OPTIMAL MONETARY POLICY WITH AND WITHOUT DEBT

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Optimal Monetary Policy with and without Debt*

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Abstract

We derive optimal monetary policy rules when government debt may be a constraint for the monetary authority. We focus on an environment where fiscal policy is exogenous, setting taxes according to a rule that specifies the tax rate as a function of lagged debt. In the case where taxes do not adjust sufficiently to ensure the solvency of debt, then the monetary authority is burdened by debt sustainability. Under this scenario, optimal monetary policy is a 'passive money rule', setting the interest rate to weakly respond to inflation. We characterize analytically the optimal inflation coefficients under alternative specifications of the central bank loss function, using a simple Fisherian model, but also the canonical New Keynesian model. We show that the maturity structure of debt is a key variable behind optimal policy. When debt maturity is calibrated to US data, our model predicts that a simple inflation targeting rule where the inflation coefficient is $1 - \frac{1}{\text{Maturity}}$ is a good approximation of the optimal policy.

Lastly, our framework can also nest the case where fiscal policy adjusts taxes to satisfy the intertemporal debt constraint. In this scenario optimal monetary policy is an active policy rule. We contrast the properties of active and passive policies, using the analytical optimal policy rules derived from this framework of monetary/fiscal interactions.

Keywords: Fiscal/monetary policy interactions, Fiscal theory of the price level, Ramsey policy.

JEL classification: E31, E52, E58, E62, C11.

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1 Introduction

Since the 2008-9 recession and more recently due to the spending and transfer programs launched to deal with the effects of the COVID 19 recession, governments in developed economies accumulated large stocks of debt. High debt levels require bold fiscal adjustments to ensure the solvency of government budgets. However, in many cases it is questionable whether fiscal authorities will be able/willing to generate the large required surpluses to finance debt.[1]

At high debt levels, and when fiscal authorities may not be able/willing to adjust taxes sufficiently, debt may become an important constraint for monetary policy. Under such circumstances, ensuring debt solvency becomes a task burdening the monetary authority and inflation needs to be used to finance debt. How should policy be then designed to be optimal?

A sizable literature has studied optimal policy in the 3-equation New Keynesian model (see e.g. Woodford (2001b); Giannoni and Woodford (2003); Orphanides and Williams (2007); Svensson (1999, 2003); Woodford (2003); Giannoni (2014) among numerous others), that is in the case where monetary policy is not burdened with debt substainability. These papers have used the baseline New Keynesian framework to develop practically relevant recommendations for the design of interest rate rules, the desirable response of the main instrument of monetary policy to macroeconomic conditions.

Another strand of literature has analyzed optimal policy in environments where inflation responds to debt. Using the so called *Ramsey approach* to optimal policy, Chari and Kehoe (1999); Siu (2004); Schmitt-Grohé and Uribe (2004); Faraglia, Marcet, Oikonomou, and Scott (2013); Lustig, Sleet, and Yeltekin (2008); Leeper and Zhou (2021) among others, have solved policy problems in which a Ramsey planner can simultaneously set inflation and fiscal variables (taxes) to satisfy the intertemporal solvency of debt.

From this second class of models, however, it is not easy to derive transparent conclusions concerning the conduct of monetary policy. Firstly, because bringing together monetary and fiscal policies under one authority makes it difficult to disentangle which of the implications of the models concern monetary policy and which do not.[2] Secondly, and most importantly, because in the optimal policy equilibrium in these models macroeconomic variables become functions of current and lagged values of the Lagrange multiplier attached to the consolidated budget constraint, the object that defines the dynamics of debt. The multipliers are state variables and it is not easy to derive an interest rate rule that expresses the nominal rate solely as a function of macroeconomic conditions, without involving Lagrange multipliers.

This is evidently an important limitation of the analysis. Lagrange multipliers cannot be observed in practice, and therefore solving the Ramsey first order conditions (expressing the path of the nominal rate as a function of the multipliers) cannot provide any meaningful guidance to monetary policy.

This paper makes progress with characterizing optimal interest rate rules in an environment where

[1]Even prior to the COVID crisis, the US had a debt to GDP ratio exceeding 100 percent and was projected to rise. At the same time, on the fiscal side, there was no announced increase in tax rates to stabilize debt. In light of this, several papers have built models based on the assumption that taxes do not adjust to ensure fiscal solvency and investigating the effects on the macroeconomy (see Bianchi and Melosi (2017, 2019) and the recent strand of the literature on monetary/fiscal interactions we summarize below).

In the Euro area, similar conditions held after the 2010-11 debt crisis, the main concern being that debt has been explosive in some countries, and sustainable in others. This has led to a development of a considerable academic and policy literature on 'fiscal divergence'.

These problems are obviously more relevant today due to the effects of COVID on debt levels in OECD economies.

[2]In many of these models the planner will use the tax schedule not only to generate surpluses and finance debt, but also to 'manipulate interest rates' and hence change the real costs of financing debt (see Faraglia, Marcet, Oikonomou, and Scott, 2019; Faraglia et al., 2019). Taxes will therefore affect real and nominal interest rates but so does monetary policy, operating through inflation and output targets. In effect, the optimal interest rate path is jointly determined by the fiscal/monetary policies.

the monetary authority may have to take into account debt sustainability. We utilize a standard New Keynesian model augmented with a fiscal block, the consolidated budget constraint and a rule that sets taxes as a function of the lagged value of debt. We solve an optimal policy problem in this linear quadratic framework assuming, as in much of the literature, that the central bank may seek to minimize inflation, output and interest rate variability.

In Section 2 we show that the nominal interest rate in this model can be expressed as the sum of two components: i) the optimal interest rate rule obtained from the standard 3-equation New Keynesian model (when debt is not a relevant constraint); and ii) a term expressing the influence of debt, and which is the sum of current and lagged Lagrange multipliers attached to the consolidated budget.

In Sections 3 and 4 we strive to derive interest rate rules that express the nominal rate as a function of macroeconomic conditions only, that is to substitute out the Lagrange multipliers or to identify conditions under which the multipliers are zero and so the optimal policy we can derive from our model becomes that of the standard New Keynesian benchmark.

We firstly show that our model admits two types of optimal policy equilibria: In one case, when taxes adjust strongly to debt (in broadly used terminology fiscal policy is 'passive'), then the Lagrange multipliers equal 0, and the debt constraint is slack. In the second case, when taxes do not adjust to debt, (fiscal policy is 'active') then the debt constraint is binding; the multipliers are not zero and exert an influence on the optimal policy.

We then focus on the latter scenario (though we contrast our results with the former). We ask, is there an equivalent writing of the optimal interest rate rule where the right hand side variables are macroeconomic variables only and Lagrange multipliers can be dropped? Our key finding is that such rules do exist and they are *passive money rules*: the nominal rate responds weakly to inflation, the output gap etc (e.g. Leeper, 1991).

Over a wide range of specifications of the model parameters and of the central bank's objective function we can characterize these rules analytically. We first use a simplistic Fisherian model (as in e.g. Cochrane (2001); Sims (2013) among others) assuming that the central bank only seeks to minimize the volatility of inflation. We then show that the optimal policy is a simple inflation targeting rule that tracks the real interest rate and the inflation coefficient equals δ, the decay factor of the coupon payments on government debt.

This result is intuitive. Under passive monetary policy, a higher inflation coefficient implies more persistence, given that inflation is a backward looking process. When the maturity of debt is short (i.e. $\delta = 0$) persistence is undesirable, since only short term price changes can contribute towards making debt sustainable when a shock has occurred. With long maturity (δ close to 1) it is optimal for inflation to revert to target at the same rate as the coupon payments on government debt decay. This enables to spread efficiently the costs of inflation over time.

Our analysis then departs from this simplistic setup to consider more plausible specifications of the model, considering a central bank that desires to smooth output and the nominal rate and assuming that the real interest rate is not exogenous, i.e. as in the canonical New Keynesian model. Smoothing output (or the nominal rate) adds inertia to the optimal policy rule, which makes inflation a more persistent process, for any maturity of debt. Our analytical results characterize the optimal coefficients in these cases, as functions of the model parameters.

In the canonical model, when the real interest rates are not exogenous but are a function of output growth, optimal policy also takes the form of a simple inflation targeting rule. The optimal inflation coefficient is now not only a function of the average maturity of debt but also accounts for the indirect effects of inflation through output on the real bond prices, since prices matter for the intertemporal solvency of debt. We provide a simple formula showing the dependence of the optimal rule on the parameters that account for the direct and indirect channels of inflation.

Using our model we can also easily characterize analytically, for each of the different parameterizations that we consider, the optimal policy when taxes adjust to debt and satisfy fiscal solvency. In this case, optimal interest rate rules are active, the maturity of debt or any other object related to the consolidated budget constraint is not relevant (since the constraint itself is also not relevant). Besides illuminating the contrast between interest rate rules under active and passive fiscal policies, this exercise demonstrates that our model nests the two equilibria of active/passive policies defined in Leeper (1991). The contribution here is that we derive optimal monetary policy rules, whereas in Leeper (1991) and the rest of the literature that used his influential framework interest rate rules are ad-hoc.

Section 4 then investigates further the optimal monetary policy under active fiscal policy, deriving analytically the impulse responses of inflation and interest rates to shocks. This helps us highlight the key channels of transmission of policy in the various versions of the model considered. In the final paragraph of Section 4 we highlight an important implication of these analytical results. We show that when the maturity of debt in the model is calibrated to the US data, then a simple inflation targeting rule that sets the inflation coefficient equal to δ provides a very good approximation of optimal policies in the canonical model and when the central bank can have the dual objective to stabilize both output and inflation. For this practically relevant case the indirect effects on bond prices turn out not matter much and the objective to smooth output lines up with the objective to spread the distortions of inflation across periods. Therefore, a simple transparent rule where the average maturity of debt is the only relevant moment, works pretty well.

Finally, Section 5 briefly argues that these results hold regardless of whether taxes are lump sum or they are distortionary, referring the reader to the appendix for the complete derivations.

Our work is related to a large literature that studies the interactions between monetary and fiscal policies in macroeconomic models (e.g. Sargent, Wallace et al., 1981; Leeper, 1991; Sims, 1994; Woodford, 1994, 1995, 2001a; Cochrane, 1998, 2001; Schmitt-Grohé and Uribe, 2000; Bassetto, 2002; Eggertsson, 2008; Canzoneri, Cumby, and Diba, 2010; Del Negro and Sims, 2015; Reis, 2016; Jarociński and Maćkowiak, 2018; Leeper and Leith, 2016; Kumhof, Nunes, and Yakadina, 2010; Bi and Kumhof, 2011; Benigno and Woodford, 2007; among many others).[3] As discussed previously, particularly related is the framing of the monetary/fiscal interactions in Leeper (1991), in terms of two distinct regimes: In one regime fiscal policy is 'active' (taxes do not ensure the solvency of debt) and monetary policy is 'passive', and in the second regime (active monetary/passive fiscal) the opposite holds, taxes adjust strongly to satisfy the intertemporal debt constraint. For any other configuration of policies, (i.e. active/active or passive/passive) the model does not have a unique stable rational expectations equilibrium. Building on this influential framework Bianchi and Ilut (2017); Bianchi and Melosi (2017, 2019); Leeper, Traum, and Walker (2017), have constructed rich DSGE models. Bianchi and Ilut (2017); Bianchi and Melosi (2017, 2019), and Davig and Leeper (2007), among others, have considered the case where policy can fluctuate across regimes.

Our framework can be seen as an extension of this important model of monetary/fiscal interactions to optimal monetary policy. We solve an optimal monetary policy program under active/passive fiscal policies and derive (respectively) passive/active optimal interest rate rules. Thus, our solution gives rise to the two stable equilibria defined in Leeper (1991), and there is no room here for any lack of coordination between the two policies that could bring about multiple equilibria (as for example in the case of a passive/passive regime).[4] Furthermore, we consider the two fiscal regimes to be permanent, that is we do not allow for random switches in fiscal policy. We believe, however, that the framework proposed here can be applied to study regime fluctuations, something that we intend

[3]See in particular Leeper and Leith (2016) for a very comprehensive overview of this literature focusing on the interactions between monetary and fiscal policy.

[4]This has to do with the assumed structure of the policy program. The fiscal authority 'commits' to a tax rule and this is internalized by the optimizing monetary authority. Essentially, monetary policy acts like a 'Stackelberg leader' (though this term is not fully correct since we do not model optimal fiscal policy here). Given this framing, it is not surprising that optimal monetary policy leads to unique equilibria.

to consider in future work.

This work is also broadly related to a considerable literature studying optimal policy with debt. A large strand of this literature has studied the properties of optimal distortionary taxes and debt issuance in real models (e.g. Lucas and Stokey (1983); Aiyagari, Marcet, Sargent, and Seppälä (2002); Angeletos (2002); Buera and Nicolini (2004); Faraglia et al. (2019) among many others). Lucas and Stokey (1983) were the first to study this problem in a complete financial market setting, that is assuming that debt is issued in state contingent securities. Aiyagari et al. (2002) and Marcet and Scott (2009) instead focused on the case where the optimizing government issues only non-state contingent debt, a short term bond. Faraglia et al. (2019) extend this approach to modelling long term government bonds. Angeletos (2002) and Buera and Nicolini (2004) considered models with a rich enough maturity structure of non-state contingent assets so that effectively the optimizing government can complete the market and use this framework to draw relevant conclusions regarding the optimal maturity structure of debt.

Our approach to modelling optimal policy is methodologically similar to the approach taken in these papers. We assume incomplete markets as Aiyagari et al. (2002), Marcet and Scott (2009), and Faraglia et al. (2019) do, however, we also consider limiting scenarios where the debt maturity structure is such that markets are 'effectively complete'. Even though here taxes are exogenous and the policy instead sets distortionary inflation to make debt solvent, many of the features of the optimal allocation are common with optimal taxation models. We therefore frequently refer to well known results from this literature to explain our findings.

From the second strand of this related literature studying optimal policy with debt- the papers on Ramsey monetary/fiscal policies previously referenced- the work of Leeper and Zhou (2021) is most closely related to ours. Leeper and Zhou (2021) utilize a linear quadratic framework broadly similar to the one employed here, to study how the optimal mix of inflation and taxes varies with the debt maturity and with the relative importance attached to smoothing inflation v.s. smoothing output fluctuations in the objective of the planner. While their paper makes considerable progress with deriving analytical results in this context, their interest is not to derive optimal monetary policy rules, as we do in this paper. Thus our findings are complementary.

Lastly, in recent work, Chafwehé, de Beauffort, and Oikonomou (2022) use the simplistic Fisherian model we present in Section 3 to derive optimal interest rate rate rules in a model with active fiscal policy. Their paper experiments with alternative ways of modelling government bonds, most notably considering separately the case where governments can engage in debt buybacks and the case where they cannot. We use a standard setup of modelling government debt (as in e.g; Woodford, 2001a; Bianchi and Ilut, 2017; Leeper and Zhou, 2021 and others) and moreover, relative to Chafwehé et al. (2022), our derivations extend to more plausible calibrations of the New Keynesian model with debt.

2 Theoretical Framework

We consider an optimal policy equilibrium where a Ramsey planner (the Fed) sets the path of macroeconomic variables, interest rates, inflation and output, subject to the dynamic equations that define the competitive equilibrium. Our framework is a standard New Keynesian model, with monopolistically competitive firms operating technologies which are linear in the labour input and setting prices subject to adjustment costs as in Rotemberg (1982). The model is augmented with a fiscal block, the consolidated budget constraint and a tax rule that determines the response of taxes to the lagged value of government debt. We first derive our results under the assumption that taxes are lump sum.

Since this is a standard setup we will describe the competitive equilibrium using the equations of the log-linear model. We leave it to the appendix to characterize the household and firm optimal behavior from the (background) non-linear model.

2.1 The model

Let $\hat{x}$ denote the log deviation of variable x from its steady state value, $\overline{x}$. The model equations are the following:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t - \kappa_2 \hat{G}_t + \beta E_t \hat{\pi}_{t+1}, \tag{1}$$

where $\kappa_1 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}(\gamma_h + \sigma\frac{\overline{Y}}{\overline{C}}) > 0$, and $\kappa_2 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\sigma\frac{\overline{G}}{\overline{C}} > 0$,

$$\hat{i}_t = E_t\left(\hat{\pi}_{t+1} - \hat{\xi}_{t+1} + \hat{\xi}_t - \sigma\left[\frac{\overline{Y}}{\overline{C}}(\hat{Y}_t - \hat{Y}_{t+1}) - \frac{\overline{G}}{\overline{C}}(\hat{G}_t - \hat{G}_{t+1})\right]\right), \tag{2}$$

$$\overline{p}_\delta \overline{b}\hat{b}_{t,\delta} + \overline{p}_\delta \overline{b}\hat{p}_{t,\delta} = -\overline{s}\hat{s}_t + (1+\delta\overline{p}_\delta)\overline{b}\left(\hat{b}_{t-1,\delta} - \hat{\pi}_t\right) + \delta\overline{p}_\delta \overline{b}\hat{p}_{t,\delta}, \tag{3}$$

$$\overline{p}_\delta \hat{p}_{t,\delta} = \beta(1+\overline{p}_\delta\delta)E_t\left[-\sigma\left(\frac{\overline{Y}}{\overline{C}}(\hat{Y}_{t+1} - \hat{Y}_t) - \frac{\overline{G}}{\overline{C}}(\hat{G}_{t+1} - \hat{G}_t)\right) - \hat{\pi}_{t+1} + \hat{\xi}_{t+1} - \hat{\xi}_t\right] + \beta\delta\overline{p}_\delta E_t\hat{p}_{t+1,\delta}\Big]. \tag{4}$$

(1) is the Phillips curve at the heart of our model. $\hat{\pi}_t$ represents inflation and $\hat{Y}_t$ is the output gap. $\hat{G}_t$ denotes government spending in t. Parameters $\eta < 0$ and $\theta > 0$ govern the elasticity of substitution across the differentiated (monopolistically competitive) goods produced in the economy and the degree of price stickiness, respectively.[5]

σ denotes the inverse of the intertemporal elasticity of substitution and γ_h is the inverse of the Frisch elasticity of labour supply. These objects influence the slope of the Phillips curve, κ_1, through their influence on the response of hours/output to changes in marginal costs (wages).

(2) is the standard log-linear IS-Euler equation which prices a short term nominal asset. $\hat{\xi}$ is a standard preference shock which affects the relative valuation of current vs. future utility by the household. A drop in $\hat{\xi}$ makes the household relatively patient, willing to substitute current for future consumption.[6]

(3) is the consolidated budget constraint. The LHS of this equation represents the value of debt issued in period t. The leading term, $\hat{b}_{t,\delta}$ denotes the quantity of real net government bonds issued in t and held by the private sector, whereas the second term, $\hat{p}_{t,\delta}$, represents the price of the newly issued debt in deviation from steady state. When $\delta > 0$ debt is issued in a perpetuity bond that pays decaying coupons.[7] When $\delta = 0$ only short bonds are issued by the government. On the RHS of (3) we have the government's surplus ($\overline{s}\hat{s}_t = \overline{\tau}\hat{\tau}_t - \overline{G}\hat{G}_t$, where τ denotes taxes) and the real value of debt that was issued in $t-1$ (remaining terms).

Finally, equation (4) defines the recursive formula that determines the price of debt in period t. Iterating forward this equation and substituting the equilibrium price in (3) and rearranging, it is

[5] θ is the parameter that governs the magnitude of price adjustment costs in the standard quadratic cost function of Rotemberg (1982). When θ equals zero prices are fully flexible.

[6] Below we refer to $\hat{\xi}_t$ also as the demand shock and to $\hat{G}_t$ simply as the spending shock. Obviously both $\hat{\xi}_t$ and $\hat{G}_t$ are demand innovations, but we will reserve this term for $\hat{\xi}_t$.

We focus on these two types of shocks as firstly fiscal shocks are clearly important in the context of a paper on fiscal inflation, and secondly, changes in real interest rates driven by demand shocks have been recently shown an important source of fluctuations for government budgets. For example, de Lannoy, Bhandari, Evans, Golosov, and Sargent (2022) argue that such shocks are an important source of risk that debt management should ward off to ensure the intertemporal solvency of US government debt. Here the focus is on the inflation consequences of the shocks however.

Finally, it should also be noted that the formulae that we will derive below will continue being relevant when we assume additional sources of shocks, e.g. shocks to government transfers, or cost-push types of shocks. We discuss this further below.

[7] In this case we can assume that short term debt is in zero net supply.

possible to write the consolidated constraint as:

$$\frac{\beta\bar{b}}{1-\beta\delta}\hat{b}_{t,\delta}+\bar{b}\sum_{j=1}^{\infty}\beta^{j}\delta^{j-1}\left[E_t\left(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j}-\frac{\overline{G}}{\overline{C}}\hat{G}_{t+j})-\sum_{l=1}^{j}\hat{\pi}_{t+l}+\hat{\xi}_{t+j}\right)\right]$$

$$=-\bar{s}\hat{S}_t-\bar{b}\sigma\left(\frac{\overline{Y}}{\overline{C}}\hat{Y}_t-\frac{\overline{G}}{\overline{C}}\hat{G}_t\right)+\bar{b}\hat{\xi}_t \tag{5}$$

$$+\frac{\bar{b}}{1-\beta\delta}(\hat{b}_{t-1,\delta}-\hat{\pi}_t)+\delta\bar{b}\sum_{j=1}^{\infty}\beta^{j}\delta^{j-1}E_t\left(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j}-\frac{\overline{G}}{\overline{C}}\hat{G}_{t+j})-\sum_{l=1}^{j}\hat{\pi}_{t+l}+\hat{\xi}_{t+j}\right)$$

where $\hat{S}_t$ denotes the surplus scaled by marginal utility in deviation from the steady state and

$$\bar{s}\hat{S}_t\equiv\left[-\overline{G}\left(\hat{G}_t(1+\sigma\frac{\overline{G}}{\overline{C}})-\sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_t+\hat{\xi}_t\right)+\bar{\tau}\left(\hat{\tau}_t-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_t-\frac{\overline{G}}{\overline{C}}\hat{G}_t)+\hat{\xi}_t\right)\right]$$

2.1.1 Tax rule

Fiscal policy is assumed to follow a standard rule that links the level of taxes to the face value of debt outstanding:

$$\hat{\tau}_t=\phi_{\tau,b}\hat{b}_{t-1,\delta} \tag{6}$$

Coefficient $\phi_{\tau,b}$ is a crucial object. Below we will separately consider cases where taxes adjust strongly to debt, so that debt becomes solvent through fiscal policy and cases where taxes do not adjust sufficiently to debt, in which case the solvency of debt will be (endogenously) ensured by monetary policy. In standard terminology, fiscal policy is passive in the former case and active in the latter (e.g. Leeper (1991)).

To derive our results analytically, we will assume that when fiscal policy is 'active', then $\phi_{\tau,b}=0$. This assumption is also made by Bianchi and Ilut (2017) and Bianchi and Melosi (2017). For passive fiscal policy we will set $\phi_{\tau,b}>\widetilde{\phi_{\tau}}$ where $\widetilde{\phi_{\tau}}$ is a threshold value defined below as a function of model parameters.

2.2 Optimal Policy

The central bank sets inflation, output and interest rate sequences to maximize the following objective function:

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^{t}E_0\left\{\hat{\pi}_t^2+\lambda_Y\hat{Y}_t^2+\lambda_i\hat{i}_t^2\right\} \tag{7}$$

for $\lambda_Y,\lambda_i\geq 0$.[8] Maximization of (7) is subject to the dynamic equations (1), (2) and (5) and given the tax rule (6).[9]

As it is standard we solve for optimal policies with a Lagrangian. Letting $\psi_{\pi,t}$ be the multiplier attached to the Phillips curve constraint, $\psi_{i,t}$, and $\psi_{gov,t}$ the analogous multipliers attached to the

[8] We adopt a standard loss function for the central bank, as in Giannoni and Woodford (2003). Our results below are presented in terms of parameters λ_Y,λ_i and we do not consider that these weights have to necessarily coincide with the weights that would derive from a quadratic approximation of the household's welfare function.

[9] Given optimal policies we can use (4) to solve for $\hat{p}_{t,\delta}$. In other words, we do not have to keep track of the bond price in the optimal policy program.

Euler equation and the consolidated budget respectively, the first order conditions for the optimum are given by:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} - \frac{\psi_{i,t-1}}{\beta} + \frac{\bar{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} = 0 \tag{8}$$

$$-\lambda_Y \hat{Y}_t - \psi_{\pi,t}\kappa_1 + \sigma\frac{\overline{Y}}{\overline{C}}(\psi_{i,t} - \frac{\psi_{i,t-1}}{\beta}) + \sigma\frac{\overline{Y}}{\overline{C}}\bar{b}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} + \omega_Y\psi_{gov,t} = 0 \tag{9}$$

$$-\lambda_i \hat{i}_t + \psi_{i,t} = 0 \tag{10}$$

$$\frac{\bar{b}}{1-\beta\delta}\left(\psi_{gov,t} - E_t\psi_{gov,t+1}\right) + \phi_{\tau,b}\overline{\tau}E_t\psi_{gov,t+1} = 0 \tag{11}$$

where $\omega_Y \equiv \sigma\frac{\overline{Y}}{\overline{C}}(\overline{G} - \overline{\tau})$.[10]

(8) is the FONC with respect to $\hat{\pi}_t$; (9), (10), (11) are first order conditions with respect to $\hat{Y}_t$, $\hat{i}_t$ and $\hat{b}_{t,\delta}$ respectively.

2.2.1 Interpreting the first order conditions

To inspect these optimality conditions, let us consider first the case where $\psi_{gov,t} = 0$ for all t. Notice that this corresponds to the case where the consolidated budget constraint exerts no influence on optimal policy and it will later be shown as an endogenous outcome of the model under passive fiscal policy. Equations (8) - (10) reduce to:

$$\begin{aligned} -\hat{\pi}_t + \Delta\psi_{\pi,t} - \frac{\psi_{i,t-1}}{\beta} &= 0 \\ -\lambda_Y \hat{Y}_t - \psi_{\pi,t}\kappa_1 + \sigma\frac{\overline{Y}}{\overline{C}}(\psi_{i,t} - \frac{\psi_{i,t-1}}{\beta}) &= 0 \\ -\lambda_i \hat{i}_t + \psi_{i,t} &= 0 \end{aligned} \tag{12}$$

System (12) can be solved to derive the optimal policy outcome defined in Giannoni and Woodford (2003), in the context of the 3-equation NK model.[11] As is well known, this optimal plan is inertial, featuring the lagged Lagrange multipliers $\psi_{i,t-1}, \psi_{\pi,t-1}$ due to the presence of expectations in the Euler equation and the Phillips curve constraints. Giannoni and Woodford (2003) have shown that these optimality conditions can be rearranged to derive a policy rule expressing the target nominal rate as a function of macroeconomic conditions. We will later revisit their derivation.

Consider now the case where $\psi_{gov,t} \neq 0$. The first order conditions (8) and (9) feature additional terms, the weighted averages of the current and lagged values of the multiplier on the consolidated budget. What do these terms capture? Shocks to government spending or to demand will impact the value of debt and the deficit. When the debt constraint becomes important for the optimal allocation, inflation and output adjust to satisfy the constraint and ensure the solvency of debt. The

[10]Note that we solve for optimal policy from a *timeless perspective*. We thus do not consider (for example) the usual initial allocation problem whereby the planner may inflate away public debt at the beginning of the horizon.

As is well known, solving for optimal policies from a timeless perspective, requires to introduce additional constraints on the initial allocation (e.g Giannoni and Woodford, 2003), or the program can be stated in terms of an objective function that accounts explicitly for the lagged Lagrange multipliers at the beginning of the planning horizon (e.g Faraglia et al. (2019)). To avoid introducing explicitly all these elements we do not state the Lagrangian here.

[11]Since we imposed that the consolidated budget constraint has no influence, optimal policy is equivalent to maximizing objective (7) subject to the Phillips curve and the Euler equation. This is essentially the problem considered by Giannoni and Woodford (2003).

final terms in equations (8) and (9) essentially capture changes in $\hat{\pi}_t$ and $\hat{Y}_t$ driven by shocks being filtered through the consolidated budget constraint.

To clarify this further, consider the intertemporal budget constraint that can be obtained by iterating forward on (5):

$$E_t \sum_{j=0}^{\infty} \beta^j \bar{s} \hat{S}_{t+j} = \frac{\bar{b}}{1-\beta\delta} \hat{b}_{t-1,\delta} + \bar{b} \sum_{j=0}^{\infty} \beta^j \delta^j E_t \left[-\sigma \left(\frac{\overline{Y}}{\overline{C}} \hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}} \hat{G}_{t+j} \right) - \sum_{l=0}^{j} \hat{\pi}_{t+l} + \hat{\xi}_{t+j} \right] \tag{13}$$

The intertemporal constraint (13) links the present value of the government's surplus (LHS) to the real value of debt outstanding in t (RHS). Note also that (13) is equivalent to (5) in terms of the optimal policy.[12] Consider the impact of a shock which lowers the LHS of (13) relative to the RHS. This may, for example, occur following a shock which increases spending. In response to such a shock, the constraint tightens, and the value of the multiplier ψ_{gov} then increases. To satisfy the constraint, the monetary authority may need to engineer a drop in the real payout of debt (the RHS of (13)) either through increasing inflation and/or adjusting output when $\sigma > 0$. Moreover, since optimal policy features full commitment it is feasible to adjust both the current and future values of these variables. When debt is long term, $\delta > 0$, promising higher future inflation (output) in response to the shock, will be optimal. The lagged values of the multipliers in the date t optimality conditions capture promises made by the planner to change inflation and output in t in response to shocks which have occurred in the past.

The multiplier under active fiscal policy. Consider now the case where fiscal policy is active and $\phi_{\tau,b} = 0$. Then (11) becomes:

$$\psi_{gov,t} = E_t \psi_{gov,t+1}$$

The multiplier thus evolves according to a random walk.

This result is standard in models of optimal policy under *incomplete markets*. Aiyagari et al. (2002); Faraglia et al. (2019) consider models where a Ramsey planner sets optimally the path of distortionary taxes to finance spending shocks. In Schmitt-Grohé and Uribe (2004); Lustig et al. (2008); Faraglia et al. (2013); Leeper and Zhou (2021) the planner can set simultaneously taxes and distortionary inflation to satisfy the intertemporal budget constraint. In both cases, the multiplier on the debt constraint measures the burden of the distortions, and follows a random walk because the planner wants to spread evenly the costs over time.

The principle also applies here. Though we have assumed that taxes are exogenous to the planning program, inflation is distortionary (because of the Phillips curve and the objective (7)). The random walk property means that optimal policy seeks to spread the burden of the distortions due to inflation evenly across periods.

2.2.2 Optimal interest rate policy

Combining the first order conditions (8)-(10) and following the argument of Giannoni and Woodford (2003) we can derive analytically the optimal interest rate rule from the Ramsey program. The following proposition summarizes the result:

Proposition 1. *The optimal interest rate policy is:*

$$\hat{i}_t = \mathcal{T}_t + \mathcal{D}_t \tag{14}$$

[12] See for example Aiyagari et al. (2002).

$$\mathcal{T}_t \equiv \phi_\pi \hat{\pi}_t + \phi_Y \Delta \hat{Y}_t + \phi_i \hat{i}_{t-1} + \frac{1}{\beta}\Delta \hat{i}_{t-1}$$

with $\phi_\pi = \frac{\kappa_1 \overline{C}}{\lambda_i \sigma \overline{Y}}$, $\phi_i = (1 + \frac{\kappa_1}{\beta\sigma}\frac{\overline{C}}{\overline{Y}})$ *and* $\phi_Y = \frac{\lambda_Y \overline{C}}{\sigma \lambda_i \overline{Y}}$. *Moreover,*

$$\mathcal{D}_t = -\frac{\overline{C}}{\overline{Y}}\frac{\kappa_1}{\lambda_i \sigma}\frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} - \frac{\overline{b}}{\lambda_i}\sum_{l=0}^{\infty}\delta^l\left(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\right) - \frac{\overline{C}\omega_Y}{\overline{Y}\sigma\lambda_i}\Delta\psi_{gov,t}$$

Proof See online appendix.

Equation (14) shows that there are two distinct components in the interest rate rule. The first, $\mathcal{T}_t$, is the standard robustly optimal rule derived in Giannoni and Woodford (2003). It links interest rates to inflation, output growth and lagged values of the interest rate. The impact of these variables on $\hat{i}_t$ depends on the weights λ_i, λ_Y that capture the output and interest rate stabilization objectives of the central bank and on the structural parameters σ, κ_1 and β.

The second component, the term $\mathcal{D}_t$, measures the impact of changes in the value of $\psi_{gov,t}$. As discussed previously, the lagged values of the multiplier are the promises made by the planner to alter inflation and output following shocks that have hit the consolidated budget in the past. These terms enter in $\mathcal{D}_t$ because changes in inflation and output will influence the path of the nominal interest rate.

3 Optimal Monetary Policy and Interactions with Fiscal Policy

The rule derived in Proposition 1 can implement the optimal policy equilibrium. We can thus leave aside the first order conditions for $\hat{\pi}_t, \hat{Y}_t, \hat{i}_t$ and focus on the properties of (14) to characterize optimal monetary policy. As we saw, the optimal policy rule is the rule of Giannoni and Woodford (2003) when $\psi_{gov,t} = \mathcal{D}_t = 0$ (when the debt constraint is not important for the optimal allocation) but features the additional term $\mathcal{D}_t$, when $\psi_{gov,t} \neq 0$ and policy responds to the debt constraint.

How important is the influence of $\mathcal{D}_t$ on the interest rate policy? Does rule (14) imply a drastically different response of the nominal interest rate to macroeconomic conditions than the optimal policy when $\mathcal{D}_t = 0$? Moreover, is there a version of the model (a range of values of the parameters) for which we get $\mathcal{D}_t = 0$ as an endogenous outcome?

Our analytical results in this section establish the following properties: First, we show that the model admits two types of equilibria. In one case, when fiscal policy is passive, we have $\mathcal{D}_t = 0$. Then, debt sustainability is ensured by taxes and inflation does not need to adjust to make debt solvent. In the opposite case, when fiscal policy is active we have in equilibrium $\mathcal{D}_t \neq 0$ and fluctuations of inflation are driven also by debt sustainability.

Second, we show that in the case where $\mathcal{D}_t \neq 0$, we can derive an optimal monetary policy rule which expresses the nominal rate as a function of macroeconomic variables only, without involving the Lagrange multipliers on the consolidated budget. The resulting rule is a passive money rule, the nominal rate responds weakly to macroeconomic variables, to inflation, lagged interest rates etc.

We provide analytical formulae expressing the optimal coefficients on macroeconomic variables for several calibrations of the model. We firstly consider a simplistic setup, a Fisherian economy in which the central bank seeks to stabilize inflation only (e.g. Cochrane, 2001). This amounts to assuming $\lambda_Y = \lambda_i = \sigma = 0$. We then relax these assumptions to consider cases where the central bank explicitly targets output and the interest rate, and to also consider the canonical New Keynesian model where $\sigma > 0$.

Finally, note that in the cases that we will consider, the optimal monetary policy rules when fiscal policy is passive, will not necessarily be of the functional form $\mathcal{T}$ in Proposition 1. We will therefore also derive optimal policy rules for passive policy to illuminate the differences with the active fiscal scenario. We present our results as two separate equilibria, one in which the monetary/fiscal regime is active/passive and one in which it is passive/active. This framing essentially follows Leeper (1991).

3.1 A simplistic Fisherian model

Let us first assume that $\lambda_Y = \lambda_i = \sigma = 0$. Under this calibration, output and interest rate stabilization are not objectives of the central bank and the real interest rate is exogenous (driven only by shocks to demand). This Fisherian setup is also employed by Aiyagari et al. (2002); Cochrane (2001); Davig and Leeper (2007); Cochrane (2018); Sims (2013); Faraglia et al. (2019); Bouakez, Oikonomou, and Priftis (2018); Bianchi and Melosi (2019).

3.1.1 Fiscal Policy

We begin by characterizing the equilibrium multiplier ψ_{gov} under passive and active fiscal policies. When $\lambda_Y = \lambda_i = \sigma = 0$ we can reduce the dimensionality of the equilibrium system of equations by dispensing with some of the variables and multipliers. Effectively, the system that needs to be resolved can be reduced to the consolidated budget constraint and the first order condition for $\hat{b}_{t,\delta}$, (11).

To simplify the algebra let us also assume $\delta = 0$ and that shocks to demand and spending are i.i.d.[13] Then, inflation will evolve according to:

$$\hat{\pi}_t = \overline{b}\Delta\psi_{gov,t} \tag{15}$$

Using this expression to substitute inflation out from the consolidated budget and using the Phillips curve to substitute output, together with the tax rule (6), we get:

$$\hat{b}_{t,\delta} - \overline{b}E_t\Delta\psi_{gov,t+1} + \frac{1}{\beta\overline{b}}\left((\overline{s}-\overline{b})\hat{\xi}_t - \overline{G}\hat{G}_t\right) = \frac{1}{\beta}\left[1 - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}}\right]\hat{b}_{t-1,\delta} - \overline{b}\Delta\psi_{gov,t} \tag{16}$$

Equation (16) together with the first order condition (11) which defines the process of $\psi_{gov,t}$, form the system of equations that needs to be resolved to find the optimal allocation.

Active Fiscal Policy. Consider first $\phi_{\tau,b} = 0$. Since from (11) $\psi_{gov,t}$ is a random walk we can write (16) as:

$$\hat{b}_{t,\delta} + \frac{1}{\beta}\widetilde{\chi}_t = \frac{1}{\beta}\hat{b}_{t-1,\delta} - \overline{b}\Delta\psi_{gov,t} \tag{17}$$

where $\widetilde{\chi}_t \equiv \frac{1}{\overline{b}}\left((\overline{s}-\overline{b})\hat{\xi}_t - \overline{G}\hat{G}_t\right)$.

(17) defines a difference equation with an unstable root $\frac{1}{\beta}$. It can be solved forward to give:

$$\hat{b}_{t-1,\delta} = \sum_{j\geq 0}\beta^j E_t\left[\widetilde{\chi}_{t+j} + \overline{b}\Delta\psi_{gov,t+j}\right] = \widetilde{\chi}_t + \overline{b}\Delta\psi_{gov,t} \tag{18}$$

[13] At the end of this subsection we generalize the results to $\delta \geq 0$. Moreover, we will derive most of our analytical results in Sections 3 and 4 assuming i.i.d shocks for simplicity, but at the end of Section 4 we will show that it is simple to extend our analytical formulae to the case of persistent shocks.

where the second equality makes use of the assumption that shocks are i.i.d and the random walk property of the multiplier. Clearly, (18) can be consistent with the random walk if and only if $\hat{b}_{t,\delta} = 0$ for all t. In equilibrium we then have:

$$\Delta\psi_{gov,t} = -\frac{1}{\bar{b}}\widetilde{\chi}_t$$

and from (15) $\hat{\pi}_t = -\widetilde{\chi}_t$.

Passive Fiscal Policy. Now assume $\phi_{\tau,b}$ is positive and its value exceeds some threshold $\widetilde{\phi}_\tau > 0$. $\psi_{gov,t}$ evolves according to:

$$\psi_{gov,t} = \left(1 - \frac{\overline{\tau}\phi_{\tau,b}}{\bar{b}}\right)E_t\psi_{gov,t+1} \tag{19}$$

and solving (19) forward we get $\psi_{gov,t} = 0$ for all t[14].

Inflation will be zero at all times. The consolidated budget constraint then becomes:

$$\hat{b}_{t,\delta} + \frac{1}{\beta}\widetilde{\chi}_t = \frac{1}{\beta}\left(1 - \frac{\overline{\tau}\phi_{\tau,b}}{\bar{b}}\right)\hat{b}_{t-1,\delta} \tag{20}$$

Equation (20) can be used to characterize the threshold $\widetilde{\phi}_\tau$. It defines a stable debt process (consistent with the equilibrium where debt will not explode) if $\left(1 - \frac{\overline{\tau}\phi_{\tau,b}}{\bar{b}}\right) < \beta$. It must thus be:

$$\phi_{\tau,b} > (1-\beta)\frac{\bar{b}}{\overline{\tau}} \equiv \widetilde{\phi}_\tau \tag{21}$$

The model admits two types of equilibria: In one case, when fiscal policy is active we have that $\psi_{gov,t} \neq 0$. Inflation then responds to shocks that hit the consolidated budget constraint as equation (15) suggests. In the second scenario, fiscal policy adjusts taxes to debt, so that condition (21) holds. Then, taxes ensure the intertemporal solvency of debt and $\psi_{gov,t} = 0$. The debt constraint is slack and it is not necessary to use inflation to adjust the real value of debt.

The above derivations can be easily extended to the case where $\delta > 0$. The algebra is then a bit more cumbersome and so we simply highlight the main result with the following Proposition:

Proposition 2. *If $\phi_{\tau,b}$ satisfies*

$$\phi_{\tau,b} > \frac{(1-\beta)}{(1-\beta\delta)}\frac{\bar{b}}{\overline{\tau}} \equiv \widetilde{\phi}_\tau$$

then $\psi_{gov,t}, \mathcal{D}_t = 0$. If $\phi_{\tau,b} = 0$ then $\psi_{gov,t}, \mathcal{D}_t \neq 0$.

Finally, note that Proposition 2 does not only hold under the calibration $\lambda_Y = \lambda_i = \sigma = 0$ considered here. It will continue to hold when these parameters are considered positive in subsequent sections.

[14]Note that we will focus on cases where $\left(1 - \frac{\overline{\tau}\phi_{\tau,b}}{\bar{b}}\right) > 0$. Scenarios where taxes respond too strongly to debt and violate this condition are implausible.

3.1.2 Optimal Monetary Policy Rules

We now turn to the optimal monetary policy rules that can implement the Ramsey outcome when $\lambda_Y = \lambda_i = \sigma = 0$. Moreover, we now let $\delta \geq 0$. Under these assumptions we cannot derive an optimal policy rule directly from the first order conditions; however, we can verify that the following simple inflation targeting rule can implement the allocation under optimal policy:

$$\hat{i}_t = \hat{\xi}_t + \widetilde{\phi}_\pi \hat{\pi}_t \tag{22}$$

Consider first the case where fiscal policy is passive, i.e. $\phi_{\tau,b} \geq \widetilde{\phi}_\tau$. As we showed previously, in this equilibrium optimal policy sets $\hat{\pi}_t = 0$. From the Phillips curve we find $\hat{Y}_t = 0$. It is easy to show that the interest rate rule (22) can implement this outcome. Combining (22) with the Euler equation we get the following difference equation in inflation:

$$\hat{\pi}_t \widetilde{\phi}_\pi = E_t \hat{\pi}_{t+1}$$

Standard results yield $\hat{\pi}_t = 0$ as the unique solution if and only if $\widetilde{\phi}_\pi > 1$.

Consider now the case of active fiscal policy. In equilibrium inflation and the interest rate will be given by the following expressions:

$$\hat{\pi}_t = \frac{\overline{b}}{1-\beta\delta} \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l} \qquad \hat{i}_t = \hat{\xi}_t + \frac{\overline{b}\delta}{1-\beta\delta} \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l} \tag{23}$$

A rule of the form (22) can implement this outcome if the following holds:

$$\widetilde{\phi}_\pi \frac{\overline{b}}{1-\beta\delta} \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l} = \delta \frac{\overline{b}}{1-\beta\delta} \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l} \tag{24}$$

hence $\widetilde{\phi}_\pi = \delta \in [0, 1]$.

We highlight these results with the following Proposition:

Proposition 3. *Assume $\lambda_Y = \sigma = \lambda_i = 0$. The optimal policy is a rule of the form (22). In the case where fiscal policy is passive, the optimal monetary policy sets $\widetilde{\phi}_\pi > 1$. In the case where fiscal policy is active, the optimal inflation coefficient is $\widetilde{\phi}_\pi = \delta \in [0, 1]$. Optimal monetary policy is then 'passive'.*

Several comments are in order. First, note that this is an example which shows clearly the analogy between our framework and the equilibria with active/passive monetary/fiscal policies defined in Leeper (1991). Whereas in Leeper (1991) and in the rest of the literature that used this influential framework, monetary policy is modelled using ad hoc interest rate rules, here the policy rules are optimal, in the sense that commitment to rule (22) can implement the optimal allocation when the parameter values are as stated in Proposition 3. Moreover, simple inflation targeting rules as in (22) are commonly used in the literature. The analytical derivations we provide here show the conditions under which (22) is optimal in terms of the objective of the planner and the fundamental model parameters.

According to Proposition 3, in the case of passive fiscal policy the optimal allocation can be implemented with any rule that satisfies the Taylor principle (sets $\widetilde{\phi}_\pi > 1$). In contrast, in the active fiscal scenario, optimal monetary policy constrains the parameter $\widetilde{\phi}_\pi$ to equal δ, the rate at which the coupon payments on debt decay. The average debt maturity ($\frac{1}{1-\delta}$) becomes a key variable determining the reaction to inflation. A higher maturity of debt entails a stronger reaction of the nominal rate to inflation.

Why is this the desired response to inflation? It is quite simple to show that following a shock to either spending or demand, inflation will jump initially and then monotonically revert to steady state, at rate $\widetilde{\phi}_\pi$.[15] When inflation is used to satisfy the intertemporal budget, it is then optimal that the rate at which it reverts to zero is exactly the rate at which debt payments decay. With short term debt all of the response of inflation should be concentrated to the period when a shock occurs; future inflation will not contribute anything towards making debt sustainable and a policy setting a positive inflation coefficient couldn't be optimal. Analogously, when $\delta > 0$ making inflation persistently react to the shock enables to spread the burden of fiscal inflation efficiently.[16]If the inflation coefficient exceeded (fell short of) δ then the inflation persistence will be too high (too low), in the sense that inflation would be substantially above 0 (close to 0) even when the coupon payments are nearly 0 (are high).[17]

Finally, under both active and passive fiscal policies, the optimal interest rate rule tracks the real interest rate. This is a standard feature of optimal monetary policy, which enables to eliminate the demand shock from the Euler equation. When fiscal policy is passive, this leads to the zero inflation/divine coincidence outcome in response to demand shocks. In the case of active fiscal policy inflation will not be zero, since the demand shock is also filtered through the government budget constraint, as we will later illustrate. Real interest rate tracking will be a feature of all versions of the model we will consider subsequently.

3.2 Alternative Calibrations

We now extend the above analysis to the case where $\lambda_Y, \lambda_i, \sigma$ are not constrained to equal zero. In order to continue with analytical results and to transparently demonstrate how each of the parameters affects the optimal monetary policy rule under passive and active fiscal policies, we consider 3 cases separately, allowing one of the parameters at a time to be positive, the other two being set to 0.

[15]This can be easily seen by combining the Euler equation and the interest rate rule. To simplify, assume that a shock hits in t and thereafter no further shock occurs. We then have $\hat{\pi}_{t+\bar{t}+1} = \widetilde{\phi}_\pi \hat{\pi}_{t+\bar{t}}$ for $\bar{t} \geq 0$ which for $\widetilde{\phi}_\pi < 1$ can be solved backwards up to the initial condition $\hat{\pi}_t$. $\hat{\pi}_t$ is pinned down by the intertemporal debt constraint.

[16]See also Lustig et al., 2008; Leeper and Zhou, 2021.

[17]Rule (22) defines the optimal monetary policy as a simple inflation targeting rule with real interest rate targeting. However, it should be noted that this rule is not unique in the sense that other specifications of the interest rate reaction function, can deliver the same outcome. For example, consider a rule of the form

$$\hat{i}_t = \hat{\xi}_t + \widetilde{\phi}_\pi \hat{\pi}_t + \widetilde{\phi}_Y \hat{Y}_t \tag{25}$$

where the nominal rate responds to both output and inflation. We can show that (25) can implement the optimal policy outcome under active fiscal policy provided that $\widetilde{\phi}_\pi + \frac{\widetilde{\phi}_Y}{\kappa_1}(1-\beta\delta) = \delta$. When this condition holds, then inflation will again decay at rate δ following a shock to the economy (the analogous condition to get the passive fiscal policy equilibrium under rule (25) is the familiar $\widetilde{\phi}_\pi + \frac{\widetilde{\phi}_Y}{\kappa_1}(1-\beta) > 1$).

Though (25) is an optimal interest rate rule in this model, we prefer to work with the simple inflation targeting rule in (22) for two reasons: First, because it is less appealing to assume that monetary policy targets output in a model where output fluctuations do not contribute towards making debt sustainable (i.e. when $\sigma = 0$); second, rule (22) leads to a more transparent policy recommendation, setting the inflation coefficient equal to δ, relative to (25) where the analogous condition involves estimating additional parameters β and κ_1. For all models considered below we focus on the simplest possible rules.

Finally, note that the optimal policy in this subsection could be stated as in Proposition 1, a rule that satisfies $\hat{i}_t = \mathcal{T}_t + \mathcal{D}_t$. We can write $\mathcal{T}_t = \hat{\xi}_t + (1+\delta)\hat{\pi}_t$ and $\mathcal{D}_t = -\frac{\bar{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l}$. Then, substituting out the term $\mathcal{D}_t$ we arrive to the passive rule defined in Proposition 3.

From now on we find convenient to state our results as in Proposition 3, directly characterizing the optimal rules when we have substituted out the Lagrange multipliers.

Our aim in this subsection is to derive the interest rate rules, not to provide an in depth characterization of the mechanisms driving optimal monetary policy in each of the model versions considered. The analytical formulae we derive here will be used in Section 4, where we carry out an analysis of the dynamic adjustment of the economy following shocks to spending and demand. We will then clarify the key forces behind optimal policy.

3.2.1 Case 1: $\lambda_i > 0,\ \lambda_Y, \sigma = 0$.

Consider first the case where $\lambda_i > 0$. The following defines the optimal monetary policy rules that emerge from this model.

Proposition 4. *Assume $\lambda_i > 0$ and $\lambda_Y = \sigma = 0$. The optimal interest rate rules are:*
i)

$$\hat{i}_t = \frac{\hat{\xi}_t}{1+\lambda_i/\beta} + \widetilde{\lambda}_1 \hat{\pi}_t + \widetilde{\lambda}_2 \hat{i}_{t-1}$$

with $\widetilde{\lambda}_1 + \widetilde{\lambda}_2 > 1$ and $\widetilde{\lambda}_2 = \frac{\lambda_i}{\beta}\widetilde{\lambda}_1$, when $\phi_{\tau,b} > \widetilde{\phi}_\tau$ and $\psi_{gov,t} = 0$ (passive fiscal policy).
ii)

$$\hat{i}_t = \frac{\hat{\xi}_t}{1+\lambda_i/\beta} + \frac{\delta}{1+\lambda_i/\beta}\hat{\pi}_t + \frac{\delta\lambda_i/\beta}{1+\lambda_i/\beta}\hat{i}_{t-1}$$

when $\phi_{\tau,b} = 0$, $\psi_{gov,t} \neq 0$ (active fiscal policy).

Proof: See appendix.

According to Proposition 4 optimal interest rates follow inertial rules, $\hat{i}_t$ reacts to inflation and to the lagged value of the nominal rate. This is clearly in accordance with the planner's objective when $\lambda_i > 0$. Moreover, the model continues to admit the two equilibria under passive/active monetary/fiscal policies. When fiscal policy is passive, the condition $\widetilde{\lambda}_1 + \widetilde{\lambda}_2 > 1$ tells us that monetary policy is active (satisfies the Taylor principle). In this case an x percent increase in inflation in period t will lead to a cumulative adjustment of the nominal rate of $x\frac{\widetilde{\lambda}_1}{1-\widetilde{\lambda}_2} > 1$. In contrast, when fiscal policy is active then monetary policy is passive, the cumulative response of the nominal rate to a rise in inflation in t is $\frac{\delta}{1+\lambda_i/\beta(1-\delta)}x \leq \delta x$.

3.2.2 Case 2: $\lambda_Y > 0,\ \lambda_i, \sigma = 0$.

Next, assume that the planner's objective is to smooth both inflation and the output gap, focusing on the case where $\lambda_Y > 0$ but $\lambda_i, \sigma = 0$. Under these assumptions, combining (8) and (9) we can show that inflation satisfies the following condition:

$$-\hat{\pi}_t - \frac{\lambda_Y}{\kappa_1}\Delta\hat{Y}_t + \frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k \Delta\psi_{gov,t-l} = 0 \tag{26}$$

Moreover, using the Phillips curve to substitute out aggregate output and after rearranging we obtain the following :

$$E_t\hat{\pi}_{t+1} - (1+\frac{1}{\beta}+\widetilde{\kappa})\hat{\pi}_t + \frac{1}{\beta}\hat{\pi}_{t-1} = -\zeta_t \tag{27}$$

where $\widetilde{\kappa} = \frac{\kappa_1^2}{\lambda_Y \beta}$ and $\zeta_t \neq 0$ (=0) when fiscal policy is active (passive). Inflation thus follows a second order difference equation; for brevity we define the forcing term ζ_t in the appendix.

Consider the equilibrium where fiscal policy is passive. Equation (27) can be written as:

$$E_t \hat{\pi}_{t+1} = (\widetilde{\lambda}_1 + \widetilde{\lambda}_2)\hat{\pi}_t - \widetilde{\lambda}_1 \widetilde{\lambda}_2 \hat{\pi}_{t-1}$$

where $\widetilde{\lambda}_{1,2}$ are the roots of the characteristic polynomial in (27):

$$\widetilde{\lambda}_{1,2} = \frac{1}{2}\left((1 + \frac{1}{\beta} + \widetilde{\kappa}) \pm \sqrt{(1 + \frac{1}{\beta} + \widetilde{\kappa})^2 - \frac{4}{\beta}}\right)$$

Since one of the roots is unstable (say $\widetilde{\lambda}_2$) and the other root is stable, the interest rate rule that implements the optimal allocation

$$\hat{i}_t = \hat{\xi}_t + (\widetilde{\lambda}_1 + \widetilde{\lambda}_2)\hat{\pi}_t - \widetilde{\lambda}_1 \widetilde{\lambda}_2 \hat{\pi}_{t-1} \tag{28}$$

defines an active monetary policy.

Turning to the active fiscal policy scenario we can show that the optimal interest rate rule is:

$$\hat{i}_t = \hat{\xi}_t + (\widetilde{\lambda}_1 + \delta)\hat{\pi}_t - \widetilde{\lambda}_1 \delta \hat{\pi}_{t-1} - \delta \frac{\widetilde{o}}{\widetilde{\lambda}_2 - 1} \Delta\psi_{gov,t} \tag{29}$$

where $\widetilde{o} > 0$ is defined in the appendix for brevity.

Proposition 5 summarizes these results:

Proposition 5. *Assume $\lambda_Y > 0$ and $\lambda_i = \sigma = 0$. The Ramsey optimal interest rate rule is given by (28) when fiscal policy is passive and $\psi_{gov,t} = 0$. It is given by (29) when fiscal policy is active and $\psi_{gov,t} \neq 0$.*

Proof: See appendix.

The result in Proposition 5 and in particular the active fiscal scenario deserves a brief comment. Note that the systematic response of the nominal rate to current and lagged inflation in (29) indeed defines a passive monetary policy. Since $\widetilde{\lambda}_1 + \delta - \widetilde{\lambda}_1 \delta < 1$ an x per cent rise in inflation will lead to a less than x per cent increase in the nominal rate. Thus, this case also conforms with the principle that when fiscal policy is active, optimal monetary policy can be expressed as a passive money rule. However, notice also that the Lagrange multiplier $\psi_{gov,t}$ continues to appear on the RHS of (29); we did not (yet) derive a solution where the nominal rate is only a function of the real rate, $\hat{\xi}_t$, and current and lagged inflation.

The final term on the RHS of (29), $-\delta \frac{\widetilde{o}}{\widetilde{\lambda}_2 - 1} \Delta\psi_{gov,t}$ is a *stochastic intercept*. It basically introduces a temporary innovation to the interest rate rule, whenever a shock hits the economy leading to $\Delta\psi_{gov,t} \neq 0$. For example, assume that spending increases in t and so $\Delta\psi_{gov,t} > 0$. According to (29) and since $-\delta \frac{\widetilde{o}}{\widetilde{\lambda}_2 - 1} < 0$, optimal policy will keep the nominal rate slightly lower in t than the value implied by the systematic component of the interest rate rule. This effect concerns only period t, since as we saw $\Delta\psi_{gov,t}$ is an i.i.d variable.

It turns out that in equilibrium $\Delta\psi_{gov,t}$ can be written as a function of the two shocks (see Section 4 for the analytical expression). Thus, we could replace the stochastic intercept with the shocks, or even some linear combination of macroeconomic variables can substitute out $\Delta\psi_{gov,t}$.[18] We choose not to expand on this here. In Section 4, when we will evaluate the model, we will explore in detail the stochastic intercept term and investigate its significance for optimal policy. We will then show that over plausible calibrations of the model this term exerts a small influence on optimal policy.

[18] In the latter case, the macroeconomic variables cannot be part of the systematic response component, they will only be relevant in period t, when the shock hits, but not in other periods.

3.2.3 Case 3: $\sigma > 0$, $\lambda_Y = \lambda_i = 0$

Finally, consider the case where $\sigma > 0$. To simplify the formulae we will derive in this subsection let us assume that only demand shocks can hit the economy, setting also $\overline{G} = 0$ and $\overline{Y} = \overline{C}$. The appendix extends our derivations to the case of spending shocks.

Combining the FONC of the planner's program we now get:

$$\hat{\pi}_t = \frac{\bar{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} + \frac{\sigma}{\kappa_1}\bar{b}\sum_{l=0}^{\infty}\delta^l\Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\Big) - \frac{\sigma\overline{\tau}}{\kappa_1}\Delta\psi_{gov,t} \tag{30}$$

which expresses inflation as a function of the multipliers.

It is evident that optimal inflation is zero in the equilibrium where $\psi_{gov,t} = 0$ for all t and the debt constraint is not relevant. A simple inflation targeting rule as in (22) can implement this outcome insofar as $\widetilde{\phi}_\pi > 1$.

Consider now the case where $\psi_{gov,t} \neq 0$ under active fiscal policy. Using the Phillips curve to eliminate aggregate output we can write the Euler equation as:

$$\hat{i}_t = \frac{\sigma}{\kappa_1}E_t\Big(\hat{\pi}_{t+1} - \beta\hat{\pi}_{t+2} - \hat{\pi}_t + \beta\hat{\pi}_{t+1}\Big) + E_t\hat{\pi}_{t+1} + \hat{\xi}_t$$

and using (30) it is simple to show that:

$$E_t\hat{\pi}_{t+1} = \delta\hat{\pi}_t - \frac{\sigma}{\kappa_1}(\bar{b} - \delta\overline{\tau})\Delta\psi_{gov,t} \quad \text{and} \quad E_t\hat{\pi}_{t+2} = \delta E_t\hat{\pi}_{t+1}$$

With appropriate substitutions (see appendix) we obtain the following expression for the interest rate rule:

$$\hat{i}_t = \hat{\xi}_t + \Big(\delta + \frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)\Big)\hat{\pi}_t - \widetilde{\omega}\Delta\psi_{gov,t} \tag{31}$$

where $\widetilde{\omega} > 0$ is derived in the appendix and $\Big(\delta + \frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)\Big) < 1$ so that (31) defines a passive monetary policy.

The nominal rate thus again follows a simple rule which tracks the demand shock $\hat{\xi}_t$, and responds to inflation. As is evident from (31) the optimal coefficient on inflation is a function of debt maturity, but also parameters κ_1 (the slope of the Phillips curve) and σ the (inverse of the IES) influence the value of the coefficient. We will later explain in detail how these terms are relevant, but basically these parameters pertain to effects that changes in output have on the intertemporal debt constraint, when we assume $\sigma > 0$. These effects are internalized by optimal policy through the term $\frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta-1)$.

Notice also that as in the previous paragraph, the optimal rule features a stochastic intercept, the final term on the RHS of (31). The loading on this term is negative and therefore when $\Delta\psi_{gov,t} > 0$ (for example due to a positive spending shock) optimal policy will keep the nominal interest rate lower in t than what is implied by the systematic component of the rule. We again leave it to Section 4 to explain this term.

Finally, it will prove useful for our subsequent analysis to state here the optimal policy rule in the case where fiscal shocks can hit the economy and $\overline{G} > 0$.

Proposition 6. *Assume $\sigma > 0$, $\lambda_Y = \lambda_i = 0$ and $\overline{G} > 0$. The optimal monetary policy rule under active fiscal policy is:*

$$\hat{i}_t = \hat{r}_t^n + \Big(\delta + \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)\Big)\hat{\pi}_t - \underbrace{f\Delta\psi_{gov,t}}_{\text{Stochastic Intercept}} \tag{32}$$

where $\hat{r}^n_t \equiv \hat{\xi}_t + \sigma(\frac{\overline{G}}{\overline{C}} - \frac{\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}})\hat{G}_t$.

Proof: See appendix.

For brevity we give the explicit formula for coefficient f in the appendix. (32) is similar to (31), the main difference is that the leading term is now also a function of the spending shock. The term $\hat{r}^n_t$ denotes the *natural rate of interest* obtained under flexible prices.[19]

3.3 Optimal Policy Rules: Further Examples

We have now derived optimal interest rate rules considering three separate cases, when the central bank desires to smooth output or the nominal rate, and when intertemporal elasticity of substitution is finite as in the canonical New Keynesian model. Though considering separately each of these cases will enable us later on (in Section 4) to transparently show how each of the parameters affects optimal policy, the reader may be wondering whether it is feasible to combine the above formulae or to derive interest rate rules in cases where (say) two of these parameters are simultaneously positive. Even if this is feasible, the algebra involved will be extremely cumbersome and the solution may not feature any new elements relative to the scenarios we considered. Thus, instead of attempting to get further analytical results, we utilized the numerical solution of the model to characterize the optimal monetary policy rule under active fiscal policy, in the case where λ_i, σ are positive and $\lambda_Y \geq 0$. For brevity, we present our experiments in the appendix and only provide a brief account of our findings here.

To approximate the optimal interest rate rules we matched the impulse responses of the Ramsey model with those of a model in which monetary policy follows interest rate rules such as the ones we derived in the previous subsections. That is, simple inflation targeting rules, that may also feature interest rate lags. We found that such rules can deliver a perfect match with the Ramsey model and moreover the coefficients on inflation, lagged interest rates, etc were such that monetary policy is passive.

These experiments also revealed an important difference with the optimal policy in the standard New Keynesian model (the rule of Giannoni and Woodford (2003) when we assume $\lambda_i, \lambda_Y, \sigma > 0$). Whereas in the robustly optimal policy of Giannoni and Woodford (2003), the interest rate rule is superinertial (the second order difference equation for $\hat{i}_t$ contains an explosive root), under active fiscal policy we can obtain a close approximation of the Ramsey outcome with inertial rules featuring only up to one lag of the nominal rate, similar to the analytical results of the previous paragraphs.[20]

In Section 4 we will resume this analysis focusing on the case where $\lambda_Y, \sigma > 0$ (the canonical New Keynesian model when we assume that the central bank has a dual objective of stabilizing inflation

[19] Analogously, the rule in the passive fiscal scenario is now of the form:

$$\hat{i}_t = \hat{r}^n_t + \widetilde{\phi}_\pi \hat{\pi}_t$$

where again $\widetilde{\phi}_\pi > 1$. Since this is easy to verify we left it outside Proposition 6, but state it here for completeness.

[20] We can (for example) show analytically that when $\lambda_i, \sigma > 0$ but $\lambda_Y = 0$ and debt is short term, a rule of the form

$$\hat{i}_t = \text{Shock} + \phi_\pi \hat{\pi}_t \tag{33}$$

where 'Shock' is a function of spending and demand shocks capturing the natural interest rate (as in the above examples), can fit perfectly the Ramsey solution. Contrast this with the optimal rule under passive fiscal policy, object $\mathcal{T}$ in Proposition 1 to see how the consolidated budget constraint can change optimal interest rate policy.

Note also that with $\delta = 0$ the finding that (33) can fit the Ramsey solution when $\lambda_i > 0$, conforms with the result we derived in Proposition 4 where the lagged interest rate is included only when $\delta > 0$. We will return to explain this property later on.

and the output gap, a very commonly used setup of optimal policy). There we will also show that for a realistic calibration of the model a simple inflation targeting rule can approximate very well the Ramsey solution.

3.4 Optimal Policy when Debt is a Shock Absorber

We close this section by providing a further analytical result, deriving the optimal interest rate rule under active fiscal policy in a limiting scenario where $\psi_{gov,t} = 0$. Our argument is rooted on a recent literature on optimal debt management in macroeconomic models[21] which shows that under certain conditions regarding the debt maturity structure, debt can absorb shocks that hit the consolidated budget and no adjustment of either inflation or fiscal variables is necessary to satisfy the intertemporal constraint. We will show that even in this limiting case, when shocks that hit the economy do not tighten the intertemporal budget constraint (and so $\psi_{gov,t} = 0$ for t), optimal monetary policy under constant taxes can be again represented as a passive money rule.

To simplify, let us continue with the model of subsection 3.2.3 assuming $\sigma > 0$ and only demand shocks can hit the economy. We consider $\delta = 1$, the limit when long bonds are consols. We show in the appendix that, under these assumptions, inflation, output and debt are at steady state for all t.

To grasp the intuition behind this result, notice that when long bonds are consols the consolidated budget (in format analogous to equation (3)) can be written as:

$$\overline{b}\overline{p}_1(\hat{b}_{t,1} + \hat{p}_{t,1}) = (1 + \overline{p}_1)\overline{b}(\hat{b}_{t-1,1} - \hat{\pi}_t) + \overline{b}\overline{p}_1\hat{p}_{t,1} - \overline{R}\hat{R}_t \tag{34}$$

where b_1 now represents the quantity of the consol and $\hat{p}_1$ is the corresponding price. The latter evolves according to:

$$\hat{p}_{t,1} = -\hat{\xi}_t - E_t\big[\hat{\pi}_{t+1} + \sigma(\hat{Y}_t - \hat{Y}_{t+1}) + \beta\hat{p}_{t+1,1}\big].$$

Clearly, the terms $\overline{b}\overline{p}_1\hat{p}_{t,1}$ on the LHS and RHS of (34) cancel out and $\hat{\xi}_t$ can be dropped from this equation. The disturbance $\hat{\xi}_t$ changes the price of newly issued debt, and changes, in proportion, the market value of outstanding debt, which acts as a shock absorber. Under optimal policy we have $\psi_{gov,t} = 0$ and inflation, output do not need to adjust to finance the shock, regardless of the specification of fiscal policy.

In the appendix we show that optimal monetary policy again can be represented as a simple inflation targeting rule of the form (22). The following Proposition formalizes our findings:

Proposition 7 *Consider the case where $\sigma > 0$ and $\lambda_i = \lambda_Y = \overline{G} = 0$. Assume also that $\delta = 1$ (i.e. long bonds are consols). We then have $\psi_{gov,t} = 0$ under both active and passive fiscal policies. Optimal monetary policy sets the nominal rate according to (22) and:*

i) Under passive fiscal policy it sets $\widetilde{\phi}_\pi > 1$

ii) Under active fiscal policy it sets $\widetilde{\phi}_\pi \leq 1$.

Proof: See appendix.

i) is simply a repetition of the result in subsection 3.2.3. ii) states that in the active fiscal case, optimal monetary policy sets the inflation coefficient to any value less than or equal to 1. Interestingly, the optimal rule derived in subsection 3.2.3 (i.e. equation (31)) predicts that in the limit when $\delta = 1$

[21] See for example Angeletos (2002); Buera and Nicolini (2004); Faraglia et al. (2019); Bhandari, Evans, Golosov, and Sargent (2017).

the optimal policy is $\hat{i}_t = \hat{\xi}_t + \hat{\pi}_t$ (the stochastic intercept term can be dropped since, as we saw, $\Delta\psi_{gov,t} = 0$). However, because in this model shocks exert no influence, all that monetary policy needs to do is to support the zero inflation outcome. This can in turn be accomplished with any passive interest rate policy.

Proposition 7 can be extended in several meaningful ways to find analogous results when $\lambda_i, \lambda_Y > 0$ and when spending shocks can hit the economy.[22] In all these cases we can derive a passive interest rate rule to implement the optimal policy equilibrium.

4 Going deeper into optimal policy

We now build the analytical results of the previous section to go deeper into the mechanics of optimal policy, unraveling the various channels, under the different cases considered. To do so we complement our derivations with new formulae characterizing the transmission of the spending and demand shocks on the macroeconomy, focusing in particular on the dynamic response of inflation to the shocks. We also present numerical simulations of the model to study the responses of other macroeconomic variables and to consider cases where parameters $\lambda_i, \sigma, \lambda_Y$ are simultaneously positive.

Finally, throughout this section we focus on the case of active fiscal policy. The passive fiscal model, besides having been very well investigated in the literature, was previously shown to lead to zero inflation, under most of the versions we considered. Thus, it is simple for the reader to compare the properties of the equilibrium under passive and active fiscal policies.

4.1 Impulse Responses

4.1.1 Output smoothing ($\lambda_Y \geq 0$)

Consider first the Fisherian model of Section 3 ($\sigma = 0$) and let us first focus on the case where the planner can have the dual objective of stabilizing inflation and the output gap. We therefore assume $\lambda_Y \geq 0$ but $\lambda_i = 0$. Consider shocks at date t which change the values of $\{\hat{G}_t, \hat{\xi}_t\}$ assuming that after t there are no further shocks to the economy.

In the appendix we show the following dynamic response of inflation to one off shocks:

$$\hat{\pi}_t = \frac{\widetilde{\lambda}_2}{\widetilde{\lambda}_2 - 1}\widetilde{o}\Delta\psi_{gov,t}$$

$$\hat{\pi}_{t+j} = \widetilde{o}\frac{(\delta^{j+1} - \widetilde{\lambda}_1^{j+1})}{\delta - \widetilde{\lambda}_1}\Delta\psi_{gov,t} + \widetilde{o}\frac{\widetilde{\lambda}_1^j}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t}, \qquad j \geq 1 \tag{35}$$

where $0 < \widetilde{\lambda}_1 < 1, \widetilde{\lambda}_2 > 1$ and $\widetilde{o}$ were defined previously.[23] Moreover,

$$\Delta\psi_{gov,t} = \widetilde{\psi}(\overline{G}\hat{G}_t + (\overline{b} - \overline{s})\hat{\xi}_t) \tag{36}$$

[22] The case $\lambda_Y > 0$ is simple to illustrate. Since in the equilibrium studied above output is always equal to target, any value $\lambda_Y > 0$ would again lead us to Proposition 7. Assuming that spending shocks can hit the economy however, would probably require a change in the modelling of government debt. For example Angeletos (2002) and Buera and Nicolini (2004) consider the case where debt is issued in two zero coupon bonds and show that the portfolio that can absorb fiscal shocks is one where the long bond debt position is several times GDP and short term debt is negative. The returns to this portfolio cannot be replicated with the decaying coupon model we assume here. Finally, when $\lambda_i > 0$ letting $\delta = 1$ would not be enough for debt to fully absorb the demand shocks. Under interest rate smoothing inflation and output will not be zero and it turns out that to insulate the budget constraint from the shock, the portfolio needs to be tilted even more towards long term debt. See below for an analytical derivation supporting this claim.

[23] This formula assumes the most plausible scenario $\widetilde{\lambda}_1 \neq \delta$.

where $\widetilde{\psi} = \frac{(1-\beta\delta)(1-\beta\delta\widetilde{\lambda}_1)}{\bar{b}\widetilde{o}(\frac{1}{1-\beta\delta^2}+\frac{1}{\widetilde{\lambda}_2-1})} > 0$. Let us use these formulae to discern how a shock to demand/spending will impact inflation.

According to (36) a positive spending shock in t will yield $\Delta\psi_{gov,t} > 0$. This is not surprising; the increase in spending reduces the present value of government revenues and the consolidated budget constraint tightens. The planner needs to increase inflation to make debt solvent.

(35) shows that this will have a persistent effect on inflation. Persistence derives from two sources: First, it derives from the maturity of debt, with higher coefficient δ implying more persistence and second, it derives from the objective to smooth output through time (coefficient $\widetilde{\lambda}_1$ is increasing in λ_Y).

The rationale behind the first channel was stated previously. Assume for simplicity that $\lambda_Y = 0$. We can then show that $\widetilde{\lambda}_1 = 0$, $\widetilde{\lambda}_2 \to \infty$ and $\widetilde{o} = \frac{\bar{b}}{1-\beta\delta}$. Then, the above formulae tell us that inflation will display persistence equal to δ. A shock to spending will increase inflation on impact and subsequently $\hat{\pi}$ converges towards the target value at the same rate at which the coupon payments on debt decay. As discussed previously, this path spreads the costs of inflation efficiently.

In addition to this channel, output stabilization also contributes to persistence. Assume that $\delta = 0$ and so debt is only short term. Then,

$$\hat{\pi}_{t+j} = \widetilde{o}\frac{\widetilde{\lambda}_2}{\widetilde{\lambda}_2 - 1}\widetilde{\lambda}_1^j \Delta\psi_{gov,t}, \qquad j \geq 0$$

and inflation decays at rate $\widetilde{\lambda}_1$. We further have $\frac{d\widetilde{\lambda}_1}{d\lambda_Y} > 0$ and in the limit, when $\lambda_Y \to \infty$, $\widetilde{\lambda}_1 \to 1$.

Even in the presence of short term debt, the deviations of inflation from target can be very persistent, depending on the desire to smooth output fluctuations. The intuition for this property is simple: Making inflation respond only in period t (the optimal policy under $\delta = 0$ and no output smoothing) will entail a large contemporaneous response of output to the shock. When a smooth path of output is desired inflation needs to adjust gradually to the shock.

Let us now investigate how the output smoothing objective affects the *magnitude of the response* of inflation to the shock. A stark result is that when $\delta = 0$, coefficient λ_Y has no bearing on the level of inflation in t.[24] In contrast, when $\delta > 0$, then a stronger incentive to smooth output implies a smaller initial response of inflation to the shock. To understand these properties notice first that, in this Fisherian model, the path of output does not matter at all for fiscal sustainability. Since real bond prices and the intertemporal surplus are not functions of output, it is only inflation that can adjust to make debt sustainable. When $\delta = 0$ all of the burden of the adjustment falls on period t inflation. Thus, more inflation persistence, when $\lambda_Y > 0$, will not change the level of inflation in t required to satisfy the intertemporal debt constraint. In contrast, with long term debt, a more persistent response of inflation will imply a larger fall in the real payout of debt following the spending shock, and the increase in inflation in t required to satisfy the intertemporal budget constraint will be smaller.

It may thus seem that when the maturity of debt is long an explicit output smoothing objective will complement the inflation smoothing motive of the planner. However, this is not so. Once again output has nothing to do with satisfying the intertemporal debt constraint, and inflation persistence driven by output stabilization may not be desirable in terms of the inflation smoothing objective.

[24] To see this, combine (35) and (36), evaluated at $\delta = 0$. We get

$$\hat{\pi}_t = \frac{\overline{G}\hat{G}_t + (\bar{b} - \bar{s})\hat{\xi}_t}{\bar{b}}$$

which in independent of $\widetilde{\lambda}_1$ and $\widetilde{\lambda}_2$ and hence also independent of λ_Y.

The term $\widetilde{o}\frac{(\delta^{j+1}-\widetilde{\lambda}_1^{j+1})}{\delta-\widetilde{\lambda}_1}\Delta\psi_{gov,t}$ in (35) reveals this property. This term is essentially a correction in terms of the persistence of the inflation process relative to the second term in (35) which is persistence driven purely by output smoothing. Whenever $\delta < \widetilde{\lambda}_1$ then persistence deriving from smoothing output is too high and the first term will frontload inflation to match more closely the payment profile of debt. In contrast, if $\delta > \widetilde{\lambda}_1$ then the first term will add persistence, the planner targets a flatter path of inflation than what is dictated by the objective to smooth output.

Let us now turn to the demand shock and in particular let us assume that in period t the economy is hit by a negative shock, i.e. $\hat{\xi}_t < 0$. Noting that $\overline{s} < \overline{b}$ when $\delta < 1$,[25] equation (36) tells us that the shock will make $\Delta\psi_{gov,t}$ turn negative, therefore inflation will drop following the shock.

To understand why a negative $\hat{\xi}_t$ shock is deflationary, note that it has two opposing effects on the intertemporal consolidated budget. On the one hand, the shock will increase the market value of debt outstanding in t through increasing real long bond prices. On the other hand, it will also increase the present value of surpluses that compensate for debt. When $\delta < 1$ the second effect dominates and the shock needs to be 'financed' with deflation for the intertemporal budget to hold. When long bonds are consols, $\delta = 1$, the two effects will cancel out and inflation will be zero, as was shown in subsection 3.4.

The previous discussion concerning how parameters λ_Y and δ affect the path of inflation continues to apply to the $\hat{\xi}$ shock.

Graphical impulse response analysis. We complement the above formulae with plots showing the impulse responses of macroeconomic variables to the spending and demand shocks under different calibrations of λ_Y and δ. Table 1 reports the assumed numerical values of the model's parameters and the notes of the table briefly explain our calibration targets.

The top panel of Figure 1 shows the responses to the spending shock when $\lambda_Y = 0$. From left to right we plot the response of inflation, output and the nominal interest rate. The blue, red and black lines represent the case of active fiscal policy under $\delta = 0, 0.5, 0.95$ respectively. (For comparison, the cyan line shows the passive fiscal policy outcome.) The bottom panel of the Figure plots the same responses when $\lambda_Y = 0.5$.

The graphs are qualitatively consistent with our previous analytical results. Consider the behavior of the interest rate shown in the top panels. When debt is short term and the planner only cares about smoothing inflation, the nominal rate is kept constant after the spending shock. With long term debt optimal policy increases the interest rate with inflation, the optimal response is equal to δ.

Introducing the objective to stabilize output exerts a significant influence on the path of macroeconomic variables but only when debt is short or medium term. Under long term debt (i.e. $\delta = 0.95$) setting λ_Y to 0 or to 0.5 makes only a small difference to the path of inflation and the nominal rate. This property is intuitive. With long debt inflation rises (almost) permanently to absorb the shock, and the path of output is guaranteed to be smooth regardless of λ_Y.

Figure 2 shows the same objects as Figure 1 for the case of a negative demand shock. The responses of the variables have flipped sign, but the patterns are essentially the same as in Figure 1.

4.1.2 Interest rate smoothing

We now turn to the case where the planner's loss function includes the objective to smooth the nominal interest rate, letting (as before) $\sigma = \lambda_Y = 0$. In the appendix we derive the following

[25] From the steady state intertemporal budget we have $\frac{\overline{s}}{1-\beta} = \frac{\overline{b}}{1-\beta\delta}$ (the present value of the surplus equals the value of debt outstanding). Thus $\overline{s} \leq \overline{b}$ with equality only when $\delta = 1$.

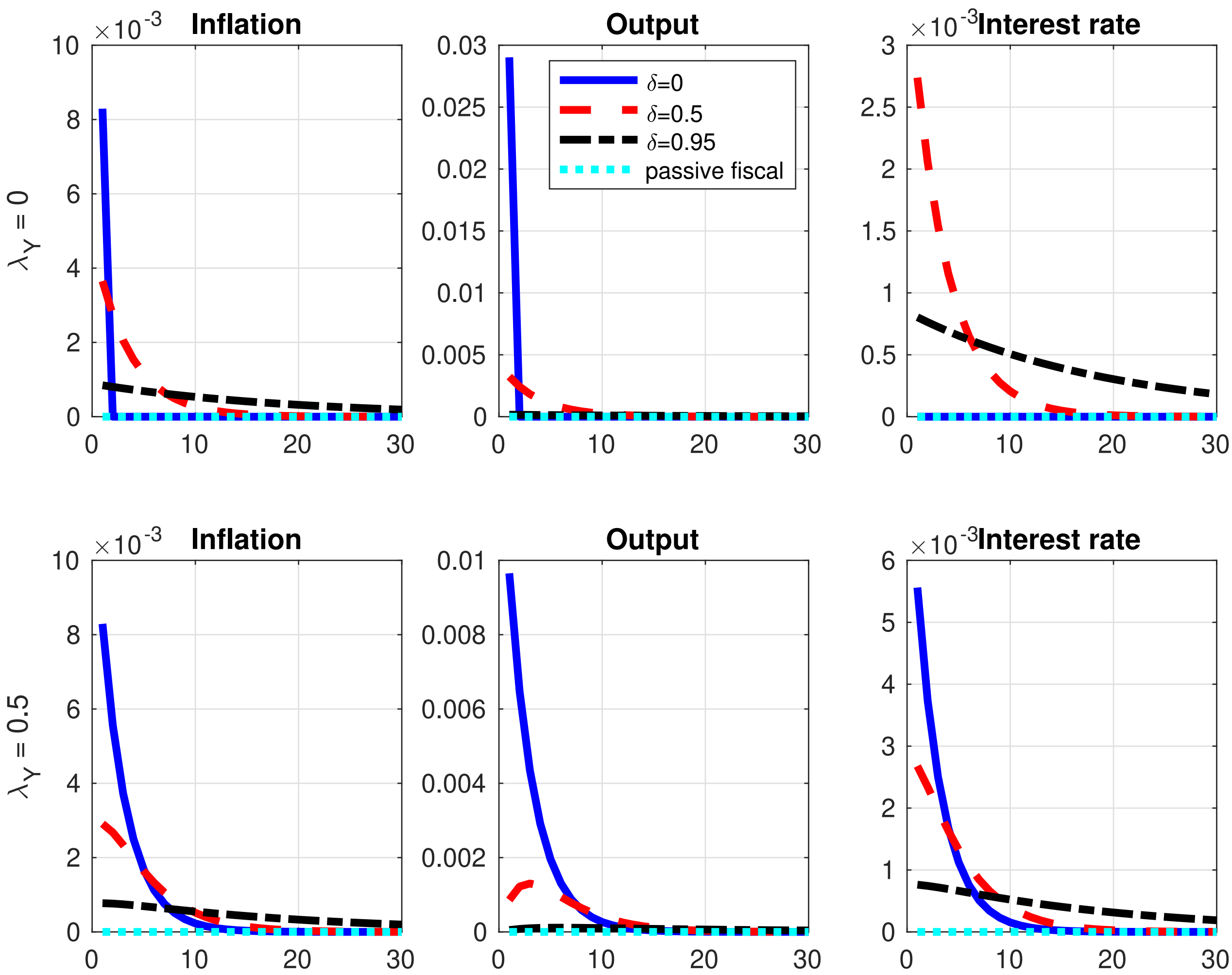
Table 1: **Calibration**

Parameter	Value	Label
β	0.995	Discount factor
λ_Y	$\{0,\ 0.5\}$	Loss function - weight on output
λ_i	$\{0,\ 0.5\}$	Loss function - weight on interest rate
θ	17.5	Price Stickiness
η	-6.88	Elasticity of Demand
σ	$\{0,\ 1\}$	Inverse of IES
γ_h	1	Inverse of Frisch Elasticity
$\overline{b}$	2.4	SS debt Level
$\overline{\tau}$	0.11	SS tax Rate
$\overline{Y}$	1	SS output
$\overline{G}$	0.1	SS gov. spending

Notes: The table reports the values of model parameters assumed in the numerical examples considered in Sections 3 and 4 of the paper. β notes the discount factor chosen to target a steady state (annual) real interest rate of 2 percent. Parameter η is calibrated to target markups of 17 percent in steady state. θ governs the price adjustment cost and is calibrated as in Schmitt-Grohé and Uribe (2004). The steady state level of debt is assumed equal to 60 percent of GDP (at annual horizon), and the level of public spending is 10 percent of aggregate output which is normalized to unity in steady state. The value of the tax rate is such that the steady state government budget constraint holds. We further assumed that inflation at steady state is 0.

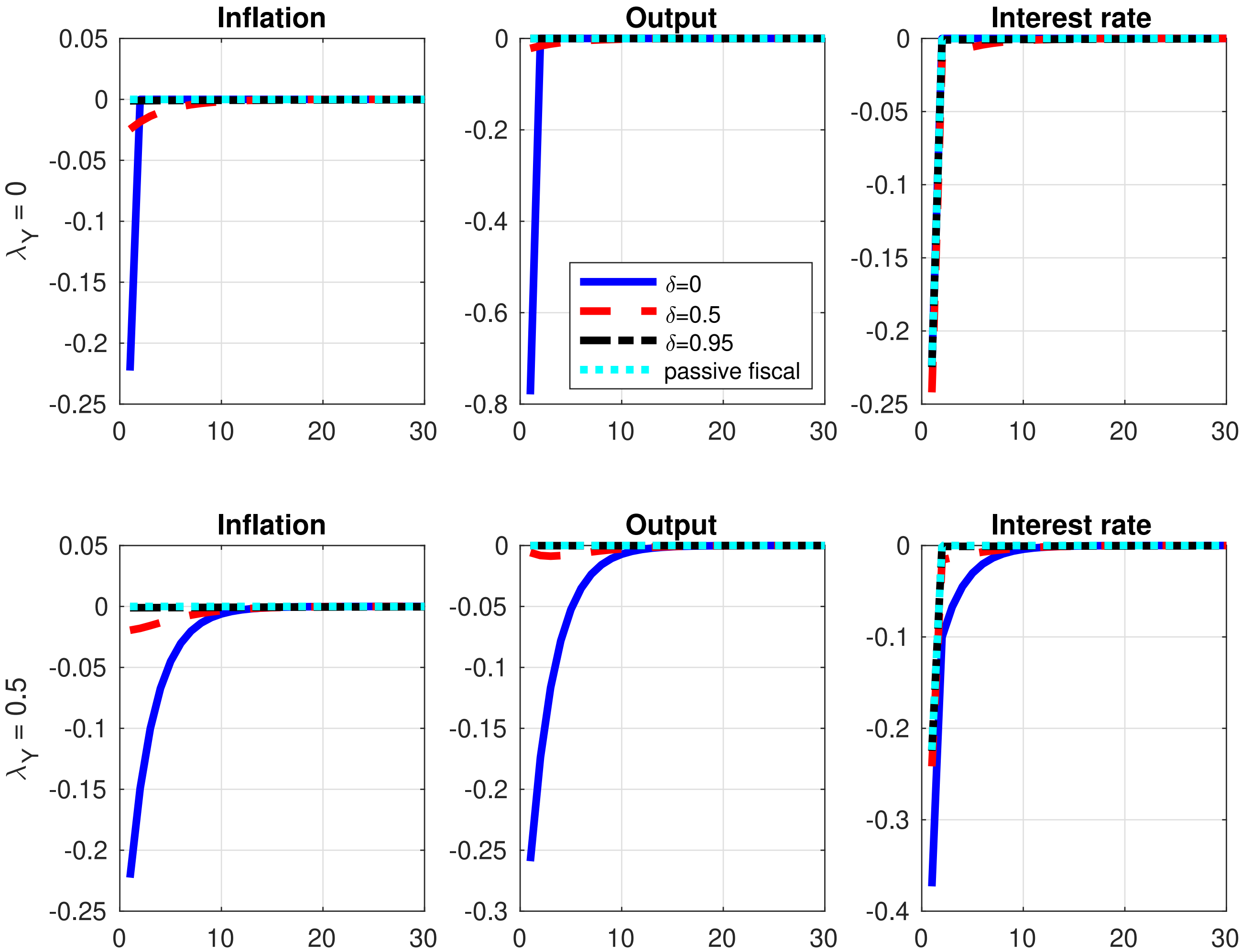
These parameter values are held constant throughout the numerical experiments of Section 4. Parameters λ_Y, λ_i and σ (the relative weights on output and interest rates stabilization and the inverse of the IES respectively) vary across experiments. We set $\sigma = 1$ as our baseline in the canonical New Keynesian model. Moreover, we consider $\lambda_Y \in \{0, 0.5\}$ and $\lambda_i \in \{0, 0.5\}$.

Figure 1: **Impulse response functions, G shock**



Notes: The figure displays the impulse responses of inflation, output, and the nominal interest rate following a government spending shock, in the case where $\lambda_i = \sigma = 0$. Top panels assume $\lambda_Y = 0$, while in the bottom panels we set $\lambda_Y = 0.5$. In each plot, the solid blue line depicts impulse responses in the case where government debt is short term ($\delta = 0$); the dashed red lines and dash-dotted black lines plot the responses of variables when $\delta = 0.5$ and $\delta = 0.95$, respectively. The dotted cyan line considers the case where fiscal policy is passive ($\phi_{\tau,b} > \tilde{\phi}_\tau$).

Figure 2: **Impulse response functions, ξ shock**



Notes: The figure displays the impulse responses of inflation, output, and the nominal interest rate following a negative preference shock (ξ), in the case where $\lambda_i = \sigma = 0$. Top panels assume $\lambda_Y = 0$, while in the bottom panels we set $\lambda_Y = 0.5$. In each plot, the solid blue line depicts impulse responses in the case where government debt is short term ($\delta = 0$); the dashed red lines and dash-dotted black lines plot the responses of variables when $\delta = 0.5$ and $\delta = 0.95$, respectively. The dotted cyan line considers the case where fiscal policy is passive ($\phi_{\tau,b} > \tilde{\phi}_\tau$).

formula characterizing the response of inflation to the two shocks:

$$\hat{\pi}_{t+j} = \left\{ \begin{array}{ll} \frac{\bar{b}}{1-\beta\delta}\Delta\psi_{gov,t} & j=0 \\ \\ -\frac{\lambda_i/\beta}{1+\lambda_i/\beta}\hat{\xi}_t + \frac{\delta}{1+\lambda_i/\beta}\frac{\bar{b}}{1-\beta\delta}\Delta\psi_{gov,t} & j=1 \\ \\ \frac{\delta^j}{1+\lambda_i/\beta}\frac{\bar{b}}{1-\beta\delta}\Delta\psi_{gov,t} & j\geq 2 \end{array} \right\} \tag{37}$$

$$\Delta\psi_{gov,t} = \left[\overline{G}\hat{G}_t + \left(\bar{b} - \bar{s} + \frac{\bar{b}}{1-\beta\delta}\frac{\beta\delta\lambda_i/\beta}{1+\lambda_i/\beta}\right)\hat{\xi}_t\right] \tag{38}$$

Smoothing interest rates has two important effects on optimal policy. First, it makes optimal inflation frontloaded. Following the shock in spending we again have $\Delta\psi_{gov,t} > 0$; however, inflation does not monotonically decline at rate δ throughout the entire path. In period $t+1$ inflation is $\frac{\delta}{1+\lambda_i/\beta}$ times $\hat{\pi}_t$. This is due to the inertial response of interest rates to inflation.

Second, assuming that λ_i is positive introduces an additional source of inflation volatility, the term $-\frac{\lambda_i/\beta}{1+\lambda_i/\beta}\hat{\xi}_t$ in period $t+1$ inflation. Under interest rate smoothing $\hat{i}_t$ does not fully track the real rate $\hat{\xi}_t$. Therefore the shock does not only impact macroeconomic variables through the debt constraint, but also through the Euler equation, the standard demand channel. This effect is concentrated in period $t+1$ since we have assumed an i.i.d shock.

Graphs. The top panels of Figures 3 and 4 plot the responses of macroeconomic variables to the two shocks. We assumed $\lambda_i = 0.5$ to produce these graphs. Moreover, in the bottom panels we assumed that the planner also desires to smooth output setting $\lambda_Y = 0.5$.

Consider first the top panel of Figure 3. It is obvious that the desire to smooth interest rates affects optimal policy when the maturity of debt is long.[26] Consistently with formula (37), inflation is now frontloaded, displaying a sharp drop between t and $t+1$ when $\delta > 0$. The response of output is analogous, displaying a sharp increase in the period the shock occurs.

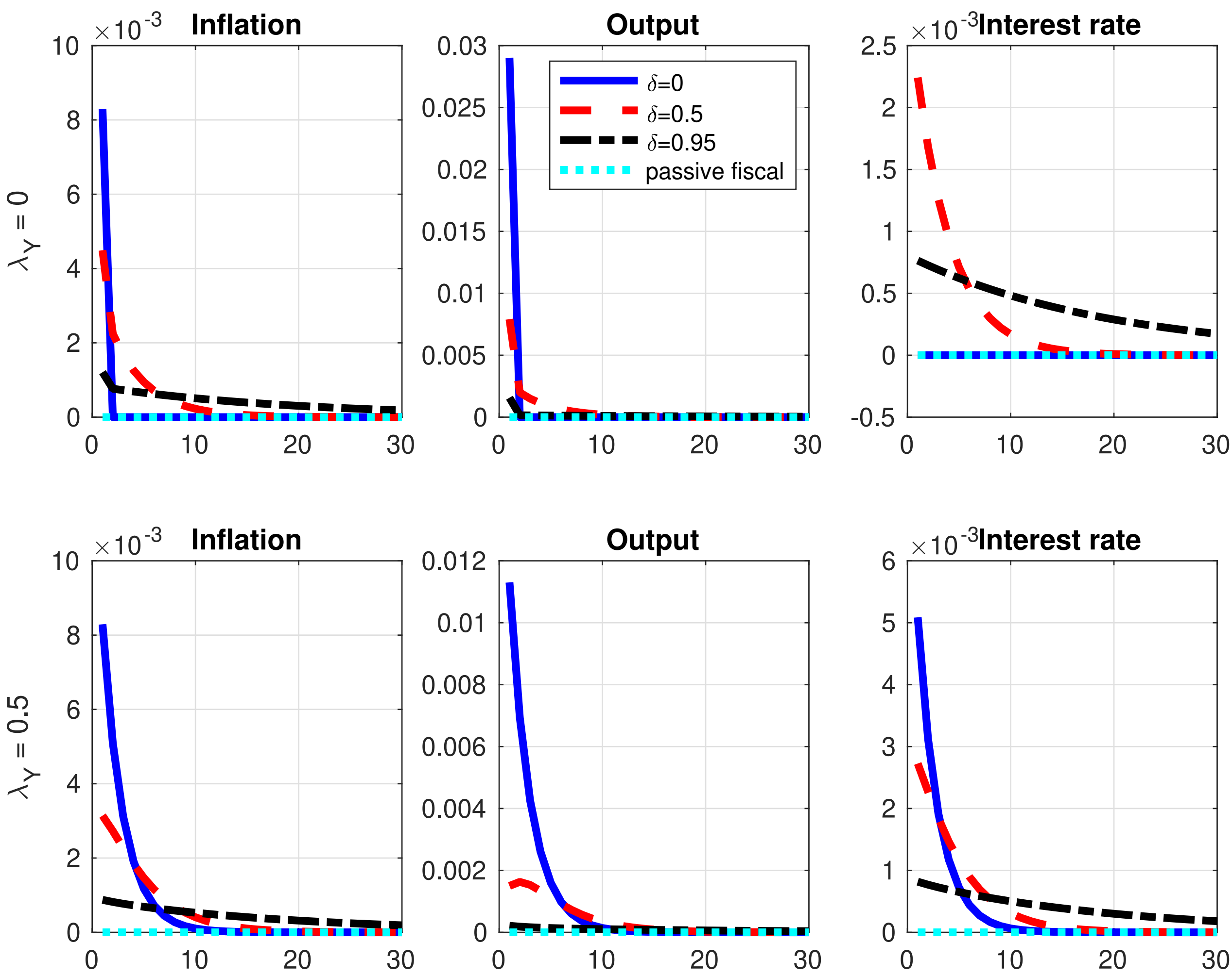
The bottom panel shows that these properties do not carry over to the case where $\lambda_Y = 0.5$. Introducing output smoothing in the policy objective restores the smooth paths of inflation and output we had seen in previous graphs. This is an interesting property: It suggests that the objective to smooth output fluctuations overrides the objective to smooth the nominal rate, in this calibrated version of the model.

Let us turn to the responses to the demand shock, shown in Figure 4. In the top panel of the Figure we see that inflation turns positive in period $t+1$ and is negative in all other periods. Therefore, the negative $\hat{\xi}_t$ shock implies (as before) that $\Delta\psi_{gov,t} < 0$ (implying that a drop in the price level is needed to satisfy the debt constraint) but the positive impact of the demand shock on inflation dominates in $t+1$.

Moreover, assuming long term debt reduces the variability of inflation. Note that with interest rate smoothing this is not an obvious finding, and therefore it is worth commenting on. Recall that long term debt stabilizes inflation through two channels: First, it enables to spread inflation over a longer term, relying on both current and future price changes to adjust the real value of debt when the shock hits and second, debt acts as a shock absorber when changes in real long bond prices compensate for the change in the present value of the government's surplus.

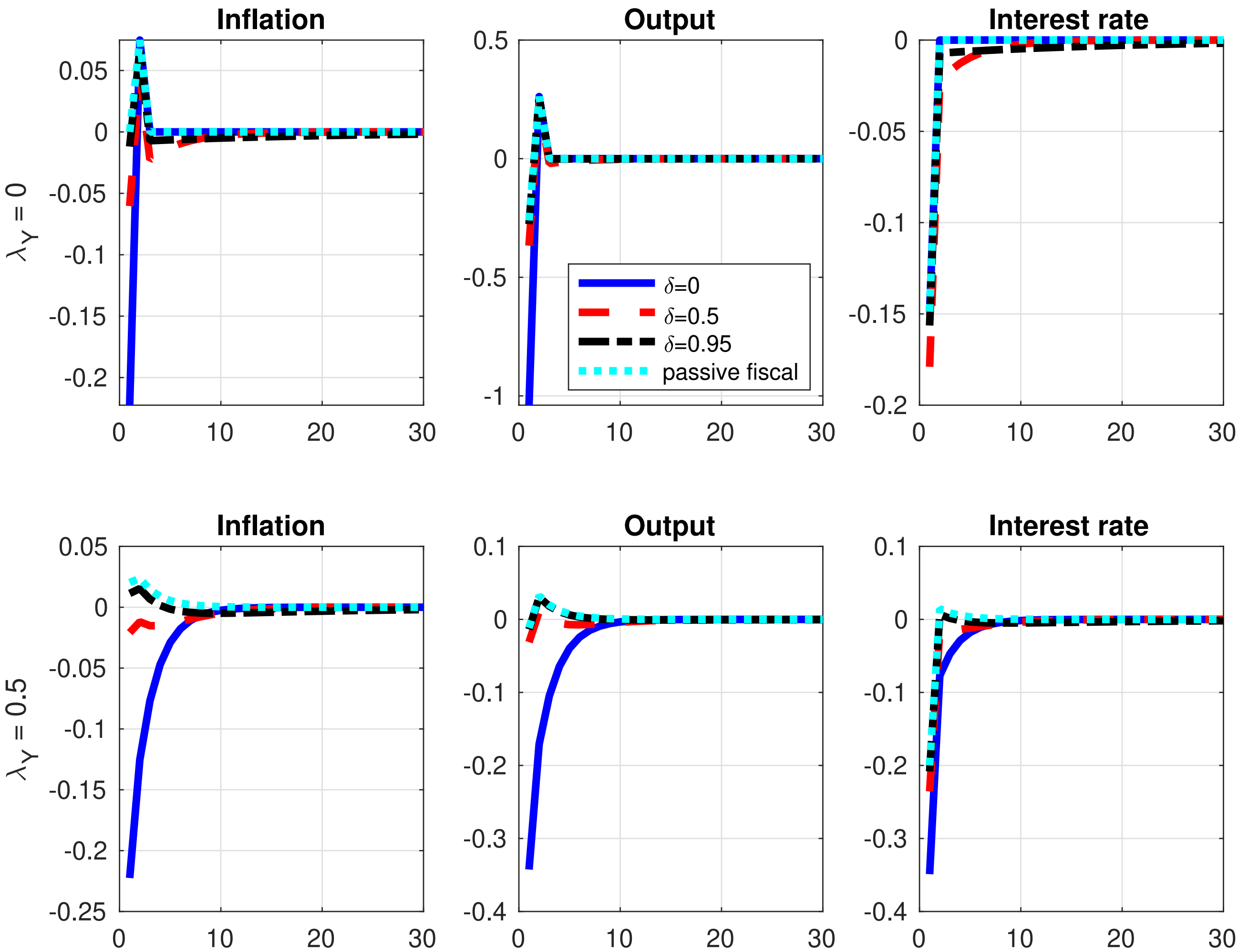
[26] With short term debt, the optimal policy keeps the nominal rate constant in response to a fiscal shock and so there is perfect interest rate smoothing. We thus get the same responses independent of λ_i.

Figure 3: **Impulse response functions with interest rate smoothing (G shock)**



Notes: The figure displays the impulse responses of inflation, output, and the nominal interest rate following a government spending shock, in the case where $\lambda_i = 0.5$ and $\sigma = 0$. Top panels assume $\lambda_Y = 0$, while in the bottom panels we set $\lambda_Y = 0.5$. In each plot, the solid blue line depicts impulse responses in the case where government debt is short term ($\delta = 0$); the dashed red lines and dash-dotted black lines plot the responses of variables when $\delta = 0.5$ and $\delta = 0.95$, respectively. The dotted cyan line considers the case where fiscal policy is passive ($\phi_{\tau,b} > \tilde{\phi}_\tau$).

Figure 4: **Impulse response functions with interest rate smoothing (ξ shock)**



Notes: The figure displays the impulse responses of inflation, output, and the nominal interest rate following a negative demand shock (ξ), in the case where $\lambda_i = 0.5$ and $\sigma = 0$. Top panels assume $\lambda_Y = 0$, while in the bottom panels we set $\lambda_Y = 0.5$. In each plot, the solid blue line depicts impulse responses in the case where government debt is short term ($\delta = 0$); the dashed red lines and dash-dotted black lines plot the responses of variables when $\delta = 0.5$ and $\delta = 0.95$, respectively. The dotted cyan line considers the case where fiscal policy is passive ($\phi_{\tau,b} > \tilde{\phi}_\tau$).

With interest rate smoothing the first channel may not reduce the variability of inflation, and in fact it may even increase it. Since the price level must drop for the intertemporal budget to hold, however inflation is positive in $t+1$, issuing long debt (setting $\delta > 0$) gives a higher weight to $\hat{\pi}_{t+1}$ and so inflation must be more negative in other periods to satisfy the intertemporal constraint. This is not what we find in Figure 4. Therefore, it must be that the second channel (debt is a shock absorber) is more powerful.[27]

Finally, consider the bottom part of Figure 4. Assuming $\lambda_Y > 0$ again produces a much smoother response of inflation to the shock, most notably inflation does not switch sign between periods t and $t+1$. Interestingly, with long debt maturity inflation is positive when the shock hits and remains positive for a few periods, before turning persistently negative around period $t+10$. When debt is long term the planner can leverage on future deflation to satisfy the intertemporal debt constraint. In contrast, when debt is short or medium term inflation must be negative throughout the entire path for the intertemporal budget to hold.[28]

4.1.3 The canonical New Keynesian model ($\sigma > 0$)

We now turn to the case where $\sigma > 0$. In the appendix we show that the following formula characterizes the path of inflation under active fiscal policy and $\lambda_Y = \lambda_i = 0$:

$$\hat{\pi}_{t+j} = \left\{ \begin{array}{ll} \left[\frac{\overline{b}}{1-\beta\delta} + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\overline{b}-\overline{s})\right]\Delta\psi_{gov,t} & j=0 \\ \\ \left[\frac{\overline{b}}{1-\beta\delta}\delta^j - \frac{\sigma}{\kappa_1}(1-\delta)\delta^{j-1}\frac{\overline{Y}}{\overline{C}}\overline{b}\right]\Delta\psi_{gov,t} & j \geq 1 \end{array} \right\} \tag{40}$$

where

$$\Delta\psi_{gov,t} = \widetilde{\epsilon}\left[(\overline{G} + (\overline{b}-\overline{s})\sigma\frac{\overline{G}}{\overline{C}})\hat{G}_t + (\overline{b}-\overline{s})\hat{\xi}_t\right] \tag{41}$$

and $\widetilde{\epsilon} > 0$.[29]

Let us explain this formula starting from the impact effect of a shock on inflation. In (40) there are two main channels driving period t inflation: The term $\frac{\overline{b}}{1-\beta\delta}$ measures the *direct effect* on the real payout of all outstanding debt. An increase in inflation in t lowers the real value of the entire stream of payments, i.e. $\overline{b}, \overline{b}\beta\delta, \overline{b}(\beta\delta)^2, ...$

The second term represents the *indirect effect* of inflation, through output, on the intertemporal budget constraint. Since we now assume $\sigma > 0$ the path of output affects the constraint through two channels: First, through impacting real bond prices it impacts the real value of debt (this is measured by the term $\overline{b}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}$); second, output also affects the value of the government's surplus (the term $\overline{s}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}$).

[27] Equation (38) can be used to show that in order to fully absorb the shock δ must exceed 1. To see this, note that setting

$$\overline{b} = \frac{\overline{s}}{1 + \frac{1}{1-\beta\delta}\frac{\beta\delta\lambda_i/\beta}{1+\lambda_i/\beta}} < \overline{s} \tag{39}$$

is required to have $\Delta\psi_{gov,t} = 0$. From the steady state intertemporal budget we have: $\overline{b} = \frac{1-\beta\delta}{1-\beta}\overline{s}$. Thus, $\overline{b} < \overline{s}$ can only hold when $\delta > 1$.

[28] Issuing long debt is preferable also in this case. The reader can be sure of this by noting that the black dotted line is closest to the cyan dashed line that represents the passive fiscal outcome. Obviously, under passive fiscal policy we have $\Delta\psi_{gov,t} = 0$.

[29] The expression for $\widetilde{\epsilon}$ is cumbersome and for the sake of the exposition is left to the appendix.

Next, consider inflation after t. Again we have two distinct channels: The direct effect of inflation $\frac{\bar{b}}{1-\beta\delta}\delta^j$ (the magnitude of which decays at the coupon rate since $\hat{\pi}_{t+j}$ affects the real value of payments to be made in $t+j$ and thereafter) and the indirect effect through output, $-\frac{\sigma}{\kappa_1}(1-\delta)\delta^{j-1}\frac{\overline{Y}}{\overline{\overline{C}}}\bar{b}$.

Let us focus on the new indirect output channel in t and after t and note that what is particularly striking here is that whereas in t the output term contributes positively to the variability of inflation (i.e. it holds that $\bar{b} > \bar{s}$) after t the sign switches and the effect becomes negative.

To understand this consider again the intertemporal budget constraint (13). Assume $\delta = 0$ and for simplicity let us also assume that only spending shocks can hit the economy. Then, (13) can be written as:[30]

$$-\overline{G}\hat{G}_t + \bar{s}\frac{\overline{G}}{\overline{\overline{C}}}\sigma\hat{G}_t - \bar{s}\frac{\overline{Y}}{\overline{\overline{C}}}\sigma\hat{Y}_t + E_t\sum_{j=1}^{\infty}\beta^j\bar{s}\hat{S}_{t+j} = \bar{b}\hat{b}_{t-1,\delta} - \bar{b}\left[\sigma\left(\frac{\overline{Y}}{\overline{\overline{C}}}\hat{Y}_t - \frac{\overline{G}}{\overline{\overline{C}}}\hat{G}_t\right) + \hat{\pi}_t\right] \tag{42}$$

With short term debt the RHS of the constraint decreases in period t inflation and output. The LHS also decreases in output, but the effect on the RHS dominates.[31] Thus, following a positive spending shock in t, the planner will increase inflation and hence output to satisfy the constraint. The dependence of the importance of this channel on parameter κ_1 follows from the Phillips curve.

Notice however that even though it is only date t variables that matter here for fiscal solvency, equation (40) suggests that inflation will continue responding to the shock in $t+1$ but it will turn negative.[32] This is so because of the Phillips curve: to engineer a bigger increase in date t output, it is optimal to let prices fall in $t+1$. Basically, the planner is trying to shift some of the burden of the adjustment from inflation to output. Reducing output volatility is not an objective and, moreover, the distortions stemming from inflation are convex and so the planner is willing to tolerate a slightly negative inflation rate in $t+1$ in exchange for a smaller inflation rate in t.

When debt is long term the effect of output on debt solvency is not concentrated in t. The RHS of the inter-temporal budget will also depend on output in $t+1, t+2$, since real long bond prices will depend on output. This effect is negative and so the planner will target a path of output so that

[30] We have imposed that taxes are constant.

[31] Note that for brevity we wrote the term $E_t\sum_{j=1}^{\infty}\beta^j\bar{s}\hat{S}_{t+j}$ on the left hand side of (42). This term, though clearly a function of future output, can be expressed as a function of current output and inflation.

Recall that $\bar{s}\hat{S}_{t+j} = -\bar{s}\sigma\frac{\overline{Y}}{\overline{\overline{C}}}\hat{Y}_{t+j} - (\overline{G} - \bar{s}\sigma\frac{\overline{G}}{\overline{\overline{C}}})\hat{G}_{t+j} + \bar{s}\hat{\xi}_{t+j}$. Then, dropping the shocks, as these will cancel out due to the i.i.d processes, we can focus on $E_t\sum_{j=1}^{\infty}\beta^j\bar{s}\hat{S}_{t+j} = -E_t\sum_{j=1}^{\infty}\beta^j\bar{s}\sigma\frac{\overline{Y}}{\overline{\overline{C}}}\hat{Y}_{t+j}$. Using the Phillips curve we have:

$$-E_t\sum_{j=1}^{\infty}\beta^j\bar{s}\sigma\frac{\overline{Y}}{\overline{\overline{C}}}\hat{Y}_{t+j} = -\bar{s}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}}\beta E_t\hat{\pi}_{t+1} = -\bar{s}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}}(\hat{\pi}_t - \kappa_1\hat{Y}_t + \kappa_2\hat{G}_t)$$

which makes evident that only current output and inflation influence this term.

Intuitively, output exerts and influence here, because it affects the real interest rates and hence the value of the given stream of surpluses $\bar{s}$. It is an effect analogous to what Leeper and Zhou (2021) label a 'discount factor' effect of policy. Whereas in their model this channel concerns both inflation and tax policies, here it is a pure inflation effect.

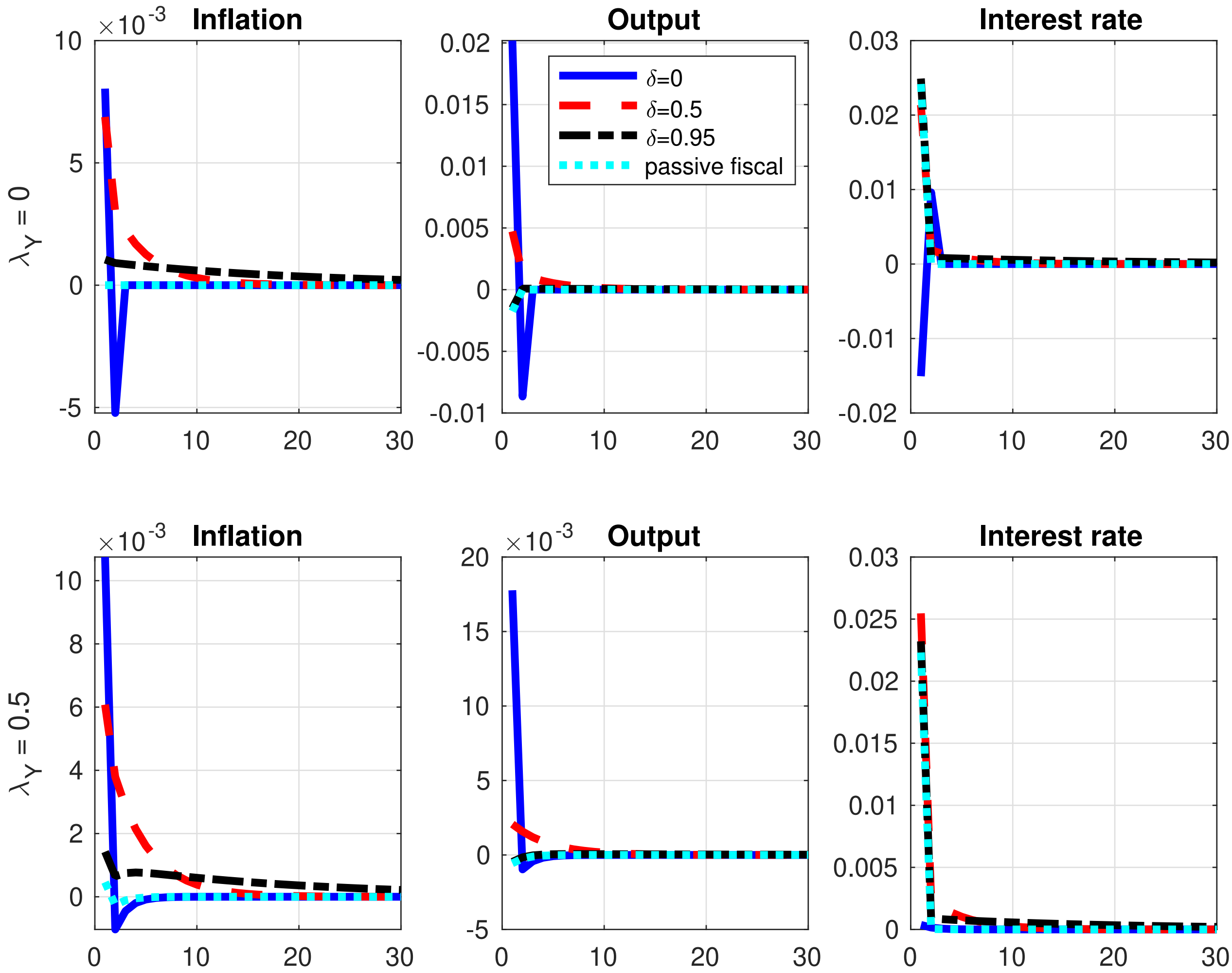
Notice also that with these derivations, on the LHS of (42) we will get:

$$-\sigma\frac{\overline{Y}}{\overline{\overline{C}}}\hat{Y}_t + E_t\sum_{j=1}^{\infty}\beta^j\bar{s}\hat{S}_{t+j} = -\bar{s}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}}(\hat{\pi}_t + \kappa_2\hat{G}_t)$$

This term captures the indirect effect of inflation, when we use the Phillips curve to substitute out output.

[32] Equation (40) suggests that $\hat{\pi}_{t+1} = -\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}}\bar{b}\Delta\psi_{gov,t}$

Figure 5: **Impulse response functions with $\sigma > 0$ (G shock)**



Notes: The figure displays the impulse responses of inflation, output, and the nominal interest rate following a government spending shock, in the case where $\lambda_i = 0$ and $\sigma = 1$. Top panels assume $\lambda_Y = 0$, while in the bottom panels we set $\lambda_Y = 0.5$. In each plot, the solid blue line depicts impulse responses in the case where government debt is short term ($\delta = 0$); the dashed red lines and dash-dotted black lines plot the responses of variables when $\delta = 0.5$ and $\delta = 0.95$, respectively. The dotted cyan line considers the case where fiscal policy is passive ($\phi_{\tau,b} > \tilde{\phi}_\tau$).

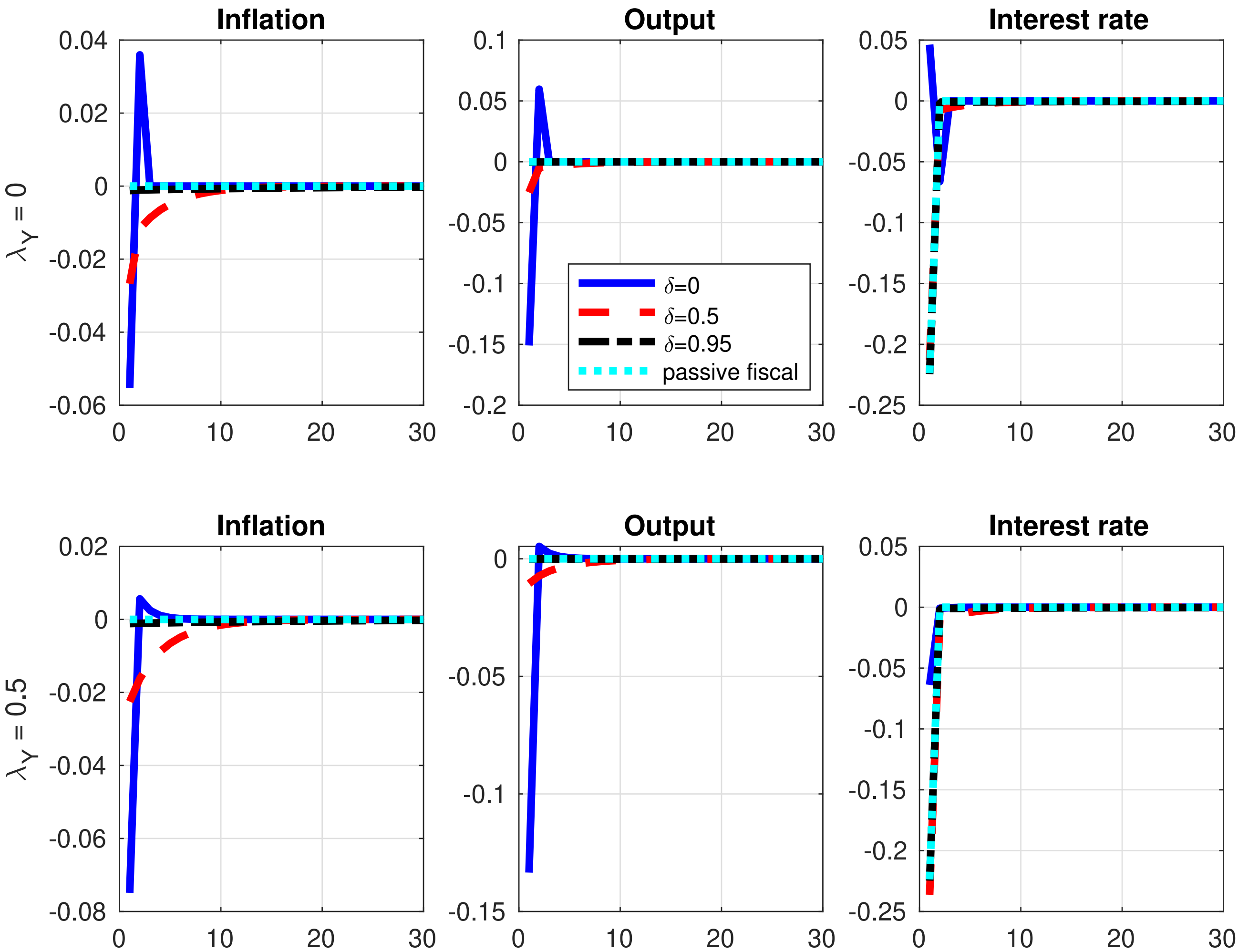
the change in the prices absorbs part of the shock. Following a positive shock to spending, it now becomes optimal to target a higher output level after t to engineer a sharper drop in bond prices.

Note that this basically explains why the term $-\frac{\sigma}{\kappa_1}(1-\delta)\delta^{j-1}\frac{\overline{Y}}{\overline{C}}\overline{b}$ becomes less important as δ increases. The planner will not allow output to fall in $t+1$ to strengthen the increase in output at date t. This also applies to periods after t.

The direct and indirect channels of inflation are present in the optimal interest rate rule we derived for this model, in Proposition 6. The optimal inflation coefficient is equal to $\delta + \frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)$ echoing the principle that the optimal policy is mindful not only of the direct impact of inflation on the real debt, but also of the output impact. Moreover, when debt is long term then the indirect output channel becomes less important in the interest rate rule. For example, when δ is close to 1, $\frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)$ will approximately be 0, and the optimal policy can be approximated by a simpler rule setting the inflation coefficient equal to δ. We will return to this implication of the model below.

Graphs. Figures 5 and 6 plot the usual IRFS. Focus on the top panel in Figure 5 to continue

Figure 6: **Impulse response functions with $\sigma > 0$ (ξ shock)**



Notes: The figure displays the impulse responses of inflation, output, and the nominal interest rate following a negative demand shock (ξ), in the case where $\lambda_i = 0$ and $\sigma = 1$. Top panels assume $\lambda_Y = 0$, while in the bottom panels we set $\lambda_Y = 0.5$. In each plot, the solid blue line depicts impulse responses in the case where government debt is short term ($\delta = 0$); the dashed red lines and dash-dotted black lines plot the responses of variables when $\delta = 0.5$ and $\delta = 0.95$, respectively. The dotted cyan line considers the case where fiscal policy is passive ($\phi_{\tau,b} > \tilde{\phi}_\tau$).

studying the effect of the spending shock when $\lambda_Y = 0$. As can be seen from the Figure, when debt is short term, output and inflation increase contemporaneously with the shock and subsequently drop in $t+1$. This is obviously consistent with equation (40). The nominal interest rate drops initially and turns positive in $t+1$. From equation (32), we know that this is due to monetary policy operating through the indirect output effect, when $\delta = 0$.

With long maturity debt ($\delta = 0.95$) optimal policy targets a smooth path of inflation and output. The nominal rate rises when the shock hits (responding to $\hat{r}_t^n$ in (32)) and subsequently reacts to inflation only; the inflation coefficient is now approximately equal to δ.

These results continue being relevant in the case $\lambda_Y > 0$ studied in the bottom panels of Figures 5 and 6. In this case the indirect output channel becomes somewhat less important under short term debt, since targeting a smoother path makes increasing output in t to absorb the spending shock (Figure 5) or reducing it to absorb the preference shock (Figure 6) less desirable. Nonetheless, the indirect effect through output remains an active channel of policy.

4.2 Optimal Policy with Simpler Rules

We have now analyzed the key forces behind optimal monetary policy in various versions of the model. As we have seen, assuming that the monetary authority desires to smooth inflation and output introduces inertia in the optimal interest rates, $\hat{i}_t$ needs to respond not only to current but also to lagged inflation. Moreover, in the canonical model with $\sigma > 0$ where optimal monetary policy set the interest rate as a function of current inflation only, the optimal inflation coefficient reflected both the direct effect of inflation on real debt and the indirect effect through output on bond prices and on the debt constraint. Finally, our results in Section 3 demonstrated that in both of these cases optimal interest rate rules featured stochastic intercepts, terms involving the multiplier $\Delta\psi_{gov,t}$.

In this paragraph we examine more closely these results focusing on a plausibly calibrated value of δ and show that even the simplest inflation targeting rule setting the inflation coefficient equal to δ can approximate the Ramsey outcomes very well. Our argument is built in two layers; first, we demonstrate that stochastic intercepts are not essential and without them the paths of inflation, output and interest rates are close to their optimal policy analogues. Second, we show that when $\delta = 0.95$ (implying a 5 year average maturity of debt, a reasonable calibration for the US data) then not accounting for indirect output effects or lagged inflation does not change dramatically the behavior of macroeconomic variables. For brevity, we focus only on the case of spending shocks, however, our results also apply to demand driven fluctuations.[33]

4.2.1 The (un)importance of stochastic intercepts.

Let us first consider the model of subsection 3.2.2 and in particular equation (29) characterizing optimal policy when $\lambda_Y > 0$. Using also the analytical formula for $\Delta\psi_{gov,t}$ in equation (36) we can write:

$$-\delta \frac{\widetilde{o}}{\widetilde{\lambda}_2 - 1} \Delta\psi_{gov,t} = -\delta \frac{\widetilde{o}}{\widetilde{\lambda}_2 - 1} \widetilde{\psi} \overline{G} \hat{G}_t$$

Consider a positive fiscal shock. In response to this shock optimal policy will keep the nominal rate slightly lower in t than the value implied by $(\widetilde{\lambda}_1 + \delta)\hat{\pi}_t - \widetilde{\lambda}_1 \delta \hat{\pi}_{t-1}$ to target a more gradual response

[33] Note also that we focus on the case of no interest rate smoothing. Assuming a dual objective to stabilize inflation and output is most common in the literature and it would also correspond to the objective we could derive as an approximation to the household's utility. Though, as discussed, our purpose is not interpreting as such the central bank's loss function, being able to include this case seems important.

For the interest rate smoothing scenario, the numerical results of subsection 3.3 and in the appendix provide a glimpse into how the interest rate responds to macroeconomic variables when $\lambda_Y, \lambda_i, \sigma > 0$.

of output to the shock. Note that this is driven only by the desire to smooth output when debt is long term. When $\lambda_Y = 0$ and/or $\delta = 0$ the stochastic intercept does not influence optimal policy.[34]

Figure 7 shows the responses of macroeconomic variables to the spending shock. The solid blue lines are the Ramsey outcomes, the dashed red lines are the solution to a model where monetary policy sets interest rates as in (29) but without the stochastic intercept. The top panel corresponds to short term debt. The middle and bottom panels set $\delta = 0.5$ and $\delta = 0.95$ respectively. As expected, the stochastic intercept has no bearing on the outcomes in the case of short debt. However, it is also noteworthy that even when we assume long term debt the differences between the Ramsey outcome and the model without the intercept are insignificant. We therefore conclude that in this model the stochastic intercept is not an important aspect of optimal policy.

Next, consider the model of subsection 3.2.3 (setting $\sigma > 0$) focusing on equation (32), noting that we can again express the stochastic intercept $-f\Delta\psi_{gov,t}$ as a function of the spending shock (e.g. equation (41)). Once again this term will keep the nominal rate lower in t following a positive innovation to government spending. The effect of this channel is revealed in Figure 8 which plots the IRFS under optimal policy and in a model where monetary policy is set according to

$$\hat{i}_t = \hat{r}_t^n + \left(\delta + \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)\right)\hat{\pi}_t$$

The top, middle and bottom panels vary the maturity of debt δ.

Notice that when debt is short or medium term the stochastic intercept does affect optimal policy. Most notably, in the case where $\delta = 0$ the interest rate rule without the intercept predicts that output and inflation increase when the shock hits, but they will not fall in the next period as is the case under the optimal policy. This leads to a smaller increase in output in t which is compensated by a larger response of inflation contemporaneously to the shock, to satisfy the intertemporal budget constraint. Thus, the stochastic intercept enables to reduce the variability of inflation by relying more on a stronger response of output to make debt solvent, when $\lambda_Y = 0$.[35]

However, as is evident from the bottom panel of the Figure, this channel is not at all important when $\delta = 0.95$.

4.2.2 Simple inflation targeting rules

Therefore, simple rules as in (29) and (40) but without the stochastic intercepts can approximate well the optimal policy outcome under the plausible calibration of the debt maturity structure, $\delta = 0.95$. We now insist on this finding to investigate whether even simpler interest rate rules in which the central bank may target only inflation and the inflation coefficient is equal to δ also perform well.

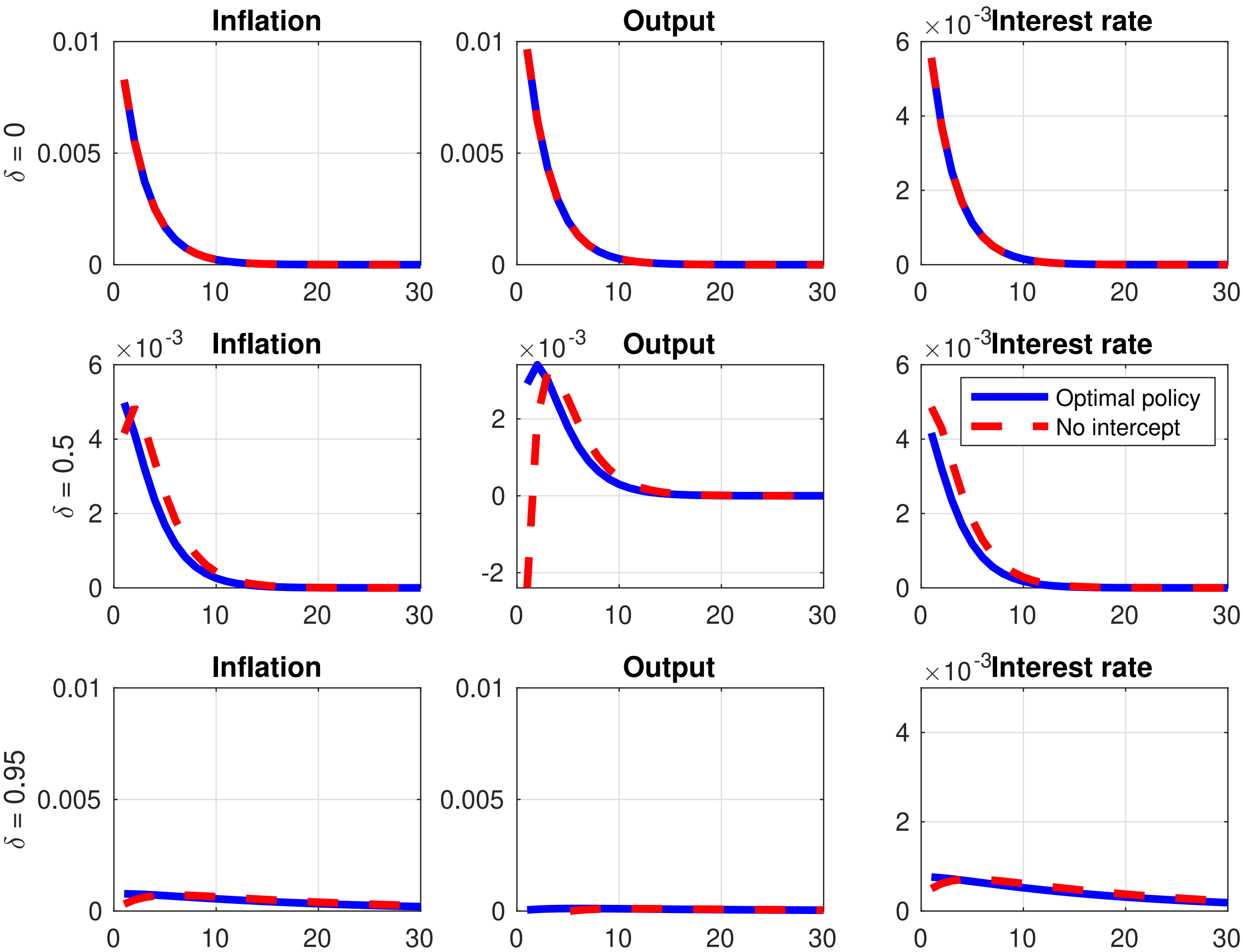
Consider first the case where $\sigma = 1$ (our baseline value for this parameter) and $\lambda_Y = 0$. The top panel of Figure 9 shows the responses under the optimal policy and in the model where the interest rate rule is $\hat{i}_t = \hat{r}_t^n + \delta\hat{\pi}_t$, assuming in both cases a 5 year average maturity of debt. As can be seen from the Figure the impulse responses almost completely overlap.

The intuition for this finding should be clear from our previous discussion. When debt is long term, indirect output effects exert only a small influence on optimal policy, the main driving force is the direct impact of inflation on the real value of debt. For our assumed parameter values we get that $\delta + \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1) = 0.9484$ which is indeed very close to $\delta = 0.95$. Thus, a simpler rule

[34] Evidently, when $\delta = 0$ there is no margin for the planner to smooth the response of output by trading off less inflation in t for more inflation in the future, since only date t inflation can satisfy the intertemporal budget constraint.

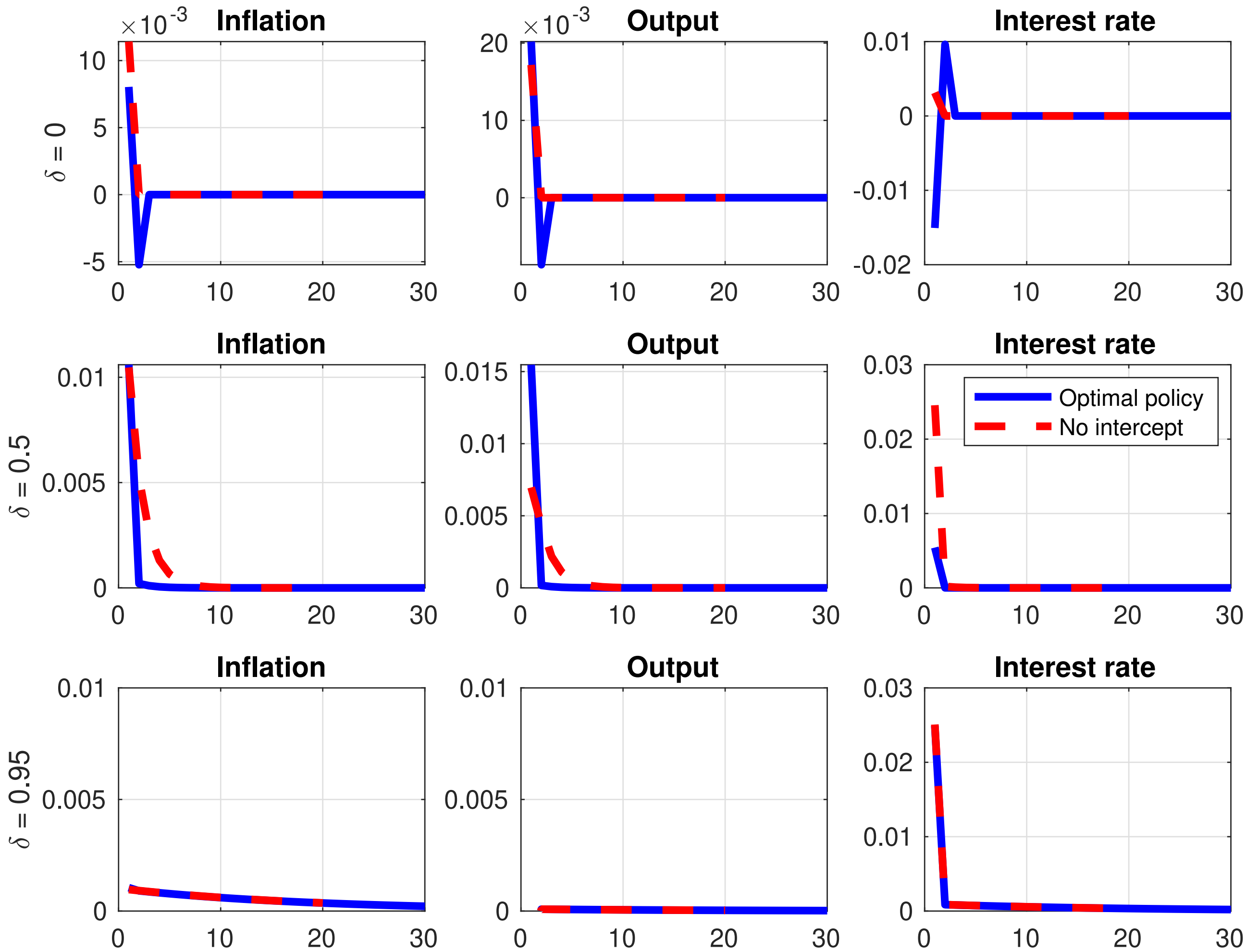
[35] It is interesting to note that even though including the term $-f\Delta\psi_{gov,t}$ can change the optimal path of inflation significantly with short term debt, the gain in terms of the loss function will not be significant. This is intuitive since shifting the burden of the fiscal adjustment to output requires to engineer deflation in $t+1$. This also impinges a loss.

Figure 7: **The role of stochastic intercepts: $\lambda_Y > 0, \sigma = 0$.**



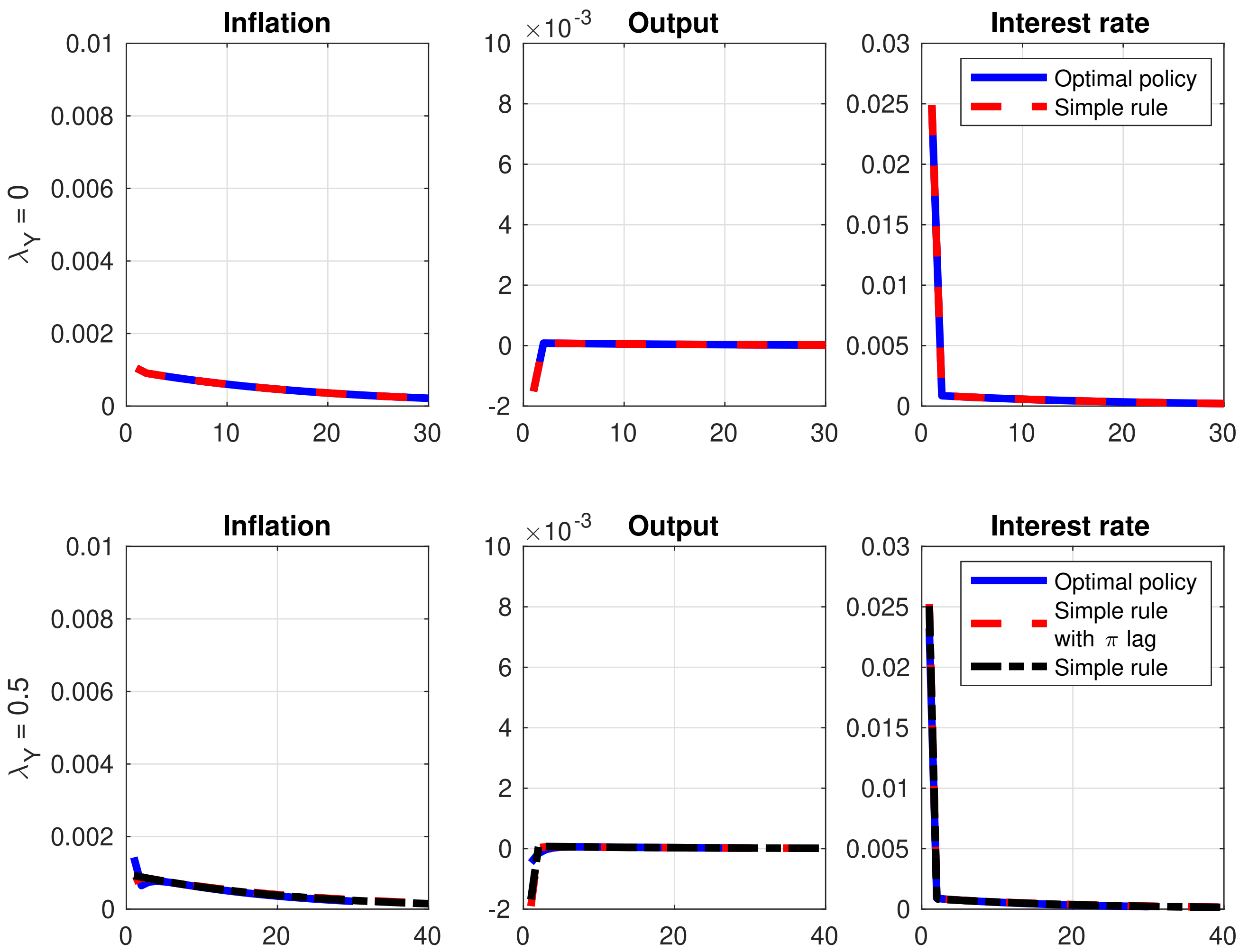
Notes: The figure plots the impulse responses of inflation, output, and the nominal interest rate following a government spending shock, in the full Ramsey solution (solid lines) and in a model without stochastic intercepts (dashed lines). We assume $\lambda_Y = 0.5$ and $\sigma = 0$. The top panel shows the case of short term debt, the middle and bottom panels set $\delta = 0.5, 0.95$ respectively.

Figure 8: **The role of stochastic intercepts:** $\lambda_Y = 0, \sigma > 0$.



Notes: The figure plots the impulse responses of inflation, output, and the nominal interest rate following a government spending shock, in the full Ramsey solution (solid lines) and in a model without stochastic intercepts (dashed lines). We assume $\lambda_Y = 0$ and $\sigma = 1$. The top panel shows the case of short term debt, the middle and bottom panels set $\delta = 0.5, 0.95$ respectively.

Figure 9: **Simple rules vs Ramsey outcome.**



Notes: The figure compares the optimal policy impulse responses of inflation, output, and the nominal interest rate with simple inflation targeting rules. We assume $\delta = 0.95$ in all graphs. The top panel focuses on the case where $\sigma = 1$ and $\lambda_Y = 0$. The solid line is the optimal policy and the dashed line assumes that the inflation coefficient is equal to δ. The bottom panel assumes $\sigma = 1$ and $\lambda_Y = 0.5$. In this case a 'Simple Rule with Inflation Lags' shock, is a rule of the form $\hat{i}_t = \hat{r}_t^n + (\widetilde{\lambda}_1 + \delta)\hat{\pi}_t - \widetilde{\lambda}_1 \delta \hat{\pi}_{t-1}$. The 'Simple Rule' is $\hat{i}_t = \hat{r}_t^n + \delta \hat{\pi}_t$.

that only accounts for the maturity of debt to determine the inflation coefficient, is very close to the fully optimal policy.

Next, consider the case where $\sigma = 1$ and $\lambda_Y = 0.5$. This is shown in the bottom panel of the Figure. The solid blue line shows the optimal policy outcome, whereas the red dashed line is the impulse response when monetary policy is set according to

$$\hat{i}_t = \hat{r}_t^n + (\widetilde{\lambda}_1 + \delta)\hat{\pi}_t - \widetilde{\lambda}_1 \delta \hat{\pi}_{t-1}$$

where $\widetilde{\lambda}_1$ is defined as in subsection 3.2.2. We label this case 'Simple Rule with Inflation Lags' in the Figure. Then, the dashed black line ('Simple Rule') assumes that interest rates follow:

$$\hat{i}_t = \hat{r}_t^n + \delta \hat{\pi}_t$$

The three cases produce similar impulse responses. (Remarkably the simplest policy, setting the inflation coefficient to δ, is even closer to the optimal policy outcome). Intuitively, since debt is long term, the objective to smooth output aligns with the objective to smooth inflation distortions over time. Thus, simply targeting a persistent path of inflation is enough to approximate the fully optimal outcome.

We therefore conclude that when the average debt maturity is calibrated to the US data, the optimal policy in the canonical model and with a dual objective to stabilize inflation and output can be closely approximated by a simple inflation targeting rule with coefficient δ.

4.3 Alternative Shock Structures

We derived, for tractability, our analytical results assuming that spending and demand shocks are i.i.d. Our findings can be easily generalized to the case where spending and demand are first order autoregressive processes. It is indeed possible to show that the interest rate rules we derived in Section 3 are still relevant in this case. For example, consider the model of subsection 3.2.2. With persistent shocks the optimal policy will follow a rule

$$\hat{i}_t = \hat{\xi}_t(1-\rho_\xi) + (\widetilde{\lambda}_1 + \delta)\hat{\pi}_t - \widetilde{\lambda}_1 \delta \hat{\pi}_{t-1} - \delta \frac{\widetilde{o}}{\widetilde{\lambda}_2 - 1} \Delta \psi_{gov,t} \tag{43}$$

where ρ_ξ denotes the demand shock persistence. The values of $\widetilde{\lambda}_1, \widetilde{\lambda}_2, \widetilde{o}$ are the same as before. Evidently, to account for persistent shocks we only need to adjust the leading (real interest rate) term, the optimal coefficients on current and lagged inflation will not need to be changed.

Moreover, in the version of the model studied in subsection 3.2.3 the optimal rule will become:

$$\hat{i}_t = \underbrace{\hat{\xi}_t(1-\rho_\xi) + \sigma\Big(\frac{\overline{G}}{\overline{\overline{C}}} - \frac{\sigma\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}}\Big)\hat{G}_t(1-\rho_G)}_{\hat{r}_t^n} + \left(\delta + \frac{\overline{Y}}{\overline{\overline{C}}}\frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)\right)\hat{\pi}_t - f\Delta\psi_{gov,t} \tag{44}$$

where now ρ_G is the persistence parameter of spending. Once again the inflation coefficient is unaffected by shock persistence.

The principle behind these changes is the following: The leading terms in (43) and (44) have to account for shock persistence since monetary policy uses these terms to eliminate the shocks from the Euler equation. The optimal inflation coefficients are however determined by the debt constraint and the persistence of the shocks will not matter for how inflation will optimally adjust to satisfy this constraint. Shock persistence will affect only the size of the response of inflation and output.

Finally, note that its is also easy to generalize our findings to consider alternative types of shocks, for example considering shocks to government transfers (a common modelling assumption in related

DSGE models) or cost push shocks. Again, the optimal rules will not change in the presence of these alternative shock types. Transfers shocks will only impact the consolidated budget, similar to spending shocks in the Fisherian model we presented. Cost push shocks will be filtered through the consolidated budget and the Phillips curve, parameter λ_Y will determine how much output and inflation stabilization is desired. The usual tradeoff is already present in the policy rules we derived.

5 Distortionary Taxation

Before concluding the paper in the next section we briefly discuss the implications of replacing the assumption that taxes are lump sum with distortionary taxation. Distortionary taxes do not change any of the conclusions we drew from our previous analysis. The model continues to admit two types of equilibria, when fiscal policy is active/passive and moreover, the optimal monetary policy rules are very similar to the rules we derived previously.

More specifically, we show in the appendix that the threshold for passive fiscal policy is now given by:

$$\widetilde{\phi}_\tau \equiv \frac{\overline{b}(1-\beta)}{\overline{r}(1-\beta\delta)\left(\frac{1}{1-\overline{\tau}^d} - \frac{\overline{\tau}^d}{1-\overline{\tau}^d}(1+\frac{1}{\gamma_h+\sigma\frac{\overline{Y}}{\overline{C}}})\right)} \tag{45}$$

where $\overline{r}$ denotes the steady state revenue of the government and $\overline{\tau}^d$ is the distortionary tax rate. Parameters γ_h, σ become important for the threshold $\widetilde{\phi}_\tau$ since they determine the responses of output to shocks and consequently the responses of fiscal revenue.

To show how optimal monetary policy rules change in the presence of distortionary taxes we consider the simplest scenario $\sigma = \lambda_Y = \lambda_i = 0$. We can show that when $\phi_{\tau,b} > \widetilde{\phi}_\tau$ the optimal monetary policy is again a rule of the form (22) and the inflation coefficient exceeds unity. In the active fiscal scenario the optimal interest rate rule is:

$$\hat{i}_t = \xi_t + \delta\hat{\pi}_t - \delta\frac{\overline{r}(1+\gamma_h)}{\kappa_1}\Delta\psi_{gov,t} \tag{46}$$

(see appendix).

Notice that relative to Proposition 3 which characterized the analogous rule under lump sum taxes, assuming distortionary taxation introduces an additional element to policy, the final term on the RHS of (46).

This term relates to the effect of output on fiscal revenue. An increase in inflation will increase current output and increase the government surplus. In response to, say, a fiscal shock that needs to be financed via higher inflation, the planner will also internalize the new revenue effect. According to (46) this will induce to keep the nominal rate slightly lower in t as is indicated by the intercept $-\delta\frac{\overline{r}(1+\gamma_h)}{\kappa_1}\Delta\psi_{gov,t}$.

Evidently, this effect is valid only when debt is long term. With short debt, only period t inflation can finance debt and the magnitude of the response of inflation is pinned down by the intertemporal budget. When $\delta > 0$, then inflation will be slightly more frontloaded in t and decay at rate δ from $t+1$ onwards. The planner tolerates a less smooth inflation path in order to reduce the overall volatility of inflation, taking advantage of the higher revenue that relaxes the intertemporal constraint.

It turns out that this effect is not substantial. One can therefore, drop the stochastic intercept term from the optimal policy rule and end up with very similar dynamics for macroeconomic variables. For the sake of brevity, we leave it to the appendix to plot the impulse response functions for this model. We also derive optimal policy rules for each of the other versions of the model we considered. We prove that our previous results continue to hold.

6 Conclusion

We presented a framework of optimal monetary policy when the central bank may need to take into account the government budget constraint and is thus concerned with the solvency of debt. Our model is tractable and enables us to derive optimal interest rate rules analytically. We exploited this to show how (under various specifications of the model) the monetary policy optimally reacts to inflation. A key result is that when the debt maturity is calibrated to the US data, then a simple inflation targeting rule setting the inflation coefficient equal to $1 - \frac{1}{\text{Maturity}}$ brings us close to the fully optimal policy outcome.

A couple of extensions of this work seem to us fruitful for future research. First, using the framework we proposed to extend the analysis to the case of regime fluctuations, seems a meaningful next step. Second, our analytical results were derived in the baseline New Keynesian framework (augmented with a fiscal block) and we have not explored optimal policy in environments featuring wage rigidities, inflation inertia or the zero lower bound constraint. It would be interesting to extend the approach of this paper to models including these features.

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Appendix

A Proofs

A.1 Proofs of Propositions and Derivations in Sections 3 and 4

We provide proofs for the Propositions made in text. We also derive the analytical formulae shown in Section 4 (the impulse responses of inflation).

A.1.1 Proof of Proposition 1.

From the first order conditions of the planner's program we have:

$$\kappa_1 \Delta\psi_{\pi,t} = -\lambda_Y \Delta\hat{Y}_t + \sigma\frac{\overline{Y}}{\overline{C}}\left(\lambda_i \Delta\hat{i}_t - \frac{\lambda_i \Delta\hat{i}_{t-1}}{\beta}\right)$$
$$+\sigma\frac{\overline{Y}}{\overline{C}}\bar{b}_\delta \sum_{l=0}^{\infty}\delta^l(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}) + \omega_Y \Delta\psi_{gov,t}$$

and the FONC with respect to inflation becomes

$$-\hat{\pi}_t + \frac{1}{\kappa_1}\left[-\lambda_Y \Delta\hat{Y}_t + \sigma\frac{\overline{Y}}{\overline{C}}\left(\lambda_i \Delta\hat{i}_t - \frac{\lambda_i \Delta\hat{i}_{t-1}}{\beta}\right)\right.$$
$$\left.+\sigma\frac{\overline{Y}}{\overline{C}}\bar{b}_\delta \sum_{l=0}^{\infty}\delta^l(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}) + \omega_Y \Delta\psi_{gov,t}\right] - \frac{\lambda_i \hat{i}_{t-1}}{\beta} + \frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} = 0 \tag{47}$$

Rearranging the FONC with respect to $\hat{i}$ we get:

$$\frac{\sigma\lambda_i}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\hat{i}_t = \hat{\pi}_t + \frac{\lambda_Y \Delta\hat{Y}_t}{\kappa_1} + \left(\frac{\sigma\lambda_i}{\kappa_1}\frac{\overline{Y}}{\overline{C}} + \frac{\lambda_i}{\beta}\right)\hat{i}_{t-1} + \frac{\sigma\lambda_i\frac{\overline{Y}}{\overline{C}}\Delta\hat{i}_{t-1}}{\kappa_1\beta}$$
$$-\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}_\delta\sum_{l=0}^{\infty}\delta^l(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}) + \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} + \frac{\bar{b}_\delta}{(1-\beta\delta)}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} \tag{48}$$

Rearranging (48) we can easily get the optimal interest rate rule in Proposition 1.■

A.1.2 Proof of Proposition 4 and derivations for Case 1: $\lambda_i > 0$, $\lambda_Y, \sigma = 0$.

Proof of Proposition 4.

Consider $\lambda_i \geq 0$ and $\lambda_Y = \sigma = 0$ and the case of active monetary policy. Conjecture that optimal policy is a rule of the form

$$\hat{i}_t = \theta\hat{\xi}_t + \widetilde{\lambda}_1\hat{\pi}_t + \lambda_2\hat{i}_{t-1} \tag{49}$$

From the FONC of the Ramsey program, inflation satisfies:

$$\hat{\pi}_t = -\frac{\lambda_i}{\beta}\hat{i}_{t-1} \tag{50}$$

and therefore $\hat{i}_t = \frac{1}{1+\frac{\lambda_i}{\beta}}\hat{\xi}_t$.

Now use (49) and the Euler equation to get:

$$\hat{i}_t = \frac{1}{1-\widetilde{\lambda}_2 L}\left(\theta\hat{\xi}_t + \widetilde{\lambda}_1\hat{\pi}_t\right) = \hat{\xi}_t + E_t\hat{\pi}_{t+1}$$

and when $\widetilde{\lambda}_1 + \widetilde{\lambda}_2 > 1$ we have

$$\hat{\pi}_t = E_t\sum_{j\geq 0}\frac{1}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)^j}\left(\frac{1-\theta}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\hat{\xi}_{t+j} - \frac{\widetilde{\lambda}_2}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\hat{\xi}_{t+j-1}\right) = \frac{1-\theta}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\hat{\xi}_t - \frac{\widetilde{\lambda}_2}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\hat{\xi}_{t-1} - \frac{\widetilde{\lambda}_2}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)^2}\hat{\xi}_t$$

where the final equality uses the assumption that shocks are i.i.d. From this equation, we can easily derive the conditions on parameters $\theta, \widetilde{\lambda}_1, \widetilde{\lambda}_2$ stated in the Proposition.

Now consider the case of passive monetary policy. Optimal inflation satisfies:

$$\hat{\pi}_t = -\frac{\lambda_i}{\beta}\hat{i}_{t-1} + \frac{\bar{b}}{1-\beta\delta}\sum_{l\geq 0}\delta^l\Delta\psi_{gov,t-l}$$

From the Euler equation we have:

$$\begin{aligned}\hat{i}_t = \hat{\xi}_t + E_t\left(-\frac{\lambda_i}{\beta}\hat{i}_t + \frac{\bar{b}}{1-\beta\delta}\sum_{l\geq 0}\delta^l\Delta\psi_{gov,t-l+1}\right) &= \hat{\xi}_t - \frac{\lambda_i}{\beta}\hat{i}_t + \frac{\delta\bar{b}}{1-\beta\delta}\sum_{l\geq 0}\delta^l\Delta\psi_{gov,t-l}\\ &= \hat{\xi}_t - \frac{\lambda_i}{\beta}\hat{i}_t + \delta\left(\hat{\pi}_t + \frac{\lambda_i}{\beta}\hat{i}_{t-1}\right)\end{aligned}$$

Thus :

$$\hat{i}_t = \frac{1}{1+\frac{\lambda_i}{\beta}}\hat{\xi}_t + \frac{\delta}{1+\frac{\lambda_i}{\beta}}\hat{\pi}_t + \frac{\frac{\lambda_i}{\beta}}{1+\frac{\lambda_i}{\beta}}\hat{i}_{t-1}$$

■

Additional derivations: The path of optimal inflation.
Consider now the dynamic response of inflation to shocks in period t. In the active monetary policy case we have:

$$\begin{aligned}\frac{1}{1-\widetilde{\lambda}_2 L}\left(\frac{\hat{\xi}_t}{1+\lambda_i/\beta} + \widetilde{\lambda}_1\hat{\pi}_t\right) &= \hat{\xi}_t + E_t\hat{\pi}_{t+1}\\ \hat{\pi}_t = \frac{1}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\left(\frac{\lambda_i/\beta\hat{\xi}_t}{1+\lambda_i/\beta} - \widetilde{\lambda}_2\hat{\xi}_{t-1}\right) &+ \frac{1}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}E_t\hat{\pi}_{t+1} =\\ -\frac{\widetilde{\lambda}_2}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\hat{\xi}_{t-1} + \frac{1}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\underbrace{\left(\frac{\lambda_i/\beta}{1+\lambda_i/\beta} - \frac{\widetilde{\lambda}_2}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\right)}_{=0}\hat{\xi}_t &= -\frac{\widetilde{\lambda}_2}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\hat{\xi}_{t-1}\end{aligned}$$

Therefore in the case of a single shock in period t we get:

$$\hat{\pi}_{t+j} = 0 \quad \text{for} \quad j \neq 1 \qquad \text{and} \quad \hat{\pi}_{t+1} = -\frac{\widetilde{\lambda}_2}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\hat{\xi}_t$$

Now consider the passive monetary policy equilibrium. Combining the interest rate rule and the Euler equation, and dropping expectations, we get:

$$(\hat{\pi}_{t+1}+\hat{\xi}_t)(1-\frac{\delta\lambda_i/\beta}{1+\lambda_i/\beta}L)=\hat{\xi}_t\frac{1}{1+\lambda_i/\beta}+\frac{\delta}{1+\lambda_i/\beta}\hat{\pi}_t$$

or

$$\hat{\pi}_{t+1}=\delta\hat{\pi}_t-\frac{\lambda_i/\beta}{1+\lambda_i/\beta}\hat{\xi}_t+\frac{\delta\lambda_i/\beta}{1+\lambda_i/\beta}\hat{\xi}_{t-1} \tag{51}$$

To derive the reaction of inflation to the shocks in t we use the intertemporal consolidated budget constraint. Under the assumed parameter values this can be written as:

$$-\overline{G}\hat{G}_t+\overline{s}\hat{\xi}_t=-\overline{b}\sum_{j\geq 0}(\beta\delta)^j\sum_{l=0}^{j}\hat{\pi}_{t+l}+\overline{b}\hat{\xi}_t$$

(To derive this equation we assumed no shocks before t and $\hat{b}_{t-1,\delta}=0$. The focus here is to derive the impulse response of inflation to a one off shock).

Generalizing (51) to $t+j$ we can easily show that $\hat{\pi}_{t+1}=\delta^j\hat{\pi}_t$ when $j>1$. With that we can write the intertemporal constraint as:

$$-\overline{G}\hat{G}_t+\ \hat{\xi}_t=-\frac{\overline{b}}{(1-\beta\delta)(1-\beta\delta^2)}\hat{\pi}_t+\frac{\beta\delta\overline{b}}{(1-\beta\delta)}\frac{\lambda_i/\beta}{1+\lambda_i/\beta}\hat{\xi}_t+\overline{b}\hat{\xi}_t$$

Solving for $\hat{\pi}_t$ and using (51) we can solve for the entire path of inflation. Consider first the response to the spending shock:

$$\hat{\pi}_{t+j}=\delta^j(1-\beta\delta)(1-\beta\delta^2)\frac{\overline{G}\hat{G}_t}{\overline{b}}$$

and clearly the impulse response is independent of λ_i. For the demand shock we have:

$$\hat{\pi}_t=\frac{(1-\beta\delta)(1-\beta\delta^2)}{\overline{b}}\left(\overline{b}-\overline{s}+\frac{\beta\delta\overline{b}}{(1-\beta\delta)}\frac{\lambda_i/\beta}{1+\lambda_i/\beta}\right)\hat{\xi}_t$$

$$\hat{\pi}_{t+1}=\delta\frac{(1-\beta\delta)(1-\beta\delta^2)}{\overline{b}}\left(\overline{b}-\overline{s}+\frac{\beta\delta\overline{b}}{(1-\beta\delta)}\frac{\lambda_i/\beta}{1+\lambda_i/\beta}\right)\hat{\xi}_t-\frac{\lambda_i/\beta}{1+\lambda_i/\beta}\hat{\xi}_t$$

$$\hat{\pi}_{t+j}=\delta^j\frac{(1-\beta\delta)(1-\beta\delta^2)}{\overline{b}}\left(\overline{b}-\overline{s}+\frac{\beta\delta\overline{b}}{(1-\beta\delta)}\frac{\lambda_i/\beta}{1+\lambda_i/\beta}\right)\hat{\xi}_t$$

A.1.3 Proof of Proposition 5 and derivations for Case 2: $\lambda_i=0$ and $\lambda_Y>0$

The FONC for inflation and output combined give us the following condition:

$$-\hat{\pi}_t-\frac{\lambda_Y}{\kappa_1}\Delta\hat{Y}_t+\frac{\overline{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-l}=0$$

Using the Phillips curve, we can write:

$$-\hat{\pi}_t-\frac{\lambda_Y}{\kappa_1^2}(\hat{\pi}_t-\beta E_t\hat{\pi}_{t+1})+\frac{\lambda_Y}{\kappa_1^2}(\hat{\pi}_{t-1}-\beta E_{t-1}\hat{\pi}_t)+\frac{\overline{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-l}=0$$

Define:

$$\zeta_t \equiv (\hat{\pi}_t - E_{t-1}\hat{\pi}_t) + \frac{\kappa_1^2}{\beta\lambda_Y}\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k \Delta\psi_{gov,t-l} = 0$$

Then, inflation evolves according to:

$$E_t\hat{\pi}_{t+1} - (1 + \frac{1}{\beta} + \frac{\kappa_1^2}{\lambda_Y\beta})\hat{\pi}_t + \frac{1}{\beta}\hat{\pi}_{t-1} = -\zeta_t \tag{52}$$

We will now resolve the above difference equation. Letting $\widetilde{\kappa} = \frac{\kappa_1^2}{\lambda_Y\beta}$, the characteristic polynomial is $\lambda^2 - (1 + \frac{1}{\beta} + \widetilde{\kappa})\lambda + \frac{1}{\beta}$. The two roots are:

$$\widetilde{\lambda}_{1,2} = \frac{1}{2}\left((1 + \frac{1}{\beta} + \widetilde{\kappa}) \pm \sqrt{(1 + \frac{1}{\beta} + \widetilde{\kappa})^2 - \frac{\widetilde{4}}{\beta}}\right)$$

It is simple to show that one root is stable and one unstable. Let $\widetilde{\lambda}_1$ denote the stable root. (52) can be written as:

$$\hat{\pi}_t = \frac{1}{\widetilde{\lambda}_2}E_t\hat{\pi}_{t+1} + \frac{1}{\widetilde{\lambda}_2}\frac{1}{1-\widetilde{\lambda}_1 L}\zeta_t = \frac{1}{\widetilde{\lambda}_2}\frac{1}{1-\widetilde{\lambda}_1 L}\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\zeta_{t+j} \tag{53}$$

(for the usual boundary condition that inflation does not explode).

Let us compute the term

$$\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\zeta_{t+j} = \sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\left[(\hat{\pi}_{t+j} - E_{t+j-1}\hat{\pi}_{t+j}) + \widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t+j-l}\right]$$

When $\Delta\psi_{gov,t} \neq 0$ (in an equilibrium with active fiscal policy), the final term on the RHS is

$$\widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\left[\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t+j-l}\right] = \widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t}$$

(this follows from the random walk property of the multiplier). Moreover, it clearly holds that:

$$\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t(\hat{\pi}_{t+j} - E_{t+j-1}\hat{\pi}_{t+j}) = \hat{\pi}_t - E_{t-1}\hat{\pi}_t$$

Putting everything together and using (53) we have:

$$\hat{\pi}_t = \widetilde{\lambda}_1\hat{\pi}_{t-1} + \frac{1}{\widetilde{\lambda}_2}(\hat{\pi}_t - E_{t-1}\hat{\pi}_t) + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t} \tag{54}$$

Proof of Proposition 5.

To derive the interest rate rules use (85). Consider first the the case where $\Delta\psi_{gov,t} = 0$ (passive fiscal policy).

Then,

$$E_t\hat{\pi}_{t+1} = \widetilde{\lambda}_1\hat{\pi}_t + \frac{1}{\widetilde{\lambda}_2}E_t(\hat{\pi}_{t+1} - E\pi_{t+1}) = \widetilde{\lambda}_1\hat{\pi}_t$$

and clearly $E_{t-1}\hat{\pi}_t = \widetilde{\lambda}_1 \hat{\pi}_{t-1}$. Then since $\zeta_t = 0$ from (52) optimal inflation solves

$$\hat{\pi}_{t+1} - (\widetilde{\lambda}_1 + \widetilde{\lambda}_2)\hat{\pi}_t + \widetilde{\lambda}_1\widetilde{\lambda}_2\hat{\pi}_{t-1} = 0 \tag{55}$$

(expectations can be dropped because inflation is clearly not random). The unique solution is $\hat{\pi}_t = 0$ for all t. Standard arguments imply uniqueness of the equilibrium when:

$$\hat{i}_t = \hat{\xi}_t + (\widetilde{\lambda}_1 + \widetilde{\lambda}_2)\hat{\pi}_t - \widetilde{\lambda}_1\widetilde{\lambda}_2\hat{\pi}_{t-1}$$

Now consider the case where $\Delta\psi_{gov,t} \neq 0$. From (85) we have

$$E_t\hat{\pi}_{t+1} = \widetilde{\lambda}_1\hat{\pi}_t + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\underbrace{E_t\frac{1}{1-\delta L}\Delta\psi_{gov,t+1}}_{=\frac{\delta}{1-\delta L}\Delta\psi_{gov,t}}$$

and also

$$E_{t-1}\hat{\pi}_t = \widetilde{\lambda}_1\hat{\pi}_{t-1} + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{\delta}{1-\delta L}\Delta\psi_{gov,t-1}$$

We thus have

$$\hat{\pi}_t - E_{t-1}\hat{\pi}_t = \frac{1}{\widetilde{\lambda}_2}(\hat{\pi}_t - E_{t-1}\hat{\pi}_t) + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\Delta\psi_{gov,t}$$
$$\rightarrow \hat{\pi}_t - E_{t-1}\hat{\pi}_t = \frac{\widetilde{\lambda}_2}{\widetilde{\lambda}_2 - 1}\frac{\widetilde{\kappa}}{\lambda_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\Delta\psi_{gov,t}$$

It is now easy to derive the optimal interest rate rule. Let $\widetilde{o} = \frac{\widetilde{\kappa}}{\lambda_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}$

$$\hat{i}_t = \hat{\xi}_t + E_t\hat{\pi}_{t+1} = \hat{\xi}_t + \widetilde{\lambda}_1\hat{\pi}_t + \widetilde{o}\frac{\delta}{1-\delta L}\Delta\psi_{gov,t} = \hat{\xi}_t + \widetilde{\lambda}_1\hat{\pi}_t + \delta\left(\hat{\pi}_t - \widetilde{\lambda}_1\hat{\pi}_{t-1} - \frac{\widetilde{o}}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t}\right)$$

■

Additional derivations: The path of optimal inflation.

We now provide additional derivations for the impulse responses shown in Section 4.

The case where fiscal policy is passive is trivial since inflation and output are at steady state in all periods. Consider the case where fiscal policy is active. Focus on the case a shock in either spending or preferences can hit the economy in t and all shocks before or after t are 0. Thus, $\Delta\psi_{gov,t} \neq 0$ but $\Delta\psi_{gov,t+j} = 0$ for $j \neq 0$. Moreover, let $\hat{b}_{t-1} = 0$. The intertemporal consolidated budget in this model can be written as:

$$-\overline{G}\hat{G}_t + (\bar{s} - \bar{b})\hat{\xi}_t = -\bar{b}\sum_{j\geq 0}(\beta\delta)^j\sum_{l=0}^{j}\hat{\pi}_{t+l}$$

From the above derivations we can show that:

$$\hat{\pi}_t = \widetilde{\lambda}_1 \hat{\pi}_{t-1} + \frac{1}{\widetilde{\lambda}_2 - 1}\widetilde{o}\Delta\psi_{gov,t} + \widetilde{o}\Delta\psi_{gov,t}$$

(given that $\Delta\psi_{gov,t-1} = \Delta\psi_{gov,t-2} = ...0.$) Also,

$$\hat{\pi}_{t+1} = \widetilde{\lambda}_1 \hat{\pi}_t + \widetilde{o}\delta\Delta\psi_{gov,t} = \widetilde{\lambda}_1 \hat{\pi}_{t-1} + \widetilde{o}(\delta + \widetilde{\lambda}_1)\Delta\psi_{gov,t} + \widetilde{o}\frac{\widetilde{\lambda}_1}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t}$$

$$...$$

$$\hat{\pi}_{t+l} = \widetilde{\lambda}_1^{l+1}\hat{\pi}_{t-1} + \widetilde{o}\left(\delta^l + \delta^{l-1}\widetilde{\lambda}_1 + \delta^{l-2}\widetilde{\lambda}_1^2 + ... + \delta\widetilde{\lambda}_1^{l-1} + \widetilde{\lambda}^l\right)\Delta\psi_{gov,t} + \widetilde{o}\frac{\widetilde{\lambda}_1^l}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t}$$

Noting that $\left(\delta^l + \delta^{l-1}\widetilde{\lambda}_1 + \delta^{l-2}\widetilde{\lambda}_1^2 + ... + \delta\widetilde{\lambda}_1^{l-1} + \widetilde{\lambda}^l\right) = \frac{\left(\delta^{l+1} - \widetilde{\lambda}_1^{l+1}\right)}{\delta - \widetilde{\lambda}_1}$ and also that by assumption $\hat{\pi}_{t-1} = 0$, we can write the consolidated budget constraint as:

$$-\overline{G}\hat{G}_t + (\overline{s} - \overline{b})\hat{\xi}_t = -\overline{b}\widetilde{o}\Delta\psi_{gov,t}\sum_{j\geq 0}(\beta\delta)^j\sum_{l=0}^{j}\left(\frac{\widetilde{\lambda}_1^l}{\widetilde{\lambda}_2 - 1} + \frac{\left(\delta^{l+1} - \widetilde{\lambda}_1^{l+1}\right)}{\delta - \widetilde{\lambda}_1}\right)$$

For brevity let us skip the algebra. The final result is:

$$\Delta\psi_{gov,t} = (1-\beta\delta)(1-\beta\delta\widetilde{\lambda}_1)\frac{\left(\overline{G}\hat{G}_t + (\overline{b} - \overline{s})\hat{\xi}_t\right)}{\overline{b}\widetilde{o}(\frac{1}{1-\beta\delta^2} + \frac{1}{\widetilde{\lambda}_2 - 1})}$$

Now use the above derivations to derive the impulse responses. We have:

$$\hat{\pi}_t = \frac{\widetilde{\lambda}_2}{\widetilde{\lambda}_2 - 1}\widetilde{o}\Delta\psi_{gov,t}$$

$$\hat{\pi}_{t+j} = \widetilde{o}\frac{\left(\delta^{j+1} - \widetilde{\lambda}_1^{j+1}\right)}{\delta - \widetilde{\lambda}_1}\Delta\psi_{gov,t} + \widetilde{o}\frac{\widetilde{\lambda}_1^j}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t}, \quad j \geq 1$$

The case $\lambda_Y = 0$. Now let us calculate the limit when $\lambda_Y = 0$. In this case we have $\widetilde{\kappa} \to \infty$ and it is easy to show that $\widetilde{\lambda}_1 \to 0$ and $\widetilde{\lambda}_2 \to \infty$. Also: $\lim_{\lambda_Y \to 0}\widetilde{o} = \lim_{\lambda_Y \to 0}\frac{\widetilde{\kappa}}{\lambda_2}\frac{\overline{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\lambda_2}} = \frac{\overline{b}}{1-\beta\delta}$. We thus have:

$$\hat{\pi}_{t+j} = \frac{\overline{b}}{1-\beta\delta}\delta^j\Delta\psi_{gov,t}, \quad j \geq 0$$

and

$$\Delta\psi_{gov,t} = (1-\beta\delta)^2(1-\beta\delta^2)\frac{\left(\overline{G}\hat{G}_t + (\overline{b} - \overline{s})\hat{\xi}_t\right)}{\overline{b}^2}$$

A.1.4 Proof of Proposition 6 and derivations for Case 3: $\sigma > 0$ $\lambda_i, \lambda_Y = 0$.

As in the main text we assume $\lambda_Y = \lambda_i = 0$. The FONC for inflation are:

$$\hat{\pi}_t = \frac{\bar{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} + \sigma\frac{\overline{Y}}{\overline{C}\kappa_1}\bar{b}\sum_{l=0}^{\infty}\delta^l\Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\Big) + \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} \tag{56}$$

In text we assumed $\overline{G} = 0$. To find the optimal interest rate rule in the case where fiscal policy is active we combined the Euler equation and the Phillips curve:

$$\hat{i}_t = \sigma\Big(E_t\hat{Y}_{t+1} - \hat{Y}_t\Big) + E_t\hat{\pi}_{t+1} + \hat{\xi}_t = \frac{\sigma}{\kappa_1}E_t\Big(\hat{\pi}_{t+1} - \beta\hat{\pi}_{t+2} - \hat{\pi}_t + \beta\hat{\pi}_{t+1}\Big) + E_t\hat{\pi}_{t+1} + \hat{\xi}_t$$

We can now derive $E_t\hat{\pi}_{t+1}$ as follows:

$$E_t\hat{\pi}_{t+1} = \frac{\bar{b}\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} + \delta\frac{\sigma}{\kappa_1}\bar{b}\sum_{l=0}^{\infty}\delta^l\Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\Big) - \frac{\sigma}{\kappa_1}\bar{b}\Delta\psi_{gov,t} =$$
$$\delta\hat{\pi}_t - \delta\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} - \frac{\sigma}{\kappa_1}\bar{b}\Delta\psi_{gov,t}$$

Moreover, it is simple to show that

$$E_t\hat{\pi}_{t+2} = \delta^2\hat{\pi}_t - \delta^2\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} - \delta\frac{\sigma}{\kappa_1}\bar{b}\Delta\psi_{gov,t}$$

Making use of this we get:

$$\hat{i}_t = \hat{\xi}_t + \Big(\delta + \frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)\Big)\hat{\pi}_t - (\delta\frac{\omega_Y}{\kappa_1} + \frac{\sigma}{\kappa_1}\bar{b})(1 + \frac{\sigma}{\kappa_1} + \beta\frac{\sigma}{\kappa_1}(1-\delta))\Delta\psi_{gov,t}$$

Now let us turn to the passive fiscal policy case. Since the shock is a demand shock, we have that $\hat{\pi}_t = \hat{Y}_t = 0$ (divine coincidence). The interest rate rule that can implement this outcome is $\hat{i}_t = \hat{\xi}_t + \phi_\pi\hat{\pi}_t$ where $\phi_\pi > 1$.

Proof of Proposition 6: Optimal interest rate rule when $\overline{G} > 0$. For completeness let us also find the optimal rule when $\overline{G} > 0$.

$$\hat{i}_t = \sigma\frac{\overline{Y}}{\overline{C}}\Big(E_t\hat{Y}_{t+1} - \hat{Y}_t\Big) + E_t\hat{\pi}_{t+1} + \hat{\xi}_t + \sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t$$
$$= \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}E_t\Big(\hat{\pi}_{t+1} - \beta\hat{\pi}_{t+2} - \hat{\pi}_t + \beta\hat{\pi}_{t+1}\Big) + E_t\hat{\pi}_{t+1} + \hat{\xi}_t + \sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t - \frac{\sigma\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\hat{G}_t$$

Moreover, now

$$E_t\hat{\pi}_{t+1} = \frac{\bar{b}\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} + \delta\frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}\bar{b}\sum_{l=0}^{\infty}\delta^l\Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\Big) - \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}\bar{b}\Delta\psi_{gov,t} =$$
$$\delta\hat{\pi}_t - \delta\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\Delta\psi_{gov,t}$$

and

$$E_t\hat{\pi}_{t+2} = \delta^2\hat{\pi}_t - \delta^2\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} - \delta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\Delta\psi_{gov,t}$$

We therefore get:

$$\hat{i}_t = \hat{\xi}_t + \sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t - \frac{\sigma\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\hat{G}_t + \left(\delta + \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)\right)\hat{\pi}_t - f\Delta\psi_{gov,t} \tag{57}$$

where $f = \left(1 + \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}(1+\beta-\beta\delta)\right)\left(\delta\frac{\omega_Y}{\kappa_1} + \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}\overline{b}\right)$ ■

The optimal path of inflation.

The LHS of the consolidated budget constraint under the parameter values assumed here, is:

$$\begin{aligned}\sum_{j\geq 0}\beta^j\overline{s}\hat{S}_{t+j} &= \sum_{j\geq 0}\beta^j\left[-\overline{G}\hat{G}_{t+j} + (\overline{\tau}-\overline{G})\sigma\frac{\overline{G}}{\overline{C}}\hat{G}_{t+j} - (\overline{\tau}-\overline{G})\sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} + (\overline{\tau}-\overline{G})\hat{\xi}_{t+j}\right] \\ &= -\overline{G}\hat{G}_t + (\overline{\tau}-\overline{G})\sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t + (\overline{\tau}-\overline{G})\hat{\xi}_t - \sum_{j\geq 0}\beta^j\left[(\overline{\tau}-\overline{G})\sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j}\right]\end{aligned}$$

Consider the last term. We have:

$$\begin{aligned}-(\overline{\tau}-\overline{G})\sigma\frac{\overline{Y}}{\overline{C}}\sum_{j\geq 0}\beta^j\left[\hat{Y}_{t+j}\right] &= -(\overline{\tau}-\overline{G})\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\sum_{j\geq 0}\beta^j\left[\hat{\pi}_{t+j} + \kappa_2\hat{G}_{t+j} - \beta\hat{\pi}_{t+j+1}\right] \\ &= -(\overline{\tau}-\overline{G})\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\left(\hat{\pi}_t + \kappa_2\hat{G}_t\right)\end{aligned}$$

We therefore get:

$$\sum_{j\geq 0}\beta^j\overline{s}\hat{S}_{t+j} = -\overline{G}\hat{G}_t + (\overline{\tau}-\overline{G})\sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t + (\overline{\tau}-\overline{G})\hat{\xi}_t - (\overline{\tau}-\overline{G})\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\left(\hat{\pi}_t + \kappa_2\hat{G}_t\right) \tag{58}$$

The RHS of the intertemporal constraint is:

$$\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}) - \sum_{l=0}^{j}\hat{\pi}_{t+l}\right) + \bar{b}\hat{\xi}_t$$

Let us separately derive each of the components. The first is:

$$\begin{aligned}\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j})\right) &= -\sigma\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j}\right) + \sigma\bar{b}\frac{\overline{G}}{\overline{C}}\hat{G}_t \\ &\quad -\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(\hat{\pi}_{t+j} + \kappa_2\hat{G}_{t+j} - \beta\hat{\pi}_{t+j+1}\right) + \sigma\bar{b}\frac{\overline{G}}{\overline{C}}\hat{G}_t \\ &= -\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(\hat{\pi}_{t+j} - \beta\hat{\pi}_{t+j+1}\right) + \sigma\bar{b}\hat{G}_t\left(\frac{\overline{G}}{\overline{C}} - \frac{\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\right)\end{aligned}$$

We can now substitute out the term $\hat{\pi}_{t+j} - \beta\hat{\pi}_{t+j+1}$ using formula (40) in text. Let $\nu = \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\bar{b} - (\overline{\tau} - \overline{G}))$ we can write (40) as:

$$\hat{\pi}_{t+j} - \beta\hat{\pi}_{t+j+1} = \left\{ \begin{array}{ll} \left[\frac{\bar{b}}{1-\beta\delta}(1-\beta\delta) + \nu - \beta\frac{\sigma}{\kappa_1}(1-\delta)\frac{\overline{Y}}{\overline{C}}\bar{b}\right]\Delta\psi_{gov,t} & j=0 \\ \left[\frac{\bar{b}}{1-\beta\delta}\delta^j - \frac{\sigma}{\kappa_1}(1-\delta)\delta^{j-1}\frac{\overline{Y}}{\overline{C}}\bar{b}\right](1-\beta\delta)\Delta\psi_{gov,t} & j\geq 1 \end{array} \right\}$$

Making use of this result we can write:

$$-\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(\hat{\pi}_{t+j} - \beta\hat{\pi}_{t+j+1}\right) = -\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\left[\frac{\bar{b}}{1-\beta\delta}(1-\beta\delta) + \nu - \beta\frac{\sigma}{\kappa_1}(1-\delta)\frac{\overline{Y}}{\overline{C}}\bar{b}\right]\Delta\psi_{gov,t}$$
$$-\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}(1-\beta\delta)\Delta\psi_{gov,t}\sum_{j\geq 1}(\beta\delta)^j\left[\frac{\bar{b}}{1-\beta\delta}\delta^j - \frac{\sigma}{\kappa_1}(1-\delta)\delta^{j-1}\frac{\overline{Y}}{\overline{C}}\bar{b}\right] =$$
$$-\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\Delta\psi_{gov,t}\left\{(1-\beta\delta)\left[\frac{\bar{b}}{1-\beta\delta}\frac{\beta\delta^2}{1-\beta\delta^2} - \frac{\sigma}{\kappa_1}(1-\delta)\frac{\overline{Y}}{\overline{C}}\bar{b}\frac{\beta\delta}{1-\beta\delta^2}\right] + \left[\frac{\bar{b}}{1-\beta\delta}(1-\beta\delta) + \nu - \beta\frac{\sigma}{\kappa_1}(1-\delta)\frac{\overline{Y}}{\overline{C}}\bar{b}\right]\right.$$
$$= -\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\Delta\psi_{gov,t}\left\{\left[\frac{\bar{b}}{1-\beta\delta^2} + \nu - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}(1-\delta)\left((1-\beta\delta)\frac{\beta\delta}{1-\beta\delta^2} + \beta\right)\right]\right\}$$

Consider now the term:

$$-\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(\sum_{l=0}^{j}\hat{\pi}_{t+l}\right) = \left[-\frac{\bar{b}}{1-\beta\delta}\nu - \frac{\bar{b}^2}{1-\beta\delta}\sum_{j\geq 0}(\beta\delta)^j\frac{1-\delta^{j+1}}{1-\delta} + \frac{\sigma}{\kappa_1}(1-\delta)\frac{\overline{Y}}{\overline{C}}\bar{b}^2\sum_{j\geq 1}(\beta\delta)^j\frac{1-\delta^j}{1-\delta}\right]\Delta\psi_{gov,t} =$$
$$-\left[\frac{\bar{b}}{1-\beta\delta}\nu + \frac{\bar{b}^2}{(1-\beta\delta)^2(1-\beta\delta^2)} - \frac{\sigma}{\kappa_1}(1-\delta)\frac{\overline{Y}}{\overline{C}}\frac{\bar{b}^2(\beta\delta)^2}{(1-\beta\delta)(1-\beta\delta^2)}\right]\Delta\psi_{gov,t}$$

We can thus write the intertemporal constraint as:

$$-\left[\mu_1 + \mu_2 - \overline{s}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\left(\frac{\bar{b}}{1-\beta\delta} + \nu\right)\right]\Delta\psi_{gov,t} = -\overline{G}\hat{G}_t + \left(\overline{s} - \bar{b}\right)\sigma\left(\frac{\overline{G}}{\overline{C}} - \frac{\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\right)\hat{G}_t + \left(\overline{s} - \bar{b}\right)\hat{\xi}_t \quad (59)$$
$$\mu_1 = \left[\frac{\bar{b}}{1-\beta\delta}\nu + \frac{\bar{b}^2}{(1-\beta\delta)^2(1-\beta\delta^2)} - \frac{\sigma}{\kappa_1}\left(1-\delta\right)\frac{\overline{Y}}{\overline{C}}\frac{\bar{b}^2(\beta\delta)^2}{(1-\beta\delta)(1-\beta\delta^2)}\right]$$
$$\mu_2 = \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\left\{\left[\frac{\bar{b}}{1-\beta\delta^2} + \nu - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\left(1-\delta\right)\left((1-\beta\delta)\frac{\beta\delta}{1-\beta\delta^2} + \beta\right)\right]\right\}$$

where $\overline{s} = \overline{\tau} - \overline{G}$.

Finally, consider the $\hat{G}_t$ terms in (59). We can write the ratio $\frac{\kappa_2}{\kappa_1} = \frac{\sigma\frac{\overline{G}}{\overline{C}}}{\overline{Y}^2(\gamma_h + \sigma\frac{\overline{Y}}{\overline{C}})}$ and so:

$$-\overline{G}\hat{G}_t + \left(\overline{s} - \bar{b}\right)\sigma\left(\frac{\overline{G}}{\overline{C}} - \frac{\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\right)\hat{G}_t = -\overline{G}\hat{G}_t + \left(\overline{s} - \bar{b}\right)\sigma\frac{\overline{G}}{\overline{C}}\left(1 - \frac{1}{\overline{Y}^2(\gamma_h + \sigma\frac{\overline{Y}}{\overline{C}})}\frac{\overline{Y}}{\overline{C}}\right)\hat{G}_t$$

A.1.5 Proof of Proposition 7

We will first prove that the equilibrium under optimal policies features 0 inflation when $\delta = 1$. Then we will show that this outcome can be implemented with a rule of the form $\hat{i}_t = \hat{\xi}_t + \widetilde{\phi}_\pi\hat{\pi}_t + \widetilde{\phi}_Y\hat{Y}_t$.

This is a more general rule than the one considered in Proposition 7 where we focused on the case $\widetilde{\phi}_Y = 0$. The conditions we derive below nest this case as well.

For clarity, we write here the system of equations that needs to be resolved under the parameter values assumed. We have:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \beta E_t \hat{\pi}_{t+1}, \tag{60}$$

$$\hat{i}_t = \hat{\xi}_t + E_t\Big(\hat{\pi}_{t+1} - \sigma\Big(\hat{Y}_t - \hat{Y}_{t+1})\Big) \tag{61}$$

$$\bar{b}_1 \bar{p}\Big(\hat{b}_{t,1} + \hat{p}_t\Big) = \Big(1 + \bar{p}\Big)\bar{b}_1\Big(\hat{b}_{t-1,1} - \hat{\pi}_t\Big) + \bar{b}_1 \bar{p}\hat{p}_t - \bar{\tau}\hat{\tau}_t \tag{62}$$

$$\hat{p}_t = \Big(-\hat{\xi}_t - E_t\Big(\hat{\pi}_{t+1} + \sigma\Big(\hat{Y}_t - \hat{Y}_{t+1})\Big)\Big) + \beta \hat{p}_{t+1} \tag{63}$$

where $\bar{p} = \frac{\beta}{1-\beta}$ denotes the steady state of the consol.

Under parameters $\lambda_i = 0$ $\lambda_Y, \sigma > 0$ the first order conditions assuming active fiscal policy give us:

$$-\hat{\pi}_t - \frac{\lambda_Y}{\kappa_1}\Delta \hat{Y}_t + \frac{\sigma}{\kappa_1}\bar{b}_1 \sum_{l=0}^{\infty}\Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\Big) + \omega_Y \Delta\psi_{gov,t} + \frac{\bar{b}_1}{1-\beta}\sum_{l=0}^{\infty}\Delta\psi_{gov,t-l} = 0 \tag{64}$$

$$\psi_{gov,t} - E_t\psi_{gov,t+1} = 0 \tag{65}$$

We are required to solve the system of equations (60), (62), (63), (64) and (65) together with the fiscal rule. [36] (62) can be written as:

$$\hat{b}_{t,1} = \frac{1}{\beta}\Big(\hat{b}_{t-1,1} - \hat{\pi}_t\Big) - \frac{\bar{\tau}\phi_{\tau,b}}{\bar{b}_1\bar{p}}\hat{b}_{t-1,1} \tag{66}$$

since $\hat{p}_t$ drops from the LHS and RHS of (66) it is also obvious that (63) can be treated as a residual in this equilibrium system. The complete system is now (60) (66), (64) and (65). Moreover, since ξ_t does not appear anywhere in these equations, it is obvious that the solution is $\hat{\pi}_t = \hat{Y}_t = \hat{b}_{t,1} = \psi_{gov,t} = 0$. It can be shown that this solution is unique.

In the passive fiscal model we know from subsection 3.2.3 that $\hat{\pi}_t = \hat{Y}_t = 0$.

Let us now show that these outcomes can be implemented with an inflation targeting rule. Now the system of equations that we need to resolve is (60), (66) together with

$$\widetilde{\phi}_\pi \hat{\pi}_t + \widetilde{\phi}_Y \hat{Y}_t = E_t\Big(\hat{\pi}_{t+1} - \sigma\Big(\hat{Y}_t - \hat{Y}_{t+1})\Big) \tag{67}$$

$$\hat{p}_t = -\widetilde{\phi}_\pi \hat{\pi}_t - \widetilde{\phi}_Y \hat{Y}_t - \hat{\xi}_t + \beta \hat{p}_{t+1} \tag{68}$$

In matrix form we can write this system as:

$$AE_t z_{t+1} = Bz_t + C\hat{\xi}_t \tag{69}$$

[36] (61) can be satisfied ex post since $\hat{i}_t$ is a 'slack' variable.

where $z_t = \left(\hat{\pi}_t, \hat{Y}_t, \hat{p}_t, \hat{b}_{t-1}\right)$ and

$$A = \begin{bmatrix} \beta & 0 & 0 & 0 \\ 1 & \sigma & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -\kappa_1 & 0 & 0 \\ \widetilde{\phi}_\pi & \widetilde{\phi}_Y + \sigma & 0 & 0 \\ \widetilde{\phi}_\pi & \widetilde{\phi}_Y & 1 & 0 \\ -\frac{1}{\beta} & 0 & 0 & \frac{1}{\beta} - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}_1\overline{p}} \end{bmatrix}$$

and $C = [0, 0, 1, 0]'$. The dynamics of (69) are governed by the eigenvalues of $A^{-1}B$.

$$A^{-1}B = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa_1}{\beta} & 0 & 0 \\ \frac{\widetilde{\phi}_\pi}{\sigma} - \frac{1}{\sigma\beta} & \frac{\kappa_1}{\sigma\beta} + \frac{\widetilde{\phi}_Y}{\sigma} + 1 & 0 & 0 \\ \frac{\widetilde{\phi}_\pi}{\beta} & \frac{\widetilde{\phi}_Y}{\beta} & \frac{1}{\beta} & 0 \\ 0 & 0 & \frac{1}{\beta} - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}_1\overline{p}} & \end{bmatrix}$$

The characteristic polynomial is:

$$\left(\frac{1}{\beta} - \lambda\right)^2 \left(\frac{\kappa_1}{\sigma\beta} + \frac{\widetilde{\phi}_Y}{\sigma} + 1 - \lambda\right)\left(\frac{1}{\beta} - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}_1\overline{p}} - \lambda\right) + \left(\frac{1}{\beta} - \lambda\right)\left(\frac{1}{\beta} - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}_1\overline{p}} - \lambda\right)\frac{\kappa_1}{\beta}(\frac{\widetilde{\phi}_\pi}{\sigma} - \frac{1}{\sigma\beta}) = 0$$

Rearranging we get:

$$\left(\frac{1}{\beta} - \lambda\right)\left(\frac{1}{\beta} - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}_1\overline{p}} - \lambda\right)\left[\left(\frac{1}{\beta} - \lambda\right)\left(\frac{\kappa_1}{\sigma\beta} + \frac{\widetilde{\phi}_Y}{\sigma} + 1 - \lambda\right) + \frac{\kappa_1}{\beta}\frac{\widetilde{\phi}_\pi}{\sigma}\right] = 0$$

The solution is unique if we get 3 eigenvalues outside the unit circle. Suppose that fiscal policy is 'passive' so that $\frac{1}{\beta} - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}_1\overline{p}} < 1$. Then the 2 eigenvalues that we can obtain from solving the second order polynomial in the square bracket above have to exceed 1. The polynomial is:

$$\lambda^2 - \lambda\Bigg(\underbrace{\frac{1}{\beta} + \frac{\kappa_1}{\sigma\beta} + \frac{\widetilde{\phi}_Y}{\sigma}}_{\chi_1} + 1\Bigg) + \underbrace{\frac{1}{\beta}\left(\frac{\widetilde{\phi}_Y}{\sigma} + 1\right) + \frac{\kappa_1}{\beta}\frac{\widetilde{\phi}_\pi}{\sigma}}_{\chi_2} = 0$$

Focusing on positive values for parameters $\widetilde{\phi}_\pi, \widetilde{\phi}_Y$ both roots exceed 1 if:

$$\frac{(\chi_1 + 1) - \sqrt{(\chi_1 + 1)^2 - 4\chi_2}}{2} > 1$$

or

$$(\chi_1 - 1) > \sqrt{(\chi_1 + 1)^2 - 4\chi_2} \rightarrow (\chi_1 - 1)^2 > (\chi_1 + 1)^2 - 4\chi_2 \rightarrow 4\chi_2 > (\chi_1 + 1)^2 - (\chi_1 - 1)^2 = 4\chi_1$$

The condition $\chi_2 > \chi_1$ holds when

$$\frac{1}{\beta}\left(\frac{\widetilde{\phi}_Y}{\sigma} + 1\right) + \frac{\kappa_1}{\beta}\frac{\widetilde{\phi}_\pi}{\sigma} > \frac{1}{\beta} + \frac{\kappa_1}{\sigma\beta} + \frac{\widetilde{\phi}_Y}{\sigma}$$

or

$$\frac{\widetilde{\phi}_Y(1-\beta)}{\kappa_1} + \widetilde{\phi}_\pi > 1 \tag{70}$$

In the case where fiscal policy is active it is easy to check that a unique equilibrium requires the opposite inequality to hold in (70). Finally, it is easy to verify that in either of these cases inflation, debt and output are always at steady state. ■

A.2 Matching Impulse Responses under Active Fiscal Policy

In this section we conduct the experiments described in paragraph 3.3 to derive numerically optimal interest rate rules when at least two of parameters λ_i, λ_Y and σ can be positive. As discussed in text, we do this by matching the impulse response functions of Ramsey model with those of a model where monetary policy follows an interest rate rule whose parameters are set to approximate the Ramsey outcome. For simplicity we will consider only the case of spending shocks. What we will show here can be easily extended to demand shocks.

Let us first derive an analytical result. Assume that $\lambda_i, \sigma > 0$ but $\lambda_Y = 0$. Assume further that debt is short term, $\delta = 0$. We will show that a simple inflation targeting rule of the form: $\hat{i}_t = \phi_G \hat{G}_t + \phi_\pi \hat{\pi}_t$ can fit perfectly the Ramsey impulse response function.

Combining the first order conditions under the assumed parameter values gives:

$$-\hat{\pi}_t + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\lambda_i \Delta\hat{i}_t - \frac{\lambda_i}{\beta}\Delta\hat{i}_{t-1}) + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}(\Delta\psi_{gov,t} - \Delta\psi_{gov,t-1}) + \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} - \frac{\lambda_i \hat{i}_{t-1}}{\beta} + \bar{b}\Delta\psi_{gov,t} = 0$$

Using the interest rate rule along with the initial conditions $\hat{i}_{t-1} = \hat{i}_{t-2}, \Delta\psi_{gov,t-1} = 0$ and the fact that $\Delta\psi_{gov,t} = \epsilon\hat{G}_t$ we can write:

$$-\hat{\pi}_t + \frac{\sigma\lambda_i}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\phi_G\hat{G}_t + \phi_\pi\hat{\pi}_t) + (\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b} + \frac{\omega_Y}{\kappa_1} + \bar{b})\epsilon\hat{G}_t = 0$$

Next the first order condition in $t+1$ becomes:

$$-\hat{\pi}_{t+1} + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\lambda_i\Delta\hat{i}_{t+1} - \frac{\lambda_i}{\beta}\hat{i}_t) - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\epsilon\hat{G}_t - \frac{\lambda_i\hat{i}_t}{\beta} = 0$$

or

$$\hat{\pi}_{t+1}\underbrace{(1 - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\lambda_i\phi_\pi)}_{\zeta_1} = -\underbrace{(\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\epsilon + \lambda_i(\frac{1}{\beta} + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\frac{1}{\beta}))\phi_G)}_{\zeta_2}\hat{G}_t - \underbrace{\lambda_i(\frac{1}{\beta} + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\frac{1}{\beta}))\phi_\pi}_{\zeta_3}\hat{\pi}_t$$

Finally, inflation in $t+2$ solves:

$$-\hat{\pi}_{t+2} + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\lambda_i\Delta\hat{i}_{t+2} - \frac{\lambda_i}{\beta}\Delta\hat{i}_{t+1}) - \frac{\lambda_i\hat{i}_{t+1}}{\beta} = 0$$

or

$$\hat{\pi}_{t+2}\zeta_1 = -\zeta_3\hat{\pi}_{t+1} + \frac{\lambda_i}{\beta}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\phi_\pi\hat{\pi}_t + \phi_G\hat{G}_t) = \zeta_3(\frac{\zeta_3}{\zeta_1}\hat{\pi}_t + \frac{\zeta_2}{\zeta_1}\hat{G}_t) + \frac{\lambda_i}{\beta}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\phi_\pi\hat{\pi}_t + \phi_G\hat{G}_t)$$

Dropping the shocks for simplicity and using the policy rule the Euler equation in t can be written as:

$$\phi_\pi\hat{\pi}_t = \left(\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta) + 1\right)\hat{\pi}_{t+1} - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\hat{\pi}_t - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\hat{\pi}_{t+2}$$

We thus get:

$$\phi_\pi = -\left(\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta) + 1\right)\frac{\zeta_3}{\zeta_1} - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}} - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\frac{\zeta_3^2}{\zeta_1^2} + \frac{\lambda_i}{\beta}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\frac{\phi_\pi}{\zeta_1})$$

a nonlinear equation in coefficient ϕ_π. Solving this we will obtain the value that makes the inflation targeting rule fit the impulse response function of the Ramsey model.

When we assume $\delta > 0$ or $\lambda_Y > 0$ the approximate policy rule may feature lags of interests rates. Our next experiments use the numerical impulse responses and the baseline calibration of the model to determine the optimal coefficients in the interest rate rule.

In particular, we assume that fiscal policy is active and that monetary policy sets the nominal interest rate according to:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t + \phi_i \hat{i}_{t-1} + \phi_{\Delta i} \Delta \hat{i}_{t-1} \tag{71}$$

Evidently, the RHS of (71) has the same functional form as $\mathcal{T}$ in Proposition 1, and object $\mathcal{D}_t$ has been omitted. We will estimate the values of coefficients $\phi_\pi, \phi_Y, \phi_i, \phi_{\Delta i}$ so that the model when interest rates follow (71) approximates the optimal policy model.

Table 2 shows the estimates of the coefficients.[37] We consider several calibrations of the model: The table is split in 3 sub-tables. The first two correspond to the active fiscal policy model. We set $\delta = 0.5$ in the left, and $\delta = 0.95$ in the middle. Moreover, in the top panel of each subtable we let $\lambda_Y = 0$, in which case we also constrain ϕ_Y to be equal to 0, whereas the bottom panel assumes $\lambda_Y = 0.5$, thus setting the weight to output stabilization in the policy objective to be half of the weight attached to inflation stabilization. Each of the columns of the subtables corresponds to a different calibration of the pair λ_i, σ.

For comparison, the right part of the Table shows the coefficients in the passive fiscal model corresponding to each calibration considered. Finally, the assumed values for the remaining model parameters are reported in Table 1 (see the notes of that table for a brief discussion of the calibration).

Several results stand out. First, notice that assuming active fiscal policy reduces substantially the values of coefficients ϕ_i and $\phi_{\Delta i}$ relative to the case of passive policy. As is well known (see Giannoni and Woodford, 2003), the optimal policy rule in the passive fiscal model is *super-inertial*; the coefficients are such that the rule contains an explosive root. In contrast, under active fiscal policy both roots of the second order difference equation defined by the interest rate rule are stable.

Second, the estimated coefficients ϕ_π, ϕ_Y are also smaller in magnitude than their passive fiscal counterparts. This property is in line with our previous findings.[38] Third, these coefficients do vary (as expected) as we vary the values of parameters $\sigma, \lambda_i, \lambda_Y$, but also it is evident that a key determinant of the estimates is the debt maturity. This is easily noticeable in the top panels of the left and middle sub-tables. Assuming $\sigma = 1$ and $\lambda_i = 0.5$ yields an estimate ϕ_π of 0.21 when $\delta = 0.5$ and 0.43 when $\delta = 0.95$. Moreover, when we assume $\lambda_i = 1$ we have -0.10 and 0.52 for $\delta = 0.5$

[37] We produced these estimates through matching the impulse responses of the optimal policy model. We focused on matching the responses to the spending shock only. Note that (71) could produce a perfect fit in some of the cases reported in Table 2. In other cases this was not so and to improve the fit we added the spending shock as an additional argument to the rule. We therefore estimated

$$\hat{i}_t = \phi_G \hat{G}_t + \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t + \phi_i \hat{i}_{t-1} + \phi_{\Delta i} \Delta \hat{i}_{t-1}$$

This turned out being useful when $\delta = 0.95$. Presumably, we would need to add further lags of inflation, output and interest rates in (71) to avoid introducing $\hat{G}_t$. We did not want to extend to further lags, as we wanted to maintain as closely as possible the structure of Proposition 1. Coefficient ϕ_G was found to be small in magnitude and we do not report it in the Table.

[38] It is however not necessary to have ϕ_π, ϕ_Y larger in the passive fiscal model. Under a super-inertial rule, monetary policy is active for any positive values of coefficients ϕ_π, ϕ_Y. Thus ϕ_π, ϕ_Y could even be smaller than in the active fiscal scenario.

To claim that the estimated rules in the left and middle sub-tables define passive monetary policies, we have to jointly consider the estimated values of ϕ_π, ϕ_Y, ϕ_i and $\phi_{\Delta i}$. These have to satisfy the usual condition that the nominal interest rate weakly responds to the deviations of the variables from targets. This requirement is obviously met, otherwise the Blanchard-Kahn condition would not be satisfied and the model solution could not be found.

and $\delta = 0.95$ respectively.[39] Thus, higher debt maturity yields a stronger reaction of monetary policy to inflation, implying a more persistent response of inflation to the shock. This feature can be easily understood based on our previous discussion regarding optimal policy in the active fiscal policy model.

These patterns continue to hold for many alternative calibrations of the model, which for brevity we do not show here.

[39] It is not paradoxical to get a negative inflation coefficient from these regressions. As we have previously seen, for the case $\sigma > 0$ and $\lambda_i = 0$ the optimal inflation coefficient is $\delta + \frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)$ which can be negative if δ is not of high value. (We discuss this property in detail below.) This applies also to the case $\sigma, \lambda_i > 0$.

Table 2: **Model implied Taylor rules**

Active FP $\delta = 0.5$

	$\sigma = 1$		$\sigma = 2$	
	$\lambda_i = 0.5$	$\lambda_i = 1$	$\lambda_i = 0.5$	$\lambda_i = 1$
ϕ_π	0.214	-0.102	-0.131	-0.262
ϕ_Y	0	0	0	0
ϕ_i	0.378	0.273	0.254	0.052
ϕ_{Δ_i}	0.006	0.067	0.003	0.001
ϕ_π	0.533	0.516	0.449	0.352
ϕ_Y	0.367	0.233	0.234	0.072
ϕ_i	0.165	0.21	0.222	0.208
ϕ_{Δ_i}	0.025	0.032	-0.008	-0.034

Active FP $\delta = 0.95$

	$\sigma = 1$		$\sigma = 2$	
	$\lambda_i = 0.5$	$\lambda_i = 1$	$\lambda_i = 0.5$	$\lambda_i = 1$
ϕ_π	0.43	0.522	0.4	0.266
ϕ_Y	0	0	0	0
ϕ_i	0.518	0.478	0.547	0.682
ϕ_{Δ_i}	0	0.005	0	0
ϕ_π	0.618	0.458	0.502	0.356
ϕ_Y	0.448	0.333	0.393	0.216
ϕ_i	0.335	0.493	0.447	0.593
ϕ_{Δ_i}	-0.024	-0.035	-0.022	-0.051

Passive FP

	$\sigma = 1$		$\sigma = 2$	
	$\lambda_i = 0.5$	$\lambda_i = 1$	$\lambda_i = 0.5$	$\lambda_i = 1$
ϕ_π	1.65	0.82	0.82	0.41
ϕ_Y	0	0	0	0
ϕ_i	1.83	1.83	1.41	1.41
ϕ_{Δ_i}	1.005	1.005	1.005	1.005
ϕ_π	1.65	0.82	0.82	0.41
ϕ_Y	0.90	0.45	0.45	0.225
ϕ_i	1.83	1.83	1.41	1.41
ϕ_{Δ_i}	1.005	1.005	1.005	1.005

Notes: The tables report model implied Taylor rules $\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_Y \Delta \hat{Y}_t + \phi_i \hat{i}_{t-1} + \phi_{\Delta_i} \Delta \hat{i}_{t-1}$. The left table assumes that fiscal policy is active, setting $\delta = 0.5$. The middle table also considers active fiscal policy but sets $\delta = 0.95$. The right table reports the coefficients of the Giannoni and Woodford (2003) optimal rule, under passive fiscal policy. In the top panels of each table we assume $\lambda_Y = 0$ and in the bottom panels $\lambda_Y = 0.5$. The values of parameters λ_i, σ corresponding to each of the columns of the tables are stated at the top. The remaining model parameters are assigned the values reported in Table 1.

B Optimal Policies with Distortionary Taxation

We now present analytical results for the case where taxes are distortionary. Under this assumption the two model equations that need to be changed are the Phillips curve and the government budget constraint. We now have

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t - \kappa_2 \hat{G}_t + \kappa_3 \hat{\tau}_t + \beta E_t \hat{\pi}_{t+1}, \tag{72}$$

where $\kappa_1 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}(\gamma_h + \sigma\frac{\overline{Y}}{\overline{C}}) > 0$, $\kappa_3 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\frac{\overline{\tau}^d}{(1-\overline{\tau}^d)} \geq 0$, $\kappa_2 \equiv -\frac{(1+\eta)}{\theta\overline{Y}}\sigma\frac{\overline{G}}{\overline{C}} > 0$, and where $\overline{\tau}^d$ denotes the steady state distortionary tax rate.

Moreover, now the surplus of the government becomes a function of output and we have:

$$\overline{s}\hat{S}_t \equiv \left[-\overline{G}\left(\hat{G}_t(1+\sigma\frac{\overline{G}}{\overline{C}}) - \sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_t + \hat{\xi}_t\right) + \overline{r}\left(\hat{R}_t + \hat{\xi}_t\right)\right]$$

$\overline{r}$ is the steady state revenue of the government, and $\hat{R}_t$ denotes the revenue scaled by marginal utility. We have:

$$\hat{R}_t = \left((1+\gamma_h)\hat{Y}_t + \frac{\hat{\tau}_t}{1-\overline{\tau}^d}\right)\mathcal{I}_{\overline{\tau}^d>0}$$

We continue to assume that fiscal policy is given by (6).

Optimal policy solves the following system of equations:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} - \frac{\psi_{i,t-1}}{\beta} + \frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} = 0 \tag{73}$$

$$-\lambda_Y \hat{Y}_t - \psi_{\pi,t}\kappa_1 + \sigma\frac{\overline{Y}}{\overline{C}}(\psi_{i,t} - \frac{\psi_{i,t-1}}{\beta}) + \sigma\frac{\overline{Y}}{\overline{C}}\overline{b}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} + \omega_Y \psi_{gov,t} = 0 \tag{74}$$

$$-\lambda_i \hat{i}_t + \psi_{i,t} = 0 \tag{75}$$

$$\frac{\beta\overline{b}}{1-\beta\delta}\left(\psi_{gov,t} - E_t\psi_{gov,t+1}\right) - \beta\phi_{\tau,b}E_t\left(\kappa_3\psi_{\pi,t+1} - \overline{r}\frac{d\hat{R}_t}{d\hat{\tau}_t}\psi_{gov,t+1}\right) = 0 \tag{76}$$

where now $\omega_Y \equiv \overline{G}\sigma\frac{\overline{Y}}{\overline{C}} + \overline{r}(1+\gamma_h)$.

Note that (73) to (75) are essentially the same as (8) to (10) derived in text under the assumption that taxes are lump sum. Rearranging these first order conditions (following the steps of the proof of Proposition 1) it is again possible to express interest rates under as the sum of $\mathcal{T}_t$ and $\mathcal{D}_t$.

B.1 The two equilibria under distortionary taxes

It is possible to show that this model admits two equilibria under active and passive fiscal policies. However, we now need to separately treat the cases where $\lambda_Y = 0$ and $\lambda_Y > 0$. As we explain below in the case $\lambda_Y > 0$ we need to modify the objective of the planner slightly assuming that the target is the natural level of output, not the steady state level.

B.1.1 $\lambda_Y = 0$

Assume first that $\lambda_Y = 0$. (76) can be written as;

$$\frac{\beta\overline{b}}{1-\beta\delta}\left(\psi_{gov,t} - E_t\psi_{gov,t+1}\right) - \beta\phi_{\tau,b}E_t\left(\kappa_3\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\overline{b}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l+1} + \frac{\kappa_3\omega_Y}{\kappa_1}\psi_{gov,t+1} - \frac{\overline{r}}{1-\overline{\tau}^d}\psi_{gov,t+1}\right) = 0 \tag{77}$$

which can be written as

$$\widetilde{\eta}_1\Bigg(\psi_{gov,t} - E_t\psi_{gov,t+1}\Bigg) - \beta\phi_{\tau,b}E_t\Bigg(\kappa_3\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\overline{b}\frac{\Delta\psi_{gov,t+1}}{1-\delta L} + \widetilde{\eta}_2\psi_{gov,t+1}\Bigg) = 0$$

where $\widetilde{\eta}_2 < 0$ to be at the upward sloping part of the Laffer curve.

The above can be rearranged into a second order difference equation:

$$-\Bigg(\widetilde{\eta}_1 + \beta\phi_{\tau,b}(\widetilde{\eta}_2 + \kappa_3\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\overline{b})\Bigg)E_t\psi_{gov,t+1} + \Bigg(\widetilde{\eta}_1(1+\delta) + \beta\phi_{\tau,b}(\widetilde{\eta}_2\delta + \kappa_3\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\overline{b})\Bigg)\psi_{gov,t} - \delta\widetilde{\eta}_1\psi_{gov,t-1} = 0 \tag{78}$$

or

$$E_t\psi_{gov,t+1} - \Bigg(1 - (1-\delta)\widetilde{\eta}_2\frac{\beta\phi_{\tau,b}}{\widetilde{\eta}_3} + \frac{\delta\widetilde{\eta}_1}{\widetilde{\eta}_3}\Bigg)\psi_{gov,t} + \frac{\delta\widetilde{\eta}_1}{\widetilde{\eta}_3}\psi_{gov,t-1} = 0 \tag{79}$$

where $\widetilde{\eta}_3 = \Bigg(\widetilde{\eta}_1 + \beta\phi_{\tau,b}(\widetilde{\eta}_2 + \kappa_3\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\overline{b})\Bigg)$. It is easy to show that the characteristic polynomial has two roots, one stable and one unstable. The unique non-explosive solution is thus $\psi_{gov,t} = 0$.

We can now derive the threshold value $\phi_{\tau,b}$. From the first order condition of inflation it is easy to show that when $\psi_{gov,t} = 0$, $\hat{\pi}_t = 0$ for all t. With distortionary taxation output evolves according to:

$$\hat{Y}_t = -\frac{\kappa_3}{\kappa_1}\hat{\tau}_t + \frac{\kappa_2}{\kappa_1}\hat{G}_t$$

The Euler equation then gives us:

$$\hat{i}_t = \hat{\xi}_t + \sigma\frac{\overline{Y}}{\overline{C}}E_t\Bigg(-\frac{\kappa_3}{\kappa_1}\Delta\hat{\tau}_{t+1} + \frac{\kappa_2}{\kappa_1}\Delta\hat{G}_{t+1}\Bigg) - \sigma\frac{\overline{G}}{\overline{C}}E_t\Delta\hat{G}_{t+1}$$

Analogously, the price of long term bonds evolves according to:

$$\hat{p}_t = -\hat{\xi}_t - \sigma\frac{\overline{Y}}{\overline{C}}E_t\Bigg(-\frac{\kappa_3}{\kappa_1}\Delta\hat{\tau}_{t+1} + \frac{\kappa_2}{\kappa_1}\Delta\hat{G}_{t+1}\Bigg) + \sigma\frac{\overline{G}}{\overline{C}}E_t\Delta\hat{G}_{t+1} + \beta\delta E_t\hat{p}_{t+1}$$

For simplicity, let us suppress the shocks (This does not change anything with regard to the threshold $\widetilde{\phi}_\tau$). Using the above expression we can then write:

$$\hat{p}_t = \sigma\frac{\overline{Y}}{\overline{C}}\frac{\kappa_3}{\kappa_1}\phi_{\tau,b}(\hat{b}_t - \hat{b}_{t-1}) + \beta\delta E_t\hat{p}_{t+1} \tag{80}$$

Moreover, the consolidated budget constraint (again without shocks) can written as:

$$\beta\overline{b}\Bigg(\hat{b}_t + \hat{p}_t\Bigg) = \overline{b}\hat{b}_{t-1} + \beta\delta\overline{b}\hat{p}_t + \overline{r}(1-\beta\delta)\phi_{\tau,b}\Bigg(\frac{\kappa_3}{\kappa_1}(1+\gamma_h+\sigma\frac{\overline{Y}}{\overline{C}}) - \frac{1}{1-\overline{\tau}^d}\Bigg)\hat{b}_{t-1} \tag{81}$$

(80) and (81) form the system of equations that needs to be resolved.

$$\underbrace{\begin{bmatrix}\beta\delta & \widetilde{\epsilon}\\ 0 & \beta\overline{b}\end{bmatrix}}_{\equiv A}\begin{pmatrix}E_t\hat{p}_{t+1}\\ \hat{b}_t\end{pmatrix} = \underbrace{\begin{bmatrix}1 & \widetilde{\epsilon}\\ \overline{b}\beta(\delta-1) & \overline{b}+\widetilde{\chi}\end{bmatrix}}_{\equiv B}\begin{pmatrix}\hat{p}_t\\ \hat{b}_{t-1}\end{pmatrix}$$

where $\widetilde{\chi} \equiv \overline{R}(1-\beta\delta)\phi_{\tau,b}\Bigg(\frac{\kappa_3}{\kappa_1}(1+\gamma_h+\sigma\frac{\overline{Y}}{\overline{C}}) - \frac{1}{1-\overline{\tau}^d}\Bigg)$ and $\widetilde{\epsilon} \equiv \sigma\frac{\overline{Y}}{\overline{C}}\frac{\kappa_3}{\kappa_1}\phi_{\tau,b}$.

Then

$$A^{-1}B = \frac{1}{\det(A)} \begin{bmatrix} \beta\bar{b} - \widetilde{\epsilon}\beta\bar{b}(\delta-1) & \widetilde{\epsilon}(\beta-1)\bar{b} - \widetilde{\epsilon}\widetilde{\chi} \\ \beta^2\delta\bar{b}(\delta-1) & \beta\delta(\bar{b}+\widetilde{\chi}) \end{bmatrix}$$

The characteristic equation is:

$$\lambda^2 - \frac{\lambda}{\det(A)}\Big(\beta\bar{b} + \widetilde{\epsilon}\beta\bar{b}(1-\delta) + \beta\delta(\bar{b}+\widetilde{\chi})\Big) + \frac{1}{\det(A)^2}\Big(\beta^2\delta\bar{b}(\bar{b}+\widetilde{\chi}) + \widetilde{\epsilon}\beta^3\delta(1-\delta)\bar{b}^2\Big)$$

The smallest root is:

$$\lambda_1 = \frac{1}{2}\frac{1}{\det(A)}\Bigg[\Big(\beta\bar{b} + \widetilde{\epsilon}\beta\bar{b}(1-\delta) + \beta\delta(\bar{b}+\widetilde{\chi})\Big) -$$
$$-\sqrt{\Big(\beta\bar{b} + \widetilde{\epsilon}\beta\bar{b}(1-\delta) + \beta\delta(\bar{b}+\widetilde{\chi})\Big)^2 - 4\Big(\beta^2\delta\bar{b}(\bar{b}+\widetilde{\chi}) + \widetilde{\epsilon}\beta^3\delta(1-\delta)\bar{b}^2\Big)}\Bigg]$$

It is easy to show that $\lambda_1 = 1$ when $\bar{b} + \widetilde{\chi} = \beta\bar{b}$, or

$$\phi_{\tau,b} = \frac{\bar{b}(1-\beta)}{\bar{r}(1-\beta\delta)\Big(\frac{1}{1-\bar{\tau}^d} - \frac{\kappa_3}{\kappa_1}(1+\gamma_h+\sigma\frac{\overline{Y}}{\overline{C}})\Big)} \equiv \widetilde{\phi}_\tau$$

which is the expression shown in Section 5 in text.

Moreover, we can show that λ_1 is monotonically decreasing in $\phi_{\tau,b}$ and $\lambda_1 < 1$ when $\phi_{\tau,b} > \widetilde{\phi}_\tau$. Finally, the largest root always exceeds 1. Thus the unique stable equilibrium is attained when $\phi_{\tau,b} > \widetilde{\phi}_\tau$.

B.1.2 $\lambda_Y > 0$

Consider now the case where $\lambda_Y > 0$ maintaining that $\lambda_i = 0$. Now an equilibrium under passive fiscal policy where $\psi_{gov,t} = 0$ may not exist. To see this, note that (76) can now be written as:

$$\frac{\beta\bar{b}}{1-\beta\delta}\Big(\psi_{gov,t} - E_t\psi_{gov,t+1}\Big) - \beta\phi_{\tau,b}E_t\Big(-\frac{\kappa_3}{\kappa_1}\lambda_Y\hat{Y}_{t+1} + \kappa_3\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l+1} + \frac{\kappa_3\omega_Y}{\kappa_1}\psi_{gov,t+1}$$
$$-\frac{\bar{r}}{1-\bar{\tau}^d}\psi_{gov,t+1}\Big) = 0$$

Following the notation of the previous subsection this can be rearranged into:

$$E_t\psi_{gov,t+1} - \Big(1 - (1-\delta)\widetilde{\eta}_2\frac{\beta\phi_{\tau,b}}{\widetilde{\eta}_3} + \frac{\delta\widetilde{\eta}_1}{\widetilde{\eta}_3}\Big)\psi_{gov,t} + \frac{\delta\widetilde{\eta}_1}{\widetilde{\eta}_3}\psi_{gov,t-1} = -\frac{\beta\phi_{\tau,b}\kappa_3\lambda_Y}{\kappa_1\widetilde{\eta}_3}\Big(E_t\hat{Y}_{t+1} - \delta\hat{Y}_t\Big)$$

Notice that the forcing term $E_t\hat{Y}_{t+1} - \delta\hat{Y}_t$ on the RHS of the above equation precludes a solution where $\psi_{gov,t} = 0$. To see this suppose that indeed $\psi_{gov,t}$ were equal to 0. Then, it should also be that $E_t\hat{Y}_{t+1} - \delta\hat{Y}_t = 0$. From the first order conditions of inflation and output we would get: $\hat{\pi}_t = \frac{\lambda_Y}{\kappa_1}(\hat{Y}_t - \hat{Y}_{t-1})$. Using this and the Phillips curve we can derive a difference equation in aggregate output with forcing term $-\frac{\kappa_3}{\kappa_1}\hat{\tau}_t + \frac{\kappa_2}{\kappa_1}\hat{G}_t$. Aggregate output will generally not equal zero, and $E_t\hat{Y}_{t+1} - \delta\hat{Y}_t$ will not be zero either.

To interpret the above, notice that when taxes are distortionary, they become a cost push shock in the Phillips curve which drives the inflation output tradeoff in the equilibrium with active fiscal policy. Thus, the planner will always attempt to use the tax schedule, targeting the path of debt, in order to smooth the shock and thereby smoothing the inflation output tradeoff. This makes the government debt constraint relevant, independent of the value of $\phi_{\tau,b}$. [40]

Finally note that the above problem may not arise if we adopt an alternative specification of the objective to smooth output and in particular if we assume that optimal policy attempts to stabilize output not around the constant steady state value, but around the natural level. In this case, when taxes deviate from steady state the planner will not attempt to restore the steady state level of output, using distortionary inflation.

To show this let us assume for simplicity $\sigma = 0$ and the output objective is $\lambda_Y(\hat{Y}_t - Y_t^n)^2$ where $\hat{Y}_t^n = -\frac{\overline{\tau}^d}{\gamma_h(1-\overline{\tau}^d)}\hat{\tau}_t$ is the natural level of output.

It is trivial to show that the model now admits an equilibrium where $\psi_{gov,t} = 0$. The first order condition for bonds can be written as:

$$\frac{\beta\overline{b}}{1-\beta\delta}\bigg(\psi_{gov,t} - E_t\psi_{gov,t+1}\bigg) - \beta\phi_{\tau,b}E_t\bigg(-\frac{\kappa_3}{\kappa_1}\lambda_Y(\hat{Y}_{t+1} - \hat{Y}_{t+1}^n) + \underbrace{(\frac{\kappa_3}{\kappa_1}\overline{\tau}^d(1+\gamma_h) - \overline{r}\frac{1}{1-\overline{\tau}^d})}_{f_1}\psi_{gov,t+1}\bigg) = 0 \tag{82}$$

where $f_1 < 0$ otherwise the economy is on the wrong side of the Laffer curve. This in turn gives:

$$\psi_{gov,t} = \underbrace{(1 + \frac{\beta\phi_{\tau,b}f_1}{\beta\overline{b}}(1-\beta\delta))}_{<1}E_t\psi_{gov,t+1} - \frac{\beta\phi_{\tau,b}}{\beta\overline{b}}(1-\beta\delta)\frac{\kappa_3}{\kappa_1}\lambda_Y E_t(\hat{Y}_{t+1} - \hat{Y}_{t+1}^n) \tag{83}$$

The solution $\psi_{gov,t} = 0$ is guaranteed if $\hat{Y}_{t+1} = \hat{Y}_{t+1}^n$. Note that this property will hold when output is stabilized around the natural level and fiscal policy is passive. We will then have an inflation rate equal to 0 for all t.

The threshold $\widetilde{\phi}_\tau$ can be found using the budget constraint. Leaving out the shocks (as we previously did) we can express this as:

$$\frac{\beta\overline{b}}{1-\beta\delta}\hat{b}_{t,\delta} + \overline{r}((1+\gamma_h)\hat{Y}_t^n + \frac{\hat{\tau}_t}{1-\overline{\tau}^d}) = \frac{\overline{b}}{1-\beta\delta}\hat{b}_{t-1,\delta}$$

and so using the definition of $\hat{Y}_t^n$ above and the fiscal rule we have:

$$\hat{b}_{t,\delta} = \frac{1}{\beta}\bigg[1 - \overline{r}\frac{(1-\beta\delta)\phi_{\tau,b}}{\overline{b}(1-\overline{\tau}^d)}(1 - \frac{1+\gamma_h}{\gamma_h}\overline{\tau}^d)\bigg]\hat{b}_{t-1,\delta}$$

Quite evidently we now have

$$\widetilde{\phi}_\tau = \frac{(1-\beta)}{1-\beta\delta}\frac{\overline{b}}{\overline{r}}\frac{(1-\overline{\tau}^d)}{(1-\frac{1+\gamma_h}{\gamma_h}\overline{\tau}^d)}$$

which is the formula shown in text assuming $\sigma = 0$. It is possible to extend the above to the case $\sigma > 0$ and recover the expression for $\widetilde{\phi}_\tau$ in text.

[40] In solving this model numerically, however, we found that increasing $\phi_{\tau,b}$ can bring the model very close to the 3 equation NK model, where debt is not a constraint for monetary policy. Thus from a practical standpoint, optimal monetary policy effectively becomes active even though this is not possible to show analytically.

B.2 Optimal policy rules with distortionary taxation.

We first derive the formulae shown in Section 5 of the paper for the simple Fisherian model assuming that the planner's objective only features inflation stabilization. Then we solve for optimal policies under the alternative versions considered in Section 3.

Simple Fisherian policies. With distortionary taxes and assuming $\sigma = \lambda_Y = \lambda_i = 0$ optimal inflation is determined by the following condition:

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} + \frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} = 0$$

where $\omega_Y \equiv \overline{R}(1+\gamma_h)$. Using the Euler equation in this model we then have that:

$$\hat{i}_t = \hat{\xi}_t + E_t\hat{\pi}_{t+1} = \hat{\xi}_t + E_t(\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t+1} + \frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l+1}) = \hat{\xi}_t + \delta\frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l}$$

Thus,

$$\hat{i}_t = \hat{\xi}_t + \delta(\hat{\pi}_t - \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t})$$

which is the result we showed in text.

The canonical model, $\sigma > 0$. Now consider the case where $\sigma > 0$. In this case, optimal inflation is still described by

$$\hat{\pi}_t = \frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} + \sigma\frac{\overline{Y}}{\overline{C}\kappa_1}\overline{b}\sum_{l=0}^{\infty}\delta^l\left(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\right) + \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} \tag{84}$$

i.e. the condition we derived in subsection A.1.4. The difference is that under distortionary taxes $\omega_Y = \overline{G}\sigma\frac{\overline{Y}}{\overline{C}} + \overline{R}(1+\gamma_h)$.

Essentially, all the derivations of subsection A.1.4 can be repeated here. We will therefore get the same interest rate rule, i.e. equation (57).

Interest rate smoothing. Consider now the case where $\lambda_i > 0$ and $\sigma = \lambda_Y = 0$. We now have

$$\hat{\pi}_t = -\frac{\lambda_i}{\beta}\hat{i}_{t-1} + \frac{\overline{b}}{1-\beta\delta}\sum_{l\geq 0}\delta^l\Delta\psi_{gov,t-l} + \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t}$$

From the Euler equation we have:

$$\hat{i}_t = \hat{\xi}_t + E_t\left(-\frac{\lambda_i}{\beta}\hat{i}_t + \frac{\overline{b}}{1-\beta\delta}\sum_{l\geq 0}\delta^l\Delta\psi_{gov,t-l+1} + \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t+1}\right) = \hat{\xi}_t - \frac{\lambda_i}{\beta}\hat{i}_t + \frac{\delta\overline{b}}{1-\beta\delta}\sum_{l\geq 0}\delta^l\Delta\psi_{gov,t-l}$$
$$= \hat{\xi}_t - \frac{\lambda_i}{\beta}\hat{i}_t + \delta\left(\hat{\pi}_t + \frac{\lambda_i}{\beta}\hat{i}_{t-1} - \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t}\right)$$

Thus :

$$\hat{i}_t = \frac{1}{1+\frac{\lambda_i}{\beta}}\hat{\xi}_t + \frac{\delta}{1+\frac{\lambda_i}{\beta}}\hat{\pi}_t + \frac{\frac{\lambda_i}{\beta}}{1+\frac{\lambda_i}{\beta}}\hat{i}_{t-1} - \frac{1}{1+\frac{\lambda_i}{\beta}}\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t}$$

Output stabilization. Finally let $\lambda_Y > 0$ and $\sigma = \lambda_i = 0$. Optimal inflation obeys the following

$$-\hat{\pi}_t - \frac{\lambda_Y}{\kappa_1^2}(\hat{\pi}_t - \beta E_t\hat{\pi}_{t+1}) + \frac{\lambda_Y}{\kappa_1^2}(\hat{\pi}_{t-1} - \beta E_{t-1}\hat{\pi}_t) + \frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t} + \frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-l} = 0$$

We now define:

$$\zeta_t \equiv (\hat{\pi}_t - E_{t-1}\hat{\pi}_t) + \frac{\kappa_1^2}{\beta\lambda_Y}\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-l} + \frac{\kappa_1^2}{\beta\lambda_Y}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t} = 0$$

So that inflation again evolves according to:

$$E_t\hat{\pi}_{t+1} - (1 + \frac{1}{\beta} + \frac{\kappa_1^2}{\lambda_Y\beta})\hat{\pi}_t + \frac{1}{\beta}\hat{\pi}_{t-1} = -\zeta_t$$

Solving the second order polynomial we once again obtain:

$$\hat{\pi}_t = \frac{1}{\widetilde{\lambda}_2}E_t\hat{\pi}_{t+1} + \frac{1}{\widetilde{\lambda}_2}\frac{1}{1-\widetilde{\lambda}_1 L}\zeta_t = \frac{1}{\widetilde{\lambda}_2}\frac{1}{1-\widetilde{\lambda}_1 L}\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\zeta_{t+j}$$

Then,

$$\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\zeta_{t+j} = \sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\left[(\hat{\pi}_{t+j} - E_{t+j-1}\hat{\pi}_{t+j}) + \widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t+j} + \widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t+j-l}\right] =$$

$$\hat{\pi}_t - E_{t-1}\hat{\pi}_t + \widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t} + \widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t}$$

and so

$$\hat{\pi}_t = \widetilde{\lambda}_1\hat{\pi}_{t-1} + \frac{1}{\widetilde{\lambda}_2}(\hat{\pi}_t - E_{t-1}\hat{\pi}_t) + \frac{1}{\widetilde{\lambda}_2}\widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t} + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t} \tag{85}$$

We can once again show that

$$E_t\hat{\pi}_{t+1} = \widetilde{\lambda}_1\hat{\pi}_t + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\underbrace{E_t\frac{1}{1-\delta L}\Delta\psi_{gov,t+1}}_{=\frac{\delta}{1-\delta L}\Delta\psi_{gov,t}}$$

and

$$\hat{\pi}_t - E_{t-1}\hat{\pi}_t = \frac{\widetilde{\lambda}_2}{\widetilde{\lambda}_2 - 1}\frac{\widetilde{\kappa}}{\lambda_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\Delta\psi_{gov,t}$$

The optimal interest rate rule is given by:

$$\hat{i}_t = \hat{\xi}_t + (\widetilde{\lambda}_1 + \delta)\hat{\pi}_t - \delta\widetilde{\lambda}_1\hat{\pi}_{t-1} - \frac{\delta}{\widetilde{\lambda}_2}\widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t} - \delta\frac{\widetilde{o}}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t}$$

which is of the same form as the rule under lump sum taxes, however, now the stochastic intercept is $-\frac{\delta}{\widetilde{\lambda}_2}\widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t} - \delta\frac{\widetilde{o}}{\widetilde{\lambda}_2-1}\Delta\psi_{gov,t}$ instead of just $-\delta\frac{\widetilde{o}}{\widetilde{\lambda}_2-1}\Delta\psi_{gov,t}$.

Finally, note that under output stabilization, in general the model will not admit two equilibria under active and passive fiscal policies when taxes are distortionary. The reason is that even when ϕ_τ is of high value, since taxes now appear in the Phillips curve, the budget constraint will always be relevant for a planner that cares about stabilizing output. (Otherwise, if output stabilization is not in the objective, then aggregate output can absorb time variation in taxes to smooth inflation). Therefore we will not generally obtain $\psi_{gov,t} = 0$ endogenously in this model, though as ϕ_τ increases the equilibrium will indeed resemble an active monetary/passive fiscal scenario.

In order to get two equilibria in this model we need to specify a different output stabilization objective, assuming that monetary policy accounts for the deviations of output from its efficient level. We will not consider this case explicitly here.

Figure 10 displays the impulse response functions to a government spending shock obtained from the model with distortionary labor taxes.

B.3 The nonlinear model.

We present here the nonlinear equations for the simplistic NK model used in the paper.

Households Households maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \right)$$

subject to

$$P_t C_t + P_{t,s} B_{t,S} + P_{t,L} B_{t,\delta} \leqslant (1-\tau_t) W_t h_t - P_t T_t + P_t D_t + B_{t-1,S} + (1 + \delta P_{t,L}) B_{t-1,\delta}$$

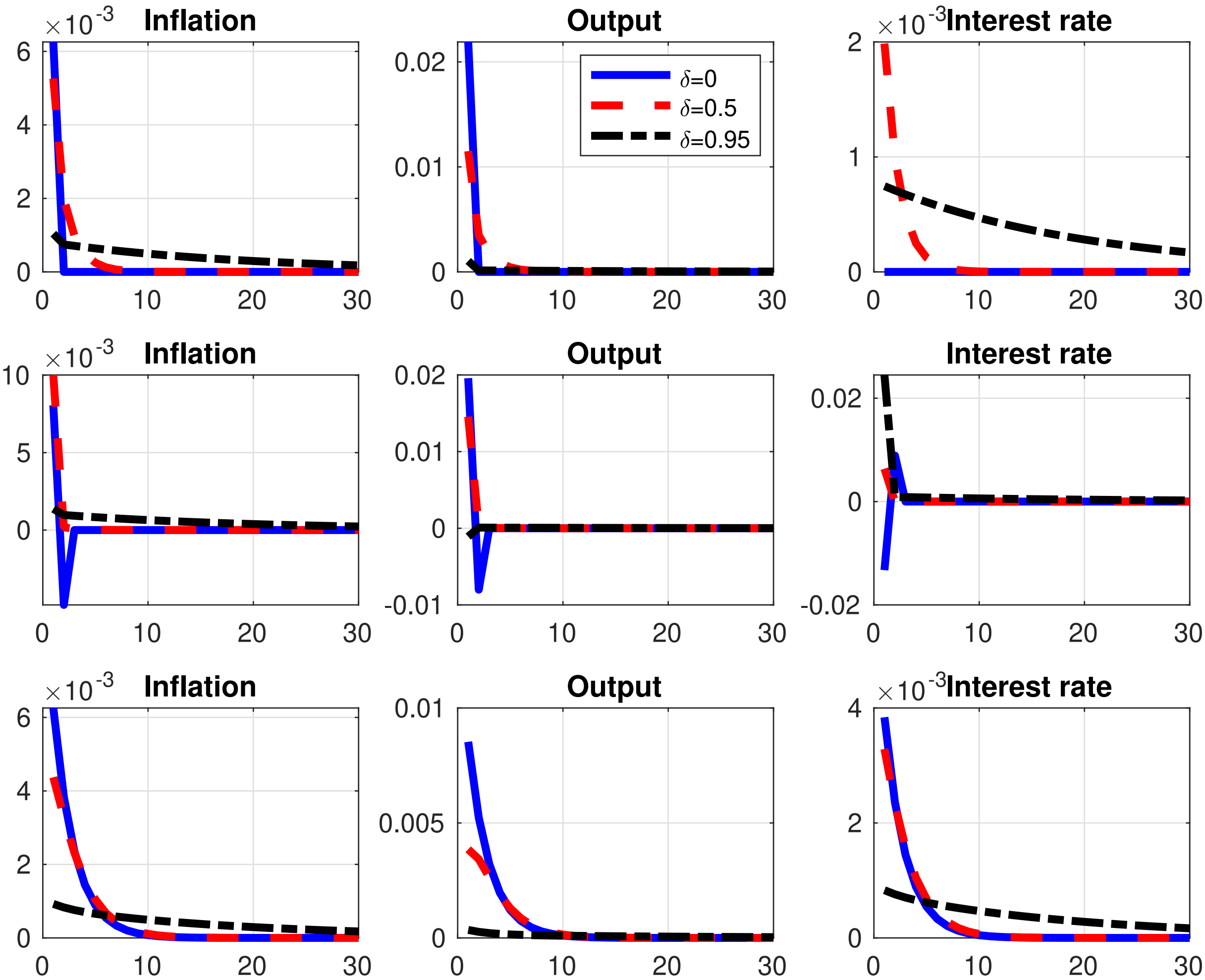
where C_t denotes consumption and h_t denotes hours worked. D_t represents firms' profits redistributed to households, and P_t denotes the aggregate price level. $B_{t,\delta}$ is a long-term government bond, a perpetuity with coupon payments decaying at the rate $0 \leqslant \delta < 1$ and price $P_{t,L}$. $B_{t,s}$ denotes short-term bonds with price is $P_{t,S}$. We assume that short debt is in zero net supply. ξ_t is the preference shock. $0 \leq \tau_t \leq 1$ denotes the (distortionary) tax rate on labour, and T_t the level of lump-sum taxes.

The first order conditions of the household's problem are:

$$\begin{aligned} P_{t,s}\xi_t C_t^{-\sigma} &= \beta E_t \xi_{t+1} \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \\ P_{t,L}\xi_t C_t^{-\sigma} &= \beta E_t \xi_{t+1} \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}(1 + \delta P_{t+1,L}) \\ h_t^{\gamma_h} C_t^{\sigma} &= (1-\tau_t)\frac{W_t}{P_t} \end{aligned}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

Figure 10: **Impulse response functions with distortionary taxes**



Notes: The figure displays the impulse responses of inflation, output, and the nominal interest rate following a government spending shock, in the model with distortionary labor taxes. Top panels assume $\sigma = \lambda_Y = \lambda_i = 0$, mid-panels set $\sigma = 1$ and $\lambda_Y = \lambda_i = 0$, and bottom panels are for the case $\sigma = 0$ and $\lambda_i = \lambda_Y = 0.5$. In each plot, the solid blue line depicts impulse responses in the case where government debt is short term ($\delta = 0$); the dashed red lines and dash-dotted black lines plot the responses of variables when $\delta = 0.5$ and $\delta = 0.95$, respectively.

Firms Production takes place in monopolistically competitive firms which operate technologies with labour as the sole input. The final good is a CES aggregate of the intermediate goods $Y_t(j)$:

$$Y_t = \Big(\int_0^1 Y_t(j)^{\frac{1+\eta}{\eta}} dj \Big)^{\frac{\eta}{1+\eta}} \tag{86}$$

where η governs the elasticity of substitution between differentiated goods. Firms set prices to maximize profits subject to the demand curve

$$Y_t(j) = \Big(\frac{P_t(j)}{P_t} \Big)^{\eta} Y_t \tag{87}$$

and given price adjustment costs, modelled as in Rotemberg (1982). The dynamic profit maximization program is:

$$\max_{P_t(j)} \quad E_t \sum_{s=0}^{\infty} Q_{t,t+s} \Big(\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - \frac{W_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - AC_{t+s}(j) \Big)$$

$$\text{s.t.} \quad Y_{t+s}(j) = \Big(\frac{P_{t+s}(j)}{P_{t+s}} \Big)^{\eta} Y_{t+s} \tag{88}$$

$$AC_{t+s}(j) = \frac{\theta}{2} \Big(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - \pi \Big)^2 Y_{t+s} \tag{89}$$

where $Q_{t,t+s} \equiv \beta^s$ is the discount factor of households and W_{t+s} is the wage rate, that is equal to the marginal cost of production. (89) is the quadratic adjustment costs incurred by firms when resetting their price.

Focusing on a symmetric equilibrium the first order condition from the firm's dynamic program gives the following non-linear Phillips Curve:

$$\theta(\pi_t - \pi)\pi_t = 1 + \eta(1 - \frac{W_t}{P_t}) + \beta\theta E_t \frac{C_t^{\sigma}}{C_{t+1}^{\sigma}} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - \pi)\pi_{t+1} \tag{90}$$

The firms' technology is linear in labour and thus $Y_t(j) = h_t(j)$ where $j \in [0, 1]$ denotes the generic firm.

Fiscal policy Government spending G_t evolves exogenously according to an AR(1) process in logs:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t}$$

where $\hat{g}_t \equiv \log G_t - \log \overline{G}$.

The (distortionary) labor tax is set by the fiscal authority according to the simple rule described in text (in log-linear form). The flow government budget constraint can be written as:

$$P_{t,L} b_{t,\delta} = (1 + \delta P_{t,L}) \frac{b_{t-1,\delta}}{\pi_t} + G_t - \tau_t h_t w_t - T_t \tag{91}$$

where $b_{t,\delta} \equiv \frac{B_{t,\delta}}{P_t}$ denotes real long-term government debt.

We consider two alternatives regarding the fiscal instrument available to the government. In the first case, $\tau_t = 0$ for all t and the government only makes use of lump-sum taxes T_t. In the second, $T_t = 0$ and only distortionary taxes are used. In each case, we denote the log-deviation of the instrument from its steady-state value as $\hat{\tau}_t$, and this instrument evolves according to the rule presented in equation (6).

Log-linearization Making use of the labor supply condition $h_t^{\gamma_h} C_t^{\sigma} = (1-\tau_t)\frac{W_t}{P_t}$, as well as the resource constraint $h_t = Y_t = C_t + G_t$ to dispense with W_t, C_t and h_t, we get the following linear New Keynesian Phillips Curve:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \kappa_2 \hat{\tau}_t - \kappa_3 \hat{G}_t + \beta E_t \hat{\pi}_{t+1} \tag{92}$$

where κ_1, κ_2 and κ_3 are defined in text. Note that when the fiscal instrument is the lump-sum tax, $\kappa_2 = 0$.

Defining $i_t \equiv -\log P_{t,S}$, log-linearizing the Euler equation for short bonds (and again making use of the resource constraint $Y_t = C_t + G_t$) we get:

$$\hat{i}_t = E_t\left(\hat{\pi}_{t+1} - \hat{\xi}_{t+1} + \hat{\xi}_t - \sigma\left[\frac{Y}{C}(\hat{Y}_t - \hat{Y}_{t+1}) - \frac{G}{C}(\hat{G}_t - \hat{G}_{t+1})\right]\right) \tag{93}$$

Finally, making use of the Euler equation for long bonds and iterating forward and making use of the resource constraint we get $P_{t,L} = \sum_{j\geq 1} E_t \beta^j \delta^{j-1} \frac{\xi_{t+j}}{\xi_t} \frac{(Y_{t+j} - G^{-\sigma}_{t+j)}}{(Y_t - G^{-\sigma}_{t)} \prod_{l=1}^{j} \pi_{t+l}}$. Using this to substitute out $P_{L,t}$ from the government budget constraint we obtain:

$$\sum_{j\geq 1} E_t \beta^j \delta^{j-1} \frac{\xi_{t+j}}{\xi_t} \frac{(Y_{t+j} - G_{t+j})^{-\sigma}}{(Y_t - G_t)^{-\sigma} \prod_{l=1}^{j} \pi_{t+l}} b_{t,\delta} =$$
$$\left(1 + \delta \sum_{j\geq 1} E_t \beta^j \delta^{j-1} \frac{\xi_{t+j}}{\xi_t} \frac{(Y_{t+j} - G_{t+j})^{-\sigma}}{(Y_t - G_t)^{-\sigma} \prod_{l=1}^{j} \pi_{t+l}}\right) \frac{b_{t-1,\delta}}{\pi_t} - S_t$$

where $S_t = \tau_t h_t w_t - G_t$ or $S_t = T_t - G_t$ depending on whether the fiscal instrument is the labour tax or the lump-sum tax. Log-linearizing this equation we get:

$$\frac{\beta \bar{b}}{1-\beta\delta} \hat{b}_{t,\delta} + \bar{b} \sum_{j=1}^{\infty} \beta^j \delta^{j-1} \left[E_t \left(-\sigma(\frac{\overline{Y}}{\overline{C}} \hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}} \hat{G}_{t+j}) - \sum_{l=1}^{j} \hat{\pi}_{t+l} + \hat{\xi}_{t+j} \right) \right]$$
$$= -\overline{S}\hat{S}_t - \bar{b}\sigma\left(\frac{\overline{Y}}{\overline{C}}\hat{Y}_t - \frac{\overline{G}}{\overline{C}}\hat{G}_t\right) + \bar{b}\hat{\xi}_t \tag{94}$$
$$+ \frac{\bar{b}}{1-\beta\delta}(\hat{b}_{t-1,\delta} - \hat{\pi}_t) + \delta \bar{b} \sum_{j=1}^{\infty} \beta^j \delta^{j-1} E_t \left(-\sigma(\frac{\overline{Y}}{\overline{C}} \hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}} \hat{G}_{t+j}) - \sum_{l=1}^{j} \hat{\pi}_{t+l} + \hat{\xi}_{t+j} \right)$$

which is the equation stated in the main text.

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