PARTIAL DE-ANNUITIZATION OF PUBLIC PENSIONS V.S. RETIREMENT AGE DIFFERENTIATION. WHICH IS BEST TO ACCOUNT FOR LONGEVITY DIFFERENCES?

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Partial De-Annuitization of Public Pensions v.s. Retirement Age Differentiation. Which is Best to Account for Longevity Differences?

Vincent Vandenberghe*

Abstract

Extensive research by demographers and economists has shown that longevity differs across socioeconomic status (SES), with low-educated or low-income people living, on average, shorter lives than their better-endowed and wealthier peers. Therefore, a pension system with a unique retirement age is a priori problematic. The usual policy recommendation to address this problem is to differentiate the retirement age by SES. This paper explores the relative merits of `partial` de-annuitization of public pensions as a way of addressing the (imperfectly assessed) inequality of longevity.

Keywords: Pension Policy, Longevity Difference, Equity, Annuitization, Retirement Age Differentiation

JEL Codes: H55, J26, J14

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1 Introduction

The length of life of individuals (longevity hereafter) is correlated with socio-demographic characteristics: on average women outlive men, and low-income individuals live, on average, significantly shorter lives than their better-endowed and wealthier peers (Chetty et al., 2016; Olshansky et al., 2012). Therefore, a pension system with a unique retirement age (or uniform contribution or replacement rates) is a priori problematic. Unaccounted longevity differences in contributory pension systems amount to taxing short-lived people and subsidising their long-lived peers (Ayuso et al., 2016), potentially distorting labour supply. Also, the social gradient in life expectancy reduces the progressivity of public pensions in those countries (e.g. the US) where the replacement rate is a negative function of earned income (Bosworth et al., 2016; Bommier et al., 2011). Some would even argue that longevity difference makes public pensions regressive (Piketty and Goldhammer, 2015).

We show is this paper that unaccounted longevity differences violate the most basis definition of equity/actuarial fairness under both a Bismarckian (i.e. fully contributory) or a Beveridgian pension system.[1] One of the usual policy recommendation to address these problems is to differentiate the retirement age by socio-economic status (SES hereafter) (Ayuso et al., 2016; Leroux et al., 2015). Related proposals – but that turn out to be functionally equivalent – recommend differentiating contributions or replacement rates (Bismarck) or the amount of the instalment (Beveridge) based on expected longevity differences. This paper argues that there might be another, possibly more effective, option. We call it `partial` de-annuitization of public pensions.

Annuitization is a common (implicit) feature of most, if not all, public pension systems organised on a pay-as-you go basis (PAYG).[2] But in principle, if we leave aside liquidity and transition issues, nothing prevents imagining a public PAYG pension scheme where `part` of the sums earmarked for someone are paid upfront (i.e. at beginning of the retirement spell) as a lump sum.[3] In the universe of fully funded pensions systems, including public

[1]It is common to distinguish Bismarckian and Beveridgean pension regimes. Bismarckian ones are contributory and, in that sense, work-related. Benefits are paid *prorata* the duration and level of contributions. This is a basic feature of the first fully-fledged public pension scheme introduced by German Chancellor Bismarck in 1889. By contrast, Beveridgean pensions (in reference to the British economist W. Beveridge who presided over the design the British system) are non-contributory and distribute basic universal benefits and so provide a (generally small minimal) pension to all, in particular those who do not qualify for a contributory pension (e.g. because they never worked...).

[2]A system in which pensions are explicitly financed by contributions levied from current workers.

[3]Until 2016 in the UK, the (small) contributory segment of the PAYG public pension system (the Additional State Pension) offered a one off lump-sum payment option.

or publicly sponsored ones, that option is available, and sometimes explicitly related to the perspective of a short life. Examples that we are aware of comprise

- the Netherlands. From 2022 onward, the Dutch Government will allow lump sum payments equal to a maximum of 10% of the accumulated capital when reaching the retirement date, under occupational pension plans.
- the US, State of New York where the public sector employees can upon retirement fill a form to receive a "Partial Lump Sum Payment" corresponding to 5 to (max) 25% of the accumulated capital, with "a reduced lifetime monthly benefit based on the remainder".
- the UK, a pension Commencement Lump Sum (PCLS) representing max 25% of the capital can be withdrawn (tax free) before the age of 55 `if you are in poor health`.
- Canada, British Colombia, with the Public Service Pension Plan you may receive a lump-sum payment in lieu of a monthly pension if you have `an illness or disability that has shortened your life expectancy`.

In what follows, partial de-annuitization will not be an option but considered as automatic and universal (i.e. applicable to all pensioners). But the key intuition will remain the same as in the above UK or Canadian examples: if longevity varies and is a source of inequality, paying part of the accrued pension rights when (all) prospective pensioners are still alive is a way to minimize pension-related lifetime inequalities. The idea is echoing the notion of `reverse retirement` introduced by Ponthiere (2020) who considers a model where individuals start their life in retirement (and thus "all" receive their pension) before moving to work. What follows should be seen as a much milder, but more realistic, version of that thought-provoking idea. Ours is more related to the notion of `front-loaded` benefits in pension economics (Palmer, 2000).[4]

Our realism partially stems from the fact that we are not so much interested in the absolute level of lifetime equity gains that can be achieved via de-annuitization. It is almost tautological that full de-annuitization is very effective in dealing with longevity differences.

[4] In Sweden, for instance, the individual replacement rate from the contributory public pension is higher at the beginning of the retirement spell.

But also that it would annihilate pensions' capacity to cover the longevity risk.[5] Our perspective in this paper is that of the `relative` performance of de-annuitization vs retirement age differentiation (and by analogy, vs differentiation policies targeting either the contributory phase or the payout phase of pensions (Sanchez-Romero et al., 2020)). The key question of this paper is how much de-annuitization is needed to match the equity gains delivered by retirement age differentiation?

Of course, for obvious budgetary reasons, introducing de-annuitization (and the upfront payment of a lump sum to all) implies a reduction of the value of the pension annuity. Answering the question of `how much` de-annuitization is needed is thus also a way to quantify the propensity of partial de-annuitization to come at the expense one of the key objectives of annuitization i.e. insuring individuals against the risk of longevity. That risk – and the underlying shortsightedness of individuals – is regularly mentioned in the literature as a justification for the State to impose a minimal degree of annuitization of the pension capital (Barr and Diamond, 2006). The results presented in this paper show that the reduction of the annuity needed to match the equity gains achieved via extensive retirement age differentiation (up to 200 different ages) is quite small. Using US data assembled by Chetty et al. (2016), we estimate the reduction to range from 4 to 5%. This suggests a limited risk of significantly eroding pensions' monthly payment adequacy[6] and their capacity to insure the risk of longevity. Also, partial de-annuitization is intrinsically less costly to implement than retirement age differentiation, and it is not prone to misreporting and moral hazard.

Note that throughout this paper will we consider that retirement age(s) or the degree of de-annuitization are decided paternalistically by the State. Such a perspective partially reflects the European context underpinning this paper, where retirement is still largely driven by State-edicted rules. This said, we also consider the `political economy` of the proposal i.e. that of the number of people who could support it.

From a normative point of view, we will consider throughout the paper that all realised longevity differences matter. This means that we subscribe to `ex post` egalitarianism when it comes to dealing with longevity inequalities (Fleurbaey et al., 2016).

The rest of the paper is organized as follows. Section 2 exposes a simple framework to assess the gains from retirement age differentiation. Section 3 does the same thing for the idea of de-annuitization and exposes how the two approaches are logically related. Section 4 exposes the longevity data we use. Section 5 presents the key numerical results of the paper.

[5] The risk that individuals outlive their money, dying in poverty or burdening relatives.
[6] Their ability to support a basic acceptable standard of living.

Section 6 examines the political economy of partial de-annuitization considering the number of winners vs losers. Section 7 concludes.

2 A simple framework to assess the equity gains of retirement age differentiation

We consider a world where longevity varies significantly across individuals forming a cohort ($l_i, i = 1 \dots N$) and in a way that is related with observable SES category j, with $j = 1 \dots k$ and n_j the number of individuals forming category j.[7] The full distribution of longevity is unknown to the planner/pension minister. Her knowledge is limited to the `correlation` between SES and longevity. Equivalently, the planner can only differentiate treatment (retirement age) based on the $j = 1 \dots k$ SES group to which individuals belong.

Throughout the paper, we consider the two canonical versions of pay-as-you-go (PAYG) public pensions: the fully contributory Bismarckian version where benefits are indexed on contributions, and the Beveridgian one where every individual receives the same pension.

2.1 Bismarckian contributory pension scheme

The problem of policymakers under such a regime is to equalise the ratio of lifetime pensions benefits to lifetime pension contributions, i.e. equalising their pension rate of return (rr). This is the very definition of `actuarial fairness` in pensions. Abstracting from education length differences, career breaks, wage growth, demographic changes or discount and indexation rates, and considering that retirement age is uniform, that rate of return writes,

$$\begin{gathered} rr_{i,j}(\boldsymbol{ra}) = S_{i,j}(\boldsymbol{ra}) \frac{(l_{i,j} - \boldsymbol{ra})\, \delta\, w_{i,j}}{\boldsymbol{ra}\, \eta\, w_{i,j}} = S_{i,j}(\boldsymbol{ra}) \frac{(l_{i,j} - \boldsymbol{ra})}{\boldsymbol{ra}} \theta \\ \text{where } \theta \equiv \frac{\delta}{\eta} \end{gathered} \tag{1}$$

Centrally defined reference retirement age is $\boldsymbol{ra}$ and lifetime benefits are equal to the time spent in retirement times the annuity $\delta w_{i,j}$ where δ is the replacement rate and $w_{i,j}$ is the individual level of earnings. Note that people can die before reaching retirement age.

[7] Logically we have that $\sum_j^k n_j = N$.

So we have the dummy variable $S_{i,j}(\boldsymbol{ra}) = 0$ if $l_{i,j} \leq \boldsymbol{ra}$ and $S_{i,j}(\boldsymbol{ra}) = 1$ otherwise. By definition of a Bismarckian system, lifetime contribution corresponds to the duration of the career (here, the retirement age) that multiplies the annual contributions at a rate η. We define $\theta \equiv \frac{\delta}{\eta}$ as the (uniform) rate of replacement for each euro of contribution.[8]

An alternative way of expressing reference retirement age is $\boldsymbol{ra} \equiv \alpha \boldsymbol{l}$ where $\boldsymbol{l}$ is the unique reference longevity and $0 \leq \alpha \leq 1$ is the share of life the reference person is supposed to spend working. Reference retirement age and reference longevity are thus isomorphic formulations in our setting.

$$rr_{i,j}(\boldsymbol{ra}) = S_{i,j}(\boldsymbol{ra}) \frac{(l_{i,j} - \alpha \boldsymbol{l})}{\alpha \boldsymbol{l}} \theta \tag{2}$$

The equalisation of lifetime rates of return across individuals ($\forall i, j : rr_{i,j} = rr$) can be achieved via the full individualisation of the retirement age, or, equivalently, via the use of each individual's longevity $l_{i,j}$ when defining the retirement age.[9] For any value of α, if the retirement age is fully individualised (i.e. $ra_{i,j} \equiv \alpha l_{i,j}, \forall i, j$), we verify

$$rr = \frac{(1 - \alpha)}{\alpha} \theta \tag{3}$$

Note that, by definition, if the retirement age is fully individualized, and if $\alpha < 1$,[10] $S_{i,j}(ra_{i,j})$ is always equal to 1. In other words, nobody dies before reaching his fully individualised retirement age.

Using a uniform reference retirement age (i.e. $\boldsymbol{ra} \equiv \alpha \boldsymbol{l}$) introduces a `gap` between the fair rate of return and the actual one

$$gap_{i,j}(\boldsymbol{ra}) \equiv rr - rr_{i,j}(\boldsymbol{ra}) = \theta \Big[\frac{1 - \alpha}{\alpha} - S_{i,j}(\boldsymbol{ra}) \, \frac{l_{i,j} - \boldsymbol{ra}}{\boldsymbol{ra}} \Big] \tag{4}$$

or equivalently when the reference retirement age is differentiated by SES category (i.e. $\boldsymbol{ra}_j \equiv \alpha \boldsymbol{l}_j$), the (presumably smaller) gap is

[8] In reality, with PAYG, θ is also driven by the evolution of relative size of the generations of (old) pensioners vs (younger) contributors, and by the wage/productivity gains that have occurred between the contributory and the payout years (Aaron, 1966).

[9] The other one, that we will not discuss systematically is to differentiate θ by SES and make it inversely proportional to expected longevity.

[10] Some part of life goes to retirement.

$$gap_{i,j}(\boldsymbol{ra}_j) \equiv rr - rr_{i,j}(\boldsymbol{ra}_j) = \theta\Big[\frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra}_j)\,\frac{l_{i,j} - \boldsymbol{ra}_j}{\boldsymbol{ra}_j}\Big] \tag{5}$$

A graphical representation of what happens under uniform vs differentiated retirement age appears on Figure 1. The lower part of the graph represents the distribution of longevity, while the upper part depicts the (lifetime) rate of return. The first-best situation amounts to ensuring that every person gets a return corresponding to the horizontal green line. The actual/realised rate of return corresponds to the red line. People who die before the retirement age get a zero (lifetime) return. Beyond that point, the return rises at a rate $\frac{\theta}{\boldsymbol{ra}}$.[11] Under uniform retirement age, only those whose longevity coincides with reference longevity ($\boldsymbol{l}$) get the first best. Under differentiated retirement age (2 different retirement ages are depicted in Figure 1), the number of people who get on the green line a priori rises. However, situations synonymous with "undershooting" or "overshooting" still abound.

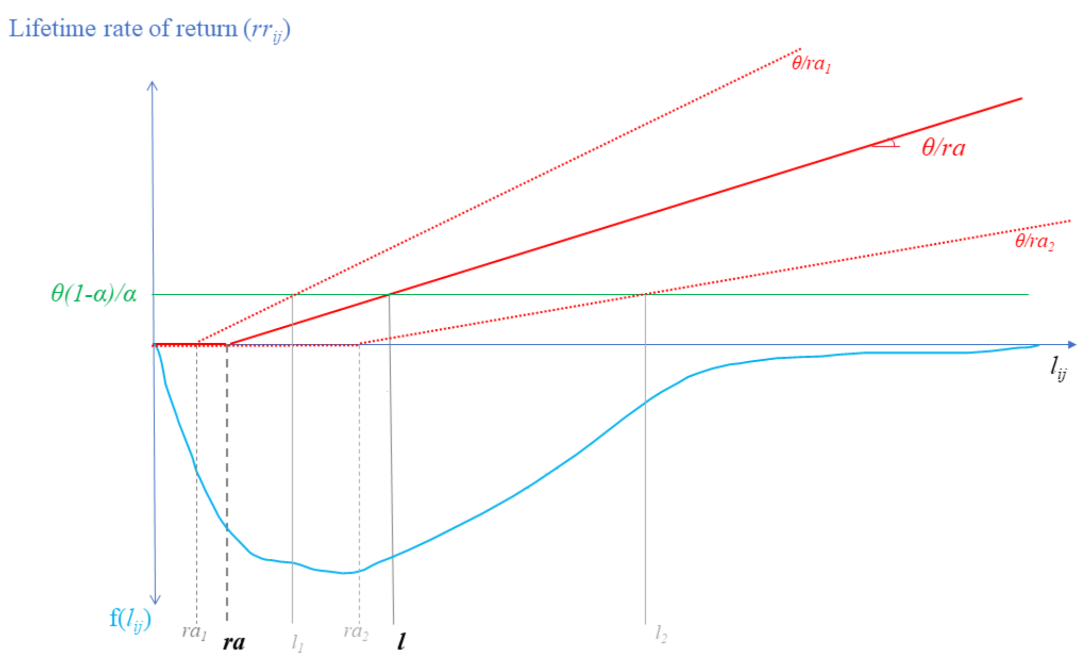

Figure 1: Bismarckian pension under uniform vs diff. ret age

[11]It is worth stressing that we here get a sense of what would happen if, instead of differentiating the age or retirement, policy markers were to differentiate either the contribution rate η or the replacement rate δ by SES. That would amount to differentiating $\theta \equiv \delta/\eta$, thus the slope of the red curve, while keeping the retirement age unchanged.

Following Baurin (2021), the overall reduction in the propensity of retirement policy to deviate from the full individualised first best can be expressed as a index[12] $\in [0,1]$ where the numerator aggregates the (absolute) values of the individual equity gaps under retirement age differentiation by SES $j = 1 \dots k$, and the denominator does the same when there is no differentiation. Note that the index being a ratio, constant θ cancel out. In the numerical simulations underpinning results of Section 5 the differentiated reference retirement ages/longevities correspond to averages by SES category j.

$$I^{Bismarck} \in [0,1] = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j}\left|\frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra}_j)\,\frac{l_{i,j}-\boldsymbol{ra}_j}{\boldsymbol{ra}_j}\right|}{\sum_{j=1}^{k}\sum_{i=1}^{n_j}\left|\frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra})\,\frac{l_{i,j}-\boldsymbol{ra}}{\boldsymbol{ra}}\right|} \quad (6)$$
$$\text{with } \boldsymbol{ra} \equiv \alpha \boldsymbol{l}\ ;\ \boldsymbol{ra}_j \equiv \alpha \boldsymbol{l}_j$$

Note that we assume that the planner absorbs the net budgetary deficit or surplus generated by the move from uniform to differentiated retirement. In Appendix 8.1, we examine the implications for the above equity index when we impose that θ changes in order to ensure a strict (cohortal) budgetary equivalence between the two policies. Compare to uniform retirement, we tentatively show that less money is going for differentiated retirement. This means a higher θ for the numerator of the index. The consequences are that the unaccounted gaps in terms of years in retirement are "priced" higher, and this translates into significantly fewer (monetary) equity gains generated by retirement age differentiation.

2.2 Beveridgian pension scheme

By definition, a Beveridgian pension system would rather aim at equalising lifetime pension benefits ($B_{i,j}$).

$$B_{i,j}(\boldsymbol{ra}) \equiv b\, S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra}) \quad (7)$$

In the above expression, b is the standard uniform annual/monthly pension (which is independent of earning $w_{i,j}$ and contributions) that multiplies the time spent in retirement. Again, people can die before reaching retirement age. So we have the dummy variable $S_{i,j}(\boldsymbol{ra}) = 0$ if $l_{i,j} \leq \boldsymbol{ra}$ and $S_{i,j}(\boldsymbol{ra}) = 1$ otherwise. Equalisation of lifetime benefits

[12] To be precise, our index deviates from that of Baurin (2021) in the sense that is `monetarised`. Gaps are expressed in monetary terms and not just in terms of years.

($\forall i, j : B_{i,j} = B$) can only be achieved via full-individualisation of retirement age, or the corresponding reference longevity ($ra_{ij} \equiv l_{i,j} - \omega$), where ω is the reference number of years spent in retirement.

$$B \equiv b(l_{i,j} - ra_{i,j}) = b\,(l_{i,j} - l_{i,j} + \omega) = b\,\omega \tag{8}$$

Note again that $S_{i,j}(\boldsymbol{ra}) = 1$ if there is perfect individualisation and if $\omega > 0$. We logically assume ω is the time spent in retirement by the person whose longevity is equal to the reference longevity ($\boldsymbol{l}$) under a uniform retirement policy ($\omega = \boldsymbol{l} - \boldsymbol{ra} = (1-\alpha)\boldsymbol{l}$).[13]

Key is that the use of a uniform retirement age/longevity reference ($\boldsymbol{ra}$) leads to lifetime benefits gaps

$$gap_{i,j}(\boldsymbol{ra}) \equiv B - B_{i,j}(\boldsymbol{ra}) = b\Big[\omega - S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra})\Big] \tag{9}$$

or equivalently when the reference retirement age is differentiated by SES category ($\boldsymbol{ra}_j$), the (presumably smaller) gap is

$$gap_{i,j}(\boldsymbol{ra}_j) \equiv B - B_{i,j}(\boldsymbol{ra}_j) = b\Big[\omega - S_{i,j}(\boldsymbol{ra}_j)(l_{i,j} - \boldsymbol{ra}_j)\Big] \tag{10}$$

Again, we can produce a graphical representation of what happens under uniform vs differentiated (Figure 2). The first-best situation amounts to ensuring that every person receives the green line in terms of (lifetime) benefits ($b\omega$) where ω is the number of years spent in retirement by the reference pensioner with longevity $\boldsymbol{l}$. Actual/realised lifetime benefits correspond to the red line. People who die before the retirement age receive no benefits. Beyond that point, lifetime benefits rise at a rate b.[14] Under uniform retirement age, only individuals whose longevity coincides with reference $\boldsymbol{l}$ get the first best. Under differentiated retirement age (2 different retirement ages are depicted in Figure 2), the number of people who get on the green line a priori rises. But note again that situations synonymous with "undershooting" or "overshooting" are still very frequent.

[13] Thus the equalising retirement age can also be written as $ra_{i,j} = l_{i,j} - \boldsymbol{l} + \boldsymbol{ra}$.

[14] Here again we get a sense of what would happen if, instead of differentiating the age or retirement, policy markers were to differentiate pension instalment b by SES. That would amount to differentiating the slope of the red curve to increase the chance of crossing the green line.

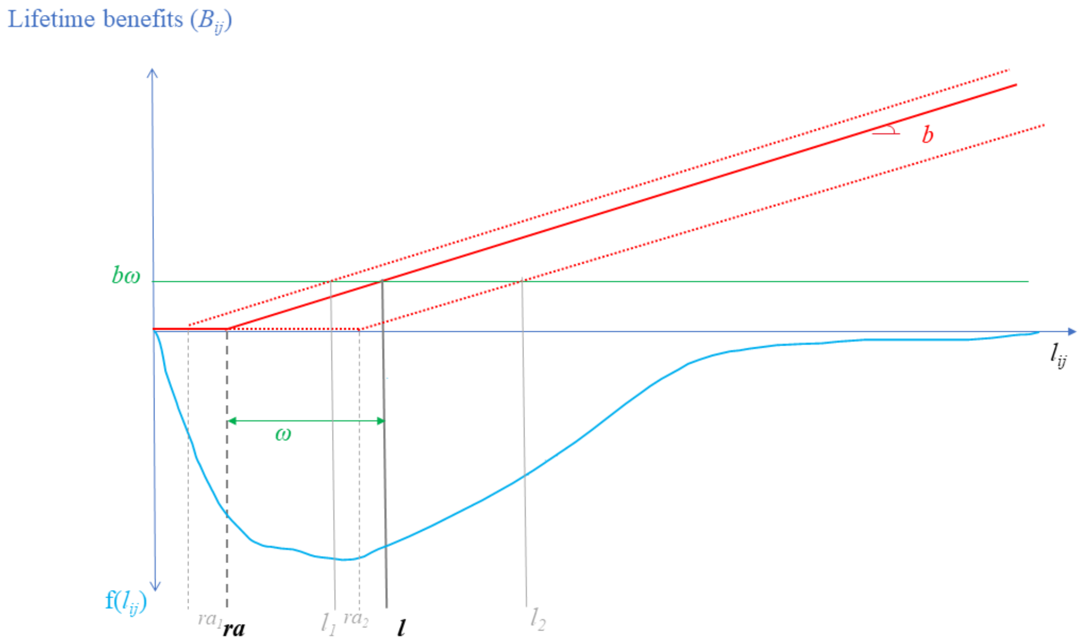

Figure 2: Beveridgian pension under uniform vsdiff. ret age

In more analytical terms, the reduction in the overall propensity of retirement policy to deviate from the fully individualised first best can be expressed as the following index

$$I^{Beveridge} \in [0,1] = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j}|\omega - S_{i,j}(\boldsymbol{ra}_j)\,(l_{i,j} - \boldsymbol{ra}_j)|}{\sum_{j=1}^{k}\sum_{i=1}^{n_j}|\omega - S_{i,j}(\boldsymbol{ra})\,(l_{i,j} - \boldsymbol{ra})|} \quad \text{with } \boldsymbol{ra} \equiv \boldsymbol{l} - \omega = \alpha \boldsymbol{l} \quad \boldsymbol{ra}_j \equiv \boldsymbol{l}_j - \omega \tag{11}$$

Note again that we assume the planner absorbs the net deficit or surplus generated by the move from uniform to differentiated retirement. In Appendix 8.1, we examine in greater details the impact on the above equity index when we let b change to ensure a strict (cohortal) budgetary equivalence. We show that slightly less money is spent when going for differentiated retirement (the magnitude of the gain stemming from retirement differentiation is less important under Beveridge than Bismarck). This means a moderately higher b to the numerator of the inequity index. Unaccounted longevity gaps (in years) are “priced” slightly higher, and this results into slightly fewer equity gains generated by retirement age differentiation.

To sum up, minimising both the Bismarckian and Beveridgian inequity indices (equ. 11;6) depends on the social planner being able to match the full distribution of longevity across individuals $l_{i,j}$ i.e., the different values of the horizontal axis forming the pale blue longevity distribution on Figures 1;2. If she can only go for tagging (Akerlof, 1978) i.e. use $j = 1 \dots k < N$ proxies $\boldsymbol{l}_j$ that are simply correlated to realised longevity $l_{i,j}$ to differentiate treatment, and if unaccounted longevity differences are important and matter, then both policies should translate into values of our indices that are relatively close to 1. We will show in Section 5 simulation results illustrating this using US data on longevity heterogeneity.

3 De-annuitization

We now consider Bismarckian and Beveridgian pension schemes with some de-annuitization: i.e. with an upfront lump-sum payment LS.[15] With the Bismarckian pension, the rate of return becomes

$$rr_{i,j} = \frac{LS_{i,j} + S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra})\,\delta'\,w_{i,j}}{\boldsymbol{ra}\,\eta\,w_{i,j}} = \mu + S_{i,j}(\boldsymbol{ra})\frac{(l_{i,j} - \boldsymbol{ra})}{\boldsymbol{ra}}\theta' \tag{12}$$

with the (logically lower) annuity corresponding here to a lower replacement rate i.e. $\delta' < \delta$, $\theta' \equiv \frac{\delta'}{\eta} < \theta$, and $\mu \equiv \frac{LS_{i,j}}{\boldsymbol{ra}\,\eta\,w_{i,j}}$, $\forall\, i, j$ the "guaranteed" rate of return as a uniform fraction of lifetime contributions.[16] In Appendix 8.2 we show that the value of μ that is compatible with the budgetary constraint (i.e. same sums spent on a cohort with and without de-annuitization) is

$$\mu = (\theta - \theta')\,\frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})\,w_{i,j}}{N\boldsymbol{ra}\overline{w}} \tag{13}$$

where the term that post-multiplies $(\theta - \theta')$ is the (wage weighted) ratio of the number years spent in retirement to the number of years spent contributing. Note that $\mu > 0$ if

[15] We leave aside complications stemming from those who did not survive until prime age i.e. the moment from which longevity heterogeneity is considered conceptually or empirically (i.e. 40 hereafter).

[16] Note that, unlike lifetime benefits, potential lifetime contributions $(\boldsymbol{ra}\,\eta\,w_{i,j})$ are fully known by the State. So there is no problem fully individualising LS to achieve uniformity across individuals in terms of guaranteed rate of return μ.

$\theta' < \theta$.

The interesting point is what happens with the inequity gap indices when θ is reduced to θ'. The building blocks of the Bismarckian version of that index consist (to the numerator) of the gaps between the fair rate of return and the one actually achieved via the policy envisaged. With de-annuitization (assuming a unique retirement age), the gaps become

$$\begin{aligned} gap_{i,j}(\boldsymbol{ra}, \mu, \theta') &= \Big[(\mu + \theta' \frac{1-\alpha}{\alpha}) - (\mu + \theta' S_{i,j}(\boldsymbol{ra})\, \frac{l_{i,j} - \boldsymbol{ra}}{\boldsymbol{ra}})\Big] \\ &= \theta' \Big[\frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra})\, \frac{l_{i,j} - \boldsymbol{ra}}{\boldsymbol{ra}}\Big] \end{aligned} \tag{14}$$

The index capturing the gains achieved via de-annuitization (the reference policy being one with no de-annuitization and uniform retirement age) now writes:

$$\begin{aligned} I^{Bismarck} \in [0,1] &= \frac{\theta' \sum_{j=1}^{k} \sum_{i=1}^{n_j} |\frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra})\, \frac{l_{i,j} - \boldsymbol{ra}}{\boldsymbol{ra}}|}{\theta \sum_{j=1}^{k} \sum_{i=1}^{n_j} |\frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra})\, \frac{l_{i,j} - \boldsymbol{ra}}{\boldsymbol{ra}}|} \\ &= \frac{\theta'}{\theta} \\ \text{with } \boldsymbol{ra} &\equiv \alpha \boldsymbol{l} \end{aligned} \tag{15}$$

The gain achieved via de-annuitization is strictly proportional to the reduction of the annuity ($\frac{\theta'}{\theta} < 1$).[17] And an interesting numerical exercise, based on actual longevity data, is to compute the gains that can be achieved via retirement age differentiation. This will provide a certain value of the index $I^{Bismark} < 1$, from which we can infer the corresponding value of $\frac{\theta'}{\theta}$ (and thus also of μ) ensuring the `same` fairness improvement. Thus, quantifying the gains that can be achieved via retirement age differentiation – as we do in Section 5 – amounts to computing the degree de-annuitization that will provide exactly the same pension fairness gains.

It is straightforward to show that a similar equivalence can be established between retirement age differentiation and partial de-annuitization of Beveridgian pension benefits. This time, the lump sum LS paid upfront is uniform and writes:

$$B_{i,j} \equiv LS + b'\, S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra}) = LS + b'\, S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{l} + \omega) \tag{16}$$

[17] Strictly speaking, the (reduced) annuity is $\delta' w_{i,j}$. But δ' is directly related to θ' as $\theta' = \frac{\delta'}{\eta}$. Thus the new annuity becomes $\theta' \eta w_{i,j}$.

with a logically lower annuity $b' < b$. In appendix 8.2 we show that the value of LS that is compatible with the budgetary constraint (i.e. same sums spent on a given cohort with and without de-annuitization) is

$$LS = (b - b')\, \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})}{N} \tag{17}$$

where the term that post-multiplies $(b - b')$ that is just the average number years spent in retirement. Note that $LS > 0$ if $b' < b$

The key point is again to consider what happens with the indices exposed above when b is reduced to b'. The building blocks of the Beveridgian version of that index consist (to the numerator) of the gaps between the fair annuity and the one actually achieved via the policy envisaged. With de-annuitization (assuming again a unique retirement age), the gaps become

$$\begin{aligned} gap_{i,j}(\boldsymbol{ra}, LS, b') &= (LS + b'\omega) - (LS + b'\, S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra})) \\ &= b' \left[\omega - S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra})\right] \end{aligned} \tag{18}$$

Hence the index capturing the gains achieved via de-annuitization (the reference policy still being one with no de-annuitization and uniform retirement age) writes

$$\begin{aligned} I^{Beveridge} \in [0,1] &= \frac{b'\ \sum_{j=1}^{k} \sum_{i=1}^{n_j} |\omega - S_{i,j}(\boldsymbol{ra})\ (l_{i,j} - \boldsymbol{ra}_j)|}{b \sum_{j=1}^{k} \sum_{i=1}^{n_j} |\omega - S_{i,j}(\boldsymbol{ra})\ (l_{i,j} - \boldsymbol{ra})|} \\ &= \frac{b'}{b} \\ \text{with}\ \ \boldsymbol{ra} &\equiv \alpha \boldsymbol{l} \end{aligned} \tag{19}$$

So the gains achieved via de-annuitization are strictly proportional to the reduction of the annuity $(\frac{b'}{b} < 1)$.

4 Data construction

The data used to analyse partial de-annuitization vs retirement age differentiation are from the US. They consist of a `simulation` of the full distribution of longevity across a cohort of N individuals with different socio-demographic background $(l_{i,j}; i = 1 \dots n_j; j = 1 \dots k)$ who have survived until prime age. As its core, the simulation rests on the (unavailable to us) mortality rates assembled by Chetty et al. (2016). The underlying micro data comprises a sample of 1.4 billion observations from anonymised tax records, covering the years 1999 to 2014. Mortality data start at age 40 and are available either by gender, US state of residence and income quartile; or by gender and income percentile. We retain the gender income version of the Chetty data. More precisely, we use the (publicly available) parameters of Gompertz functions they provide for each gender X income cell j, alongside the number of people in the US population belonging to these cells $(n_j, j = i \dots k)$.[18] The parameters of the Gompertz function capture the expected differences of mortality between categories j. Whereas the predicted values delivered by each Gompertz function j provides the "within" category distribution of mortality rates for each (potential) age of death.[19] These mortality rates by age can then be multiplied by the number of individuals forming each cell j to know the number of individuals whose longevity is equal to $40, 41, \dots, 120$.

In Figure 3 we display the Gompertz-generated distribution of longevity for men belonging to the lowest income percentile of the US male population vs the equivalent distribution for women forming the highest income percentile of the female population. Average longevity (corresponding to the dashed vertical line) is different. The expected/average longevity gap between the two categories is larger than 16 years (88.7 v.s. 71.9 years). Still, quite many women forming the upper income percentile die before and after the average age of 88.7, and similarly for low-income men whose tentative (differentiated) reference longevity would be set at a much lower level.[20]

[18]A Gompertz function is sigmoid which describes growth (here mortality) as being slowest at the start and end of a given period (respectively age 40 and age 120 with the Chetty data). The right-hand or future value asymptote of the function is approached much more gradually by the curve than the left-hand or lower valued asymptote. This is in contrast to the simple logistic function in which both asymptotes are approached by the curve symmetrically. The Gompertz is a special case of the generalised logistic function that has proved adequate to describe human mortality as an (accelerating) function of age.

[19]The latter is know by demographers as a life table (Chiang1984life).

[20]Remember that reference retirement ages are defined as $\boldsymbol{ra}_j \equiv \alpha\, \boldsymbol{l}_j$ with $\alpha < 1$ the share of life spent working.

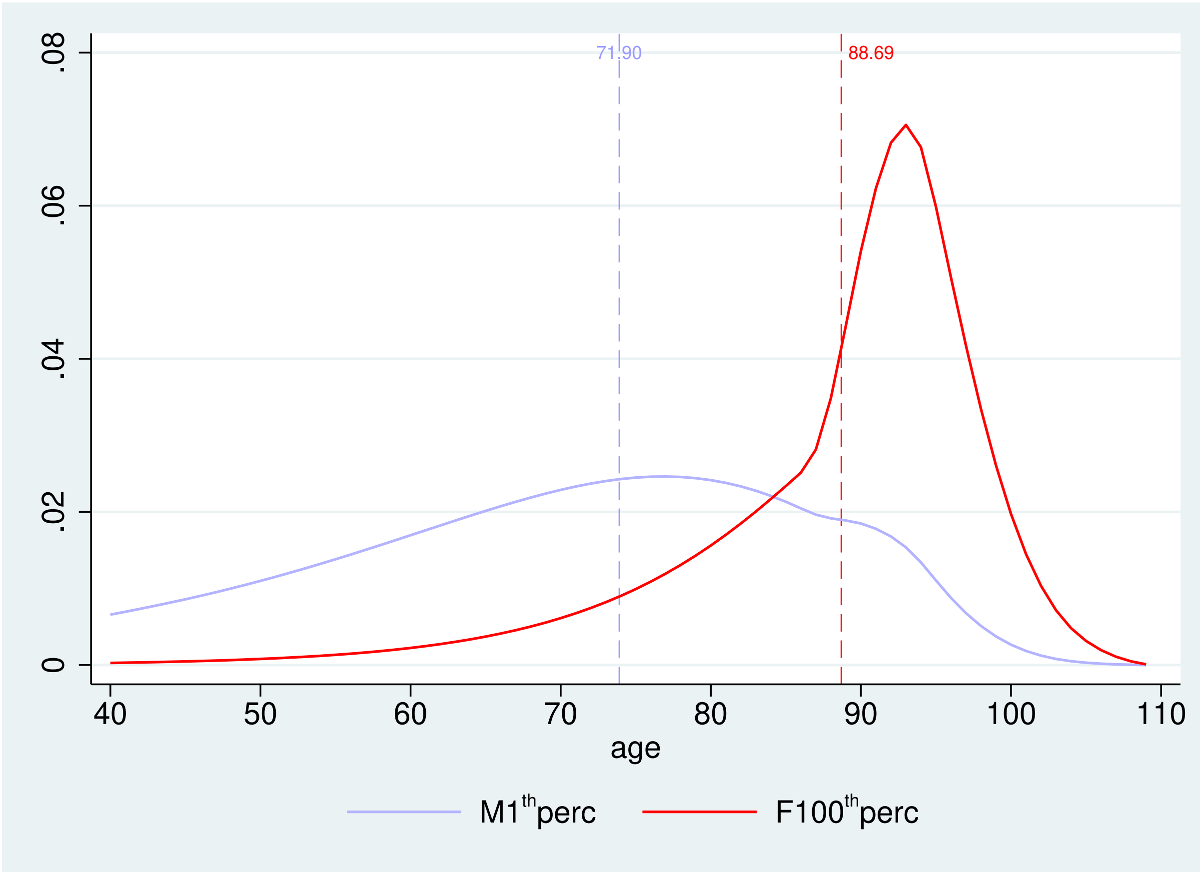

Figure 3: Distribution longevity: men lowest income percentile vs women highest percentile

5 Numeric simulation results

We start by assuming that our the reference longevity (i.e. $\boldsymbol{l}$) underpinning the uniform retirement policy is the average longevity in the Chetty data. Without loss of generality, and for the sake of clarity, the value of the share of life spent in employment α is chosen so that the corresponding uniform retirement age is equal to 65. Here after, the results for the retirement age policy are centred on that age of 65 ($\boldsymbol{ra} = 65$).

Given the Chetty data documenting longevity differences only past the age of 40, the minimal retirement age under retirement age differentiation is 40. And with de-annuitization, the lump-sum payment is also assumed to intervene at 40. By definition, it consists of paying a lump-sum to all whose longevity is considered, and that is only feasible before the first individual dies, thus here at 40. Also, given the data used, the two policies examined here (equally) ignore the problem of the very short-lived, i.e. those who don't survive up to the age of 40. Note that the hypothetical use of a data set documenting longevity only from the age of 50 or 55 would simply inflate the number of individuals who are de facto not compensated, but without affecting our key analytical results markedly. Remember that our prime interest is to assess de-annuitization needed to match whatever can be achieved via retirement age differentiation, and by that we also mean the extent to which that policy ignores some short-lived. Remember also that the major factor driving our results is the (in)ability of the planner to match the full distribution of longevity using a few proxies. Whether it is the post 40, 50 or 55 distribution does not matter much.

Our results consist of the simulated values of the gains generated by retirement age differentiation in terms of inequity gap indexes exposed above, one for the Biskmarckian system and one for the Beveridgean one. In both cases we estimate numerically the gains achieved by resorting to 200 different reference retirement ages/longevities (i.e $j = 1 \ldots 200$, corresponding to 2 genders X 100 income percentiles).

The differentiated retirement ages we use are visible in Figures 4, 5. They correspond to each of our SES category j's average longevity (multiplied by α with Bismarck, or minus ω with Beveridge). The equity gains achieved via differentiation are reported on top of Table 2. We see values of .963 and .964 for (respectively) the Biskmarckian and Beveridgian schemes. These show the degree of de-annuitization required to generate an equivalent gain in terms of pension fairness. We see it is relatively limited: 4 % points of reduction of θ (Bismarck)[21] or of the basic pension b (Beveridge) would be enough to generate the same equity gains are extensive retirement age differentiation across 200 gender$\times$ income categories; with retirement ages ranging from 56.5 to 69.7 (Bismarckian) or 54.2 to 71 (Beveridge).

[21] And thus of the annuity as the latter is $\theta' \eta w_{i,j}$.

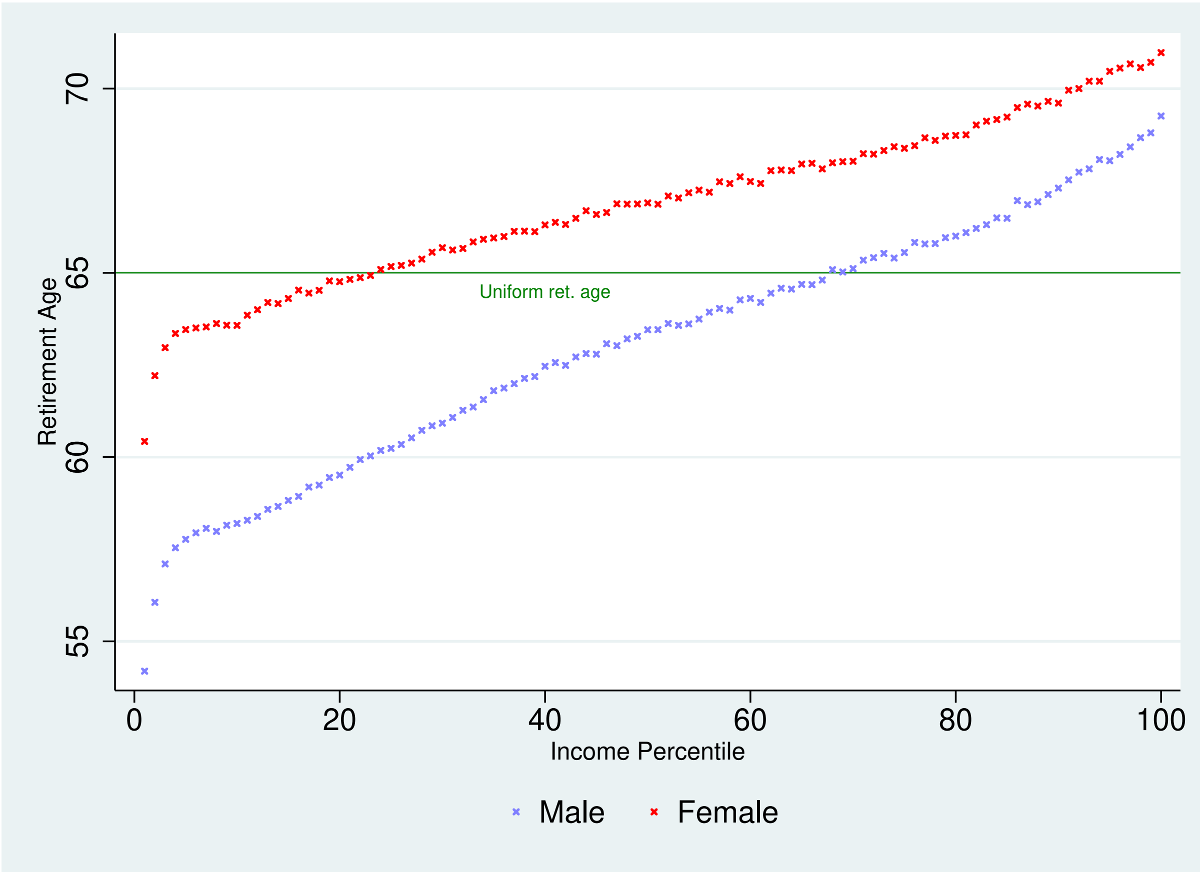

Figure 4: Bismarckian differentiated pension ages

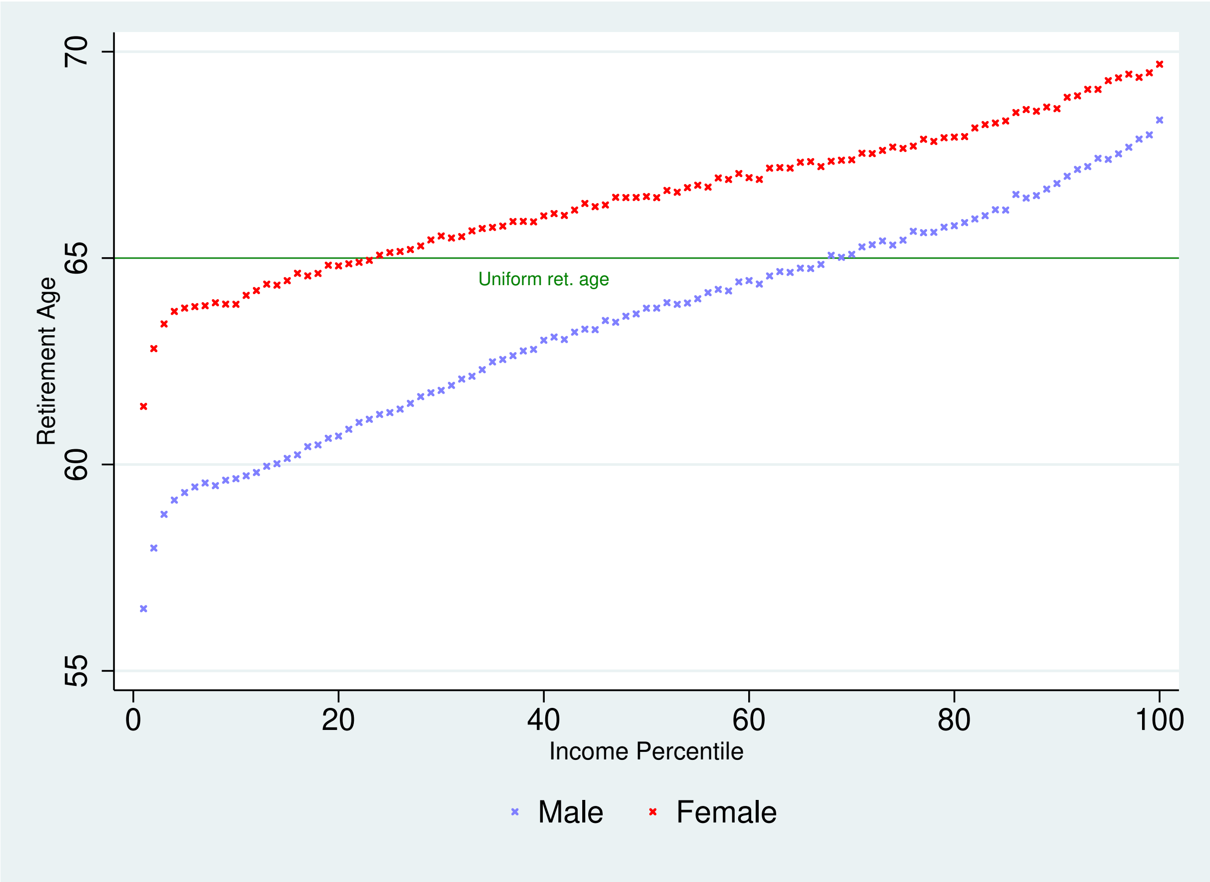

Figure 5: Beveridgian differentiated pension ages

Table 1: Numeric results: differentiated retirement ages and equity indices [without budget neutrality condition]

	Pension regime			
	Bismark		Beveridge	
	F	M	F	M
Equity Index	0.963		0.964	
Inc. perc.				
1	61.407	56.507	60.428	54.192
10	63.881	59.656	63.576	58.200
20	64.812	60.688	64.760	59.513
30	65.535	61.795	65.681	60.922
40	66.022	63.010	66.301	62.467
50	66.491	63.785	66.898	63.455
60	66.948	64.456	67.479	64.307
70	67.381	65.090	68.029	65.114
80	67.931	65.784	68.730	65.997
90	68.620	66.807	69.607	67.300
100	69.697	68.345	70.976	69.256
Ref. ret. age	65	65	65	65

6 Winners, losers

In this section, we explore the question of the support/opposition that partial de-annuitization might encounter. Our approach is quite simple. We consider that the support for de-annuitization depends on the share of short-lived individuals who get a higher lifetime rate or return (or benefits with a Beveridgian system). On the Figure 6, it corresponds to all the individuals whose longevity is inferior to l^*.

Algebraically and numerically, one can show that the "indifferent" pensioner is not influenced by the degree of de-annuitization. The blue point in Figure 6 and the corresponding longevity l^* are `fixed points`. Whatever the intensity of de-annuitization, there is always the same share of pensioners who gain from de-annuitization vs uniform retirement age. Proof of this is in Appendix 8.3.[22]

[22] The same cannot be said about how much, in monetary terms, is gained and lost because of various degrees of de-annuitization. More de-annuitization means more monetary gains(losses) for those who gain(lose). The headcount approach and the monetary one are not strictly equivalent.

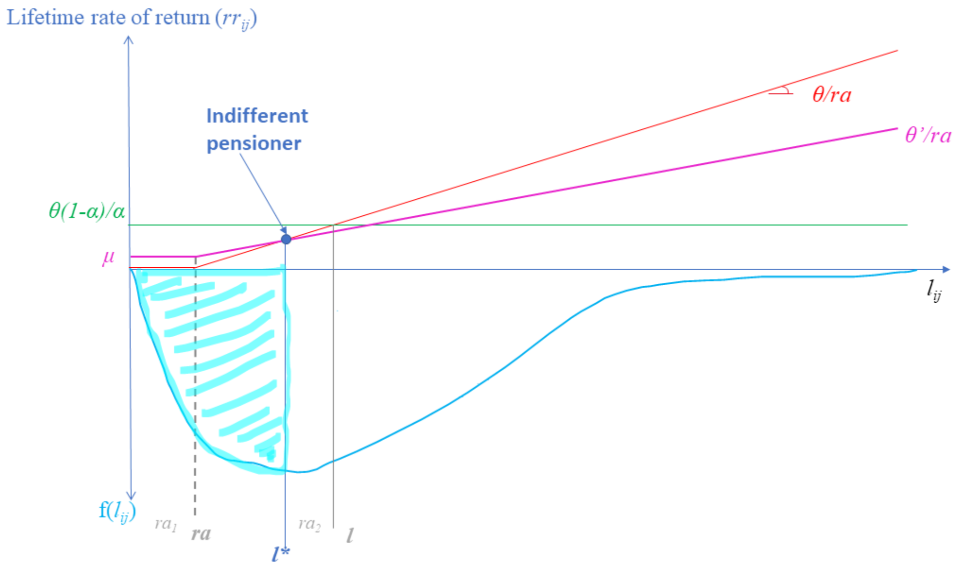

Figure 6: Partial de-annuitisation: winners vs losers

This said, it is important to stress that the value l^* varies with the reference retirement age ($\boldsymbol{ra}$). In Figure 7 we report simulation results on the share of (relatively short-lived) pensioners who would gain from de-annuitization. It is clearly a rising function of the reference retirement age. In policy terms this means that the support for de-annuitization is likely to rise the more policy-markers increase the (unique) retirement age.

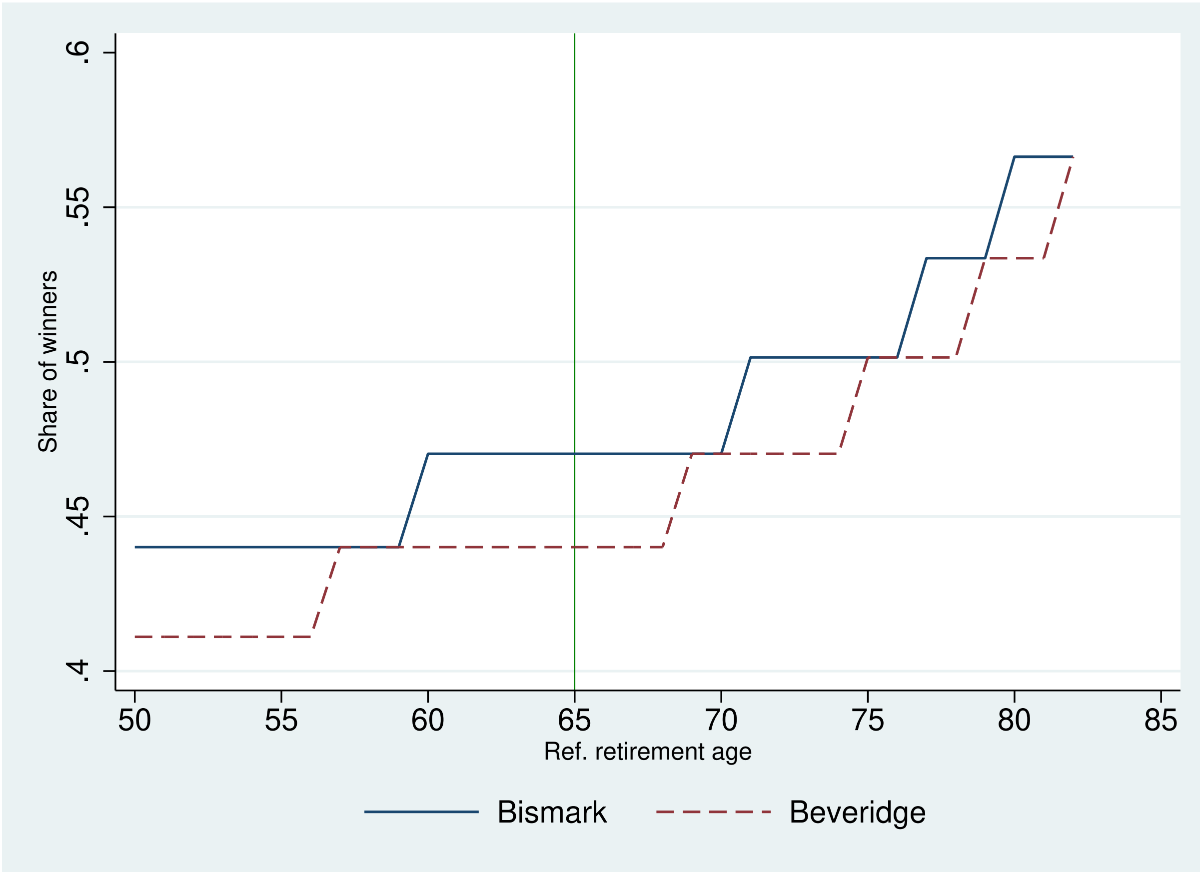

Figure 7: Partial de-annuitisation: share of winners and ref. retirement age

7 Concluding remarks

In the US context, the dominant view among economists is that people are "under-annuitised" for their privately provided and funded pension. By that, our colleagues mean they are underinsured against the risk of outliving their pension capital. And one challenge is to understand the so-called `annuity puzzle` i.e. the fact that annuities are rarely purchased (voluntarily) despite the longevity insurance they provide. The key problem seems to be that people are too pessimistic about their longevity (O'Dea and Sturrock, 2020).

In this paper, we adopt the opposite stance, as we plead in favour of the de-annuitization of pensions, even PAYG public ones. But the underlying context is quite different. In Europe and for public pensions organised on a PAYG basis, full and mandatory annuitization is the (unquestioned) rule. Another difference with the US debate is that our starting point is not the risk of poverty at (very) old age but the risk of inequality inherent to full annuitization when the length of life varies a lot across individuals. The latter problem is getting more and more attention among economists, but the focus is only on differentiation of treatment based on expected longevity differences across socio-demographic groups (Ayuso et al., 2016) or occupations. The parameters of differentiation investigated in the literature comprise the retirement age, and also the replacement rate or the contribution rate during the pension build-up phase (Biskmarckian pensions), or simply the amount of the basic pension (Beveridgian pensions).

What we show in this paper is that partial de-annuitization of PAYG pensions would be as effective at addressing the inequalities and inefficiencies generated by longevity differences. If all (or most) longevity differences matter from a normative point of view[23], for both the Bismarckian and Beverigdian versions of public PAGY pensions, we show that a modest de-annuitization – 4 to 5 % point reduction of the annuity – would be enough to match the equality gains recorded via extensive (but costly to implement and prone to misreporting or moral hazard) retirement age differentiation.[24] These small numbers also support the idea that partial de-annuitization might not compromise the longevity insurance role of public pensions; the very one that pushes our US colleagues to recommend more annuitization.

Acknowledgement

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8 Appendix

8.1 Budget-neutral retirement age differentiation

Hereby, we identify the conditions ensuring that the sums spent on a cohort are the same with retirement age differentiation and uniform retirement age. We also report the equity gains achieved by retirement age differentiation when that budget equivalence condition is imposed.

[23] Because they are unrelated to risky lifestyles.

[24] De-annuitization amounts to paying a lump sum at a certain age to everyone. As is well documented in public economics, these types of payments are exempt from risk of misreporting or disincentive to perform (here earn a lower wage so that one gets classified as a short-lived persons entitled to early retirement).

We show that, in particular for Bismarckian pensions, equity gains are lower and potentially negative. This is because of the interplay between `price` vs `years in retirement` effects, with the former eroding the gains derived from the latter. Retirement age differentiation can generate savings, essentially because less is spent on top earners. If those savings are used to increase b (Beveridge) or θ (Bismarck) to balance a cohort's budget then each (smaller) retirement age gap (the building clocks of our equity index) is `priced` at a higher rate. Note that the key take-home message remains : retirement age differentiation's capacity to reduce pension-related lifetime inequalities is limited and is dominated by partial de-annuitization. Remember that for the latter, we systematically integrate the condition of budget equivalence/neutrality.

Retirement age differentiation potentially changes the sums spent on a cohort. To ensure strict budgetary equivalence, the Beveridgian planner should use an annuity b^{rad} such that

$$\begin{aligned} b^{rad}\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra}_j)(l_{ij}-\boldsymbol{ra}_j) &= b\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{i,j}-\boldsymbol{ra}) \\ \frac{b^{rad}}{b} &= \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{i,j}-\boldsymbol{ra})}{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra}_j)(l_{i,j}-\boldsymbol{ra}_j)} \end{aligned} \tag{20}$$

The budget balancing b^{rad} is thus inversely proportional to the change (possibly reduction) of aggregate time spent in retirement due to introducing different retirement ages.

The Bismarkian planner's equivalent problem is a bit more complex. She needs the ratio of benefits to contribution to be equivalent to what it is under uniform retirement. Formally we need

$$\frac{BEN^{rad}}{CONT^{rad}} / \frac{BEN}{CONT} = 1$$

where

$$\begin{aligned}
BEN^{rad} &\equiv \delta^{rad} \sum_{j=1}^{k} \sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra}_j)(l_{i,j} - \boldsymbol{ra}_j)\, w_{i,j} \\
BEN &\equiv \delta \sum_{j=1}^{k} \sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra})\, w_{i,j} \\
CONT^{rad} &\equiv \eta^{rad} \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[S_{i,j}(\boldsymbol{ra}_j)\boldsymbol{ra}_j \;+ (1 - S_{i,j}(\boldsymbol{ra}_j))\, \boldsymbol{l_{i,j}} \right] w_{i,j} \\
CONT &\equiv \eta \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[S_{i,j}(\boldsymbol{ra})\boldsymbol{ra} + (1 - S_{i,j}(\boldsymbol{ra}))\, \boldsymbol{l_{i,j}} \right] w_{i,j}
\end{aligned} \tag{21}$$

or equivalently

$$\frac{\theta^{rad}}{\theta} = \frac{WYIR(\boldsymbol{ra})}{WYIR(\boldsymbol{ra}_j)} \frac{WYIE(\boldsymbol{ra}_j)}{WYIE(\boldsymbol{ra})}$$

where

$$\begin{aligned}
\theta^{rad} &= \frac{\delta^{rad}}{\eta^{rad}}; \theta = \frac{\delta}{\eta} \\
WYIR(\boldsymbol{ra}_j) &\equiv \sum_{j=1}^{k} \sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra}_j)(l_{i,j} - \boldsymbol{ra}_j)\, w_{i,j} \\
WYIR(\boldsymbol{ra}) &\equiv \sum_{j=1}^{k} \sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra})\, w_{i,j} \\
WYIE(\boldsymbol{ra}_j) &\equiv \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[S_{i,j}(\boldsymbol{ra}_j)\boldsymbol{ra}_j \;+ (1 - S_{i,j}(\boldsymbol{ra}_j))\boldsymbol{l}_{i,j} \right] w_{i,j} \\
WYIE(\boldsymbol{ra}) &\equiv \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[S_{i,j}(\boldsymbol{ra})\boldsymbol{ra} + (1 - S_{i,j}(\boldsymbol{ra}))\boldsymbol{l}_{i,j} \right] w_{i,j}
\end{aligned} \tag{22}$$

So θ^{rad} should be inversely proportional to the change of the (wage-weighted)[25] years spent in retirement ($WYIR$) and employment ($WYIE$). If retirement age differentiation

[25]In our simulations, we have assumed (average) by SES pension-relevant) wages with a gradient of 1 (lowest income percentile) to 4 (highest income percentile), and a .2 gender wage gap.

leads to fewer years spent in retirement[26] (fraction 1 to the rhs of equ. (22) is < 1) and simultaneously more years spent working (fraction 2 >1) then the planner can finance $\theta^{rad} > \theta$. This means that each years-in-retirement gap in equ.(6) can be priced at a higher rate, contributing to a lower level of equity gains. Using the Chetty data, with uniform retirement age of 65, we report our estimates of the likely equity gains of retirement age differentiation with and without the budget neutrality condition. Results suggest that budget neutrality reduces the equity performance of retirement age differentiation. It even completely disappears in the case of Bismarckian pensions.

Table 2: Numeric results: differentiated retirement ages and equity indices [with budget neutrality condition]

	Pension regime			
	Bismark		Beveridge	
	F	M	F	M
Equity Index	0.963		0.964	
Equity Index **with adjustment**	1.015		0.969	
Inc. perc.				
1	61.407	56.507	60.428	54.192
10	63.881	59.656	63.576	58.200
20	64.812	60.688	64.760	59.513
30	65.535	61.795	65.681	60.922
40	66.022	63.010	66.301	62.467
50	66.491	63.785	66.898	63.455
60	66.948	64.456	67.479	64.307
70	67.381	65.090	68.029	65.114
80	67.931	65.784	68.730	65.997
90	68.620	66.807	69.607	67.300
100	69.697	68.345	70.976	69.256
Ref. ret. age	65	65	65	65

8.2 Budget-neutral de-annuitization

Here, we identify the conditions for de-annuitization to generate overall benefits for a cohort matching what is spent under uniform retirement age.

We start with partial de-annuitization of Bismarckian pensions, where a uniform fraction ($\mu \equiv \frac{LS_{i,j}}{\boldsymbol{ra}\,\eta\, w_{i,j}}$) of lifetime contributions (i.e. the "guaranteed" part of the return) is

[26]We talk here about the `overall` number of years.

handed over to every retiree at the beginning of the retirement spell. For a cohort, budget equivalence/neutrality is achieved if the lump-sum payment $LS_{i,j}$ and the reduced annuity, calculated with a lower replacement rate δ', verify

$$\begin{aligned}
\sum_{j=1}^{k}\sum_{i=1}^{n_j} LS_{i,j} + \sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})\,\delta' w_{i,j} &= \sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})\,\delta w_i \\
\sum_{j=1}^{k}\sum_{i=1}^{n_j} \mu\,\boldsymbol{ra}\,\eta w_{i,j} + \sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})\,\delta' w_{i,j} &= \sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})\,\delta w_{i,j} \\
\mu\eta N\overline{w} &= (\delta-\delta')\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})\,w_{i,j} \\
\mu &= (\theta-\theta')\,\frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})\,w_{i,j}}{N\boldsymbol{ra}\overline{w}} \\
\mu &= (\theta-\theta')\,\frac{WYIR(\boldsymbol{ra})}{WYIE(\boldsymbol{ra})}
\end{aligned} \tag{23}$$

where $WYIR(\boldsymbol{ra})$ is the overall (wage-weighted) number of years spent in retirement and $WYIR(\boldsymbol{ra})$ is the overall (wage-weighted) number of years spent in employment.[27]

In a Beveridgian scheme, the uniform lump-sum payment LS and the reduced annuity b' must verify

$$\begin{aligned}
\sum_{j=1}^{k}\sum_{i=1}^{n_j} LS + \sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})\,b' &= \sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})\,b \\
LS\,N &= (b-b')\,\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra}) \\
LS &= (b-b')\,\frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})}{N} \\
LS &= (b-b')\,AYIR(\boldsymbol{ra})
\end{aligned} \tag{24}$$

where $AYIR(\boldsymbol{ra})$ is the average number of years spent in retirement

[27] In our simulations, we have assumed (average) by SES (pension-relevant) wages with a gradient of 1 (lowest income percentile) to 4 (highest income percentile), and a .2 gender wage gap.

8.3 De-annuitization and the indifferent pensioner

Here we characterize algebraically the `indifferent` retiree (Figure 6). This is the person who is (or should be) indifferent[28] between what he gets under partial de-annuitization and under uniform retirement. With a Bismarckian system, that person has longevity l^* such that

$$\frac{\theta(l^* - \boldsymbol{ra})}{\boldsymbol{ra}} = \mu + \theta' \frac{(l^* - \boldsymbol{ra})}{\boldsymbol{ra}}$$
$$\text{with, given the budget neutrality condition} \tag{25}$$
$$\mu = (\theta - \theta') \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})\, w_{i,j}}{N\boldsymbol{ra}\overline{w}}$$

After some simple algebraic transformations we get

$$l^* = \frac{\mu\, \boldsymbol{ra}}{(\theta - \theta')} + \boldsymbol{ra}$$
$$\text{with from the budget neutrality condition} \tag{26}$$
$$\frac{\mu\, \boldsymbol{ra}}{(\theta - \theta')} = \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})\, w_{i,j}}{N\overline{w}} = \text{constant}|\boldsymbol{ra}$$

In other words, l^* is independent of de-annuitization parameters μ and $\theta' < \theta$. So the intensity of de-annuitization has no impact on the longevity identifying the indifferent retiree. Note, however, that l^* is a function of (uniform) retirement age ($\boldsymbol{ra}$).

Similarly, with Beveridgian pension system, the indifferent pensioner has longevity l^* that verifies

$$b(l^* - \boldsymbol{ra}) = LS + b'(l^* - \boldsymbol{ra})$$
$$\text{with, given the budget neutrality condition} \tag{27}$$
$$LS = (b - b') \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})}{N}$$

[28] Assuming perfect foresightedness.

After some simple algebraic transformations we get

$$l^* = \frac{LS}{(b-b')} + \boldsymbol{ra}$$

with, given the budget neutrality condition (28)

$$\frac{LS}{(b-b')} = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})}{N} = \text{constant}|\boldsymbol{ra}$$

where, again, l^* turns out to be independent of de-annuitization parameters LF and $b' < b$. So the intensity of de-annuitization has no impact on the longevity identifying the indifferent retiree. But note again that l^* is a function of (uniform) retirement age ($\boldsymbol{ra}$).

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