SPEED OF CONVERGENCE IN A MALTHUSIAN WORLD: WEAK OR STRONG HOMEOSTASIS?

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Speed of Convergence in a Malthusian World: Weak or Strong *Homeostasis*?*

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Abstract

Standard Malthusian models predict that a productivity or population shock modify income per capita in the short run. In the long run, however, population pressures make income per capita gradually come back to its steady state. I investigate the duration of this short-run fluctuation, estimating the speed of convergence of Malthusian economies to their GDP per capita and population steady-states. To do so, I first build and calibrate a Malthusian model capturing explicitly the idea that marriages are postponed (advanced) and fertility potential of couples reduced (augmented) during depressions (expansions). I then also run β-convergence regressions on historical panel data. I find consistent evidence of weak *homeostasis*, with a half-life of about one century. It implies that early modern data may display high persistence without necessarily rejecting the Malthusian hypothesis.

Keywords: Convergence, Homeostasis, Malthusian dynamics, Preventive check, Marriage, Fertility, Malthusian model, β-convergence

JEL Codes: N10, N13, N33, O10, O47

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In four centuries [1300-1700], the [French] population only increased by 2 million persons in all! And some say less! [...] Thus, an extraordinary ecological equilibrium is revealed. Of course, it did not exclude possibly prodigious, but always temporary, upheavals and negative fluctuations in its time like those experienced by animal population.

Emmanuel Le Roy Ladurie (1977), Motionless History.

1 Introduction

One of the most central prediction of the Malthusian theory is that standards of living were *stagnant* before the onset of industrialization. Stagnation however does not literally mean *constant*, or flat, per capita income. In fact, any shock striking a Malthusian economy generates fluctuations, or volatility, in the standards of living, namely *temporary* or *non*-sustained economic growth. Indeed, a simple Malthusian model predicts that a positive shock on the technology level – say the introduction of better cultivation techniques – increases income per capita in the short run only; in the long run, population increases and the economy returns to its initial level of income per capita. This is the so-called "Malthusian trap" mechanism, that has been recognized as one of the major obstacles to achieve sustained economic growth during millennia (Kremer, 1993; Galor and Weil, 2000; Hansen and Prescott, 2002; Clark, 2007; Ashraf and Galor, 2011; Galor, 2011).

While the existence of the Malthusian trap mechanism is widely accepted, previous literature has found mixed evidence about its exact strength. A first group of studies finds evidence of a weak Malthusian trap, known as weak *homeostasis*[1], with slow convergence rates of several centuries (Lee and Anderson, 2002; Crafts and Mills, 2009; Fernihough, 2013; Bouscasse et al., 2021). On the other hand, Madsen et al. (2019) find evidence of a strong Malthusian trap or strong *homeostasis*, with fast convergence rates (few decades).

[1] *Homeostasis* comes from the Greek *homoios* "similar" and *stasis* "steady", meaning "staying the same". In demography, it refers to a population equilibrium maintained by density-dependent checks (Lee, 1987).

In this article, I reinvestigate the question of the strength of the Malthusian trap by examining the speed of convergence of Malthusian economies to their steady state – i.e. the time it takes them to go back to their steady state after a shock. I argue that fluctuations in Malthusian times are, by nature, *long*, as a shock is absorbed through demographic fluctuations which take time to unfold. For example, the introduction of better cultivation techniques increases income per capita and, in turn, people are expected to marry younger and to start having children earlier in life. Fertility will accordingly increase, but slowly, and not by a enormous margin. Similarly, the increase in per capita income allows to concentrate more resources on the same number of individuals, slowly improving survival chances and lowering mortality. These are the so called *preventive* and *positive* checks, originally argued by Malthus (1798) himself. This means that any shock to a Malthusian economy is likely to take generations to disappear.

To investigate this conjecture, I first build an overlapping-generations Malthusian growth model including both preventive and positive checks as means of population adjustment. In particular, agents first choose to marry (or not), influencing the extensive margin of fertility, and then choose the number of children within marriage, influencing the intensive margin of fertility. Both choices depend on income per capita, in a Malthusian fashion. I show that the speed of convergence of a Malthusian economy to its steady state depends on four parameters: the land share of output and the elasticities of fertility, marriage and survival with respect to income per capita. I calibrate the model for England and show that, under plausible parameter values, the speed of convergence indicates weak *homeostasis*, with a half-life of about a century. Using elasticity values estimated in the literature, I find further evidence of weak *homeostasis*, of about the same magnitude, for Scandinavian and European countries.

Second, I systematically confront the model predictions with the data, using β-convergence regressions *à la* Barro and Sala-i Martin (1992). Using the latest version of the Maddison Project historical GDP per capita series and simulated GDP per capita data series from Lagerlöf (2019), I empirically confirm weak *homeostasis*, in the same magnitude as predicted by my model. Endogeneity issues are addressed using an internal instrument approach (GMM) and controlling for the State History Index of Borcan et al. (2018). Measurement error issues are dealt using several strategies, including exclusion of the most uncertain part of the data, time averages, and time-interacted regressors. Next, I run the same regressions using McEvedy et al.'s (1978) historical population series and Reba et al.'s (2016) historical urban population series, confirming weak *homeostasis* and its magnitude.

This article contributes to the growing literature examining the existence and strength of the Malthusian trap (Lee and Anderson, 2002; Nicolini, 2007; Crafts and Mills, 2009; Kelly and Gráda, 2012; Fernihough, 2013; Møller and Sharp, 2014; Lagerlöf, 2015; Madsen et al., 2019; Cummins, 2020; Jensen et al., 2021). For instance, Fernihough (2013) finds evidence of weak *homeostasis* in Northern Italy using VAR methods. Similarly, Jensen et al. (2021) investigate Malthusian dynamics in Denmark using also a VAR methodology. I contribute to this literature in two main respects. First, rather than analyzing the case of one specific country using time series methods, I use a panel of Malthusian countries and a β-convergence model that exploits the within country variations in the lagged income per capita or population levels to estimate the speed of convergence. This article is the first, to my knowldege, to provide evidence of weak *homeostasis* in a panel of Malthusian economies. Second, using the most comprehensive and up to date panel data available to study Malthusian economies, I am able to characterize, for the first time, the full distribution of convergence speed during the Malthusian period. I show that most of the countries were characterized by weak *homeostasis*, while highlighting significant differences in the strength of the Malthusian trap. The closest article to mine is Madsen et al. (2019), which find strong *homeostasis* for a panel of 17 countries (900-1870). The main difference between the two articles lies in the approach to the data and the statistical model. While Madsen et al. (2019) rely on largely interpolated data from heterogeneous historical sources and use a SUR model, I take the data as given and use the techniques and remedies developed in the empirical growth literature, such as fixed-effects models and internal instruments (GMM).

This article also adds to the literature studying Malthusian dynamics in an overlapping-generations frameworks. The existing overlapping-generations Malthusian frameworks consider the intensive margin of fertility as the only channel through which population adjusts (Ashraf and Galor, 2011; Lagerlöf, 2019). I build on these previous models by incorporating, for the first time, marriage as an explicit channel through which the population adjusts, as originally argued by Malthus (1798) himself. The marriage channel allows me to model the extensive margin of fertility, as unmarried individuals typically did not have children in the Malthusian era. I can therefore model richer Malthusian population and convergence dynamics.

Finally, this article relates to the literature deriving the speed of convergence in growth models. Working in continuous time, Irmen (2004) and Szulga (2012) find that the speed of convergence of a Malthusian economy depends on the land share of output and the elasticities of the birth rate and death rate to income per capita. I contribute to this literature by showing that the elasticity of the marriage rate to income per capita also matters to characterize the speed of convergence. In a modern context, this article relates also to the seminal work of Barro (1991) and Barro and Sala-i Martin (1992).

The rest of this article is organized as follows. Section 2 presents my Malthusian growth model. Section 3 presents my calibration exercise, discussing the parameters I use and presenting my simulations. I also derive the speed of convergence implied by my model and discuss it in relation to the literature. Section 4 describes my empirical strategy and the data I use to estimate the speed of convergence. Section 5 presents and discusses my empirical results. Section 6 concludes.

2 Theoretical Framework

To describe the key mechanisms behind the dynamics of GDP per capita and population at the Malthusian epoch, I first build a theoretical model. I consider an overlapping-generations economy with time modelled as discrete and going from zero to infinity, and where agents live two periods. In the first period of their life, they are inactive children entirely supported by their parents; they make no decisions. In the second period of their life, they work, earn an income and make decisions about consumption, marriage and fertility.

I deviate from textbook Malthusian models by modelling explicitly marriage, celibacy and childlessness decisions. In brief, that means that I am considering both the *extensive* margin of fertility, i.e. whether or not an individual marries and can have children, and the *intensive* margin of fertility, i.e. variations in individual's number of surviving children within marriage. Those two elements are crucial in my model as they directly affect the speed at which a Malthusian economy returns to its steady state after a shock. They are in line with empirical studies showing the importance of the so called *preventive* checks, advocated by Malthus (1798) himself, in affecting fertility. Indeed, Cinnirella et al. (2017) show that real wages affect negatively birth spacing within marriage and the time of marriage and first child in England for the period 1540-1850. Cummins (2020) finds similar results with a negative effect of living standards on the age at first marriage in France between 1650 and 1820. de La Croix et al. (2019) show that singleness and childlessness are key elements to take into account when estimating reproductive success in pre-industrial times. Therefore, modelling both the *extensive* and *intensive* margins of fertility appears crucial to a rigorous analysis of population dynamics during the Malthusian era.

I model childlessness and celibacy together, leaving the possibility to procreate only to married agents. This is fully consistent with historical studies showing very low illegitimate birth rates in pre-industrial Europe (Hajnal, 1965; Segalen and Fine, 1988; Wrigley et al., 1989). Marriage offers the opportunity for agents to gain utility from another source than just pure consumption.[2] On the other hand, the disutility of marriage is represented by a search cost that agents need to pay in order to match with a partner.[3] Agents are assumed to be heterogeneous in their search cost, which is exogenously given. At the beginning of their adult life, agents draw a search cost λ_i with $\lambda_i \sim \mathcal{U}(1, b)$ with b being the maximum of the uniform distribution. Agents maximize their utility and therefore a marriage occurs only if the utility of being married is superior to the utility of being single. Within marriage, I let the agent's fertility depend on his income per capita, according to the standard Malthusian theory and empirical evidence (Cinnirella et al., 2017; de La Croix et al., 2019; Cummins, 2020).

Preferences and Budget Constraints.— The utility of a married agent i of generation t is defined *à la* Baudin et al. (2015):

$$U_{i,t}^M = \ln c_t + \gamma \ln (n_t + \nu) - \ln \lambda_i \ , \tag{1}$$

where c_t denotes consumption, $\gamma > 0$ is a child preference parameter, n_t is the number of surviving children, $\nu > 0$ allows for childlessness as the individual utility remains defined when $n_t = 0$ and λ_i is the utility cost of marriage.

It follows that the utility of an unmarried agent of generation t is given by:

$$U_{i,t}^S = \ln c_t + \gamma \ln (\nu) \ . \tag{2}$$

Agents allocate their income between consumption and child rearing such that we have the following budget constraint:

$$c_t = y_t - f(n_t) \ , \tag{3}$$

[2]This means that parents only care about the quantity of surviving children, as in a standard Malthusian model.
[3]Alternatively, one can think the cost as representing a dowry that agents need to pay in order to marry.

where y_t is agent's income, $f(n_t)$ is the cost of having n_t children in terms of goods.

A convenient functional form for $f(\cdot)$ capturing both the idea of childlessness ($f(0) = 0$) and allowing for potentially non-constant returns to scale in the production of children is the following one:

$$f(n_t) = q(n_t + \nu)^{1/\delta} - q\,\nu^{1/\delta}\ , \tag{4}$$

with $q > 0$ being unitary cost of a child and $\delta > 0$ a parameter influencing the degree of return to scale in child production.

Fertility.— Maximizing (1) subject to (4), I obtain the optimal fertility behaviour of a married agent of generation t:

$$n_t = \kappa \cdot \left(y_t + q\,\nu^{1/\delta}\right)^{\delta} - \nu\ \equiv n_t(y_t), \tag{5}$$

where $\kappa = \left(\frac{q}{\gamma\delta} + q\right)^{-\delta}$. Thus, in accordance with Malthusian theory, the number of surviving children within marriage depends positively on income per capita ($\partial n_t/\partial y_t > 0$).

Marriage.— An agent is indifferent between being married and single if utility is the same in both situations. I define $\overline{\lambda}$ as the draw from the search cost distribution that makes an agent indifferent between being married and single. The condition for an agent to be married is: $\lambda_i < \overline{\lambda}$ with $\lambda_i \sim \mathcal{U}(1, b)$. I can therefore compute the probability for an agent of generation t to be married as:

$$p_t = P(\lambda_i < \overline{\lambda}) = \frac{\overline{\lambda}(y_t) - 1}{b - 1} \equiv p_t(y_t)\ , \tag{6}$$

where b is the maximum of a uniform distribution and the threshold draw $\overline{\lambda}$ depends on an individual's income.[4] Since I work at the generation level, p_t is also equivalent to the marriage rate in that Malthusian economy. In the rest of the article, I will use p_t as the marriage rate.

[4] The full expression of $\overline{\lambda}$ is available in Section A of the Appendix

Thus, in accordance to the idea of Malthus (1798), an increase in income lowers the age at marriage, resulting in a higher marriage rate at the generation level in our model ($\partial p_t/\partial y_t > 0$).

Production.— Total output in period t is given by:

$$Y_t = (A_t T)^{\alpha} L_t^{1-\alpha} \ , \tag{7}$$

where A_t is a land-augmenting technology factor, T is total land area, L_t is the size of the labour force that is equivalent to the adult population in my analysis and $\alpha \in (0,1)$ is the land share of output.

I assume that workers are self-employed and earn an income equal to the output per worker in t. Using (7) and normalizing land area to unity ($T = 1$), we obtain:

$$y_t = \left(\frac{A_t}{L_t}\right)^{\alpha} \ . \tag{8}$$

Following Lagerlöf (2019), I consider sustained but constant growth in land productivity. The technological level in period t is given by:

$$A_t = A_0(1+g)^t \ , \tag{9}$$

where A_0 is the initial technological level and g is an exogenously given and constant rate of technological progress.

Mortality.— Malthus (1798) and the Malthusian theory assert that population adjusts via the so called *positive* and *preventive* checks. My model includes the two types of Malthusian population adjustment: (i) *preventive* checks, as both the decision to marry and the number of kids within marriage result from agents' optimization, and (ii) *positive* checks as I model the survival rate of adult agents as directly depending on their income in the following way:

$$s_t = min(\overline{s}, \ \underline{s} \, y_t^{\phi}) \ , \tag{10}$$

where $\overline{s}$ is the maximal survival rate, $\underline{s}$ is a parameter calibrated to target an initial survival rate and ϕ is the elasticity of the survival rate to income per capita. Thus, in accordance with the Malthusian theory, adult's survival is increasing along income as long as $\overline{s} > \underline{s}\ y_t^{\phi}$ since $\underline{s} > 0$ and $\phi > 0$.

Population Dynamics.— The size of the population of the next generation $t+1$ is given by:

$$L_{t+1} = n_t\ p_t\ s_t\ L_t\ . \tag{11}$$

Income per capita Dynamics.— Forwarding (8) to period $t+1$ and using (8), (9) and (11), I obtain a first-order difference equation giving the income per capita of the next generation:

$$y_{t+1} = \left(\frac{1+g}{n_t(y_t)\ p_t(y_t)\ s_t(y_t)} \right)^{\alpha} \cdot y_t \equiv \psi(y_t)\ . \tag{12}$$

Steady State.— Expression (12) is a non-monotonic function of y with a unique inflexion point. Provided that the initial income per capita y_0 is not too low, it is possible to demonstrate that $\psi(y_t)$ has a unique and globally stable interior steady-state implicitly defined by:

$$y^* \equiv \left(\frac{1+g}{n(y^*)\ p(y^*)\ s(y^*)} \right)^{\alpha} = 1\ . \tag{13}$$

Proof. See Section A of the Appendix.

3 Quantitative Analysis

In this section, I analyse the speed of convergence of a representative Malthusian economy using my theoretical model. I start by discussing the identification of the parameters that I use to calibrate my Malthusian model. I then discuss the simulation results of my calibration exercise. Finally, I derive the speed of convergence implied by my model and discuss it with respect to the literature.

3.1 Identification of the Parameters

In order to simulate the evolution of a representative Malthusian economy and study its speed of convergence, I first set the value of some parameters *a priori*, while some others are set to match some target following an exact identification procedure. I focus on England as the literature already provides a rich array of parameter values for that economy during the Malthusian period. Table 1 summarizes and explains my calibration strategy.

Table 1: Benchmark Parameter Values

Parameter	Value	Interpretation and comments
t	25	Number of years per generation. Fixed *a priori*
γ	1	Preference for children. Fixed *a priori*
q	1	Unitary cost of a child. Fixed *a priori*
δ	0.09	Gives preventive checks-income per capita elasticity of 0.22. Fixed *a priori*
ϕ	0.13	Gives positive checks-income per capita elasticity of 0.13. Fixed *a priori*
α	0.5	Land share of output. Fixed *a priori*
g	0.023	Rate of technological progress per generation. Fixed *a priori*
$\underline{s}$	0.196	Minimum of the survival rate. To match $s^* = 0.71$
ν	0.33	Child quantity preference parameter. To match $n^* = 1.62$
b	5.96	Maximum of the search cost distribution. To match $p^* = 0.89$

Notes: See text for more details on the sources.

First, the length of a period or generation t is fixed at 25 years, meaning that an agent is living at most 50 years in my model.[5] This is in line with life expectancy figures in pre-industrial England as reported by Wrigley et al. (1997). Life expectancy at the age of 20 was as high as 33-34 years on the period 1550-1799. Conditional on their survival until the age of 20, Malthusian agents have therefore good chances to reach the age of 50. This is also in line with the evidence on the so-called European Marriage Pattern (EMP) from Hajnal (1965). Indeed, the EMP is characterized by a late age of first marriage for women (between age of 24 and 26) and low illegitimacy birth rates. In my setting, agents marry and procreate only in the second period of their life, that is to say between age of 25 and 50 as indicated by the EMP.

Next, I normalize γ and q, respectively the agent's preference for children and the cost of

[5] de la Croix and Gobbi (2017) make a similar assumption in a modern context with developing economies.

raising a child, to one.

Elasticity parameters δ and ϕ are particularly important in my setting, as they directly affect the speed of convergence in my model (see Section 3.3). Since I am working at the generation level, those parameters represent respectively the long-run elasticities of the preventive checks (fertility and marriage) and the long-run elasticities of the positive checks (survival) to income per capita. The empirical literature on Malthusian dynamics provide various estimates of such long-run elasticities based on wage, Crude Birth Rate (CBR), Crude Marriage Rate (CMR) and Crude Death Rate (CDR) time-series. Such estimates are available for England (Lee, 1981; Lee and Anderson, 2002; Klemp, 2012; Møller and Sharp, 2014), Northern-Italy (Fernihough, 2013), Scandinavian countries (Lagerlöf, 2015; Klemp and Møller, 2016) and Germany (Pfister and Fertig, 2020).

For England, long-run elasticities range from 0.12 to 0.32 for the preventive checks and from 0.08 to 0.22 for the positive checks. I set $\delta = 0.09$ and $\phi = 0.13$ in my benchmark specification to match the mean of the long-run elasticities provided by the aforementioned literature for England. This corresponds to a long-run elasticity of 0.22 for the preventive checks and 0.13 for the positive checks. In my model, the value of the long-run elasticity of the positive checks is directly given by ϕ, as equation (10) corresponds to the unit-elastic case. For the long-run elasticity of the preventive checks, I fix δ such that the sum of the elasticities of fertility and marriage with respect to income per capita is equal to the targeted value (see Section B of the Appendix for more details).

Setting $\delta < 1$ means that my model consider decreasing returns to scale in the production of children, while most standard Malthusian models assume constant returns to scale ($\delta = 1$).[6] As pointed out by Lagerlöf (2019), we may interpret decreasing returns to scale in the production of children as stemming from an implicit production function for child survival featuring two inputs: parental time devoted to each child and each child's food intake. More

[6] See, for instance, Ashraf and Galor (2011).

children automatically yields less time per child, leading to an increase in the per-child amount of the consumption good necessary to ensure the survival of each child. Furthermore, the aforementioned empirical literature consistently finds values well below unity for the long-term elasticities of the preventive and positive checks. For instance, using exogenous cross-county variations in Swedish harvest between 1816 and 1856, Lagerlöf (2015) finds long-run elasticities of fertility, marriage and mortality of 0.1, 0.16 and -0.09, respectively.

The land share of output (α) for England is set at 0.5, corresponding to its estimated long-run value for the Malthusian period (Federico et al., 2020).

In standard Malthusian models with constant technological progress, total population at the steady state is not constant. In fact, (13) shows that population grows at the same pace as technology; this is a necessary condition to keep income per capita constant at the steady state. Consequently, g is calibrated using 25-years average population growth using Campbell et al. (2015) data for the period 1270-1675.

Consider next the three remaining parameters, $\underline{s}$, ν and b that are calibrated to match respectively the steady-state survival rate for adults (s^*), agent's steady-state fertility (n^*) and the steady-state marriage rate (p^*) following an exact identification procedure. The first target s^* is set to 0.71 as in Wrigley (1968). This corresponds to the survival rate of population of 25 years old until the age of 50 for the period 1538-1624 in England. The second target p^* is set to 0.89 which corresponds to a percentage of never married women of 11% as reported by Dennison and Ogilvie (2014) for England. This figure is the average of the percentage of never married women for England across 45 historical studies and is also very close to the value reported in the seminal study of Wrigley et al. (1989). Knowing the two first targets, the third target n^* is given by the steady-state condition in (13). I also set the steady-state level of income per capita y^* at 20,108 (2013 British pounds). This corresponds to the 1300-1325 average GDP per capita of England cumulated over one generation (25 years) using Campbell et al. (2015) data. I adjust the initial level of technology A_0 in (9) to reach the desired level of y^*.

3.2 Simulation Results

Before looking at the speed of convergence in itself, this section shows the overall ability of my model to reproduce Malthusian dynamics. To do so, I simulate a Black Death alike shock killing 60% of the population at $t = 5$. The size of the population shock is taken from Campbell et al. (2015) and corresponds to the lowest population level observed in England after the Black Death to take into account the diffusion process of the plague and its multiple resurgences. This figure is also consistent with Benedictow et al. (2004), finding an overall mortality of 62.5% for England. Figure 1 shows the evolution of income per capita (y_t), fertility (n_t), the marriage rate (p_t) and the survival rate (s_t) under our benchmark parametrization across 20 generations.

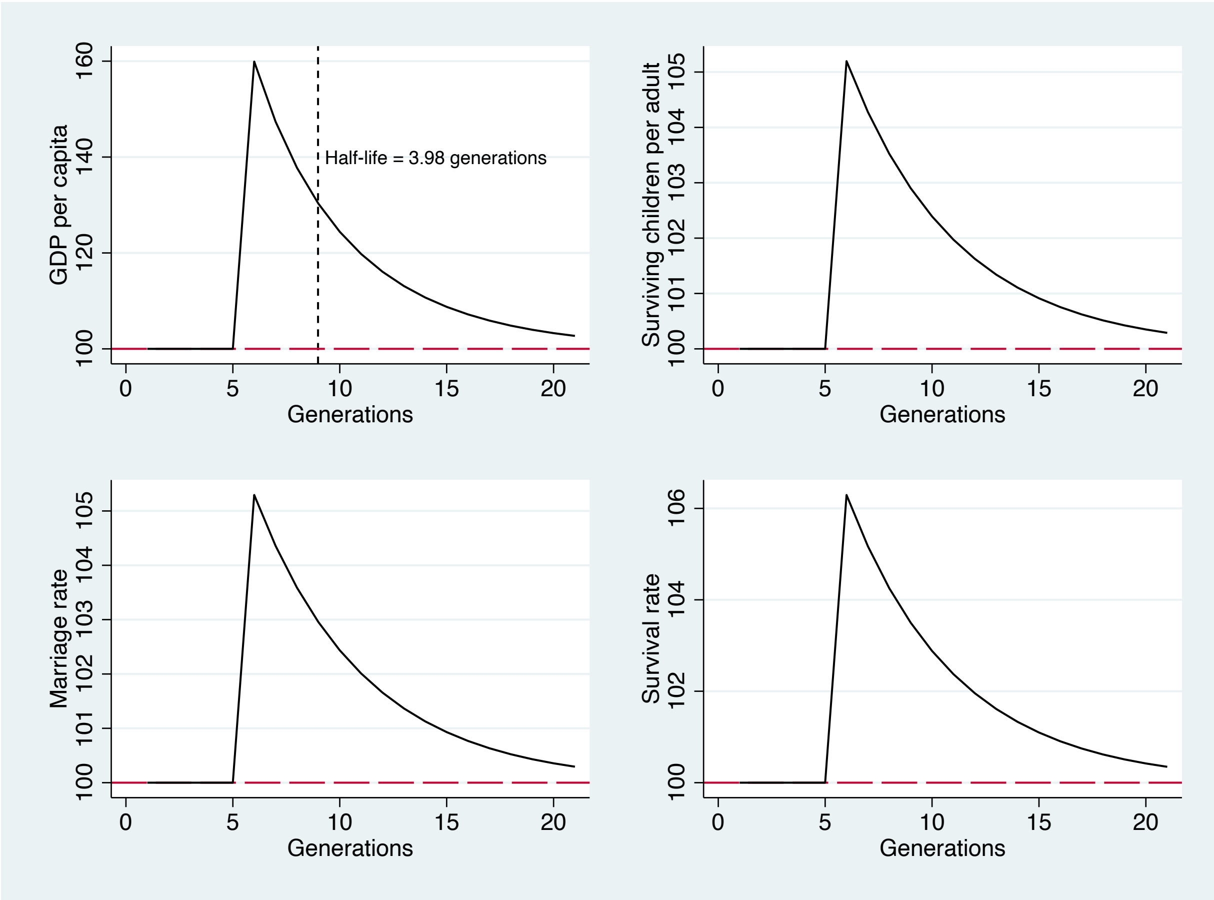
Standard Malthusian theory predicts that an exogenous negative shock on the population level (or Black Death) increases income per capita in the short run only.[7] After the shock, population increases and the economy gradually converges back to its steady state such that, at the long-run, the income per capita is constant. This is, by construction, what I observe in my model. Figure 1 shows that, right after the plague onset, the surviving agents enjoy indeed a temporarily higher level of income per capita. Those better material conditions mean that agents have better chances to survive, they marry more and are able to raise more surviving children inside marriage. This translates into faster population growth, which in turn triggers the convergence process of income per capita to its steady state. In Figure 1, I also display the half-life of convergence for my benchmark specification. The half-life is about 4 generations, meaning that any shock on the English Malthusian economy is persistent across several generations (see Section 3.3 for a complete discussion on the speed of convergence).

Figure 2 evaluates the ability of my model to replicate the dynamic of income per capita after the Black Death, using English historical GDP per capita data from Campbell et al. (2015). To do so, I first extract the cyclical component in the data using an Hodrick–Prescott filter.[8] This

[7] Jedwab et al. (2022) find evidence that the Black Death was indeed a plausibly exogenous shock to the European economy.

[8] I set the smoothing parameter to 500 given that I use generations of 25 years.

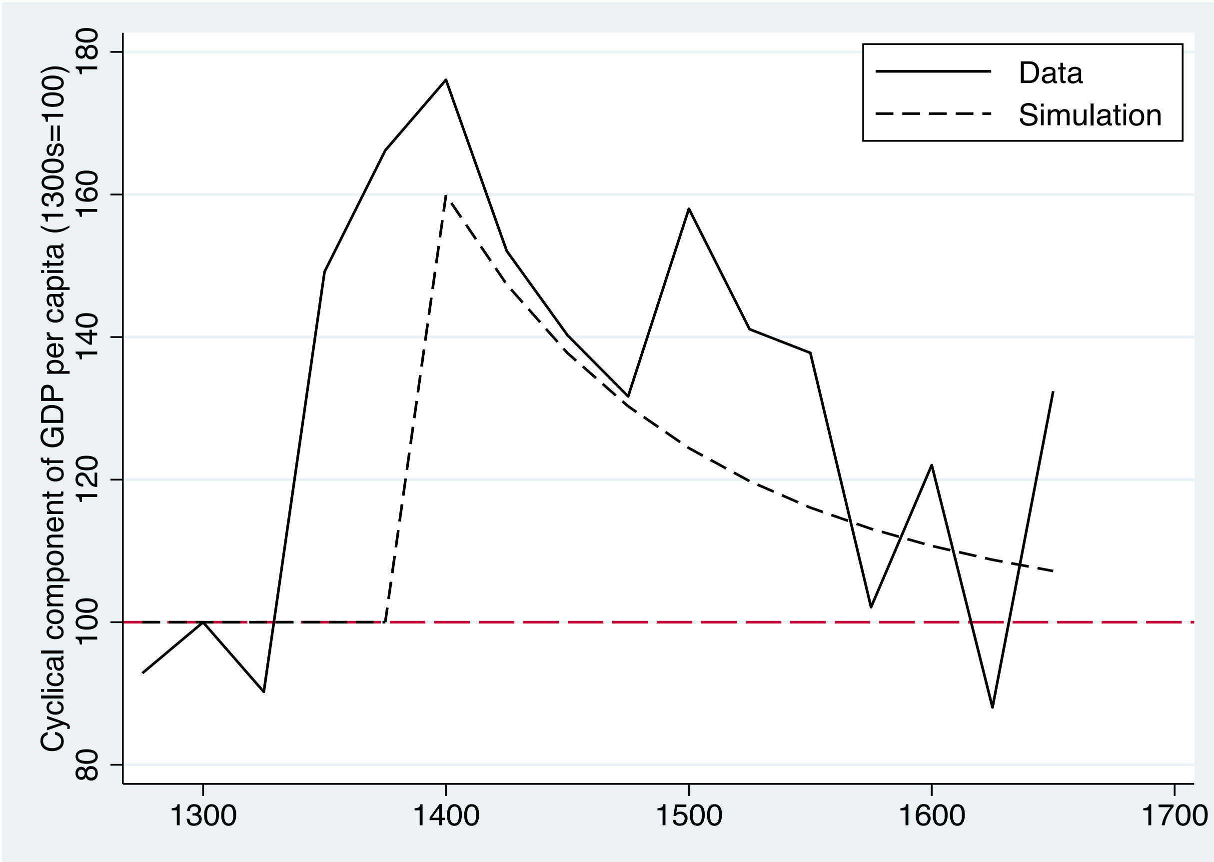
Figure 1: Responses of the English Malthusian Economy to a Black Death



Notes: This figure plots the response of income per capita (top-left panel), fertility (top-right panel), marriage (bottom-left panel) and survival (bottom-right panel) to a Black Death alike shock, killing 60% of the population at $t = 5$.

is necessary as my model analyses the dynamic of convergence to a unique and fixed steady state. On the contrary, fluctuations in the data might reflect changes in the position of the Malthusian steady state, as well as transitionnal dynamics. As argued by North and Thomas (1973) and Acemoglu and Robinson (2012), the Black Death might have affected the steady state of the English economy through institutional changes. Figure 2 shows that my model generates a path for GDP per capita very similar to the cyclical component of the data in the years following the Black Death. This is remarkable as the transitional dynamic in my model is only governed by the long-run elasticities provided by the aforementioned empirical literature.

Figure 2: GDP per capita Dynamic after the Black Death – Simulated Path vs. Data



Notes: This figure plots the cyclical component of GDP per capita after the Black Death from Campbell et al. (2015) (solid line) and from our benchmark simulation (dashed line). The cyclical component is obtained using an Hodrick-Prescott filter with a smoothing parameter of 500. We normalize data on the period 1300-1325, last period before the occurrence of the Black Death in England (1348).

3.3 The Speed of Convergence

In my model, the speed at which GDP per capita converges to its steady state is given by:

$$\beta^* = \alpha(\epsilon_{n_t} + \epsilon_{p_t} + \epsilon_{s_t}) \ , \tag{14}$$

where ϵ_{n_t}, ϵ_{p_t} and ϵ_{s_t} are the elasticities of fertility, marriage and survival with respect to income per capita. See Section C in the Appendix for additional details on the derivation of the speed of convergence. It is hence possible to fully characterize the speed of convergence of a Malthusian economy using only elasticities. Similar results are found by Irmen (2004) and Szulga (2012) in continuous time.

Table 2 gives the speed of convergence for different parametrizations of my model. Under my benchmark parametrization, the speed of convergence is about 17% per generation. Convergence to the steady state is hence slow: it takes about four generations, or one century, for the English Malthusian economy to absorb half of a shock. This is in line with much of the literature, finding evidence of weak *homeostatsis* (Lee, 1993; Lee and Anderson, 2002; Crafts and Mills, 2009; Fernihough, 2013; de la Croix and Gobbi, 2017; Bouscasse et al., 2021; de la Croix and Gobbi, 2022). Using equation (14), I also compute the speed of convergence for Denmark, Norway, Sweden and European Malthusian societies thanks to the long-run elasticities provided by Galloway (1988), Lagerlöf (2015) and Klemp and Møller (2016). I find half-lives ranging between 48 and 115 years, pointing once again towards weak *homeostasis*. All in all, our benchmark falls exactly at the median of the half-lives reported in Table 2 (mean of 126; standard deviation of 101). Looking at the studies focused on England, our benchmark appears close to the lower tail of the half-lives found in the literature. In particular, I am very close to Lee and Anderson (2002) who find an half-life of 107 years for the period 1540-1870.

To see under which conditions my model can generate a speed of convergence compatible with strong *homeostasis* for England, I consider three deviations from my benchmark parametrization. The logic is to gradually push the long-run elasticities towards the upper-bounds provided by the literature. Doing so, I am able to compute the highest speed of convergence that the English Malthusian economy can achieve under plausible elasticity values. I find that the lowest half-life for England is 54 years. This is almost twice as fast as my benchmark, but still well above strong *homeostasis*, represented by a half-life of 30 years or less as in Madsen et al. (2019). This gives further evidence that any shock on a Malthusian economy is persistent over several generations, even if the Malthusian trap mechanism remains binding in the long run.

Table 2: Speed of Convergence in our Model and in the Literature

Country	Value of Parameters	$\mid \beta \mid$	Half-Life (years)	Comments
<u>This Study:</u>				
England	$\delta = 0.09$, $\phi = 0.13$ and $\alpha = 0.5$	0.175	99	Benchmark specification
England	$\delta = 0.11$, $\phi = 0.22$ and $\alpha = 0.5$	0.273	64	With upper-bound elasticity values
England	$\delta = 0.09$, $\phi = 0.13$ and $\alpha = 0.6$	0.21	83	With the upper-bound land share of output
England	$\delta = 0.11$, $\phi = 0.22$ and $\alpha = 0.6$	0.327	54	With upper-bound elasticity and land share of output
Denmark (1821-1890)			84	Using (14), $\alpha = 0.5$ and elasticities reported by Klemp and Møller (2016)
Norway (1775-1853)			91	Using (14), $\alpha = 0.5$ and elasticities reported by Klemp and Møller (2016)
Sweden (1775-1873)			48	Using (14), $\alpha = 0.5$ and elasticities reported by Klemp and Møller (2016)
Sweden (1816-1870)			99	Using (14), $\alpha = 0.5$ and elasticities reported by Lagerlöf (2015)
Europe (1540-1870)			115	Using (14), $\alpha = 0.5$ and elasticities reported by Galloway (1988)
<u>Estimated Half-Lives from Other Studies:</u>				
England (1541-1870)			431	Crafts and Mills (2009)
Sub-Saharan Africa (1990)			198	de la Croix and Gobbi (2022)
England (1250-1870)			150	Bouscasse et al. (2021)
Northern Italy (1650-1881)			112	Fernihough (2013)
England (1540-1870)			107	Lee and Anderson (2002)
Developing countries (1990)			100	de la Croix and Gobbi (2017)
England (1540-1870)			70	Lee (1993)
17 countries (1470-1870)			29	Madsen et al. (2019); article claiming strong *homeostasis*

Notes: See text for more details on parameter values and sources.

4 Empirical Framework

In this section, I start by presenting the data I use to estimate the speed of convergence of Malthusian economies. Next, I detail my main estimating equation and discuss the potential threats to my identification strategy.

4.1 Data

In my analysis, I use two kinds of datasets: (i) GDP per capita series (either historical or simulated) and (ii) historical population series (either total or urban population). Historical GDP per capita series come from the Maddison Project Database (Bolt and Van Zanden, 2020). Building on the pioneering work of Maddison (2003), the Maddison Project provides standardized historical GDP per capita series running over several centuries. These series are regularly updated and enriched by researchers in the field of historical national accounting. To limit measurement error issues, I focus on the period 1000 CE - 1800 CE and consider only countries with good data availability – i.e. GDP per capita data available every year or every ten years before 1800 CE. Following these two criteria, twelve Malthusian economies are considered, including core (eg. Italy, England, China) and more peripheric (e.g. Mexico, Poland, Sweden) Malthusian economies. Simulated GDP per capita series come from Lagerlöf (2019). Lagerlöf (2019) shows that a Malthusian model with stochastic and accelerating growth in land productivity is able to match the moments of historical GDP per capita series presented in Fouquet and Broadberry (2015). Simulations are available for 1,000 model economies and 501 years, making it very useful to circumvent the lack of GDP per capita data inherent to the pre-industrial period. From an econometric point of view, it corresponds to an ideal setting where both the cross-sectional and the time dimensions are large, limiting the bias of the different estimators on the speed of convergence.

Historical population series come from various sources. First, considering total population figures, I use McEvedy et al.'s (1978) data. Population series from that source have been widely used to address various questions in the comparative development literature, with most of the contributions exploiting cross-country variations over a few years (Acemoglu et al., 2001; Nunn, 2008; Nunn and Qian, 2011; Ashraf and Galor, 2011, 2013).[9] My objective on the other hand is to exploit within-country population changes, and I therefore coded McEvedy et al.'s (1978) data in its full panel dimension. Despite its wide use in the litterature, McEvedy et al.'s (1978) data are also heavily criticized, mostly for measurement error issues (Guinnane, 2021). In order to mitigate this problem, I use only a specific time frame and set of countries. First, I consider only the period between 1000 CE and 1750 CE. It corresponds to a period recognized as Malthusian and avoid the sizeable uncertainty on population figures surrounding the end of the Roman Empire and the Early Middle Ages. Second, within the selected period, I keep only countries for which population figures are reported with the maximum frequency – i.e. every century before 1600 CE and every half-century after 1600 CE. Following those two criteria, eighteen countries are considered with a majority of European countries. Turning to historical urban population series, I use Reba et al. (2016) who compiled and geocoded Chandler's (1987) and Modelski's (2003) figures. In particular, the database provides population level for cities all around the world from 3700 BC to 2000 CE. I apply the same procedure as for the Maddison Project's data or McEvedy et al. (1978)'s data, namely I first select urban population levels within the 1000 CE - 1800 CE period.[10] Then, I focus on cities with a good data availability – i.e. cities with a population figure available at least for seven half-centuries (out of the seventeen potentially available) between 1000 CE and 1800 CE.[11]

[9]For example, Ashraf and Galor (2011) use McEvedy et al.'s (1978) data as dependent variable for three periods: 1 CE, 1000 CE and 1500 CE.

[10]When both Chandler and Modelski estimates are available for the same city and year, we take the average between the two figures. This is the case for 20 cities, only for year 1000 CE.

[11]That threshold corresponds to the median of data availability.

4.2 Empirical Strategy

To empirically assess the speed of convergence of a Malthusian economy to its steady state, I rely on standard β-convergence models. Such models have been extensively used in the growth literature to quantify the speed at which modern economies converge to their steady state (Barro, 1991; Barro and Sala-i Martin, 1992; Islam, 1995; Caselli et al., 1996; Barro, 2015). More recently, that framework has also been used in the Malthusian context (Madsen et al., 2019).

My main specification is the following dynamic panel:

$$\frac{\ln(y_{i,t}) - \ln(y_{i,t-\tau})}{\tau} = \beta \ln(y_{i,t-\tau}) + \gamma' \mathbf{X}_{\mathrm{i,t}} + \delta_t + \alpha_i + \varepsilon_{i,t} \ , \tag{15}$$

where $i = 1, ..., N$ indicates my unit of analysis which can be either a country or a city and $t = 1, ..., T$ corresponds to a year. The left-hand side corresponds to the growth rate of my variable of interest y, which can be either GDP per capita or population levels. The parameter τ indicates the number of years between two available y in the data, such that my dependent variable is always the average annual growth rate of y between period $t - \tau$ and t. To handle gaps, measurement error and to avoid spurious changes in the data, I target a minimal τ of 50 years.[12] The right-hand side is composed of the lagged dependent variable $y_{i,t-\tau}$, a vector of control variables $\mathbf{X}_{\mathrm{i,t}}$, time fixed effects δ_t, country or city fixed effects α_i and an idiosyncratic error term $\varepsilon_{i,t}$.

My coefficient of interest is β, which corresponds to the average annual speed at which an economy converges to its steady state. Obtaining unbiased estimates of the speed of convergence is challenging in many ways. First, one might challenge the inclusion of country fixed effects in my regressions. Country fixed effects are indeed traditionally viewed as a solution to the omitted variable bias, as they control for all time-invariant characteristics affecting long-run economic

[12] It means that when the data are available at a lower frequency than 50 years, we compute 50 years averages for that variable. It corresponds to two generations in our theoretical model, or the complete lifetime of a Malthusian agent.

development such as geography, climate, culture. Not including country fixed effects in a β-convergence model will hence irremediably bias downwards the speed of convergence, unless the set of time-varying explanatory variables $\mathbf{X}_{i,t}$ is rich enough. However, as highlighted by Barro (2015), country fixed effects are themselves a source of upward bias in the measurement of convergence speed, referred to as the Hurwicz-Nickell bias (Hurwicz, 1950; Nickell, 1981). Barro (2015) shows in particular that the Hurwicz-Nickell bias tends to zero as the overall sample length in years tends to infinity, meaning that the bias can be substantial in the modern growth context where the time dimension rarely exceeds 50 years. In a Malthusian context though, the advantages of using country fixed effects are heightened, while the associated risks are dampened. Indeed, the scarcity of available time-varying control variables in the case of a large sample and long time frame renders the country fixed effects crucial to neutralize the omitted variable bias. On the other hand, the risk of a sizeable Hurwicz-Nickell bias is greatly mitigated by the large overall sample length, since my Malthusian analysis spans over centuries.

Even if the advantages of using country fixed effects might overweight their disadvantages in the Malthusian context, β-convergence models can still be plagued by the presence of the endogeneity bias. Country fixed effects cannot capture time-varying steady-state determinants, such as institutional changes, which can jointly determine current economic growth and past levels of economic development. To directly address that issue, I include *Statehist* and its squared level as control variables (Borcan et al., 2018). *Statehist* is an index retracing state development every half-century from 3500 BCE until today, and is therefore a suitable control for broad institutional changes. Moreover, my analysis always includes time fixed effects in order to control for global changes in the steady-state determinants, such as the spread of new technologies or climatic changes.[13]

To address further the endogeneity bias, I provide, when possible, results using the Arellano

[13]For instance, most of our analysis spans from the 11th to the 19th century, meaning that we are capturing both the effect of the Medieval Warm Period and the Little Ice Age with time fixed effects, assuming that the effect of those climatic events was, on average, the same for each country.

and Bond (1991) GMM estimator (AB) and the Blundell and Bond (1998) GMM estimator (BB). The AB estimator uses a GMM estimation procedure where all the variables are taken in first-difference and lagged levels are used to instrument the endogenous regressors. This procedure was first used by Caselli et al. (1996) in the growth context to address both the Hurwicz-Nickell bias and the endogeneity of regressors. The BB estimator builds on AB, exploiting additional moment conditions which use lagged first differences of the regressors to instrument the levels of the endogenous variables. Obviously, AB and BB are not panacea and the literature on dynamical panel has identified several issues in their use. AB's main issue is the problem of weak instruments, which is known to bias β estimates towards their LSDV counterparts (Hauk and Wacziarg, 2009). BB oftenly corrects for the weak instrument problem, but requires a stationarity assumption to deliver consistent results. In particular, BB requires that the country fixed effects are uncorrelated with lagged differences in the dependent variable – i.e. $E(\alpha_i(\Delta \ln y_{i,t-s} - \Delta \ln y_{i,t-r})) = 0$ for all r and s. Even if the stationarity condition does not seem to hold in practice, it has been demonstrated that BB delivers systematically lower biases in the modern growth context than AB under weak instruments (Hauk and Wacziarg, 2009).Monte Carlo simulations in the modern growth context have further shown that within-country estimators (LSDV, AB and BB) perform better in estimating the "true" speed of convergence than the between or the random effects estimators when the endogeneity bias on the steady-state determinants is severe (Hauk Jr, 2017). Measurement error on the other hand is better dealed using the between or the random effects estimator (Hauk and Wacziarg, 2009). Considering the endogeneity bias stemming from omitted variables as the most serious threat to our analysis, I therefore choose to rely on within-country estimators (LSDV, AB and BB) to estimate the speed of convergence.

To address measurement error in the lagged dependent variable, I implement nevertheless several strategies. First, as already mentioned above, I systematically avoid using the most uncertain population or GDP per capita data by excluding pre-1000 CE figures. Indeed, as pointed recently by Guinnane (2021) for population figures, we simply "do not know the population" going that far back in the past where standardized and systematic censuses were not operated. Population and output measures between 1000 CE and 1800 CE contain also a sizeable part of uncertainty. However, local censuses, parish registers or proxy variables such as urbanization are increasingly available on that period, reducing the overall measurement error. I also only consider countries or cities with the best, or at least above median, data coverage for the considered time periods (see Section 4.1 for more details). Second, I compute 50-year averages when the data are available at a lower frequency to avoid spurious changes in the considered variables, and to focus on long-run dynamics. Third, in presence of classical measurement error in the regressors, AB and BB can in principle correct for it, as they are based on an internal instrumentation strategy to estimate coefficients. Non-classical measurement error, such as systematic differences in the GDP per capita or population levels across countries, are taken into account via country fixed effects. This is the case for instance if rich countries report systematically, but consistently through time, lower errors than poor countries. Time fixed effects can also deal with common time-varying measurement error, as for instance increased uncertainty in population figures moving away from 1800 CE. Finally, as a further robustness check for non-classical measurement error, I also systematically perform LSDV estimations with year-interacted lagged dependent variables. By that mean, I can in principle take into account any time varying differences in measurement correlated with initial population or GDP per capita levels.

5 Results

In this section, I present my empirical estimates of the speed of convergence for various Malthusian economies. I start by presenting my results using historical and simulated income per capita data. Then, I present my results using total and urban population historical data.

5.1 Convergence with GDP per capita Data

In Table 3, I present my results based on OLS and LSDV estimations of equation (15) using Maddison Project's data (Bolt and Van Zanden, 2020).[14] Table 3 shows that Malthusian economies take at least several generations to absorb a shock, revealing a pattern of weak *homeostasis*.

Table 3: Speed of Convergence using GDP per capita Data from the Maddison Project

Sample Used:	Full			Europe		
	OLS (1)	LSDV (2)	LSDV (3)	OLS (4)	LSDV (5)	LSDV (6)
log(GDPpc)	-0.0006	-0.0057**	-0.0057***	0.0000	-0.0046**	-0.0046**
	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	No	Yes	Yes	No	Yes	Yes
Statehist	No	No	Yes	No	No	Yes
Observations	85	85	85	69	69	69
adj. R-sq	-0.01	0.16	0.14	-0.05	0.11	0.08
Half-Life	1197	122	121	-18766	150	152
Half-Life 95% C.I.	[-434,252]	[422,71]	[391,72]	[-356,370]	[587,86]	[663,86]

Notes: This table presents estimates of the speed of convergence using GDP per capita data from the Maddison Project. Columns 1-3 present results using the full sample of countries we selected from the Maddison Project's data and columns 4-6 show results focusing on European countries. Standard errors clustered at the country level are in parentheses. * p<0.1, ** p<0.05, *** p<0.01.

Starting with the most parsimonious specification, with only time fixed effects as controls, column 1 reveals that the lagged dependent variable coefficient is not statistically different from zero. This is not really surprising as the omitted variable bias is substantial here, driving the lagged dependent coefficient towards zero. Moreover, as my model suggests, Malthusian economies should display conditional convergence rather than absolute convergence, as the steady-state position of each economy depends on its characteristics.[15]

[14] In that case, GMM estimations are not reported due to the lack of observational units. Indeed, as advised by Roodman (2009), a useful rule of thumb to avoid weak instrument issues in GMM estimations is to keep the total number of instruments below the number of observational units. It is not possible with the sample we consider from the Maddison Project data as we have eleven countries and fifteen instruments in the most parsimonious possible instrumentation, resulting in unitary Hansen test p-values.

[15] From the steady-state condition in (13), it is clear that two economies with for instance different rates of

Adding country fixed effects, column 2 reveals a negative and significant relationship between GDP per capita growth and its initial level, pointing toward conditional convergence of Malthusian economies. The estimated coefficient implies a half-life of 122 years ($\ln(2)/0.0057$), with a 95% confidence interval giving half-lives between 422 years and 71 years. Therefore, the most comprehensive and up-to-date historical GDP per capita series are consistent with weak *homeostasis* of Malthusian economies, as it takes at least several generations to absorb a shock. Compared to other studies, my results fall between Fernihough (2013) and Bouscasse et al. (2021), who find half-lives of 112 and 150 years respectively. However, my results are in great contrast with Madsen et al. (2019), who find a half-life of 29 years for income per capita and conclude in favor of strong *homeostasis* of Malthusian economies.[16] In addition, since LSDV typically delivers an overestimate of the speed of convergence and OLS an underestimate of it, the true β should lie between OLS and LSDV. This means that the "true" half-life should be consistent with weak *homeostasis* only, as the upper bound of the half-life in that case is about 120 years.

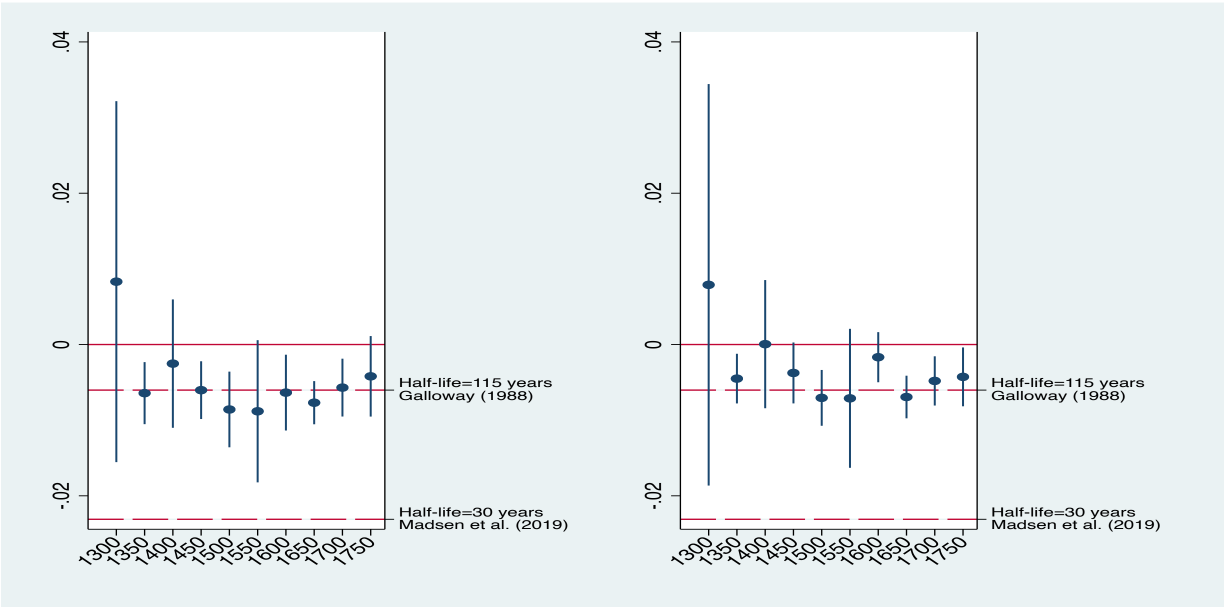
To limit the omitted variable bias, column 3 adds *Statehist* and its squared level as controls. The speed of convergence is almost unaffected, as the reported half-life is now slightly higher at 121 years. As a robustness check, columns 4 to 6 replicate the analysis restricting our sample to European countries, which gives similar results. In particular, column 6 indicates a slower speed of convergence, with a half-life of 152 years, confirming the weak *homeostasis* pattern found in the previous columns. I find no significant differences in the estimated speed of convergence between the two samples of countries.

As an additional robustness check, Figure 3 displays LSDV estimations of columns 3 and 6, adding an interaction term between time fixed effects and the initial level of GDP per capita.

technological progress g will not converge to the same steady state.

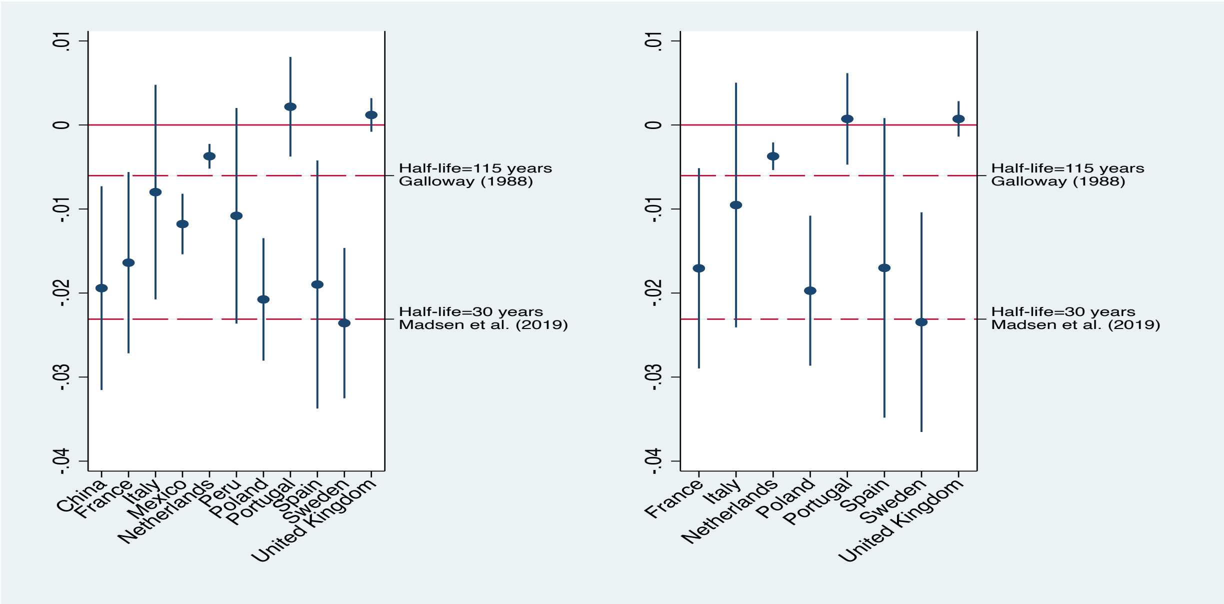
[16]Note that my article has several methodological differences with respect to Madsen et al. (2019). First, they use seemingly unrelated regression (SUR) models – a random effects family estimator – while we use within-country estimators (LSDV, AB and BB). Second, they rely on interpolated data coming from heterogeneous sources for GDP per capita and population data, while we I take the data as given.

Figure 3: Speed of Convergence per period using Maddison Project Data



Notes: This figure reports estimates of the speed of convergence using GDP per capita data from the Maddison Project. It corresponds to the LSDV estimations in column 3, Table 3 (left panel) and in column 6, Table 3 (right panel), adding year-interacted lagged GDP per capita levels as controls. 95% confidence intervals are reported.

Figure 4: Speed of Convergence per country using Maddison Project Data



Notes: This figure reports estimates of the speed of convergence using GDP per capita data from the Maddison Project. It corresponds to the LSDV estimations in column 3, Table 3 (left panel) and in column 6, Table 3 (right panel), adding country-interacted lagged GDP per capita levels as controls. 95% confidence intervals are reported.

This allows to look at the heterogeneity of the speed of convergence through time, and check the possible influence of non-classical measurement errors. Whether considering the full or the European sample of countries, the vast majority of the estimated coefficients are not statistically different from a half-life of 115 years, as found for Europe using the long-run elasticities of Galloway (1988). Overall, the point estimates are stable in magnitude and, in majority, statistically different from zero at the 5% level. This indicates a clear and stable pattern of weak *homeostasis* during the Malthusian period. On the contrary, strong *homeostasis*, as represented by the highest half-life found in Madsen et al. (2019) (about 30 years), is always rejected at the 5% level.

Turning to the heterogeneity of the speed of convergence by country, Figure 4 displays the point estimates of LSDV estimations of column 3 and 6 adding an interaction term between country fixed effects and the initial level of GDP per capita. Figure 4 reveals mixed results as some countries are found compatible with weak *homeostasis* (eg the Netherlands), and some other countries rather lean towards strong *homeostasis* (eg Poland). Some countries, like France or Spain, are even found to be compatible with both types of *homeostasis*. However, the precision of my estimates is clearly an issue in that specification. As displayed on Figure 4, confidence intervals are generally large, due to a lack of variation in the data for some countries.

In Table 4, I present my results based on OLS, LSDV and GMM estimations of equation (15) using Lagerlöf's (2019) simulated data. That dataset has the great advantage of reproducing the same moments than Maddison Project's data, while possessing a far larger time and cross-sectional dimension. This is useful for estimating with greater precision the spectrum of plausible half-lives in Malthusian economies. Consistent with the weak *homeostasis* displayed when using historical GDP per capita series from the Maddison Project, Table 4 reveals half-lives ranging from about three to one century.

The LSDV estimates imply a half-life of 133 years, with a 95% confidence interval giving half-lives between 141 and 126 years. As expected, the speed of convergence is now estimated

Table 4: Speed of Convergence using simulated GDP per capita Data from Lagerlöf (2019)

	OLS (1)	LSDV (2)	GMM-AB (3)	GMM-BB (4)
log(GDPpc)	-0.0019*** (0.000)	-0.0052*** (0.000)	-0.0063*** (0.002)	-0.0047*** (0.001)
Time FE	Yes	Yes	Yes	Yes
Country FE	No	Yes	Yes	Yes
Observations	10000	10000	9000	10000
adj. R-sq	0.09	0.18	.	.
AR(7)			0.17	0.18
Hansen			0.22	0.23
Diff. Hansen			.	0.21
Instruments			13	15
Half-Life	363	133	110	146
Half-Life 95% C.I.	[403,330]	[141,126]	[212,75]	[250,103]

Notes: This table presents estimates of the speed of convergence using simulated GDP per capita data from Lagerlöf (2019). Columns 3 and 4 display Arellano and Bond (1991) and Blundell and Bond (1998) GMM estimations, using the seventh and further lagged values of GDP per capita as instruments. We use a collapse matrix of instruments and report instrument count. The AR(7) row reports the p-value of a test for no seventh-order correlation in the residuals. Standard errors clustered at the country level are in parentheses. * p<0.1, ** p<0.05, *** p<0.01.

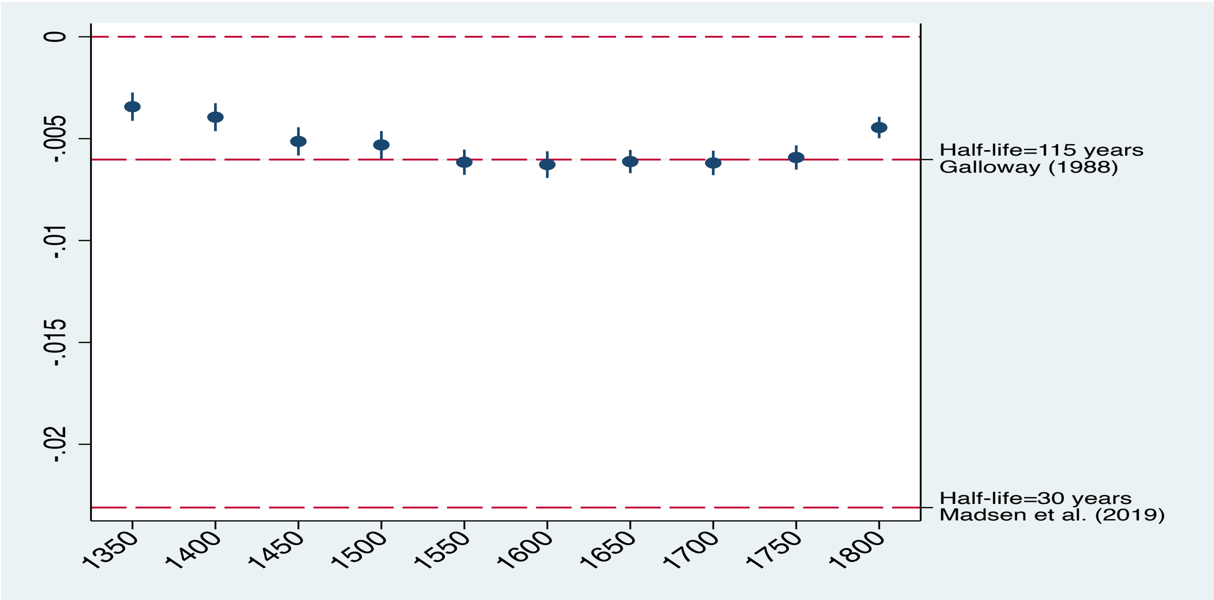
with much more precision, while falling in the wide confidence intervals of our previous results in Table 3. Note that the Hurwicz-Nickell bias is very unlikely to affect my estimates in that case, as this is an ideal setting where both the time and the sample size are very large ($N = 1000$ and $T = 500$). Interestingly, the point estimate is found very close to my previous LSDV estimation in Table 3, columns 3, indicating that the Hurwicz-Nickell bias is indeed not substantial in the Malthusian context. A comparison with Maddison Project's data is fully relevant here, as Lagerlöf's (2019) simulated data come from a Malthusian model which is found to match the moments of the six historical GDP per capita series presented in Fouquet and Broadberry (2015). The original series presented in Fouquet and Broadberry (2015) are still part of the latest Maddison Project database for some countries (eg Holland and Italy) or are updated versions using the same methodology (eg England and Sweden).

Columns 3 and 4 display AB and BB GMM results. As highlighted by Monte Carlo simulations in the modern growth context (Hauk and Wacziarg, 2009; Hauk Jr, 2017), BB is likely to deliver better estimates of the speed of convergence compared to AB in presence of weak instruments; the second best estimator in that context is LSDV. Column 3 reveals AB estimates of the speed of convergence that are out of the plausible bound given by OLS and LSDV, which is recognized as a sign of weak instruments in the litterature. In those conditions, BB is the prefered GMM estimator. Column 4 shows BB estimation results with a half-life of about 146 years, pointing again towards weak *homeostasis*. The 95% confidence interval gives half-lives between 250 and 103 years. In terms of post-estimation tests, I first reject the null hypothesis of seventh-order serial correlation in the residuals (AR(7) test), meaning that using the seventh (and greater) lag of GDP per capita as instruments does not violate the exclusion restriction. Second, I reject both the null hypothesis of the Hansen test and the difference in Hansen test for all GMM instruments, indicating that the moment conditions are globally satisfied.

Figure 5 investigates the time heterogeneity of the speed of convergence. All the coefficients are statistically different from zero and are very precisely estimated, thanks to the large time and sample size in Lagerlöf (2019). The speed of convergence is fairly stable over time. Half of the estimated coefficients cannot reject a half-life of 115 years at the 5% level, as found for Europe using the long-run elasticities of Galloway (1988). Moreover, all the remaining coefficients indicate a slower speed of convergence, which is again a clear sign of weak *homeostasis* of Malthusian economies.

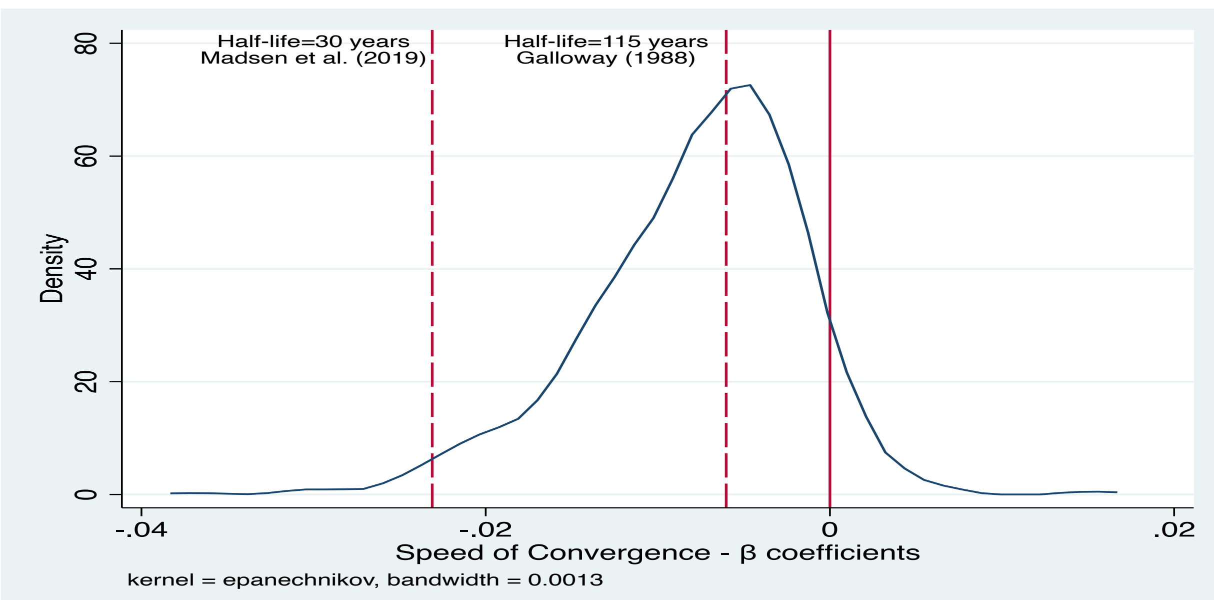
Investigating the cross-sectional heterogeneity of the speed of convergence, Figure 6 displays the kernel density of the estimated speed for the 1000 simulated Malthusian economies in Lagerlöf (2019). Thanks to the large sample size, I can visualize the whole spectrum of the possible speed of convergence during Malthusian times. Consistent with the literature and my results, it appears that the mode of the distribution is very close from a half-life of 115 years, as found for Europe using the long-run elasticities of Galloway (1988). Strong *homeostasis*, represented by a half-life of 30 years or less, is found much less likely as it is close to the lower-tail of the distribution. Figure E-4 in the Appendix delivers the point estimates along with their

Figure 5: Speed of Convergence per period using Lagerlöf's (2019) simulated data



Notes: This figure reports estimates of the speed of convergence by period using simulated GDP per capita data from Lagerlöf (2019). It corresponds to the LSDV estimation in column 2, Table 4, adding year-interacted lagged GDP per capita levels as controls. 95% confidence intervals are reported.

Figure 6: Speed of Convergence per country using Lagerlöf's (2019) simulated data



Notes: This figure reports the kernel density of the estimated speed of convergence by country using simulated GDP per capita data from Lagerlöf (2019). It corresponds to the LSDV estimation in column 2, Table 4, adding country-interacted lagged GDP per capita levels as controls.

95% confidence intervals for the 200 first simulated economies from Lagerlöf (2019).

5.2 Convergence with Population Data

In Table 5, I present my results based on OLS, LSDV and GMM estimations of equation (15) using McEvedy et al.'s (1978) population data. Consistent with the predictions of our theoretical model (see Section C of the Appendix for more details), population converges to its Malthusian steady state at a similar pace than GDP per capita and displays weak *homeostasis*.

Table 5: Speed of Convergence using Total Population Data from McEvedy et al. (1978)

	OLS (1)	LSDV (2)	LSDV (3)	GMM-AB (4)	GMM-BB (5)
log(Population)	-0.000***	-0.005***	-0.006***	-0.009***	-0.004*
	(0.000)	(0.001)	(0.001)	(0.003)	(0.002)
Time FE	Yes	Yes	Yes	Yes	Yes
Country FE	No	Yes	Yes	Yes	Yes
Statehist	No	No	Yes	Yes	Yes
Observations	180	180	180	162	180
adj. R-sq	0.48	0.60	0.61	.	.
AR(2)				0.69	0.36
Hansen				0.94	0.99
Diff. Hansen				.	0.87
Instruments				18	22
Half-Life	4414	147	125	73	167
Half-Life 95% C.I.	[12873,2663]	[224,109]	[231,86]	[215,44]	[-1176,78]

Notes: This table presents estimates of the speed of convergence using total population data from McEvedy et al. (1978). Columns 4 and 5 display Arellano and Bond (1991) and Blundell and Bond (1998) GMM estimations, using the second to fourth lagged values of total population as instruments. We treat $Statehist$ and its squared level as endogenous, using the same set of lags as instruments. We use a collapse matrix of instruments and report instrument count. The AR(2) row reports the p-value of a test for no second-order correlation in the residuals. Standard errors clustered at the country level are in parentheses. * p<0.1, ** p<0.05, *** p<0.01.

Controlling for time and country fixed effects, column 2 reveals a negative and highly significant relationship between population growth and its initial level. The implied half-life is about 147 years, which is in line with my previous results using historical GDP per capita series. The 95% confidence interval indicates half-lives between 224 and 109 years, which stays clearly in the range of weak *homeostasis*.Dealing with omitted variable issues, column 3 adds *Statehist* and its squared level as controls. Convergence tends to be faster, with a half-life of about 125 years. However, I do not find significant differences in the speed of convergence between columns 2 and 3. This last result is close from my previous LSDV estimations using GDP per capita series in Table 3, column 3, and Table 4, column 2, showing again evidence of weak *homeostasis*.

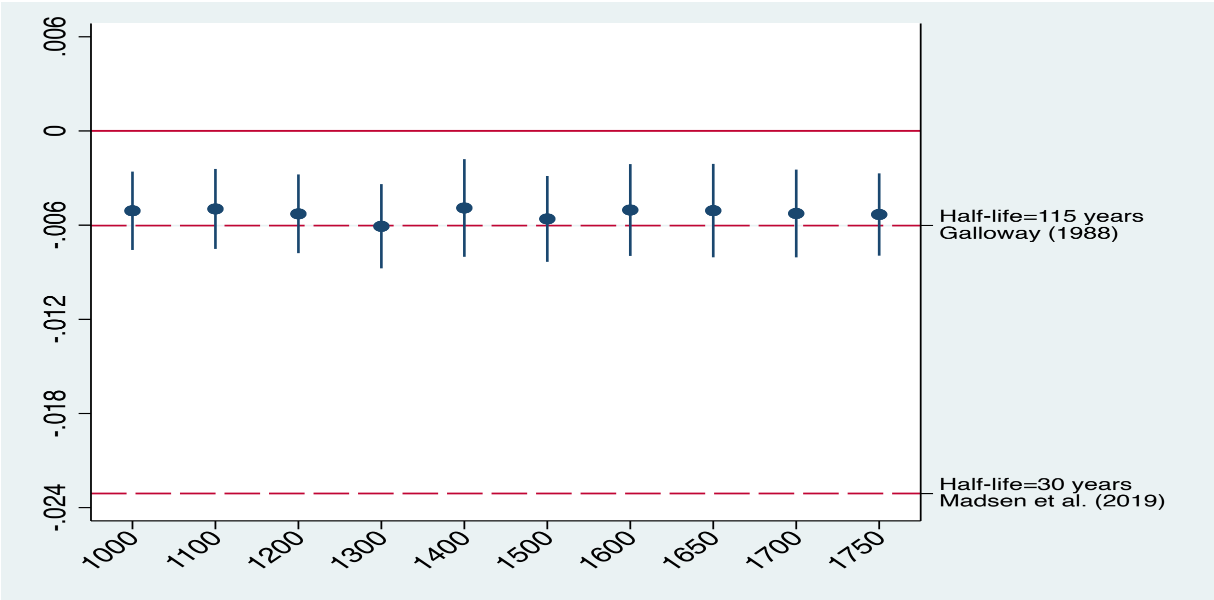
Columns 4 and 5 use GMM estimation procedures. Starting with the AB GMM estimation, column 4 reveals a faster speed of convergence compared to our LSDV results, with a half-life of 73 years. On the contrary, BB GMM estimation in column 5 displays much slower speed of convergence than LSDV, with a half-life of 167 years. Clearly, AB estimation of the speed of convergence falls out of the plausible bounds provided by OLS and LSDV, symptomatic of a weak instruments problem. In those conditions, the BB and LSDV estimations appear to be more reliable, pointing again towards weak *homeostasis*. However, further caution is needed in interpreting the GMM results as Hansen test's p-values are close to unity. This comes from the fact that the cross-sectional dimension is small using McEvedy et al.'s (1978) data compared to the number of instruments.[17] This is the well known "too many instruments problem", highlighted by Roodman (2009).

Figure 7 displays the point estimates of my LSDV estimation in column 3, adding year-interacted initial population levels. All the estimated coefficients are found compatible with a half-life of 115 years, as found for Europe using the long-run elasticities of Galloway (1988). The point estimates are fairly stable in magnitude, and are all statistically different from zero. This indicates, again, a clear pattern of weak *homeostasis* during the whole Malthusian period. Strong *homeostasis*, on the contrary, is fully rejected.

Figure 8 investigates the cross-country heterogeneity of the speed of convergence. As in Figure 7, strong *homeostasis* is clearly rejected since all the estimated coefficients reject a half-life of 30 years or less as in Madsen et al. (2019). In particular, the estimated half-life for England and Wales is 90 years and is not statistically different from my benchmark result of 100 years.

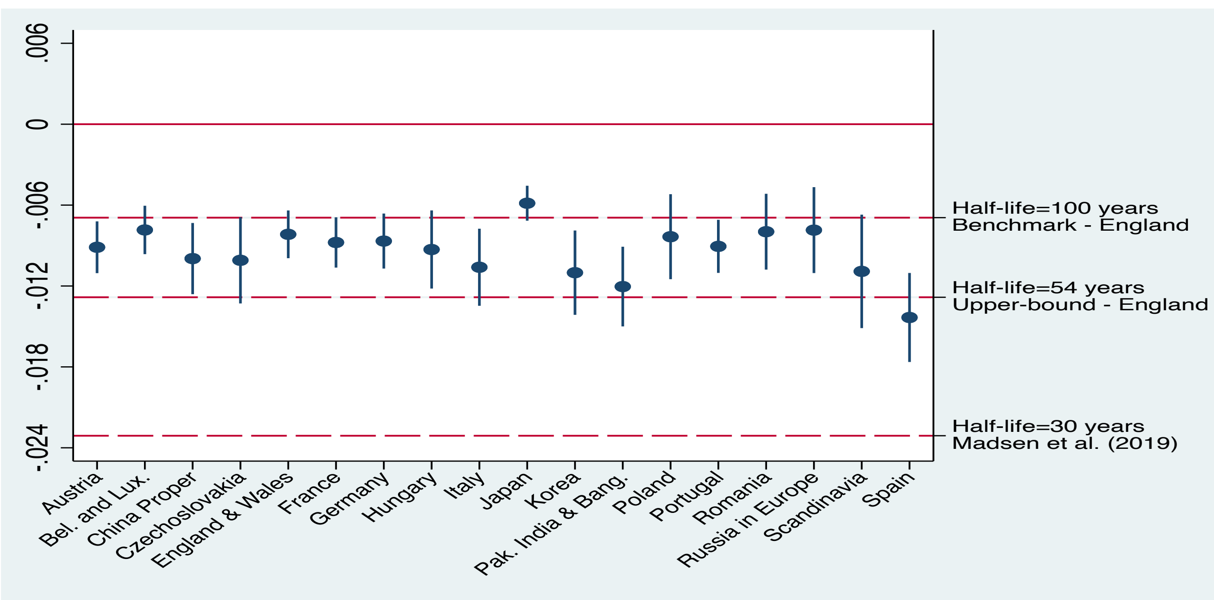
[17]In that case, our sample is composed of 18 countries. For example, in column 4 we instrument all right hand-side variables with their three first lags, amounting to 22 instruments.

Figure 7: Speed of Convergence per period using Population Data from McEvedy et al. (1978)



Notes: This figure reports estimates of the speed of convergence by period using total population data from McEvedy et al. (1978). It corresponds to the LSDV estimation in column 3, Table 5, adding year-interacted lagged total population levels as controls. 95% confidence intervals are reported.

Figure 8: Speed of Convergence per country using Population Data from McEvedy et al. (1978)



*Notes:*This figure reports estimates of the speed of convergence by country using total population data from McEvedy et al. (1978). It corresponds to the LSDV estimation in column 3, Table 5, adding country-interacted lagged total population levels as controls. 95% confidence intervals are reported.

On the contrary, the upper-bound half-life of 54 years for England is clearly rejected, giving further evidence on the strength of the Malthusian trap in England. I am also able to confirm that England is converging faster than the average European economy, as a test of equality with a half-life of 115 years rejects the null hypothesis. Some Malthusian economies, such as Spain or Korea, have significantly higher speed of convergence. However, it is not clear whether this pattern reflects the influence of measurement error or the existence of stronger Malthusian forces. Reassuringly, all those economies are found compatible with the upper-bound half-life of England, that can represent in that case one of the highest speed of convergence that a Malthusian economy can reach. This is still in line with weak *homeostasis* as it means that two generations are needed to absorb half of a shock.

In Table 6, I present my results based on OLS, LSDV and GMM estimations of equation (15) using urban population data from Reba et al. (2016). My results confirm the weak *homeostasis* pattern found using GDP per capita or total population historical series, with half-lives of about one century.

Starting with city-level population data, column 2 reveals a negative and highly significant relationship between urban population growth and the initial level of urban population, conditional on time and city fixed effects. The corresponding half-life is about 95 years, with a 95% confidence interval indicating half-lives between 155 and 65 years.

Columns 3 and 4 display AB and BB GMM estimations. The AB estimation in column 3 shows a speed of convergence falling within the OLS-LSDV bound, with a half-life of 115 years. This is further confirmed by the BB GMM estimation in column 4, which delivers a very similar speed of convergence compared to AB, with a half-life of 119 years. However, I fail to pass the AR(3) test in both specifications, suggesting a violation of the exclusion restriction for the set of considered internal instruments.

Turning to country-level estimations, column 6 reveals a negative and highly significant relationship between urban population growth and its initial level, conditional on time and

Table 6: Speed of Convergence using Urban Population Data from Reba et al. (2016)

Observational Unit:	City				Country				
	OLS (1)	LSDV (2)	GMM-AB (3)	GMM-BB (4)	OLS (5)	LSDV (6)	LSDV (7)	GMM-AB (8)	GMM-BB (9)
log(Population)	-0.003***	-0.007***	-0.006***	-0.006***	-0.004***	-0.007***	-0.008***	-0.007**	-0.005*
	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.003)	(0.003)
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
City FE	No	Yes	Yes	Yes	No	No	No	No	No
Country FE	No	No	No	No	No	Yes	Yes	Yes	Yes
Statehist	No	No	No	No	No	No	Yes	Yes	Yes
Observations	1706	1706	1239	1706	509	509	509	411	509
adj. R-sq	0.08	0.22	.	.	0.16	0.24	0.26	.	.
AR(3)			0.06	0.07				0.11	0.12
Hansen			0.12	0.12				0.56	0.91
Diff. Hansen			.	0.02				.	0.99
Instruments			29	31				21	25
Half-Life	258	95	115	119	192	94	84	97	133
Half-Life 95% C.I.	[500,174]	[155,68]	[253,74]	[325,73]	[270,149]	[133,73]	[118,64]	[2707,49]	[-698,61]

Notes: This table presents estimates of the speed of convergence using urban population data from Reba et al. (2016). Columns 1-4 present results using city-level data and columns 5-9 show results using urban population aggregated at the country-level. Columns 3 and 4 and columns 8 and 9 display Arellano and Bond (1991) and Blundell and Bond (1998) GMM estimations. In the case of city-level estimates using GMM, we use the third and further lagged values of urban population as instruments. In the case of country-level GMM estimates, we use the third and fourth lagged values of urban population as instruments. When included, we treat *Statehist* and its squared level as endogenous, using the same set of lags as instruments. We use a collapse matrix of instruments and report instrument count. The AR(3) row reports the *p*-value of a test for non third-order correlation in the residuals. Standard errors clustered at the city level in columns 1-4 and at the country level in columns 5-9 are in parentheses. * $p<0.1$, ** $p<0.05$, *** $p<0.01$.

country fixed effects. The half-life is almost identical to the previous LSDV estimation using city-level data in column 2. Column 7 adds *Statehist* and its squared level as control variables in order to reduce the omitted variable bias. The estimated speed of convergence is now faster with a half-life of 84 years.

In columns 8 and 9, I perform GMM estimations at the country level. I find half-lives of 97 and 133 years respectively. The AB GMM estimation falls within the OLS-LSDV bounds. Contrary to the previous GMM estimations in columns 3 and 4, I am now passing the AR(3) test in both cases. Moreover, the Hansen test and the difference in Hansen test reject their null hypothesis, sign that the moment conditions are globally satisfied.[18] This is a clear indication that the instruments and the moment conditions used are valid.

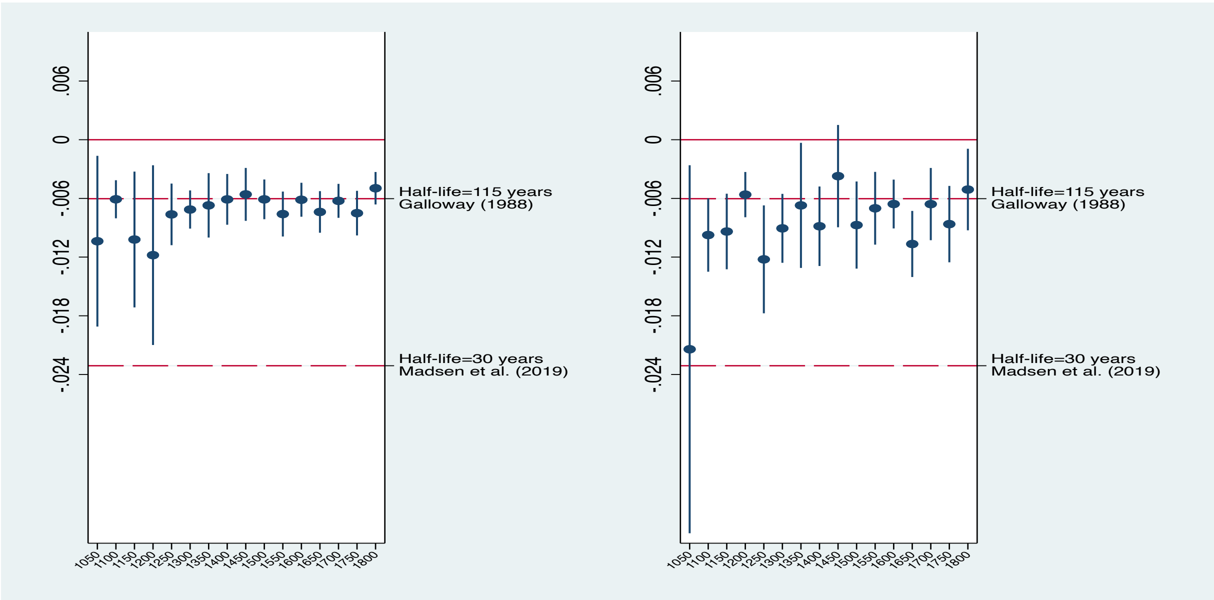
Overall, my results using historical urban population data clearly confirm the weak *homeostasis* pattern found in the previous sections. Most of the half-lives are close to one century, and the smallest half-life found is about 84 years.

Figure 9 explores the time heterogeneity of the speed of convergence, both for my city-level and country-level estimations. In both cases, weak *homeostasis* is confirmed. This is particularly striking for the city-level data where all the point estimates starting from 1250 CE onward cannot reject a half-life of 115 years at the 5% level. On the other hand, strong *homeostasis* is always rejected at the level of 5%, except for the first period of the country-level data.

Figure 10 plots the kernel density of the estimated speed of convergence for a sample of 185 cities. A half-life of 115 years, as found for Europe using the long-run elasticities of Galloway (1988), is very close to the mode of the distribution. Moreover, the distribution is also more concentrated around that given half-life than my previous estimates with Lagerlöf's (2019) simulated data (Figure 6), giving more precise evidence in favor of weak *homeostasis*. On the contrary, strong *homeostasis* is much less likely. Figure E-5 of the Appendix delivers the point estimates along with their 95% confidence intervals for the 185 cities in our sample.

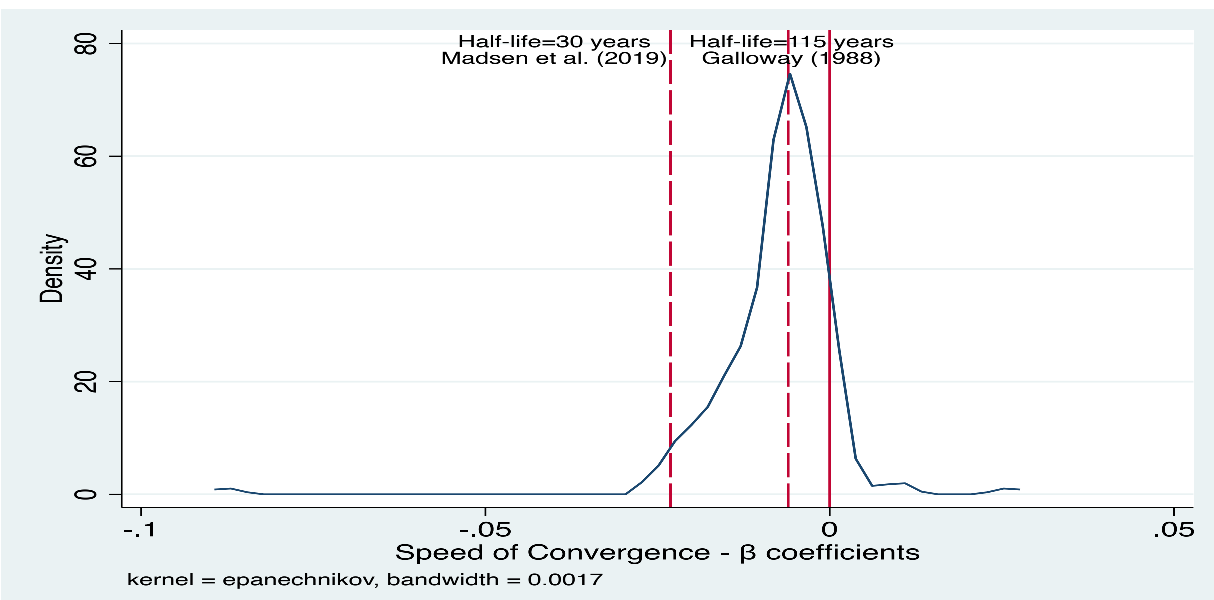
[18] Despite close to unity Hansen test's p-values, the total number of instruments is well below the number of observational units, with 25 instruments for 47 countries in column 9.

Figure 9: Speed of Convergence per period using Data from Reba et al. (2016)



Notes: This figure reports estimates of the speed of convergence by period using urban population data from Reba et al. (2016). It corresponds to the LSDV estimations in column 2, Table 6 (left panel) and in column 7, Table 6 (right panel), adding year-interacted lagged urban population levels as controls. 95% confidence intervals reported.

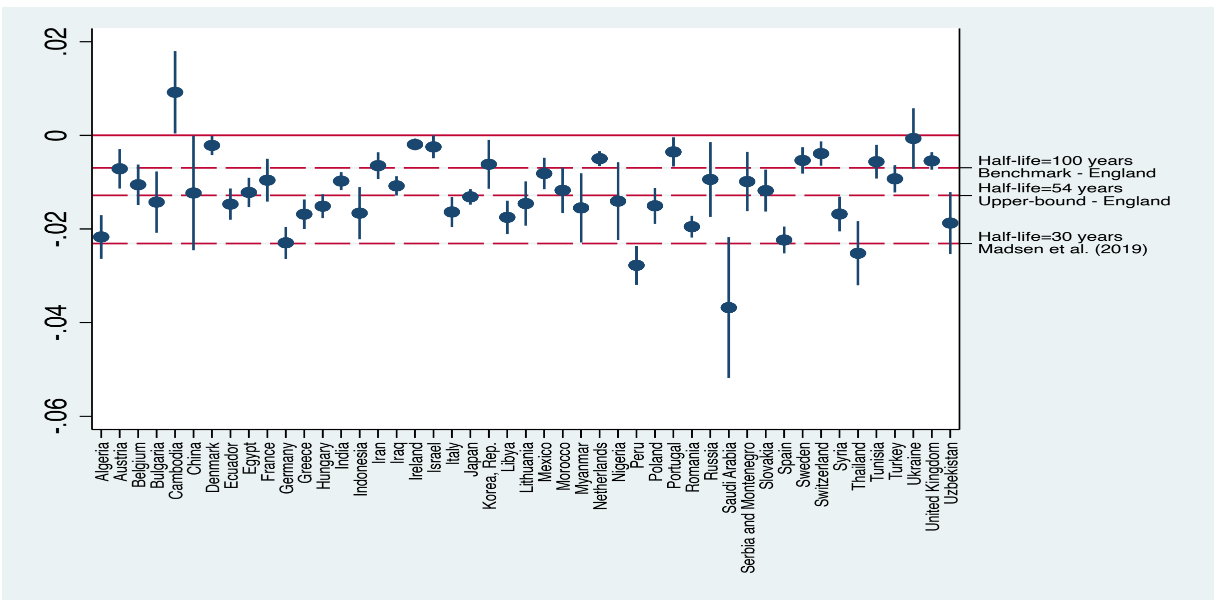
Figure 10: Speed of Convergence per city using Data from Reba et al. (2016)



Notes: This figure reports the kernel density of the estimated speed of convergence by city using urban population data from Reba et al. (2016). It corresponds to the LSDV estimation in column 2, Table 6, adding city-interacted lagged urban population levels as controls.

Figure 11 shows the cross-country heterogeneity of the speed of convergence using urban population data. Along with the city-level results, Figure 11 shows that very few countries fail to reject strong *homeostasis* at the 5% level. It is important to note that countries known to have good quality urban population data, such as the Netherlands or the United Kingdom, are estimated with great precision. In particular, the estimated speed of convergence for the United Kingdom is not statistically different from the half-life of one century found with our benchmark parametrization for England.

Figure 11: Speed of Convergence per country using Data from Reba et al. (2016)



Notes: This figure reports estimates of the speed of convergence by country urban population data from Reba et al. (2016). It corresponds to the LSDV estimations in column 7, Table 6, adding country-interacted lagged urban population levels as controls. 95% confidence intervals are reported.

6 Conclusion

How long can living standards deviate from their long-run equilibrium after a shock in a Malthusian world? This article proposes to look at the speed of convergence of Malthusian economies to answer this question. I proceed in two steps. First, I build an overlapping-generations

Malthusian growth model, putting the emphasize on the channels through which population adjusts to its standards of living. In particular, I follow the idea of Malthus (1798), including both preventive and positive checks as means of population adjustment. Agents first choose to marry (or not), influencing the extensive margin of fertility, and then choose the number of children within marriage, influencing the intensive margin of fertility. Both choices depend on income per capita, in a Malthusian fashion. I then derive the speed of convergence implied by my model, showing that it depends only on the land share of output and the elasticities of fertility, marriage and survival with respect to income per capita. I also perform a calibration exercise, showing that under plausible parameter values the speed of convergence indicates weak *homeostasis* for England, with a half-life of about a century. Second, I systematically confront the model predictions with the data using β-convergence regressions *à la* Barro and Sala-i Martin (1992). I first focus on historical and simulated GDP per capita data series, showing that they exhibit weak *homeostasis* in the same magnitudes as revealed by our model. I then use historical total and urban population series to estimate the speed of convergence, finding similar results regarding the pattern of weak *homeostasis* and its magnitude. I address the endogeneity issues using an internal instrument approach (GMM) and controlling for the State History Index of Borcan et al. (2018). Measurement error issues are dealt using several strategies, including the exclusion of the most uncertain part of the data, time averages and time-interacted regressors.

Overall, my results highlight the validity of the Malthusian theory and the Malthusian trap mechanism to explain the thousands of years of stagnation that humanity faced before the Industrial Revolution. In this perspective, weak *homeostasis* help us in understanding the multiple episodes of non-sustained growth episodes – but lasting for decades – in pre-industrial times, referred as “golden ages” or “efflorescences” in the literature (Goldstone, 2002). Similarly, different patterns of *homeostasis* can help explain why common economic shocks can lead to a long-lasting divergence in the standards of living between two Malthusian economies. In this perspective, my work already highlights significant differences between the speed of convergence of certain Malthusian economies. The study of the determinants, cultural or institutional, and the consequences, in the context of the Little or Great Divergence, of these differences in the degree of *homeostasis* is a fruitful area for future research.

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Appendix

A Proof

Proposition 1 (Unique and Globally Stable Interior Steady State). *If income per capita evolves according to $\psi(y_t)$, then the economy converges to a unique and globally stable interior steady state, providing that the initial income per capita y_0 is not too low.*

Recall that the time path of GDP per capita is governed by the following first-order difference equation:

$$\psi(y_t) = \left(\frac{1+g}{n_t(y_t)\ p_t(y_t)\ s_t(y_t)}\right)^{\alpha} \cdot y_t$$
$$\psi(y_t) = \left(\frac{1+g}{\left(\kappa^{-\delta}\ \left(y_t + q\ \nu^{1/\delta}\right)^{\delta} - \nu\right) \cdot \frac{(\overline{\lambda}(y_t)-1)}{(b-1)} \cdot \overline{s}\ y_t^{\phi}}\right)^{\alpha} \cdot y_t\ , \tag{1}$$

with $\overline{\lambda}(y_t) = \exp\left(\ln(1-\frac{q}{\kappa}) + (1+\delta\gamma)\ln(y_t + q\ \nu^{1/\delta}) - \delta\gamma\ln(\kappa) - \ln(y_t) - \gamma\ln(\nu)\right)$.

First, it is clear that my model does not possess a corner steady-state as:

$$n_t(\underline{y}) = 0 \Leftrightarrow \underline{y} = \frac{q}{\gamma\delta} \cdot \nu^{1/\delta} > 0\ , \tag{2}$$

since q, δ, γ and ν are strictly positive.

Turning to the first and second derivative of $\psi(y_t)$, we have:

$$\psi'(y_t) = \left(\frac{(1+g)}{n_t(y_t)\ p_t(y_t)\ s_t(y_t)}\right)^{\alpha}\ \left(1 - \alpha\left(\epsilon_{n_t} + \epsilon_{p_t} + \epsilon_{s_t}\right)\right)\ , \tag{3}$$

$$\psi''(y_t) = \left(\frac{(1+g)}{n_t(y_t)\ p_t(y_t)\ s_t(y_t)}\right)^{\alpha} \cdot \left[-\alpha\left(\frac{n'}{n} + \frac{p'}{p} + \frac{s'}{s}\right) \cdot (1 - \alpha(\epsilon_{n_t} + \epsilon_{p_t} + \epsilon_{s_t})) - \alpha(\epsilon'_{n_t} + \epsilon'_{p_t} + \epsilon'_{s_t})\right]\ , \tag{4}$$

with $\epsilon_{n_t} = n'_t(y_t)/n_t \cdot y_t$, $\epsilon_{p_t} = p'_t(y_t)/p_t \cdot y_t$ and $\epsilon_{s_t} = s'_t(y_t)/s_t \cdot y_t$ elasticities of the income per capita to fertility, marriage and survival respectively.

The sufficient condition for a unique and globally stable steady state is the concavity of $\psi(y_t)$ – i.e. $\psi'(y_t) > 0$ and $\psi''(y_t) < 0$ – for a sequence of $y_t \in [\underline{y}, +\infty[$. Turning to the first derivative in (18), we directly see that $\psi'(y_t) > 0 \Leftrightarrow \alpha\left(\epsilon_{n_t} + \epsilon_{p_t} + \epsilon_{s_t}\right) < 1$ since $g > 0$ and $n_t(y_t)$, $p_t(y_t)$ and $s_t(y_t)$ are strictly monotonically increasing and positively valued concave functions for all $y_t > \underline{y}$. Elasticities of $n_t(y_t)$, $p_t(y_t)$ and $s_t(y_t)$ with respect to y_t are therefore monotonically decreasing convex functions with $\lim_{y_t \to \underline{y}} \epsilon = +\infty$ and $\lim_{y_t \to +\infty} \epsilon = 0^+$. It means that we have a unique turning point of $\psi(y_t)$ on the sequence $[\underline{y}, ..., +\infty[$ and that the condition $\psi'(y_t) > 0$ is satisfy as y_t goes to infinity.

Turning to the second derivative in (19), the condition for the concavity of $\psi(y_t)$ is that $\psi''(y_t) < 0 \Leftrightarrow \alpha \left(\frac{n'}{n} + \frac{p'}{p} + \frac{s'}{s}\right)(1 - \alpha(\epsilon_{n_t} + \epsilon_{p_t} + \epsilon_{s_t})) > \alpha(\epsilon'_{n_t} + \epsilon'_{p_t} + \epsilon'_{s_t})$. This also holds for sufficiently high values of y_t. In my benchmark specification, it turns that $\psi''(y_t) < 0$ for initial conditions $y_0 > 0.58 \cdot 10^{-3}$. Therefore, for any initial condition $y_0 > 0.58 \cdot 10^{-3}$ the economy converges to a unique, positive and globally stable steady state, and the level of GDP per capita at the steady state is given by (13). In the benchmark parametrization, I set the initial condition y_0 at 20,108 following Campbell et al. (2015), ensuring that I am analysing the behaviour of a representative Malthusian economy on the strictly concave part of $\psi(y_t)$.

B Elasticities

In my model, the extent to which Malthusian agents adjust their fertility, marriage behaviour and endure mortality is a key element in determining the speed of convergence to the steady state (see Section 3.3). These adjustments are governed by elasticity parameters that I group, according to the Malthusian theory, in two categories: (i) the preventive checks and (ii) the positive checks. In both cases, I calibrate elasticity parameters to match the mean of the long-run elasticities provided by the literature for the preventive and the positive checks for England (Lee, 1981; Lee and Anderson, 2002; Klemp, 2012; Møller and Sharp, 2014). See Appendix B of Klemp and Møller (2016) for a summary of the long-run elasticity values I use for England.

The preventive checks consider how marriage and fertility within marriage adjust to variations in the standards of living. In my model, I have a single parameter δ influencing the reaction of marriage and fertility to income per capita. I set $\delta = 0.09$ in my benchmark specification, such that the sum of the long-run elasticities of fertility and the marriage rate with respect to income per capita equals 0.22.

The positive checks consider how survival (or mortality) adjusts when income per capita varies. The parameter governing the elasticity of the survival rate with respect to income per capita in my model is ϕ. I set $\phi = 0.13$ in my benchmark simulation to match a long-run elasticity value of 0.13 for the positive check.

Table B-1 provides the estimated long-run elasticities of fertility, marriage and survival with respect to income per capita coming from my benchmark and three alternative parametrizations. I use it to systematically check whether the parameters δ and ϕ matches their targets.

Table B-1: Estimated Long-Run Elasticities from our Simulations

Dependent Variable:	log($Fertility_t$)				log($Survival_t$)				log($Marriage_t$)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
log(y_t) – Benchmark	0.1080***				0.1300***				0.1100***			
	(0.000)				(0.000)				(0.000)			
log(y_t) – with $\delta = 0.11$, $\phi = 0.22$ and $\alpha = 0.5$		0.1553***				0.2200***				0.1627***		
		(0.000)				(0.000)				(0.000)		
log(y_t) – with $\delta = 0.09$, $\phi = 0.13$ and $\alpha = 0.5$			0.1079***				0.1300***				0.1099***	
			(0.000)				(0.000)				(0.000)	
log(y_t) – with $\delta = 0.11$, $\phi = 0.22$ and $\alpha = 0.6$				0.1551***				0.2200***				0.1624***
				(0.000)				(0.000)				(0.000)
Observations	21	21	21	21	21	21	21	21	21	21	21	21

Notes: This table presents estimates of the long-run elasticities of fertility, survival rate and marriage rate with respect to income per capita based on our different simulations. Standard errors robust against heteroskedasticity are in parentheses. * $p<0.1$, ** $p<0.05$, *** $p<0.01$.

C Derivation of the Speed of Convergence

Taking a first-order Taylor expansion of $\psi(y_t)$ around y^*, we have:

$$\begin{aligned} \psi(y_t) &\approx \psi(y^*) + \psi'(y^*) \cdot (y_t - y^*) \\ y_{t+1} &\approx y_t - \alpha(\epsilon_{n_t} + \epsilon_{p_t} + \epsilon_{s_t}) \cdot (y_t - y^*) \ . \end{aligned} \tag{5}$$

It follows that GDP per capita growth rate at the neighbourhood of the steady state is:

$$g^y \equiv \frac{y_{t+1} - y_t}{y_t} \approx -\beta^* \cdot (\ln y_t - \ln y^*) \ , \tag{6}$$

with $\beta^* = \alpha(\epsilon_{n_t} + \epsilon_{p_t} + \epsilon_{s_t})$ the speed of convergence to the steady-state.

In my model, population is not constant at the steady state but rather growth at the same pace as technology. To analyse the speed of convergence to the population steady state, I first need to express labour L_t in terms of effective units:

$$\widehat{L}_t \equiv \frac{L_t}{A_t} \tag{7}$$

Recall equation (8), we can express effective units of labour as:

$$\widehat{L}_t = y_t^{-1/\alpha} \ . \tag{8}$$

Taking the logarithm of (28) and highlighting growth rates, we have:

$$g^{\widehat{L}} = \frac{\partial \ln \widehat{L}_t}{\partial t} = -\frac{1}{\alpha} \frac{\partial \ln y_t}{\partial t} = -\frac{1}{\alpha} g^y \ . \tag{9}$$

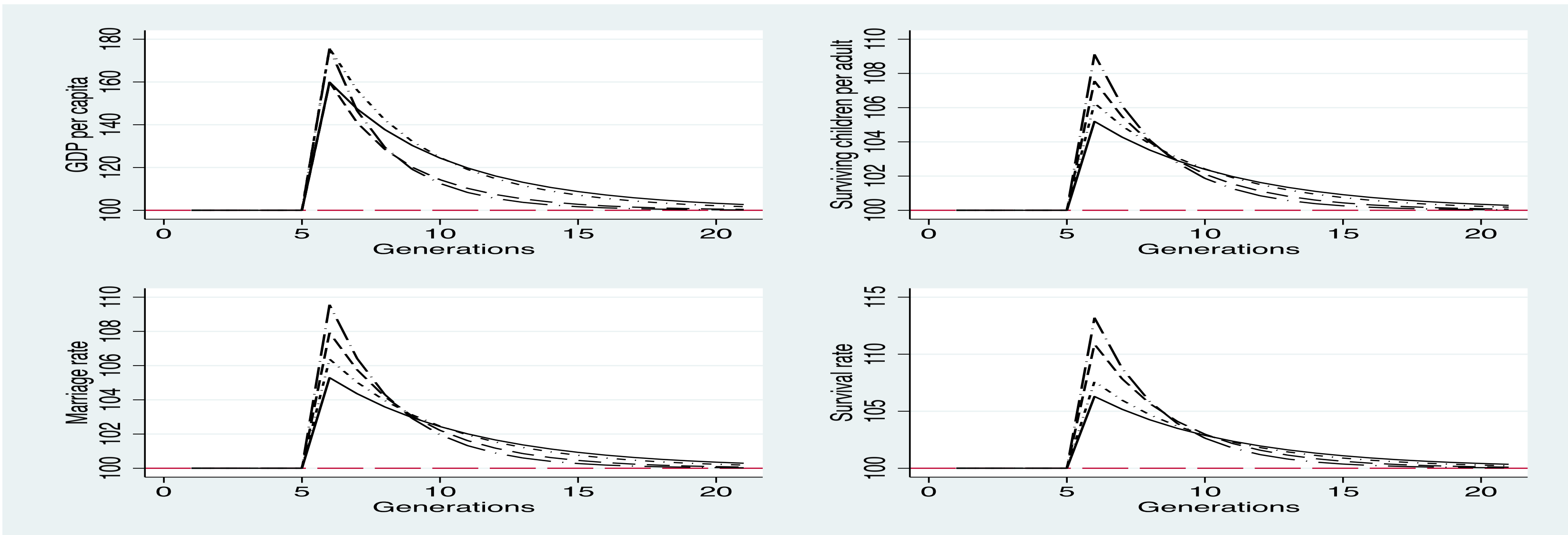
Using (29) and (26), we have:

$$\begin{aligned} g^{\widehat{L}} &\approx -\beta^* \ -\frac{1}{\alpha} \cdot (\ln y_t - \ln y^*) \\ g^{\widehat{L}} &\approx -\beta^* \cdot (\ln \widehat{L}_t - \ln \widehat{L}^*) \ . \end{aligned} \tag{10}$$

It means that in a Malthusian economy, effective unit of labour converges to its steady-state at the same pace than GDP per capita. Consequently, once technological progress and the size of land is hold constant, population data can be used to estimate the speed of convergence of a Malthusian economy.

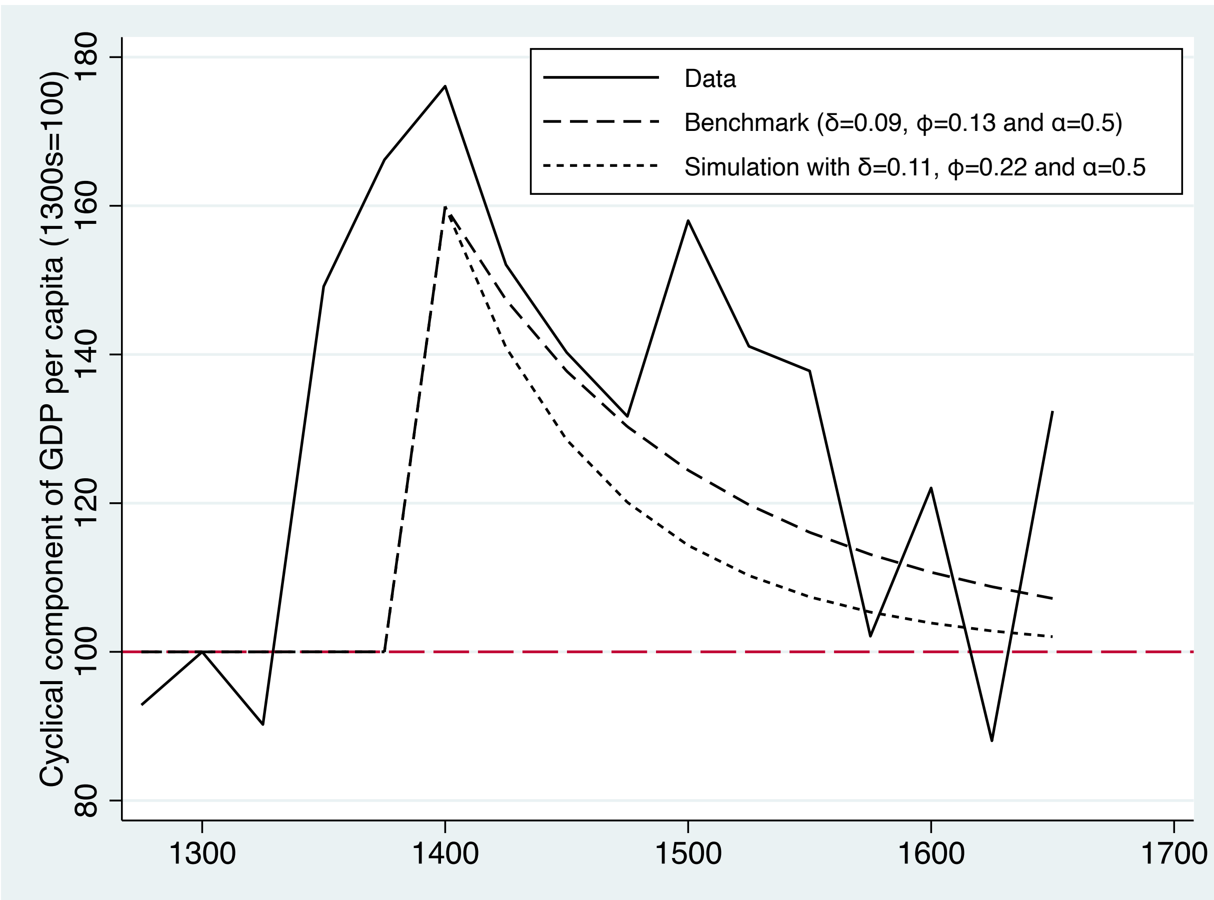
D Comparaison between the Four Parametrizations

Figure D-1: Responses of the English Malthusian Economy to a Black Death – the Four Parametrizations



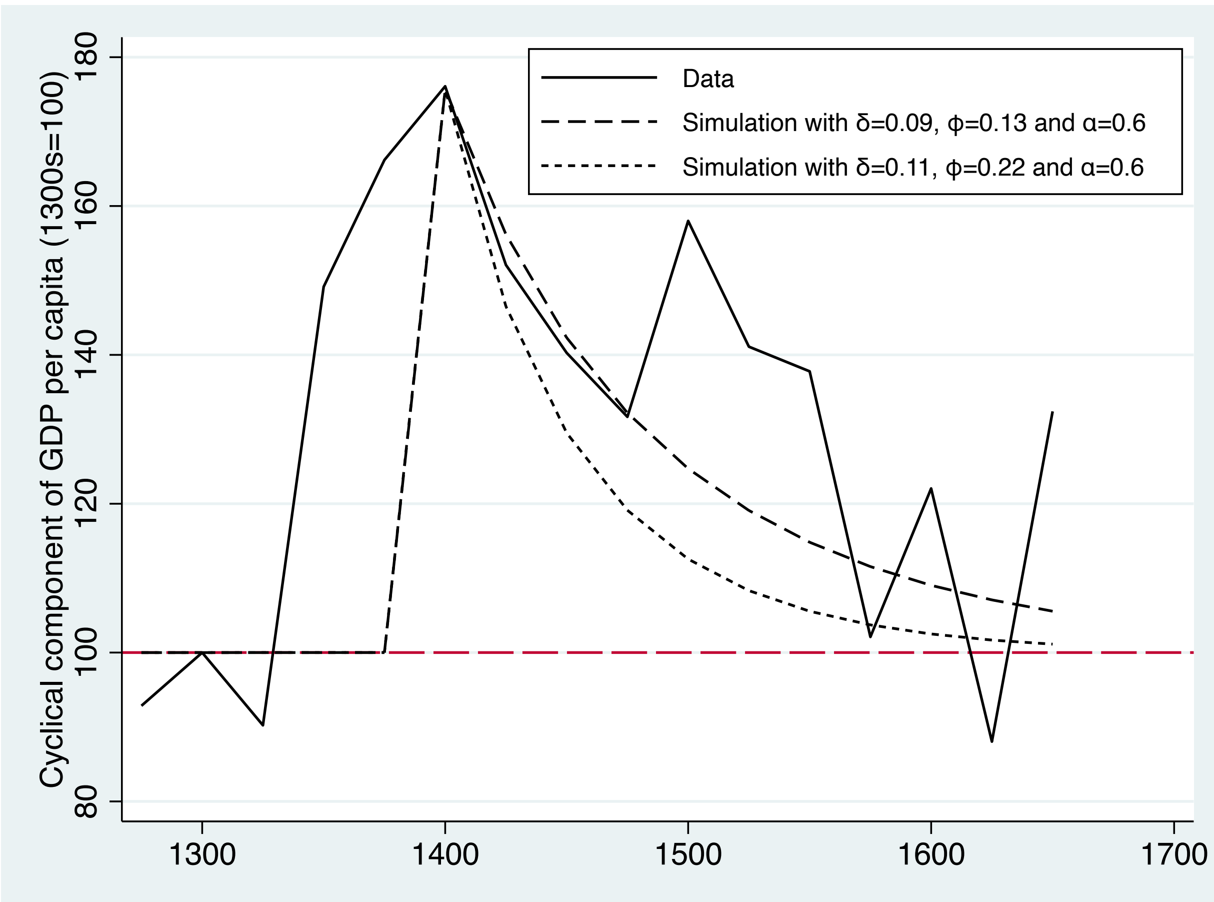
Notes: This figure plots the response of income per capita (top-left panel), fertility (top-right panel), marriage (bottom-left panel) and survival (bottom-right panel) to a Black Death alike shock, killing 60% of the population at $t = 5$. It displays our benchmark simulation (solid line), our simulation with upper-bound elasticity values (dashed line), our simulation with the upper-bound land share of output (short dashed and dotted line) and our simulation with both upper-bound elasticities and land share of output (long dashed and dotted line).

Figure D-2: GDP per capita Dynamic after the Black Death – alternative calibration 1



Notes: This figure plots the cyclical component of GDP per capita after the Black Death from Campbell et al. (2015) (solid line), our benchmark simulation (dashed line) and our simulation using upper-bound elasticities (short dashed line). The cyclical component is obtained using an Hodrick-Prescott filter with a smoothing parameter of 500. We normalize data on the period 1300-1325, last period before the occurrence of the Black Death in England (1348).

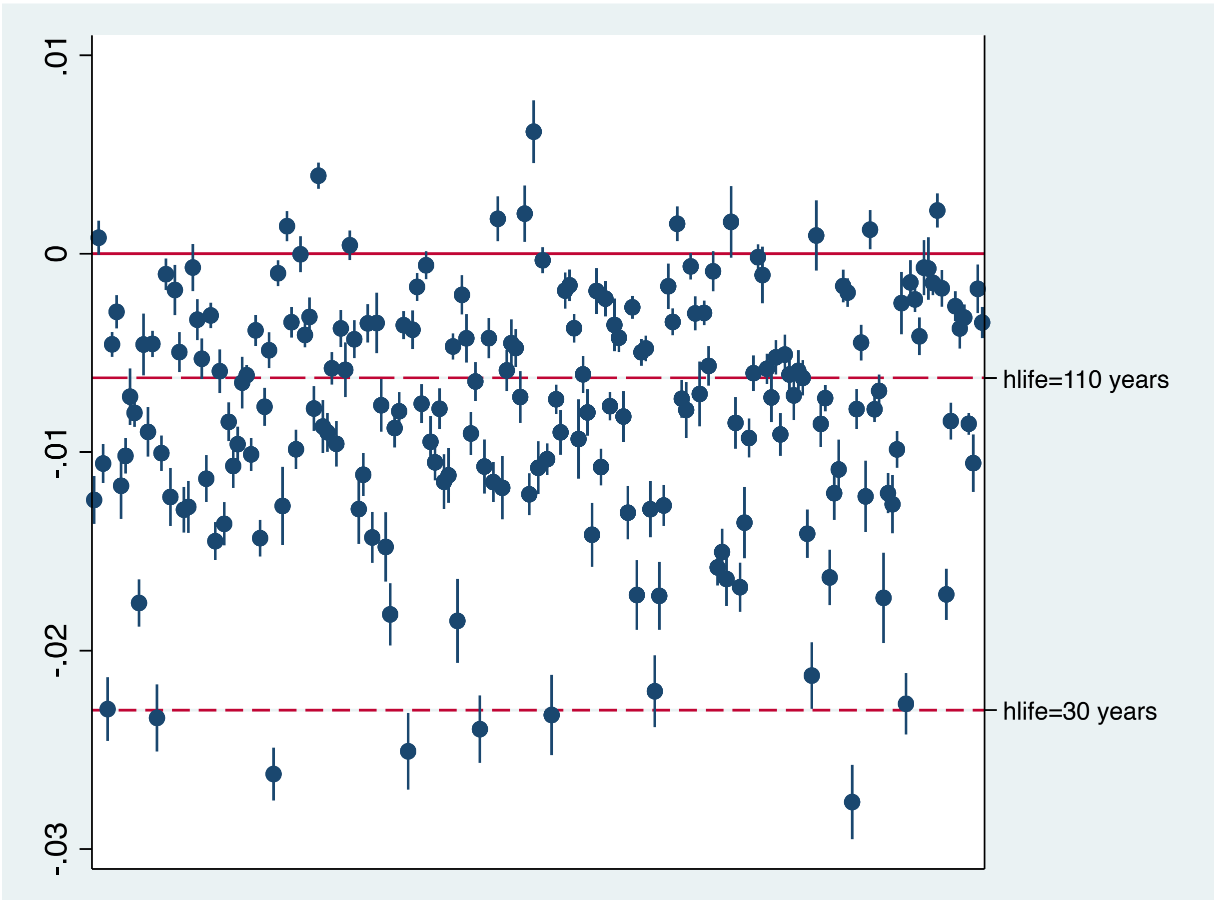
Figure D-3: GDP per capita Dynamic after the Black Death – alternative calibration 2



Notes: This figure plots the cyclical component of GDP per capita after the Black Death from Campbell et al. (2015) (solid line), our simulation using the upper-bound land share of output (dashed line) and our simulation using both upper-bound elasticities and land share of output (short dashed line). The cyclical component is obtained using an Hodrick-Prescott filter with a smoothing parameter of 500. We normalize data on the period 1300-1325, last period before the occurrence of the Black Death in England (1348).

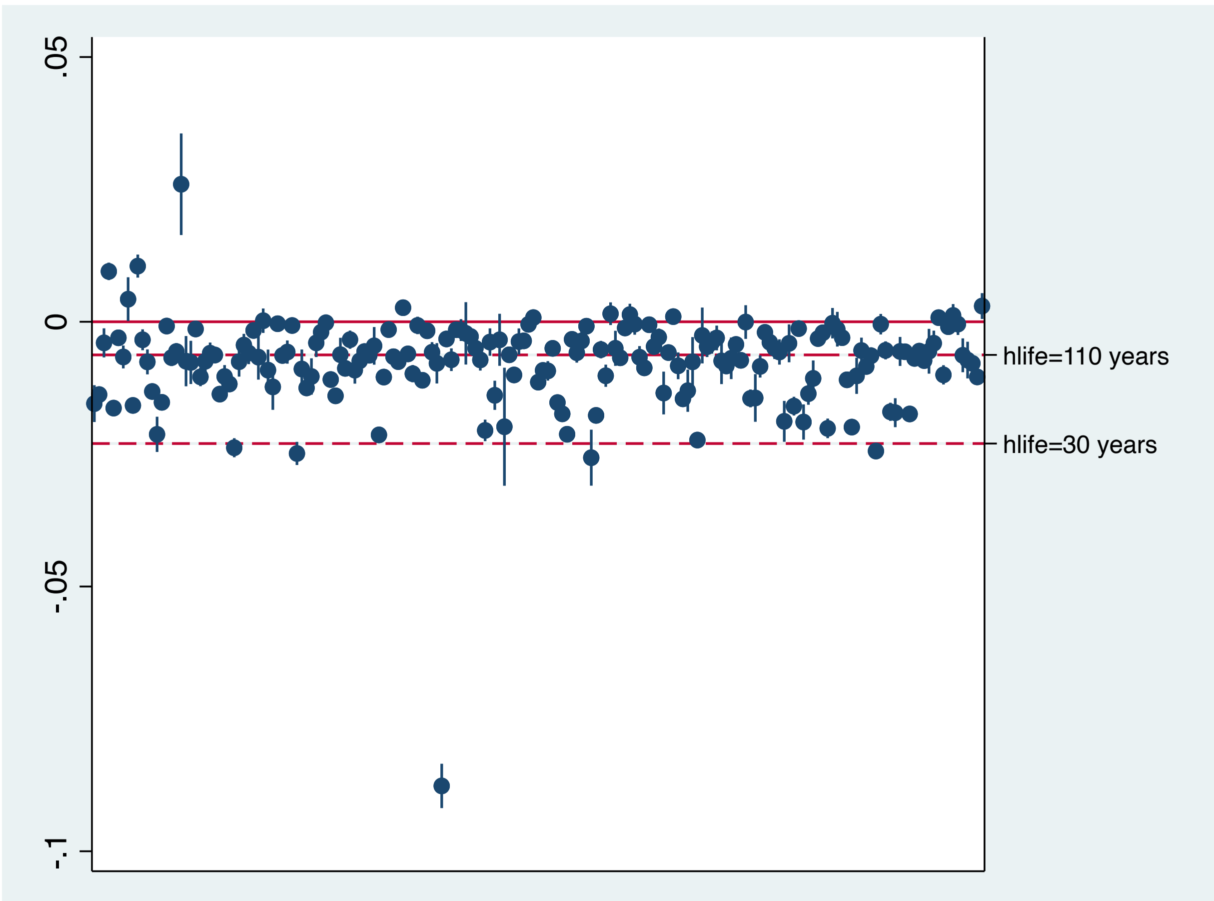
E Additional Results

Figure E-4: Speed of Convergence for the 200 first Malthusian Economies in Lagerlöf (2019)



Notes: This figure plots the estimated speed of convergence for the 200 first simulated economies from Lagerlöf (2019). It corresponds to the LSDV estimation in Table 4, column 2, adding country-interacted lagged GDP per capita as controls. 95% confidence intervals are reported.

Figure E-5: Speed of Convergence per city using Data from Reba et al. (2016)



Notes: This figure plots the estimated speed of convergence for the 185 cities in our sample using data from Reba et al. (2016). It corresponds to the LSDV estimation in Table 6, column 2, adding city-interacted lagged population levels as controls. 95% confidence intervals are reported.

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