition 1 (Decentralized Steady State Equilibrium). *Under Assumptions 1, 2, 3 and 4, there exists a unique* $c^* > 0$ *and* $x^* > \bar{\bar{x}}$ *satisfying* $c = 1/f(x)$ *and* $c = g(x)$*, for all policy parameters* $\phi, \theta \in (-1, 1)$*. Moreover,* $\partial x^*/\partial \phi > 0$ *and* $\partial x^*/\partial \theta < 0$*.*

Given that, under the above assumptions, $1/f(x) > 0$ is increasing, $g(x)$ is decreasing, $g(\infty) = -\infty$ and $1/f(\bar{\bar{x}}) < g(\bar{\bar{x}})$, the first part of Proposition 1 is straightforward. Figure 2 illustrates this equilibrium. How strong are the above assumptions? Though we cannot provide a definite answer, we show in Section 6 that a standard model parametrization respects them and therefore leads to a unique equilibrium to the right of $\bar{\bar{x}}$.

Next, we assess how changes in the two policy parameters affect the decentralized equilibrium. The left panel in Figure 2 shows how the $1/f(x)$ and $g(x)$ curves react to an increase in the prevention subsidy ϕ. Raising ϕ boosts the marginal return $f(x)$ of health investment (equation (9)), shifting the curve $1/f(x)$ downwards. The intuition is straightforward. $1-\phi$ is the relative price of health investment. As a result, an increase in ϕ boosts the latter's marginal return, as fewer resources are required to obtain the same benefit. In addition, ϕ has no effect on the budget curve $g(x)$ (equation (10)), as it is just a transfer among newborn individuals. Combining these two insights leads to a straightforward conclusion: raising the prevention subsidy promotes health investment x^*, at the expense of consumption c^*.

The right panel in Figure 2 shows that an increase in θ shifts $1/f(x)$ upwards. Indeed, as long as higher health investment lowers the number of individuals facing LTC in the second period (which is the case when $x^* > \bar{\bar{x}}$), then increasing the care subsidy θ reduces the marginal return $f(x)$ of health investment. In other words: higher θ's shield individuals from LTC costs, making health investment less attractive. Moreover, a higher θ moves $g(x)$ downwards as it unambiguously implies higher taxes. As a result, a higher care subsidy reduces x^* while having an ambiguous effect on c^*.

The second part of Proposition 1 summarizes these policy effects and Section 6 investigates how far relationships between health investment and the two subsidies are observable in cross-country data.

3.4. **Alternative Scenarios.** As noted earlier, health investment has a twofold impact: it raises the probability $\epsilon(x)$ of surviving the first period and, conditional on surviving, lowers the probability $\pi(x)$ of incurring LTC costs in the second period. The resulting product $\Lambda(x)$ represents the number of dependent individuals. In this subsection, we close down one channel at a time and explore how these hypothetical scenarios affect the decentralized equilibrium.

No longevity channel. Suppose that prevention does not affect longevity but only frailty. This type of scenario is found in Marchiori and Pierrard (2023). Mathematically, longevity becomes exogenous and is given by $\epsilon(x) = \bar{\epsilon} \in (0, 1)$, leading to $\Lambda(x) = \bar{\epsilon}\pi(x)$. In this case, the dependency function $\Lambda(x)$ is always decreasing and convex for all $x \in (0, \infty)$ and $\bar{\bar{x}}$ becomes irrelevant (put differently, we are always to the right of $\bar{\bar{x}}$). However, we observe that Assumption 3 is more difficult to satisfy and that Assumption 4 is always violated. Importantly, this does not mean that a unique decentralized equilibrium does not exist, but simply that we can no longer guarantee this equilibrium for all (θ, ϕ) between -1 and 1.

No frailty channel. Suppose now that prevention does not affect frailty but only longevity. This type of scenario is found in Leung and Wang (2010). Mathematically, frailty becomes exogenous and is given by $\pi(x) = \bar{\pi} \in (0, 1)$, leading to $\Lambda(x) = \bar{\pi}\epsilon(x)$. In this case, the dependency function $\Lambda(x)$ is always increasing and concave. Therefore, we are always to the left of $\bar{\bar{x}}$ and it is impossible to be in the neighborhood of the Ramsey equilibrium (in fact we see in the next section that a Ramsey equilibrium does not exist in this case). However, it is straightforward to show that $g(x)$ is always decreasing and that we easily obtain $1/f(0) < g(0)$. In words, a unique decentralized equilibrium is easy to generate but it will never be optimal, whatever the policy. We come back to this discussion later.

4. Optimal Ramsey Policy

As discussed in Section 3, the decentralized equilibrium (c^*, x^*) is not optimal, for two externalities violate the necessary conditions of the Fundamental Welfare Theorems: the pension externality and the dependency externality. Therefore, we now turn our attention to the Ramsey problem, i.e. a government choosing (ϕ^R, θ^R) to maximize steady state welfare $W = \log(c) + \tilde{v}\epsilon(x)d$, subject to the decentralized steady state equations (Ramsey, 1927).[6] Define $\tilde{d} = \epsilon(x)d$. The Ramsey problem is

$$\max_{\phi,\theta,x,c,\tilde{d},\lambda_1,\lambda_2,\lambda_3} \log(c)+\tilde{v}\tilde{d}+\lambda_1\left(f(x;\theta,\phi)c-1\right)+\lambda_2\left(g(x;\theta)-c\right)+\lambda_3\left(A(1-\mu\Lambda(x))-c-x-\tilde{d}\right)$$

Here $\tilde{d} = \epsilon(x)d$ represents total consumption in the second period; $f(x;\theta,\phi)$ and $g(x;\theta)$ are defined in equations (9) and (10) respectively; and the λ's are Lagrange multipliers. The last constraint ensures market clearing.[7] The next proposition characterizes the Ramsey equilibrium and the implied corollary discusses its existence.

Proposition 2 (Ramsey Steady State Equilibrium)**.** *The solution to the Ramsey maximization program verifies*

$$(a)\quad \frac{1}{c^R} = \tilde{v}\,, \qquad\qquad (b)\quad 1 = -\mu A\Lambda'(x^R)\,,$$

$$(c)\quad c^R + x^R = A\left(1-\gamma\epsilon(x^R) - \theta^R\mu\Lambda(x^R)\right)\,, \qquad (d)\quad \theta^R = A\gamma\epsilon'(x^R) + \phi^R\,.$$

Corollary 1 (Existence of the Ramsey Steady State Equilibrium)**.** *A unique Ramsey steady state equilibrium exists if and only if $\Lambda'(\bar{\bar{x}}) \leq -1/(\mu A)$ and $x^R \geq \bar{\bar{x}}$.*

Proposition 2 conveys some important insights. First, equation (a) balances the marginal utility of consumption in the first period with its discounted counterpart in the second period. Because of the linear utility assumption in the second period, consumption by newborn individuals only depends on the structural parameter $\tilde{v}$ and not on the policy instruments (see Section 5 for an extension with concave utility in the second period).

Second, equation (b) reveals how the government sets health investment so that increasing it by one unit lowers the dependency cost by the same amount. Figure 3 helps understand this equation. A Ramsey equilibrium exists if and only if $\Lambda'(\bar{\bar{x}}) \leq -1/(\mu A)$; that is, if the slope of the dependency curve $\Lambda(x)$ is sufficiently negative for some x. Crucially, assuming existence, equation (b) has two solutions: one lies to the left of $\bar{\bar{x}}$ and one to the right of $\bar{\bar{x}}$.

[6]In Appendix B, we solve the dynamic Ramsey problem and we show that when the weight on all generations is 1, the problem is equivalent to solving the steady state Ramsey problem (see for instance Krueger et al., 2021, for a similar point). We optimize with respect to two policy instruments because we have two externalities. Using a single instrument would be sub-optimal, while adding a third instrument (for instance γ) would leave it undetermined.

[7]We implicitly assume that health investment is strictly positive, meaning we rule out a solution $x^R = 0$ in which individuals would only live in the first period.

However, the Ramsey equilibrium, x^R, maximizes welfare, $W(x^R)$, if and only if $W''(x^R) < 0$. Since immediate computations yield $W''(x^R) = -\tilde{v}A\mu\Lambda''(x^R)$, the maximum occurs when $\Lambda''(x^R) > 0$. In other words, the Ramsey health investment equilibrium x^R is unique and lies to the right of $\bar{\bar{x}}$. Corollary 1 summarizes these conditions.

Third, equation (d) ensures the incentives to over- and under-invest in health cancel each other out. Indeed, the left-hand side, θ^R, captures the incentives to under-invest in health in the first period, since when θ^R is positive, it reduces the relative price of care below one. In contrast, the right-hand side represents the two incentives to over-invest in health: the pension scheme funded by the next generation and the prevention subsidy driving the relative price of health investment below one. By choosing the optimal allocation of resources, the government removes all incentives to over- and under-invest in health. Importantly, equation (d) implies that $\theta^R > \phi^R$, as θ^R serves as the sole instrument offsetting the incentives to over-invest in health arising from the pension scheme and the prevention subsidy.

Lastly, two additional observations deserve further comment. The first relates to a point made earlier: at the Ramsey equilibrium, $(1/f(x^R))' > 0$ and $g'(x^R) < 0$. Therefore, in the neighborhood of x^R, the effect of changes in ϕ and θ are as shown in Figure 2. The second observation links our Ramsey problem to the golden rule, here denoted by x^{gr}, which maximizes total steady state consumption, $c + \epsilon(x)d = A(1 - \mu\Lambda(x)) - x$. Choosing x to maximize total consumption $c + \epsilon(x)d$ immediately leads to our equation (b). In our setup, the level of health investment prescribed by the golden rule thus coincides with that determined by the Ramsey problem. Moreover, the Ramsey problem goes one step further by optimally allocating available resources across generations.

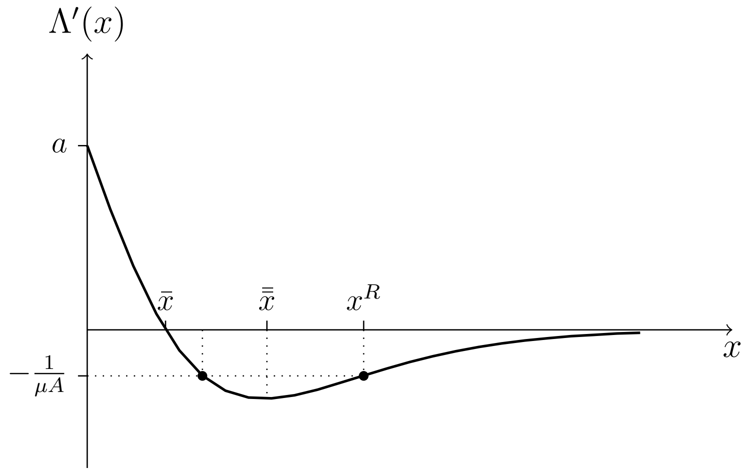
4.1. **Alternative Scenarios.** We here briefly discuss the Ramsey equilibrium when we shut down the longevity and the frailty channels.

No longevity channel. Suppose $\epsilon(x) = \bar{\epsilon} \in (0, 1)$. Then equation (d) in Proposition 2 simplifies to $\theta^R = \phi^R$. Indeed, the incentive to over-invest due to the pension externality is no longer present and θ^R no longer needs to be strictly above ϕ^R. With only the frailty channel operating, the two health policy instruments are complements (their respective reactions to any parameter change must be the same).

No frailty channel. Now, suppose $\pi(x) = \bar{\pi} \in (0, 1)$. Then equation (b) in Proposition 2 is $-\mu A\bar{\pi}\epsilon'(x^R) = 1$, which does not admit any solution (see Corollary 1). Therefore, a Ramsey equilibrium does not exist without frailty channel.

4.2. **Local Stability of the Ramsey Equilibrium.** To check the local stability of the Ramsey equilibrium, we linearize the decentralized equilibrium around the Ramsey steady state and we show that the slope $|\partial\hat{x}_t/\partial\hat{x}_{t-1}| < 1$, where $\hat{x}_t = (x_t - x^R)/x^R$. This means that for any x_{t-1} in the neighborhood of the Ramsey steady state, x_t will progressively

Figure 3. Ramsey Equilibrium



Notes. $\Lambda(x) = \epsilon(x)\pi(x)$ is the dependency function and the properties of $\epsilon(x)$ and $\pi(x)$ are given in Assumption 1. $\bar{x}$ and $\bar{\bar{x}}$ are such that $\Lambda'(\bar{x}) = 0$ and $\Lambda''(\bar{\bar{x}}) = 0$. x^R is the Ramsey equilibrium as defined by equation (b) in Proposition 2.

converge to x^R. Since c_t can be expressed as a function of x_t, the convergence of x_t implies the convergence of c_t. We summarize this property in Proposition 3 and illustrate the proof in Figure 4.

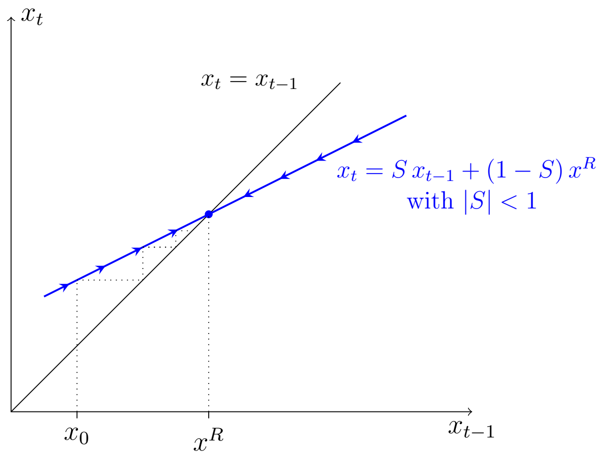
Proposition 3. *Under Assumptions 1 and 2, the Ramsey steady state as defined in Proposition 2 is locally stable.*

Proof. See Appendix C. □

4.3. **Comparative Statistics.** Table 1 studies how changes in model parameters shape the Ramsey equilibrium, as deduced from Proposition 2. An increase in the replacement rate $(\Delta^+\gamma)$ leaves the optimal allocation of resources unchanged. However, this is not true in the decentralized equilibrium, where a more generous pension system penalizes first-period consumption through a higher tax burden.[8] To compensate, the policy maker needs to reduce the generosity of the health system to lower the tax burden. Moreover, a higher γ magnifies the pension externality, i.e. the over-investment in health. As a result, the reduction in θ^R must be lower than the reduction in ϕ^R, which results in an increase in the gap between θ^R and ϕ^R. In our model, the optimal Ramsey policy therefore suggests a tradeoff between

[8]Mathematically, equations (9) and (10) imply that $\partial f(x)/\partial\gamma > 0$ and $\partial g(x)/\partial\gamma < 0$, which lowers consumption c^*.

FIGURE 4. Local Stability around Ramsey Equilibrium



Notes. The curve $x_t = Sx_{t-1} + (1-S)x^R$ is the linear approximation of the true non-linear Ramsey solution $x_t = F(x_{t-1})$. Since $|S| < 1$, the solution will converge to x^R for any x_0 in the neighborhood of x^R. The charts assumes a positive S. When S is negative, the convergence is oscillatory.

public expenditures in pensions and in health. In Section 6, we check whether this optimal tradeoff can be found in cross-country OECD data.

Faced with an increase in costs through higher LTC needs ($\Delta^+\mu$), the government seeks to raise health investment, which enlarges the gap with the decentralized level. Indeed, as explained before, μ magnifies the dependency externality and therefore under-investment in x. To boost health investment, the government lowers the subsidy to long-term care. We cannot analytically determine the impact on the prevention subsidy but, in any case, it cannot decrease more than the subsidy to long-term care, because its effect would be totally offset. In other words, the gap between θ^R and ϕ^R must decrease.

Similarly, an increase in labor productivity (Δ^+A), and hence output, also leads to higher health investment. Nonetheless, the specific policy settings to reach the new allocation of resources (more health investment) is no longer determined by our general setup, as it depends on the particular functional forms for longevity, $\epsilon(x)$, and dependency, $\Lambda(x)$. Finally, an increase $\Delta^+\tilde{v}$ in the discounted marginal utility of consumption in the second period (assuming survival) will lead the Ramsey government to transfer resources from the young to the elderly. This involves raising the care subsidy, θ^R, and also the prevention subsidy ϕ^R, to boost the number of individuals surviving the first period.

5. Discussion

This section introduces private savings in order to finance consumption when old, provides an extension involving a concave utility in the second period, shows that our model may

Table 1. Effects of Parameters on Ramsey Equilibrium

	x^R	c^R	ϕ^R	θ^R	$\theta^R - \phi^R$
$\Delta^+\gamma$	0	0	-	-	+
$\Delta^+\mu$	+	0	?	-	-
Δ^+A	+	0	?	?	?
$\Delta^+\tilde{v}$	0	-	+	+	0

Notes. γ is the pension replacement rate, μ determines the dependency cost, A is total production and $\tilde{v}$ represents the preference for future utility.

also address informal long-term care, and presents alternative assumptions regarding the longevity and frailty functions.

5.1. **Fully-Funded Pension System.** The pay-as-you-go (PAYG) pension scheme is prevalent in developed nations (OECD, 2018, see for instance). In our analysis with a PAYG system, health investment creates a pension externality for future generations by influencing their tax burden. As a result, it leads to over-investment in health.

Consider now a fully-funded pension scheme, commonly discussed in textbooks (see for instance Acemoglu, 2009). This replaces the pension externality with a standard saving externality. Specifically, individuals ignore that their saving decisions affect the labor income of future generations, resulting in insufficient investment in capital. Therefore, moving from PAYG to fully-funded pensions introduces a new variable – capital – in the analysis. It also replaces over-investment in health with under-investment in capital. Appendix D outlines a simple OLG model featuring fully-funded pensions, demonstrating that capital in the decentralized equilibrium is lower than in the Ramsey equilibrium.

Therefore, in a model with only savings, two externalities co-exist (our usual dependency externality and the new one related to capital) and two policy instruments are still needed to restore the optimal equilibrium. In a model with savings and a PAYG pension, we have three externalities (the two aforementioned and the pension externality) and three policy instruments are needed (for instance θ, ϕ and the replacement rate γ).

5.2. **Concave Utility.** The quasi-linear utility function used in Section 2 allows us to obtain closed-form solutions for the decentralized and the Ramsey steady state equilibrium (see for instance Koskela et al., 2002; Chien and Wen, 2022, for other papers with quasi-linear preferences). We now relax this assumption, considering instead a concave utility function for both periods. Formally, we now maximize

$$\log c_t + \beta\epsilon(x_t)\,\mathbb{E}_t\,\bar{v}\frac{d_{t+1}^{1-\sigma}}{1-\sigma}.$$

Here $\sigma \in [0,1)$, where $\sigma = 0$ brings us back to the quasi-linear problem.[9] We impose $\sigma < 1$ to avoid negative second period utility; that is, a 'preference for death' scenario (see for instance Dragone and Strulik, 2020, for a discussion). The private budget constraints (2) and (3) along with the public budget constraint (6) remain unchanged. As before, the decentralized steady state equilibrium results from the intersection of the first order condition $c = 1/f(x)$ and the budget constraint $c = g(x)$. While $g(x)$ is still given by equation (10), $f(x)$ becomes

$$f(x) = \frac{\tilde{v}A^{1-\sigma}\left((\epsilon'(x) - \Lambda'(x))\gamma^{1-\sigma} + \Lambda'(x)(\gamma - (1-\theta)\mu)^{1-\sigma}\right)}{(1-\phi)(1-\sigma)} . \qquad (11)$$

Unfortunately, we cannot determine the impact of σ on the $f(x)$ curve.

As for the Ramsey government's problem, equations from Proposition 2 now become

$$\begin{aligned}
\frac{1}{c^R} &= \tilde{v}\left(A(\gamma - (1-\theta^R)\mu)\right)^{-\sigma} , \\
\frac{1}{c^R} &= f(x^R;\theta^R,\phi^R) , \\
c^R + x^R &= g(x^R;\theta^R) , \\
-\mu A\Lambda'(x^R)\theta^R &= A\gamma\epsilon'(x^R) + \phi^R ,
\end{aligned}$$

where $f(.)$ and $g(.)$ are given by expressions (11) and (10), respectively. It is easy to show that when $\sigma = 0$, the above system of equations boils down to Proposition 2. However, strict concavity in the second-period utility function generates additional relationships between variables which complicate analysis. Therefore, in Section 6, we calibrate our model to provide a numerical illustration of the role of concavity.

5.3. **Informal Long-Term Care.** LTC is frequently provided informally: it is unpaid and offered by close relatives, often wives and daughters. These care-giving responsibilities can substantially reduce labor force participation and income (Vangen, 2021; Carrino et al., 2023). We now adapt our simple model to capture this phenomenon. Suppose LTC is entirely informal and is supplied by the young generation with a negative impact on their labor supply. Assume, moreover, that the time spent providing informal care is proportional to the number of dependent individuals. Consequently, 'productive' labor supply is no longer

[9] An alternative could be to consider a double-CES utility function $\frac{c_t^{1-\sigma}}{1-\sigma} + \beta\epsilon(x_t)\bar{v}\frac{d_{t+1}^{1-\sigma}}{1-\sigma}$. However, in this case, our quasi-linear problem no longer corresponds to a special case of this double-CES.

$n_t = 1$ but $n_t = 1 - \mu\Lambda(x_{t-1})$. As before, the representative firm maximizes profit $A\, n_t - w_t n_t$, yielding $A = w_t$.

The household inelastically supplies n_t unit of time and the budget constraints in the first and second periods are

$$\begin{aligned} c_t + (1-\phi)x_t &= (1-\tau_t)w_t n_t\,, \\ d_{t+1} &= \gamma w_t\,. \end{aligned}$$

Using the definition of n_t, the maximization of (1) with respect to x_t gives

$$\frac{1-\phi}{c_t} = \tilde{v}\epsilon'(x_{t-1})\gamma w_t. \tag{12}$$

Using the government budget constraint $\tau_t w_t n_t = \phi x_t + \epsilon(x_{t-1})\gamma w_{t-1}$ as well as the firm's first-order condition $A = w_t$, we finally obtain

$$c_t + x_t = A(1 - \mu\Lambda(x_{t-1}) - \gamma\epsilon(x_{t-1}))\,, \tag{13}$$

$$c_t + x_t + \tilde{d}_t = A(1 - \mu\Lambda(x_{t-1}))\,. \tag{14}$$

Equations (12) to (14) represent the decentralized equilibrium discussed in Section 2 when $\theta = 1$. Therefore, we can analyze the informal care scenario through the lens of our simple model just by setting $\theta = 1$. For example, computing the optimal Ramsey policy gives the optimal prevention subsidy rate[10]

$$\phi^R = \underbrace{-\mu A\Lambda'(x^R)}_{\substack{>0 \\ \text{dependency ext.}}} \underbrace{-A\gamma\epsilon'(x^R)}_{\substack{<0 \\ \text{pension ext.}}}\,.$$

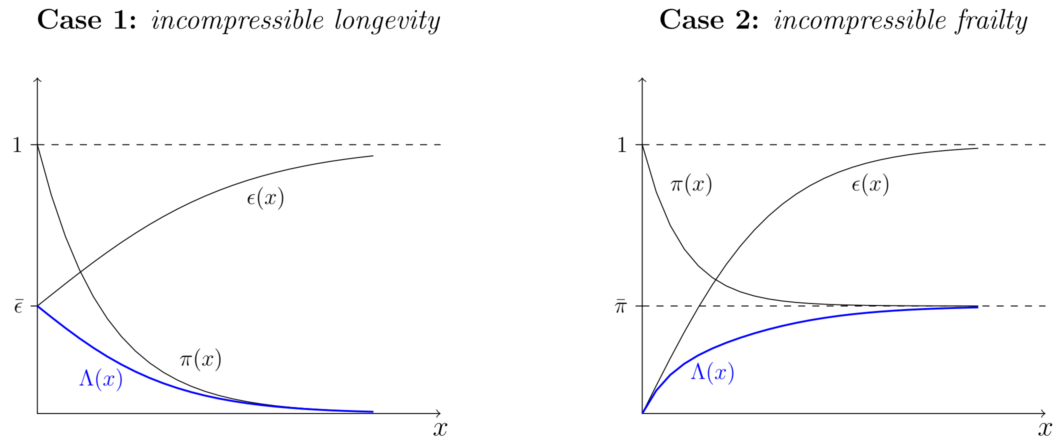
The right-hand side comprises two terms: a term representing the dependency externality, which requires a positive subsidy to fight under-investment, and a term representing the pension externality, which requires a negative subsidy to fight over-investment. We provide a numerical illustration in Section 6.

5.4. **Longevity and Frailty Functions.** Assumption 1 stipulates that both longevity and frailty functions range between 0 and 1. This means that with zero health investment, the probability of survival is nil, and with infinite health investment, the probability of frailty (conditional on surviving) is also nil. We here question these assumptions and introduce the concepts of incompressible longevity and incompressible frailty.

First, we suppose that everyone is born with innate health capital, which ensures a strictly positive probability of surviving, even with no health investment (incompressible longevity,

[10] In practice, we solve the Ramsey problem as outlined in Section 4, setting $\theta = 1$ in functions $f(.)$ and $g(.)$, thus not maximizing with respect to θ. The two other solutions to the Ramsey problem with informal care are $(1-\phi^R)/c^R = \tilde{v}A\gamma\epsilon'(x^R)$ and $x^R + c^R = A(1 - (\mu\Lambda(x^R) + \gamma\epsilon(x^R)))$. We use these equations in our numerical illustration in the next section.

FIGURE 5. Alternative Longevity and Frailty Functions



Notes. $\epsilon(x)$ and $\pi(x)$ are the longevity and frailty functions, respectively, whose properties are given in Assumption 1, except that $\epsilon(0) = \bar{\epsilon} > 0$ in Case 1 (incompressible longevity) and $\pi(\infty) = \bar{\pi} > 0$ in Case 2 (incompressible frailty). $\Lambda(x) = \epsilon(x)\pi(x)$ is the resulting dependency function. Moreover, we assume that $\bar{\epsilon}$ is sufficiently high ($\bar{\epsilon} > \max[a/b, (2ab - \epsilon''(0))/\pi''(0)]$) to imply a decreasing and convex dependency (Case 1), and that $\bar{\pi}$ is sufficiently high ($\bar{\pi} > -\lim_{x\to\infty} \pi'(x)/\epsilon'(x)$) to imply an increasing and concave dependency (Case 2).

see for instance Leung and Wang, 2010, for a similar function). Mathematically, we impose the same restrictions on $\epsilon(x)$ as in Assumption 1, except that $\epsilon(0) = \bar{\epsilon} \in (0, 1)$ (see Case 1 in Figure 5). We can prove that if $\bar{\epsilon} > \max[a/b, (2ab - \epsilon''(0))/\pi''(0)]$, then the resulting dependency function $\Lambda(x)$ is decreasing and convex everywhere. As a result, the Ramsey health investment x^R can take any possible value and is no longer restricted to values higher than a given threshold ($\bar{\bar{x}}$ in Corollary 1).

Second, we impose that prevention cannot fully remove frailty but at best compress it to a short period before death. To introduce this incompressible frailty (also often named morbidity compression in the related literature, see for instance Howdon and Rice, 2018), we impose the same restrictions on $\pi(x)$ as in Assumption 1, except that $\pi(\infty) = \bar{\pi} \in (0, 1)$ (see Case 2 in Figure 5). Again, we can prove that if $\bar{\pi} > -\lim_{x\to\infty} \pi'(x)/\epsilon'(x)$, then the resulting dependency function is increasing and concave everywhere. In other words, an interior Ramsey equilibrium does not exist in this case.

Obviously, we could combine incompressible longevity and incompressible frailty. Depending on their relative strengths, the dependency function $\Lambda(x)$ may be convex, concave, or mixed (as in Figure 1).

6. Parametric Illustration and Data

This section reports numerical illustrations using a calibrated version of our model and compares our results with OECD data.

6.1. **Model Calibration.** To numerically illustrate some features of our model, we select the following functional forms satisfying Assumption 1

$$\epsilon(x) = \tanh(ax),$$
$$\pi(x) = \exp(-bx),$$

where $a, b > 0$. Next, we make specific assumptions about parameter values, setting $A = 18$; $a = 3.5$; $b = 1.8$; $\gamma = 0.52$; $\mu = 0.13$; and $\tilde{v} = 0.13$. This parametrization satisfies Assumptions 2, 3 and 4 and therefore ensures that a unique decentralized equilibrium exists for all θ and ϕ in the interval $(0, 1)$, with $x^* > \bar{\bar{x}}$ (Proposition 1). In addition, the parametrization respects the condition stated in Corollary 1 which allows for a Ramsey equilibrium. Finally, we suppose that the policy instruments in the decentralized equilibrium equal their Ramsey counterparts, $\phi = \phi^R$ and $\theta = \theta^R$, which implies $c^* = c^R$ and $x^* = x^R$. The calibration leads to $\phi^R \approx 20\%$ and $\theta^R \approx 80\%$ as well as $c/A \approx 40\%$, $x/A \approx 5\%$, $d/A \approx 50\%$. Thus, LTC expenditures over A represents $1 - (c + x + d)/A = 5\%$. Since in our model A equals total output, none of these values are unreasonable.

Two points here deserve further comment. First, our parametrization is somewhat arbitrary and alternative parameter combinations, consistent with the different conditions, could be used without affecting our discussion. Second, the numerical levels of ϕ^R and θ^R should not be taken at face value: what matters is their relative levels and how they react to parameter changes. For instance, we assume the same technology to produce consumption and prevention goods. Instead, we could consider different productivity in the production of prevention goods. This would change the relative price of prevention and hence the optimal levels of the policy parameters but leave all our insights unchanged (see Appendix E for a thorough discussion).

6.2. **Comparative Statistics.** In Section 4, we determine analytically how changes to parameters affect the Ramsey equilibrium, but some changes have ambiguous effects (see Table 1). Panel (A) in Table 2 revisits this exercise with the calibrated model and therefore resolves (conditional on the calibration) the previous ambiguities. These results appear in the shaded cells in Panel (A) and we only comment them below. $\Delta^+\mu$ magnifies the dependency externality, which generates more under-investment (our numerical illustration focuses on the parameter space where $\Lambda'(x) < 0$). As a result, the subsidy to prevention should increase. We also see that richer countries ($\Delta^+ A$) should adopt more generous health policies. Since prevention should be higher, the subsidy to prevention should increase more than the

Table 2. Effects of Parameters on Ramsey Equilibrium with the Calibrated Model

(a) Baseline model

	x^R	c^R	ϕ^R	θ^R	$\theta^R - \phi^R$
$\Delta^+\gamma$	0	0	-	-	+
$\Delta^+\mu$	+	0	+	-	-
Δ^+A	+	0	+	+	-
$\Delta^+\tilde{v}$	0	-	+	+	0

(b) Concave utility model

	x^R	c^R	ϕ^R	θ^R	$\theta^R - \phi^R$
$\Delta^+\gamma$	-	-	-	-	+
$\Delta^+\mu$	+	-	+	-	-
Δ^+A	+	+	+	+	-
$\Delta^+\tilde{v}$	+	-	+	+	-

Notes. We use the calibration $A = 18$, $a = 3.5$, $b = 1.8$, $\gamma = 0.52$, $\mu = 0.13$, and $\tilde{v} = 0.13$; where γ is the pension replacement rate, μ determines the dependency cost, A is total production and $\tilde{v}$ represents the preference for the future utility. In Panel (A), the shaded cells are results which can only be shown numerically (conditional on the calibration). In Panel (B), the shaded cells indicate effects which are absent in the baseline model.

subsidy to long-term care. Finally, the optimal subsidies ϕ^R and θ^R are complements in most cases. That is, they react similarly to a structural change in the economy (the only exception being a change in μ).

Interestingly, the two externalities in the model can generate all possible combinations of under-/over-investment and under-/over-consumption. Remember that the calibration exercise supposes the policy instruments in the decentralized equilibrium are equal to their Ramsey counterparts, $\phi = \phi^R$ and $\theta = \theta^R$. Therefore, $c^* = c^R$ and $x^* = x^R$. Now, assume an increase in μ. This magnifies the dependency externality, which triggers under-investment. At the same time, the change raises taxes, which produces under-consumption. Obviously, a decrease in μ would imply opposite results: over-investment and over-consumption. Now suppose an increase in A. This results in under-investment and over-consumption. Indeed, while consumption is independent of A in the optimal equilibrium, so that health investment captures the increase in disposable income, this is not the case in the decentralized equilibrium. Without government intervention, newborn individuals divide their newly gained income between consumption and health investment, leading to under-investment and over-consumption. Again, a decrease in A would generate opposite results: over-investment and under-consumption.

6.3. **Concavity and Informal Care.** Section 5 presented an extension with concave utility in the second period of life. Unfortunately, we could not determine unambiguously how $\sigma > 0$ affects the $f(x)$ curve and the Ramsey equilibrium. Therefore, we use the calibrated model to show that increasing concavity (while leaving the other parameters unchanged) reduces x^* and increases x^R, thus generating under-investment. As a result, the optimal policy response will lower the gap between θ^R and ϕ^R. Moreover, Panel (B) in Table 2 reports the usual comparative statistics with a concave utility in the second period.[11] In our baseline model with quasi-linear utility (Panel (A)), changes in parameters do not affect the Ramsey equilibrium of some variables (for instance a change in γ has no effect on x^R and c^R). This is no longer the case with a concave utility function in the second period. Shaded cells in Panel (B) highlight these effects which are absent in the baseline model. With concave utility, prevention decreases in γ (higher income tomorrow) and increases in $\tilde{v}$ (more weight on the future). Consumption lowers with γ and μ (higher taxes), and rises with A (higher income). Finally, since x^R increases in $\tilde{v}$, the gap between the optimal policy parameters decreases. All these results are intuitive.

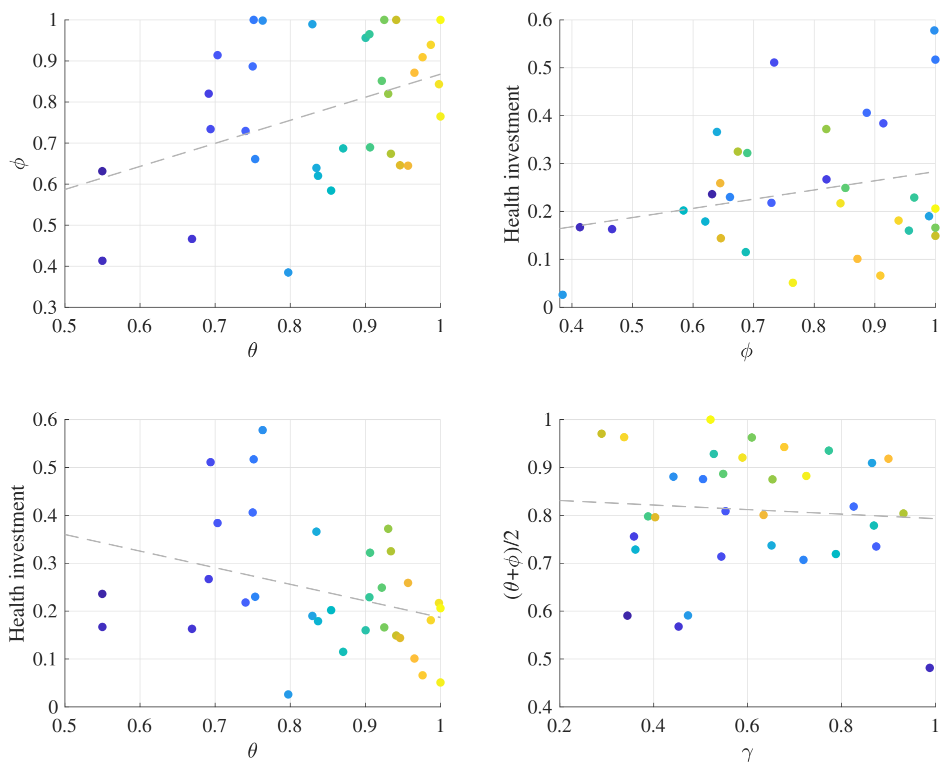
Finally, in Section 5, informal care is introduced by imposing $\theta = 1$. Let us therefore move θ from the calibrated value of 0.8 to 1. Recall that Proposition 1 revealed that a higher θ reduced x^*. Our numerical simulations show that imposing $\theta = 1$ further reduces x^R (the Ramsey equilibrium under informal care is given by Equations (13)-(14) and those in Footnote 10). As a result, imposing $\theta = 1$ leads to over-investment in health, so that the optimal distance between θ and ϕ^R must increase (according to our calibration, we show that ϕ^R increases but less than θ).

6.4. **Empirical Evidence.** This subsection provides cross-country empirical evidence, suggesting our results align well with observed health-related patterns across OECD countries in 2018. For each OECD country, we calculate the key ratios needed in the model: the prevention subsidy (ϕ) is measured by public preventive care divided by total preventive care; the care subsidy (θ) is public LTC divided by total LTC; the health investment ratio x/A is calculated as the GDP share of total preventive care expenditure; and the replacement rate (γ) is the net pension replacement rate at the average wage (refer to Appendix F for detailed information on the data). Figure 6 visually depicts the relationships between these empirical equivalents, where each dot corresponds to an OECD country.

Beginning with the top left panel, a positive correlation appears between the two health subsidies. This observation is consistent with the results of our earlier numerical exercise, where we found that ϕ and θ were in general complements in the vicinity of the Ramsey equilibrium. Moving to the top right panel, there appears to be a weak positive correlation between the prevention subsidy and health investment. The bottom left panel reveals a negative link between the care subsidy and health investment. These findings are in line with Proposition 1. Lastly, the bottom right panel suggests a negative correlation between the generosity of the pension scheme and that of health subsidies. Once again, this observation squares well with our results concerning the Ramsey equilibrium, as the first line in Table 1 confirms.

[11] Increasing σ may substantially change both the decentralized and Ramsey equilibria, so we set a small $\sigma = 0.025$. Increasing σ to more standard values is possible if we change the other parameters accordingly.

Figure 6. Selected Health-Related Patterns across OECD Economies



Notes. Each dot represents an OECD country in 2018 (health data) or 2022 (pension data). See Appendix F for details on the data and the computation of the empirical counterparts of θ (care subsidy), ϕ (prevention subsidy), health investment and γ (pension replacement rate).

7. Concluding Remarks

We have used a simple model to shed light on a difficult subject: how the interplay among preventive health investment, longevity and aging-related frailty shapes optimal health policy. Despite its simplicity, our model encapsulates a wide array of effects, including two externalities (one linked to the pension scheme and the other to healthcare costs), two channels through which preventive health investment impacts these externalities (longevity and aging-related frailty) and two policy instruments (subsidies to care and to prevention). As a result, the model yields some interesting, even surprising, findings. For example, in our model, setting the care subsidy higher than the prevention subsidy is always optimal. Furthermore, the optimal levels of these subsidies are inversely related to the generosity of the pension scheme, as measured by the replacement rate.

Despite its simplified nature, we hope that our model turns out to be a useful guide to analyzing more realistic setups in the future. A quantitative approach could provide a promising avenue for future research. For instance, returning to the findings mentioned above, how much higher should the care subsidy be compared to the prevention subsidy, and how does this difference vary with key structural parameters such as the discount rate or the productivity growth in medical technology? Also, by how much does the generosity of the pension scheme affect the optimal health subsidies? Needless to say, answering these questions requires large numerical models to address capital accumulation, empirically-credible functional forms, and fiscal policy.

Another related avenue for research stems from our finding that health investment has an ambiguous effect on dependency because the increase in longevity may raise the number of elderly individuals requiring long-term care. Formulating effective health policy requires determining whether this effect prevails over the reduction in frailty. Hence, empirical analysis could help understand which country characteristics, including income per capita and the shape of the population pyramid, determine which effects dominates the impact of health investment on dependency.

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Appendix A. Long-Term Care Production Sector

We assume a 2-sector economy. The first sector produces goods y_t according to the linear labor intensive technology $y_t = An_t^g$, where A is the level of productivity and n_t^g is labor. The firm in the goods sector maximizes its profit $An_t^g - w_t^g n_t^g$, which gives $A = w_t^g$ with w_t^g the wage in the goods sector.

The second sector produces LTC services according to the linear labor intensive technology $\mu\Lambda(x_{t-1}) = n_t^c$, where the level of productivity is normalized to 1 and n_t^c is labor. The firm in the good sector maximizes the profit $p_t\mu\Lambda(x_{t-1}) - w_t^c n_t^c$, where p_t is the relative price of the LTC services and w_t^c the wage in the LTC sector, which gives $p_t = w_t^c$.

The household supplies inelastically one unit of time ($1 = n_t^g + n_t^c$) but chooses how to split this unit between the goods sector and the LTC sector. The budget constraints for the first and second periods are as follows

$$
\begin{aligned}
c_t + (1-\phi)x_t &= (1-\tau_t)(w_t^g n_t^g + w_t^c(1-n_t^g)) \,, \\
d_{t+1} + p_{t+1}\pi(x_t)(1-\theta)\mu &= \gamma(w_t^g n_t^g + w_t^c(1-n_t^g)) \,,
\end{aligned}
$$

and the maximization of (1) with respect to n_t^g gives $w_t^g - w_t^c = 0$. Together with the two firm first-order conditions, we finally get $A = w_t^g = w_t^c = p_t$, $n_t^g = 1 - \mu\Lambda(x_{t-1})$ and $y_t = c_t + x_t + \epsilon(x_{t-1})d_t = A(1 - \mu\Lambda(x_{t-1}))$. It is then straightforward to show this equilibrium is equivalent to the one we present in Section 2. The equivalence obviously relies to the fact that (i) in both sectors we do not have capital and production is linear in labor, and (ii) the labor supply is perfect substitute across sector.

Appendix B. The Ramsey Problem

The Ramsey government chooses a policy $\{\phi, \theta\}$ to maximize the welfare of all (present and future) generations $\sum_{t=0}^{\infty} \rho^t W_t$. $\rho \in (0, 1]$ is the discount rate or, equivalently, ρ^t is the weight associated to generation t; and $W_t = \log c_t + \tilde{v}\tilde{d}_{t+1}$ is the lifetime utility of generation t as in the decentralized equilibrium. The maximization is subject to the three decentralized equilibrium conditions

$$
\begin{aligned}
\frac{1-\phi}{c_t} &= \tilde{v}A\left(\gamma\epsilon'(x_t) - (1-\theta)\mu\Lambda'(x_t)\right) \,, \\
c_t + x_t &= A\left(1 - \gamma\epsilon(x_{t-1}) - \theta\mu\Lambda(x_{t-1})\right) \,, \\
c_t + x_t + \tilde{d}_t &= A(1 - \mu\Lambda(x_{t-1})) \,.
\end{aligned}
$$

Note that to avoid any issues of time-inconsistency, we assume that the government credibly commits to the policies ϕ and θ. The solution of this maximization under constraints, with respect to $\{\phi, \theta, c_t, x_t, \tilde{d}_{t+1}\}$ is

$$\begin{aligned} \frac{1}{c_t} &= \frac{\tilde{v}}{\rho}, \\ 1 &= -\rho\mu A\Lambda'(x_t). \end{aligned}$$

Furthermore, ϕ, θ and $\tilde{d}_t$ are directly inferred from the three constraints.

First, we see that the optimal c_t and x_t are constant through time, as well as $\tilde{d}_t$. Second, when $\rho = 1$ (no discount or equal weight to all generations), the solution presented here is equivalent to the steady state Ramsey solution displayed in Proposition 2. Third, when $\rho < 1$, optimal c and x are lower than the optimal c^R and x^R from the steady state Ramsey.

Appendix C. Proof of Proposition 3

The linearization of the decentralized equilibrium (7)-(8) around any pair (x, c) gives

$$-\frac{1-\phi}{c}\,\hat{c}_t = \tilde{v}\,A\,x\,\hat{x}_t\left(\epsilon''(x)\gamma - (1-\theta)\mu\Lambda''(x)\right),$$

$$\hat{c}_t = -\frac{x}{c}\left(\hat{x}_t + A\,\hat{x}_{t-1}(\gamma\epsilon'(x) + \theta\mu\Lambda'(x))\right),$$

where $\hat{x}_t = (x_t - x)/x$ and $\hat{c}_t = (c_t - c)/c$. After eliminating $\hat{c}_t$ from the above equations, it is straightforward to compute the slope

$$S = \frac{\partial \hat{x}_t}{\partial \hat{x}_{t-1}} = \frac{(1-\phi)A\left(\gamma\epsilon'(x) + \theta\mu\Lambda'(x)\right)}{\tilde{v}\,A\,c^2\left(\epsilon''(x)\gamma - (1-\theta)\mu\Lambda''(x)\right) - (1-\phi)}.$$

Around the Ramsey steady state, that is assuming that $(x, c) = (x^R, c^R)$ and $(\theta, \phi) = (\theta^R, \phi^R)$ as given by Proposition 2, the slope becomes

$$S|_{Ramsey} = \frac{-\phi^R\,(1-\phi^R)}{\frac{A}{\tilde{v}}\underbrace{\left(\epsilon''(x^R)\gamma - (1-\theta^R)\mu\Lambda''(x^R)\right)}_{<0} - (1-\phi^R)}.$$

The slope around the Ramsey steady state can be negative (when ϕ^R is negative) or positive (when ϕ^R is positive) but in any case we have that the absolute value is below 1, since the absolute value of the numerator is in $[0, 1-\phi^R)$ and the denominator is lower than $-(1-\phi)$. Note that if ϕ^R is negative, the convergence is oscillatory.

Appendix D. Fully-Funded Pensions

Here, we develop a simple OLG model with savings but without health investment. The first generation solves the following problem

$$\begin{aligned} &\text{Maximize} && \log c_t + \tilde{v}d_{t+1}, \\ &\text{subject to} && c_t + s_t = w_t - t_t, \\ & && d_{t+1} = R_{t+1}s_t + t_{t+1}, \end{aligned}$$

where s_t is saving, R_{t+1} the gross return on saving and t_t a transfer from the young to the old generation (a negative t therefore reverse the transfer from the old to the young). We consider t_t as the policy parameter. Other notations are as in the paper. The firm maximizes $A\, s_{t-1}^{\alpha} - w_t - R_t s_{t-1}$. We see that previous saving is the current capital and that the elasticity of output to capital is $\alpha \in (0,1)$. Solving the household and the firm problem and taking the steady state gives

$$\frac{1}{c} = \tilde{v}\, A\, \alpha\, s^{\alpha-1}\,, \tag{16}$$

$$c = A\,(1-\alpha)\, s^{\alpha} - s - t\,, \tag{17}$$

$$A\, s^{\alpha} = c + s + d\,. \tag{18}$$

The first equation is the Euler equation (choice between consumption and saving), the second equation is the budget constraint of the first generation, and the last equation is the final good constraint.

We then consider a Ramsey planner at the steady state. The planner maximizes $\log c + \tilde{v} d$ with respect to $\{c, d, s, t\}$ under the constraints (16) to (18). We can show that the steady state saving in the Ramsey equilibrium must satisfy

$$s^R - A\alpha\left(s^R\right)^{\alpha} + \frac{1-\alpha}{A\alpha\,\tilde{v}}\left(s^R\right)^{1-\alpha} - \frac{1-\alpha}{\tilde{v}} = 0\,,$$

whereas, combining equations (16) and (17), steady state saving in the decentralized equilibrium must satisfy

$$s^* - A(1-\alpha)\,(s^*)^{\alpha} + \frac{1}{A\alpha\,\tilde{v}}\,(s^*)^{1-\alpha} + t = 0\,.$$

We assume no transfer in the decentralized equilibrium (t=0) and, to get a closed-form expression, we impose $\alpha = 1/2$ and $A^2\tilde{v} > 4$. We then obtain

$$\left(s^R\right)^{\frac{1}{2}} = \frac{\frac{A^2\tilde{v}-4}{2A\tilde{v}} + \sqrt{\left(\frac{A^2\tilde{v}-4}{2A\tilde{v}}\right)^2 + \frac{2}{\tilde{v}}}}{2}\,,$$

$$\left(s^*\right)^{\frac{1}{2}} = \frac{A^2\tilde{v}-4}{2A\tilde{v}}\,.$$

We immediately see that $s^R > s^*$ (under-saving). As a result, a Ramsey planner should impose a transfer from the old to the young, i.e. $t < 0$.

Appendix E. Relative Productivity in the Production of Prevention Goods

Let us assume an intermediate firm producing intermediate goods y_t with a linear labor intensive technology $y_t = A\, n_t$, paying a wage w_t, and selling these intermediate goods to a final firm, at a price p_t. The maximization program is

$$\max_{n_t} p_t A\, n_t - w_t n_t\,,$$

which gives $p_t A = w_t$. Then we have a final firm buying the intermediate goods and transforming them into consumption goods and prevention goods. The relative productivity to produce prevention goods is q_t and the relative price of prevention goods is p_t^x. The maximization program is

$$\max_{c_t, x_t, y_t} c_t + p_t^x x_t - p_t y_t + \lambda_t (y_t - c_t - x_t / q_t)\,,$$

where λ_t is the shadow price associated to the production constraint. We obtain $\lambda_t = p_t = 1$ and $p_t^x = 1/q_t$. It is straightforward to show that, as before, the decentralized equilibrium is given by the intersection of the two curves

$$\begin{aligned} c &= 1/(q\, f(x))\,, \\ c &= g(x) + x\,(q-1)/q\,, \end{aligned}$$

where $f(.)$ and $g(.)$ are still given by equations (9) and (10). Moreover, easy computations produce the Ramsey equilibrium

$$\begin{aligned} 1/c^R &= \tilde{v}\,, \\ 1/q &= -\mu A \Lambda'(x^R)\,, && (19) \\ c^R + x^R/q &= A\left(1 - \gamma\epsilon(x^R) - \theta^R \mu \Lambda(x^R)\right)\,, \\ \theta^R &= A\gamma q \epsilon'(x^R) + \phi^R\,. && (20) \end{aligned}$$

Having $q_t \neq 1$ does not change our results. For instance, equation (19) still requires a downward sloping $\Lambda(.)$ to get a Ramsey equilibrium and equation (20) still implies that $\theta^R > \phi^R$ and that the two policy parameters tend to be equal when the longevity channel vanishes.

Appendix F. Data

All health data are from the OECD dataset on 'Health expenditure and financing, Current expenditure on health (all functions)' for the year 2018 (pre-Covid) and for 33 OECD countries (data for Chile, Colombia, Mexico, New-Zealand and Turkey are not available).

- Health investment corresponds to 'Preventive care (all financing schemes)', as a percentage of the GDP.
- ϕ corresponds to 'Preventive care (Government/compulsory schemes)' divided by 'Preventive care (all financing schemes)'.
- θ corresponds to 'Long-term care (Government/compulsory schemes)' divided by 'Long-term care (all financing schemes)'.

Pension data are from the OECD dataset on 'Pensions at a glance' for the year 2022 (latest available data) and for the same 33 OECD countries.

- γ corresponds to the 'Net pension replacement rate, Male, 1.00 of AW'.

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