OPTIMAL FISCAL AND MONETARY POLICY WITH PREFERENCE OVER SAFE ASSETS

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Abstract

This paper investigates optimal fiscal and monetary policy in a New-Keynesian model with preferences over safe assets (POSA). Relative to a model with standard preferences, a Ramsey planner facing POSA uses inflation more actively to absorb the effects of fiscal and demand shocks despite inflation being costly. The optimal response of inflation to the shocks thus departs from the traditional prescription observed in standard New-keynesian models with sticky prices in which inflation volatility is near zero. Moreover, under POSA taxes are not as smooth as under standard preferences and are more front-loaded, an outcome that brings the model closer to optimal policy under flexible prices. With POSA, debt issuance depresses the liquidity premium, reducing revenues collected by the government and tightening the budget constraint. Therefore the planner is much less willing to issue debt in response to (say) a fiscal shock, which explains the excess tax volatility observed.

These results do not dramatically change when private capital is introduced to the economy, the planner stills finds optimal to use inflation to absorb the shocks. Moreover in spite of the fact that debt issuance is lower private investment is still crowded out under POSA due to the higher distortionary taxes. Finally, the planner faced with POSA outperforms the New-Keynesian planner (with standard preferences) in terms of stabilizing the economy to a negative demand disturbance, but underperforms in terms of managing the government spending shock. The negative demand shock increases the demand for government debt and relaxes the tradeoff facing the planner. The opposite holds in the case of a spending shock.

Keywords: optimal fiscal and monetary policy, bonds in the utility function, distortionary taxes, liquidity premium

JEL Classification: E31, E52, E62, H21

1 Introduction

Since the 1980s public debt as percentage of GDP has been showing an upward trend in most developed economies. This trend in government liabilities has been observed along with a persistent drop in the return on these assets as well as slow economic growth. Most developed countries do not seem to face any significant risk of having a debt crisis in spite of the considerable growth of debt. One plausible explanation for this is that the demand for public debt has been growing too, due to particular services provided by government debt and which are valued by private investors. Indeed a large number of recent papers (see below for an extensive review of the literature) argue that public debt besides a store of value, provides liquidity and safety to private investors and is also valuable as collateral. As a counterpart for providing those services, government bonds pay a lower return than private assets, this in turn implies an extra revenue for the government as the provider of those services.

From the perspective of debt management, fiscal and monetary policies this must surely have implications. First, the additional revenue that a government can enjoy will relax the intertemporal solvency condition of debt, implying that a lower intertemporal surplus can be targeted to finance outstanding liabilities. Second, over the business cycle, debt management, fiscal and monetary policies will probably face different tradeoffs when debt provides services to private investors, and this is the main focus of this paper. Aiming to analyze these differences, we investigate optimal policy when government bonds provide liquidity to private agents and compare this with optimal policy in a model where they do not. To capture liquidity services we assume, as many recent papers do, that bonds enter in the utility function, thus explicitly assuming that agents have 'preferences over safe assets' (POSA).

In this environment, the classical tradeoff between issuing debt to tax in the future vs. taxing today changes drastically. Issuing debt to smooth taxes across time causes a drop in the convenience yield (a sharper rise in the interest rate) and consequently lower revenues for the government collects. This may induce a lower debt issuance and more frontloaded tax distortions in the model with POSA.

Moreover, under sticky prices the planner faces an additional tradeoff, between using inflation today to tax nominal private assets and financing shocks with taxes and debt issuances. When the path of taxation is more frontloaded under POSA, the optimal use of inflation may also change, since inflation will relieve the burden from distortionary taxation.

We analyse these tradeoffs when the economy can experience shocks to government spending that increase the deficit and to demand that induce agents to postpone consumption and increase desired savings.

Our experiments show that when faced with the tradeoffs implied by POSA, the planner prescribes different policies relative to the New-keynesian model with standard preferences. Under POSA, it becomes optimal to use unanticipated inflation to a greater extend in order to tax nominal private wealth (reduce the real payout of public debt), and indeed changes in the prices level absorb a larger part of the shock. The optimal path of inflation is indeed more volatile significantly departing from the near zero volatility of inflation typically obtained in models with sticky prices and non-state contingent debt. Moreover, the tax rate becomes more frontloaded, than under standard preferences a result which proves the reluctance of the planner to issue debt when debt services are valued by private investors.

It is well known, (see for example Aiyagari et al. (2002)) that under optimal Ramsey policy, taxes evolve according to a random walk. In response to a positive spending shock, taxes will permanently increase, this in turn means that public debt will also be permanently higher. This is the celebrated tax smoothing á la Barro result. Though under POSA the tax schedule does have a permanent component implying higher debt in the long run, there is also a mean reverting component which is akin to models with flexible prices or state contingent debt (as in Chari and Kehoe (1999) or Lucas and Stokey (1983)).

We firstly demonstrate these properties in a model in which public debt is the only asset available to private investors. In other words in the baseline model there is no capital. The fact that debt provides liquidity/safety services implies that the price of debt is higher than the household's discount factor and is a function of the real debt level. When debt increases the premium drops, the elasticity is governed by the curvature of the utility function with response to debt. The differences between optimal policy under standard preferences and under POSA turn out to depend a lot on this curvature parameter. The higher the curvature, the more the debt issuance suppresses the premium of government bonds and the more reluctant the planner is to issue debt. This increases the volatility of taxes and inflation in response to shocks.

We then extend the model to include private capital as an alternative asset available to households. This in turn allows to interpret the liquidity/safety premium as the wedge between the return on real capital and the real return obtained from public debt. When a fiscal shock hits the economy, the planner continues to use inflation and taxes more under POSA. Surprisingly, POSA preferences imply also a stronger crowding out effect of the shock on private investment, even though the shock induces a drop in the liquidity premium. We could expect the opposite to hold in this model: Since debt and capital carry different returns a fiscal shock may not induce a strong reaction of the return to capital, but only a reduction in the liquidity premium, which ought to stabilize private investment. We obtain more corwding out, however, under optimal policy and this is only due to the excess volatility of taxes and inflation which reduce the real resources available to the private sector to invest. The return to investment increases more under POSA and this implies a larger drop in private investment.

In response to a negative demand shock, the tradeoff facing the planner improves. Since this shock increases desired savings investment can be crowded in spite of the fact that taxes will increase to finance the shock.

As a final experiment we contrast POSA and standard preferences in a model where monetary and fiscal policies are not jointly optimal (as is usually assumed in the Ramsey literature), and rather monetary policy is independent policy and follows a Taylor rule that sets the nominal interest rate as a function of macroeconomic conditions, inflation and output. Otherwise fiscal variables are set optimally by the planner.

In this version of the model we find that the differences between POSA and standard preferences are much smaller. In line with previous results in the literature, standard preferences produce a larger volatility of inflation and taxes that are serially uncorrelated (giving up on the assumption that monetary policy is optimal typically has an important effect on the optimal allocation in this class of models). Analogously POSA preferences produce serially uncorrelated taxes and slightly more responsive inflation to the shocks. The differences are then not considerable. From this experiment we conclude that assuming an unconstrained and optimal monetary policy is key to get considerable effects on policy when public debt provides liquidity/safety services.

2 Literature review

This paper is related to the considerable literature on optimal fiscal and monetary policies with incomplete financial markets (non-contingent debt), for example Aiyagari et al. (2002), Schmitt-Grohé and Uribe (2004), Faraglia et al. (2013) and Lustig et al. (2008). Two common results in this literature are firstly that that debt and taxes evolve according to random walks and secondly when prices are sticky and hence inflation is distortionary the optimal adjustment to shocks burdens mostly taxes, inflation does not adjust to make debt solvent. In this paper, we maintain the assumption that government debt is non-state contingent but assume that it debt is net wealth for private investors (its value exceeds that of the present value of dividends). Our key finding is that in such an environment a benevolent Ramsey planner finds optimal to use inflation to a much greater extent and do not necessarily increase permanently in response to shocks, rather the optimal path is frontloaded. This brings optimal policy in this model closer to the flexible price benchmark analyzed in Chari et al. (1991).

There is a growing literature that acknowledges the importance of public debt in providing liquidity services and this of course is relevant here. Firstly in the relevant finance literature Krishnamurthy and Vissing-Jorgensen (2012) showed that larger supply of public debt suppresses the premium of Treasuries over private debt instruments. Like in this paper their modelling assumes that public debt provides utility to private investors.

Second, a couple of recent papers study optimal policy when debt is net wealth analysing the implications for tax smoothing, as we do here. Angeletos et al. (2020) builds a model in when debt can be used as collateral in transactions. In a special case of their model in which preferences are linear in consumption, the authors are able to derive an equivalence to a bonds in utility framework and analytically derive implications for optimal fiscal policy using the nonlinear model. In their model the Ramsey planner does not utilize inflation to stabilize debt.

Here, we take a more reduced form approach to a similar policy problem Angeletos et al. (2020) and our results are quantitative derived from a linear model. However, we do not need constrain the model to quasi-linear preferences and consider both taxes and inflation as policy instruments available to the planner. Moreover, though the focus of Angeletos et al. (2020) lies primarily in characterizing transitional dynamics to the Ramsey steady state, here the focus is mainly on the business cycle properties of taxes, debt and inflation. The two approaches thus complement one another.

Canzoneri et al. (2016) study optimal Ramsey policy in a model with cash, credit and bond goods. Public debt can be used to purchase bond goods and so it facilitates transactions for the private sector. However, in the baseline model debt is state contingent and the authors consider how the introduction of an illiquid public debt instrument affects optimal policy allocations. The main result is that with such a bond, the optimality of the Friedman rule and that of perfect tax smoothing holds. In the absence of the illiquid asset, full tax smoothing is not optimal as this leads to excess volatility of the liquid public bonds supply.

The result is similar to the one obtained here, however the framing of the paper is different. We consider the Ramsey solution under POSA (only liquid debt) but do not assume complete financial markets (state-contingent assets). This has implications regarding the timing of debt taxes and inflation; under incomplete markets these variables show considerable persistence through time, one of the points made by this paper is that this result hinges also on the liquidity services of public debt. Moreover, whereas Canzoneri et al. (2016) build a very stylized model to derive analytical results, we explore the quantitative aspects of optimal policy in a more standard New-Keynesian model and also consider a version in which capital is available to investors. The assumption of state contingent assets is thus replaced with a more realistic asset structure, this makes the environment considered in this paper more relevant empirically.

On a more positive side (papers that do not study optimal policies) are Rannenberg (2021) or Bonam (2020) Berentsen and Waller (2018) and Bayer et al. (2020). Rannenberg (2021) shows that with POSA, the fiscal multiplier of a permanent increase in government expenditures is larger when interest rates are at the zero lower bound, however finds only a small difference when interest rates are positive and follow a Taylor rule. Bonam (2020) and Berentsen and Waller (2018) study the effect of the "convenience yield" on the fiscal theory of the price level. Lastly, Bayer et al. (2020) analyses the liquidity channel of fiscal policy showing both empirically and theoretically that there less crowding out of investment. An expansionary fiscal policy lowers the liquidity premium over private illiquid capital and does not reduce investment. As discussed previously we obtain the opposite result under optimal distortionary taxation.

The paper proceeds as follows. Section 3 contains the model environment while 3.4 defines the private equilibrium and the liquidity premium. Section 4 describes the optimal policy problems under commitment and the calibration. Section 5.3 contains the numerical results, while section 7 concludes.

3 The baseline model with Capital

3.1 Households/ Firms and the Government

We model an economy in which households derive utility from holding safe assets as in Rannenberg (2019). Safe assets are short-term government bonds. To save space we will consider in our baseline model that households can also invest in physical capital, a non-liquid asset. We will however also separately consider experiments in which private capital is not available to households, or, equivalently, the capital stock is held constant through time.

Including short term government bonds in the utility function can be motivated in several ways. As in Krishnamurthy and Vissing-Jorgensen (2012) it could be capturing (explicitly) household preferences for safe and liquid assets. Second, a few recent papers have shown that the properties of models of heterogeneous agents, are similar to those of models in which wealth enters in the utility (e.g. Kaplan and Violante (2018) and Hagedorn (2018)). In heterogenous agents models households value safe short term debt since they use this asset to accumulate precautionary savings. Finally, Angeletos et al. (2020) showed in a model in which public debt can serve as collateral, that bonds in utility can arise endogenously under certain assumptions regarding household preferences. The same applies to Canzoneri et al. (2016) who consider a model where households can use bonds to purchase 'bond goods'.

The model is otherwise essentially a standard New-Keynesian model. We assume that firms with price adjustment cost as in Rotemberg (1982), which gives rise to a standard NEw-Keynesian Phillips curve. The government collects revenues through a distortionary labor tax, levied on labour income. It issues debt to smooth taxes across time, financing its primary deficit.

Households

The economy is inhabited by identical households with the following preferences:

$$E_t \sum_{t=0}^{\infty} \beta^t \left(z_t \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_b \frac{b_t^{1-\sigma_b}}{1-\sigma_b} \right) \tag{1}$$

where β denotes the usual discount factor, c_t is consumption, n_t are hours worked and b_t is the real value of risk-free short-term government bonds.[1] We assume χ, σ, φ, χ_b, σ_b are all positive parameters.

z_t is an exogenous shock to the preferences for consumption. A drop in z_t makes the household value future consumption more, relative to current consumption. Notice that this is a different approach to modelling a demand shock than many other papers. It is common in the literature, to model demand shocks as shocks to the discount factor (thus a shock enters multiplicatively to utility and concerns all arguments, not just C). In models of optimal policy such discount factor shocks turn out have only a small effect on the economy, since monetary policy can very effectively track the real rate and neutralize them. Modelling a demand shock like z_t on the other hand will have non-trivial effects on the economy. This type of shock will change the intertemporal preferences for consumption but also the intratemporal allocation between consumption, leisure and bonds. A lower z_t will also imply that the household desires more leisure and enjoys a relatively higher utility from liquid wealth. Both seem plausible for a household that desires to postpone consumption. Finally, note that Faraglia et al. (2013), among others model demand shocks in this way.

Households have at the beggining of period t, b_{t-1} units of real bonds and k_t units of physical capital. They receive a wage for supplying labor and a rent payment from supplying capital to firms. The household budget constraint in real terms can be written as:

$$c_t + b_t + I_t = (1-\tau_t) w_t n_t + r_t^k k_t + D_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + T_t \tag{2}$$

where w_t is the real wage, r_t^k is the real return rate for non-liquid private capital units, D_t and T_t are dividends coming from firm ownership and lump-sum transfers collected by the government. The household also pays τ_t distortionary labor taxes to the government which together with the wage and the rental rate from capital are taken as given by the household. We denote by π_t the gross rate of inflation (P_t/P_{t-1} when P_t is the price of the final good produced in the economy) and R_t is the nominal gross interest rate on public debt.

Private capital accumulation is subject to the following law of motion, which allows for investment adjustment costs:

[1]Thus households value real debt (the ratio of the dollar value of debt they hold divided by the general price level) as opposed to nominal bonds. This is a standard modelling assumption.

$$k_{t+1} = \left(1 - S\left(\frac{It}{I_{t-1}}\right)\right) I_t + (1-\delta)k_t$$

where δ is the depreciation rate and the function $S(.)$ satisfies $S'(.) \geq 0$; $S''(.) \geq 0$, $S(1) = 0$; $S'(1) = 0$, standard properties for adjustment cost functions.

Notice that adjustment costs effectively make private capital non-liquid. Thus the household derives liquidity services from debt (and this changes the utility function) and invests also in non-liquid capital. Both assumptions are needed. Adjustment costs are necessary to have non-trivial investment dynamics. Bonds in utility imply that the supply of bonds affects the Euler equation for short term debt and as will be shown later this will have a big effect on the equilibrium under optimal policy.

Firms

The economy produces a final good that can be used for consumption and investment. The firm that produces this final good is assumed to operate in a perfectly competitive market. The technology is a standard CES function whose inputs are intermediate products:

$$Y_t = \left(\int_0^1 Y_t(j)^{1-1/\theta} dj\right)^{\frac{\theta}{\theta-1}}$$

where $Y_t(j)$ is the quantity of intermediate good j used in the production of the final good and $\theta \geq 1$ denotes the elasticity of substitution between intermediate goods.

Intermediate goods firms have market power, since $Y_t(j)$ are differentiated products. Denoting by $P_t(j)$ the price of the intermediate goods, standard arguments can be applied to show that the demand for j is given by:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t$$

defining a downward sloping demand curve.

The intermediate differentiated good $Y_t(j)$ is producing using labor and capital. Production function for the firm reads,

$$Y_t(j) \geq F(n_t(j), k_t(j)) = n_t(j)^a k_t(j)^b$$

with $a + b = 1$, which is a standard Cobb-Douglas function with constant returns to scale. Firms set prices subject adjustment costs á la Rotenberg. The costs function for firm j is :

$$\Sigma_t(j) = \frac{\psi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t$$

which implies a resources costs when the price level in t differs from the price in $t-1$. $\psi \geq 0$ governs the magnitude of these price adjustment costs.

Dividends payed out by firms are given by:

$$D_t(j) = (1 + \text{Sub})\frac{P_t(j)}{P_t}Y_t(j) - w_t n_t(j) - r_t^k k_t(j) - \Sigma_t(j)$$

Note finally, that we assume that firm production is subsidized, to eliminate inefficiencies due to monopolistic competition (firm market power). The subsidy Sub $= \frac{1}{(\theta-1)}$ is financed through a nondistortionary tax on the household budgets (object T).

Government

Government spending is exogenous to the model and it is uncertain. We assume that households do not derive utility from the level of government expenditures.

To finance expenditures, the government can levy distortionary labor income taxes and issue short-term nominal debt. Let B_t be the the nominal value of debt. The government budget constraint in real terms can be written as:

$$b_t \equiv \frac{B_t}{P_t} = \frac{R_{t-1}}{\pi_t}b_{t-1} + (G_t^c - \tau_t w_t n_t) \tag{3}$$

Where G_t^c stands for public consumption, an exogenous process and a source of uncertainty in the economy.

We next define the optimization problems of households and firms.

3.2 Household program

Households choose sequences for consumption, next period capital, bonds, labor and investment $\{c_t, n_t, b_t, k_{t+1}, I_t\}$ to maximize utility 1 subject to the budget constraint, the law of motion for capital and a No-Ponzi game condition. As usual, households take government policies, wages, rental rate for capital and prices as given. We can state the household program as follows:

$$\max_{\{b_t,k_{t+1},c_t,n_t,I_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left(z_t \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_b \frac{b_t^{1-\sigma_b}}{1-\sigma_b} \right)$$

$st.$

$$k_{t+1} = \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t + (1-\delta) k_t$$

$$c_t + b_t + I_t = (1-\tau_t) w_t n_t + r_t^k k_t + D_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + T_t$$

Solving the above program with standard techniques, yields the following conditions characterizing the optimum:

$$(1-\tau_t) w_t = \frac{\chi n_t^{\varphi}}{z_t c_t^{-\sigma}} \tag{4}$$

$$z_t c_t^{-\sigma} = \beta E_t \left[z_{t+1} \frac{R_t c_{t+1}^{-\sigma}}{\pi_{t+1}} \right] + \chi_b b_t^{-\sigma_b} \tag{5}$$

$$1 = \left[1 - S\left(\frac{I}{I_{t-1}}\right) - S'\left(\frac{I}{I_{t-1}}\right) \frac{I}{I_{t-1}} \right] q_t + \beta E_t \left[\frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} S_I' \left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 q_{t+1} \right] \tag{6}$$

$$q_t = \beta E_t \left[\frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} ((1-\delta) q_{t+1} + r_{t+1}^k) \right] \tag{7}$$

which together with the household budget constraint, the law of accumulation for private capital and the usual boundary (No-Ponzi game) condition form the system of equations that needs to be resolved to determine the optimal allocation.

Let us briefly discuss the above equations. 4 is the optimal usual labor supply condition; at the margin the household equates the utility cost of exerting additional work effort to the benefit it receives, stated in terms of the net wages and the increment in utility derived from an increase in consumption.

Equation 5 is the Euler equation for bonds. Notice that since the household derives utility from the real quantity of bonds, b_t, this is now different than the usual condition equating the utility cost of savings $z_t c_t^{-\sigma}$ with the discounted marginal utility benefit of gaining return R next period (in other words the term $\beta E_t \left[z_{t+1} \frac{R_t c_{t+1}^{-\sigma}}{\pi_{t+1}} \right]$). What is different, obviously, is the term $\chi_b b_t^{-\sigma_b}$ on the RHS of 5. The household in this model will not hold government debt only for its return properties, but also for deriving utility (services, see previous discussion) from debt. Equation 5 then equates the cost of savings, to the benefit measured both in terms of the return and the services that government bonds yield to the household.

Equations 6 and 7 are to be considered jointly. 6 is the first order condition with respect to investment and 7 is an Euler equation for the optimal capital accumulation. Object q_t is the price of installed capital whose dynamics are defined in 6. Then, 7 has the usual interpretation, whereby the household equates the cost q_t of acquiring an extra unit of capital to the (appropriately discounted) benefit $((1-\delta)q_{t+1} + r_{t+1}^k)$, given that capital pays a dividend r_{t+1}^k and can be sold at price q_{t+1} tomorrow.

To explore further these conditions it is worthwhile constrasting the Euler equations for debt and for capital. On the basis of these objects we can then define the liquidity/safety premium for government debt over private capital, the latter can be seen as a non-liquid/non-safe asset in this model.

For simplicity and to make concepts clear, we will momentarily abstract from investment adjustment costs. This enables to get rid of equation 6, and focus only on equations 5 and 7, since trivially $q_t = 1$. What we will then derive here, will also be relevant for the quantitative model of Section 5 where the presence of investment costs is motivated by the observation that without them one cannot get a realistic hump shaped response of investment to shocks (see for example Christiano et al. (2011)).

Assuming $q_t = 1$, we can define the gross real rent of capital as $R^k_t = r^k_t + 1 - \delta$. Merging the Euler equations 5 and 7, we arrive to the following equation on the basis of which we can define the liquidity/safety premia in this version of the model:

$$\beta E_t[z_{t+1}c_{t+1}^{-\sigma}R^k_{t+1}] - \beta E_t\left[\frac{z_{t+1}c_{t+1}^{-\sigma}R_t}{\pi_{t+1}}\right] = \chi_b b_t^{-\sigma_b}$$

Then standard results, enable us to write this as:

$$\beta E_t[z_{t+1}c_{t+1}^{-\sigma}]E_t[R^k_{t+1}] + \beta Cov_t(z_{t+1}c_{t+1}^{-\sigma}, R^k_{t+1}) - \beta E_t[z_{t+1}c_{t+1}^{-\sigma}]E_t\left[\frac{R_t}{\pi_{t+1}}\right] + \beta Cov_t\left[z_{t+1}c_{t+1}^{-\sigma}, \frac{R_t}{\pi_{t+1}}\right] = \chi_b b_t^{-\sigma_b} \tag{8}$$

where the covariance terms capture the asset returns covariances with the marginal utility of consumption. Rearranging this expression we can the excess return of capital over government debt as:

$$\left(E_t R^k_{t+1} - E_t\frac{R_t}{\pi_{t+1}}\right) = \frac{1}{\beta E_t(z_{t+1}c_{t+1}^{-\sigma})}\left[-\beta Cov_t\left(z_{t+1}c_{t+1}^{-\sigma}, R^k_{t+1}\right) + \beta Cov_t\left(z_{t+1}c_{t+1}^{-\sigma}, \frac{R_t}{\pi_{t+1}}\right) + \chi_b b_t^{-\sigma_b}\right] \tag{9}$$

According to 9, the difference in the expected rates of return of capital and debt (the latter defined as the real return, scaled by inflation) is proportional to the difference between the two covariance terms leading the RHS of 9, as proportional also to the marginal utility derived from bond holdings.

Note that in a standard fashion we can utilize the leading difference in the covariance to define a safety premium for bonds. When the return to capital covaries positively with the marginal utility of consumption $z_{t+1}c_{t+1}^{-\sigma}$ then the asset is a poor hedge against consumption risk. A low return is delivered when the consumption is low (and/or when the z is high so that the household does not desire to postpone consumption) and so private capital should compensate the households with a high gross return R^k in expectation. Analogously, the term $Cov_t\left(z_{t+1}c_{t+1}^{-\sigma}, \frac{R_t}{\pi_{t+1}}\right)$ measures the effectiveness of holding government bonds for consumption smoothing purposes. If this covariance is negative then the real return to government debt is low in times of high consumption needs. This increases the expected return $E_t\frac{R_t}{\pi_{t+1}}$ required to compensate the investor for bearing the risk.

In this non-linear model therefore the covariances can be interpreted as a premium to safety of government bonds when the second covariance is less negative than the first. Such a property will normally hold in standard models, though it is perhaps worth mentioning that an entire literature has been devoted to explain why the equity premium predicted by the models is typically too low relative to the data. (For the sake of brevity we will not revisit the arguments here.)

Furthermore, note that the last term on the RHS of 9 also contributes towards a positive difference in expected returns $E_tR_{t+1}^k - E_t\frac{R_t}{\pi_{t+1}}$. In this non-linear version of the model (when the covariance terms measure relative safety) this term can be interpreted as a liquidity premium: Government debt provides services to the private sector (modelled here in reduced form through a direct utility effect) whereas private capital does not provide any services. Then, our modelling assumptions tell as that the higher is the supply of debt, the smaller is the premium and hence the difference between the expected returns of the two assets. This pattern is consistent with recent empirical evidence (e.g. Krishnamurthy and Vissing-Jorgensen (2012)).

Importantly, let us note that up to a first order approximation of this model, the covariance terms will be equal to 0. We can show that

$$\left[R^{k*}U_c^*\right]E_t[\tilde{R}_{t+1}^k] - \left[\frac{R^*U_c^*}{\pi^*}\right][\tilde{R}_t - E_t[\tilde{\pi_{t+1}}]] = \frac{U_b^*}{\beta}[-\sigma_b\tilde{b_t} + \sigma E[\tilde{c_{t+1}}] - E[\tilde{z_{t+1}}]]$$

where we let tildes denote that a variable is in log-deviation from its steady state value and U_c^* and U_b^* are, respectively, the marginal utility of consumption and government bonds at the steady-state.

On the LHS of the above equation we have the differences in the expected real returns of the two assets, and on the RHS we have the supply of government debt and the marginal utility of consumption. Notice that since the covariances will not appear in this linear equation then we have to interpet the RHS of this equation as reflecting both liquidity and safety premia for bonds. This is a shortcut taken by many papers. [2] Moreover, since without the bonds in utility assumption we made here, private capital and government

[2]For example, in the recent QE literature, where the focus is on long term over short term government bonds, the excess risk of holding long term debt is also considered to be a linear function of the relative long bond supply. The modelling assumption here is similar.

It is perhaps worth discussing how a safety premium can vary with the supply of bonds. Note that if indeed bonds are safer than stocks then an increase in their relative supply should make the households consumption stream less volatile. This could in debt would have the same expected return in the linear model, parameters U_b^*, σ_b could be calibrated so that the difference of expected returns in the model is equal to the equity premium one finds in the data. We return to this below.

3.3 Firms problem

Let us now turn to the firm's program. Consider a generic producer in the intermediate sector indexed by j. This firm chooses prices and inputs to maximize the discounted flow of net profits (dividends):

$$E_t \sum_{s=0}^{\infty} \beta^t \frac{z_{t+s} U_{c,t+s}}{z_t U_{C,t}} \left[(1 + \text{Sub}) \frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - w_{t+s} N_{t+s}(j) - r^k_{t+s} k_{t+s}(j) - \frac{\psi}{2} \left(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - 1 \right)^2 Y_{t+s} \right]$$

where the appropriate discount factor is the households asset pricing kernel and the firm's profits are net of price adjustment costs. Moreover, object Sub $= 1/(\theta - 1)$ is a subsidy correcting for the steady state distortions from monoposlitc competition in the goods market.[3]

Optimization for firm j is subject to the following equations:

$$Y_{t+s}(j) = \left(\frac{P_{t+s}(j)}{P_{t+s}} \right)^{-\theta} Y_{t+s}$$
$$Y_{t+s}(j) \geq F(n_{t+s}(j), k_{t+s}(j))$$

For brevity we will note derive explicitly the first order conditions from this program.

It is well known, that in the context of this model where firms bear a resource cost of adjusting prices the equilibrium is symmetric. This will also be the focus here. Thus all firms will solve the same program and we can, hereafter, drop j from notation. Imposing the symmetric equilibrium, we can show that the firm's optimality conditions can be rearranged to obtain the following condition governming the dynamics of inflation in the model:

$$\pi_t(\pi_t - 1) = \frac{\theta}{\psi}(mc_t - 1) + -\beta E_t \left(\frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - 1) \pi_{t+1} \right)$$

which is essentially the Phillips curve at the heart of our model. $\pi_t = (P_t/P_{t-1})$ denotes the gross inflation rate. Variable mc_t is the marginal cost of production. In terms of the standard cost minimization program solved by the firms, mc_t reflects the marginal cost of using labour to produce an additional unit of output $\frac{w_t}{F_{N,t}}$ (where F_N denotes the marginal product of that factor) and the cost of producing with capital $\frac{r_t^k}{F_{K,t}}$

turn mean that the household is more willing to bear the additional risk (measured in terms of the covariance with consumption) of holding stocks, or commands a lower risk premium for doing so. If this non-linear effect is translated to the linear model, then the naturally, the risk premium will be considered a function of b.

[3]Note that this is a standard element in the context of optimal policy in linear quadratic models, which is the framework that we will use here.

(F_K is the marginal product of K). At the optimum firms will equate $\frac{w_t}{F_{N,t}}$ and $\frac{r_t^k}{F_{K,t}}$. (See appendix A for the complete derivations leading up to this equation).

3.4 Definition of the Competitive equilibrium

Having described the firms and households programs we now provide the definition of the competitive equilibrium in this economy. In standard fashion we define the equilibrium for a given sequence of policy variables, taxes and interest rates (and also given the exogenous random realizations of the government spending process). The sequence of policy variables will then be determined as the outcome of the optimzation program of a benevolent planner, that will select policies to implement the welfare maximizing competitive equilibrium. We will later turn to the formal description of that program.

Definition CE: Given exogenous process for the preference shock z_t, and the government spending shock G_t^c, given initial levels for the public debt ,b_{-1} and private capital, k_{-1}, and government policies $\{\tau_t, R_t\}$ a competitive equilibrium is a sequence $\{c_t, n_t, y_t, \pi_t, w_t, b_t, r_t^k, I_t, k_t\}_{t=0}^{\infty}$ such that: (1) $\{c_t, n_t, b_t, k_t, I_t\}_{t=0}^{\infty}$ solves the household problem given prices and government policies, (2) $\{\pi_t\}_{t=0}^{\infty}$ derives from the the optimal pricing program for the firms, (3) the government budget constraint is satisfied and (4) the goods and factor markets clear.

Thus, the following conditions need to hold for a competitive equilibrium:

$$
\begin{aligned}
&(1-\tau_t)w_t = \frac{\chi n_t^{\varphi}}{z_t c_t^{-\sigma}} \\
&z_t c_t^{-\sigma} = \beta E_t\left[z_{t+1}\frac{R_t c_{t+1}^{-\sigma}}{\pi_{t+1}}\right] + \chi_b b_t^{-\sigma_b} \\
&1 = \left[1 - S_I\left(\frac{I_t}{I_{t-1}}\right) - S_I'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right]q_t + \beta E_t\left[\frac{z_{t+1}}{z_t}\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}S_I'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 q_{t+1}\right] \\
&q_t = \beta E_t\left[\frac{z_{t+1}}{z_t}\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}((1-\delta)q_{t+1} + r_{t+1}^k)\right] \\
&mc_t = 1 + \frac{\psi}{\theta}\left[\pi_t(\pi_t - 1) - \beta E_t\left(\frac{z_{t+1}}{z_t}\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\frac{Y_{t+1}}{Y_t}(\pi_{t+1}-1)\pi_{t+1}\right)\right] \\
&k_{t+1} = (1-\delta)k_t + \left(1 - S_I\left(\frac{I_t}{I_{t-1}}\right)\right)I_t \\
&Y_t = F(n_t, k_t) = n_t^a k_t^b \\
&b_t \equiv \frac{B_t}{P_t} = \frac{R_{t-1}}{\pi_t}b_{t-1} + G_t^c - \tau_t w_t n_t \\
&\left(1 - \frac{\psi}{2}\pi_t^2\right)Y_t = c_t + I_t + G_t^c
\end{aligned}
$$

where for the sake of the exposition we repeated equations derived previously. The final equation in the above system is the economy wide resource constraint, stating that total output net of price adjustment costs can be used to finance, consumption, investment and government expenditures.

4 Optimal fiscal and monetary policy

4.1 The Ramsey problem

We now present the optimal policy problem that the planner solves. As standard in the literature on optimal policy, we assume that a benevolent Ramsey planner will set policy variables to select from the set of competitive equilibria defined previously, the one that maximizes household utility 1. Notice that this practically means that optimal policy will involve treating the competitive equilibrium equations in Definition CE, as constraints in the planner's program. However, applying standard results, we can reduce the dimensionality of the program, leave aside some of the variables and corresponding constraints when the latter will be slack in optimization. For example, substituting out wages, using the labour supply condition, we have one less constraint to treat. Analogously variable r_t^k can be be substituted out of the system when we make use of the Euler equation for capital. Given w and r^k will be recovered as residuals (when we have solved the planning program we can use the labour supply and Euler equations to find their exact values) the marginal cost mc will also not need to be explicitly considered in optimization.

Formally, after these substitutions, we can state the planner's program as:

$$\max_{\{c_t,n_t,k_{t+1},I_t,q_t,R_t,b_t,\tau_t,\pi_t\}} = E_0 \sum_{t=0} \beta^t \Bigg\{ \left(z_t \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_b \frac{b_t^{1-\sigma_b}}{1-\sigma_b} \right)$$

$st.$

$$\left[z_t c_t^{-\sigma} - \beta E_t \left[z_{t+1} \frac{c_{t+1}^{-\sigma} R_t}{\pi_{t+1}} \right] - \chi_b b_t^{-\sigma_b} \right]$$

$$\left[\frac{-\chi n_t^{\varphi}}{(1-\tau_t) z_t c_t^{-\sigma} F_n(k_t,n_t)/n_t} - 1 - \frac{\psi}{\theta} \left[\pi_t(\pi_t - 1) - \beta E_t \left(\frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{F(k_{t+1},n_{t+1})}{F(k_t,n_t)} (\pi_{t+1}-1)\pi_{t+1} \right) \right] \right]$$

$$\left[\left(1 - \frac{\psi}{2} \pi_t^2 \right) F(k_t,n_t) - c_t - I_t - G_t^c \right]$$

$$\left[(1-\delta) k_t + \left(1 - S_I \left(\frac{I_t}{I_{t-1}} \right) \right) I_t - k_{t+1} \right]$$

$$\left[\left[1 - S_I \left(\frac{I_t}{I_{t-1}} \right) - S_I' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] q_t + \beta \frac{z_{t+1}}{z_t} \frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} S_I' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 q_{t+1} - 1 \right]$$

$$\left[\beta \frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} ((1-\delta) + \frac{-\chi n_{t+1}^{\varphi}}{(1-\tau_{t+1}) z_{t+1} c_{t+1}^{-\sigma} F_n(k_{t+1},n_{t+1})/n_{t+1}} F_k(k_{t+1},n_{t+1})) - 1 \right]$$

$$\left[b_t - \frac{R_{t-1}}{\pi_t} b_{t-1} - G_t^c + \tau_t \left[\frac{-\chi n_t^{\varphi}}{(1-\tau_t) z_t c_t^{-\sigma}} \right] n_t \right] \Bigg\}$$

Then to solve the above program we solve the following standard optimization problem with the Lagrangian:

$$\mathcal{L} = E_0 \sum_{t=0} \beta^t \Bigg\{ \left(z_t \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_b \frac{b_t^{1-\sigma_b}}{1-\sigma_b} \right)$$
$$+\phi_{1,t}\left[z_t c_t^{-\sigma} - \beta E_t \left[z_{t+1} \frac{c_{t+1}^{-\sigma} R_t}{\pi_{t+1}} \right] - \chi_b b_t^{-\sigma_b} \right]$$
$$+\phi_{2,t}\left[\frac{-\chi n_t^{\varphi}}{(1-\tau_t) z_t c_t^{-\sigma} F_n(k_t, n_t)/n_t} - 1 - \frac{\psi}{\theta}\left[\pi_t(\pi_t - 1) - \beta E_t \left(\frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{F(k_{t+1}, n_{t+1})}{F(k_t, n_t)} (\pi_{t+1} - 1)\pi_{t+1} \right) \right] \right]$$
$$+\phi_{3,t}\left[\left(1 - \frac{\psi}{2}\pi_t^2 \right) F(k_t, n_t) - c_t - I_t - G_t^c \right]$$
$$+\phi_{4,t}\left[(1-\delta)k_t + \left(1 - S_I\left(\frac{I_t}{I_{t-1}} \right) \right) I_t - k_{t+1} \right]$$
$$+\phi_{5,t}\left[\left[1 - S_I\left(\frac{I_t}{I_{t-1}} \right) - S_I'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] q_t + \beta \frac{z_{t+1}}{z_t} \frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} S_I'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 q_{t+1} - 1 \right]$$
$$+\phi_{6,t}\left[\beta \frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \left((1-\delta) + \frac{-\chi n_{t+1}^{\varphi}}{(1-\tau_{t+1}) z_{t+1} c_{t+1}^{-\sigma} F_n(k_{t+1}, n_{t+1})/n_{t+1}} F_k(k_{t+1}, n_{t+1}) \right) - 1 \right]$$
$$+\phi_{7,t}\left[b_t - \frac{R_{t-1}}{\pi_t} b_{t-1} - G_t^c + \tau_t \left[\frac{-\chi n_t^{\varphi}}{(1-\tau_t) z_t c_t^{-\sigma}} \right] n_t \right] \Bigg\}$$

where ϕ_i denote the Lagrange multipliers we attach to the constraints and we further asset $\phi_{i,-1},\ i = 1, ..., 7$ are given.

For brevity, the first-order conditions for the problem are stated in appendix B.

4.2 The log-linear system of equations.

To solve for optimal policies we will resort to a log-linear approximation of the first order conditions and the constraints. The expressions for the log-linear competitive equilibrium constraints are given by;

$$-\sigma c^{*-\sigma} \tilde{c}_t + \tilde{z}_t = c^{*-\sigma} \beta R^* [E_t \tilde{z_{t+1}} + \tilde{R}_t - \sigma E_t \tilde{c}_{t+1} - E_t \tilde{\pi}_{t+1}] - \sigma_b b^* \tilde{b}_t$$
$$\tilde{\pi}_t = \frac{\theta}{\psi}\left(\frac{\tau^*}{1-\tau^*}\tilde{\tau}_t + \varphi \tilde{n}_t + \sigma \tilde{c}_t - (a\tilde{n}_t + b\tilde{k}_t) + \tilde{n}_t - \tilde{z}_t \right) + \beta E_t[\tilde{\pi_{t+1}}]$$
$$a\tilde{n}_t + b\tilde{k}_t = \frac{c^*}{y^*}\tilde{c}_t + \frac{i^*}{y^*}\tilde{i}_t + \frac{g^{c,*}}{y^*}\tilde{g}_t^c$$
$$\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \delta \tilde{i}_t$$
$$\tilde{i}_t = \frac{1}{1+\beta}\tilde{i_{t-1}} + \frac{\beta}{1+\beta}E_t[\tilde{i_{t+1}}] + \frac{1}{\omega(1+\beta)}\tilde{q}_t$$
$$\tilde{q}_t = E_t \tilde{z_{t+1}} - \tilde{z}_t + \beta(1-\delta)E_t[\tilde{q}_{t+1}] + \sigma[E_t[\tilde{c}_{t+1}] - \tilde{c}_t] +$$
$$+\left[1 - \beta(1-\delta) \right] E_t \left[\frac{\tau^*}{1-\tau^*}\tilde{\tau_{t+1}} + \varphi \tilde{n_{t+1}} + \sigma \tilde{c_{t+1}} - (a\tilde{n}_{t+1} + b\tilde{k}_{t+1}) + \tilde{n_{t+1}} - \tilde{z_{t+1}} + a\tilde{n}_{t+1} + b\tilde{k}_{t+1} - \tilde{k}_{t+1} \right]$$
$$b^*\tilde{b}_t = R^* b^* [\tilde{R_{t-1}} - \tilde{\pi}_t + \tilde{b}_{t-1}] + g^{c,*}\tilde{g}_t^c - \tau^* \frac{\chi n^{*\varphi}}{(1-\tau^*)c^{*\sigma}} n^* \left(\tilde{\tau}_t + \frac{\tau^*}{1-\tau^*}\tilde{\tau}_t + \varphi \tilde{n}_t + \sigma \tilde{c}_t - \tilde{z}_t + \tilde{n}_t \right)$$

where these equations represent, the (log-linear) Euler equation, the Phillips curve the reosource constraints and the law of motion of capital. We then have the optimality conditions for invest and the Euler equation for capital and finally the government budget constraint.

Analogously, the linearized the first order conditions derived from the Lagrangian are given by:

$$c_t: \ U_c^*(-\sigma\tilde{c_t}+\tilde{z_t})+U_{cc}^*\hat{\phi_{1,t}}-R^*U_{cc}^*\hat{\phi_{1,t-1}}+\sigma(1/c^*)\hat{\phi_{2,t}}-\phi_3^*\hat{\phi_{3,t}}+n^*\frac{\chi n^{*\varphi}}{(1-\tau^*)c^{*\sigma}}\tau^*\sigma(1/c^*)\hat{\phi_{7,t}}$$

$$+\beta((1-\delta)+F_k^*)(-U_{cc}^*/U_c^*)\hat{\phi_{6,t}}-((1-\delta)+F_k^*)(-U_{cc}^*/U_c^*)\hat{\phi_{6t-1}}+\hat{\phi_{6,t-1}}(-U_{cc}^*/U_c^*)F_k^*=0$$

$$n_t: \varphi U_n^*\tilde{n_t}+(1+\varphi-a)\frac{1}{n^*}\hat{\phi_{2,t}}+\phi_3^*Y_n^*(\hat{\phi_{3,t}}-\tilde{n_t}+a\tilde{n_t}+b\tilde{k_t})+((1+\varphi)\frac{F_k^*}{n^*})\hat{\phi_{6,t-1}}$$

$$+(\tau^*\frac{\chi n^{*\varphi}}{(1-\tau^*)c^{*\sigma}}n^*(1+\varphi))\hat{\phi_{7,t}}=0$$

$$I_t: \ -\hat{\phi_{3,t}}+\hat{\phi_{4,t}}-\omega(1+\beta)\tilde{i_t}+\omega\tilde{i_{t-1}}+\beta\omega E_t\tilde{i_{t+1}}+\beta\omega/(\phi_4^*I^*)E_t\hat{\phi_{5,t+1}}+\omega/(\phi_4^*I^*)\hat{\phi_{5,t-1}}-$$

$$-\omega(1+\beta)/(\phi_4^*I^*)\hat{\phi_{5,t}}=0$$

$$q_t: \ \hat{\phi_{5,t}}-\hat{\phi_{6,t}}+(1-\delta)\hat{\phi_{6,t-1}}=0$$

$$k_{t+1}: \ -\beta(b/k^*)E_t\hat{\phi_{2,t+1}}+\phi_3^*\beta F_k^*E_t(\tilde{\phi_{3,t+1}}+a\tilde{n_{t+1}}+b\tilde{k_{t+1}}-\tilde{k_{t+1}})-\phi_4^*\hat{\phi_{4,t}}+\beta(1-\delta)\phi_4^*E_t\hat{\phi_{4,t+1}}$$

$$-\hat{\phi_{6,t}}((1-\beta(1-\delta))/k^*)=0$$

$$\pi_t: \ (R^*/\beta)\hat{\phi_{1,t-1}}+(\psi/\theta)(\hat{\phi_{2,t-1}}-\hat{\phi_{2,t}})-\phi_3^*Y^*\psi\hat{\pi}_t=0$$

$$R_t: \ -\beta U^c\hat{\phi_{1,t}}-\beta b^*E_t\hat{\phi_{7,t+1}}=0$$

$$b_t: \ -\sigma_b U_b^*\tilde{b_t}-U_{bb}^*\hat{\phi_{1,t}}+\hat{\phi_{7,t}}-\beta R^*E_t\hat{\phi_{7,t+1}}=0$$

$$\tau_t: \ (1/(1-\tau^*))\hat{\phi_{2,t}}+(F_k^*/(k^*(1-\tau^*)))\hat{\phi_{6,t-1}}+n^*\frac{\chi n^{*\varphi}}{(1-\tau^*)c^{*\sigma}}((2-\tau^*)/(1-\tau^*))\hat{\phi_{7,t}}=0$$

where for convenience, in front of each equation we have added the variable to which the FONC corresponds. As previously, we use $\tilde{x_t}$ to indicate that variable x is in log-deviation relative to the steady state, x^*. Variables denoted with $\hat{x_t}$ are in level deviations from steady state. We use both types of approximations in levels and in logs, since some of the variables (e.g. the multipliers) could be zero in the steady state (i.e some of the constraints are not relevant for the steady state allocation. This is a standard property of Ramsey programs).

It is perhaps too costly in terms of space to inspect in detail each and every one of the above equations. Let us therefore focus on a couple of noteworthy features of the optimal allocation, to conserve space. Note that a central object in this class of models is the multiplier attached to the government budget constraint. We can write:

$$-\sigma_b U_b^*\tilde{b_t}-U_{bb}^*\hat{\phi_{1,t}}+\hat{\phi_{7,t}}-\vartheta E_t\hat{\phi_{7,t+1}}=0 \tag{10}$$

which is a difference equation determining the dynamics of this variable. U^*_{bb} is the second order derivative of utility with respect to government bonds evaluated at the steady state. Moreover, for simplicity, we wrote $\vartheta \equiv \beta R$.

Consider first the standard model without POSA preferences; In this case we have that $\vartheta = 1$ and $U^*_b, U^*_{bb} = 0$. Then the above equation reduces to the well-known result in the literature on Ramsey fiscal policy under incomplete markets in which the multiplier attached to the government budget constraint, ϕ_7, follows a random walk (see Schmitt-Grohé and Uribe (2004), Aiyagari et al. (2002) and Faraglia et al. (2013) among others). The intuition behind this well known property is the following: Since the planner issued debt to smooth tax distortions over time, then she will optimally target to increase taxes permanently following a shock (to either spending or preferences). Making taxes permanently increase, essentially means that debt should also follow a random walk, hence the process for the multiplier.

In the presence of POSA, this equation changes. There is now an additional term reflecting the contribution of debt to household utility (the leading term in the above equation) and another term (measured by the multiplier ϕ_1) which captures the change in the price of government debt for a given change in the quantity of bonds outstanding. Finally, (besides these two forcing terms) the stochastic difference equation does not imply that ϕ_7 will need to follow a random walk, since also $\vartheta < 1$. Let us briefly explain why this is so. First, it is obvious that the planner needs to account for the direct effect that b has on household utility. Second, the term multiplied by ϕ_1 basically captures the effect of a change in the bond quantity of the price of bonds. As discussed previously, the risk/liquidity premium in this model is a function of b, with higher debt supply implying a smaller premium. Thus the planner also accounts for the effect of b on the price.

Notice that if we had these leading two terms were set equal to 0, and the FONC could be written as $\hat{\phi_{7,t}} - \vartheta E_t \hat{\phi_{7,t+1}} = 0$ then it would be simple to show that $\hat{\phi_{7,t}} = 0$ for all t. The government budget is then not a relevant constraint for the planning program; to understand why this may be so, note that since debt trades at a higher average price than β, then hypothetically, financing government debt could be done simply through issuing more debt. The budget constraint is thus slack.

However, since increasing the debt supply will entail a drop in the price of debt (thus an increase in the real rate) and because debt is an argument of utility (and hence the planner may not target a debt level that maximizes rents in terms of government budget) we will in general have $\hat{\phi_{7,t}} \neq 0$.

In short, the dynamics of debt, taxes and the multiplier under POSA preferences are going to be completely different from the usual Ramsey policy analogues in the standard model. We next calibrate and solve the models to discern how these differences translate into different paths for macroeconomic and policy variables.

5 Quantitative experiments

5.1 Calibration

We first describe the calibration of the model's parameters.

We analyze the optimal response of the economy to two types of shocks: a government spending shock and a negative preference shock. For both shocks, we compare the response of the economy under optimal policy with POSA and the response with standard preferences (NOPOSA) to assess the effect of introducing liquidity preferences on optimal fiscal policy.

We calibrate the model with quarterly data from the US. We assume the economy is at the steady-state in period 0. We assume a zero inflation steady-state, thus $\pi^* = 1$. Table 1 in appendix E contains the calibrated parameters and their counterparts in the data. We fix the debt to output ratio to be 60%, a standard value in the literature on optimal fiscal policy. The the labour income tax is 24.7% at steady-state. Government spending is set to 20% of output broadly consistent with the data on the US economy for the period 1987-2018.

[Table 1 approximately here]

Moreover, the gross nominal interest rate, R^* at the steady state is set to 1.005. In annual terms, this gives an interest rate of 2,02%. With NOPOSA, this value for the interest rate implies $\beta = 1/R^*$ is equal to 0.995. For the POSA model we set the value for ϑ in 0.99 following Campbell et al. (2017). Then $\beta = \frac{\vartheta}{R^*} = 0.9851$.

Given these calibrated values, we can compute the value for parameter χ_b at the steady-state. We obtain χ_b equal to 0.0261 when $\sigma_b = 0.2$ and equal to 0.03 when $\sigma_b = 1$.

As discussed previously we will present results from a model where households can purchase government bonds and invest in private capital, but also explore a model where investments in private capital are ruled out and government debt is the only asset available to households. The above calibration is then used for both models. [4]

The curvature parameter σ_b is set to 0.2 in the baseline calibration of the model as in Rannenberg (2021). We will also experiment with $\sigma_b = 1$. Finally, the intertemporal elasticity of substitution is set to 1, and nverse Frisch elasticity is also 1 following much of the related literature. We choose the value for χ such

[4] With the calibrated values for β and R^*, we get private investment equal to 17.3% of output in the model with NOPOSA, at steady state. This is consistent with US data extracted from the Federal Reserve Bank of St. Louis for the period 1987-2018. We set the depreciation rate for private capital to be 0.02. In the model with POSA, when $\vartheta = 0.99$ we get a rati of investment over output equal to 16% not far from the data average.

Note that, Rannenberg (2021) calibrates ϑ to be 0.96 in his model, which gives $\beta = 0.9557$. If we set chose these values instead then ratio I/Y falls to 10%, much lower than the 17.3% observed in data. We show in the appendix the results do not change if we consider a model without capital and $\vartheta = 0.96$ as in Rannenberg (2021) that the household spends 1/3 of hours working at the steady state.

Given these parameter values the POSA model predicts an excess return of capital over government debt equal to 4.2% (at the annual horizon). Damodaran (2015) computes the implied equity premium as the return differential on S&P500 and 10-year Treasury bond, to be equal to 4.1% in the US for the period 1987-2018. Thus our model is consistent with this crucial data moment.

The production technology is Cobb-Douglas with constant returns to scale. We assume that labor intensity is 0.64, while the elasticity with respect to private capital is 0.36 (see Christiano et al. (2005)). The elasticity of substitution between intermediate inputs is 6, so the steady-state mark-up is 20% (before correction by the subsidy). The parameter governing price adjustment costs, ψ is set to be 200, and is calibrated together with θ to have a slope of the linearized Phillips curve equal to 0.03. Following Christiano et al. (2005), investment adjustment costs are of the form:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

and we set the parameter ω to 5.5 following Rannenberg (2019). The presence of adjustment costs shapes the decisions of the planner regarding investment, which in this model is a vehicle for savings and most important a substitute, although imperfect, for government bonds.

Finally, government purchases G_t^c and the shock to preferences over private consumption, z_t are exogenous and follow AR(1) process of the form:

$$log(z_t) = \rho_z log(z_{t-1}) + \epsilon_t, \;\; \epsilon_t \sim \mathcal{N}(0, \sigma_z^2)$$

$$log(g^c) = \rho_g log(g_{t-1}^c) + \nu_t, \;\; \nu_t \sim \mathcal{N}(0, \sigma_{g^c}^2)$$

where ρ_z and ρ_g are set to 0.8, thus both of them persistent process. We assume ϵ_t and ν_t are i.i.d. disturbances with a standard deviation of 1%. Since the paper is rather normative, we do not aim to match the data but instead to analyze optimal fiscal policy in the context of persistent shocks.

5.2 Quantitative results: the economy without capital

5.2.1 Fiscal expenditure shock

We first focus on the economy without capital. Figure 1 shows the response of the economy to an unexpected increase of 1 percentage point in government spending. The blue line depicts the optimal response under standard preferences, the red line and the green dashed lines are the responses with POSA for $\sigma_b = 0.2$ and $\sigma_b = 1$, respectively. For all the models considered, the government spending shock increases the interest rate, thus crowding out private consumption. Output and hours increase driven by standard wealth effects on labour supply.

[Figure 1 approximately here]

Note that the increase in government spending is financed partly by newly issued debt and partly through higher labour income taxes and higher inflation (the inflation tax channel). The extent to which the planner makes use of debt vs taxes to finance the shock differs across the models. Firstly, taxes are more volatile with POSA than under NOPOSA and therefore a larger fraction of the shock is financed through the adjustment of taxes. Secondly, the path of taxes differs across models, with the POSA model predicting a more frontloaded response of the tax schedule to the shock.

Clearly the driving force behind these differences is the fact that with POSA the planner will resist increasing government debt considerably after the shock. This is evident from the response of real bonds shown in the bottom left panel. The reason is twofold; firstly, debt is an argument in utility and so a large reaction to the shock increases the supply of bonds too much relative to the target level implied by the household utility; moreover, since increasing bond supply lowers the bond price (increases the nominal interest rate) the government has to pay a higher cost of financing debt.

These forces are not present in the NOPOSA model debt and taxes essentially follow a random walk.[5] With POSA, taxes are frontloaded, and do not follow a random walk.

It is also worth noting that the responses vary with σ_b as expected. Higher σ_b implies a more rapid increase in the cost of issuing debt when the debt level rises, and this provides the incentive to rely more on taxation to finance the shock. Analogously, assuming a higher σ_b implies that the household is less willing to tolerate large swings in the real value of debt and the response of debt is more muted also for this reason.

The top right panel in 1 shows the liquidity/safety premium's response to the shock. Consistent what we said previously, the premium falls more in the case where $\sigma_b = 1$ in spite of the fact that the bond supply changes less.

Finally, let us discuss how inflation reacts to the shock. Under all models considered, inflation increases after the increase in spending, but the rise of inflation will only last for one model period. This is not surprising. Since debt is short-term, the only margin via which inflation can erode the real value of debt and stabilize the government's budget following the shock, is the contemporaneous one. Had we assumed

[5] To be precise the process is not a pure random walk. There is an additional effect which increases taxes when the shock hits. In Faraglia et al. (2016) this impact is called an interest rate manipulation effect, the planner attempts to alleviate part of the debt burden by using taxes to change the real bond prices. This channel should also be operative in the POSA model.

that government bonds are of longer maturity, then the inflation channel could be used over the longer term by the planner (see for example Leeper et al. (2021), Chafwehé et al. (2022) among others).

Nevertheless we see a significant difference between the POSA and NOPOSA models in terms of how the planner will make use of inflation, even though the differences are shortlived. In particular, assuming POSA and $\sigma_b = 1$ leads to a rise of inflation in response to the shock which is about 6 times larger than the analogous rise under NOPOSA. This is an interesting finding. Many papers on optimal monetary/fiscal policies have highlighted that in the presence of sticky prices the government will not resort to inflation to finance fiscal shocks and instead will utilize debt and taxes to do so (e.g. Schmitt-Grohé and Uribe (2004), Faraglia et al. (2013)). The rationale behind this finding, is that the costs of inflation in standard New Keyenesian models (when the Phillips curve is calibrated to match the US data) are considerably higher than the implicit costs of using distortionary taxes intertemporally. We find however that this result does not hold necessarily with POSA. The planner is willing to bear the higher costs of inflation to avoid having to increase debt considerably. The welfare costs of adjusting debt are therefore also a relevant dimension of the planner's tradeoff.

Finally note that although the volatility of inflation is higher under POSA, we cannot claim here that much of the burden of the fiscal adjustment falls on inflation. As the figure showed, the adjustment of taxes was indeed much stronger in this model where government debt is short term. Had this exercise considered government bonds of long maturity then inflation could be a more crucial margin since the planner could adjust the real value of debt by committing to increase prices not only contemporaneously with the shock but also in the longer term. It would then be interesting to see how fiscal inflation differs in the long run between POSA and NOPOSA. We leave this to future work.

5.2.2 Demand shock

We now consider the response of the economy to a negative demand shock under optimal policy. Figure 2 shows the behavior of macroeconomic variables to the shock. Both, with and without POSA, private consumption and hours fall as a consequence of the shock. As explained previously, a negative shock to preferences implies two effects: First, the households will desire to postpone consumption and second, hours worked will fall, the shock acts as a labour wedge shock in the labour supply condition. This triggers simultaneously a reduction in output and consumption.

[Figure 2 approximately here]

How does the planner react to the shock? From Figure 2 we see that the shock leads to a reduction in taxes, inflation and the debt level. Note that this may seem odd given that the shock leads in a recession, and thus to a drop in the revenue of the government. However, simultaneously, the shock lowers the real interest rate and through this, it increases the present value of surpluses that finance government debt. Since the rise in the value of the future surpluses is larger, the planner will optimally reduce taxes and inflation in response to the shock (see Chafwehé et al. (2022)).[6]

We are again interested in the differences between POSA and NOPOSA. As is shown clearly in the Figure, the differences noted in the previous paragraph remain; POSA entails a much stronger reaction to the shock through taxes and inflation and these differences increase in σ_b. At the same time, whereas taxes in the NOPOSA model evolve according to a random walk under POSA the optimal tax schedule is more frontloaded, echoing the incentive of the planner to not let debt get far off target. Thus we obtain a similar adjustment of taxes and inflation to the preference shock as we did to the spending shock.

We next turn to see whether these similarities will also apply to the case where households can invest in private capital as well as in government debt.

5.3 The economy with private capital

Let us first focus on the case of a rise in spending. Note that in the economy with capital, a government spending shock will not only crowd consumption out, but it will also exert a negative effect on private investment. In the standard New Keynesian model where bonds and capital earn effectively the same rate of return, this is evident. A rise in the real interest rate following a shock to spending will induce households to reduce consumption and induce them also to not invest.

With POSA this margin becomes even more interesting to consider: Since the return of short term government debt differs from the return to capital accumulation, the adverse spending shock will force the government to issue debt and this may only lead to a reduction in the liquidity/safety premium, without impinging a big effect on the price of capital. Under this condition a spending shock may exert a weaker effect on investment than in the NOPOSA model (see for example Bayer et al. (2020)) .

We find the opposite under optimal policy, however. Though the liquidity premium does indeed react to the shock (middle left panel), investment drops by more under POSA. We attribute this to the response of policy variables which is again different across the models. Once again, the POSA model predicts considerable tax volatility and also inflation is more responsive to the shock. Under NOPOSA we obtain a weaker reaction of both taxes and inflation. Our previous findings thus continue to hold.

[Figure 3 approximately here]

[6] It turns out that for preference shocks the maturity of debt being issued is important. For example assuming long term debt, means that long bond prices will increase simultaneously with the governments surplus. This can then shield the budget constraint of the government and the direct effects on output can dominate. In this case we could even observe a rise in taxes following the negative demand shock.

Let us now turn to the demand shock and notice that this adds a further interesting margin to account for. Given that this shock induces households to postpone consumption, it strengthens the incentive to save. As such, we anticipate to see private consumption and investment comove negatively, and this is indeed confirmed by figure 12. As before, consumption drops after the shock, however, now private investment increases.

Notice that this additional effect (relative to the no capital model studied in the previous subsection) leads to a complete revision of the ordering of the reaction of taxes to the shock, in terms of tax volatility. We now see that taxes adjust differently under POSA and NOPOSA and moreover while they decrease under POSA, they slightly increase under NOPOSA.

What explains this? Note that as discussed previously a negative preference shock not only leads to a drop in consumption and hours worked, it also makes investing in government bonds relatively more attractive. The price of debt then increases as investors are willing to be given a lower return to hold the asset, which now yields additional utility services to them. In a model without capital, the optimizing government has the incentive to lower taxes in order to keep the supply of debt roughly constant. But in the presence of capital, an alternative asset available for savings, the drop in the returns on government debt makes capital relatively more attractive and a larger part of the additional savings is channeled in purchases of investment goods. The planner thus cannot easily target a nearly constant path of debt and prefers to let the bond supply decrease following the shock. In case of POSA, the adjustment debt supply does not fall that much (it does so by much less under $\sigma_b = 1$) while with NOPOSA the sharper drop in the bond supply entails an increase in taxes when the shock occurs.

This effect is reinforced by the change in the liquidity premium, which falls with POSA. This fall due to the direct effect of the shock in preferences (z_t) which affect directly the liquidity premium and by the cut in debt, which raises marginal utility of debt. All in all, this increases the revenues collected by the government allowing it to cut taxes, at least initially, which ended up by stimulating private consumption and investment.

Nevertheless, taxes are lower in the long run which is consistent with the fact that real debt is also lower. As discussed previously, a negative preference shock slackens the budget constraint by making the present value of the governments surplus exceed the real value of outstanding debt. The government thus has to lower taxes eventually and also make inflation turn negative to satisfy the intertemporal budget. Remarkably, inflation continues to respond more to the shock under POSA than under NOPOSA.

[Figure 12 approximately here]

6 Extensions

In this section we study further the previous results and perform robustness check. First, we keep analyzing joint optimal monetary and fiscal policy, but, as a robustness check, we allow for different degrees of price stickiness thus inflation becomes more costly for the planner. We will show, that no matter the degree of price stickiness, optimal inflation and tax rate are more volatility under POSA preferences. This result is also robust to changes in the main parameters enclosed by POSA preferences: ϑ and σ_b.

In a second experiment, we contrast POSA with NOPOSA, but instead of analyzing jointly optimal fiscal and monetary policy, we let the planner to choose optimal fiscal policies while monetary policy behaves as a Taylor rule type. We will confirm that tax and inflation are way less persistent under both models, while there is little difference in the optimal path for the main variables under POSA and NOPOSA.

6.1 Degree of price adjustment costs

The literature on optimal policies under sticky prices, such as Schmitt-Grohé and Uribe (2004), shows that the planner makes little use of inflation to stabilize the economy against fiscal shocks when inflation is costly. The previous section shows with POSA, this prescription is no longer observed and the planner uses inflation as a non distorting tax on nominal wealth to avoid debt issuance.

To stress previous result, we perform a sensitivity analysis with respect to the degree of price adjustment costs, ψ. We do so for the version of the economy without capital, as the result could be easily extended to an economy with capital. A lower value for the parameter ψ indicates that is less costly for the firm to adjust prices, meaning the economy shows a higher degree of price flexibility.

[Figure 5 approximately here]

[Figure 6 approximately here]

Figure 5 and 6 in the appendix E, shows the optimal response of the economy without capital to a government expenditure shock and a negative demand shock for different degrees of price stickiness. The blue line is the baseline economy without POSA preferences, the red line is the baseline economy with POSA, in which the value for $\psi = 200$, and the dashed green line is an economy with POSA, with a value for $\psi = 20$, thus ten times lower. The latter implies a value for the slope of the linear version of the Phillips curve of 0.3, so ten times higher than the one assumed in the baseline economy.

When looking at the response of the economy to a government consumption shock depicted in figure 5, we can draw two main observations. First, both inflation and taxes are more volatile than in the case of NOPOSA, thus the planner uses more intensively the tax rate and inflation to absorb the value of the shock when preferences are of POSA type. Secondly, the higher the value for the parameter ψ, the more the planner decides to use taxes and not inflation. For instance, we can see that as soon as the government expenditures jumps, the planner in the baseline economy increases the tax rate by 0.5% above the level of the steady state. However, when we drop the value for the parameter ψ, and price setting becomes less costly for the firms, the planner increases taxes by much less.

Conversely, we observe the opposite result for inflation. With a lower value for the price adjustment cost parameter, the planner relies even more on inflation to stabilize the economy when facing an unexpected increase in government consumption. Thus we can see the planner exchanges instruments one with another to manage the costs of shocks. Nevertheless, the planner makes use of inflation way more in the case of POSA, than in the case of NOPOSA.

Figure 6 shows a similar picture for the negative demand shock. Tax rate falls when the demand shocks arrive, and they do it by more in the case of an economy with a higher degree of price stickiness (red line). On the contrary, when the economy shows a relatively higher degree of price flexibility, the planner relies on negative inflation to manage the shock, as is the case in the economy depicted by the dashed green line.

6.2 Optimal fiscal policy with a Taylor rule

Consider now that monetary policy cannot be chosen arbitrarily by the planner, rather it has to follow an interest rate rule setting the nominal interest rate as a function of inflation the output gap etc. In this scenario we may consider that monetary policy is independent and commits to an instrument rule, the fiscal authority which again is represented by the Ramsey planner has to account for this rule in optimization.This type of optimal policy scenario has been considered for example by Leeper and Leith (2016) .

We saw previously that since debt is short term in the model, it becomes optimal to use inflation only contemporaneously with the shock. The intuition was that inflation after the shock has occurred will not contribute anything towards making debt sustainable and on the other hand inflation is costly in the model, so the optimal policy keeps a constant price level after the shock.

Implementing such a path for inflation, may not be easy when the nominal interest rate is tied down by an instrument rule. In the previous sections the planner was able to set interest rates independently from inflation and it is unlikely that optimal policies could be described by a simple rule setting the nominal rate to respond to the deviation of inflation from its target, as commonly independent central banks have been using. In the POSA model, this interaction between an independent monetary authority and an optimizing fiscal authority may be even more interesting, if we consider that the nominal interest rate will influence the liquidity /safety premium of government debt, a margin that directly relates to household utility.

We now consider as an additional constraint to the planners program a rule of the form

$$R_t = \left[\left(\frac{\pi_t}{\pi^*} \right)^{\phi_\pi} \left(\frac{Y_t}{Y^*} \right)^{\phi_y} \right] \tag{11}$$

where we set $\phi_\pi > 1$ and $\phi_y \geq 0$ as is commonly assumed in the literature. Starred variables are the target levels of inflation and output and these are the steady-state values.

The Ramsey problem and its first order conditions are similar to the problem described in section 4, but now the planner takes into account the Taylor rule. For brevity, we do not state here the problem, but refer the reader to appendix D, which contains the non-linear Ramsey problem and the associated first order conditions.

To study the responses to the fiscal and the negative preference shocks we use the same calibration as before. We further assign the value of 1.5 to the inflation coefficient ϕ_π (a common assumption in the literature) and let 0.15 be the coefficient ϕ_y. Under this calibration the modified Ramsey program yield a determinate equilibrium solution to the system of first order conditions.

Consider first the case of a fiscal shock in the model without capital. Figure 7 shows the responses. Notice that now, the shapes of the responses of policy variables to the shock are similar across the POSA and NOPOSA models. Indeed under both scenarios taxes are serially uncorrelated, the random walk properly no long holds even for NOPOSA. It has been noted that assuming an independent monetary policy has a consierable effect of how the government will time taxes Leeper and Leith (2016). To understand this intuitively, note that distortionary taxation not only affects the government's budget constraint in the model but also it exerts an influence on the Phillips curve (through the marginal cost) and further more it affects aggregate output and consumption thus affecting also the real rate of interest. Thus, a government that is constrained to not tolerate a deviation of inflation from target or a deviation of output from its respective target level, is obliged to also constrain the response of taxes to the shock. (It is then not surprising that the only effects present are contemporaneous to the shock, as inflation and taxes will need to adjust anyway to satisfy the government budget constraint, and as we have seen for inflation this is only through a contemporaneous channel.)

Remarkably, however, we continue to find a higher volatility of inflation and taxes under POSA.

[Figure 7 approximately here]

Consider now the case of the negative preference shock shown in Figure 8. Essentially the policy variables react in the same manner (though the sign is opposite of course). We again find that the planner limits the response of taxes (essentially) to the period the shock occurs, even though POSA continues to give more volatility to taxes and inflation. Once again the constraint imposed by the instrument rule, affects the optimal policy solution profoundly.

[Figure 8 approximately here]

[Figures 9 and 10 approximately here]

Lastly, the same finding emerges from the models with capital shown in Figures 9 and 10. We thus conclude that assuming an explicit interest rate rule has a considerable bearing on optimal policies. Note that the results shown here concern only one calibration of the interest rate rule. Admittedly, it would also be important to consider different functional forms for the instrument rule for example allowing for interest rate inertia or (perhaps more importantly) assuming that the monetary authority targets the deviation of output not from its steady state level, but from the efficient level which accounts for distortionary taxes. In these cases the policy rule could allow the planner more room to manipulate taxes intertemporally and this could bring back some of the results highlighted in the previous subsections. We have not done so, because specifying rules that target efficient output, though optimal in some contexts, is generically not a realistic assumption. In any case this paragraph confirms a previous finding in the literature, that the Ramsey policy can be constrained by an independent monetary authority, also in the case of POSA. Whether there are plausible specifications of monetary policies under which this constraint is not significant remains to be explored.

7 Conclusion

In this paper we studied optimal monetary and fiscal policy when households has preferences over government bonds. When this is the case, there is a wedge between the household discount factor and the nominal interest rate allowing the government to collect extra revenues from providing liquidity services. In other words, the government can borrow cheaper than in the standard New-Keynesian model due to the presence of the liquidity premium.

In such a world, the planner faces different forces in addition to the tax smoothing behavior proposed by Barro: debt issuance allows to smooth taxes, though at the same time reduces the liquidity premium and constrains the government. This force prevents the planner from issuing larger levels of debt to finance an unexpected raise in government spending. Instead, the planner uses inflation as a non-distortionary tax at period zero to absorb most of the shock when monetary policy is optimal. These behaviour departs from the "near zero" volatility of inflation usually present in an environment with sticky prices.

Moreover, due to the tensions found by the planner at the moment of issuing debt, its become more difficult to smooth taxes and tax distortions are front-loaded making the tax rate more volatile than in the standard model. As a consequence of the high distortionary taxes and inflation, the crowding-out of consumption and investment is higher under POSA. This is a strike difference from models using non-distortionary taxes to model the effect of government expenditure shocks under POSA.

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A Appendix: Firm's problem

Production takes place in monopolistically competitive firms j which operates technologies with labor and public capital. The final good is a CES aggregate of the intermediate goods, $Y_t(j)$ of the form.

$$Y_t = \left(\int_0^1 Y_t(j)^{1-1/\theta} dj \right)^{\frac{\theta}{\theta-1}}$$

Intermediate goods seeks to set prices and labor input in order to maximize profits and subject to the following demand curve for each intermediate input j,

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t$$

and the price adjustment costs, which are given by,

$$\Sigma_t(j) = \frac{\psi}{2} \left(\frac{P_t(j)}{P_t} - 1 \right)^2 Y_t$$

Consider the cost minimization problem for the firm, the firm chooses labor such that,

$$\min W_t n_t - r_t k_t$$

subject to,

$$F(n_t(j), k_{t-1}(j)) \geq \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t$$

The Lagrangian for this problem can be written as,

$$\mathcal{L} = -W_t n_t(j) - r_t k_t(j) + \mu_t(j) \left(F(n_t(j), k_{t-1}(j)) - \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t \right)$$

The FONC is:

$$W_t = \mu_t(j)F_n(n_t(j), k_{t-1}(j))$$

$$r_t = \mu_t(j)F_k(n_t(j), k_{t-1}(j))$$

Define $\mu_t(j) = \frac{W_t}{F_n(n_t(j),k_{t-1}(j))} = \frac{r_t}{F_k(n_t(j),k_{t-1}(j))}$ is the marginal cost for the firm j, $mc_t(j)$

Considering that $F(n_t, k_{t-1})$ is a Cobb-Douglas we can write the marginal productivity of labor for each firm j as $F_n(n_t(j), k_{t-1}(j)) = \frac{\alpha Y_t(j)}{n_t(j)}$ and the marginal productivity of capital as $F_k(n_t(j), k_{t-1}(j)) = \frac{(1-\alpha)Y_t(j)}{k_t}$. The latter allow us to write, $w_t = mc_t\alpha\frac{Y_t}{N_t}$ and $r_t = mc_t(1-\alpha)\frac{Y_t}{k_t}$.

Total costs given by $w_tN_t(j) + r_tk_t(j)$ can be written as $mc_t(j)\alpha Y_t(j) + mc_t(j)(1-\alpha)Y_t(j) = mc_t(j)Y_t(j)$

Considering now the problem in which the firm seeks prices, $P_t(j)$ to maximize profits,

$$\max_{P(j)} E_t \sum_{l=0}^{\infty} \beta^t \frac{U_{c,t+l+1}}{U_{c,t+l}} \left[(1+sub)\frac{P_{t+l}(j)}{P_{t+l}}Y_{t+l}(j) - \frac{mc_{t+l}(j)}{P_{t+l}}Y_{t+l}(j) - \frac{\psi}{2}\left(\frac{P_{t+l}(j)}{P_{t+l-1}(j)} - 1\right)^2 Y_{t+l} \right]$$

subject to the constraints

$$Y_{t+l}(j) = \left(\frac{P_{t+l}(j)}{P_{t+l}}\right)^{-\theta} Y_{t+l}$$

$$Y_{t+l}(j) \geq F(n_{t+l}(j), k_{t+l}(j))$$

$U_{c,t+l+1}/U_{c,t+l} = z_{t+l+1}c_{t+l+1}^{-\sigma}/z_{t+l}c_{t+l}^{-\sigma}$ is the discount factor of the households that own the shares of firm j.

The firm's first order condition with respect to $P_t(j)$ are given by,

$$(1+sub)\frac{1}{P_t}Y_t\left(\frac{P_t(j)}{P_t}\right)^{-\theta} + (1+sub)\frac{P_t(j)}{P_t^2}Y_t(-\theta)\left(\frac{P_t(j)}{P_t}\right)^{-\theta-1} + \theta\frac{mc_t(j)}{P_t}Y_t - \psi\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)\frac{Y_t}{P_{t-1}(j)}$$
$$+ \beta E_t\frac{\epsilon_{t+1}c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}}\psi\left(\frac{P_{t+1}(j)}{P_t(j)} - 1\right)\frac{P_{t+1}(j)}{P_t^2(j)}Y_{t+1} = 0$$

Imposing a symmetric equilibrium in which all firms set the same price, $P_t(j) = P_t$, all the firms consider the same productivity and real wage:

$$(1+s)\frac{Y_t}{P_t} - (1+sub)\theta\frac{Y_t}{P_t} + \theta\frac{mc_t}{P_t}Y_t - \psi\left(\frac{P_t}{P_{t-1}} - 1\right)\frac{Y_t}{P_{t-1}} + \beta E_t\frac{\epsilon_{t+1}c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}}\psi\left(\frac{P_{t+1}}{P_t} - 1\right)\frac{P_{t+1}}{P_t^2}Y_{t+1} = 0$$

Multiply by P_t and dividing by Y_t:

$$(1+sub)1-(1+sub)\theta-\psi\left(\frac{P_t}{P_{t-1}}-1\right)\frac{P_t}{P_{t-1}}+\theta mc_t+\beta E_t\frac{\epsilon_{t+1}c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}}\psi\left(\frac{P_{t+1}}{P_t}-1\right)\frac{P_{t+1}}{P_t}\frac{Y_{t+1}}{Y_t}=0$$

Define gross inflation rate, π_t as P_t/P_{t-1}, then

$$\begin{aligned}
(1+sub)(1-\theta)+\theta mc_t-\psi\left[(\pi_t-1)\pi_t-\beta E_t\frac{\epsilon_{t+1}c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}}(\pi_{t+1}-1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]=0\\
\theta mc_t=(\theta-1)(1+sub)+\psi\left[(\pi_t-1)\pi_t-\beta E_t\frac{\epsilon_{t+1}c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}}(\pi_{t+1}-1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]\\
mc_t=\frac{(1+sub)(\theta-1)}{\theta}+\frac{\psi}{\theta}\left[(\pi_t-1)\pi_t-\beta E_t\frac{\epsilon_{t+1}c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}}(\pi_{t+1}-1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]
\end{aligned}$$

Replacing by marginal cost by the definition and making $sub=1/(\theta-1)$ we get the same equation as in text

$$\frac{w_t}{F_{N,t}}=1+\frac{\psi}{\theta}\left[(\pi_t-1)\pi_t-\beta E_t\frac{z_{t+1}c_{t+1}^{-\sigma}}{z_t c_t^{-\sigma}}(\pi_{t+1}-1)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]$$

B Appendix: FONC for the Ramsey planner problem. Joint optimal fiscal and monetary policy

The first-order condition for the problem described in section 4 are:

Consumption, C_t:

$$\begin{aligned}
0=E_t\Bigg\{&z_tU_{C,t}+\left(z_t\phi_{1,t}-\beta^{-1}\phi_{1,t-1}\beta\frac{R_{t-1}}{\pi_t}\right)\Big(U_{CC,t}(.)\Big)-\phi_{2,t}\left(\frac{U_{cc,t}}{U_{c,t}}\right)mc_t\\
&-\frac{\psi}{\theta}\left(\frac{U_{CC,t}(.)}{U_{C,t}(.)}\right)\left(\phi_{2,t}\beta\frac{U_{C,t+1}(.)}{U_{C,t}(.)}\frac{Y_{t+1}}{Y_t}\pi_{t+1}(\pi_{t+1}-1)-\beta^{-1}\phi_{2,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}\frac{Y_t}{Y_{t-1}}\pi_t(\pi_t-1)\right)-\phi_{3,t}\\
&+\phi_{6,t}\beta\frac{U_{C,t+1}(.)}{U_{C,t}(.)}\Big((1-\delta)q_{t+1}+mc_{t+1}F_{K,t+1}\Big)\left(-\frac{U_{CC,t}(.)}{U_{C,t}(.)}\right)\\
&-\beta^{-1}\phi_{6,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}\Big((1-\delta)q_t+mc_tF_{K,t}\Big)\left(-\frac{U_{CC,t}(.)}{U_{C,t}(.)}\right)+\beta^{-1}\phi_{6,t-1}\beta\frac{U_{C,t}}{U_{C,t-1}}\left(-\frac{U_{cc,t}}{U_{c,t}}\right)mc_tF_{K,t}\\
&+\phi_{7,t}\tau_tN_tmc_t\left(\frac{-U_{CC}}{U_C}\right)\Bigg\}
\end{aligned}\tag{12}$$

Labor, N_t:

$$\begin{aligned}0 = E_t\Bigg\{ & U_{N,t} + \phi_{2,t}\left(\frac{U_{NN,t}}{U_{N,t}} - \frac{F_{NN,t}}{F_{N,t}}\right)mc_t \\ & - \frac{\psi}{\theta}\left(\phi_{2,t}\beta\frac{U_{C,t+1}(.)}{U_{C,t}(.)}\frac{Y_{t+1}}{Y_t}\pi_{t+1}(\pi_{t+1}-1) - \beta^{-1}\phi_{2,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}\frac{Y_t}{Y_{t-1}}\pi_t(\pi_t-1)\right)\left(\frac{F_{N,t}}{Y_t}\right) \\ & + \phi_{3,t}F_{N,t}\left(1-\psi\frac{\pi_t^2}{2}\right) + \beta^{-1}\phi_{6,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}mc_tF_{K,t}\left(\frac{U_{NN,t}}{U_{N,t}} - \frac{F_{NN,t}}{F_{N,t}} + \frac{F_{KN,t}}{F_{K,t}}\right) \\ & + \phi_{7,t}\tau_t\left(1 + N_t mc_t\left(\frac{U_{NN,t}}{U_{N,t}} - \frac{F_{NN,t}}{F_{N,t}}\right)\right)\Bigg\}\end{aligned} \tag{13}$$

Private investment, I_t

$$\begin{aligned}0 = E_t\Bigg\{ & -\phi_{3,t} + \phi_{4,t}\left[1 - S_I\left(\frac{I_t}{I_{t-1}}\right) - \left(\frac{I_t}{I_{t-1}}\right)S_I'\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta\phi_{4,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 S_I'\left(\frac{I_{t+1}}{I_t}\right) \\ & - \phi_{5,t}\left[\frac{q_t}{I_{t-1}}\left[2S_I'\left(\frac{I_t}{I_{t-1}}\right) + \left(\frac{I_t}{I_{t-1}}\right)S_I''\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta\frac{U_{C,t+1}(.)}{U_{C,t}(.)}\frac{q_{t+1}}{I_t}\left(\frac{I_{t+1}}{I_t}\right)^2\left[2S_I'\left(\frac{I_{t+1}}{I_t}\right) + \left(\frac{I_{t+1}}{I_t}\right)S_I''\left(\frac{I_{t+1}}{I_t}\right)\right]\right] \\ & + \beta\phi_{5,t+1}\frac{q_{t+1}}{I_t}\left(\frac{I_{t+1}}{I_t}\right)\left[2S_I'\left(\frac{I_{t+1}}{I_t}\right) + \left(\frac{I_{t+1}}{I_t}\right)S_I''\left(\frac{I_{t+1}}{I_t}\right)\right] \\ & + \beta^{-1}\phi_{5,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}\left(\frac{I_t}{I_{t-1}}\right)\frac{q_t}{I_{t-1}}\left[2S_I'\left(\frac{I_{t+1}}{I_t}\right) + \left(\frac{I_{t+1}}{I_t}\right)S_I''\left(\frac{I_{t+1}}{I_t}\right)\right]\Bigg\}\end{aligned} \tag{14}$$

Private capital,K_{t+1}

$$\begin{aligned}0 = E_t\Bigg\{ & -\beta\phi_{2,t+1}\left(\frac{F_{NK,t+1}}{F_{N,t+1}}\right)mc_{t+1} - \\ & -\frac{\psi}{\theta}\left(\beta\phi_{2,t+1}\beta\frac{U_{C,t+2}(.)}{U_{C,t+1}(.)}\frac{Y_{t+2}}{Y_{t+1}}\pi_{t+2}(\pi_{t+2}-1) - \phi_{2,t}\beta\frac{U_{C,t+1}(.)}{U_{C,t}(.)}\frac{Y_{t+1}}{Y_t}\pi_{t+1}(\pi_{t+1}-1)\right)\left(\frac{F_{K,t+1}}{Y_{t+1}}\right) + \\ & + \beta\phi_{3,t+1}\left(1-\psi\frac{\pi_{t+1}^2}{2}\right)F_{K,t+1} - \phi_{4,t} + \beta(1-\delta)\phi_{4,t+1} - \phi_{6,t}\beta\frac{U_{C,t+1}(.)}{U_{C,t}(.)}\left(\frac{F_{NK,t+1}}{F_{N,t+1}} - \frac{F_{KK,t+1}}{F_{K,t+1}}\right)mc_{t+1}F_{K,t+1}\Bigg\}\end{aligned} \tag{15}$$

Price of private capital,q_t

$$\begin{aligned}0 = E_t\Bigg\{ & \phi_{5,t}\left[1 - S_I\left(\frac{I_t}{I_{t-1}}\right) - \left(\frac{I_t}{I_{t-1}}\right)S_I'\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta^{-1}\phi_{5,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}\left(\frac{I_t}{I_{t-1}}\right)^2 S_I'\left(\frac{I_t}{I_{t-1}}\right) \\ & -\phi_{6,t} + \beta^{-1}\phi_{6,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}(1-\delta)\Bigg\}\end{aligned} \tag{16}$$

Interest rate,R_t

$$0 = E_t\left\{-\phi_{1,t}\beta U_{C,t+1}\frac{1}{\pi_{t+1}} - \beta\phi_{7,t+1}\frac{b_t}{\pi_{t+1}}\right\} \tag{17}$$

Inflation rate,π_t

$$0 = E_t\Bigg\{\beta^{-1}\frac{\phi_{1,t-1}R_{t-1}}{\pi_t^2}\frac{U_{C,t}(.)}{U_{C,t-1}(.)} - \frac{\psi}{\theta}\Bigg[\phi_{2,t} - \beta^{-1}\phi_{2,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}\frac{Y_t}{Y_{t-1}}\Bigg](1+2\pi_t) - \phi_{3,t}\psi Y_t\pi_t + \phi_{7,t}\frac{R_{t-1}}{\pi_t^2}b_{t-1}\Bigg\} \tag{18}$$

Public debt ,b_t

$$0 = E_t\Bigg\{ - \sigma_b U_B b_t - \phi_{1,t}U_{BB,t} + \phi_{7,t} - \beta\phi_{7,t+1}\frac{R_t}{\pi_{t+1}}\Bigg\} \tag{19}$$

taxes ,τ_t

$$0 = E_t\Bigg\{\phi_{2,t}\Bigg[\frac{1}{(1-\tau_t)}\Bigg]mc_t + \beta^{-1}\phi_{6,t-1}\beta\frac{U_{Ct}}{U_{Ct-1}}\frac{F_{K,t}}{(1-\tau)}mc_t + \phi_{7,t}N_t w_t\Bigg[\frac{2-\tau}{1-\tau}\Bigg]mc_t\Bigg\} \tag{20}$$

Where the production function, real wages and marginal cost are defined as:

$$Y_t = F(k_t, K_G, N_t)$$
$$w_t = \frac{-U_{N,t}}{(1-\tau_t)U_{C,t}}$$
$$mc_t = \frac{w_t}{F_{N,t}}$$

And have been substituted out in the log-linear version in the main text for exposition motives.

C Appendix: Flexible prices steady-state

From the first order conditions for the households, the euler equation for short term bonds in the steady-state reads:

$$
\begin{aligned}
c^{*-\sigma} &= \beta \frac{R^* c^{*-\sigma}}{\pi^*} + \chi_b (b^*)^{-\sigma_b} \\
1 &= \beta \frac{R^*}{\pi^*} + \chi_b \frac{b^{*-\sigma_b}}{c^{*-\sigma}}
\end{aligned}
$$

Considering a zero inflation steady state ($\pi^* = 1$),

$$
\begin{aligned}
1 &= \beta R^* + \chi_b \frac{b^{*-\sigma_b}}{c^{*-\sigma}} \\
\beta R^* &= 1 - \chi_b \frac{b^{*-\sigma_b}}{c^{*-\sigma}} < 1
\end{aligned}
$$

We can perform the same transformation as Campbell et al. (2017) and Rannenberg (2021) and express the euler equation for bonds as:

$$
\tilde{c}_t = \vartheta[\tilde{R}_t - E_t \tilde{\pi}_{t+1}] - \vartheta E_t \tilde{c}_{t+1} + (1 - \vartheta) \frac{\sigma_b}{\sigma} \tilde{b}_t \tag{21}
$$

whenever $\vartheta = \beta R^*$. In absence of POSA, $\vartheta = 1$ and the equation is the standard Euler equation for bonds.

From the household first order condition for labor, eliminating time subscripts, imposing $n^* = 1/3$ we arrive to the following:

$$
\begin{aligned}
\chi \frac{n^{*\varphi}}{(1 - \tau^*) c^{*-\sigma}} &= \alpha \frac{y^*}{n^*} \\
\chi &= \alpha \frac{y^* c^{*-\sigma} (1 - \tau^*)}{n^{*1+\varphi}}
\end{aligned}
$$

From the FONC for the firm w.r.t labor, we can obtain the wage by replacing the production function and the optimal subsidy:

$$
\begin{aligned}
mc_t &= \frac{w_t}{F_{N,t}} \\
w^* &= mc^* \alpha \frac{Y^*}{n^*} \\
w^* &= \frac{mc^* \alpha Y^*}{(1 - \tau^*) n^*}
\end{aligned}
$$

Given the assumption for the investment adjustment costs, $S(1) = 0$ and $S'(1) = 0$, from the FONC from the households, in the steady-state we have:

$$1 = [1 - S(1) - S'(1)]q^* + \beta E\left[\frac{U'^*}{U'^*} S'(1) q\right]$$
$$q^* = 1$$

Substituting last result in the last FONC for the household,

$$1 = \beta \left[\frac{U'^*}{U'^*}(1-\delta) + mc^* F_k^*\right]$$
$$F_k^* = \frac{1 - \beta(1-\delta)}{\beta mc^*}$$

Calibrating hours in the steady-state to be 1/3 of disposable time, we can use last equation together with the production function to solve for K in the steady state.

$$(1-\alpha) N^{\alpha} K^{-\alpha} = \frac{1 - \beta(1-\delta)}{\beta}$$

which in the steady state is:

$$K^* = \left[\frac{1 - \beta(1-\delta)}{\beta} \frac{N^{-\alpha}}{1-\alpha}\right]^{-1/\alpha}$$

and using the production function:

$$Y^* = N^{\alpha} K^{1-\alpha}$$

Using Law of motion for private capital, we can recover investment:

$$I^* = \delta K^*$$

An initial steady-state level of debt, and assuming a zero inflation steady state can be obtained from the government budget constraint for given values of government consumption. We impose the value for g^c from the steady-state according to data, while labor hours are set to 1/3.

$$b^* = \frac{\tau^* w^* n^* - g^{c,*}}{R^*}$$

For the level of the tax rate at the steady state, we calibrate a value so that debt to output ratio is 60% in the steady-state (on a quarterly calibration this correspond to 240% of output). From the govt budget constraint, first replacing the price for the bond:

$$\tau_t w_t n_t - g_t^c = b_t - q_t b_t$$

$$\tau_t w_t n_{tt} - g_t^c = (1-\beta) b_t$$

Replace w_t for the value in the steady state:

$$\tau_t \frac{mc_t \alpha y_t}{(1-\tau) n_t} n_t - g_t^c = (1-\beta) b_t$$

$$\tau_t = \frac{(1-\tau_t)}{mc_t \alpha y_t} (1-\beta) b_t + g_t^c$$

$$\tau_t = \frac{\frac{1}{mc_t \alpha y_t}\left[(1-\beta) b_t + g_t^c\right]}{\left[1 + \frac{1}{mc_t \alpha y_t}(1-\beta) b_t + g_t^c\right]}$$

So in the steady state, the tax rate can be the one to sustain certain level of debt to output ratio, calibrated to 60%

$$\tau^* = \frac{\frac{1}{\alpha mc^*}\left[(1-\beta)\frac{b^*}{y^*} + \frac{g^{c,*}}{y^*}\right]}{\left[1 + \frac{1}{\alpha mc^*}(1-\beta)\frac{b^*}{y^*} + \frac{g^{c,*}}{y^*}\right]}$$

In a flexible price steady-state, the planner is not constrained by the Phillips curve, thus the equation and the multiplier can be dropped from the system, thus $\phi_2 = 0$. Now, we impose a zero inflation steady-state such that,

$$\pi^* = 1$$

D Appendix: Ramsey planner problem with a Taylor rule

In this section we describe the Ramsey planner problem when the economy can accumulate private capital and it is constrained by a Taylor rule for the nominal interest rate. When consider an economy without capital, one just need to remove the constraints linked to private capital: the Euler equation for capital, the household first order condition with respect to investment and the law of motion for private capital, together with the variables I_t, k_t, q_t, r_t^k

Consider $H_t = \{c_t, n_t, k_{t+1}, I_t, R_t, \pi_t, b_t, \tau_t, q_t\}$, the vector of control variables. The Lagrangian for a competitive equilibrium with monetary policy following a Taylor rule:

$$
\begin{aligned}
&\max_{\{H_t\}} L_0 = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ U(c_t, n_t, b_t, z_t) \\
&+\phi_{1,t}\left[z_t U_{C,t} - \beta E_t \left[z_{t+1} \frac{U_{C,t+1} R_t}{\pi_{t+1}} \right] - U_{b,t} \right] \\
&+\phi_{2,t}\left[mc_t - 1 - \frac{\psi}{\theta}\left(\pi_t(\pi_t - 1) - \beta \frac{z_{t+1}}{z_t} \frac{U_{C,t+1}(.)}{U_{C,t}(.)} \frac{Y_{t+1}}{Y_t} \pi_{t+1}(\pi_{t+1} - 1) \right) \right] \\
&+\phi_{3,t}\left[\left(1 - \frac{\psi}{2}\pi_t^2 \right) Y_t - C_t - I_t - G_t^c \right] \\
&+\phi_{4,t}\left[(1-\delta)k_t + I_t - k_{t+1} \right] \\
&+\phi_{5,t}\left[\left[1 - S_I\left(\frac{I_t}{I_{t-1}} \right) - S_I'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] q_t + \beta \frac{z_{t+1}}{z_t} \frac{U_{C,t+1}(.)}{U_{C,t}(.)} S_I'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 q_{t+1} - 1 \right] \\
&+\phi_{6,t}\left[\beta \frac{z_{t+1}}{z_t} \frac{U_{C,t+1}}{U_{C,t}} ((1-\delta) + mc_{t+1} F_{K,t+1}) - 1 \right] \\
&+\phi_{7,t}\left[R_t - (\pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \right] \\
&+\phi_{8,t}\left[b_t - \frac{R_{t-1}}{\pi_t} b_{t-1} - G_t^c + \tau_t w_t N_t \right] \Bigg\}
\end{aligned}
$$

Where the production function, real wages and marginal cost are defined as:

$$
\begin{aligned}
Y_t &= F(k_t, n_t) \\
w_t &= \frac{-U_{N,t}}{(1-\tau_t) z_t U_{C,t}} \\
mc_t &= \frac{w_t}{F_{N,t}}
\end{aligned}
$$

The first-order condition for the problem are:

Consumption, C_t:

$$
\begin{aligned}
0 = E_t\Big\{ & z_t U_{C,t} + \Big(z_t\phi_{1,t} - \beta^{-1}\phi_{1,t-1}\beta\frac{R_{t-1}}{\pi_t}\Big)\Big(U_{CC,t}(.)\Big) - \phi_{2,t}\Big(\frac{U_{cc,t}}{U_{c,t}}\Big)mc_t \\
& - \frac{\psi}{\theta}\Big(\frac{U_{CC,t}(.)}{U_{C,t}(.)}\Big)\Big(\phi_{2,t}\beta\frac{U_{C,t+1}(.)}{U_{C,t}(.)}\frac{Y_{t+1}}{Y_t}\pi_{t+1}(\pi_{t+1}-1) - \beta^{-1}\phi_{2,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}\frac{Y_t}{Y_{t-1}}\pi_t(\pi_t-1)\Big) - \phi_{3,t} \\
& + \phi_{6,t}\beta\frac{U_{C,t+1}(.)}{U_{C,t}(.)}\Big((1-\delta)q_{t+1} + mc_{t+1}F_{K,t+1}\Big)\Big(-\frac{U_{CC,t}(.)}{U_{C,t}(.)}\Big) \\
& - \beta^{-1}\phi_{6,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}\Big((1-\delta)q_t + mc_tF_{K,t}\Big)\Big(-\frac{U_{CC,t}(.)}{U_{C,t}(.)}\Big) + \beta^{-1}\phi_{6,t-1}\beta\frac{U_{C,t}}{U_{C,t-1}}\Big(-\frac{U_{cc,t}}{U_{c,t}}\Big)mc_tF_{K,t} \\
& + \phi_{8,t}\tau_tN_tmc_t\Big(\frac{-U_{CC}}{U_C}\Big)\Big\}
\end{aligned} \tag{22}
$$

Labor, N_t:

$$
\begin{aligned}
0 = E_t\Big\{ & U_{N,t} + \phi_{2,t}\Big(\frac{U_{NN,t}}{U_{N,t}} - \frac{F_{NN,t}}{F_{N,t}}\Big)mc_t \\
& - \frac{\psi}{\theta}\Big(\phi_{2,t}\beta\frac{U_{C,t+1}(.)}{U_{C,t}(.)}\frac{Y_{t+1}}{Y_t}\pi_{t+1}(\pi_{t+1}-1) - \beta^{-1}\phi_{2,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}\frac{Y_t}{Y_{t-1}}\pi_t(\pi_t-1)\Big)\Big(\frac{F_{N,t}}{Y_t}\Big) \\
& + \phi_{3,t}F_{N,t}\Big(1-\psi\frac{\pi_t^2}{2}\Big) + \beta^{-1}\phi_{6,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}mc_tF_{K,t}\Big(\frac{U_{NN,t}}{U_{N,t}} - \frac{F_{NN,t}}{F_{N,t}} + \frac{F_{KN,t}}{F_{K,t}}\Big) \\
& - \phi_{7,t}\phi_y(\pi_t)^{\phi_\pi}\Big(\frac{Y_t}{Y}\Big)^{\phi_y}\frac{F_{N,t}}{Y_t} + \phi_{8,t}\tau_t\Big(1 + N_tmc_t\Big(\frac{U_{NN,t}}{U_{N,t}} - \frac{F_{NN,t}}{F_{N,t}}\Big)\Big)\Big\}
\end{aligned} \tag{23}
$$

Private investment, I_t

$$
\begin{aligned}
0 = E_t\Big\{ & -\phi_{3,t} + \phi_{4,t}\Big[1 - S_I\Big(\frac{I_t}{I_{t-1}}\Big) - \Big(\frac{I_t}{I_{t-1}}\Big)S_I'\Big(\frac{I_t}{I_{t-1}}\Big)\Big] + \beta\phi_{4,t+1}\Big(\frac{I_{t+1}}{I_t}\Big)^2 S_I'\Big(\frac{I_{t+1}}{I_t}\Big) \\
& - \phi_{5,t}\Big[\frac{q_t}{I_{t-1}}\Big[2S_I'\Big(\frac{I_t}{I_{t-1}}\Big) + \Big(\frac{I_t}{I_{t-1}}\Big)S_I''\Big(\frac{I_t}{I_{t-1}}\Big)\Big] + \beta\frac{U_{C,t+1}(.)}{U_{C,t}(.)}\frac{q_{t+1}}{I_t}\Big(\frac{I_{t+1}}{I_t}\Big)^2\Big[2S_I'\Big(\frac{I_{t+1}}{I_t}\Big) + \Big(\frac{I_{t+1}}{I_t}\Big)S_I''\Big(\frac{I_{t+1}}{I_t}\Big)\Big]\Big] \\
& + \beta\phi_{5,t+1}\frac{q_{t+1}}{I_t}\Big(\frac{I_{t+1}}{I_t}\Big)\Big[2S_I'\Big(\frac{I_{t+1}}{I_t}\Big) + \Big(\frac{I_{t+1}}{I_t}\Big)S_I''\Big(\frac{I_{t+1}}{I_t}\Big)\Big] \\
& + \beta^{-1}\phi_{5,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}\Big(\frac{I_t}{I_{t-1}}\Big)\frac{q_t}{I_{t-1}}\Big[2S_I'\Big(\frac{I_{t+1}}{I_t}\Big) + \Big(\frac{I_{t+1}}{I_t}\Big)S_I''\Big(\frac{I_{t+1}}{I_t}\Big)\Big]\Big\}
\end{aligned} \tag{24}
$$

Capital,K_{t+1}

$$
\begin{aligned}
0 = E_t\Bigg\{ & -\beta\phi_{2,t+1}\left(\frac{F_{NK,t+1}}{F_{N,t+1}}\right)mc_{t+1}- \\
& -\frac{\psi}{\theta}\left(\beta\phi_{2,t+1}\beta\frac{U_{C,t+2}(.)}{U_{C,t+1}(.)}\frac{Y_{t+2}}{Y_{t+1}}\pi_{t+2}(\pi_{t+2}-1)-\phi_{2,t}\beta\frac{U_{C,t+1}(.)}{U_{C,t}(.)}\frac{Y_{t+1}}{Y_t}\pi_{t+1}(\pi_{t+1}-1)\right)\left(\frac{F_{K,t+1}}{Y_{t+1}}\right)+ \\
& +\beta\phi_{3,t+1}\left(1-\psi\frac{\pi_{t+1}^2}{2}\right)F_{K,t+1}-\phi_{5,t}+\beta(1-\delta)\phi_{4,t+1} \\
& -\phi_{6,t}\beta\frac{U_{C,t+1}(.)}{U_{C,t}(.)}\left(\frac{F_{NK,t+1}}{F_{N,t+1}}-\frac{F_{KK,t+1}}{F_{K,t+1}}\right)mc_{t+1}F_{K,t+1}- \\
& -\phi_{7,t+1}\phi_y(\pi_{t+1})^{\phi_p i}\left(\frac{Y_{t+1}}{Y}\right)^{\phi_y}\frac{F_{K,t+1}}{Y_{t+1}}+\beta\phi_{8,t+1}\tau_{t+1}N_{t+1}mc_{t+1}\left(\frac{F_{NK,t+1}}{F_{N,t+1}}\right)\Bigg\}
\end{aligned}
\tag{25}
$$

Price of private capital,q_t

$$
\begin{aligned}
0 = E_t\Bigg\{ & \phi_{5,t}\left[1-S_I\left(\frac{I_t}{I_{t-1}}\right)-\left(\frac{I_t}{I_{t-1}}\right)S_I'\left(\frac{I_t}{I_{t-1}}\right)\right]+\beta^{-1}\phi_{5,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}\left(\frac{I_t}{I_{t-1}}\right)^2 S_I'\left(\frac{I_t}{I_{t-1}}\right) \\
& -\phi_{6,t}+\beta^{-1}\phi_{6,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}(1-\delta)\Bigg\}
\end{aligned}
\tag{26}
$$

Interest rate,R_t

$$
0 = E_t\left\{-\phi_{1,t}\beta U_{C,t+1}\frac{1}{\pi_{t+1}}+\phi_{7,t}-\beta\phi_{8,t+1}\frac{b_t}{\pi_{t+1}}\right\}
\tag{27}
$$

Inflation rate,π_t

$$
\begin{aligned}
0 = E_t\Bigg\{ & \beta^{-1}\frac{\phi_{1,t-1}R_{t-1}}{(\pi_t)^2}\frac{U_{C,t}(.)}{U_{C,t-1}(.)}-\frac{\psi}{\theta}\left[\phi_{2,t}-\beta^{-1}\phi_{2,t-1}\beta\frac{U_{C,t}(.)}{U_{C,t-1}(.)}\frac{Y_t}{Y_{t-1}}\right](1+2\pi_t)-\phi_{3,t}\psi Y_t\pi_t \\
& -\phi_{7,t}\phi_\pi\left(\frac{Y_{t+1}}{Y}\right)^{\phi_y}(1+\pi_t)^{\phi_\pi-1}+\phi_{8,t}\frac{R_{t-1}}{(\pi_t)^2}b_{t-1}\Bigg\}
\end{aligned}
\tag{28}
$$

Public debt ,b_t

$$
0 = E_t\left\{-\sigma_b U_B b_t-\phi_{1,t}U_{BB,t}+\phi_{8,t}-\beta\phi_{8,t+1}\frac{R_t}{\pi_{t+1}}\right\}
\tag{29}
$$

taxes ,τ_t

$$
0 = E_t\left\{\phi_{2,t}\left[\frac{1}{(1-\tau_t)}\right]mc_t+\beta^{-1}\phi_{6,t-1}\beta\frac{U_{Ct}}{U_{Ct-1}}\frac{F_{K,t}}{(1-\tau)}mc_t+\phi_{8,t}N_t w_t\left[\frac{2-\tau}{1-\tau}\right]mc_t\right\}
\tag{30}
$$

The linear version of the model is similar to the previous one, but one needs to add the Linear version of the Taylor rule, and consider the multiplier ϕ_8 attached to it.

E Appendix: tables and figures

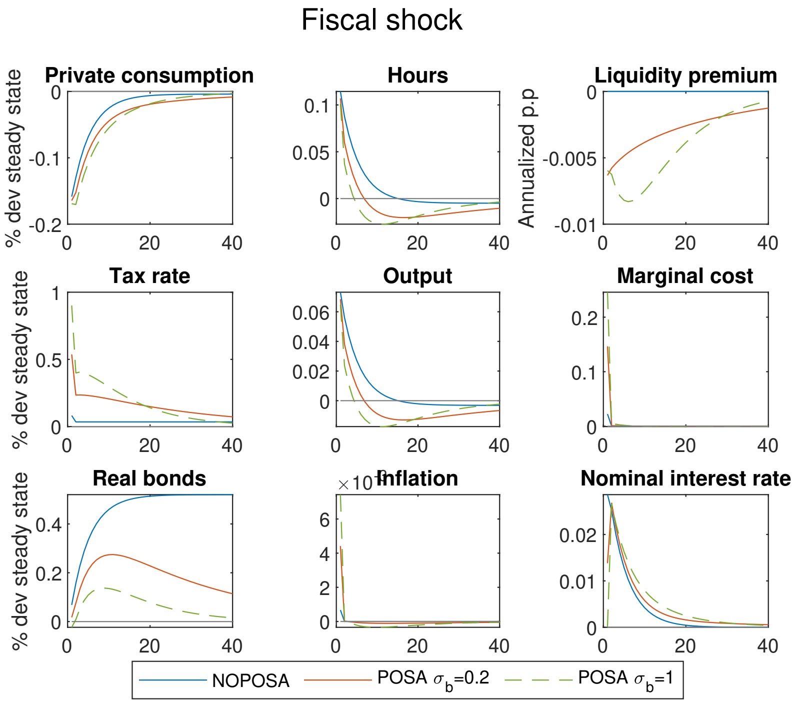
Table 1: Model parameters

Description	Parameter	POSA	NOPOSA	Source/Target
Preferences parameters				
Discount factor	β	0.985	0.99	r=0.005
Consumption risk aversion	σ	1		standard
Inverse Frisch elasticity	φ	1		standard
Curvature safe assets	σ_b	0.2; 1	-.-	Rannenberg (2021)
Discounting wedge	ϑ	0.99	1	Campbell et al. (2017)
Firms				
Labor intenstity	a	0.64		Christiano et al. (2005)
Price adjustment cost	ψ	200		target Phillips curve slope 0.03
Elasticity substitution intermediate goods	θ	6		target mark-up 20%
Depreciation rate	δ	0.02		target $K/4Y = 2.7\%$
Private investment adjustment costs	ω	5.5		Rannenberg (2021)
Taylor rule				
Taylor rule inflation	ϕ_π	1.5		Woodford (2011)
Taylor rule output	ϕ_y	0.15		
Government				
Government expenditure share	g/y	20%		US data
Government debt to output	$b/4y$	60%		US data

Notes: The table shows the main calibration parameters to meet the data in steady-state considering standard preferences, ie. NOPOSA. US data for the period 1987-2018 from the Federal Reserve Bank of St. Louis

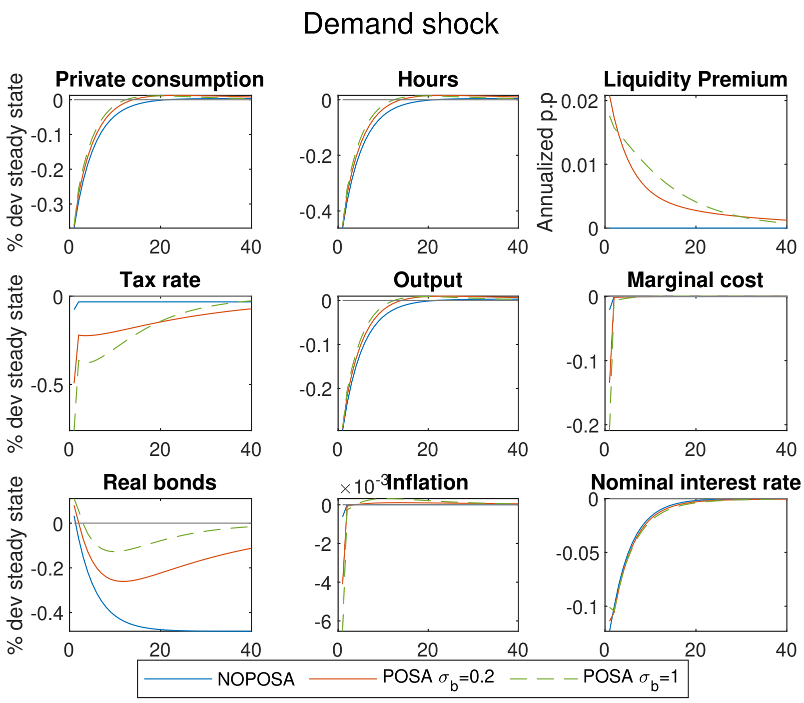
E.1 Figures

Figure 1: Optimal fiscal and monetary policy in response to a fiscal shock



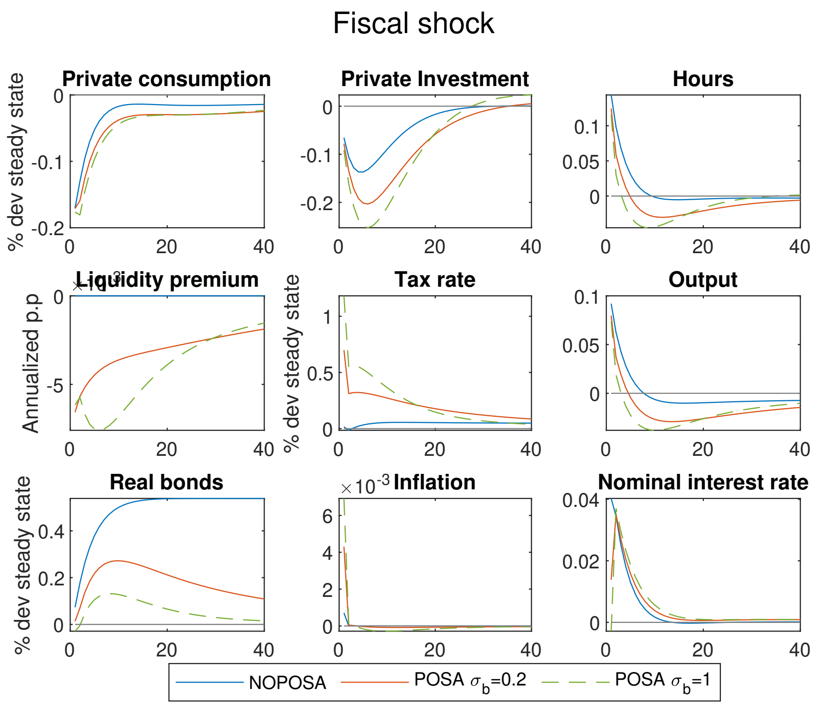
Note: the figure shows the optimal response of the economy to a government spending shock, which increases expenditures by 1% above the steady-state level in an economy without capital. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

Figure 2: Optimal fiscal and monetary policy in response to a demand shock



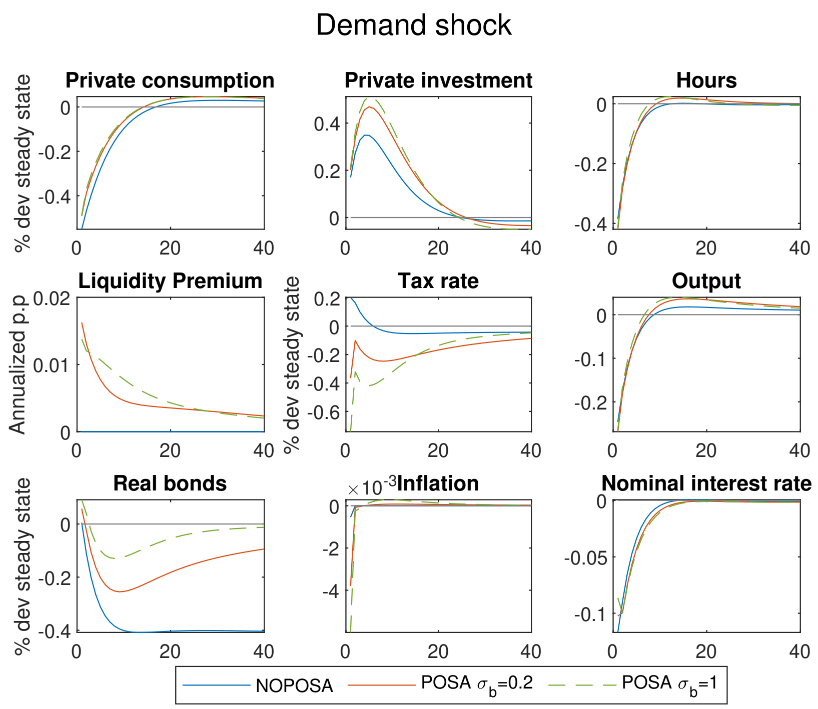
Note: the figure shows the optimal response of the economy to a negative demand shock z_t reducing consumption by 1% in an economy without capital. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$.

Figure 3: Optimal monetary and fiscal response to a government spending shock



Note: the figure shows the optimal response of the economy to a negative demand shock z_t reducing consumption in 1% in an economy with private capital. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

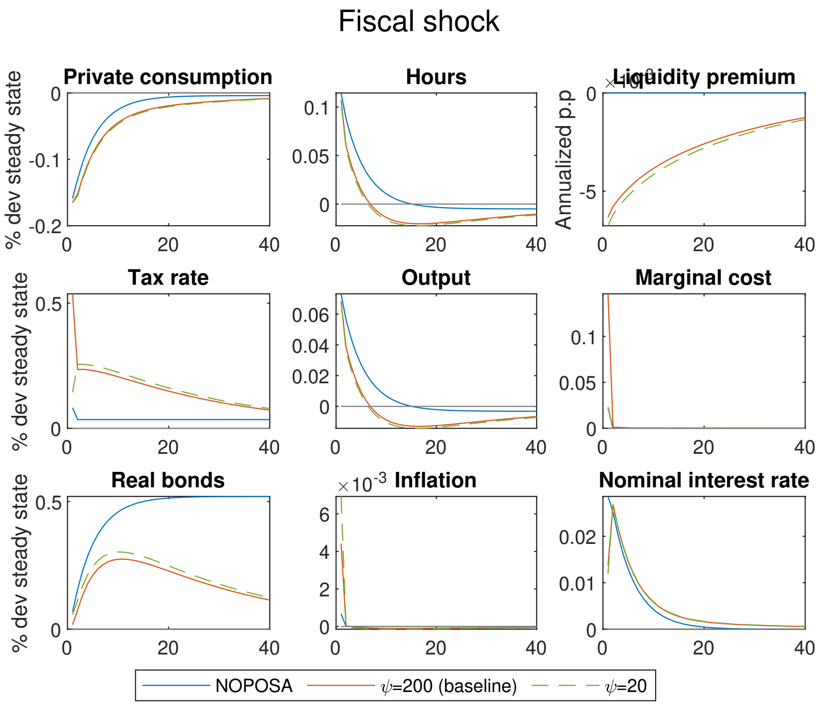
Figure 4: Optimal monetary and fiscal response to a demand shock



Note: the figure shows the optimal response of the economy to a government spending shock, which increases expenditures by 1% above the steady-state level in an economy with private capital. The blue line is the economy with standard preferences. TThe red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

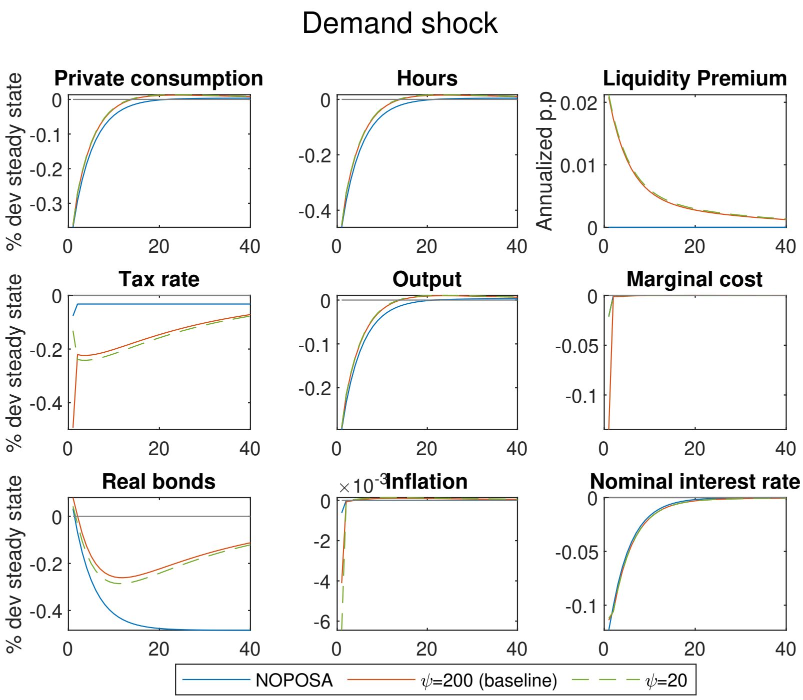
E.2 Extension: Degree of price stickiness

Figure 5: Degree of price stickiness and optimal inflation and tax rate volatility



Note: the figure shows the optimal response of the economy to a government expenditure shock of 1% in an economy without capital. The blue line stands for the economy under NOPOSA (standard model). The red line is the baseline economy with $\psi = 200$ and the green dashed line is the economy with $\psi = 20$ with a slope for the Phillips curve of 0.3, ten times larger than the baseline economy

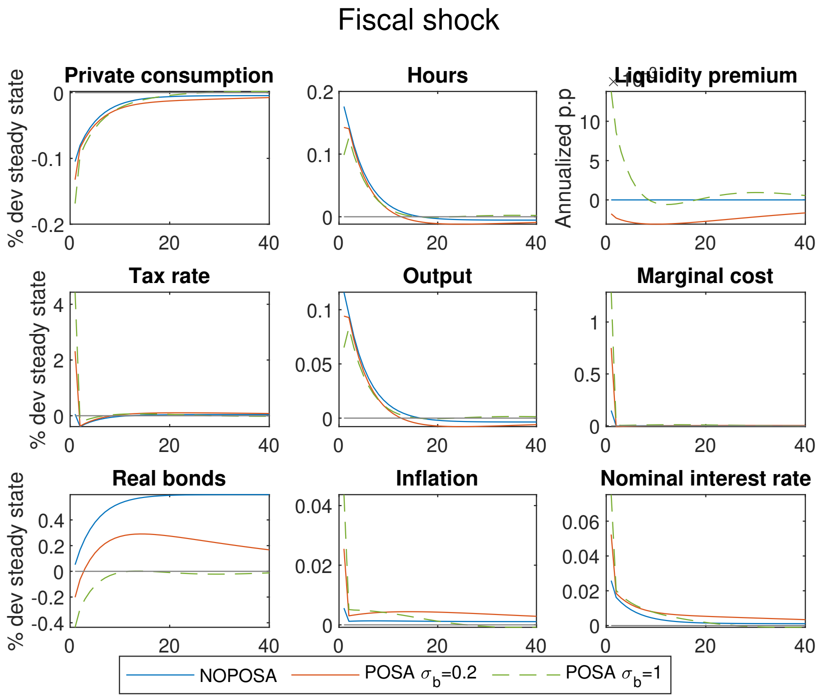
Figure 6: Degree of price stickiness and optimal inflation and tax rate volatility



Note: the figure shows the optimal response of the economy to a negative demand shock z_t reducing consumption in 1% in an economy without capital. The blue line stands for the economy under NOPOSA (standard model). The red line is the baseline economy with $\psi = 200$ and the green dashed line is the economy with $\psi = 20$ representing a slope for the Phillips curve of 0.3, ten times larger than the baseline economy

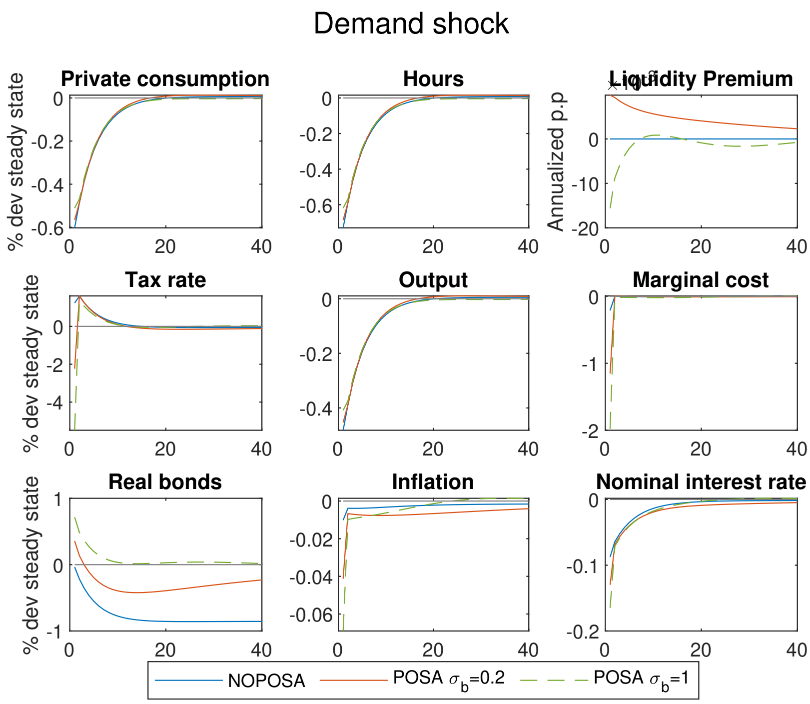
E.3 Extension: Economy with Taylor rule

Figure 7: Optimal fiscal policy in response to a fiscal shock - Taylor rule



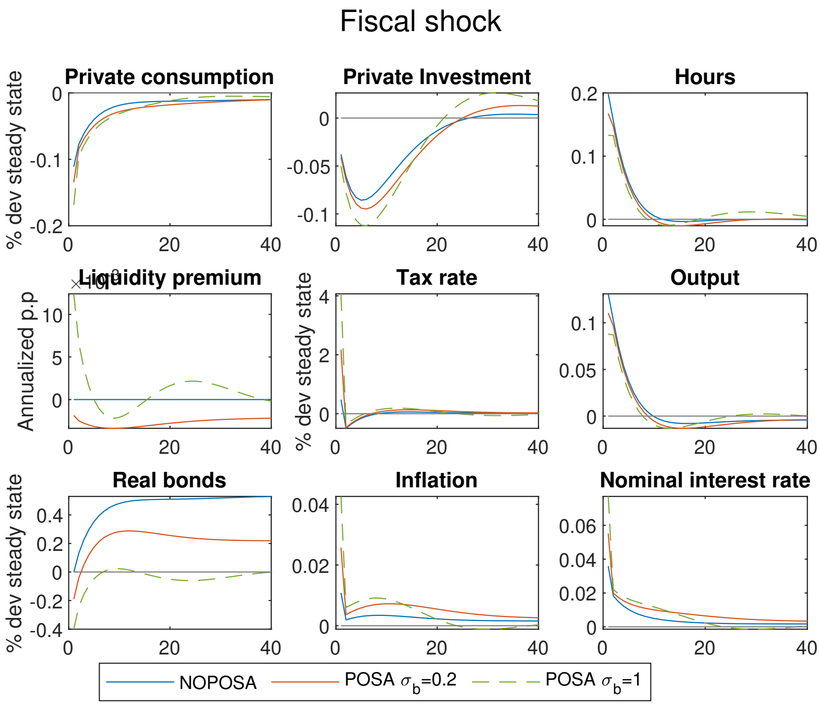
Note: the figure shows the optimal response of the economy to a government spending shock, increasing expenditures by 1% above the steady-state level in an economy without capital. Nominal interest rate follows a Taylor rule of the form $\tilde{R}_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t$. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

Figure 8: Optimal fiscal policy in response to a demand shock-Taylor rule



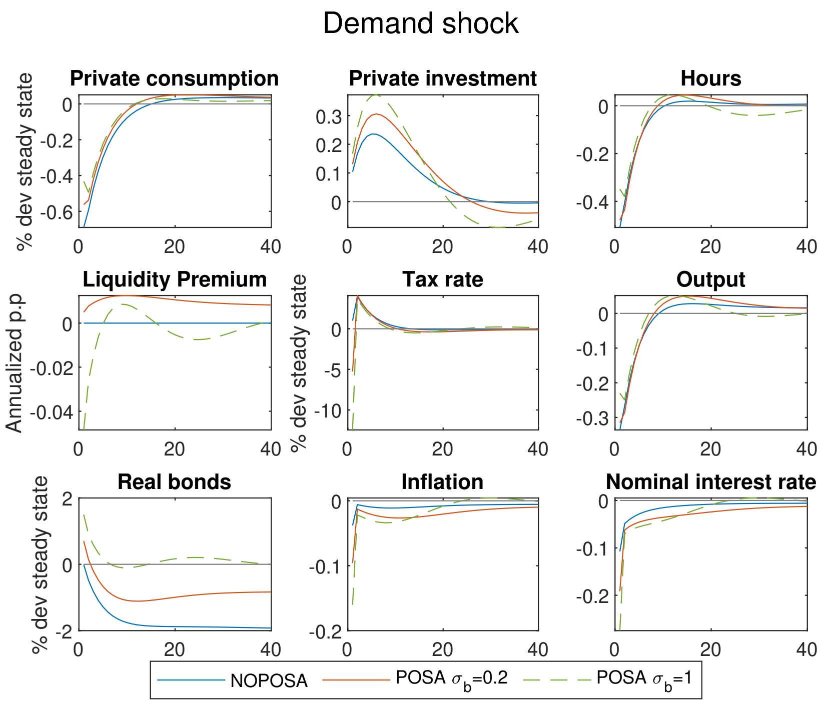
Note: the figure shows the optimal response of the economy to a negative demand shock z_t reducing consumption by 1% in an economy without capital. Nominal interest rate follows a Taylor rule, $R_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t$. The blue line is the economy with standard preferences.The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

Figure 9: Optimal response to a fiscal shock: Taylor rule



Note: the figure shows the optimal response of the economy to a government spending shock, which increases expenditures by 1% above the steady-state level in an economy with private capital. Nominal interest rate follows a rule of the form $\tilde{R}_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t$. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

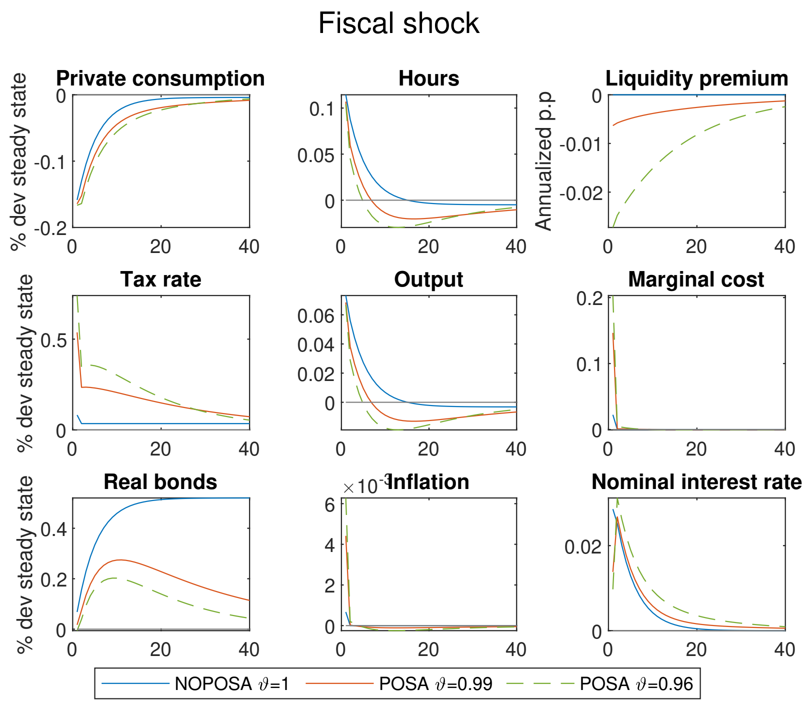
Figure 10: Optimal fiscal policy in response to a demand shock-Taylor rule



Note: the figure shows the optimal response of the economy to a negative demand shock z_t reducing consumption by 1% in an economy with capital. Nominal interest rate follows a Taylor rule, $R_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t$. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

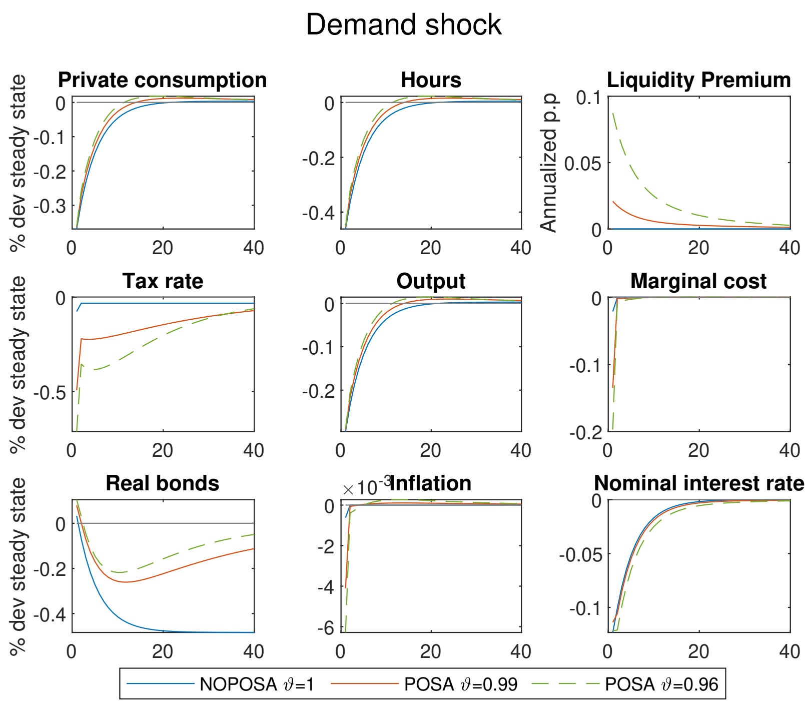
E.4 Model with no capital and $\vartheta = 0.96$

Figure 11: Optimal monetary and fiscal response to a fiscal shock with different levels of ϑ



Note: the figure shows the optimal response of the economy to a government spending shock, which increases expenditures by 1% above the steady-state level in an economy without private capital. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\vartheta = 0.99$ and $\vartheta = 0.96$

Figure 12: Optimal monetary and fiscal response to a demand shock



Note: the figure shows the optimal response of the economy to a government spending shock, which reduces consumption by 1% below the steady-state level in an economy without private capital. The blue line is the economy with standard preferences. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\vartheta = 0.99$ and $\vartheta = 0.96$

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