OPTIMAL TAX AND TRANSFERS WITH HOUSEHOLD HETEROGENEITY

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Optimal Taxes and Transfers with Household Heterogeneity*

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Abstract

We investigate the properties of optimal fiscal policy in a framework where household heterogeneity is accounted for. The Ramsey planner chooses (distortionary) labor taxes and transfers to maximize aggregate welfare in a two-agent economy. We contrast the properties of optimal labor taxes in our model to the ones obtained in the representative agent counterpart. We first show that the presence of household heterogeneity introduces an additional source of fluctuations in the optimal tax rate, as varying taxes allows the planner to use transfers for redistributive purposes. We then show that, depending on the assumptions that are made on how transfer receipts are distributed among households, and the type of shocks hitting the economy, the structure of government bond markets becomes more or less important in shaping the dynamics of the Ramsey allocation. In some cases, the presence of transfers brings the incomplete markets allocation close to the one in which the planner has access to state-contingent claims. We finally show that the presence of heterogeneity and optimal transfers helps bring the behaviour of fiscal variables in the Ramsey model closer to their counterpart in US data.

Keywords: Fiscal policy, Household heterogeneity, Optimal taxation, Transfers.

JEL classification: E32, E62, H21, H23, H31.

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1 Introduction

The importance of heterogeneity at the household level in shaping economic fluctuations is the focus of a flourishing literature in macroeconomics. While many authors have studied the positive aspects of heterogeneity to analyze its impact on the cyclical properties of key macroeconomic aggregates, little work has been done that combines the cycle with the normative aspects of policy. This paper intents to fill this gap, by providing an analysis of optimal fiscal policies in models where households are heterogeneous in terms of labor productivity and holdings of financial assets.

The fiscal tools available to governments can broadly be seen as serving two main goals. First, deficit financing can be used along the business cycle to stabilize macroeconomic fluctuations; second, governments can use fiscal instruments to redistribute resources between economic agents and provide social insurance. While many papers in the optimal fiscal policy (OFP) literature have studied these two components in isolation, few authors have considered them at the same time. Indeed, broadly speaking, the OFP literature can be divided in two parts. On the one hand, several papers develop representative agent models in which fiscal instruments are used to mitigate the impact of aggregate shocks on macroeconomic variables[1]. These papers are therefore mostly concerned with the stabilization role of fiscal policy. On the other hand, numerous works have developed frameworks featuring household heterogeneity and in which the fiscal authority sets policy variables so as to redistribute resources between agents[2]. This line of work is therefore mostly concerned with the redistribution component of policy, and it usually relies on models where aggregate shocks are absent. In this paper, we consider a framework that weds these two approaches. We introduce a business cycle model which relies on a mild degree of heterogeneity, and consider fiscal instruments that allow the planner to redistribute resources across agents.[3] This implies a potential conflict between the stabilization of aggregate variables and the redistribution of market resources, and we are interested in looking at the trade-offs facing the fiscal authority.

The set of questions we want to address in this paper are the following. First, we intend to

[1]Two seminal papers in this literature are the work of Lucas and Stokey (1983) and Aiyagari, Marcet, Sargent, and Seppälä (2002).

[2]See, for instance, Conesa and Krueger (2006); Conesa, Kitao, and Krueger (2009) and Heathcote, Storesletten, and Violante (2017).

[3]In this way our paper relates to the consequent literature using the so-called TANK model to study the impact of household heteorogeneity on the macroeconomy (following the work of Galí, López-Salido, and Vallés, 2007 and Bilbiie, 2008). Note, however, that most of the work that has been done using this type of models has focused on monetary policy, while we are interested in the role of fiscal policy.

understand the properties of the optimal tax schedule when heterogeneity is accounted for. In particular, we want to investigate how the volatility of optimal labor taxes is impacted by the presence of household heterogeneity and the availability of transfers as a fiscal instrument for the Ramsey planner. We study to which extent variations in transfers rather than distortionary taxes are used to finance deficits, when it comes at the cost of increasing inequality in consumption and hours worked between households. In order to assess whether heterogeneity matters for the design of the optimal policy, we contrast our results to the ones obtained in the representative agent framework typically used in the literature. Later in the paper, we investigate whether our simple model can reproduce key stylized facts related to the conduct of fiscal policy in the US. In particular, we assess whether the cyclical properties of output, deficits, transfers, and government debt, are better matched by our two-agent framework than by its representative agent counterpart, when we calibrate the model parameters plausibly.

In our model, *savers*, or *Ricardian* households, have access to financial markets and save by accumulating government bonds. *Non-savers*, or *hand-to-mouth* households, do not have access to financial assets and consume their entire disposable income in every period. Households also differ in terms of their labor productivity, and we allow these differences to move over the business cycle (in other words, we allow for non-trivial interactions between aggregate and 'household-specific' shocks). Using this framework, we study the properties of optimal fiscal policy under commitment, where a benevolent government sets distortionary labor taxes and issues debt to finance government spending and transfers, so as to maximize aggregate welfare in the economy. Transfers are restricted to follow an empirically-motivated targeting rule which specifies their repartition between agents, and implies a co-movement between the transfers received by each household. The level of transfers is also set optimally by the planner, and we do not impose restrictions on their value. Therefore a government that prioritizes to smooth the burden from distortionary taxation can set transfers negative, thus relying on lump-sum taxes to finance deficits.

We first analyze the steady-state properties of the model, in which the planner chooses labor taxes and transfers to maximize aggregate welfare, for given levels of government expenditures and government debt. We provide a condition for the optimal long-run tax rate on labor to be positive, and show that it is not always optimal to set the labor tax rate to zero and finance the entire deficit in lump-sum. The optimal labor tax is positive when heterogeneity between households is sufficiently high, and when the design of transfers allows for sufficient redistribution between agents. In this case, increasing the tax rate and letting transfers be positive, allows the planner to redistribute across agents.

We then study the properties of optimal taxes and transfers in the presence of shocks, and look at the cyclical properties of the Ramsey allocation. We thus extend the analysis of Lucas and Stokey (1983) and Aiyagari et al. (2002, AMSS) to a two-agent setting. We derive an analytical expression for the optimal labor tax schedule, showing that tax volatility can be attributed to two components/sources. The first is related to market incompleteness, since as in AMSS we assume that debt is not state contingent. We show that transfers enable the planner to reduce tax volatility through (for example) lowering transfers when the deficit is high. This tends to bring the economy closer to a complete market outcome (e.g. Lucas and Stokey, 1983). The second component of tax volatility derives from heterogeneity in consumption and hours worked between households. Tax volatility is shown to be a function of the cyclical behavior of variables that summarize consumption and hours inequality. Using our simplistic model we investigate the relative importance of these two sources of volatility. We show that the quantitative impact of heterogeneity is much weaker than the impact arising from incomplete markets.

In the last part of the paper, we investigate the potential of an optimal fiscal policy model with heterogeneity to match the US data. We do so mainly to gain insights on the relative importance of heterogeneity and incomplete markets in shaping the behavior of fiscal variables and on the necessity of explictly modeling heterogeneity in models of optimal fiscal policy. We use a simulated method of moments (SMM) estimator to choose the parameter values that minimize the distance between moments generated by the model and those observed in the data. We find that the estimated model does a good job in matching the empirical properties of fiscal variables and for plausible parameter values. Our estimates suggest that a key driver of fluctuations in model variables are the shocks affecting the relative labor productivity of hand-to-mouth agents. In response to these shocks, the planner implements a policy which implies co-movement between transfers, deficits and GDP that is very similar to the analogous moments in the data. At the estimated parameters values, we find that the optimal allocation features less tax volatility than in a representative agent, incomplete markets model.

Related Literature Our paper contributes to the literature in several ways. First, we consider a model where household heterogeneity is accounted for using a two-agent setup, along the lines of Campbell and Mankiw (1989), Galí et al. (2007) and Bilbiie (2008).[4] This class of models has widely been used to look at the positive aspects of monetary and fiscal policies;[5] we take a normative stance and study the properties of optimal fiscal policy implemented by a planner that operates under full commitment and has access to two types of instruments: distortionary labor taxes and transfers.

Our model also builds on the seminal contributions of Lucas and Stokey (1983) and Aiyagari et al. (2002). These two papers study the cyclical properties of Ramsey policies when, respectively, the planner has access to a full set of contingent securities (financial markets are complete), and when the planner can only issue one-period risk free bonds, such that markets are incomplete.[6] Scott (2007) and Marcet and Scott (2009) show that the co-movement between deficits and government debt is negative under complete markets, while it is positive under incomplete markets. Moreover, they show that Ramsey models with complete markets imply no persistence in government debt while incomplete markets models imply very persistent levels.[7] Empirically, the behaviour of fiscal variables seems to favor incomplete markets, as the observed correlation between deficits and the market value of debt is positive. However, the persistence of government debt under incomplete markets that is derived in these models overshoots the data counterpart (see Marcet and Scott (2009)). In this paper, we show that, when household heterogeneity is accounted for, and transfers are introduced, the obtained persistence of government debt gets closer to what is observed in the data.

While models featuring household heterogeneity have been widely used to study the long-run properties of the optimal tax schedule (see Aiyagari (1995), Conesa et al. (2009), Heathcote et al. (2017) and references therein), little work has been done on Ramsey policies in frameworks that integrate both heterogeneous agents and aggregate shocks. Notable exceptions are Werning (2007), Bassetto (2014), Bhandari, Evans, Golosov, and Sargent (2017, 2018), and Le Grand

[4]A growing literature studies the impact of household heterogeneity on macroeconomic outcomes in models where agents face idiosyncratic risks. The seminal contribution of Kaplan, Moll, and Violante (2018) studies monetary policy in their so-called HANK model. Other examples of papers this literature include Krueger, Mitman, and Perri (2016), Auclert (2019), McKay and Reis (2016b), Werning (2015). Debortoli and Galí (2017) provide a useful comparison between the predictions obtained from TANK versus HANK models.

[5]Bilbiie and Ragot (2017),Challe et al. (2017) and Bilbiie (2018) use this framework to study optimal monetary policies.

[6]The implications of the maturity of the government debt, i.e. the existence of both short and/or long term bonds, on the properties of optimal fiscal policy are well understood. See, for instance, Angeletos (2002), Buera, Nicolini et al. (2004), and Faraglia, Marcet, Oikonomou, and Scott (2019a,b).

[7]The literature has also explored the role of monetary policy in stabilizing government debt. It has been shown that, in the presence of nominal rigidities, inflation volatility has too big welfare costs to become an appropriate margin to stabilize debt. See e.g. Schmitt-Grohé and Uribe (2004) and Faraglia, Marcet, Oikonomou, and Scott (2013).

and Ragot (2017). Le Grand and Ragot (2017) and Bhandari et al. (2018) develop numerical algorithms that approximate Ramsey allocations in economies with heterogeneous agents, uninsurable idiosyncratic risks, and aggregate shocks. They apply their novel methods to study, respectively, optimal capital taxation, and jointly optimal monetary and fiscal policies. Compared to these papers, ours adopts a simpler 2-agent structure, and focuses on the optimal use of labor taxes and transfers under complete and incomplete government bond markets, in a model without capital and nominal rigidities. Our simpler setup enables us to derive analytical formulae that shed light on the trade-offs facing the government. Bassetto (2014), Werning (2007) and Bhandari et al. (2017) study optimal policies in settings closely related to ours. The novelty in our paper lies in its focus on the role that transfers and market (in)completeness play for the conduct of optimal fiscal policy.

Finally, given the emphasis we give to transfers and their role for social insurance along the business cycle, our paper is also related to Oh and Reis (2012), who study the stabilizing role of transfers in the Great Recession in a model with heterogeneous households and idiosyncratic risk; McKay and Reis (2016a,b) use a similar framework to look at the role of automatic stabilizers in the business cycle. Transfers in our model can also be used by the planner as a social insurance tool which has a role in mitigating the impact of aggregate shocks. However, contrarily to these papers, we focus on the joint use of transfers and other fiscal instruments such as debt and labor taxes and the implied trade-offs between providing insurance and minimizing labor tax distortions.

The remainder of the paper is organized as follows. In Section 2, we provide key stylized facts regarding the behaviour of fiscal variables in the US, using both macro and micro data sources. Section 3 describes the building blocks of the model that we use in the subsequent analysis. Section 4 and 5 contain the results obtained from the model, with a focus on the steady-state properties and the dynamics implied by the Ramsey allocation. Section 6 concludes.

2 Fiscal policy in the United States

In this section we present facts on the cyclical and distributional properties of fiscal variables in the US, focusing on the behaviour of transfers. The objective is twofold. First, we want to provide a clear definition of the notion of transfers that we will use throughout the paper. Second, we want to shed light on the data properties of transfers that we will use to justify the way we design transfers in the theoretical model presented in the next section. We first use aggregate data to describe the main cyclical properties of transfers. Then, we make use of household-level data to analyze the redistributive aspects of policy.

We use data from the NIPA tables to look at the aggregate properties of transfers. In this paper, we will focus on the components of transfers which are targeted towards households. We therefore construct our aggregate series accordingly, and define transfers as the sum of unemployment insurance payments, and other social benefits such as food stamps and income assistance programs. A complete description of our data series is provided in Appendix C.[8] We now study the cyclical properties of our transfer series, and its relation with other components of the US federal budget. We summarize the observed properties with the following :

Fact 1: *Transfers are counter-cyclical, and are strongly correlated with primary deficits.*

In Table 1 we provide the correlation matrix of the cyclical components of transfers, primary deficits, and GDP. From this table we can observe that transfers and GDP have a negative correlation (-0.45). This observed counter-cyclicality of transfers can be attributed to the fact that, in recessions, governments use social insurance schemes to alleviate the drop in income that households suffer, and therefore (partially) insure them against fluctuations. We will further investigate this insurance component while analysing transfers using micro data sources.

[Table 1 approximately here]

Another noteworthy feature is the strong positive correlation between transfers and deficits observed from the table. It suggests that transfers are an important component of deficits, or at least that the government is not reducing transfers when deficits are high.

Figure 1 illustrates the negative correlation between GDP and transfers and the positive correlation between transfers and deficits. It shows that transfers are not used by the government to stabilize deficits. On the contrary, they are one of the most important components enabling public finances to fulfil their automatic stabilization role.

[Figure 1 approximately here]

[8] Oh and Reis (2012) show that the fiscal expansion during the Great Recession was also mostly driven by increases in social transfers. While unemployment benefits and income assistance programs explain a large share of the increase in transfers, social security and Medicare expenditures also increased significantly during the period 2007-2009.

We now provide empirical evidence on the behaviour of transfers in the cross-section. We use data from the Consumer Expenditure Survey (CEX), a quarterly survey of consumption expenditure by US households conducted by the Bureau of Labor Statistics. Besides detailed consumption data, the survey provides information on labor earnings and transfers received by households that are interviewed. We make use of these records to construct data series that summarize the behaviour of these variables across different household groups. Details on the construction of our series are provided in Appendix C.

To perform our analysis, we proceed as follows. We first classify households according to the value of their yearly earnings, in order to divide them in two groups. The first group contains households at the bottom 30% of the income distribution, while the other group contains the remaining 70%.[9] Then, we aggregate data series across households in each group to look at how the empirical properties of income and transfers vary across income groups.[10]

We define individual transfers in a similar way as for the aggregate series described above. The aggregate transfer series obtained from the micro database shares the same cyclical properties as the macro data series used previously.

Let us now turn to the second stylized fact:

Fact 2: *Transfers are unevenly targeted towards low-income households.*

In Figures 2 and 3 we plot, respectively, the evolution over time of the value of per capita transfers for each of the two household groups defined above, and the total share of transfers received by households in the low-income group.[11] We can observe from these figures that on average, the value of transfers directed towards households at the bottom 30% of the income distribution is greater than the one received by households in the high income group. Indeed, as depicted in Figure 3, while hand-to-mouth agents represents 30% of the population, they receive about two third of total transfers: the average value for this share is 67% for our sample period. This

[9] In the theoretical framework developed in the next section, we distinguish between *hand-to-mouth* and *Ricardian households*. The first group is defined as households which do not have access to a savings technology and therefore consume their entire disposable income in every period. As the data we use do not contain information on wealth and asset holdings, we use labor earnings as a proxy for these variables, and assign households to subgroups according to this criterion.

[10] While CEX data are collected at a quarterly frequency, households only report the values of earnings and transfers received over the preceding 12 months. Hence, the quarterly series we get for these variables have some inertia, and are not as volatile as they should be, which does not allow us to study the dynamics of earnings and income inequality in greater detail. However, it is sufficient to get a broad overview of the dynamics of transfers at the micro level.

[11] In Appendix C, we provide the transfer series by income quintile and show that the relation between income and transfer receipts described here is similar using this level of aggregation.

confirms that transfers have a redistributive aspect, and that low-income individuals are more likely to benefit from them.

[Figures 2 and 3 approximately here]

Fact 3: *Transfers directed towards low and high income groups are strongly correlated.*
To finish this section, we want to emphasize the strong correlation between the cyclical component of transfers across groups. The correlation coefficient between the two series plotted in Figure 2 is equal to 0.79, implying a huge co-movement of the transfer series across the two considered income groups. This observation will be used in the next section to justify the assumption made in our theoretical model of a unique transfer that is split between households, rather than separate transfer processes targeting individual households independently.

3 A simple two-agent model

In this section, we consider a simple two-agent model to outline the key features of optimal fiscal policy when household heterogeneity is accounted for. We first lay out a model where the government can only issue one period risk-free debt, and markets are incomplete. Then, we set up a model in which the planner issues state-contingent claims and markets are complete. Contrasting these two cases will allow us to study the implications of the market structure on the optimal behaviour of taxes and transfers, and characterize its interplay with heterogeneity.

Two types of households populate the economy: a fraction $1 - \lambda$ of agents are Ricardian, or *savers*, and have access to financial markets: they can accumulate government bonds to smooth consumption over time. The remaining fraction λ of agents are hand-to-mouth, or *non-savers*; they do not have access to the savings technology and consume their entire disposable income every period. Labor supply is chosen optimally by each household. There is no capital, and production is linear in total labor effort. We consider uncertainty deriving from three sources: shocks to government spending, to aggregate productivity, and individual productivity shocks relevant only for hand-to-mouth agents.

In what follows, we describe in detail the main building blocks of the model.

3.1 Model description

Government: The government has to finance an exogenous stream of spending denoted g_t, and transfers T_t, which are chosen optimally. Transfers are allowed to be negative, in which case they are essentially lump-sum taxes. To generate revenues, the government also taxes labor at the rate $0 \leqslant \tau_t \leqslant 1$. The latter is also chosen optimally by the Ramsey planner. Finally, the government issues debt, which takes the form of one-period discount government bonds. Current bond issuances are denoted b_t, and their price is q_t. The intertemporal government budget constraint is written as:

$$b_{t-1} = \tau_t a_t n_t - g_t - T_t + q_t b_t \tag{1}$$

where a_t and n_t denote respectively aggregate productivity and total labor supply. These objects are described below.

Modelling transfers: One of the goals of this paper is to explore the effect of transfers and their design on the properties of optimal fiscal policy. As we will show, the assumptions we make on the transfer schedule have important implications for the outcomes of the Ramsey allocation. It is therefore crucial to model transfers in an appropriate and empirically-relevant way. For instance, we could allow for household-specific transfers, that could be denoted T_t^i, for agent $i = h, s$, where h and s denote respectively hand-to-mouth and Ricardian households. The government would choose their optimal value and total transfers would simply equal their weighted sum: $T_t = \lambda T_t^h + (1-\lambda) T_t^s$. However, such an assumption is unappealing for two reasons. First, letting the planner choose household-specific transfer levels would allow her to reach the first-best allocation, thereby leading to trivial results. In such allocation, labor taxes would be set to zero and, as is shown below, the planner would choose transfers to equate marginal utilities of consumption across households.

Second, household-specific transfers, which imply that individual transfers can potentially evolve very differently across households, are not a good approximation of the behaviour of transfers observed in the data. This would violate the last stylized fact of the previous section, according to which individual transfers are strongly correlated across households.

For the above reasons, we choose to model transfers as a unique process T_t. However, to make it consistent with our second stylized fact (according to which transfers are unevenly targeted towards a subset of households), we introduce a targeting rule, which specifies the share of transfers going towards each of the two households populating the economy. More precisely, we assume that a share $\omega \in [0,1]$ of total tranfers is targeted towards hand-to-mouth agents. The per capita transfers to hand-to-mouth households and savers are therefore respectively given by $T_t^h = \frac{\omega}{\lambda} T_t$ and $T_t^s = \frac{1-\omega}{1-\lambda} T_t$. Therefore, when $\omega = \lambda$, transfers are equally distributed among the population ($T_t^h = T_t^s = T_t$), and whenever $\omega > \lambda$ transfers are targeted towards hand-to-mouth agents ($T_t^h > T_t^s$).[12] Although our targeting rule is simple and abstracts from many elements characterizing the design of transfers in the US, we believe that our assumptions enable us to develop a workable approximation of the actual transfer schedule.

Ricardian agents: Ricardian households choose consumption, labor supply, and holdings of government bonds to maximize their expected lifetime utility, which is expressed as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big(u(c_t^s) - v(n_t^s) \Big) \tag{2}$$

where c_t^s and n_t^s denote respectively the consumption and the labor effort exerted by the agent at time t. Period utility is assumed to be separable between consumption and labor. We make common assumptions on the functions $u(\cdot)$ and $v(\cdot)$, namely $u' > 0$, $u'' < 0$, and $v' > 0$, $v'' > 0$.

The budget constraint of savers, which has to be satisfied in every time period t, can be expressed as:

$$c_t^s + q_t b_t^s = (1 - \tau_t) a_t n_t^s + b_{t-1}^s + \frac{1-\omega}{1-\lambda} T_t \tag{3}$$

This constraint states that to finance consumption and the accumulation of newly issued government bonds (b_t^s denotes the quantity of bonds held by savers), the household uses its labor income net of taxes, transfers, and the payoff on government bonds issued in $t-1$. Note that the coefficient $\frac{1-\omega}{1-\lambda}$ appearing in front of aggregate transfers T_t is consistent with our targeting rule described above.

[12] Throughout the paper, we assume that ω is exogenous, and constant over time. While we believe that allowing the planner to optimally choose ω is an interesting exercise, we abstract from this in our model.

First order conditions for hand-to-mouth agents give:

$$\frac{v^s_{n,t}}{u^s_{c,t}} = a_t(1-\tau_t) \tag{4}$$

$$q_t = \beta\frac{E_t u^s_{c,t+1}}{u^s_{c,t}} \tag{5}$$

where $u_c \equiv \frac{\partial u}{\partial c}$ and $v_n \equiv \frac{\partial v}{\partial n}$ denote, respectively, the marginal utility of consumption, and the marginal disutility of work. The first equation is a standard labor supply condition, equating the marginal rate of substitution between leisure and consumption to (net) labor income. The second equation is the usual Euler equation, which also constitutes the pricing condition for government bonds and has to be accounted for by the social planner.

Hand-to-mouth agents: Non-savers do not have access to financial markets and are therefore constrained in each period to consume their entire disposable income. They choose labor supply optimally to maximize their period utility $u(c^h_t) - v(n^h_t)$, subject to the budget constraint:

$$c^h_t = (1-\tau_t)\theta^h_t a_t n^h_t + \frac{\omega}{\lambda}T_t \tag{6}$$

Variable θ^h_t denotes an exogenous shock to the relative productivity of hand-to-mouth agents. If $\theta^h_t < 1$, hand-to-mouth agents are less productive than Ricardian Households; this might provide incentives to the planner to tilt transfers towards hand-to-mouth agents.

The introduction of shocks to the relative productivity of hand-to-mouth households is useful to capture the following properties. First, on average the revenue of hand-to-mouth households is lower than the revenue of savers in the data. Second, the cyclical properties of their income are also likely to differ from the ones of savers. In the Appendix, we show that in our data, the earnings of households at the bottom of the distribution are more volatile than the ones of richer agents.[13]

Utility maximization by hand-to-mouth agents gives the labor supply condition:

$$\frac{v^h_{n,t}}{u^h_{c,t}} = \theta^h_t a_t(1-\tau_t) \tag{7}$$

[13] We also show in the Appendix that the correlation between the revenues of low-income households and GDP is higher than the one obtained for high-income households: in other words, the revenues of low-income agents is more pro-cyclical. While we believe that the implications of this property for optimal fiscal policy is an interesting issue, we abstract from this in the current version of the paper.

Equilibrium We define aggregate consumption and hours worked, respectively, as follows:

$$\begin{aligned} c_t &= \lambda c_t^h + (1-\lambda) c_t^s \\ n_t &= \lambda \theta_t^h n_t^h + (1-\lambda) n_t^s \end{aligned}$$

Equilibrium in the market for government debt implies $b_t^s = \frac{1}{1-\lambda} b_t$.

We can use the government budget constraint, and the individual budget constraints of both households to obtain the economy-wide resource constraint:

$$c_t + g_t = a_t n_t \tag{8}$$

3.2 Optimal policy

3.2.1 The Ramsey problem

The Ramsey planner maximizes aggregate welfare, which we define as the weighted sum of individual expected lifetime utility functions:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \Big[\lambda \Big(u(c_t^h) - v(n_t^h) \Big) + (1-\lambda) \Big(u(c_t^s) - v(n_t^s) \Big) \Big] \tag{9}$$

The Ramsey planner maximizes (9) subject to her inter-temporal budget constraint (1), households optimality conditions (4), (5) and (7), the resource constraint (8), and the budget constraint of hand-to-mouth agents (6).

The Lagrangian associated to the planner's optimization program is:

$$\begin{aligned} \mathcal{L} \;=\; & E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \lambda \Big[u(c_t^h) - v(n_t^h) \Big] + (1-\lambda) \Big[u(c_t^s) - v(n_t^s) \Big] \qquad (10) \\ & + \Psi_t^{IM} \Big[- u_{c,t}^s b_{t-1} + (u_{c,t}^s a_t - v_{n,t}^s)(\lambda \theta_t^h n_t^h + (1-\lambda) n_t^s) - u_{c,t}^s (g_t + T_t) + \beta u_{c,t+1}^s b_t \Big] \\ & + \Psi_t^1 \Big[a_t \lambda \theta_t^h n_t^h + (1-\lambda) n_t^s a_t - \lambda c_t^h - (1-\lambda) c_t^s - g_t \Big] + \Psi_t^2 \Big[\frac{v_{n,t}^s}{u_{c,t}^s} - \frac{1}{\theta^h} \frac{v_{n,t}^h}{u_{c,t}^h} \Big] \\ & + \Psi_t^3 \Big[- u_{c,t}^s c_t^h + v_{n,t}^s \theta_t^h n_t^h + \frac{\omega}{\lambda} u_{c,t}^s T_t \Big] \end{aligned}$$

where we made use of Ricardian agents' Euler equation and labor supply condition to substitute away the bond price q_t and the labor tax rate τ_t, thereby making use of the primal approach to solve for optimal policies (see Lucas and Stokey, 1983). The optimality conditions associated to the planner's problem are the following:

$$
\begin{aligned}
c_t^h &: \lambda u_{c,t}^h - \lambda \Psi_t^1 + \frac{\Psi_t^2}{\theta_t^h} \frac{v_{n,t}^h}{(u_{c,t}^h)^2} u_{cc,t}^h - \Psi_t^3 u_{c,t}^s = 0 \\
c_t^s &: (1-\lambda) u_{c,t}^s + \Psi_t^{IM} \Big[u_{cc,t}^s a_t n_t - u_{cc,t}^s (g_t + T_t) \Big] + (\Psi_{t-1}^{IM} - \Psi_t^{IM}) u_{cc,t}^s b_{t-1} \\
&\quad -\Psi_t^1 (1-\lambda) - \Psi_t^2 \frac{v_{n,t}^s}{(u_{c,t}^s)^2} u_{cc,t}^s + \Psi_t^3 (-u_{cc,t}^s c_t^h + \frac{\omega}{\lambda} T_t u_{cc,t}^s) = 0 \\
n_t^h &: -\lambda v_{n,t}^h + \Psi_t^{IM} (u_{c,t}^s a_t - v_{n,t}^s) \lambda \theta_t^h + \Psi_t^1 a_t \lambda \theta_t^h - \frac{\Psi_t^2}{\theta_t^h} \frac{v_{nn,t}^h}{u_{c,t}^h} + \Psi_t^3 v_{n,t}^s \theta_t^h = 0 \\
n_t^s &: -(1-\lambda) v_{n,t}^s + \Psi_t^{IM} \Big[(1-\lambda)(u_{c,t}^s a_t - v_{n,t}^s) - v_{nn,t}^s n_t \Big] + \Psi_t^1 (1-\lambda) a_t + \Psi_t^2 \frac{v_{nn,t}^s}{u_{c,t}^s} + \Psi_t^3 v_{nn,t}^s \theta_t^h n_t^h = 0 \\
T_t &: -\Psi_t^{IM} u_{c,t}^s + \Psi_t^3 \frac{\omega}{\lambda} u_{c,t}^s = 0 \\
b_t &: -E_t \Psi_{t+1}^{IM} u_{c,t+1}^s + \Psi_t^{IM} E_t u_{c,t+1}^s = 0
\end{aligned}
$$

3.2.2 The first-best allocation as a useful benchmark

In Appendix B we show that the first-best allocation in our two-agent model can be characterized by the following conditions:

$$
\frac{v_{n,t}^h}{\theta_t^h u_{c,t}^h} = \frac{v_{n,t}^s}{u_{c,t}^s} = a_t
$$

$$
u_{c,t}^h = u_{c,t}^s
$$

The first equalities provide a well-known optimality condition which can also be derived from representative agent models: it states that the marginal rate of substitution between consumption and leisure must be equal to the marginal product of labor, which is equal to a_t for Ricardian households, and $\theta_t^h a_t$ for hand-to-mouth households. This allocation can be attained by the planner if it is possible for her to set $\tau_t = 0$ for all t (which is the case if, for instance, she only makes use of variations in transfers to finance fiscal deficits), and therefore eliminate tax distortions.

The second condition equates marginal utilities of consumption across agents. It therefore states that the first-best allocation features complete consumption equality, given the assumptions made on the function $u(\cdot)$. In the framework considered here, it is in some cases possible for the planner to make this condition hold by using transfers to redistribute resources across agents.

However, as such a policy often comes at the cost of increasing the level and volatility of labor taxes, this in turn prevents the planner from getting close to the first set of conditions mentioned above. This therefore creates a trade-off between efficiency and redistribution; one of the goals of this paper is to investigate the nature and the resolution of such a trade-off by the Ramsey planner.

3.3 Complete vs. incomplete markets

In this section, we briefly outline how the complete markets version of the model looks like. The Ramsey problem under complete markets is similar to the one which has been described above for the incomplete market case; the main difference lies in the government budget constraint.

In the model presented in the previous section, the state of the economy at time t can be represented by the object $s^t = \{s_0, s_1, ..., s_t\} = \{s^{t-1}, s_t\}$, where s_t denotes the state vector at time t. We assume that shocks are Markovian, implying that the evolution of exogenous processes can be represented by a density function of the form $f(s^t|s_{t-1})$.[14]

For the sake of exposition, in this section we rewrite model variables as $x_t = x(s^t)$ for any variable x. This way, we explicitly acknowledge the mapping between the state object s^t and the value taken by any variable x at time t. Using this notation, we can rewrite the government budget constraint under incomplete markets, specified in (1), as:

$$b(s^{t-1}) = \overline{S}(s^t) + q(s^t)b(s^t) \tag{11}$$

where $\overline{S}_t \equiv \tau_t a_t n_t - g_t - T_t$ is the primary surplus at time t. Using Ricardian agents' Euler equation (5) and iterating equation (11) forward, we can express the inter-temporal budget constraint as follows:

$$u_c^s(s^t)b(s^{t-1}) = E_0 \sum_{j=0}^{\infty} \beta^j \overline{S}(s^{t+j}) u_c^s(s^{t+j}) \tag{12}$$

This equation has to hold for any t (acknowledging that $b(s^{-1}) = b_{-1}$: the initial value of government debt is taken as given by the government); this expression is very standard: it states that the inherited value of government debt (in units of marginal utility) has to be equal to the

[14]As stated above, we consider three types of shocks in this model version: shocks to government spending, to aggregate productivity, and to the relative productivity of hand-to-mouth households.

discounted sum of future primary surpluses.

When financial markets are complete, the government issues in each period t, a portfolio of state-contingent bonds $b^{CM}(s^{t+1})$ at prices $q^{CM}(s^{t+1})$. The dependence of b^{CM} on the state vector in $t+1$ makes it clear that the payoff on government debt depends on the state of the economy when bond payments are due. In this scenario, the government budget constraint can be written as follows:

$$b^{CM}(s^t) = \overline{S}^{CM}(s^t) + \int_{s^{t+1}} q^{CM}(s^{t+1}) b^{CM}(s^{t+1})\, ds^{t+1} \tag{13}$$

From savers' first order conditions (see Appendix B), we obtain the pricing equation:

$$u_c^{s,CM}(s^t) q^{CM}(s^{t+1}) = \beta f(s^{t+1}|s_t) u_c^{s,CM}(s^{t+1}) \tag{14}$$

This equation has to hold for any s^{t+1}. Combining equations (13) and (14), and iterating forward, we get:

$$u_c^{s,CM}(s^t) b(s^t) = E_t \sum_{j=0}^{\infty} \beta^j \overline{S}^{CM}(s^{t+j}) u_c^{s,CM}(s^{t+j}) \quad \text{for } t \geqslant 1 \tag{15}$$

$$u_{c,0}^{s,CM} b_{-1} = E_0 \sum_{t=0}^{\infty} \beta^t \overline{S}^{CM}(s^t) u_c^{s,CM}(s^t) \quad \text{for } t = 0 \tag{16}$$

where $E_t x(s^{t+j}) \equiv \int_{s^{t+j}} x(s^{t+j}) f(s^{t+j}|s_t)\, ds^{t+j}$ is the conditional expectation of variable x at time $t+j$ based on the information set at time t. As mentioned earlier, in any period $t \geqslant 1$, the state-contingent nature of government debt makes debt payments b^{CM} a function of the current state vector s^t. It follows that the constraint (15) is slack (and therefore irrelevant for the Ramsey planner) for $t \geqslant 1$, as the government can freely choose the value of $b(s^t)$ which makes the constraint non-binding. In period 0, however, the inherited the level of government debt is b_{-1} is given; the planner has no influence on this value. This makes equation (16) the only binding constraint that has to be part of the planner's constraint set.

It follows from the above discussion that when markets are complete, only the inter-temporal budget constraint in the initial period $t = 0$ needs to hold; this is a standard property of optimal fiscal policy models with complete financial markets (see Lucas and Stokey, 1983). The full set of constraints that have to hold in this model version is therefore constituted of equations (4), (6), (7), (8) and (16). A full description of the Ramsey program under complete markets and the associated first order conditions are provided in Appendix B.

3.4 Calibration and functional forms

We describe here the parameter values and functional forms that will be used in the subsequent analysis and numerical simulations. The period utility function is separable between consumption and leisure, and takes the form:

$$u(c) - v(n) = \log(c) - \frac{n^{1+\phi}}{1+\phi} \tag{17}$$

The three shock processes are all assumed to follow AR(1) process in logs:

$$\log\left(\frac{x_t}{x}\right) = \rho_x \log\left(\frac{x_{t-1}}{x}\right) + \epsilon_{x,t} \tag{18}$$

for $x = g, a, \theta^h$. Shocks $\epsilon_{x,t}$ are iid with mean zero and standard deviations σ_x.

[Table 2 approximately here]

The parameter values are summarized in Table 2. The model horizon is annual, and we set the discount factor to $\beta = 0.95$. The inverse Frisch elasticity ϕ is set to unity. The share of hand-to-mouth agents is $\lambda = 0.3$, a value which is standard in the literature.[15]

We assume a steady-state debt-to-gdp ratio of 60%, and set the steady-state value of government expenditures to 15% of output.

Given the crucial role played by the individual productivity of constrained agents θ^h and the share of transfers going to these agents ω, we will allow for different values of these parameters in the next section. This will allow us to study how these parameters affect the properties of the Ramsey allocation. However, when not stated otherwise, we consider the benchmark case of $\theta^h = 1$, i.e. the scenario under which the two agents are equally productive in the steady-state. As for ω, there are two cases that we focus on: the case where $\omega = \lambda$ (transfers are evenly distributed among households, and can therefore be considered as purely lump-sum); and the case where $\omega = 1$ (all the transfers are targeted towards hand-to-mouth agents). We will present results for these two cases throughout the rest of the paper.

[15] This is also consistent with the empirical evidence provided in Kaplan and Violante (2014), according to which around 30% of US households can be considered as being hand-to-mouth.

4 Steady-state analysis

In this section we study the steady-state properties of the model. We first analyze the set of equilibrium outcomes that are attainable to the planner for the combinations of taxes and transfers satisfying the budget constraint; this gives us a sense of the objective of the planner. We then solve for the optimal long-run allocation, and study how taxes and transfers vary as a function of key model parameters.

4.1 Long-run tradeoffs

We assume that the government inherits a long-run level of government debt b_{-1}; the steady-state level of government spending is given and equal to g. In the steady-state, the government budget constraint is:[16]

$$(1-\beta)b_{-1} = \tau n - g - T \tag{19}$$

where dropping time subscripts from the variables denotes the steady-state. When choosing taxes and transfers, the planner faces a trade-off between efficiency and redistribution. On the increasing side of the Laffer curve, a higher labor tax rate τ increases tax revenues, thereby allowing the government to raise transfers T, which potentially has a redistributive role. However, this comes at the cost of distorting labor supply, and therefore reducing aggregate output. In standard models featuring a representative agent, the planner will always choose to set the distortionary tax on labor to zero, and to finance the entire deficit through lump-sum taxes (negative transfers). We study here whether the same property holds when agents are heterogeneous.

Figure 4 displays steady-state consumption inequality, measured as the ratio of hand-to-mouth to Ricardian households' consumption ($\frac{c^h}{c^s}$), as a function of the labor tax rate (τ). The first best allocation (which, as mentioned before, is the allocation which sets the labor tax rate to 0 and the consumption ratio to 1), is depicted in the figure by a red cross.

[Figure 4 approximately here]

We can see from the left panel of the figure (which assumes $\theta^h = 1$) that increases in the labor

[16] Note that the steady-state allocations under complete and incomplete markets, for a given level of inherited government debt, are the same. It implies, among others, that the steady-state multipliers on the government budget constraint are equal ($\psi^{IM} = \psi^{CM} = \psi$), and so are the optimal labor tax rate and transfers.

tax rate have almost no effect on consumption inequality if $\omega = \lambda$. In such a scenario transfers are evenly targeted between households, and as a result, a tax-financed increase in transfers does not allow for redistribution. However, when $\omega = 1$, an increase in transfers financed by an increase in the labor tax rate has a strong impact on the consumption ratio $\frac{c^h}{c^s}$, and such a policy allows the planner to reduce consumption inequality. Note that, as there is little ex-ante heterogeneity between households (the two agents are equally productive), the first-best is almost attainable by the planner when $\omega = \lambda$. Moreover, in such a case, increases in the tax rate *increase* consumption inequality, so there is no trade-off between efficiency and redistribution and under this parameterization (as will be shown below), the optimal allocation features a zero labor tax rate.

When hand-to-mouth households are less productive than Ricardian households (as in the right panel of Figure 4, for which we assume $\theta^h = 0.7$), a similar picture arises: increases in the labor tax rate affect consumption inequality only if transfers are unevenly distributed among households (which is the case when $\omega = 1$). However, the first best solution is now unreachable at any value of ω. Under an evenly distributed transfer rule, the planner never achieves to equalize consumption between households, while it requires a large labor tax rate to ensure consumption equality between households when transfers are targeted towards hand-to-mouth households.

Figure 4 shows that in some cases, increases in transfers (financed by positive labor taxes) allow the planner to reduce the consumption gap between households. Nevertheless, because such a policy implies efficiency losses (as higher labor taxes distort labor supply), we might wonder whether such a policy can be welfare improving. In the following section, we show under which conditions it is indeed optimal for the planner to set a positive labor tax rate.

4.2 Optimal taxes and transfers

The following proposition provides the condition under which it is optimal for the planner to set a positive labor tax.

Proposition 1: Given an inherited level of government debt b_{-1} and in the absence of stochastic shocks, the optimal long-run tax rate of labor is strictly positive if:

$$\frac{c^*}{c^{s*}} + \omega\Big(c^* - (1-\beta)b_{-1}\Big)\left(\frac{1}{c^{h*}} - \frac{1}{c^{s*}}\frac{\theta^h(1+\phi\frac{c^{h*}}{n^{h*}})}{\theta^h + \phi\frac{c^{h*}}{n^{h*}}}\right) > 1 \tag{20}$$

where variables with a "$*$" are evaluated at the allocation implied by setting $\tau = 0$: for any x, $x^* = x|_{\tau=0}$.

Proof: The above proposition essentially provides the condition under which, in the steady-state allocation, aggregate welfare is increasing in τ, when $\tau = 0$. More formally, it provides an expression for $\frac{dU}{d\tau}|_{\tau=0} > 0$, which is obtained by totally differentiating the system of steady-state model equations.

In the steady-state allocation, the competitive equilibrium can be represented by:

$$\begin{gathered}(1-\beta)b_{-1} = \tau(\lambda n^h + (1-\lambda)n^s) - g - T \\ c^h = (1-\tau)\theta^h n^h + \frac{\omega}{\lambda}T \\ c^s = (1-\beta)b_{-1} + (1-\tau)n^s + \frac{1-\omega}{1-\lambda}T \\ (n^h)^\phi c^h = \theta^h(1-\tau) \\ (n^s)^\phi c^s = 1-\tau\end{gathered}$$

This gives us a system of 5 equations in $(c^h, c^s, n^h, n^s, T, \tau)$. Totally differentiating the equations, we obtain a system in $(dc^h, dc^s, dn^h, dn^s, dT, d\tau)$, that we can rearrange to get:

$$\begin{aligned} dn^h &= \left(\tau\lambda\theta^h + \phi\frac{\lambda}{\omega}\frac{c^h}{n^h} + \frac{\lambda}{\omega}(1-\tau)\theta^h - \tau\frac{\lambda\theta^h(1+\frac{c^h}{n^h}\phi)}{1+\phi\frac{c^s}{n^s}}\right)^{-1}\left(\frac{\lambda}{\omega}\theta^h n^h - n + \frac{\tau}{1-\tau}\frac{c}{1+\phi\frac{c^s}{n^s}} - \frac{\lambda}{\omega}\frac{c^h}{1-\lambda}\right)d\tau \\ dn^s &= \left((1-\lambda)(1+\phi\frac{c^s}{n^s})\right)^{-1}\left(-\lambda\theta^h(1+\phi\frac{c^h}{n^h})dn^h - \frac{c}{1-\tau}d\tau\right) \\ dc^h &= -\frac{c^h}{1-\tau}d\tau - \phi\frac{c^h}{n^h}dn^h \\ dc^s &= -\frac{c^s}{1-\tau}d\tau - \phi\frac{c^s}{n^s}dn^s \end{aligned} \tag{21}$$

Totally differentiating aggregate welfare gives:

$$dU = \lambda(u_c^h\, dc^h - v_n^h\, dn^h) + (1-\lambda)(u_c^s\, dc^s - v_n^s\, dn^s) \tag{22}$$

Making use of the system (21) in (22) , setting $\tau = 0$ and rearranging, we get the result stated in expression (20). ■

Proposition 1 gives us the condition under which setting positive labor taxes is welfare improving. In the next paragraphs, we describe three specific cases which help interpreting this result. Then, we study the effect of two specific parameters, the individual productivity of constrained agents (θ^h) and the share of transfers going to these agents (ω), on the optimal tax rate.

Representative agent: The first special case we consider is the representative agent version of the model, under which both distortionary taxes and lump-sum transfers (or taxes) are available. In such a setting, it is well-known that Ricardian equivalence holds and the planner chooses to entirely finance deficits at any time using lump-sum taxes, therefore setting $\tau = 0$. This can be seen by setting, $c^{s*} = c^{h*} = c^*$, and $\theta^h = 1$ in the above expression. In this case, the left-hand size of the condition is equal to one, and it is therefore sub-optimal to set $\tau > 0$.

Two-agent, evenly distributed transfers: Turning to the two-agent case, we assume first that $\omega = \lambda$, $\theta^h = 1$, $b_{-1} = 0$ and $g > 0$. In such a case, the two households have the same amount of resources available in the steady-state, as (i) they are equally productive, (ii) no agent holds assets, and (iii) transfers affect each agent in the same way. The assumption of positive government spending implies a need for the government to generate resources by taxation to satisfy its budget constraint. In such a scenario, we can show that, as in the representative agent case, the LHS of the expression stated in Proposition is equal to 1, and therefore the optimal labor tax is zero. It reflects the fact that, when the two households have similar characteristics, it is optimal to finance fiscal deficits with lump-sum taxes, if such an instrument is available to the planner. Because the two agents have similar characteristics, the optimal allocation is the one that maximizes aggregate consumption and output; this allocation is obtained by setting $\tau = 0$ and financing the entire deficit lump-sum.

Two-agent, transfers directed towards HTM households: As a third example, we still assume that $\theta^h = 1$, $b_{-1} = 0$ and $g > 0$, but we now set $\omega = 1$, such that transfers are fully targeted towards hand-to-mouth households. In this case, the LHS of the above condition boils down to $\frac{c^*}{c^{h*}}$. We can easily show from the steady-state model equations that, in such a scenario, $c^{s*} > c^{h*}$, and then $\frac{c^*}{c^{h*}} > 1$. Then, according to Proposition 1 the optimal long-run tax is positive. This is the case because, when the labor tax rate is zero, government expenditures are entirely financed by a lump-sum tax on hand-to-mouth agents, which reduces their consumption level, while Ricardian households are not impacted by such a tax. The planner thus finds it optimal to spread the tax burden across households by setting a positive labor tax rate. However, the optimal allocation does not equalize consumption/hours and welfare levels across household types: the efficiency costs associated with higher distortionary taxes imply that it is not optimal for the planner to fully wipe out any level of cross-sectional inequality.

[Figure 5 approximately here]

In Figure 5, we display aggregate steady-state welfare as a function of the labor tax, for the parametrization just described. We can see from the Figure that the tax rate which maximizes welfare (as shown by the red circle) is approximately 15%. At this value, the transfer is still negative, but is such that the costs associated to government spending are more equally shared between households.

4.2.1 Effect of key parameters

We now solve for the steady-state of the Ramsey allocation, and look at the properties of the optimal tax schedule. We are primarily interested in identifying the effect of key model parameters on the behaviour of fiscal variables. We set the parameters to their baseline values, as described in Section 3 and summarized in Table 2, and then look at the impact of changing one parameter at a time on the behaviour of the steady-state labor tax. The results of this exercise are displayed in Figure 6.

[Figure 6 approximately here]

Panel (a) shows the effect of ω, the parameter targeting the share of transfers going to hand-to-mouth households, on the long-run optimal tax rate. As we can see from the Figure, the effect of ω on the optimal tax rate is non-monotonic: at low values of ω (when most of the transfer is directed towards Ricardian agents), the optimal tax rate is decreasing with ω, and reaches zero for values which are close to λ. Then, as ω starts increasing again, the tax rate becomes higher. This result emerges because, when transfers are evenly distributed among households (ω is close to λ), increasing them do not allow for much redistribution across households. Therefore, it is optimal to finance the deficit by decreasing transfers, which allows the planner to reduce the labor tax rate and thereby minimize its distortionary effects.[17]

In Panel (b) of the figure we look at the effects of the relative productivity of constrained agents (θ^h). The solid blue line provides results for the case where $\omega = \lambda$ (evenly distributed transfers), while the dashed black line is for the case where $\omega = 1$ (transfers targeted towards HTM agents).

[17] As it has been showed formally above, when the transfer is entirely lump-sum ($\omega = \lambda$), lump-sum taxes finance spending needs and the optimal labor tax is zero.

In this case too, the optimal labor tax is non-monotonic. At low values of θ^h, the optimal tax rate is positive: it is optimal for the planner to increase labor taxes to finance an increase in transfers and bring the consumption of hand-to-mouth agents closer to the one of Ricardian households, even if it comes with an efficiency cost. At high levels of θ^h, the optimal labor tax is increasing when $\omega = \lambda$, while it stays at zero when $\omega = 1$. In the former case, the relative productivity of hand-to-mouth agents becomes so high that it is optimal to redistribute resources towards Ricardian households by increasing labor taxes and transfers. In the latter case, redistribution towards savers is impossible, hence there is no tax response when $\omega = 1$ (dashed line).

Panels (c) and (d) display the results of changes in steady-state debt levels and government spending. Unsurprisingly, we observe that, when spending needs increase (higher values of b and g), the optimal labor tax increases.

We have seen throughout this section that, in the long-run allocation, when heterogeneity is sufficiently high, it is optimal to redistribute across households through setting $\tau > 0$, even though this comes at the cost of higher tax distortions. In the next section we introduce dynamics and study the properties of the Ramsey allocation out of the steady-state.

5 Optimal responses to shocks

In the previous section, we showed how the Ramsey allocation departs from the first-best in the long-run. We now analyze the cyclical properties of the model when the economy is hit by random shocks. We first present theoretical results which allow us to discuss the properties of the optimal labor tax schedule. In particular, we study the link between fluctuations in consumption/hours heterogeneity between households, market (in)completeness, and the volatility of labor taxes in the Ramsey allocation. Then, we rely on numerical simulations to study the response of key variables to the stochastic shocks present in the model.[18]

5.1 Heterogeneity, market (in)completeness, and optimal labor taxes

We first analyze the properties of the optimal labor tax schedule.[19] We start with the following proposition, which characterizes the optimal labor tax rate in the two-agent model presented

[18]Throughout the paper, we solve the model using linear perturbation techniques around the initial steady-state.

[19]We focus on taxes and not transfers, as it allows us to directly compare our results to the ones obtained in representative agent models that are common in the Ramsey literature, and in which the planner is usually not allowed to choose transfers optimally.

above:

Proposition 2: When the planner has access to distortionary taxes and transfers, and financial markets are incomplete, the optimal labor tax rate can be expressed as:

$$\tau_t = 1 - \frac{(H_t^c)^{-1}\Big[1 + (\Psi_t^{IM} - \Psi_{t-1}^{IM})\frac{b_{t-1}}{c_t^s}\Big]}{H_t + (1+\phi)\Big[1 - \frac{\lambda}{\omega}\theta_t^h \frac{h_t^n}{H_t^n}\Big]\Psi_t^{IM}} \tag{23}$$

where $h_t^c \equiv \frac{c_t^h}{c_t^s}$; $h_t^n \equiv \frac{n_t^h}{n_t^s}$; $H_t^c \equiv \lambda h_t^c + 1 - \lambda$; $H_t^n \equiv \lambda\theta_t^h h_t^n + 1 - \lambda$; $H_t \equiv (\lambda\theta_t^h \frac{h_t^n}{h_t^c} + 1 - \lambda)/H_t^n$.

When markets are complete, the multiplier associated with the budget constraint is constant ($\Psi_t^{IM} = \Psi \; \forall \; t$), and the expression becomes:

$$\tau_t = 1 - \frac{(H_t^c)^{-1}}{H_t + (1+\phi)\Big[1 - \frac{\lambda}{\omega}\theta_t^h \frac{h_t^n}{H_t^n}\Big]\Psi}$$

Proof: We can rewrite the first order conditions given in section 3.1 (rearranging and using $\frac{v_{n,t}^s}{u_{c,t}^s} = \frac{v_{n,t}^h}{\theta_h u_{c,t}^h} = a_t(1-\tau)$) as follows:

$$\begin{aligned}
\Psi_t^2 a_t(1-\tau_t)\frac{u_{cc,t}^h}{u_{c,t}^h} &= -\lambda u_{c,t}^h + \lambda\Psi_t^1 + \Psi_t^3 u_{c,t}^s \\
\Psi_t^2 a_t(1-\tau_t)\frac{u_{cc,t}^s}{u_{c,t}^s} &= (1-\lambda)u_{c,t}^s - \Psi_t^1(1-\lambda) - u_{cc,t}^s\Psi_t^3(c_t^h - \frac{\omega}{\lambda}T_t) \\
&\quad + \Psi_t^{IM} u_{cc,t}^s(a_t n_t - g_t - T_t) - u_{cc,t}^s b_{t-1}\Big[\Psi_t^{IM} - \Psi_{t-1}^{IM}\Big] \\
\Psi_t^2 a_t(1-\tau_t)\frac{v_{nn,t}^h}{v_{n,t}^h} &= -\lambda v_{n,t}^h + \Psi_t^{IM}(u_{c,t}^s a_t - v_{n,t}^s)\lambda\theta_t^h + \Psi_t^1 a_t\lambda\theta_t^h + \Psi_t^3 v_{n,t}^s\theta_t^h \\
\Psi_t^2 a_t(1-\tau_t)\frac{v_{nn,t}^s}{v_{n,t}^s} &= (1-\lambda)v_{n,t}^s - \Psi_t^{IM}\Big[(1-\lambda)(u_{c,t}^s a_t - v_{n,t}^s) + v_{nn,t}^s n_t\Big] \\
&\quad - \Psi_t^1(1-\lambda)a_t - \Psi_t^3 v_{nn,t}^s\theta_t^h n_t^h
\end{aligned} \tag{24}$$

Merging the expressions of the system (24), and using functional forms given in (17), we get:

$$\left[\frac{c_t^h}{c_t^s}\lambda+1-\lambda\right]\left[\frac{n_t^h}{n_t^s}\left(\lambda\frac{v_{n,t}^h}{u_{c,t}^h}u_{c,t}^h-\Psi_t^{IM}\lambda\theta_t^h a_t\tau_t u_{c,t}^s-u_{c,t}^s\Psi_t^3\frac{v_{n,t}^s}{u_{c,t}^s}\theta_t^h a_t\right)+(1-\lambda)\frac{v_{n,t}^s}{u_{c,t}^s}u_{c,t}^s\right.$$
$$\left.-\Psi_t^{IM}\left((1-\lambda)a_t\tau_t u_{c,t}^s-\frac{v_{n,t}^s v_{nn,t}^s}{n_t^s v_{n,t}^s}n_t n_t^s\right)-\Psi_t^3 n_t^s\frac{v_{nn,t}^s}{v_{n,t}^s}\theta_t^h a_t\frac{n_t^h}{n_t^s}v_{n,t}^s\right] =$$
$$\left[\frac{n_t^h}{n_t^s}a_t\theta_t^h\lambda+(1-\lambda)a_t\right]\left[\frac{c_t^h}{c_t^s}(\lambda u_{c,t}^h-\Psi^3 u_{c,t}^s)+(1-\lambda)u_{c,t}^s+\Psi_t^{IM}u_{cc,t}^s(c_t-T_t)\right.$$
$$\left.-\left(\Psi_t^{IM}-\Psi_{t-1}^{IM}\right)u_{cc,t}^s b_{t-1}-\Psi_t^3 u_{cc,t}^s(1-\tau_t)a_t\theta_t^h n_t^h\right]$$

Dividing both sides by $u_{c,t}^s$ and a_t, we get:

$$\left[\frac{c_t^h}{c_t^s}\lambda+1-\lambda\right]\left[\frac{n_t^h}{n_t^s}\left(\lambda\theta_t^h(1-\tau)\frac{u_{c,t}^h}{u_{c,t}^s}-\Psi_t^{IM}\lambda\theta_t^h\tau_t-\Psi_t^3\theta_t^h(1-\tau_t)\right)+(1-\lambda)(1-\tau_t)\right.$$
$$\left.-\Psi_t^{IM}\left((1-\lambda)\tau_t-\phi(1-\tau_t)(\lambda\frac{n_t^h}{n_t^s}\theta_t^h+1-\lambda)\right)-\Psi_t^3\phi\theta_t^h\frac{n_t^h}{n_t^s}(1-\tau_t)\right] =$$
$$\left[\frac{n_t^h}{n_t^s}\theta_t^h\lambda+1-\lambda\right]\left[\frac{c_t^h}{c_t^s}(\lambda\frac{u_{c,t}^h}{u_{c,t}^s}-\Psi_t^3)+(1-\lambda)-\Psi_t^{IM}(\lambda\frac{c_t^h}{c_t^s}+1-\lambda-\frac{T_t}{c_t^s})\right.$$
$$\left.+\left(\Psi_t^{IM}-\Psi_{t-1}^{IM}\right)\frac{b_{t-1}}{c_t^s}+\Psi_t^3\left(\frac{c_t^h}{c_t^s}-\frac{\omega}{\lambda}\frac{T_t}{c_t^s}\right)\right] \tag{25}$$

Rearranging (25) and using $\Psi_t^3=\frac{\lambda}{\omega}\Psi_t^{IM}$, we get the tax expression given in (23). ■

As we can see from equation (23), when markets are incomplete changes in the optimal tax rate reflect changes in two types of variables: Ψ_t^{IM}, which denotes the multiplier on the government budget constraint (1) in the Lagrangian associated with the Ramsey problem, and variables related to household heterogeneity ($h_t^c, h_t^n, H_t^c, H_t^n, H_t$). Notice that in the absence of heterogeneity these elements are all equal to one. In contrast, under complete markets, Ψ is constant and only $h_t^c, h_t^n, H_t^c, H_t^n, H_t$ affect the optimal allocation.[20]

In order to study the effects of heterogeneity on the optimal tax schedule, we provide the counterpart of equation (23) in the representative agent case, where only distortionary taxes are available to the planner.[21] In the Appendix, we show that in this case the labor tax expression

[20]Note that, as mentioned above, the steady-state allocation is the same under complete and incomplete markets. Under incomplete markets, the multiplier Ψ is time-varying. When markets are complete, this multiplier is constantly equal to its steady-state value (see Aiyagari et al., 2002).

[21]Introducing lump-sum transfers/taxes in such a framework would imply a trivial response of fiscal variables: labor taxes would be constant at zero, and lump-sum taxes would finance the inter-temporal budget of the government, thereby allowing the government to complete the market.

is:

$$\tau_t^R = 1 - \frac{1 + (\Psi_t^R - \Psi_{t-1}^R)\frac{b_{t-1}}{c_t}}{1 + (1+\phi)\Psi_t^R} \tag{26}$$

under incomplete markets, and $\tau^R = 1 - \frac{1}{1+(1+\phi)\Psi^R}$ when markets are complete.[22] Therefore, it turns out that the tax rate in the representative agent model is a special case of our two-agent economy, in which the calibration is such that all agents are Ricardian and there is no more heterogeneity in the model.[23]

The above property is useful to decompose the effect of heterogeneity and incomplete markets on the optimal labor tax. To simplify the exposition, we log-linearize expressions (23) and (26), to obtain:

$$\hat{\tau}_t^R \quad = \quad \tau\hat{\Psi}_t^R - \hat{\psi}_t^R \tag{27}$$

for the representative agent case, and

$$\hat{\tau}_t \quad = \quad \underbrace{\tau\hat{\Psi}_t - \hat{\psi}_t}_{\text{(a) IM}} + \underbrace{(1-\tau)(1-H^cH)\hat{\Psi}_t}_{\text{(b) IM + Heterogeneity}} + \underbrace{\hat{H}_t^c + (1-\tau)H^cH\hat{H}_t + \left[1 - H^c\left(1-\tau\right)\left(H + (1+\phi)\Psi\right)\right]\left(\hat{\theta}_t^h + \hat{h}_t^n - \hat{H}_t^n\right)}_{\text{(c) Heterogeneity}} \tag{28}$$

for the two-agent version. Variables with hats denote log-deviations from steady state ($\hat{x}_t = \log(\frac{x_t}{x})$)[24], and variables without time subscripts denote steady-state values. We also define $\psi_t \equiv 1 + (\Psi_t - \Psi_{t-1})\frac{b_{t-1}}{c_t^s}$, and similarly for ψ_t^R in the representative agent case.

We can distinguish two main components in the characterization of labor tax fluctuations given by equation (28).[25] The first component (elements a and b) is related to market incompleteness, and summarizes the co-movement between the labor tax and the multiplier on the government budget constraint Ψ_t. Note that, when comparing the equation with its representative agent version in (27), we can see that the introduction of heterogeneity implies an additional term related to market incompleteness (the component b). It implies that heterogeneity can am-

[22]The constant optimal tax rate on labor is a well-known property of this class of model when the elasticity of labor supply is constant, as is the case under the functional form for individual preferences stated in (17).

[23]The tax rate in the representative agent model can be obtained from equation (23) by setting $\lambda = 0$, $h_t^c = h_t^n = H_t^c = H_t^n = H_t = 1$, and $c_t^s = c_t$.

[24]With the exception of $\hat{\tau}_t \equiv -\log\left(\frac{1-\tau_t}{1-\tau}\right)$. This way, $\hat{\tau}_t$ represents the approximated change in the tax rate in percentage points, rather the percentage change from its steady-state value. This makes results easier to interpret and facilitates the comparison between the representative agent and two-agent versions of the model, because these models do not feature the same steady-state tax rate.

[25]We should stress here that expressions (23) and (28) do not constitute optimal tax reaction functions for the fiscal authority. Instead, they describe relationship between taxes and other model variables which are satisfied along the Ramsey equilibrium path, and in this sense only they help to shed light of the mechanisms at play in the model.

plify/dampen the effects of market incompleteness on tax volatility, depending on the sign of $1 - H^c H$. However, for plausible parameter values, this effect is likely to be small.

The second component, summarized in the term c, shows that the tax rate in the Ramsey allocation is also affected by variables related to household heterogeneity. It means that consumption and hours dispersion between households introduces an additional source of tax volatility in the two-agent model, compared to the representative agent counterpart.

Together, these two forces (market incompleteness and heterogeneity) imply that, for labor taxes to have low volatility, two conditions must be met. First, it is required that shocks affecting government deficits can be financed in a non-distorting way, i.e. through changes in lump-sum taxes, or through the accumulation of state-contingent claims when markets are complete. These two policies allow the government to finance its deficit while having little impact on households' welfare. Therefore, the volatility of the multiplier on the government budget constraint (Ψ_t) is low and little variations in labor taxes arise from elements (a) and (b) of expression (28).

Second, it must be that shocks can be financed through a policy that limits variations in consumption and hours dispersion between households. In our model, the impact of fiscal policy on heterogeneity is influenced by the design of the sharing rule for transfers (the value of the parameter ω): if $\omega \approx \lambda$, variations in transfers imply little redistribution between households. In this case, variations in transfers are an efficient way to finance deficits in response to shocks that have little impact on heterogeneity, as they do not distort labor supply and do not exacerbate fluctuations in heterogeneity terms. For this reason, variations in transfers provide a good hedge against the aggregate shocks present in the model (government spending and TFP shocks). For values of ω away from λ, transfers imply redistribution and can therefore not be used in such a way. However, in this case they are efficient in bringing down fluctuations in heterogeneity arising from shocks that affect households unequally, such as the shocks to the productivity of hand-to-mouth households (θ^h). To shed light on these properties, the next section studies the response of taxes to shocks under different model specifications.

5.2 Impulse responses

Aggregate shocks Figure 7 displays the response of taxes to the two aggregate shocks present in the model (government spending, and TFP shocks), for the parametrization in which $\omega = \lambda$, i.e. where transfers are evenly distributed between the two households. We also provide in this figure the decomposition of the tax rate following equation (28). The main insight emerging from the figure is the following. Comparing the representative agent model (dashed black line) to the two-agent model (solid blue line), we observe that the tax response to any aggregate shock is stronger in the model featuring a representative agent. As explained above, fluctuations in the tax rate can be decomposed in two parts: a component related to market incompleteness, and a component related to heterogeneity. For each shock, we display the contributions of these two elements in the middle and bottom panels of Figure 7. As can be seen from the figure, the main difference between the representative agent model and the two-agent model lies in the weaker variation in the component related to market incompleteness. This result comes from the fact that, in the two-agent setting, the planner can use variations in transfers to finance her inter-temporal budget, and thus does not have to rely entirely on labor tax fluctuations. Because the use of transfers does not generate distortions, the shocks have a smaller effect on the Ψ multiplier, and we observe in the middle panels of the figure that the tax component linked to market incompleteness responds less strongly to shocks in the two-agent framework. This implies that the planner is able to rely on fluctuations in transfers to bring the economy closer to the complete market allocation. Indeed, the solid blue lines (depicting the response of variables in the incomplete markets version of the two-agent framework), are close to the dotted cyan lines, which provide the response of variables in the complete markets version. Moreover, the planner can implement this transfer policy at low costs because, as can be seen from the bottom panels of the figure, the policy response is also associated with weak responses in the component associated with heterogeneity. Therefore, the policy implemented by the planner allows her to contain fluctuations in heterogeneity, while financing the deficit in a non-distorting way.

[Figure 7 approximately here]

The case of government spending shocks is a good illustration of this result. Following such a shock, the response of labor taxes is mute when $\omega = \lambda$, because financing the impact of the shock with transfers only allows the planner to spread the implied fiscal costs equally between households, in such a way that the dispersion in consumption and hours worked between them is left unaffected. Financing government spending shocks with transfers (or, in this case, lump-sum taxes) therefore implies that there is no variation in the multiplier Ψ (because transfers are non-distortive), and no variations in the variables summarizing heterogeneity. Both of these forces translate into no change in the tax rate following the shock.

To shed light on the mechanisms outlined in the previous paragraphs, we also provide in Figure 7 the tax response for a version of the two-agent model where only distortionary taxes are available.[26] The results are depicted in the dotted red lines. We can see that, in this setup, the tax response to shocks is similar to the one obtained in the representative agent model. This shows that, when transfers are not available and the labor tax is the main instrument allowing the government to finance the government budget, the resulting tax volatility implied by market incompleteness is stronger.

[Figure 8 approximately here]

In Figure 8 we consider the case where transfers are fully targeted towards hand-to-mouth households ($\omega = 1$). We can see from the figure that in this model version, the tax response is stronger than in the case where $\omega = \lambda$, which was displayed in the previous figure. This result arises from the fact that, under the present calibration, fluctuations in transfers cannot be used to finance the government budget without impacting heterogeneity between households. As a result, the planner is not able to use transfers to bring the economy closer to the complete markets allocation, while keeping consumption and hours dispersion close to constant. This can be seen from comparing the solid blue lines with the dotted cyan lines which depict the response under complete markets: contrarily to the case where transfers are evenly distributed across households, when $\omega = 1$ the response of taxes under incomplete markets is much stronger than in the complete markets counterpart. Unevenly targeted transfers makes the policy of absorbing the shocks solely through variations in transfers sub-optimal. As a result, a bigger share of the inter-temporal budget is financed through changes in the labor tax rate, which becomes more volatile. We can see from Figure 8 that, following government spending and technology shocks, the policy response implies little changes in variables describing heterogeneity, and makes fluctuations in the second component of taxes (bottom panels) very low. Indeed, most of the tax volatility implied by the shocks comes from the impact of market incompleteness (middle panels).

Shocks to hand-to-mouth productivity In Figure 9 we study the response of key variables to a positive shock to the individual productivity of hand-to-mouth households (θ^h). The top panels display the case where $\omega = \lambda$, while the bottom panels are obtained setting $\omega = 1$. The key takeaway from this figure is the following. We observe that in the two cases, and both

[26] In this case, the tax expression, as stated in Proposition 2 and equation (28) is slightly altered. We provide the tax expression in this model version in Appendix B.

under complete and incomplete financial markets, transfers drop following the shock. This can be explained by two factors: first, as hand-to-mouth agents become more productive, they rely less on transfers to finance their consumption, which leaves some room for the planner to reduce them; second, reducing transfers allows the planner to generate a negative wealth effect for these agents, which incentives them to increase their labor supply precisely at the time when their productivity is above average, thus generating efficiency gains. We observe that the fall in transfers is higher when $\omega = \lambda$: because in this case transfers are evenly distributed between agents, the planner needs to engineer a bigger reduction to produce the desired effect on hand-to-mouth agents' budget constraint.

[Figure 9 approximately here]

Turning to the response of the labor tax rate, we can see from the figure that, as was the case for aggregate shocks, the component related to market incompleteness is still the one explaining most of the tax change. Indeed, under complete markets (dotted cyan lines), the response of the labor tax is much weaker than under incomplete markets (solid blue lines), and the response of the incomplete markets component of the tax rate is much stronger than the one summarizing the effect of heterogeneity. Therefore, even though the shock considered here affects heterogeneity in a non-negligible way (even after accounting for the optimal transfer response), this effect does not have much influence on the volatility of labor taxes. This brings us to the conclusion that, in the model presented here, market incompleteness is the main component influencing the volatility of labor taxes, no matter which shock is considered.

Comparing the case where $\omega = \lambda$ to the one where $\omega = 1$, it can be observed that, under incomplete markets, the sign of the labor tax response is opposite: it increases in the first case, and decreases in the second. This is the case because, when variations in transfers affects the two households equally ($\omega = \lambda$), the total drop in transfers for HTM agents is weaker. This implies a lower negative wealth effect on these households, which then decide to increase their labor supply by a lesser amount. All in all, the aggregate labor supply drops, and so do labor tax revenues. This has the effect of tightening the government budget constraint (the multiplier Ψ increases), and calls for an increase in the labor tax rate. When transfers are fully targeted towards HTM households ($\omega = 1$), the drop in transfers is big enough to induce a labor supply response which has the effect of increasing total labor effort, and tax revenues. It loosens the government budget constraint, and implies a fall in the labor tax.

Let us also stress that, in the two cases considered in Figure 9, when markets are incomplete, the initial tax response when the shock occurs overshoots its response in the subsequent periods. This property is known in the optimal fiscal policy literature under the name of *interest rate twisting* (see e.g. Faraglia et al., 2019a): the Ramsey planner uses fluctuations in the tax rate to influence households' consumption path and therefore manipulate bond prices to influence borrowing conditions and use government debt as an active fiscal policy tool. In our model, because the bond is priced by Ricardian households, the planner seeks to influence this price through changing the consumption path of these agents only. Following a shock, she therefore uses the proper combination of labor taxes and transfers to induce a kink in the consumption process of Ricardian agents, while keeping the consumption path of hand-to-mouth households as smooth as possible.

Discussion To end this section, we want to stress the importance of the transfer sharing rule, summarized by the parameter ω, in shaping the optimal behavior of the labor tax rate. This parameter specifies the share of total transfers going towards hand-to-mouth households. As discussed above, the impact of ω on optimal policy depends on the type of shocks hitting the economy. When shocks affect individual households in a similar way (which is the case for government spending and productivity shocks in our model), the labor tax rate remains constant only when ω is close to λ. In this case, the planner uses transfers to finance its deficit, and such a policy does not impact consumption and hours heterogeneity. In contrast, when ω gets away from λ, as in our example assuming $\omega = 1$, transfers are redistributive and financing deficits using them can be welfare detrimental, as such a policy exacerbates consumption and hours heterogeneity. In this case the optimal policy features more tax volatility.

When the economy is hit by household-specific shocks (such as a shock to θ^h in our model), labor taxes are also less volatile when $\omega \approx \lambda$. However, in this case consumption heterogeneity between household is greater, as fiscal instruments do not allow the planner to reduce household heterogeneity.

5.3 Matching the empirical properties of transfers

In this section we assess whether our optimal fiscal policy model is able to match some of the key moments that summarize the business cyclical properties of macroeconomic and fiscal variables in the US. In particular, we study the co-movement between transfers, deficits, output and the market value of debt.

We are particularly interested in the ability of our model to match the following data properties. First, the negative correlation between transfers and GDP, and the positive correlation between transfers and the primary deficit, which have been described in Section 2. Being able to generate counter-cyclical transfers with our model will be possible (i) if there is less need for redistribution in expansions, meaning that transfers can be decreased without negatively impacting households' welfare; and (ii) if the government is willing to consolidate its budget in an expansion, and thereby decides to decrease transfers to generate fiscal surpluses. To generate the positive correlation between transfers and deficits, it is key that variations in transfers are not used to compensate for rising deficits implied, for instance, by a rise in government spending. Transfers should, on the contrary, be one of the main drivers behind changes in primary deficits.

Second, we are interested in the co-movement between deficits and the market value of debt (positive in the US data). Marcet and Scott (2009, MS) analyze the ability of simple optimal fiscal policy models to reproduce key facts related to the behaviour of government debt. They show that, in representative agent models, when markets are complete, the market value of debt falls in response to shocks that force the deficit to increase. Moreover, the optimal portfolio pays out more than the actual income shock to compensate for higher future deficits when shocks are persistent. This is what MS call the "over-insurance" effect. Hence, the co-movement between deficits and debt is negative when the government can issue state-contingent securities, which is at odds with what is observed empirically. In the incomplete markets case, the market value of debt cannot decrease when deficits rise, and the government issues more debt to absorb the shock. Therefore, there is a positive correlation between deficits and debt in the representative agent, incomplete markets benchmark.

Third, we are interested in the persistence of the market value of government debt. Throughout this section, we follow MS and use the *k-variance* to measure the persistence of government debt. The k-variance of a random variable x at horizon k is defined as:

$$Var_k(x) = \frac{Var(x_t - x_{t-k})}{kVar(x_t - x_{t-1})} \text{ for } k = 1, 2, ...$$

This object is a measure of the persistence of a random variable: when a process reverts to its mean, its k-variance converges to zero; otherwise it takes a higher value. MS show that Ramsey models with complete markets imply no persistence of debt (the k-variance quickly converges to zero, which is at odds with the actual behaviour of US government debt), while incomplete markets models imply very persistent levels of debt, such that the k-variances obtained from these models usually overshoot their data counterpart.

In the previous sections, we have already seen that, in our two-agent, incomplete markets setting, the use of transfers to finance the government budget allows the planner to reduce tax distortions and brings the optimal allocation close to its complete markets counterpart. Such a property is both a threat and an opportunity for the ability of our model to reproduce the moments of interest. On the one hand, it might prevent the model to generate the desired co-movements between deficits, output and the market value of debt that are usually generated in a representative agent framework with incomplete markets. However, on the other hand, it might help the incomplete markets model to generate a lower persistence of debt, thereby overcoming the overshooting feature outlined by MS. The object of this section is therefore to study whether our two-agent model can help bringing the persistence of debt closer to its empirical counterpart while also doing a good job matching the other properties mentioned above.

The next subsection makes use of the impulse response functions of key fiscal variables to each of the shocks present in the model, under the calibration described in Table 2, to shed light on the ability of the model to match the moments described above. We then make use of the simulated method of moments to choose the model parameters in order to match a broad vector of moments computed from US data.

5.3.1 Impact of shocks on the cyclical properties of fiscal variables

Figure 10 depicts the impulse response of the main fiscal variables, consumption and output for each of the three shocks present in the model, in the incomplete markets version of the framework with $\omega = \lambda$ (top panels), and $\omega = 1$ (bottom panels). This allows us to analyze the impact of shocks on the co-movement between variables, as described above.

Government spending shocks: The solid blue lines in Figure 10 depict the responses to a positive government spending shock. As described in the previous section, when the shock hits, households work more and the planner reduces transfers to finance the shock. When transfers are evenly distributed between households ($\omega = \lambda$), the combined effect of increased hours and decreased transfers enables the deficit to remain constant. Hence, while the co-movement between transfers and output is negative following a g shock, the co-movement between deficit and output is almost null.

When transfers are directed towards hand-to-mouth households only ($\omega = 1$), the deficit cannot be insulated from the shock. Indeed because of their effect on heterogeneity, the planner is not willing to use transfers to absorb the shock entirely. Therefore, it is sub-optimal to use variations in transfers to bring the economy close to the complete markets allocation. As such, government spending shocks imply positive co-movements between deficits and output and between deficits and the market value of government debt, the former being at odds with what we see in the data.

[Figure 10 approximately here]

TFP shocks: Dashed red lines of Figure 10 describe the response of variables to a positive TFP shock. When $\omega = \lambda$, transfers, deficits and output increase on impact. Then, as the shock fades away, transfers and deficits continue to increase, while output decreases to return to its steady-state value. Hence, the co-movement between these variables is positive.

When $\omega = 1$, transfers remain approximately constant. Then, there is no co-movement between transfers and output, and between transfers and deficits. Note that, in this case, the market value of debt and deficits have a positive co-movement, which is not the case when $\omega = \lambda$.

Shocks to the productivity of HTM agents: Dotted black lines of Figure 10 depict the response of variables to an increase in the productivity of hand-to-mouth households. Following the shock, we observe a big drop in transfers and an important increase in output. This is true for the two values of ω that are considered. Overall, output and transfers co-move negatively, and output and deficits too. We can also see from the graph that the response of deficits and the market value of debt is negative, implying a positive co-movement between the two variables, as is the case in the US data.

From this analysis of the impulse responses of fiscal variables to various shocks, we can draw two important conclusions. First, when transfers are unevenly targeted towards hand-to-mouth households ($\omega > \lambda$), our model produces moments that are getting closer to their empirical counterpart. Second, it appears that the only shock that enables the model to jointly match the empirical moments discussed in the preceding sections is the shock to the productivity of hand-to-mouth households θ^h. Hence, the values of ω and the properties of the θ^h shock are likely to be key to allow the model to match the behaviour of US data moments, a task that we undertake in the following section.

5.3.2 Choosing model parameters to target empirical moments

To investigate the ability of our simple model to produce moments which are close to their empirical counterpart, we estimate some of the parameters using the simulated method of moments (SMM). In particular, we choose the share of transfers going to HTM households (ω), the inverse Frisch elasticity (ϕ), the steady-state productivity of hand-to-mouth agents (θ^h), as well as the AR coefficients and standard deviations of our three shock processes, to minimize the quadratic distance between the moments generated by the model and those observed in US data. We make use of quarterly US data for the period 1960Q1-2017Q3 to compute our moments.[27] We target 15 moments related to the cyclical properties of fiscal variables and heterogeneity between households (see Table 3 for the full list of moments). Our estimation methodology is explained in more detail in Appendix B.

As the data moments are computed at a quarterly frequency, we assume a model period of one quarter. We set the discount factor to $\beta = 0.995$ (this implies a 2% steady-state annual real interest rate) and $\lambda = 0.3$, as was the case before. We set the steady-state levels of government spending-to-gdp and the market value of annual government debt-to-GDP to their empirical average of respectively 7.36% and 35.08%.

[Table 3 approximately here]

Our results are presented in Table 3. The upper part of the table displays the estimated values of model parameters. One result is worth mentioning here. The value we obtain for the parameter targeting the share of transfers going to HTM households is $\omega = 0.82$. This is higher than λ, which was calibrated to 0.3, which implies that most of the transfers are targeted towards hand-to-mouth households. We also observe that in steady-state transfers are positive (transfers-to-gdp are equal to 15.6%), which means that under the obtained parametrization, the planner chooses to redistribute resources towards HTM agents, which helps reducing consumption inequality. The value of ω is not far from its data counterpart (the average value of transfers targeted to households at the bottom 30% of the income distribution) which, as we

[27] The sample period is reduced to 1984Q1-2017Q3 for the cross-sectional data, as the dataset we use only covers this timespan. The datasets we use are the ones described in Section 2: we make use of NIPA tables to compute aggregate statistics, and the CEX to get cross-sectional moments.

mentioned in Section 2, is equal to 0.67. No data on the cross-sectional distribution of transfers were used to compute the targeted moments, so we see this as a way to stress the ability of our simple model to match key empirical facts.

In the lower part of Table 3, we compare the moments obtained from our model to the ones observed in the US data. We observe that our simple model does surprisingly well in matching the targeted moments. Indeed, each of the simulated correlation has the appropriate sign, with the exception of the correlation between the market value of debt and deficits, which is almost zero in the model. The absolute deviations of most of the model moments to their data counterpart are very small. Of particular interest are the aggregate fiscal moments described in Section 2: the correlation between transfers, deficits and GDP, which are displayed in the first three lines of the table. We see that, for these moments, the fit is particularly good. To understand this result, we provide in Figure 11 the variance decomposition of some of the key variables when the parameters are set to their estimated values. We can see from the figure that more than 90% of the variation in the labor tax rate, transfers, and consumption heterogeneity, is explained by the shocks to hand-to-mouth agents' productivity (θ_t^h). These shocks account for about 50% of variations in output, while TFP shocks account for about 46% of the fluctuations in this variable. The importance of shocks to the productivity of hand-to-mouth households in the estimated model explain why the model does so well in generating a co-movement of fiscal variables which is very close to the one observed in the US. Indeed, we have seen in the previous section that such a shock generates the desired co-movement between deficits, transfers, and GDP; then, given the prevalence of fiscal variables in the moments that we target, it is with little surprise that the estimated process for this shock is such that it plays an important role.

[Figures 11 and 12 approximately here]

In Figure 12 we display the k-variance of the market value of government debt in our model and in the data, up to an horizon of 40 quarters. The solid blue line depicts the k-variance for the baseline 2-agent, incomplete markets model, using our estimated parameters; the dotted black line provides the empirical counterpart, computed using US data.[28] We observe from the figure that the model generates a k-variance which implies too little persistence in the debt

[28]To compute the k-variance of US government debt, we use data for the period 1960Q1-2007Q4. This way, our sample period resembles the one used by Marcet and Scott (2009), which helps us comparing our results to the ones described in their paper.

process compared to its empirical counterpart. This is in contrast with representative agents models, which typically overshoot this statistics, as reported in Marcet and Scott (2009). The k-variance obtained in the representative agent version of the model is depicted by the dashed-dotted green lines in the figure.[29] These results confirm the fact that in our 2-agent framework, the use of transfers allow the planner to bring the economy closer to the complete markets allocation, thereby implying a lower persistence of government debt. Note that there is still a stark contrast between the baseline results, and the ones obtained assuming complete financial markets, which are displayed in the dashed red line of Figure12, and in which we observed that the k-variance of debt is below one at all horizons, and quickly reaches zero.

6 Conclusions

In this paper we extend some results of the optimal fiscal policy literature by considering a model which introduces a small degree of heterogeneity between households, and an additional instrument to the Ramsey planner: (un)targeted transfers. We make the following contributions.

First, we provide the condition under which the optimal labor tax is strictly positive in the steady-state allocation. We show that, when heterogeneity in consumption and hours between households is sufficiently high, and/or when transfers are targeted towards a given type of agent, it might be optimal for the planner to set positive labor taxes in order to free up resources and increase the value of transfers. Such a policy partially removes inequality between households, which, as we show, can be optimal even though it comes at the cost of reducing aggregate consumption and output.

Second, we derive analytical expressions for the optimal dynamic response of labor taxes under complete and incomplete financial markets. We find that the main driver of labor tax fluctuations comes from the incomplete markets assumption rather than household heterogeneity. When markets are incomplete, fluctuations in transfers are used by the planner to bring the economy closer to the complete markets allocation, featuring a very low tax volatility. The allocation gets closer to complete markets when the transfer rule is not targeted towards an agent type, and shocks are aggregate. When transfers are directed towards a given type of house-

[29] We use the same model parameters as in our two-agent model to compute the statistics in the representative agent framework. Note that there are only two shocks (government spending and TFP) in this model version. We simply ignore the θ^h shock, and do not reestimate the variance of the shock processes to obtain our results.

holds, the planner faces a trade-off between minimizing labor tax distortions and reducing consumption and hours heterogeneity. Hence, it becomes less optimal to keep the labor tax close to constant.

Finally, in order to investigate the ability of our simple model to reproduce empirical moments, we study the joint behaviour of key macro and fiscal variables and estimate some of the model parameters using the simulated method of moments (SMM). We show that heterogeneity, and more specifically shocks to the productivity of hand-to-mouth agents, is key for the model to be able to reproduce the cyclical behaviour of transfers, deficits, and output.

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Appendix

A Tables and figures

TABLE 1: **Corr. matrix of fiscal variables in the US**

	GDP	Deficit	Transfers
GDP	1	-	-
Deficit	-0.7961	1	-
Transfers	-0.4462	0.6841	1

Notes: Data are from the NIPA database and cover the period 1984Q1-2013Q1. The frequency is quarterly. All variables are expressed in per-capita terms and are de-trended using the HP-Filter (Smoothing parameter: 1,600). Transfers correspond to our restricted definition, which covers unemployment benefits and other income assistance programs (see Section 2). Taking a broader definition of transfers - that is, government social benefits to persons - leads to even more counter-cyclical transfers. In this case, the correlation between GDP and transfers amounts to -0.7592.

TABLE 2: **Calibrated parameter values**

Parameter	Description	Value
β	Discount factor	0.95
λ	Share of hand-to-mouth households	0.30
ϕ	Inverse Frisch elasticity	1
g/y	Steady-state government spending to gdp	0.15
b/y	Steady-state government debt to gdp	0.6
θ^h	Steady-state productivity of HTM agents	1
ω	Share of transfers targeted to HTM agents	$\{\lambda; 1\}$

Notes: The table provides the assumed parameter values in the baseline specification of the model presented in Section 3.

TABLE 3: **Estimated parameters and implied moments**

Estimated parameters:

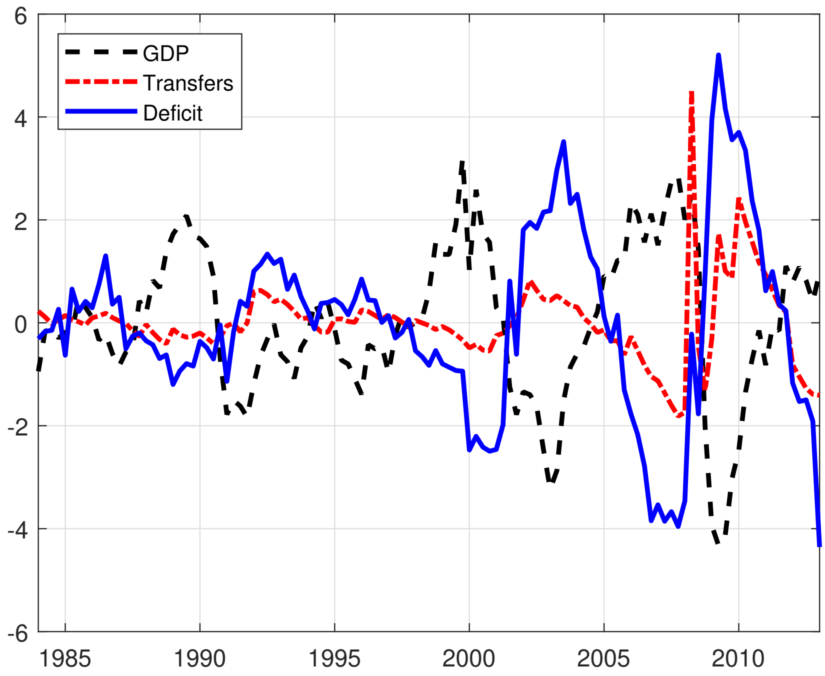
Parameter	Description	Value
ω	Share of transfers targeted to HTM agents	0.82043
ϕ	Inverse Frisch elasticity	1.9004
θ^h	Steady-state productivity of HTM agents	0.57591
Shocks		
ρ_g	AR coeff. gov. spending	0.7139
ρ_a	AR coeff. total factor productivity	0.93072
ρ_θ	AR coeff. relative productivity of HTM agents	0.60146
σ_g	Std, gov. spending	0.074173
σ_a	Std, total factor productivity	0.0052298
σ_θ	Std, relative productivity of HTM agents	0.052671

Implied moments:

	Data	Model
$corr(T, y)$	-0.6323	-0.59263
$corr(def, y)$	-0.631	-0.60137
$corr(T, def)$	0.7329	0.82561
$std(y)$	0.0146	0.016852
$std(g/y)$	0.0379	0.10109
$corr(mv/y, def)$	0.5065	-0.08311
$corr(g_t, g_{t-1})$	0.6836	0.68526
$corr(y_t, y_{t-1})$	0.8623	0.72385
$cor(mv_t, mv_{t-1})$	0.8752	0.98709
$mean(\theta^h n^h/n^s)$	0.2089	0.46055
$std(\theta^h n^h/n^s)$	0.0724	0.10122
$corr(\theta^h_t n^h_t/n^s_t, \theta^h_{t-1} n^h_{t-1}/n^s_{t-1})$	0.5085	0.57832
$mean(h^c)$	0.5582	0.88074
$std(h^c)$	0.0331	0.0045294
$corr(h^c_t, h^c_{t-1})$	0.5044	0.61323

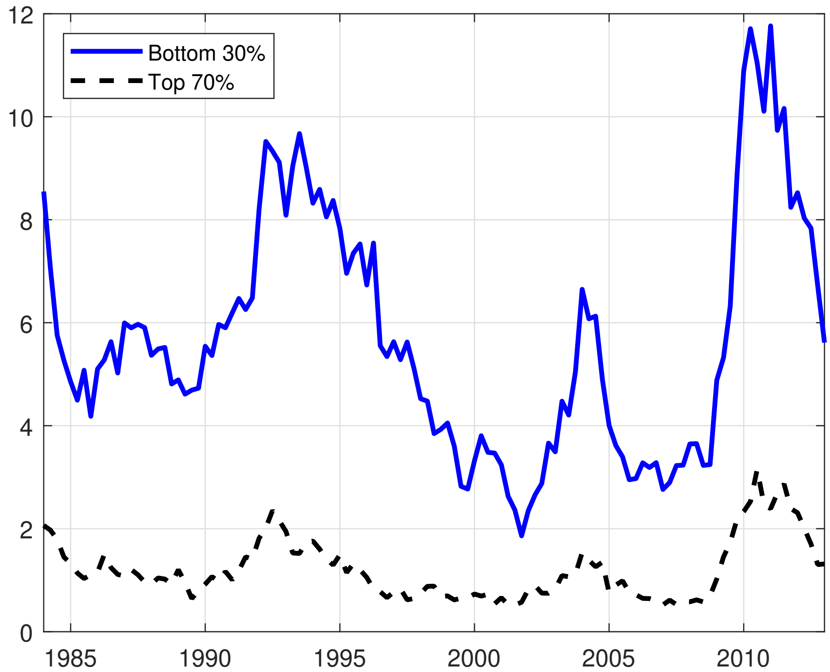
Notes: The table presents the results obtained in our Simulated Method of Moments (SMM) exercise. We choose the model parameters to match the set of moments presented in the bottom part of the table (the left column displays the values obtained from the US data for the period 1960Q1-20017Q3), while the right columns presents the values obtained from the model). The top part of the displays the values of estimated parameters. The remaining parameters are set to the value displayed in Table 2.

Figure 1: **Cyclical behaviour of fiscal data in the US**



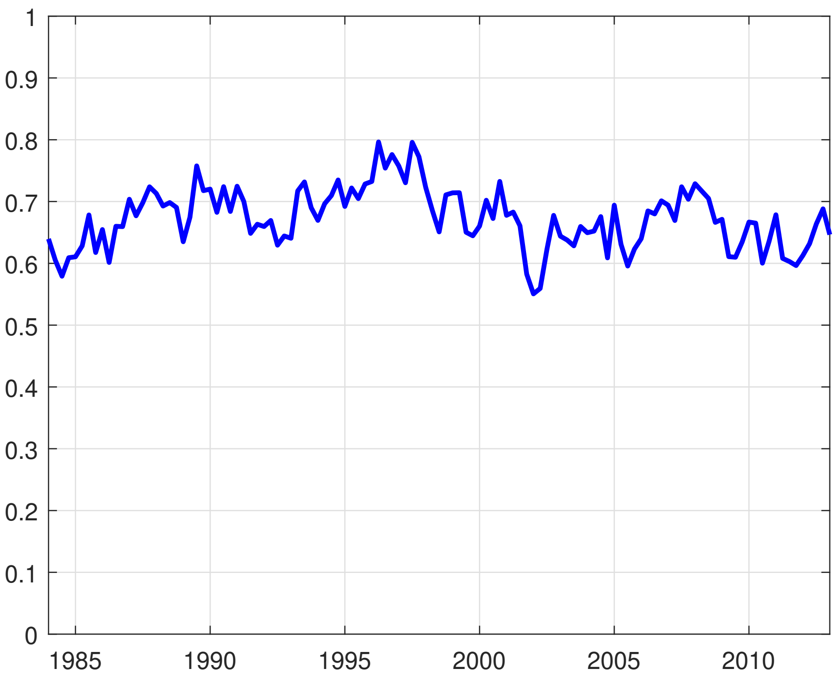
Notes: The figure plots the cyclical behaviour of real GDP (dashed black line), real deficit (solid blue line), and real transfers (dashed red line), at the quarterly frequency. Data are from the NIPA database. Variables are in per-capita terms and de-trended with HP-Filter.

FIGURE 2: **Transfers towards household income groups**



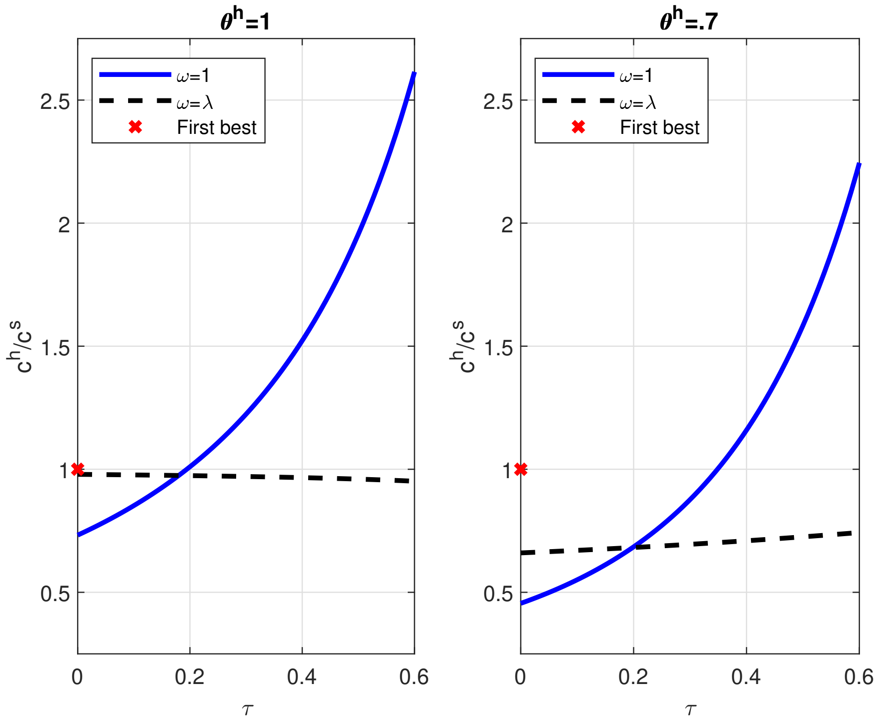
Notes: The figure plots the behaviour of transfers per capita for households at the bottom 30% of the income distribution (solid blue line), and the top 70% (dashed black line), at the quarterly frequency. Household-level data is aggregated from the CEX database.

FIGURE 3: **Share of transfers targeted towards low-income households**



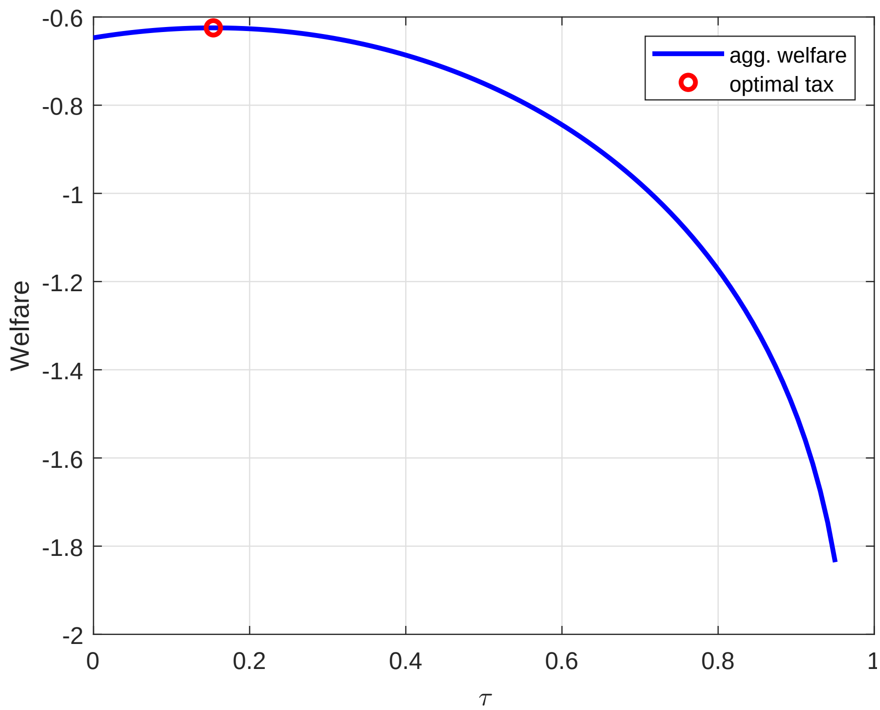
Notes: The figure plots the evolution of the share of total transfers targeted towards households at the bottom 30% of the income distribution (solid blue line), at the quarterly frequency. The series was constructed using household-level data from the CEX.

FIGURE 4: **Steady-state consumption inequality**



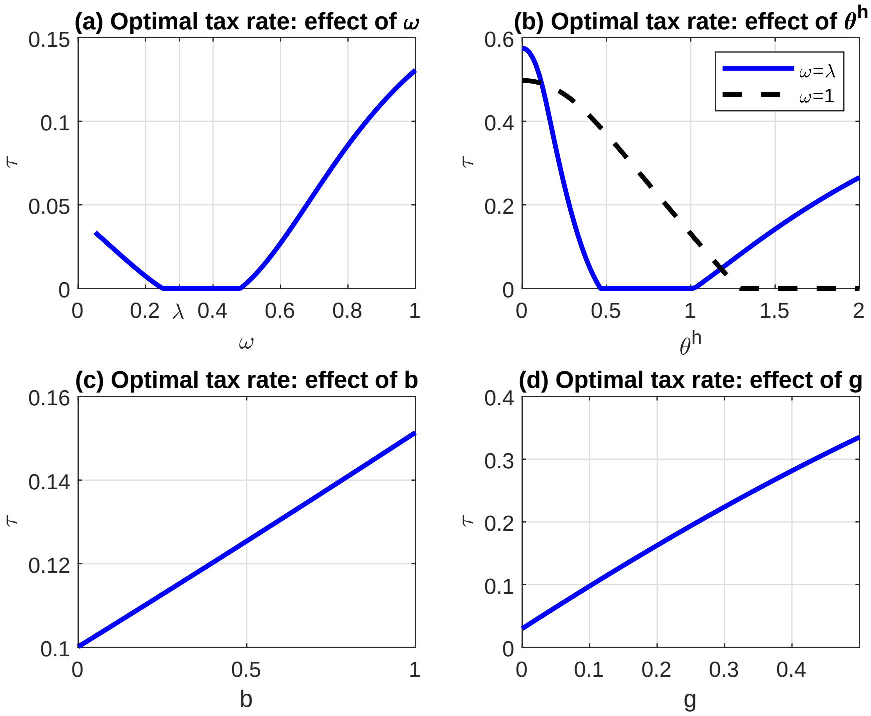
Notes: The figure depicts the steady-state consumption inequality, measured as the ratio of hand-to-mouth over Ricardian households' per capita consumption levels (c^h/c^s), as a function of the labor tax rate. The red cross depicts the first-best allocation. The dashed black line plots consumption inequality for the case of evenly targeted transfers ($\omega = \lambda$). The solid blue line presents consumption inequality for the case of transfers fully targeted towards hand-to-mouth households ($\omega = 1$).

Figure 5: **Steady-state welfare**



Notes: The solid blue line plots the value of aggregate welfare U in the steady-state of the model presented in Section 3, as a function of the labor tax rate. The red circle depicts the optimal the tax rate and associated welfare level. To produce this figure, we set $\theta^h = 1$, $\omega = 1$, and $b_{-1} = 0$. The remaining parameters are set to the baseline values provided in Table 2.

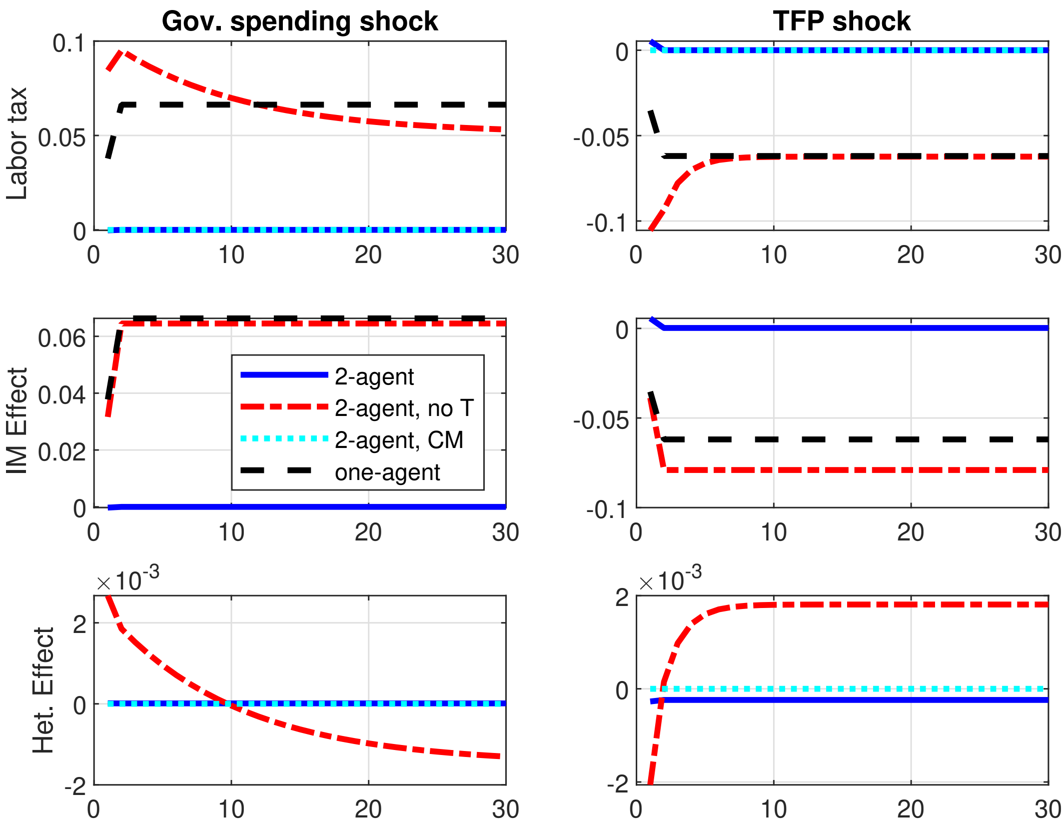
FIGURE 6: **Optimal long-run taxes: effect of key parameters**



Notes: The figure analyses the effect of key model parameters on the optimal labor tax rate in the steady-state of the baseline model. Panel (a) displays the effect of ω, the share of transfers targeted towards hand-to-mouth agents. Panel (b) depicts the effect of θ^h, the long-run value of the relative productivity of hand-to-mouth agents. Pnael (c) and (d) present, respectively, the effect of the long-run levels of the debt-to-gdp and government spending-to-gdp ratios. The remaining parameters are set to their baseline values, as presented in Table 2.

FIGURE 7: **Labor tax response under incomplete markets: one vs. two-agent models (1)**

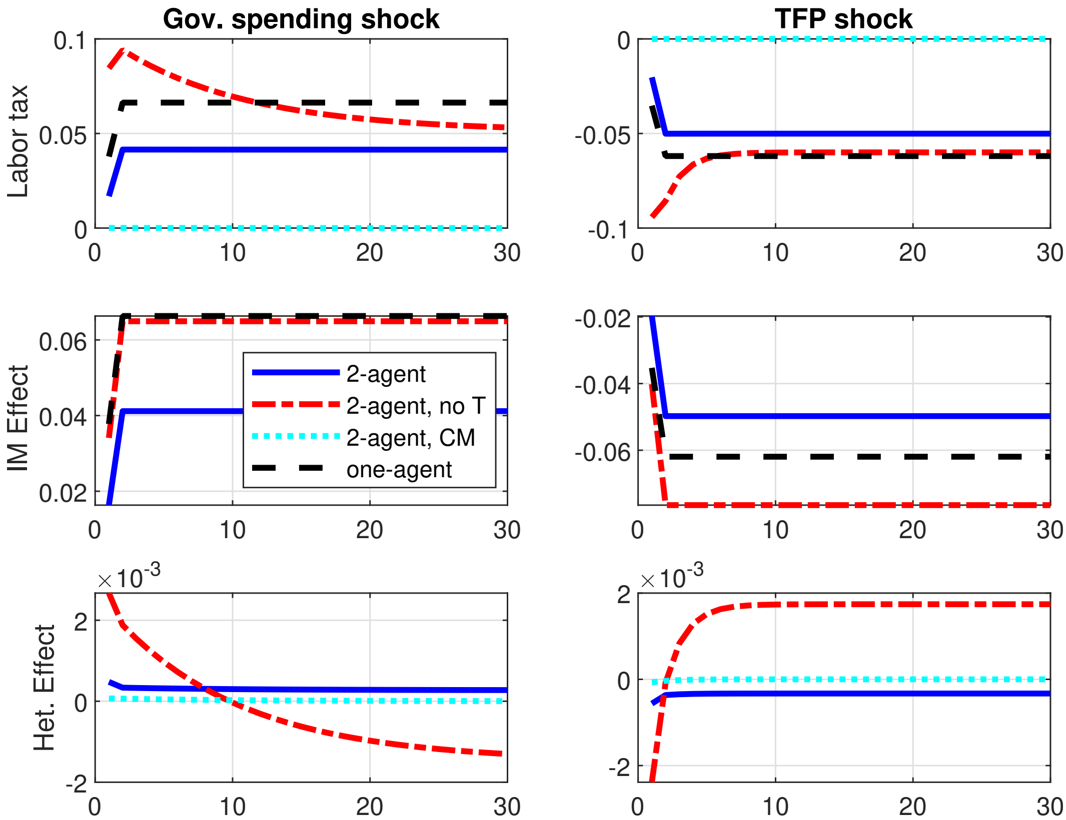
Case 1: $\omega = \lambda$



Notes: Response of the labor income tax to government spending (left panels) and TFP shocks (right panels), for the case of evenly targeted transfers ($\omega = \lambda$). The value of each shock is normalized to one. The top panels present the response of the optimal labor tax (in deviation from steady-state). The middle and bottom panels depict, respectively, the component of taxes related to market incompleteness and heterogeneity, following the decomposition presented in equation (28). The solid blue lines are for the baseline two-agent model under incomplete markets, and the dotted cyan lines for the complete markets counterpart. The dashed red lines display the responses for a 2-agent model version where only labor taxes are available to the planner. Finally, the dashed black lines present the results for a standard representative agent model without transfers.

Figure 8: **Labor tax response under incomplete markets: one vs. two-agent models (2)**

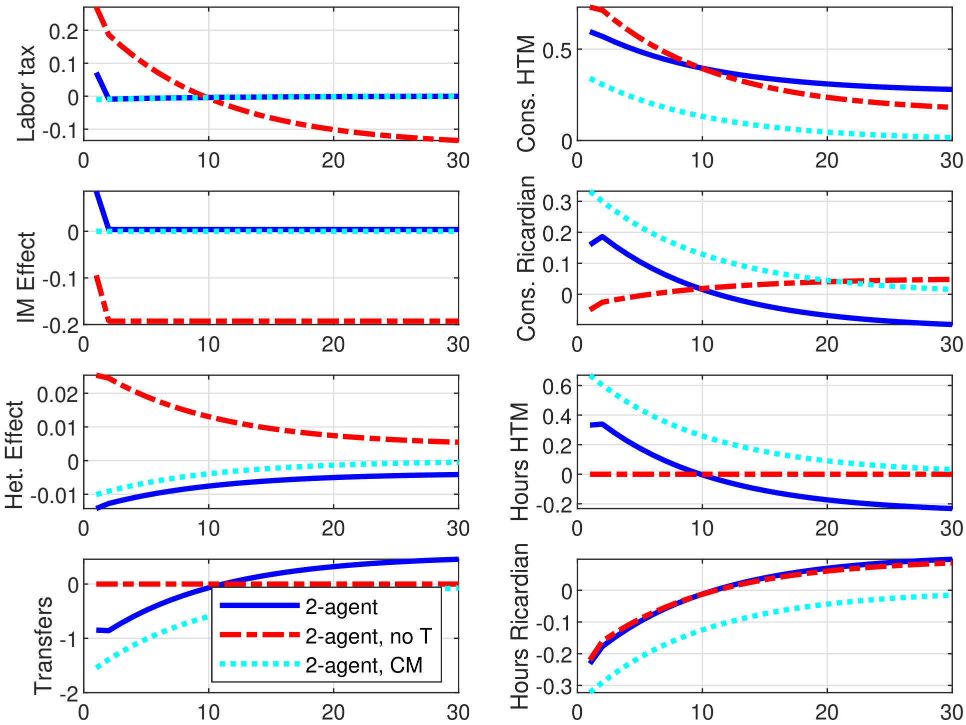
Case 2: $\omega = 1$



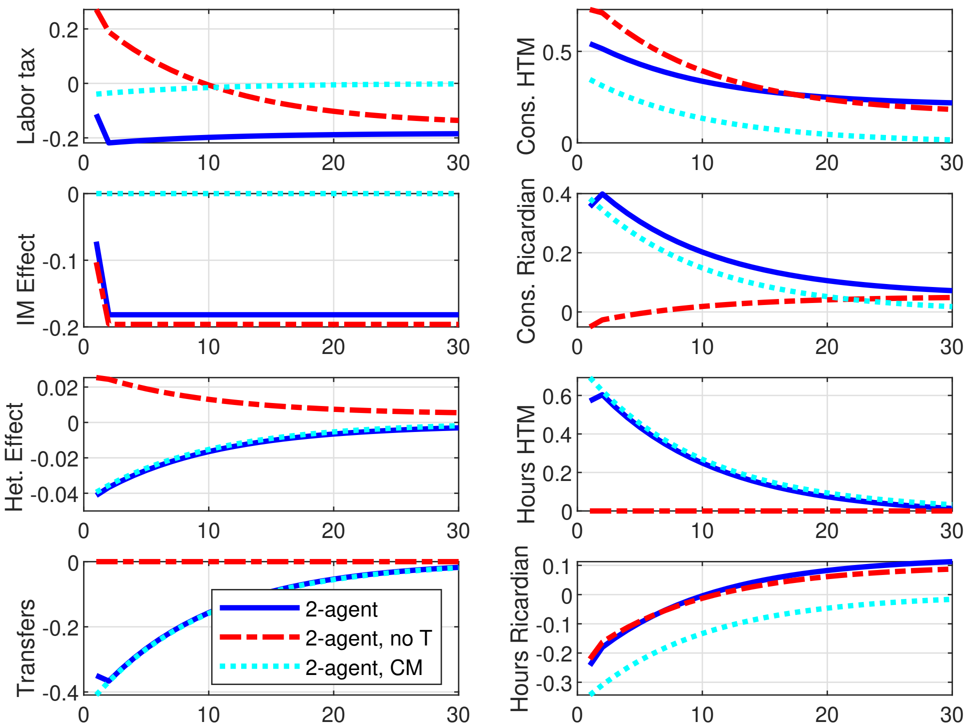
Notes: Response of the labor income tax to government spending (left panels) and TFP shocks (right panels), for the case where transfers are targeted towards hand-to-mouth agents ($\omega = 1$). The value of each shock is normalized to one. The top panels present the response of the optimal labor tax (in deviation from steady-state). The middle and bottom panels depict, respectively, the component of taxes related to market incompleteness and heterogeneity, following the decomposition presented in equation (28). The solid blue lines are for the baseline two-agent model under incomplete markets, and the dotted cyan lines for the complete markets counterpart. The dashed red lines display the responses for a 2-agent model version where only labor taxes are available to the planner. Finally, the dashed black lines present the results for a standard representative agent model without transfers.

Figure 9: **IRFs under incomplete markets: Shock to HTM productivity**

Case 1: $\omega = \lambda$



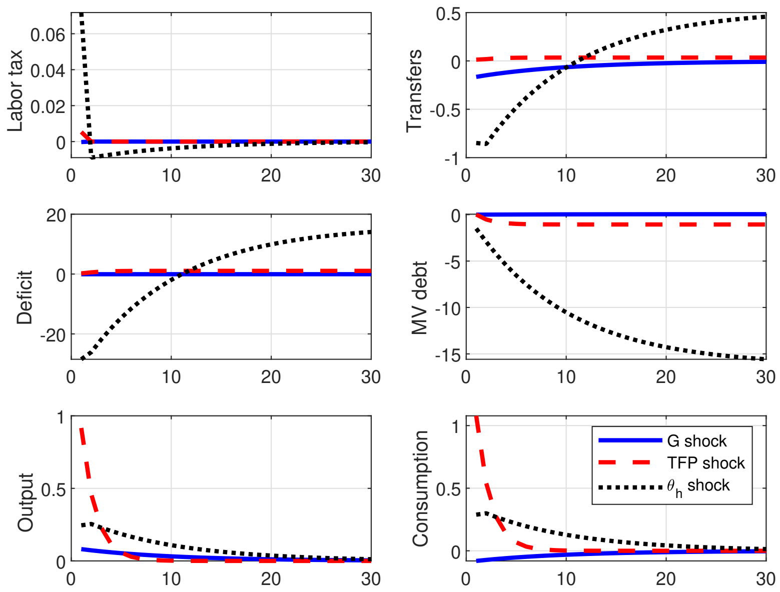
Case 2: $\omega = 1$



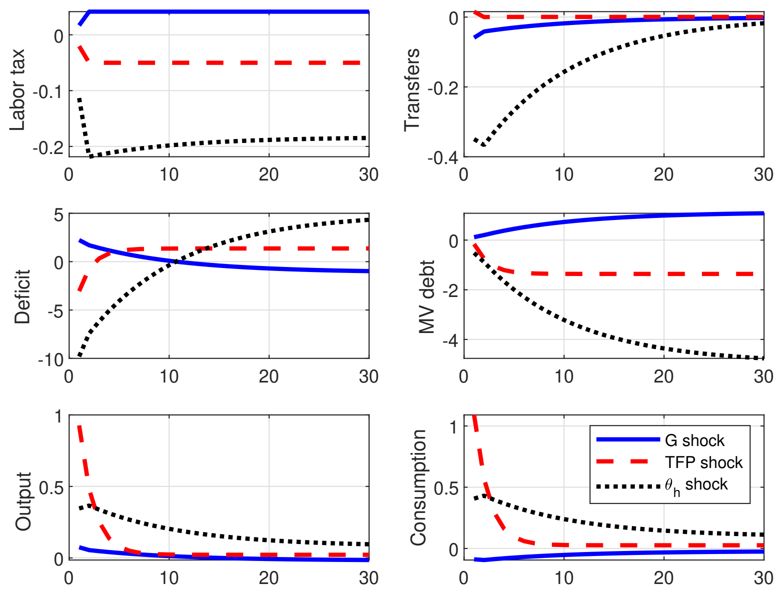
Notes: Response of key model variables to a θ^h shock, for the case of evenly targeted transfers ($\omega = \lambda$, top graphs) and transfers towards hand-to-mouth agents only ($\omega = 1$, bottom graphs). The value of the shock is normalized to one. The solid blue lines are for the baseline two-agent model under incomplete markets, and the dotted cyan lines for the complete markets counterpart. The dashed red lines display the responses for a 2-agent model version where only labor taxes are available to the planner.

FIGURE 10: **IRFs under incomplete markets: fiscal variables**

(a) **Case 1:** $\omega = \lambda$

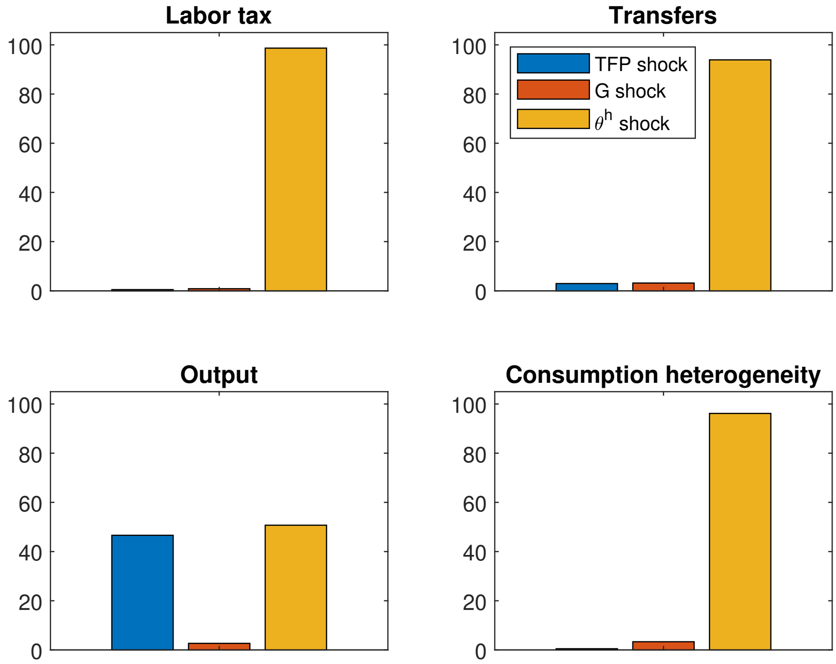


(b) **Case 2:** $\omega = 1$



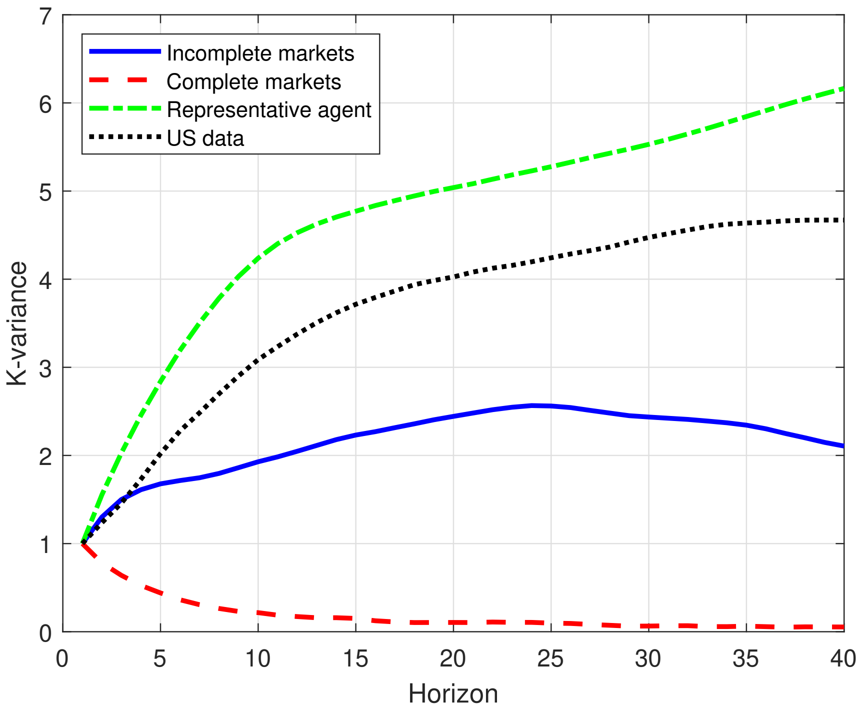
Notes: The figure presents the impulse response functions of key variables to the three shocks present in the model. The top panels display results for the case where $\omega = \lambda$, while the bottom panels set $\omega = 1$. The value of the remaining model parameters are displayed in Table 2. The solid blue lines display the responses to a government spending shock, the dashed red lines to the TFP shock, and the dotted black lines present results to a θ^h shock. The values taken by the shock is normalized to one in the three cases.

Figure 11: **SMM results: Variance decomposition**



Notes: The figure displays the variance decomposition of labor taxes, transfers, output and consumption heterogeneity, for the model estimated with the SMM (the parameter values are displayed in Table 3).

Figure 12: **SMM results: k-Variance of debt**



Notes: The figure displays the behaviour of the k-variance for the market value of debt in our estimated model. The solid blue line depicts the obtained values for our baseline 2-agent model with incomplete markets. The dashed red line provides the analogous result when we assume complete markets, and the dashed-dotted green line displays the representative agent counterpart, where only distortionary taxes are available. The dotted black line shows the series obtained from the US data in the period 1960Q1-2007Q4.

B Model appendix

B.1 Complete markets

In this section we provide additional details on the complete markets model studied in the main text and presented in Section 3.3. In this model version the Ricardian households' budget constraint can be written as:

$$c_t^s + \int_{s^{t+1}} q^{CM}(s^{t+1}) b^{s,CM}(s^{t+1})\, ds^{t+1} = (1-\tau_t) a_t n_t^s + b^{s,CM}(s^t) + T_t^s \tag{29}$$

The first order conditions give:

$$\frac{v_{n,t}^s}{u_{c,t}^s} = 1 - \tau_t \tag{30}$$

$$u_c^s(s^t) q^{CM}(s^{t+1}) = \beta f(s^{t+1}|s_t) u_c^s(s^{t+1}) \tag{31}$$

The second equation gives the pricing condition for each state contingent security $b^{CM}(s^{t+1})$. This equation can be aggregated over states s^{t+1} to obtain the usual Euler equation:

$$q_t^{CM} = \beta \int_{s^{t+1}} \frac{u_{c,t+1}^s}{u_{c,t}^s} f(s^{t+1}|s_t) ds^{t+1}$$

The Lagrangian associated to the Ramsey program (the complete markets equivalent of equation (10)) can be written as:

$$\begin{aligned}
\mathcal{L}^{CM} \quad = \quad & E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \lambda \Big[u(c_t^h) - v(n_t^h) \Big] + (1-\lambda) \Big[u(c_t^s) - v(n_t^s) \Big] \\
& + \Psi^{CM} \Big[- u_{c,0}^s b_{-1} + (u_{c,t}^s a_t - v_{n,t}^s)(\lambda \theta_t^h n_t^h + (1-\lambda) n_t^s) - u_{c,t}^s (g_t + T_t) \Big] \\
& + \Psi_t^1 \Big[a_t \lambda \theta_t^h n_t^h + (1-\lambda) n_t^s a_t - \lambda c_t^h - (1-\lambda) c_t^s - g_t \Big] + \Psi_t^2 \Big[\frac{v_{n,t}^s}{u_{c,t}^s} - \frac{1}{\theta^h} \frac{v_{n,t}^h}{u_{c,t}^h} \Big] \\
& + \Psi_t^3 \Big[- u_{c,t}^s c_t^h + v_{n,t}^s \theta_t^h n_t^h + \frac{\omega}{\lambda} u_{c,t}^s T_t \Big]
\end{aligned}$$

The associated first order conditions are:

$$
\begin{aligned}
c_t^h &: \quad \lambda u_{c,t}^h - \lambda \Psi_t^1 + \frac{\Psi_t^2}{\theta_t^h} \frac{v_{n,t}^h}{(u_{c,t}^h)^2} u_{cc,t}^h - \Psi_t^3 u_{c,t}^s = 0 \\
c_t^s &: \quad (1-\lambda) u_{c,t}^s + \Psi^{CM} \Big[u_{cc,t}^s a_t n_t - u_{cc,t}^s (g_t + T_t) \Big] \\
&\quad -\Psi_t^1 (1-\lambda) - \Psi_t^2 \frac{v_{n,t}^s}{(u_{c,t}^s)^2} u_{cc,t}^s + \Psi_t^3 (-u_{cc,t}^s c_t^h + \frac{\omega}{\lambda} T_t u_{cc,t}^s) = 0 \\
n_t^h &: \quad -\lambda v_{n,t}^h + \Psi^{CM} (u_{c,t}^s a_t - v_{n,t}^s) \lambda \theta_t^h + \Psi_t^1 a_t \lambda \theta_t^h - \frac{\Psi_t^2}{\theta_t^h} \frac{v_{nn,t}^h}{u_{c,t}^h} + \Psi_t^3 v_{n,t}^s \theta_t^h = 0 \\
n_t^s &: \quad -(1-\lambda) v_{n,t}^s + \Psi^{CM} \Big[(1-\lambda)(u_{c,t}^s a_t - v_{n,t}^s) - v_{nn,t}^s n_t \Big] + \Psi_t^1 (1-\lambda) a_t + \Psi_t^2 \frac{v_{nn,t}^s}{u_{c,t}^s} + \Psi_t^3 v_{nn,t}^s \theta_t^h n_t^h = 0 \\
T_t &: \quad -\Psi^{CM} u_{c,t}^s + \Psi_t^3 \frac{\omega}{\lambda} u_{c,t}^s = 0
\end{aligned}
$$

B.2 Representative agent

The representative agent model is a specific case of our two agent model where $\lambda = 0$. As such, the Lagrangian associated to the planner's optimization program in the incomplete markets setting is:

$$
\begin{aligned}
\mathcal{L} \;=\; & E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ u(c_t) - v(n_t) + \Psi_t^{IM} \Big[-u_{c,t} b_{t-1} + (u_{c,t} a_t - v_{n,t}) n_t) - u_{c,t} g_t + \beta u_{c,t+1} b_t \Big] \\
& + \Psi_t^1 \Big[a_t n_t - c_t - g_t \Big] \Big\}
\end{aligned}
$$

where we make use of the representative agents' Euler equation $q_t = \beta \frac{E_t u_{c,t+1}}{u_{c,t}}$ and labor supply condition $(1-\tau_t) a_t = \frac{v_{n,t}}{u_{c,t}}$ to substitute away the bond price q_t and the labor tax rate τ_t.

The optimality conditions associated to the planner's problem are the following:

$$
\begin{aligned}
c_t &: \quad u_{c,t} + \Psi_t^{IM} u_{cc,t} \Big[-b_{t-1} + a_t n_t - g_t \Big] + \Psi_{t-1}^{IM} u_{cc,t} b_{t-1} - \Psi_t^1 = 0 \\
n_t &: \quad -v_{n,t} + \Psi_t^{IM} \Big[(u_{c,t} a_t - v_{n,t}) - v_{nn,t} n_t \Big] + \Psi_t^1 a_t = 0 \\
b_t &: \quad -E_t \Psi_{t+1}^{IM} u_{c,t+1} + \Psi_t^{IM} E_t u_{c,t+1} = 0
\end{aligned}
$$

Eq. (26) is derived as follows. First, divide the above two first FOC by $u_{c,t}$ and make use of the labor supply condition to get:

$$\begin{aligned}
1-\Psi_t^{IM}-\left(\Psi_t^{IM}-\Psi_{t-1}^{IM}\right)\frac{u_{cc,t}}{u_{c,t}}b_{t-1}-\frac{\Psi_t^1}{u_{c,t}} &= 0 \\
\tau_t-1+\Psi_t^{IM}\left[\tau_t-\frac{v_{nn,t}}{v_{n,t}}n_t(1-\tau_t)\right]+\frac{\Psi_t^1}{u_{c,t}} &= 0
\end{aligned}$$

Then, using the functional form described in Eq. (17) and substituting away Ψ_t^1 using the above equations leads to Eq. (26).

B.3 Two-agent framework: first-best allocation

The first best allocation in a two-agent framework is defined as the result of the maximization process of aggregate welfare subject to the feasibility constraint of the economy, i.e. the resource constraint. As such, the Lagrangian associated to the planner's optimization program is:

$$\begin{aligned}
\mathcal{L} &= E_0\sum_{t=0}^{\infty}\beta^t\Big\{\lambda\Big(u(c_t^s)-v(n_t^s)\Big)+(1-\lambda)\Big((u(c_t^h)-v(n_t^h)\Big) \\
&\quad -\Psi_t^{RC}\Big[\lambda c_t^h+(1-\lambda)c_t^s+g_t-\lambda a_t\theta_t^h n_t^h-(1-\lambda)a_t n_t^s\Big]\Big\}
\end{aligned}$$

The first order conditions associated to the planner's problem are the following:

$$c_t^s \;:\; u_{c,t}^s=\Psi_t^{RC} \qquad\qquad n_t \;:\; v_{n,t}^s=\Psi_t^{RC}a_t$$

$$c_t^h \;:\; u_{c,t}^h=\Psi_t^{RC} \qquad\qquad n_t \;:\; v_{n,t}^h=\Psi_t^{RC}\theta_t^h a_t$$

They imply that:

$$u_{c,t}^s = u_{c,t}^h \tag{32}$$

$$\frac{v_{n,t}^s}{u_{c,t}^s} = a_t \tag{33}$$

$$\frac{v_{n,t}^h}{u_{c,t}^h} = \theta_t^h a_t \tag{34}$$

In a decentralized economy, Eq (32) implies that hand-to-mouth households' consumption is equal to Ricardian households' consumption in the case of separable utility, as is assumed throughout the paper. Eq (33) and (34) indicate that the planner prefers to set labor taxes to zero. It also states that $\theta_t^h v_{n,t}^s=v_{n,t}^h$. Therefore, when $\theta_t^h<1$, hand-to-mouth households work less than Ricardian households.

B.4 Additional results

Proposition 2B: When the planner has access to distortionary taxes only and markets are incomplete, the optimal labor tax rate satisfies:

$$\tau_t = 1 - \frac{(H_t^c)^{-1}\Big[1 + (\Psi_t^{IM} - \Psi_{t-1}^{IM})\frac{b_{t-1}}{c_t^s}\Big](1-\lambda)}{\Big[H_t H_t^n + (1-\lambda)(1+\phi)\Psi_t^{IM}\Big] - (H_t^n - (1-\lambda))\Big[(1+\phi)(h_t^c)^{-1} - \frac{\phi}{H_t^c}\Big(1 + (\Psi_t^{IM} - \Psi_{t-1}^{IM})\frac{b_{t-1}}{c_t^s}\Big)\Big]} \tag{35}$$

where $h_t^c \equiv \frac{c_t^h}{c_t^s}$; $h_t^n \equiv \frac{n_t^h}{n_t^s}$; $H_t^c \equiv \lambda h_t^c + 1 - \lambda$; $H_t^n \equiv \lambda\theta_t^h h_t^n + 1 - \lambda$; $H_t \equiv (\lambda\frac{h_t^n}{h_t^c} + 1 - \lambda)/H_t^n$.

When markets are complete, the multiplier Ψ_t^{IM} is constant ($\Psi_t^{IM} = \Psi\ \forall\, t$), and the expression becomes:

$$\tau_t = 1 - \frac{(H_t^c)^{-1}(1-\lambda)}{\Big[H_t H_t^n + (1-\lambda)(1+\phi)\Psi\Big] - (H_t^n - (1-\lambda))\Big[(1+\phi)(h_t^c)^{-1} + \frac{\phi}{H_t^c}\Big]}$$

Proof: Using the equations of the system (24) and the functional forms given in (17), while setting $T_t = 0\ \forall t$, we can define Ψ^3 as follows:

$$\Psi^3 = \frac{1}{(1+\phi)\theta^h h_t^n}\Big[H_t^n\Big(H_t - \frac{1}{H^c(1-\tau_t)}\Big) + \Psi_t^{IM}\Big(\frac{H_t^n\tau_t}{1-\tau_t} + \psi\frac{n}{n^s}\Big) - \frac{b_{t-1}}{c_t^s}(\Psi_t^{IM} - \Psi_{t-1}^{IM})\frac{1}{H_t^c(1-\tau_t)}\Big]$$

Replacing Ψ^3 by this expression in equation (25) leads to the tax expression given in equation (35).

Log-linearizing the equation for the incomplete market case leads to:

$$\begin{aligned}\hat{\tau}_t \quad = \quad & \Big[\frac{(1-\tau)}{(1-\lambda)}\Big(H^n - (1-\lambda)\Big)\phi - 1\Big]\hat{\psi}_t + (1-\tau)(1+\phi)H^c\Psi\hat{\Psi}_t \\ & + \hat{H}_t^c + \frac{(1-\tau)}{(1-\lambda)}H^n H^c H(\hat{H}_t + \hat{H}_t^n) - \frac{(1-\tau)}{(1-\lambda)}H^n\Big((1+\phi)\frac{H^c}{h^c} - \phi\Big)\hat{H}_t^n + \frac{(1-\tau)}{(1-\lambda)}\Big(H^n - (1-\lambda)\Big)\Big((1+\phi)\frac{H^c}{h^c}\hat{h}_t^c - \phi\hat{H}_t^c\Big)\end{aligned}$$

.

B.5 Simulated method of moments

In Section 5.3.2 of the paper, we choose the model parameters to target empirical moments. In particular, we make use of the simulated method of moments (SMM), in which the structural model parameters are chosen to minimize the distance between the moments computed from numerical simulations of the model, and those observed in the data. More formally, we choose the vector of parameters which minimizes a quadratic loss function, thereby obtaining the estimator $\hat{\theta}$ that satisfies:

$$\hat{\theta} = \underset{\theta \in \Theta}{\operatorname{argmin}} \ \Big(m(\theta) - m_{\text{us}}\Big)' \mathbf{W} \Big(m(\theta) - m_{\text{us}}\Big)$$

where $\mathbf{W}$ is a weighting matrix, which we set to the identity matrix in our estimation routine. m_{us} is the vector of moments computed from the US data, and $m(\theta)$ is a function mapping model parameters θ to a vector of simulated moments. We make use of a non-linear optimization routine to solve for the above problem.

C Data Appendix

In this section we provide details on the construction of the data series used in Section 2, when we provide stylized facts for the US economy, and in Section 4 and 5 when we target empirical moments with our theoretical model.

C.1 Macro data

All aggregate data, i.e. government spending, transfers, deficits and the market value of government debt, are for the U.S and are observed at a quarterly frequency. They are computed based on National Income and Products Account (NIPA) collected by the Bureau of Economic Analysis. Real values are obtained using the GDP deflator.

All variables are in per capita terms. Each variable is divided by an index of the US population, constructed from the 'Pop Civilian noninstitutional population aged 16 years and over' series from the US Bureau of Labor Statistics.

Output is measured as real output per capita. We deflate nominal GDP (Table 1.1.5, Line 1) with the GDP deflator, and divide it with the population index.

Government spending is defined as the sum of consumption expenditure (Table 3.2 Line 25), gross government investment (Table 3.2 Line 45), net purchases of non-produced assets (Table 3.2 Line 47), minus consumption of fixed capital (Table 3.2 Line 48).

Tranfers: In this paper, we adopt a narrow definition of transfers. We are mainly interested in the components of transfers targeted towards households, and reflecting social insurance. We therefore define transfers as a subset of government social benefits to persons[30] (provided in

[30] Government social benefits to persons are composed of: social security benefits (Table 2.1 Line 18) Medicare

section 2 of the NIPA): we compute them as the sum of unemployment insurance (Table 2.1 Line 21) and other benefits [31] (Table 2.1 Line 23).

Deficits are defined as government expenditures (Table 3.2 Line 42) minus government receipts (Table 3.2 Line 39) and interest payments (Table 3.2 Line 32).

The market-value of debt-to-GDP is taken from the Dallas Fed. We use series on the market value of marketable treasury debt. To construct quarterly series we use the stock of debt in the first month of each quarter.

C.2 Cross-sectional data

In order to construct our series on consumption, income and transfer receipts across household types, we make use of the Consumer Expenditure Survey (CEX).

The CEX consists of two separate surveys collected for the Bureau of Labor Statistics by the Census Bureau that provide detailed information about household consumption expenditures. It consists of a rotating panel of households that are selected to be representative of the US population every quarter. Each household is interviewed for a maximum of four consecutive quarters. However, we treat each wave as cross sectional.

Income: Each household reports information on income, hours worked and taxes paid over the twelve-month period preceding the interview. We compute the **earnings** of each household as the sum of wages and salaries plus two thirds of business and farm income earned by that household. **Income before taxes** (money income) includes the sum of wages, salaries, business and farm income earned by each member plus household financial income (including interest, dividends and rents) plus private transfers (including private pensions, alimony and child support) plus public transfers (including social security, unemployment compensation, welfare and food stamps). **Income after taxes** (disposable income) is computed as money income minus personal taxes (including federal, states and local income taxes), property taxes and other taxes such as vehicle personal property taxes.

and Medicaid benefits (Table 2.1 Line 19 and 20, respectively), unemployment insurance (Table 2.1 Line 21), veterans' benefits (Table 2.1 Line 22) and other benefits (Table 2.1 Line 23).

[31] Other benefits include the main income assistance programs such as Supplemental Nutrition Assistance Program, Black lung benefits, Supplemental security income, and Direct relief. They also include housing subsidies and some education and childcare assistance programs.

Transfers: The CEX provides information about the following categories of private and public transfers received by the households:

1. **SSI**: Supplemental Security Income.
2. **WLF**: Amount received from public assistance or welfare including money received from job training grants such as Job Corps.
3. **UNEMP**: Amount received from unemployment compensation.
4. **FDSTMP**: Annual value of food stamps.
5. **OTHR**: Amount of income received from any other source such as Veteran's Administration (VA) payments, unemployment compensation, child support, or alimony.

These categories cover the amounts perceived in the past 12 months.

To comply with the narrow definition of transfers adopted in this paper, we define the individual transfer series as the sum of UNEMP and FDSTMP.

Real values: As is the case for aggregate variables, real values are obtained using the GDP deflator.

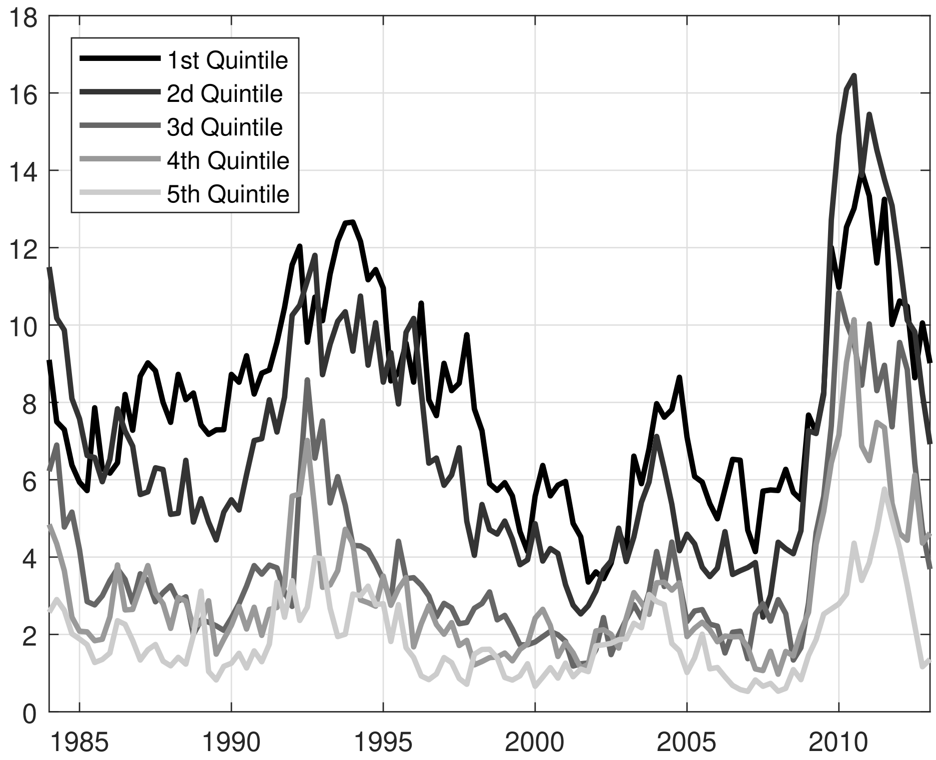
C.3 Transfers along income quintiles

To complement the results displayed in Figure 3 (plotting the share of transfers towards households at the bottom 30% and top 70% of the income distribution), we show in Figure 13 the same figure along income quintiles. We can see from the figure that the per capita amount of transfers received is decreasing along income quintile, thereby confirming the results displayed in the main text.

Indeed, most of the transfers are directed towards low income households. In particular, hand-to-mouth households receive on average more than 90% of Food Stamps benefits and around 40% of unemployment benefits [32].

[32]Moreover, hand-to-mouth households received in average more than 90% of the total amount of the public assistance and welfare programs benefits, more than 80% of the total amount of SSI benefits and around 60% of other transfers

FIGURE 13: **Average transfers per household along the income distribution**



Notes: The figure plots the behaviour of transfers per capita for households belonging to each quintile of the income distribution at the quarterly frequency. Transfers to households of the first quintile of the income distribution (that is the bottom 20%) are drawn with the darker black line, while transfers to households of the fifth quintile of the income distribution (that is the top 20%) are drawn in the lightest gray line. Data is from the CEX database.

C.4 Inequality and the business cycle:

We provide here more details on how inequality evolves over the business cycle according to CEX data.

Earnings of hand-to-mouth households appear to be slightly more pro-cyclical than the earnings of Riccardian households. Indeed, as shown in Table 4 the correlation between *hand to mouth* households' earned income and GDP amounts to 0.34, while it amounts to 0.13 for Ricardian households. Moreover, the volatility of the (log-)earnings of hand-to-mouth households appears to be four times higher than the one associated with the earnings of Riccardian households.

TABLE 4: **Correlation between Earnings and GDP**

x	**Corr(x,GDP)**	**Std(x)**
Earned Income, bottom 30%	0.34	0.05
Earned Income, top 70%	0.13	0.01

Notes: The table displays the correlation between the cyclical components of households log-earnings and GDP for low and high income households. The table also displays the standard deviation of households earnings for both groups. Data on GDP are from the NIPA database. Data on Earned Income come from the CEX. Since income data are accounted as the amount perceived over the twelve-month period preceding the interview in the CEX, GDP is computed as a moving average over 4 consecutive quarters. All variables are logged, and de-trended with the HP-Filter.

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