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Wage Rigidities in a Quantitative Spatial Economy: Commuting and Local Unemployment

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Abstract

In this paper we build a quantitative spatial general equilibrium model to study the geographical variation in unemployment rates in the presence of wage rigidities and when workers are allowed to commute from residence to workplace. Calibrating the model on Belgian district data, we find that, were workers' location choice driven only by preferences for amenities, workers would relocate away from the center of the country, generating a less concentrated spatial distribution of economic activity. We also explore the role of unemployment insurance in determining the location choices of workers. We find that when the risk of unemployment is fully insured, workers relocate to districts with initially high unemployment rates, therefore accentuating the spatial misallocation of labor. Removing unemployment insurance would instead not generate significant changes in the spatial distribution of workers. To gauge the magnitude of wage distortions, we compare the observed gross wage levels with the counterfactual market-clearing wages. Removing wage rigidities would generate significant gains in local and total GDP (+3%) and modest gains in the average real net labor income per resident (+1%). Lastly, we determine the level of the employers' social contribution rate that would allow to achieve full employment in all districts. We find that the optimal social contribution rate should be 24%, 12 percentage points lower than the observed rate, while at the same time it would increase fiscal revenue by 1.5%.

JEL Classification: J3, J5, J61, R12, R23

Keywords: wage regulation, spatial equilibrium, labor mobility, commuting, local unemployment.

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1 Introduction

The economic effects of wage rigidities, such as the minimum wage, have been the subject of an extensive theoretical and empirical literature.[1] The neoclassical model of the labor market predicts that, when a binding minimum wage is introduced in the market, the level of employment would be constrained by the labor demand, thus generating unemployment. Within this framework, Boeri et al. (2021) have recently argued that the centralized wage bargaining system typical of many European countries, while successful in compressing nominal wage inequality within a country, could create costly distortions across cities and regions. According to the authors, in the presence of geographical differences in productivity, national wage equalization would lead to higher unemployment and real wage income in the least productive locations. Here we argue that the results of Boeri et al. (2021) crucially depend on the degree of the geographical mobility of workers.

In this paper we build a quantitative spatial equilibrium model to study the geographical variation in unemployment rates in the presence of wage rigidities and when workers are allowed to commute. Building on Monte et al. (2018), our model allows for an arbitrary number of locations characterized by heterogeneous local productivities and housing supplies. Workers are geographically mobile and have heterogeneous preferences for workplace and residence locations. In line with the work of Boeri et al. (2021), we introduce unemployment in the model by assuming exogenous wage rigidities, which induce workers to account for unemployment risk in their choice of where to live and work.

We calibrate the model using Belgium as a case study. Belgium presents interesting features such as a strict centralized wage regulation and a relatively small geographical size which allows for easy commuting between almost anywhere within the country. Nevertheless, since these are common characteristics of many European countries, the conclusions provided in this paper could similarly apply to other contexts.

In the first part of the paper, we document the institutional framework of Belgium and the spatial distribution of relevant economic variables such as unemployment, productivity, and income. We begin by describing the collective bargaining system of Belgium, which effectively restraints wage determination in many industries. This institutional aspect will be accounted for in the model under the form of exogenous lower bound constraints on wages. We stress here that our goal is not to uncover the causes of spatial unemployment differentials, but to study the quantitative effects of wage rigidities and commuting on the spatial variation in unemployment rates. Thus, while modelling an endogenous centralized wage bargaining system could be an important line of research,

[1]The impact of minimum wages on unemployment is a most debated topic among economists. The recent survey of the U.S. literature by Neumark and Shirley (2021) finds that, contrary to what is often argued: "There is a clear preponderance of negative estimates in the literature [...]. This evidence of negative employment effects is stronger for teens and young adults and more so for the less educated". On the other hand, Manning (2021), in his review of the literature, argues that the evidence is elusive and there are no clear positive nor negative effects of minimum wages on employment.

we will abstract from it in this paper.

We then proceed discussing how productivity, unemployment, labor income, and commuting are geographically distributed across the country at the district level. We show the existence of a wide North-South gap in productivity and unemployment, whereas various proxies of labor income tend to be concentrated around the center of the country. In particular, we emphasize the negative correlation between productivity and unemployment, which is to be expected in the presence of a strong wage compression mechanism determined by the collective bargaining system. Similarly, as wages cannot fully reflect local productivities, the correlation between nominal labor income and productivity is in general low. We find an even smaller but positive correlation between real labor income and productivity when we adjust for the local cost of living. This is in contrast with the findings of Boeri et al. (2021), who show a sizeable negative correlation between real wage income and productivity in Italy, also characterized by strong wage compression. We reconcile this discrepancy by looking at the large shares of workers commuting across districts. While in Boeri et al. (2021) the relatively low real wage income in more productive locations is explained by higher housing prices due to internal inflows of migrants, here we argue in favor of a high ease of commuting, which allows workers to work in locations relatively distant from their place of residence, hence leaving housing prices unaffected.

In the second part of the paper, we build and calibrate the model on Belgian data, which we then use to perform several quantitative exercises. In the calibration, we show how to recover the local unemployment rates by workplace given data on the commuting probabilities of workers and on the observed local unemployment rates by residence. In fact, in the model the unemployment rate is defined at the level of the local labor market as the share of local labor supply that is not employed in that market, however, in the data unemployed workers are counted at their place of residence. We show that in our model the residential unemployment rate can be expressed as a weighted average of the unemployment rate by workplace, where the weights are determined by the commuting probabilities to workplace conditional on residence. Thus, given the data, the unemployment rate by workplace can be recovered by simply solving a system of linear equations.

In the quantitative section of the paper, we start by studying the importance of commuting in shaping the spatial distribution of local unemployment rates. We investigate the role of the ease of commuting, which incorporates preferences for amenities and commuting costs, and the role of the expected net labor income, which accounts for unemployment risk and insurance.

We show analytically that in this model the heterogeneity in the ease of commuting is a necessary condition to generate geographical variation in the unemployment rate by residence. Next, we shut down the effect of expected net labor income to investigate the role of the ease of commuting in determining workers' location choices. We find that, were commuting flows only determined by the ease of commuting, most of workers would relocate away from to the center towards the districts nearby the borders of the country. As a consequence of the wage rigidities, this counterfactual spatial reallocation of labor would determine a massive increase in residential unemployment rates in the more remote locations due to the increase in their local labor supply. This counterfactual scenario generates large differences from what is observed in the data and highlights the importance of expected labor income in the location choice of workers, especially at the center of Belgium where higher wages are usually observed.

We turn next to quantifying the amount of wage distortions in the economy, by removing the wage rigidities and by calculating the counterfactual level of gross wages that would allow each local labor market to clear. In this scenario, the market-clearing wage would be on average 2% lower than the observed wage level, generating an expansion in GDP by 3%. However, although nominal wages are adjusted downward, we find modest effects on real net labor income per resident, which in some districts would increase by around 1%. This effect is almost entirely attributable to the increase in employment, whereas housing prices would barely change.

At last we perform a policy experiment in which we ask what would be the optimal employers' social contribution rate that would allow to achieve full employment in every local labor market. We find that the optimal social contribution rate would be 12 percentage points below the observed rate. Nevertheless, the reduction in the social contribution rate would actually increase fiscal revenue by 1.4%, as the decrease in social contributions would be overcompensated by the decrease in unemployment benefits payments. Similarly, a more moderate decrease in the social contribution rate by 7 percentage points would optimize fiscal revenue which would increase by 1.9%, while at the same time reducing the aggregate unemployment rate to 1.5%.

The most closely related papers to ours are Boeri et al. (2021) and Monte et al. (2018). Boeri et al. (2021) study the effect of collective wage bargaining by providing a theoretical two-region spatial equilibrium model with a binding minimum wage constraint in the least productive of the two regions. The authors provide interesting stylized facts in support of the predictions of their model. In particular, under wage rigidities, less productive locations exhibit higher non-employment rates, but lower housing prices and higher real wage income in comparison to more productive locations. The main mechanism generating these patterns is the residential location choice of workers. As workers migrate towards more productive locations with low non-employment risk, housing prices will increase, nominal wage income will decrease, and, as a consequence, real income will be negatively affected. On the other hand, housing prices in less productive locations will decrease, however, the binding minimum wage will prevent the nominal wage income to fall, thus increasing real wage income. Boeri et al. (2021) conclude that collective wage bargaining, while inducing nominal wage equalization, generates an inefficient allocation of labor and tends to increase real wage income inequality across locations. Importantly, their work omits several features that may dampen the quantitative implications of their model. In their theoretical model, workers are perfectly mobile, have no preferences for location, live and work in the same place, and unemployment insurance is absent. In comparison, in this paper we emphasize the importance of commuting and unemployment benefits as insurance against unemployment risk, therefore mitigating the necessity of residential relocation. By including these additional insurance channels, fewer workers would relocate to the more productive locations, and as a result, housing prices would not be subject to upward pressure. Hence, real income inequality may be quantitatively lower than what their model suggests. Overall, our results point towards a milder effect of the removal of wage rigidities on GDP, nominal wages, and real income, compared to those found in Boeri et al. (2021).

While we share the same research interest as Boeri et al. (2021), our approach follows that of Monte et al. (2018), who study the heterogeneous effects of local productivity shocks on local employment in a quantitative spatial equilibrium model. In their framework, heterogeneity in local employment elasticities arises from the general equilibrium effects induced by the commuting choices of workers. In this paper we extend their quantitative model by including unemployment under the assumption of the existence of exogenous wage rigidities.

More broadly, our paper is related to several literatures. First, our research is similar in spirit to the papers in the recently emerging literature in quantitative spatial economics.[2] For example, Caliendo et al. (2017) calibrate a full-fledged quantitative spatial general equilibrium model of the US states to study the propagation of location- and industry-specific productivity shocks to other parts of the country and their aggregate effects. Fajgelbaum et al. (2018) build a spatial general equilibrium featuring a detailed characterization of the US tax system, finding sizeable gains from harmonizing taxation across states. On a smaller scale, Ahlfeldt et al. (2015) and Heblich et al. (2020) study the quantitative effects of agglomeration forces on the organization of economic activity within a city. Our paper share the same methodological approach of this literature, however it introduces inefficiencies in the labor markets, albeit in a simplified way.

Second, our research relates to the literature on spatial misallocation.[3] For example, Hsieh and Moretti (2019) quantify the aggregate costs of spatial labor misallocation across US cities induced by restrictions on housing supply. The authors find that cities with high productivity have adopted stricter restrictions on the supply of new housing unites, which limit the number of workers having access to such highly productive workplaces.

[2]See Redding and Rossi-Hansberg, 2017 for a review.

[3]See Restuccia and Rogerson (2017) and the references therein for a survey of the literature on misallocation.

According to the authors, removing these constraints would allow workers to relocate, thus increasing the aggregate US production by about 36%. Yet, if commuting allowed workers to access these highly productive locations from relatively long distances, relaxing the constraint on housing would not generate such large efficiency gains. To our knowledge, Boeri et al. (2021) is the only paper addressing the effects of wage rigidities on spatial misallocation, thus our paper constitutes an early contribution to this literature along this dimension.

Other recent papers have studied unemployment in local labor markets using a job search approach. Manning and Petrongolo (2017) estimate a spatial job search model and provide evidence of the reduced geographical extent of labor markets in the UK, as the attractiveness of jobs to applicants sharply decays with distance. As job applicants search in multiple locations around their place of residence, local labor markets tend to have a strong overlapping structure, which cannot be captured by predefined administrative units. Marinescu and Rathelot (2018) adopt a similar approach to study the contribution of the geographic mismatch between job seekers and vacancies on aggregate unemployment in the US. They find that job seekers are relatively close to vacancies on average, hence an efficient geographical reallocation of labor would only have a modest effect on aggregate unemployment. Finally, Bilal (2021) studies the spatial unemployment differentials across cities in France focusing on the location choice of employers. He finds that most of the variation in unemployment rates is mostly explained by variation in the job destruction rates, which, in his theoretical model, is rationalized by the sorting of more productive employers into locations with few vacancies per job seeker to fill vacancies more rapidly. Compared to this literature, our approach includes the residential location choice of workers, which allows to fully capture the general equilibrium effects generated in the housing market. In addition, the modularity of our theoretical framework allows for a practical and straight-forward calibration procedure, which, in turn, makes the model easily extendable to accommodate more features as more data become available.

The remainder of the paper is structured as follows. Section 2 provides an overview of the institutional framework of Belgium and the working of its centralized wage bargaining system. Section 3 documents the spatial heterogeneity of the Belgian economy. Section 4 develops the model. Section 5 discusses how to take the model to the data and the relevant data sources. Section 6 presents the results of the quantitative analysis. Section 7 concludes.

2 Wage Regulation in Belgium

In this section we provide an overview of the institutional framework of Belgium as well as a description of the wage setting mechanisms currently in place. In particular, we explain how wages are determined via a multi-tiered collective bargaining system and discuss the constraints arising from wage regulation.

Belgium is a federal state organized in three regions, Brussels-Capital, Flanders, and Wallonia, and three communities, Flemish, French, and German. Administrative power is distributed among the federal government, the regions, and the communities. The federal government has jurisdiction over subjects such as justice, defense, social security, and public finance. Regions retain territorial competence over economic and employment policies. Communities administer social, cultural, and educational activities. Among other differences, the regions adopt official languages according to the community they host: Dutch in Flanders, French in Wallonia, and both French and Dutch in Brussels.[4]

Wages are strictly regulated according to an institutionalized bargaining system, encompassing a large number of collective agreements. By royal decree, these collective agreements cover all workers, independently of the unionization of the worker (Plasman, 2015). As a consequence, even if only about 50% of all workers are unionized, 96% of them have the right to collectively bargain.[5]

Wage bargaining is organized on three tiers. At the first tier, the National Labour Council (NLC), an institution equally composed of representatives of employers and employees, determines the minimum wage legally binding at the national level. As any agreement reached by the NLC, the national minimum wage applies to all workers of all sectors.[6] On the other hand, the Central Economic Council (CEC) determines the minimum and maximum allowed growth rate of gross wages (i.e., the *wage norm*) compatible with the stability and the competitiveness of the economy.[7] While not legally binding, the *wage norm* is closely followed in lower tier negotiations (Plasman, 2015), effectively imposing a strict wage compression mechanism.

At the second tier, wages are bargained at the sectoral level in Joint Committees (*Commissions Paritaires*), freely formed entities constituted of both employers and employees representatives in equal share.[8] There are about one hundred active Joint Committees and sub-committees in existence, each representing a sector or a branch of activity

[4]The German-speaking community comprises few municipalities located in the Liège province in Wallonia.

[5]From the OECD database. Trade union density: https://stats.oecd.org/Index.aspx?DataSetCode=TUD. Collective bargaining coverage: https://stats.oecd.org/Index.aspx?DataSetCode=CBC.

[6]The public sector and apprenticeships are excluded from the national statutory minimum wages and are covered by specific agreements.

[7]The upper and lower bounds are based on the wage growth rate of the three main trade partners of of Belgium, that is, Germany, the Netherlands, and France.

[8]Bargaining at this level only concerns the private sector.

(Vandekerckhove et al., 2020).[9] At this stage of the bargaining process, the Joint Committees determine the minimum wage in their respective sectors, which can only exceed the national minimum wage.

Finally, at the third tier, wages are negotiated at the firm level. While the bargained wage cannot be set below the national and sectoral minima, firms can always offer higher wages.[10] In practice, a relatively large fraction of employees is subject to the sectoral minimum wage, while the national minimum wage is seldom binding. For example, in a recent analysis of the minimum wage across Belgian industries, Vandekerckhove et al. (2020) find that in 2015 only about 3% of the workforce was paid at the national minimum wage, whilst more than 10% of employees were employed at the sectoral minimum wage.[11]

Overall, despite differences in local conditions such as productivity and infrastructures, firms across different locations face the same wage rigidities. In theory, the system does allow for some wage adjustment at a decentralized level. However, in practice, decentralized bargaining is limited because of the restrictions on wage growth. The strict regulatory constraints imposed by the bargaining system clearly constitute important frictions for the local economic environment.

3 Spatial heterogeneity of the Belgian economy

In this section we document the spatial heterogeneity of the Belgian economy. First, we provide evidence about the wide productivity and unemployment dispersion across the 43 Belgian districts, highlighting the large gap between Flanders and Wallonia. Second, we explore the geographical dispersion of various measures of labor income, discussing how commuting, housing prices, and unemployment insurance can attenuate the dispersion in real income. Third, we study the spatial distribution of workers and their commuting flows across districts, emphasizing the high mobility of labor. Finally, we conclude the section by comparing the observed patterns in the data with the theoretical predictions advanced in Boeri et al. (2021).

[9] The full list can be found at https://emploi.belgique.be/fr/themes/commissions-paritaires-et-conventions-collectives-de-travail-cct/commissions-paritaires-1.

[10] Concerning sectoral minimum wages, a principle of no derogation holds, hence there are no exceptions to the rule (Vandekerckhove et al., 2020).

[11] The analysis of Vandekerckhove et al. (2020) includes the 43 largest Joint Committees in terms of employment. The national minimum gross wage was 1,577 euros per month in 2015, about 18% below the average sectoral minimum wages.

3.1 Local Productivity and Unemployment

We begin this section by discussing the geographical variation in productivity and unemployment presented in Figure 1. As a measure of local productivity we use Total Factor Productivity (TFP), calibrated for each district as the residual of a local production function (the exact definitions of the variables, data sources, and calibration are explained in more details in Sections 4 and 5). Panel 1a shows the percentage deviation of local TFP from the cross-district mean in 2011.[12]

(a) Total Factor Productivity

(b) Unemployment Rate

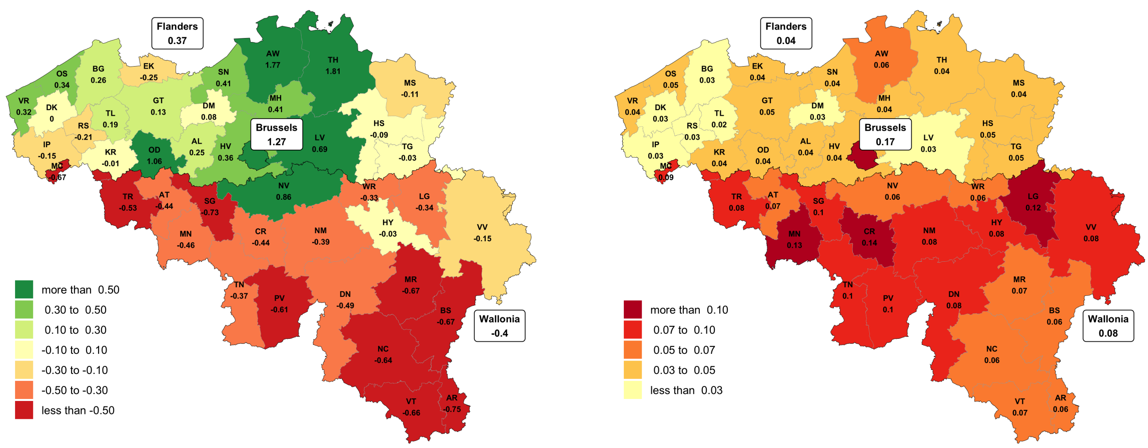


Figure 1: Local Productivity and Unemployment

Note: Panel (a): Total factor productivity, in percentage deviation from the cross-district mean. Total factor productivity has been calibrated for each district as the residual of a local production function (see Section 5 for details). Panel (b): local residential unemployment rate, obtained from ONEM. Regional values for Brussels, Flanders, and Wallonia are calculated as the simple average across all districts belonging to that region. Values are rounded to the nearest hundredth. All data pertain to the year 2011.

As the figure makes clear, there is a strong differential in local productivities between Flanders in the North and Wallonia in the South. On average, districts located in Flanders have 37% higher productivity than the mean of all districts, while in Wallonia the districts are 40% less productive than the average.[13]

Panel 1b displays a similar North-South gap in residential unemployment rates. We notice first that, while in Flanders the unemployment rate of the average districts is about 4%, in Wallonia the average unemployment rate is twice as high, at about 8%. Second, the variability of unemployment rates in Flanders is more limited, with rates ranging between 2% and 6%, whereas Wallonia exhibits a much wider dispersion, from a minimum unemployment rate of 6% and, in some districts, reaching levels of unemployment well above 10%, up to a maximum rate of 14% in Charleroi (CR).

[12] Throughout this paper, all presented data will refer to the year 2011.

[13] Differences in productivity could arise from various reasons. We abstract from this in this paper and we will consider productivities as given in the rest of the paper.

In general, from Figure 1 local productivity and unemployment appear to be negatively correlated. In this paper, we argue that wage rigidities, possibly due to the centralized wage bargaining system of Belgium described in Section 2 could be partially responsible for this observed geographical divide between North and South. The basic theoretical argument relies on the fact that, in the presence of heterogeneous labor demands, wage rigidities will generate high unemployment rates in locations where the productivity is lower. Clearly, beyond the centralized wage bargaining system, other important factors contribute to generating local unemployment, namely, the general educational level of the local workforce and the mismatch between the demand and supply of job-specific skills.[14] Nevertheless, although we acknowledge the importance of these factors, we will not address these issues in this paper.[15]

3.2 Nominal and Real Labor Income

Next, we study to which extent the North-South gap is reflected in various measures of labor income. Figure 2 shows four different measures of labor income in percentage deviation from the national cross-district mean.

Panel 2a shows the average gross wage paid to an employee working in a specific district.[16] In general, gross wages are higher in Flanders (+5%) than in Wallonia (-4%), whereas districts around Brussels generates a cluster of high paying locations, with Brussels itself being the highest paying workplace location in Belgium (+35%). The pattern of wage dispersion is similar to that of productivity shown in Figure 1a. Moreover, we notice a core-periphery structure of wage levels in Belgium that is also evident in the other three panels of Figure 2, which present measures of labor income by district of residence.

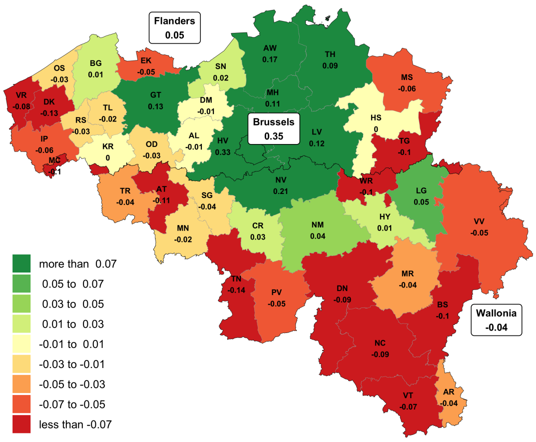
Panel 2b shows the average nominal wage income per employed resident. The difference between the two top panels is entirely attributable to cross-district commuting. Indeed, from Panel 2a to panel 2b we only change the definition of where the gross wage is measured: for a given district, in Panel 2a the gross wage earned by workers is measured at the workplace level, i.e., considering all workers working in that particular district independently of their residence, whereas in Panel 2b the gross wage income is the wage earned by all employed residents living in that district independently of where they work. Were workers working and living in the same district, there would be no difference in the two measures. To highlight the importance of commuting in mitigating the wage income gradient, consider, for example, the Walloon district of Soignies (SG), located southwest

[14] See Zimmer (2012) for some evidence on the skill mismatch in Belgium.

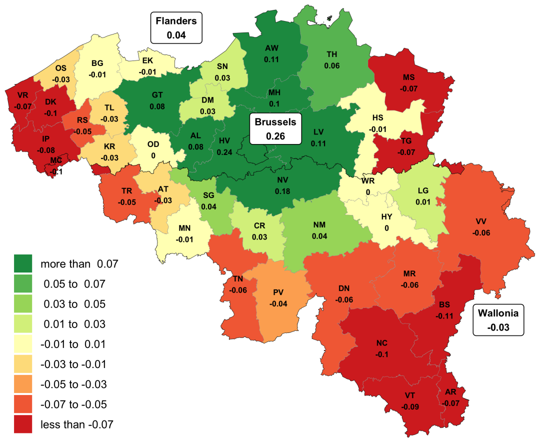
[15] In the model workers are homogeneous in terms individual productivity and skill levels. Although the model could be easily extended to the case with skill heterogeneity, we abstain from this mostly due to data limitations.

[16] The average gross wage is calculated from the National Accounts data as the ratio between the total compensation of employees and the number of employees.

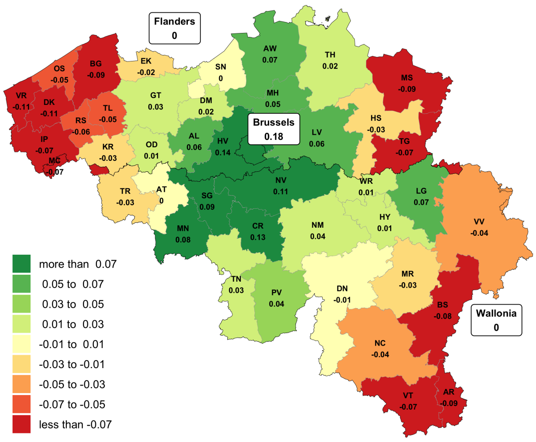
(a) Gross Wage



(b) Nominal Gross Wage Income



(c) Real Gross Wage Income



(d) Real Net Labor Income

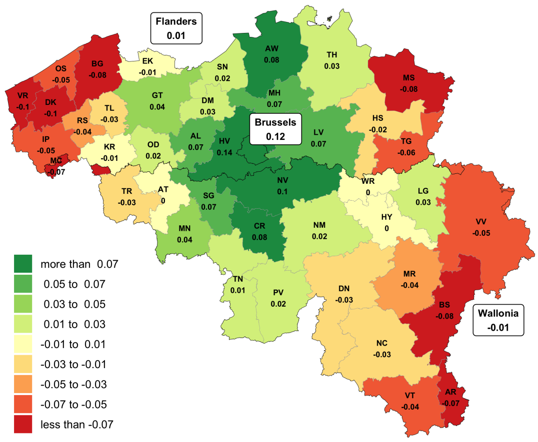


Figure 2: Labor Income

Note: Panel (a): Average paid gross wage per employee. Panel (b): Average nominal gross wage income earned by each employed resident worker. Panel (c): Average real wage income earned by each employed resident worker. Panel (d): Average real net labor income earned by each resident worker, employed or unemployed. Real net labor income includes wage income earned by employed resident workers and unemployment benefits received by unemployed resident workers. Real variables are obtained by adjusting for local housing prices. All variables are in percentage deviation from the cross-district mean. Regional values for Brussels, Flanders, and Wallonia are calculated as the simple average across all districts belonging to that region. Values are rounded to the nearest hundredth. Data have been elaborated by the authors (see Section 5 for details). All data pertain to the year 2011.

of Brussels on the border with Flanders. Employees working in Soignies earn on average a gross wage 4% below the district-mean, however, its residents are better-off than in the average district, earning a gross wage 4% above the mean. Overall, the ease of commuting, by redistributing wage income from highly productive locations to less productive ones, is a first important force of income equalization across space.

Housing prices constitute a second mechanism for reducing spatial dispersion in real incomes. Panel 2c shows the average real gross wage income per employed resident. This income measure is calculated by deflating nominal gross wage income by a local index of the cost of living, constructed from housing price data.[17] The difference with Panel 2b is noticeable: after the adjustment the gap between Flanders and Wallonia in average gross wage income has disappeared, albeit the cluster of high income districts around Brussels remains. As shown in the Appendix, the cost of living in Flanders is higher in the well-off districts around Brussels and Antwerp (AW) and in the districts located on the North Sea coast, Veurne (VR), Oostende (OS), and Brugge (BG). On the contrary, housing prices in Wallonia are relatively low, especially in the high-unemployment districts of Mons (MN), Charleroi (CR), and Liège (LG), and in the least densely populated southernmost districts. As an illustration of the effect of local housing prices, consider the differences in nominal and real gross wage income between Antwerp (AW) and Liège (LG), respectively the most populous districts of Flanders and Wallonia. Workers living in Antwerp enjoy a nominal gross wage income 12% higher than that received by Liège residents, however, after adjusting for the local cost of living, the income difference vanishes almost completely.

Finally, Panel 2d shows a broader measure of real net labor income per resident worker, which adjusts for labor taxes, includes unemployment benefits payments, and covers both employed or unemployed resident workers.[18] Relative to Panel 2c, Flanders and Wallonia exhibit slight deviations in the average real net labor income, mostly due to the wide unemployment gap discussed before. Nevertheless, the overall spatial dispersion in incomes is reduced. For example, in the Walloon districts of Mons (MN), Charleroi (CR), and Liège (LG), although low local housing prices lead to a relatively higher real gross wage income per employed resident, the real net labor income is significantly lower due to the very high fraction of unemployed residents.

Overall, all the maps of labor income presented in Figure 2 display a strong geographical cluster at the center of the country.[19] To better understand this core-periphery structure, we look next at several indicators about commuting.[20]

[17] Using the notation of the model presented in more detail in Section 4, the general local price index for residential location i is calculated as q_i^β, where q_i is the local measure of housing prices and β is the housing expenditure share. In this paper we assume that consumption goods have the same price across locations, for example if all locations produce the same good and inter-district trade costs are negligible, hence the local cost of living is mostly determined by housing prices.

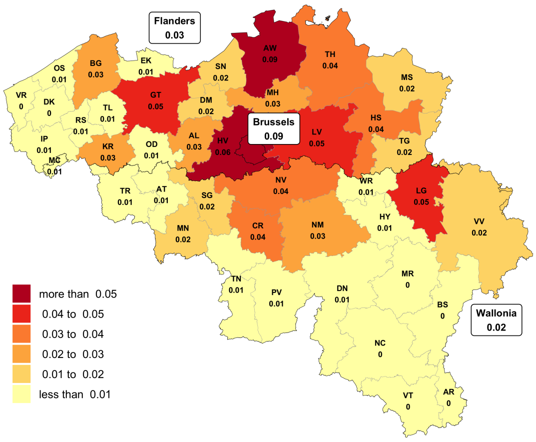
[18] Notice that the difference between Panel 2c and Panel 2d does not reflect labor taxation. Our measure of real net labor income is derived from the model, in which the labor income tax rate is proportional both to the gross wage and unemployment benefits, therefore, the effect of labor taxation cancels out when looking at the percentage mean deviations. Thus, the difference is attributable entirely to the inclusion of the unemployed in the measure.

[19] The spatial variation we observe could be attributed to spatial heterogeneity in jobs and workers. In fact Plasman et al. (2007) show that spatial differences in wages persist even after controlling for the local composition of firms and workers. We do not investigate this issue further but their results may partially alleviate some of our concerns.

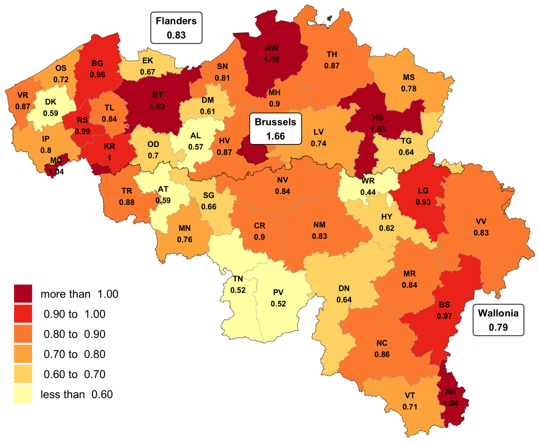
[20] See also Riguelle et al. (2007) for a more detailed investigation of the spatial structure of four urban regions in Belgium.

3.3 Commuting Flows

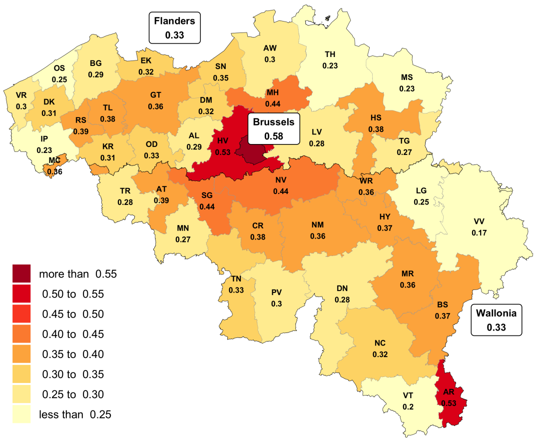
(a) Spatial Distribution of Residents



(b) Employment-to-Residents Ratio



(c) Share of Inflowing Commuters



(d) Share of Outflowing Commuters

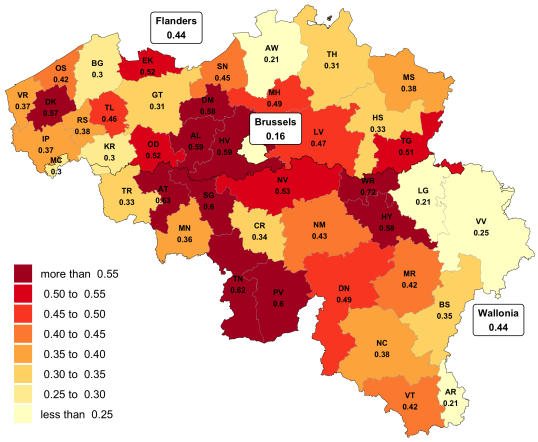


Figure 3: Labor Mobility

Note: Panel (a): Spatial distribution of workers (employed or unemployed) by district of residence. Panel (b): Ratio of local employees to local residents. Panel (c): Share of workers commuting from a different district from that of workplace. Panel (d): Share of residents commuting to a different district from that of residence. Commuters are either employed or unemployed (see model's definition in Section 4). Regional values for Brussels, Flanders, and Wallonia are calculated as the simple average across all districts belonging to that region. Values are rounded to the nearest hundredth. Data have been elaborated by the authors (see Section 5 for details). All data pertain to the year 2011.

In this subsection, we highlight the pervasiveness of commuting across Belgian districts. Throughout this subsection, our definition of commuting workers is based on a calibrated commuting matrix of the labor force. Commuting workers encompass both employed and unemployed workers, where the commuting unemployed workers can be interpreted as the number of job-seekers looking for a job in a specific district.[21] A full description of the methodology is provided later in Section 5.2.4.

[21] The commuting matrix of the labor force can be reconstructed from a combination of data on the commuting matrix of employed individuals and residential unemployment rates.

We start by examining the spatial distribution of resident workers in Figure 3a. A large fraction of residents is located in Brussels and Antwerp (AW), each accounting for 9% of the total population of workers. The rest of the population resides in the neighboring districts in Flanders, and in few other large cities such as Ghent (GT) and Liège (LG), which account for 5% of the population each. Instead, very few people live in the more remote districts of Belgium, notably on the North Sea coast and in the Ardennes in the South of Wallonia.

To understand the incidence of commuting, Panel 3b displays the ratio of local employment to the local resident population of workers, a measure defining whether the district is a net importer or exporter of workers. The map shows that the majority of districts is net exporter of workers, with only 6 out of 43 districts having an employment-to-residents ratio higher than 1.

More details about workers mobility are offered in Panels 3c and 3d, showing respectively the share of local workers commuting from a different district of residence, and the share of local residents commuting outside the district of residence. The share of inflowing commuters is similar between Flanders and Wallonia, averaging at about 33% across the districts within the respective regions. The share of inflowing commuters is comparable to the employment-to-residents ratio insofar they both provide information about local imports of workers, however, there is less variability in the share of inflowing commuters and only few locations exhibit a high fraction of outside workers. Clearly, for locations such as Brussels and Arlon (AR), where the local workforce is more than 60% of the local resident population, the share of workers commuting from outside is high and exceeds 50%. However, in other districts net importer of workers, such as Antwerp (AW) and Ghent (GT), the share of inflowing commuters is modest at about 30%, suggesting that a higher fraction of local residents choose to not commute elsewhere. This is evident in Panel 3d, showing relatively low shares of outflowing residents of 22% in Antwerp (AW) and 31% in Ghent (GT), whereas the average share is 43% in both Flanders and Wallonia. [22] In the Appendix we show that attractiveness of large workplace districts is even more evident when looking at the share of local residents commuting to Brussels.

3.4 Correlations, Dispersions, and Theoretical Predictions

In Section 2 we described the centralized wage bargaining system operating in Belgium, while throughout this Section we pointed out the differences across districts and regions along a variety of dimensions. As argued in Boeri et al. (2021), wage rigidities can generate important distortions in a highly heterogeneous spatial economy. We now combine the

[22] In particular, the closer the residence district to a larger district, the higher the share. For example, the high shares of commuting residents in the small districts of Waremme (WR) and Huy (HY) in Wallonia are mostly due to workers commuting to the nearby district of Liège (LG). A similar pattern occurs for the districts of Thuin (TN) and Philippeville (PV), exporting mostly to Charleroi (CR).

evidence accumulated so far and discuss to which extent the correlations and the relative dispersion observed in the data are in line with the predictions of Boeri et al. (2021) outlined in the Introduction.

For readability purposes, we anticipate here the model's notation for the variables mapped above: total factor productivity is denoted as A (Figure 1a); the residential unemployment rate is u^R (Figure 1b); q is a measure of average housing prices; w and y^E are, respectively, the gross wage paid at the workplace (Figure 2a) and the average nominal wage income per employed residents (Figure 2b); y^L denotes the average nominal net labor income, after accounting for unemployment benefits and labor taxes, per resident worker, that is, including both employed and unemployed residents; y^E/q^β and y^L/q^β, instead, adjust for the local cost of living q^β, where β is the housing expenditure share, and correspond to Figures 2c and 2d respectively.

The correlations between these variables are presented in Panel A of Table 1, whereas Panel B shows the ratios of their standard deviations. For better comparability, the values in Table 1 refer to the variables in percentage deviation from their respective means.

Table 1: Correlation and Relative Dispersion

Panel A: Correlation

(a) Correlation, All Districts

	A	u^R	q	w	y^E	y^E/q^β	y^L/q^β
A	1.00						
u^R	-0.13	1.00					
q	0.66	-0.39	1.00				
w	0.63	0.21	0.64	1.00			
y^E	0.63	0.30	0.59	0.93	1.00		
y^E/q^β	0.34	0.65	0.07	0.73	0.84	1.00	
y^L/q^β	0.44	0.44	0.20	0.78	0.89	0.97	1.00

(b) Correlation, Brussels omitted

	A	u^R	q	w	y^E	y^E/q^β	y^L/q^β
A	1.00						
u^R	-0.31	1.00					
q	0.63	-0.58	1.00				
w	0.58	0.01	0.60	1.00			
y^E	0.58	0.12	0.55	0.91	1.00		
y^E/q^β	0.25	0.58	-0.04	0.67	0.81	1.00	
y^L/q^β	0.38	0.36	0.12	0.75	0.89	0.97	1.00

Panel B: Relative Dispersion

(a) Relative Dispersion, All Districts

	A	u^R	q	w	y^E	y^E/q^β	y^L/q^β
A	1.00						
u^R	0.74	1.00					
q	0.29	0.40	1.00				
w	0.17	0.23	0.59	1.00			
y^E	0.14	0.19	0.47	0.79	1.00		
y^E/q^β	0.11	0.15	0.38	0.64	0.81	1.00	
y^L/q^β	0.10	0.13	0.32	0.55	0.70	0.85	1.00

(b) Relative Dispersion, Brussels omitted

	A	u^R	q	w	y^E	y^E/q^β	y^L/q^β
A	1.00						
u^R	0.71	1.00					
q	0.30	0.42	1.00				
w	0.16	0.22	0.53	1.00			
y^E	0.13	0.18	0.43	0.80	1.00		
y^E/q^β	0.11	0.15	0.36	0.68	0.85	1.00	
y^L/q^β	0.10	0.13	0.32	0.60	0.75	0.88	1.00

Note: The variables included in the table are: A, total factor productivity; u^R, residential unemployment rate; q, housing price; w, gross wage paid at the workplace; y^E, average nominal wage income per employed resident; y^E/q^β, average real wage income per employed resident; y^L/q^β, average real net labor income (after unemployment benefits and labor taxes) per resident worker (employed or unemployed). β is the housing expenditure share. All variables are in percentage deviation from the mean. Values are rounded to the nearest hundredth. Data have been elaborated by the authors (see Section 5 for details). All data pertain to the year 2011.

We begin by looking at the relationship between productivity (A), unemployment (u^R), and gross wages (w). According to the predictions in Boeri et al. (2021), wage rigidities would result in productivity having high negative correlation with unemployment and low positive correlation with wages. If there are frictions preventing wages to adjust downward, locations with low productivity levels will tend to have higher unemployment rates and relatively higher wages. On the other hand, wages in locations with high productivity will not be constrained, resulting in a higher correlation with productivity and an absence of unemployment. While the signs of the correlations in Panel A are consistent with the theory, the magnitudes are not convincing. The correlation between productivity and the unemployment rate is only -0.13, while that with wages is 0.63. Acknowledging the peculiarities of Brussels, which exhibits both very high wages and very high unemployment, we provide in Panel A(b) a robustness check by omitting Brussels from the calculations.[23] Now the magnitudes improve in the direction of the theoretical predictions, with the correlation with unemployment increasing by 18 percentage points and the one with wages decreasing by 5 percentage points [24]

The relative standard deviations in Panel B supports the pattern highlighted by the correlations.[25] The standard deviation of unemployment (u^R) is 74% that of productivity (A), while, on the other hand, the relative dispersion of gross wages (w) is only 0.17. Even if wages are correlated with productivity, the spatial dispersion is much more limited. Indeed, after the discussion on wage regulation in Section 2, we would expect a modest wage dispersion across districts, notwithstanding the heterogeneity in local productivities. Unemployment, instead, varies almost as much as productivity. We argue therefore that, while commuting can dampen the correlation between unemployment and productivity, it has only a moderate effect in mitigating the geographical variability of unemployment. In other words, although workers are free to relocate away from a slack labor market, this would not change the fact that there is a high excess supply of labor in the market.

Next, we look at the relationship between productivity and the various measures of labor income. The correlation between total factor productivity (A) and average nominal gross wage income per employed resident (y^E) is identical to that with gross wages

[23]Brussels is an influential outlier. It hosts important bureaucratic institutions such as the headquarters of the European Commission, and, therefore, it is subject to high upward wage pressure. At the same time, a high fraction of the local resident population is composed by low skilled and foreign individuals, who are more likely to be unemployed. Although we acknowledge these important characteristics, we do not control for them in our analysis.

[24]Here we point out that, while gross wages (w) and productivity (A) are both observed at the workplace location, unemployment rates measure the share of unemployed local residents. Contrarily, the strong negative correlation predicted by the theory should be observable between the unemployment rate arising *in the labor market*, which is not necessarily located where workers reside. To measure the unemployment rate by workplace, one would need to observe the labor supply in that local labor market, that is, the number of workers willing to work at the prevailing wage in the market and irrespective of their residential location. Although the measure of employed workers commuting to their workplace is observable, the local labor supply is clearly unobservable.

[25]The relative standard deviations are in general robust to the omission of Brussels.

(w). Indeed, the correlation between nominal wage income and gross wages is almost perfect, 0.93 including all districts and 0.91 omitting Brussels. However, the relative dispersion of nominal wage income is only 80% of that of gross wages. The only difference between gross wages and nominal wage income is where these variables are measured. Gross wages measure the average wage paid to employees working in a specific workplace location, whereas nominal wage income measures the average gross wage income earned by employed local residents independently of where they work. We can then attribute the difference in dispersion between the two measures entirely to the workplace location choice of workers. Therefore, commuting acts as a strong redistribution mechanism across productive locations.

By looking at the differences between nominal (y^E) and real wage income (y^E/q^β), we notice a similar redistributive effect exerted by housing prices (q). The correlation between productivity and the average real wage income per employed resident is only half that with nominal wage income, whilst the geographical dispersion of real income is slightly above 80% that of nominal wage income. Since housing prices are strongly positively correlated with productivity, the advantage of living in highly paying workplace locations is noticeably reduced, resulting in a more equally distributed real wage income across districts.

In Boeri et al. (2021) the strong positive correlation between productivity and housing prices arises from the migratory flows to the most productive locations, as workers look for workplaces with high wages and low unemployment risk. Moreover, despite the positive correlation, we would observe less dispersion in housing prices than in productivity, as workers would tend to move away from extremely expensive residential locations. Both predictions are indeed supported by the data, however, in the case of Belgium the migration mechanism is partially replaced by cross-district commuting. As workers can easily commute across districts, the benefits of fully relocating would be lower and, as a consequence, housing prices would be less affected through this channel.[26] In addition, mobility across regions is limited by natural language barriers. Although we are not aware of any study looking at interregional migration in Belgium, Persyn and Torfs (2015) estimate that a high fraction of the cost of commuting is attributable to linguistic differences. In particular, the authors show that commuting between Flanders and Wallonia is almost absent, while most of inter-regional commuting occurs with Brussels. To capture these important features of the Belgian economy, our model described below will incorporate both migration and commuting choices, while accounting for preference heterogeneity and mobility costs.

Finally, the correlation between real net labor income per resident worker (y^L/q^β),

[26]The two most geographically extreme points in Belgium, the northernmost locality of Meerle in the district of Antwerp (AW) and the southernmost locality of Torgny in the district of Virton (VT), are separated by about 290 km along the road network with a travel time of about 3 hours by car. *see Service Public Fédéral Belge*: `https://www.belgium.be/fr/la_belgique/connaitre_le_pays/geographie`.

which adjusts for unemployment benefits and proportional labor taxes, and the average real wage income per employed resident (y^E) is almost perfect and it is robust to the omission of Brussels.

Summing up this section, we underline the spatial heterogeneity of the Belgian economy. Overall, Belgium is characterized by a monocentric structure with a strong core located in Brussels. A wide gap between Flanders and Wallonia exists along multiple dimensions. Productivity is higher in the North, whereas unemployment is lower. Wages follow the geographical pattern of productivity, however, they are less dispersed, possibly because of the strict wage rigidities arising from labor market regulation. Similarly, the average real net labor income is equalized across districts, due to the combined effects of commuting, housing prices, and unemployment benefits. In the next section we build a model flexible enough to incorporate all these characteristics.

4 Model

In this section we build a quantitative spatial economic model along the line of Monte et al. (2018). We adapt their framework to include the possibility of unemployment by introducing exogenous wage rigidities in the local labor markets.

The economy consists of a set of locations $i, j \in \mathbb{N}$, where in general we denote residential locations with i and workplace locations with j. Each residential location i is endowed with a fixed supply of housing, H_i, while each workplace location j is endowed with a fixed supply of local productive structures, A_j. The economy is populated by L mobile workers, each of whom supplies a unit of labor, and N^K immobile rentiers, who own both housing units and local productive structures. Rentiers are distributed across residential locations according to the exogenous measure $\mu_i^K = N_i^K/N^K$, with $\mu_i^K \in (0, 1)$ and $\sum_{i\in\mathbb{N}} \mu_i^K = 1$, and where N_i^K is the count of rentiers living in location i.[27]

4.1 Workers

Workers are geographically mobile, have heterogeneous preferences for locations, and can either be employed or unemployed. Each worker chooses where to live and work, and the consumption of the final good and housing. Conditional on the choice of the workplace location, the worker's unemployment probability is determined by the unemployment rate, u_j, prevailing in the chosen local labor market. If employed, the worker receives the net wage, $w_j(1 - \tau_L)$, where w_j is the gross wage and τ_L is the labor income tax rate. If unemployed, the worker is partially compensated by unemployment benefits proportional to the lost net wage, $\rho w_j(1-\tau_L)$, where ρ is the replacement rate established by the social

[27] In practice, workers are identified by employees, whereas rentiers capture the remaining fraction of the adult population, that is, inactive individuals, self-employed workers, and pensioners.

security system.

Notice that, since workers are identical from the perspective of the local firm, all workers commuting to the same workplace location will be paid the same gross wage (w_j) and will face the same unemployment probability (u_j). As a consequence, the expected net labor income, y_j, for a worker commuting to workplace j is:

$$y_j = (1 - u_j)w_j(1 - \tau_L) + u_j \rho w_j(1 - \tau_L). \tag{1}$$

Given the expected net labor income, a risk-neutral worker ω chooses where to live and work by maximizing the expected indirect utility:

$$V_{ij\omega} = \frac{b_{ij\omega}}{\kappa_{ij}} \frac{y_j}{p^{1-\beta} q_i^{\beta}}, \tag{2}$$

where: p is the price of the final consumption good, which we choose as the numéraire ($p = 1$);[28] q_i is the housing rental price prevailing in residential location i; $b_{ij\omega}$ is an idiosyncratic amenities shock capturing all the omitted factors that could affect the worker's residence-workplace choice; $\kappa_{ij} \in [1, \infty)$ is an iceberg commuting cost; and β is the housing expenditure share. Following the practice in the quantitative spatial economics literature, we assume the idiosyncratic amenities shocks ($b_{ij\omega}$) are independently drawn from a Fréchet distribution:[29]

$$F_{ij}(b) = e^{-B_{ij} b^{-\epsilon}}, \quad B_{ij} > 0, \ \epsilon > 0, \tag{3}$$

where B_{ij} is the scale parameter controlling the average amenities shock, and ϵ is the shape parameter controlling the dispersion of the amenities shocks. In the Appendix we show that, given this distributional assumption, the probability of observing a worker living in location i and commuting to location j can be expressed as:[30]

$$\lambda_{ij} = \frac{B_{ij}\left(\kappa_{ij} q_i^{\beta}\right)^{-\epsilon} y_j^{\epsilon}}{\sum_{r,s \in \mathbb{N}} B_{rs}\left(\kappa_{rs} q_r^{\beta}\right)^{-\epsilon} y_s^{\epsilon}} = \frac{L_{ij}}{L}, \tag{4}$$

where L_{ij} is the corresponding count of commuting workers. Equation 4 defines a gravity

[28] Price equalization across residential locations can be obtained under the assumption frictionless trade across locations, since each firm produces the same final good. This is a reasonable assumption in the case of Belgium, given the relatively small geographical size of the country.

[29] For recent papers following this modelling strategy see Ahlfeldt et al. (2015), Monte et al. (2018), and Heblich et al. (2020). See Redding and Rossi-Hansberg (2017) for an overview of modelling choices in quantitative spatial economics.

[30] A brief sketch of the proof is provided here. Given the monotonic relationship with idiosyncratic preferences $b_{ij\omega}$, the expected indirect utility $V_{ij\omega}$ is a Fréchet random variable with scale parameter $\Psi_{ij} = B_{ij}\left(\kappa_{ij} q_i^{\beta}\right)^{-\epsilon} y_j^{\epsilon}$ and shape ϵ. The optimal residence-workplace choice yields the maximum expected indirect utility which is also Fréchet distributed with scale parameter $\Psi = \sum_{i,j \in \mathbb{N}} \Psi_{ij}$ and shape ϵ. Finally, the probability distribution of bilateral commutes between i and j is the probability that the paired locations yield the maximum expected indirect utility, that is, $\lambda_{ij} = \Pr\left(V_{ij\omega} \geq \max_{rs} V_{rs\omega}\right)$.

equation of commuting where the probability of commuting between i and j depends not only on the characteristics of residence i and workplace j ("bilateral resistance"), but also on the characteristics of all the other possible paired locations ("multilateral resistance"). The marginal distributions λ_i^R and λ_j^L determine respectively the residential population living in location i (L_i^R) and the labor supply of local labor market j (L_j):

$$\lambda_i^R = \sum_{j \in \mathbb{N}} \lambda_{ij} = \frac{L_i^R}{L}, \tag{5}$$

$$\lambda_j^L = \sum_{i \in \mathbb{N}} \lambda_{ij} = \frac{L_j}{L}. \tag{6}$$

For convenience, it is worth defining here also the conditional commuting probabilities given the location of residence:

$$\lambda_{ij|i} = \frac{\lambda_{ij}}{\lambda_i^R} = \frac{B_{ij}\Big(\kappa_{ij}\Big)^{-\epsilon} y_j^{\epsilon}}{\sum_{s \in \mathbb{N}} B_{is}\Big(\kappa_{rs}\Big)^{-\epsilon} y_s^{\epsilon}} = \frac{L_{ij}}{L_i^R}. \tag{7}$$

Finally, expected utility:

$$V = E\Big[\max_{ij} V_{ij\omega}\Big] = \Gamma\left(\frac{\epsilon - 1}{\epsilon}\right)\left[\sum_{r,s \in \mathbb{N}} B_{rs}\Big(\kappa_{rs} q_r^{\beta}\Big)^{-\epsilon} y_{rs}^{\epsilon}\right]^{\frac{1}{\epsilon}} \tag{8}$$

will be equalized across all pairs of residence and workplace locations, where $\Gamma(.)$ is the gamma function.

4.2 Production

The local representative firm produces the homogeneous final good, Y_j, by employing local productive structures, A_j, and workers, E_j, according to the following technology:

$$Y_j = A_j E_j^{\alpha_j}, \quad \alpha_j < 1, \tag{9}$$

where we allow the labor income share, α_j, to vary across locations. Profit maximization implies the following downward-sloping labor demand:

$$E_j = \left(\frac{\alpha_j A_j}{w_j(1 + \tau_{SC})}\right)^{\frac{1}{1-\alpha_j}}, \tag{10}$$

where τ_{SC} is the employers' social contribution rate. Notice that the assumption of decreasing returns to scale is crucial in this paper. Indeed, under constant return to scale the resulting labor demand would be perfectly elastic, ruling out unemployment by construction.[31] Under decreasing returns to scale, the local firm generates profits:

$$\Pi_j = (1-\alpha_j)A_j\left(\frac{\alpha_j A_j}{w_j(1+\tau_{SC})}\right)^{\frac{\alpha_j}{1-\alpha_j}}, \tag{11}$$

which are redistributed across the rentiers. We follow the approach of Caliendo et al. (2017) and assume that local rentiers own only a share $(1-\iota_i)$ of the local assets, from which they collect $(1-\iota_i)\Pi_i$. The residual profits are aggregated into a national portfolio owned by all rentiers in equal shares, which yields a per capita return equal to $\sum_{j\in\mathbb{N}}\iota_j\Pi_j/N^K$. Summing up the retained and redistributed net capital income, and accounting for capital income taxation at a rate τ^K, each residential location i receives a total net capital income equal to:

$$Y_i^{\Pi} = (1-\tau_K)\Big[(1-\iota_i)\Pi_i + \mu_i^K\sum_{j\in\mathbb{N}}\iota_j\Pi_j\Big], \tag{12}$$

where we recall $\mu_i^K = N_i^K/N^K$. Here, we point out that the distributed ownership structure will allow to rationalize the spatial distribution of disposable residential income observed in the data.

4.3 Local Labor Markets

We assume that each local labor market is subject to a workplace-specific wage constraint such that $w_j \geq \underline{w}_j$. Heterogeneity in the wage constraint is necessary in this class of models to rationalize both the existence of unemployment and the observed wage dispersion across locations. An alternative and more simplistic approach would be assuming completely exogenous wages to allow us to focus on the working of the other blocks of the model.[32] Although we do not provide direct evidence of heterogeneous wage constraints, we notice that heterogeneity can arise from differences in the industrial composition of each location, as hinted in section 2.

A binding wage constraint would determine a positive unemployment rate in the local labor market, u_j, whereas a slack constraint would allow the market to achieve full employment. In either case, employment (E_j) is determined by the labor demand in equation 10, while the labor supply (L_j) is defined in equation 6 as the number of workers commuting to local labor market j from all possible residential locations. The equilibrium in the local labor market is therefore characterized by the complementary

[31] Assuming $\alpha_j = 1$, in a competitive labor market the labor demand would pin down the wage level $w_j = A_j$, while full employment would be determined by the labor supply at that given wage. Other authors have obtained a downward-sloping labor demand under different assumptions. Boeri et al. (2021) adopt a Cobb-Douglas technology in capital and labor with constant returns to scale while introducing an exogenous and upward-sloping local supply of capital. Hsieh and Moretti (2019) extend the Cobb-Douglas production function in capital and labor with fixed commercial land.

[32] For example, Manning and Petrongolo (2017) adopt this approach in the baseline version of their spatial job search model.

slackness condition:

$$
\begin{aligned}
&E_j = (1 - u_j)L_j \\
&u_j(w_j - \underline{w}_j) = 0 \qquad (13)\\
&u_j \geq 0, \quad w_j \geq \underline{w}_j.
\end{aligned}
$$

Notice that u_j denotes the unemployment rate determined in workplace j, whereas the local unemployment rate reported in the official statistics measures the unemployed fraction of the active population residing in a particular location. Under the model's assumption that each worker chooses only one workplace location, we can define the residential unemployment rate, u_i^R, such that the number of unemployed resident in location i corresponds to the sum of all commuters who are unemployed in their labor market of choice:

$$u_i^R L_i^R = \sum_{j \in \mathbb{N}} L_{ij} u_j, \qquad (14)$$

which can be rewritten as a weighted average of the workplace unemployment rate, using the conditional commuting probabilities ($\lambda_{ij|i}$) as weights:

$$u_i^R = \sum_{j \in \mathbb{N}} \frac{L_{ij}}{L_i^R} u_j = \sum_{j \in \mathbb{N}} \lambda_{ij|i} u_j. \qquad (15)$$

Finally, given the residential unemployment rate, u_i^R, and the number of resident workers, L_i^R the total number of employed workers residing in location i can be calculated from:

$$E_i^R = L_i^R (1 - u_i^R) \qquad (16)$$

4.4 Residential Income

Each residential location i is populated by L_i^R workers and N_i^K rentiers, where L_i^R is determined by the workers' location choice in equation 5 and N_i^K is exogenous. Resident workers generate net labor income, which is composed of net wage income and unemployment benefits. Net wage income is calculated as the sum of the net wages perceived by all the employed residential workers commuting to all possible workplace destinations:

$$Y_i^E = (1 - \tau_L) \sum_{j \in \mathbb{N}} w_j L_{ij} (1 - u_j), \qquad (17)$$

and, similarly, the total amount of unemployment benefits is calculated as the sum of all benefits perceived by the remaining fraction of workers who remain unemployed in the workplace destination:

$$Y_i^U = (1 - \tau_L) \sum_{j \in \mathbb{N}} \rho w_j L_{ij} u_j. \qquad (18)$$

Thus, the total net labor income distributed to residential location i is:

$$Y_i^L = Y_i^E + Y_i^U = \sum_{j \in \mathbb{N}} L_{ij} y_j, \tag{19}$$

where we substituted the expected net labor income, y_j, from equation 1. We can define therefore the income measures per worker:

$$y_i^E = \frac{Y_i^E}{E_i^R} \tag{20}$$

$$y_i^L = \frac{Y_i^L}{L_i^R}, \tag{21}$$

corresponding respectively to net wage income per employed resident worker, y_i^E, and net labor income per resident worker, y_i^L. Local rentiers, instead, receive net profit income, Y_i^Π, from the ownership of structures, and net rents, $q_i H_i (1-\tau^K)$, from supplying housing units to workers, which sum up to constitute the total net capital income received by residential location i:

$$Y_i^K = Y_i^\Pi + q_i H_i (1 - \tau^K). \tag{22}$$

Finally, the total net income inflowing to residential location i is simply the sum of net labor and net capital income:

$$Y_i^R = Y_i^L + Y_i^K. \tag{23}$$

4.5 Final Good and Housing Markets

All workers allocate to housing a fraction β of their net labor income, independently of their employment status, while the remaining fraction is spent for consumption of the final good. On the other hand, rentiers spend the entirety of their income only for consumption goods. We can express total consumption and housing expenditure in residential location i as follows:

$$C_i = \quad (1 - \beta) Y_i^L + Y_i^K, \tag{24}$$

$$q_i H_i = \quad \beta Y_i^L, \tag{25}$$

which sum up to total net residential income (Y_i^R):

$$C_i + q_i H_i = Y_i^R. \tag{26}$$

Given the exogenous housing supply, H_i, equation 25 determines the market-clearing rental price:

$$q_i = \frac{\beta Y_i^L}{H_i}. \tag{27}$$

4.6 Government Budget Constraint

The government collects labor income taxes, T^L, capital income taxes, T^K, social contributions, SC, and it pays out unemployment benefits, UB. The residual tax revenue after unemployment benefits payments is used for public consumption of the final good, G, such that:

$$G = T^L + T^K + SC - UB, \tag{28}$$

where:

$$T^L = \tau_L \sum_{j \in \mathbb{N}} w_j E_j, \tag{29}$$

$$T^K = \tau_K \sum_{i \in \mathbb{N}} \big(\Pi_i + q_i H_i\big), \tag{30}$$

$$SC = \tau_{SC} \sum_{j \in \mathbb{N}} w_j E_j, \tag{31}$$

$$UB = \sum_{j \in \mathbb{N}} \rho w_j (1 - \tau_L) L_j u_j. \tag{32}$$

Here, government spending, G, is a residual variable which allows to close the model by satisfying the aggregate resource constraint:[33]

$$Y = C + G, \tag{33}$$

where $Y = \sum_{j \in \mathbb{N}} Y_j$, and $C = \sum_{i \in \mathbb{N}} C_i$.

In the quantitative exercises of Section 6, this feature of the model allows us to quantify the fiscal effect of a policy change in one parameter at a time by looking at the changes in the residual government spending. Notice that, on the other hand, fixing government spending and allowing either the labor or capital income tax rates to vary in order to balance the budget constraint would only result in a reduction in private consumption. For example, an increase in the labor income tax rate, τ_L, would not distort workers' location choice. This can be noted from equation 4 defining the joint commuting probabilities, λ_{ij}. Since net expected labor income, y, is proportional to $(1 - \tau_L)$ in equation 1, the direct effect of a change in τ_L would cancel out at numerator and denominator. By the same logic, the indirect effect of a change in τ_L through housing prices q would cancel out in equation 4. Analogously, a change in τ_K would only transfer resources from the rentiers to the government without affecting the allocation of labor.

[33] Since the social security budget constraint is not necessarily balanced, government spending might become negative. However, in practice, the additional fiscal revenue from labor and capital income taxes will ensure a non-negative government spending in almost all scenarios.

4.7 General Equilibrium

Given the exogenous population of workers, L, the spatial distribution of rentiers, $\{\mu_i^K\}$, and the supply of structures and housing units, $\{A_j\}$ and $\{H_i\}$, the general equilibrium of this economy is a utility value, V, a set of housing prices and gross wages, $\{q_i, w_j\}$, a set of values for local consumption expenditure, $\{C_i\}$, a set of labor supplies and unemployment rates, $\{L_j, u_j\}$, a spatial distribution of workers, $\{L_i^R\}$, and a value for government spending, G, such that, workers choose their optimal residential and workplace locations (5 and 6), expected utility is equalized across locations (8), firms maximize profits (10 and 11), net residential income is determined (17 to 23), all markets are in equilibrium (13, 24, and 25), and the aggregate resource constraint is satisfied (33).

5 Data and Calibration

5.1 Data

Our primary source of data are the National Accounts accessible from the database of the National Bank of Belgium. From the Regional Accounts we extracted district-level data on GDP, compensation of employees, and the total number of employees. From the Household Income Accounts we extracted district-level data on wages and salaries, employers' social security contributions, and total disposable income. Although both data sets pertain wages and contributions, the production and household accounts provide information at different geographical locations, respectively at workplace and residential location. Lastly, from the national decomposition of the final consumption expenditure of households, we extract the value of the total expenditure and the value of expenditure allocated to housing and utilities, which are directly used to calculate the housing expenditure share, β.[34]

Data on commuting, housing prices, and population age-structure are obtained from Statbel, the Belgian statistical office. With the exception of the data on the population age-structure, the other two data sets required an initial processing step.

The matrix of bilateral commutes is available for the entire territory from the 2011 Census of the population, disaggregated by municipality of residence and workplace. First, since our model is intended to capture internal commutes only, we remove all the counts of cross-borders commuters.[35] Second, we aggregate the counts of commuters by district of residence and workplace, since the lowest geographical unit at which all data

[34] Among the categories of housing expenditure we include imputed and actual rents, maintenance and ordinary reparation of the dwelling, and utilities (water, electricity, gas, and other fuels). We exclude furnishing, household equipment, and goods and services for routine household maintenance. All these categories are instead included in the consumption of goods and services.

[35] Cross-borders commuters accounts for 1.3% of total workers, hence their exclusion is unlikely to affect the main conclusions of the paper.

are available is the district. In addition to the counts of the commuters, the data include the coordinates of the centroids of the municipalities of residence and workplace, from which we calculate the corresponding geodesic distance. To aggregate this information at the district level, we average the bilateral distance between all municipalities within each pair of residence-workplace districts. This constitutes our final measure of geographical distance between districts, d^g_{ij}, used in the calibration of workers' preferences.[36]

Statbel provides time series about local housing prices disaggregated by district and by category of housing unit, from 1973 to 2017. Among the available categories of housing units we retain houses and flats, while we drop manors and building land. We construct our measure of housing prices, q_i, by averaging the median unitary transaction price of flats and houses, where each category of housing unit is weighted by its total surface.

Finally, the last data used in the calibration are the activity and unemployment rates the *Institut wallon de l'Évaluation, de la Prospective et de la Statistique* (IWEPS) calculates based on the Labor Force Surveys, the total amount of paid unemployment benefits and the total number of full-time insured unemployed job-seekers from the *Office National de l'Emploi* (ONEM), and the tax wedge on labor income obtained from the OECD.[37]

5.2 Calibration

This section discusses how to take the model to the data. The model is calibrated on 2011 data, since it is the only year in which all data are jointly available.[38] Most of the parameters can be uniquely recovered by rationalizing the data as an equilibrium outcome via a simple inversion of the relevant block of the model. The calibration procedure is sequential, as later calibration steps require quantities calculated in the preceding steps, and, therefore, this section will follow the order of these steps.

5.2.1 Labor Force and Rentiers

In the first step, we combine multiple data sources to calculate the size of the total labor force, L, and the spatial distribution of rentiers, μ_i^K. Since the model distinguishes between workers and rentiers on the basis of their sources of income, our definition of worker encompasses both employees and unemployed workers. Rentiers, instead, include self-employed workers and inactive individuals older than 15.[39] We start by measuring the

[36]In the literature, several papers use travel time distance between locations to account for the fact that transportation technologies may be different between several locations, where a better transport technology reduces travel time for a given geographical distance. However, Goffette-Nagot et al. (2011) have shown that for Belgium the two measures are strongly correlated, with a correlation of 0.9.

[37]OECD (2020), Tax wedge (indicator). doi: 10.1787/cea9eba3-en (Accessed on 10 September 2020)

[38]Specifically, the commuting matrix is only available from the 2011 Census.

[39]Our definition of rentiers includes also pensioners. Although we do not model pensioners explicitly, we recall that capital income is *de facto* a residual income category, after subtracting compensation of

total number of employees in the economy, $E = \sum_{j \in \mathbb{N}} E_j$, from the information available in the National Accounts. Next, we use data on the national unemployment rate from IWEPS to calculate the size of the labor force using the identity, $L = E/(1-u)$. To recover the spatial distribution of the rentiers, μ_i^K, we combine data on the age-structure of the population, the activity rates, and the number of self-employed individuals. First, given the total number of 15-64 and 65+ years old individuals, $N_i^{15\text{-}64}$ and N_i^{65+}, and the activity rate, a_i, in a given district i, we calculate the total number of 15+ years old inactive individuals as $I_i^{15+} = N_i^{15\text{-}64}(1-a_i) + N_i^{65+}$. Second, the total number of rentiers, as defined above, is calculated by summing the number of self-employed workers, E_i^{self}, and inactive individuals older than 15, that is, $N_i^K = E^{\text{self}} + I_i^{15+}$.[40] Third, we recover the spatial distribution of rentiers by normalizing, $\mu_i^K = N_i^K / N^K$, where $N^K = \sum_{i \in \mathbb{N}} N_i^K$.

5.2.2 Technology

In the second step, we use the production data from the Regional Accounts to calibrate the production function of each district. The labor income share is calculated directly from the data as the ratio of total compensation of employees to GDP, $\alpha_j = w_j(1+\tau_{SC})E_j/Y_j$, and it varies between 0.40 and 0.58 with a mean of 0.50.[41] Given the labor income share, total factor productivity is calibrated as the residual of the production function, $A_j = Y_j / E_j^{\alpha_j}$, using data on GDP and total number of employees.

5.2.3 Wages, Social Security, and Labor Income Taxation

In the third step, we recover the employers' social contribution rate, τ_{SC}, the labor income tax rate, τ_L, and the replacement rate, ρ. The social security contribution rate (τ_{SC}) is calibrated as the ratio between the total employers' social contributions and the total wages and salaries extracted from the Household Income Accounts. The resulting social contribution rate is equal to 0.364. Given τ_{SC}, gross wages, w_j, are obtained from the labor cost per worker ($w_j(1+\tau_{SC})$) extracted from the Regional Accounts data. To calculate net wages, $w_j(1-\tau_L)$, we recover the labor income tax rate (τ_L) using the tax wedge extracted from the OECD. In particular, the OECD calculates the tax wedge on labor income as the ratio between total taxes and total labor cost for an average single worker without children. Given this definition, the implied tax rate on gross wage income is calculated such that $w_j(1-\tau_L) = w_j(1+\tau_{SC})(1-\mathit{wedge})$. The resulting labor income tax rate is equal to 0.40. Finally, using the data from ONEM, the replacement rate (ρ) is

employees from GDP. See below in section 5.2.7 for more details on this point.

[40] Note that the number of self-employed workers is obtained from the National Accounts, which measure the number of workers at the workplace location. Since we cannot directly locate where self-employed workers live, here we assume that self-employed workers live and work in the same location.

[41] These values may be at odd with the standard practice in macroeconomics of setting the labor income share to 0.70. However, Karabarbounis and Neiman (2013) have shown that, across the world, the labor income share has been falling since 1975 from around 0.55 to 0.50 in 2010.

calibrated relative to the net wage using the information on total unemployment benefits payments and the total number of claimants. The replacement rate is therefore calculated as the ratio between the average unemployment benefit per claimant and the net wage calculated before, and it is equal to 0.426.

5.2.4 Unemployment Rates and Wage Constraints

In the fourth step, we recover the unemployment rate by workplace location, u_j, and the values for the wage constraints, $\underline{w}_j$. According to the model, a positive unemployment rate at the market level implies a binding wage constraint, hence we could recover the value for $\underline{w}_j$ directly from the gross wage (w_j) calculated in the previous section. In the model the unemployment rate is defined as the fraction of commuting workers remaining unemployed in the labor market of choice (including commuting workers for whom the labor market of choice is their residence district), however, in the data the unemployment rate measures the unemployed fraction of the active population residing in a particular location. To recover the unemployment rate by workplace location, we combine the observable residential unemployment rate obtained from IWEPS with the commuting matrix obtained from the 2011 Census. We start by normalizing equation 14 so that the relationship between unemployment rates can be rewritten in terms of the probabilities λ_{ij} and λ_i^R:

$$u_i^R \lambda_i^R = \sum_{j \in \mathbb{N}} \lambda_{ij} u_j. \tag{34}$$

However, instead of observing the commuting probability of the labor force (λ_{ij}), we observe only the number of commuting workers who end up being employed (E_{ij}). Nevertheless, since each worker faces the same unemployment risk in a given labor market, we can define the number of employed commuters from i to j as:

$$E_{ij} = \quad L_{ij}(1 - u_j) = L\lambda_{ij}(1 - u_j), \tag{35}$$

which, together with the definition of the number of employed resident workers E_i^R in equation 16, can be substituted in equation 34 to obtain:

$$\frac{u_i^R}{1 - u_i^R} = \sum_{j \in \mathbb{N}} \frac{E_{ij}}{E_i^R} \frac{u_j}{1 - u_j}, \tag{36}$$

defining a system of linear equations to be solved for $u_j/(1 - u_j)$, and where E_{ij}/E_i^R are the observable conditional commuting probabilities of employed workers. Since the solution of this system is not guaranteed to be non-negative, we solve the system via constrained least squares by imposing non-negativity constraints.[42] In this case, the

[42] It easy to see that a negative solution of the system would imply a negative unemployment rate by workplace.

unemployment rate is exactly equal to zero in 6 districts out of 43, which, under the lens of our model, implies that the wage constraint is not binding in these labor markets. As a consequence, for these districts we cannot infer a value for $\underline{w}_j$ from the recovered gross wage. Since these parameters cannot be identified, in the quantitative analysis we will simply assume that $\underline{w}_j$ in these markets coincides with the corresponding market-clearing wage, hence, in conclusion, in each workplace location $\underline{w}_j$ is set equal to the calculated gross wage. Finally, given the calibrated unemployment rate by workplace, we can calculate the commuting probabilities of the labor force, λ_{ij}, by inverting equation 35:[43]

$$\lambda_{ij} = \frac{E_{ij}}{(1-u_j)L}. \tag{37}$$

5.2.5 Residential Income and Its Components

In the fifth step, we decompose total residential net income, Y_i^R, which is directly observable from the Household Income Accounts, into net labor income, including net wage income and unemployment benefits, and net capital income, including rents and profits. Using the previously calibrated quantities, we calculate residential net wage income, Y_i^E, as the sum of the net wages perceived by all the employed workers living in a given residential location and commuting to all possible workplace destinations (equation 17).[44] Similarly, the total residential amount of unemployment benefits, Y_i^U, is calculated as the sum of all benefits perceived by the remaining fraction of workers who are unemployed in the workplace destination (equation 18). Residential net labor income, Y_i^L, is then calculated as the sum of the net wage income and unemployment benefits perceived by the residential workers (equation 19). Given the observable residential disposable income (Y_i^R) and the calculated net labor income (Y_i^L), net capital income, Y_i^K, can be calculated as a residual quantity from equation 22. In the following two sections we will use net labor income and net capital income to recover the values for the local housing supplies, the ownership shares, and the capital income tax rate.

5.2.6 Housing Supply

In the sixth step, we calibrate the housing supply of each district, H_i. The market clearing equation 27 allows to recover the local housing supply given the values of the housing expenditure share, β, the housing prices, q_i, and residential net labor income, Y_i^L. The housing expenditure share is 0.238 and it has been directly calculated from the data on

[43] Notice that the total counts of workers from the commuting matrix does not exactly match the counts of employees in the National Accounts. In practice, to consistently combine data from different sources we use frequencies instead of counts. See the Appendix for more details.

[44] As a sanity check, we compared the calculated residential net wage income to the actual total of wages and salaries available from the Household Income Accounts, obtaining an almost perfect fit, with an R^2 of 0.962.

final households' consumption in the National Accounts. Given our measure of local housing prices described in section 5.1 and the previously obtained net labor income, the housing supply is calibrated as $H_i = \beta Y_i^L / q_i$.[45] Given q_i and H_i, rents can be easily calculated.

5.2.7 Ownership Shares and Capital Income Taxation

In the seventh step, we complete the decomposition of capital income (Y_i^K) into profits and rents by recovering the local ownership shares in the national assets portfolio, ι_i, and the capital income tax rate, τ^K. First, for each workplace location j, produced profits can be directly calculated as the difference between GDP and compensations of employees (equation 11). Second, we recover the capital income tax rate (τ_K) from the wedge between the aggregate produced capital income, $\sum_{j \in \mathbb{N}} (\Pi_j + q_j H_j)$, and the aggregate perceived capital income calculated previously, $\sum_{i \in \mathbb{N}} Y_i^K$. The resulting capital income tax rate is equal to 0.493. Third, given profits (Π_j), the capital income tax rate (τ^K), and the spatial distribution of rentiers (μ_i^K), we can finally recover the ownership shares (ι_i) by solving the linear system defined by equation 12. The system in general is ill-conditioned, thus we recovered the parameters via regularized least squares using a very small penalty term.

5.2.8 Workers' Preferences

In the eighth and last calibration step, we calibrate the quantities governing the commuting gravity equation. Since commuting costs (κ_{ij}) and amenities (B_{ij}) enter equation 4 multiplicatively, we can only identify the composite term $\mathcal{B}_{ij} = B_{ij} \kappa_{ij}^{-\epsilon}$, which captures the ease of commuting from i to j. Given the value of ϵ, the ease of commuting can be simply recovered from the modified gravity equation:

$$\lambda_{ij} = \frac{\mathcal{B}_{ij} q_i^{-\epsilon\beta} y_j^{\epsilon}}{\sum_{r,s \in \mathbb{N}} \mathcal{B}_{rs} q_r^{-\epsilon\beta} y_s^{\epsilon}}, \tag{38}$$

which defines a linear system of equations that can be solved for $\mathcal{B}_{ij}$. Therefore, in the rest of this section, we focus on the estimation of the Fréchet shape parameter, ϵ.

We follow the two-step procedure adopted by Monte et al. (2018). In the first step, we decompose the ease of commuting into five components, such that $\mathcal{B}_{ij} = \mathbb{B}_i \mathbb{B}_j {d_{ij}^g}^{-\phi_g} e^{\phi_\ell d_{ij}^\ell} \mathbb{B}_{ij}$, where $\mathbb{B}_i$ and $\mathbb{B}_j$ are respectively residence- and workplace-specific components, d_{ij}^g is our measure of geographic distance, d_{ij}^ℓ is a dummy capturing linguistic distance (defined below), and $\mathbb{B}_{ij}$ is an orthogonal residual related to both residence and workplace. Given

[45] Since q_i and H_i enter multiplicative in equation 27, the unit of measurement for housing can be chosen arbitrarily. In practice, since q_i is constructed as the average price per housing unit, H_i measures the number of local housing units.

this decomposition, we can estimate a log-linear specification of equation 38:

$$\log \lambda_{ij} = \gamma_0 + \gamma_i + \gamma_j - \phi_g \log d_{ij}^g + \phi_\ell d_{ij}^\ell + \log \mathbb{B}_{ij}, \tag{39}$$

where the γ_i fixed effect captures housing prices (q_i) and the residence-specific component ($\mathbb{B}_i$), the γ_j fixed effect captures the expected labor income (y_j) and the workplace-specific component ($\mathbb{B}_j$), and the constant γ_0 absorbs the denominator of the gravity equation. The dependent variable is the commuting probability of the labor force (λ_{ij}) calculated before in section 5.2.4. Geographic distance (d_{ij}^g) is described in section 5.1.[46] To capture the potential linguistic barriers between regions, we include a linguistic distance dummy, d_{ij}^ℓ, equal to one if the residence and workplace locations belong to the same region or if at least one of the two districts is the Brussels-Capital region, since Brussels is a pole of attraction for both the French and the Flemish communities.[47]

Column 1 of Table 2 shows significant coefficients for both distance measures, respectively $\phi_g = 2.171$ and $\phi_\ell = 1.091$, and a very high R^2 of 0.93. For comparison, Monte et al. (2018) estimate a log-distance coefficient of 4.43 with an R^2 of 0.8 using data on commuting across US counties.[48]

In the second step, we estimate ϵ by augmenting the initial specification with expected labor income (y_j):

$$\log \lambda_{ij} = \gamma_0 + \gamma_i - 2.171 \log d_{ij}^g + 1.091\, d_{ij}^\ell + \epsilon \log y_j + \nu_{ij}, \tag{40}$$

where now, to separately identify ϵ, the workplace-specific component ($\mathbb{B}_j$) is absorbed into the error term, $\nu_{ij} = \log \mathbb{B}_j + \log \mathbb{B}_{ij}$, and the coefficients on d_{ij}^g and d_{ij}^ℓ are constrained to their previously estimated values. Since we cannot directly control for workplace fixed effects, identification of ϵ relies upon the exogeneity of expected labor income (y_j), whose variation depends on both the gross wage (w_j) and the workplace unemployment rate (u_j). According to the model, when the wage constraint is not binding, the unemployment risk is null and the gross wage is determined by the market. On the other hand, when the constraint is binding, the gross wage is exogenous and the unemployment rate is determined by the gap between labor demand and labor supply. In this latter case, we

[46] Notice that, as a result of the averaging procedure used to calculate d_{ij}^g, our measure of geographic distance also captures the cost of intra-district commuting, as districts vary in their size and number of municipalities.

[47] Belgium has three official languages, French, Dutch, and German, however, since the German-speaking community is incorporated in the district of Verviers of the French community, we do not explicitly account for this difference. Goffette-Nagot et al. (2011) follow a similar intuition in their estimation of the effects of employment accessibility on land price.

[48] Other authors in the literature have estimated the commuting gravity equation using travel time instead of log-distance. Ahlfeldt et al. (2015) estimate a coefficient of 0.07 per minute of self-reported travel time for commutes across Berlin's districts in 2008, which translates into a semi-elasticity of 4.2 per hour of travel time. Similarly, Heblich et al. (2020) find a coefficient of about 4.9 using historical commuting data across the boroughs of Greater London, where travel time is constructed as the least-cost travel time given the transport network available in 1921.

Table 2: Commuting Gravity Equation Estimation

	(1)	(2)	(3)	(4)
Dep. Var.	$\log \lambda_{ij}$	$\log \lambda_{ij}$	$\log y_j$	$\log \lambda_{ij}$
$\log d^g_{ij}$	-2.171***	-2.171		-2.171
	(0.039)	—		—
d^ℓ_{ij}	1.093***	1.093		1.093
	(0.031)	—		—
$\log y_j$		7.583***		7.084***
		(0.204)		(0.38)
$\log A_j$			0.109***	
			(0.02)	
Residence FEs	yes	yes	no	yes
Workplace FEs	yes	no	no	no
Observations	1,840	1,840	43	1,840
R^2	0.93	0.517	0.443	-

Note: This table presents the estimates of the gravity equations 39 and 40. λ_{ij} is the commuting probability of the labor force between two districts; d^g_{ij} is the average geographical distance between two districts; d^ℓ_{ij} is a dummy equal to one if the residence and workplace districts belong to the same region or if at least one of the two districts is the Brussels-Capital region; y_j and A_j are respectively the calibrated values of expected net labor income and total factor productivity. The coefficients of $\log d^g_{ij}$ and d^ℓ_{ij} in Cols. 2 and 4 are constrained to be equal to those in Col. 1. Col. 4 presents the output of a second stage regression estimated via OLS, where $\log y_j$ is the fitted value obtain from the first stage regression in Col. 3. Heteroskedasticity-robust standard errors are reported in parentheses: * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$.

would expect a positive correlation between the unemployment rate and the workplace-specific component, since, in the presence of a binding constraint, stronger preferences for a specific workplace location would determine a greater supply of workers and a higher unemployment rate. Independently of the case, expected labor income will be positively correlated with the workplace-specific component, and the OLS estimate will be biased upward. Nevertheless, we can exploit the structure of the model and use the calibrated total factor productivity (A_j) as an exogenous demand shifter. We therefore identify ϵ by instrumenting the log of the expected labor income with the log of the calibrated total factor productivity.[49]

[49] Notice that total factor productivity is correlated with the gross wage by construction. Since we calibrated A_j as the residual of the production function after recovering the labor income share (α_j), the

Columns 2 and 4 of Table 2 show respectively the OLS and TSLS estimates of equation 40, while Column 3 provides the estimates of the first stage regression.[50] Both estimates are statistically significant around a value of 7, which is not far from values found in the literature: 6.83 in Ahlfeldt et al. (2015), 3.3 in Monte et al. (2018), and 5.25 in Heblich et al. (2020). As expected, the TSLS estimate is slightly lower than the OLS, although we cannot reject the null hypothesis of the coefficients being equal.[51] In conclusion, in our final calibration we set $\epsilon = 7.084$ in line with the TSLS estimate.

6 Quantitative Analysis

In this section we use the calibrated model to perform several quantitative exercises. First, we explore the mechanics of the model by shutting down one by one the determinants of the workers residence-workplace location choice. Here the focus is on the mobility of workers and the resulting local residential unemployment rate. In particular we study the importance of the heterogeneity in the ease of commuting, the elasticity to expected real income, and the coverage of unemployment insurance. Second, we quantify the magnitude of wage distortions by relaxing the wage rigidities and calculating the counterfactual market clearing wage. Third, there is a large literature who investigates the effects of employers payroll taxes on employment outcomes and our model allows us to experiment changes of the social contribution rate (the equivalent of a payroll tax on employers) to determine the optimal zero-unemployment social contribution rate and the corresponding fiscal cost of carrying out such a policy change.

6.1 Model Mechanics: Labor Mobility and Unemployment

6.1.1 The Role of the Ease of Commuting

We begin the section by studying the role of the ease of commuting, $\mathcal{B}_{ij}$. We show that heterogeneity in the ease of commuting is a necessary condition for generating spatial dispersion in the residential unemployment rate. Starting from equation 38, we eliminate heterogeneity by setting $\mathcal{B}_{ij} = \mathcal{B}$ for all i, j. By neutralizing the role of the ease of commuting, the worker's location choice is determined only by the expected real net

following identity can be established: $\log A_j = \log Y_j - \exp(\log w_j)(1 + \tau_{SC})\frac{E_j}{Y_j} \log E_j$. This ensures the strength of the instrument by construction, while at the same time $\log A_j$ will not be a perfect predictor of $\log y_j$ due to the non-linearity of the relationship.

[50] The TSLS estimates have been obtained by regressing the log commuting probabilities on the predicted log expected income from the first stage regression in Column 3. Notice that the standard errors obtained from this procedure are biased downward, since they do not account for the estimation error of the regressor.

[51] The test statistic for the difference in coefficients $(\hat{\epsilon}_{OLS} - \hat{\epsilon}_{TSLS})/\sqrt{SE(\hat{\epsilon}_{OLS})^2 + SE(\hat{\epsilon}_{TSLS})^2}$ is equal to 1.156.

labor income (y_j/q_i^{β}):

$$\lambda_{ij} = \frac{q_i^{-\epsilon\beta} y_j^{\epsilon}}{\sum_{r,s\in\mathbb{N}} q_r^{-\epsilon\beta} y_s^{\epsilon}}. \tag{41}$$

A direct consequence is that the unemployment rate would be equalized across residential locations. First, we notice that conditional commuting probabilities given the choice of residence depend only on the expected nominal net labor income (y_j) in the workplace of choice:

$$\lambda_{ij|i} = \frac{y_j^{\epsilon}}{\sum_{s\in\mathbb{N}} y_s^{\epsilon}}, \quad \text{for all } i. \tag{42}$$

Therefore, the proportion of resident worker choosing to commute to workplace j will be the same for any two residence locations i and i'. Intuitively, conditioning on residence removes the effect of housing prices on the location choice of the worker, as also shown in equation 7. Thus, in absence of any bilateral factor, the worker's workplace location choice would be exclusively affected by the expected earnings in the chosen local labor market.

Second, as shown in equation 15, the workplace unemployment rate (u_j) and the residential unemployment rate (u_i^R) are related through the conditional commuting probabilities. In particular, the residential unemployment rate can be expressed as a weighted average of all workplace unemployment rates, where the weights equal the conditional probabilities.

Finally, as both $\lambda_{ij|i}$ and u_j do not vary across i, the residential unemployment rate is equalized across all residential locations, that is, $u_i^R = u^R$. In addition, also nominal net labor income per resident worker, y_i^L, is also equalized, since it can be similarly constructed as a weighted average of the expected nominal net labor income (y_j) with conditional commuting probabilities as weights. However, the real net labor income per resident worker (y_i^L/q_i^{β}) would not be necessarily equalized, as housing prices would be still differently affected by the heterogeneous local housing supplies.

Overall, heterogeneity in the ease of commuting is an essential feature of this model to generate heterogeneity in residential unemployment rates.

6.1.2 The Role of Expected Real Net Labor Income

Next we neutralize the effect of expected real net labor income (y_j) on the worker's location choice by setting $\epsilon = 0$. In this scenario, workers choose where to work and live only according to the ease of commuting:

$$\lambda_{ij} = \frac{\mathcal{B}_{ij}}{\sum_{r,s\in\mathbb{N}} \mathcal{B}_{rs}}. \tag{43}$$

In other words, if workers do not respond to variations in economic conditions, the labor supply in each market and the spatial distribution of workers will be fixed. Figure 4a and

4b display respectively the counterfactual percentage change in the number of resident workers and in the labor supply (at the workplace). In both maps the qualitative result is similar: workers flow away from the center of the country to more peripheral districts such as the North Sea coast or the South of Wallonia. In particular, Brussels and Antwerp (AW) lose most of their attractiveness when the effect of expected net labor income is neutralized, as more than 40% of their workers and their residents would rather relocate somewhere else.

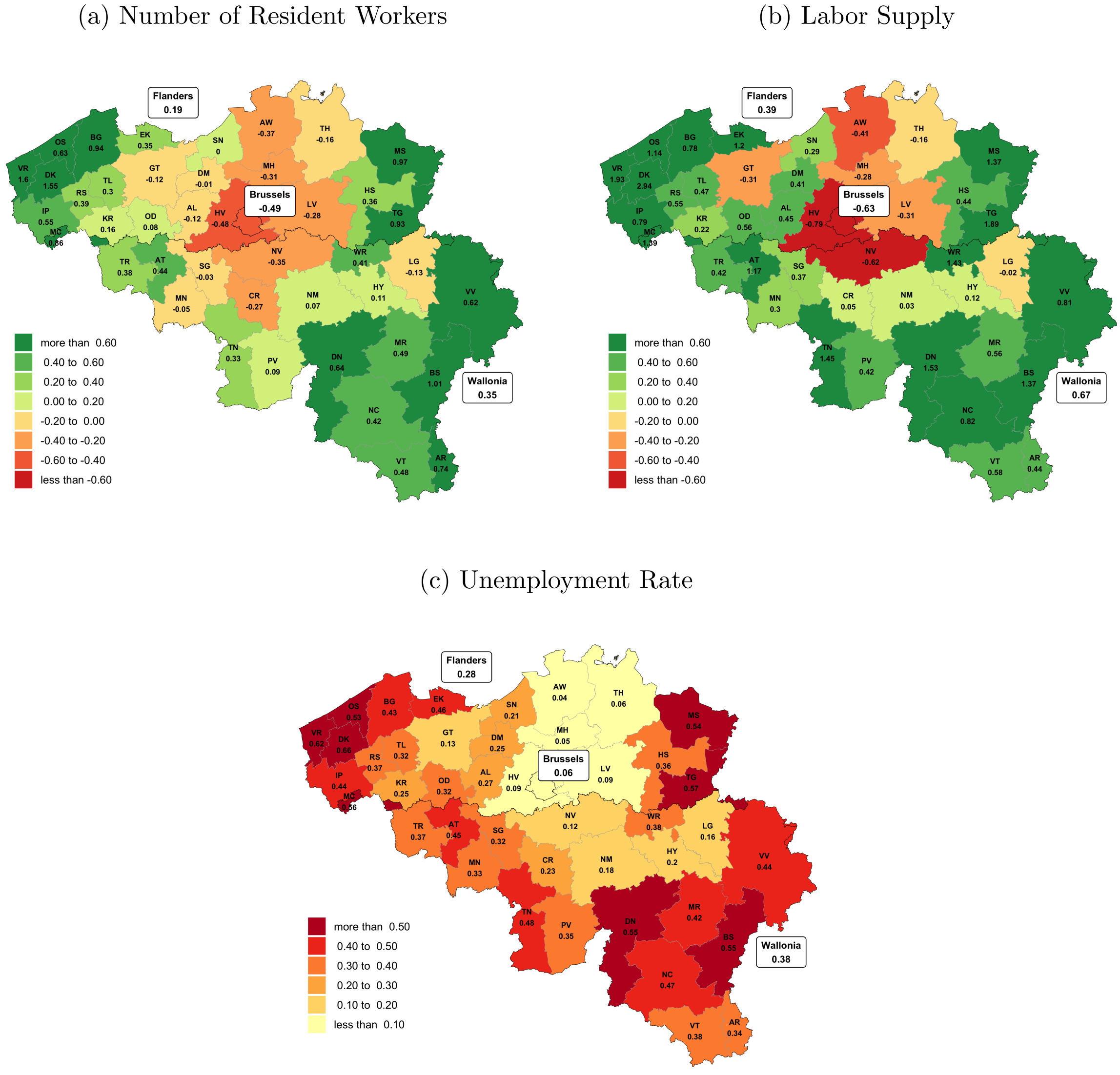


Figure 4: Fixed Commuting Flows

Note: Counterfactuals when commuting flows are determined only by the ease of commuting ($\epsilon = 0$). Panel (a): Counterfactual percentage change in the number of resident workers, λ_i^R. Panel (b): Counterfactual percentage change in the labor supply by district of workplace, λ_j^L. Panel (c): Counterfactual level of the residential unemployment rate, u_i^R. Regional values for Brussels, Flanders, and Wallonia are calculated as the simple average across all districts belonging to that region. Values are rounded to the nearest hundredth. Data have been elaborated by the authors (see main text in this Section for details).

Panel 4c maps the local unemployment rates that would arise in this counterfactual world. Unemployment rate levels in this case rise to unsustainable high levels, up to more than 50% of the residential population of workers. As explained above, workers would not take into consideration the effect of unemployment on their location choice and would cluster in the most attractive locations in terms of ease of commuting, both by residence and workplace. Therefore, given the existing wage rigidities, the excess labor supply in the most attractive workplaces would cause unemployment rates to surge. Remarkably, only Antwerp (AW) and Brussels would result having a lower residential unemployment rate than that observed in the data, as workers would be no more attracted to the higher economic standards of living of these locations, hence reducing the pressure in the labor market.

To sum up, we find that the sensitivity to expected real labor income plays a crucial role in determining the core-periphery structure of the economy. If local economic conditions were not that important to workers, we would observe a more equal distribution of workers across districts.

6.1.3 The Role of Unemployment Insurance

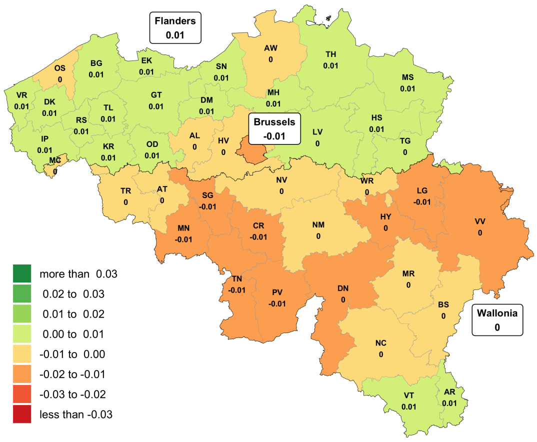
We now turn to the role of unemployment insurance in determining the location choices of workers. As unemployed workers have access to a fraction ρ of the net wage they would otherwise earn were they employed, whether this fraction is high or low affects the labor income they expect to receive from participating to a specific labor market. As shown above, workers are particularly responsive to variations in the expected real net labor income, therefore it would be reasonable to anticipate a sizeable effect of a variation in the replacement rate on the choice of the district of workplace. In general the effect of a change in the replacement rate on the expected nominal net labor income (y_j) is modulated by the level of workplace unemployment rate (u_j). The higher the workplace unemployment rate, the higher the effect on expected net labor income, and in turn, the stronger the impact on the location choice.[52] To gauge the magnitude of this effect, we therefore examine the two extreme cases, that is, when $\rho = 0$ and $\rho = 1$.

No Unemployment Insurance. First we study the effect of removing unemployment insurance by setting $\rho = 0$. Figure 5a shows the counterfactual percentage change of the number of residential workers due to the removal of unemployment insurance. Contrary to what one could expect, workers would not migrate in mass to the North to insure against the high unemployment risk in the South. The resulting changes in the number of resident workers do indeed point in the direction of a migration from Wallonia to Flanders, however the magnitude of these changes do not exceed in absolute value 1% of the local population of resident workers. We stress here that, our model does not

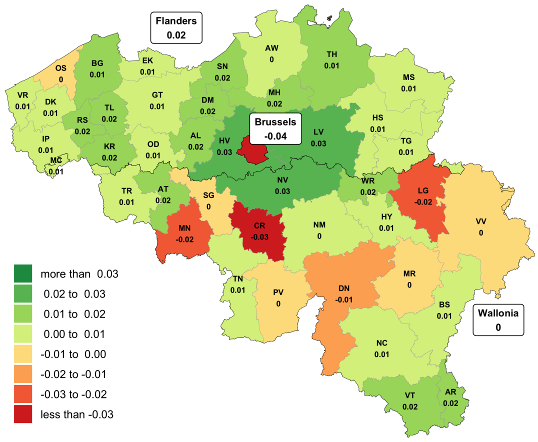
[52]This can be easily deduced from the partial derivative $\partial \log y_j / \partial \rho$.

explicitly take into account linguistic barriers, although part of this cultural constraint is reflected in the calibrated values of the ease of commuting. Were the linguistic barrier explicitly taken into account, the effect would probably be even milder.

(a) Number of Resident Workers



(b) Labor Supply



(c) Unemployment Rate

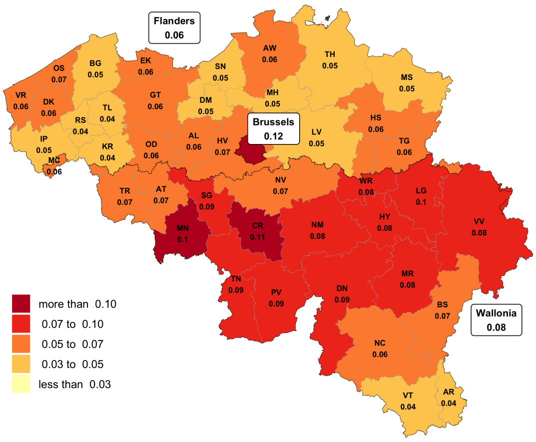


Figure 5: No Unemployment Insurance

Note: Counterfactuals under no unemployment insurance ($\rho = 0$). Panel (a): Counterfactual percentage change in the number of resident workers, λ_i^R. Panel (b): Counterfactual percentage change in the labor supply by district of workplace, λ_j^L. Panel (c): Counterfactual level of the residential unemployment rate, u_i^R. Regional values for Brussels, Flanders, and Wallonia are calculated as the simple average across all districts belonging to that region. Values are rounded to the nearest hundredth. Data have been elaborated by the authors (see main text in this Section for details).

Instead of changing residence, workers would rather change the workplace location as an alternative insurance mechanism. Panel 5b shows that the highest negative percentage change in labor supply would occur in districts with high unemployment rates. Fewer workers would commute to Brussels, Charleroi (CR), Liège (LG), and Mons (MN), while they would look for jobs in the neighboring districts. Overall, almost all districts would benefit from an increase in labor supply.

Regarding the counterfactual unemployment rates, Panel 5c displays a similar spatial distribution to that observed in the data, however, upon a closer look, the unemployment rates appear more homogeneously spread. There are indeed fewer extreme values, as now the districts of Brussels and Charleroi (CR) attain lower unemployment rates around 12%, compared to the previous respective rates of 17% and 14%. Similarly, the gap between Flanders and Wallonia is reduced due to an overall increase in the average district unemployment rate in Flanders from 4% to 6%. As for the case of the homogeneous ease of commuting, the explanation is found in the change of the local labor supply. As workers move away from high unemployment workplaces, an excess labor supply is generated in the neighboring districts given the imposed wage rigidities. The effect is accentuated in Flanders, where the labor supply increases by 2% on average.

Full Unemployment Insurance. Figure 6 shows the same counterfactuals under full unemployment insurance, that is, when $\rho = 1$. Notice that, in this case, as unemployment benefits are calculated in proportion of the net wage, the worker will always receive the full wage in the workplace of choice independently of her employment status. Therefore, as unemployment risk is no longer a factor determining the location choice, workers will prefer commuting to workplaces offering high wages. However, as discussed before, the differential effect on workers' location choices will be mostly driven by the initial level of the workplace unemployment rate, as an increase in unemployment insurance would have a much larger effect in high risk workplaces.

Panel 6a shows large residential relocation flows towards the center of the country, especially in Brussels and Charleroi (CR), with an increase in the number of resident workers of about 6% relative to the initial equilibrium. These inflows are supported by outflows in Flanders and the southernmost districts of Wallonia, which had relatively low initial unemployment rates. A similar picture is presented in Panel 6b, however, now only 4 out 43 districts arise as most attractive workplaces. These are the districts with the highest residential unemployment rates, as shown previously in Figure 1b. Overall, workers would be more inclined to work in these higher risk locations and, as a consequence, they would choose to live in the neighboring residential districts.

Panel 6c shows the map of the residential unemployment rate in the case of full insurance. As a result of workers relocation, the northern districts in Flanders and those in the south of Wallonia would achieve almost zero unemployment, with rates not exceeding 3%. On the contrary, the districts in the center of the country would exhibit extremely high levels of unemployment above 15%. The residential unemployment rate would be the highest in the most attractive workplaces of Brussels, Charleroi (CR), Mons (MN), and Liège (LG), as many workers would choose to live and work there. However, workers relocation would also impact their neighbouring districts, since many other workers would choose to live in those districts but still work in these attractive workplaces.

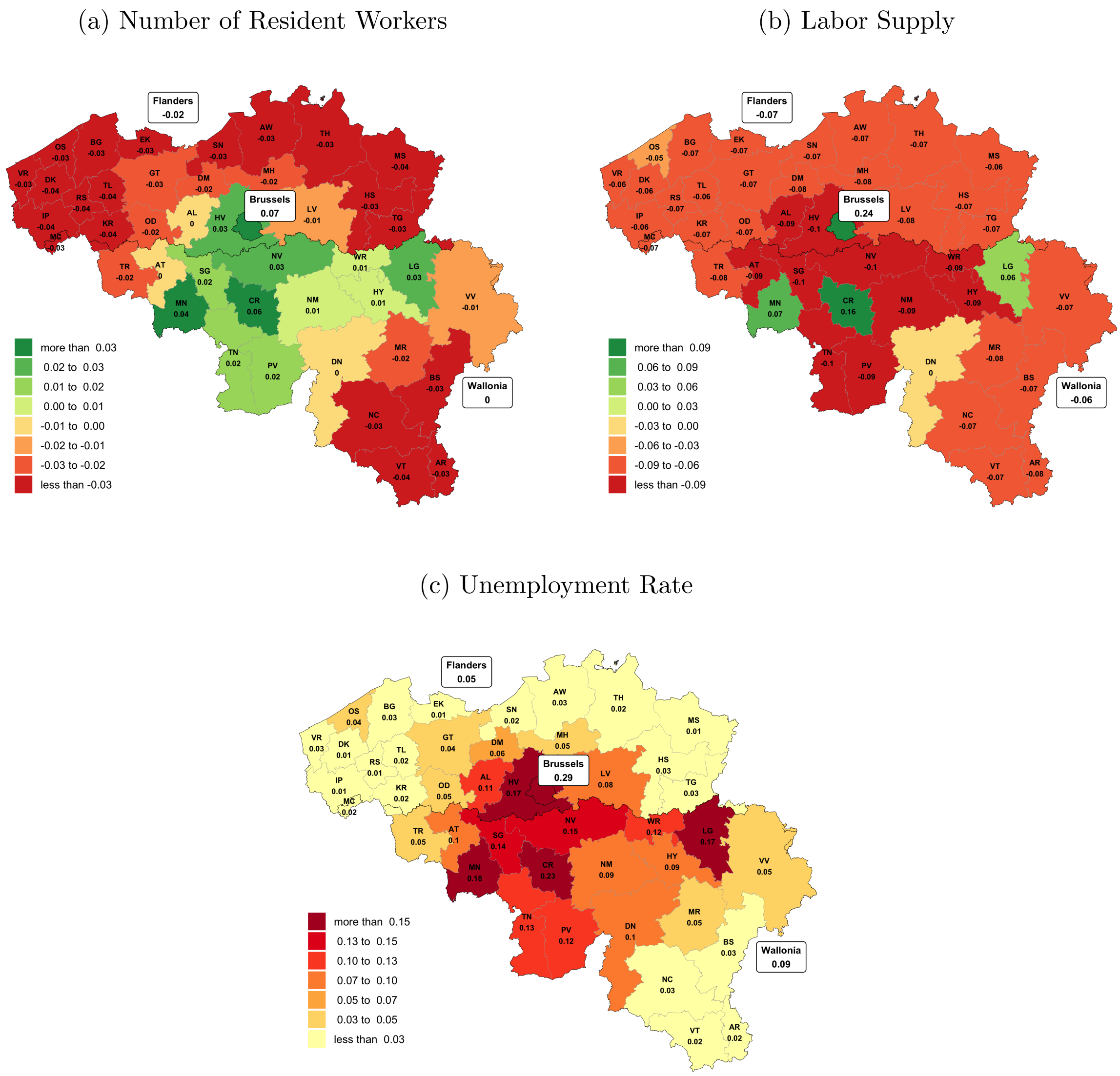


Figure 6: Full Unemployment Insurance

Note: Counterfactuals under full unemployment insurance ($\rho = 1$). Panel (a): Counterfactual percentage change in the number of resident workers, λ_i^R. Panel (b): Counterfactual percentage change in the labor supply by district of workplace, λ_j^L. Panel (c): Counterfactual level of the residential unemployment rate, u_i^R. Regional values for Brussels, Flanders, and Wallonia are calculated as the simple average across all districts belonging to that region. Values are rounded to the nearest hundredth. Data have been elaborated by the authors (see main text in this Section for details).

In summary, the degree of unemployment insurance has a strong effect on the location choice of workers. In particular, in the specific case of Belgium, a higher replacement rate would tend to accentuate the spatial misallocation of labor, as it would induce workers to relocate closer to high unemployment locations close to the center. On the other hand, a lower replacement rate would tend to spread workers away from the center and generate a more equal spatial distribution of the labor supply. Notice that the above results are obtained under the assumption that unemployment insurance is financed through cuts in government spending. As explained in Section 4, financing through either labor or capital income taxes would result in the same labor reallocation because of proportional taxation. On the other hand, if the budget constraint of the social security system had to be balanced, a change in the replacement rate would require a similar change in the employers' social contribution rate. In particular, in the full insurance scenario, the increase in overall unemployment would impose a higher contribution rate, which, in turn, would depress the local labor demand in each district, hence exacerbating the effect on local unemployment. However, this additional increase in unemployment risk would not affect further the location choice of workers, since they would be nonetheless fully insured, and the shift in the labor demands would not reduce the already constrained wages.[53] Thus, compared to what shown in Figure 6, imposing a balanced social security system would imply a higher local unemployment rate in each market, although it would not cause any labor reallocation.

6.2 Quantifying Wage Distortions

We now move to quantifying the wage distortions generated by wage rigidities. In this section we calculate the counterfactual market-clearing wages that would emerge in each local labor market in absence of wage rigidities. Figure 7 shows the percentage change in gross wages after removing the constraints.

As expected, wages would decrease the most in the districts with the highest residential unemployment rates.[54] Some districts around Brussels and the two southernmost districts in Wallonia were already close to full employment and therefore would not exhibit any change in gross wages. Wages in Flanders would be mostly similar to the observed wages in the data, while the average decrease in Wallonia would be around 3%. The greatest drop in wages would occur in Brussels, with a decrease of 9%.

Clearly, as wages are free to clear the market, the economy would achieve full employment, however, the overall effect on labor income per worker would be ex-ante less clear. Table 3 reports the cross-sectional distribution statistics for some of the most interesting variables in percentage change from the initial equilibrium. GDP (Y_j) would increase almost everywhere by about 3% due to the efficient allocation of labor. Nominal wage income per employed resident (y_i^E) would track the drop in gross wages (w_j). However,

[53] To see how a change in the employers' social contribution rate, τ_{SSC}, does not affect the labor allocation under $\rho = 1$, notice that in equation 4 the commuting probabilities, λ_{ij}, depend on the net expected labor income, y, and housing prices, q. Under $\rho = 1$, the net expected labor income (equation 1) does not depend on the unemployment rate. Moreover, if unemployment is positive in all local labor markets, then the local wage would also be constrained (equation 13), hence net expected labor income would not change due to variations in local labor demands. Insofar housing prices respond only to variations in total local labor income (equations 25 and 19), they would also be immune to increases in the employers' social contribution rate. Therefore, under full insurance, variations in social contributions would not impact the spatial allocation of labor.

[54] Actually, wages would decrease the most in districts with the highest *workplace* unemployment rate. However, due to the strict connection between commuting locations, unemployment rates by workplace and residence are highly correlated.

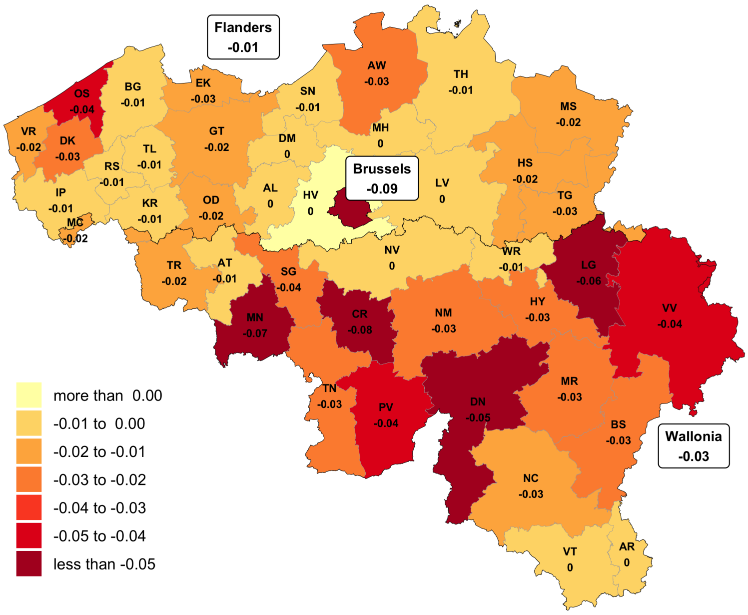


Figure 7: Market-Clearing Gross Wage

Note: Counterfactual percentage change in gross wages after removing the wage constraints ($\underline{w}_j = 0$). Regional values for Brussels, Flanders, and Wallonia are calculated as the simple average across all districts belonging to that region. Values are rounded to the nearest hundredth. Data have been elaborated by the authors (see main text in this Section for details).

due to the increase in employment, the nominal net labor income per resident (y_i^L) would increase everywhere by about 1%. Moreover, housing prices (q_i) would be mostly unaffected in the new equilibrium, and, consequently, the corresponding real measures of income (y_i^E/q_i^β and y_i^L/q_i^β) would be similar to their nominal counterparts. Finally, labor mobility would be barely affected. Workers would not change their residential location (L_i^R), while, after the drop in wages, the reallocation of the labor supply (L_j) would only be modest.

Overall, the effect of eliminating wage rigidities would allow to economy to boost its production by 3%, while at the same time there would be a marginal increase in real net labor income per resident. Finally, an obvious trade-off emerges from this counterfactual scenario, as workers would receive lower wages when employed but would never face the risk of being unemployed. In our model all workers are employed and unemployed, as the unemployment rate roughly characterizes the fraction of time the worker spends in unemployment. However, if workers were heterogeneous, that is, some workers more likely to be unemployed than others, the relaxation of the wage rigidities would generate redistribution between the two groups. We leave this interesting extension to future research.[55]

[55]Notice that there exists another trade-off concerning ex-ante and ex-post efficiency of the labor market (i.e., before and after uncertainty about a worker's labor income is resolved). In this paper, wage flexibility is necessary to achieve an efficient allocation of labor, therefore here we investigate the

Table 3: Relaxing Wage Constraints: Summary Statistics

	Min	p25	p50	p75	Max	Mean
$\%\Delta Y_j$	-0.01	0.01	0.02	0.03	0.11	0.03
$\%\Delta w_j$	-0.09	-0.03	-0.02	-0.01	0.01	-0.02
$\%\Delta y_i^E$	-0.07	-0.04	-0.02	-0.02	-0.01	-0.03
$\%\Delta y_i^L$	0.01	0.01	0.01	0.01	0.01	0.01
$\%\Delta q_i$	0.00	0.00	0.00	0.01	0.02	0.01
$\%\Delta y_i^E/q_i^\beta$	-0.08	-0.04	-0.03	-0.02	-0.01	-0.03
$\%\Delta y_i^L/q_i^\beta$	0.00	0.01	0.01	0.01	0.01	0.01
$\%\Delta L_i^R$	0.00	0.00	0.00	0.00	0.01	0.00
$\%\Delta L_j$	-0.01	-0.01	-0.01	0.00	0.02	0.00

Note: Summary statistics of counterfactual percentage changes across districts. The variables included in the table are: Y_j, gross domestic product; w_j, gross wage paid at the workplace; y_i^E, average nominal wage income per employed resident; y_i^L, average nominal net labor income per resident worker; q_i, housing price; y_i^E/q_i^β, average real wage income per employed resident; y_i^L/q_i^β, average real net labor income per resident worker. β is the housing expenditure share. All variables are in percentage change from the initial equilibrium. Values are rounded to the nearest hundredth. Data have been elaborated by the authors (see main text in this Section for details).

6.3 Policy Experiment: the Optimal Zero-Unemployment Social Contribution Rate

The last quantitative exercise is a policy experiment, in which we ask what would be the optimal social contribution rate (τ_{sc}) such that every workplace achieves full employment while retaining the wage rigidities.

A decrease in the social contribution rate determines a rightward shift in the labor demand, allowing wages to escape the constraint. However, since each district features a different level of TFP and since the local labor supply adjusts to changes in all labor markets, it is not possible to establish a priori the value of the zero-unemployment social contribution rate.

We proceed by re-calculating the equilibrium quantities over a grid of values for the social contribution rate ranging between 0 and 0.5 with increments of 0.01. The aggregate unemployment rate corresponding to each value on the grid, u, is plotted in the left panel of Figure 8. To assess the fiscal effects of this policy, we also plot the percentage change in government spending, G, in Panel 8b.

ex-post efficiency of the labor markets. Instead, another strand of the literature, less often analysed in labor economics, argues that ex-ante considerations may also be important. For example, Drèze and Gollier (1993) write that "the ex ante viewpoint suggests a high degree of ex post equality, driven by considerations of risk sharing efficiency. But ex post equalizing transfers raise issues of moral hazard, productive efficiency, [...]. Reconciling the two conflicting efficiency motivations brings us into the realm of second best analysis". Indeed, if workers are risk averse and we analyse the labor market before the uncertainty is resolved, a certain degree of wage rigidities would actually be a desirable insurance device.

(a) Unemployment Rate (b) Government Spending

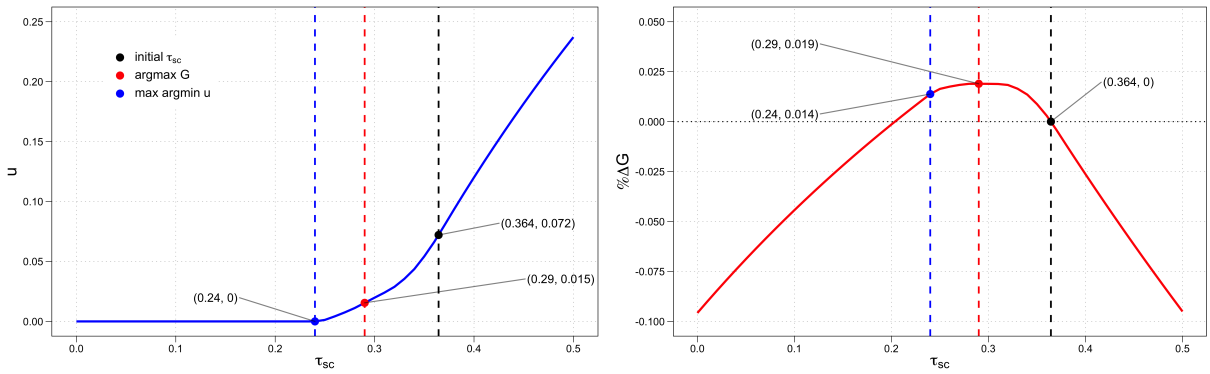


Figure 8: Optimal Social Contribution Rate

Note: Equilibrium values of unemployment and government spending for different values of the social contribution rate (τ_{sc}). Panel (a): Aggregate unemployment rate, u, in level. Panel (b): government spending, G, in percentage change from the initial equilibrium. Data have been elaborated by the authors (see main text in this Section for details).

The initial equilibrium is highlighted by the black dot in each panel, corresponding to a social contribution rate of 0.36 and an aggregate unemployment rate of 7.2%. The zero-unemployment social contribution rate is represented by the blue dot in the figure and it is achieved at the maximum social contribution rate for which unemployment is eradicated. According to our model, the social contribution rate should be reduced by about 12 percentage points down to 0.24 to achieve full employment in all districts. Although the rate would drop by one third of the initial value, the model would still predict an increase in government spending, which would rise by about 1.5%. In general, the model would predict a full range of values for the social contribution rate that would generate an increase in government spending, between 0.20 and the current level. The optimal social contribution rate which would maximize government spending would be 29%, causing public expenditure to increase by 1.9% relative to the initial equilibrium and lowering the aggregate unemployment rate to 1.5%. The implied (general equilibrium) elasticity of the unemployment rate to social contribution rate is around -3.

To better understand the fiscal effects of this policy we provide in Table 4 the percentage change in each component of the government budget constraint. For example, a one percentage point decrease in the social contribution rate from the current level of 0.36 to 0.35 would cause the total amount of social contributions to decrease by 2%. However, this negative effect would be balanced by a much larger decrease in total unemployment benefits payments by 23% as the unemployment rate drops. In addition, as more workers become employed and the economy expands its production, the government would benefits from an increase in labor taxes and capital taxes. Overall, our model suggests that even a modest decrease in the social contribution rate could generate positive effects both in terms of unemployment and fiscal revenue.

Our results relate to a broader literature on the effects of employers' payroll taxes

Table 4: Fiscal Effects of a change in the Social Contribution Rate

τ_{sc}	$\%\Delta G$	$\%\Delta T^L$	$\%\Delta T^K$	$\%\Delta SC$	$\%\Delta UB$
0.00	-0.10	0.42	0.07	-1.00	-1.00
0.05	-0.07	0.35	0.07	-0.81	-1.00
0.10	-0.04	0.29	0.06	-0.65	-1.00
0.15	-0.02	0.24	0.06	-0.49	-1.00
0.20	0.00	0.19	0.05	-0.35	-1.00
0.25	0.02	0.14	0.04	-0.22	-0.98
0.30	0.02	0.08	0.03	-0.11	-0.70
0.35	0.01	0.02	0.01	-0.02	-0.23
0.40	-0.03	-0.05	-0.03	0.04	0.63
0.45	-0.06	-0.12	-0.06	0.09	1.45
0.50	-0.10	-0.18	-0.10	0.13	2.18

Note: Percentage change in each component of the government budget constraint corresponding to different levels of the social contribution rate (τ_{sc}). The variables included in the table are: G, government spending; T^L, labor taxes; T^K, capital taxes; SC, social contributions; UB, unemployment benefits. All variables are in percentage change from the initial equilibrium. Values are rounded to the nearest hundredth. Data have been elaborated by the authors (see main text in this Section for details).

on labor market outcomes. Cahuc et al. (2018) study the effects of temporary hiring credits for firms on employment and wages in France during the Great Recession. Using a calibrated search and matching model for France they find that the hiring credits have positive effects on employment but no effects on wages[56]. Saez et al. (2019) study the effects of a large employer tax cut targeted to young workers in Sweden. While the size of the payroll tax cut is relatively close to our optimal payroll change (about -12 percentage points) the employment effects they estimate on young employment rate is relatively smaller (+ 2 percentage points). Ku et al. (2020) report qualitatively similar results for Norway[57]. The fact that we use a calibration and our elasticity integrates general equilibrium effects makes the comparison from typical studies less straightforward. For instance, discrepancies between micro and macro elasticities are well know in the labor supply literature and extensive margin macro labor supply elasticities lie in the range of 2.3 (while their micro counterpart are about 0.28) as pointed out in Chetty et al. (2011).

[56] Cahuc et al. (2018)'s elasticity of employment to total labor costs induces by the hiring credits is -4 which is in the confidence interval [-6; -2]. Note that the literature has usually estimated conditional aggregate labor demand elasticities to wages in the range [-0.75, -0.15], where -0.3 is a commonly agreed value. Unconditionnal elasticities are thought to be larger and above unity but fewer studies are available (Cahuc et al., 2014)

[57] See e.g. Benzarti and Harju (2020) or Bozio et al. (2019) for evidence that firms react to payroll taxes.

7 Conclusion

In this paper we build a quantitative spatial general equilibrium model to study the geographical variation in unemployment rates in the presence of wage rigidities and when workers are allowed to commute from residence to workplace. In particular, we calibrated the model on Belgian data to study the influence of commuting and wage rigidities on local unemployment rates. We showed that, in this class of models, accounting for preferences for amenities and commuting costs is necessary to generate variability in residential unemployment rates. We found that economic incentives are an important determinant in a worker's choice of residence and workplace. In absence of a behavioral response to local economic conditions, the spatial distribution of economic activity would be less concentrated. Workers would migrate away from districts with high real income at the center of the country as location choices being only driven by the ease of commuting. We also explored the role of unemployment insurance in determining the location choices of workers and the effect on local unemployment. We found that in the case of full unemployment insurance workers would relocate to districts with initially high unemployment rates, therefore accentuating the spatial misallocation of labor. On the other hand, when we shut down the unemployment insurance channel, we did not find any sizeable change in the residential choice of workers, while the labor supply was moderately affected. In this case, unemployment increases in districts with initially low unemployment rates (such as Flanders) and decrease in those with higher rates (such as Brussels), as workers are now looking for jobs where the probability to be employed is higher.

This paper extends the current quantitative spatial equilibrium models with unemployment and wage rigidities, which brings new results to the literature.

First, to gauge the magnitude of wage distortions, we compared the observed gross wage levels with those that would emerge if wages were allowed to clear the labor markets. According to our model, gross wages in this counterfactual world would be on average 2% lower than those observed in the data, with wages dropping more than 5% in high unemployment districts such as Brussels, Charleroi, Mons, and Liège. We also showed that removing wage ridigities would generate significant gains in local and total GDP (+3%) and modest gains in the average real net labor income per resident worker (+1%). This result also casts doubts on the large efficiency gains claimed in Boeri et al. (2021).

Second, we performed a policy experiment to determine the level of the employer's social contribution rate that would allow to achieve full employment in all districts while maintaining in place the wage rigidities. The optimal social contribution rate would fall by 12 percentage points down to 24%, however, it would still be able to increase fiscal revenue by 1.5%. A more moderate decrease in the social contribution rate to 29% would still reduce the national unemployment rate from 7.2% to 1.5%, while at the same time increasing fiscal revenue by about 1.9%.

While our model is flexible enough to capture many of the characteristics of the Belgian economy documented in this paper, it still presents several limitations. First, wage rigidities are exogenous in our model. We argued that wage rigidities could arise from the strict regulation in the Belgian centralized wage bargaining system. A more complete approach would allow for endogenous wage negotiation, allowing for a richer set of uses of the model. Second, we do not model workers' skills heterogeneity. Skills are an important variable for understanding the residential and workplace location choice of workers, as most of low skill workers tend to cluster in the largest cities. Moreover, as high unemployment risk is mostly borne by low skill workers, introducing skills heterogeneity would allow to study inequality across local labor markets[58]. Third, we do not account for the local industry composition. Firm heterogeneity and workers' profession choice can be easily introduced in this class of models, whereas the main constraint is posed by the availability of data. We believe that all these extensions form an interesting avenue for future research, both for a more accurate characterization of the specificities of the Belgian spatial economy, but also for a better understanding of local labor markets more in general.

[58]See e.g. Diamond (2016).

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Appendix

A Additional Data

In this section we provide the maps of housing prices and of the share of commuters outflowing to Brussels. These figures complement the analysis presented in Section 3 in the main text.

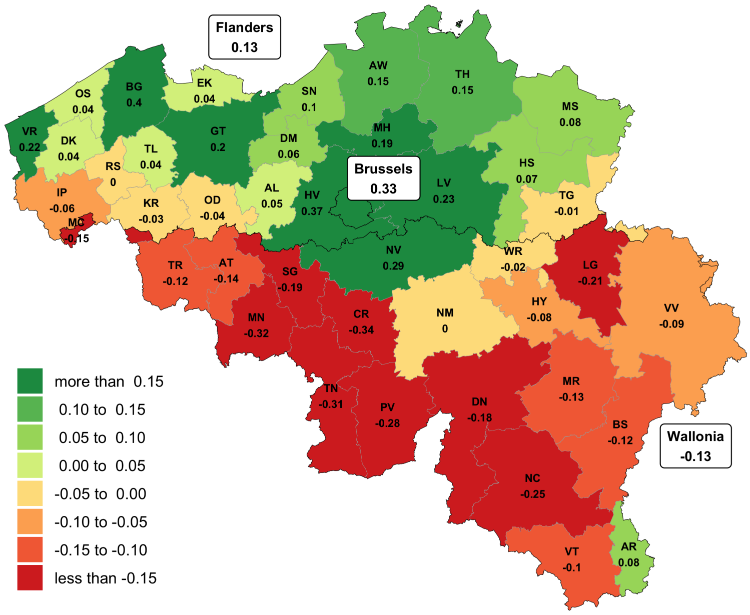

Figure 9: Housing Price

Note: Housing price in percentage deviation from the cross-district mean. Regional values for Brussels, Flanders, and Wallonia are calculated as the simple average across all districts belonging to that region. Data have been elaborated by the authors (see Section 5 for details). All data pertain to the year 2011.

Figure 9 shows the map housing prices in percentage deviation from the mean across all districts. Housing prices are on average higher in Flanders (+13%) than in Wallonia (-13%). The most expensive city where to live in is Bruges (BG), a touristic city close on the seacoast which exhibits housing prices about 40% above the cross-district mean, while Brussels and its neighborhood constitute the second most expensive cluster of districts, with housing prices about 30% higher than the average. In Wallonia housing prices are the lowest around Charleroi (CR), where housing units cost about 34% below the average, while the most expensive district in the region is Arlon (AR), due to the proximity to Luxembourg, an attractive place for firms and high skill workers.

Figure 10 shows the share of local residents commuting to Brussels. The figure displays the core-periphery structure of the commuting network, with large shares of residents commuting towards the center of the country, even from relatively distant districts. As

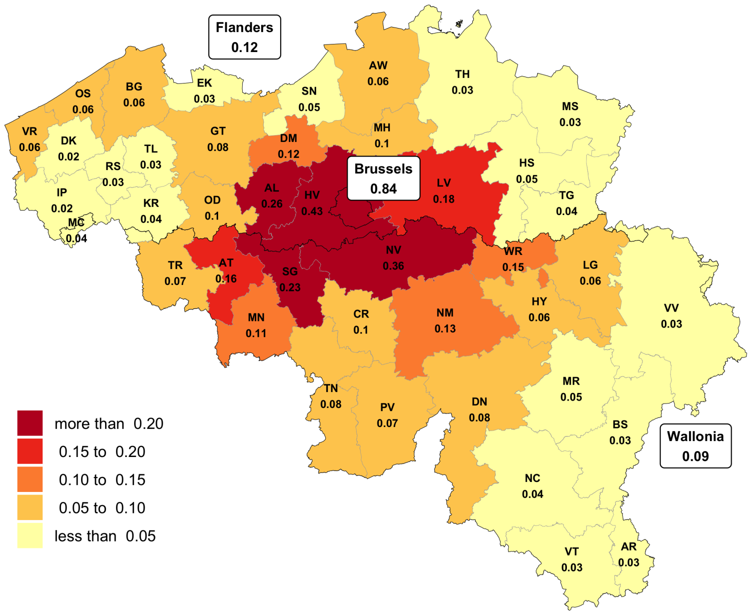

Figure 10: Share of Outflowing Commuters to Brussels

Note: Share of residents commuting to Brussels. Commuters are either employed or unemployed (see model's definition in Section 4). Regional values for Brussels, Flanders, and Wallonia are calculated as the simple average across all districts belonging to that region. Data have been elaborated by the authors (see Section 5 for details). All data pertain to the year 2011.

stated in Section 3, only 43% of workers are also resident in Brussels, while most of the workforce is drawn from neighboring districts. The share of commuters to Brussels in the closest districts hovers around 20%, and it slowly decays with distance at about 10% in the third ring of surrounding districts. Large districts, such as Ghent (GT) and Charleroi (CR), are also affected by the proximity of Brussels, exporting between 5% and 10% of their residents. Perhaps most strikingly, the share of Brussels commuters remains high at about 5% even in the most remote locations, such as Arlon (AR) in the South and Veurne (VR) on the coast.

B Model Appendix

B.1 Worker's Allocation Problem

Each worker has preferences for locations of residence and workplace and can either be employed or unemployed. Each worker chooses where to live and work, and the quantity of final good consumption, and housing.

The timing of the decision process is the following: First, the worker chooses a pair of residence and workplace locations to maximize her expected utility, taking as given the choices of all the other agents in the economy and the resulting equilibrium quantities. Second, the worker can either find a job or become unemployed, depending on the economic conditions prevailing in the chosen workplace location. Nevertheless, the worker will not be stuck forever in neither employment nor unemployment, since she will indefinitely transition across states, where the fraction of time spent in each state is determined by the unemployment rate in the chosen workplace location. Given the employment status, the worker chooses consumption of the final good and housing. Since the model is static, the worker cannot save to smooth consumption across states, however income losses are partially insured via unemployment benefits provided by the social security system.

Formally, a worker ω, living in location i, commuting to workplace j, and being in labor market status $z \in \{E, U\}$, has preferences over consumption of the final good $c^z_{ij\omega}$ and housing $h^z_{ij\omega}$:

$$U^z_{ij\omega} = \frac{b_{ij\omega}}{\kappa_{ij}} \left(\frac{c^z_{ij\omega}}{1-\beta} \right)^{1-\beta} \left(\frac{h^z_{ij\omega}}{\beta} \right)^{\beta}, \tag{44}$$

where $b_{ij\omega}$ is the worker's idiosyncratic preference for a given residence-workplace pair, and $\kappa_{ij} \in [1, \infty)$ is the iceberg commuting cost which does not vary across workers or employment status. Worker's labor income depends on her employment status: if employed ($z = E$), the worker receives the net wage, $w_j(1-\tau_L)$, where τ_L denotes the tax wedge, while if unemployed ($z = U$), she receives unemployment insurance $\rho w_j(1-\tau_L$ proportional to the gross wage, where ρ denotes the replacement rate.

The optimal allocation of consumption and housing yields indirect utility:

$$V^z_{ij\omega} = \frac{b_{ij\omega}}{\kappa_{ij}} \frac{y^z_j}{q^\beta_i}, \tag{45}$$

where q_i denote the rental price for one unit of housing. Since all workers are identical from the local firm's perspective, there is no heterogeneity in wages and unemployment probability. Therefore, the expected state of a worker commuting to workplace location j is equal to the local unemployment rate u_j prevailing in that specific labor market. Under risk-neutrality, the worker's expected indirect utility $V_{ij\omega}$ conditional on residence and workplace is linear in the expected total income y_j:

$$V_{ij\omega} = \frac{b_{ij\omega}}{\kappa_{ij}} \frac{y_j}{q^\beta_i}, \tag{46}$$

where:

$$y_j = (1-u_j)w_j(1-\tau_L) + u_j \rho w_j (1-\tau_L). \tag{47}$$

B.2 Derivation of the Commuting Probabilities

Since the expected indirect utility $V_{ij\omega}$ is monotonically increasing in the preferences parameter $b_{ij\omega}$, its distribution follows directly from that of $b_{ij\omega}$:

$$\begin{aligned}\Pr\Big(V_{ij\omega} \leq V\Big) = & \quad \Pr\left(\frac{b_{ij\omega}}{\kappa_{ij}}\frac{y_j}{q_i^\beta} \leq V\right) \\ = & \quad \Pr\left(b_{ij\omega} \leq \kappa_{ij} q_i^\beta y_j^{-1} V\right) \\ = & \quad e^{-B_{ij}\left(\kappa_{ij} q_i^\beta\right)^{-\epsilon} y_j^\epsilon V^{-\epsilon}} \\ = & \quad e^{-\Psi_{ij} V^{-\epsilon}} \\ = & \quad F_{ij}\Big(V \; ; \; \Psi_{ij}, \epsilon\Big),\end{aligned}$$

where $\Psi_{ij} = B_{ij}\big(\kappa_{ij} q_i^\beta\big)^{-\epsilon} y_j^\epsilon$ and in the third equation we substituted the Fréchet distribution of $b_{ij\omega}$, $F_{ij}\big(V \; ; \; B_{ij}, \, \epsilon\big) = e^{-B_{ij} b^{-\epsilon}}$. Each worker selects the residence-workplace pair yielding the maximum utility, which is also Fréchet distributed:

$$\begin{aligned}\Pr\left(\max_{ij} V_{ij\omega} \leq V\right) = & \quad \Pr\Big(V_{ij\omega} \leq V, \; \forall \, i, j\Big) \\ = & \quad \prod_{ij} \Pr\Big(V_{ij\omega} \leq V\Big) \\ = & \quad \prod_{ij} e^{-\Psi_{ij} V^{-\epsilon}} \\ = & \quad e^{-\left(\sum_{ij} \Psi_{ij}\right) V^{-\epsilon}} \\ = & \quad F\Big(V \; ; \; \Psi, \epsilon\Big),\end{aligned}$$

where $\Psi = \sum_{ij} \Psi_{ij}$. Finally, given the distribution of the expected indirect utility and that of its maximum, the joint commuting probabilities λ_{ij} can be obtained by noticing that the indirect expected utility for the (i, j) pair must be greater than the maximum among all the other possible pairs:

$$
\begin{aligned}
\lambda_{ij} = \quad & \Pr\left(V_{ij\omega} \geq \max_{(r,s)\neq(i,j)} V_{rs\omega}\right) \\
= \quad & \int_0^\infty \Pr\left(\max_{(r,s)\neq(i,j)} V_{rs\omega} \leq v \,\middle|\, V_{ij\omega} = v\right) f_{ij}\big(v \,;\, \Psi_{ij},\, \epsilon\big)\; \mathrm{d}v \\
= \quad & \int_0^\infty e^{-\sum_{(r,s)\neq(n,i)} \Psi_{rs} v^{-\epsilon}} \epsilon \Psi_{ij} v^{-(\epsilon+1)} e^{\Psi_{ij} v^{-\epsilon}}\; \mathrm{d}v \\
= \quad & \int_0^\infty \epsilon \Psi_{ij} v^{-(\epsilon+1)} e^{-\Psi v^{\epsilon}}\; \mathrm{d}v \\
= \quad & \frac{\Psi_{ij}}{\Psi} \int_0^\infty \epsilon \Psi v^{-(\epsilon+1)} e^{-\Psi v^{\epsilon}}\; \mathrm{d}v \\
= \quad & \frac{\Psi_{ij}}{\Psi} \int_0^\infty f\big(v \,;\, \Psi,\, \epsilon\big)\; \mathrm{d}v \\
= \quad & \frac{\Psi_{ij}}{\Psi},
\end{aligned}
$$

where in the third equation we substituted the density of $V_{ij\omega}$, $f\big(v \;;\, \Psi_{ij},\, \epsilon\big) = \epsilon\Psi_{ij} v^{-(\epsilon+1)} e^{\Psi_{ij} v^{-\epsilon}}$, and the integral in the sixth equation is the area under a Fréchet distribution with scale Ψ and shape ϵ. Substituting Ψ_{ij} and Ψ we obtain the final expression:

$$
\lambda_{ij} = \frac{B_{ij}\big(\kappa_{ij} q_i^{\beta}\big)^{-\epsilon} y_j^{\epsilon}}{\sum_{r,s\in\mathbb{N}} B_{rs}\big(\kappa_{rs} q_r^{\beta}\big)^{-\epsilon} y_s^{\epsilon}}.
$$

As shown above, the maximum utility of the indirect utility $V_{ij\omega}$ is distributed as a Fréchet random variable with scale Ψ and shape ϵ. We can therefore calculate the expected utility across the entire population of workers:

$$
\begin{aligned}
E\Big[\max_{ij} V_{ij\omega}\Big] = \quad & \int_0^\infty \epsilon \Psi v^{-\epsilon} e^{-\Psi v^{-\epsilon}}\; \mathrm{d}v \\
= \quad & -\int_\infty^0 \Psi^{\frac{1}{\epsilon}} x^{-\frac{1}{\epsilon}} e^{-x}\; \mathrm{d}x \\
= \quad & \Psi^{\frac{1}{\epsilon}} \int_0^\infty x^{-\frac{1}{\epsilon}} e^{-x}\; \mathrm{d}x \\
= \quad & \Gamma\left(\frac{\epsilon-1}{\epsilon}\right) \Psi^{\frac{1}{\epsilon}} \\
= \quad & \Gamma\left(\frac{\epsilon-1}{\epsilon}\right) \left[\sum_{s,r\in\mathbb{N}} B_{rs}\Big(\kappa_{rs} q_r^{\beta}\Big)^{-\epsilon} y_{rs}^{\epsilon}\right]^{\frac{1}{\epsilon}}
\end{aligned}
$$

where in the second equation we used the change of variable $x = \Psi v^{-\epsilon}$ with $\mathrm{d}x = -\epsilon\Psi v^{-(\epsilon+1)}\mathrm{d}v$, and $\Gamma(\cdot)$ is the gamma function.

B.3 Computational Algorithm for Finding the Equilibrium

Given the parameters $\{\beta, \epsilon, \tau_{SC}, \tau_L, \tau_K, \rho\}$ and $\{\alpha_j, \iota_j, \underline{w}_j\}$, and the exogenous variables for the total labor force L, the spatial distribution of rentiers $\{\mu_i^K\}$, the housing supply $\{H_i\}$, the total factor productivity $\{A_j\}$, and the ease of commuting $\{\mathcal{B}_{ij}\}$, guess an initial value for the commuting probabilities $\{\lambda_{ij}^{(0)}\}$. Iterate the following steps from $t = 0$ until convergence:

Step 1. Given $\lambda_{ij}^{(t)}$, calculate the labor supply:

$$L_j^{(t+1)} = \sum_{i \in \mathbb{N}} \lambda_{ij}^{(t)} L$$

Step 2. Given the labor supply, $L_j^{(t+1)}$, calculate the equilibrium in the labor market as follows:

1. Solve for the market-clearing wage $w_j^{(t+1)}$ such that $E\big(w_j^{(t+1)}\big) = L_j^{(t+1)}$:

$$w_j^{(t+1)} = \frac{\alpha_j A_j \big(L_j^{(t+1)}\big)^{\alpha_j - 1}}{(1 + \tau_{SC})}.$$

2. Check whether the minimum wage constraint is binding, that is, $w_j^{(t+1)} \geq \underline{w}_j$.

   - If $w_j^{(t+1)} \geq \underline{w}_j$, set the unemployment rate $u_j^{(t+1)} = 0$.
   - If $w_j^{(t+1)} < \underline{w}_j$, set $w_j^{(t+1)} = \underline{w}_j$ and calculate the unemployment rate:

$$u_j^{(t+1)} = \frac{L_j^{(t+1)} - \left(\frac{\alpha_j A_j}{\underline{w}_j (1+\tau_{SC})}\right)^{\frac{\alpha_j}{1-\alpha_j}}}{L_j^{(t+1)}}.$$

Step 3. Given the gross wage, $w_j^{(t+1)}$, and the unemployment rate, $u_j^{(t+1)}$, calculate the expected net labor income, $y_j^{(t+1)}$, from equation 1.

Step 4. Given the commuting probabilities, $\lambda_{ij}^{(t)}$, and the expected net labor income, $y_j^{(t+1)}$, calculate the total net labor income, $Y_i^{L(t+1)}$, from equation 19.

Step 5. Given the total net labor income for each residential location, $Y_i^{L(t+1)}$, calculate the housing price, $q_i^{(t+1)}$, from equation 25.

Step 6. Given the expected net labor income, $y_j^{(t+1)}$, and the housing price, $q_i^{(t+1)}$, re-calculate the commuting probabilities:

$$\lambda_{ij}^{(t+1)} = \frac{\mathcal{B}_{ij}\left(q_i^{(t+1)}\right)^{-\epsilon\beta}\left(y_j^{(t+1)}\right)^{\epsilon}}{\sum_{r,s\in\mathbb{N}}\mathcal{B}_{rs}\left(q_r^{(t+1)}\right)^{-\epsilon\beta}\left(y_s^{(t+1)}\right)^{\epsilon}}.$$

Step 8. Check convergence:

- Stop if $||\lambda^{(t+1)} - \lambda^{(t)}|| \leq \varepsilon$ for small ε.
- Otherwise, update using exponential smoothing with smoothing parameter η and continue:

$$\lambda_{ij}^{(t+1)} := \quad \eta\lambda_{ij}^{(t)} + (1-\eta)\lambda_{ij}^{(t+1)}.$$

C Amenities and Observed Local Characteristics

In this section we correlate the calibrated amenities with observable local characteristics. Following Diamond (2016), we collected data on 12 district-level characteristics, which we then grouped into 5 main categories: crime, health, transports, education, and parks and recreation. Among crime characteristics, we include the total number of car thefts per 10.000 cars and the total number of domestic violence per 10.000 households reports. For health characteristics we use the shares of the total local population within a 5km distance from an hospital and an elderly care facility. Similarly, for transport characteristics we include the shares of the total local population within a 5km distance of a train station and to an entry to the highway. Education characteristics are represented by the percentages of the total local population within a 5km distance of a primary school and a secondary school. For parks and recreation characteristics we use the proportion of the total area of the district dedicated to parks and gardens and that dedicated to sports and leisure activities. The crime data have been obtained from the Police official statistics, while the health, transport, education, and land use variables come from Statbel, the Belgian statistical institution.[59] Intuitively, district with higher crime statistics will reduce the amenity value of the location, while an easy access to healthcare facilities, transport connections, and schools will increase its amenity value. Similarly, a location offering more space for sport and leisure activities will have a higher amenities value. Note that, while the amenities in the model are calibrated on 2011 data, crime, transport, and education variables were collected for different years. In particular, crime characteristics are measured in 2013, the health transport and education characteristics in 2020. Our measure of amenities on the other hand is for 2011. Nevertheless, insofar local characteristics tend to persist over time, the variables used in this exercises are a good proxy to their 2011 counterparts. Finally, also notice that amenities in the model are bilateral, that is, they reflect the attractiveness of a specific workplace-residence pair, while the local characteristics are measured only at the place of residence. Therefore, in the regressions run below we use as dependent variable the average amenity value by district of residence, obtained by averaging over the workplace destinations.

Results from linear regressions are report in Table 5. We introduce each category of explanatory variables one by one from column 1 to 5. Column 6 pools all variables. Overall, introducing each category one by one produces expected correlations. The only surprise is in column 1, where "Car theft" has a positive coefficient, whereas a negative one was expected. In the regression with all the variables in column 5 the sign of this variable turns negative, albeit insignificant. On the other hand, the full regression generates a negative coefficient for "Primary school", whereas before it was strongly positive. In general, due to the limited sample size, most of the coefficients in the full regression

[59] More information can be found at https://statbel.fgov.be/fr/themes/datalab/

are imprecisely estimated. Nevertheless, and, more importantly, the adjusted R^2 in all regressions hovers around 30%, while in the full regression it reaches 62%, which suggests that our 12 variables together are able to capture a significant part of the variation in districts' amenities.

Table 5: Calibrated Amenities and Observed Local Characteristics

	(1)	(2)	(3)	(4)	(5)	(6)
Car theft	0.01**					-0.00
	(0.00)					(0.00)
Violence	-0.01***					-0.00***
	(0.00)					(0.00)
Hospital		0.57***				0.43*
		(0.18)				(0.22)
Elderly care		0.50				0.61
		(0.37)				(0.39)
Highway			0.39**			0.09
			(0.15)			(0.12)
Station			0.18			0.06
			(0.22)			(0.21)
Primary school				7.25**		-4.85
				(2.98)		(3.13)
Secondary school				0.05		0.27
				(0.25)		(0.18)
Parks and gardens					3.15**	3.78***
					(1.40)	(1.17)
Sport and leisure					60.15***	15.77
					(13.19)	(14.07)
N	43	43	43	43	43	43
adj. R^2	0.300	0.362	0.222	0.088	0.360	0.618

Note: This table presents estimates of the relationship between calibrated amenities and observed local characteristics. The dependant variable for all columns is the calibrated amenities. We introduce in each column a set of variables related to 5 amenities categories: crime, health, transports, education and parks and recreation. In Column 6 we pool all categories together. Standard errors are reported in parentheses: * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$.

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