A SOUL'S VIEW OF THE OPTIMAL POPULATION PROBLEM

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A long-standing challenge for welfare economics is to develop welfare criteria that can be applied to allocations with different population levels. Such a criterion is essential to resolve the optimal population problem, i.e., the tradeoff between population size and the welfare of each person alive. A welfare criterion that speaks to this issue inherently requires evaluating the welfare of nonexistent people, because some people exist only in some allocations but not in others. To make progress, we consider the population problem in an environment where population is variable, but there is a fixed supply of souls, who may experience multiple incarnations over time. Rather than pondering the value of nonexistence, from the souls’ perspective comparing larger or smaller populations merely involves valuing shorter or longer waits until the next incarnation. We argue that such comparisons are possible on the basis of introspection and lead to intuitive welfare criteria with attractive properties. We emphasize that one does not have to believe in reincarnation to accept the resulting criteria—rather, reincarnation serves as a metaphor to facilitate the necessary utility comparisons.
1 Introduction

Following the seminal papers by Becker (1960) and Becker and Lewis (1973), a lot of research has been devoted to economic models with endogenous fertility, where population growth is determined as a function of the optimizing choices of households. While these models allow us to better understand how fertility is determined and interacts with other economic variables, they do not make prescriptions for optimal population policy. For such a purpose we need to define a social welfare function. Unfortunately, the standard economic approach to welfare does not carry over to situations where the number of people is a choice variable. When comparing two allocations with different population sizes, one has to take a stand on how to value the utility of people who are alive in one allocation but not in the other. Implicitly, this amounts to making assumptions on the utility of not being born.  

To illustrate the difficulties involved in this step, consider two well-known existing welfare criteria: average utilitarianism and total utilitarianism. In average utilitarianism, the social welfare function is given by the average utility of everyone who is alive. Implicit in this criterion is the assumption that the utility of not being born is equal to the average utility of those who are alive. To see why this is so, compare two allocations, one in which two people are born with a utility of two each, and one where in addition to the two people with utility two a third person exists with utility equal to one. According to average utilitarianism, the first allocation is to be preferred, since average utility is higher. But no one who is alive in the first allocation is worse off in the second, we merely added a third person with utility equal to one. If we prefer the first allocation, implicitly we are stating that the world would be better off if the third person had not been born. The assumption that a new person has value only if her utility is at least equal to the average utility of the existing people seems arbitrary, and leads to some unattractive conclusions. For example, an allocation with a single person

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1Recent references for efficiency criteria with variable populations are Michel and Wigniolle (2007) and Golosov, Jones, and Tertilt (2007). Efficiency criteria only provide a partial ordering of possible allocations (by ruling out inefficient allocations) and do not define a social optimum. For social orderings with variable population, the book by Blackorby, Bossert, and Donaldson (2005) provides a comprehensive overview. Decentralization of social optima is analyzed in Nerlove, Razin, and Sadka (1985, 1986), Fan and Stark (2008), and De La Croix and Gosskeys (2009).
in existence with utility equal to one would be preferred to an arbitrarily large population with average utility \(1 - \epsilon\), no matter how small \(\epsilon\) or how large the population.

As an alternative, consider total utilitarianism (Dasgupta 1969), where the objective is to maximize total utility in the economy. Under total utilitarianism, in our first example the second allocation is to be preferred; it always improves welfare to add people with positive utility. Implicitly, therefore, we are now assuming that the utility of not being born is equal to zero. But assigning a utility of zero to unborn people is just as arbitrary as any other real number.\(^2\) With total utilitarianism, setting the utility of the unborn is therefore a key step in defining the welfare criterion, but it is not clear how that should be done despite Sophocles’ claim that “Not to be born is, beyond all estimation, best.” (Jebb 1906)

The central difficulty is that the utility of not being born cannot be determined by introspection. When we formulate social welfare functions for a fixed population size, we only need to compare the welfare of people who all are alive. At an intuitive level, we carry out such comparisons by putting ourselves in someone else’s shoes, i.e., imaging what a given person’s life would be like and how we would evaluate that person’s welfare compared to another person’s, whose welfare we can equally evaluate through such introspection. In fact, we constantly carry out such comparisons regarding our own lives. When we face life decisions, we imagine what life would be like down either path from the fork in the road. After giving it thought, we decide which path we prefer, and what we would be willing to give up to get the one rather than the other. Thinking about other people’s lives in the same way is a small step to make.

When we aim to formulate a welfare function that allows for comparisons across different population sizes, the approach of putting ourselves in other people’s shoes no longer works. Evaluating welfare now involves evaluating how we feel about someone who is born in one allocation, but not in another. No one currently alive has had the experience of never being born. Is it, then, impossible to find

\(^2\)Total utilitarianism is part of a more general class of welfare criteria known as critical-level utilitarianism, see Blackorby, Bossert, and Donaldson (1995). In this criteria adding a person is worthwhile of their utility exceeds a critical level, which happens to be zero in total utilitarianism.
an intuitive criterion, based on common human experience, on which a plausible social welfare function can be based?3

We argue that, in fact, there exists a view of the world under which the utility of the unborn can be assessed even by people who are currently alive. The key is to find an interpretation where being alive and being unborn are not mutually exclusive, but merely different states which are experienced by one and the same being over time. Such a view of the world does exist and is in fact held by a sizeable fraction of the world’s population. What is required is a world with a fixed supply of souls, who get reincarnated from time to time in different human bodies. For such a soul, determining the optimal population level does not amount to a drastic “to be or not to be” question; instead, a smaller world population merely amounts to a longer wait to the next incarnation.

The major advantage of using the souls approach is that all the relevant tradeoffs can be assessed by introspection. For example, a soul, or equivalently a human believer in reincarnation contemplating her or his own future incarnations, should have no difficulty in forming a preference over the options of being born, say, every 10,000 years as a king, or every 100 years as a farmer. Making such comparisons also helps resolve another challenge in evaluating social welfare, namely to decide on the social discount rate with which to discount the utility of future generations. The social discount rate arises implicitly from the souls’ preferences over different combinations of rates of reincarnation and the associated life experiences.

Of course, the applicability of the souls approach would be limited if it only worked for those people who actually do believe in reincarnation (even though there are many such people4). We will argue, however, that our resulting criteria should also be plausible for others who do not expect their soul to live forever. We view our approach as analogous the “veil of ignorance” metaphor used in

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3This problem is echoed in the philosophical literature where Parfit (1984) (and later Arrhenius 2000) raised a challenge as to the very possibility of meaningfully answering the question of optimal population size.

4Mulgan (2002) argues that political philosophy should be neutral to religious beliefs. Much of the literature presupposes that humans are not reborn, yet belief in rebirth underlies Hindu and Buddhist traditions espoused by millions throughout the world. According to Mulgan, these traditions cannot be easily dismissed.
traditional social choice theory for fixed populations.\textsuperscript{5} Even in traditional welfare economics, we face the problem of comparing things without an direct basis for the comparison. Every human lives just a single life, so nobody has actual experience of what it is like to be another person. Nevertheless, we approach welfare evaluations by imagining, counterfactually, that we could be someone else. We imagine a situation behind a veil of ignorance where we do not know whose life we will get to live, and in fact have an equal probability of ending up as any given person. Of course, this could never happen in actual fact. Nevertheless, this metaphor is widely agreed upon as a basis for making welfare evaluations, on the basis that humans are fundamentally similar to each other and can therefore imagine quite well what it would be like to be in someone else’s shoes. Based on this imagination, standard social choice theory develops views of what would be a good world to live in. We take the same approach of imagining a situation which, even though it may not be a technically accurate description of the world, nevertheless provides a basis for evaluating social welfare across different population sizes on the basis of human introspection.

In the following section, we introduce the economic environment and define our welfare criterion. In Section 3, we point out the crucial role played by the discounting of future incarnations in soul-based welfare, and consider alternative settings in which discounting may depend on the probability of incarnation or on the utility derived in a given incarnation. In Section 4, we demonstrate the implications of soul-based welfare criteria for the income-population tradeoff in an economy with fixed resources, and argue that our preferred criteria have attractive properties compared to leading alternatives. In Section 5, we relate our results to the challenges arising from environmental degradation faced by humanity in the near future. Section 5 concludes.

\textsuperscript{5}The use of the veil of ignorance in welfare evaluations was formalized by John Harsanyi (e.g. Harsanyi 1953, Harsanyi 1955, and Harsanyi 1977) and was popularized by Rawls (1971).
2 General Framework for Soul-Based Welfare

2.1 Souls, Individuals, and Technology

We consider social welfare in a world populated by a fixed number of souls $S$. At each point in time, a soul can be embodied or disembodied. A soul can be embodied in only one person at a time, and each person needs a soul.

This fixed supply allows us to avoid the question of the optimal number of souls, which would amount to turn back to population ethical dilemmas. To avoid those dilemmas we also need that all souls are being incarnated at least once. Otherwise, a dilemma would occur as to which soul should be brought to life. We would then be back to problems about the value of non-existence, but at the level of incarnation.

We would like to develop criteria for ranking alternative allocations that differ in how many people are alive and how well-off these people are. There are $I$ types of individuals, where the types can reflect, for example, differences in innate abilities. We frame the problem social choice problem in discrete time, from period zero to infinity, where each period corresponds to a generation.

For ease of notation, we focus on stationary allocations, in which utility for a given type of individual is constant across generations and the population size is constant. This setting is sufficient to demonstrate the main tradeoffs in evaluating social welfare, and can be easily extended.

At each date, there are $2I$ of commodities, denoted as:

$$\{x_1, \ldots, x_I, x_{I+1}, \ldots, x_{2I}\}$$

For the perspective of a soul, the variables $x_i$ with $1 \leq i \leq I$ and $0 \leq x_i \leq 1$ denote the probability of being born in a given period as a type $i$ individual. The overall probability of incarnation in a given period $q$ is therefore given by

$$q = \sum_{i=1}^{I} x_i \leq 1.$$
From an aggregate perspective, \( x_i S \) is the number of souls embodied in that period as type-\( i \) individuals, and the overall population size is \( q S \).

For a given soul, the embodiment probability \( x_i \) does not depend on the past history of the soul. Hence, when an incarnation comes to the end with the death of an individual, the soul returns to the pool from which the next candidate for embodiment is randomly picked. The length of life of an individual is exogenous.\(^6\)

The vector \( x_{i+1} \ldots x_{2I} \) represents the lifetime utilities of individual of type \( i \) with \( 1 \leq i \leq I \). As is common in social choice theory, we employ the normalization that a utility level of zero reflects neutrality, i.e., any positive utility level is a life worth living from the individual perspective (see the discussion in Broome et al. 2004). Notice that a positive utility level (i.e., a life worth living) does not imply that is socially optimal (from the souls’ perspective) for the person in question to be alive, because of the tradeoffs involved when comparing allocations with more and fewer people. With souls, the utility of not being born is not given by the neutrality level, but rather the tradeoff is between a given incarnation and a longer wait for another incarnation with a possibly higher utility level.

Given our assumptions, the commodity space \( L \) is given by \( \{ x \in ([0, 1]^I \times \mathbb{R}^I) \mid \sum_{i=1}^I x_i \leq 1 \} \). The set of feasible commodity bundles is given by a set \( X \subset L \). The set \( X \) reflects the constraints imposed by technology. In particular, any tradeoffs between population size and utility levels implied by, for example, limited natural resources and other technological constraints are enshrined in the set \( X \).

### 2.2 Soul-based Social Evaluation Ordering

Our aim is to develop a social welfare function that imposes a welfare ordering on the set \( X \). We do this by identifying social welfare with the utility of a representative soul. Souls’ preferences are represented by a utility function \( u \) that maps \( X \) into \( \mathbb{R} \). This approach allows us to map out what specific assumptions on the shape of \( u \) imply for social welfare. The appeal of alternative assumptions on \( u \), in turn, can be evaluated using our moral intuition by putting ourselves into the shoes of a soul.

\(^6\)When it can be chosen, a new tradeoff appears between the length of life and the quality of life. This is the so-called spaceship problem, see Jouvet and Ponthière (2011).
We start with just two assumptions that have intuitive appeal.

**Assumption 1.** The function \( u : X \mapsto \mathbb{R} \) satisfies:

- Souls’ utility is increasing in an individual’s utility \( x_{i+1} \) if the probability of being embodied as that individual \( x_i \) is positive.

- If all possible lives are neutral, the soul’s utility is zero: If \( x_{i+1} = 0 \) \( \forall i \in [0, I] \), then \( u(x) = 0 \).

As a consequence of Assumption 1, if \( x_{i+1} < 0 \) for all \( i \) (all lives are not worth living), then \( u(x) < 0 \).

Our proposed social evaluation ordering \( R^S \) is utilitarian in souls, i.e. it uses the sum of soul utilities to make social comparisons. We label such ordering “soul based utilitarianism,” SBU in short.

**Definition 1 (Soul-Based Utilitarianism).** The SBU social ordering \( R^S \) is given by:

\[
\forall v, w \in \mathbb{R}^S, \quad vR^S w \iff \sum_{j=0}^S v_j \geq \sum_{j=0}^S w_j
\]

Given that the number of souls is given, we can apply the usual fixed-population principles and show that our social evaluation ordering satisfies continuity, weak and strong Pareto principles, and weak inequality aversion.\(^7\)

A social planner endowed with our social evaluation ordering would maximize \( \sum_{j=0}^S u(x_j) \) subject to the constraint \( x_j \in X \).

### 2.3 Implications for Individual Welfare

While we formulate our welfare criterion at the level of souls, the criterion implicitly defines a social ordering at the level of individuals. A key test for the desirability of soul-based utilitarianism is whether the implied welfare judgements at the individuals level have attractive properties. We therefore now demonstrate that SBU satisfies a number of standard social-choice axioms.

\(^7\)See Blackorby, Bossert, and Donaldson (2005) for formal definitions of these properties and others that are used below.
Proposition 1 (SBU satisfies the Weak Pareto Criterion). Consider two allocations $w$ and $v$ with the same probabilities: $w_i = v_i$ for all $i \leq I$. Under Assumption 1, if allocation $v$ Pareto dominates for allocation $w$ at the level of individuals, then $v$ is soul-preferred to $w$. Hence $R^S$ satisfies the weak Pareto principle at the individual level.

Proof: This follows from Assumption 1 where souls’ utility is increasing in individuals’ utility.

Proposition 2 (SBU satisfies Priority for Lives Worth Living). Allocations where every individual has positive utility (above neutrality) are soul-preferred to allocations where all individuals have negative utility (below neutrality).

Proof: This follows from Assumption 1 where we have imposed that, if all embodiments generate negative utility, then soul utility would be negative too.

We do satisfy this axiom, unlike the critical level approach of Blackorby, Bossert, and Donaldson (1995), because we do not subtract anything from utility in building the social welfare function.

3 Discounting and Social Welfare

To generate more specific implications for social welfare, we have to make more specific assumptions about how souls aggregate the utility derived from the various incarnations. When it comes to evaluating allocations with different population levels, souls face a tradeoff between the frequency of incarnation and the utility derived during each incarnation. To put structure on such tradeoffs, we introduce a notion of discounting.

One may ask whether souls should discount the future at all; wouldn’t it make more sense if an ethereal being such as a soul was indifferent to the time that passes between incarnations? In our view whether or not discounting is appropriate for souls should once again rely on introspection, that is, we should work out the implications of each case and then decide which one lines up with our moral intuitions. Consider the case of no discounting in the scenario considered here were time runs on forever and we consider stationary allocations where every
generation has the same utility. In this setup, no discounting for souls implies that souls aim to maximize the average utility of their incarnations, but the probability of incarnation does not matter. For example, consider one allocation where a soul is incarnated every year with a given utility, and another allocation were the soul is incarnated only every other year with the same utility. For a soul with no discounting, these allocations yield the same utility; the additional wait time between incarnations does not affect utility. So asking whether souls should discount is to ask whether average utilitarianism is the appropriate welfare criterion. Our own moral intuitions reject average utilitarianism, because we do not feel that an allocation with a minuscule population size but high utility is better than one with slightly lower utility but for a much larger population. This means that our moral intuition takes population size into account, and this requires that souls discount the future.

Discounting may depend on the probability of being incarnated in a given period.\textsuperscript{8} It seems plausible, for example, that souls would be primarily interested in the utility they receive at their next incarnation, with the waiting time to the next incarnation being a lesser concern. Such preferences can be modeled by allowing time-varying discounting, with different rates of discounting being applied between incarnated and non-incarnated periods. The exponential-discounting case is one extreme among many possible preferences, an extreme where all periods are discounted equally and the resulting criterion is total utilitarianism. At the other extreme would be a preference where the soul does not care about the waiting time at all, and simply discounts future incarnations at a constant rate, regardless of how much time passes between incarnations. As already mentioned, such a preference ordering amounts to average utilitarianism. In between these extremes are intermediate cases that display at least some concern for the waiting time. It is in this middle ground where we will find criteria that yield intuitively plausible solutions for a number of the problems and environments discussed in the literature.

\textsuperscript{8}By doing so we connect discounting with the size of the population, as in Boucekkine, Martínez, and Ruiz-Tamarit (2018), where impatience is inversely proportional to the number of offspring. Other examples where discounting depends on expected population size are Asheim and Zuber (2016) and Piacquadio (2020).
3.1 “Time Passes Quickly between Incarnations:” Soul-Based Utilitarianism with Embodiment-Dependent Discounting

We now introduce an assumption that expresses soul utility as a discounted sum of future incarnation utility, with the discount rate potentially depending on the probability of incarnation.

**Assumption 2 (Incarnation-Dependent Discounting).** The function \( u \) has the following form:

\[
u = \sum_{t=0}^{\infty} \prod_{j=0}^{t-1} d[q] \sum_{i=0}^{I} x_i x_{i+I} = \frac{1}{1 - d[q]} \sum_{i=0}^{I} x_i x_{i+I},
\]

where \( d \) is a continuous function which maps \([0, 1]\) onto \((0, 1)\).

The term \( \sum_{i=0}^{I} x_i x_{i+I} \) represents the total utility in a given period. We denote the expected utility conditional on being incarnated by \( \bar{x} \), where:

\[
\bar{x} = \frac{\sum_{i=0}^{I} x_i x_{i+I}}{\sum_{i=0}^{I} x_i} = q^{-1} \sum_{i=0}^{I} x_i x_{i+I}.
\]

Note that our general setup does not require that souls aggregate the utility derived from different potential incarnations as expected utility; we nevertheless focus on this case from here on for concreteness and because of the well-known attractive properties of expected utility.

The function \( d \) describes how patience depends on the probability of embodiment \( q \). Notice that this discounting scheme does not define time-inconsistent preferences, as in the case with hyperbolic discounting.

The simplest case is that of a constant discount rate.

**Proposition 3 (SBU nests Total Utilitarianism).** Under Assumption 2, if \( d[q] = \bar{d} \) for all \( q \in [0, 1] \), with \( 0 < \bar{d} < 1 \), SBU is equivalent to classical utilitarianism. In that case, the utility of not being born is equal to the neutrality level.

When \( d[q] = \bar{d} \) for all \( q \in [0, 1] \) souls discount the future at a constant rate, regardless of whether they are incarnated in a given period or not. With this kind of
preferences, not being alive is equivalent, in utility terms, to being alive and deriving zero utility. Soul-based utilitarianism therefore amounts to total utilitarianism, or equivalently, to critical-level utilitarianism with a critical level of zero.

**Proposition 4 (Existence of Critical Levels).** Under Assumption 2, soul based utilitarianism satisfies existence of critical levels.

Proof: Consider two allocations $v$ and $w$ with $v_i < w_i$ for one $i \leq I$ (more persons in allocation $w$), and $v_j = w_j$ for all $j \neq i, i + I$. Then there exists an individual utility level $w_{i+I}$ such that $vIw$.

At the other extreme, souls may discount the future only while they are embodied. This is as if, from the souls’ perspective, no time passes between embodiments. In their evaluation, they pass directly from incarnation to incarnation, regardless of the actual calendar time passing between incarnations—time passes quickly when you are not incarnated. In this case, our criterion amounts to average utilitarianism. We can model this case by setting $d[q] = 1 - \gamma q$, so that the soul does not discount at all when not incarnated ($q = 0$), and discounts proportionally more in periods where the probability of incarnation $q$ is positive.

**Proposition 5 (SBU nests Average Utilitarianism).** Under Assumption 2, $d[q] = 1 - \gamma q$ with $0 < \gamma < 1$ implies that soul based utilitarianism is equivalent to average utilitarianism.

Proof: We have

$$u = \sum_{t=0}^{\infty} \Pi_{j=0}^{t-1} (1 - \gamma q) \sum_{i=0}^{I} x_i x_{i+I} = \frac{1}{\gamma q} \sum_{i=0}^{I} x_i x_{i+I} = \frac{1}{\gamma} \bar{x}.$$ 

Let us investigate whether SBU with embodiment-dependent discounting avoids the repugnant conclusion. The original formulation of the repugnant conclusion corresponds to a stationary world with utility level $x_{i+I} = \bar{x} > 0$ for all $i$ and a fixed incarnation probability $q$. The repugnant conclusion holds if for any utility level $\epsilon$ such that $0 < \epsilon < \bar{x}$, there is a $\tilde{q} > q$ such that the allocation with
\(q, \epsilon\) is socially preferred to \(q, \bar{x}\). In this case, for an arbitrarily low utility level above neutrality there is always a sufficiently large population size such that this allocation is preferred over any alternative with positive utility for a fixed number of people. The repugnant conclusion arises when population size can always be used as a substitute for lifetime utility as long as lives are above neutrality. The conclusion is referred to as repugnant given that a world overcrowded with a huge population living lives barely worth living strikes many observers as being far from socially optimal.\(^9\)

In the case of our model, there is an upper limit to the size of the population given by \(S\). We thus face the constraint \(q \leq 1\). We can formulate a weaker version of the repugnant conclusion that takes the constraint into account. We say that a soul-based criterion satisfies the repugnant conclusion if for any \(\bar{x} > 0\), \(0 < q < 1\) and \(0 < \epsilon < \bar{x}\), either there exists a \(\tilde{q} > q\) such that \(\tilde{q}, \epsilon\) is socially preferred to \(q, \bar{x}\), or given \(\epsilon > 0\), social welfare is maximized at \(q = 1\). That is, we can do better by increasing the population, or in case we hit the population constraint, we require that hitting the constraint is locally optimal. The second part of this definition implies a sufficient condition for the repugnant conclusion:

**Condition 1** (Sufficient Condition for Repugnant Conclusion). *SBU with embodiment-dependent discounting satisfies the repugnant conclusion if, keeping lifetime utilities above neutrality at some level \(\bar{x} > 0\), social welfare is maximized by setting \(q = 1\).*

We can now examine what determines whether the condition is met.

**Proposition 6** (Avoidance of the Repugnant Conclusion). *SBU avoids Condition 1 if the the discounting function \(d\) satisfies:

\[
1 - d[1] + d'[1] < 0.
\]

\(^9\)A recent discussion of the repugnant conclusion in the context of the current covid-19 pandemic and the design of the optimal lockdown policy is due to Pestieau and Ponthière (2020). The authors argue that the repugnant conclusion would apply when “for any non-maximal lockdown saving lives at the cost of reducing average utility at a given period, there exists always a stricter lockdown, which further reduces average utility, but leads to a larger aggregate welfare.” The authors show that it would be the case if the survival function is sufficiently affected by tougher lockdowns. Our setting does not directly apply to this issue as what is at stakes is the length of life of people who already exist, which we would capture through utilities \(u_{i+1}, i\), rather than the existence or non-existence of additional people.
Proof: In a stationary world with \( x_i = \bar{x} > 0 \ \forall \ i > I \) and \( q = \sum_{i=0}^{I} x_i \) we have,

\[
u = \frac{q\bar{x}}{1 - d[q]}.
\]

The derivative of social welfare with respect to \( q \) is:

\[
\frac{\partial u}{\partial q} = \frac{\bar{x}(1 - d[q] + qd'[q])}{(1 - d[q])^2}
\]

We therefore have:

\[
\frac{\partial u}{\partial q} \bigg|_{q=1} < 0 \iff 1 - d[1] + d'[1] < 0.
\]

\( 1 - d[1] + d'[1] < 0 \) guarantees that souls utility decreases around \( q = 1 \) and hence does not reach a maximum at that point.

As we already saw, SBU is switched from total towards average utilitarianism (where the repugnant conclusion does not hold) if the discount factor is linearly decreasing in the probability of incarnation. Generalizing that point, the condition \( 1 - d[1] + d'[1] < 0 \) says that the impact of the incarnation probability on discounting at \( q = 1 \) has to be sufficiently strong to avoid the repugnant conclusion.

We can also examine whether SBU is compatible with other axioms used in the social choice literature. One example is utility independence, which requires that the ranking of any two alternatives does not depend on the utility levels of people who have the same utility in both alternatives (see Blackorby, Bossert, and Donaldson 2005 for details on this axiom and others discussed below). SBU does not satisfy this axiom. To see why, suppose that there is an individual who will be born in the distant future with a very bad life (i.e., negative utility) in both allocations we consider. In that case, the soul may prefer a world with two persons and zero utility in the present to a world with just one person with positive utility. Two embodiments rather than one will help the soul “forget” about the unhappy life in the distant future (in the sense that the future is more heavily discounted). Now remove the future individual with the unhappy life. Then the soul will prefer one person with positive utility to two persons with zero utility. Hence the ranking of the alternatives depends on the future existence of the person with
the unhappy life, which violates the axiom. The same argument can be used to show that SBU does not satisfy existence independence nor independence of the existence of the dead.\(^{10}\)

To be clear, there are no social choice criteria that satisfy all common axioms; evaluating which hold and which do not helps clarify which properties a given criterion prioritizes. The most widely used class of social welfare criteria that avoids the repugnant conclusion is critical level utilitarianism, comprising both number sensitive (Blackorby, Bossert, and Donaldson 2002) and regular versions. Compared to these criteria, SBU is able to satisfy jointly the priority to lives worth living (which is violated by critical-level criteria) and the avoidance of the repugnant conclusion. Like number-dampened examples of critical-level utilitarianism (see Ng (1986)), SBU does not satisfy the negative expansion principle. Hence, in terms of underlying axioms among existing criteria the one that is most similar to SBU is number-dampened utilitarianism.

### 3.2 “Time Passes Quickly When You Are Having Fun:” Soul-Based Utilitarianism with Utility-Dependent Discounting

In the previous section, it is the fact that a soul is embodied which leads her to discount the future more heavily. This extra discount does not depend on the quality of the life during that embodiment. One might also suppose that the extent of discounting depends on the utility derived from a given incarnation. One form of such a relationship captures the notion that “time passes quickly when you are having fun;” a soul would apply little discount to the future when embodied with high utility, implying that the future is perceived to arrive quickly. In contrast, the drag of an incarnation lived close to neutrality (i.e., zero utility) would make the future appear to move far away, meaning that the soul applies more discount to what is yet to come. We can capture this notion by making discounting depend on the average utility of being incarnated in a period.\(^{11}\)

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\(^{10}\)SBU also violates the negative expansion principle. According to this principle, if we add one person with a life not worth living, the social ordering should evaluate it as worse. It is not always the case that the new situation will be evaluated as worse by SBU. The intuition is similar to the one developed above. Having a mildly bad life now may lead you to discount a very bad future life more heavily, leading to an improvement in welfare.

\(^{11}\)In doing so, we connect to a literature in which discounting is made dependent on utility
Assumption 3 (Utility-Dependent Discounting). The function $u$ has the following form:

$$u = \sum_{t=0}^{\infty} \prod_{j=0}^{t-1} d[q, \bar{x}] \sum_{i=0}^{I} x_i x_{i+1}.$$ 

Here $d[q, \bar{x}]$ is a continuous function which maps $[0, 1] \times \mathbb{R}$ onto $(0, 1)$.

We can now ask under what conditions on the shape of $d$ the SBU criterion will satisfy avoidance of the repugnant conclusion. Intuitively, what is required is that when utility levels are low, adding more people (i.e., increasing the probability of incarnation) does not increase the utility of the soul. In terms of discounting, this means that at low utility levels a higher probability of incarnation increases discounting in a way that leaves the soul’s utility constant or decreasing. In contrast, at high individual utility levels, we can allow for the soul’s utility to increase if the probability of incarnation goes up. A simple way of doing this is by combing the two cases considered above and weighing them with average utility:

$$d[q, \bar{x}] = \bar{x} d + \frac{1}{1 + \bar{x}} (1 - \gamma q) \quad (1)$$

with $0 < \bar{d} < 1$, $0 < \gamma < 1$, and where $\bar{x} = \max\{0, \bar{x}\}$.

Hence, for individual utility levels $\bar{x}$ close to zero or negative, the welfare criterion approaches the case of average utilitarianism considered above, and the repugnant conclusion will be avoided. However, for higher utility levels the criterion approaches total utilitarianism—once lives are not just barely worth living but truly enjoyable, the soul sees value in being incarnated more often, or equivalently, population size being higher. Thus, a criterion of this kind can avoid the unattractive properties of both average and total utilitarianism, and does so in a way that ties evaluation of social welfare to discounting in a way that has intuitive appeal.

Notice that the resulting criterion indeed lines up with the saying that “time (Uzawa 1968), consumption (Druegon 1996), or wages (Nourry, Sergeeva, and Venditti 2011). This literature focuses mostly on the dynamic properties of macroeconomic models with endogenous discounting, while here we investigate the implication of utility-dependent discounting across generations for optimal population levels.
passes quickly when you are having fun,” during low-utility, no-fun incarnations the future appears to be far away, which is captured through higher discounting.

In addition to avoidance of the repugnant conclusion, the resulting criterion also satisfies priority to lives worth living. Another criterion worth consideration is the negative expansion principle, which requires that adding a person with utility below neutrality (a life not worth living) to an otherwise unaffected population lowers welfare. Adding such a life clearly lowers the utility component of the soul’s utility, but its impact through discounting is ambiguous. To guarantee that the negative expansion principle holds, we can modify the discount function such that it only takes into account lives above neutrality, such as:

\[
d[q, \bar{x}] = \frac{\bar{x}}{1 + \bar{x}} \tilde{d} + \frac{1}{1 + \bar{x}} (1 - \gamma q)
\]

with \(0 < \tilde{d} < 1, 0 < \gamma < 1\), and where:

\[
\tilde{x} = \frac{\sum_{i=0}^{I} x_i \max\{0, x_{i+1}\}}{\sum_{i=0}^{I} x_i I\{x_{i+1} > 0\}}.
\]

The resulting soul utility is still increasing in \(\bar{x}\), and all the requirement of Assumptions 1 are met. Hence we still have weak Pareto and existence of critical levels. Hence, in this version of the SBU, the properties of the social welfare function are similar to restricted number-dampened utilitarianism, in the sense that the same social-choice axioms are satisfied. The advantage of our approach is to give a precise interpretation to the number-dampening assumption: being incarnated with utility close to neutrality leads the soul to discount the future more.

4 The Tradeoff between Individual Utility and Population Size under Different Social Welfare Criteria

So far, we have not put any specific assumptions on the set \(X\) of feasible commodity bundles. The essence of the optimal population problem is that this set reflects a tradeoff between population size and individual welfare, for example because of a limited supply of natural resources. We now illustrate our approach with a specific example of such a tradeoff.
4.1 Setting

Consider a stationary world with one type of individuals \( I = 1 \) and one consumption good \( c \). Total population is denoted \( P \). The probability of being embodied in a given period is thus \( q = P/S \). The utility of an individual is given by:

\[
\bar{x} = x_{I+1} = U(c),
\]

where \( U(c) \) is individual utility defined over consumption \( c \). We choose an iso-elastic utility function:

\[
U(c) = c^{1-\sigma}/(1 - \sigma) \text{ with } \sigma \in (0, 1). \tag{2}
\]

Given utility, any life with positive consumption is worth living. For a soul’s utility, we focus on the case where discounting may depend on the probability of incarnation \( q \):

\[
u = \sum_{t=0}^{\infty} \prod_{j=0}^{t-1} d[q] \sum_{i=0}^{I} x_i x_{i+I} = \frac{1}{1 - d[q]} \sum_{i=0}^{I} x_i x_{i+I}.
\]

The welfare \( u \) of a soul is then:

\[
u = \sum_{t=0}^{\infty} \prod_{j=0}^{t-1} d[q] q\bar{x} = \frac{q}{1 - d[q]} U(c) = \frac{P/S}{1 - d[P/S]} U(c).
\]

Consumption \( c \) is constrained by the Cobb-Douglas function \( Y = AL^\nu P^{1-\nu} \), where \( A \) is total factor productivity, \( L \) is a fixed factor such as land, and \( \nu \in (0, 1) \). Consumption per person is thus given by

\[
c = Y/P = AL^\nu P^{-\nu}.
\]

We now would like to determine the optimal size of population according to SBU. The answer will depend on the assumed discount function \( d[P/S] \).
4.2 Total Utilitarianism

If the discount function is a constant \( d[P/S] = \bar{d} \), the social welfare function is given by

\[
V = \frac{P/S}{(1 - \bar{d})(1 - \sigma)} (AL^{\nu}P^{\nu-\nu})^{1-\sigma}.
\]

This function is the same as the one we would have obtained assuming total utilitarianism (up to a constant). It is monotonically increasing in population \( P \), and hence the optimal \( P \) is infinite (or, more precisely, given by \( S \) if we enforce the constraint that the population at any time cannot exceed the supply of souls). The resulting consumption per person would tend towards zero. In other words, what we obtain is the repugnant conclusion: the socially optimal state is one where a huge population lives in misery with lives barely worth living.

To be clear, obtaining impoverishment of this kind is not necessary implication of total utilitarianism. For example, if we had chosen a utility function where there is a positive consumption level below which lives are not worth living (i.e., utility turns negative), such as:

\[
U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma},
\]

the optimal population would remain finite and individual utility would be strictly above the level of neutrality.

Nevertheless, the fact that the repugnant conclusion can arise at all is an argument against this welfare criterion if one agrees that the repugnant conclusion should be avoided.

4.3 Average Utilitarianism

As the next alternative, we consider the discount function is \( d[P/S] = 1 - \gamma P/S \) with \( 0 < \gamma < 1 \). The social welfare function becomes:

\[
V = \frac{1}{\gamma(1 - \sigma)} (AL^{\nu}P^{\nu-\nu})^{1-\sigma}
\]

This function is equivalent to the one obtained under average utilitarianism. It is monotonically decreasing in \( P \), and the optimal \( P \) is 0. The social optimum here is
one where all the world’s resources are used to maximize the wellbeing of a single individual in each generation. Again, this is an extreme result (at the opposite end of the scale from total utilitarianism) that goes against the moral intuition of many. The result is also robust to many perturbations of utility; all that is required is that utility is increasing in consumption throughout and does not depend on further arguments. One could obtain a positive optimal population level by introducing a bliss point in consumption or social externalities, i.e., people enjoy having some other people around. Even then, there remains the robust implication that this welfare criterion does not assign value to adding people if adding those people does not increase average utility. Consider the case, for example, where adding one person would actually make all existing people slightly better off, and the added person would obtain a happy life with utility far above neutrality. Even so, if this utility is sufficiently lower than that of the existing people that it draws down the average, this welfare criterion would prefer that person not to be born.

4.4 Soul Utilitarianism with Embodiment-Dependent Discounting

We now turn to a welfare function that represents the idea that souls will discount the future more heavily when embodied. Suppose that the function \( d[P/S] \) is given by:

\[
d[P/S] = \beta - \gamma(P/S)
\]

where \( \beta \) is the baseline discount factor and \( \gamma \) is the additional discount factor when embodied. Maximizing the souls’ welfare:

\[
u = \frac{P/S}{(1 - \beta + \gamma P/S)(1 - \sigma)} \left( AL'' P^{-\nu} \right)^{1-\sigma}
\]

yields the following optimal population level:

\[
P = S \frac{(1 - \beta)(1 - \nu(1 - \sigma))}{(1 - \sigma)\gamma \nu}.
\]

Note that this example nests the two extremes already discussed earlier:

- \( \beta = 1 \) and \( \gamma < 1 \) yields average utilitarianism, with an optimal population of zero \( P = 0 \) (i.e., a single infinitesimal being consumes all resources).
• \( \beta < 1 \) and \( \gamma = 0 \) yields total utilitarianism, with an infinite optimal population size, respectively \( P = S \) taking the limited supply of souls into account.

If we have both \( \beta < 1 \) and \( \gamma > 0 \), this SBU criterion delivers a well defined optimal population between the two extremes of average and total utilitarianism.

An arguably unattractive property of the social optimum derived here is that the optimal population level is independent of productivity. The optimal population level derives entirely from the curvature in discounting and the curvature in consumption (given technological constraints), the level of utility is irrelevant. Hence, according to this criterion if productivity increases over time, the gains should be absorbed entirely in higher individual consumption and utility, without an increase in population size.

Figure 1 illustrates this case by plotting optimal population and individual utility as a function of productivity \( A \). The optimal population level is constant at 300, and different productivity levels translate one-for-one into individuals’ utility.

The condition for avoiding the repugnant conclusion \( 1 - d[1] + d'[1] < 0 \) for this case writes as \( 1 - \beta + \gamma - \gamma = 1 - \beta < 0 \). This condition is not satisfied given our assumptions, so that we do not necessarily avoid the repugnant conclusion, in the specific sense of our setting that the optimal population level my exceed the number of souls \( S \). Other functional forms for the impact of embodiment and discounting can avoid the repugnant conclusion. For example, suppose that the function \( d[P/S] \) is given by:

\[
d[P/S] = \beta - \gamma(P/S)^2.
\]

The condition \( 1 - d[1] + d'[1] < 0 \) now writes as \( 1 - \beta + \gamma - 2\gamma = 1 - \beta - \gamma < 0 \). In this example, we can avoid the repugnant conclusion if the parameters \( \beta \) and \( \gamma \) are small enough. The basic properties of the welfare criterion are unchanged with this functional form. The first order condition to the maximization problem

\[12\]

The parameter values are \( L = 1, \sigma = \nu = 0.5, \beta = 0.99, \gamma = 0.1, S = 1000. \]
leads to the following solution:

\[ P = S \sqrt{\frac{(1 - \beta)(1 - \nu(1 - \sigma))}{\gamma(1 + \nu(1 - \sigma))}}. \]

Thus, there is still a definite optimal population level that is independent of productivity, and productivity gains will still be reflected entirely in individuals’ utility.

4.5 **Soul Utilitarianism with Utility-Dependent Discounting**

In our view, the implication of the SBU criteria above that the optimal population level is independent of productivity is unattractive. For example, if productivity falls to a level where given this population level utility is barely above neutrality,
wouldn’t it make sense to reduce the population size to give those people living a more enjoyable experience?

As discussed in Section 3.2, SBU can allow for tradeoffs of this kind if discounting is affected by the utility derived from embodiments. Consider the criterion discussed in Section 3.2 where the discount function is given by (1):

\[ d[q, \tilde{x}] = \frac{\tilde{x}}{1 + \tilde{x}} \tilde{d} + \frac{1}{1 + \tilde{x}} (1 - \gamma q). \]

For low individual utility levels (here \( \tilde{x} \)), this criterion tends towards average utilitarianism, and hence aims to maintain individual utility high even at the cost of reducing population. For high levels of individual welfare, the criterion moves...
in the direction of total utilitarianism, and places more value on the presence of additional people with happy lives.

This utility-dependent SBU criterion does not lead to a closed-form solution for the optimal population level, but optima are straightforward to compute. Figure 2 illustrates the social optima under this criterion for different productivity levels.\textsuperscript{13}

The figure shows that both optimal population and individuals’ utility vary smoothly with productivity. At low productivity levels population size is small, with utility still well above neutrality. Productivity increases are reflected both in a larger population size and in higher individual welfare. Qualitatively, this kind of outcome is what accords best with our own moral intuitions. The SBU criterion with utility-dependent discounting clarifies what preferring this outcome implies for the underlying assumptions on how to discount the experiences of future generations.

One issue to be resolved with SBU criteria is that the socially optimal allocation generally depends on the number of souls $S$, a parameter that does not have a real-world equivalent. One approach would be to set $S$ to the maximum possible population size, i.e., the “carrying capacity” of the planet. In our view, a better approach would be to treat the concept of souls less literally, and consider $S$ as a free parameter that indexes alternative SBU-optimal allocations. That is, for a given productivity level (such as the one prevailing at the current moment in time), one should consider the set of alternative combinations of population size and income per capita, and choose the one that best accords with one’s own social preferences. This will imply a specific $S$. The SBU criterion then shows how, given this $S$, optimal population should evolve as productivity changes over time.

### 4.6 Critical Level Utilitarianism

As an example of a non-SBU criterion, we can compare the optimal population level to the one prescribed by critical level utilitarianism. In that case, the social

\textsuperscript{13}The parameter values are as for Figure 1, i.e., $L = 1, \sigma = \nu = 0.5, S = 1000$. The additional parameter $d$ is set to $\bar{d} = 0.99$. 


welfare function to be maximized is:

\[ V^{CU} = P \left( (AL^\nu P^{-\nu})^{1-\sigma} - \omega \right) \]

with \( \omega > 0 \) being the critical level. Here it is as if the utility of the unborn is \( \omega \). The attraction of critical-level utilitarianism is that it avoids the extremes of both average and total utilitarianism, and the assumption of a critical level makes the underlying welfare judgement entirely transparent.

The optimal population is under critical level utilitarianism is:

\[ P = A^{\frac{1}{\nu}} L \left( \frac{1 - \nu(1 - \sigma)}{\omega(1 - \sigma)} \right)^{\frac{1}{\nu(1 - \sigma)}} \]

Hence, optimal population increases with productivity. The resulting individual utility is:

\[ \bar{x} = U(c) = \frac{\omega(1 - \sigma)}{1 - \nu(1 - \sigma)} \]

Notice that individual utility is constant in that it does not depend on productivity \( A \) or land \( L \).

Figure 3 illustrates the social optima under critical-level utilitarianism for the same setting considered above.\textsuperscript{14} We see that this welfare criterion implies an optimal level of individual consumption that is met regardless of total resources; any increase in resources is therefore fully absorbed by an increase in population, the typical characteristic of a Malthusian economy. Under this criterion and given the constraints of the economic environment considered, a sustained rise in living standards over generations, as experienced around much of the world in the past two centuries, cannot be socially optimal.

Thus, while critical-level utilitarianism avoids the extremes of average and total utilitarianism, it still yields a result that increases in productivity should not result in any improvements in living standards in the social optimum, which goes against our own moral intuition.

\textsuperscript{14}The parameter values are as for Figure 1, i.e., \( L = 1, \sigma = \nu = 0.5, \gamma = 0.1 \). The critical-level parameter \( \omega \) is set to \( \omega = 1.1 \).
Comparing critical-level utilitarianism with SBU under utility-dependent discounting, another distinction is that critical level utilitarianism has difficulty with settings where the critical level may be unattainable (due to low productivity). Critical-level utilitarianism then would imply that the optimal population size should be zero, even though some people with utility above neutrality (but below the critical level) could exist. Under SBU under utility-dependent discounting this issue does not arise, as allocations with some people above neutrality are always preferred to those with no people at all.

5 Population and Environmental Collapse

Population ethics is not just an abstract philosophical issue, but is at the center of major challenges facing humanity in the near future. The United Nations (2017)
estimate that the world population will approach 10 billion by 2050, and then peak around the beginning of the new century at about 12 billion. In times of global warming and other forms of environmental degradation, many people worry that such population growth will eventually cause environmental catastrophes that will drastically reduce living standards and may even put survival at of humanity at risk. Does the SBU welfare criterion have anything to offer to analyze this issue?

At one extreme of the debate is the idea of optimal extinction. In the normative literature on the optimal size of population, it is possible that the optimal turns out to be the corner solution of zero population. We saw examples of that with average utilitarianism, where the optimum often is an infinitesimal population that consumes all resources. Such solutions are often discarded as being uninteresting and irrelevant. However, as soon as we consider a dynamic problem, such solutions could become interesting again, since they reflect the possibility, and even the optimality, of extinction. We can at least imagine a situation where, confronted with a limited resource, a society prefers to take a maximum pleasure today and then become extinct, rather than living at a low standard for an infinite time.\(^\text{15}\) The soul-based approach to population ethics does not offer a definite view on optimal extinction. Indeed, soul-based utilitarianism aims to avoid issues of pondering the value of non-existence by instead framing the problem in terms of the frequency of incarnations. In contrast, extinction of humanity would also imply extinctions of the souls, which puts us right back at the imponderable question of the value of nonexistence. Our own moral intuition places high value on continued human existence; a theory that does not speak specifically to optimal extinction is therefore just fine with us.

Even when extinction is avoided, environmental policy still involves tradeoffs between immediate costs and potential benefits decades or even centuries in the future, accruing primarily to future generations. When evaluating such tradeoffs, a key ethical parameter is the social discount rate (see Dasgupta 2008 for an overview). Some authors, most recently Eden (2020), provide arguments for using the market discount rate as the social discount rate,\(^\text{16}\) while many others argue in

\(^{15}\)This is the program of The Voluntary Human Extinction Movement to be seen on http://www.vhemt.org/. One can even buy a sticker “thank you for not breeding”.

\(^{16}\)Giglio, Maggiori, and Stroebel (2014) provide evidence that market discount rates over very
favor of a lower social discount rate compared to market rates (e.g., Caplin and Leahy 2004 and Farhi and Werning 2007). Ramsey (1928) argued in favor of no discounting at all, and provides a method to solve for the optimal allocation in such circumstances (see De La Croix and Michel (2002), p. 91–92).

Soul-based utilitarianism does not provide a specific answer for exactly what the social discount rate should be, but offers a new way for how to think about the underlying tradeoffs. Most importantly, one cannot infer an adequate value for social discounting from the market discount rate applied by individuals, because discounting within and across incarnations are distinct concepts. If one agrees with the notion that time between incarnations is perceived to pass quickly, the social discount rate (applied across generations) should be low. Moreover, we argue that the ethical implications of utility-dependent discounting have special appeal. Agreeing with this implies that the social discount rate is not a constant, but dependent on the living standards of current and future generations. Under such a notion, preserving the individual well-being of future generations gains in urgency if this well-being is at risk, i.e., utilities decline towards neutrality. Such a notion would justify applying low social discount rates especially when the possibility of an environmental collapse with severe repercussions for human wellbeing is at stake.

6 Conclusion

A fundamental challenge for welfare economics is to develop welfare criteria that can be applied to environments with variable population. Such welfare criteria inherently require valuing nonexistent people. So far, intuitive methods for how to do this have been elusive.

We consider the population problem in an environment where population is variable, but there is a fixed supply of souls, who may experience multiple incarnations over time. Rather than pondering the value of nonexistence, from the souls’ perspective comparing larger or smaller populations merely involves valuing shorter or longer waits until the next incarnation. We argue that such comparisons are possible on the basis of introspection. long horizons are fairly low.
While the existence of souls is metaphysical, the same can be argued when one talks about lives in regular population ethics. After all, only moments are experienced by humans at any given time, and collecting moments to produce a life can already be regarded as a metaphysical step that goes beyond directly lived experience. A soul is a collection of lives just as a life is a collection of moments. Both exist as a mental operation of an individual living purely in the moment, but able to ponder outcomes and tradeoffs over a wider timescale. In this sense, the soul is a tool for reframing the population question just as a life and related concepts such as expected future utility are tools to help us think about intertemporal choices at a smaller time scale.

Reincarnation therefore serves as a metaphor to facilitate the necessary utility comparisons, but one does not have to believe in reincarnation to accept the resulting welfare principle. The soul takes into account the utility of all her future lives. It is as if a person would feel altruistic towards future people. This view is what the literature calls generalized altruism, going back to the notions of love of mankind or universal benevolence, implying that we desire the happiness of all innocent and sensible beings, embracing the immensity of the universe, present and future (see Smith 1776).

We argue in this paper that taking this approach yields criteria that have appealing properties for dealing with the basic population-income tradeoff of the population problem. Like all social welfare criteria, SBU has its own shortcomings, and finding consensus on how souls should evaluate welfare among different groups of people may be challenging. Yet, we believe that SBU can be a useful tool for formulating and defending acceptable social welfare criteria with variable population levels.

**References**


