DEBT MANAGEMENT IN A WORLD OF FISCAL DOMINANCE

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Debt management in a world of fiscal dominance *

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Abstract

We study the impact of debt maturity management in an economy where monetary policy is 'passive' and subservient to fiscal policy. We setup a tractable model, to characterize analytically the dynamics of inflation, as well as other macroeconomic variables, showing their dependence on the monetary policy rule and on the maturity of debt. Debt maturity becomes a key variable when the monetary authority reacts to inflation and the appropriate maturity of debt can restore the efficacy of monetary policy in controlling inflation. This requires debt management to focus on issuing long bonds. Moreover, we propose a novel framework of Ramsey optimal coordinated debt and monetary policies, to derive analytically the interest rate rule followed by the monetary authority as a function of debt maturity. The optimal policy model leads to the same prescription, long term debt financing enables to stabilize inflation.

Lastly, the relevance of debt maturity in reducing inflation variability is also confirmed in a medium scale DSGE model estimated with US data.

Keywords: Passive Monetary Policy, Government Debt Management, Fiscal and Monetary Policy Interactions, Bayesian estimation, Ramsey policy.

JEL: E31, E52, E58, E62, C11

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"The Federal Reserve cannot make rational decisions of monetary policy without knowing what kind of debt the Treasury intends to issue. The Treasury cannot rationally determine the maturity structure of the interest-bearing debt without knowing how much debt the Federal Reserve intends to monetise."

James Tobin (1963), 'An essay on the principles of debt management'

1 Introduction

The large rise in government debt levels observed in many countries since the 2008-9 recession and which will likely persist in the coming years as a consequence of COVID-19, raises numerous concerns regarding the conduct of monetary policy and in particular its ability to control inflation. Since it is questionable that governments will be able to generate the large required surpluses to finance this debt, resorting to inflation for this purpose, may become the only option for many advanced economies. If a such a scenario materializes, then monetary policy will become subservient to fiscal policy, giving up (at least partially) its control over inflation.

Numerous papers have studied the interactions between monetary and fiscal policies using the workhorse model of the fiscal theory of the price level (e.g. Leeper, 1991; Cochrane, 2001; Bianchi and Melosi, 2017, 2019; Bianchi and Ilut, 2017; Sims, 2011; Cochrane, 2018; Bhattarai et al., 2014; Leeper and Leith, 2016). One of the key findings in this literature is that when monetary policy becomes subservient to fiscal policy, shocks filtered through the government's budget can impact inflation, and conventional ways to react to these shocks (e.g. lowering interest rates in response to a demand contraction) will not be effective.

These important findings are supported by very elaborate models which account for numerous margins of policy and features of the way monetary and fiscal policies are conducted in practice. Yet, we believe that there is a policy margin whose effects the literature has not adequately explored and which is potentially crucial: *debt maturity management*. With few exceptions (that we discuss in detail below) the literature has abstracted from modelling explicitly the maturity of debt, assuming (mainly for simplicity) that governments issue debt in one asset, either a short or a long term bond. This is not an innocuous assumption. When debt can be issued in more than one maturity, then with the appropriate issuance of short and long bonds, debt management can insulate the government's budget from shocks, and thus reduce inflation volatility and even partially restore the efficacy of monetary policy in dealing with shocks.

In this paper we setup a tractable model to analyze how debt management can complement monetary policy in cases where the latter is subordinate to a fiscal authority that does not adjust the surplus to finance debt. Our arguments are rooted into a growing number of papers studying debt management in macroeconomic models (e.g. Angeletos, 2002; Buera and Nicolini, 2004; Lustig et al., 2008; Faraglia et al., 2016, 2019; Debortoli et al., 2017; Bhandari et al., 2017; Bouakez et al., 2018) and in which debt portfolios are chosen to enable governments to smooth distortionary taxes over time. Our paper, instead, investigates what types of debt can better enable monetary policy to stabilize inflation.

In Section 2 we setup a simplistic model consisting of the standard New Keynesian block of equations augmented with a fiscal block - the consolidated budget constraint and a policy rule which determines taxes as a function of the lagged level of debt (e.g Leeper, 1991). Our model is broadly similar to Leeper, 1991; Bianchi and Melosi, 2019, what is different is that debt can be issued in two types of bonds, long and short. Monetary policy becomes subservient to fiscal policy when the nominal interest rate adjusts weakly to inflation and when the response of taxes to debt is also weak, the standard configuration of a passive monetary/ active fiscal regime (Leeper, 1991).

Using this setup we first focus on how, depending on the maturity structure of debt, shocks to government spending and to demand are propagated through the economy; we consider two alternative specifications for the monetary policy rule. In the first case, we assume that the nominal interest rate only reacts to inflation, through the usual systematic response, and in the second case we assume that the nominal rate also reacts the movement of the real rate, through the standard 'stochastic intercept term'. For each case, we derive analytically the path of inflation, and show that it hinges crucially on whether the government finances debt long or short term, and also hinges on the specification of the monetary policy rule. We then turn to characterize optimal debt policy, to find the maturity structure that minimizes intertemporally the variability of inflation deriving from shocks to spending and demand. We establish that to deal with fiscal shocks the policymaker issues as much long debt as possible, financing the position with short term assets. To ward off with demand shocks, the optimal portfolio may feature positive short term debt, when monetary policy only reacts to inflation, but long term debt is optimal when the nominal rate tracks the real rate. Debt maturity becomes irrelevant when interest rates do not respond at all to inflation.

To understand these properties note first that under passive monetary policy, inflation becomes a backward looking process. Reacting systematically to inflation through raising the nominal rate, will generally not accomplish to reduce inflation, and rather will lead inflation to increase persistently. Issuing long term debt is optimal to finance a spending shock, as this enables to reduce the real value of debt both through current and future inflation, reducing the overall impact of the shock on inflation. Hence the optimal strategy is to issue long and finance the position with short term assets.

On the other hand, inflation dynamics following a demand contraction shock can become complex, inflation may drop initially and then (depending on debt maturity) switch sign to turn positive. This happens because a demand shock is filtered through both the government budget constraint and the Euler equation. Following a negative shock, intertemporal solvency requires that the market value of debt increases, to be equal to the value of surpluses that compensate for debt, and so inflation needs to fall. Eventually, however, the (opposite) Euler equation effect dominates and inflation rises.

Whether it is desirable to issue short or long term debt to deal with the demand shock, hinges crucially on the response of monetary policy. When the nominal interest rate only reacts to inflation, then short term debt accomplishes to shield the budget constraint from future inflation, avoiding an even larger initial drop in the price level that would otherwise be required to balance the budget intertemporally. In contrast, when monetary policy tracks the movement in the real rate, the impact of the shock on the Euler equation is muted. Then issuing long debt is optimal and in fact we find a portfolio that can fully insulate government's budget from the demand shocks, so that in equilibrium inflation is fully stabilized. This key finding suggests that the efficacy of monetary policy to control inflation can be restored by debt management, in spite of the fact that we are in a 'passive money' world.

These results emerge from a model with standard ad hoc monetary policy rules, the benchmark fiscal theory framework and this provides a useful link to the literature as well as tractable analytics to investigate the interactions between monetary policy and debt maturity. Since these interactions turn out to be non-trivial we next study how monetary policy would react to any given debt structure, when it can optimally set the nominal interest rate. In Section 3 we turn to a model where a 'Ramsey planner' chooses interest rates and the debt portfolio to minimize distortions stemming from inflation and derive analytically an optimal interest rate rule that takes the maturity of debt into account. The resulting policy rule is broadly similar to the ad hoc rules assumed in Section 2, featuring a systematic response to inflation and a stochastic intercept term, however, now key parameters are tied down by the maturity of debt. More specifically, the coefficient that governs the response of interest rates to inflation reflects the maturity of the long term bonds issued, and the 'stochastic intercept' that determines how the nominal rate reacts autonomously to spending and demand shocks also depends on maturity. The optimal debt portfolio then eliminates the dependence of monetary policy on the composition of debt through eliminating the impact of shocks on the consolidated budget. We find that to accomplish this, debt management needs to again focus on issuing long term debt.

Section 4 demonstrates the relevance of considering the maturity structure of debt as a key variable for macroeconomic stabilization under passive monetary policy, using a more realistic setup, a medium scale DSGE model. Our quantitative model extends the baseline with preferences exhibiting habit formation, and adds more shocks to the economy, in particular shocks to TFP, to markups, to government transfers, besides the spending and demand shocks. The model has a rich structure to match US data and it is broadly similar to the model of Bianchi and Ilut (2017). We estimate the quantitative model with standard Bayesian techniques using the post 1980 sample of the US historical data, when as is well known, monetary policy was not subservient to fiscal policy. We then change the policy parameters to produce a 'passive monetary/active fiscal' regime and study the propagation of shocks under different maturity structures. Our analysis reveals that not only does the maturity of debt matter for the propagation of shocks in the macroeconomy, but also that the effect of debt management policy in reducing the volatility of macroeconomic variables can be considerable. This holds in particular for fiscal shocks. According to our experiments, issuing long term debt is optimal to deal with fiscal shocks.

This paper brings several insights to the literature and is related to several strands. First, considering debt management as a policy margin that can complement monetary policy in pursuing its inflation stabilization goals, is perhaps at odds with the current institutional setup in the US and other OECD countries. According to current practice, the mandate of debt management is to finance debt at low cost given tolerable levels of rollover risk (see e.g. Blommestein and Turner, 2012) and therefore, debt managers do not pursue macroeconomic stabilization objectives as we will assume in this paper. Nevertheless, the idea that monetary policy and debt management should coordinate is not new, and dates back (at least) to the time James Tobin wrote on this subject. According to Tobin (1963) both authorities have powers to influence the entire spectrum of debt, and coordinating actions and aligning objectives seems natural.

Admittedly, changing the mandate of debt management is not a trivial shift in policy and to make the claim that it is optimal involves, at least, modelling explicitly how it would affect private sector expectations over inflation and the costs of debt issuance. [1] We will not consider any of the (possibly numerous) trade-offs in this paper. Moreover, it is worth noting, from the point of view of the model, a full alignment of objectives is perhaps not even necessary. Inflation is determined by the net debt in the hands of the private sector, and since the consolidated budget constraint is sufficient for an equilibrium, quantitative easing can implement the optimal policy outcomes that we identify. However, this property relies on the simplicity of our setting. In a world where both monetary and DM policies operate at the long end of the maturity structure and with conflicting goals, it is not clear that one authority will not undermine the actions of the other. (See e.g. Greenwood et al., 2015). If this is so, then changing the mandate of debt management to include macroeconomic stabilization goals may become important. These are important elements that we leave to future work.

Ours is not the first paper to investigate the role of maturity within the context of the fiscal theory of the price level. Cochrane (2001) was the first to introduce long debt to this model, and study the implications for inflation and optimal policy. Cochrane (2018), revisits this analysis to show that the issuance of long term debt gives rise to a 'stepping on a rake' property of monetary shocks: prices drop following a rise in the nominal interest rate but the sign of inflation is reversed subsequently. In addition, a few recent papers have studied optimal debt portfolios using models in which monetary and fiscal policies are coordinated. Lustig et al. (2008) explore how a Ramsey planner will set inflation, tax and debt issuance policies under full commitment whereas Leeper and Leith (2016) and Leeper et al. (2021) focus on equilibria without commitment. Bhattarai et al. (2015) consider the role of maturity in shaping the inflation output trade-off facing the planner

[1] For instance, when monetary policy is not subordinate of fiscal policy, then coordination between monetary and debt policies could be seen as implicit debt monetization or manipulating interest rates to reduce the cost of financing debt. Our focus however is on a passive monetary policy equilibrium, where both of these elements are already present.

during liquidity trap episodes.

These papers are related to ours, but there are some key differences. First and foremost, Ramsey models with coordinated policies cannot be easily compared to the fiscal theory of the price level model (Leeper and Leith, 2016). Typically, under Ramsey policy the planner has a dual objective to smooth taxes and inflation across time, the resulting combination of optimal policies maybe far from the 'passive monetary/active fiscal' regime which we focus on here. When we turn to study Ramsey policy in Section 3, we constrain the tax rate to be constant, thus fully focusing on an environment where only inflation can adjust to ensure debt solvency. Moreover, the optimal monetary policy rule that we obtain analytically is indeed a standard passive policy rule.

Second, most of these papers consider optimal policies in nonlinear models using global approximation methods,[2] and this implies that they cannot look at the interactions between debt and monetary policy, when the latter is specified with the simple empirically relevant rules that the DSGE literature has employed and identified from the data. In contrast, we utilize a linear model and this enables us to connect with the standard fiscal theory framework of monetary/fiscal interactions (e.g. Leeper, 1991), but also to derive sharp analytical results, and in Section 4 to apply our findings in a medium scale DSGE model.

Finally, our optimal policy model in Section 3 is broadly related to the optimal fiscal/monetary policy literature under *incomplete markets* (e.g. Aiyagari et al., 2002; Schmitt-Grohé and Uribe, 2004; Lustig et al., 2008; Faraglia et al., 2013, 2016 and many others). As in these papers, the lags of the Lagrange multiplier on the government budget constraint are state variables influencing policy through capturing the planner's commitments to a path of the policy variables that enable to adjust appropriately the real value of debt. Our analytical results showing that from complicated Ramsey policy optimality conditions we can derive simple and transparent monetary policy rules, should be of interest.

2 Theoretical Framework

We lay out our baseline model which consists of the standard New Keynesian block of equations, the consolidated budget constraint and a fiscal policy rule that sets (distortionary) taxes as a function of debt outstanding. The model draws from previous studies (e.g. Leeper, 1991; Bianchi and Melosi, 2019), the main difference is that we consider debt issued in both long and short term bonds. For brevity, we define here the competitive equilibrium equations in log-linear form. In the online appendix we describe the background non-linear model and derive the equations from the optimality conditions of the households' and firms' optimization problems. Our model in this section can also be seen as a simplified version of the model we setup in Section 4.

We let $\hat{x}$ denote the log deviation of variable x from its steady state value, $\overline{x}$. The system of the competitive equilibrium equations is the following:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \kappa_2 \hat{\tau}_t + \beta E_t \hat{\pi}_{t+1}, \tag{1}$$

[2] An exception is Cochrane (2001) who draws insights from both linear and nonlinear models.

where $\kappa_1 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\gamma_h > 0$, $\kappa_2 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\frac{\overline{\tau}}{(1-\overline{\tau})} > 0$,

$$\hat{i}_t = \hat{\xi}_t + E_t\left(\hat{\pi}_{t+1} - \hat{\xi}_{t+1}\right) \tag{2}$$

$$\overline{p}_S\overline{b}_S\hat{b}_{t,S} + \overline{p}_S\overline{b}_S\hat{p}_{t,S} + \overline{p}_\delta\overline{b}_\delta\hat{b}_{t,\delta} + \overline{p}_\delta\overline{b}_\delta\hat{p}_{t,\delta} + \frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}\left((\gamma_h+1)\hat{Y}_t + \frac{\hat{\tau}_t}{1-\overline{\tau}}\right) - \overline{G}\hat{G}_t$$
$$= \overline{b}_S(\hat{b}_{t-1,S} - \hat{\pi}_t) + \overline{p}_\delta\overline{b}_\delta(\hat{b}_{t-1,\delta} - \hat{\pi}_t) + \delta\overline{b}_\delta\overline{p}_\delta\hat{p}_{t,\delta} \tag{3}$$

$$\overline{p}_\delta\hat{p}_{t,\delta} = \sum_{j=1}^{\infty}\beta^j\delta^{j-1}\left[E_t\left(-\sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j} - \hat{\xi}_t\right)\right] \tag{4}$$

$$\overline{p}_S\hat{p}_{t,S} = \beta E_t(\hat{\xi}_{t+1} - \hat{\xi}_t - \hat{\pi}_{t+1}) \tag{5}$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t + \epsilon_{i,t} \tag{6}$$

$$\hat{\tau}_t = \phi^R_{\tau,b}\hat{D}_{t-1} \tag{7}$$

$$\left(\overline{s}_{b_\delta}(1-\overline{s}_{b_\delta})\right)\left(\hat{b}_{t,\delta} - \hat{b}_{t,S}\right) = \upsilon\hat{D}_{t-1} \tag{8}$$

(1) is the Phillips curve at the heart of our model. $\hat{\pi}_t$ denotes inflation, $\hat{Y}_t$ denotes the output gap and $\hat{\tau}_t$ is a distortionary tax levied on the labor income of households. Parameters $\eta < 0$ and $\theta > 0$ that determine the constants κ_1 and κ_2, govern the markup over marginal costs of production set by firms and the degree of price stickiness respectively.[3] β is the usual discount factor in the background nonlinear model.

(2) is the log-linear Euler equation which prices a short term nominal asset. The price of this asset is denoted $\hat{p}_{S,t}$ and it is equal to minus the nominal rate $\hat{i}_t$. $\hat{\xi}$ is a standard demand shock which in the background non-linear model reflects a disturbance to preferences changing the relative valuation of current vs. future utility by the household. In standard fashion, a drop in $\hat{\xi}$ makes the household relatively patient, willing to substitute current for future consumption and thus leading to a drop in the demand for output.

(3) is the consolidated budget constraint. We assume that debt is issued in two maturities, a short bond with real issuance denoted $\hat{b}_S$ and long bond, a perpetuity with decaying coupons, denoted $\hat{b}_\delta$ where δ is the coupon decay factor. The term $\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}\left((\gamma_h+1)\hat{Y}_t + \frac{\hat{\tau}_t}{1-\overline{\tau}}\right)$ denotes the revenues of the government from taxation. Parameter γ_h is (in the non-linear background model) the inverse of the Frisch elasticity of labor supply. $\hat{G}_t$ denotes government spending. (3) equates the value of new debt issuance and the value of the government's surplus (top line) to the real market value of debt outstanding in period t. Notice that the prices of long and short bonds are objects $\hat{p}_{t,S}$ and $\hat{p}_{t,\delta}$ defined in (4) and (5) respectively. Substituting into (3) these competitive equilibrium prices and the steady state analogues $\overline{p}_S = \beta$ and $\overline{p}_\delta = \frac{\beta}{1-\beta\delta}$. we get:

$$\beta\overline{b}_S\hat{b}_{t,S} + \beta\overline{b}_S E_t(\hat{\xi}_{t+1} - \hat{\xi}_t - \hat{\pi}_{t+1}) + \frac{\beta\overline{b}_\delta}{1-\beta\delta}\hat{b}_{t,\delta} + \overline{b}_\delta\sum_{j=1}^{\infty}\beta^j\delta^{j-1}\left[E_t\left(-\sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j} - \hat{\xi}_t\right)\right]$$
$$+\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}\left((\gamma_h+1)\hat{Y}_t + \frac{\hat{\tau}_t}{1-\overline{\tau}}\right) - \overline{G}\hat{G}_t$$
$$= \overline{b}_S(\hat{b}_{t-1,S} - \hat{\pi}_t) + \frac{\overline{b}_\delta}{1-\beta\delta}(\hat{b}_{t-1,\delta} - \hat{\pi}_t) + \delta\overline{b}_\delta\sum_{j=1}^{\infty}\beta^j\delta^{j-1}\left[E_t\left(-\sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j} - \hat{\xi}_t\right)\right] \tag{9}$$

[3] We assume price adjustment costs as in Rotemberg (1982). θ governs the magnitude of these costs. When θ equals zero, prices are fully flexible.

where (9) more succinctly summarizes the consolidated budget as an equilibrium object, since we dispensed with prices.

Monetary policy is modelled as interest rate rule (6) featuring a systematic response of the nominal interest rate to inflation, governed by parameter ϕ_π, and a disturbance term $\epsilon_{i,t}$ which represents the monetary policy shock. Note that in what follows we will consider two polar scenarios: In one case we will set $\epsilon_{i,t} = 0$ thus letting monetary policy react to inflation only through the systematic component $\phi_\pi \hat{\pi}_t$, and in the second case we will set $\epsilon_{i,t} = \hat{\xi}_t - E_t \hat{\xi}_{t+1}$ thus letting the interest rule track the movement of the real rate, the latter being determined fully by the demand shock $\hat{\xi}$. Expressing the policy rule as in (6) allows us to summarize these two cases.

Moreover, fiscal policy is rule (7) setting the tax rate as a function of the lagged face value of government debt, defined here $\hat{D}_{t-1} = \overline{b}_S \hat{b}_{S,t-1} + \frac{\overline{b}_\delta}{1-\delta} \hat{b}_{\delta,t-1}$.[4] Parameter $\phi_{\tau,b}$ measures the size of the adjustment of taxes to debt.

Finally, equation (8) specifies a rule for the share of the face value of long term debt over the total face value issued in t. $\overline{s}_{b_\delta}$ denotes the steady state share of long bonds in the government's portfolio and υ governs the response of the share to the face value $\hat{D}$.

The above equations define a model that is similar to the baseline fiscal theory framework commonly employed in the literature. As is well known, in this model parameters $\phi_{\tau,b}$ and ϕ_π are the crucial objects that determine whether monetary policy is subordinate to fiscal policy. Unlike the baseline model of the literature, our model assumes that taxes are distortionary and also that the government issues debt in two different types of assets, short and long term bonds. Since these elements appear to be novel, we now precisely characterize the regions for parameters ϕ_π and $\phi_{\tau,b}$ which give us an equilibrium where fiscal policy dominates monetary policy. The following proposition gives the result:

Proposition 1: Policy configuration

The unique equilibrium when monetary/fiscal policy is passive/active is obtained when the following is satisfied: $\phi_\pi < 1$ *and* $\phi_{\tau,b} < \frac{(\beta^{-1}-1)}{\overline{R}(1-\frac{\overline{\tau}}{\gamma_h(1-\overline{\tau})})}$.

Proof: See Appendix A.1.

Notice that as in Leeper (1991) the 'passive money' equilibrium emerges when coefficient ϕ_π is less than unity. On the other hand, the cutoff for fiscal policy $\phi_{\tau,b}$ to become 'active' is a function of steady state taxes, $\frac{1}{\gamma_h}$ and $\overline{R}$, the Frisch elasticity and the steady state revenue respectively. Since in our model taxes are distortionary, these parameters influence the behavior of hours in response to changes in taxes. Note further that $1 - \frac{\overline{\tau}}{\gamma_h(1-\overline{\tau})} > 0$ needs to hold otherwise the economy is on the wrong side of the Laffer curve. Thus, the threshold $\frac{(\beta^{-1}-1)}{\overline{R}(1-\frac{\overline{\tau}}{\gamma_h(1-\overline{\tau})})}$ is positive.

Our analysis below sets $\phi_{\tau,b} = 0$. We make this assumption to focus on a scenario where fiscal policy does not finance debt through taxation and also for tractability, to be able to derive analytical results. However, it is worth noting that under plausible values for parameters $\overline{\tau}$ and γ_h the cutoff defined in Proposition 1 will actually be close to zero, so that even assuming $\phi_{\tau,b} > 0$ would not change our results.

2.1 Debt Maturity in a passive money world

Changes in the maturity of debt will in our model exert an influence on the equilibrium, as Ricardian equivalence does not hold. Since the model is linear the crucial quantities are $\overline{b}_S, \overline{b}_\delta$, the steady state

[4] The decaying coupon bond is priced in the model exactly as a portfolio of zero coupon bonds, the principal on maturity j is δ^{j-1}. Thus the total face value is the quantity $\overline{b}_\delta \hat{b}_{\delta,t-1}$ times the principal payments $(1 + \delta + \delta^2 + ...)$.

values of the two types of debt. These are the objects that influence equilibrium inflation since, as is evident from (9), they interact with inflation in the government budget constraint. In contrast, the quantities $\hat{b}_\delta, \hat{b}_S$ (the log deviations from steady state) have no bearing on the equilibrium.[5]

Due to this property, that (zero order) steady state portfolios are only relevant, we can fully characterize macroeconomic outcomes through studying the responses of the economy to one off shocks in spending, demand and $\epsilon_{i,t}$. We now derive analytical formulae describing the dynamics of inflation, when these shocks can occur in period t, assuming that all innovations are i.i.d and no further shocks can occur thereafter. Without loss of generality we set initial debt $\hat{b}_{t-1,\delta} = \hat{b}_{t-1,S} = 0$. The following proposition provides a general formula for the path of inflation:

Proposition 2. *Assume that shocks can only occur in period t. The path of inflation is given by:*

$$\hat{\pi}_t = \eta_1 \epsilon_{i,t} + \eta_2 \hat{G}_t + \eta_3 \hat{\xi}_t$$

$$\hat{\pi}_{t+\bar{t}} = \phi_\pi^{\bar{t}-1}\Big[(1+\phi_\pi\eta_1)\epsilon_{i,t} + \eta_2\phi_\pi\hat{G}_t + (\eta_3\phi_\pi - 1)\hat{\xi}_t\Big], \quad \bar{t} = 1, 2, ...$$

where η_1, η_2, η_3 are:

$$\eta_1 \equiv -\left[\frac{\bar{b}_\delta\beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right] \Big/ \left[\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \bar{b}_S + \frac{\bar{b}_\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right]$$
$$\eta_2 \equiv \overline{G} \Big/ \left[\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \bar{b}_S + \frac{\bar{b}_\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right]$$
$$\eta_3 \equiv \left[\beta\bar{b}_S + \frac{\bar{b}_\delta\beta(1-(1-\delta)\delta\beta\phi_\pi)}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right] \Big/ \left[\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \bar{b}_S + \frac{\bar{b}_\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right]$$

and where $\overline{R} = \frac{\tau(1+\eta)\overline{Y}}{\eta}$ denotes the government's revenue in steady state.

Proof. See Appendix A.2

These expressions give us the responses as functions of the model's deep parameters, $\gamma_h, \kappa_1, \beta, \phi_\pi$... and of the portfolio $\bar{b}_S$ or $\bar{b}_\delta$. They thus reveal the impact of maturity on inflation, and also that of varying the debt level, (i.e. increasing autonomously $\bar{b}_S$ or $\bar{b}_\delta$). In our analysis we will mostly focus on the impact of maturity holding debt constant. Given this, we can further simplify the expressions for the ηs by making use of the steady state relation $\frac{\overline{S}}{1-\beta} = \bar{b}_S + \frac{\bar{b}_\delta}{1-\beta\delta}$ (equating the present value of the surplus to the value of debt in steady state) as:

$$\eta_1 \equiv -\left[\frac{\bar{b}_\delta\beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right] \Big/ \left[\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\bar{b}_\delta\phi_\pi\beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right]$$
$$\eta_2 \equiv \overline{G} \Big/ \left[\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\bar{b}_\delta\phi_\pi\beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right] \tag{10}$$
$$\eta_3 = \beta\left[\frac{\overline{S}}{(1-\beta)} + \frac{\beta\delta^2\phi_\pi\bar{b}_\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right] \Big/ \left[\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\bar{b}_\delta\phi_\pi\beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right]$$

[5]For the same reason, the specification of the debt issuance rule (8) is also not important. In other words, our framework is not suitable to talk about the optimal rebalancing of debt portfolios over the business cycle. This would require a non-linear model. It is worth noting however, that a very persistent finding in the theoretical debt management literature (see for example Angeletos, 2002; Buera and Nicolini, 2004; Lustig et al., 2008; Faraglia et al., 2019) is that optimal portfolios are constant (or roughly constant), a property that also appears to be relevant in practice. Whether or not this would hold in our framework can be explored in future work.

Notice that now the impact of varying the maturity of debt on inflation, holding the debt level constant, can be easily seen from the partial derivatives of η_1, η_2, η_3 with respect to $\bar{b}_\delta$. These expressions, then reveal a few well known properties of the passive money model, with regard to the response of inflation to shocks.

Consider the case of a monetary policy shock, $\epsilon_{i,t} > 0$, assuming that this is not yet tied down to the demand shock. If the government issues only short term debt, $\bar{b}_\delta = 0$, a rise in the nominal rate will have no effect on inflation in t (since $\eta_1 = 0$) but will increase inflation in $t+1, t+2, \ldots.$. In contrast, in the case where $\bar{b}_\delta > 0$, a rise in the policy rate, will trigger first a drop in inflation to subsequently increase inflation, since $1 + \eta_1\phi_\pi > 0$. The switching sign of inflation in response to an interest rate shock is what Sims (2011) and Cochrane (2018) refer to as 'stepping on a rake'.

Moreover, consider the case of a shock to the spending level. In the passive money model, such a shock is inflationary, and this is verified here by η_2 being positive. An increase in spending needs to be compensated by higher inflation (to make government debt solvent) and the longer is debt maturity, the smaller is the required impact of the shock in t since a persistent increase in inflation can reduce the real pay out of long term debt (e.g. Cochrane, 2001). As (10) reveals, this well known property requires $\phi_\pi > 0$ since when $\phi_\pi = 0$ the response of inflation to the spending shock will be concentrated in period t only.

A positive shock in $\hat{\xi}$, a standard innovation to demand, will increase inflation in period t (assuming that $\bar{b}_\delta$ is not too negative), but it will lower inflation in $t+1, t+2, \ldots.$ The effect of this shock is thus qualitatively similar to that of an expansionary monetary policy shock, but not quantitatively, since the coefficients η_1 and η_3 in general differ. This difference arises here because demand shocks are filtered through the government's budget whereas monetary policy shocks are not. We will later show that due to this difference in the coefficients, monetary policy's ability to mitigate demand shocks is impaired. However, we will also show, that a suitable DM strategy can restore the ability of the monetary authority to ward off $\hat{\xi}$ shocks, when portfolios can absorb the impact of these shocks from the consolidated budget.

Finally, note that knowing the path of inflation analytically enables us to obtain the path of output in the model. Combining the solution for $\hat{\pi}$ with the Phillips curve we can show that:

$$\hat{Y}_t = \frac{1}{\kappa_1}\left((\eta_1 - \beta(1+\phi_\pi\eta_1))\epsilon_{i,t} + \eta_2(1-\phi_\pi\beta)\hat{G}_t + (\eta_3 - \beta(\eta_3\phi_\pi - 1))\hat{\xi}_t\right)$$

(11)

$$\hat{Y}_{t+\bar{t}} = \phi_\pi^{\bar{t}-1}\frac{1-\phi_\pi\beta}{\kappa_1}\left((1+\phi_\pi\eta_1)\epsilon_{i,t} + \eta_2\phi_\pi\hat{G}_t + (\eta_3\phi_\pi - 1)\hat{\xi}_t\right), \quad \bar{t} \geq 1$$

which demonstrates that the same forces that influence the response of inflation to the shocks (summarized in coefficients η) also influence the path output.

2.1.1 The objective of DM

Optimal maturity management in our model consists in choosing $\bar{b}_\delta$ to minimize the volatility of inflation, given the equilibrium paths we derived in the previous paragraph. Formally, we will solve the following problem:

$$\bar{b}_\delta = \arg\min \sum_{\bar{t}\geq 0} \beta^{\bar{t}} \sigma^2_{\hat{\pi},t+\bar{t}} \tag{12}$$

where the measure of volatility is the conditional variance of inflation, before the shocks are realized. Given the previous formulae, and continuing to assume that shocks are i.i.d we can derive $\sigma^2_{\hat{\pi},t+\bar{t}}$ as:

$$\sigma^2_{\hat{\pi},t} = \eta_1^2\sigma^2_{\epsilon_i} + \eta_2^2\sigma^2_{\hat{G}} + \eta_3^2\sigma^2_{\hat{\xi}} + 2\eta_1\eta_3\sigma_{\epsilon_i,\xi}$$

(13)

$$\sigma^2_{\hat{\pi},t+\bar{t}} = \phi_\pi^{2(\bar{t}-1)}\left[(1+\phi_\pi\eta_1)^2\sigma^2_{\epsilon_i} + (\eta_2\phi_\pi)^2\sigma^2_{\hat{G}} + (\eta_3\phi_\pi - 1)^2\sigma^2_{\hat{\xi}} + 2(\eta_3\phi_\pi - 1)(1+\phi_\pi\eta_1)\sigma_{\epsilon_i,\xi}\right] \quad \bar{t} = 1, 2, ...$$

As discussed previously, we will separately consider $(\sigma^2_{\epsilon_i}, \sigma_{\epsilon_i,\xi}) \in \{(0,0), (\sigma^2_{\hat{\xi}}, \sigma^2_{\hat{\xi}})\}$ so that in one case monetary policy reacts to the demand innovation through the systematic component of the interest rate rule only, and in the other case monetary policy also responds to the demand shock, changing the nominal rate to match the movement in the real interest rate.

2.2 Demand shocks

We begin with focusing on demand shocks assuming no other policy response to these shocks, beyond the systematic response to inflation in the policy rule. Consider the case where a demand shock occurs in period t. The relevant coefficient that measures the response of inflation to the shock is η_3 in (10).

For simplicity, let us focus on the case where $\delta = 1$ (long bonds are consols). η_3 becomes

$$(14) \qquad \eta_3 = \beta\left[\frac{\overline{S}}{(1-\beta)} + \frac{\beta\phi_\pi\bar{b}_1}{(1-\beta)(1-\phi_\pi\beta)}\right] / \left[\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\bar{b}_1\phi_\pi\beta}{(1-\beta)(1-\phi_\pi\beta)}\right]$$

To investigate this formula notice first that under the assumption that the government is a debtor, so that $\overline{S} > 0$ and $\overline{R} > 0$, we have that $\eta_3 = 0$ when $\bar{b}_1 = -\frac{\overline{S}}{\beta\phi_\pi}(1-\phi_\pi\beta) < 0$. Moreover, the derivative $\frac{d\eta_3}{d\bar{b}_1}$ is strictly positive and in the limit when $\bar{b}_1$ becomes infinite, $\eta_3 = \beta$. In other words, issuing negative long term debt (equivalently positive short debt) makes inflation zero in period t, and issuing more long bonds increases the response of inflation to the demand shock.

To gain insights into why the long bond position affects inflation in this way, consider the intertemporal constraint of the government in period t. Assuming wlog that debt before the shock, in $t-1$, is at its steady state value we can write:

$$(15) \qquad \sum_{j\geq 0}\beta^j\overline{R}(\gamma_h+1)\hat{Y}_{t+j} - \overline{S}\hat{\xi}_t\frac{\beta}{1-\beta} = -\bar{b}_S\hat{\pi}_t - \bar{b}_1\sum_{j=0}^{\infty}\beta^j\left[\sum_{l=0}^{j}\hat{\pi}_{t+l}\right] - \bar{b}_1\hat{\xi}_t\frac{\beta}{1-\beta}$$

The two terms on the LHS of this equation capture the impact of the shock on the government's intertemporal surplus. The leading term on the LHS represents the effect on output which changes government revenues, whereas the second term is the effect of a shock $\hat{\xi}$ on the present discounted value of the surplus holding output constant. Analogously, the RHS of the equation tells us how the real value of debt will adjust to the shock. The first two terms measure the impact of inflation on the real payout of debt (short and long respectively), whereas the last term captures the shock impact on the real long bond price.

(15) has to hold for debt to be solvent and notice that there are two different forces that jointly ensure that this will be so. First, a standard fiscal theory argument, inflation will adjust, given debt maturity, so that (15) is satisfied and second, a debt management argument, given inflation we can find $\bar{b}_S, \bar{b}_1$ to satisfy intertemporal solvency.

Consider first how inflation responds to the shock given maturity. Notice that a negative demand shock affects the LHS of (15) in two ways. Since the shock is probably going to be contractionary, the term $\sum_{j\geq 0}\beta^j\overline{R}(\gamma_h+1)\hat{Y}_{t+j}$ is going to fall. In contrast, $-\overline{S}\hat{\xi}_t\frac{\beta}{1-\beta}$ will rise, since the shock will reduce the real interest rate at which future surpluses are discounted. Whichever of these two effects dominates will determine whether ultimately the RHS of (15) needs to rise or fall to make debt solvent.

Consider first the case where only short term debt is issued. Using $\hat{Y}_{t+j} = \frac{1}{\kappa_1}(\hat{\pi}_{t+j} - \beta\hat{\pi}_{t+j+1})$ from the Phillips curve we can write (15) as

$$\underbrace{\overline{R}\frac{(\gamma_h+1)}{\kappa_1}\hat{\pi}_t}_{=\sum_{j\geq 0}\beta^j \overline{R}(\gamma_h+1)\hat{Y}_{t+j}} \quad -\overline{S}\hat{\xi}_t\frac{\beta}{1-\beta} = -\overline{b}_S\hat{\pi}_t \tag{16}$$

and thus inflation must turn negative (equal to $\frac{\beta}{1-\beta}\frac{\overline{S}\hat{\xi}_t}{\overline{b}_S+\overline{R}\frac{(\gamma_h+1)}{\kappa_1}}$) to balance the intertemporal budget. The demand shock thus increases the value of the surplus and negative inflation is required to also increase the real value of debt.

Now suppose that both short and long bonds are issued. Going back to (15), it is evident that the final term on the RHS $-\overline{b}_1\hat{\xi}_t\frac{\beta}{1-\beta}$ exceeds 0 and thus compensates for the higher surplus on the LHS, whereas the term in the middle, $\sum_{j=0}^{\infty}\beta^j\left[\sum_{l=0}^{j}\hat{\pi}_{t+l}\right]$ will likely be of opposite sign than $\hat{\pi}_t$, since as we previously showed, inflation will switch sign in $t+1$. If this is indeed so, then issuing long bonds will imply a reduction in the real payout of long term debt after the shock, which will need to be compensated by more negative inflation in t to satisfy (15).

We can illustrate this, by considering the case where only long term debt is issued, $\overline{b}_1 = \overline{S}$. Using $\hat{\pi}_{t+\bar{t}} = \phi_\pi\hat{\pi}_{t+\bar{t}-1} - \hat{\xi}_t\mathcal{I}_{\bar{t}=1}$, we can write (15) as

$$\overline{R}\frac{(\gamma_h+1)}{\kappa_1}\hat{\pi}_t = \underbrace{-\frac{\overline{b}_1}{(1-\beta)(1-\beta\phi_\pi)}\left(\hat{\pi}_t - \hat{\xi}_t\beta\right)}_{-\overline{b}_1\sum_{j=0}^{\infty}\beta^j\left(\sum_{l=0}^{j}\hat{\pi}_{t+l}\right)} \tag{17}$$

yielding

$$\hat{\pi}_t = \frac{1}{\left(\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)(1-\beta\phi_\pi)}\right)}\frac{\beta\overline{S}}{(1-\beta)(1-\beta\phi_\pi)}\hat{\xi}_t < \frac{\beta}{1-\beta}\frac{\overline{S}\hat{\xi}_t}{\overline{R}\frac{(\gamma_h+1)}{\kappa_1}+\frac{\overline{S}}{1-\beta}} \tag{18}$$

where the final inequality is inflation under short debt issuance in (16). According to (18), focusing on issuing long bonds produces a larger drop in $\hat{\pi}_t$ than when only short debt is issued. More balanced portfolios with both short and long bonds are in between these two cases.

Moreover, notice that the finding that issuing long bonds makes inflation in t more negative (to compensate for future positive inflation) also explains why $\hat{\pi}_t = 0$ when $\overline{b}_1 = -\frac{\overline{S}}{\beta\phi_\pi}(1-\phi_\pi\beta) < 0$. When the government saves in a long term asset, the value of long term wealth will drop due to positive future inflation, so that $\hat{\pi}_t$ does not have to turn negative to increase the value of short term debt.

Consider now, the debt management argument, how given the path of inflation, the maturity structure of debt can be targeted to satisfy (15). Suppose that we can fix coefficient $\eta_3 \in (0,\beta)$ to not depend on debt maturity. Then, using $\hat{\pi}_t = \eta_3\hat{\xi}_t$ along with the previous derivations, we can show that intertemporal solvency requires:

$$\overline{R}\frac{(\gamma_h+1)}{\kappa_1}\eta_3\hat{\xi}_t - \overline{S}\hat{\xi}_t\frac{\beta}{1-\beta} = -\frac{(\overline{S}-\overline{b}_1)}{1-\beta}\eta_3\hat{\xi}_t - -\frac{\overline{b}_1}{(1-\beta)(1-\beta\phi_\pi)}\hat{\xi}_t\left(\eta_3-\beta\right) - \overline{b}_1\hat{\xi}_t\frac{\beta}{1-\beta} \tag{19}$$

We can easily find the value of $\overline{b}_1$ that satisfies this equation.

Our exercise compiles both forces as we will look for portfolios that minimize the variability of inflation when inflation simultaneously adjusts to satisfy (15). We now turn towards this optimal debt policy when shocks to demand are driving inflation.

2.2.1 The optimal DM policy

An optimal debt policy will trade off the costs of negative inflation in t with the costs of having positive inflation rates after t. Maintaining the assumption that long bonds are consols the optimal debt management program becomes:

$$\overline{b}_1^* = \arg\min \sum_{\bar{t}\geq 0} \beta^{\bar{t}} \sigma^2_{\hat{\pi},t+\bar{t}} = \arg\min \ \sigma^2_{\hat{\xi}} \left(\eta_3^2 + \frac{\beta}{1-\phi_\pi^2 \beta} (\eta_3 \phi_\pi - 1)^2 \right) \tag{20}$$

given η_3 defined in (14). The following proposition gives us the optimal coefficient $\eta_3(b_1^*)$ on inflation.

Proposition 3. *The optimal portfolio solves $\eta_3(b_1^*) = \phi_\pi \beta$ when $\phi_\pi > 0$.*

Proof. See Appendix A.3

DM will thus choose the optimal portfolio so that the impact effect of the shock on inflation is proportional to coefficient ϕ_π. This optimal policy produces the following path of inflation:

$$\hat{\pi}_t = \phi_\pi \beta \hat{\xi}_t$$

$$\hat{\pi}_{t+\bar{t}} = \phi_\pi^{\bar{t}-1} (\phi_\pi^2 \beta - 1) \hat{\xi}_t \quad \bar{t} = 1, 2, ...$$

and thus policy trades-off the fall in inflation in t (in response to a negative shock $\hat{\xi}_t$) with the subsequent rise in inflation. Completely eliminating deflation in period t is never optimal.

What portfolio implements this optimal policy? When long bonds are consols we have:

$$\overline{b}_1^* = \frac{(1-\beta)(1-\phi_\pi \beta)}{\beta \phi_\pi (\beta - \phi_\pi)} \left[\phi_\pi \overline{R} \frac{(\gamma_h + 1)}{\kappa_1} + \frac{\overline{S}(\phi_\pi - \beta)}{(1-\beta)} \right]$$

which implies that the long bond issuance could be positive or negative depending on the relative magnitude of β and ϕ_π.

To gain insight into what $\overline{b}_1^*$ is, over a plausible range of values of model parameters, we calibrate in Table 1. We set $\beta = 0.995$ (assuming a quarterly model horizon), $\theta = 17.5$ and $\eta = -6.88$ which give us a Phillips curve with slope coefficient of around 0.3 and a markup equal to 17% in steady state.[6] We normalize output to be equal to 1 in steady state and set $\overline{G}$ equal to 20 percent of $\overline{Y}$. Finally, we set the debt to GDP ratio equal to 60 percent of annual output. Table 1 reports the corresponding value of taxes that satisfy the budget constraint in steady state where inflation is zero.

Given this parameterization of the model we get $\overline{b}_1^* = -0.042$ when $\phi_\pi = .2$ ($\overline{b}_S = 10.84$). When $\phi_\pi = .5$ we obtain $\overline{b}_1^* = -0.005$ ($\overline{b}_S = 3.56$). Finally, in the case where $\phi_\pi = .95$ we have $\overline{b}_1^* = 0.007$ and $\overline{b}_S = 1.00$. The optimal policy thus requires to finance debt short term.

2.3 Monetary Policy Shocks and Demand Shocks

The previous paragraph assumed that monetary policy responds to demand shocks only through the systematic component of the Taylor rule. We now turn to study the impact of debt maturity when monetary policy responds to the demand shock through setting $\epsilon_{i,t} = \hat{\xi}_t$. It is well known, that in the standard NK model, where inflation does not have to satisfy the intertermporal budget, such a policy accomplishes to eliminate the impact of the shock, a property widely known as 'divine coincidence'.[7]

[6] These values are consistent with the rest of the literature (see for example Schmitt-Grohé and Uribe, 2004; Faraglia et al., 2013).

[7] To be precise, divine coincidence would only approximately hold under 'active' monetary policy, when $\kappa_2 \approx 0$, since we assume distortionary taxes. It would definitely hold under perfect tax smoothing, when taxes do not respond to demand shocks. In our model this would require debt management to 'complete the market' (Angeletos, 2002; Buera and Nicolini, 2004).

Table 1: **Calibration**

Parameter	Value	Label
β	0.995	Discount factor
θ	17.5	Price Stickiness
η	-6.88	Elasticity of Demand
γ_h	1	Inverse of Frisch Elasticity
$\overline{\tau}$	0.248	Tax Rate
$\overline{Y}$	1	Output
$\overline{G}$	0.2	Spending

Notes: The table reports the values of model parameters. β notes the discount factor chosen to target a steady state annual real interest rate of 2 percent. Parameter η is calibrated to target markups of 17 percent in steady state. θ is calibrated as in Schmitt-Grohé and Uribe (2004). Finally, the steady state level of debt is assumed equal to 60 percent of GDP (at annual horizon), and the level of public spending is 20 percent of aggregate output which is normalized to unity in steady state.

However, under passive money, this well known property does not generally hold. Monetary policy is not able to ward off the shock completely, as the demand disturbance exerts an influence on the consolidated budget constraint. Since inflation needs to adjust for the constraint to hold, monetary policy faces an additional constraint that needs to be satisfied.

Consider the path of inflation predicted by the model. Combining the analytical expressions in Proposition 2 we get:

$$\hat{\pi}_{t+\bar{t}} = \phi_\pi^{\bar{t}}(\eta_3 + \eta_1)\hat{\xi}_t, \quad \bar{t} = 0, 1, 2, ...$$

Notice first that now the response of inflation to the shock is monotonic. Inflation will not switch sign in $t+1$. Moreover, inflation will generally be different from zero, unless policy can set $\eta_1 = -\eta_3$ which accomplishes to fully stabilize inflation.

This condition generally fails to hold in the model. Using previous derivations we can obtain:

$$\eta_1 + \eta_3 = \beta\left[\frac{\overline{S}}{(1-\beta)} - \frac{\delta\overline{b}_\delta}{(1-\beta\delta)}\right] \Big/ \left[\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\overline{b}_\delta\phi_\pi\beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right]$$

Therefore, to fully insulate inflation from the demand shock, we need to set $\frac{\overline{S}}{(1-\beta)} = \frac{\delta\overline{b}_\delta}{(1-\beta\delta)}$. In the case where $\delta = 1$ as in the previous paragraph, this means that debt management should focus on issuing exclusively long term debt.

To understand why this is so, consider again (15), which we can now express as

$$\overline{R}\frac{(\gamma_h+1)}{\kappa_1}\hat{\pi}_t - \overline{S}\hat{\xi}_t\frac{\beta}{1-\beta} = -\overline{b}_S\hat{\pi}_t - \overline{b}_1\frac{\hat{\pi}_t}{(1-\beta)(1-\beta\phi_\pi)} - \overline{b}_1\hat{\xi}_t\frac{\beta}{1-\beta} \tag{21}$$

Recall that the final term on the LHS measures how a change in the discount rate affects the value of surpluses, whereas the final term on the RHS is the change market value of debt due to the rise of the real long bond price that follows a negative $\hat{\xi}_t$ shock. When debt management sets $\overline{b}_1 = \overline{S}$ these two terms will cancel out and evidently inflation in t will have to be zero for (21) to hold. Inflation will also be zero in all other periods.

We highlight these findings with the following proposition:

Proposition 4. *Consider the case where the monetary policy rule is of the form $\hat{i}_t = \phi_\pi\hat{\pi}_t + \hat{\xi}_t$, and the nominal interest rate responds to changes in the real rate. There exists a debt management* *strategy,* $\frac{\overline{S}}{(1-\beta)} = \frac{\delta \overline{b}_\delta}{(1-\beta\delta)}$ *that accomplishes to fully stabilize inflation. When* $\delta = 1$ *(long bonds are consols) the optimal debt management strategy is to issue long term debt only.*

2.4 Fiscal Shocks

We consider now how maturity influences the impact of a fiscal shock. From (10), coefficient η_2 is decreasing in $\overline{b}_\delta$ insofar as $\phi_\pi > 0$. Moreover, from Proposition 2, inflation continues to respond to the fiscal shock beyond period t according to $\hat{\pi}_{t+\bar{t}} = \phi_\pi^{\bar{t}} \eta_2 \hat{G}_t$. Thus, it is clear that the more long debt is issued, the lower is the effect of spending on inflation in all periods.

This result is intuitive: In our model, the real interest rate is not a function of the spending levels, and a shock to spending impacts inflation only through its effect on the consolidated budget. Then, a rise in spending reduces the government surplus, and requires an increase in inflation to bring the intertermporal budget into balance. When the monetary authority attempts to fight back inflation (but still its reaction is 'too weak', raising the nominal rate less than inflation) it only accomplishes to maintain persistently higher price level growth. With long debt, higher future inflation translates into a larger reduction in the real payout of debt and a smaller inflation rate is required in all periods to bring the intertermporal budget into balance.

2.5 Discussion: The importance of the monetary policy rule

Our analytical findings highlighted that the impact of varying debt maturity on macroeconomic outcomes depends on the specification of the monetary policy rule. As we saw, when the monetary authority only responded to inflation through the systematic component, then full inflation stabilization in response to demand shocks was not possible. In contrast, in the case where the policy rule changed the nominal rate to match the movement of the real interest rate, there was a debt management strategy that fully stabilized inflation. In both cases considered DM could complement monetary policy in pursuing the objective of reducing inflation variability.

Moreover, our analytical formulae revealed ϕ_π as another key parameter that influences the interplay between debt maturity and inflation. When $\phi_\pi = 0$, there is basically no difference between long and short term financing (of fiscal shocks) and debt maturity exerts no influence on inflation. When the nominal rate does not respond to inflation, then inflation displays no persistence and its response to shocks is concentrated in period t. Then, the effect of inflation on the real value of long and short bonds is the same.

The important lesson that we can draw from these findings is that it is not only debt policy that can help monetary policy manage inflation, but also it is important that monetary policy adopts the right kind of rule to allow debt policy to exert an influence. According to our findings, this rule should feature both a strong systematic response to inflation setting $\phi_\pi >> 0$, but also allow the nominal rate to respond to fluctuations in the real interest rate.

These are of course only partial results. We have not assumed that monetary policy is optimal, allowing it to respond to shocks and to the maturity structure of debt. A fully optimal monetary policy could turn out to follow a very different interest rate rule and analogously the optimal debt portfolios could also be different than the ones we have found in this section. We will turn to the case of optimal monetary policy in Section 3 of the paper.

2.6 Extensions

2.6.1 Optimal portfolios with simultaneous shocks

Thus far we have studied demand and spending shocks in isolation. However, since both types of shocks can hit the economy in period t, it is purposeful to discuss debt management strategies that can deal with the occurrence of both types of shocks. We saw previously that in order deal with fiscal shocks the optimal DM strategy calls for making the long bond position as large as possible, and financing with short term assets. In contrast, for demand shocks optimal long bond positions were finite. There is thus a tension when both shocks can occur simultaneously, and DM needs to find a portfolio that balances the benefit from insulating inflation against demand shocks with the benefit of dealing effectively with fiscal shocks.

In Appendix A.4 we derive the FONC of the optimal policy problem when the two types of shocks coexist. The following proposition summarizes our results.

Proposition 5. *Assume long bonds are consols. Consider the case where monetary policy reacts to the demand shock only through the systematic reaction to inflation and $\phi_\pi > 0$. The optimal portfolio solves:*

$$\eta_3 = \phi_\pi \beta + \eta_2 \frac{\overline{G}}{\beta \overline{R} \frac{(1+\gamma_h)}{\kappa_1}} \frac{\sigma^2_{\hat{G}}}{\sigma^2_{\hat{\xi}}} \tag{22}$$

The optimal issuance becomes:

$$\overline{b}_1^*(\hat{\xi}, \hat{G}) = \frac{(1-\beta)(1-\phi_\pi\beta)}{\beta\phi_\pi(1-\phi_\pi)} \left[\phi_\pi \overline{R} \frac{(\gamma_h+1)}{\kappa_1} - \frac{\overline{S}(1-\phi_\pi)}{(1-\beta)} + \frac{\overline{G}^2}{\beta^2 \frac{\overline{R}(\gamma_h+1)}{\kappa_1}} \frac{\sigma^2_{\hat{G}}}{\sigma^2_{\hat{\xi}}} \right]$$

Alternatively, in the case where monetary policy offsets the demand shock setting $\epsilon_{i,t} = \hat{\xi}_t$, the optimal portfolio is:

$$\overline{b}_1^*(\hat{\xi}, \hat{G}) = \overline{S} + \overline{G}^2 \frac{\sigma^2_{\hat{G}}}{\sigma^2_{\hat{\xi}}} \frac{1-\beta}{\beta(1-\beta\phi_\pi)} \frac{\phi_\pi}{\left(\frac{\overline{S}}{1-\beta\phi_\pi} + \overline{R}\frac{(1+\gamma_h)}{\kappa_1} \right)} \tag{23}$$

Proof: See Appendix A.4.

Proposition 5 is easily comparable to our previous analytical results. We previously saw that in the presence of only the $\hat{\xi}$ shock, the optimal portfolio is $\eta_3 = \phi_\pi \beta$ when $\epsilon_{i,t} = 0$. According to (22) we have $\eta_3 > \phi_\pi \beta$ implying that the issuance of long term debt now increases. Moreover, when monetary policy tracks the real rate, the optimal portfolio no longer sets $\overline{b}_1 = \overline{S}$, now the issuance of the long bond is higher as is illustrated by (23).

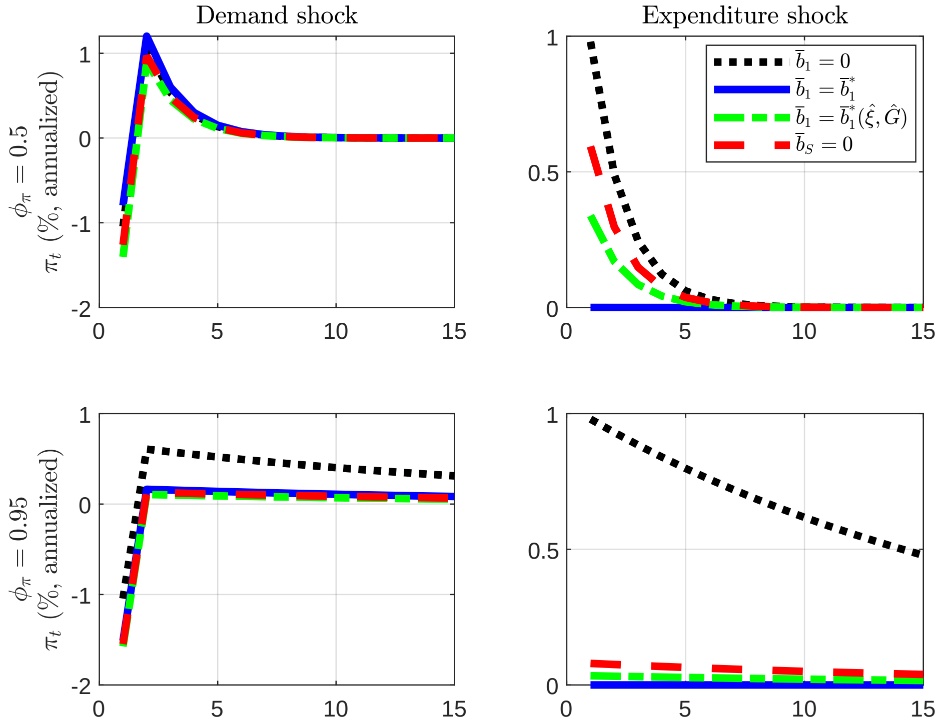
These results are of course to be expected. Spending shocks become a factor that pulls the long bond issuance towards infinity (since for spending shocks the larger the position in long debt the better). To deal with these types of shocks the policymaker has to tolerate a higher impact of demand disturbances on inflation. The larger is the relative variance of the spending shock the larger is the weight that the optimal formula attributes to it (the term $\frac{\sigma^2_{\hat{G}}}{\sigma^2_{\hat{\xi}}}$ in (22) and (23)).

Finally, note that according to (23), ϕ_π becomes a crucial parameter in the case where monetary policy directly responds to the demand shock through the stochastic intercept term $\epsilon_{i,t}$. To interpret (23), recall that when ϕ_π is close to zero, then increasing the long bond issuance will not help much in stabilizing inflation in response to the spending shock, because inflation displays very little persistence. Then, optimal policy prefers to focus on stabilizing inflation responding to the demand disturbance. High values of ϕ_π imply the opposite; it is now optimal ward off the inflationary effects of spending shocks, by issuing large amounts of long term debt.

2.6.2 Macroeconomic Volatility

Does debt management help reduce considerably macroeconomic volatility? So far we have relied on analytical expressions to find qualitative results. However, in order to suggest that DM is an important tool to mitigate the impact of shocks on inflation we need to quantitatively assess this

Figure 1: **Responses to shocks when interest rates respond to inflation only.**



Notes: The solid lines show the response of inflation under the optimal portfolio. The dotted lines represent the responses when all debt is long term, whereas the dashed lines the case where debt is short term. The dashed-dotted lines assume the optimal portfolio in Proposition 5, when spending and demand shocks can simultaneously occur. We assume a monetary policy rule of the form $\hat{i}_t = \phi_\pi \hat{\pi}_t$.

impact under alternative maturity structures. We use the parameter values reported in Table 1 and further assume $(\sigma_\xi, \sigma_G) = (0.4\%, 4.5\%)$.[8]

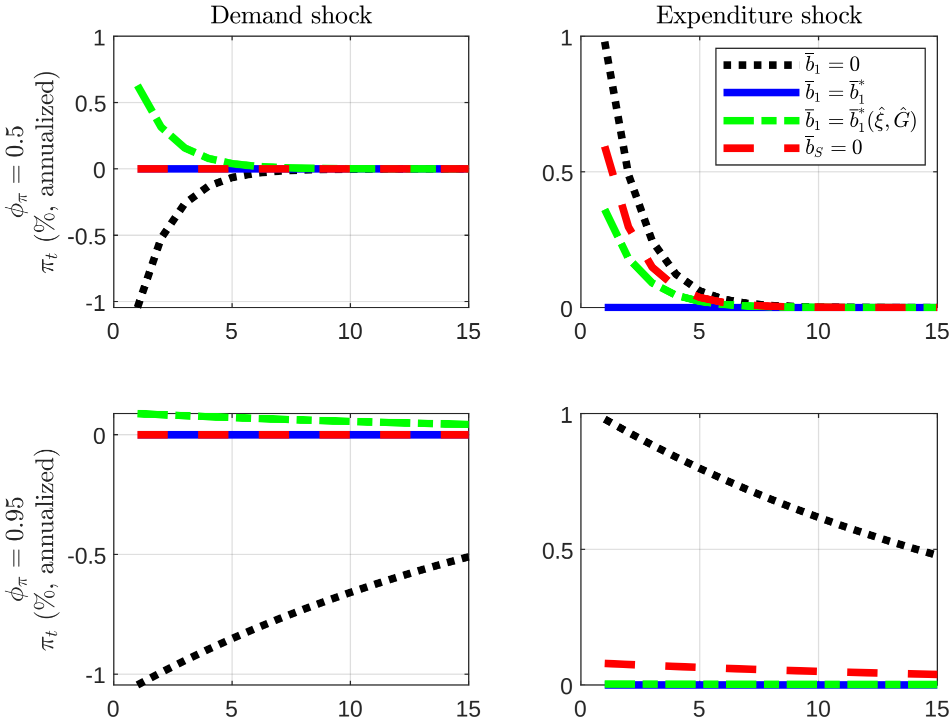
Figures 1 and 2 show the impulse responses to one standard deviation shocks in demand and spending. Figure 1 assumes $\epsilon_{t,i} = 0$ whereas in Figure 2 we let the monetary authority lower the nominal rate 1 for 1 with the real rate. The left panels consider the case of the demand shock, whereas on the right we plot the responses to a spending shock. We consider alternative maturity structures as follows: The dashed red lines assume that DM finances debt short term. The dotted lines assume only long term financing, whereas solid lines correspond to the optimal portfolios.[9] Finally, the dashed-dotted green lines set $\bar{b}_\delta$ to the optimal policy defined in Proposition 5, when both shocks can simultaneously occur.

As is evident from the figures, even with our simplistic i.i.d structure of shocks we obtain large impacts from varying the maturity of debt. For example, financing short implies that a spending shock raises inflation by 1 percent on impact whereas long term financing reduces this impact effect

[8] The value for σ_ξ is chosen so that a one standard deviation shock does not drive the level interest rate below zero. $\sigma_G = 0.045$ is a standard calibration for the variance of the spending shock.

[9] In the case of the spending shock we make $\bar{b}_\delta$ a very large number (10 times GDP).

Figure 2: **Responses to shocks when interest rates track the real rate.**



Notes: See Figure for a description of the portfolio that corresponds to each graph. We assume a monetary policy rule of the form $\hat{i}_t = \phi_\pi \hat{\pi}_t + \hat{\xi}_t$.

considerably and the larger is coefficient ϕ_π the less is the volatility displayed by inflation (e.g. bottom panels). Analogously, whereas issuing long debt only, fully stabilizes inflation following a demand shock in Figure 2, when short debt is issued, inflation drops to -1% and continues being negative for several periods along the transition.

Finally, when debt management attempts to deal with both types of shocks adopting the portfolios derived in Proposition 5, inflation turns positive following a demand contraction in Figure 2, since now the long bond issuance exceeds 100 percent of the value of the portfolio and so $\eta_1 + \eta_3 > 0$.

2.6.3 The Impact of Debt Levels and the slope of the Phillips curve

We close this section by briefly considering how our results would change if we assumed a higher initial debt level and if the slope of the Phillips curve (parameter κ_1) is lower. Both impacts of these parameter changes can be easily read off from our previous formulae. Higher debt implies higher values of $\overline{S}$ and $\overline{R}$. In the case of fiscal shocks, it remains optimal to issue as much long debt as possible, and as the expression for η_2 reveals, for any $\overline{b}_\delta$ the effect of spending shocks on inflation is now less. Intuitively, at higher debt levels, a given increase in inflation reduces more the real payout of debt and so less inflation is needed to satisfy the intertemporal constraint. In the case of demand shocks, and focusing on the case where $\epsilon_{i,t} = \hat{\xi}_t$ and $\delta = 1$ we still obtain $\overline{b}_1 = \overline{S}$. Financing long remains optimal and clearly the quantity of long bonds issued increases in the debt level.

Parameter κ_1 exerts a similar influence, appearing in the denominator of the fractions η_1, η_2, η_3.

Changes in the slope of the Phillips curve do not affect the optimal portfolios identified previously.

3 Optimal Monetary and DM policies

Our analysis thus far has relied on a model where monetary policy is summarized through an ad hoc interest rate rule. One of our key findings was that DM can restore the efficacy of monetary policy in shielding inflation from the impact of demand shocks. This was the case when debt was long term and the monetary policy rule responded to the demand shock directly, through tracking the real rate. Another important finding was that for debt management to be able to contribute in stabilizing inflation a significant systematic response of interest rates to inflation was needed.

We now abandon the assumption that monetary policy follows ad hoc interest rate rules and turn to a model where both monetary and debt policies are optimal. We do so for several reasons, including to investigate whether the above findings generalize to the case where the monetary authority can set interest rates to be optimal for any debt maturity, but also to see whether optimal policy will tie down parameter ϕ_π and the stochastic intercept term to the maturity of debt issued. Moreover, under optimal interest rates, perhaps a different maturity of debt than the one we identified previously will be desirable.

To investigate the above, we solve a Ramsey program assuming coordinated DM and monetary authorities. The 'planner' sets the path of inflation, output and interest rates, and the debt portfolio to minimize the variability of inflation in response to the spending and demand shocks and we further assume full commitment to announced state contingent paths for the policy variables. Under these assumptions, we will show that optimal monetary policy can be summarized through an interest rate rule, broadly similar to the ad hoc rule that we employed in Section 2. However, the inflation coefficient ϕ_π and the stochastic intercept will indeed reflect debt maturity. Optimal portfolios will however, be the same as before, as focusing on long term debt will enable to stabilize inflation.

3.1 Ramsey Program and Optimality

Even though we will maintain the same structure of shocks as in the previous section (i.i.d. and after t no further shocks occur), to make the analysis more general, we will first solve the optimal policy program without imposing these assumptions. The Ramsey planner in our model chooses sequences $\left\{\hat{\pi}_{t+\bar{t}}, \hat{Y}_{t+\bar{t}}, \hat{i}_{t+\bar{t}}, \hat{b}_{t+\bar{t},S}, \hat{b}_{t+\bar{t},\delta}\right\}_{\bar{t}\geq 0}$ and the steady state portfolio $\bar{b}_\delta$ to minimize the variability of inflation subject to the competitive equilibrium equations (the Euler and Phillips curve equations and the consolidated budget), assuming also that taxes are held constant.[10] Noting that since $\hat{i}_{t+\bar{t}}$ will only appear on the LHS of the Euler equation (as we do not tie down interest rates to follow a specific functional form) we can dispense with this constraint. We then can state the policy problem formally as:

$$\max_{\left\{\hat{\pi}_{t+\bar{t}}, \hat{Y}_{t+\bar{t}}, \hat{b}_{t+\bar{t},S}, \hat{b}_{t+\bar{t},\delta}\right\}_{\bar{t}\geq 0}, \bar{b}_\delta} -E_{t-1} \sum_{\bar{t}\geq 0} \beta^{\bar{t}} \hat{\pi}^2_{t+\bar{t}}$$

subject to

$$\hat{\pi}_{t+\bar{t}} = \kappa_1 \hat{Y}_{t+\bar{t}} + \beta E_{t+\bar{t}} \hat{\pi}_{t+\bar{t}+1}$$

[10]Notice that the usual resource constraint that is included in Ramsey programs is already accounted for, since we could replace consumption with output and spending.

and

$$\beta\bar{b}_S\hat{b}_{t+\bar{t},S}+\beta\bar{b}_S E_{t+\bar{t}}(\hat{\xi}_{t+\bar{t}+1}-\hat{\xi}_{t+\bar{t}}-\hat{\pi}_{t+\bar{t}+1})+\frac{\beta\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t+\bar{t},\delta}+\bar{b}_\delta\sum_{j=1}^{\infty}\beta^j\delta^{j-1}\left[E_{t+\bar{t}}\left(-\sum_{l=1}^{j}\hat{\pi}_{t+\bar{t}+l}+\hat{\xi}_{t+\bar{t}+j}-\hat{\xi}_{t+\bar{t}}\right)\right]$$
$$+\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}(\gamma_h+1)\hat{Y}_{t+\bar{t}}-\overline{G}\hat{G}_{t+\bar{t}}$$
$$=\bar{b}_S(\hat{b}_{t+\bar{t}-1,S}-\hat{\pi}_{t+\bar{t}})+\frac{\bar{b}_\delta}{1-\beta\delta}(\hat{b}_{t+\bar{t}-1,\delta}-\hat{\pi}_{t+\bar{t}})+\delta\bar{b}_\delta\sum_{j=1}^{\infty}\beta^j\delta^{j-1}\left[E_{t+\bar{t}}\left(-\sum_{l=1}^{j}\hat{\pi}_{t+\bar{t}+l}+\hat{\xi}_{t+\bar{t}+j}-\hat{\xi}_{t+\bar{t}}\right)\right]$$

given also that $\bar{b}_S=\frac{\overline{S}}{1-\beta}-\frac{\delta\bar{b}_\delta}{1-\beta\delta}$.[11]

Note that since $\bar{b}_\delta$ is chosen, the above is not a linear-quadratic program. To solve for the optimal policies, we proceed in two steps: We first hold constant $\bar{b}_\delta$ and solve a linear quadratic program to determine the optimal sequence $\left\{\hat{\pi}_{t+\bar{t}},\hat{Y}_{t+\bar{t}},\hat{b}_{t+\bar{t},S},\hat{b}_{t+\bar{t},\delta}\right\}_{\bar{t}\geq 0}$ through solving a system of first order conditions. Second, we vary $\bar{b}_\delta$ to determine the portfolio through the upper envelope defined by the optimality conditions in the first step.

3.1.1 Optimality

In the online appendix we setup a Lagrangian to derive the optimal paths of output inflation and bond quantities (in log deviation from steady state). Attach a multiplier $\psi_{\pi,t+\bar{t}}$ to the Phillips curve and $\psi_{g,t+\bar{t}}$ to the consolidated budget; the first order conditions are the following:

$$-\hat{\pi}_{t+\bar{t}}+\Delta\psi_{\pi,t+\bar{t}}+\bar{b}_S\Delta\psi_{gov,t+\bar{t}}+\frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t+\bar{t}-l}=0 \tag{24}$$

$$-\psi_{\pi,t+\bar{t}}\kappa_1+\frac{\overline{\tau}(1+\eta)}{\eta}\overline{Y}(1+\gamma_h)\psi_{gov,t+\bar{t}}=0 \tag{25}$$

$$\frac{\beta\bar{b}_\delta}{1-\beta\delta}\left(\psi_{gov,t+\bar{t}}-E_{t+\bar{t}}\psi_{gov,t+\bar{t}+1}\right)=0 \tag{26}$$

$$\beta\bar{b}_S\left(\psi_{gov,t+\bar{t}}-E_{t+\bar{t}}\psi_{gov,t+\bar{t}+1}\right)=0 \tag{27}$$

(24) is the FONC with respect to $\hat{\pi}_{t+\bar{t}}$; (25), (26) , (27) are first order conditions with respect to $\hat{Y}_{t+\bar{t}}$, $\hat{b}_{t+\bar{t},\delta}$ and $\hat{b}_{t+\bar{t},S}$ respectively.

Several comments are in order. First, note that since the model is linear (26) and (27) will not pin down an optimal rule for the share of long over short term bonds. As before, quantities $\hat{b}_{t+\bar{t},\delta}$ and $\hat{b}_{t+\bar{t},S}$ will not be important; thus, arbitrarily setting $\hat{b}_{t+\bar{t},\delta}$ to zero and financing with $\hat{b}_{t+\bar{t},S}$ is consistent with the first order conditions since both (26) and (27) define that $\psi_{gov,t+\bar{t}}$ evolves like a random walk.

Second, the random walk property of the multiplier $\psi_{gov,t+\bar{t}}$ is a standard feature in the optimal policy literature. Since financing debt impinges distortions to the economy, ours is a model of optimal policy under incomplete markets as in Aiyagari et al., 2002; Schmitt-Grohé and Uribe, 2004; Lustig et al., 2008; Faraglia et al., 2013, 2016. Whereas in these papers debt can be financed through taxes, in our model taxes are held constant and the planner uses distortionary inflation to satisfy budget solvency. In both contexts, shocks to the economy translate to changes in the excess burden of

[11]Note that we will further assume that the planner does not want to inflate away public debt at the beginning of the horizon. As usual, this will involve choosing initial conditions for the Lagrange multipliers on the constraints.

distortions, and the multiplier, which measures the magnitude of these distortions, behaves like a random walk, since the planner wants to spread evenly the costs across periods.

Third, as (24) reveals, when government debt is long term, all the lags of the multiplier enter into the state vector and influence inflation. Combining (24) and (25) to substitute out ψ_π we can obtain the following expression which pins down $\hat{\pi}_{t+\bar{t}}$ as a function of these state variables

$$\hat{\pi}_{t+\bar{t}} = \overline{R}\frac{(1+\gamma_h)}{\kappa_1}\Delta\psi_{gov,t+\bar{t}} + \bar{b}_S\Delta\psi_{gov,t+\bar{t}} + \frac{\bar{b}_\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t+\bar{t}-l} \tag{28}$$

To interpret (28), note that a shock to either demand or spending will change the value of the multiplier according to its impact on the consolidated budget. For instance, an increase in spending will tighten the constraint and thus increase the value of $\psi_{gov,t}$. According to (28) this will increase inflation on impact, but also continue exerting an influence on inflation in the future, i.e. through the term $\delta^{\bar{t}}\Delta\psi_{gov,t}$ in the final part on the RHS. When debt is long term, the planner desires to spread inflation across periods, through promising higher inflation rates in the future. This feature of optimal policy in our model is common with Lustig et al., 2008; Faraglia et al., 2013, 2016.

3.2 The Optimal Interest rate rule

We now show that optimal policy in the Ramsey model can be implemented with an interest rate rule that is similar to the ad hoc rules we employed in Section 2. We summarize this policy rule in the following proposition:

Proposition 6. *The optimal interest rate policy that implements the Ramsey outcome is:*

$$\hat{i}_{t+\bar{t}} = \delta\hat{\pi}_{t+\bar{t}} - \delta\omega_Y\Delta\psi_{gov,t+\bar{t}} + \hat{\xi}_{t+\bar{t}} \tag{29}$$

where $\omega_Y \equiv \frac{\overline{S}}{1-\beta} - \frac{\bar{b}_\delta\beta\delta}{1-\beta\delta} + \overline{R}\frac{(1+\gamma_h)}{\kappa_1}$.

Proof: See Appendix A.5.

Notice first that optimal interest rates are determined by two components. The first is a standard systematic response to inflation, $\delta\hat{\pi}_{t+\bar{t}}$, the second is the stochastic intercept term $-\delta\omega_Y\Delta\psi_{gov,t+\bar{t}} + \hat{\xi}_{t+\bar{t}}$. Let us focus first on the systematic response. Using, for comparison, the notation we employed in Section 2, we now have $\phi_\pi = \delta < 1$. Thus, the Ramsey planner fights inflation by raising the nominal rate, but the magnitude of the reaction of the interest rate is now tied down by the decaying coupon factor.

To interpret this feature, recall, that parameter ϕ_π governs the persistence of inflation. When long bonds are issued, both current and future inflation can contribute towards adjusting the real value of debt. But if the duration of long bonds is not considerable (or δ is a small number) then letting inflation deviate persistently from target (setting $\phi_\pi > \delta$) will not help in terms of making debt more sustainable. Thus, under the optimal plan, inflation distortions persist for as long as long debt coupon payments last.

Notice however, that (perhaps counterintuitively) the term $\delta\hat{\pi}_{t+\bar{t}}$ does not depend on whether the long term debt is actually issued, i.e. when $\bar{b}_\delta > 0$. We will now explain that the stochastic intercept term will compensate for this, and inflation will be frontloaded if all debt is short.

Consider the term $\delta\omega_Y\Delta\psi_{gov,t+\bar{t}}$. Notice that the maturity of debt influences it. Coefficient ω_Y is decreasing in the quantity of long bonds issued. Moreover, as we argued previously, shocks filtered through budget constraint, will induce fluctuations in $\Delta\psi_{gov,t+\bar{t}}$. For example, a positive spending shock occurring in period $t+\bar{t}$, will result in $\Delta\psi_{gov,t+\bar{t}} > 0$, since the consolidated budget tightens. Then, if debt is short term, $\omega_Y > 0$ and the planner will keep the interest rate lower in period $t+\bar{t}$ (than $\delta\hat{\pi}_{t+\bar{t}}$). In contrast, if a sufficiently high quantity of long bonds is issued then the opposite will hold, the nominal rate will be set higher when the shock hits, since now $\omega_Y < 0$.

What is the planner trying to do? Notice that lowering the nominal rate will have as effect to frontload inflation. When debt is financed short, this enables a larger reduction of real debt in response to the shock. Conversely, increasing $\hat{i}_{t+\bar{t}}$ will accomplish to spread inflation across periods, which is optimal when debt is long term.

Lastly, according to (29), the nominal rate tracks movements in the real rate. $\hat{\xi}_{t+\bar{t}}$ exerts a direct influence on the policy rule, but also an indirect influence through the intertemporal budget and $\Delta\psi_{gov,t+\bar{t}}$.

3.3 One off Shocks: The dynamic path of inflation

To investigate further these properties let us now go back to the case where shocks in t are i.i.d and there are no further shocks to the economy after t. Under these assumptions it is possible to derive analytical expressions for the multiplier ψ_{gov}. From (26) and (27) (removing conditional expectations after t) we have $\Delta\psi_{gov,t+\bar{t}} = 0$ for $\bar{t} \geq 1$ and $\Delta\psi_{gov,t} \neq 0$. Assuming further $\psi_{gov,t-1} = \psi_{gov,t-2} = \dots$ (such that in the absence of shocks optimal inflation is zero), the optimal interest rate path becomes:

$$\hat{i}_t = \delta\hat{\pi}_t - \delta\omega_Y \Delta\psi_{gov,t} + \hat{\xi}_t$$

(30)

$$\hat{i}_{t+\bar{t}} = \delta\hat{\pi}_{t+\bar{t}}$$

and we can further show that:

$$\Delta\psi_{gov,t} = \left[\left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)^2 + \frac{\overline{b}_\delta^2\beta\delta^2}{(1-\beta\delta^2)(1-\beta\delta)^2}\right]^{-1}\left[\overline{G}\hat{G}_t + \left(\frac{\overline{\beta S}}{1-\beta} - \frac{\beta\delta\overline{b}_\delta}{1-\beta\delta}\right)\hat{\xi}_t\right] \quad (31)$$

(see Appendix A.6).

(31) expresses $\Delta\psi_{gov,t}$ as a function of spending and demand shocks; the loadings on the shocks are functions of the maturity of debt. As is evident, higher long debt issuance reduces the response of the multiplier to the spending shock. Moreover, the effect of the demand shock is zero when the government sets $\frac{\beta\delta\overline{b}_\delta}{1-\beta\delta} = \frac{\overline{S}}{1-\beta}$.

Using (28) we can now show that the path of inflation is given by

$$\hat{\pi}_t = \left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)\Delta\psi_{gov,t}$$

(32)

$$\hat{\pi}_{t+\bar{t}} = \frac{\overline{b}_\delta}{1-\beta\delta}\delta^{\bar{t}}\Delta\psi_{gov,t}, \quad \bar{t} \geq 1$$

Combining (31) and (32) we can derive the impact effects (the counterparts of coefficients η in Section 2) for spending and demand shocks as:

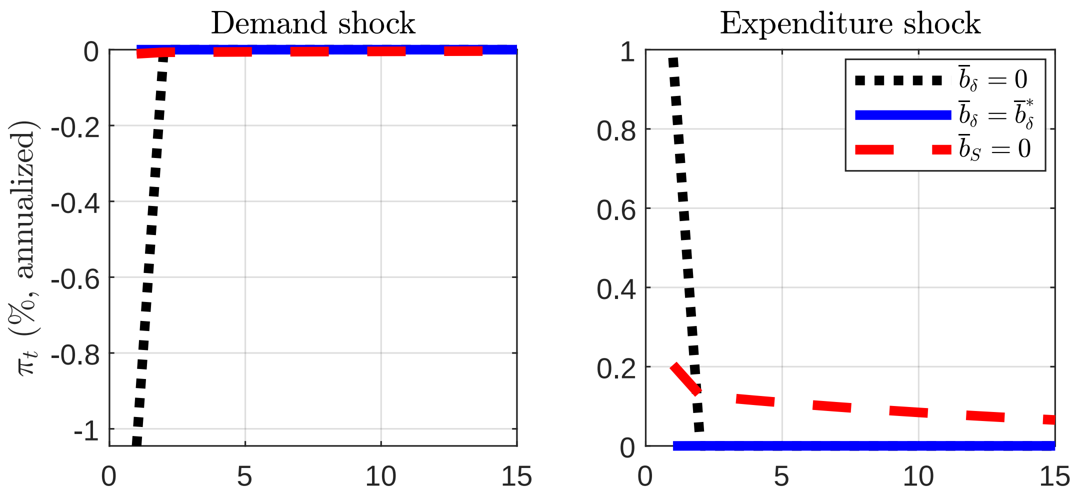
$$\eta_2^* = \left[\left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)^2 + \frac{\overline{b}_\delta^2\beta\delta^2}{(1-\beta\delta^2)(1-\beta\delta)^2}\right]^{-1}\overline{G}\left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right) \quad (33)$$

(34)

$$(\eta_1^* + \eta_3^*) = \left[\left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)^2 + \frac{\overline{b}_\delta^2\beta\delta^2}{(1-\beta\delta^2)(1-\beta\delta)^2}\right]^{-1}\left(\frac{\overline{\beta S}}{1-\beta} - \frac{\beta\delta\overline{b}_\delta}{1-\beta\delta}\right)\left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)$$

where the stars denote that monetary policy is now optimal. Consider first the response to the spending shock, η_2^*, and let us compare this to the analogous response in the model of Section 2. As is clear from (10) and (33), η_2^* is not the same as η_2 even when we set $\phi_\pi = \delta$. The terms that appear in the denominator in (10) appear also in (33), however, it is now the squares that show up in the denominator, rather than the levels. The numerators of η_2^* and η_2 are also different.

Figure 3: **Responses to shocks under optimal policy.**



Notes: The left panel shows the response of inflation to a negative demand shock when monetary policy is optimal. The dashed line assumes long term financing, whereas the dotted line assumes only short term debt. The solid line corresponds to the optimal portfolio. The right panel plots the responses to a spending shock.

Analogously, in the case of the demand shock, letting $\phi_\pi = \delta$ we have

$$\eta_1 + \eta_3 = \beta\left[\frac{\overline{S}}{(1-\beta)} - \frac{\beta\delta\bar{b}_\delta}{(1-\beta\delta)}\right] / \left[\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\bar{b}_\delta\beta\delta^2}{(1-\beta\delta)(1-\beta\delta^2)}\right]$$

where again the coefficient differs from the response under optimal policy in (33).

Where do these differences come from? Simple inspection of the monetary policy rules suggests that the s term $-\delta\omega_Y\Delta\psi_{gov,t}$ is responsible. As explained, this term determines whether the optimal policy desires to frontload/backload inflation as a function of the quantity of long bonds issued.

In Figure 3 we show the impulse responses of inflation to the demand and spending shocks under optimal policy. The dotted lines plot the responses when all of debt is short term and the dashed lines the case where all debt is long. Under short term debt both demand and spending shocks have only a temporary effect on inflation. This is the term $-\delta\omega_Y\Delta\psi_{gov,t}$ at work. In contrast, when debt is long term ($\bar{b}_S = 0$ in the figure), inflation deviates persistently from zero after the spending shock.

3.4 Optimal Maturity

The optimal maturity $\overline{b}_\delta$ is easy to find. Consider first the case of spending shocks only. We can show that the objective function becomes:

$$-E_t \sum_{\bar{t}\geq 0} \beta^{\bar{t}} \hat{\pi}^2_{t+\bar{t}} = -\left[\left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)^2 + \frac{\overline{b}^2_\delta \beta\delta^2}{(1-\beta\delta^2)(1-\beta\delta)^2}\right]^{-1} \overline{G}^2 \sigma^2_{\hat{G}}$$

and as in the model of section 2, the optimal policy is to issue as much long term debt as possible. Moreover, from (31) it is obvious that the policy that fully stabilizes inflation sets $\frac{\beta\delta\bar{b}_\delta}{1-\beta\delta} = \frac{\overline{S}}{1-\beta}$, the optimal structure of debt we identified in Section 2.

Notice, finally, that under optimal debt we will have $\Delta\psi_{gov,t} = 0$ (in the limit in the case of spending shocks). The optimal debt policy thus eliminates the dependence of the monetary policy rule on the composition of debt, through eliminating the impact of shocks on the consolidated budget. For this to happen debt management needs to again focus on issuing long term debt.

4 A Medium Scale DSGE model

Our core analysis in this paper has relied on a simplistic model to characterize transparently the interplay between debt maturity and inflation. Yet to claim that the interactions between debt and monetary polices are indeed relevant, it is important to extend our findings to a more empirically relevant setup, a medium scale DSGE model. We now work with a model with a wider set of shocks, including shocks to productivity, markups and government transfers, to study how debt policy can influence inflation in the presence of these additional disturbances. Moreover, the representative household in the model has preferences featuring habit formation, this implies more realistic adjustment of asset prices to shocks.

We first briefly describe our medium scale model, which is broadly similar to Bianchi and Ilut (2017). Then, we describe the estimation of the model and the results we get in subsection 4.2. Lastly, we will investigate the effects of maturity on equilibrium outcomes in subsection 4.3.

4.1 The Model

4.1.1 Household Preferences and Optimality

The economy is populated by a single household with preferences of the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\log(C_t - \Omega C^a_{t-1}) - \chi \frac{h_t^{1+\gamma_h}}{1+\gamma_h}\right)$$

where C_t denotes the consumption of the household, ΩC^a_{t-1} is an external habit stock, where $0 < \Omega < 1$ and C^a_{t-1} denotes the average level of consumption in $t-1$. The household derives disutility from exerting labor effort h_t. Parameters χ and γ_h govern the household's preferences over leisure. ξ_t is a preference shifter which impacts the relative discounting of current and future utility flows.

The household maximizes utility subject to the flow budget constraint:

$$P_tC_t + P_{t,\delta}B_{t,\delta} + P_{t,S}B_{t,S} = (1-\tau_t)W_th_t + P_tTr_t + B_{t-1,S} + (1+\delta P_{t,\delta})B_{t,\delta} + P_tDiv_t$$

$B_{t,\delta}$ is a long government bond. $P_{t,\delta}$ is the price of the asset. $B_{t,S}$ denotes the quantity of short term (one-period) debt and $P_{t,S}$ its price at issuance.

W_t denotes the nominal wage and P_t is the price level. Div_t is real dividends paid by monopolistically competitive firms and Tr_t denotes lump-sum transfers given to the household by the fiscal authority. The household maximizes utility subject to the flow budget constraint.

The household maximizes utility subject to the budget constraint. The first order conditions for long and short bonds and the labor supply condition are given by:

$$\frac{1}{C_t - \Omega C^a_{t-1}} = \beta R_t E_t \frac{1}{\pi_{t+1}} \frac{1}{C_{t+1} - \Omega C^a_t} \tag{35}$$

$$\frac{1}{C_t - \Omega C^a_{t-1}} P_{L,t} = \beta E_t \frac{1 + \delta P_{L,t+1}}{\pi_{t+1}} \frac{1}{C_{t+1} - \Omega C^a_t} \tag{36}$$

$$\chi h_t^{\gamma_h}(C_t - \Omega C^a_{t-1}) = (1 - \tau_t)\frac{W_t}{P_t} \tag{37}$$

4.1.2 Firms, Production and the Phillips curve

We assume that output is produced by a continuum of monopolistically competitive firms which operate technologies with labor as the sole input. Aggregate output is produced by a representative, perfectly competitive, final-good producer that aggregates the intermediate products of firms according to $Y_t = \left(\int_0^1 Y_t(j)^{\frac{1+\eta_t}{\eta_t}} dj\right)^{\frac{\eta_t}{1+\eta_t}}$. η_t is a (time varying) parameter that governs the elasticity of substitution. The production function of the generic good firm j is $Y_t(j) = A_t h_t(j)^{1-\alpha}$; A_t denotes the level of TFP in the economy. We assume that the growth rate of A_t (expressed in logarithms) follows an AR(1) process:

$$\ln\left(\frac{A_t}{A_{t-1}}\right) \equiv a_t = (1 - \rho_a)\gamma + \rho_a a_{t-1} + \epsilon_{a,t}$$

Parameter γ denotes the steady-state growth rate of A_t.

Firms face price adjustment costs as in Rotemberg (1982). The cost function of firm j is: $AC_t(j) = \frac{\theta}{2}(\frac{P_t(j)}{P_{t-1}(j)} - \pi)^2 Y_t$. $\theta \geq 0$ again governs the degree of price stickiness. π is the steady state level of gross inflation.

The profit maximization problem of the generic firm j is defined by :

$$\begin{aligned} \max_{P_t(j)} \quad & E_t \sum_{s=0}^{\infty} Q_{t,t+s}\Big(\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - \frac{MC_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - AC_{t+s}(j)\Big) \\ \text{s.t.} \quad & Y_{t+s}(j) = \Big(\frac{P_{t+s}(j)}{P_{t+s}}\Big)^{\eta_t} Y_{t+s} \\ & AC_{t+s}(j) = \frac{\theta}{2}\Big(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - \overline{\pi}\Big)^2 Y_{t+s} \end{aligned}$$

where $Q_{t,t+s} \equiv \beta^s E_t \frac{C_t - \Omega C^a_{t-1}}{C_{t+s} - \Omega C^a_{t+s-1}}$ is the household's discount factor and MC_{t+s} denotes marginal costs of production. $AC_{t+s}(j)$ is the quadratic price adjustment cost incurred by the firm.

In the online appendix we show that solving this problem and imposing a symmetric equilibrium, gives rise to the following (non-linear) New-Keynesian Phillips curve:

$$\theta(\pi_t - \overline{\pi})\pi_t = (1 + \eta_t)(1 - \frac{MC_t}{P_t}) + \beta\theta E_t \frac{C_t - \Omega C^a_{t-1}}{C_{t+1} - \Omega C^a_t} \frac{Y_{t+1}}{Y_t}(\pi_{t+1} - \overline{\pi})\pi_{t+1}.$$

MC_t denotes marginal costs of production.

4.1.3 Fiscal and Monetary Policy

The government levies distortionary taxes and issues debt to finance spending G_t and transfers Tr_t. The flow budget constraint of the government is:

$$P_{t,S}B_{t,S} + P_{t,\delta}B_{t,\delta} = B_{t-1,S} + (1 + \delta P_{t,\delta})B_{t-1,\delta} + P_t(G_t + Tr_t) - \tau_t W_t h_t + \Lambda_t \tag{38}$$

Notice that following Bianchi and Ilut (2017) we augment the flow budget with an exogenous shock variable Λ_t capturing features of government finances that we have left outside the model.[12] $\tau_t W_t h_t$ denotes the fiscal revenues of the government.

In the online appendix, we rewrite all model equations, expressing variables as ratios over nominal GDP and take a log linear approximation around the non-stochastic steady state. To define the policy rules of the fiscal and the monetary authorities, and the stochastic processes for transfers and the shocks we will now use the log-linear format.

Labour income taxes are set according to:

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1-\rho_\tau)\left[\phi_{\tau,b}\hat{D}_{t-1} + \phi_{\tau,y}(\hat{Y}_t - \hat{Y}_t^n) + \phi_{\tau,g}(g^{-1}\hat{g}_t + \hat{tr}_t)\right] + \epsilon_{\tau,t} \tag{39}$$

where again $\hat{D}_{t-1}$ denotes the face value of debt issued in $t-1$, $\hat{Y}_t$ is output and $\hat{g}_t$ is the log deviation of scaled spending, $\frac{1}{1-\frac{G}{Y}_t}$, from its steady state value. Y_t^n is the natural level of output, that obtains under flexible prices.

Notice that in (39) labour income taxes are not only allowed to adjust to debt, but also to the cycle, through the term $\phi_{\tau,y}(\hat{Y}_t - \hat{Y}_t^n)$, and we further allow for spending and transfers to influence $\hat{\tau}_t$.

Transfers in the model evolve according to:

$$\hat{tr}_t = \rho_{tr}\hat{tr}_{t-1} + (1-\rho_{tr})\phi_{tr,y}(\hat{Y}_t - \hat{Y}_t^n) + \epsilon_{tr,t} \tag{40}$$

where ρ_{tr} governs the persistence of transfers and $\phi_{tr,y}$ measures the response to the output gap. Government spending (as a fraction of GDP) follows a simple AR(1) process:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t} \tag{41}$$

The shocks $\epsilon_{\tau,t}$, , $\epsilon_{tr,t}$ and $\epsilon_{g,t}$ are all assumed to be i.i.d.

Monetary policy is assumed to set the nominal rate according to:

$$\hat{i}_t = \rho_r \hat{i}_{t-1} + (1-\rho_r)\Big(\phi_\pi \hat{\pi}_t + \phi_y(\hat{Y}_t - \hat{Y}_t^n)\Big) + \epsilon_{i,t} \tag{42}$$

Finally, the remaining shocks are all assumed to follow first order autoregressive processes (see online appendix).

4.2 Estimation

We estimate parameter values and stochastic processes so that the model matches the data observations from the US economy. As it is well known, since the beginning of the 1980s, US monetary policy was not subservient to fiscal policy. The passive regime is more likely to have prevailed in the 1970s and before (see e.g. Bianchi and Ilut, 2017). Since our model is not designed to deal with regime fluctuations, we thus have to choose either to estimate the model under the assumption that monetary policy is 'active' using post 1980 data, and then set policy parameters to produce a passive regime, or estimate the model with observations prior to 1980 which would allow us to identify these policy parameters directly from the data.

We choose to do the former. The main reason is that we do not see why, if the passive monetary regime is to resurface in the future, the interest rate rule coefficients have to be equal to the values that fit the pre 1980 data. In our experiments below we will treat these coefficients as free parameters, considering various specifications of the passive monetary policy rule. Moreover, using the more

[12] Note that this shock is also necessary in order to estimate the model since debt, spending and transfers will be treated as observables.

recent observations allows us to obtain more accurate estimates of structural parameters that are likely to have shifted over the decades.[13].

Our sample is 1980:Q1 - 2008Q4. We truncate the sample to the 4th quarter of 2008 since the short term nominal rate in the US reached zero in the first quarter or 2009. Dealing with the non-linearities implied by the non-negativity constraint on the nominal interest rate, in estimation is beyond the scope of this exercise.

We include the following variables in the estimation of the model: Real GDP growth, GDP deflator inflation, the federal funds rate, federal revenues as a fraction of GDP, total government expenditures (including transfers) to GDP, the market value of debt to GDP ratio and finally government spending (for consumption and investment) to GDP. The details on the sources of these variables and the measurement equations that are employed to link data variables with their model counterparts are spelled out in the online appendix.

To proceed with estimation we first select prior distributions for the parameters we wish to estimate and pick values for parameters that we want to fix in estimation. Table 2 summarizes the calibrated values of the parameters that we fix and the right side of Table 3 reports our choice of prior distributions for the parameters we estimate with Bayesian techniques. The priors are in line with previous papers in the literature (see e.g. Bianchi and Ilut, 2017).

Table 2: **Calibrated parameters**

	Parameter	Value
y	steady state output (normalization)	1
$1-\alpha$	labor share	0.66
δ	decaying rate of coupon bonds	0.95
η	demand Elasticity	-7.66
γ_h	inverse of Frisch elasticity	1

Notes: The table reports model parameters whose values we fix in estimation. See text for details.

We fix the values of the labor share, α, the elasticity of labor supply, $\frac{1}{\gamma_h}$ and the demand elasticity parameter, η. We assume $\alpha = 0.66$ and $\gamma_h = 1$. Moreover, η is chosen so that that markups are 15 percent in steady state. Finally, we normalize the steady-state value of output to unity. Parameter δ is also set exogenously and we assume $\delta = 0.95$.[14]

The left side of Table 3 reports the posterior estimates of the model parameter distributions. Our estimates are reasonably close to the analogous objects reported in Bianchi and Ilut (2017) for the post 1980s sample. For example, according to both our estimates and theirs, the monetary policy rule displays a strong reaction of the nominal interest rate to inflation, and the estimated mean of the response to output is around .6. The parameter that governs interest rate smoothing exceeds 0.9. Moreover, the estimated response of taxes to the lagged value of debt is relatively low ($\phi_{\tau,b} = 0.064$ at the mean of the posterior distribution). Though with this value fiscal policy is active, debt is close to being an explosive process.

Other key parameters, e.g. the slope of the New Keynesian Phillips curve (which is relatively flat - the mean estimate being $\kappa = 0.009$) and the habit parameter (Ω, which is centered around roughly 0.5) are also in line with the results reported in Bianchi and Ilut (2017).

[13]Most notably, parameters related to the Phillips curve have shifted as recent literature advocates (see Del Negro et al., 2015)

[14]Note that, to simplify, we allow for only long term debt in the estimation. As discussed, in the linear model the behavior of bond portfolios over the cycle does not matter, and moreover, under active monetary policy the steady state portfolios will not impact (directly) inflation.

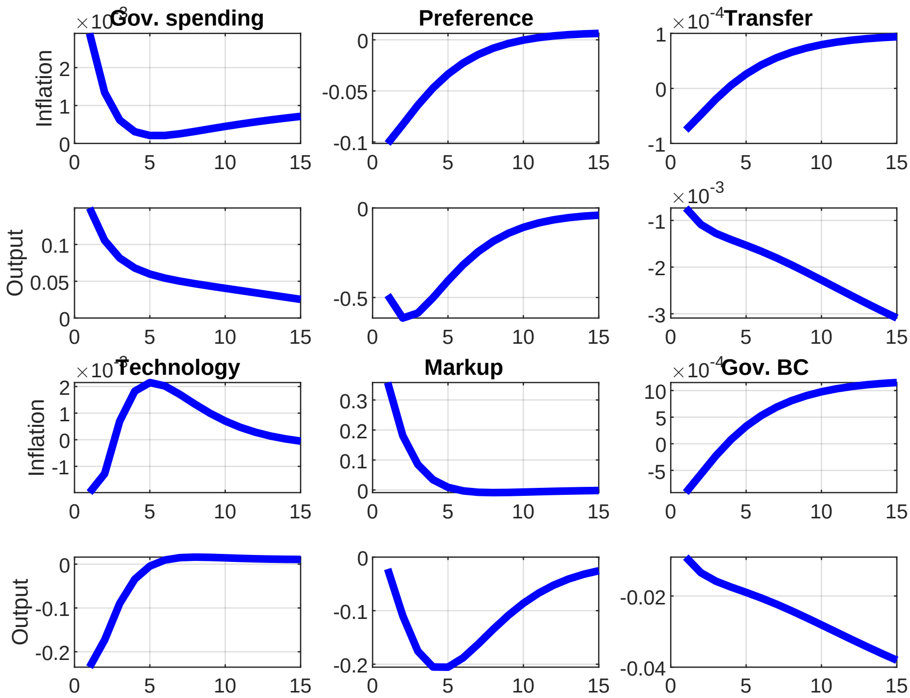
Table 3: **Prior and posterior distributions**

	Parameter	Posterior		Prior		
		mean	90 % interval	distrib	par A	par B
Quarterly trends and ss values						
100γ	trend growth	0.361	[0.29 ; 0.433]	G	0.4	0.05
$100 \log \pi$	inflation	0.607	[0.516 ; 0.692]	G	0.5	0.05
$100(\beta^{-1}-1)$	discount rate	0.139	[0.051 ; 0.228]	G	0.25	0.1
g	g-to-gdp	1.07	[1.06 ; 1.08]	N	1.06	0.04
$b_L/4$	annual mv debt-to-gdp	0.223	[0.17 ; 0.282]	N	0.25	0.05
tax	taxes-to-gdp	0.043	[0.041 ; 0.046]	N	0.045	0.0025
Households and firms						
Ω	habits	0.501	[0.407 ; 0.593]	B	0.8	0.1
κ	slope nkpc	0.009	[0.001 ; 0.017]	G	0.3	0.15
Monetary policy						
ϕ_π	taylor, inflation	2.264	[1.69 ; 2.802]	N	2.5	0.3
ϕ_y	taylor; output	0.649	[0.467 ; 0.844]	G	0.4	0.1
ρ_r	i.r. smoothing	0.964	[0.954 ; 0.975]	B	0.5	0.2
Fiscal rules						
$\phi_{\tau,b}$	tax response to b	0.064	[0.037 ; 0.09]	G	0.07	0.02
$\phi_{\tau,y}$	tax response to y	0.307	[-0.021 ; 0.641]	N	0.4	0.2
$\phi_{\tau,g}$	tax response to g	0.491	[0.17 ; 0.811]	N	0.5	0.2
$\phi_{tr,y}$	tr response to y	-0.641	[-0.771 ; -0.5]	N	-0.4	0.2
ρ_{tr}	tr smoothing	0.212	[0.131 ; 0.293]	B	0.2	0.05
ρ_τ	tax smoothing	0.969	[0.95 ; 0.99]	B	0.5	0.2
Shocks, persistence						
ρ_η	markup	0.563	[0.483 ; 0.637]	B	0.5	0.2
ρ_ξ	preference	0.954	[0.935 ; 0.973]	B	0.5	0.2
ρ_a	tfp	0.299	[0.162 ; 0.439]	B	0.5	0.2
ρ_g	gov. spending	0.976	[0.959 ; 0.993]	B	0.5	0.2
ρ_λ	gov b.c	0.155	[0.038 ; 0.265]	B	0.5	0.2
Shocks, standard deviations						
σ_τ	taxes	3.603	[2.607 ; 4.632]	IG	1	2
σ_g	gov. spending	3.742	[3.342 ; 4.144]	IG	1	1
σ_η	markup	0.227	[0.202 ; 0.251]	IG	1	1
σ_ξ	preference	0.178	[0.156 ; 0.199]	IG	1	1
σ_a	tfp	0.894	[0.691 ; 1.096]	IG	1	1
σ_m	mon. policy	0.088	[0.078 ; 0.096]	IG	0.5	0.5
σ_λ	gov b.c	0.371	[0.313 ; 0.43]	IG	0.5	0.5
σ_{tr}	transfers	0.285	[0.247 ; 0.324]	IG	0.5	0.5

Notes: The table reports the prior and posterior distributions of the estimated parameters. The first column reports the mean of the posterior of each parameter, obtained from Monte-Carlo simulations of the posterior distribution using the MH algorithm. The second column reports the 90% HPD intervals obtained from the same draws. The third column indicates the assumed prior distribution (B: beta, G: gamma, IG: inverse gamma, N: normal). The fourth and fifth columns report the first and second moments of the priors.

Finally, before turning to the main focus of this exercise, which is to evaluate how debt maturity affects the properties of inflation under passive monetary, we study the impulse responses to shocks in the active monetary policy regime that we have estimated.

Figure 4: **Responses to shocks in the estimated model under active monetary policy.**



Notes: The graphs show impulse responses to one standard deviation shocks from the posterior distributions of the medium scale model. For each of the shocks in the model we plot inflation and output responses. Debt is only long term, as we assume in estimation.

Figure 4 plots the responses of inflation and output to demand, TFP, markup shocks and shocks to fiscal variables[15] and notice that these responses are now measured in percentage points. Therefore, 1 is a 1 percent increase of a variable relative to the balanced growth path, 0.1 is a 0.1 percent increase, etc. As is evident from the figure shocks related to technology and to fiscal variables exert essentially no influence on inflation (though they do impact the output gap) and it is rather shocks to markups and demand shocks driving inflation volatility. This is to be expected. Since monetary policy is not concerned with debt sustainability in this model fiscal shocks have effectively no bearing on inflation. On the other hand, the fact that markup shocks are key is a common finding in the literature.[16]

4.3 The role of debt maturity in the medium scale DSGE model

Our theoretical analysis explored how the maturity of debt influences the properties of inflation. In some cases, we were able to show that choosing the right maturity of debt completely insulated inflation from the impact of random shocks, and restored the ability of monetary policy to control inflation. Though it is possible to repeat this type of analysis in the medium scale model of this

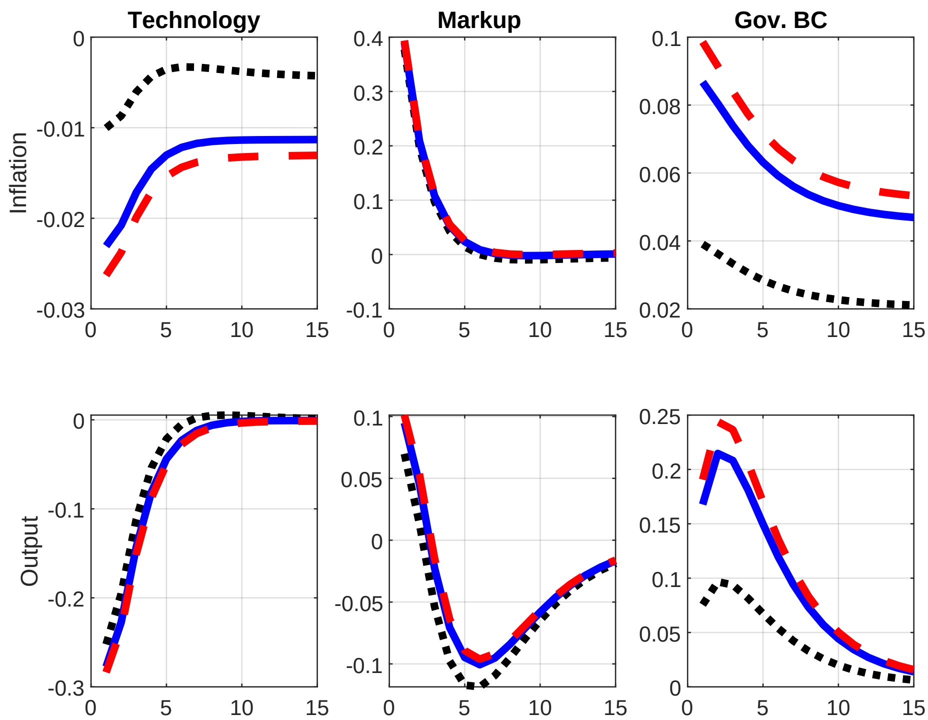
[15]We leave outside the figure the tax shock and the shock to the interest rate rule, since we will also later not consider them in the passive monetary policy model.

[16]See, for example, Fratto and Uhlig (2020).

section, since we now have many shocks, we choose to focus on a set of simpler experiments, studying how three alternative debt management strategies ('only short', 'only long', 'borrow long and save in short') impact the dynamic adjustment of inflation to the shocks. The aim throughout this section is to verify, in the medium scale model, that debt maturity is important for inflation variability.

To bring the economy to the passive regime we assume first that taxes are constant through time and second, we assume that monetary policy follows rules of the form (42) but now parameters are such that the nominal interest rate responds weakly to inflation. In Figures 5 and 6 we set $\phi_\pi = 0.9$ leaving the remaining parameters of the monetary policy rule be equal to the reported means of the posterior distributions in Table 3. Each of the figures shows impulse responses to three types of shocks. The top panels trace the responses of inflation whereas the bottom panels concern the adjustment of output. The dashed red lines show impulse responses when all debt is short term. The solid (blue) lines assume that all debt is long term and the dotted (black) lines set long term debt to be 10 times larger than the total value of debt financing the position with short term savings.

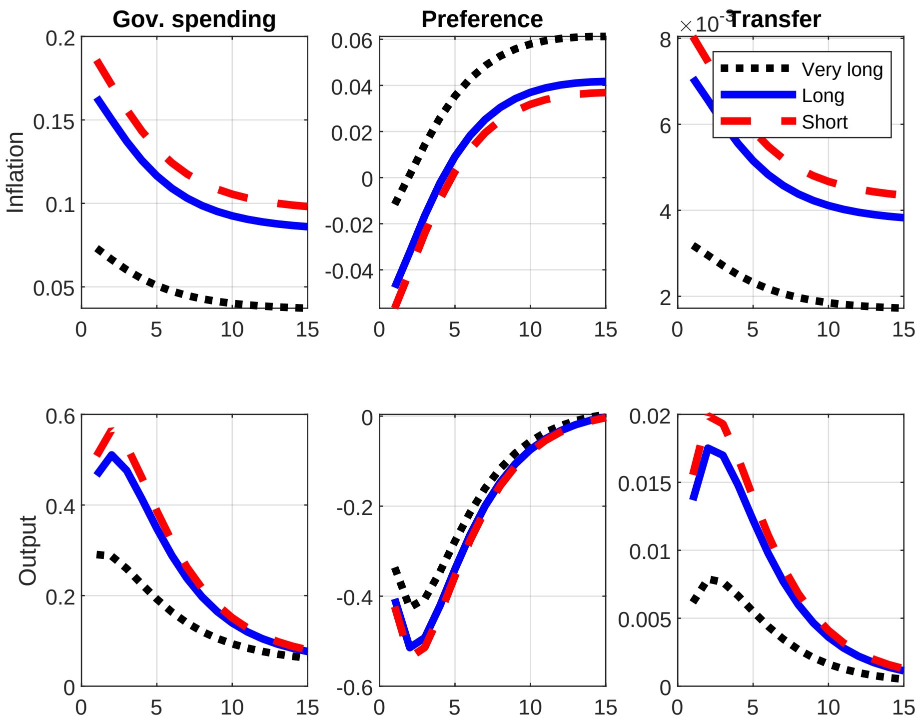
Figure 5: **Responses to shocks in the medium scale model I.**



Notes: The figure plots the impulse responses of inflation and output to TFP, markup shocks and shocks to the government budget Λ. The dashed lines correspond to the case where all debt is short term, the solid lines to the case where all debt is long and the dashed-dotted lines assume that long term debt is 10 times the value of total debt.

Consider first Figure 5 which covers shocks to TFP, markups and shocks to the government budget (Λ). Notice that even though the impact of TFP shocks is not large, it is clearly evident that shorter debt maturity increases the magnitude of the response of inflation to the shock. In contrast, debt maturity does not seem to matter for the propagation of markup shocks, which essentially lead to the same impulse responses in Figure 5 as in Figure 4 under the active policy regime.

Figure 6: **Responses to shocks in the medium scale model II.**



Notes: The figure plots the impulse responses of inflation and output to spending, demand (preference) and transfer shocks. The dashed lines correspond to the case where all debt is short term, the solid lines to the case where all debt is long and the dashed-dotted lines assume that long term debt is 10 times the value of total debt.

In contrast to the active regime, where markup shocks are the most important driver of inflation, under passive policy inflation displays a strong reaction to fiscal shocks. This is clearly shown in the right panel of Figure 5 but also in Figure 6 where we plot the reaction of inflation to $\hat{G}$ shocks in the left panel. Moreover, for both types of shocks the maturity structure of debt impacts the response of inflation, and the longer is maturity, the less is the response. Under a very long maturity structure (when the long bond issuance is 10 times the value of debt) we obtain for both shocks a response of inflation that is less than half of the response under short term debt. This prediction is obviously in line with our theoretical analysis where we found that issuing large amounts of long term debt is optimal to deal with fiscal shocks.

Finally, consider the middle panel of Figure 6, where we plot the response to a negative demand shock. When debt is only short, the demand shock causes a drop in inflation and a recession initially, however, a few periods down the line, inflation turns positive. This also happens with long term debt. Because of the switch in sign, debt maturity cannot mitigate the response of inflation to the shock, it could only be chosen to balance the cost of initial deflation with the cost of future inflation, as we saw in Section 2 of the paper.

To reiterate debt management is effective in stabilizing inflation in the face of fiscal shocks, which drive most of the excess inflation volatility under passive monetary policy. Thus, according to our findings it is preferable to issue large amounts of long bonds.

The online appendix extends further this analysis. We experiment with interest rate rules that feature no interest rate smoothing, and also consider rules that track the real interest rate. In these cases as well, large long bond issuances reduce the impact of fiscal shocks and it is preferable to tilt the maturity structure towards long debt.

5 Conclusion

We have provided a tractable framework to think about the interactions between debt maturity and inflation when monetary policy is subservient to fiscal policy. Our analytical results showed that these interactions are non-trivial and that debt management can complement monetary policy in pursuing its objective to control inflation. We drew analogous insights from a model where monetary and debt policies are jointly optimal. A particularly interesting analytical finding we obtained from this model, is that optimal monetary policy can be summarized in a simple interest rate rule that clarifies how interest rate policies depend on the maturity of debt issued.

Lastly, using a medium scale DGSE model we have tested whether indeed debt maturity matters when monetary policy is passive and found large effects when we looked at fiscal shocks and their impact on inflation.

Future work could apply these insights to models with regime fluctuations when monetary policy can transition between the active and passive regimes. In this case, debt maturity could exert a significant influence even in the active policy scenario. Moreover, such an exercise could be useful to pin down the optimal maturity when there is a temporary switch in the regime to e.g. monetize part of debt.

Of course a meaningful next step would also be to make the model non-linear, such that it can rely on more realistic yield curves, when inflation risk premia matter. Finally, in a non-linear model with regime fluctuations one could consider whether changing the mandate of debt management to align objectives with monetary policy as we assumed in this paper, is indeed desirable.

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Appendices

Appendix A Derivations from the analytical model

A.1 Proof of proposition 1

Let $\hat{V}_t = \Omega_S \hat{b}_{t,S} + \Omega_\delta \hat{b}_{t,\delta}$ be the log deviation of the total market value of debt from its steady state value where $\Omega_S \equiv \beta b_S$ and $\Omega_\delta \equiv \frac{\beta b_\delta}{1-\beta\delta}$. According to rule (7), the tax rate responds to the market value of debt.[17] Then, the system of equations period-to-period can be written as:

$$
\begin{aligned}
\hat{V}_t - \Omega_S \hat{i}_t + \Omega_\delta (1-\delta)\hat{p}_{t,\delta} \\
-\beta^{-1}\hat{V}_{t-1} - \beta^{-1}(\Omega_S + \Omega_\delta)\hat{\pi}_t + \frac{\tau(1+\eta)\overline{Y}}{\eta}\left([1-\gamma_h]\hat{y}_t + \frac{\hat{\tau}}{1-\tau}\right) - G\hat{G}_t = 0 \\
\hat{p}_{t,\delta} + \hat{i}_t - \beta\delta E_t \hat{p}_{t+1,\delta} = 0 \\
\hat{i}_t - E_t(\hat{\pi}_{t+1} - \hat{\xi}_{t+1}) - \hat{\xi}_t = 0 \\
\hat{\pi}_t - \kappa_1 \hat{y}_t - \kappa_2 \hat{\tau}_t - \beta E_t \hat{\pi}_{t+1} = 0
\end{aligned}
$$

where $\hat{p}_{t,\delta}$ denotes the price of the long-term bond. Applying substitutions with rules (7) and (6), the system can be reduced to the following three equations:

$$
\begin{aligned}
\hat{V}_t - \Omega_S \epsilon_{i,t} + \Omega_\delta(1-\delta)\hat{p}_{t,\delta} - \left(\beta^{-1} - \frac{\tau\overline{Y}(1+\eta)(\kappa_1 - (1-\tau)(1+\gamma_h)\kappa_2)}{\eta(1-\tau)\kappa_1}\phi^R_{\tau,b}\right)\hat{V}_{t-1} - G\hat{G}_t \\
+\left(\beta^{-1}(\Omega_S + \Omega_\delta) + \frac{\tau(1+\eta)\overline{Y}(1+\gamma_h)}{\eta\kappa_1} - \Omega_S\phi_\pi\right)\hat{\pi}_t - \frac{\tau(1+\eta)\overline{Y}(1+\gamma_h)}{\eta\kappa_1}\beta E_t\hat{\pi}_{t+1} = 0 \\
\hat{p}_{t,\delta} + \phi_\pi\hat{\pi}_t + \epsilon_{i,t} - \beta\delta E_t\hat{p}_{t+1,\delta} = 0 \\
\phi_\pi\hat{\pi}_t + \epsilon_{i,t} - E_t\hat{\pi}_{t+1} + E_t\hat{\xi}_{t+1} - \hat{\xi}_t = 0
\end{aligned}
$$

Let's denote $\Omega \equiv \Omega_S + \Omega_\delta$ and $\Phi \equiv \frac{\tau(1+\eta)\overline{Y}(1+\gamma_h)}{\eta\kappa_1}\beta$. For any variable x_t, define the rational expectations errors by $\eta^x_t \equiv \hat{x}_t - E_{t-1}\hat{x}_t$ and replace x_t with $E_{t-1}x_t + \eta^x_t$. Assuming i.i.d. shocks, we can then

[17] Notice that the precise definition of $\hat{V}_t$ does not matter for the dynamics of other macroeconomic variables provided appropriate scaling.

write the system in matrix form as

$$
\begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta\delta & 0 \\ -\Phi & 0 & 1 \end{bmatrix} \begin{bmatrix} E_t\hat{\pi}_{t+1} \\ E_t\hat{p}_{t+1,\delta} \\ \hat{V}_t \end{bmatrix}
$$

$$
= \begin{bmatrix} \phi_\pi & 0 & 0 \\ \phi_\pi & 1 & 0 \\ \left(\Omega_S\phi_\pi - \beta^{-1}\Omega - \Phi\beta^{-1}\right) & -\Omega_\delta(1-\delta) & \left(\beta^{-1} - \frac{\kappa_1-(1-\tau)(1+\gamma_h)\kappa_2}{(1-\tau)(1+\gamma_h)\beta}\Phi\phi^R_{\tau,b}\right) \end{bmatrix} \begin{bmatrix} E_{t-1}\hat{\pi}_t \\ E_{t-1}\hat{p}_{t,\delta} \\ \hat{V}_{t-1} \end{bmatrix}
$$

$$
+ \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ \Omega_S & G & 0 \end{bmatrix} \begin{bmatrix} \epsilon_{i,t} \\ \hat{G}_t \\ \hat{\xi}_t \end{bmatrix} + \begin{bmatrix} \phi_\pi & 0 \\ \phi_\pi & 1 \\ \left(\Omega_S\phi_\pi - \beta^{-1}\Omega - \Phi\beta^{-1}\right) & -\Omega_\delta(1-\delta) \end{bmatrix} \begin{bmatrix} \eta^\pi_t \\ \eta^{p_\delta}_t \end{bmatrix}
$$

Or, more compactly:

$$AZ_t = BZ_{t-1} + CX_t + DY_t$$

where X_t is the vector of exogenous variables. Now, premultiply each side by:

$$
A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (\beta\delta)^{-1} & 0 \\ \Phi & 0 & 1 \end{bmatrix}
$$

The resulting system reads

$$
\begin{bmatrix} E_t\hat{\pi}_{t+1} \\ E_t\hat{p}_{t+1,\delta} \\ \hat{V}_t \end{bmatrix}
$$

$$
= \begin{bmatrix} \phi_\pi & 0 & 0 \\ \phi_\pi(\beta\delta)^{-1} & (\beta\delta)^{-1} & 0 \\ \left(\Phi\phi_\pi + \Omega_S\phi_\pi - \beta^{-1}\Omega - \Phi\beta^{-1}\right) & -\Omega_\delta(1-\delta) & \left(\beta^{-1} - \frac{\kappa_1-(1-\tau)(1+\gamma_h)\kappa_2}{(1-\tau)(1+\gamma_h)\beta}\Phi\phi^R_{\tau,b}\right) \end{bmatrix} \begin{bmatrix} E_{t-1}\hat{\pi}_t \\ E_{t-1}\hat{p}_{t,\delta} \\ \hat{V}_{t-1} \end{bmatrix}
$$

$$
+ \begin{bmatrix} 1 & 0 & -1 \\ (\beta\delta)^{-1} & 0 & 0 \\ \Phi + \Omega_S & G & -\Phi \end{bmatrix} \begin{bmatrix} \epsilon_{i,t} \\ \hat{G}_t \\ \hat{\xi}_t \end{bmatrix} + \begin{bmatrix} \phi_\pi & 0 \\ \phi_\pi(\beta\delta)^{-1} & (\beta\delta)^{-1} \\ \left(\Phi\phi_\pi + \Omega_S\phi_\pi - \beta^{-1}\Omega - \Phi\beta^{-1}\right) & -\Omega_\delta(1-\delta) \end{bmatrix} \begin{bmatrix} \eta^\pi_t \\ \eta^{p_\delta}_t \end{bmatrix}
$$

The system has three eigenvalues:

$$
\begin{aligned}
e_1 &= \phi_\pi \\
e_2 &= (\beta\delta)^{-1} \\
e_3 &= \beta^{-1} - \frac{\tau\overline{Y}(1+\eta)(\kappa_1 - (1-\tau)(1+\gamma_h)\kappa_2)}{\eta(1-\tau)\kappa_1}\phi^R_{\tau,b}
\end{aligned}
$$

Moreover, there are two non-predetermined variables, $\mathbb{E}_t\hat{p}_{t+1,\delta}$ and $\mathbb{E}_t\hat{\pi}_{t+1}$. For determinacy, we hence need two of the eigenvalues to lie outside the unit circle. For any non-explosive maturity of debt, we have that $\delta \leq 1$ and thus, $e_2 > 1$. Consequently, we need either $\phi_\pi > 1$ or $\phi_{\tau,b} <$

$\frac{\eta(1-\tau)\kappa_1(\beta^{-1}-1)}{\tau(1+\eta)\overline{Y}(\kappa_1-(1-\tau)(1+\gamma_h)\kappa_2)}$. ■

A.2 Proof of proposition 2

Assume that $\phi_{\tau,b} = 0$ and $\phi_\pi < 1$. Combining the Euler equation with the monetary policy rule (and assuming no shocks after period t) we get:

$$\hat{\pi}_{t+\bar{t}} = \phi_\pi \hat{\pi}_{t+\bar{t}-1} + (\epsilon_{i,t} - \hat{\xi}_t)\mathcal{I}_{\bar{t}=1} \tag{43}$$

From the Phillips curve we have

$$\hat{Y}_{t+\bar{t}} = \frac{1}{\kappa_1}\left(\hat{\pi}_{t+\bar{t}} - \beta\hat{\pi}_{t+1+\bar{t}}\right) = \frac{1}{\kappa_1}(1-\beta\phi_\pi)\hat{\pi}_{t+\bar{t}} - \frac{\beta}{\kappa_1}(\epsilon_{i,t} - \hat{\xi}_t)\mathcal{I}_{\bar{t}=1}$$

Given these expressions to characterize inflation in t we use the date t intertemporal budget constraint of the government. We can write the constraint as:

$$\sum_{j\geq 0}\beta^j E_t\left(\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}((\gamma_h+1)\hat{Y}_{t+j} + \hat{\xi}_{t+j}) - \overline{G}(\hat{G}_{t+j} + \hat{\xi}_{t+j})\right) = \overline{b}_S(\hat{b}_{t-1,S} - \hat{\pi}_t) + \frac{\overline{b}_\delta}{1-\beta\delta}\hat{b}_{t-1,\delta} +$$
$$\overline{b}_\delta E_t \sum_{j=0}^{\infty}\beta^j\delta^j\left[-\sum_{l=0}^{j}\hat{\pi}_{t+l}\right] + \hat{\xi}_t(\overline{b}_\delta + \overline{b}_S)$$

The RHS can be written as:

$$\overline{b}_S(\hat{b}_{t-1,S} - \hat{\pi}_t) + \frac{\overline{b}_\delta}{1-\beta\delta}\hat{b}_{t-1,\delta} - \overline{b}_\delta\sum_{j=0}^{\infty}\beta^j\delta^j\left[\frac{(1-\phi_\pi^{j+1})}{1-\phi_\pi}\hat{\pi}_t + \frac{(1-\phi_\pi^j)}{1-\phi_\pi}(\epsilon_{i,t} - \hat{\xi}_t)\right] + \hat{\xi}_t(\overline{b}_\delta + \overline{b}_S)$$

The LHS is

$$\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}(\gamma_h+1)\frac{1}{\kappa_1}\hat{\pi}_t - \overline{G}\hat{G}_t + \left(\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta} - \overline{G}\right)\hat{\xi}_t$$

Simplifying further the RHS can be written as:

$$\overline{b}_S(\hat{b}_{t-1,S} - \hat{\pi}_t) + \frac{\overline{b}_\delta}{1-\beta\delta}\hat{b}_{t-1,\delta} - \frac{\overline{b}_\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\hat{\pi}_t - \frac{\overline{b}_\delta\beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}(\epsilon_{i,t} - \hat{\xi}_t) + \hat{\xi}_t(\overline{b}_\delta + \overline{b}_S)$$

Equating LHS and RHS letting $\overline{R} = \overline{\tau}(1+\eta)\frac{\overline{Y}}{\eta}$ and noting that the steady state surplus can be written as $\left(\frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta} - \overline{G}\right) = (1-\beta)\overline{b}_S + \frac{1-\beta}{1-\beta\delta)}\overline{b}_\delta$ and rearranging we get:

$$\left[\overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \overline{b}_S + \frac{\overline{b}_\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right]\hat{\pi}_{t}, =$$
$$-\left[\frac{\overline{b}_\delta\beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right]\epsilon_{i,t} + \overline{G}\hat{G}_t + \left[\beta\overline{b}_S + \frac{\overline{b}_\delta\beta(1-(1-\delta)\delta\beta\phi_\pi)}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}\right]\hat{\xi}_t$$

It is easy to obtain coefficients η_1, η_2 and η_3 from the above expression. From (43) it is possible to characterize the entire path of inflation. ■

A.3 Proof of Proposition 3.

The FONC of the minimisation program (12) with only demand shocks is:

$$\left(\eta_3 + \frac{\phi_\pi \beta}{1-\phi_\pi^2 \beta}(\eta_3 \phi_\pi - 1)\right)\frac{d\eta_3}{d\bar{b}_1}\sigma_{\hat{\xi}}^2 = 0$$

When $\phi_\pi = 0$, it holds that $\frac{d\eta_3}{d\bar{b}_1} = 0$. The maturity of debt is irrelevant.

In the case where $\phi_\pi > 0$ we have that $\frac{d\eta_3}{d\bar{b}_1} \neq 0$ and thus, the FONC imposes that $\eta_3 + \frac{\phi_\pi \beta}{1-\phi_\pi^2 \beta}(\eta_3 \phi_\pi - 1) = 0$ which corresponds to $\eta_3 = \beta \phi_\pi$ ■

A.4 Proof of Proposition 5.

Let's consider first the case where the interest rate respond to the demand shock only through its endogenous response to inflation. The FONC for the combined shock is:

$$\eta_2 \eta_2' \sigma_G^2 \left(1 + \frac{\beta \phi_\pi^2}{1-\beta\phi_\pi^2}\right) + \eta_3' \sigma_\xi^2 \left(\eta_3 + \frac{\beta \phi_\pi (\eta_3 \phi_\pi - 1)}{1-\beta\phi_\pi^2}\right) = 0$$

where $\eta_2' \equiv \frac{d\eta_2}{d\bar{b}_1}$ and $\eta_3' \equiv \frac{d\eta_3}{d\bar{b}_1}$. Rearranging this expression, we obtain:

$$\eta_3 = \beta\phi_\pi - \eta_2 \frac{\eta_2'}{\eta_3'} \frac{\sigma_G^2}{\sigma_\xi^2} \tag{44}$$

Notice that η_2 and η_3 can be written as:

$$\eta_2 = \frac{\overline{G}}{\Lambda}$$
$$\eta_3 = \frac{\beta\left(\Lambda - \frac{\overline{R}(\gamma_h+1)}{\kappa_1}\right)}{\Lambda}$$

where $\Lambda \equiv \overline{R}\frac{(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{(1-\beta)} + \frac{\bar{b}_\delta \phi_\pi \beta\delta}{(1-\beta\delta)(1-\phi_\pi\beta\delta)}$. And thus,

$$\eta_2' = \frac{-\Lambda'_{b_\delta}\overline{G}}{\Lambda^2}$$
$$\eta_3' = \frac{\frac{\beta\overline{R}(\gamma_h+1)}{\kappa_1}\Lambda'_{b_\delta}}{\Lambda^2}$$

Substituting these derivatives in (44) allows to easily recover the formula in the text.

Next, let us consider the case of monetary policy responding one-to-one to the demand shock i.e.

$\epsilon_{i,t} = \hat{\xi}_t$. Since this implies that $\sigma_{\epsilon_i,\hat{\xi}} = \sigma^2_{\hat{\xi}}$, the FONC becomes:

$$\eta_1\eta_1'\sigma_G^2 + (\eta_2\eta_2' + \eta_3\eta_3' + \eta_1'\eta_3 + \eta_3'\eta_1)\sigma_\xi^2 + \frac{\beta\phi_\pi\eta_2\eta_2'}{1-\beta\phi_\pi}\sigma_G^2 + \left(\frac{(1+\phi_\pi\eta_1)\eta_1'}{1-\beta\phi_\pi} + \frac{(\eta_3\phi_\pi - 1)\eta_3'}{1-\beta\phi_\pi}\right)\beta\phi_\pi\sigma_\xi^2$$
$$= \frac{\phi_\pi\eta_1 - \eta_3\phi_\pi - \eta_1'\eta_3\phi_\pi^2 - \eta_3'\eta_1\phi_\pi^2}{1-\beta\phi_\pi}\beta\sigma_\xi^2$$

Simplifying this expression, we arrive to:

$$\eta_1 + \eta_3 = -\frac{\eta_2\eta_2'}{\eta_1' + \eta_3'}\frac{\sigma_G^2}{\sigma_\xi^2}$$

where $\eta_1' \equiv \frac{d\eta_1}{d\bar{b}_1}$.

Focus first on the LHS. Notice that η_1 can be written as:

$$\eta_1 = -\frac{\left[\Lambda - \left(\frac{\overline{R}(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)\right]\frac{1}{\phi_\pi}}{\Lambda}$$

and hence, we have:

$$LHS = \eta_1 + \eta_3 = \frac{\beta}{(1-\beta)\Lambda}(\overline{S} - \bar{b}_\delta)$$

Let us now turn to the RHS. The derivative of η_1 is:

$$\eta_1' = -\frac{\left(\frac{\overline{R}(\gamma_h+1)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)\frac{\Lambda'_{b_\delta}}{\phi_\pi}}{\Lambda^2}$$

Hence,

$$\eta_1' + \eta_3' = \frac{\left(\frac{\overline{R}(\gamma_h+1)}{\kappa_1}(\beta\phi_\pi - 1) - \frac{\overline{S}}{1-\beta}\right)\frac{\Lambda'_{b_\delta}}{\phi_\pi}}{\Lambda^2}$$

The RHS is thus:

$$RHS = -\frac{\overline{G}^2\phi_\pi}{\left(\frac{\overline{R}(\gamma_h+1)}{\kappa_1}(1-\beta\phi_\pi) + \frac{\overline{S}}{1-\beta}\right)\Lambda}$$

Equating the LHS and the RHS and rearranging, one can easily retrieve the formula in the text. ■

A.5 Proof of Proposition 6.

Rewriting equation (28) with the lag operator gives:

$$(1-\delta L)\hat{\pi}_{t+\bar{t}} = \frac{(1-L)(1-\delta L)(1+\gamma_h)}{\kappa_1}\bar{R}\psi_{gov,t+\bar{t}} + (1-L)(1-\delta L)\bar{b}_S\psi_{gov,t+\bar{t}} + \frac{1-L}{1-\beta\delta}\bar{b}_\delta\psi_{gov,t+\bar{t}}$$

Forward this equation one period and use the backshift operator $BE_{t+\bar{t}}x_{t+\bar{t}} = E_{t+\bar{t}}x_{t+\bar{t}-1}$ to obtain:

$$
\begin{aligned}
(1-\delta B)E_{t+\bar{t}}\hat{\pi}_{t+\bar{t}+1} = {} & \frac{(1-B)(1-\delta B)(1+\gamma_h)}{\kappa_1}\bar{R}E_{t+\bar{t}}\psi_{gov,t+\bar{t}+1} \\
& + (1-B)(1-\delta B)\bar{b}_S E_{t+\bar{t}}\psi_{gov,t+\bar{t}+1} + \frac{1-B}{1-\beta\delta}\bar{b}_\delta E_{t+\bar{t}}\psi_{gov,t+\bar{t}+1}
\end{aligned}
$$

Since $\psi_{gov,t}$ is a random walk, we have that $(1-B)E_t\psi_{gov,t+1} = 0$. Hence, the previous equation boils down to:

$$
E_{t+\bar{t}}\hat{\pi}_{t+\bar{t}+1} = \delta\hat{\pi}_{t+\bar{t}} - \delta\left(\frac{(1+\gamma_h)}{\kappa_1}\bar{R} + \bar{b}_S\right)\Delta\psi_{gov,t+\bar{t}} \tag{45}
$$

Plugging this expression in the Fisher equation $\hat{i}_{t+\bar{t}} = E_{t+\bar{t}}\hat{\pi}_{t+1+\bar{t}} + \hat{\xi}_{t+\bar{t}}$ gives the optimal interest rule in the text. ■

A.6 Derivation of equation (31) in the text

Since we assume that no further shock hits the economy after period t then $\Delta\psi_{gov,t+j} = 0$, $j \geq 1$. Moreover setting $\psi_{gov,t-1} = \psi_{gov,t-2} = \ldots$ we can write (28) as

$$
\hat{\pi}_t = \left(\overline{R}\frac{(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)\Delta\psi_{gov,t}
$$

Moreover, combining this equation with (45) and extrapolating for any generic period gives the following path of future inflation

$$
\hat{\pi}_{t+\bar{t}} = \frac{\overline{b}_\delta}{1-\beta\delta}\delta^{\bar{t}}\Delta\psi_{gov,t} \tag{46}
$$

Next, consider the intertemporal government budget constraint:

$$
\begin{aligned}
& \sum_{j=0}^{\infty}\beta^j\Big(\bar{R}[(1+\gamma_h)\hat{y}_{t+j} + \hat{\xi}_{t+j}] - \bar{G}[\hat{G}_{t+j} + \hat{\xi}_{t+j}]\Big) \\
& \qquad = \bar{b}_S(\hat{b}_{t-1,S} - \hat{\pi}_t) + \frac{\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t-1,\delta} - \bar{b}_\delta\sum_{j=0}^{\infty}\beta^j\delta^j\sum_{l=0}^{j}\hat{\pi}_{t+l} + (\bar{b}_S + \bar{b}_\delta)\hat{\xi}_t
\end{aligned}
$$

Plugging (46) in the Phillips Curve, we have:

$$
\hat{y}_t = \frac{\hat{\pi}_t}{\kappa_1} - \frac{\beta\overline{b}_\delta}{\kappa_1(1-\beta\delta)}\delta\Delta\psi_{gov,t}
$$

and, for any $j > 0$,

$$
\hat{y}_{t+j} = \frac{\overline{b}_\delta}{\kappa_1(1-\beta\delta)}\delta^j\Delta\psi_{gov,t} - \frac{\beta\overline{b}_\delta}{\kappa_1(1-\beta\delta)}\delta^{j+1}\Delta\psi_{gov,t}
$$

Hence:

$$\sum_{j=0}^{\infty} \beta^j \hat{y}_{t+j} = \frac{\hat{\pi}_t}{\kappa_1}$$

The LHS of the intertemporal budget constraint can be written as:

$$\frac{\overline{R}(1+\gamma_h)}{\kappa_1}\hat{\pi}_t - \overline{G}\hat{G}_t + \overline{S}\hat{\xi}_t$$

Let's focus now on the RHS. The following decomposition applies:

$$- \bar{b}_\delta \sum_{j=0}^{\infty} \beta^j \delta^j \sum_{l=0}^{j} \hat{\pi}_{t+l} = -\bar{b}_\delta \sum_{j=0}^{\infty} \beta^j \delta^j \hat{\pi}_t - \frac{\bar{b}_\delta^2}{1-\beta\delta} \Delta\psi_{gov,t} \sum_{j=1}^{\infty} \beta^j \delta^j \sum_{l=1}^{j} \delta^l$$
$$= -\frac{\bar{b}_\delta}{1-\beta\delta}\hat{\pi}_t - \frac{\bar{b}_\delta^2}{1-\beta\delta}\Delta\psi_{gov,t} \sum_{j=0}^{\infty} \beta^j \delta^{j+1} \frac{1-\delta^j}{1-\delta} = \left(\bar{b}_S - \frac{\overline{S}}{1-\beta}\right)\hat{\pi}_t - \frac{\bar{b}_\delta^2 \beta\delta^2}{(1-\beta\delta)^2(1-\beta\delta^2)}\Delta\psi_{gov,t}$$

where the last equality stems from the steady state identity $\frac{\bar{b}_\delta}{1-\beta\delta} = \bar{b}_S - \frac{\overline{S}}{1-\beta}$. Equating the LHS and the RHS (assuming as before that initial debt is zero), we find:

$$\left(\frac{\overline{R}(1+\gamma_h)}{\kappa_1} + \frac{\overline{S}}{1-\beta}\right)\hat{\pi}_t = \overline{G}\hat{G}_t + (\bar{b}_S + \bar{b}_\delta - \overline{S})\hat{\xi}_t - \frac{\bar{b}_\delta^2 \beta\delta^2}{(1-\beta\delta)^2(1-\beta\delta^2)}\Delta\psi_{gov,t}$$

Using (28) to substitute for $\hat{\pi}_t$ and rearranging gives the expression in the text.

Appendix B Lagrangian of the optimal policy problem

Let $\psi_{\pi,t+\bar{t}}$ and $\psi_{g,t+\bar{t}}$ be the Lagrange multipliers attached to the Phillips curve and the consolidated budget, respectively. We formulate the Lagrangian as

$$\mathcal{L} = E_t \sum_{\bar{t}=0}^{\infty} \beta^{\bar{t}} \Bigg\{ \hat{\pi}_{t+\bar{t}} + \psi_{\pi,t+\bar{t}}\left[\hat{\pi}_{t+\bar{t}} - \kappa_1 \hat{Y}_{t+\bar{t}} - \beta E_{t+\bar{t}}\hat{\pi}_{t+\bar{t}+1}\right]$$
$$+ \psi_{g,t+\bar{t}}\Bigg[\beta\bar{b}_S\hat{b}_{t+\bar{t},S} + \beta\bar{b}_S E_{t+\bar{t}}(\hat{\xi}_{t+\bar{t}+1} - \hat{\xi}_{t+\bar{t}} - \hat{\pi}_{t+\bar{t}+1}) + \frac{\beta\bar{b}_\delta}{1-\beta\delta}\hat{b}_{t+\bar{t},\delta}$$
$$+ \bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1}\left(E_{t+\bar{t}}\left(-\sum_{l=1}^{j} \hat{\pi}_{t+\bar{t}+l} + \hat{\xi}_{t+\bar{t}+j} - \hat{\xi}_{t+\bar{t}}\right)\right) + \frac{\overline{\tau}(1+\eta)\overline{Y}}{\eta}(\gamma_h + 1)\hat{Y}_{t+\bar{t}} - \overline{G}\hat{G}_{t+\bar{t}}$$
$$-\bar{b}_S(\hat{b}_{t+\bar{t}-1,S} - \hat{\pi}_{t+\bar{t}}) - \frac{\bar{b}_\delta}{1-\beta\delta}(\hat{b}_{t+\bar{t}-1,\delta} - \hat{\pi}_{t+\bar{t}}) - \delta\bar{b}_\delta \sum_{j=1}^{\infty} \beta^j \delta^{j-1}\left(E_{t+\bar{t}}\left(-\sum_{l=1}^{j} \hat{\pi}_{t+\bar{t}+l} + \hat{\xi}_{t+\bar{t}+j} - \hat{\xi}_{t+\bar{t}}\right)\right)\Bigg]\Bigg\}$$

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