OPTIMAL FISCAL POLICY AND THE FISCAL THEORY OF THE PRICE LEVEL

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Abstract

This paper studies optimal fiscal policy in the context of a DSGE model in which the optimizing government can issue nominal non-contingent debt and is subject to an independent monetary policy setting the nominal interest rate according to an inflation targeting rule. The fiscal authority can stabilize the economy having several tools at its disposal, including government consumption, public investment and distortionary taxes. We focus on the case where the monetary authority sets the nominal interest rate to respond weakly to inflation, the so called passive money regime Leeper (1991). We compare the outcome of optimal fiscal policy in that case with the polar opposite, when the monetary authority aggressively responds to inflation.

It is well known that in standard DGSE models (without an optimizing government and when fiscal policy follows ad hoc rules) switching from an active to a passive monetary regime, brings about a considerable increase in macroeconomic volatility, mainly due to inflation fluctuations reflect debt sustainability. We ask whether optimal fiscal policy would choose to set fiscal variables irresponsibly when inflation fiscal debt in the passive money case. We find that the answer is no. The differences in the optimal policy allocation under active/passive monetary policies are small when the fundamental disturbances that hit the economy are standard demand shocks.

We show that this result holds both under full commitment and in the case where time-consistent policy cannot commit to the future path of its fiscal variables. In both cases changing the monetary regime from active to passive only has a small effect on equilibrium outcomes.

Keywords: passive monetary policy, optimal fiscal policy, time-consistent equilibrium, public investment

JEL Classification: E31, E52, E62, H21, H54

1 Introduction

One of the main consequences of the Great Recession and the COVID-19 crisis has been the increase in public debt observed in most of the developed economies. In the US, for instance, figure 1 shows a sharp increase in the ratio of debt to GDP, which surpassed the 100% on December 2020. At the same time, as depicted in figure 2, the US has been experimenting a rapid surge in inflation rate, which has been observed along with a weak reaction in the key policy rate.

A theoretical interpretation for the scenario described by figures 1 and 2 can be found in the Fiscal Theory of the Price Level (FTPL). According to this theory, if forward-looking agents believe that government debt is not entirely backed by future tax revenues, debt issuance becomes inflationary (Cochrane (2001), Cochrane (2021)). Moreover, a weak reaction of monetary policy to inflation has a pervasive effect on the economy, exacerbating inflation and output volatility (Sims (2011), Leeper et al. (2010)). [1]

[Figure 1 approximately here]

[Figure 2 approximately here]

Much of the literature on the FTPL studies the interactions between monetary and fiscal policies assuming ad-hoc rules for interest rates and taxes respectively. In his influential paper Leeper (1991) considers two types of equilibria. In one case monetary policy is active (it responds aggressively to inflation) and fiscal policy is passive (adjusts taxes to make debt solvent intertemporally) in another case monetary/fiscal policies are passive/active, inflation adjusts to make debt solvent and taxes are (roughly) constant through time. Following Leeper (1991), numerous papers have extended this framework, a literature which we summarize below.

A well known property of this benhcmark model is that switching from the active/passive (monetary/fiscal) scenario to the passive/active regime, brings about a considerable increase in macroeconomic volatility. Shocks that hit the economy are filtered through the consolidated budget constraint and this produces additional variability in macroeconomic aggregates, inflation and output.

This paper asks whether this conclusion is robust towards assuming that a benevolent government can set fiscal variables optimally and monetary policy is prepared to back the optimal plan by tolerating that the inflation deviates from target, not responding aggressively to that deviation. We ask in particular, whether in this case where monetary policy has committed to be passive (setting accordingly the inflation coefficient

[1] Along with purely theoretical contributions, a growing literature has been trying to explain past inflationary process in the US, such as the '70s as the result of the interaction between an active fiscal policy and a passive monetary policy. See, for instance Bianchi (2013), Bianchi and Ilut (2017)

in the interest rate rule to be below unity) fiscal policy is bound to choose a path of fiscal variables such that the volatility of inflation increases considerably.

To answer this question we use a standard New Keyensian model augmented with a fiscal block and assuming that the optimizing government has at it disposal a rich set of instruments, including distortionary labour income taxes but also public consumption and public investment. The fundamental shock driving the business cycle is a standard innovation to demand. Finally, monetary policy is assumed to follow a simple inflation targeting rule.

Using this setup, we consider separately the case where the fiscal authority can commit to the entire path of fiscal variables (the full commitment allocation) and the time-consistent optimal policy where the planner cannot commit to the path of future taxes and spending levels. Our key finding is that under both of these well known setups of optimal policy, the equilibrium does not change considerably when monetary policy is passive, relative to the case where it is active, and explicitly targets inflation.

We measure welfare by doing a second order approximation to the utility function as in Benigno and Woodford (2012), and we analyze optimal fiscal policy when monetary policy is conducted by a Taylor rule for the nominal interest rate. We compare the properties of the business cycle when monetary policy is active and passive by allowing for different responsiveness of the Taylor rule to inflation. When monetary policy is active, we set the coefficient of the Taylor rule responding to inflation in 1.5 as in Woodford (2001) and the majority of the literature on New-Keynesian DSGE models.[2]. When monetary policy is passive, we perform the analysis under two values for the coefficient of the Taylor rule targeting inflation: 0.95 and 0.7. The first value is close to the level in which monetary policy becomes active and follows estimations from DSGE models for the period starting in 2005 as in Bianchi (2013). The second is an arbitrary value in which monetary policy reacts even less to inflation and serves as a more strict scenario for passive monetary policy.

Under commitment, the optimal path for most of the variables in response to a negative demand shock is similar regardless interest rate response to inflation. The results comes from the planner's ability to set optimal distortionary taxes and debt. Taking as reference the economy under rules, with active fiscal policy, taxes do not respond to debt, thus the government resorts on inflation to relax the budget constraint. When the government can optimally set the tax rate, taxes moves accordingly to finance the fiscal deficit, while public debt falls by more. Meanwhile, government consumption and investment increase to close the output gap under passive and active monetary policy. As in Bouakez et al. (2019), the planner resorts more in public investment to boost the economy, a result possible linked to the higher multiplier of public investment relative to public consumption when the time-to-build public capital is set to zero (see Ramey

[2]We used the value from Woodford (2001) despite some works estimate the coefficient to be well above 1.5 (Carvalho (2019), Mehra and Minton (2007)) during the Greenspan era, a period in which monetary policy was considered to be active

(2020)). The more noticeably difference between active and passive monetary policy when fiscal policy is optimal is the response of inflation, which picks up faster when monetary policy is passive.

Considering the previous result, we study optimal fiscal policy in a time-consistent environment. An experiment, we do mainly for two reasons. First, a time-consistent planner is, in general, less able to spread the burden of taxation across time, leading to higher volatility. Secondly, from a more empirical perspective, one can think of fiscal stimulus plans, such the ones approved during the Great Recession or the COVID-19 recession, as the result of time-consistent policies. Indeed, in a time-consistent equilibrium, the planner chooses uncontingent policies from which future policy makers will not deviate from, given the path for state variables and under the assumption no future shocks will hit the economy. The stimulus packages approved for instance, during the Great Recession or the COVID-19 recession, could resemble the outcome of time-consistent decisions, if one accepts that they were the result of an agreement between political parties who agree not to revise once in power.

The main result does not change with discretion and the volatility of macro aggregates is very similar under active and passive monetary policy when the economy is hit by a negative demand shock. Nevertheless, optimal policies under discretion are qualitatively very different from the ones prescribed by a committed planner. After the arrival of the shock, the planner cuts distortionary taxes and public investment while increases public consumption and debt. By increasing debt and cutting public capital, the planner increases inflation expectations. An increase in debt raises concerns about repayment, and private sector expects higher inflation to relax the government budget constraint. A cut in public capital, increases future real marginal cost and prices, thus the planner relies on the supply-side of such a policy. [3]. Last result is new to the literature, as this is the first paper analyzing optimal public capital as a stabilization tool in a time-consistent environment.

We find our main result, the differences in optimal policies allocations are small under active/passive monetary policy, to be robust when the economy is hit by a liquidity premium shock and when monetary policy targets not only inflation but also output. Finally, we confront the economy to a shock in preferences for government consumption expenditure, so government spending becomes quasi exogenous in the model. We find that private consumption falls by more under active monetary policy due to higher nominal interest rates.

[3]See Bouakez et al. (2017) and Bouakez et al. (2019) for a discussion regarding the supply side and demand side effects of government investment

2 Related Literature

This paper is related to the literature on optimal fiscal policy and the Fiscal Theory of the Price Level (FTPL). This theory argues that since the real value of government debt must equal the sum of real future fiscal surplus, if current debt is not perceived to be backed by future taxes, inflation may raise (Cochrane (2001), Cochrane (2021)) to relax the budget constraint of the government.

In much of the literature on the FTPL, such as Cochrane (2001), Sims (2011), Cochrane (2001), among others, monetary and fiscal policy obeys ad-hoc rules. Monetary policy is usually characterized by a Taylor rule in which nominal interest rate reacts weakly to inflation, while fiscal policy is resumed by a rule in which taxes or fiscal surplus reacts to lagged values of debt. Within this framework, the FTPL could be characterized using Leeper (1991)s' terminology by an Active Fiscal Policy/Passive Fiscal policy mix. This literature has shown that the response of the economy to demand or fiscal shocks under passive and active monetary policy could be very different. Although, we can not compare directly the results with rules and optimal fiscal policy, we try instead to highlight the role for optimal taxation when monetary policy is passive. In other words, we show that the common assumption of using rules to model monetary and fiscal policy interactions is not innocuous at the moment of describing the economic behaviour.

The paper is also related to the literature on the interaction between monetary and fiscal policy. Along these lines, there is strand of the literature analyzing joint optimal fiscal and monetary policy, such as Schmitt-Grohé and Uribe (2004), Leith and Wren-Lewis (2013), Eggertsson (2011). In all this papers, the planner has the ability to manipulate the bond price by setting optimally the interest rate. In this paper, in turn, the planner rely on taxes and debt to stabilize the economy foregoing price stability and allowing instead a higher and persistent inflation rate. This environment can capture, for instance, the institutional arrangement in which an independent central bank has its different objectives from those of the government. It can also describe an environment in which the government has little influence in bond prices, for instance, when the country has debt management units unable to control fiscal deficit. In such a world, shocks have an impact on the government budget constraint affecting inflation. We show, that if fiscal policy is optimal, there is little difference at the time of stabilizing the economy with respect to an active Taylor rule.

Jia (2020) analyzes fiscal policy under passive monetary policy rules, though the paper is more concerned with the implications for debt maturity. Jia (2020) also derives optimised simple rules to mimic the outcome of a joint optimal monetary and fiscal policy choice. In this paper, we do not address joint optimal policies, which are extensively reviewed by the literature, but we focus on comparing the business cycle properties under active and passive monetary policy when the planner can optimally chose a thorough set of fiscal variables such as government consumption, investment and distortionary taxes.

Finally, this paper contributes also to the literature studying public investment as a stabilization tool. This is the first paper to consider public capital as a stabilization tool in a time-consistent environment. The recently approved fiscal packages in Europe and the US gave raise to a literature, such as Leeper et al. (2010), Drautzburg and Uhlig (2015) or Coenen et al. (2013) analyzing the effectiveness of public capital to boost the economy. None of these papers deals with optimal government investment. Bouakez et al. (2019) investigates optimal public investment as a stabilization tool, though it does analyzes neither the way to finance such a policy nor the behaviour of optimal policies under discretion as this paper analyzes. Another strand of the literature on public capital is concerned on the properties of optimal public capital for long-term growth (Ambler and Paquet (1996), Glomm and Ravikumar (1994)), but not as a tool for stabilizing the economy in response to macroeconomic shocks. [4]

Section 3 contains the model environment while 4 defines the a private equilibrium. Section 5 describes the optimal policy problems under commitment and no-commitment. Section 6 contains the numerical results for the main model. In section 7, we consider several extensions: A Taylor rule tracking inflation and output, a liquidity premium shock increasing the demand for bonds and a shock in preferences for government consumption in which public expenditure becomes quasi-exogenous. Finally, section 8 concludes.

3 The economy

The framework is a standard New-Keynesian model, with imperfect competition in which firms sets prices as in Rotemberg (1982). Households choose labor to supply to firms, consumption and savings. The model is enriched with a large fiscal block including public consumption, from which households enjoy utility, and public investment, which serves as an external input for the monopolistically competitive firm.

The government issues non-state contingent nominal long-term bonds and collects revenues from a distortionary tax, levied on labor income. We model an economy with long-term bonds to capture the maturity structure in developed economies. In line with the literature on optimal fiscal policy, we assume that the government does not have access to lump-sum transfers, which makes the optimal policy program non-trivial as Ricardian equivalence does not hold.

Monetary policy follows a Taylor rule for the nominal interest rate targeting inflation only as in e.g. Leeper (1991).

The presence of nominal rigidities and distortionary taxes precludes an inefficient steady-state which we correct by a subsidy in the same way as Leith and Wren-Lewis (2013) and Matveev (2021). The derivation

[4] Azzimonti et al. (2009) analyzes optimal public investment in a time-consistent environment, though there is no debt, nor nominal rigidity.

of the efficient steady-state and the subsidy is in Appendix A.

3.1 The environment

Households

The economy is inhabited by identical agents who have the following preferences:

$$E_t \sum_{t=0}^{\infty} \beta^t \left(\epsilon_t \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_g \frac{G_t^{c\,1-\sigma}}{1-\sigma} \right) \tag{1}$$

where β dentotes the usual discount factor, c_t is consumption, n_t are hours worked and G_t^c is a non-storable public good. We assume, σ, φ, χ and χ_g are all positve parameters.

ϵ_t is a shock reducing private consumption. Notice that this is a different approach to modelling a demand shock than many other papers. It is common in the literature, to model demand shocks as shocks to the discount factor (thus a shock enters multiplicatively to utility and concerns all arguments, not just C). However, modelling a demand shock like ϵ_t will have non-trivial effects on the economy because the shock will change the intertemporal preferences for consumption but also the intratemporal allocation between consumption and leisure and between consumption and the public good. A lower ϵ_t will also imply that the household desires to work less hours, which seems plausible for a household that desires to postpone consumption. Moreover, the shock does not multiply the period preferences the public good G_t^c, which means that the optimal provision of the public good, will change depending on the realized value of ϵ_t. We will discuss this in detail later on.

Note also that this modelling approach follows Faraglia et al. (2013), who study optimal monetary and fiscal policy in a model where government spending is also assumed to be an exogenously given random variable. Therefore, relative to them we use a setup where public spending is endogenous, and is part of the optimal policy program we will setup below. Moreover, in section 7.2 we will follow a different approach and model the demand shock as a liquidity premium shock as in for example Smets and Wouters (2007). Using this alternative setup we will show that our results are robust.

Households enter each period t with $B_{t-1,S}$ units of a short-term bond and $B_{t-1,L}$ units of a long-term bond. We consider that both types of debt are risk-free assets, meaning that debt is default free. Households receive wages from supplying labor and rental payments for owning firms. The household budget constraint can be written as:

$$P_t c_t + q_{t,S} B_{t,S} + q_{t,L} B_{t,L} = (1-\tau_t) W_t n_t + P_t D_t + B_{t-1,S} + (1+\rho q_{t,L}) B_{t-1,L} + T_t \tag{2}$$

where W_t is the nominal wage and τ_t are labor taxes, both taken as given by households. D_t are dividends.[5] We denote with $q_{t,S}$ the price for the short-term government bond and $q_{t,L}$ the price for $B_{t,L}$, a perpetuity with coupon payments decaying at rate $0 \leq \rho \leq 1$ as in Woodford (2001). When $\rho = 0$, we assume the government issues only short-term debt.

Firms

The economy produces a final good that can be used for consumption and investment. The firm that produces this final good is assumed to operate in a perfectly competitive market. The technology is a standard CES function whose inputs are intermediate products:

$$Y_t = \left(\int_0^1 Y_t(z)^{1-1/\theta} dz \right)^{\frac{\theta}{\theta-1}}$$

where $Y_t(z)$ is the quantity of intermediate good z used in the production of the final good and $\theta \geq 1$ denotes the elasticity of substitution between intermediate goods.

Firms producing intermediate goods have market power, every firm produces a differentiated good, $Y_t(z)$. Denoting by $P_t(z)$ the price of the generic intermediate goods, we can use standard arguments to show that the demand for $Y_t(z)$ is:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\theta} Y_t \tag{3}$$

The generic firm operating in the intermediate goods sector uses labor and public capital to produce output. Notice that this is non-standard for the literature on optimal fiscal policy which typically abstracts from modelling public capital.

We consider public capital in this paper because we seek to model government policy decisions as realistically as possible. Public investment is obviously an important part of government policy; for example the recent stimulus packages for the Great Recession and the COVID-19 recession in the US entailed large increases of public investment.

This paper considers public capital to be an externality to production, in other words, a input that is not chosen by firms but by the government. In particular the production function is a Cobb-Douglas with

[5] T_t are lump-sum transfers by the government. Recall that we will not treat T_t as a policy instrument for the optimizing government. T_t is used here because to derive a second order approximation to the household's welfare (see below) we must subsidize firms to reach the efficient level of production. T_t will be assumed to finance that subsidy.

Moreover, a further reason for subsidizing firm production is the following: Leith and Wren-Lewis (2013) showed that when considering time-consistent policies, the presence of a non-efficient steady-state creates incentives for the planner to distort the steady state by reducing debt. The subsidy allows a positive amount of sustainable debt at the steady-state. We thus follow Leith and Wren-Lewis (2013) to derive the subsidy.

constant return to scale as in Bouakez et al. (2019):

$$Y_t(z) \leq F(N_t(z), K_{G,t-1}(z)) = N_t^{\alpha}(z) K_{G,t-1}^{1-\alpha}(z) \tag{4}$$

Needless to say, the inclusion of public capital in the production function will have non-trivial effects on the macroeconomy and on firm supply and household demand decisions. An increase in public capital boosts aggregate demand and household wealth and consumption. It also increases marginal productivity of labor, reducing marginal cost and prices.

Firms set prices subject to adjustment costs as in Rotemberg (1982). The cost function for firm z is given by,

$$\Sigma_t(z) = \frac{\psi}{2}\left(\frac{P_t(z)}{P_{t-1}(z)} - 1\right)^2 Y_t$$

which implies a resource cost when the price in t differs from the price in $t-1$. Parameter $\psi \geq 0$ governs the magnitude of the price adjustment cost.

Dividends payed out by the firm are given by:

$$D_t(z) = \frac{P_t(z)}{P_t} Y_t(z) - w_t N_t(z)(1-s) - \Sigma_t(z)$$

where s is a time-invariant employment subsidy used to eliminate distortions arising at the steady-state due to monopolistic competition and distortionary taxation as in Leith and Wren-Lewis (2013) and Matveev (2021). This subsidy is constant over time and as discussed previously it is financed with lump-sum transfer, T_t.

Government

The government is able to spend in a non-storable public good G_t^c and in public capital $K_{G,t}$. To finance its expenditures, the government can levy distortionary taxes and issue long-term nominal debt.

We will assume that short-term debt is in zero net supply, so we focus only on long-term debt. Let $b_{t,L}$ be the real value for the stock of long-term debt, the budget constraint in real terms can be written as:

$$q_t b_t = (1 + \rho q_t)\frac{b_{t-1}}{\pi_t} + (G_t^i + G_t^c - \tau_t w_t n_t) + s(w_t n_t - w^* n^*) \tag{5}$$

Where G_t^i and G_t^c stands for expenditures in public investment and consumption respectively. We denote by π_t the gross inflation rate, (P_t/P_{t-1}). The last term, $sw_t n_t - sw^* n^*$ is the deviation of the subsidy from its level at the steady-state.

Finally, the law of motion for public capital is standard. For simplicity, we do not allow for delays linked to the accumulation of public capital as in Kydland and Prescott (1982), or Bouakez et al. (2019). Instead, we assume that capital is build immediately. For simplicity, we also abstract from investment adjustment costs. The law of motion for public capital is given by:

$$K_{G,t} = G_t^i + (1 - \delta_G) K_{G,t-1} \tag{6}$$

where $0 < \delta_G < 1$ is the depreciation rate.

Monetary policy

Monetary policy is assumed to follow a simple inflation targeting rule of the form:

$$R_t = (\pi_t / \pi^*)^{\phi_\pi} \tag{7}$$

Coefficient ϕ_π is a crucial object. As is well known, when $\phi_\pi > 1$ (the Taylor principle is satisfied), monetary policy is 'active' e.g. Leeper (1991). Under this condition, in equilibrium inflation is independent of government debt and is only driven by shocks to the Euler equation and the Phillips curve. In contrast, in the case where $\phi_\pi < 1$ then policy is passive and inflation is set to satisfy the intertemporal solvency of debt. In this equilibrium fiscal variables can affect inflation also due to their influence on the consolidated budget constraint.

In section 7, we show that the main result of the paper, namely the volatility of aggregate variables is similar under active and passive monetary policy, also holds if the nominal interest rate reacts react to output in addition to inflation.

4 Competitive equilibrium

We now define the competitive equilibrium for this economy given the sequence of government policies $\{G_t^c, G_t^i, \tau_t, B_{t,S}, B_{t,L}, K_{G,t}, R_t\}$ and the path of the exogenous demand shock $\{\epsilon_t\}$. We first derive the optimality conditions of households and firms. Subsequently using the optimality conditions we provide a formal definition of the equilibrium.

4.1 Household problem

Households choose sequences $\{c_t, n_t, B_{t,S}, B_{t,L}\}$ to maximize utility subject to their budget constraints and the usual boundary No-Ponzi condition. As usual, households take government policies, wages and prices as given. The generic household's program (where the relevant vector of state variables at period t is $\{K_{G,t-1}, b_{L,t-1}, b_{S,t-1}, \epsilon_t\}$) can be written as:

$$V(B_{L,t-1}, B_{S,t-1}, K_{G,t-1}, \epsilon_t) = \max_{\{n_t, c_t, B_{L,t}, B_{S,t}\}} \left\{ U(c_t, n_t, G_t^c) + \beta E[V_{t+1}(B_{L,t}, B_{S,t}, K_{G,t}, \epsilon_{t+1})] \right\}$$

$st.$

$$P_t c_t + q_{t,S} B_{t,S} + q_{t,L} B_{t,L} = (1-\tau_t) W_t n_t + P_t D_t + B_{t-1,S} + (1+\rho q_{t,L}) B_{t-1,L} + T_t$$

Standard arguments imply that optimality conditions for households are given by

$$\frac{W_t}{P_t}(1-\tau) = \frac{\chi n_t^{\varphi}}{\epsilon_t c_t^{-\sigma}} \tag{8}$$

$$q_{t,S}\epsilon_t c_t^{-\sigma} = \beta E_t \left[\epsilon_{t+1} \frac{c_{t+1}^{-\sigma}}{\pi_{t+1}} \right] \tag{9}$$

$$q_{t,L}\epsilon_t c_t^{-\sigma} = \beta E_t \left[\epsilon_{t+1} \frac{c_{t+1}^{-\sigma}}{\pi_{t+1}} (1 + \delta q_{t+1,L}) \right] \tag{10}$$

Equation 8 is the optimal labor supply condition while equations 9 and 10 are the Euler equations for short and long-term bonds respectively. Note also that for brevity we expressed these optimality conditions in terms of real debt holdings. π_{t+1} denotes the gross inflation rate. Thus equation 9 gives us the usual indifference condition between investing an additional unit of income in the short term asset and receiving the appropriately discounted random payoff $\frac{1}{\pi_{t+1}}$. Equation 10 in turn gives us the payoff of investing in the long term bond for one period and then acquiring the value of the investment, measured in terms of the resale price $q_{t+1,L}$ and discounted by inflation. In appendix C we derive the complete problem for the household.

4.2 Firm's problem

Each firm z in the intermediate sector sets prices and inputs to maximize the present value of dividends, appropriately discounted using the household real discount factor. We therefore have the following program:

$$\max_{P(z),N(z)} E_t \sum_{l=0}^{\infty} \beta^t \frac{U_{c,t+l+1}}{U_{c,t+l}} \left[\frac{P_{t+l}(z)}{P_{t+l}} Y_{t+l}(z) - (1-s) w_{t+l} N_{t+l}(z) - \frac{\psi}{2} \left(\frac{P_{t+l}(z)}{P_{t+l-1}(z)} - 1 \right)^2 Y_{t+l} \right]$$

subject to the constraints

$$Y_{t+l}(z) = \left(\frac{P_{t+l}(z)}{P_{t+l}} \right)^{-\theta} Y_{t+l}$$

$$Y_{t+l}(z) \geq F(N_{t+l}(z), K_{G,t+l})$$

$U_{c,t+l+1}/U_{c,t+l} = \epsilon_{t+l+1} c_{t+l+1}^{-\sigma} / \epsilon_{t+l} c_{t+l}^{-\sigma}$ is the discount factor of the households that own the shares of firm z.

Note that we will assume a symmetric equilibrium. Thus all firms will solve the same program and we can drop z from the notation from now on. The first order conditions with respect to prices P_t, labor, N_t are the following:

$$\frac{w_t(1-s)}{F_{N,t}} - \frac{\theta - 1}{\theta} = \frac{\psi}{\theta} \left[\pi_t(\pi_t - 1) - \beta E_t \left(\frac{\epsilon_{t+1}}{\epsilon_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - 1) \pi_{t+1} \right) \right]$$

$$- w_t(1-s) + \mu_t F_{N,t} = 0$$

$$Y_t = F(N_t, K_{G,t})$$

where we made use of the fact that all firms set the same price level in the symmetric equilibrium (equal by definition to the aggregate price level) and thus for brevity we expressed optimality with respect to P as a function of gross inflation $\pi_t = (P_t/P_{t-1})$ (appendix C derives the complete problem for the firm). $\mu_t = \frac{w_t(1-s)}{F_{N,t}}$ is the real marginal cost of production, w_t are real wages, and $F_{N,t}$ is the marginal productivity of labor. Note that the first equation is the standard Phillips curve, determining the inflation output tradeoff in the model.

4.3 Definition of the competitive equilibrium

Definition CE: Given an exogenous process for the shock, ϵ_t, and initial levels of public debt ,B_{-1}, and public capital ,K_{G-1}, a private sector equilibrium for this economy is a sequence of stochastic processes $\{c_t, n_t, y_t, \pi_t, w_t, b_t, k_{Gt}, g_t^i, g_t^c, \tau_t, R_t\}_{t=0}^{\infty}$ such that: (1) $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ solves the household problem for given prices and government policies, (2) $\{\pi_t\}_{t=0}^{\infty}$ conforms the optimal pricing for the firms, (3) the government budget constraint is satisfied, (4) the Taylor rule for Monetary policy is satisfied and (5) the market for goods and labor are clear.

The above is formalized by the following conditions:

$$
\begin{aligned}
&(1-\tau_t)\frac{W_t}{P_t} = \frac{\chi n_t^{\varphi}}{\epsilon_t c_t^{-\sigma}} \\
&q_{t,L}\epsilon_t C_t^{-\sigma} = \beta E_t\left[\epsilon_{t+1}\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right](1+\delta q_{t+1,L}) \\
&q_{t,S}\epsilon_t C_t^{-\sigma} = \beta E_t\left[\epsilon_{t+1}\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right] \\
&\frac{w_t(1-s)}{F_{N,t}} - \frac{\theta-1}{\theta} = \frac{\psi}{\theta}\left[\pi_t(\pi_t-1) - \beta E_t\left(\frac{\epsilon_{t+1}}{\epsilon_t}\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\frac{Y_{t+1}}{Y_t}(\pi_{t+1}-1)\pi_{t+1}\right)\right] \\
&K_{G,t} = (1-\delta_g)K_{G,t-1} + G_t^i \\
&R_t = (\pi_t/\pi^*)^{\phi_\pi} \\
&q_{t,L}b_{t,L} = G_t^i + G_t - \tau_t w_t n_t + (1+\delta q_{t,L})\frac{b_{L,t-1}}{\pi_t} + s(w_t n_t - w^* n^*) \\
&Y_t = C_t + G_t^c + G_t^i + \frac{\psi}{2}(\pi_t-1)^2 Y_t
\end{aligned}
$$

together with the condition $b_{t,S} = 0$ stating that short term debt is in zero net supply.

A few comments are worth making. First note that we repeated equations we derived previously from the households and firms programs for convenience. Second, the last line represents the economy wide resource constraint, expressing that output is divided between private and public consumption C_t and G_t^c respectively, public investment G_t^i and the resource costs of inflation $\frac{\psi}{2}(\pi_t-1)^2Y_t$. To derive this equation we combined the household and government budget constraints. Therefore, we dispensed with the household constraint in the definition of the equilibrium.

Last, we imposed that private assets are equal to the debt issued by the government, which is necessary for market clearing.

5 Optimal fiscal policy

We now present the optimal policy program that the government solves. To analyze optimal fiscal policy, we linearize model equations and derive a second order approximation to the utility function around an efficient steady-state. This is a standard approach to optimal policy (as in e.g. Benigno and Woodford (2012)). Using the linearized model we consider both the case where optimal policy operates under commitment and the optimal time consistent policy under discretion. We refer the reader to appendix B for the complete derivation of the welfare function and the constraint set of the government in each case.

5.1 The Welfare Criterion

In the appendix we show the second order approximation of household utility delivers the following policy objective function:

$$-E_0 \sum_{t=0}^{\infty} \beta^t L_t$$

where

$$L_t = \omega_{g^i} \frac{\tilde{g_t^i}^2}{2} + \omega_y \frac{\tilde{y_t}^2}{2} + \omega_\pi \frac{\tilde{\pi_t}^2}{2} + \omega_{g^c} \frac{\tilde{g_t^c}^2}{2} + \omega_{k_g-1} \frac{\tilde{k_{G,t-1}}^2}{2} + \omega_{yg^i} \tilde{y}_t \tilde{g_t^i} + \omega_{yg^c} \tilde{y}_t \tilde{g_t^c} + \omega_{g^i g^c} \tilde{g_t^i} \tilde{g_t^c} + \omega_{yk_g} \tilde{y}_t \tilde{k_{G,t-1}}$$
$$+ \omega_{\epsilon g^i} \tilde{g_t^i} \tilde{\epsilon}_t + \omega_{\epsilon g^c} \tilde{g_t^c} \tilde{\epsilon}_t + \omega_{\epsilon y} \tilde{y}_t \tilde{\epsilon}_t$$

where the variables are expressed in deviation from the efficient steady state (we use a tilde to denote this) and the coefficients ω are given by:

$$\omega_{g^i} = \frac{g^{i,*}}{c^{*\sigma}} \left[g^{i,*} - \frac{g^{i,*}\sigma}{c^*} \right]$$
$$\omega_y = -\frac{y^*}{c^{*\sigma}} \left[\frac{\sigma y^*}{c^*} + \frac{1+\varphi}{\alpha} - 1 \right]$$
$$\omega_\pi = -\frac{y^* \psi}{c^{*\sigma}}$$
$$\omega_{g^c} = -\sigma \left[\frac{g^{c,*2}}{c^{*,\sigma+1}} + \frac{g^{c,*}}{c^{*\sigma}} \right]$$
$$\omega_{yg^i} = \frac{\sigma g^{i,*} y^*}{c^{*1+\sigma}}$$
$$\omega_{y\epsilon} = \frac{y^*}{c^\sigma}$$

$$\omega_{yg^c} = \frac{\sigma g^{c,*} y^*}{c^{*1+\sigma}}$$
$$\omega_{g^i g^c} = -\frac{\sigma g^{c,*} g^{i,*}}{c^{*1+\sigma}}$$
$$\omega_{\epsilon g^i} = -\frac{g^{i,*}}{c^{*,\sigma}}$$
$$\omega_{yk_g} = -\frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma}\alpha}$$
$$\omega_{g^c\epsilon} = -\frac{g^{c,*}}{c^{*\sigma}}$$
$$\omega_{k_g-1} = -\frac{(\alpha-1)y^*}{c^{*\sigma}} \left[\frac{(\alpha-1)}{\alpha}(1+\varphi) - 2 \right] + \frac{k_g^{*2}}{c^{*\sigma}}(1-\delta)$$

where the complete derivation of these expressions can be found in the Appendix.

As is evident L is indeed a second order objective function, expressing preferences of the household for smoothing output inflation, (the usual arguments), and also smoothing public consumption and investment. Clearly, public consumption affects welfare directly whereas public investment and capital exert an indirect influence on welfare through affecting the production possibilities of the economy. The coefficients corresponding to these terms measure the magnitude of these indirect effects on household utility.

Finally, it is interesting to note that the cross terms ($\tilde{y}_t \tilde{g_t^i}$, $\tilde{y}_t \tilde{k_{t-1}}$ etc) suggests the presence of a degree of complementarity (substitutability) between the arguments of the objective.

5.2 The Ramsey planner under commitment

We first consider the case of optimal policy under commitment. To solve this program we utilize the methodological approach of Aiyagari et al. (2002), Schmitt-Grohé and Uribe (2004) and other papers solving for optimal Ramsey policy equilibria under incomplete financial markets.

As is typical in the literature we dispense with some of the variables of the competitive equilibrium and build the constraint set of the government using only the sufficient implementability conditions. We thus use the primal approach and substitute out from the government budget constraint taxes and the bond prices. Then, given the optimal policy allocation, these variables can be found to satisfy the first order condition for labor and the bond price Euler equation. We can therefore dispense also with these equations as constraints.

Making these substitutions, the log-linear government budget constraint can be written as:

$$\begin{aligned} q^*b^*\tilde{b_t} &= q^*b^*\beta^{-1}(\tilde{b_{t-1}} - \tilde{\pi_t}) + q^*b^*(1-\rho)(-\sigma\tilde{c_t} + \tilde{\zeta_t} + E_t[\sigma\tilde{c_{t+1}} + \tilde{\pi_{t+1}} - \epsilon_{t+1}] - \\ &- \beta\rho E_t[\tilde{q_{t+1}}]) + g^{i,*}\tilde{g_t^i} + g^{c,*}\tilde{g_t^c} - \tau^*n^*w^*\left(\tilde{n_t} + \tilde{w_t} + \frac{1-\tau^*}{\tau^*}\left(\tilde{w_t} - \varphi\tilde{n_t} - \sigma\tilde{c_t} + \epsilon_t\right)\right) \end{aligned} \tag{11}$$

For simplicity, we also substitute out private consumption by using the aggregate resource constraint of the economy.

$$\tilde{c_t} = \frac{y^*}{c^*}\tilde{y_t} - \frac{g^{i,*}}{c^*}\tilde{g_t^i} - \frac{g^{c,*}}{c^*}\tilde{g_t^c}$$

We solve for optimal policies using the second order approximation of the utility function around the steady-state defined avobe which features output and public capital. Hence, we substitute out labor by the production function. and the marginal costs by the FONC for the firms, keeping real wage in the system.

The planner chooses output, inflation, short-term interest rate, government consumption and investment, public capital, wages and public debt. In other words, the vector of control variables for the planner's problem is $\{y_t, \pi_t, R_t, g_t^i, g_t^c, k_{G,t+1}, w_t, b_{t,\rho}\}$. The planner problem can be written using a Lagrangian as:

$$
\begin{aligned}
\max_{\{y_t,\pi_t,R_t,g_t^i,g_t^c,k_{G,t+1},w_t,b_{t,\rho}\}} &= \sum_{t=0}^{\infty}\beta^t E_0\Bigg\{-L_t \\
&+\phi_{1,t}\Bigg[\tilde{R}_t+\sigma(\frac{y^*}{c^*}\tilde{y_t}-\frac{g^{i,*}}{c^*}\tilde{g_t^i}-\frac{g^{c,*}}{c^*}\tilde{g_t^c})-\tilde{\epsilon_t}-E_t[\sigma(\frac{y^*}{c^*}\tilde{y_{t+1}}-\frac{g^{i,*}}{c^*}\tilde{g_{t+1}^i}-\frac{g^{c,*}}{c^*}\tilde{g_{t+1}^c})+\tilde{\pi_{t+1}}-\tilde{\epsilon_{t+1}}]\Bigg] \\
&+\phi_{2,t}\Bigg[\tilde{\pi}_t-\frac{\theta}{\psi}\Bigg(\tilde{w}_t-\tilde{y_t}+\frac{\tilde{y_t}-(1-\alpha)\tilde{k_{G,t}}}{\alpha}\Bigg)-\beta E_t[\tilde{\pi_{t+1}}]\Bigg] \\
&+\phi_{3,t}\Bigg[\tilde{k}_{G,t}-(1-\delta_G)\tilde{k}_{G,t-1}-\delta_G\tilde{g^i}_t\Bigg] \\
&+\phi_{4,t}\Bigg[\tilde{R}_t-\phi_\pi\tilde{\pi}_t\Bigg] \\
&+\phi_{5,t}\Bigg[q^*b^*\tilde{b_t}-q^*b^*\beta^{-1}\tilde{b_{t-1}}+q^*b^*\beta^{-1}\tilde{\pi}_t-q^*b^*(1-\rho)\Bigg(-\sigma(\frac{y^*}{c^*}\tilde{y_t}-\frac{g^{i,*}}{c^*}\tilde{g_t^i}-\frac{g^{c,*}}{c^*}\tilde{g_t^c})+\tilde{\epsilon}_t \\
&+E_t[\sigma(\frac{y^*}{c^*}\tilde{y_{t+1}}-\frac{g^{i,*}}{c^*}\tilde{g_{t+1}^i}-\frac{g^{c,*}}{c^*}\tilde{g_{t+1}^c})+\tilde{\pi_{t+1}}-\tilde{\epsilon_{t+1}}]-\beta\rho E_t[\tilde{q_{t+1}}]\Bigg)-g^{i,*}\tilde{g_t^i}-g^{c,*}\tilde{g_t^c} \\
&+\tau^*n^*w^*\Bigg(\frac{\tilde{y_t}-(1-\alpha)\tilde{k_{G,t}}}{\alpha}+\tilde{w}_t+\frac{1-\tau^*}{\tau^*}\Bigg(\tilde{w}_t-\varphi\frac{\tilde{y_t}-(1-\alpha)\tilde{k_{G,t}}}{\alpha}-\sigma(\frac{y^*}{c^*}\tilde{y_t}-\frac{g^{i,*}}{c^*}\tilde{g_t^i}-\frac{g^{c,*}}{c^*}\tilde{g_t^c})+\tilde{\epsilon}_t\Bigg)\Bigg)\Bigg]\Bigg\}
\end{aligned}
$$

Where $\phi_{i,t}\ \forall i=1,..,5$ are Lagrange multipliers attached to the constraints.

It is straightforward to show that the following first order conditions derive from the Lagrangian:

$$
\begin{aligned}
\tilde{y}_t :&-\omega_y\tilde{y}_t-\omega_{yg^i}\tilde{g_t^i}-\omega_{yg^c}\tilde{g_t^c}-\omega_{yk_g}\tilde{k_{G,t-t}}-\omega_{\epsilon y}\tilde{\epsilon}_t+\sigma\frac{y^*}{c^*}\Bigg(\phi_{1,t}-\beta^{-1}\phi_{1,t-1}\Bigg)+\phi_{2,t}\frac{\theta}{\psi} \\
&+\frac{\beta b^*}{1-\beta\rho}(1-\rho)\sigma\frac{y^*}{c^*}\Bigg(\phi_{5,t}-\beta^{-1}\phi_{5,t-1}\Bigg)+\phi_{5,t}\Bigg[w^*n^*\tau^*\Bigg(\frac{1}{\alpha}-\frac{1-\tau^*}{\tau^*}\Bigg(\frac{\varphi}{\alpha}+\sigma\frac{y^*}{c^*}\Bigg)\Bigg)\Bigg]=0 \\
\tilde{\pi}_t :&-\omega_\pi\tilde{\pi}_t-\beta^{-1}\phi_{1,t-1}+\phi_{2,t}-\phi_{2,t-1}-\phi_{4,t}\phi_\pi+\frac{b^*}{1-\beta\rho}(\phi_{5,t}-(1-\rho)\phi_{5,t-1})=0 \\
\tilde{R}_t :&\phi_{1,t}+\phi_{4,t}=0 \\
\tilde{g_t^i} :&-\omega_{g^i}\tilde{g_t^i}-\omega_{yg^i}\tilde{y}_t-\omega_{g^cg^i}\tilde{g_t^c}+\sigma\frac{g^{i,*}}{c^*}\Bigg(\beta^{-1}\phi_{1,t-1}-\phi_{1,t}\Bigg)-\phi_{3,t}\delta_G \\
&+\frac{\beta b^*}{1-\beta\rho}(1-\rho)\sigma\frac{g^{i,*}}{c^*}\Bigg(\beta^{-1}\phi_{5,t-1}-\phi_{5,t}\Bigg)+\phi_{5,t}\Bigg(-g^{i,*}+n^*w^*(1-\tau^*)\sigma\frac{g^{i,*}}{c^*}\Bigg)=0 \\
\tilde{g_t^c} :&-\omega_{g^c}\tilde{g_t^c}-\omega_{yg^c}\tilde{y}_t-\omega_{g^cg^i}\tilde{g_t^i}+\sigma\frac{g^{c,*}}{c^*}\Bigg(\beta^{-1}\phi_{1,t-1}-\phi_{1,t}\Bigg)+ \\
&\frac{\beta b^*}{1-\beta\rho}(1-\rho)\sigma\frac{g^{c,*}}{c^*}\Bigg(\beta^{-1}\phi_{5,t-1}-\phi_{5,t}\Bigg)+\phi_{5,t}\Bigg(-g^{c,*}+n^*w^*(1-\tau^*)\sigma\frac{g^{c,*}}{c^*}\Bigg)=0 \\
\tilde{w}_t :&-\frac{\theta}{\psi}\phi_{2,t}+\phi_{5,t}\Bigg[w^*n^*\tau^*\Bigg(1+\frac{1-\tau^*}{\tau^*}\Bigg)\Bigg]=0 \\
\tilde{k_{G,t}} :&-\beta E_t(\omega_{k_g-1}\tilde{k_{G,t}}+\omega_{yk_g}\tilde{y_{t+1}})+\phi_{3,t}-(1-\delta_G)\beta E_t[\phi_{3,t+1}]+ \\
&+\beta E_t\phi_{5,t+1}\Bigg(-n^*w^*\tau^*\frac{(1-\alpha)}{\alpha}+(1-\tau^*)n^*w^*\frac{\varphi(1-\alpha)}{\alpha}\Bigg)=0 \\
\tilde{b_{t,L}} :&\frac{\beta b^*}{1-\beta\rho}(\phi_{5,t}-E_t\phi_{5,t+1})=0
\end{aligned}
$$

Note that for convenience next to each of the FONC we have written the corresponding variable.

A couple of remarks are important here. Note that the above system of first order conditions (together with the constraints) form the system of equations that needs to be resolved (numerically) to find the optimal policy equilibrium. As usual in the case of full commitment policies, the first order conditions feature lags and leads of Lagrange multipliers. The lagged multipliers reveal a well known property of this class of models, 'history dependence' (see Aiyagari et al. (2002)). Since in the constraint set we have expectations of future economic variables (e.g. inflation) then optimal policy makes promised about the future to manipulate expectations. The lags of the multipliers capture essentially those promises that were made in past periods about the period t allocation.

Moreover, note that the lead terms that appearing in the system above, concern the optimality conditions for debt and public capital. These conditions can be read as the Euler equations that derive under optimal public policy. For example, consider the last line of the system, which can be simplified to $\phi_{5,t} = E_t\phi_{5,t+1}$. According to this condition, the multiplier on the consolidated budget constraint, $\phi_{5,t}$, evolves as a pure random walk under optimal policy. This property is standard (e.g. Aiyagari et al (2002)). Under optimal policy, the multiplier measures the burden to society from distortionary taxation (or distortionary inflation) that finances debt and the random walk property means that the planner desires to spread the distortions evenly across periods.

The optimality condition with respect to capital has an analogous interpretation.

Finally, note that the above system can be easily solved with standard techniques for solving linear stochastic systems of difference equations. Since there is nothing new here, we do not provide a description of the numerical procedure used.

5.3 Time-consistent problem

The previous section described the optimal policy under full commitment. We now consider the opposite scenario, assuming that the optimizing government cannot commit to future policies at all, in other words we solve the problem of time-consistent optimal policy. There are several reasons why we want to consider both policy environments in this paper. First, it is well known that for a class of models similar to this one under commitment Ramsey policy makes much less use of inflation to finance debt than under no-commitment. For example Faraglia et al. (2013) study optimal monetary and fiscal policy in a Ramsey model with exogenous spending and conclude that inflation's role in stabilizing is only limited in the commitment case, whereas it is considerably more important in the time consistent allocation. Though this is not exactly the environment we have here (we have to extend this finding to the case of endogenous public consumption and investment) we motivate the time consistent scenario based on their finding. Second, the assumption that fiscal policy cannot commit to future allocations can perhaps be seen as extreme, but so is the full commitment assumption. In reality governments may be able to partially commit to policies for a short horizon (enact a spending plan that determines the path of fiscal variables for a few months or years) but they probably cannot commit to an entire path of state contingent policies (over the infinite horizon) as is the case in the full commitment model laid out previously. We thus follow much of the literature in considering the no-commitment scenario, to study policy under the two polar opposite assumptions, being aware that reality may be somewhere in between.

As it is standard in the literature we solve for optimal policy under no-commitment focusing on Markov perfect equilibria where the actions of the government and the private sector are assumed to be well behaved functions of a vector of state variables determining the relevant payoffs. We assume that this state vector is $(K_{G,t-1}, b_{t-1,L}, \epsilon_t)$ $((\widetilde{k_{G,t-1}}, \widetilde{b_{t-1,L}}, \tilde{\epsilon_t})$ in terms of the notation of the linearized model). Note also that in a Markov perfect equilibrium, private sector expectations are to be replaced by a linear functions of $((\widetilde{k_{G,t}}, \widetilde{b_{t,L}}, \tilde{\epsilon_t})$ as we elaborate below. [6]

We now provide the definition of the optimal policy Markov Perfect Equilibrium.

Definition: Markov Perfect Equilibrium The equilibrium consists of a value function V,and decision rules for quantities $\{y_t, \pi_t\}$ and policy variables $\{R_t, g_t^i, g_t^c, k_{G,t}, b_{t,L}, \tau_t\}$ which are all functions of the state vector $(\widetilde{k_{G,t-1}}, \widetilde{b_{t-1,L}}, \tilde{\epsilon_t})$ and such that for given initial conditions $\widetilde{k_{G,-1}}, \widetilde{b_{-1,L}}$, the quantities and policies solve the following problem:

$$
\begin{aligned}
&\max_{\{y_t,\pi_t,R_t,g_t^i,k_{G,t},w_t,b_t\}} V(\widetilde{k_{G,t-1}}, \widetilde{b_{t-1}}, \epsilon_t) = -L_t + \beta E_t V(\widetilde{k_{G,t}}, \tilde{b_t}, \widetilde{\epsilon_{t+1}}) \\
&+ \phi_{1,t}\left[\tilde{R}_t + \sigma(\frac{y^*}{c^*}\tilde{y_t} - \frac{g^{i,*}}{c^*}\tilde{g_t^i} - \frac{g^{c,*}}{c^*}\tilde{g_t^c}) - \tilde{\epsilon_t} + h_1(\widetilde{k_{G,t}}, \tilde{\epsilon_t}, \tilde{b_t})\right] \\
&+ \phi_{2,t}\left[\tilde{\pi_t} - \frac{\theta}{\psi}\left(\tilde{w_t} - \tilde{y_t} + \frac{\tilde{y_t} - (1-\alpha)\widetilde{k_{G,t}}}{\alpha}\right) - \beta h_2(\widetilde{k_{G,t}}, \tilde{\epsilon_t}, \tilde{b_t})\right] \\
&+ \phi_{3,t}\left[\tilde{k}_{G,t} - (1-\delta_G)\tilde{k}_{G,t-1} - \delta_G \tilde{g^i}_t\right] \\
&+ \phi_{4,t}\left[\tilde{R}_t - \phi_\pi \tilde{\pi}_t\right] \\
&+ \phi_{5,t}\Bigg[q^* b^* \tilde{b_t} - q^* b^* \beta^{-1}\widetilde{b_{t-1}} + q^* b^* \beta^{-1}\tilde{\pi}_t - q^* b^*(1-\rho)\Bigg(- \sigma(\frac{y^*}{c^*}\tilde{y_t} - \frac{g^{i,*}}{c^*}\tilde{g_t^i} - \frac{g^{c,*}}{c^*}\tilde{g_t^c}) + \tilde{\epsilon_t} \\
&- h_1(\widetilde{k_{G,t}}, \tilde{\epsilon_t}, \tilde{b_t}) - \beta\rho h_3(\widetilde{k_{G,t}}, \tilde{\epsilon_t}, \tilde{b_t})\Bigg) - g^{i,*}\tilde{g_t^i} - g^{c,*}\tilde{g_t^c} \\
&+ \tau^* n^* w^*\left(\frac{\tilde{y_t} - (1-\alpha)\widetilde{k_{G,t}}}{\alpha} + \tilde{w_t} + \frac{1-\tau^*}{\tau^*}\left(\tilde{w_t} - \varphi\frac{\tilde{y_t} - (1-\alpha)\widetilde{k_{G,t}}}{\alpha} - \sigma(\frac{y^*}{c^*}\tilde{y_t} - \frac{g^{i,*}}{c^*}\tilde{g_t^i} - \frac{g^{c,*}}{c^*}\tilde{g_t^c}) + \tilde{\epsilon_t}\right)\right)\Bigg]\Bigg\}
\end{aligned}
$$

[6] It should be noted that New-Keynesian models under time-consistent policies can lead to multiple equilibria even in the linear quadratic setup, as pointed out by Blake and Kirsanova (2012). According to Blake and Kirsanova (2012) there are two types of equilibria: the "point-in-time", which arises as a consequence of different decisions of the private sector for a given government policy, and the ones arising from different discretionary rules. We do not address this here.

As before, we use $\phi_{i,t}$ for $i = 1, .., 5$ to denote the multipliers attached to the constraints.

Note that the objects included in the above definition are the variables that appear in the sufficient implementability conditions for a competitive equilibrium under optimal time consistent policies. Therefore, using the same primal approach as in the commitment model of the previous section we have substituted out the labour input and the marginal cost of firms. Having solved for the optimal policy equilibrium these quantities can be easily recovered from:

$$\tilde{n_t} = \frac{(\tilde{y}_t - (1-\alpha)\tilde{k_{G,t-1}})}{\alpha}$$
$$\tilde{mc_t} = \tilde{w}_t - \tilde{y}_t + \tilde{n_t}$$

which are the production function and the firm's optimality condition with respect to labor respectively.

Finally, let us explain the terms denoted with an h that appear in the constraint set in the above program. As is well known, under time-consistent public policies the government cannot make promises for future variables. Therefore terms containing expectations about future variables, such as $E_t\tilde{\pi}_{t+1}$ or $E_t\tilde{y}_{t+1}$, need to be replaced by functions of the state vector, to acknowledge that the only influence that optimal policy can have on expectations is through setting future state variables.

Define the expectations contained in Euler equations for short and long-term bonds and the Phillips curve as $h_1(.), h_2(.), h_3(.)$ respectively. Since we are assuming a Markov Perfect Equilibrium, these expectations are functions of the state variables, $\tilde{k_{G,t}}, \tilde{\epsilon}_t, \tilde{b_t}$.

We therefore have the following definitions:

$$h_{1,t}(\tilde{k_{G,t}}, \tilde{\epsilon}_t, \tilde{b_t}) = E_t\Bigg[-\sigma\left(\frac{y^*}{c^*}\tilde{y_{t+1}} - \frac{g^{i,*}}{c^*}\tilde{g^i_{t+1}} - \frac{g^{c,*}}{c^*}\tilde{g^c_{t+1}}\right) - \tilde{\pi_{t+1}} + \tilde{\epsilon_t}\Bigg]$$
$$h_{2,t}(\tilde{k_{G,t}}, \tilde{\epsilon}_t, \tilde{b_t}) = E_t[\tilde{\pi_{t+1}}]$$
$$h_{3,t}(\tilde{k_{G,t}}, \tilde{\epsilon}_t, \tilde{b_t}) = E_t[\tilde{q_{t+1}}]$$

Finally, at the equilibrium obviously, the future variables $\tilde{q_{t+1}}, \tilde{g^c_{t+1}}$ etc, are also functions of the state vector as the definition of the Markov equilibrium makes clear. These functions, as well as the functions h are assumed to be well behaved (continuously differentiable). In this linear quadratic program we obviously assume linear functions of the state variables.

Solving the value function above gives us the following set of first order conditions in the time consistent optimal policy model:

$$
\begin{aligned}
\tilde{y_t} :& -\omega_y\tilde{y_t} - \omega_{yg^i}\tilde{g^i_t} - \omega_{yg^c}\tilde{g^c_t} - \omega_{yk_g}\tilde{k_{G,t-t}} - \omega_{\epsilon y}\tilde{\epsilon_t} + \sigma\frac{y^*}{c^*}\phi_{1,t} + \phi_{2,t}\frac{\theta}{\psi} \\
&+ \phi_{5,t}\left[\frac{\beta b^*}{1-\beta\delta}(1-\rho)\sigma\frac{y^*}{c^*} + w^*n^*\tau^*\left(\frac{1}{\alpha} - \frac{1-\tau^*}{\tau^*}\left(\frac{\varphi}{\alpha} + \sigma\frac{y^*}{c^*}\right)\right)\right] = 0 \\
\tilde{\pi_t} :& -\omega_\pi\tilde{\pi_t} + \phi_{2,t} - \phi_{4,t}\phi_\pi - \frac{b^*}{1-\beta\rho}\phi_{5,t} = 0 \\
\tilde{R_t} :& \phi_{1,t} + \phi_{4,t} = 0 \\
\tilde{g^i_t} :& -\omega_{g^i}\tilde{g^i_t} - \omega_{yg^i}\tilde{y_t} - \omega_{g^cg^i}\tilde{g^c_t} - \sigma\frac{g^{i,*}}{c^*}\phi_{1,t} - \phi_{3,t}\delta_G \\
&+ \phi_{5,t}\left(-\frac{\beta b^*}{1-\beta\delta}(1-\rho)\sigma\frac{g^{i,*}}{c^*} - g^{i,*} + n^*w^*(1-\tau^*)\sigma\frac{g^{i,*}}{c^*}\right) = 0 \\
\tilde{g^c_t} :& -\omega_{g^c}\tilde{g^c_t} - \omega_{yg^c}\tilde{y_t} - \omega_{g^cg^i}\tilde{g^i_t} - \sigma\frac{g^{c,*}}{c^*}\phi_{1,t} \\
&+ \phi_{5,t}\left(-\frac{\beta b^*}{1-\beta\delta}(1-\rho)\sigma\frac{g^{c,*}}{c^*} - g^{c,*} + n^*w^*(1-\tau^*)\sigma\frac{g^{c,*}}{c^*}\right) = 0 \\
\tilde{k_{G,t}} :& \beta E_t\frac{\partial V(.)}{\partial\tilde{b_t}} - \phi_{1,t}\frac{\partial h_1(.)}{\partial\tilde{k_{g,t}}} - \phi_{2,t}\beta\frac{\partial h_2(.)}{\partial k_{G,t}} + \phi_{3,t} - \phi_{5,t}\frac{\beta b^*}{1-\beta\rho}(1-\rho)\left(\frac{\partial h_1(.)}{\partial\tilde{k_{g,t}}} - \beta\rho\frac{\partial h_3(.)}{\partial\tilde{k_{g,t}}}\right) = 0 \\
\tilde{w_t} :& -\frac{\theta}{\psi}\phi_{2,t} + \phi_{5,t}\left[w^*n^*\tau^*\left(1 + \frac{1-\tau^*}{\tau^*}\right)\right] = 0 \\
\tilde{b_{t,\rho}} :& \beta E_t\frac{\partial V_{t+1}}{\partial\tilde{b_t}} - \phi_{1,t}\frac{\partial h_1(.)}{\partial\tilde{b_t}} - \phi_{2,t}\beta\frac{\partial h_2(.)}{\partial\tilde{b_t}} + \phi_{5,t}\frac{\beta b^*}{1-\beta\rho}\left(1 - (1-\rho)\left(\frac{\partial h_1(.)}{\partial\tilde{b_t}} - \beta\rho\frac{\partial h_3(.)}{\partial\tilde{b_t}}\right)\right) = 0
\end{aligned}
$$

Using the envelope theorem, we can replace for the derivative of the value function with respect to the states variables: $\tilde{k_{G,t}}$ and $\tilde{b_t}$ in the first order condition for debt and public capital

$$
\begin{aligned}
E_t\frac{\partial V_{t+1}}{\partial\tilde{k_{G,t}}} &= -\beta E_t(\omega_{k_g-1}\tilde{k_{G,t}}) - (1-\delta_G)\beta E_t\phi_{3,t+1} \\
&+ \beta E_t\phi_{5,t+1}\left(-n^*w^*\tau^*\frac{(1-\alpha)}{\alpha} + (1-\tau^*)n^*w^*\frac{\varphi(1-\alpha)}{\alpha}\right) = 0
\end{aligned}
$$

$$
E_t\frac{\partial V_{t+1}}{\partial b_t} = \frac{\partial L_t}{\partial b_t} = -\frac{\beta b^*}{1-\beta\rho}E_t\phi_{5,t+1}
$$

and as a consequence, we can write the first order condition for public capital, $\tilde{k_{G,t}}$, and government debt, $\tilde{b_t}$ as

$$
\begin{aligned}
\tilde{k_{G,t}} :& -\beta E_t(\omega_{k_g-1}\tilde{k_{G,t}}) - (1-\delta_G)\beta E_t\phi_{3,t+1} + \beta E_t\phi_{5,t+1}\left(-n^*w^*\tau^*\frac{(1-\alpha)}{\alpha} + (1-\tau^*)n^*w^*\frac{\varphi(1-\alpha)}{\alpha}\right) \\
&- \phi_{1,t}\frac{\partial h_1(.)}{\partial\tilde{k_{g,t}}} - \phi_{2,t}\beta\frac{\partial h_2(.)}{\partial k_{G,t}} + \phi_{3,t} - \phi_{5,t}\frac{\beta b^*}{1-\beta\rho}(1-\rho)\left(\frac{\partial h_1(.)}{\partial\tilde{k_{g,t}}} - \beta\rho\frac{\partial h_3(.)}{\partial\tilde{k_{g,t}}}\right) = 0 \\
\tilde{b_{t,\rho}} :& -\frac{\beta b^*}{1-\beta\rho}E_t\phi_{5,t+1} - \phi_{1,t}\frac{\partial h_1(.)}{\partial\tilde{b_t}} - \phi_{2,t}\beta\frac{\partial h_2(.)}{\partial\tilde{b_t}} + \phi_{5,t}\frac{\beta b^*}{1-\beta\rho}\left(1 - (1-\rho)\left(\frac{\partial h_1(.)}{\partial\tilde{b_t}} - \beta\rho\frac{\partial h_3(.)}{\partial\tilde{b_t}}\right)\right) = 0
\end{aligned}
$$

It is worthwhile to briefly compare this system of first order conditions with the analogous system we derived in the previous subsection, under commitment. Note first, in contrast to the commitment case the system now does not feature any lagged multipliers. Instead the FONC features the partial derivatives of the functions h with respect to their arguments. The interpretation is however analogous. Whereas under commitment the lags were capturing the promises that the planer made with regard to future macroeconomic variables, the derivatives h capture the planner's incentive to manipulate expectations by setting the path of the next periods state variables. Under commitment the lags of the multipliers were state variables of the model, here (by the definition of the equilibrium) the state vector is $(\tilde{k_{G,t-1}}, \tilde{b_{t-1,L}}, \tilde{\epsilon}_t)$.

Note also that just like in the commitment model the system of FONC in the time consistent solution features expectations for the multipliers, $\phi_{i,t+1}$. Again these are related to the intertemporal choices the planner has to make with respect to public capital and debt. In the time consistent solution these expectations can also be expressed as functions of the state vector.

Lastly, note that solving for optimal policies in this case is not trivial (as it was under commitment). To find the solution we need to parameterize the conditional expectations and guess the values of the coefficients of the linear functions. Then, an iterative procedure needs to be followed, to update these coefficients until they converge (do not change with additional iterations). More details on this solution technique can be found in appendix D.

6 Quantitative experiments.

6.1 Calibration

Table 1 in appendix E contains the main values for the parameters used to perform the quantitative experiments. We consider each period as a quarter in time and we calibrate the parameters for the US economy. We set β, the usual discount factor in 0.99, thus the interest rate has an annual value of 4%.

[Table 1 approximately here]

We assume the production function is a Cobb-Douglas. Output elasticity to public capital is set to 0.06, which is within the range of 0.03 - 0.1 estimated by the literature on public capital (see Ramey (2020) for a review). Labor intensity is set in 0.94. According to data from the Federal Reserve Bank of St. Louis, total government expenditure for the US as percentage of GDP for the period 1990-2018 has been at around 20%, from which a 4.5% belongs to public investment. So, we let government consumption G_t^c to be 0.15 of output at the steady-state. δ, the depreciation rate for public capital, is set to be 0.03 which, given β, α and the ratio for government consumption to GDP targets public investment expenditures as percentage of total government expenditure at around 24% which is the figure observed in the data. This value for δ is close to Bouakez et al. (2019), Leeper et al. (2010) and Drautzburg and Uhlig (2015) who sets the value in 0.02.

Elasticity of substitution for intermediate goods θ is set to be 6, in line with a mark-up of 20% in the steady state. The parameter ψ governing the price adjustment cost is set to 200, such that the slope for the linear Phillips curve (θ/ψ) is 0.03, as in Bouakez et al. (2019). Gross steady state inflation is assumed to be 1, meaning a zero inflation steady-state.

We set risk aversion, σ, to 1.5 and the inverse Frisch elasticity, φ to 2. Both Drautzburg and Uhlig (2015) and Leeper et al. (2010) estimates DSGE models with public capital and debt using preferences with habit formations with respect to consumption in the previous period and without the public good. They set the mean for risk aversion in 1.5 and 1.75 respectively and the inverse of the Frisch elasticity in 2.

We set the debt level to be 60% of steady-state output as much of the literature on optimal fiscal policy. Together with government spending as percentage of output, this figure gives a labor tax rate in the steady-state close to 17%. We fix ρ, the decaying coupon factor for long-term debt to be 0.95. This allows to match an average duration for the public debt of 16 quarters, a close figure to that of developed economies.

We calibrate the autocorrelation parameter of the preference shock to be 0.9 and its variance in 0.07, so output falls by around to 2% with respect to the steady state at the peak of the recession when monetary policy is active. This is close to the maximum drop registered between two quarters in US GDP: 2.2% between Q3-2008 and Q4-2008. We assume the shock evolves as:

$$log(\epsilon_t) = \rho_\epsilon log(\epsilon_{t-1}) + \nu_t, \;\; \nu_t \sim \mathcal{N}(0, \sigma_\nu^2)$$

To analyze the economy under the two regimes, active and passive monetary policy, we allow for different values for the response of the interest rate to inflation. When monetary policy is active, the parameter of the Taylor rule is fixed in 1.5. This value is set according to Woodford (2001) and is the most frequent value used by the literature when calibrating a Taylor rule for the US economy, even though some empirical works estimates the coefficient well above 1.5 (Mehra and Minton (2007); Carvalho (2019)) in the post-Volcker era (1979-2007).

When considering a passive regime for monetary policy, we consider two values for the parameter ϕ_π: 0.7 and 0.95. The empirical evidence points out the pre-Volcker era, 1955-1979, as a period with weak response of interest rate to inflation and estimates the coefficient to be between 0.8 and 0.9 (Carvalho (2019)). In DSGE model estimated via bayesian methods, such as Bianchi (2013), the coefficient is close to 0.9 when monetary policy is passive. Thus we choose two values, 0.95 and 0.7. The first one is a value close to the estimation above when monetary policy is passive. The second one is an arbitrary value below the estimated one, which makes the interest rate even less responsive to inflation.

6.2 Optimal policies under commitment

Figure 3 in appendix E shows the optimal response of the economy to a negative consumption shock. The blue line shows the economy under an active monetary policy rule while the red and orange lines shows optimal policies under passive monetary policy rule with $\phi_\pi = 0.95$ and $\phi_\pi = 0.7$, respectively. As can be seen in the figure, the optimal path for private consumption, labor and output are very similar in both regimes, though it is not the case for inflation, the interest rates and public debt.

[Figure 3 approximately here]

The shock reduces private consumption, reducing output and hours. The optimizing planner uses government consumption and investment in response to the shock by increasing both variables above the level of the efficient steady-state. This can be seen under both regimes, active and passive monetary policy.

Clearly, the planner pursues such a policy as a way of closing the output gap after the shock arrives. Public investment boosts aggregate demand like public consumption does, but it also stimulates aggregate supply by increasing public capital, an input for production. Public investment grows from 4.5% of output in steady state to a peak of almost 6%. Meanwhile, public consumption raises from 15% of steady state output to a peak of 15.5%. The government relies more on investment than in government consumption: public investment climbs from 23% of total government expenditure in the steady-state to almost 28%.

The shock affects directly private consumption, but since it does not affects public consumption (in the utility function, the shock ϵ only affects c_t) it changes the optimal provision of public consumption by altering the marginal rate of substitution between public and private consumption. The first order condition for the planner with respect to public consumption, which we re-stated here for exposition purposes, is given by,

$$-\omega_{g^c}\tilde{g}_t^c - \omega_{yg^c}\tilde{y}_t - \omega_{\epsilon g^c}\tilde{\epsilon}_t + \sigma\frac{g^{c,*}}{c^*}\left(\beta^{-1}\phi_{1,t-1} - \phi_{1,t}\right) +$$
$$\frac{\beta b^*}{1-\beta\rho}(1-\rho)\sigma\frac{g^{c,*}}{c^*}\left(\beta^{-1}\phi_{5,t-1} - \phi_{5,t}\right) + \phi_{5,t}\left(-g^{c,*} + n^*w^*(1-\tau^*)\sigma\frac{g^{c,*}}{c^*}\right) = 0$$

In presence of a balanced budget, the multiplier $\phi_{5,t} = 0$ and the planner will try to equate the marginal loses of increasing public consumption given by the terms $-\omega_{g^c}\tilde{g}_t^c - \omega_{yg^c}\tilde{y}_t - \omega_{\epsilon g^c}\tilde{\epsilon}_t$ with the welfare gains in terms of private consumption given by the the Euler equation and its respective multiplier ϕ_1. When distortionary taxes and debt are included, the planner needs also to balance the changes in the budget constraint of the government due to changes in tax and debt to finance government expenditures. Since in this model, the planner is able to choose optimal the tax rate, it needs to take into account the distortions from taxes. [7]

The increase in government expenditures is financed by an increase in distortionary taxes under both regimes, which came due to the ability of the planner to choose taxes optimally. From the perspective of the planner, a higher tax rate increases marginal costs and wages helping to reduce deflationary pressures. The tax rate increases from 17% in the steady-state to a maximum of 20% in both regimes. This result for the tax rate is very different from the response we normally observe in models with fiscal rule for taxes and passive monetary policy. Under ad-hoc rules, the FTPL poses that taxes react too little to changes in debt (or does not react at all), so the government relies on inflation to relax the budget constraint. With optimal taxes, we do not observe this behaviour in presence of passive monetary policy. On the contrary, under optimal policies, the planner can adjust taxes in the same way as under active monetary policy, helping the government to increase public expenditure and reduce output and consumption volatility.

Meanwhile, the government pursues a debt reduction scheme under both scenarios, active and passive monetary policy. The market value of debt falls from the 60% of output at the steady-state to nearly 50% when $\phi_\pi = 0.95$ and to 55% when $\phi_\pi = 0.7$. With $\phi_\pi = 1.5$ debt to output falls to 55%. The optimal debt reduction is larger under passive monetary policy, and the effect is long lasting due to more persistent policies and higher inflation rates. Figure 3 shows that inflation picks up faster when monetary policy is passive, though interest rates reacts weakly so the real interest rate becomes higher and the bond price lower reducing further the market value of debt in the passive monetary policy regime.

The mechanism identified by the FTPL, in which a passive monetary policy regime leads to higher inflation as a way of closing the output gap is still visible, even with optimal fiscal policy. Nevertheless, the planner rely less on inflation than in the case with rules, due to the ability to choose taxes in an optimal way.

6.3 Optimal policies under discretion

In this section, we turn to analyze optimal policies when the planner can not commit. Figure 4 in appendix E shows the response of the economy to a negative shock on private consumption (preferences shock). The blue line corresponds to the active monetary policy regime while the red and orange lines shows the economy under passive monetary policy regime, for $\phi_\pi = 0.95$ and $\phi_\pi = 0.7$ respectively. As was the case under commitment, the main conclusion of the paper, namely the volatility of macro aggregates is similar

[7] A similar condition can be observed from government investment

under active and passive monetary policy, still holds when the planner can not commit to its policies. Indeed, if so, under discretion the difference between the two regimes tends to be even smaller than under commitment. This could be due to the fact that the planner can not commit to policies, which reduces its ability to spread the effects of the shock over time.

As was the case under commitment, the shock reduces private consumption and hours, increasing the desire to save. As a consequence, hours and output fall and inflation drops. The interest rate follows inflation due to the presence of a Taylor rule and falls as well. The fall in consumption, hours and output is slightly more pronounced with passive monetary policy (red and orange lines), though the difference is negligible.

As it can not commit to its policies, the ability of the planner to spread the costs of the shock over time is reduced, so much of the shock is absorbed by the economy as soon as the shock arrives. Discretion leads to policies which are considerable more volatile than the the ones prescribed under commitment. Inflation, for instance, falls by 4%, while it was nearly 0.2% under commitment. Output falls by 3.7% under discretion, while under commitment it does so by 2%.

From the figures, we can see that the path for consumption, hours, and output are very similar regardless of monetary policy. Under discretion, the planner will rely on public debt and public capital to affect private sector expectations about the future path for inflation and output. Although, this time, the planner will try to avoid the deflationary scenario due to the shock. By issuing long-term bonds, the government influences the private sector, who expects higher levels of inflation in the future will be needed to close the budget constraint of the government (a similar mechanism to that of Eggertsson (2011)). This is specially the case for passive monetary policy, due to the weak responsiveness of the interest rate. Market value for public debt increases from 60% of steady-state output to a 67% when monetary policy is active, but surpasses the 70% when monetary policy is passive.

On the other hand, the planner cuts initially public capital and government investment falls by near 40% with respect to its steady state level when monetary policy is passive and by 35% when monetary policy is active. This, of course, means the planner relies on the supply side effect of government investment. As noted by the literature on public capital, the latter has mainly two effects: a supply side and a demand side effect. On the one hand, public capital is an input for production, so an increase in public capital raises marginal productivity of labor (the private input), increasing output and reducing prices. This would be the supply side effect we can see in figure 4 . At the same time, an increase in public investment has a wealth effect over households, increasing private expenditure and prices. This would be the demand side effect we saw under commitment.

Despite both effects are present, under commitment, the planner raises public capital, while under discretion, the result is the opposite. The reason is that under commitment the planner respects the future chosen path for public capital and can commit to future drops in government investment to avoid future increases in prices when the economy recovers. This effect is absent when the planner renegate on past promises, so the time-consistent planner uses public capital to fight deflation back as soon as the shock arrives. By reducing public capital, the planner increases future marginal costs, conducting future prices increases by the firm due to an input supply shortage. Importantly, we abstract from time-to-build, the delay in government investment execution, so the supply side effect of public capital comes to play immediately.

[Figure 4 approximately here]

At the same time, the planner increases public consumption as an effort to close the output gap. Government consumption reaches 17% of output, so 2 p.p. above from the steady-state. Reducing public investment and increasing public consumption shifts the optimal composition of public expenditure under discretion towards public consumption, unlike the commitment case. Under discretion, public consumption represents around 17% of steady-state output at the peak of the crisis; 2 p.p. above the level observed in steady state. On the other hand, public investment as a share of the steady-state output declines from 4.5% to around 3% at the peak of the crisis before raising to 5.5% as soon as inflation picks up. As percentage of total public expenditure, government investment declines from 23% in the steady state to 15% at the peak of the crisis. [8] As inflation picks up, the planner can increase public capital and investment to boost the economy again. Deflation is actually short-lived, while output and private consumption take more time to return to the steady-state. Thus the planner can increase public investment as long as deflation remains under control.

Contrary to the commitment allocation, the planner cuts distortionary labor taxes, stimulating labor supply. Taxes falls from 17% in the steady-state to levels close to zero in both scenarios: active and passive monetary policy, but falling slightly more under the latter. Under both scenarios, the planner increases fiscal deficit, which would be financed by debt, raising inflation expectations by the private sector.

So, despite the prescribed policies under discretion are different from the commitment scenario, the main result of the paper does not change. The volatility of macro aggregates, namely private consumption, hours an output, are similar under passive and active monetary policy.

[8] Note that by cutting government investment, the government creates some space for increasing government consumption and cutting taxes, while keep debt in a sustainable path

7 Extensions

In the previous section we show that when the economy is hit by a negative preference shock over consumption, there is small differences in the optimal equilibrium allocations under active/passive monetary policy. We also show, the result is preserved if the planner is not able to commit to its policies. In this section, we seek to determine if this result holds under different scenarios.

We first allow for a Taylor rule targeting not only inflation, as was in the previous section, but also output. Secondly, we allow for other demand shocks well studied in the literature: a shock in preference for government consumption, which makes public consumption expenditure quasi exogenous, and a liquidity premium shock, which affects the return for government bonds.

7.1 Taylor rule targeting inflation and output

Consider now a Taylor rule targeting not only inflation, but also output. This form of the Taylor rule considers two objectives for monetary policy as it seeks to stabilize inflation and reduce output volatility. It is also a rule for monetary policy widely used in the literature on DSGE models.

We can write the new Taylor rule as,

$$\tilde{R}_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t$$

With active monetary policy, $\phi_\pi = 1.5$, the Taylor principle is valid, so the stable equilibrium requires a combination of the parameters satisfying the well-known following condition (with κ being the slope of the Phillips curve with respect to real marginal cost),

$$\kappa(\phi_\pi - 1) + \phi_y(1 - \beta) \geq 0$$

Meanwhile, when monetary policy is passive, the sufficient condition to have a determined equilibrium becomes the following

$$\phi_\pi + \frac{\phi_\pi(1 - \beta)}{\kappa} \leq 1$$

With passive monetary policy, we set ϕ_π at 0.95 and 0.7, thus considering the last value (0.7), the condition for a stable equilibrium requires values for ϕ_y lower than 0.3. We set ϕ_y at 0.1. Note this value also satisfies the condition for a stable equilibrium when monetary policy is active.

Figure 5 shows the result from previous section holds when the Taylor rule targets inflation and output and the planner can commit to its policies. The negative shock reduces private consumption, hours and output, hence inflation enters negative territory and output gap becomes negative. This time, the interest rate reacts to inflation and output falling by more than in the previous case, helping to stabilize better the economy. Market value of debt initially raises financing the increase in public consumption and investment.

[Figure 5 approximately here]

When the planner can not commit to its policies, the main result of the paper does not change and 6 shows there is no major differences in the optimal response of the economy under passive/active monetary policy when the Taylor rule responds to output and inflation. Qualitatively, the the main difference is that the time-consistent planner increases public capital promoting economic recovery. Output, of course, falls by much less than with a Taylor rule targeting only inflation.

[Figure 6 approximately here]

7.2 Liquidity premium shock

This section considers a liquidity premium, a shock usually considered by the literature able to mimic relatively well the effects of a negative demand shock. We will show that with this shock, the path for optimal policies such as private consumption, hours and output is similar regardless the response of monetary policy to inflation.

To introduce the shock consider the budget constraint of the household from the main section,

$$P_t c_t + \zeta_t q_{t,S} B_{t,S} + \zeta_t q_{t,L} B_{t,L} = (1-\tau_t) W_t n_t + P_t D_t + B_{t-1,S} + (1+\rho q_{t,L}) B_{t-1,L} + T_t$$

We add ζ_t, an exogenous shock to the returns on government bonds (short and long-term). We interpret this shock as a liquidity premium shock similar to Smets and Wouters (2007). A negative shock on ζ_t increase the returns on government bonds, making them more attractive. Private consumption and hours fall as a consequence of the shock

To perform the quantitative experiment, we assume the shock evolves as,

$$log(\zeta_t) = \rho_\zeta log(\zeta_{t-1}) + \nu_t, \;\; \nu_t \sim \mathcal{N}(0, \sigma_\nu^2)$$

We calibrate the shock to have a persistence, $\rho_\zeta = 0.8$ and a variance $\sigma_\nu = 0.01$, so we observe consumption dropping by the same magnitude that in the experiment of the main section.

The household problem is similar to the one in the previous section 4, but the optimality conditions for the households are now given by:

$$(1-\tau_t)w_t = \frac{\chi n_t^{\varphi}}{c_t^{-\sigma}}$$
$$q_{t,S}\zeta_t C_t^{-\sigma} = \beta E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right]$$
$$q_{t,L}\zeta_t C_t^{-\sigma} = \beta E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right](1+\delta q_{t+1,L})$$

while the rest of the constraints satisfying a competitive equilibrium does not change.

Log-Linear version of this equations are given by

$$\tilde{w_t} = \varphi\tilde{n_t} + \sigma\tilde{c_t} + \tfrac{\tau^*}{(1-\tau^*)}\tilde{\tau}_t$$
$$\tilde{R_t} = -\sigma\tilde{c_t} + \tilde{\zeta_t} + E_t[\sigma\widetilde{c_{t+1}} + \widetilde{\pi_{t+1}}]$$
$$\widetilde{q_{t,L}} = -\tilde{R_t} + \beta\rho E_t[\widetilde{q_{t+1,L}}]$$

Different from equations defining a competitive equilibrium in section 4, the condition for labor supply does not carry the shock, while the Euler equation for short term bond does not carry the value for the shock in the next period $t+1$.

There is a change in the preferences due to the absence of the shock over private consumption from section 3, so the second order approximation to the utility function also changes. The terms considering the interactions between the consumption shock and the main variables g_t^i,g_t^c, y_t and $k_{G,t}$ are no longer there. When the shock is included, the loss function can be written as [9]:

$$-E_0\sum_{t=0}^{\infty}\beta^t L_t$$

$$L_t = \omega_{g^i}\frac{\tilde{g_t^i}^2}{2} + \omega_y\frac{\tilde{y_t}^2}{2} + \omega_\pi\frac{\tilde{\pi_t}^2}{2} + \omega_{g^c}\frac{\tilde{g_t^c}^2}{2} + \omega_{k_g-1}\frac{\widetilde{k_{G,t-1}}^2}{2} + \omega_{yg^i}\tilde{y_t}\tilde{g_t^i} + \omega_{yg^c}\tilde{y_t}\tilde{g_t^c} + \omega_{g^ig^c}\tilde{g_t^i}\tilde{g_t^c} + \omega_{yk}\tilde{y_t}\widetilde{k_{G,t-1}} \quad (12)$$

The problem for the planner stays similar to those described in section 5 and 5.3, but now the planner

[9] We refer the reader to appendix B for a full description of the utility function.

will minimize function 12 subject to the new set of first order conditions for the household, the government budget constraint and the Phillips curve, whose linearized version do not change with respect to the ones already presented in section 5 and 5.3.

Figure 5 shows the response of the economy to a liquidity premium shock under commitment, where we can see there is no difference regarding monetary policy. The shock reduces consumption and labor, because households demand larger levels of government debt. This creates a demand driven recession that translated into a drop in output and inflation.

As was the case with the negative consumption shock, the planner increases government expenditure (public consumption and investment) to close the output gap, while cuts public debt and increases the tax rate. Figure 7 shows that optimal path for consumption, output and hours are very similar regardless the response of the interest rate to inflation.

[Figure 7 approximately here]

This is not the case for the case of inflation and public debt, which shows striking differences depending on the response of the interest rate on inflation. Inflation rate picks up fast with passive monetary policy, while the market debt of public debt adjust by more.

[Figure 8 approximately here]

The IRF to a liquidity premium shock under discretion is depicted in figure 8. As was the case under a preference shock, the government cuts the tax rate and government investment as a way of increasing marginal cost and prices. Meanwhile it increases government spending to close the output gap. The fiscal deficit is financed by debt issuance. Nevertheless, within a time-consistent framework, there is almost no difference regarding the response of monetary policy to inflation.

7.3 Shock to preferences on government consumption

In this section, we explore a shock to preferences over the consumption of public goods. Consider the preferences,

$$E_t \sum_{s=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \xi_t \frac{\chi_g G_t^{c,1-\sigma}}{1-\sigma} \right)$$

where ξ_t is a shock to preferences on government consumption expenditure. Being positive, the shock increases households demand for public goods and government consumption expenditure, resembling an expansionary fiscal shock. In this way government consumption expenditure become quasi exogenous, because, besides the shock, the planner can still choose in some degree the optimal level of government consumption expenditure.

To include the shock in the planner problem we change the welfare loss function. There is now an interaction term accounting for the shock and the government consumption expenditure. The welfare loss function can be written as follows,

$$-E_0 \sum_{t=0}^{\infty} \beta^t L_t$$

$$L_t = \omega_{g^i} \frac{\tilde{g_t^i}^2}{2} + \omega_y \frac{\tilde{y_t}^2}{2} + \omega_\pi \frac{\tilde{\pi_t}^2}{2} + \omega_{g^c} \frac{\tilde{g_t^c}^2}{2} + \omega_{k-1} \frac{\tilde{k_{t-1}}^2}{2} + \omega_{yg^i} \tilde{y}_t \tilde{g_t^i} + \omega_{yg^c} \tilde{y}_t \tilde{g_t^c} + \omega_{g^i g^c} \tilde{g_t^i} \tilde{g_t^c} + \omega_{yk} \tilde{y}_t \tilde{k_{t-1}} + \omega_{\xi g^c} \tilde{g_t^c} \tilde{\xi}_t \quad (13)$$

where the last term, $\omega_{\xi g^c} \tilde{g_t^c} \tilde{\xi}_t$ shows the interaction between the shock and the government consumption expenditure. We assume the shock evolves as,

$$log(\xi_t) = \rho_\xi log(\xi_{t-1}) + \nu_t, \;\; \nu_t \sim \mathcal{N}(0, \sigma_\nu^2)$$

We calibrate the parameters, $\rho_\xi = 0.9$ and $\sigma_\nu = 0.07$, so the shock has the same variance and persistence as the preference shock on private consumption analysed in the main section. This calibration is not far from the one used in the literature on optimal fiscal policy, such as Faraglia et al. (2013) or Schmitt-Grohé and Uribe (2004), to calibrate government spending shocks that does not provide utility to household. For instance in Faraglia et al. (2013) the government spending shock is modeled as an auto correlated process with a coefficient ρ_ξ of 0.9 and a standard deviation, σ_ν, of 0.08. So despite the nature of the shock we are modelling is not the same as in Faraglia et al. (2013) or Schmitt-Grohé and Uribe (2004), we calibrate it in a close way.

[Figure 9 approximately here]

Under this scenario, the economy shows some difference in the optimal path for private consumption when considering active and passive monetary policy. However, output, labor and the rest of fiscal instruments shows minimum difference. Both under commitment and discretion, the optimal path for private consumption falls by more when monetary policy is active due to a higher nominal interest rate.

Figure 9 in the appendix E shows the optimal response for the main economic variables after the shock hits the economy when monetary policy is active (blue line) and passive (red and orange lines). The response of the economy is similar to the classic "fiscal shock" usually analysed in the literature on optimal fiscal policy: an increase in government consumption expenditure crowds out private consumption, while hours, output and inflation increases. To counteract the effect on prices, the planner raises the interest rates and reduces (although minimally) the tax rate and government investment. Public debt, on the other hand increases persistently.

[Figure 10 approximately here]

Like the case under commitment, under discretion (figure 10 in the appendix E) the positive shock in government consumption crowds out private consumption expenditure, which falls, especially when monetary policy is active. The shock is expansionary, and hours, output and inflation increase. In a Markov Perfect Equilibrium, the planner can affect expectations about future variables by manipulating public debt and public capital.

This time the shock is inflationary, so the government reduces public debt, fueling expectations for low inflation rates in the future. Public capital, on the contrary raises, so the planner leverages on the supply-side effect of public capital to reduce future marginal costs by increasing supply in the economy. To contain the fiscal deficit due to higher public expenditure, the time-consistent planner increases distortionary taxes.

Perhaps with an exogenous increase in government consumption, the model shows some differences between active and passive monetary policy, specially regarding private consumption. However, this differences are not as wide as we can see with rules as in Sims (2011). One reason is of course, the ability of the planner to optimally choose taxes to finance the expenditures, which is not seen in the case of active fiscal policy rule and passive monetary policy. The other reason is the presence of government consumption in the household utility function. Under this scenario, welfare is not totally reduced by an increase in government expenditures as it happens when the latter is considered to be a waste.

8 Conclusion

In this paper we analyze optimal fiscal policy in a New-Keynesian model when monetary policy is passive in the sense of Leeper (1991). We conclude that when fiscal policy is optimal, there is little change in consumption and output volatility in comparison to the active monetary policy regime, thoroughly analyzed by the literature. We endow the planner with several fiscal tools such as public consumption and investment, distortionary taxes and long-term bonds. Monetary policy follows a Taylor rule targeting inflation to acknowledge for the fact that central banks in developed economy shows some degree of independence.

With ad-hoc rules for monetary and fiscal policy, the literature on the FTPL has shown that within the passive monetary/active fiscal policy regime, the tax rate reacts weakly to changes in debt, letting inflation to do most of the job at the time of debt repayment. As a consequence, output and consumption volatility is higher due to the effects of higher inflation and interest rate. In this paper, we show that if the planner can optimally choose the tax rate, the importance of this mechanism is reduced and taxes ensures debt repayment. The planner uses also public consumption and investment to close the output gap and compensate for the fall in private consumption after the negative demand shock.

We found the same result under discretion: the planner chooses optimally the path for state variables (public capital and debt) and tax rates so there is practically no difference between the active and passive monetary policy regime. We show that if taxes and debt can be optimally chosen by a planner, the way in which monetary policy is set does not affect the path for private consumption and output when the economy faces demand shocks.

Qualitatively, results under commitment and discretion are very different at the moment of using the fiscal tools to fight back a demand driven recession. While in the commitment allocation the stimulus the government increases both, public consumption and investment, under discretion, public capital drops to increase future prices. The way in which the stimulus is financed is considerably different under both scenarios. While a committed planner increases taxes, cuts public debt as percentage of output and promises a future reduction of fiscal deficit, a no committed planner increases debt and cuts public capital to manipulate inflation expectations.

Finally, we show the following result holds in a scenario in which the Taylor rule tracks inflation and output and the economy is hit by a liquidity premium shock. Perhaps if there is some difference between the two models, is when there is a fiscal shock. Nevertheless, differences in optimal policies are not of the magnitude observed with rules.

References

Aiyagari, S., Marcet, A., Sargent, T., and Seppälä, J. (2002). Optimal taxation without state contingent debt. *Journal of Political Economy*, 110(6):1220–1254.

Ambler, S. and Paquet, A. (1996). Fiscal spending shocks, endogenous government spending, and real business cycles. *Journal of Economic Dynamics and Control*, 20(1):237 – 256.

Azzimonti, M., Sarte, P.-D., and Soares, J. (2009). Distortionary taxes and public investment when government promises are not enforceable. *Journal of Economic Dynamics and Control*, 33(9):1662 – 1681.

Benigno, P. and Woodford, M. (2012). Linear-quadratic approximation of optimal policy problems. *Journal of Economic Theory*, 147(1):1 – 42.

Bianchi, F. (2013). Regime switches, agents' beliefs, and post-world war ii u.s. macroeconomic dynamics. *The Review of Economic Studies*, 80(2 (283)):463–490.

Bianchi, F. and Ilut, C. (2017). Monetary/fiscal policy mix and agents' beliefs. *Review of Economic Dynamics*, 26:113–139.

Blake, A. P. and Kirsanova, T. (2012). Discretionary Policy and Multiple Equilibria in LQ RE Models. *The Review of Economic Studies*, 79(4):1309–1339.

Bouakez, H., Guillard, M., and Roulleau-Pasdeloup, J. (2017). Public investment, time to build, and the zero lower bound. *Review of Economic Dynamics*, 23:60 – 79.

Bouakez, H., Guillard, M., and Roulleau-Pasdeloup, J. (2019). The optimal composition of public spending in a deep recession. *Journal of Monetary Economics*.

Carvalho, Carlos, F. N. T. T. (2019). Taylor rule estimations by ols. *Federal Reserve Bank of San Francisco Working Paper*.

Cochrane, J. H. (2001). Long-term debt and optimal policy in the fiscal theory of the price level. *Econometrica*, 69(1):69–116.

Cochrane, J. H. (2021). The fiscal roots of inflation. *Review of Economic Dynamics*.

Coenen, G., Straub, R., and Trabandt, M. (2013). Gauging the effects of fiscal stimulus packages in the euro area. *Journal of Economic Dynamics and Control*, 37(2):367 – 386. Fiscal Policy in the Aftermath of the Financial Crisis.

Drautzburg, T. and Uhlig, H. (2015). Fiscal stimulus and distortionary taxation. *Review of Economic Dynamics*, 18(4):894 – 920.

Eggertsson, G. (2011). What fiscal policy is effective at zero interest rates? *NBER Macroeconomics Annual, 2010*, 25:59–112.

Faraglia, E., Marcet, A., Oikonomou, R., and Scott, A. (2013). The impact of debt levels and debt maturity on inflation. *The Economic Journal*, 123(566):F164–F192.

Glomm, G. and Ravikumar, B. (1994). Public investment in infrastructure in a simple growth model. *Journal of Economic Dynamics and Control*, 18(6):1173 – 1187.

Hansen, J. (2018). Optimal monetary policy with capital and a financial accelerator. *Journal of Economic Dynamics and Control*, 92:84 – 102.

Jia, P. (2020). The macroeconomic impact of monetary-fiscal policy in a "fiscal dominance" world. *Macroeconomic Dynamics*, 24(3):670–707.

Kydland, F. E. and Prescott, E. C. (1982). Time to build and aggregate fluctuations. *Econometrica*, 50(6):1345–1370.

Leeper, E. M. (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics*, 27(1):129–147.

Leeper, E. M., Walker, T. B., and Yang, S.-C. S. (2010). Government investment and fiscal stimulus. *Journal of Monetary Economics*, 57(8):1000 – 1012.

Leith, C. and Wren-Lewis, S. (2013). Fiscal sustainability in a new keynesian model. *Journal of Money, Credit and Banking*, 45(8):1477–1516.

Matveev, D. (2021). Time-consistent management of a liquidity trap with government debt. *Journal of Money, Credit and Banking*, 53(8):2129–2165.

Mehra, Y. P. and Minton, B. (2007). A taylor rule and the greenspan era. *FRB Richmond Economic Quarterly*, 93(3):229–250.

Ramey, V. (2020). The macroeconomic consequence of infrastructure investment. *NBER*.

Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy*, 90(6):1187–1211.

Schmitt-Grohé, S. and Uribe, M. (2004). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory*, 114(2):198–230.

Sims, C. A. (2011). Stepping on a rake: The role of fiscal policy in the inflation of the 1970s. *European Economic Review*, 55(1):48–56. Special Issue on Monetary and Fiscal Interactions in Times of Economic Stress.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American Economic Review*, 97(3):586–606.

Woodford, M. (2001). Fiscal requirements for price stability. *Journal of Money, Credit and Banking*, 33(3):669–728.

Woodford, M. (2011). Simple analytics of the government expenditure multiplier. *American Economic Journal: Macroeconomics*, 3(1):1–35.

A Appendix: First Best and flexible price steady state

First best

In this scenario, the planner is choosing $\{C_t, N_t, K_{G,t}, G^i_t, G^c_t\}$.

$$\max_{\{C_t,N_t,K_{G,t},G^i_t,G^c_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ U(C_t, N_t, G^c_t)$$
$$+\lambda_{1,t} \Big[F(N_t, K_{G,t-1}) - C_t - G^c_t - G^i_t \Big]$$
$$+\lambda_{2,t} \Big[(1-\delta_g) K_{G,t-1} + G^i_t - K_{G,t} \Big] \Big\}$$

The efficient allocation is the solution to the following set of FONC:

$$0 = U_{c,t} - \lambda_{1,t}$$
$$0 = -U_{n,t} + \lambda_{1,t} F_N(N_t, K_{G,t-1})$$
$$0 = U_{g,t} - \lambda_{1,t}$$
$$0 = -\lambda_{1,t} + \lambda_{2,t}$$
$$0 = \beta\lambda_{1,t} F_K(K_{G,t}, N_t) - \lambda_{2,t} + \beta E_t \lambda_{2,t+1}(1-\delta_g)$$
$$0 = F(N_t, K_{G,t-1}) - C_t - G^c_t - G^i_t$$
$$0 = (1-\delta_g) K_{G,t-1} + G^i_t - K_{G,t}$$

Steady-state Using 14, we can get the following:

$$F_K(K_G, N) = \frac{1-\beta(1-\delta)}{\beta}$$

Calibrating hours in the steady-state to be 1/3 of disposable time, we can use pat equation together with the production function to solve for K_G in the steady state.

$$(1-\alpha) N^{\alpha} K_G^{-\alpha} = \frac{1-\beta(1-\delta)}{\beta}$$

which in the steady state is:

$$K^*_G = \left[\frac{1-\beta(1-\delta)}{\beta} \frac{N^{-\alpha}}{1-\alpha} \right]^{-1/\alpha}$$

and using the production function:

$$Y^{*} = N^{\alpha} K_{G}^{1-\alpha}$$

Using equation 14, we can recover government investment (where δ_g is taken from the literature):

$$G^{i,*} = \delta_g K_G^{*}$$

Using 14, the value for σ and φ from the literature, we can recover χ, so the household devotes 1/3 of her disposable time working:

$$\chi \frac{n^{*\varphi}}{c^{*-\sigma}} = \alpha \frac{y^{*}}{n^{*}}$$
$$\chi = \alpha \frac{y^{*} c^{*-\sigma}}{n^{*1+\varphi}}$$

We can combine 14 and 14 to find for χ_g

$$\chi_g = \frac{c^{-\sigma}}{g^{-\sigma}}$$

Finally, we calibrate the value for $G^{c,*}$ in the steady state to be 0.2, hence using aggregate resource constraint we can recover C^{*}.

Flexible price steady-state

In a flexible price steady-state, the planner is not constrained by the Phillips curve, thus the equation can be dropped from the set of constraints along with the attached multiplier ϕ_2. The euler equation for short term debt has to hold, hence dropping time subscripts from euler equation and in a zero inflation steady state and assuming $\zeta^{*} = 1$, $\pi^{*} = 1$:

$$\frac{1}{c^{\sigma}} q_{t,S} = \beta \frac{1}{c^{\sigma}} \frac{1}{\pi^{*}}$$
$$q_S = \frac{1}{R^{*}} = \beta$$
$$R^{*} = \frac{1}{\beta}$$

From the euler equation from long term bonds

$$
\begin{aligned}
&\frac{1}{c^\sigma} q_{t,L} = \beta \frac{1}{c^\sigma} \frac{1}{\pi^*} (1 + \delta q_{t,L}) \\
&q_L = \frac{\beta}{1 - \beta\delta} \\
&R_L^* = \frac{1 + \delta q_L^*}{q_L^*}
\end{aligned}
$$

From the profit maximization problem of the firm, the marginal costs are

$$
mc_t = 1 - \frac{1}{\theta}
$$

The behavior of firms imply they will produce until marginal cost equals marginal revenues, and substituing the definition from marginal cost in the the household FONC for labor, we can obtain the optimal subsidy for labor

$$
\begin{aligned}
&log(\mu_t) = -mc_t \\
&mc^* = \left(1 - \frac{1}{\theta}\right) \\
&\left(1 - \frac{1}{\theta}\right) = \frac{(1 - s)}{(1 - \tau_t)} \chi \frac{n_t^\varphi}{c_t^{-\sigma}} \frac{\alpha Y_t}{n_t} \\
&(1 - s^*) = \left(1 - \frac{1}{\theta}\right)(1 - \tau^*)
\end{aligned}
$$

Taxing the households accordingly we arrive to the same labor supply as in the efficient steady state. From the first order condition with respect to labor, eliminating time subscripts:

$$
\begin{aligned}
\chi \frac{n^{*\varphi}}{c^{*-\sigma}} &= \alpha \frac{y^*}{n^*} \\
\chi &= \alpha \frac{y^* c^{*-\sigma}}{n^{*1+\varphi}}
\end{aligned}
$$

Note that the equation is the same one obtained in the first best equilibrium.

From the FONC for the firm w.r.t labor, we can obtain the wage by replacing the production function and the optimal subsidy:

$$
\begin{aligned}
mc_t &= \frac{w_t(1-s)}{F_{N,t}} \\
w^* &= \frac{mc^*}{(1-s^*)}\alpha\frac{Y^*}{n^*} \\
w^* &= \frac{\alpha Y^*}{(1-\tau^*)n^*}
\end{aligned}
$$

Taking the households budget constraint, and dividing through price P_t, and imposing short term bonds are in zero net supply

$$
\begin{aligned}
&P_t c_t + b_{t,L} + q_{t,S}B_{t,S} = (1-\tau_t)W_t n_t + q_t D_t + (1+\rho q_{t,L})B_{t-1,L} + B_{t-1,S} + T_t \\
&c_t + b_{t,L} = (1-\tau_t)W_t n_t + D_t + (1+\rho q_{t,L})b_{t-1,L} + T_t \\
&c_t + b_{t,L} = (1-\tau_t)W_t n_t + D_t + (1+\rho q_{t,L})b_{t-1,L} + T_t
\end{aligned}
$$

replacing dividens and the function for $Y_t(z)$

$$
\begin{aligned}
&c_t + b_{t,L} = (1-\tau_t)W_t n_t + (1+\rho q_{t,L})b_{t-1,L} + \frac{P_t(z)}{P_t}Y_t(z) - w_t N_t(z)(1-s) - \frac{\psi}{2}\left(\frac{P_t(z)}{P_{t-1}(z)} - 1\right)^2 Y_t + T_t \\
&c_t + b_{t,L} = W_t n_t(s-\tau_t) + (1+\rho q_{t,L})b_{t-1,L} + \frac{P_t(z)}{P_t}\left(\frac{P_t(z)}{P_t}\right)^{-\theta} Y_t - \frac{\psi}{2}\left(\frac{P_t(z)}{P_{t-1}(z)} - 1\right)^2 Y_t + T_t
\end{aligned}
$$

Imposing a symmetric equilibrium in which each firm chooses the same price, $P_t(z) = P_t$

$$
c_t + b_{t,L} = W_t n_t(s-\tau_t) + (1+\rho q_{t,L})b_{t-1,L} + Y_t - \frac{\psi}{2}\Big(\pi_t - 1\Big)^2 Y_t + T_t
$$

replacing c_t with aggregate resource constraint and canceling the terms we arrive to the govt budget constraint and the subsidy in the steady state

$$
\begin{aligned}
&Y_t - G_t^i - G_t^c - \frac{\psi}{2}\Big(\pi_t - 1\Big)^2 Y_t + b_{t,L} = W_t n_t(s-\tau_t) + (1+\rho q_{t,L})b_{t-1,L} + Y_t - \frac{\psi}{2}\Big(\pi_t - 1\Big)^2 Y_t + T_t \\
&-G_t^i - G_t^c + b_{t,L} = W_t n_t(s-\tau_t) + (1+\rho q_{t,L})b_{t-1,L} + T_t \\
&W_t n_t s = T_t
\end{aligned}
$$

For the steady-state value of the tax rate, we calibrate a value so that debt to gdp ratio is 60% of output in the steady-state and we consider the value for wages in the steady state. From the govt budget constraint, first replacing the price for the long term bond:

$$\tau_t w_t n_t - g_t^i - g_t^c = (1 + \rho q_{t,L}) b_{t,L} - q_{t,L} b_{t,L}$$
$$\tau_t w_t n_t - g_t^i - g_t^c = \left(\frac{1-\beta}{1-\beta\rho}\right) b_{t,L}$$

Replace w_t for the value in the steady state:

$$\tau_t \frac{\alpha y_t}{(1-\tau) n_t} n_t - g_t^i - g_t^c = \left(\frac{1-\beta}{1-\beta\rho}\right) b_{t,L}$$
$$\tau_t = \frac{(1-\tau_t)}{\alpha y_t}\left(\frac{1-\beta}{1-\beta\rho}\right) b_{t,L} + g_t^i + g_t^c$$
$$\tau_t = \frac{\frac{1}{\alpha y_t}\left[\left(\frac{1-\beta}{1-\beta\rho}\right) b_{t,\delta} + g_t^i + g_t^c\right]}{\left[1 + \frac{1}{\alpha y_t}\left(\frac{1-\beta}{1-\beta\rho}\right) b_{t,L} + g_t^i + g_t^c\right]}$$

So in the steady state, the tax rate can be the one to sustain certain level of debt to output ratio, calibrated to 60%

$$\tau^* = \frac{\frac{1}{\alpha}\left[\left(\frac{1-\beta}{1-\beta\rho}\right)\frac{b_L^*}{y^*} + \frac{g^{*,i}}{y^*} + \frac{g^{c,*}}{y^*}\right]}{\left[1 + \frac{1}{\alpha}\left(\frac{1-\beta}{1-\beta\rho}\right)\frac{b_L^*}{y^*} + \frac{g^{i*}}{y^*} + \frac{g^{c,*}}{y^*}\right]}$$

B Appendix: Second order approximation to the utility function

In this appendix we derive the second order approximation for the utility function (see Benigno and Woodford (2012)). We first replace consumption by the aggregate resource constraint and labor by the production function. Variables denoted with a $\tilde{x}$ represents the percentage deviation from the steady-state, while variables denoted as x^* denoted the steady-state value for variable x.

The utility function is:

$$E_t \sum_{s=0}^{\infty} \beta^t \left(\epsilon_t \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_g \frac{G_t^{c,1-\sigma}}{1-\sigma} \right)$$

Taking the first term on consumption and substituing by the aggregate resource constraint the:

$$E_t \sum_{s=0}^{\infty} \beta^t \left(\epsilon_t \frac{[Y_t(1-\psi/2(\pi_t-1)^2) - G_t^i - G_t^c]^{1-\sigma}}{1-\sigma} \right)$$

taking derivatives:

$$F_\pi = -\psi(\pi_t - 1) y_t c_t^{-\sigma} \epsilon_t$$

$$F_y = c_t^{-\sigma} \left(1 - \frac{\psi}{2}(\pi_t - 1)^2\right) \epsilon_t$$

$$F_{g^i} = -c^{-\sigma} \epsilon_t$$

$$F_{g^c} = -c^{-\sigma} \epsilon_t$$

$$F_{\epsilon_t} = \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$F_{yy} = -\sigma c_t^{-\sigma-1} \left(1 - \frac{\psi}{2}\left(\pi_t - 1\right)^2\right)^2 \epsilon_t$$

$$F_{\pi\pi} = -\frac{y^* \psi}{c^{*,\sigma}}$$

$$F_{g^i g^i} = -\sigma c_t^{-\sigma-1} \epsilon_t$$

$$F_{g^c g^c} = -\sigma c_t^{-\sigma-1} \epsilon_t$$

$$F_{\epsilon\epsilon} = 0$$

$$F_{\pi y} = \psi(\pi_t - 1) y_t \sigma c_t^{-\sigma-1} \epsilon_t \left[1 - \frac{\psi}{2}(\pi_t - 1)^2\right]$$

$$F_{y g^i} = \left(1 - \frac{\psi^2}{2}(\pi_t - 1)^2\right)^2 \epsilon_t \sigma c_t^{-\sigma-1}$$

$$F_{y g^c} = \left(1 - \frac{\psi^2}{2}(\pi_t - 1)^2\right)^2 \epsilon_t \sigma c_t^{-\sigma-1}$$

$$F_{\pi g^i} = \psi(\pi_t - 1) y_t \sigma c_t^{-\sigma-1} \epsilon_t$$

$$F_{\pi g^c} = \psi(\pi_t - 1) y_t \sigma c_t^{-\sigma-1} \epsilon_t$$

$$F_{g^i \pi} = \sigma c_t^{-\sigma-1} \epsilon_t (-\psi(\pi_t - 1) y_t)$$

$$F_{g^c \pi} = \sigma c_t^{-\sigma-1} \epsilon_t (-\psi(\pi_t - 1) y_t)$$

$$F_{g^i y} = \sigma c_t^{-\sigma-1} \epsilon_t \left(1 - \frac{\psi}{2}(\pi_t - 1)\right)$$

$$F_{g^c y} = \sigma c_t^{-\sigma-1} \epsilon_t \left(1 - \frac{\psi}{2}(\pi_t - 1)\right)$$

$$F_{g^c g^i} = -\sigma c_t^{-\sigma-1} \epsilon_t$$

$$F_{g^i g^c} = -\sigma c_t^{-\sigma-1} \epsilon_t$$

$$F_{g^i \epsilon} = -c_t^{-\sigma}$$

$$F_{g^c \epsilon} = -c_t^{-\sigma}$$

$$F_{\epsilon g^c} = -c_t^{-\sigma}$$

$$F_{\epsilon g^i} = -c_t^{-\sigma}$$

$$F_{\epsilon y} = \left(1 - \frac{\psi}{2}(\pi_t - 1)^2\right) c_t^{-\sigma}$$

$$F_{y \epsilon} = \left(1 - \frac{\psi}{2}(\pi_t - 1)^2\right) c_t^{-\sigma}$$

$$F_{\epsilon \pi} = -\psi(\pi_t - 1) c_t^{-\sigma} y_t \epsilon_t$$

$$F_{\pi \epsilon} = -\psi(\pi_t - 1) c_t^{-\sigma} y_t$$

$$F_{y \pi} = \epsilon_t \psi(\pi_t - 1) \left[\sigma c^{-\sigma-1} y_t \left(1 - \frac{\psi}{2}(\pi_t - 1)^2\right) - c_t^{-\sigma}\right]$$

Considering that in the steady-state $\pi_t = 1$ and $\epsilon_t = 1$, the second order approximation is:

$$
\begin{aligned}
F = & \frac{y^*}{c^{*\sigma}}\tilde{y}_t - \frac{g^{i,*}}{c^{*\sigma}}\tilde{g^i_t} - \frac{\sigma y^{*2}}{c^{*1+\sigma}}\frac{1}{2}\tilde{y}_t{}^2 - \frac{y^*\psi}{c^{*\sigma}}\frac{1}{2}\tilde{\pi}_t{}^2 - \frac{\sigma g^{i,*2}}{c^{*1+\sigma}}\frac{1}{2}\tilde{g^i_t}^2 + \frac{\sigma g^{i,*}y^*}{c^{*1+\sigma}}\tilde{y}_t\tilde{g^i_t} + \frac{\sigma g^{c,*}y^*}{c^{*1+\sigma}}\tilde{y}_t\tilde{g^c_t} \\
& - \frac{\sigma g^{c,*}g^{i,*}}{c^{*1+\sigma}}\tilde{g^i_t}\tilde{g^c_t} - \frac{g^{i,*}}{c^{*,\sigma}}\tilde{g^i_t}\tilde{\epsilon}_t + \frac{y^*}{c^{*,\sigma}}\tilde{y}_t\tilde{\epsilon}_t - \frac{g^{c,*}}{c^{*,\sigma}}\tilde{g^c_t} - \sigma\frac{g^{c,*2}}{c^{*,\sigma+1}}\frac{1}{2}\tilde{g^c_t}^2 - \frac{g^{c,*}}{c^{*,\sigma}}\tilde{g^c_t}\tilde{\epsilon}_t + t.i.p
\end{aligned}
$$

Taking disutility from labor and replacing by the production function

$$E_t\sum_{s=0}^{\infty}\beta^t\left(\chi\frac{(y_t^{\frac{1}{\alpha}}k_{g,t-1}^{\frac{\alpha-1}{\alpha}})^{1+\varphi}}{1+\varphi}\right)$$

taking derivatives:

$$
\begin{aligned}
H_y &= \frac{1}{\alpha}n_t^{1+\varphi}\frac{\chi}{y_t} \\
H_{k_g} &= \frac{\alpha-1}{\alpha}n_t^{1+\varphi}\frac{\chi}{k_{g,t-1}} \\
H_{yy} &= \frac{1}{\alpha}n_t^{1+\varphi}\frac{\chi}{y_t^2}\left[\frac{1}{\alpha}(\varphi+1)-1\right] \\
H_{k_gk_g} &= \frac{\alpha-1}{\alpha}n_t^{1+\varphi}\frac{\chi}{k_{g,t-1}^2}\left[\frac{\alpha-1}{\alpha}(\varphi+1)-1\right]
\end{aligned}
$$

$$
\begin{aligned}
H_{yk_g} &= (1+\varphi)\frac{1}{\alpha}\frac{\alpha-1}{\alpha}n_t^{1+\varphi}\frac{\chi}{k_{g,t-1}y_t} \\
H_{k_gy} &= (1+\varphi)\frac{1}{\alpha}\frac{\alpha-1}{\alpha}n_t^{1+\varphi}\frac{\chi}{k_{g,t-1}y_t}
\end{aligned}
$$

At the steady state, total hours worked are 1/3 of disposable time, hence $\chi = \frac{\alpha y^* c^{*\sigma}}{n^{*1+\varphi}}$, making a second order approximation to the disutility of labor

$$
\begin{aligned}
H = & \frac{y^*}{c^{*\sigma}}\tilde{y}_t + \frac{(\alpha-1)y^*}{c^{*\sigma}}\tilde{k_{g,t-1}} + \frac{y^*}{c^{*\sigma}}\left[\frac{(1+\varphi)}{\alpha}-1\right]\frac{1}{2}\tilde{y}_t{}^2 + \frac{(\alpha-1)y^*}{c^{*\sigma}}\left[\frac{(\alpha-1)}{\alpha}(1+\varphi)-1\right]\frac{1}{2}\tilde{k_{g,t-1}}^2 \\
& + \frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma}\alpha}\tilde{y}_t\tilde{k_{g,t-1}} + t.i.p
\end{aligned}
$$

Taking government consumption

$$E_t\sum_{s=0}^{\infty}\beta^t\left(\chi_g\frac{G_t^{c,1-\sigma}}{1-\sigma}\right)$$

$$
\begin{aligned}
G_g &= \chi_g G_t^{-\sigma} \\
G_{gg} &= -\chi_g\sigma G_t^{-\sigma-1}
\end{aligned}
$$

$$G = \chi_g g^{c,*-\sigma+1}\tilde{g_t^c} - \chi_g \sigma g^{c,-\sigma+1}\frac{1}{2}\tilde{g_t^c}^2 + t.i.p$$

At the steady state: $\chi_g = \frac{c^{-\sigma}}{g^{-\sigma}}$, hence the equation becomes

$$G = \frac{g^{c,*}}{c^{*,\sigma}}\tilde{g_t^c} - \frac{g^{c,*}\sigma}{c^{*,\sigma}}\frac{1}{2}\tilde{g_t^c}^2 + t.i.p$$

Putting the terms together: $F + G - H$

$$\begin{aligned}
U = {} & \frac{y^*}{c^{*\sigma}}\tilde{y_t} - \frac{g^{i,*}}{c^{*\sigma}}\tilde{g_t^i} - \frac{\sigma y^{*2}}{c^{*1+\sigma}}\frac{1}{2}\tilde{y_t}^2 - \frac{y^*\psi}{c^{*\sigma}}\frac{1}{2}\tilde{\pi_t}^2 - \frac{\sigma g^{i,*2}}{c^{*1+\sigma}}\frac{1}{2}\tilde{g_t^i}^2 + \frac{\sigma g^{i,*}y^*}{c^{*1+\sigma}}\tilde{y_t}\tilde{g_t^i} + \frac{\sigma g^{c,*}y^*}{c^{*1+\sigma}}\tilde{y_t}\tilde{g_t^c} \\
& - \frac{\sigma g^{c,*}g^{i,*}}{c^{*1+\sigma}}\tilde{g_t^i}\tilde{g_t^c} - \frac{g^{i,*}}{c^{*,\sigma}}\tilde{g_t^i}\tilde{\epsilon_t} + \frac{y^*}{c^{*,\sigma}}\tilde{y_t}\tilde{\epsilon_t} - \frac{g^{c,*}}{c^{*,\sigma}}\tilde{g_t^c} - \frac{\sigma g^{c,*2}}{c^{*,\sigma+1}}\frac{1}{2}\tilde{g_t^c}^2 - \frac{g^{c,*}}{c^{*\sigma}}\tilde{g_t^c}\tilde{\epsilon_t} + \frac{g^{c,*}}{c^{*\sigma}}\tilde{g_t^c} - \frac{\sigma g^{c,*}}{c^{*\sigma}}\frac{1}{2}\tilde{g_t^c}^2 \\
& - \frac{y^*}{c^{*\sigma}}\tilde{y_t} - \frac{(\alpha-1)y^*}{c^{*\sigma}}\tilde{k_{g,t-1}} - \frac{y^*}{c^{*\sigma}}\left[\frac{(1+\varphi)}{\alpha} - 1\right]\frac{1}{2}\tilde{y_t}^2 \\
& - \frac{(\alpha-1)y^*}{c^{*\sigma}}\left[\frac{(\alpha-1)}{\alpha}(1+\varphi) - 1\right]\frac{1}{2}\tilde{k_{g,t-1}}^2 - \frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma}\alpha}\tilde{y_t}\tilde{k_{g,t-1}} + t.i.p
\end{aligned}$$

Canceling some terms,

$$\begin{aligned}
U = {} & -\frac{g^{i,*}}{c^{*\sigma}}\tilde{g_t^i} - \frac{\sigma y^{*2}}{c^{*1+\sigma}}\frac{1}{2}\tilde{y_t}^2 - \frac{y^*\psi}{c^{*\sigma}}\frac{1}{2}\tilde{\pi_t}^2 - \frac{\sigma g^{i,*2}}{c^{*1+\sigma}}\frac{1}{2}\tilde{g_t^i}^2 + \frac{\sigma g^{i,*}y^*}{c^{*1+\sigma}}\tilde{y_t}\tilde{g_t^i} + \frac{\sigma g^{c,*}y^*}{c^{*1+\sigma}}\tilde{y_t}\tilde{g_t^c} - \frac{\sigma g^{c,*}g^{i,*}}{c^{*1+\sigma}}\tilde{g_t^i}\tilde{g_t^c} - \frac{g^{i,*}}{c^{*,\sigma}}\tilde{g_t^i}\tilde{\epsilon_t} \\
& - \frac{\sigma g^{c,*2}}{c^{*,\sigma+1}}\frac{1}{2}\tilde{g_t^c}^2 - \frac{\sigma g^{c,*}}{c^{*\sigma}}\frac{1}{2}\tilde{g_t^c}^2 - \frac{g^{c,*}}{c^{*\sigma}}\tilde{g_t^c}\tilde{\epsilon_t} + \frac{y^*}{c^{*,\sigma}}\tilde{y_t}\tilde{\epsilon_t} \\
& - \frac{(\alpha-1)y^*}{c^{*\sigma}}\tilde{k_{g,t-1}} - \frac{y^*}{c^{*\sigma}}\left[\frac{(1+\varphi)}{\alpha} - 1\right]\frac{1}{2}\tilde{y_t}^2 - \frac{(\alpha-1)y^*}{c^{*\sigma}}\left[\frac{(\alpha-1)}{\alpha}(1+\varphi) - 1\right]\frac{1}{2}\tilde{k_{g,t-1}}^2 \\
& - \frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma}\alpha}\tilde{y_t}\tilde{k_{g,t-1}} + t.i.p.
\end{aligned}$$

There are some linear terms remaining, taking a second order approximation to the law of motion for capital and reordering, we can replace for government investment (see Hansen (2018), which indeed is an application of Benigno and Woodford (2012)) [10]:

$$\tilde{g_t^i} = \left[\tilde{k_{g,t}} + k_g^*\frac{\tilde{k_{g,t}}^2}{2} - (1-\delta)\left(\tilde{k_{g,t-1}} + k_g^*\frac{\tilde{k_{g,t-1}}^2}{2}\right)\right]\frac{1}{\delta} - g^{*,i}\frac{\tilde{g_t^i}^2}{2}$$

[10] Hansen (2018) proposes a New-Keynesian model with private capital and Calvo pricing. They derive a second order approximation as in Benigno and Woodford (2012) for the utility function using a second order approximation for the aggregate resource constraint and the law of motion for capital to eliminate linear terms from the objective function: consumption, and investment. In this problem there is only one linear variable, which is public investment.

Replacing in the second order approximation for the utility function:

$$
\begin{aligned}
U = &-\frac{g^{i,*}}{c^{*\sigma}}\left[\left[\tilde{k}_{g,t} + k_g^* \frac{\tilde{k}_{g,t}^{2}}{2} - (1-\delta)\left(\tilde{k}_{g,t-1} + k_g^* \frac{\tilde{k}_{g,t-1}^{2}}{2}\right)\right]\frac{1}{\delta} - g^{i*}\frac{\tilde{g}_t^{i\,2}}{2}\right] - \frac{\sigma y^{*2}}{c^{*1+\sigma}}\frac{1}{2}\tilde{y}_t^{2} - \frac{y^*\psi}{c^{*\sigma}}\frac{1}{2}\tilde{\pi}_t^{2} \\
&- \frac{\sigma g^{i,*2}}{c^{*1+\sigma}}\frac{1}{2}\tilde{g}_t^{i\,2} - \frac{\sigma g^{i,*}y^*}{c^{*1+\sigma}}\tilde{y}_t\tilde{g}_t^i + \frac{\sigma g^{c,*}y^*}{c^{*1+\sigma}}\tilde{y}_t\tilde{g}_t^c + \frac{\sigma g^{c,*}g^{i,*}}{c^{*1+\sigma}}\tilde{g}_t^i\tilde{g}_t^c - \frac{g^{i,*}}{c^{*,\sigma}}\tilde{g}_t^i\tilde{\zeta}_t - \frac{\sigma g^{c,*2}}{c^{*,\sigma+1}}\frac{1}{2}\tilde{g}_t^{c\,2} - \frac{\sigma g^{c,*}}{c^{*\sigma}}\frac{1}{2}\tilde{g}_t^{c\,2} - \frac{g^{c,*}}{c^{*\sigma}}\tilde{g}_t^c\tilde{\zeta}_t \\
&- \frac{(\alpha-1)y^*}{c^{*\sigma}}\tilde{k}_{g,t-1} - \frac{y^*}{c^{*\sigma}}\left[\frac{(1+\varphi)}{\alpha} - 1\right]\frac{1}{2}\tilde{y}_t^{2} - \frac{(\alpha-1)y^*}{c^{*\sigma}}\left[\frac{(\alpha-1)}{\alpha}(1+\varphi) - 1\right]\frac{1}{2}\tilde{k}_{g,t-1}^{2} \\
&- \frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma}\alpha}\tilde{y}_t\tilde{k}_{g,t-1} + \frac{y^*}{c^{*,\sigma}}\tilde{y}_t\tilde{\epsilon}_t + t.i.p.
\end{aligned}
$$

Focusing on the linear terms....

$$
-\frac{g^{i,*}}{c^{*\sigma}}\left[\tilde{k}_{g,t} - (1-\delta)\left(\tilde{k}_{g,t-1}\right)\right]\frac{1}{\delta} - \frac{(\alpha-1)y^*}{c^{*\sigma}}\tilde{k}_{g,t-1}
$$

In the steady state $g^{i,*} = \delta k_g^*$, production function is a Cobb-Douglas so $F_{k_g} = \frac{1-\beta(1-\delta)}{\beta} = \frac{(1-\alpha)y^*}{k_g^*}$,

$$
-\frac{k_g^*}{c^{*\sigma}}\left[\tilde{k}_{g,t} - (1-\delta)\tilde{k}_{g,t-1} + (\alpha-1)y^*\tilde{k}_{g,t-1}\right]
$$

recalling the summation [11]

$$
\begin{aligned}
&-\frac{k_g^*}{c^{*\sigma}}\sum_{t=0}^{\infty}\beta^t\left[\tilde{k}_{g.t} - (1-\delta)\tilde{k}_{g,t-1} + (\alpha-1)y^*\tilde{k}_{g,t-1}\right] \\
&-\frac{k_g^*}{c^{*\sigma}}\sum_{t=0}^{\infty}\beta^t\left[\tilde{k}_{g,t} - \beta^{-1}\tilde{k}_{g,t-1}\right] \\
&-\frac{k_g^*}{c^{*\sigma}}k_{g,0}
\end{aligned}
$$

hence there is no linear terms in the second order approximation

Adding and substracting: $\frac{(\alpha-1)y^*}{c^{*\sigma}}k_{g,t-1}^2$,

[11] Expanding the summation one can see that, in the second line, it adds up to $k_{g,0}$

$$U = \frac{g^{i,*2}}{c^{*\sigma}}\frac{\tilde{g_t^i}^2}{2} - \frac{\sigma y^{*2}}{c^{*1+\sigma}}\frac{1}{2}\tilde{y_t}^2 - \frac{y^*\psi}{c^{*\sigma}}\frac{1}{2}\tilde{\pi_t}^2 - \frac{\sigma g^{i,*2}}{c^{*1+\sigma}}\frac{1}{2}\tilde{g_t^i}^2 - \frac{\sigma g^{i,*}y^*}{c^{*1+\sigma}}\tilde{y_t}\tilde{g_t^i} + \frac{\sigma g^{c,*}y^*}{c^{*1+\sigma}}\tilde{y_t}\tilde{g_t^c} + \frac{\sigma g^{c,*}g^{i,*}}{c^{*1+\sigma}}\tilde{g_t^i}\tilde{g_t^c} - \frac{g^{i,*}}{c^{*,\sigma}}\tilde{g_t^i}\tilde{\epsilon_t}$$
$$- \left[\frac{\sigma g^{c,*2}}{c^{*,\sigma+1}} + \frac{\sigma g^{c,*}}{c^{*\sigma}}\right]\frac{1}{2}\tilde{g_t^c}^2 - \frac{g^{c,*}}{c^{*\sigma}}\tilde{g_t^c}\tilde{\epsilon_t} - \frac{y^*}{c^{*\sigma}}\left[\frac{(1+\varphi)}{\alpha} - 1\right]\frac{1}{2}\tilde{y_t}^2 + \frac{y^*}{c^{*,\sigma}}\tilde{y_t}\tilde{\epsilon_t}$$
$$- \frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma}\alpha}\tilde{y_t}\tilde{k_{g,t-1}} - \frac{(\alpha-1)y^*}{c^{*\sigma}}\left[\frac{(\alpha-1)}{\alpha}(1+\varphi) - 1\right]\frac{1}{2}\tilde{k_{g,t-1}}^2 - \frac{k_g^{*2}}{c^{*\sigma}}\frac{\tilde{k_{g,t}}^2}{2}$$
$$+ \frac{k_g^{*2}}{c^{*\sigma}}(1-\delta)\frac{1}{2}\tilde{k_{g,t-1}}^2 - \frac{(\alpha-1)y^*}{c^{*\sigma}}k_{g,t-1}^2 + \frac{(\alpha-1)y^*}{c^{*\sigma}}k_{g,t-1}^2 + t.i.p.$$

Then:

$$-\frac{k_g^*}{c^{*\sigma}}\left[\tilde{k_{g,t}^2} - (1-\delta)\tilde{k_{g,t-1}^2} + (\alpha-1)y^*\tilde{k_{g,t-1}^2}\right]$$

recalling the summation

$$-\frac{k_g^*}{c^{*\sigma}}\sum_{t=0}^{\infty}\beta^t\left[\tilde{k_{g,t}^2} - (1-\delta)\tilde{k_{g,t-1}^2} + (\alpha-1)y^*\tilde{k_{g,t-1}^2}\right]$$

Which as before can be proved independent to policy, hence:

$$U = \frac{g^{i,*2}}{c^{*\sigma}}\frac{\tilde{g_t^i}^2}{2} - \frac{\sigma y^{*2}}{c^{*1+\sigma}}\frac{1}{2}\tilde{y_t}^2 - \frac{y^*\psi}{c^{*\sigma}}\frac{1}{2}\tilde{\pi_t}^2 - \frac{\sigma g^{i,*2}}{c^{*1+\sigma}}\frac{1}{2}\tilde{g_t^i}^2 - \frac{\sigma g^{i,*}y^*}{c^{*1+\sigma}}\tilde{y_t}\tilde{g_t^i} + \frac{\sigma g^{c,*}y^*}{c^{*1+\sigma}}\tilde{y_t}\tilde{g_t^c} + \frac{\sigma g^{c,*}g^{i,*}}{c^{*1+\sigma}}\tilde{g_t^i}\tilde{g_t^c} - \frac{g^{i,*}}{c^{*,\sigma}}\tilde{g_t^i}\tilde{\epsilon_t}$$
$$- \left[\frac{\sigma g^{c,*2}}{c^{*,\sigma+1}} + \frac{\sigma g^{c,*}}{c^{*\sigma}}\right]\frac{1}{2}\tilde{g_t^c}^2 - \frac{g^{c,*}}{c^{*\sigma}}\tilde{g_t^c}\tilde{\epsilon_t} - \frac{y^*}{c^{*\sigma}}\left[\frac{(1+\varphi)}{\alpha} - 1\right]\frac{1}{2}\tilde{y_t}^2 - \frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma}\alpha}\tilde{y_t}\tilde{k_{g,t-1}}$$
$$- \frac{(\alpha-1)y^*}{c^{*\sigma}}\left[\frac{(\alpha-1)}{\alpha}(1+\varphi) - 2\right] + \frac{y^*}{c^{*,\sigma}}\tilde{y_t}\tilde{\epsilon_t} + t.i.p$$

Last equation can be written as

$$L_t = \omega_{g^i}\frac{\tilde{g_t^i}^2}{2} + \omega_y\frac{\tilde{y_t}^2}{2} + \omega_\pi\frac{\tilde{\pi_t}^2}{2} + \omega_{g^c}\frac{\tilde{g_t^c}^2}{2} + \omega_{k_g-1}\frac{\tilde{k_{g,t-1}}^2}{2} + \omega_{yg^i}\tilde{y_t}\tilde{g_t^i} + \omega_{yg^c}\tilde{y_t}\tilde{g_t^c} + \omega_{g^ig^c}\tilde{g_t^i}\tilde{g_t^c} + \omega_{\epsilon g^i}\tilde{g_t^i}\tilde{\epsilon_t}$$
$$+ \omega_{yk_g}\tilde{y_t}\tilde{k_{g,t-1}} + \omega_{\epsilon g^c}\tilde{g_t^c}\tilde{\epsilon_t} + \omega_{\epsilon y}\tilde{y_t}\tilde{\epsilon_t}$$

where:

$$\omega_{g^i} = \frac{g^{i,*}}{c^{*\sigma}}\left[g^{i,*} - \frac{g^{i,*}\sigma}{c^*}\right]$$

$$\omega_y = -\frac{y^*}{c^{*\sigma}}\left[\frac{\sigma y^*}{c^*} + \frac{1+\varphi}{\alpha} - 1\right]$$

$$\omega_{kg} = -\frac{k_g^{*2}}{c^{\sigma *}}$$

$$\omega_\pi = -\frac{y^*\psi}{c^{*\sigma}}$$

$$\omega_{g^c} = -\sigma\left[\frac{g^{c,*2}}{c^{*,\sigma+1}} + \frac{g^{c,*}}{c^{*\sigma}}\right]$$

$$\omega_{yg^i} = \frac{\sigma g^{i,*}y^*}{c^{*1+\sigma}}$$

$$\omega_{yg^c} = \frac{\sigma g^{c,*}y^*}{c^{*1+\sigma}}$$

$$\omega_{\epsilon y} = \frac{y^*}{c^{*,\sigma}}$$

$$\omega_{g^i g^c} = -\frac{\sigma g^{c,*}g^{i,*}}{c^{*1+\sigma}}$$

$$\omega_{\epsilon g^i} = -\frac{g^{i,*}}{c^{*,\sigma}}$$

$$\omega_{yk_g} = -\frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma}\alpha}$$

$$\omega_{\epsilon k_g} = -\frac{(\alpha-1)y^*}{c^{*\sigma}}$$

$$\omega_{g^c\epsilon} = -\frac{g^{c,*}}{c^{*\sigma}}$$

$$\omega_{k_g-1} = -\frac{(\alpha-1)y^*}{c^{*\sigma}}\left[\frac{(\alpha-1)}{\alpha}(1+\varphi) - 1\right] + \frac{k_g^{*2}}{c^{*\sigma}}(1-\delta)$$

C Appendix: Households and firm's problem

We now state the household's and firms problems considering preferences defined in the main text.

Households: Household maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\epsilon \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_g \frac{G_t^{c,1-\sigma}}{1-\sigma} \right)$$

subject to,

$$P_t c_t + q_{t,S} + q_{t,L} B_{t,L} = (1-\tau_t) W_t n_t + P_t D_t + B_{t-1,S} + (1+\rho q_{t,L}) B_{t-1,L} + T_t \tag{14}$$

where C_t denotes consumption, n_t denotes hours worked. D_t represents firm's profits redistributed to households, and P_t denotes the aggregate price level. $B_{t,L}$ is a long-term government bond, a perpetuity with coupons payment decaying at rate $0 \leq \rho < 1$ and price $q_{t,L}$. $B_{t,S}$ denotes a short-term government bond with price $q_{t,S}$. We assume that short-term debt is in zero net supply. ϵ_t is a preference shock affecting private consumption.

The first order conditions for the household's problem are:

$$\begin{aligned} q_{t,s} \epsilon_t c_t^{\sigma} &= \beta E_t \epsilon_{t+1} \tfrac{c_{t+1}^{\sigma}}{\pi_{t+1}} \\ q_{t,L} \epsilon_t c_t^{\sigma} &= \beta E_t \epsilon_{t+1} \tfrac{c_{t+1}^{\sigma}}{\pi_{t+1}} (1 + \rho q_{t+1,L}) \\ (1-\tau_t) \tfrac{W_t}{P_t} &= \tfrac{\chi n_t^{\varphi}}{\epsilon_t c_t^{-\sigma}} \end{aligned}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate

Firms

Production takes place in monopolistically competitive firms z which operates technologies with labor and public capital. The final good is a CES aggregate of the intermediate goods, $Y_t(z)$ of the form.

$$Y_t = \left(\int_0^1 Y_t(z)^{1-1/\theta} dz \right)^{\frac{\theta}{\theta-1}}$$

Intermediate goods seeks to set prices and labor input in order to maximize profits and subject to the following demand curve for each intermediate input z,

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\theta} Y_t$$

and the price adjustment costs, which are given by,

$$\Sigma_t(z) = \frac{\psi}{2} \left(\frac{P_t(z)}{P_t} - 1 \right)^2 Y_t$$

Consider the dynamic profit maximization problem, in which the firm chooses price $P_t(z)$,

$$\max_{P(z)} E_t \sum_{l=0}^{\infty} \beta^t \frac{U_{c,t+l+1}}{U_{c,t+l}} \left[\frac{P_{t+l}(z)}{P_{t+l}} Y_{t+l}(z) - (1-s) \frac{mc_{t+l}}{P_{t+l}} Y_{t+l}(z) - \frac{\psi}{2} \left(\frac{P_{t+l}(z)}{P_{t+l-1}(z)} - 1 \right)^2 Y_{t+l} \right]$$

subject to the constraints

$$Y_{t+l}(z) = \left(\frac{P_{t+l}(z)}{P_{t+l}} \right)^{-\theta} Y_{t+l}$$

$$Y_{t+l}(z) \geq F(N_{t+l}(z), K_{G,t+l})$$

$U_{c,t+l+1}/U_{c,t+l} = \epsilon_{t+l+1} c_{t+l+1}^{-\sigma} / \epsilon_{t+l} c_{t+l}^{-\sigma}$ is the discount factor of the households that own the shares of firm z.

The firms's first order condition with respect to $P_t(z)$ are given by,

$$\frac{1}{P_t} Y_t \left(\frac{P_t(z)}{P_t} \right)^{-\theta} + \frac{P_t(z)}{P_t^2} Y_t(-\theta) \left(\frac{P_t(z)}{P_t} \right)^{-\theta-1} + \theta \frac{(1-s) mc_t(z)}{P_t} Y_t - \psi \left(\frac{P_t(z)}{P_{t-1}(z)} - 1 \right) \frac{Y_t}{P_{t-1}(z)}$$
$$+ \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} \psi \left(\frac{P_{t+1}(z)}{P_t(z)} - 1 \right) \frac{P_{t+1}(z)}{P_t^2(z)} Y_{t+1} = 0$$

Imposing a symmetric equilibrium in which all firms set the same price, $P_t(z) = P_t$, all the firms consider the same productivity and real wage:

$$\frac{Y_t}{P_t} - \theta\frac{Y_t}{P_t} + \theta\frac{mc_t(1-s)}{P_t}Y_t - \psi\left(\frac{P_t}{P_{t-1}} - 1\right)\frac{Y_t}{P_{t-1}} + \beta E_t\frac{\epsilon_{t+1}c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}}\psi\left(\frac{P_{t+1}}{P_t} - 1\right)\frac{P_{t+1}}{P_t^2}Y_{t+1} = 0$$

Multiply by P_t and dividing by Y_t:

$$1 - \theta - \psi\left(\frac{P_t}{P_{t-1}} - 1\right)\frac{P_t}{P_{t-1}} + \theta(1-s)mc_t + \beta E_t\frac{\epsilon_{t+1}c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}}\psi\left(\frac{P_{t+1}}{P_t} - 1\right)\frac{P_{t+1}}{P_t}\frac{Y_{t+1}}{Y_t} = 0$$

Define gross inflation rate, π_t as P_t/P_{t-1}, then

$$\begin{aligned}
1 - \theta + \theta(1-s)mc_t - \psi\left[(\pi_t - 1)\pi_t - \beta E_t\frac{\epsilon_{t+1}c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}}(\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] = 0 \\
\theta(1-s)mc_t = \theta - 1 + \psi\left[(\pi_t - 1)\pi_t - \beta E_t\frac{\epsilon_{t+1}c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}}(\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] \\
(1-s)mc_t = \frac{\theta - 1}{\theta} + \frac{\psi}{\theta}\left[(\pi_t - 1)\pi_t - \beta E_t\frac{\epsilon_{t+1}c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}}(\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]
\end{aligned}$$

Replacing by marginal cost by the definition, we get the same equation as in text

$$(1-s)\frac{w_t}{F_{N,t}} = \frac{\theta - 1}{\theta} + \frac{\psi}{\theta}\left[(\pi_t - 1)\pi_t - \beta E_t\frac{\epsilon_{t+1}c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}}(\pi_{t+1} - 1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]$$

D Appendix: Numerical strategy for the time-consistent problem

We first solve the linearized version of the model under commitment using Dynare. By doing this, we obtain the policy function for each endogenous variable. Afterwards, we parameterize each variable contained in the expectations described in section 5.3 as linear polynomials of the state variables. Notice that since we are solving a linear model, the derivatives of each variable contained in the expectations term with respect to the state variables are directly the coefficients from the policy function.

We then solve the set of equations for the time-consistent model by using as initial guess for the parameters contained in the expectations, the coefficients from the policy function under commitment. We then update the parameters until the change in the parameters with the previous iteration become less than $1e^{-6}$. To sum up:

- Run a linear version of the model under commitment to get the initial guess for the coefficients of the policy function
  1. The model is linear so policy functions would be of the form $x_t = \alpha_1 s_{t-1} + \alpha_2 s_{t-1} + \alpha_3 z_{t-1}$, where x_t is the endogenous variable, s_t z_t are the endogenous and exogenous state variables
  2. Parameterize the conditional expectations contained in the time-consistent problem as linear function of the state variables. Eg: $E_t c_{t+1} = \alpha_1 E_t b_t + \alpha_2 E_t kg_t + \alpha_3 E_t \zeta_{t+1}$, under consistency, is : $E_t c_{t+1} = \alpha_1 b_t + \alpha_2 kg_t + \alpha_3 \rho \zeta_t$
  3. Once the expectations are parameterized solve the system for the time-consistent problem using Dynare and the coefficients from the policy function obtained in the commitment problem
  4. Update the coefficients until the change with previous iteration is less than $1e^{-6}$

Notice that a linear solution would be of the form:

$$c_t = a_k(k_{G,t-1} - k_G^*) + a_b(b_{t-1} - b^*) + a_z(z_t - z^*) + a_e e_t$$

$$E_t[c_{t+1} - c^*] = E[a_k(k_{G,t} - k_G^*) + a_b(b_{t-1} - b^*) + a_z(z_{t+1} - z^*) + a_e e_{t+1}]$$

$$Ee_{t+1} = 0$$

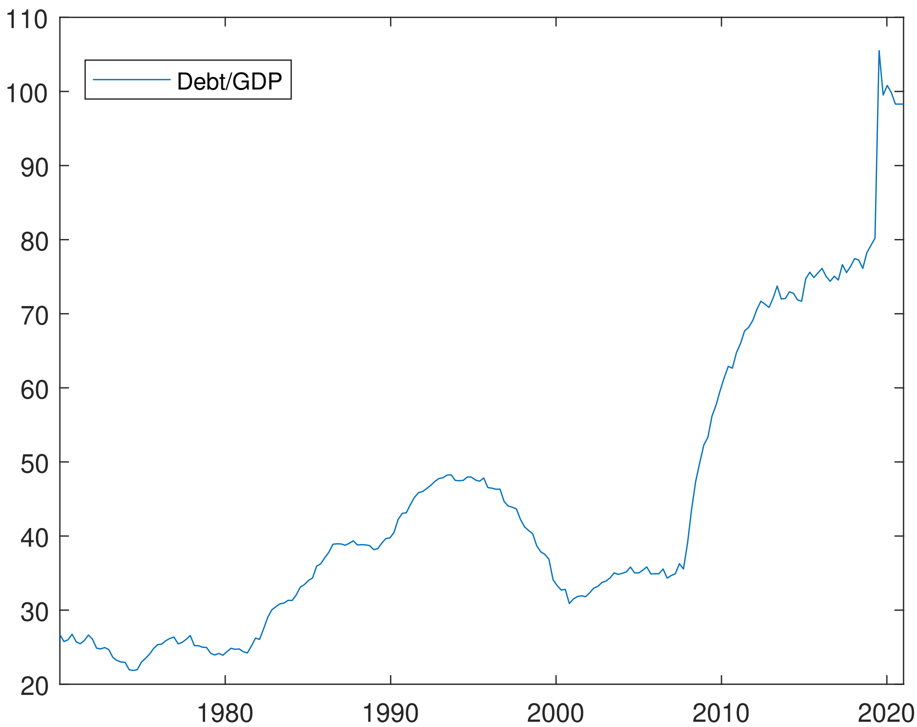
$$E_t[\tilde{c_{t+1}}] = a_k \tilde{k_{g,t}} + a_b \tilde{b_t} + a_z \rho \tilde{z}_t$$

E Appendix: Tables and figures

Table 1: Model parameters

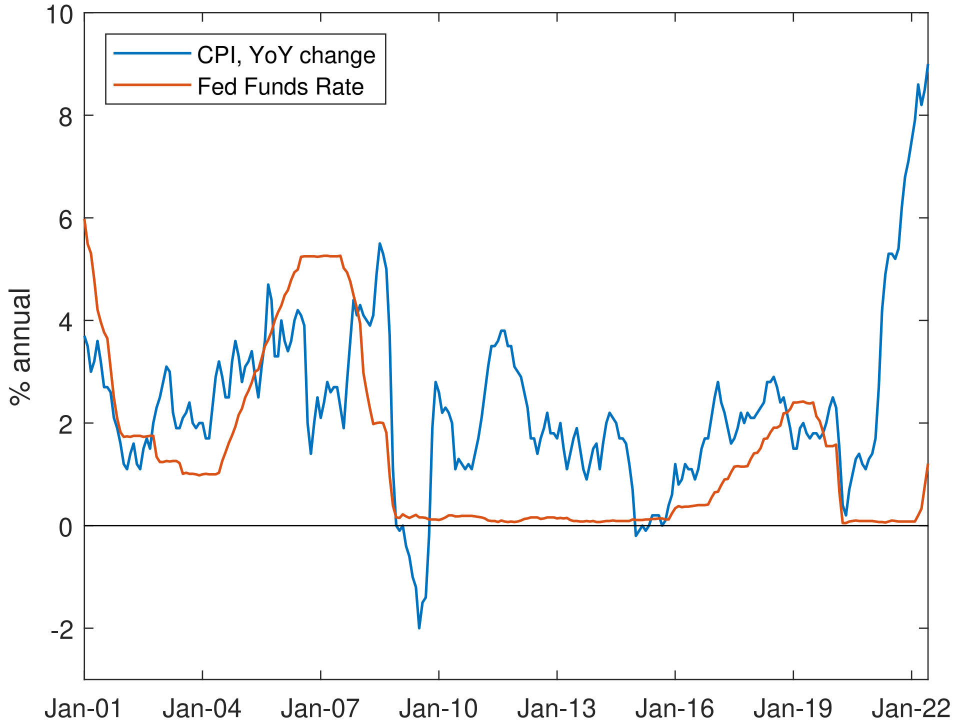
Description	Parameter	Value	Source/Target
Discount factor	β	0.99	r=0.04
Risk aversion	σ	1.5	Leeper et al. (2010)
Frisch elasticity	φ	2	Leeper et al. (2010)
Labor intenstity	a	0.94	Leeper et al. (2010)
Elasticity of substitution intermediate goods	θ	6	target mark-up 20%
Price-adjustment cost parameter	ψ	200	target Phillips curve slope 0.03
Autocorrelation preference shock	ρ_ϵ	0.9	
Autocorrelation liquidity premium shock	ρ_ζ	0.8	
Autocorrelation preference government consumption shock	ρ_z	0.9	
Decaying coupon rate	ρ	0.95	target debt maturity 16 quarters
Depreciation rate	δ	0.03	target $g^{i,*}/y^* = 4.5\%$
Monetary policy			
Feedback to inflation: active	ϕ_π	1.5	Woodford (2011)
passive		0.95	see text
passive		0.7	see text
Government			
Debt to output	$b_L^*/4y^*$	60%	US data
Public consumption to output	$g^{c,*}/y^*$	15%	US data

Figure 1: US public debt held by the public as % of GDP



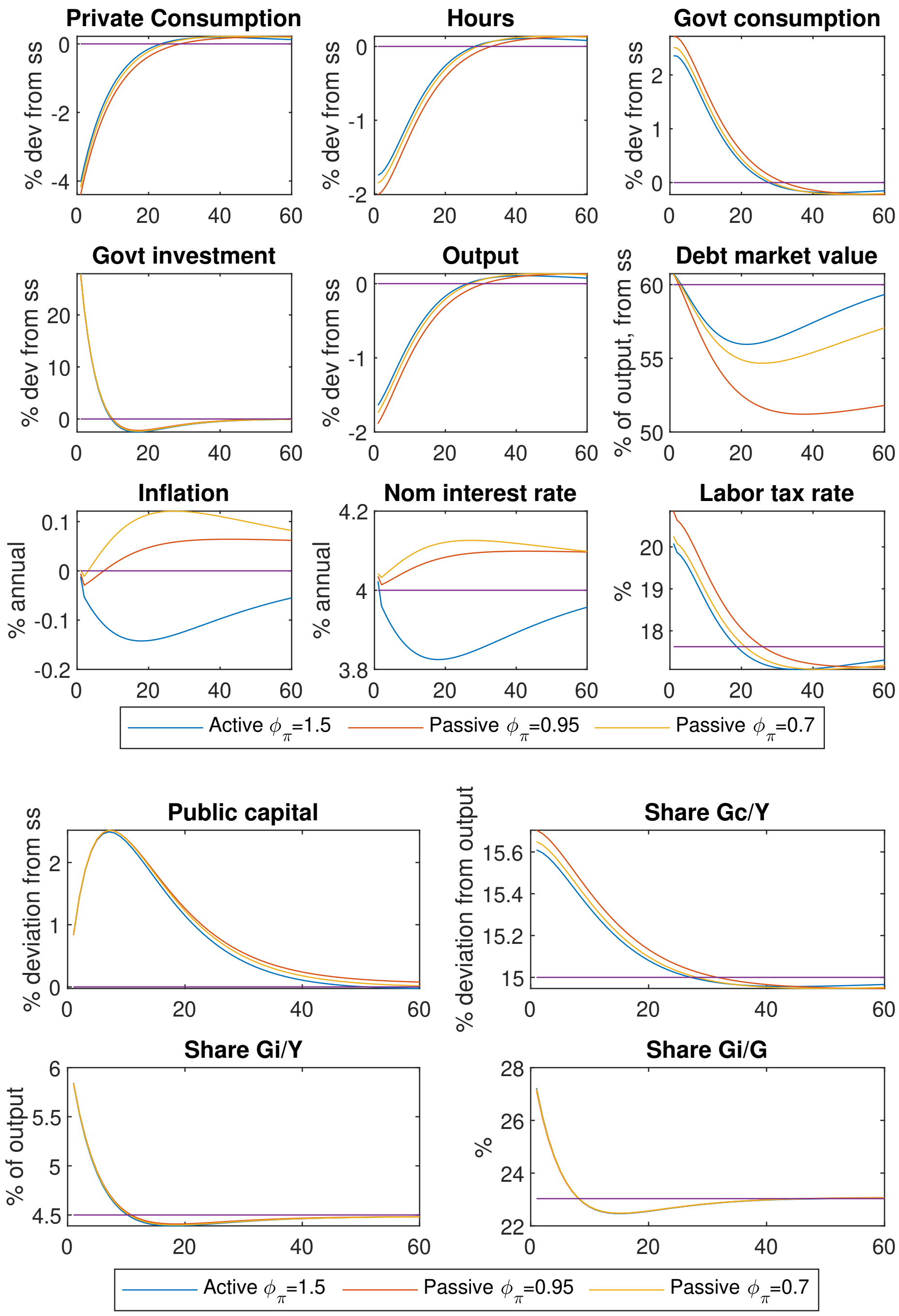
Note: the figure shows US public debt held by the public as percentage of GDP. Source: Federal Reserve Bank of St. Louis

Figure 2: US inflation YoY (%) and FED funds interest rate



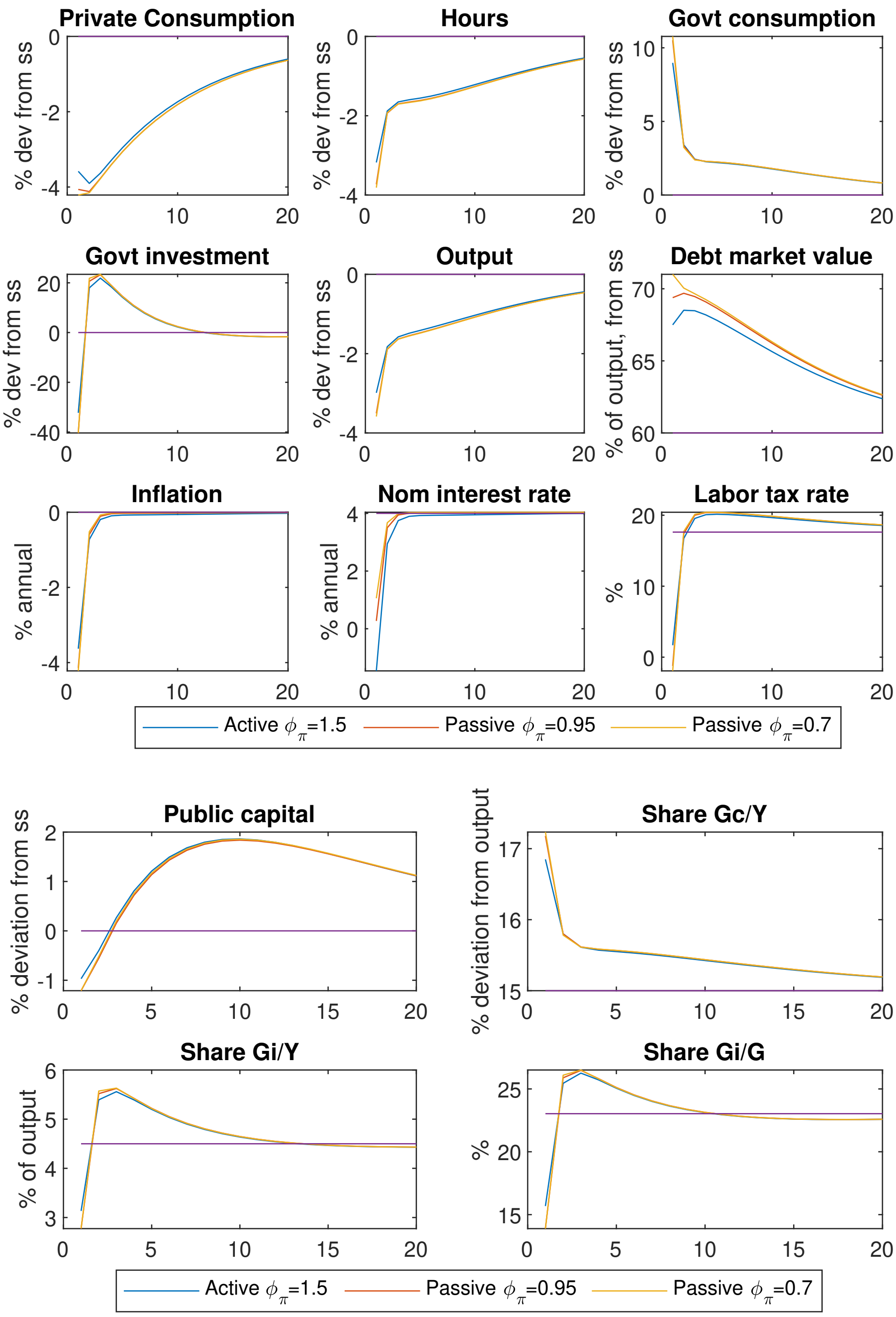
Note: the figure shows the annual change of the CPI for the US (in percentage) as measured by the Bureau of Labor Statistics and the Effective Federal Funds interest rate. Source: Bureau of Labor Statistics and Federal Reserve Bank of St. Louis

Figure 3: Ramsey optimal response to a negative preference shock on private consumption: Active vs Passive monetary policy



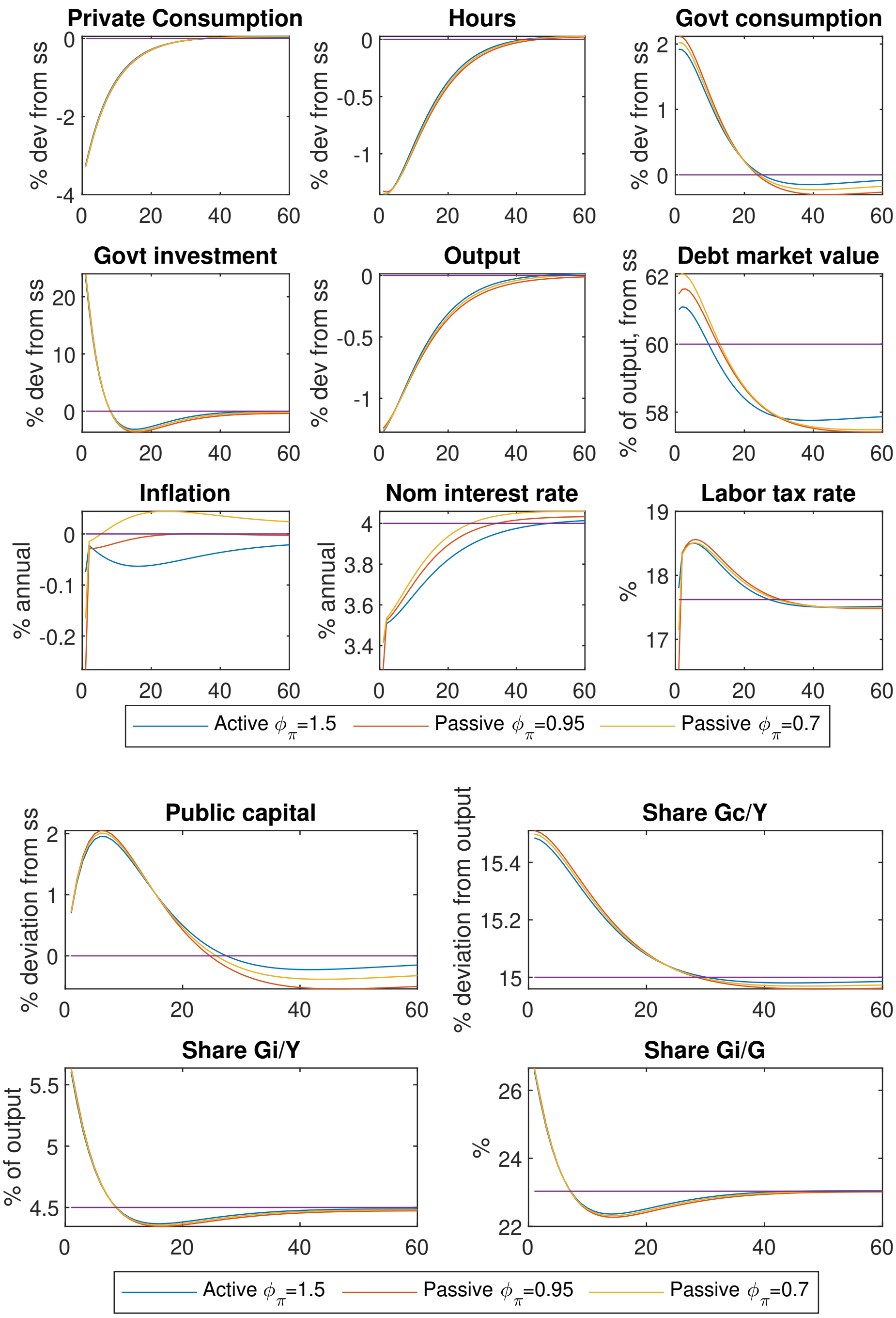
Note: the figure shows the optimal response of the main macroeconomic variables to a negative consumption shock for a committed Ramsey planner. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red red and the orange lines represents the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.7$) ($\phi_\pi = 0.95$) respectively

Figure 4: Time-consistent response to a negative preference shock on private consumption: Active vs passive monetary policy



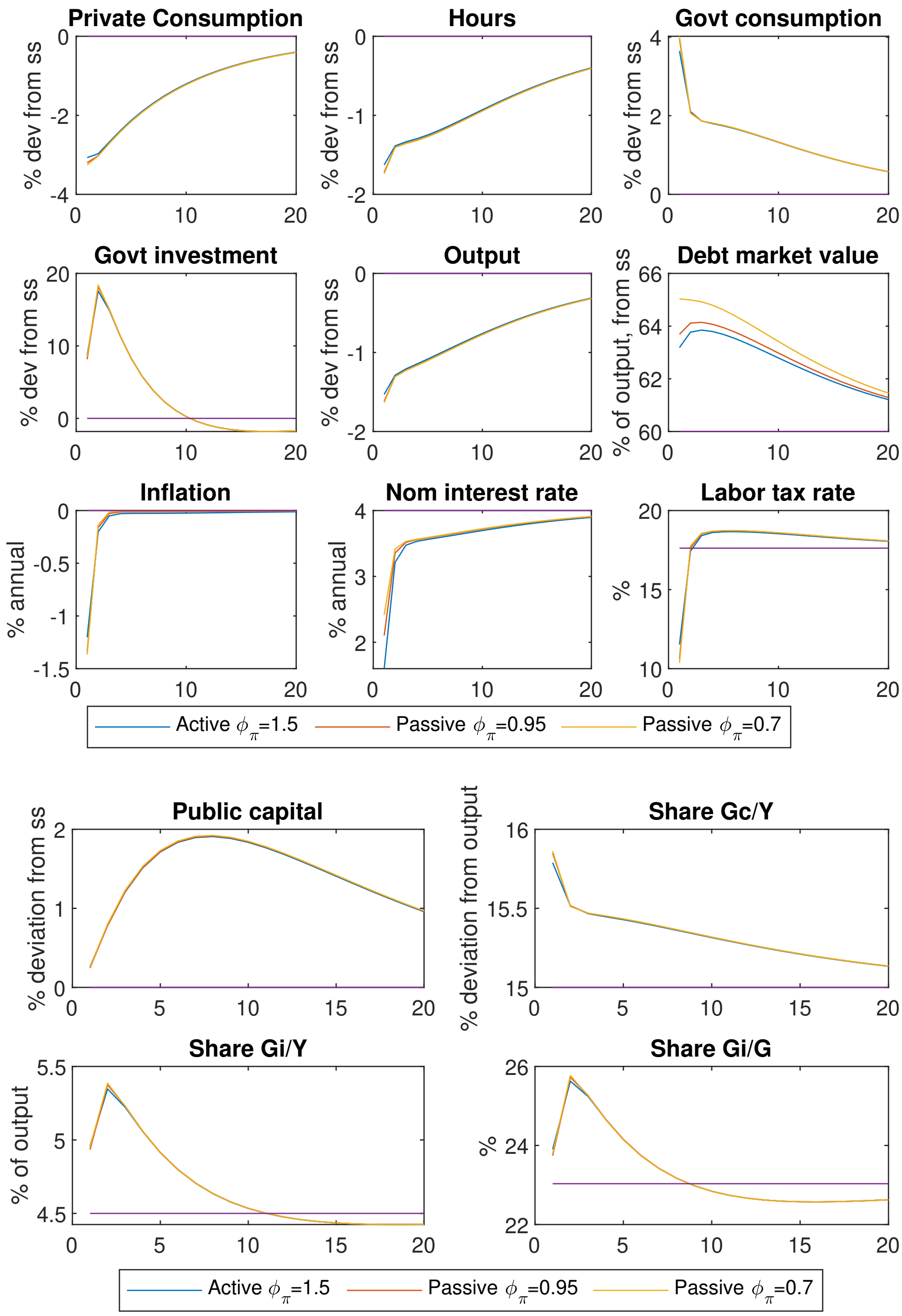
Note: the figure shows the optimal response of the main macroeconomic variables to a negative consumption shock for a time-consistent planner. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red red and the orange lines represents the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.7$) ($\phi_\pi = 0.95$) respectively

Figure 5: Commitment: Optimal response to a negative preference shock on private consumption-Taylor rule targeting inflation and output



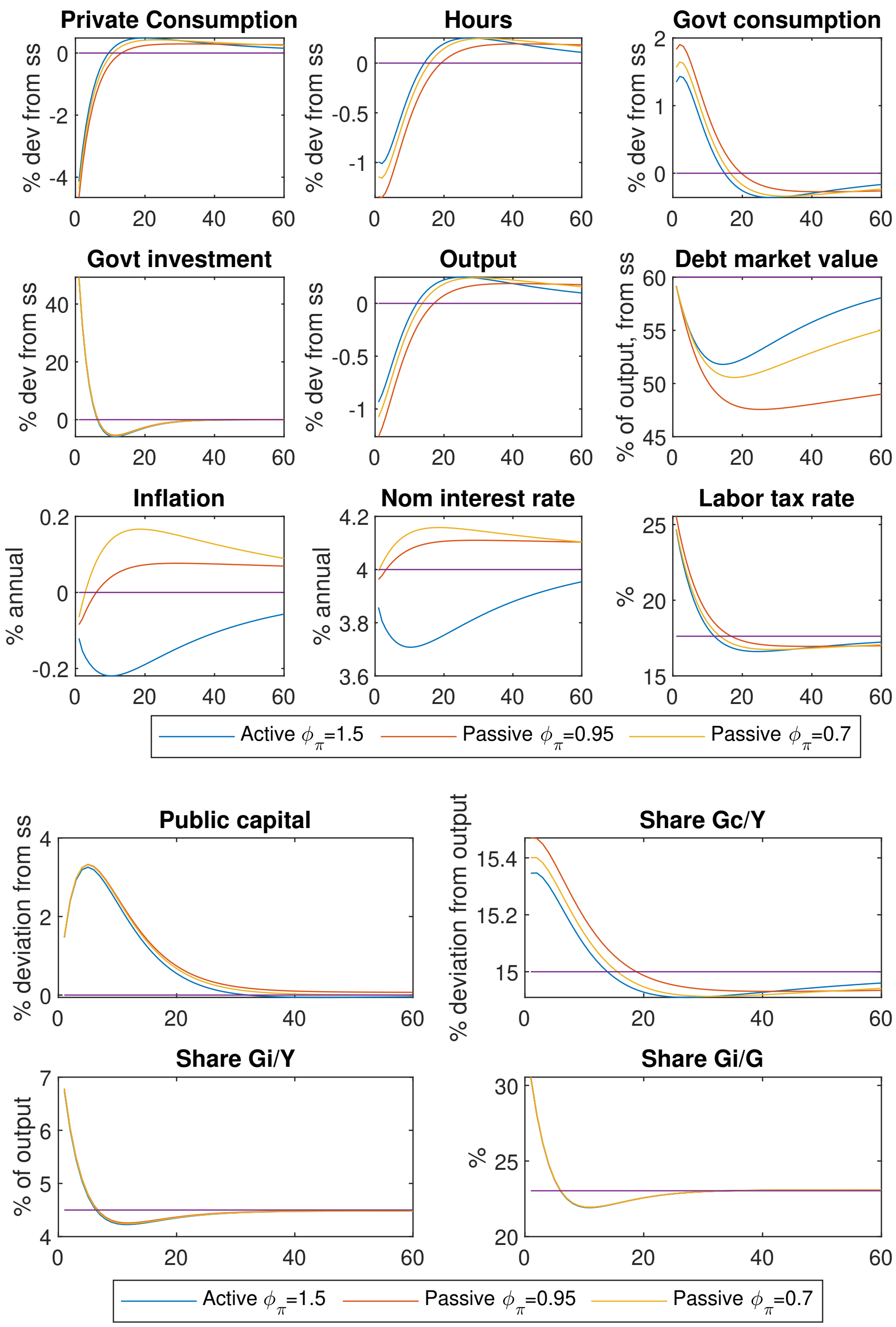
Note: the figure shows the optimal response of the main macroeconomic variables to a liquidity premium shock for a committed Ramsey planner. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red red and the orange lines represents the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.7$) ($\phi_\pi = 0.95$) respectively.In all cases, $\phi_y = 0.1$

Figure 6: Discretion: Optimal response to a negative preference shock on private consumption-Taylor rule targeting inflation and output



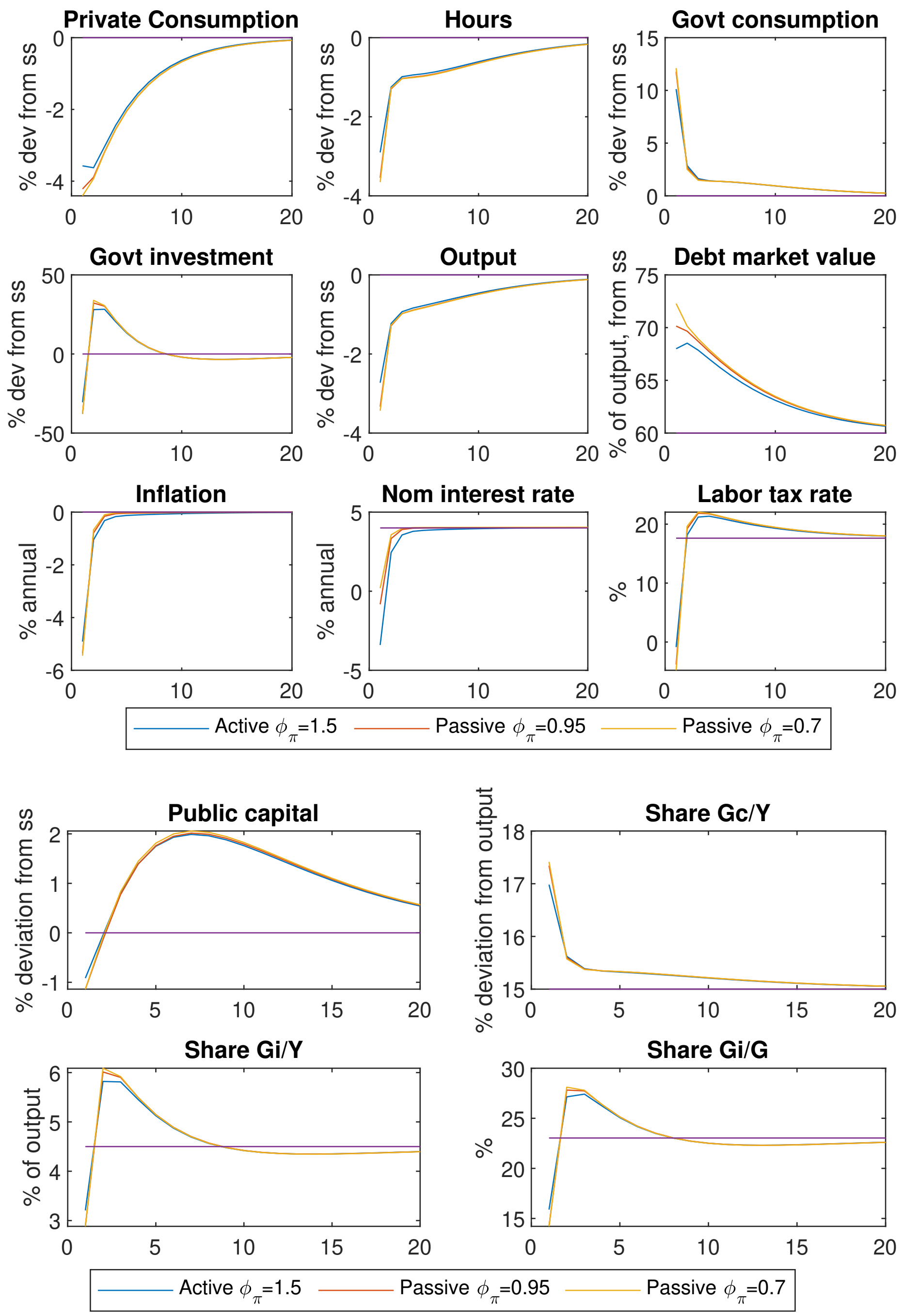
Note: the figure shows the optimal response of the main macroeconomic variables to a negative consumption shock for a time-consistent planner. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red red and the orange lines represents the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.7$) ($\phi_\pi = 0.95$) respectively. In all cases, $\phi_y = 0.1$

Figure 7: Commitment: Optimal response to a liquidity premium shock



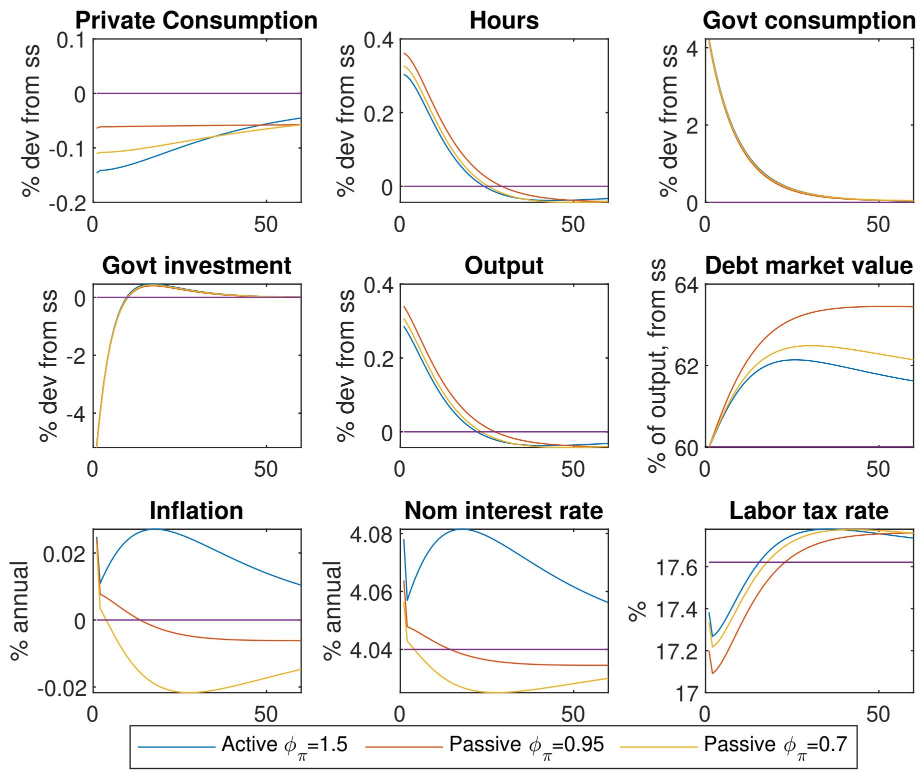
Note: the figure shows the optimal response of the main macroeconomic variables to a liquidity premium shock for a committed Ramsey planner. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red red and the orange lines represents the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.7$) ($\phi_\pi = 0.95$) respectively

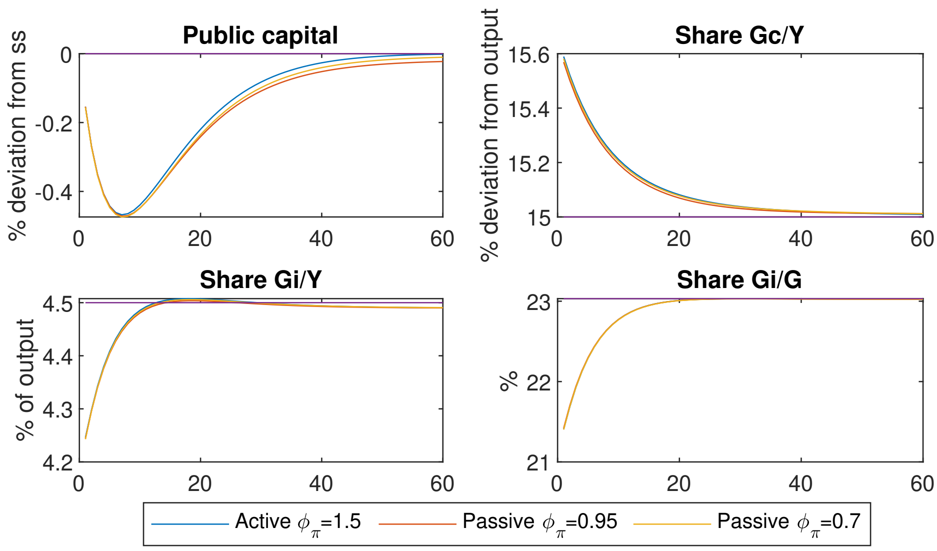
Figure 8: Discretion: Optimal response to a liquidity premium shock



Note: the figure shows the optimal response of the main macroeconomic variables to a negative consumption shock for a time-consistent planner. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red red and the orange lines represents the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.7$) ($\phi_\pi = 0.95$) respectively

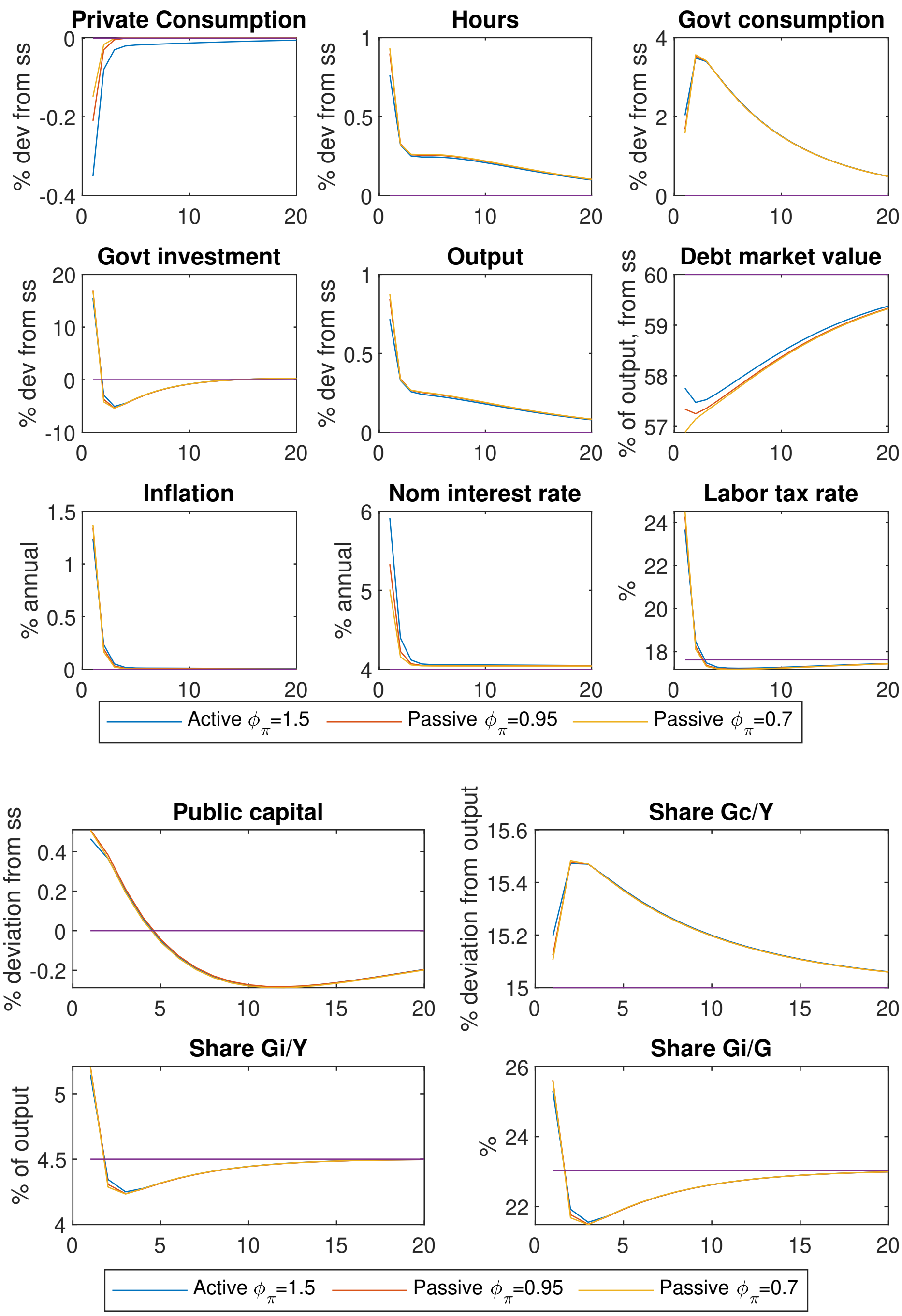
Figure 9: Commitment: Optimal response to a positive shock in preferences for government consumption





Note: the figure shows the optimal response of the main macroeconomic variables to a shock in preferences for government consumption. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red red and the orange lines represents the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.7$) ($\phi_\pi = 0.95$) respectively

Figure 10: Discretion: Optimal to a positive shock in preferences for government consumption



Note: the figure shows the optimal response of the main macroeconomic variables to a shock in preferences for government consumption. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red red and the orange lines represents the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.7$) ($\phi_\pi = 0.95$) respectively

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