OPTIMAL MONETARY POLICY RULES IN THE FISCAL THEORY OF THE PRICE LEVEL

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LIDAM Discussion Paper IRES
2022 / 26

UCLouvain

IRES | Louvain Institute of Data Analysis and Modeling in economics and statistics

Optimal Monetary Policy Rules in the Fiscal Theory of the Price Level*

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November 29, 2022

Abstract

In the fiscal theory of the price level, inflation and debt dynamics are determined jointly. We derive optimal monetary policy rules that can approximate the Ramsey outcome in this environment. When the government issues a portfolio of bonds of different maturities and *buys it back* every period the optimal interest rate response to inflation is a simple, transparent function of the average debt maturity. This policy exploits the maturity structure to minimize the intertemporal variability of inflation in response to fiscal shocks. We then turn to the more realistic scenario of *no buyback* assuming that the government does not repurchase and reissue debt in every period. In the case where debt is only long term, the optimal policy equilibrium features oscillations in inflation and simple inflation targeting rules may lead to explosive inflation dynamics. Issuing both short and long bonds rules out oscillations and allows simple rules to approximate the Ramsey outcome closely.

Underlying these results is the ability of the optimizing policy authority to smooth distortions stemming from inflation across periods. When debt is short term or it is bought back in every period, the planner can spread evenly the distortions over time. Under no repurchases, this ability is lost.

Keywords: Monetary Policy, Fiscal Theory, Optimal Interest Rates, Government Debt Maturity, Ramsey policy.

JEL: E31, E52, E58, E62, C11.

*We received helpful comments from Xavier Debrun, Anastasios Karantounias, Robert Kollmann, Jochen Mankart, Francesca Monti, Sebastian Schmidt, Raf Wouters and seminar participants in UCLouvain. de Beauffort is grateful to the National Bank of Belgium for funding. The views expressed are those of the authors and do not necessarily reflect those of the National Bank of Belgium, or the European Commission.

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1 Introduction

A considerable literature has analyzed optimal monetary policy in environments where inflation is used to stabilize government debt dynamics. Chari and Kehoe (1999) first considered this in the context of Ramsey optimal policy assuming an economy with flexible prices where debt is issued in a short term bond. Their approach has been subsequently extended to sticky price models (Schmitt-Grohé and Uribe, 2004 and Siu, 2004) and to models in which debt can be both short and long term (e.g. Lustig et al., 2008; Faraglia et al., 2013; Sims, 2013; Leeper and Zhou, 2021 among others).

In these models optimal policy is the solution to the first order conditions of the Ramsey program, which involves the Lagrange multipliers of current and past consolidated budget constraints, the objects that define the dynamics of debt. These multipliers are state variables; they summarize the impact of shocks that have hit the economy and thus also impacted debt. In the optimal policy equilibrium macroeconomic variables (such as inflation and output) as well as the nominal interest rate become functions of the multipliers.

This approach to optimal policy has thus the limitation that it yields a path of the nominal rate, the main instrument of monetary policy, defined on the basis of variables that are not observed in practice. Even though Lagrange multipliers in these models can be expressed as functions of the histories of economic shocks, a policy instrument that depends explicitly on shocks is also not practical, in that it involves the non trivial task of identifying current and past shocks.

Practically relevant policy rules specify the path of the nominal rate as a function of macroeconomic variables (inflation, output, etc). These are the types of rules that we estimate in medium scale DSGE models and have also been considered by a large number of papers studying optimal policy in the context of the baseline New Keynesian model (where inflation does not respond to the debt aggregate).[1] We would like to know whether in a model where inflation and debt dynamics are determined jointly (in the fiscal theory of the price level framework) such rules can approximate the optimal policy and if so, what is the appropriate coefficient on inflation (or on other relevant macroeconomic variables) in the interest rate rule.

Our baseline model is a simplistic Fisherian - New Keynesian framework, which allows us to characterize optimal policy analytically, experimenting with various modelling assumptions regarding the structure of debt. The model is augmented with the consolidated budget constraint and moreover, to isolate our focus on the role of inflation in stabilizing debt, we assume that taxes are constant, assuming also that the government's surplus fluctuates according to an exogenous shock to spending. Our framework is thus broadly similar to that of Cochrane (2001).[2]

In Section 2 we begin by laying out our theoretical model. Following the optimal policy literature cited above, we model long term bonds with *full buybacks*, that is assuming that debt is repurchased one period after issuance. We revisit Ramsey optimal policy in this framework since this is the benchmark by which the optimality of the interest rate rules that we will consider will be measured, and explaining the key forces that determine the path of inflation under Ramsey is important. The simplicity of our framework enables us to characterize the Ramsey solution analytically, under any maturity structure of debt.

Section 3 then turns to the analysis of optimal interest rate rules. Our substantive finding is that commitment to a rule that sets the nominal interest rate only as a function of inflation is sufficient to approximate the Ramsey outcome very well and, under specific debt maturity structures, it even delivers effectively the same outcome as Ramsey policy. One does not need to include many lags of inflation (or other macroeconomic variables) in the policy rule, a property that seems surprising given the dependency of Ramsey policy on the history of Lagrange multipliers and shocks.

[1] See for example Giannoni and Woodford (2003a,b); Giannoni (2014), among numerous others.

[2] See also Sims (2013); Bouakez et al. (2018); Bianchi and Melosi (2019); Aiyagari et al. (2002); Davig and Leeper (2006), for analogous Fisherian models in various policy contexts.

Moreover, over the broad range of alternative maturity structures of debt that we consider in our Fisherian model, the optimal inflation coefficient is given by a simple formula of the average debt maturity. The coefficient is $1-\frac{1}{\text{Maturity}}$. A higher maturity leads to a stronger reaction of the nominal interest rate to inflation.

To understand this result, note first that, as in any other fiscal theory model, the inflation coefficient must be between 0 and 1 in order to have a unique stable equilibrium. This is obviously the case here. As is well known, in this type of environment, inflation becomes a backward looking process and raising the nominal rate will not accomplish a lower inflation rate, rather it will make inflation increase persistently. This is desirable when the average maturity of debt is long, since it enables to spread inflation over more periods and stabilize debt. In contrast, in the case where debt is short, making inflation respond over the longer term to fiscal shocks is wasteful, because it is only short term price growth that can contribute towards debt stability. Under the optimal policy, therefore, the inflation coefficient is zero when debt is only short term and it is strictly positive when both short and long bonds are issued. In the case of a flat maturity structure (equivalently debt is issued in a consol that pays fixed coupons) the coefficient becomes equal to one.

These findings turn out to hold independently of the types of shocks that hit the economy and their stochastic processes. The key driver of optimal inflation dynamics in our model is the average debt maturity, not the source of fluctuations in the economy.

In Section 4 we turn towards a modelling assumption for long term bonds that departs significantly from the canonical modelling found in the literature. All papers mentioned previously assume that the entire stock of long bonds is repurchased one period after issuance. This assumption is made for tractability (keeping track of many lags of debt in the state vector is not easy) however it is not in line with observed practices in the US and elsewhere. Faraglia et al. (2019) provide extensive evidence that the US Treasury does not buy back their debt prior to maturity. The Quantitative easing program run by the Federal reserve since the 2008-9 recession can be seen as a partial buyback of long term government bonds.

When we consider *no buy back* as the modelling assumption for long term debt we find strikingly different implications for optimal policy. Under no debt repurchases and when debt is a zero coupon bond of maturity N, optimal inflation features oscillations of periodicity N which persist forever. Simple policy rules that specify the nominal rate as a function of current inflation will not work; these types of rules do not lead to stable equilibria, even when the inflation coefficients are between 0 and 1.

To flex out the intuition behind the first result (that oscillations occur in this model) we provide a simple analytical example assuming $N=2$. A negative shock to the surplus in period t will need to be compensated with higher inflation to reduce the real payout of debt that matures in t, that is debt that has been issued in $t-2$. However, higher period t inflation will also impact the real value of debt that has not matured, debt issued in $t-1$. This impact will destabilize the intertemporal debt constraint in period $t+1$ (when this debt has to be redeemed) and for the constraint to hold, it must be that inflation drops in $t+1$. These effects persist indefinitely, even though the government continues to issue new debt in every period.

A simple inflation targeting rule cannot mitigate the N cycle as it does not pin down a unique stable equilibrium path. Instability is a worse outcome; arbitrary inflation oscillations may occur and their magnitude can grow over time. In the no buyback model, a unique stable equilibrium can be reached when the policy rule pins down the growth of the price level between $t-N$ and t, in other words when it determines the sum of inflation rates $\hat{\pi}_t+\hat{\pi}_{t-1}+\ldots+\hat{\pi}_{t-N+1}$ (in terms of the notation in our log linear model). The interest rate rules that can deliver this are highly impractical, featuring $N-1$ lags and leads of inflation to determine the sum.

The key element that lies behind the strikingly different results we get out of the buyback and no buyback models is the optimal policy's ability to spread the distortions of inflation across time. When we assume that all debt is repurchased one period after being issued, the distortions can be evenly spread across periods. Under no repurchases this is not so, and simple interest rate rules cannot approximate optimal policy. We frame this result using the Lagrange multiplier on the consolidated budget, which measures the distortions under optimal policy. Under buyback this multiplier follows a random walk, implying evenly spread distortions. Under no buyback, it follows a cycle of periodicity N implying uneven distortions.[3]

Our final experiments in Section 4 explore whether the above results carry over to cases where the government issues any maturity structure of debt, not only a zero coupon bond of maturity N under no buyback. It turns out that key to enable to evenly spread distortions across periods, is to issue positive amounts of short term bonds. When the maturity of some of the debt issued, coincides with the periodicity of the model (1 quarter) then simple inflation targeting rules work. When this condition does not hold, oscillations become unavoidable.

Section 5 considers several extensions of the baseline model. First, we depart from the baseline where we assumed that the policy objective is to minimize the volatility of fiscal inflation, to consider the case where output stabilization also becomes a goal of optimal policy. Also in this case we can obtain analytically policy rules with inflation coefficients that are simple functions of debt maturity and approximately deliver the Ramsey outcome. Second, we argue that our findings do not only concern the Fisherian economy that we have employed for analytical convenience, but also carry over to the canonical New Keynesian model, where the real interest rate is endogenously affected by aggregate output and spending. We derive a simple formula for the optimal inflation coefficient, as a function of the average maturity of debt, and other parameters of the model that measure the effect of output on the real interest rates and the intertemporal solvency of debt. The key result of this paper, that the Ramsey outcome can be approximated through a simple inflation targeting rule thus also holds in the canonical model. Moreover, our results concerning the no buyback also continue to hold. Lastly, we show that our results would also hold if we had assumed that taxes are not constant through time, an assumption that we made for analytical tractability. A final section concludes the paper.

Our paper is related to a vast literature of optimal policy models. First, numerous papers have studied optimal policy under commitment to an interest rate rule in the context of the baseline New Keynesian model, to identify simple rules that can approximate Ramsey outcomes. See Giannoni and Woodford (2003a,b); Giannoni (2014) among others. We apply the arguments of these papers to optimal policy in the fiscal theory of the price level framework.

Second, many papers have studied policy assuming that an optimizing government chooses taxes to finance debt in the context of real models (e.g. Lucas and Stokey, 1983; Aiyagari et al., 2002; Marcet and Scott, 2009; Faraglia et al., 2016 and others). Lucas and Stokey (1983) assume that the government can issue debt in state contingent instruments, whereas Aiyagari et al. (2002); Marcet and Scott (2009); Faraglia et al. (2016) assume 'incomplete markets' letting debt be issued only in a single bond, long or short term. Our optimal Ramsey policy framework also assumes incomplete markets, and thus our approach is methodologically similar to Aiyagari et al. (2002); Marcet and Scott (2009); Faraglia et al. (2016); however, we allow debt to be issued in multiple assets of different maturity. Importantly, Faraglia et al. (2016) consider a distinction between buyback and no buyback in a real economy showing that under no buyback the Ramsey solution features tax oscillations. Our result in Section 4 that inflation oscillations are unavoidable when repurchases of long term nominal bonds are ruled out, is rooted into their analysis.

Relatedly, a literature on public debt management using real models with distortionary taxes has explored how debt portfolios can be designed to ensure the intertemporal solvency of debt,

[3] Importantly, as we argue, this result extends to cases where debt is callable, it can be repurchased a few periods after being issued, or the central bank can repurchase long bonds that have been issued some time ago, or even in the presence of multiple long bonds and no buyback. In all these cases the multiplier follows a cycle of periodicity > 1.

absorbing shocks that hit the government budget. Angeletos (2002) and Buera and Nicolini (2004) both assuming that debt is repurchased, reach the conclusion that when governments focus on issuing long term debt, then taxes will not need to adjust to shocks to government spending.[4] Faraglia et al. (2019) show that this will not hold in a model where debt repurchases are ruled out. Under no buyback it becomes optimal to issue a mix of both short term and long term bonds. Our result that issuing positive amounts of short bonds removes inflation oscillations and restores the optimality of simple interest rate rules is inspired by their finding.

Moreover, ours is a paper on the fiscal theory and so the considerable literature studying the interactions between monetary and fiscal policies in macroeconomic models is related to our work (e.g. Sargent et al., 1981; Leeper, 1991; Sims, 1994; Woodford, 1994, 1995, 2001; Cochrane, 1998, 2001; Schmitt-Grohé and Uribe, 2000; Bassetto, 2002; Eggertsson, 2008; Canzoneri et al., 2010; Del Negro and Sims, 2015; Reis, 2016; Bianchi and Melosi, 2017; Bianchi and Ilut, 2017; Bianchi and Melosi, 2019; Davig and Leeper, 2007; Jarociński and Maćkowiak, 2018; Benigno and Woodford, 2007; Chen et al., 2022; Bi and Kumhof, 2011; Kumhof et al., 2010 among others). See also Leeper and Leith, 2016 for a very comprehensive survey. Within this context, papers that study optimal policy using the linear quadratic framework (for example Cochrane (2001), Leeper et al. (2021), Benigno and Woodford (2007)) are particularly relevant. Cochrane (2001) explores the optimal debt management policy under various modelling assumptions for long bonds, to investigate how alternative bond issuance strategies affect the path inflation in a Fisherian model with flexible prices. Benigno and Woodford (2007) study the optimal paths of inflation under various fiscal policy regimes (including the case where taxes are held constant). Most of their results are built on the assumption that debt is short term, though the authors also consider the case where debt maturity becomes a shock absorber, as in Angeletos (2002) and Buera and Nicolini (2004). Leeper and Zhou (2021) solve a Ramsey problem assuming that debt is issued in a perpetuity and make progress with deriving analytical results. Our baseline is a simpler (Fisherian) setup which allows us to experiment with alternative assumptions regarding the debt structure, including the no buyback assumption whose implications we explore in Section 4. Our analytical formulae complement those of Leeper and Zhou (2021) and Benigno and Woodford (2007).

Lastly, Chafwehé et al. (2022) derive optimal interest rate rules from the Ramsey policy assuming that taxes follow an exogenous rule, distinguishing between passive and active tax policies. Some of the results we show in Section 5 are based on that paper. However, Chafwehé et al. (2022) focus only on the case where debt is issued in a perpetuity bond that pays decaying coupons. We consider a wider set of assumptions regarding the maturity structure of debt, and emphasize the importance of modelling long bonds with buybacks or with no buyback, which Chafwehé et al. (2022) do not consider.

2 Theoretical Framework and Optimal Ramsey Policy

Our baseline model is a Fisherian economy featuring sticky prices and a fiscal block, the consolidated budget constraint and taxes which can either be lump sum or distortionary (levied on labour income). For simplicity, we will derive analytical results assuming that taxes are constant through time.[5] We

[4] In the context of models of optimal inflation (e.g. Lustig et al., 2008; Sims, 2013; Leeper and Zhou, 2021) a similar prediction obtains, long term enables to reduce inflation's reaction to shocks.

[5] Note that constant taxes is a common assumption in the context of the fiscal theory literature, (e.g. Bianchi and Melosi, 2017; Bianchi and Ilut, 2017; Cochrane, 2001 among others). As discussed previously this assumption is not restrictive. We can derive the key results shown below, assuming that taxes (mildly) adjust to the deviation of debt from a target value (so that the solvency of debt is not fully ensured by taxes as is standard in the context of the fiscal theory) or that taxes are set optimally along with inflation as is common in many papers in the Ramsey literature. We will briefly consider the second alternative in Section 5.

will further assume that the surplus of the government fluctuates over time due to exogenous shocks to the spending level. Since taxes are constant, spending shocks can only be financed through changes in the inflation rate that adjust the real market value of government debt. The model is thus a standard laboratory of the fiscal theory, as in e.g. Cochrane (2001).

Since this is a well known setup, for brevity, we will define here the competitive equilibrium equations in log-linear form. In the appendix we describe the background non-linear model and derive the equations from the optimality conditions of the households' and firms' optimization problems.[6]

We let $\hat{x}$ denote the log deviation of variable x from its steady state value, $\overline{x}$. The system of the competitive equilibrium equations is the following:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \beta E_t \hat{\pi}_{t+1}, \tag{1}$$

where $\kappa_1 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\gamma_h > 0$.

$$\sum_{k=1}^{\infty} \overline{p}_k \overline{b}_k (\hat{b}_{t,k} + \hat{p}_{t,k}) = -\overline{S}\hat{S}_t + \overline{b}_1(\hat{b}_{t-1,1} - \hat{\pi}_t) + \sum_{k=2}^{\infty} \overline{p}_k \overline{b}_k (\hat{b}_{t-1,k} + \hat{p}_{t,k-1} - \hat{\pi}_t) \tag{2}$$

where

$$\overline{S}\hat{S}_t = -\overline{G}\hat{G}_t + \overline{R}(1+\gamma_h)\hat{Y}_t$$

$$-\hat{p}_{t,1} = \hat{i}_t = E_t \hat{\pi}_{t+1} \tag{3}$$

$$\overline{p}_k \hat{p}_{t,k} = -\beta^k \sum_{l=1}^{k} E_t \hat{\pi}_{t+l} \tag{4}$$

(1) is the Phillips curve at the heart of our model. $\hat{\pi}_t$ represents inflation and $\hat{Y}_t$ is the output gap. Parameters $\eta < 0$ and $\theta > 0$ govern the elasticity of substitution across the differentiated (monopolistically competitive) goods produced in the economy and the degree of price stickiness respectively.[7] Parameter γ_h is the inverse of the Frisch elasticity of labor supply.

(2) is the consolidated budget constraint. The LHS of this equation represents the value of debt issued in period t. We assume that debt can be issued in bonds of maturities k where $k = 1, 2, ...$ and which pay zero coupons. $\overline{b}_k$ denotes the (steady state) quantity of the k bond. The price of the bond is denoted $\hat{p}_{t,k}$ ($\overline{p}_k$ in steady state).

The first term on the RHS of (2) is the government's surplus ($\overline{S}\hat{S}_t$). $\hat{G}_t$ is the spending of the government and $\overline{R}$ denotes the (steady state) revenue due to distortionary taxation. When all revenue derives from distortionary taxation we have $\overline{S} = \overline{R} - \overline{G}$. In contrast, if taxes are 100 percent lump sum then $\overline{R} = 0$. Notice also that since taxes are assumed constant, changes in revenue can only derive from fluctuations in output $\hat{Y}_t$, as long as $\overline{R} > 0$.

The second term on the RHS of (2) is the real value of debt that was issued in $t-1$ and repurchased in t. $\overline{p}_k \overline{b}_k (\hat{b}_{t-1,k} + \hat{p}_{t,k-1} - \hat{\pi}_t)$ denotes the value of debt of maturity k issued in $t-1$. This debt is priced in t as $k-1$ maturity debt and the corresponding price is $\hat{p}_{t,k-1}$.

[6]It is perhaps useful to mention that the Fisherian economy is a standard New Keynesian model in which household preferences a quasilinear (linear in consumption). This setup is not uncommon in the literature. Besides Cochrane (2001) see also Aiyagari et al. (2002); Faraglia et al. (2013); Sims (2013); Bouakez et al. (2018); Davig and Leeper (2006) and others.

[7]θ is the parameter that governs the magnitude of price adjustment costs in the standard quadratic cost function of Rotemberg (1982). When θ equals zero prices are fully flexible.

Equations (3) and (4) define the prices of k bonds. (3) is the log-linear IS-Euler equation. Since our setup is Fisherian, the real rate is exogenous and constant. The (log of the) short term nominal interest rate, $\hat{i}_t$, thus equals $-\hat{p}_{1,t}$. (4) defines the formula that determines the price of debt of any k in period t. A long term bond issued in t promises one unit of income in $t+k$. The price is the real value of this claim of income, adjusted according to expected inflation between periods $t+1$ to $t+k$. The steady state price satisfies $\overline{p}_k = \beta^k$ where $\beta < 1$ denotes the standard household discount factor.

2.1 Ramsey Optimal Policies

We first consider the Ramsey policy equilibrium. We assume that a benevolent planner chooses sequences $\left\{\hat{\pi}_t, \hat{Y}_t, \hat{i}_t, \hat{b}_{t,k}, \hat{p}_{t,k}\right\}_{t\geq 0}$ subject to the competitive equilibrium equations to maximize the following objective:[8]

$$-\frac{1}{2}E_0\sum_{t\geq 0}\beta^t\hat{\pi}_t^2 \tag{5}$$

To simplify this problem, we take the standard approach of dispensing with prices and equations. Substituting (4) into (2) we obtain the following expression for the consolidated budget constraint

$$\sum_{k=1}^{\infty}\beta^k\overline{b}_k(\hat{b}_{t,k}-\sum_{l=1}^{k}E_t\hat{\pi}_{t+l}) = -\overline{S}\hat{S}_t + \overline{b}_1(\hat{b}_{t-1,1}-\hat{\pi}_t) + \sum_{k=2}^{\infty}\beta^{k-1}\overline{b}_k(\hat{b}_{t-1,k}-\sum_{l=0}^{k-1}E_t\hat{\pi}_{t+l}) \tag{6}$$

which is independent of bond prices. Moreover, noting that, given the optimal path of inflation, $\hat{i}_t$ can be set to satisfy (3) and $\hat{p}_{t,k}$ set to satisfy (4), we can drop these equations from the constraint set.

Finally, we can further simplify, noticing that the portfolio $\hat{b}_{t,k}$ will not be uniquely defined in this program. Letting $\overline{d}\hat{d}_t \equiv = \sum_{k=1}^{\infty}\beta^{k-1}\overline{b}_k\hat{b}_{t,k}$ be the value of debt issued in t and bought back in $t+1$, evaluated at steady state prices, we let the planner choose $\hat{d}_t$ to maximize (5).

The optimal policy solves:

$$\max_{\{\hat{\pi}_t,\hat{Y}_t,\hat{d}_t\}_{t\geq 0}} -E_0\frac{1}{2}\sum_{t\geq 0}\beta^t\hat{\pi}_t^2$$

subject to (1) and

$$\beta\overline{d}\hat{d}_t - \sum_{k=1}^{\infty}\beta^k\overline{b}_k\sum_{l=1}^{k}\hat{\pi}_{t+l} + \overline{R}\left(\gamma_h+1\right)\hat{Y}_t - \overline{G}\hat{G}_t = \overline{d}\hat{d}_{t-1} - \sum_{k=1}^{\infty}\beta^{k-1}\overline{b}_k\sum_{l=0}^{k-1}\hat{\pi}_{t+l} \tag{7}$$

[8] As in Cochrane (2001), our baseline model assumes that the planner focuses on minimizing the variability of inflation. Notice that in the Fisherian model considered here this may be seen as an appropriate criterion, since in the background non-linear model, household preferences are *quasilinear*. In the appendix we derive a second order approximation of the household's utility and show that, for certain parameterizations of the model, we do obtain (5).

Notice, however, that our analysis will not be constrained by the household's preferences to derive the objective function of the monetary authority. We consider that the planner's loss function need not coincide with a welfare based objective. In Section 5 we will experiment with alternative specifications of the loss function, while maintaining the Fisherian setup. We will also study optimal policy in the canonical New Keynesian model assuming an inflation stabilization objective.

2.1.1 Optimality

As it is standard, we solve for the optimal policies using a Lagrangian.[9] Attach a multiplier $\psi_{\pi,t}$ to the Phillips curve and $\psi_{gov,t}$ to the consolidated budget. The first order conditions of the Ramsey program are:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} + \sum_{k=1}^{\infty} \bar{b}_k \sum_{l=1}^{k} \beta^{k-l} \Delta\psi_{gov,t-l+1} = 0 \tag{8}$$

$$-\psi_{\pi,t}\kappa_1 + \overline{R}(1+\gamma_h)\psi_{gov,t} = 0 \tag{9}$$

$$\psi_{gov,t} - E_t\psi_{gov,t+1} = 0 \tag{10}$$

where (8), (9) and (10) are the FONC with respect to $\hat{\pi}_t$, $\hat{Y}_t$ and $\hat{d}_t$ respectively.

To inspect these optimality conditions, combine (8) and (9) to substitute out $\psi_{\pi,t}$ and obtain the following expression for $\hat{\pi}_t$

$$\hat{\pi}_t = \overline{R}\frac{(1+\gamma_h)}{\kappa_1}\Delta\psi_{gov,t} + \sum_{k=1}^{\infty} \bar{b}_k \sum_{l=1}^{k} \beta^{k-l} \Delta\psi_{gov,t-l+1} \tag{11}$$

According to (11), under optimal policy, inflation becomes a weighted average of current and lagged values of the growth of the multiplier ψ_{gov}. From (10), the latter object evolves according to a random walk.

Note that these are standard outcomes of optimal Ramsey policy (e.g. Aiyagari et al. (2002); Schmitt-Grohé and Uribe (2004); Lustig et al. (2008); Faraglia et al. (2013, 2016), among others). Debt and deficit fluctuations in our model can only be financed through distortionary inflation and the multiplier ψ_{gov}, which measures the burden of the distortions, follows a random walk because the planner desires to spread the burden evenly across periods.

Moreover, to clarify the dependence of inflation on the current and lagged values of ψ_{gov} let us iterate forward on constraint (7) to obtain the *intertemporal consolidated budget constraint* as:

$$E_t \sum_{j=0}^{\infty} \beta^j \overline{S}\hat{S}_{t+j} = \bar{d}\hat{d}_{t-1} - \sum_{k=1}^{\infty} \beta^{k-1}\bar{b}_k \sum_{l=0}^{k-1} E_t\hat{\pi}_{t+l} \tag{12}$$

(12) links the present discounted value of the fiscal surplus (LHS) to the real value of debt outstanding in t (RHS). It is equivalent to (7) in terms of the Ramsey policy.[10] Consider a spending shock which lowers the LHS of (12) relative to the RHS. In response to such a shock the constraint tightens, and the value of the multiplier ψ_{gov} increases. To satisfy the constraint the planner needs to engineer a drop in the real payout of debt, the last term of the RHS of (12). This requires to increase current inflation and also promise to increase future inflation, the latter in the case where long term debt has been issued. The lagged terms $\Delta\psi_{gov,t-l+1}$ in (11) capture the promises made by the planner to adjust inflation in response to past shocks.

Finally, using (3) and (11) we can obtain the following expression for the nominal rate under Ramsey policy:

$$\hat{i}_t = E_t\hat{\pi}_{t+1} = \sum_{k=2}^{\infty} \bar{b}_k \sum_{l=2}^{k} \beta^{k-l} \Delta\psi_{gov,t+2-l} \tag{13}$$

which reveals that the nominal rate also is a function of the state variables $\Delta\psi_{gov,t-l+1}$.

[9]Following numerous papers, we assume a *timeless perspective*. As is well known, solving for optimal policies under this assumption, requires to introduce additional constraints on the initial allocation (e.g Woodford, 2003), or the program can be stated in terms of an objective function that accounts explicitly for the lagged Lagrange multipliers at the beginning of the planning horizon (e.g Faraglia et al. (2016)). To avoid introducing explicitly all these elements we do not state the Lagrangian here.

[10]See for example Aiyagari et al. (2002).

2.2 Optimal Inflation

With the optimality conditions we can go very far towards characterizing analytically key features of optimal policy. We now study the properties of inflation under alternative maturity structures $\overline{b}_k$. To simplify the algebra, we will assume that spending $\hat{G}_t$ follows an i.i.d process. This will prove to be without any loss of generality for the results that follow.

2.2.1 One Maturity

We first consider the case where debt is being issued in one maturity only. Specifically, let us assume that all debt is issued in maturity N zero coupon bonds, or $\overline{b}_N > 0$ and $\overline{b}_k = 0$ for $k \neq N$. Focusing first on this simple scenario enables to transparently characterize the forces that drive inflation under optimal policy but it is also rather common in the literature. For example Schmitt-Grohé and Uribe (2004) set $N = 1$ (assuming that all government debt is short term) whereas Faraglia et al. (2013) consider the case of long term debt, $N > 1$. Moreover, Lustig et al. (2008) solve a non-linear model, in which a Ramsey planner can choose a portfolio of multiple assets (maturities 1 to N, say) along with inflation.[11] In their model it becomes optimal to issue debt only in the longest maturity available, the quantities of shorter maturity bonds are optimally zero. Thus the N zero coupon bond we focus on here could be seen as an approximation of the optimal policy of Lustig et al. (2008).

The following proposition characterizes the path of optimal inflation in this model:

Proposition 1. *Assume that the government issues debt in a single N period bond. Optimal inflation under Ramsey is given by:*

$$\hat{\pi}_t = \sum_{j=0}^{N-1} \eta_{-j} \hat{G}_{t-j} \tag{14}$$

where

$$\eta_{-j} = \begin{cases} \dfrac{\widetilde{f}\,\overline{G}}{\left[\widetilde{f}^2 + (\beta^{N-1}\overline{b}_N)^2 \left(\frac{1-\frac{1}{\beta^N}}{1-\frac{1}{\beta}} - 1\right)\right]} & \text{for } j = 0 \\[2em] \dfrac{\beta^{N-j-1}\overline{b}_N \overline{G}}{\left[\widetilde{f}^2 + (\beta^{N-1}\overline{b}_N)^2 \left(\frac{1-\frac{1}{\beta^N}}{1-\frac{1}{\beta}} - 1\right)\right]} & \text{for } j = 1, 2, ..., N-1 \end{cases} \tag{15}$$

$$\widetilde{f} = \left(\frac{\overline{R}}{\kappa_1}(1+\gamma_h) + \beta^{N-1}\overline{b}_N\right) > 0$$

Proof: See appendix.

According to Proposition 1, inflation is a weighted average of the current and past ($N-1$ lags) shocks to spending. Since coefficients η_{-j} are positive, following a positive spending shock in t inflation will rise on impact and will remain above zero until period $t + N - 1$. The optimal path of inflation may be frontloaded in the sense that if $\overline{R} > 0$ (taxes are distortionary) then $\widetilde{f}$ exceeds $\beta^{N-j-1}\overline{b}_N$ and the impact of the shock on inflation in t is larger than in other periods. Otherwise, since β is plausibly close to 1, the rate of inflation will be roughly constant through time.

Turning to the effect of maturity, N, note first that the term $\beta^{N-1}\overline{b}_N$ is such that in steady state the value of debt equals the present value of the surplus. Therefore, $\beta^{N-1}\overline{b}_N = \frac{\overline{S}}{1-\beta}$ is independent of

[11] More precisely, Lustig et al. (2008) also assume that bond quantities cannot be negative, ruling out that the government can purchase private short-term assets. A similar solution to this optimal portfolio problem was obtained by Nosbusch (2008).

N. Debt maturity N affects the coefficients η_{-j} through the term $\left(\frac{1-\frac{1}{\beta^N}}{1-\frac{1}{\beta}}-1\right)$ in the denominator. Longer maturity increases this term, thus lowering the coefficients η_{-j}, however, the response of inflation to the shock is now spread over more periods. When $N=1$ we have $\eta_0=\frac{\overline{G}}{\overline{f}}$ and all of the response of inflation is concentrated in t. As N grows towards infinity, we obtain $\eta_{-j}\approx 0$ and inflation will permanently increase in response to the shock.

To understand the above properties, notice first that when debt is of maturity N, setting a positive inflation rate after $t+N-1$ will not contribute towards satisfying the intertemporal constraint (12), it would be wasteful from the point of view of fiscal solvency. This explains why only the first $N-1$ lags of spending shocks matter for inflation. Moreover, higher N reduces the coefficients η_{-j} because following a positive spending shock that lowers the intertemporal surplus, satisfaction of (12) can be achieved through a smaller adjustment in inflation in any given period, when inflation adjusts over more periods.[12]

Furthermore, to see why inflation may be frontloaded and coefficient η_0 is larger when taxes are distortionary, notice that when $R>0$ the government's surplus is a function of output. Then inflation has a direct impact on the LHS of the intertemporal constraint (12). In particular, we have:

$$E_t\sum_{j=0}^{\infty}\beta^j\overline{S}\hat{S}_{t+j}=E_t\sum_{j=0}^{\infty}\beta^j\left(\overline{R}(1+\gamma_h)\hat{Y}_{t+j}-\overline{G}\hat{G}_{t+j}\right)$$

From the Phillips curve $\hat{Y}_{t+j}=\frac{1}{\kappa_1}\left(\hat{\pi}_{t+j}-\beta E_{t+j}\hat{\pi}_{t+j+1}\right)$ and we can write:

$$E_t\sum_{j\geq 0}\beta^j\overline{R}(\gamma_h+1)\hat{Y}_{t+j}=\frac{\overline{R}}{\kappa_1}(\gamma_h+1)\hat{\pi}_t$$

which reveals that higher inflation in t will increase the present value of revenues from distortionary taxation.

Through making inflation higher in t, the planner enables a smaller drop in the intertemporal surplus, following a positive spending shock, which in turn reduces the increase in inflation in periods $t+1,t+2,t+3,...,t+N-1$ required to satisfy (12), assuming that debt is long term.[13]

Impulse responses. Figure 1 plots the response of inflation to a positive spending shock under different values of N. Table 1 reports the numerical values of the model's parameters assumed to construct the figure and the notes of the table briefly explain our calibration targets.

The top panel of Figure 1 assumes that taxes are distortionary. Notice that even though prices are quite sticky in the model, the incentive to frontload inflation is not particularly strong. Moreover, the longer is the maturity, the less felt is the initial 'blip' in inflation. When $N=10$ the resulting path of inflation is basically the same as the analogous object in the top panel of Figure 1 in which taxes are lump sum and the incentive to frontload inflation is not present.

[12] See e.g. Lustig et al. (2008); Faraglia et al. (2013); Sims (2013); Leeper and Zhou (2021).

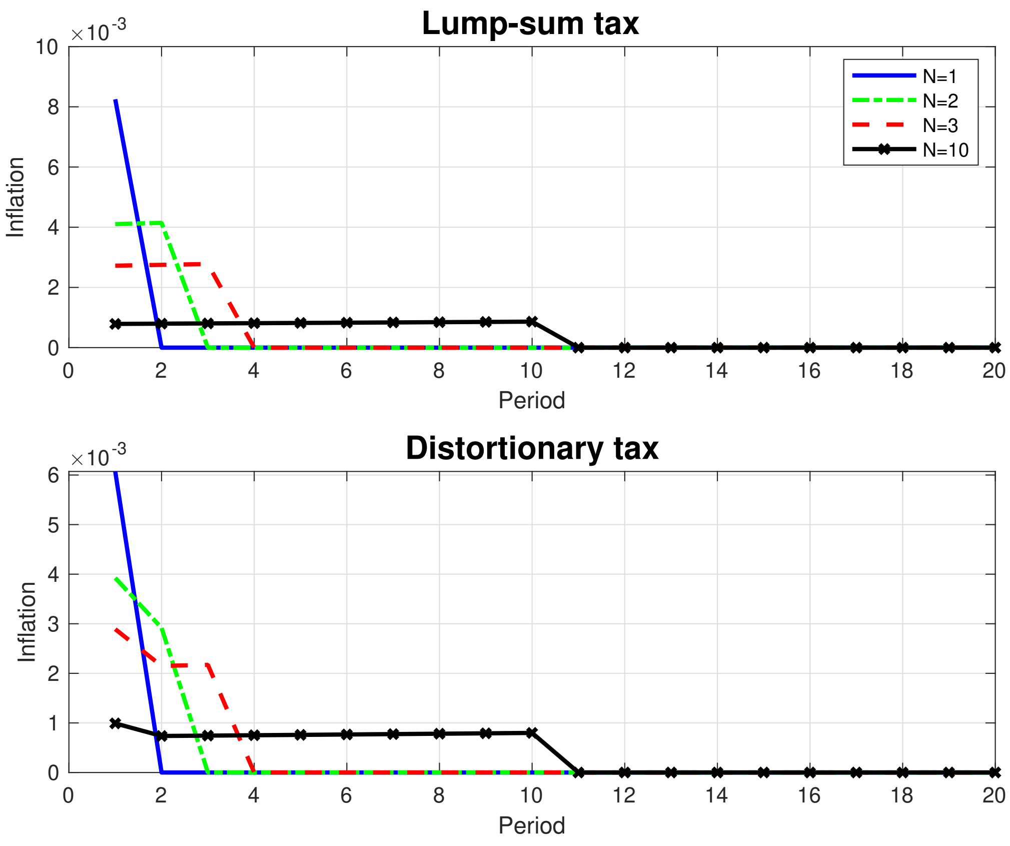
[13] Besides $\overline{R}>0$ a further condition that needs to be satisfied for this channel to be important, is that prices are sticky, the slope of the Phillips curve coefficient, κ_1, should not be large. If prices are quite flexible, then frontloading inflation will not impact output and the government's revenue. In contrast, under sticky prices, a change in inflation can impinge a significant effect on output. Note that this is the only channel through which the degree of price stickiness can exert an influence on the path of inflation. In contrast to the Ramsey literature (e.g. Schmitt-Grohé and Uribe (2004); Faraglia et al. (2013); Lustig et al. (2008); Leeper and Zhou (2021) and others) where it is typically assumed that the planner can finance debt either through inflation or through taxes and a small slope coefficient κ_1 shifts the optimal policy mix towards more taxation, here the planner only has one instrument. Stickier prices will not reduce inflation volatility, rather they will increase the volatility when $\overline{R}>0$ and frontloading inflation becomes optimal.

Table 1: **Calibration**

Parameter	Value	Label
β	0.995	Discount factor
θ	17.5	Price Stickiness
η	-6.88	Elasticity of Demand
γ_h	1	Inverse of Frisch Elasticity
$\overline{Y}$	1	Steady State Output
$\overline{G}$	0.1	Steady State Spending

Notes: The table reports the values of model parameters. The model period is one quarter. β denotes the discount factor chosen to target a steady state annual real interest rate of 2 percent. Parameter η is calibrated to target markups of 17 percent in steady state. θ determines the cost of adjusting prices and is calibrated as in Schmitt-Grohé and Uribe (2004). Coefficient γ_h, the inverse of the Frisch elasticity of labour supply, equals 1, a standard value assumed in the macro literature. Finally, the steady state level of debt is assumed equal to 60 percent of GDP (at annual horizon, 240 percent at the quarterly horizon assumed here), and the level of public spending is 10 percent of aggregate output, which is normalized to unity in steady state.

Figure 1: **Responses to a spending shock: Optimal Ramsey policy.**



Notes: The figure plots the path of optimal inflation in response to a shock that increases spending by 20% (from 10% of GDP to 12% of GDP). The top panel shows the case of distortionary taxes and the bottom panel assumes lump sum taxation. Each response corresponds to a different debt maturity. See legend of the Figure.

2.2.2 Multiple Maturities

Let us now consider the general case where instead of assuming only one maturity, all maturities $\bar{b}_k$ can be issued. In the appendix we derive the following formula for optimal inflation in this model:

Proposition 2. *Assume that the government issues maturities* $\{\bar{b}_k\}_{k\geq 1}$. *Optimal inflation under Ramsey is given by:*

$$\hat{\pi}_t = \sum_{j=0}^{t} \eta_{-j} \hat{G}_{t-j}$$

where

$$\eta_{-j} = \begin{cases} \dfrac{\widetilde{f}\,\overline{G}}{\left[\widetilde{f}^2 + \sum_{k=2}^{\infty} \beta^{k-1}\lambda_k^2\right]} & \text{for } j = 0 \\ \\ \dfrac{\lambda_j \overline{G}}{\left[\widetilde{f}^2 + \sum_{k=2}^{\infty} \beta^{k-1}\lambda_k^2\right]} & \text{for } j \geq 1 \end{cases} \tag{16}$$

$$\lambda_j = \frac{1}{\beta}(\lambda_{j-1} - \bar{b}_{j-1}) \quad \text{and} \quad \lambda_1 = \sum_{k=1}^{\infty} \beta^{k-1}\bar{b}_k \tag{17}$$

$$\widetilde{f} = \left(\frac{\overline{R}}{\kappa_1}(1+\gamma_h) + \sum_{k=1}^{\infty} \beta^{k-1}\bar{b}_k\right)$$

Proof: See appendix.

The recursive formula in Proposition 2 enables to easily calculate the optimal inflation path for any maturity structure of debt. To inspect the formula, consider first coefficient η_0. The numerator term $\widetilde{f}$ is determined by two forces. First, $\frac{\overline{R}}{\kappa_1}(1+\gamma_h)$ again measures the impact of inflation on the present value of the surplus when taxes are distortionary, and second, the term $\sum_{k=1}^{\infty} \beta^{k-1}\bar{b}_k$ measures the effect of period t inflation on the real payout of total government debt, long and short bonds outstanding. Coefficients η_{-j} are then determined by objects λ_j. According to (17) λ_j will be positive insofar as not all debt is of maturity less than or equal to j and be 0 otherwise.

A standard modelling assumption found in the literature (e.g Angeletos, 2002; Buera and Nicolini, 2004 and Faraglia et al., 2019) is to assume that the government issues debt in two assets, one short bond of 1 period maturity and one long term asset of maturity N. We then have $\bar{b}_1, \bar{b}_N \neq 0$ and $\bar{b}_k = 0$ for $k \neq 1, N$ and the path of the λs is given by:

$$\lambda_j = \frac{1}{\beta^{j-1}}\beta^{N-1}\bar{b}_N, \quad j = 2, ..., N-1$$

$$\lambda_N = \frac{1}{\beta}(\lambda_{N-1} - \bar{b}_N) = 0$$

and $\lambda_{N+1} = \lambda_{N+2} = ... = 0$.

We can then compute the second term in the denominator of η_{-j} as:

$$\sum_{j=2}^{\infty} \beta^{j-1}\lambda_j^2 = \sum_{j=2}^{N-1} \frac{1}{\beta^{j-1}}(\beta^{(N-1)}\bar{b}_N)^2 = (\frac{\overline{S}}{1-\beta} - \bar{b}_1)^2 \frac{1}{1-\beta}(\frac{1}{\beta^{N-1}} - 1)$$

where the last equality derives from the steady state intertemporal constraint, $\frac{\overline{S}}{1-\beta} = \overline{b}_1 + \beta^{N-1}\overline{b}_N$.

The above condition suggests that $\sum_{j=2}^{\infty} \beta^{j-1}\lambda_j^2$ will be higher the more tilted is the portfolio towards long term debt (the smaller $\overline{b}_1$ is, the larger $\frac{\overline{S}}{1-\beta} - \overline{b}_1$ will be). Then the coefficients η_{-j} become smaller in magnitude and the inflation response to a spending shock weakens. In fact, when the government can issue a very large amount of the long term asset, financing its position through negative debt (savings) in the short term bond, we can have that $\eta_{-j} \approx 0$.

This result resembles the finding of Angeletos (2002); Buera and Nicolini (2004) that issuing long term bonds enables to absorb fiscal shocks by exploiting the variability of long bond prices. Whereas in Angeletos (2002) and Buera and Nicolini (2004) this happens because real long bond prices drop following a spending shock, here the planner can leverage on the persistent increase in inflation which yields a drop in the nominal long bond price after the shock.

Another interesting case, and one to which we will later turn when we study optimized interest rate rules, is when the government issues a portfolio in which the shares of $\overline{b}_k$ bonds decay at constant rate δ, i.e. $\overline{b}_k = \delta^{k-1}\overline{b}$. Equivalently, debt is a perpetuity that pays decaying coupons (e.g. Cochrane, 2001; Leeper and Zhou, 2021). The formula in Proposition 2 then gives us:

$$\eta_0 = \frac{\widetilde{f}\,\overline{G}}{\left[\widetilde{f}^2 + \frac{\overline{b}_\delta^2 \beta\delta^2}{(1-\beta\delta^2)(1-\beta\delta)^2}\right]}$$

$$\eta_{-j} = \frac{\frac{\overline{b}}{1-\beta\delta}\delta^j \overline{G}}{\left[\widetilde{f}^2 + \frac{\overline{b}_\delta^2 \beta\delta^2}{(1-\beta\delta^2)(1-\beta\delta)^2}\right]} \quad , j = 1, 2, ..$$

which illustrates that the coefficients η_{-j}, $j \geq 1$, that capture the response of inflation to past spending shocks, decrease at rate δ. Moreover, it is easy to show that a higher value of δ, reduces the magnitude of these coefficients.

Finally, we will also consider the case where the portfolio shares are given by $\overline{b}_k = \overline{b}e^{-\widetilde{\lambda}}\frac{\widetilde{\lambda}^{k-1}}{(k-1)!}$ for $k = 1, 2, ...$ We can then show that

$$\lambda_j = e^{-\widetilde{\lambda}}\overline{b}\frac{(\widetilde{\lambda})^{j-1}}{(j-1)!}$$

suggesting that the coefficients η_{-j}, $j \geq 1$ vary according to $\frac{(\widetilde{\lambda})^{j-1}}{(j-1)!}$ and thus may not decay monotonically, for high values of $\widetilde{\lambda}$.

3 Rules rather than Ramsey

Ramsey policies give rise to the best outcome given the competitive equilibrium conditions of the model. However, from a practical standpoint, a Ramsey policy may be difficult to implement, when it leads to a path of interest rates that depends on the Lagrange multipliers (i.e. the one derived in equation (13)) and which is not directly related to macroeconomic variables such as inflation, output, or lagged values of interest rates. In practice, policies implemented by central banks are informed by macroeconomic conditions as they are summarized by these macro variables, and a large literature has been devoted to studying how to design interest rate rules which are simple functions of inflation, output, etc, in the baseline New Keynesian model.

We now consider a model where monetary policy sets the nominal interest rate as a function of macroeconomic variables only. We begin assuming a simple inflation targeting rule:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t. \tag{18}$$

where the coefficient ϕ_π will be set optimally to maximize objective (5). Such exercises (assuming commitment to a rule and optimizing over parameters) have been considered many times in the context of the standard New Keynesian model. We carry out this exercise in the context of the fiscal theory.

Our key finding in this section is that the optimal Ramsey policy can be very closely approximated, even by simple rules of the form (18). Committing to a rule that sets the nominal interest rate only as a function of current inflation is sufficient, and we do not need to include many lags of inflation (or other variables) in the policy rule, a property that seems surprising given the dependency of Ramsey policy on the history of Lagrange multipliers.

Moreover, for the Fisherian model we consider here, we find that the optimal inflation coefficient in rule (18) is given by:

$$\phi_\pi^* \approx 1 - \frac{1}{\text{maturity}}$$

Our results reveal a simple and transparent relation between the optimal coefficient and the maturity of debt which will prove to hold across a wide range of alternative maturity structures.

3.1 Optimal policy with a simple policy rule

To solve for the optimal policy rule we proceed in two steps. We first characterize the equilibrium under a generic value for parameter ϕ_π, then we compute the optimal coefficient ϕ_π as a function of the debt maturity.

3.1.1 One Maturity

Consider first the case where debt is issued in a single maturity N. Under (18) the equilibrium is a solution to the following system of equations:

$$\beta^N \overline{b}_N(\hat{b}_{t,N} - \sum_{l=1}^{N} E_t\hat{\pi}_{t+l}) = -\overline{S}\hat{S}_t + \sum_{k=2}^{\infty} \beta^{N-1}\overline{b}_N(\hat{b}_{t-1,N} - \sum_{l=0}^{N-1} E_t\hat{\pi}_{t+l})$$
$$\phi_\pi \hat{\pi}_t = E_t\hat{\pi}_{t+1}$$
$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \beta E_t\hat{\pi}_{t+1}$$

Consider the second equation which is obtained by substituting out the nominal rate from the Euler equation using (18). When $\phi_\pi > 1$ this equation has an unstable root and can be solved forward to give us a unique solution $\hat{\pi}_t = 0$, for all t. From the Phillips curve it also holds that $\hat{Y}_t = 0$. Then, inflation will not satisfy the consolidated budget constraint and intertemporal solvency will fail.

In the case where $0 \le \phi_\pi \le 1$ the model has a unique equilibrium where inflation is not zero. The intertemporal budget constraint will pin down inflation.

These are standard results of course. Monetary policy needs to be 'passive' (e.g. Leeper, 1991) for the equilibrium to be unique in a model where taxes do not adjust to ensure the solvency of the government's budget. Allowing for a richer maturity structure will not change this property.

Solving the above system of equations, it is easy to show that equilibrium inflation is given by:

$$\hat{\pi}_t = \sum_{j=0}^{t} \frac{\phi_\pi^{j-1}\overline{G}}{\xi}\hat{G}_{t-j} \tag{19}$$

where $\xi = \overline{R}\frac{1+\gamma_h}{\kappa_1} + \beta^{N-1}\overline{b}_N\frac{1-\phi_\pi^N}{1-\phi_\pi}$.

Note that there are two ways in which ϕ_π influences this solution. First, (19) states that inflation will display persistence ϕ_π, a higher inflation coefficient will translate into a more persistent process of inflation. This follows easily from $\phi_\pi \hat{\pi}_t = E_t \hat{\pi}_{t+1}$, which defines a backward looking process. Second, ϕ_π also influences the denominator of (19). When $N > 1$ a higher ϕ_π implies higher ξ. When $N = 1$, ξ does not depend on ϕ_π.

Why is this so? With long term debt, inflation can contribute towards stabilizing debt up to period $t + N - 1$. A higher ϕ_π will make inflation more persistent in response to the spending shock, and a smaller increase in inflation is needed (in each period) to satisfy the intertemporal debt solvency condition. This explains why the denominator of (19) increases in ϕ_π. When debt is short term, however, then only inflation in t can absorb the shock, and the persistence of inflation will not matter for debt solvency.

This finding hints at how the optimal inflation coefficient will be influenced by debt maturity in this model. In the case where $N = 1$ optimal policy should set a constant interest rate, as letting $\phi_\pi > 0$ will lead inflation to persistently deviate from target, without contributing anything towards debt sustainability. Conversely, if $N > 1$, then persistence of inflation will be desirable, as it will spread the burden of inflation over more periods reducing overall losses.[14]

Formally, the optimal policy solves:[15]

$$\max_{\phi_\pi} -\frac{1}{2} E_0 \sum_{t \geq 0} \beta^t \hat{\pi}_t^2 = \max_{\phi_\pi} -\frac{1}{2} \frac{\overline{G}^2 \sigma_G^2}{\xi^2} \frac{1}{1 - \beta \phi_\pi^2}$$

The first order condition is:

$$-\frac{1}{2} \frac{\overline{G}^2 \sigma_G^2}{\xi^2} \frac{1}{1 - \beta \phi_\pi^2} \left[\frac{\beta \phi_\pi}{1 - \beta \phi_\pi^2} - \beta^{N-1} \overline{b}_N \frac{1}{\xi (1 - \phi_\pi)^2} \left(1 + (N-1) \phi_\pi^N - N \phi_\pi^{N-1} \right) \right] = 0 \qquad (20)$$

Let us focus on the case where $\overline{R} = 0$ assuming for simplicity (but also consistently with the calibration reported in Table 1) that $\beta \approx 1$. Then, the optimal coefficient ϕ_π^* solves:

$$\frac{\phi_\pi (1 - \phi_\pi^N)}{1 + \phi_\pi} = \left(1 + (N-1) \phi_\pi^N - N \phi_\pi^{N-1} \right) \qquad (21)$$

When $N = 1$ the optimum is $\phi_\pi^* = 0$. When $N = 2$ we have $\phi_\pi^* = \frac{1}{2}$. For higher N the LHS of (21) defines a concave function which equals 0 when ϕ_π is either 0 or 1. The RHS of (21) defines a strictly downward sloping function, which is equal to 1 at $\phi_\pi = 0$ and 0 at $\phi_\pi = 1$. The LHS is equal to the RHS at a unique $\phi_\pi \in (0, 1)$ which defines the optimum.[16]

The solution cannot be characterized analytically for general N. It holds however that $\phi_\pi^* \approx 1 - \frac{1}{N}$. One way to show this is by comparing the implied paths of inflation in response to a shock to spending. This is done in the top panels of Figure 2. We plot the responses of inflation for different N, when the inflation coefficient is equal to ϕ_π^* (crossed / black lines) and when it is equal to $1 - \frac{1}{N}$ (dashed / red lines). The left panel in the figure sets $N = 4$. In the middle we assume $N = 20$ corresponding to an average maturity of 5 years. Finally, on the right panel we let $N = 28$ (7 year average maturity).[17]

[14]See Leeper and Leith (2016) for a numerical experiment with ad hoc rules of the form (18) where this result emerges.

[15]We denoted σ_G^2 the variance of $\hat{G}_t$.

[16]It can also be easily seen that in the case where $N = \infty$ (21) gives a unique solution $\phi_\pi^* = 1$.

[17]We chose these three values for the following reasons: First, as was made evident from the previous derivations in the case where $N = 1$ the Ramsey outcome coincides with the outcome under rule $\hat{i}_t = 1 - \frac{1}{N} = 0$. Thus, in this case we get coincidence (trivially so, since there are no dynamics in inflation, its response is only contemporaneous to

As it is evident, the responses are very similar. Effectively, $\phi_\pi = 1 - \frac{1}{N}$ is a very good approximation of optimal policy.[18]

3.1.2 A comparison with Ramsey and Ramsey implied rules.

For this model, the Ramsey policy first order conditions can be rearranged to yield an interest rate rule that targets current inflation. Using the Euler equation and (11), assuming one bond of maturity N, we get:

$$\hat{i}_t = E_t\hat{\pi}_{t+1} = \frac{1}{\beta}\bar{b}_N \sum_{l=1}^{N-1} \beta^{N-l}\Delta\psi_{gov,t-l+1}$$

and then using (11) to replace the weighted sum of the multipliers, we obtain:

$$\hat{i}_t = \frac{1}{\beta}\left(\hat{\pi}_t - \widetilde{f}\Delta\psi_{gov,t} - \bar{b}_N\Delta\psi_{gov,t-N+1}\right) \tag{22}$$

According to (22), the nominal rate is a function of inflation (with coefficient $\frac{1}{\beta}$) and of the stochastic intercept terms $\Delta\psi_{gov,t}$ and $\Delta\psi_{gov,t-N+1}$. Moreover, since making interest rate policy contingent on the multipliers $\psi_{gov,t}$ does not seem practically relevant, we could express the multipliers as a function of $\hat{G}$. Under Ramsey policy it holds that :

$$\Delta\psi_{gov,t} = \frac{\overline{G}}{\left[\widetilde{f}^2 + (\beta^{N-1}\bar{b}_N)^2\left(\frac{1-\frac{1}{\beta^N}}{1-\frac{1}{\beta}} - 1\right)\right]}\hat{G}_t$$

and so it follows that (22) expresses the nominal rate as a function of inflation and of spending in t and $t - N + 1$.

It thus seems that a system of equations comprising the Phillips curve, the Euler equation, the consolidated budget and assuming that monetary policy follows (22) (when $\Delta\psi_{gov}$ is substituted out of the system) will reproduce the Ramsey policy outcome. However, this is not so. The problem is that policy rule (22) will lead to an explosive solution. Since the inflation coefficient exceeds unity, (22) defines an 'active' monetary policy (Leeper, 1991).[19] In contrast, a simple rule of the form (18) leads to a unique stable solution.

the spending shock and pinned down by the intertemporal budget) and so we utilize as our short term debt scenario a value N where the Ramsey and rule based outcomes are not equivalent in terms of the responses of inflation. Setting $N = 4$ makes debt maturity equal to 1 year, however, letting $N = 2, 3$ or $N = 5, 6...$ does not change our results.

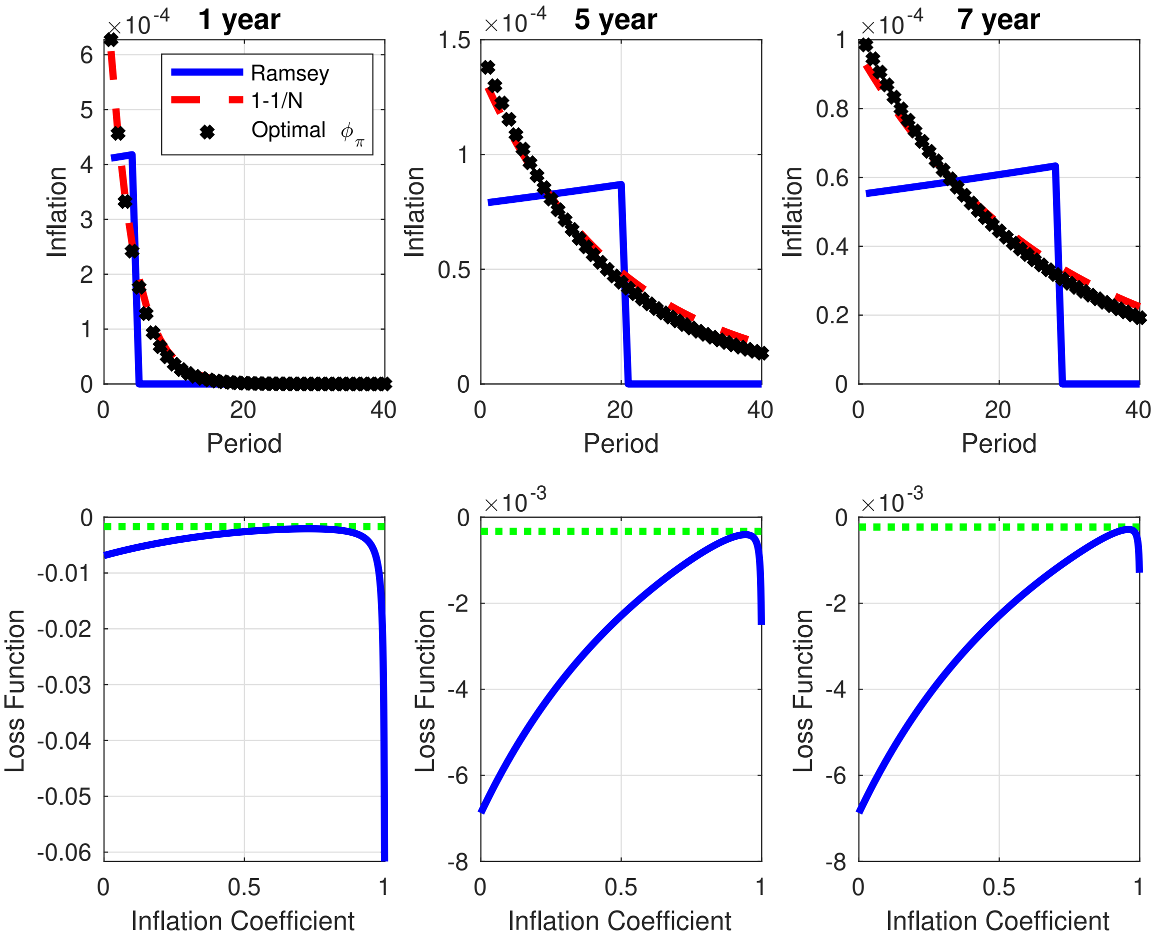
Moreover, $N = 20$ means that maturity is of five years and this is a commonly assumed value for average maturity in the US economy. Lastly, $N = 28$ (7 year maturity) is the long term asset considered in Lustig et al. (2008). The reader should also note that for any maturity longer than that, the Ramsey outcomes can get arbitrarily close to the rule based outcome in terms of the losses from inflation volatility (see our analysis below), the principle being that very long maturities entail a (nearly) permanent deviation of inflation from target and indeed this can be accomplished by rule based policy when $\phi^*_\pi \approx 1$.

[18] In the appendix, we repeat this exercise assuming distortionary taxes. We then recover ϕ^*_π as a solution to (20). Again we find that $1 - \frac{1}{N}$ is very close to ϕ^*_π.

[19] Intuitively, when we substitute out from the model $\Delta\psi_{gov,t}$ we also lose the random walk condition $E_t\Delta\psi_{gov,t+1} = 0$. Then, the dynamic system has too many unstable roots. Otherwise, it would be possible to solve for a unique equilibrium using (22) without substituting out the multiplier and without dropping the martingale condition.

Note also that the claim here is not that (22) is a unique interest rate policy that can implement the Ramsey outcome. Given the model structure it is plausible that there are policy rules that make the nominal rate a function of (many) lags of inflation and possibly deliver a unique equilibrium. Non-uniqueness of *robustly optimal rules* is a standard feature of the New-Keynesian model (see Giannoni and Woodford, 2003a).

Figure 2: **Outcomes under Ramsey and Inflation Targeting Rules.**



Notes: The top panels plots the path of inflation in response to a shock under Ramsey and inflation targeting rules. Debt is a zero coupon bond of maturity N. The solid blue line is the Ramsey inflation response. The dashed red line sets $\phi_\pi = 1 - \frac{1}{N}$. In the crossed black line coefficient ϕ_π solves (20).
The bottom panels show the loss function under Ramsey (dashed-dotted green line) and under rule based policy for a range of values of ϕ_π (solid blue line).
We assumed $N = 4, 20, 28$ in the left, middle and right graphs, respectively.

The top panels of Figure 2 plot the responses of inflation under Ramsey together with the rule based policy we previously studied.[20] Clearly, the inflation paths do not coincide for $N = 4, 20, 28$ considered in the Figure. The rule based policy prescribes a monotonic path for inflation whereas under Ramsey, inflation is roughly flat for N periods and then abruptly becomes 0. The rule based outcome would coincide with Ramsey policy outcome only when $N = 1$ or when $N \to \infty$.

Given this result, it may seem that an alternative inflation targeting rule, one that could set inflation to respond to the shock for $N-1$ periods, and subsequently change drastically the nominal rate so that inflation returns to target (or close to target), will provide a better approximation of the Ramsey policy than the simple rule that sets $\phi_\pi = 1 - \frac{1}{N}$. In this sense, the Ramsey implied policy (22) can serve as a useful benchmark. For example, since $\beta \approx 1$ we can perhaps set policy according to

$$\hat{i}_t = \hat{\pi}_t - \frac{1}{\beta}\widetilde{f}\frac{\overline{G}}{\left[\widetilde{f}^2 + (\beta^{N-1}\overline{b}_N)^2\left(\frac{1-\frac{1}{\beta^N}}{1-\frac{1}{\beta}} - 1\right)\right]}\hat{G}_t - \frac{1}{\beta}\overline{b}_N\frac{\overline{G}}{\left[\widetilde{f}^2 + (\beta^{N-1}\overline{b}_N)^2\left(\frac{1-\frac{1}{\beta^N}}{1-\frac{1}{\beta}} - 1\right)\right]}\hat{G}_{t-N+1} \quad (23)$$

i.e. set the inflation coefficient to 1 instead of $\frac{1}{\beta}$ to obtain a stable equilibrium.

Ultimately, whether or not it is worthwhile devising a more elaborate interest rate policy based on the Ramsey outcome requires to evaluate the loss function (5) under the Ramsey policy equilibrium and under the rule based policy (18). If the differences are small then there is little margin to improve on the outcome of the rule based policy.

In the bottom panels of Figure 2 we plot the loss function under the policy rule (18) for $\phi_\pi \in [0, 1]$. The Ramsey outcome is represented with the dashed-dotted /green line. Note that when $\phi_\pi = 1 - \frac{1}{N}$ the differences are minuscule. Thus, even though the inflation paths of the two models differ, this does not translate to a significant loss under rule based policy. We obtain a similar finding for many other calibrations of N. Finally, we have computed the loss function under the interest rate rule (23). The losses were several orders of magnitude larger than Ramsey.

The results we showed in this section continue to hold when we assume distortionary taxes instead of lump sum taxation. For brevity we study this case in the appendix.

3.1.3 Multiple maturities: decaying payment profiles

Let us now turn to the case where more than one maturity can be issued. The optimal coefficient ϕ_π solves:

$$\left[\frac{\beta\phi_\pi}{1-\beta\phi_\pi^2}\left(\overline{R}\frac{1+\gamma_h}{\kappa_1} + \sum_{k=1}^{\infty}\beta^{k-1}\overline{b}_k\frac{1-\phi_\pi^k}{1-\phi_\pi}\right) - \sum_{k=1}^{\infty}\beta^{k-1}\overline{b}_k\frac{1}{(1-\phi_\pi)^2}\left(1+(k-1)\phi_\pi^k - k\phi_\pi^{k-1}\right)\right] = 0 \quad (24)$$

To derive an analytical solution let us first assume that debt is a perpetuity that pays decaying coupons, $\overline{b}_k = \delta^{k-1}\overline{b}$. The following Proposition gives the optimal policy rule:

Proposition 3: *Assume that $\overline{b}_k = \delta^{k-1}\overline{b}$ and $\overline{R} = 0$. The optimal interest rate rule is*

$$\hat{i}_t = \delta\hat{\pi}_t \quad (25)$$

Proof: See appendix.

[20] The solid blue lines in the top panels, which show the Ramsey outcome are essentially the responses shown in Figure 1 but here we focus on different N

Under the assumed maturity structure, the average maturity (of the face value of debt) is $\frac{1}{1-\delta}$. Thus Proposition 3 confirms the principle that the optimal policy rule sets the inflation coefficient equal to $1 - \frac{1}{\text{Maturity}}$. To understand why this is optimal here, note again that when $\phi_\pi = \delta$ then inflation displays first order autocorrelation equal to δ. Thus, following a positive spending shock inflation will rise and gradually revert back towards 0 at this rate. Assume that the planner had set $\phi_\pi > \delta$ thus making inflation a more persistent process. Then, inflation would be high even when the coupon payments on debt outstanding in t have become low, which implies a higher cost of inflation without bringing any significant benefit in terms of stabilizing debt. Conversely, in the case where $\phi_\pi < \delta$ inflation becomes too frontloaded. A higher persistence would then enable to spread the costs more efficiently. Making inflation decay at the same rate as the coupons is the optimal policy.[21]

How does this rule based policy fare against the Ramsey outcome? In the appendix we show that the optimal Ramsey plan admits the following solution for the equilibrium interest rate:

$$\hat{i}_t = \delta \hat{\pi}_t - \delta \overline{R} \frac{(1+\gamma_h)}{\kappa_1} \Delta \psi_{gov,t} \tag{26}$$

Clearly, in the case $\overline{R} = 0$ as we assumed in Proposition 3, the nominal interest rate under Ramsey policy is simply equal to $\delta \hat{\pi}_t$. In other words, this is the *robustly optimal rule* that we can recover from solving the Ramsey first order conditions (e.g. Giannoni and Woodford, 2003a).

When $\overline{R}\frac{(1+\gamma_h)}{\kappa_1}$ is not zero (i.e. when revenues derive from distortionary taxes), (26) defines a response of the nominal rate to a positive spending shock which has a lower intercept in t. As discussed previously, this accomplishes to make inflation slightly higher in t and increase output to raise the surplus. Afterwards, inflation will decay monotonically at rate δ. However, this effect is not significant. It turns out that even in this case $\phi_\pi^* \approx \delta$ for the calibration assumed in Table 1.

We thus conclude that under the maturity structure assumed in this paragraph, a simple inflation targeting rule approximately delivers the Ramsey outcome.

3.1.4 Alternative maturity structures with multiple assets.

Assuming a debt structure of the form $\overline{b}_k = \delta^{k-1}\overline{b}$ provides a good approximation of outstanding payments on US government debt. However, this need not be the case in other advanced economies, and it is worthwhile exploring whether the result that simple inflation targeting rules can get close to the Ramsey outcome, generalizes to other scenarios with multiple assets.[22] Of course, this would be infeasible to show for any arbitrary (hypothetical) maturity structure. We thus focus on a couple of alternatives that preserve tractability.

We firstly assume $\overline{b}_k = \overline{b} e^{-\widetilde{\lambda}} \frac{\lambda^{k-1}}{(k-1)!}$, for $k = 1, 2, \ldots$. Notice that under this assumption $\overline{b}_k$ will not generally decay monotonically in k, and for high values of parameter $\widetilde{\lambda}$, the maturity distributions are (approximately) centered around the average debt maturity, $\widetilde{\lambda} + 1$. This is a useful setup to study, in particular for maturity structures more tilted towards long term debt and featuring less short bonds than in the US where, as discussed previously, monotonically decaying payments are a good approximation.

[21] This also applies in the case where long bonds are consols ($\delta = 1$). Then $\phi_\pi^* = 1$ and inflation becomes a random walk. The formula $\phi_\pi^* = 1 - \frac{1}{\text{Maturity}}$ continues to hold since in this case the maturity of debt is infinite.

[22] Indeed Euro area governments tend to issue much less short term debt than the US does. For example, in Germany the share of short term debt (defined empirically as all debt that is of maturity less than one year, and including short term payments of long term bonds) over total debt varies between 10 and 20 percent whereas longer maturities have higher shares in the government portfolio (see Equiza-Goni et al., 2023). In the US, the average share of short term debt exceeds 40 percent (see Faraglia et al., 2019). The decaying coupon assumption may thus be suitable for US data but not for German data.

Under the 'Poisson' debt structure, and further assuming lump sum taxes and $\beta \approx 1$, (24) can be written as:

$$\frac{\phi_\pi}{1+\phi_\pi}\left(1-e^{-\widetilde{\lambda}(1-\phi_\pi)}\right)=1+\widetilde{\lambda}\phi_\pi^2 e^{-\widetilde{\lambda}(1-\phi_\pi)}-\widetilde{\lambda}\phi_\pi e^{-\widetilde{\lambda}(1-\phi_\pi)}-e^{-\widetilde{\lambda}(1-\phi_\pi)}$$

(see appendix). It is simple to show using this equation that the optimal inflation coefficient is equal to 0 when $\widetilde{\lambda}=0$ (short debt only) and becomes 1 when $\widetilde{\lambda}$ tends to infinity. Otherwise, even with this simple debt structure, it is not possible to solve the above equation explicitly and find the inflation coefficient as a function of $\widetilde{\lambda}$.

The solid blue line in the top panel of Figure 3 shows the numerical solution. Compare this to the green dotted line that plots the usual formula $1-\frac{1}{1+\widetilde{\lambda}}$. As is evident from the graph the optimal inflation coefficients are approximately $1-\frac{1}{1+\widetilde{\lambda}}$.

The dashed red and black crossed lines in the figure, set the value of β as in our baseline calibration. The red line continues to assume lump sum taxation, whereas the black line corresponds to the case where taxes are distortionary. Note that again the difference in terms of the optimal inflation coefficients are small. We thus continue to find that the formula $1-\frac{1}{\text{maturity}}$ is a close approximation of optimal inflation targeting policy.

More importantly, the simple inflation targeting rule, again provides a very close approximation of the Ramsey outcome in terms of the loss from inflation volatility. To conserve space we show this with a graph in the appendix, considering maturities of 1, 5 and 7 years as we did in Figure 2. Though the responses of inflation under Ramsey are different than under the rule based policy, these differences do not exert a significant impact in terms of the policy objective.[23]

Next, we consider the case where the debt portfolio comprises of a mixture of perpetuities that have different discount factors δ. Barrett et al. (2021) argue that modelling debt payment profiles in this way, allowing for two (or three) decaying coupon bonds of different maturities, enables to explain the bulk of the time variation of payments of US government debt. Let us assume that:

$$\bar{b}_k=\bar{b}\sum_{i=1}^{M}\omega_i\delta_i^{k-1}$$

so that payments of maturity k are the coupons of M bonds and where ω_i represents the weight attached to the bond with profile δ_i in the government's portfolio. According to Barrett et al. (2021) even setting $M=2$ can fit the US data very well and to simplify we adopt this value.

Even so, deriving an analytical formula is not easy and once again we solve (24) numerically. The bottom panel of Figure 3 sets $\delta_1=0.75$, an average maturity of 4 quarters for the first bond, and $\delta_2=0.975$ which gives an average maturity of 10 years for the long term asset.[24] On the horizontal axis we have plotted the weight ω, the share of the long bond in the portfolio. The solid blue is the optimal coefficients in the model with lump sum taxes and the red line assumes distortionary taxation. The green dotted line again plots $1-\frac{1}{\text{Maturity}}$.[25] Finally, the arrow points to the value of ω

[23]Intuitively, the Poisson model compiles the forces of both the zero coupon and the decaying coupon models. For example assuming $\widetilde{\lambda}+1=20$ (5 year average maturity) implies only a small amount debt outstanding at the short end of the maturity structure and most of debt is concentrated around the mean. Then, the Ramsey planner will find optimal to initially set inflation as in the zero coupon model (a response that is roughly flat over time) and when the bulk of debt is close to redemption (i.e. after 15 quarters or so) then inflation will start to monotonically decay towards 0.

Given our previous findings, the result that a simple rule approximates this Ramsey outcome, is therefore not surprising.

[24]This provides a good approximation of the data since a large share of the outstanding debt in the US is concentrated at maturities between 1 month and 1 year, but also long bonds of maturity equal to 10 years (or more) are being issued.

[25]See the notes of the table for how we have calculated average maturity in this model.

that gives an average maturity of 5 years, the US data calibration. Evidently, the optimal coefficients continue being close to $1 - \frac{1}{\text{Maturity}}$.

To close this paragraph we note that the simple inflation targeting rule continues providing a very good approximation of the Ramsey outcome in terms of the policy objective also in this model. To conserve space we prove this claim in the appendix.

3.2 Alternative Rules

The previous subsections showed that a simple inflation targeting rule can deliver effectively the same outcome as Ramsey policy when the policy objective focuses on minimizing the variability of fiscal inflation. When the debt payment profiles decay at constant rate δ (an assumption that fits well the US data) we obtained an equivalence of the two types of policies. Under alternative debt structures a rule based policy can approximate closely the Ramsey outcome in terms of the loss function.

We now briefly investigate whether assuming a different interest rate rule (drawing from the set of commonly used rules in the literature) can alter these findings. In particular, we consider rules in which the nominal rate can respond to both inflation and output and inertial rules, when the interest rate tracks the first order lag. Would assuming these alternative policies enable us to better match the Ramsey responses of inflation?

In the context of our simple Fisherian model, it turns out, that these alternative rules will basically lead to exactly the same outcome as the simple inflation targeting rule. To see this, consider first the case where the policy rule is of the form:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t \tag{27}$$

Combining this rule with the Euler equation and the Phillips curve, we can show that inflation dynamics evolve according to:

$$E_t \hat{\pi}_{t+1} = \frac{\phi_\pi + \frac{\phi_Y}{\kappa_1}}{1 + \beta \frac{\phi_Y}{\kappa_1}} \hat{\pi}_t$$

It is obvious that the sufficient condition to have an equilibrium in this model is $\phi_\pi + \frac{\phi_Y}{\kappa_1}(1-\beta) \leq 1$, the standard configuration of the parameters for which monetary policy is passive. Moreover, solving the optimal policy program, finding coefficients ϕ_π, ϕ_Y to minimize the variability of inflation can be trivially shown to yield:

$$\frac{\phi_\pi + \frac{\phi_Y}{\kappa_1}}{1 + \beta \frac{\phi_Y}{\kappa_1}} \approx 1 - \frac{1}{\text{Maturity}}$$

and showing that including an explicit output target will not yield any improvement relative to the simple inflation targeting rule is also trivial.

Next, we consider a rule with interest rate inertia

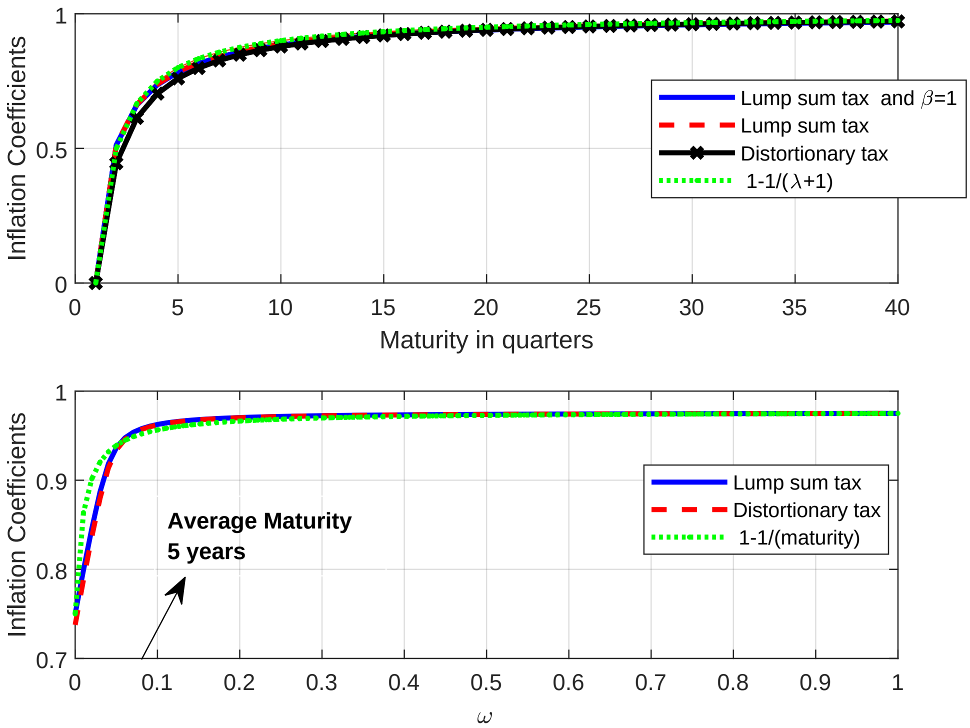
$$\hat{i}_t = \rho \hat{i}_{t-1} + (1-\rho)\phi_\pi \hat{\pi}_t$$

Following the same logic as above, it is simple to show that now it is optimal to set

$$(\rho + (1-\rho)\phi_\pi) \approx 1 - \frac{1}{\text{Maturity}}$$

(which defines a positive inflation coefficient in the case where $\rho < 1 - \frac{1}{\text{Maturity}}$). Then, inflation will display exactly the same dynamics as under our baseline policy rule, independent of ρ. We thus once again obtain the previous outcome.

Figure 3: **Optimal Inflation Coefficients: Alternative Maturity Structures with Multiple assets**



Notes: The top panel plots the optimal inflation coefficients under a Poisson payment structure of government debt. The solid blue line assumes that taxes are lump sum and sets $\beta = 1$. The dashed red line sets $\beta = 0.995$ as in the baseline calibration of the model. The crossed black line corresponds to the case of distortionary taxes and finally, the green dotted line is the benchmark $1 - \frac{1}{\text{maturity}}$.
The bottom panel of the Figure plots optimal inflation coefficients in the case where debt is issued in two decaying coupon bonds with coefficients δ_1, δ_2 respectively. ω is the weight of type 2 debt in the government's portfolio. The figure plots the optimal inflation coefficients that solve equation (24) as a function of the weight ω. The solid blue line assumes lump sum taxes whereas the red dashed line, distortionary taxation. The green line is again the benchmark $1 - \frac{1}{\text{Maturity}}$. The average maturity has been computed using the formula $\frac{(1-\omega)\frac{1}{(1-\delta_1)^2}+\omega\frac{1}{(1-\delta_2)^2}}{(1-\omega)\frac{1}{1-\delta_1}+\omega\frac{1}{1-\delta_2}}$.

3.3 Alternative Shock Structures

We have for simplicity considered a model where government spending shocks drive all fluctuations in the surplus assuming further that shocks to government spending are i.i.d. It is however, simple to extend our analysis, assuming persistent shocks in spending or other shocks driving fluctuations.

The case of persistent spending is simple to analyze. Since in our Fisherian model, the spending shock is only filtered through the government budget constraint (it does not influence the Phillips curve or the Euler equation) persistence will only matter through the ultimate effect of the shock on the present value of the surplus. A more persistent shock may have a larger impact, but the optimal path of inflation will not change either under Ramsey or under an optimized rule based policy. Only the magnitude of the response of inflation will change. Thus, all the formulae derived previously will continue to hold.

Next, consider the case of a shock to demand, assuming a disturbance that changes the real interest rate (up to now we have assumed the latter to be constant).[26] Such a shock will influence both the government budget constraint and the Euler equation.

In this setting we can show that an optimized rule will be of the form:

$$\hat{i}_t = \hat{r}_t + \phi_\pi \hat{\pi}_t$$

(where r_t denotes the real rate and can be assumed a persistent process) setting also $\phi_\pi \approx 1 - \frac{1}{\text{Maturity}}$.[27] The principle behind this policy is the following: Tracking the real interest rate, enables to eliminate the shock from the Euler equation, and then using the systematic response to inflation, optimize along the maturity structure of debt.

Lastly, assuming other sources of fluctuations, for example introducing random government transfers to the model (another commonly made assumption in the context of the fiscal theory) will not affect at all the formulae we derived previously. In our simplistic Fisherian model, shocks to spending and transfers deliver effectively the same effects.

The key driver of optimal inflation dynamics in our model is the maturity structure of debt, and thus far we showed that simple interest rate rules can bring us very close to the best equilibrium outcome, the Ramsey policy. We next consider an alternative modelling setup for long term government bonds, in which this result may not hold.

4 No buy back

Thus far we modelled long bonds following the bulk of the literature and assuming repurchases of long debt one period after it has been issued. Though this assumption is commonly made as a simplification it is obvious that it is at odds with the observed practices of both governments and central banks. The stock of long term debt is not repurchased continuously, to be replaced by new long debt.

Abandoning this assumption of full buybacks, requires to identify an alternative setup for the modelling of long term debt. This might entail allowing for partial buybacks of long bonds prior to maturity (e.g. Quantitative Easing) or even ruling out repurchases altogether and assuming that debt is redeemed at maturity.

[26] In standard fashion, such a demand disturbance can be introduced in the model through a discount factor /preference shock in the background non-linear model.

[27] In the appendix we prove this explicitly in the case of decaying payment profiles. We obtain:

$$\hat{i}_t = \hat{r}_t + \delta \hat{\pi}_t$$

Both of these assumptions fit US policy in the post WWII era. Faraglia et al. (2019) provide evidence that buybacks by the US Treasury have been very rare during this period, the Treasury typically redeemed long term debt at maturity.[28] On the other hand, the Federal reserve bought considerable amounts of long term bonds as part of the Quantitative easing program launched in the decade following the 2008-9 crisis and also intervened in secondary bond markets to buy long term assets during the so called Operation Twist in the early 1960s. Since ours is a monetary model featuring the consolidated budget constraint, it is appropriate to think of these episodes as partial buybacks of long bonds.

Our analysis in this section adopts the first alternative and thus we rule out buybacks from the outset and assume that the long bonds are always redeemed at maturity. We do so for simplicity (modeling partial buybacks involves much more notation), but also because having covered the cases of full buybacks in sections 2 and 3 and no buyback in this section, it becomes possible to think of the intermediate scenario of partial buybacks.

Our results in this section demonstrate that no buyback can become an important friction for optimal monetary policy under the fiscal theory. When we assume that debt is only long term (a zero coupon bond) optimal Ramsey policy gives rise to oscillations in inflation that persist forever. Simple inflation targeting interest rate rules lead to explosive inflation dynamics and thus cannot approximate the Ramsey policy equilibrium.

The key element of the model that lies behind these strikingly different implications of the no buyback model, relative to the full repurchase model we studied in the previous sections, is that without buybacks the optimal policy's ability to spread the distortions of inflation across time is impaired. Whereas as we saw, under full repurchases the Ramsey planner issues debt to spread the distortions evenly across periods (the Lagrange multiplier ψ_{gov} follows a random walk), under no buyback and in the case of zero coupon long bonds, this is not a feasible outcome. The multiplier follows instead a cycle of periodicity N implying uneven distortions for $N > 1$.

We then turn to evaluate whether this result applies to any maturity structure of debt, not only to zero coupon long term bonds. Our key finding is that a necessary condition to avoid oscillations under no buyback is that the government focuses not only on issuing long term debt, but issues positive amounts of both short and long term bonds. Under this condition, it is possible to spread distortions evenly across periods even when repurchases are ruled out.

4.1 Optimal Ramsey policy without repurchases

We begin by characterizing optimal policy under no buyback. The only equation of the model that needs to be modified to rule out repurchases is the consolidated budget constraint. We now have

$$\sum_{k=1}^{\infty} \beta^k \overline{b}_k (\hat{b}_{t,k} - \sum_{l=1}^{k} E_t \hat{\pi}_{t+l}) + \overline{S}\hat{S}_t = \overline{b}_1 (\hat{b}_{t-1,1} - \hat{\pi}_t) + \sum_{k=2}^{\infty} \overline{b}_k (\hat{b}_{t-k,k} - \sum_{l=0}^{k-1} \hat{\pi}_{t-l}) \tag{28}$$

Note that (28) differs from the constraint under buy back (equation (6)) only with respect to the last term on the RHS. This term now measures the real value of debt that has reached maturity and is redeemed in t. Thus, $\hat{b}_{t-k,k}$ denotes debt that was of maturity k in period $t-k$. The real payout of this debt in t depends on the realized inflation between periods $t-k+1$ and t.

Notice also that the RHS of (28) does not represent the entire market value of debt that has been issued by the government. Since debt is not redeemed prior to maturity, there are bonds that

[28] A noteworthy exception is the 2001 buyback program. Moreover, until the 1980s a sizable fraction of long term debt outstanding was in callable bonds. This type of debt can be bought back prior to maturity, within a certain call window which however starts long after the bond has been issued. For example, a callable 15 year bond can be bought back 2 years before it matures. Callable bonds are thus approximately redeemed at maturity, their payment profiles are similar to those of non-callable bonds (see Faraglia et al. (2019)).

haven't yet matured in the government's portfolio. These objects will show up in future consolidated constraints and also show up in the intertemporal budget constraint which equates the value of debt to the present value of surpluses. The latter object can be written as:

$$E_t \sum_{j=1}^{\infty} \beta^j \overline{S}\hat{S}_{t+j} = \overline{b}_1(\hat{b}_{t-1,1} - \hat{\pi}_t) + \sum_{k=2}^{\infty} \overline{b}_k \left(\sum_{i=1}^{k} \beta^{k-i} (\hat{b}_{t-i,k} - E_t \sum_{l=1}^{k} \hat{\pi}_{t-i+l}) \right) \tag{29}$$

4.1.1 Optimal policy with one N bond.

For simplicity, let us first consider the Ramsey program under the assumption that the government issues only one N period bond. The optimal policy solves:

$$\max_{\{\hat{\pi}_t, \hat{Y}_t, \hat{b}_{t,N}\}_{t\geq 0}} -E_0 \frac{1}{2} \sum_{t\geq 0} \beta^t \hat{\pi}_t^2$$

subject to (1) and

$$\beta^N \left(\hat{b}_{t,N} - E_t \sum_{l=1}^{N} \hat{\pi}_{t+l} \right) + \overline{R}(\gamma_h + 1)\hat{Y}_t - \overline{G}\hat{G}_t = \overline{b}_N \left(\hat{b}_{t-N,N} - \sum_{l=0}^{k-1} \hat{\pi}_{t-l} \right) \tag{30}$$

The first order conditions are:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} + \overline{b}_N \sum_{l=1}^{N} \beta^{N-l} (\psi_{gov,t+N-l} - \psi_{gov,t-l}) = 0 \tag{31}$$

$$-\psi_{\pi,t}\kappa_1 + \overline{R}(1+\gamma_h)\psi_{gov,t} = 0 \tag{32}$$

$$\psi_{gov,t} - E_t \psi_{gov,t+N} = 0 \tag{33}$$

and with appropriate substitutions we get the following expression for optimal inflation:

$$\hat{\pi}_t = \overline{R}\frac{1+\gamma_h}{\kappa_1} \Delta\psi_{gov,t} + \overline{b}_N \sum_{l=1}^{N} \beta^{N-l} \Delta^N E_t \psi_{gov,t+N-l} \tag{34}$$

where $\Delta^N \psi_{gov,t+N} = \psi_{gov,t+N} - \psi_{gov,t}$

There are several noteworthy features. First, note that the multiplier $\psi_{gov,t}$ no longer follows a random walk. As equation (33) shows, $\psi_{gov,t}$ is equated to the expected value of the period $t+N$ multiplier, which gives us the standard martingale condition only when $N = 1$ (debt is short term). Second, according to (34) inflation is no longer only a function of the current and lagged values of the multiplier; also future multipliers exert an influence on the path of inflation.

To explain these properties we assume for simplicity $N = 2$. Iterating forward equation (30) gives:

$$E_t \sum_{j=0}^{\infty} \beta^{2j} \overline{S}\hat{S}_{t+2j} = \overline{b}_2(\hat{b}_{t-2,2} - \hat{\pi}_t - \hat{\pi}_{t-1}) \tag{35}$$

In (35) the market value of debt outstanding in t compensates for the surplus in $t, t+2, t+4, \ldots$. Consider an i.i.d shock $\hat{G}_t$ that lowers the LHS of (35). Then, since $\hat{\pi}_{t-1}$ is predetermined, only $\hat{\pi}_t$ can adjust to reduce the real value of maturing debt and ensure satisfaction of the constraint.

Future inflation (in particular inflation in $t+1$ when we assume a two period asset) does not help with making the debt solvent in t. The analogue of (35) in $t+1$ is:

$$E_{t+1}\sum_{j=0}^{\infty}\beta^{2j}\overline{S}\hat{S}_{t+1+2j}=\overline{b}_2\left(\hat{b}_{t-1,2}-\hat{\pi}_{t+1}-\hat{\pi}_t\right) \tag{36}$$

Evidently, the shock $\hat{G}_t$ has no impact on this intertemporal constraint. $\hat{\pi}_{t+1}$ will respond to the (expected) surplus sequence $\hat{S}_{t+1}, \hat{S}_{t+3}, \hat{S}_{t+5}, \dots$ and not to $\hat{S}_t, \hat{S}_{t+2}, \hat{S}_{t+4}, \dots$.

Generically, since following a positive shock to spending (35) will tighten, but not necessarily (36), these constraints will affect the solution differently and so the associated Lagrange multipliers will differ. The fact that ψ_{gov} follows a cycle of 2 periods in the model can be understood in terms of this property. To show how it affects the optimal path of inflation, we consider the impulse response with respect to a shock occurring in period t and assuming no shock will hit the economy thereafter. Conditional expectations can then be dropped and we can derive analytically the path of inflation. We have:

Proposition 4: *Assume $N=2$ and consider a spending shock in period t assuming no further shock thereafter. Optimal inflation in the Ramsey program under no buyback is:*

$$\hat{\pi}_{t+\bar{t}}=\begin{cases}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\overline{\psi}+\overline{b}_2(\overline{\psi}+\beta\underline{\psi}) & \bar{t}=0\\ \frac{\overline{R}}{\kappa_1}(1+\gamma_h)\underline{\psi}+\overline{b}_2\underline{\psi} & \bar{t}=1\\ \\ \frac{\overline{R}}{\kappa_1}(1+\gamma_h)(\overline{\psi}-\underline{\psi})I_{\bar{t}=\text{even}}+\frac{\overline{R}}{\kappa_1}(1+\gamma_h)(\underline{\psi}-\overline{\psi})I_{\bar{t}=\text{odd}} & \bar{t}>1\end{cases} \tag{37}$$

where $\underline{\psi}\neq\overline{\psi}$ denote the values of the Lagrange multipliers, $\psi_{gov,t+j}=\overline{\psi}$ for $j=0,2,4,\dots$ and $\psi_{gov,t+j}=\underline{\psi}$ for $j=1,3,5,\dots$.

Proof: See appendix.

The appendix provides an analytical formula for $\underline{\psi},\overline{\psi}$.

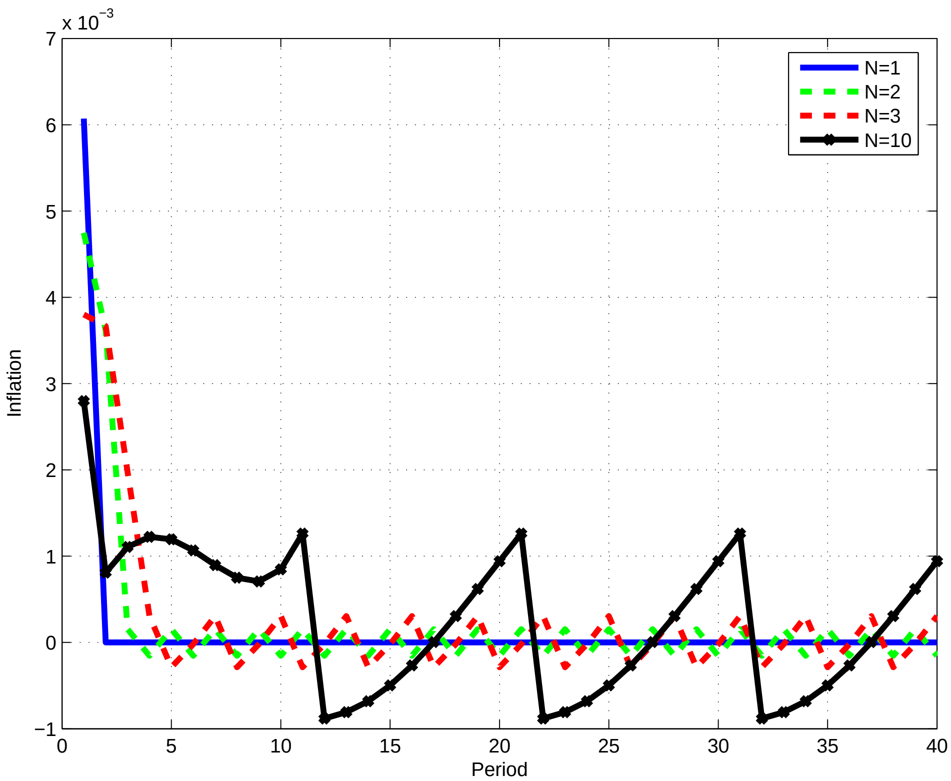
There are two key messages in Proposition 4 that are worth highlighting: First, inflation in $t+1$ will generally not equal zero. Second, inflation will persist from period $t+2$ onwards and follow a 2 period cycle. Generically, $\underline{\psi}<\overline{\psi}$ and so inflation will be positive when $\bar{t}$ is even (i.e. in periods $t+2, t+4, ..$ etc) and negative when $\bar{t}$ is odd (periods $t+3, t+5, ..$ etc). From equations (35) and (36) it is clear why this is so. $\hat{\pi}_t$ will adjust to satisfy (35), however, since $\hat{\pi}_t>0$ when a positive shock has hit, $\hat{\pi}_{t+1}$ must turn negative to satisfy (36). Then, $\hat{\pi}_{t+2}$ will be positive again to satisfy the intertemporal constraint in $t+2$ and subsequently, $\hat{\pi}_{t+3}$ will have to compensate for this, in order to satisfy the constraint in $t+3$. This process goes on indefinitely.

4.1.2 The effect of maturity.

In Figure 4 we use our baseline calibration to solve the optimal policy equilibrium under different values of N.[29] Consider first the cases where debt is long term, i.e. $N\geq 2$. As is evident from the Figure, optimal inflation follows a N period cycle which starts N periods after the shock hits. The pattern highlighted in Proposition 4 thus holds more generally, across all $N\geq 2$ considered in the Figure. Moreover, the Figure shows that the longer the maturity is, the larger is the volatility

[29] Period t is period 1 in the graph.

Figure 4: **Responses to the spending shock under no buyback.**



Notes: The figure plots the path of optimal inflation in response to a shock that increases spending by 20% (from 10% of GDP to 12% of GDP) under various maturity structures and assuming no debt repurchases.

displayed by inflation. Thus, issuing long term debt under no buyback does not enable to spread the burden of inflation through time. In the buyback model of Section 2, the opposite property held.

Finally, consider the case where $N = 1$ (solid blue line). Debt is short term and so repurchasing is coincident to redeeming debt at maturity; the dynamics of inflation are effectively the same as the analogous dynamics in the model of Section 2.

4.2 Interest Rate Rules under No Buy Back

We have seen that when long term bonds are redeemed at maturity, the optimal policy gives rise to oscillations in inflation that persist indefinitely. The longer is the maturity of debt the larger are the fluctuations in inflation after period N.

These features emerge from the Ramsey solution and so clearly they represent the best possible competitive equilibrium outcome given the set of constraints that define the equilibrium and the assumptions that we made in the model. Nonetheless, an equilibrium outcome in which inflation fluctuates periodically may not be desirable from a practical standpoint. First, because the interest rate rule that can implement this outcome will likely be a complicated function of macroeconomic conditions and debt dynamics, and thus not conform with the principle that a policy rule should be a simple, transparent function of macroeconomic variables. Second, foreseeable inflation oscillations may be disruptive in various contexts, including in financial markets or in terms of firms' pricing decisions. Third, oscillations may imply that the nominal interest rate will periodically be at its effective lower bound.

Our purpose here is not to acknowledge these issues explicitly; we want to investigate whether alternative outcomes in which inflation does not feature considerable oscillations are available when a policy rule of the form (18) is implemented by the monetary authority.

We reach a very negative result: In a model where debt repurchases are ruled out and government debt is long term, a policy rule (18) does not yield a unique non-explosive solution. The appendix proves the following Proposition:

Proposition 5. *Consider the no buyback model where monetary policy follows (18), the consolidated budget constraint is given by (30), together with the Phillips curve and the Euler equation. The dynamic system has $N + 1$ eigenvalues outside the unit circle for N forward looking variables for all $\phi_\pi \in [0, 1]$. Thus, there is no non-explosive solution.*

Proof: See appendix.

This result will also hold for $\phi_\pi \notin [0, 1]$. We focus on the usual region where monetary policy is passive, however, assuming (say) $\phi_\pi > 1$ will only add another unstable root to the system. Given that the dynamic system is unstable, it is evident that inflation is an explosive process, a far worse outcome than the stable oscillations we had previously. An equilibrium in which inflation does not feature oscillations and monetary policy follows a simple rule as in (18) is not available.

The result in Proposition 5 suggests that in the presence of long term debt and no buyback, the usual property that a unique stable equilibrium obtains in the fiscal theory when the Taylor principle is violated (i.e. $\phi_\pi < 1$) will not hold. This will also apply if interest rates are set contingently on current output or lagged interest rates. For example, a policy rule (27) will not deliver a non-explosive solution even when $\phi_\pi + \frac{\phi_Y}{\kappa_1}(1 - \beta) \leq 1$. This finding should be of interest.

4.2.1 Stability through odd rules

Key to the above result is that policy rule (18) does not pin down the lagged inflation terms on the RHS of (30), $\hat{\pi}_t, \hat{\pi}_{t-1}, ..., \hat{\pi}_{t-N+1}$.

It turns out that to obtain a stable equilibrium in this model, policy has to be specified in such way so that $z^t_{t-N+1} \equiv \hat{\pi}_t + \hat{\pi}_{t-1} + ... + \hat{\pi}_{t-N+1}$ solves a stable difference equation. Thus, a rule that gives

$$z^{t+N}_{t+1} = \epsilon z^t_{t-N+1} + \text{Forcing Terms}$$

will work provided that ϵ lies within the unit circle.

For simplicity, let us focus on the case $N = 2$. A policy rule that can work is of the form:

$$\hat{i}_t = \phi_\pi(\hat{\pi}_t + \hat{\pi}_{t-1}) - E_t\hat{\pi}_{t+2} \tag{38}$$

which determines z as the non-explosive solution to

$$E_t(\hat{\pi}_{t+2} + \hat{\pi}_{t+1}) = \phi_\pi(\hat{\pi}_t + \hat{\pi}_{t-1})$$

This solution will not avoid oscillations in inflation. The higher is ϕ_π, the larger will be the oscillations in inflation, and the more persistent z will be. This is shown in Figure 5 which plots the responses of inflation to a spending shock under various values of parameter ϕ_π along with the optimal Ramsey policy. Notice that no matter the size of the coefficient ϕ_π, the initial response of inflation is the same (close to 0.1 in the graph). Future expected inflation oscillations do not add anything in terms of making debt more sustainable when the shock hits, but also do not worsen the equilibrium outcome. Debt issuance will adjust to ensure satisfaction of future intertemporal constraints. These properties suggest that in this model it should be optimal to set $\phi_\pi \approx 0$. This would also be (partially) consistent with Proposition 4, which showed that z becomes zero, three periods after the shock in the Ramsey policy.

4.3 Is the result general (and the role of short term bonds)?

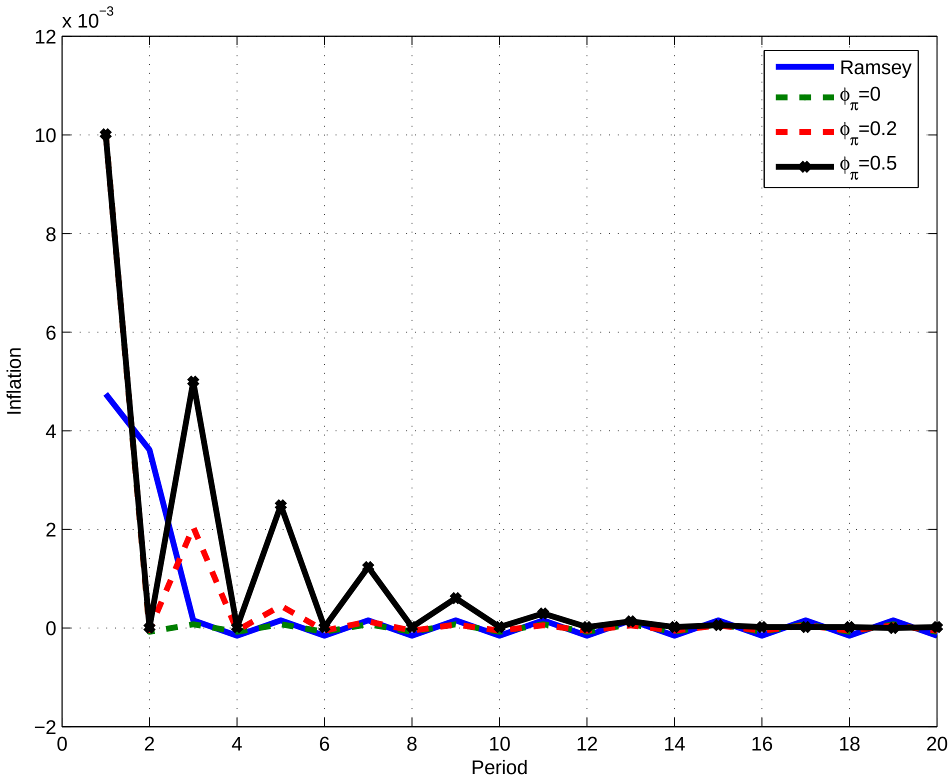
Policies of the form (38) produce a unique stable equilibrium (albeit one featuring oscillations) but we cannot claim that they are obviously practically relevant. Not necessarily because they prescribe targeting future inflation; rather the issue is that at long debt maturity N, a stable equilibrium path is attained when the nominal interest rate tracks $N - 1$ leads and N lags of inflation, to pin down z. A high N makes the number of variables that interest rates must respond to, very large. Such a policy would be in practice difficult to implement as well as to interpret.

Rather than insisting on finding a simpler rule, one that relies on fewer variables to deliver a non-explosive solution, we now take a different perspective and consider the role of the debt maturity structure in determining the policy rule. In particular, we ask: Are there maturity structures such that a simple rule of the form (18) gives us a stable equilibrium under no buyback? Under such debt structures what is the optimal inflation coefficient and does the optimal policy provide a good approximation of the Ramsey outcome?

Note that in answering these questions we will also be investigating the generality of the result of the previous paragraph, that under no repurchases optimal policy cannot spread the distortions of inflation evenly, to alternative assumptions about the maturity structure. The previous example focused on a single zero coupon N bond, to develop analytical insights into how no buyback affects feasible allocations, but this assumption can be seen as too restrictive.

It turns out that Propositions 4 and 5 can be extended in several meaningful ways, considering richer maturity structures than a single N bond. For example, we can consider cases where the government issues positive amounts of debt in long term bonds of maturities $\{\underline{k}, \underline{k} + 1, ..., N\}$ and still show that optimal Ramsey policy will feature oscillations, in this case a cycle of periodicity $\underline{k} > 1$. Moreover, we can allow the government to repurchase part or all of the stock of long term debt, a few periods after it has been issued and still get oscillations in equilibrium. For example,

Figure 5: **Responses to the spending shock under no buyback.**



Notes: The figure plots the path of inflation in response to a shock that increases spending by 20% (from 10% of GDP to 12% of GDP). We assume $N = 2$ and no debt repurchases. $\phi_\pi = 0, 0.2, 0.5$ are the assumed values of the coefficient in (38). 'Ramsey' stands for the optimal Ramsey policy outcome.

consider a long-term bond of maturity N assuming it can be repurchased periods j after issuance. Let also $1 < j < N$ so that the bond is not repurchased right after issuance (in the next period) and it is also not redeemed at maturity. In this model optimal policy will feature fluctuations of periodicity j.[30]

Given these observations as well as our previous analytical results, it becomes evident that if there is any chance to obtain an equilibrium without oscillations under no buyback, then the portfolio of the government should not only feature long term bonds but also positive amounts of short term debt. With short bonds, no buyback is coincident with buyback and the optimal policy will again be able to spread the distortions of inflation evenly across periods. We now illustrate this property formally and also show that simple interest rate rules approximately give us the Ramsey outcome.

Let us first go back to Ramsey policy equilibrium and consider that $\bar{b}_1, \bar{b}_2, ..., \bar{b}_N > 0$. To simplify our derivations we fix the portfolio shares assuming that $\hat{b}_{t,1} = \hat{b}_{t,2} = ... = \hat{b}_t$. We also assume the following debt structure: $\bar{b}_1 = \bar{b}_2, ..., \bar{b}_{N-1} = \bar{b} \leq \bar{b}_N = \tilde{\bar{b}}$.

The last assumption imposes that the bond quantities are $\bar{b}$ for maturities 1 to $N-1$ and then $\tilde{\bar{b}}$ for maturity N. This nests the case of a flat maturity structure when $\bar{b} = \tilde{\bar{b}}$, which becomes a consol bond when $N \to \infty$. In the case where $\bar{b} < \tilde{\bar{b}}$ the government issues debt in a long bond that pays constant coupons equal to $\frac{\bar{b}}{\tilde{\bar{b}}}$ and the principal can be normalized to 1. In all these cases some short maturity debt is being issued.

Under these assumptions the budget constraint is:

$$\begin{aligned} \hat{b}_t\left(\bar{b}\sum_{j=1}^{N-1}\beta^j + \beta^N\tilde{\bar{b}}\right) - \bar{b}\sum_{j=1}^{N-1}\beta^j\sum_{l=1}^{j}E_t\hat{\pi}_{t+l} \quad &- \quad \tilde{\bar{b}}\beta^N\sum_{l=1}^{N}E_t\hat{\pi}_{t+l} + \overline{S}\hat{S}_t \\ &= \quad \bar{b}\sum_{j=1}^{N-1}\left(\hat{b}_{t-j} - \sum_{l=0}^{j-1}\hat{\pi}_{t-l}\right) + \beta^N\tilde{\bar{b}}\left(\hat{b}_{t-N} - \sum_{l=0}^{N-1}\hat{\pi}_{t-l}\right) \end{aligned}$$

The first order condition of the Ramsey program with respect to $\hat{b}_t$ is given by:

$$\psi_{gov,t}\left(\bar{b}\sum_{j=1}^{N-1}\beta^j + \beta^N\tilde{\bar{b}}\right) - \left[\bar{b}\left(\sum_{j=1}^{N-1}\beta^j E_t\psi_{gov,t+j}\right) + \tilde{\bar{b}}\beta^N E_t\psi_{gov,t+N}\right] = 0 \tag{39}$$

which implies non-trivial dynamics linking the current value of the multiplier with the expected future values up to period $t+N$.

To analyze these dynamics let us revisit the previous example where a shock hits the economy in t and no further shock is expected thereafter. We can then derive the following Nth order difference equation that determines $\psi_{gov,t}$:

$$\psi_{gov,t+N} + \frac{\bar{b}}{\tilde{\bar{b}}}\left(\frac{1}{\beta}\psi_{gov,t+N-1} + \frac{1}{\beta^2}\psi_{gov,t+N-2} + ... + \frac{1}{\beta^{N-1}}\psi_{gov,t+1}\right) - \left(\frac{\bar{b}}{\tilde{\bar{b}}}\frac{1-\frac{1}{\beta^N}}{1-\frac{1}{\beta}} + 1\right)\psi_{gov,t} = 0 \tag{40}$$

The above equation has one root that is equal to 1. In the case where $N = 2$ there is another real

[30] Under this modelling assumption debt is issued in a *callable bond* with a call option (starting) at j. In practice callable bonds can be bought back at different dates, within a call window that typically starts close to maturity. (For example, ten year callable debt can be repurchased within 2 years before it matures.) However, the US Treasury always bought back this debt at the start of the call window (see Faraglia et al., 2019). Thus assuming that debt can only be bought in period j is not restrictive.

root, equal to $-(1+\frac{\bar{b}}{\tilde{\bar{b}}}\frac{1}{\beta}) < -1$. Let us focus on this case for simplicity.[31] We then have that

$$\psi_{gov,t+2}(1-L)\left(1+(1+\frac{\bar{b}}{\tilde{\bar{b}}}\frac{1}{\beta})L\right)=0$$

where L denotes the lag operator. Notice that when $\bar{b}=0$ this gives

$$\Delta\psi_{gov,t+2}(1-L)=0 \rightarrow \psi_{gov,t+2}=\psi_{gov,t}$$

and as before we obtain a 2 period cycle. However, in the case where $\bar{b}>0$ we can write

$$-\frac{1}{(1+\frac{\bar{b}}{\tilde{\bar{b}}}\frac{1}{\beta})}\Delta\psi_{gov,t+2}=\Delta\psi_{gov,t+1}$$

Assuming a bounded process, or $\lim_{j\to\infty}(-(1+\frac{\bar{b}}{\tilde{\bar{b}}}\frac{1}{\beta}))^j\Delta\psi_{gov,t+j}=0$, delivers the following:

$$\Delta\psi_{gov,t+1}=0$$

The random walk property of the multiplier is restored.

This is an important property. Recall that in the Ramsey model with a single long bond, inflation oscillations under no buyback resulted from the fact that inflation in period t would compensate for changes in the value of the surplus in periods $t, t+2, t+4, ..$ but not for the surplus in periods $t+1, t+3, ...$ Then, $\hat{\pi}_t$ would absorb the shock $\hat{G}_t$, and this would ensure satisfaction of (35), but it would also perturb (36) so that $\hat{\pi}_{t+1}$ needs to adjust to satisfy the constraint. The effect carried over to other periods. In terms of the Ramsey program, this then meant that (35) and (36) impact the solution differently, or generically $\psi_{gov,t}\neq\psi_{gov,t+1}$. Proposition 4 showed that inflation oscillations can be described in terms of these multipliers.

The fact that $\psi_{gov,t}=\psi_{gov,t+1}$ when short debt is issued implies that inflation oscillations will not occur. We can show that following a shock in $\hat{G}_t$ inflation will evolve according to:

$$\hat{\pi}_{t+\bar{t}}=\left\{\begin{array}{ll}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\overline{\psi}+\overline{b\psi}+\widetilde{\overline{b\psi}}(1+\beta) & \bar{t}=0\\ \widetilde{\overline{b\psi}} & \bar{t}=1\\ \\ 0 & \bar{t}>1\end{array}\right\}$$

where $\overline{\psi}$ now denotes the difference of the Lagrange multiplier between the pre shock value and the value after the shock.

Quite evidently, the response of inflation to the shock resembles the analogous object in the buyback model of Section 2.

4.3.1 Why are short term bonds so important?

Another way of saying that (35) and (36) influence differently the Ramsey solution under no buyback and only long term debt, is to say that we cannot add up these constraints in the Ramsey program. If the Lagrange multipliers were equal then solving the Ramsey program using the intertemporal constraint

$$E_t\sum_{j=0}^{\infty}\beta^j\overline{S}\hat{S}_{t+j}=\overline{b}_2\left(\hat{b}_{t-2,2}-\hat{\pi}_t-\hat{\pi}_{t-1}\right)+\overline{b}_2\beta\left(\hat{b}_{t-1,2}-E_t\hat{\pi}_{t+1}-\hat{\pi}_t\right) \tag{41}$$

[31] Otherwise for $N>2$ some of the roots of the characteristic equation are complex and moreover it is difficult to factor the characteristic polynomial.

would suffice. Generally, equation (41) is not sufficient for an optimal policy equilibrium under no buyback (see Faraglia et al., 2016). Instead (35) and (36) are both important implementability conditions, along with the Philips curve.

When short term debt is issued however, the intertemporal constraint (the analogue of equation (41))

$$E_t \sum_{j=0}^{\infty} \beta^j \overline{S} \hat{S}_{t+j} = \bar{b}_2 \Big(\hat{b}_{t-2,2} - \hat{\pi}_t - \hat{\pi}_{t-1} \Big) + \bar{b}_2 \beta \Big(\hat{b}_{t-1,2} - E_t \hat{\pi}_{t+1} - \hat{\pi}_t \Big) + \bar{b}_1 \Big(\hat{b}_{t-1,1} - \hat{\pi}_t \Big) \tag{42}$$

becomes sufficient. The intuition is that the short bond issuance can adjust to satisfy (42) in $t+1$ (and all future periods) given a smooth path of inflation following the shock in spending. Thus, under no buyback, the planner can use short term debt to smooth inflation over time, and satisfy the standard intertemporal solvency condition (42).

These results stand out as particularly relevant for the literature studying optimal fiscal inflation policy under a Ramsey planner. A well known feature of these models is that when debt is long term, inflation can more effectively absorb shocks to spending, enabling the planner to minimize distortions by committing to inflate away public debt over a long horizon (e.g. Lustig et al., 2008; Sims, 2013; Leeper and Zhou, 2021 among others).[32] Issuing debt in the longest maturity available, enables to fully exploit this channel (e.g. Lustig et al., 2008).

However, these predictions do not carry over to the no buyback model. As we showed, when only long term bonds are issued, then inflation features excess volatility and oscillations that increase in N (e.g. Figure 4). Focusing on long term debt does not enable to smooth inflation over time.

Finally, note that our findings in this paragraph are similar to Faraglia et al. (2019) who show, in a Ramsey model of optimal taxation, that no buyback generates the incentive to issue short term debt. Their non-linear model predicts an optimal debt structure in which short bonds make up for roughly half of the total market value of debt issued. Here, we used a simpler (linear) model, which does not pin down an optimal portfolio. We leave this to explore in future research.

4.3.2 Inflation targeting rules under no buyback.

Issuing short term debt brings us back to our previous findings regarding the optimality of simple interest rate rules. We can show that (18) once again leads to a unique non-explosive solution, and delivers a close approximation of the Ramsey policy. Instead of working through all previous model versions, let us focus here on the case where the maturity structure is flat and $N \to \infty$. Then (39) becomes:

$$\frac{1}{1-\beta} \psi_{gov,t} = \sum_{j=1}^{\infty} \beta^j E_t \psi_{gov,t+j} = (1-\beta)\beta E_t \frac{1}{1-\beta L^{-1}} \psi_{gov,t+1}$$

which again gives the random walk. The following Proposition defines the optimal interest rate policy in this model:

Proposition 6. *Assume no buyback and long bonds are consols. The optimal path of $\hat{i}_t$ under Ramsey is given by:*

$$\hat{i}_t = \hat{\pi}_t - \frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t}$$

which is the same optimal monetary policy as in the buyback model.

[32] This prediction was confirmed in our buyback model. Consider again Proposition 1. Coefficients η_{-j} become 0 as $N \to \infty$.

Proof: See appendix.

Clearly, in the case where $\overline{R} = 0$ the optimal policy is a rule (18) where the inflation coefficient is $1 - \frac{1}{\text{Maturity}}$ in this case where the average maturity is infinite. The coefficient will be approximately one when $\overline{R} > 0$.

Finally, note that Proposition 6 can be extended to the case of decaying coupon payments. The proof that we provide in the appendix begins from this assumption and shows the more general result.

5 Extensions: Output Stabilization, the Canonical Model and Optimal Fiscal Policy

The main takeaway from the previous sections is that optimal policies can be implemented through simple inflation targeting rules. We utilized a simplistic setup, assuming that the central bank seeks to stabilize inflation and considering a Fisherian model in which the real interest rate is exogenous and not a function of output.

We now show that our results carry over to alternative setups of policy, when the central bank pursues a dual objective of stabilizing inflation and the output gap and when the real rate is a function of output as in the canonical New Keynesian model. Finally, we also briefly consider a jointly optimal monetary/fiscal policy program, and show that our results continue to hold in this case where taxes are not constant.

Our results in this section focus, for simplicity, on zero coupon N bonds, and on decaying coupon δ bonds.

5.1 Output stabilization.

Consider first the case where the objective of policy is to stabilize both inflation and the output gap and let us maintain the assumption that the real interest rate is exogenous as in our baseline Fisherian model. The objective of the planner is given by:

$$-\frac{1}{2}E_0 \sum_{t\geq 0} \beta^t (\hat{\pi}_t^2 + \lambda_Y \hat{Y}_t) \tag{43}$$

where λ_Y is the relative weight attached to stabilizing output around its target steady state level.

It is simple to show that optimal policy (when taxes are lump sum) will set:

$$\hat{\pi}_t + \frac{\lambda_Y}{\kappa_1}\Delta\hat{Y}_t = \sum_{k=1}^{\infty} \bar{b}_k \sum_{l=1}^{k} \beta^{k-l}\Delta\psi_{gov,t-l+1} = \begin{cases} \bar{b}_N \sum_{l=0}^{N-1} \beta^{N-l-1}\Delta\psi_{gov,t-l} \\ \\ \frac{\bar{b}}{1-\beta\delta} \sum_{l=0}^{\infty} \Delta\psi_{gov,t-l} \end{cases} \tag{44}$$

where the last equality states the formulae for the N and the δ bonds separately.

As is evident from (44) optimal inflation is now a function of output growth $\Delta\hat{Y}$ as well as the multipliers $\Delta\psi_{gov}$. Using the Phillips curve we can obtain:

$$\Delta\hat{Y}_t = \frac{1}{\kappa_1}\Big(\hat{\pi}_t(1+\beta) - \beta E_t\hat{\pi}_{t+1} - \hat{\pi}_{t-1}\Big) - \zeta_t$$

where $\zeta_t \equiv \frac{\beta}{\kappa_1}(\hat{\pi}_t - E_{t-1}\hat{\pi}_t)$ can be written as a function of the spending shock under optimal policy. Using this result, we can arrive to the following second order difference equation governing the dynamics of inflation in this model:

$$E_t\hat{\pi}_{t+1} - (1 + \frac{1}{\beta} + \widetilde{\kappa})\hat{\pi}_t + \frac{1}{\beta}\hat{\pi}_{t-1} = \mu_t \tag{45}$$

where the forcing term μ_t is a function of ζ_t and the Lagrange multipliers (see appendix).

The two roots of (45) are:

$$\nu_{1,2} = \frac{1}{2}\left((1 + \frac{1}{\beta} + \widetilde{\kappa}) \pm \sqrt{(1 + \frac{1}{\beta} + \widetilde{\kappa}) - \frac{4}{\beta}}\right) \tag{46}$$

and it is easy to show that one root is (say ν_1) is stable, and one root is unstable.

Let us now turn to the optimal interest rate rule. Note that the above conditions seem to suggest that a simple rule that can implement the Ramsey outcome is:[33]

$$\hat{\imath}_t = (\nu_1 + \nu_2)\hat{\pi}_t - \nu_1\nu_2\hat{\pi}_{t-1} + \mu_t$$

However, repeating the arguments we made in subsection 3.1.2, we can show that the fact that ν_2 lies outside the unit circle, implies that no stable equilibrium can be reached in this model.

We thus again need to turn to an inflation targeting rule of the form (18) and find the optimal coefficient ϕ_π, or even, for this model in which Ramsey inflation evolves according to a second order difference equation, a rule in which both current and lagged inflation exert an influence on the nominal rate provides a better approximation of optimal policy.

We focus on the second scenario and consider rules of the form

$$\hat{i}_t = (\nu_1 + \phi_\pi)\hat{\pi}_t - \nu_1\phi_\pi\hat{\pi}_{t-1} \tag{47}$$

setting the coefficient ϕ_π to maximize the objective (43).

Consider first the single bond model. Equilibrium inflation is given by

$$\hat{\pi}_t = \sum_{j=0}^{t} \frac{\phi_\pi^{j+1} - \nu_1^{j+1}}{\beta^{N-1}\overline{b}_N \sum_{l=0}^{N-1}(\phi_\pi^{l+1} - \nu_1^{l+1})}\overline{G}\hat{G}_{t-j}$$

(see appendix). Using the Phillips curve it is then easy to obtain an analogous closed form solution for output.

In the appendix we derive the objective (43), and the first order condition defining the optimal coefficient ϕ_π. The formulae are cumbersome and they do not always admit an analytical solution, and so we will not show them here, however, one analytical result that is worth highlighting, concerns the optimal coefficient under short term debt. When $N = 1$ we get $\phi_\pi^* = 0$ which essentially means that $\hat{i}_t = \nu_1\hat{\pi}_t$ is the optimal interest rate rule.

Recall that when the planner stabilizes inflation only, the optimal interest rate rule features no systematic reaction to inflation. Here, in contrast, it is optimal for $\hat{i}_t$ to respond to inflation and setting the inflation coefficient to be equal to $\nu_1 > 0$.

What explains this difference? When output smoothing becomes an objective, a policy that features no persistence in inflation, under short term debt, is not optimal because concentrating inflation in the period the shock hits results in high output volatility. In contrast, making inflation persistent, enables to smooth output across time, and delivers a better outcome in terms of the objective (43), in spite of the fact that inflation persistence does not contribute anything towards stabilizing debt. A higher weight λ_Y increases persistence, and in the limit when $\lambda_Y \to \infty$ (equivalently when the

[33]This follows easily from (45) and the Euler equation.

planner only targets output) we have $\nu_1 \to 1$. Inflation will then permanently rise in response to a positive spending shock.

These properties carry over to the case $N > 1$. When maturity is long however, making inflation a persistent process is desirable also from the point of view of reducing inflation variability. Thus the goals of smoothing inflation and output line up.

A simple inflation targeting rule of the form (47) can approximate the Ramsey solution closely also in this model. Rather than showing these results here, we leave them for the appendix and instead turn to the more realistic case where debt payments decay at rate δ. The following Proposition shows the optimal interest rate rule for this model.

Proposition 7: *Assume $\bar{b}_k = \bar{b}\delta^{k-1}$ and Ramsey policy maximizes objective (43). Then, the optimal interest rate rule that implements the Ramsey outcome is:*

$$\hat{i}_t = (\nu_1 + \delta)\hat{\pi}_t - \nu_1 \delta \hat{\pi}_{t-1} + \text{Stochastic Intercept} \tag{48}$$

Proof: See Chafwehé et al. (2022).

Analogously to our previous findings in the decaying payment profiles model, a rule based policy is an optimal policy in the sense of Ramsey and sets $\phi_\pi^* = \delta$. The 'stochastic intercept term' in (48) is once again a function of $\Delta\psi_{gov,t}$ (and can be written as a function of the spending shock). In the full blown Ramsey solution this term captures the incentive of the planner to hold the nominal rate at a slightly lower level, contemporaneously with a positive spending shock. The effect concerns only the period that the shock occurs and one can show that it does not matter much for the resulting optimal paths of inflation and output. Therefore, omitting this term and focusing on a simple rule setting the nominal interest rate as a function of current and lagged inflation, is sufficient to approximate the Ramsey outcome very well. For brevity we relegate these derivations and numerical results to the appendix.

5.2 The canonical New Keynesian model

Our results in the previous sections were derived assuming a simplistic Fisherian setup in which the real interest rate is exogenous. This enabled us to derive transparent interest rate rules under various maturity structures, including to investigate the effects of debt buybacks on optimal policy. Naturally the reader will be wondering whether the main result of this paper, that simple interest rate rules can approximate the Ramsey outcome (under no buyback when short bonds are being issued) will continue to hold in the canonical New Keynesian model when the real interest rate becomes a function of output growth and spending. We argue that it will.

Obviously, investigating the canonical model in detail, repeating the previous derivations, requires a lot more space than this robustness subsection provides; it requires a separate paper! Chafwehé et al. (2022) have taken up the task of characterizing optimal rules in this model, focusing on the decaying payment profile structure and assuming as is common in the literature, debt buybacks. We will summarize in a few lines their findings here, and also provide additional numerical results in the appendix to show optimal policies in the case of N bonds under buyback and no buyback.

A crucial difference between our Fisherian setup and the canonical model is that in the latter, output fluctuations contribute towards making debt sustainable whereas in the former this is not so. Consider again the intertemporal budget constraint in the Fisherian model, equation (12). The RHS of this equation features only inflation. The LHS will depend on output when we assume distortionary taxes, but as we explained previously, this does not have a significant effect on optimal policy.

In the canonical model, the analogue of equation (12) is:

$$E_t \sum_{j=0}^{\infty} \beta^j \overline{S}\hat{S}_{t+j} = \frac{\bar{b}}{1-\beta\delta}\hat{b}_{t-1} + \bar{b}\sum_{j=0}^{\infty}(\beta\delta)^j E_t\left[-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}) - \sum_{l=0}^{j}\hat{\pi}_{t+l}\right] \tag{49}$$

where we assume a decaying debt profile structure. Parameter σ denotes the inverse of the intertemporal elasticity of substitution. $\overline{C}, \overline{Y}$ are the steady state levels of consumption and output respectively.

The RHS of the constraint is a function of output. When a spending shock hits, changing the present value of the surplus on the LHS, then both changes in output and inflation can absorb the shock and ensure satisfaction of (49).

In the presence of this additional output channel, the optimal policy will not only target inflation to make debt sustainable, but also target output, accounting simultaneously for the indirect effect of inflation on output, through the Phillips curve.[34]

It is however possible to derive a simple inflation targeting rule in this model which expresses the nominal interest rate as a function current inflation. In particular, we have

$$\hat{i}_t = r_t^n + \left(\delta + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1-\delta)(\beta\delta-1)\right)\hat{\pi}_t + \text{Stochastic Intercept}$$

A couple of lines are needed to explain this formula. First, notice that the leading term r_t^n is the natural rate of interest, the real rate obtained under flexible prices. This can be expressed as a function of the spending shock and it holds that $r_t^n = \sigma\frac{\overline{G}}{\overline{C}}(1-\frac{\sigma}{\gamma_h+\sigma\frac{\overline{Y}}{\overline{C}}})\hat{G}_t$[35]. Basically, through tracking the real rate, the optimal policy accomplishes to eliminate the shock from the Euler equation. Then, the spending shock can only affect the economy through the debt constraint and the Phillips curve.

Second, the optimal inflation coefficient is now not simply equal to δ: there is an additional term that depends on parameters $\delta, \beta, \kappa_1, \sigma$. This term measures the indirect effect of inflation (through output) on the consolidated budget constraint. A positive spending shock leads to an increase in inflation that transmits to output through the Phillips curve. Higher output will increase the real interest rates, leading to a drop in bond prices. This also contributes towards stabilizing government debt, through lowering the RHS of (49). Clearly, assuming $\sigma = 0$ brings us back to the Fisherian setup we considered previously.

Our purpose here is not to evaluate this model in detail, only to show that optimal policy can be approximated by simple inflation targeting rules in the canonical model. We refer the reader to Chafwehé et al. (2022) for a further analysis of optimal policy when $\sigma > 0$. In the appendix, we present numerical results from this model as well as for the case where the government issues one N bond.[36] We continue to find that inflation targeting rules effectively deliver the Ramsey outcome. Finally, we revisit our results in Section 4 showing that N bonds under no buyback induce instability under the optimal policy equilibrium.

[34]Note that output will also influence the LHS in (49) since the real interest rates influences the present value of government surpluses. Optimal policy will also internalize this effect, see Leeper and Zhou, 2021 and Chafwehé et al. (2022) among others for this additional discounting channel.

[35]For a persistent shock the expression is modified to $r_t^n = \sigma\frac{\overline{G}}{\overline{C}}(1-\frac{\sigma}{\gamma_h+\sigma\frac{\overline{Y}}{\overline{C}}})\hat{G}_t(1-\rho_G)$, where ρ_G is the first order autocorrelation coefficient.

[36]It is interesting to note that in the case of N bonds the optimal inflation coefficient becomes (approximately) $\left(1-\frac{1}{N}+\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\frac{1}{N}(\beta(1-\frac{1}{N})-1)\right)$, that is analogous to the optimal coefficient we derived for δ bonds.

5.3 Optimal Taxation and Inflation

Before concluding the paper in the next paragraph we briefly discuss an additional set of results derived in the appendix.

Throughout the paper we have assumed, for analytical tractability, that taxes are constant through time. Though this is a standard assumption in the context of the fiscal theory (see for example Bianchi and Ilut, 2017; Bianchi and Melosi, 2017), assuming that taxes respond to shocks will not affect our results. We make this point in the appendix, using the model of Sections 2 and 3 but letting the Ramsey planner optimally sets distortionary taxation along with inflation and output as in Siu (2004); Schmitt-Grohé and Uribe (2004); Faraglia et al. (2013); Sims (2013); Leeper and Zhou (2021).[37]

Following Sims (2013) we assume the following objective function for the policy maker:

$$-\frac{1}{2}E_0\sum_{t\geq 0}\beta^t\left(\hat{\pi}_t^2+\lambda_\tau\hat{\tau}_t^2\right) \tag{50}$$

The optimal policy sets taxes according to:

$$\lambda_\tau\hat{\tau}_t=\psi_{gov,t}\frac{\widetilde{c}}{\lambda_\tau}$$

for $\widetilde{c}$ a function of model parameters, which for brevity defined is in the appendix. Optimal taxes thus respond only to the multiplier $\psi_{gov,t}$ following a random walk (a standard property of optimal policy models) for $0<\lambda_\tau<\infty$. The first order conditions for inflation, output and government debt do not change and so we can repeat previous steps and derive optimal interest rate rules in this model.[38] The only way in which the availability of taxes as an additional policy instrument affect our previous results, is through impacting the magnitude of the response of inflation to the shocks. When the welfare costs of tax distortions are less than the costs due to inflation (that is when λ_τ is low and the degree of price stickiness θ is high), the planner will opt for using taxes to adjust the intertemporal surplus (e.g. Siu, 2004; Schmitt-Grohé and Uribe, 2004; Faraglia et al., 2013). When the opposite holds, relying more on inflation to finance debt becomes optimal (e.g. Sims, 2013; Leeper and Zhou, 2021). The time-path of inflation will not change.

6 Conclusion

We studied optimized interest rate rules in the context of the fiscal theory of the price level. Our key result is that simple inflation targeting rules can approximate closely the Ramsey outcome. The optimal inflation coefficient depends on the average debt maturity. We derive simple formulae showing this dependence. We also investigate how departing from the canonical modelling of long bonds found in the literature, that is by making the empirically grounded assumption of no repurchases of long term debt, affects our results. Under no buyback, simple inflation targeting rules work only when the government issues a portfolio with positive amounts of both short and long term bonds. Otherwise, the optimal policy equilibrium features excess volatility of inflation, which takes the form of persistent oscillations. Contrarily to previous studies that ignore the no buy-back constraint, we conclude that short-term debt has an important role to play in the stabilization of inflation.

[37] Obviously, with lump sum taxation the optimal policy can completely insulate inflation from fiscal shocks by making the consolidated budget constraint slack.

[38] This may seem more involved here, because fiscal policy is set jointly with inflation, but we can solve for the tax schedule under Ramsey analytically as a function of spending and use this as a tax policy in the rule based model. Alternatively, proving that the optimal rules deliver effectively the same outcome as Ramsey can be done by adding the interest rate rule as a constraint to the Ramsey program and comparing the policy objective with the unconstrained Ramsey solution. Such exercises are not uncommon in the literature (see Leeper and Leith (2016) among others).

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Appendices

Appendix A Proofs of propositions

A.1 Proof of proposition 1

Optimal inflation is given by:

$$\hat{\pi}_t = \overline{R}\frac{(1+\gamma_h)}{\kappa_1}\Delta\psi_{gov,t} + \overline{b}_N\left(\beta^{N-1}\Delta\psi_{gov,t} + \beta^{N-2}\Delta\psi_{gov,t-1} + ... + \Delta\psi_{gov,t-N+1}\right) \tag{51}$$

We can write the intertemporal consolidated budget constraint ((12) in text) as

$$\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\hat{\pi}_t - \overline{G}\hat{G}_t = \overline{b}_N\hat{b}_{t-1,N} - \beta^{N-1}\overline{b}_N E_t\left(\hat{\pi}_t + \hat{\pi}_{t+1} + ... + \hat{\pi}_{t+N-1}\right)$$

Letting $\omega \equiv \frac{\overline{R}}{\kappa_1}(1+\gamma_h)$, replacing (51) into the constraint and using the random walk property of the multiplier we get:

$$\omega\left[\omega\Delta\psi_{gov,t} + \overline{b}_N\left(\beta^{N-1}\Delta\psi_{gov,t} + \beta^{N-2}\Delta\psi_{gov,t-1} + ... + \Delta\psi_{gov,t-N+1}\right)\right] - \overline{G}\hat{G}_t =$$
$$\overline{b}_N\hat{b}_{t-1,N} - \beta^{N-1}\overline{b}_N\Bigg[\underbrace{\omega\Delta\psi_{gov,t} + \overline{b}_N\left(\beta^{N-1}\Delta\psi_{gov,t} + \beta^{N-2}\Delta\psi_{gov,t-1} + ... + \Delta\psi_{gov,t-N+1}\right)}_{\hat{\pi}_t}$$
$$+\underbrace{\overline{b}_N\left(\beta^{N-2}\Delta\psi_{gov,t} + ... + \Delta\psi_{gov,t-N+2}\right)}_{E_t\hat{\pi}_{t+1}} + ... + \underbrace{\overline{b}_N\Delta\psi_{gov,t}}_{E_t\hat{\pi}_{t+N-1}}\Bigg]$$

Noting that in the absence of any shock in t the lagged terms on the LHS and RHS of the above equation must cancel out for the intertemporal constraint to hold, we have:

$$\omega\left[\omega\Delta\psi_{gov,t} + \overline{b}_N\beta^{N-1}\Delta\psi_{gov,t}\right] - \overline{G}\hat{G}_t = -\beta^{N-1}\overline{b}_N\left[\omega\Delta\psi_{gov,t} + \overline{b}_N\Delta\psi_{gov,t}\left(1+\beta+...+\beta^{N-1}\right)\right]$$

Thus:

$$\left[\left(\omega + \overline{b}_N\beta^{N-1}\right)^2 + (\beta^{N-1}\overline{b}_N)^2\left(\frac{\frac{1}{\beta^N}-1}{\frac{1}{\beta}-1}\right)\right]\Delta\psi_{gov,t} = \overline{G}\hat{G}_t$$

Using this result in (51) delivers the expression in Proposition 1. ■

A.2 Proof of proposition 2

The proof is analogous to that of Proposition 1. We can write the intertemporal constraint (12) as

$$\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\hat{\pi}_t - \overline{G}\hat{G}_t = \overline{d}\hat{d}_{t-1} - \sum_{k=1}^{\infty}\beta^{k-1}\overline{b}_k\hat{\pi}_t - \sum_{k=2}^{\infty}\beta^{k-1}\overline{b}_k\sum_{l=1}^{k-1}E_t\hat{\pi}_{t+l} \tag{52}$$

Using the random walk property of the multiplier we have

$$E_t\hat{\pi}_{t+l} = \sum_{k=l}^{\infty}\overline{b}_k\sum_{i=l+1}^{k}\beta^{k-i}\Delta\psi_{gov,t+l-i+1} \tag{53}$$

Noting that all lagged terms $t-1, t-2, ...$ will cancel out in the intertemporal constraint, we get the following expression that pins down $\Delta\psi_{gov,t}$.

$$\Delta\psi_{gov,t}\left(\frac{\overline{R}}{\kappa_1}(1+\gamma_h) + \sum_{k=1}^{\infty}\beta^{k-1}\overline{b}_k\right)^2 + \Delta\psi_{gov,t}\sum_{k=2}^{\infty}\beta^{k-1}\overline{b}_k\sum_{l=1}^{k-1}\sum_{i=l+1}^{\infty}\overline{b}_i\beta^{i-(l+1)} = \overline{G}\hat{G}_t \tag{54}$$

Moreover we have:

$$\begin{aligned}\sum_{k=2}^{\infty}\beta^{k-1}\overline{b}_k\sum_{l=1}^{k-1}\sum_{i=l+1}^{\infty}\overline{b}_i\beta^{i-(l+1)} &= \sum_{k=2}^{\infty}\beta^{k-1}\overline{b}_k\sum_{l=2}^{k}\sum_{i=l}^{\infty}\overline{b}_i\beta^{i-l} = \sum_{k=2}^{\infty}\beta^{k-1}\overline{b}_k\sum_{l=2}^{\infty}\mathcal{I}_{l\leq k}\sum_{i=l}^{\infty}\overline{b}_i\beta^{i-l} \\ &= \sum_{l=2}^{\infty}\beta^{l-1}\underbrace{\left(\sum_{k=l}^{\infty}\beta^{k-l}\overline{b}_k\right)}_{\lambda_l}\underbrace{\left(\sum_{i=l}^{\infty}\overline{b}_i\beta^{i-l}\right)}_{\lambda_l} = \sum_{l=2}^{\infty}\beta^{l-1}\lambda_l^2\end{aligned}$$

To see the last equality, that $\sum_{k=l}^{\infty}\beta^{k-l}\overline{b}_k \equiv \lambda_l$ use the definition of λ in text. We stated that:

$$\begin{aligned}\lambda_1 &= \sum_{j=1}\beta^{j-1}\overline{b}_j = \frac{\overline{S}}{1-\beta} \\ \lambda_2 &= \frac{1}{\beta}\left(\lambda_1 - \overline{b}_1\right) = \frac{1}{\beta}\left(\sum_{j\geq 1}\beta^{j-1}\overline{b}_j - \overline{b}_1\right) = \sum_{j\geq 2}\beta^{j-2}\overline{b}_j \\ \lambda_3 &= \frac{1}{\beta}\left(\lambda_2 - \overline{b}_2\right) = \frac{1}{\beta}\left(\sum_{j\geq 2}\beta^{j-2}\overline{b}_j - \overline{b}_2\right) = \sum_{j\geq 3}\beta^{j-3}\overline{b}_j\end{aligned}$$

and so on. With these formulae, (52) becomes

$$\Delta\psi_{gov,t}\left(\widetilde{f}^2 + \sum_{l=2}^{\infty}\beta^{l-1}\lambda_l^2\right) = \overline{G}\hat{G}_t$$

The coefficients η_{-j} follow easily from the above. ■

A.3 Proof of Proposition 3

With the assumptions of Proposition 3 the first order condition can be written as:

$$\left[\frac{\beta\phi_\pi}{1-\beta\phi_\pi^2}\left(\sum_{k=1}^{\infty}\beta^{k-1}\delta^{k-1}\frac{1-\phi_\pi^k}{1-\phi_\pi}\right)-\sum_{k=1}^{\infty}\beta^{k-1}\delta^{k-1}\frac{1}{(1-\phi_\pi)^2}\left(1+(k-1)\phi_\pi^k-k\phi_\pi^{k-1}\right)\right]=0 \quad (55)$$

Expanding the sums and using the geometric formula we get

$$\frac{\beta\phi_\pi}{1-\beta\phi_\pi^2}\frac{1}{1-\phi_\pi}\left(\frac{1}{1-\beta\delta}-\frac{\phi_\pi}{1-\phi_\pi\beta\delta}\right)=\frac{1}{(1-\phi_\pi)^2}\left[\frac{1}{1-\beta\delta}+\frac{\phi_\pi^2\beta\delta-1}{(1-\beta\delta\phi_\pi)^2}\right]$$

This reduces to

$$\frac{\phi_\pi}{1-\beta\phi_\pi^2}=\frac{\delta}{(1-\beta\delta\phi_\pi)}$$

It is obvious that $\phi_\pi=\delta$ is the solution. ■

A.4 Proof of proposition 4

Assume that the economy is hit by a shock in t and after there are no more shocks. To simplify, assume initial conditions $\psi_{gov,t-1}=\psi_{gov,t-2}=...=0$ and $\hat{b}_{t-1,2},\hat{b}_{t-2,2}=0$. Assume further that $\hat{\pi}_{t-1}=0$. Optimal Ramsey inflation satisfies:

$$\hat{\pi}_{t+\bar{t}}=\overline{R}\frac{(1+\gamma_h)}{\kappa_1}\Delta\psi_{gov,t+\bar{t}}+\bar{b}_2\left(\beta(\psi_{gov,t+1+\bar{t}}-\psi_{gov,t-1+\bar{t}})+(\psi_{gov,t+\bar{t}}-\psi_{gov,t-2+\bar{t}})\right) \quad (56)$$

From $\psi_{gov,t+\bar{t}}=\psi_{gov,t+\bar{t}+2}$ we define:

$$\overline{\psi}=\psi_{gov,t}=\psi_{gov,t+2}=\psi_{gov,t+4}=...$$
$$\underline{\psi}=\psi_{gov,t+1}=\psi_{gov,t+3}=\psi_{gov,t+5}=...$$

We then have the following path for inflation:

$$\hat{\pi}_t=\overline{R}\frac{(1+\gamma_h)}{\kappa_1}\overline{\psi}+\bar{b}_2\left(\overline{\psi}+\beta\underline{\psi}\right)$$
$$\hat{\pi}_{t+1}=\overline{R}\frac{(1+\gamma_h)}{\kappa_1}(\underline{\psi}-\overline{\psi})+\bar{b}_2\left(\underline{\psi}+\beta\overline{\psi})\right)$$
$$\hat{\pi}_{t+2}=\hat{\pi}_{t+4}=...=\overline{R}\frac{(1+\gamma_h)}{\kappa_1}(\overline{\psi}-\underline{\psi})+\bar{b}_2(1-\beta)\left(\overline{\psi}-\underline{\psi}\right)$$
$$\hat{\pi}_{t+3}=\hat{\pi}_{t+5}=...=-\overline{R}\frac{(1+\gamma_h)}{\kappa_1}(\overline{\psi}-\underline{\psi})-\bar{b}_2(1-\beta)\left(\overline{\psi}-\underline{\psi}\right)$$

To verify that indeed $\underline{\psi}\neq\overline{\psi}$ use the intertemporal budget constraints. The following two objects are sufficient for an equilibrium:

$$-\overline{b}_2\hat{\pi}_t = \sum_{j\geq 0}\beta^{2j}\overline{R}(1+\gamma_h)\hat{Y}_{t+2j} - \hat{G}_t \tag{57}$$

$$-\overline{b}_2(\hat{\pi}_t + \hat{\pi}_{t+1}) = \sum_{j\geq 0}\beta^{2j}\overline{R}(1+\gamma_h)\hat{Y}_{t+2j+1} \tag{58}$$

Using the Phillips curve we can write the first condition as:

$$-\overline{b}_2\hat{\pi}_t = -\overline{b}_2\left[\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\overline{\psi} + \overline{b}_2\left(\overline{\psi} + \beta\underline{\psi}\right)\right] = \frac{\overline{R}(1+\gamma_h)}{\kappa_1}\left[(\hat{\pi}_t - \beta\hat{\pi}_{t+1}) + \beta^2(\hat{\pi}_{t+2} - \beta\hat{\pi}_{t+3}) + ...\right] - \hat{G}_t$$

Letting $\omega = \frac{\overline{R}(1+\gamma_h)}{\kappa_1}$, we can simplify this equation:

$$-\overline{b}_2\left[\omega\overline{\psi} + \overline{b}_2\left(\overline{\psi} + \beta\underline{\psi}\right)\right] = \omega\ \left[(\omega + \overline{b}_2)\overline{\psi} + \overline{b}_2\beta\underline{\psi} - \beta(\omega + \overline{b}_2)\underline{\psi}\right] + \frac{\beta^2}{1-\beta}\omega^2(\overline{\psi} - \underline{\psi}) - \hat{G}_t$$

Rearranging we get

$$-\underbrace{\left[(\omega + \overline{b}_2)^2 + \frac{\beta^2}{1-\beta}\omega^2\right]}_{\epsilon_1}\overline{\psi} - \beta\underbrace{\left[\overline{b}_2^2 - \frac{\omega^2}{1-\beta}\right]}_{\epsilon_2}\underline{\psi} = -\hat{G}_t$$

Moreover, rather than using the second intertemporal constraint, we use the sum of the two constraints. This gives us:

$$-\overline{b}_2\hat{\pi}_t - \overline{b}_2\beta(\hat{\pi}_t + \hat{\pi}_{t+1}) = \omega\hat{\pi}_t - \hat{G}_t$$

or

$$-\underbrace{(\overline{b}_2(1+\beta) + \omega)\left[(\omega + \overline{b}_2)\right]}_{\epsilon_3}\overline{\psi} - \underbrace{\overline{b}_2\beta\left[2\omega + \overline{b}_2(2+\beta)\right]}_{\epsilon_3}\underline{\psi} = -\hat{G}_t$$

Solving the two equations gives:

$$\overline{\psi} = \frac{\epsilon_4 - \epsilon_2}{\epsilon_4\epsilon_1 - \epsilon_2\epsilon_3}, \qquad \underline{\psi} = \frac{\epsilon_1 - \epsilon_3}{\beta(\epsilon_4\epsilon_1 - \epsilon_2\epsilon_3)}$$

Generically $\overline{\psi} \neq \underline{\psi}$. ■

A.5 Proof of Proposition 5

For simplicity, we prove the Proposition for $N = 2$. The proof is analogous in the case $N > 2$.

Assume that monetary policy sets $\hat{i}_t = \phi_\pi\hat{\pi}_t$. We will show that for all ϕ_π the equilibrium is explosive.

The model equations are the following:

$$\begin{gathered}\hat{\pi}_t = \kappa_1 \hat{Y}_t + \beta E_t \hat{\pi}_{t+1} \\ \overline{b}_2 \beta^2 \Big(\hat{b}_{t,2} - E_t(\hat{\pi}_{t+1} + \hat{\pi}_{t+2}) \Big) + \overline{R} \Big(\gamma_h + 1 \Big) \hat{Y}_t - \overline{G} \hat{G}_t = \overline{b}_2 \Big(\hat{b}_{t-2,2} - \hat{\pi}_t - \hat{\pi}_{t-1} \Big) \\ \phi_\pi \hat{\pi}_t = E_t \hat{\pi_{t+1}}\end{gathered} \tag{59}$$

Substituting out the Phillips curve and using the last equation to substitute out expectations we have:

$$\begin{gathered}\overline{b}_2 \beta^2 \Big(\hat{b}_{t,2} - \phi_\pi (1+\phi_\pi) \hat{\pi}_t \Big) + \overline{R} \frac{\Big(\gamma_h + 1\Big)}{\kappa_1} \hat{\pi}_t (1 - \beta \phi_\pi) - \overline{G} \hat{G}_t = \overline{b}_2 \Big(\hat{b}_{t-2,2} - \hat{\pi}_t - \hat{\pi}_{t-1} \Big) \\ \phi_\pi \hat{\pi}_t = E_t \hat{\pi_{t+1}}\end{gathered}$$

In matrix form this system can be written as:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \beta^2 \overline{b}_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{A} \begin{bmatrix} E_t \hat{\pi}_{t+1} \\ \hat{\pi}_t \\ \hat{b}_{t,2} \\ \hat{b}_{t-1,2} \end{bmatrix} = \underbrace{\begin{bmatrix} \phi_\pi & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \beta^2 \overline{b}_2 \phi_\pi (1+\phi_\pi) - \overline{R} \frac{\gamma_h+1}{\kappa_1}(1-\beta\phi_\pi) & -\overline{b}_2 & 0 & \overline{b}_2 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{B} \begin{bmatrix} \hat{\pi}_t \\ \hat{\pi}_{t-1} \\ \hat{b}_{t-1,2} \\ \hat{b}_{t-2,2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \overline{G} \\ 0 \end{bmatrix} \hat{G}_t$$

Stability and determinacy of the equilibrium can be studied by computing the eigenvalues of $A^{-1}B$. We have

$$A^{-1}B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\beta^2 \overline{b}_2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \phi_\pi & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \beta^2 \overline{b}_2 \phi_\pi (1+\phi_\pi) - \overline{R} \frac{\gamma_h+1}{\kappa_1}(1-\beta\phi_\pi) & -\overline{b}_2 & 0 & \overline{b}_2 \\ 0 & 0 & 1 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} \phi_\pi & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \phi_\pi (1+\phi_\pi) - \overline{R} \frac{\gamma_h+1}{\kappa_1} \frac{(1-\beta\phi_\pi)}{\beta^2 \overline{b}_2} & -\frac{1}{\beta^2} & 0 & \frac{1}{\beta^2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Thus

$$\det \Big(A^{-1}B - \lambda I_{4\times4} \Big) = (\phi_\pi - \lambda)(-\lambda)(\lambda^2 - \frac{1}{\beta^2})$$

There are 4 eigenvalues: $\phi_\pi, 0, \pm\frac{1}{\beta}$. Thus when $0 \leq \phi_\pi \leq 1$, two eigenvalues are in absolute value greater than 1 and we have 1 forward looking variable. There is thus no stable equilibrium in this model.

Finally, note that it is obvious that system (59) which features 2 forward looking variables ($E_t\hat{\pi}_{t+1}$ and $E_t\hat{\pi}_{t+2}$) has 3 unstable roots. ■

A.6 Proof of Proposition 6

Consider a model where the government does not buyback debt and debt is structured so that cash flows are $1, \delta, \delta^2, ...$

We can write the budget constraint as:

$$\bar{b}\beta\sum_{k=1}^{\infty}\beta^{k-1}\delta^{k-1}\left(\hat{b}_t - \sum_{l=1}^{k}E_t\hat{\pi}_{t+l}\right) = -\overline{S}\hat{S}_t + \bar{b}\sum_{k=1}^{\infty}\delta^{k-1}\left(\hat{b}_{t-k} - \sum_{l=0}^{k-1}\hat{\pi}_{t-l}\right)$$

The first order conditions of the Ramsey program are now:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} + \frac{\bar{b}}{1-\delta}\sum_{k=0}^{\infty}(\beta\delta)^k E_t\psi_{gov,t+k} - \frac{\bar{b}}{1-\beta\delta}\sum_{k=1}^{\infty}\delta^{k-1}\psi_{gov,t-k} = 0 \tag{60}$$

$$-\psi_{\pi,t}\kappa_1 + \overline{R}\left(1+\gamma_h\right)\psi_{gov,t} = 0 \tag{61}$$

$$\bar{b}\beta\sum_{k=1}^{\infty}(\beta\delta)^{k-1}\left(E_t\psi_{gov,t+k} - \psi_{gov,t}\right) = 0 \tag{62}$$

Consider the last of equation. This can be written as:

$$\psi_{gov,t} = (1-\beta\delta)E_t\frac{\psi_{gov,t+1}}{1-\beta\delta L^{-1}} \rightarrow \psi_{gov,t} = E_t\psi_{gov,t+1}$$

From this result we can combine (60) and (61) into:

$$-\hat{\pi}_t + \frac{\overline{R}}{\kappa_1}\left(1+\gamma_h\right)\Delta\psi_{gov,t} + \frac{\bar{b}}{1-\delta}\psi_{gov,t}\sum_{k=0}^{\infty}(\beta\delta)^k - \frac{\bar{b}}{1-\beta\delta}\sum_{k=1}^{\infty}\delta^{k-1}\psi_{gov,t-k} = 0$$

The final two terms can be written as:

$$\frac{\bar{b}}{1-\delta}\psi_{gov,t}\sum_{k=0}^{\infty}(\beta\delta)^k - \frac{\bar{b}}{1-\beta\delta}\sum_{k=1}^{\infty}\delta^{k-1}\psi_{gov,t-k} = \frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k(\psi_{gov,t} - \psi_{gov,t-k-1})$$
$$= \frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k(\Delta\psi_{gov,t} + \Delta\psi_{gov,t-1} + ... + \Delta\psi_{gov,t-k+1}) =$$

and therefore

$$-\hat{\pi}_t + \frac{\overline{R}}{\kappa_1}\left(1+\gamma_h\right)\Delta\psi_{gov,t} + \frac{1}{1-\beta\delta}\frac{\bar{b}}{1-\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-k} = 0 \tag{63}$$

Note that essentially the above condition is the same as in the buyback model. To see this note first that the quantity $\bar{b}$ is not the same under buyback and no buyback. In particular from the steady state budget constraints we have we have:

$$\frac{\beta\overline{b}^{BB}}{1-\beta\delta} + \overline{S} = \frac{\overline{b}^{BB}}{1-\beta\delta} \rightarrow \frac{\overline{b}^{BB}}{1-\beta\delta} = \frac{\overline{S}}{1-\beta}$$

and

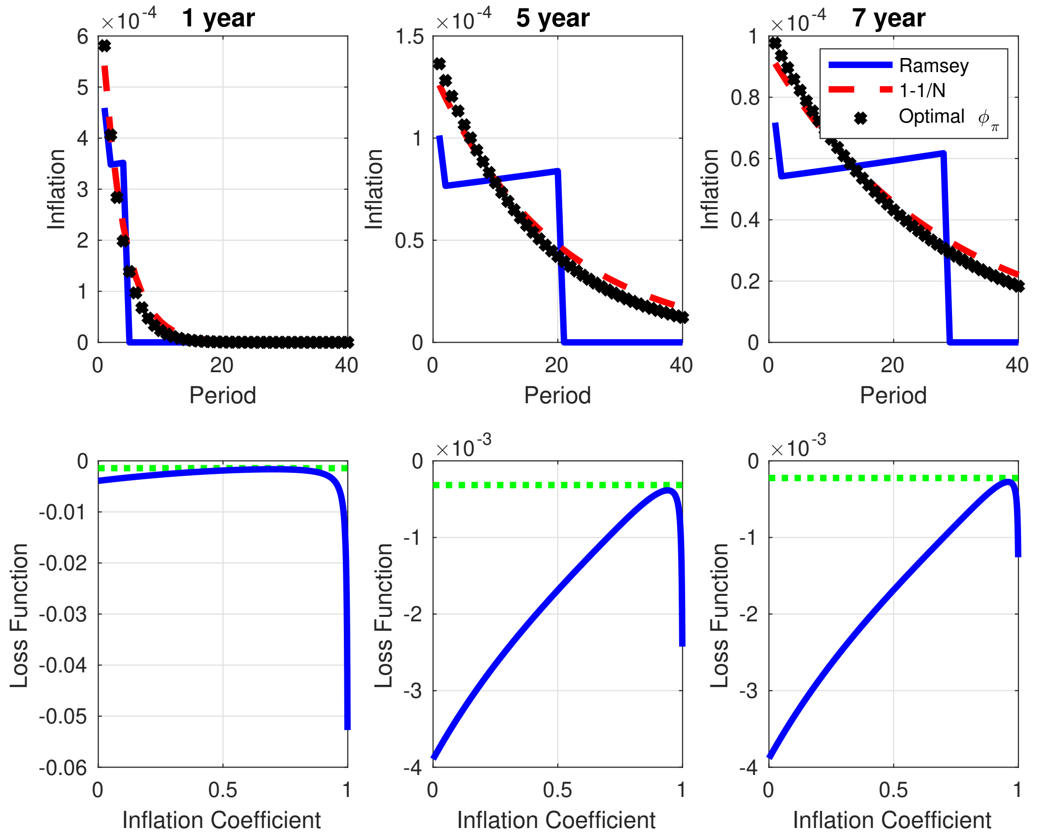
$$\frac{\beta\overline{b}^{NBB}}{1-\beta\delta} + \overline{S} = \frac{\overline{b}^{NBB}}{1-\delta} \rightarrow \frac{\overline{b}^{NBB}}{(1-\beta\delta)(1-\delta)} = \frac{\overline{S}}{1-\beta}$$

where superscripts BB, NBB denote the buyback and nobuyback cases respectively. Therefore $\frac{\overline{b}^{NBB}}{(1-\delta)} = \overline{b}^{BB}$ and so (63) can be written as

$$-\hat{\pi}_t + \frac{\overline{R}}{\kappa_1}\left(1+\gamma_h\right)\Delta\psi_{gov,t} + \frac{\overline{b}^{BB}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-k} = 0 \tag{64}$$

which proves the equivalence with the buyback model. It is now easy to show the equivalence of the optimal interest rate policy with the buyback model. ■

Figure 6: **Outcomes under Ramsey and Inflation Targeting Rules – distortionary taxes.**



Notes: The top panels plots the path of inflation in response to a shock under Ramsey and inflation targeting rules, when taxes are distortionary. Debt is a zero coupon bond of maturity N. The solid blue line is the Ramsey inflation response. The dashed red line sets $\phi_\pi = 1 - \frac{1}{N}$. In the crossed black line coefficient ϕ_π is the one minimizing the planner's loss function.
The bottom panels show the loss function under Ramsey (dashed-dotted green line) and under rule based policy for a range of values of ϕ_π (solid blue line).
We assumed $N = 4, 20, 28$ in the left, middle and right graphs, respectively.

Appendix B Nonlinear model and Additional Results

B.1 Distortionary taxes

We complete the analysis of subsection 3.1.2 by showing results from the model with distortionary taxes. Figure 6 is analogous to Figure 2 shown in text but assumes tax taxes are distortionary. The top panels show the responses of inflation under Ramsey and rule based policies. The blue line is the Ramsey policy whereas the dashed red line sets the inflation coefficient to $1-\frac{1}{N}$ and the crossed black line corresponds to the optimal inflation coefficient. Note that the red and black lines essentially overlap proving that $1 - \frac{1}{N}$ is a good approximation of the optimal rule based policy.

The bottom panels in the Figure show the loss function under Ramsey (green horizontal line) and rule based policy (blue line) as a function of the inflation coefficient. The gain from switching to Ramsey policy from an optimized inflation targeting rule is negligible.

B.2 A model with real interest rate fluctuations

We now consider a model where that real rate fluctuates according to an exogenous stochastic process. We show that the optimal Ramsey policy under decaying coupons admits an interest rate rule with an inflation coefficient equal to δ.

Let $\hat{r}_t$ denote the real interest rate. We assume that fluctuations in $\hat{r}_t$ occur due to demand shocks . Let $\hat{\xi}_t$ denote the demand shock. Standard modelling of the Euler/Fisher equation gives:

$$\hat{i}_t = \underbrace{\hat{r}_t}_{\hat{\xi}_t - E_t\hat{\xi}_{t+1}} + E_t\hat{\pi}_{t+1}$$

The consolidated budget constraint (6 in text) is

$$\sum_{k=1}^{\infty} \beta^k \overline{b}_k \left(\hat{b}_{t,k} + E_t(\hat{\xi}_{t+k} - \hat{\xi}_t) - \sum_{l=1}^{k} E_t\hat{\pi}_{t+l} \right) = -\overline{S}\hat{S}_t + \overline{b}_1(\hat{b}_{t-1,1} - \hat{\pi}_t) + \sum_{k=2}^{\infty} \beta^{k-1}\overline{b}_k \left(\hat{b}_{t-1,k} + E_t(\hat{\xi}_{t+k-1} - \hat{\xi}_t) - \sum_{l=0}^{k-1} \hat{\pi}_{t+l} \right)$$

It is simple to show that the Ramsey policy leads to the same FONC for inflation and output and debt as in the baseline model of Section 2 (equations (8) to (10)). The optimal inflation rate is given again by (11). When the debt structure is $\overline{b}_k = \delta^{k-1}\overline{b}$ we get

$$\hat{\pi}_t = \overline{R}\frac{(1+\gamma_h)}{\kappa_1}\Delta\psi_{gov,t} + \frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l}$$

Combining the Fisher equation and the random walk property of $\psi_{gov,t}$ we have:

$$\hat{i}_t - \hat{r}_t = E_t\hat{\pi}_{t+1} = \frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l E_t\Delta\psi_{gov,t-l+1} = \delta\frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} = \delta\left(\hat{\pi}_t - \overline{R}\frac{(1+\gamma_h)}{\kappa_1}\Delta\psi_{gov,t}\right)$$

In the case where $\overline{R}\frac{(1+\gamma_h)}{\kappa_1} \approx 0$ the nominal rate is set according to

$$\hat{i}_t = \hat{r}_t + \delta\hat{\pi}_t$$

as was claimed in text.

B.3 Multiple Maturities Optimal Inflation coefficients: Further Results

We now go back to the model of Section 3 and use equation (24) to derive further analytical results under alternative debt maturity structures. Consider first the case where $\overline{b}_k = \overline{b}e^{-\widetilde{\lambda}}\frac{\widetilde{\lambda}^{k-1}}{(k-1)!}$, $k = 1, 2, ...$ In other words, the debt payment profiles are assumed to follow a Poisson distribution and the average maturity is $\widetilde{\lambda} + 1$.

Assuming that taxes are lump sum and exploiting the fact that $\beta \approx 1$, (24) becomes:

$$\left[\frac{\phi_\pi}{1+\phi_\pi}\left(\sum_{k=1}^{\infty} e^{-\widetilde{\lambda}}\frac{\widetilde{\lambda}^{k-1}}{(k-1)!}(1-\phi_\pi^k)\right) - \sum_{k=1}^{\infty} e^{-\widetilde{\lambda}}\frac{\widetilde{\lambda}^{k-1}}{(k-1)!}\left(1+(k-1)\phi_\pi^k - k\phi_\pi^{k-1}\right)\right] = 0$$

The LHS is:

$$\frac{\phi_\pi}{1+\phi_\pi}\left(\sum_{k=1}^{\infty} e^{-\widetilde{\lambda}}\frac{\widetilde{\lambda}^{k-1}}{(k-1)!}(1-\phi_\pi^k)\right) = \frac{\phi_\pi}{1+\phi_\pi}\left(1-\sum_{k=1}^{\infty} e^{-\widetilde{\lambda}}\frac{\widetilde{\lambda}^{k-1}}{(k-1)!}\phi_\pi^k\right) = \frac{\phi_\pi}{1+\phi_\pi}\left(1-e^{-\widetilde{\lambda}}e^{\widetilde{\lambda}\phi_\pi}\phi_\pi\right)$$

where the last equality uses the fact that $\sum_{k=1}^{\infty} e^{-\widetilde{\lambda}\phi_\pi}\frac{(\widetilde{\lambda}\phi_\pi)^{k-1}}{(k-1)!} = 1$ as the sum of the pdf of a Poisson distribution with parameter $\widetilde{\lambda}\phi_\pi$.

The RHS can be written as:

$$\sum_{k=1}^{\infty} e^{-\widetilde{\lambda}}\frac{\widetilde{\lambda}^{k-1}}{(k-1)!}\left(1+(k-1)\phi_\pi^k - k\phi_\pi^{k-1}\right) = 1 + \sum_{k=1}^{\infty} e^{-\widetilde{\lambda}}\frac{\widetilde{\lambda}^{k-1}}{(k-1)!}(k-1)\phi_\pi^k$$
$$-\sum_{k=1}^{\infty} e^{-\widetilde{\lambda}}\frac{\widetilde{\lambda}^{k-1}}{(k-1)!}(k-1)\phi_\pi^{k-1} - \sum_{k=1}^{\infty} e^{-\widetilde{\lambda}}\frac{\widetilde{\lambda}^{k-1}}{(k-1)!}\phi_\pi^{k-1} = 1 + \widetilde{\lambda}\phi_\pi^2 e^{-\widetilde{\lambda}}e^{\widetilde{\lambda}\phi_\pi} - \widetilde{\lambda}\phi_\pi - e^{-\widetilde{\lambda}}e^{\widetilde{\lambda}\phi_\pi}$$

The optimal inflation coefficient thus solves:

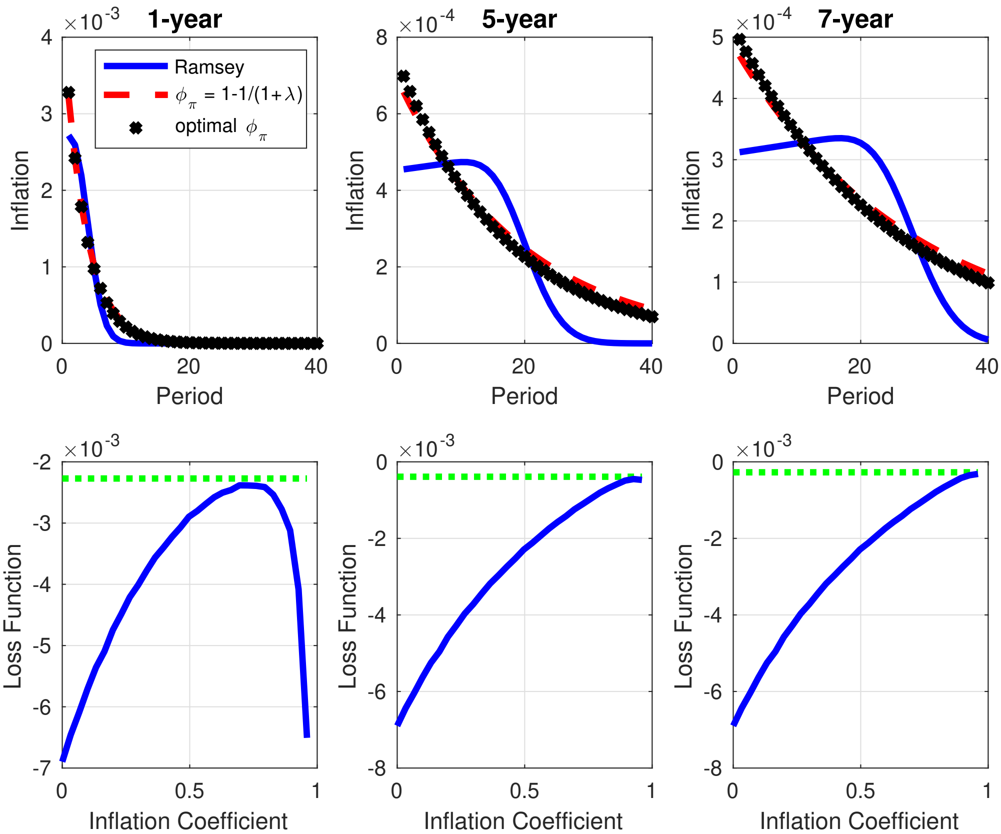
$$\frac{\phi_\pi}{1+\phi_\pi}\left(1-e^{-\widetilde{\lambda}}e^{\widetilde{\lambda}\phi_\pi}\phi_\pi\right) = 1 + \widetilde{\lambda}\phi_\pi^2 e^{-\widetilde{\lambda}}e^{\widetilde{\lambda}\phi_\pi} - \widetilde{\lambda}\phi_\pi e^{-\widetilde{\lambda}}e^{\widetilde{\lambda}\phi_\pi} - e^{-\widetilde{\lambda}}e^{\widetilde{\lambda}\phi_\pi} \tag{65}$$

It is easy to verify that the solution in the case where debt is short term, $\widetilde{\lambda} = 0$ is $\phi_\pi = 1$.

Figure 7 shows the responses of inflation under Ramsey and rule based policies (top panels). The solid blue lines are the Ramsey policy. The dashed lines is a rule setting $\phi_\pi = 1 - \frac{1}{1+\widetilde{\lambda}}$ and the crossed black line is the optimal rule. Clearly the red and black IRFS almost completely overlap suggesting that the optimal inflation coefficient is approximately $1 - \frac{1}{1+\widetilde{\lambda}}$. Moreover, as noted in the main text, it is easy to notice that the Ramsey policy in the Poisson model compiles the forces of both the zero coupon and the decaying coupon models. The Ramsey planner finds optimal to initially set inflation as in the zero coupon model (a response that is roughly flat over time) and when the bulk of debt is close to redemption then inflation will start to monotonically decay towards 0.

The bottom panels show the loss function outcomes. As in both the zero coupon and the decaying coupon models the gains of switching from an optimized rule policy to the full Ramsey policy are minuscule.

Figure 7: **Outcomes under Ramsey and Inflation Targeting Rules – Poisson maturity structure.**



Notes: The top panels plots the path of inflation in response to a shock under Ramsey and inflation targeting rules, when taxes are distortionary. The debt maturity profile is assumed to follow a Poisson distribution. In the left panels, we set the average maturity to one-year ($\widetilde{\lambda} = 3$), the middle panels assume an average maturity of 5 years ($\widetilde{\lambda} = 19$), and the right panel 7 years ($\widetilde{\lambda} = 27$). The solid blue line is the Ramsey inflation response. The dashed red line sets $\phi_\pi = 1 - \frac{1}{1+\widetilde{\lambda}}$. In the crossed black line coefficient ϕ_π is the one minimizing the planner's loss function.
The bottom panels show the loss function under Ramsey (dashed-dotted green line) and under rule based policy for a range of values of ϕ_π (solid blue line).

B.4 Closing the output gap objective

B.4.1 Decaying coupon bonds

Assume now that the objective of the planner is

$$-\frac{1}{2}E\sum_{t\geq 0}\beta^t\left(\hat{\pi}_t^2+\lambda_Y\hat{Y}_t^2\right)$$

Also assume first that debt pays decaying coupons. We can easily show that the FONC from the Ramsey program will yield the following condition for inflation.

$$-\hat{\pi}_t-\frac{\lambda_Y}{\kappa_1}\Delta\hat{Y}_t+\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t}+\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-l}=0$$

Using the Phillips curve, we obtain the following

$$-\hat{\pi}_t-\frac{\lambda_Y}{\kappa_1^2}(\hat{\pi}_t-\beta E_t\hat{\pi}_{t+1})+\frac{\lambda_Y}{\kappa_1^2}(\hat{\pi}_{t-1}-\beta E_{t-1}\hat{\pi}_t)+\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t}+\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-l}=0$$

Define:

$$\chi_t=(\hat{\pi}_t-E_{t-1}\hat{\pi}_t)+\frac{\kappa_1\overline{R}}{\beta\lambda_Y}(1+\gamma_h)\Delta\psi_{gov,t}+\frac{\kappa_1^2}{\beta\lambda_Y}\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-l}=0$$

where the term $\hat{\pi}_t-E_{t-1}\hat{\pi}_t$ will be a linear function of $\hat{G}_t$. Then

$$E_t\hat{\pi}_{t+1}-(1+\frac{1}{\beta}+\frac{\kappa_1^2}{\lambda_Y\beta})\hat{\pi}_t+\frac{1}{\beta}\hat{\pi}_{t-1}=-\chi_t \tag{66}$$

We will now resolve the above difference equation. Letting $\widetilde{\kappa}=\frac{\kappa_1^2}{\lambda_Y\beta})$ the characteristic polynomial is $\nu^2-(1+\frac{1}{\beta}+\widetilde{\kappa})\nu+\frac{1}{\beta}$.

Skipping a few steps, the two roots are:

$$\nu_{1,2}=\frac{1}{2}\left((1+\frac{1}{\beta}+\widetilde{\kappa})\pm\sqrt{(1-\frac{1}{\beta}-\widetilde{\kappa})^2+4\frac{\widetilde{\kappa}}{\beta}}\right)$$

It is simple to show that one root is stable and one unstable. Let ν_1 denote the stable root. (66) can be written as:

$$\hat{\pi}_t=\frac{1}{\nu_2}E_t\hat{\pi}_{t+1}+\frac{1}{\nu_2}\frac{1}{1-\nu_1L}\chi_t=\frac{1}{\nu_2}\frac{1}{1-\nu_1L}\sum_{j\geq 0}\frac{1}{\nu_2^j}E_t\chi_{t+j} \tag{67}$$

(for the usual boundary condition that inflation does not explode).

Let us compute the term

$$\sum_{j\geq 0}\frac{1}{\nu_2^j}E_t\chi_{t+j}=\sum_{j\geq 0}\frac{1}{\nu_2^j}E_t\left[\epsilon\hat{G}_{t+j}+\widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t+j}+\widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t+j-k}\right]$$

The final term on the RHS is

$$\widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\sum_{j\geq 0}\frac{1}{\nu_2^j}E_t\left[\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t+j-k}\right]=\widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\nu_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t}$$

(this follows from the random walk property of the multiplier). Also it is simple to show that

$$\sum_{j\geq 0}\frac{1}{\nu_2^j}E_t\left[\epsilon\hat{G}_{t+j}+\widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t+j}\right]=\epsilon\hat{G}_t+\widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t}$$

Putting everything together we get

$$\hat{\pi}_t=\nu_1\hat{\pi}_{t-1}+\frac{1}{\nu_2}\left(\epsilon\hat{G}_t+\widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t}+\widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\nu_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t}\right)$$

To derive the interest rate rule we can compute

$$E_t\hat{\pi}_{t+1}=\nu_1\hat{\pi}_t+\frac{1}{\nu_2}\left(\epsilon\underbrace{E_t\hat{G}_{t+1}}_{=0}+\widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\underbrace{E_t\Delta\psi_{gov,t+1}}_{=0}+\widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\nu_2}}\underbrace{E_t\frac{1}{1-\delta L}\Delta\psi_{gov,t}}_{=\frac{\delta}{1-\delta L}\Delta\psi_{gov,t}}\right)$$

which gives us:

$$\hat{i}_t=E_t\hat{\pi}_{t+1}=\nu_1\hat{\pi}_t+\delta\left(\hat{\pi}_t-\nu_1\hat{\pi}_{t-1}-\frac{\epsilon}{\nu_2}\hat{G}_t-\widetilde{\kappa}\frac{\overline{R}}{\kappa_1}\frac{(1+\gamma_h)}{\nu_2}\Delta\psi_{gov,t}\right)$$

The term labeled Stochastic Intercept in Proposition 7 corresponds to $-\epsilon\hat{G}_t$

B.4.2 Zero coupon bonds

For the case of zero coupon N bonds our approximate rule is (47) and we now solve for the optimal coefficient ϕ_π. Combining the rule and the Euler equation and assuming further to simplify that only one shock can hit the economy in t we can write:

$$\hat{\pi}_{t+j}-\nu_1\hat{\pi}_{t+j-1}\equiv\widetilde{\hat{\pi}}_{t+j}=\phi_\pi\widetilde{\hat{\pi}}_{t+j-1}=...=\phi_\pi^j\widetilde{\hat{\pi}}_t\rightarrow\hat{\pi}_{t+j}=\sum_{k=0}^{j}\nu_1^k\phi_\pi^{j-k}\widetilde{\hat{\pi}}_t=\frac{\phi_\pi^{j+1}-\nu_1^{j+1}}{\phi_\pi-\nu_1}\widetilde{\hat{\pi}}_t$$

Using this formula we can characterize the impulse response of inflation to a one off shock though solving the intertemporal constraint:

$$\overline{G}\hat{G}_t = \overline{b}_N \beta^{N-1} \sum_{j=0}^{N-1} \hat{\pi}_{t+j} = \overline{b}_N \beta^{N-1} \sum_{j=0}^{N-1} \frac{\phi_\pi^{j+1} - \nu_1^{j+1}}{\phi_\pi - \nu_1} \widetilde{\hat{\pi}}_t$$

where we assumed lump sum taxes. Noting that to characterize the impulse response we can also set $\widetilde{\hat{\pi}}_t = \hat{\pi}_t$ (i.e. lagged inflation can be set to 0) we get:

$$\hat{\pi}_t = \frac{\overline{G}\hat{G}_t}{\overline{b}_N \beta^{N-1} \sum_{j=0}^{N-1} \frac{\phi_\pi^{j+1} - \nu_1^{j+1}}{\phi_\pi - \nu_1}}$$

Taking this result into account we get

Consider first the single bond model. Equilibrium inflation is given by

$$\hat{\pi}_t = (\nu_1 + \phi_\pi)\hat{\pi}_{t-1} - \nu_1 \phi_\pi \hat{\pi}_{t-2} + \frac{\overline{G}\hat{G}_t}{\overline{b}_N \beta^{N-1} \sum_{j=0}^{N-1} \frac{\phi_\pi^{j+1} - \nu_1^{j+1}}{\phi_\pi - \nu_1}}$$

This can be easily be resolved to yield

$$\hat{\pi}_t = \sum_{k=0}^{t} \frac{\phi_\pi^{k+1} - \nu_1^{k+1}}{\beta^{N-1}\overline{b}_N \sum_{j=0}^{N-1}(\phi_\pi^{j+1} - \nu_1^{j+1})} \overline{G}\hat{G}_{t-k}$$

the expression shown in text.

Given this result aggregate output can be found using the Phillips curve as:

$$\hat{Y}_t = \frac{1}{\kappa_1}\left[\sum_{k=0}^{t} \frac{\phi_\pi^{k+1} - \nu_1^{k+1}}{\beta^{N-1}\overline{b}_N \sum_{j=0}^{N-1}(\phi_\pi^{j+1} - \nu_1^{j+1})} - \beta \sum_{k=0}^{t} \frac{\phi_\pi^{k+2} - \nu_1^{k+2}}{\beta^{N-1}\overline{b}_N \sum_{j=0}^{N-1}(\phi_\pi^{j+1} - \nu_1^{j+1})}\right] \overline{G}\hat{G}_{t-k} =$$
$$\frac{1}{\kappa_1}\left[\sum_{k=0}^{t} \frac{\phi_\pi^{k+1}(1-\beta\phi_\pi) - \nu_1^{k+1}(1-\beta\nu_1)}{\beta^{N-1}\overline{b}_N \sum_{j=0}^{N-1}(\phi_\pi^{j+1} - \nu_1^{j+1})}\right] \overline{G}\hat{G}_{t-k}$$

We can now write the policy objective as:

$$-\frac{1}{2}\overline{G}^2 \sigma_G^2 \left\{ \sum_{t\geq 0} \beta^t \left[\sum_{k=0}^{t} \frac{\overbrace{(\phi_\pi^{k+1} - \nu_1^{k+1})^2 + \frac{\lambda_Y}{\kappa_1^2}(\phi_\pi^{k+1}(1-\beta\phi_\pi) - \nu_1^{k+1}(1-\beta\nu_1))^2}^{\Omega_{k+1}}}{\left(\beta^{N-1}\overline{b}_N \sum_{j=0}^{N-1}(\phi_\pi^{j+1} - \nu_1^{j+1})\right)^2} \right] \right\} =$$
$$= -\frac{1}{2} \frac{\overline{G}^2 \sigma_G^2}{\left(\beta^{N-1}\overline{b}_N \sum_{j=0}^{N-1}(\phi_\pi^{j+1} - \nu_1^{j+1})\right)^2} \sum_{k\geq 0} \frac{\beta^k}{1-\beta} \Omega_{k+1}$$

Notice that it is possible to expand the squares in this formula and take first order conditions to find the optimum. The resulting expression will be cumbersome and for simplicity we show the numerical solution. This is done in Figure 8 where we plot the usual IRFS (top panel) and the loss function bottom panel for $N = 4, 20, 28$. It is worthwhile briefly discussing our findings. Consider the middle panel which plots a 5 year maturity. The optimal response of inflation can be divided in two regions. First from impact to around quarter 17 we see that inflation follows a standard process is nearly flat. This is essentially the same property as the one we had in the zero coupon model without output smoothing. Subsequently, inflation starts to gradually adjust towards to zero. (In contrast without output smoothing inflation suddenly would drop to zero in period 20). This is the output smoothing objective exerting an influence on the optimal policy. If inflation fell suddenly to zero when the debt matured, then output would spike one period before maturity.[39] To avoid the sudden increase of output, the planner tolerates a gradual decrease in inflation.

The dashed red line shows the outcome under the rule based policy. It is clear, that a simple rule cannot track the Ramsey path of inflation (this is not surprising a the 0 coupon bond) but it is clear that the principles behind Ramsey policy are also present here. The differences in terms of the planners objective between the Ramsey policy and the rule based policy are however small as is indicated by the bottom panels.

B.5 The canonical New Keynesian model

Most of our analytical results in the main text were derived in a Fisherian model assuming that the real interest rate is exogenous. We now turn to the canonical New Keynesian model, assuming that the real rate is a function of output and spending. In this paragraph we state the first order conditions of the Ramsey program under buyback and no-buyback. We also present the numerical results we referred to in text. Finally note that we will not derive here analytically the interest rate rules shown in text since these derivations can be found in Chafwehé et al. (2022).

Let $\sigma \geq 0$ denote the inverse of the intertemporal elasticity of substitution. The Fisherian model considered in text corresponds to the case where $\sigma = 0$.

The New Keynesian Phillips curve under $\sigma > 0$ is given by:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t - \kappa_2 \hat{G}_t + \beta E_t \hat{\pi}_{t+1}$$

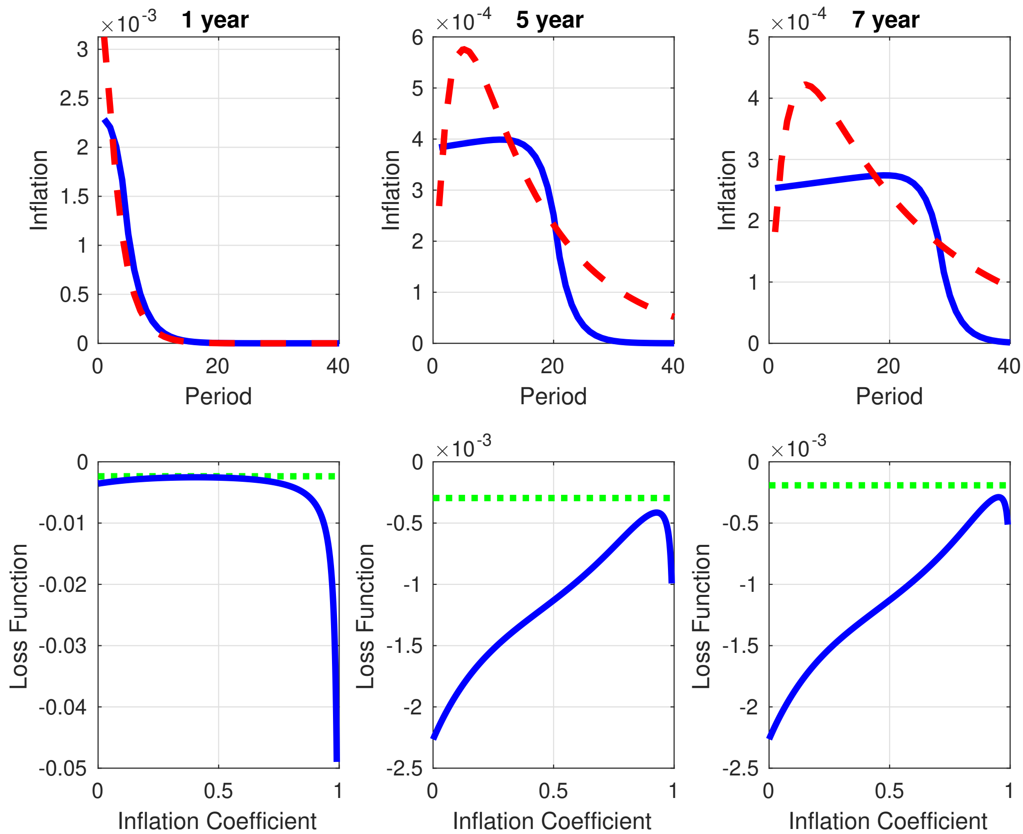
where $\kappa_1 = -(1+\eta)\frac{\overline{Y}}{\theta}(\gamma_h + \sigma\frac{\overline{Y}}{\overline{C}})$, $\kappa_2 = -(1+\eta)\frac{\overline{Y}}{\theta}\sigma\frac{\overline{G}}{\overline{C}}$ and we assumed that taxes are constant.

Zero Coupons. Consider the case of one zero coupon bond of maturity N. The government budget constraint under buyback now is:

$$\beta^N \overline{b}_N \left(\hat{b}_{t,N} - \sigma E_t(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+N} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+N}) - E_t \sum_{i=1}^{N} \hat{\pi}_{t+i} \right) + \overline{R}(1+\gamma_h)\hat{Y}_t - \overline{G}\hat{G}_t - \sigma(\overline{T} - \overline{G})(\frac{\overline{Y}}{\overline{C}}\hat{Y}_t - \frac{\overline{G}}{\overline{C}}\hat{G}_t)$$
$$= \beta^{N-1}\overline{b}_N \left(\hat{b}_{t-1,N} - \sigma E_t(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+N-1} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+N-1}) - E_t \sum_{i=0}^{N-1} \hat{\pi}_{t+i} \right)$$

[39]For the initial 17 quarters output would not change since the path of inflation is increasing at $\frac{1}{\beta}$.

Figure 8: **Outcomes under Ramsey and Inflation Targeting Rules – Output smoothing.**



Notes: The top panels plots the path of inflation in response to a shock under Ramsey and inflation targeting rules, when the planner wants to stabilize both inflation and the output gap ($\lambda_Y > 0$) . Debt is a zero coupon bond of maturity N. The solid blue line is the Ramsey inflation response. The dashed red line sets $\phi_\pi = 1 - \frac{1}{N}$. In the crossed black line coefficient ϕ_π is the one minimizing the planner's loss function. The bottom panels show the loss function under Ramsey (dashed-dotted green line) and under rule based policy for a range of values of ϕ_π (solid blue line). We assumed $N = 4, 20, 28$ in the left, middle and right graphs, respectively.

where again $\overline{R}$ denotes the revenue from distortionary taxes and $\overline{T}$ is the lump sum tax in steady state.[40]

Under no buyback the constraint becomes:

$$\beta^N \overline{b}_N \left(\hat{b}_{t,N} - \sigma E_t(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+N} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+N}) - E_t \sum_{i=1}^{N} \hat{\pi}_{t+i} \right) + \overline{R}(1+\gamma_h)\hat{Y}_t - \overline{G}\hat{G}_t - \sigma(\overline{T} - \overline{G})(\frac{\overline{Y}}{\overline{C}}\hat{Y}_t - \frac{\overline{G}}{\overline{C}}\hat{G}_t)$$
$$= \overline{b}_N \left(\hat{b}_{t-N,N} - \sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_t - \frac{\overline{G}}{\overline{C}}\hat{G}_t) - \sum_{i=0}^{N-1} \hat{\pi}_{t-i} \right)$$

We can now derive the optimality conditions from the planner's program. In the buyback model we have the following conditions:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} + \overline{b}_N \sum_{l=1}^{N} \beta^{N-l} \Delta\psi_{gov,t-l+1} = 0$$
$$-\lambda_Y \hat{Y}_t - \psi_{\pi,t}\kappa_1 + \left[\overline{R}\left(1+\gamma_h\right) - \sigma(\overline{T} - \overline{G})\frac{\overline{Y}}{\overline{C}} \right]\psi_{gov,t} - \sigma\frac{\overline{Y}}{\overline{C}}\overline{b}_N(\psi_{gov,t-N} - \psi_{gov,t-N+1}) = 0$$
$$\psi_{gov,t} - E_t\psi_{gov,t+1} = 0$$

and in the case of no buyback we have:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} + \overline{b}_N E_t \sum_{l=1}^{N} \beta^{N-l} \left(\psi_{gov,t+N-l} - \psi_{gov,t-l} \right) = 0$$
$$-\lambda_Y \hat{Y}_t - \psi_{\pi,t}\kappa_1 + \left[\overline{R}\left(1+\gamma_h\right) - \sigma(\overline{T} - \overline{G})\frac{\overline{Y}}{\overline{C}} \right]\psi_{gov,t} - \sigma\frac{\overline{Y}}{\overline{C}}\overline{b}_N(\psi_{gov,t-N} - \psi_{gov,t}) = 0$$
$$\left(\psi_{gov,t} - E_t\psi_{gov,t+N} \right) = 0$$

Optimized interest rate rule.

We revisit the derivations of the model of Section 3. We consider an interest rate rule of the form:

$$\hat{i}_t = r_t^n + \phi_\pi \hat{\pi}_t \tag{68}$$

where r_t^n denotes the natural interest rate, consistent with flexible prices. (The expression is given in the main text).

Combining the Euler equation with the interest rate rule and using the Phillips curve to substitute out aggregate output we get:

$$r_t^n + \phi_\pi \hat{\pi}_t = \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}} E_t(\hat{\pi}_{t+1} - \beta\hat{\pi}_{t+2}) - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}} E_t(\hat{\pi}_t - \beta\hat{\pi}_{t+1}) + E_t\hat{\pi}_{t+1} + \underbrace{\sigma(\frac{\overline{G}}{\overline{C}} - \frac{\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}})\hat{G}_t}_{r_t^n} \tag{69}$$

[40] To derive the above equation we scaled the budget constraint by the marginal utility of consumption and then we divided the RHS and LHS by the steady state marginal utility.

Thus inflation solves the following homogeneous difference equation.

$$(\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}} + \phi_\pi)\hat{\pi}_t - (1 + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}}(1+\beta))E_t\hat{\pi}_{t+1} + \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}}E_t\hat{\pi}_{t+2} = 0 \tag{70}$$

or

$$(\frac{1}{\beta} + \frac{\kappa_1}{\sigma\beta}\frac{\overline{C}}{\overline{\overline{Y}}}\phi_\pi)\hat{\pi}_t - (\frac{\kappa_1}{\sigma\beta}\frac{\overline{C}}{\overline{\overline{Y}}} + 1 + \frac{1}{\beta})E_t\hat{\pi}_{t+1} + E_t\hat{\pi}_{t+2} = 0 \tag{71}$$

The two eigenvalues are:

$$\lambda_{1,2} = \frac{1}{2}\left(\frac{\kappa_1}{\sigma\beta}\frac{\overline{C}}{\overline{\overline{Y}}} + 1 + \frac{1}{\beta} \pm \sqrt{(\frac{\kappa_1}{\sigma\beta}\frac{\overline{C}}{\overline{\overline{Y}}} + 1 + \frac{1}{\beta})^2 - 4(\frac{1}{\beta} + \frac{\kappa_1}{\sigma\beta}\frac{\overline{C}}{\overline{\overline{Y}}}\phi_\pi)}\right) \tag{72}$$

A unique stable equilibrium requires that $\phi_\pi < 1$ so that one root is stable. Letting λ_1 be the stable root, equilibrium inflation satisfies

$$E_t\hat{\pi}_{t+1}(1-\lambda_1 L)(\frac{1}{\lambda_2}L^{-1} - 1) = 0$$

and we thus have:

$$E_t\hat{\pi}_{t+1} = \lambda_1\hat{\pi}_t$$

Consider now the intertemporal government budget constraint under buyback. Letting $\hat{b}_{t-1,N} = 0$ to characterize the IRF we have:

$$E_t\sum_{j\geq 0}\beta^j\left(\overline{R}(1+\gamma_h)\hat{Y}_{t+j} - \overline{G}\hat{G}_{t+j} - \sigma(\overline{T}-\overline{G})(\frac{\overline{Y}}{\overline{\overline{C}}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{\overline{C}}}\hat{G}_{t+j})\right) =$$

$$\left[\frac{\overline{R}}{\kappa_1}(1+\gamma_h) - \frac{\sigma}{\kappa_1}(\overline{T}-\overline{G})\frac{\overline{Y}}{\overline{\overline{C}}}\right]\hat{\pi}_t - \left(\overline{G} + \sigma(\overline{T}-\overline{G})(\frac{\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}} - \frac{\overline{G}}{\overline{\overline{C}}}) - \overline{R}(1+\gamma_h)\frac{\kappa_2}{\kappa_1}\right)\hat{G}_t$$

$$= -\beta^{N-1}\overline{b}_N\left(\sigma E_t(\frac{\overline{Y}}{\overline{\overline{C}}}\hat{Y}_{t+N-1} - \frac{\overline{G}}{\overline{\overline{C}}}\hat{G}_{t+N-1}) + E_t\sum_{i=0}^{N-1}\hat{\pi}_{t+i}\right) = -\beta^{N-1}\overline{b}_N\left(\sigma\frac{\overline{Y}}{\overline{\overline{C}}}\frac{1}{\kappa_1}(1-\beta\lambda_1)\lambda_1^{N-1} + \frac{1-\lambda_1^N}{1-\lambda_1}\right)\hat{\pi}_t$$

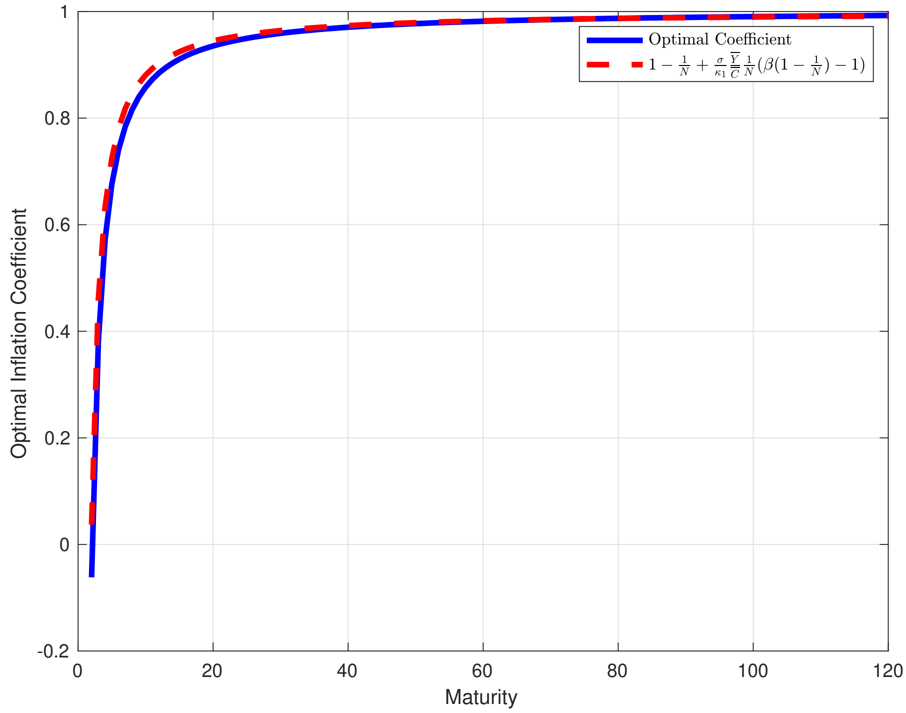
when $N > 1$.

Focusing for simplicity on the case of lump sum taxation we have:

$$\underbrace{\left[\beta^{N-1}\overline{b}_N\left(\sigma\frac{\overline{Y}}{\overline{\overline{C}}}\frac{1}{\kappa_1}(1-\beta\lambda_1)\lambda_1^{N-1} + \frac{1-\lambda_1^N}{1-\lambda_1}\right) - \frac{\sigma}{\kappa_1}(\overline{T}-\overline{G})\frac{\overline{Y}}{\overline{\overline{C}}}\right]}_{\omega(\lambda_1)}\hat{\pi}_t = \widetilde{\chi}\hat{G}_t$$

It is thus evident that inflation in t can be written as: $\hat{\pi}_t = \sum_{j=0}^{t}\lambda_1^j\frac{\widetilde{\chi}}{\omega(\lambda_1)}\hat{G}_{t-j}$. From this we can derive:

Figure 9: **Optimal Inflation Coefficients in the Canonical Model: One zero coupon bond.**



Notes: The figure plots the optimal inflation coefficients that solve (73) as a function of maturity N (solid/blue line) along with the coefficients $1-\frac{1}{N}+\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\frac{1}{N}(\beta(1-\frac{1}{N})-1)$ (dashed/red line).

$$-\frac{1}{2}E\sum_{t\geq 0}\beta^t(\sum_{j=0}^{t}\lambda_1^j\frac{\widetilde{\chi}}{\omega(\lambda_1)}\hat{G}_{t-j})^2 = -\frac{1}{2(1-\beta)}(\frac{\widetilde{\chi}}{\omega(\lambda_1)})^2\sigma_G^2\frac{1}{1-\beta\lambda_1^2}$$

Optimal policy sets the inflation coefficient to minimize the above objective function.

The first order conditions give:

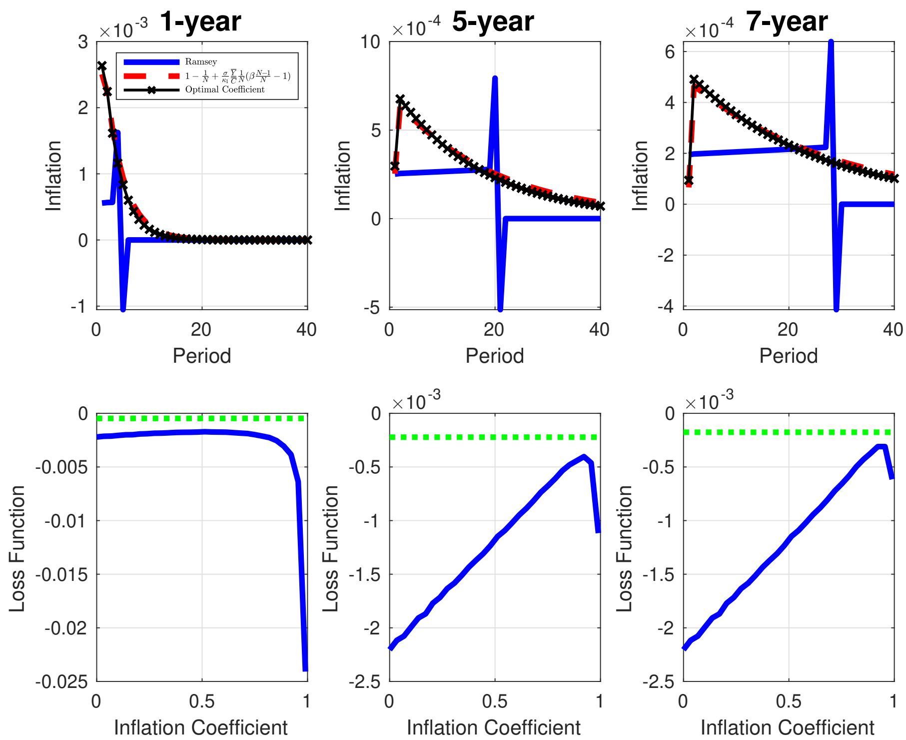
$$-\frac{\beta}{2}(\frac{\widetilde{\chi}}{\omega(\lambda_1)})^2\sigma_G^2\frac{1}{1-\beta\lambda_1^2}\left[\frac{2\beta\lambda_1}{1-\beta\lambda_1^2}-\frac{2}{\omega(\lambda_1)}\frac{d\omega(\lambda_1)}{d\lambda_1}\right]=0$$

$$\left[\frac{\beta\lambda_1}{1-\beta\lambda_1^2}-\frac{1}{\omega(\lambda_1)}\beta^{N-1}\overline{b}_N\left(\sigma\frac{\overline{Y}}{\overline{C}}\frac{1}{\kappa_1}((N-1)\lambda_1^{N-2}-\beta N\lambda_1^{N-1})+\frac{1+(N-1)\lambda_1^N-N\lambda_1^{N-1}}{(1-\lambda_1)^2}\right)\right]=0 \tag{73}$$

Note that in the case $\sigma=0$ the above gives us equation (20) in text. (We then have $\lambda_1=\phi_\pi$).

Figure 9 plots the optimal inflation coefficients we obtain from solving (73) as a function of maturity N. The dashed red line in the Figure plots the function '$1-\frac{1}{N}+\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\frac{1}{N}(\beta(1-\frac{1}{N})-1)'$. This is essentially the same inflation coefficient as in the decaying coupon bond model (i.e. when $1-\frac{1}{N}=\delta$.) As can be seen from the Figure this formula fits very well the solution to (73).

Figure 10: **Impulse responses and loss function outcomes in the canonical New-Keynesian model: Zero coupon bonds.**



Notes: The figure plots the paths of inflation under Ramsey (solid blue lines) under the optimal inflation targeting rule (crossed-black lines) and the inflation targeting rule with the coefficient $1-\frac{1}{N}+\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\frac{1}{N}(\beta(1-\frac{1}{N})-1)$ (dashed/red line). The bottom panel shows the difference between the Ramsey policy (horizontal line) and the rule based policy, in terms of the loss function.

Figure 10 shows the usual comparison of outcomes between rule based policy and Ramsey policy in the N bond model. As is evident from the Figure, the optimal responses of inflation between the two models do not overlap, however, the loss from switching to an optimized rule based policy is small (bottom panel). Moreover, the optimal Ramsey policy has the following noteworthy feature: Inflation is roughly constant until period $N-2$, then in the period $N-1$ it jumps and subsequently becomes negative for one period. These properties of inflation is matched with analogous responses of aggregate output. In particular since in this model output affects the solvency of debt, the planner will promise higher output in period $N-1$ to reduce the real payout of long term debt outstanding. This is a standard interest rate twisting channel of optimal policy (see e.g. Faraglia et al. (2013, 2016); Leeper et al. (2021)). Since output increases in $N-1$ so does inflation, according to the Phillips curve. In addition, in period N when neither inflation nor output matter (any more) for fiscal solvency, the planner will promise lower inflation. This is again due to the Phillips curve. Decreasing inflation in N enables a larger increase in output in $N-1$ so that output relative to inflation, bears a larger part of the fiscal adjustment. This is optimal because stabilizing output is not an objective of the planner.

When we assume a dual objective, we indeed find that the responses of inflation are smoother. Importantly, we once again obtain that a simple inflation targeting rule approximates the Ramsey outcome. For brevity we do not show these simulations here.[41]

Decaying coupons. The case where coupons decay at rate δ and the optimal policy rules are studied in Chafwehé et al. (2022). For brevity we refer the reader to that paper for the derivations of the formulae shown in text. The impulse responses obtained in this case are depicted in Figure 11.

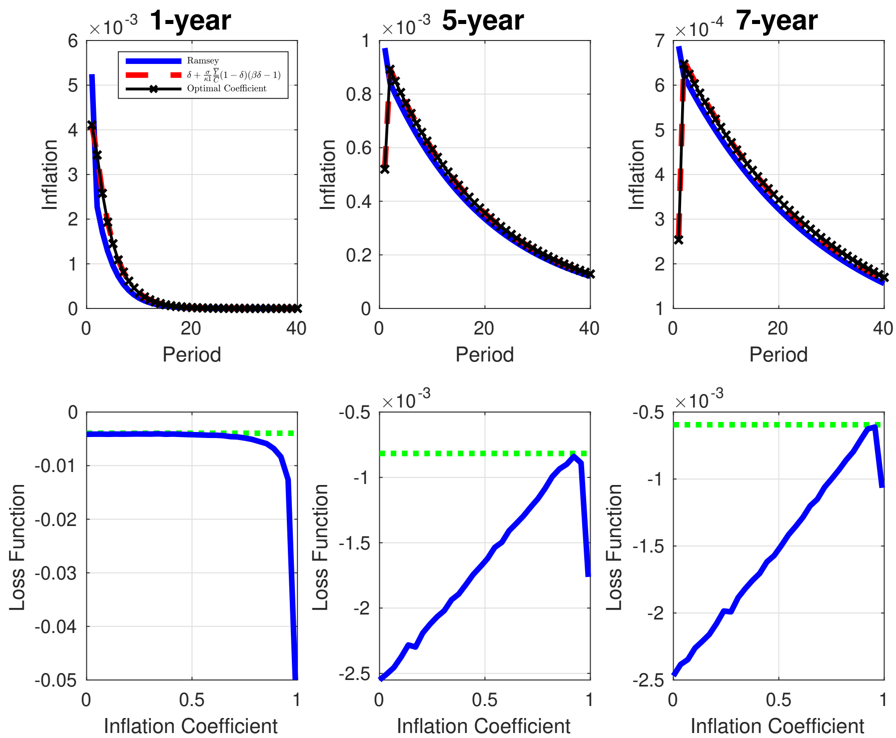
No buyback. We can confirm all of our results for the no buyback model, assuming $\sigma > 0$. First, Figure 12 shows the responses of inflation under Ramsey policy in the zero coupon bond model and assuming lump sum taxes. As is evident from the Figure in this optimal policy equilibrium, inflation displays a cycle of periodicity N. Second, it is simple to show (the proof is omitted for brevity) that simple inflation targeting rules produce an explosive solution. Third we can show that under no buyback, issuing short term debt brings us back to the buyback case.

We prove this explicitly. The budget constraint under no buyback and decaying coupons is:

$$\overline{b}\beta \sum_{k=1}^{\infty} \beta^{k-1}\delta^{k-1}\left(\hat{b}_t - \sigma(\frac{\overline{Y}}{\overline{\overline{C}}}\hat{Y}_{t+k} - \frac{\overline{G}}{\overline{\overline{C}}}\hat{G}_{t+k}) - \sum_{l=1}^{k} E_t\hat{\pi}_{t+l}\right) + \overline{R}(1+\gamma_h)\hat{Y}_t - \overline{G}\hat{G}_t - \sigma(\overline{T} - \overline{G})(\frac{\overline{Y}}{\overline{\overline{C}}}\hat{Y}_t - \frac{\overline{G}}{\overline{\overline{C}}}\hat{G}_t)$$
$$= \overline{b}\sum_{k=1}^{\infty}\delta^{k-1}\left(\hat{b}_{t-k} - \sigma(\frac{\overline{Y}}{\overline{\overline{C}}}\hat{Y}_t - \frac{\overline{G}}{\overline{\overline{C}}}\hat{G}_t) - \sum_{l=0}^{k-1}\hat{\pi}_{t-l}\right)$$

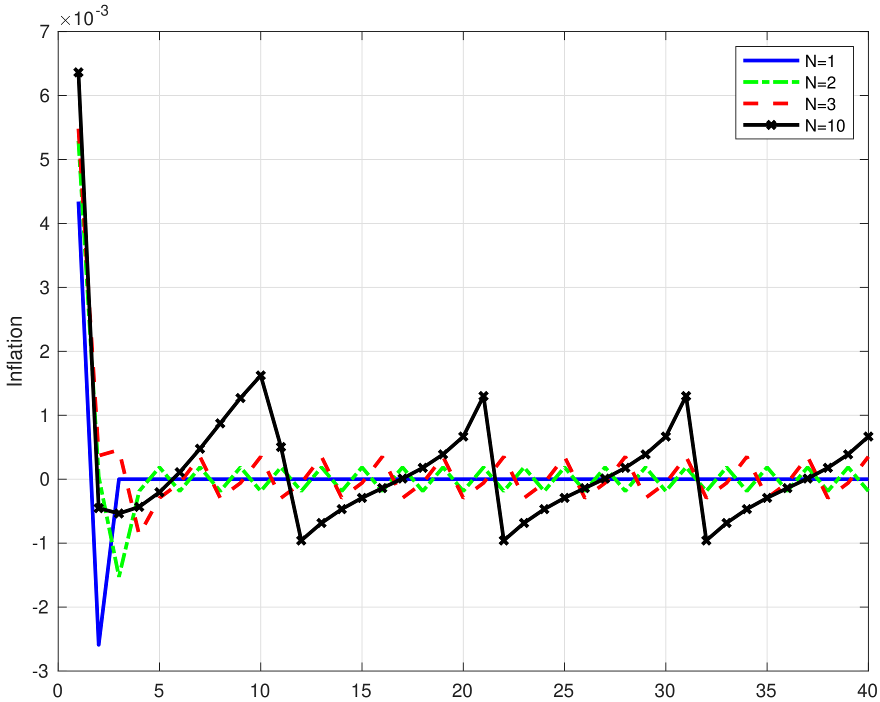
[41] See Chafwehé et al. (2022) for this scenario.

Figure 11: **Impulse responses and loss function outcomes in the canonical New-Keynesian model: Decaying coupon bonds.**



Notes: The figure plots the paths of inflation under Ramsey (solid blue lines) under the optimal inflation targeting rule (crossed-black lines) and the inflation targeting rule with the coefficient $\delta + \frac{\sigma}{\kappa 1}\frac{\overline{Y}}{\overline{C}}(1-\delta)(\beta\delta - 1)$ (dashed/red line). The bottom panel shows the difference between the Ramsey policy (horizontal line) and the rule based policy, in terms of the loss function.

Figure 12: **Responses to the spending shock under no buyback in the New-Keynesian model.**



Notes: The figure plots the path of optimal inflation in response to a shock that increases spending by 20% (from 10% of GDP to 12% of GDP) under various maturity structures and assuming no debt repurchases, in the canonical New-Keynesian model assuming $\sigma = 1$.

The first order conditions of the Ramsey program are now:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} + \frac{\bar{b}}{1-\delta}\sum_{k=0}^{\infty}(\beta\delta)^k E_t\psi_{gov,t+k} - \frac{\bar{b}}{1-\beta\delta}\sum_{k=1}^{\infty}\delta^{k-1}\psi_{gov,t-k} = 0$$

$$-\lambda_Y\hat{Y}_t - \psi_{\pi,t}\kappa_1 + \left[\overline{R}\Big(1+\gamma_h\Big) - \sigma(\overline{T}-\overline{G})\right]\psi_{gov,t} - \bar{b}\sigma\frac{\overline{Y}}{\overline{C}}(\sum_{k=1}^{\infty}\delta^{k-1}\psi_{gov,t-k} - \sum_{k=1}^{\infty}\delta^{k-1}\psi_{gov,t}) = 0$$

$$\bar{b}\beta\sum_{k=1}^{\infty}(\beta\delta)^{k-1}\Big(E_t\psi_{gov,t+k} - \psi_{gov,t}\Big) = 0$$

Once again the last equation can be written as:

$$\psi_{gov,t} = (1-\beta\delta)E_t\frac{\psi_{gov,t+1}}{1-\beta\delta L^{-1}} \rightarrow \psi_{gov,t} = E_t\psi_{gov,t+1}$$

Given that the random walk property holds in this model we can write:

$$\frac{\bar{b}}{1-\delta}\sum_{k=0}^{\infty}(\beta\delta)^k E_t\psi_{gov,t+k} - \frac{\bar{b}}{1-\beta\delta}\sum_{k=1}^{\infty}\delta^{k-1}\psi_{gov,t-k} = \frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}(\delta)^k\psi_{gov,t} - \frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\psi_{gov,t-k-1}$$

$$= \frac{\bar{b}}{(1-\beta\delta)(1-\delta)}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-k}$$

$$\bar{b}\sigma\frac{\overline{Y}}{\overline{C}}(\sum_{k=1}^{\infty}\delta^{k-1}\psi_{gov,t-k} - \sum_{k=1}^{\infty}\delta^{k-1}\psi_{gov,t}) = -\bar{b}\sigma\frac{\overline{Y}}{\overline{C}}\sum_{k=1}^{\infty}\delta^{k-1}(\psi_{gov,t} - \psi_{gov,t-k}) =$$

$$= -\bar{b}\sigma\frac{\overline{Y}}{\overline{C}}\sum_{k=1}^{\infty}\delta^{k-1}(\Delta\psi_{gov,t} + \Delta\psi_{gov,t-1} + \Delta\psi_{gov,t-k+1}) = -\frac{\bar{b}}{1-\delta}\sigma\frac{\overline{Y}}{\overline{C}}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-k}$$

We can therefore write the FONC as follows:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} + \frac{\bar{b}}{(1-\beta\delta)(1-\delta)}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-k} = 0$$

$$-\lambda_Y\hat{Y}_t - \psi_{\pi,t}\kappa_1 + \left[\overline{R}\Big(1+\gamma_h\Big) - \sigma(\overline{T}-\overline{G})\right]\psi_{gov,t} + \frac{\bar{b}}{1-\delta}\sigma\frac{\overline{Y}}{\overline{C}}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-k} = 0$$

These optimality conditions are indeed equivalent to those of the buyback model. Once again the steady state debt level is different across the two models and it holds that $\bar{b}^{BB} = \frac{\bar{b}^{NBB}}{1-\delta}$. The optimal policy rule we showed in text for the buyback model, fully applies to the no buyback case.

B.6 Non-linear model equations

In this section we derive the log-linear equations describing the model economy presented in Section 2, starting from the non-linear equations representing its competitive equilibrium. As mentioned in the main text, we consider a standard New-Keynesian model with quasi-linear preferences, that is augmented with a fiscal block.

Households Households maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(C_t - \chi \frac{h_t^{1+\gamma_h}}{1+\gamma_h} \right)$$

subject to

$$P_t C_t + \sum_{k=1}^{\infty} P_{t,k} B_{t,k} \leqslant (1-\tau) W_t h_t - T + P_t D_t + B_{t-1,1} + \sum_{k=2}^{\infty} P_{t,k-1} B_{t-1,k}$$

where C_t denotes consumption and h_t denotes hours worked. D_t represents firms' profits redistributed to households, and P_t denotes the aggregate price level. $B_{t,k}$ denotes government bonds of maturity $k = 1, 2, ...$ issued at time t. These bonds are risk-free and deliver one unit of the (nominal) consumption good in period $t+k$. They are traded at price $P_{t,k}$. τ is the distortionary tax on labour, and T the lump-sum tax imposed on households. As mentioned in the main text, both are assumed to be constant in our fiscally-led economy.

The first order conditions of the household's problem are:

$$P_{t,1} = \beta E_t \frac{1}{\pi_{t+1}} \tag{74}$$

$$P_{t,k} = \beta E_t \frac{1}{\pi_{t+1}} P_{t+1,k-1} \tag{75}$$

$$h_t^{\gamma_h} = (1-\tau_t) \frac{W_t}{P_t} \tag{76}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

Firms Production takes place in monopolistically competitive firms which operate technologies with labour as the sole input. The final good is a CES aggregate of the intermediate goods $Y_t(j)$:

$$Y_t = \Big(\int_0^1 Y_t(j)^{\frac{1+\eta}{\eta}} dj \Big)^{\frac{\eta}{1+\eta}} \tag{77}$$

where η governs the elasticity of substitution between differentiated goods. Firms set prices to maximize profits subject to the demand curve

$$Y_t(j) = \Big(\frac{P_t(j)}{P_t} \Big)^{\eta} Y_t \tag{78}$$

and given price adjustment costs, modelled as in Rotemberg (1982). The dynamic profit maximization program is:

$$\max_{P_t(j)} \quad E_t \sum_{s=0}^{\infty} Q_{t,t+s}\Big(\frac{P_{t+s}(j)}{P_{t+s}}Y_{t+s}(j) - \frac{W_{t+s}(j)}{P_{t+s}}Y_{t+s}(j) - AC_{t+s}(j)\Big)$$

$$\text{s.t.} \quad Y_{t+s}(j) = \Big(\frac{P_{t+s}(j)}{P_{t+s}}\Big)^{\eta} Y_{t+s} \tag{79}$$

$$AC_{t+s}(j) = \frac{\theta}{2}\Big(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - \pi\Big)^2 Y_{t+s} \tag{80}$$

where $Q_{t,t+s} \equiv \beta^s$ is the discount factor of households and W_{t+s} is the wage rate, that is equal to the marginal cost of production. (80) is the quadratic adjustment costs incurred by firms when resetting their price.

Focusing on a symmetric equilibrium the first order condition from the firm's dynamic program gives the following non-linear Phillips Curve:

$$\theta(\pi_t - \pi)\pi_t = 1 + \eta(1 - \frac{W_t}{P_t}) + \beta\theta E_t \frac{Y_{t+1}}{Y_t}(\pi_{t+1} - \pi)\pi_{t+1} \tag{81}$$

The firms' technology is linear in labour and thus $Y_t(j) = h_t(j)$ where $j \in [0,1]$ denotes the generic firm.

Fiscal policy The flow government budget constraint can be written as:

$$\sum_{k=1}^{\infty} P_{t,k} b_{t,k} = \frac{b_t}{\pi_t} + \sum_{k=2}^{\infty} P_{t,k-1}\frac{b_{t-1,k}}{\pi_t} + G_t - \tau h_t w_t - T \tag{82}$$

where $b_{t,k} \equiv \frac{B_{t,k}}{P_t}$ denotes the real value in t of government bonds with matuirty k, τ is the distortionary tax on labour, and T the lump-sum tax imposed on households. As mentioned above, both of these instrument are assumed to stay constant under the assumptions described in the main text. G_t denotes government spending, which is exogenous and is assumed to follow an i.i.d process.

Log-linear model Making use of the labor supply condition $h_t^{\gamma_h} = (1-\tau)\frac{W_t}{P_t}$, as well as the resource constraint $h_t = Y_t = C_t + G_t$ to dispense with W_t, C_t and h_t, we get the following linear New Keynesian Phillips Curve:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \beta E_t \hat{\pi}_{t+1} \tag{83}$$

where κ_1 is defined in text.

Defining $i_t \equiv -\log P_{t,1}$, log-linearizing the Euler equation for short bonds we get the Fisher equation described in text:

$$\hat{i}_t = E_t \hat{\pi}_{t+1} \tag{84}$$

Log-linearizing the Euler equation (75) for bonds with maturity $k > 1$, we get:

$$\hat{p}_{t,k} = E_t(\hat{p}_{t+1,k-1} - \hat{\pi}_{t+1}) \tag{85}$$

Iterating forward we get the equation displayed in text.

Log-linearizing equation (82) and using the primary surplus expression $S_t = \tau w_t h_t + T - G_t$, we get the intertemporal budget constraint (2).

B.7 Optimal policy problem

In the optimal policy problem we solve in Section 2, the Ramsey planner minimizes its loss function subject to the constraints defining the competitive equilibrium of the economy, as derived in the previous section. The Lagrangian associated to this problem is:

$$\begin{aligned} \mathcal{L} &= E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ -\frac{1}{2}\hat{\pi}_t^2 + \psi_{\pi,t}\Big(\hat{\pi}_t - \kappa_1 \hat{Y}_t - \beta \hat{\pi}_{t+1}\Big) \\ &+ \psi_{gov,t}\Big(\beta \bar{d}\hat{d}_t - \sum_{k=1}^{\infty} \beta^k \bar{b}_k \sum_{l=1}^{k} \hat{\pi}_{t+l}) + \overline{R}\Big(\gamma_h + 1\Big)\hat{Y}_t - \overline{G}\hat{G}_t - \bar{d}\hat{d}_{t-1} + \sum_{k=1}^{\infty} \beta^{k-1}\bar{b}_k \sum_{l=0}^{k-1} \hat{\pi}_{t+l}\Big)\Big\} \end{aligned} \tag{86}$$

The first order conditions of this problem are provided in the main text.

B.8 Social loss function for a specific case

Assume that the setady state is efficient. This can be guaranteed by introducing a constant employment subsidy that cancels out distortions from labor taxes and monopolistic competition at the steady state (see eg Leith and Wren-Lewis, 2013). This subsidy would modify some steady state quantities such as $\bar{R}$ and κ_1 but would be without loss of generality regarding our qualitative results. A second-order approximation of the representative households' utility around that efficient steady state gives

$$U(C_t, Y_t) \approx \bar{C}\hat{c}_t + \frac{1}{2}\bar{C}\hat{c}_t^{\,2} - \chi \bar{Y}^{1+\gamma_h}\hat{y}_t - \frac{1}{2}\chi(1+\gamma_h)\bar{Y}^{1+\gamma_h}\hat{y}_t^2$$

At the efficient steady state, we have $\chi = \bar{Y}^{-\gamma_h}$ and thus we can write

$$U(C_t, Y_t) \approx \bar{C}\hat{c}_t + \frac{1}{2}\bar{C}\hat{c}_t^{\,2} - \bar{Y}\hat{y}_t - \frac{1}{2}(1+\gamma_h)\bar{Y}\hat{y}_t^2$$

Next, a second-order approximation of the resource constraint $C_t + G_t = Y_t\Big(1 - \frac{\theta}{2}(\pi_t - 1)^2\Big)$ gives

$$RC(C_t, G_t, \pi_t, Y_t) \approx \bar{C}\hat{c}_t + \frac{1}{2}\bar{C}\hat{c}_t^{\,2} + \bar{G}\hat{G}_t + \frac{1}{2}\bar{G}\hat{G}_t^{\,2} + \frac{1}{2}\theta\bar{Y}\bar{\pi}^2\hat{\pi}_t^2 = \bar{Y}\hat{y}_t + \frac{1}{2}\bar{Y}\hat{y}_t^{\,2}$$

Solving for $\bar{C}\hat{c}_t + \frac{1}{2}\bar{C}\hat{c}_t^{\,2}$, substituting in the approximated utility and using $\bar{\pi} = 1$, we find

$$U(C_t, Y_t) \approx -\frac{1}{2}\Big(\hat{\pi}_t^2 + \lambda_Y\hat{y}_t^{\,2}\Big) + tip$$

with $\lambda_Y \equiv \frac{\gamma_h}{\theta}$. Hence, when the cost of price adjustment, θ, goes to infinity, social welfare is a quadratic measure of inflation only. Otherwise, both inflation and output enter in the objective (Leeper and Zhou (2021) find a similar result in their online appendix when considering a distorted steady state).

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ISSN 1379-244X D/2022/3082/26

IRES | Louvain Institute of Data Analysis and Modeling in economics and statistics