THE LIMITED POWER OF SOCIOECONOMIC STATUS TO PREDICT LONGEVITY: IMPLICATIONS FOR PENSION POLICY

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The Limited Power of Socioeconomic Status to Predict Longevity: Implications for Pension Policy*

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Abstract

Differences of life expectancy across socioeconomic status are well-documented and many economists argue that they should be taken into account when designing pension systems. This paper analyses the relevance of using socioeconomic characteristics to differentiate retirement age. Using US mortality rate assembled by Chetty et al. (2016), we simulate the longevity distribution both across and within socioeconomic status. Then, we analyze the power of socioeconomic status to predict individuals' longevity. Results suggest that socioeconomic status has relatively limited predictive power, due to the huge within status longevity variance.

Keywords:
Pension policy, Pension progressivity, Longevity, Tagging
JEL: D63, H55, J14, J18

1. Introduction

It is now well established that life expectancy differs across socioeconomic categories, with low-educated or low-income people living, on average, significantly shorter live than their better-endowed and wealthier peers. This issue has recently received considerable critical attention for its impact on the pension system (OECD, 2018). Many analysts question the fairness of policies that uniformly raise the retirement age, neglecting the importance of life expectancy differences between workers. The General Secretary of the Trades Union Congress

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in the UK (Brendan Barber) said: "We remain opposed to helping pay for more generous state pensions by increasing the state pension age. This means that the poor and those with stressful jobs will end up paying for better pensions of the better off with longer life expectancies" (Whitehouse and Zaidi, 2008, p. 8) and Piketty (2019) criticized the recent French pension reform proposal for "taking no account of social inequalities in life expectancy". Indeed, if people at a higher level in the income distribution tend to live longer, an implicit and unwitting redistribution will be done in their favor, due to the longer duration of perceived retirement benefits. This issue has been thoroughly investigated in the economic literature, which recommends to consider retirement age according to the socioeconomic status. However, a limitation of such policy is that it focuses on differences in life expectancy across social groups but it ignores the differences in longevity within social groups. While the life expectancy is the number of years someone can expect to live, his longevity is the number of years he actually lives.

This paper analyzes the importance of the longevity differences across and within socioeconomic status. First, using the US mortality rates assembled by Chetty et al. (2016), we simulate the longevity distribution both across and within each socioeconomic group. We do so to analyze the relevance of the heterogeneity within each socioeconomic category in addition to the one across. Our longevity distributions start at age 40 because Chetty et al. (2016) provide mortality rates from that age. For convenience, we use the term "population" to describe our simulations starting at age 40. Second, we propose that the social planner's objective is to consider a retirement age proportional to an individual's longevity. Let us assume that an individual should retire having lived 80 % of his life. This figure approximately matches reality (retirement age around 65 and life expectancy around 80), but any other percentage would not have changed the argumentation. If the social planner could only set one retirement age, he would use the average longevity and set the (unique) retirement age at 68. However, only 78 % of the 10^{th} income percentile population would still be alive at that age. This is problematic. Ideally, the social planner should be able to differentiate the retirement age of every individual. Nevertheless, this solution is clearly not realistic. First, it requires to know *in advance* the longevity of each individual. Second, this solution would be very costly to implement. As a consequence, our social planner will "tag" individuals to infer their ages at death. Tagging consists of using a characteristic to differentiate a policy among individuals (see the seminal paper of Akerlof, 1978). Longevity is a function of different factors such as income, sex, lifestyle and genes (see e.g. Passarino et al., 2016). We assume that our social planner will only use sex, income and/or US state of residence to tag individuals for his retirement policy. Data on sex, income and state are easier to come by while data on genes are less available.[1] Those criteria are also chosen with regard to the pension literature, which concentrates on socioeconomic characteristics. Let us take a simple example by considering tagging only by income. If the social planner could differentiate the retirement age by income percentile, he would set it at 65 for the 10^{th} percentile and at 73 for the 90^{th}. However, there exists a huge variance within each percentile. Compared to a unique retirement age of 68, there is an increase of 5 % of the individuals in the 10^{th} percentile who are able to retire; but, there is also a decrease of 5 % of those in the 90^{th} who are able to do so. Therefore, the problem of having a retirement age set at 80 % of the individual's life is not solved. Figure 1 illustrates this problem graphically. The main contribution of this paper is to show that differentiating the retirement age by socioeconomic status is far from perfect because it still leaves a huge heterogeneity of longevity within each category. To the best of our knowledge, there is no empirical paper addressing the problem of within group longevity heterogeneity in pension policy and, with the noteworthy exception of Pestieau and Racionero (2016), not much work either in the theoretical literature.

This paper proceeds as follows. Section 2 provides a review of the empirical and theoretical literature. In Section 3, we construct a "gap index", measuring the relevance of the tag(s) used by the social planner. More secifically, it computes the distance between the proportional retirement age of an individual (e.g. 80 % of his longevity) and the retirement age set by the social planner with the help of his tag(s). Section 4 explains our simulations. Section 5 presents our results. A key finding is that differentiating the retirement age by state, sex and income quartile reduces the gap index by only 5 %; primarily due to enormous longevity variance within each socioeconomic group. This finding is robust to several specifications of the index, which suggests that "tagging" has limited power for pension policy. In Section 6, we discuss the underlying normative principles behind our paper and then, conclude.

2. Literature review

A considerable amount of literature has been published on a social gradient in life expectancy. In particular, various studies have shown it, as well as its evolution, in the case of the United States. Meara et al. (2008) found an increase in the gap of life expectancy between low- and high-educated white men, aged 25, from 6.2 to 7.8 years between 1990 and 2000. Using a more granular approach, Olshansky et al. (2012) reported that, in 2008, Black Americans with less than 12 years of education have a life expectancy at birth of 14.2 years lower compared to White Americans with 16 or more years of education. Recently, Chetty et al. (2016) reassessed the life expectancy gap and showed that a man at the top

[1] Moreover, we may wonder about the pros and cons of having a state using genes in its policy. This is an highly ethical debate that we do not address (and need to address) in this paper.

income quartile has gained, on average, 0.2 years of life expectancy (at 40) each year between 2001 and 2014, whereas the gain for a man at the bottom quartile was only, on average, 0.08 years. According to their results, there is a life expectancy gap of 14.6 years for men and 10.1 years for women between the top 1 % and the bottom 1 % of the income distribution. Data from other countries confirm a life expectancy differential across socioeconomic groups over the world; see e.g. Huisman et al. (2004) for Europe.

The existence of a socioeconomic gradient has led economists to investigate how this impacts on the progressivity of the pension system. Overall evidence is that it makes the system less progressive. Bommier et al. (2011) have shown that the progressivity of the French pension system is reduced by one quarter to one half when taking into account the social gradient. Whitehouse and Zaidi (2008) performed the same analysis for several countries in Europe with an impact ranging from staying progressive (in e.g. Norway), becoming neutral (in e.g. Germany) or even becoming regressive (in e.g. Poland). Contrary to the previous study, Haan et al. (2019) suggested that the German pension system becomes regressive, using data from cohorts born between 1926 and 1949. There is a lack of consensus on the impact on the progressivity and this is explained by the differences in assumptions, methods and cohorts being analysed.

Policy recommendations from the empirical literature have proposed either that the retirement age should vary by socioeconomic status (see e.g. Ayuso et al., 2016), or that the retirement benefit should be linked to the remaining life expectancy (Breyer and Hupfeld, 2010). Although it was not clearly identified, either approaches follow the "tagging" principle introduced by Akerlof (1978). According to Akerlof (1978), policy should be based on "tag", which help to identify the needy. However, tagging is far from being perfect as people could end up missclassified (e.g. a rich individual tagged as poor). For example, the state could tag lone-parent as more needy; although some lone-parent are wealthy. Parsons (1996) discussed the design of social security system with imperfect tagging (i.e. when people could be missclassified). He proposed to create a scheme such that people have an incentive to self-reveal their type if they are mistagged. Another way is an individual screening; however, it has been shown that this latter could lead to strong errors (see e.g. Benitez-Silva et al., 2004, who showed that 20 % of US disability recipients are missclassified). In our context, individuals cannot know in advance their longevity so this rules out the use of a self-revealed approach. Therefore, policy makers are left to find a tag which provides relevant information about longevity. This problem also relates to the one of "statistical discrimination" developed by Phelps (1972). He explained in his seminal paper that a decision maker could use observable characteristics (e.g. education) to infer unobservable ones (e.g. ability). However, by doing so, the decision maker discriminates "statistically" people with high ability who are belonging to a less-educated group. The same problem arises with tagging in public policy, where the state could discriminate an individual by missclassifying it.

In the recent years, the longevity differential has attracted attention in the theoretical literature (see e.g. Leroux et al., 2015, for a survey). The main emphasis has been put on the optimal policy to compensate the short-lived individuals. The usual conclusion is to differentiate the retirement age between short- and long-lived individuals. However, this literature does not consider the use of a proxy (such as income) to identify the type of an individual; but considers that the longevity is either perfectly known or a private information or a random variable. To the best of our knowledge, there is only one theoretical paper dealing with this issue. Pestieau and Racionero (2016) considered a framework with two jobs (a harsh and a soft job) with a mix of short- and long-lived individuals working in those jobs (they assume that the harsh job has a higher proportion of short-lived individuals). They showed the shortcomings of establishing the retirement age based on the profession as a proxy for longevity. They considered different frameworks and showed that heterogeneity in longevity within each profession makes always impossible to equalize utility between individuals. For example, a maximin criterion on the utility of the short-lived individuals in both occupations leads to differences in utility between the long-lived individuals in the soft and harsh professions. At the end, they conclude that a pension system "should be sufficiently flexible to separate the lucky (in terms of life expectancy) individuals from the unlucky ones within each occupation" (Pestieau and Racionero, 2016, p. 201).

Although extensive research has been carried out on the impact of the socioeconomic gradient on the pension system progressivity, no single study has analyzed *in details* the possibility of a policy solution. Empirical literature has proposed to differentiate the retirement age according to life expectancy by socioeconomic status, but failed to notice the relevance of the longevity distribution. In this paper, we analyze in depth the relevance of tagging by socioeconomic status, when taking into account the longevity distribution. Our paper could be seen as the first empirical investigation of the problem raised by Pestieau and Racionero (2016).

3. Gap index

We assess the relevance of using socioeconomic characteristics to differentiate the retirement age. To do so, we construct a gap index consisting of the sum of deviations of each individual between his proportional retirement age and the one of the social planner. The proportional retirement age is a given percentage of the individual's longevity and the one of the social planner could differ according to the socioeconomic group to which the individual belongs. We use the terms "gap" and "deviations" interchangeably. If the tags were a perfect predictor of the individual's longevity, the social planner would be able to differentiate perfectly the retirement age and the gaps would not exist.

However, the achievement of this goal is clearly utopic due to the longevity distribution within each socioeconomic status. The longevity distribution will always generate some unwitting deviations between the retirement age set by the social planner and the proportional ones. The economic literature has identified regressive transfers inside pension system. Those transfers are due to the longer duration of perceived retirement benefits. Poor individuals die earlier than richer ones, making the system regressive/less progressive. It is important to notice that if the social planner would know the longevity of each individual (such that he would make no deviations), he would be able to avoid transfer from a poor individual to a richer one. The magnitude of those transfers depends on many of the features of the pension system; but, they fundamentally stem from the social planner's ex ante ignorance of the individuals' longevity. This ignorance is represented by the deviations between the proportional retirement age and the one(s) set by the social planner.

We assume that society is composed of a set of agents with exogenous and heterogeneous longevity depending on US state of residence, sex and income percentile. The social planner is egalitarian, so he would like everyone to spend the same share of his life in retirement. Consequently, the retirement age policy is, for each individual, to retire at α times his longevity.

Definition 1. The retirement age policy is, for each individual i, to retire at α times his longevity.

$$Retirement\, age_i = \alpha \times longevity_i$$

This retirement age policy follows the recommendation of the European Commission (2012) to link, at the national level, the retirement age to the life expectancy and echoes the current concerns, raised e.g. by Piketty (2019), that pension reforms unilaterally raising the retirement age are unfair because they ignore the social gradient in life expectancy. Some theoretical economic models have also shown that the retirement age should be a constant proportion of the length of life (see e.g. Bloom et al., 2007). Some readers could be puzzled by the retirement age policy proposed. Why should the retirement age necessarily be proportional? Our answer, at this point in this paper, is that the proportional retirement age is more convenient to present our results. We could adopt other policies (and we will discuss it in Section 6); nevertheless, it appears useful to use a simpler context to exhibit our approach at first. Let us consider that the social planner would like to produce a retirement scheme that equalizes as much as possible the share of life spent in pension (so, it amounts to individualising the retirement age using the full distribution of actual longevities). Therefore, the social planner's strategy consists of moving away from a unique retirement age regime and use several retirement ages (based on tags like e.g. sex and/or income percentile) that accounts for the heterogeneity in longevity in the society.

In order to introduce our index, let us first consider a simplified example. Society consists of n members, living a life that varies in length, who can be tagged into two subsets (H and L) of identical size. Subset H enjoys a high life expectancy with half of its member living 4.5 periods and the other half 3.5. Subset L has a lower life expectancy with half of its member living 2.5 periods and the other half 1.5. As explained before, the social planner has for policy that individuals should spend a share α of their life working. Consequently, he is willing to introduce different retirement ages that reflect the longevity of the different groups. However, he cannot divide further the different subsets due to his incapacity to distinguish the short- vs long-lived within each group. The use of subset-specific retirement ages is still a better arrangement than the use of a unique one. In the first case, the social planner would set the retirement age at $\alpha \times 3$.[2] This will create a total sum of deviations between the retirement age and the fair retirement ages of $\alpha\, n$. In contrast, he would set the retirement age at $\alpha \times 4$ for subset H and at $\alpha \times 2$ for subset L, generating a total sum of deviations of $0.5\,\alpha\, n$. Our index will measure the percentage of *improvement* (i.e. the diminution of the gaps) between those two policies; so the decrease in the sum of gaps between the proportional retirement age and the one(s) of the social planner. In this simple example, it is equal to $\frac{0.5\,\alpha\, n}{\alpha\, n}$ suggesting a gain of 50 %. This amounts to saying that the mean deviation is reduced by half. It is crucial to notice that our index measures the *improvement* between the worst policy (only one retirement age) and the one proposed (e.g. a different one for each income percentile). The value of α does not play any role; but it facilitates the explanation.

Let's us now generalize to a society constituted of a set of individuals i who could be split into j mutually exclusive subsets defined by a vector of socioeconomic characteristics (such as income percentile). Their longevity (i.e. age at death) is denoted $m_{i,j}$. Our gap index is formulated as follows:

$$I(\mathbf{m}) = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} |\alpha\, m_{i,j} - \alpha\,\mu_j(\mathbf{m_j})\,|^{\beta}}{\sum_{j=1}^{k}\sum_{i=1}^{n_j} |\alpha\, m_{i,j} - \alpha\,\mu(\mathbf{m})\,|^{\beta}} \tag{1}$$

$$s.t. \quad \mu(\mathbf{m}) \in \underset{\mu(\mathbf{m})}{\arg\min} \sum_{j=1}^{k}\sum_{i=1}^{n_j} |\alpha\, m_{i,j} - \alpha\,\mu(\mathbf{m})\,|^{\beta} \tag{2}$$

$$\mu_j(\mathbf{m_j}) \in \underset{\mu_j(\mathbf{m_j})}{\arg\min} \sum_{i=1}^{n_j} |\alpha\, m_{i,j} - \alpha\,\mu_j(\mathbf{m_j})\,|^{\beta}, \forall j \tag{3}$$

The index is the sum, over each individual, of the absolute deviations between the proportional retirement age ($\alpha\, m_{i,j}$) and the retirement age as chosen

[2]The retirement age is set at $\alpha \times 3$ because it minimizes the sum of the deviations between the retirement age (i.e. $\alpha \times 3$) and the proportional ones ($\alpha \times 4.5$; $\alpha \times 3.5$; $\alpha \times 2.5$ and $\alpha \times 1.5$); more details will be provided in the general formulation below.

by the social planner $(\alpha\,\mu_j\,(\mathbf{m_j}))$ to the power β (representing gap aversion), divided by the sum of deviations in case of a unique retirement age $(\alpha\,\mu\,(\mathbf{m}))$. Individuals retire at $\alpha\,\mu\,(\mathbf{m})$ in case of a unique retirement age and at $\alpha\,\mu_j\,(\mathbf{m_j})$ for those in the subset j when the social planner chooses a subset specific retirement age. Constraint (2) (resp. (3)) ensures that the retirement age set by the social planner is the optimal one in the case of a unique (resp. each of the different) retirement age. It can be shown (see Appendix A) that the pair $(\alpha\,\mu\,(\mathbf{m})\,;\alpha\,\mu_j\,(\mathbf{m_j}))$ corresponds to the median longevity if β is equal to 1 and to the mean longevity for a value of β of 2. Above 2, the most appropriate values do not correspond to any usual statistical moment and are identified numerically.[3] The parameter β represents the social planner's degree of aversion to the gap between the proportional retirement age and the one chosen by the social planner. A value of 1 corresponds to an absence of aversion and a higher degree of aversion is reflected by a higher value of β. No gap aversion amounts to saying that a policy generating 10 deviations of 1 is not more appropriate than one producing one deviation of 10. This is ethically questionable as the first moderately affects 10 individuals whereas the second imposes a large cost to one individual. Values of $\beta > 1$ solves this issue by weighting more the larger deviations. The properties of our index are:

- The index is relative to the default option of a unique retirement age and is below (resp. above) 1 for a better policy (resp. worse);
- The index has a value of zero if each individual has his retirement age set optimally (i.e. $\alpha\,m_{i,j} = \alpha\,\mu_j\,(\mathbf{m_j})\,, \forall\, i, j$);
- The index does not depend on α;
- The index puts more weight on the short-lived if and only if $\beta > 1$. In particular, the higher β is, the more extreme deviations matter;
- The index is invariant to the total population size;
- The index is invariant with respect to the length of longevity (multiplicative and additive);
- The index respects the anonymity principle.

4. Data construction

The next section presents estimates of our index based on US simulation of longevity for several tagging policies. We first need to explain how we simulate our data. The computation of the index calls for a complete distribution of longevity of a population, which could be split into subsets based on some

[3]In practice, we use Stata_{16} to compute the deviations brought by every possible retirement age and select the most suitable one.

socioeconomic characteristics (such as income). We have used the mortality rates assembled by Chetty et al. (2016) for the US to simulate it. Their data are available either by sex and income percentile or by US state of residence, sex and income quartile. Following the life table techniques of Chiang (1984), we simulate the longevity distribution of our population (see Appendix B for more details). Figure 2 displays two longevity distributions; the dashed line corresponds to women in the 20^{th} income percentile while the solid line corresponds to women in the 80^{th} percentile. Simulations start at age 40 because Chetty et al. (2016) provide mortality rates from that age. More deaths occur at a younger age for the women in the 20^{th} percentile. This is in accordance with the mortality gradient, which describes the fact that people in lower socioeconomic status tend to die earlier than richer people. Two stylised facts emerge. First, the 20^{th} percentile distribution is less negatively skewed, illustration of the negative impact of a lower income on life expectancy. Second, the distribution of longevity is more disperse for the 20^{th} percentile than it is for the 80^{th} percentile, reflecting the well-known demographic fact of a higher longevity dispersion for lower socioeconomic categories (see e.g. van Raalte et al., 2011).

5. Results

5.1. No gap aversion ($\beta = 1$)

We will now empirically analyze the values of our index with US data for several policies. The pension policies will be based on state, sex and income rank. One can raise the question of a potential problem of moral hazard. For example, people could decide to move to another state to benefit from a lower retirement age. Sex is naturally not prone to moral hazard. The problem is reduced for income if the policy uses percentiles (i.e. a measure of relative income). It is easier to place oneself below/above a particular threshold of income than in a particular place in the income distribution (e.g. at the 80^{th} percentile). We come back to this point after presentations of our results. State is naturally prone to moral hazard. However, this is not an issue in our study because it is not a powerful tag (see results below). Therefore, there is no need to discuss in depth its moral hazard problem. We present it more as an example in a first best framework.

As explained before, it is assumed that each individual should spend a fraction $(1-\alpha)$ of his life in retirement. The aim of this paper is not to discuss the particular value that α should take and we normalized it at 1 in the tables and figures. Therefore, the reader should multiply the values in tables and figures by α to have a retirement age. The social planner tries to adjust with the upmost precision an appropriate retirement age to fulfill his goal. We can observe in Figure 3 that the life expectancy is lower for men than for women and, therefore, imposing a unique retirement age, regardless of sex, appears inadequate. Nevertheless, and this is *the crucial point*, the effect of differentiating the retirement age would be ambiguous. Figure 4 shows the longevity distribution of men and two different policies: the solid line corresponds to the unique retirement age and the dashed one to the differentiated retirement age. The change from the unique retirement age to the differentiated one reduces the deviations of the people who will die sooner than expected; for example, those located in the point A. The distance between them and the dashed line is less than the distance with the solid line. However, it will lead to a higher deviations for the people who will live longer, for example those located in the point B. The solid line is indeed closer to their position than the dashed one.

We will first assume that our social planner has no gap aversion (i.e. β equals 1). The median longevity for the whole population is 85 and the unique retirement age is therefore set at $\alpha \times 85$. The policy recommendation of the empirical literature has often been to differentiate the retirement age by income. The literature never analyzes the heterogeneity within group as if the group were homogeneous. Our results should be interpreted with that point in mind. If the decrease of the index is important (say 50 %) when the retirement age is differentiated by income, then the shortcomings of analyzing only across categories is justified. However, if the index decreases only by some percentages point, the shortcomings of analyzing only across categories is less easily justified. Let us start by simulating a tag based on income percentile. Table 1 displays the difference of retirement age for people in the 10^{th}, 20^{th}, 30^{th}, 40^{th}, 50^{th}, 60^{th}, 70^{th}, 80^{th}, 90^{th}, 100^{th}. The poorest people can retire $\alpha \times 4$ years before the retirement age that would have prevailed in the case of a unique retirement age whereas the richest people would end their working life $\alpha \times 6$ years later. So, the difference of retirement age between the poor in the 10^{th} percentile and the rich in the 90^{th} income percentile is $\alpha \times 10$. Nevertheless, this calculus does not bring more information than the previous papers (e.g. Ayuso et al., 2016) about differentiating the retirement age by socioeconomic status. The aim of this paper is to take into account the overall longevity distribution by computing our index. We can see in Table 2 that our index reveals that 96.80 % of the deviations remain after having differentiated the retirement age by income percentile. This means that the mean deviation between the proportional retirement age and the ones of the social planner is decreased by only 3.20 % compared to a situation without any tags. The usual solution of the empirical literature (differentiating the retirement age by socioeconomic status) has never analyzed the relevance of the heterogeneity within group. We see that the faith put in an income-based differentiated retirement policy might be inappropriate, as only 3.20 % of the deviations disappears with such a policy. The missclassification impacts differently the members within the society. The deviations of the short-lived in the bottom income percentile are now smaller as the life expectancy on which the retirement age is calculated is now closer to their mortality ages. However, the deviations of the long-lived is now higher. The contrary arises for the top income percentile. The only deviations that are unaffected are those of the individuals with a longevity comprised between the unique and the differentiated retirement ages, as a change from one to the other does not lead to any change in the deviations for them. Two conclusions could emerge either the decrease seems too weak and one concludes that tagging is not an interesting idea, or the decrease looks sufficiently important and therefore, one concludes in the opposing direction. To our opinion, this assessment is highly subjective. One could argue that even a small diminution is always worthy, whereas another could disagree by saying that a huge effect is required for a policy to be implemented. Nevertheless, our index remind that the groups are far from being homogeneous. A further differentiation by sex and by income percentile leads to a change of 4.96 %.

The small diminution of the index is due to the high variance of longevity within groups. To better sustain this statement, we have decomposed the variance of the longevity as well as a Theil[4] index (when the retirement ages are differentiated by sex and income percentile). The decomposition of the Theil index tells us that 95.22 % is within group and the one of the variance gives us a figure of 92.35 %. Those numbers are not surprising as van Raalte et al. (2012) have shown that differences of longevity between education status (elementary, lower secondary, higher secondary and tertiary) cannot explain more than 4 % of the overall variance, the rest being within group differences. In a policy paper, Deaton (2002) already pointed out that health policy should not be targeted directly at the social gradient in health, but rather at sick people, due to the high variance of health among individuals forming a particular socioeconomic group. However, one could wonder if creating more socioeconomic categories would have provided a better index. In order to answer this question, we have plotted the index (see Figure 5) for 1 to 10 income categories.[5] We see that the marginal improvement is decreasing and, that the gain is closed to zero after 8 income percentile brackets. Therefore, further differentiating the retirement age should have a very small effect as we are already converging. This suggests that if one would really differentiate the retirement ages, he does not need to consider many of them. This is an interesting point for the problem of moral hazard. An individual could try to change his place in the income distribution to modify his retirement age. However, if the social planner uses 8 income percentile brackets (instead of the 100 percentiles), it is relatively more difficult for an individual to change from one category to another.

We finish this section by illustrating a tagging policy based on state, which is represented in Figure 6. The map shows that the retirement ages vary a lot between states. People in Nevada could retire at $\alpha \times 83$ whereas those in Minesotta could end their working lives at only $\alpha \times 87$. Nevertheless, the decrease of our index is small as its value stays at 99.59 %. If we differentiate

[4] We use the formulas provided in the Additional File from van Raalte et al. (2012).

[5] If there are two categories, the retirement age is different for the people in the 1-50$^{\text{th}}$ and the 51-100$^{\text{th}}$ income percentiles. If there are three categories, the retirement age is different for the people in the 1-33$^{\text{th}}$, 34-66$^{\text{th}}$ and the 67-100$^{\text{th}}$ income percentiles. And so on.

further the retirement age based on state, sex and income quartile, we arrive at a value of 95.08 %.

5.2. Impact of risk aversion ($\beta \to 10$)

We have assumed in our previous computation that β is equal to 1 (i.e. the social planner is not gap averse). The sensitivity of our result to his degree of aversion deserves attention. Table 3 displays the index as well as the unique and some differentiated retirement ages. A value above 2 is already quite extreme (1 deviation of 10 equals 100 deviations of 1), and almost implausible; nevertheless, computing the values of the index and of the retirement ages above 2 remains interesting to notice the tendency implied by a growing β. Let us first consider β equal to 2. The unique retirement age decreases to $\alpha \times 83$ (compared to $\alpha \times 85$ with $\beta = 1$) and the ones tag by gender to $\alpha \times 81$ for men (compared to $\alpha \times 83$ with $\beta = 1$) and $\alpha \times 85$ for women (compared to $\alpha \times 88$ with $\beta = 1$). However, although retirement ages have changed, the values of the index do not improve that much, as they are now at 97.64 % for a retirement age differentiated by sex (compared to 98.32 % with $\beta = 1$), 92.35 % by sex and by percentile (compared to 95.04 % with $\beta = 1$), 99.53 % by state (compared to 99.59 % with $\beta = 1$) and 92.57 % by state, sex and income quartile (compared to 95.08 % with $\beta = 1$). This indicates that, with a reasonable value of gap aversion, differentiating the retirement age does not lead to significant improvement(s). A system with 200 different retirement ages (by sex and percentile) reduces our index by only 7.65 %. The root of the problem remains in the shape of the distribution, which is too spread for the mean to be meaningful.

Three interesting results come into view in Table 3. First, a higher aversion implies a lower index. Second, the retirement age decreases as β increases. Finally, a higher aversion reduces the gap between the different retirement ages (e.g. between the one of the 25^{th} percentile and the one of the 75^{th} percentile). This stylized fact is noticeable in the comparison between Figure 6 and 7. Whereas Figure 6 displays a gap of $\alpha \times 4$ years across the different retirement ages, Figure 7 shows a difference of only $\alpha \times 1$ year. The explanation of the first point is rather straightforward. The unique retirement age generates more deviations (and also more extreme) at the denominator of (1) than the differentiated retirement ages at the numerator of (1). By consequence, as β increases, the denominator of the index grows more than its numerator; as a result, the index diminishes as the gap aversion grows. This point is also quite intuitive; a very gap averse social planner will care more about deviations from the situation of equal treatment and, therefore, be more prone to establish several retirement ages. The second feature (the diminution of the retirement age) stems from the fact that i) by definition of gap aversion, larger deviations matter more than small ones ii) and that these large deviations are more frequent on the left-hand side of the distribution than on the right hand side (see Figure 8). The third point (the decrease of the gap between the retirement ages) is due to the difference of longevity dispersion between the income percentiles. Figure

9 shows that the distribution characterising the longevity of men belonging the 25^{th} percentile is more dispersed that the distribution of the longevity of women in the 75^{th} income percentile. It is well-documented in demography that lower income percentiles display more longevity dispersion (see e.g. van Raalte et al., 2011). By consequence, the change of retirement age for low income is smaller due to the importance of right-hand side deviations. As a result, as one retirement age decreases more than the other, the gap between the retirement ages drops.

5.3. Different weights for positive versus negative deviations

So far, we have shown that the value of β does impact the index and the different retirement ages. However, we have not investigated the possibility that the social planner cares more about negative deviations than about positive ones. This assumes that mistagging should have a different value depending if the individual retires too late or too soon. It is indeed likely that people prefers to end their working life earlier than later. We will therefore modify (1) as follows:

$$I\left(\mathbf{m}\right)=\frac{\left(\sum_{j=1}^{k}\sum_{i=1}^{n_j}|\alpha m_{i,j}-\alpha\mu_j(\mathbf{m_j})|^{\beta}|\,m_{i,j}\leq\mu_j(\mathbf{m_j})\right)+\left(\sum_{j=1}^{k}\sum_{i=1}^{n_j}\sigma|\alpha m_{i,j}-\alpha\mu_j(\mathbf{m_j})|^{\beta}|\,m_{i,j}\geq\mu_j(\mathbf{m_j})\right)}{\left(\sum_{j=1}^{k}\sum_{i=1}^{n_j}|\alpha m_{i,j}-\alpha\mu(\mathbf{m})|^{\beta}|\,m_{i,j}\leq\mu(\mathbf{m})\right)+\left(\sum_{j=1}^{k}\sum_{i=1}^{n_j}\sigma|\alpha m_{i,j}-\alpha\mu(\mathbf{m})|^{\beta}|\,m_{i,j}\geq\mu(\mathbf{m})\right)} \tag{4}$$

The difference between (1) and (4) is the presence of σ that weights differently positive and negative deviations. In some sense, σ can be related to a sort of poverty line, which purpose is to "partition the population into two groups that we want to treat differently" (Cowell, 2016, p. 49). A value below 1 means that deviations corresponding to those who retire too early are less important than the deviations reflecting the situation of individuals who retire too late. Table 4 shows the effect of various σ on the retirement age when β equals 1. The retirement age decreases with a diminishing σ. This result is quite intuitive; as the social planner cares more about the deviations of those who retire too late than too early, he lowers the retirement age. The extreme result is obtained when he does not care at all about the deviations of those who retire too early ($\sigma = 0$). In this case, the retirement age is set at $\alpha \times 40$ with no deviation on the left; all are on the right, but he does not care about them. This situation is obviously not realistic. The second question is the impact of σ on the index. The effect is rather small as Table 5 suggests. This comes from the fact than the structure of the distribution is not altered by the weighting parameter σ. This parameter changes the retirement age and the weight on some deviations; but the behaviour of the index remains primarily driven by the very high dispersion within each category.

5.4. Truncation

We have shown that our results are robust to extra weight put on later retirees or to an higher degree of gap aversion. The last specification that we consider is to remove the simulations below a certain age threshold. One might raise doubts about calculating the index based on the whole distribution. Retirement can only happen at a certain age and, therefore, one might argue that the presence of people dying at, e.g., 40 should not be taken into account. Table 6 shows the computation of the index with different level of truncation. Truncation at 40 is our benchmark result from previous section (simulations start at that age). Those at 55, 60 and 70 are some arbitrary value below which the dead "should not be relevant". The table shows that our results are robust to such a left-hand side truncation. Although, the retirement age increases (see Table 7) as we do not care about people below a certain threshold, the values of our index are relatively stable.

In this section, we have shown the behavior of our index with various specifications. The improvement of the pension system, without gap aversion, was small and it took 200 different retirement ages to decrease the index by 5 %. However, gap aversion did not change the conclusion much as long as a very high level was not assumed. Weighting differently positive and negative deviations did not alter the index either. Truncation at a given threshold did not modify substantially our results. Together those different specifications show that our results are robust. The present paper uses data from the United States, which are known to have higher health inequality than, for example, France or Belgium (see e.g. Delavande and Rohwedder, 2011). Therefore, the relatively weak effect found in this paper is likely to be even weaker with French/Belgian data.

6. Discussion

Section 5 has presented our results without properly discussing the normative principles underpinning our paper. This section aims at filling that void. Three important points should be raised: the choice of a proportional retirement age, the choice of an ex post or an ex ante setting and the choice to shape retirement policy with respect to individuals or to groups.

6.1. Proportional retirement age

The policy choice made in this paper is a proportional retirement age. One might doubt its relevance. For example, different jobs have different level of penibility[6] and another policy would be to differentiate the value of α according to penibility. This remark is important; but it should be pointed out that it does not invalidate our result. It makes them even stronger. The small decrease in the index was due to the longevity heterogeneity with a proportional retirement age. If one wants to differentiate α based on the level of penbility, a second level of heterogeneity will be added. The same job could be painful in one firm, but

[6]For a discussion of the level of job penibility across working sector, see Baurin and Hindriks (2019).

not in another. At the end, our results will be stronger because the unobserved heterogeneity (longevity and job penibility) is likely to increase. Overall, this suggests that differentiating the retirement age by socioeconomic status is not a very relevant policy. We now turn to the choice of using an ex post or an ex ante framework.

6.2. Ex post or ex ante?

An important choice to make is the use of an ex ante or ex post framework. An ex ante framework assesses situation based on expectations (e.g. the probability of a lottery) while an ex post assesses based on realizations (e.g. the results of the lottery). In our context, ex ante implies using life expectancy (i.e. an expectation) and ex post implies using longevity (i.e. a realization). The existing empirical literature is quite ambiguous as to which of the perspectives should be adopted. It is not uncommon to read policy recommendations that are intrinsically ex ante (focusing on life expectancy) from papers that are based on ex post mortality data. Several papers in the theoretical literature emphasize the importance of using an ex post approach. The starting argument is that "at the end of the day, what matters is what people achieved, not what they expected to achieve" (Fleurbaey et al., 2016, p. 201). Let us explain the fundamental flaw of using an ex ante approach when it comes to differentiate the retirement age. Assume that a social planner is egalitarian. He wants to differentiate the retirement age because of differences of life expectancy. Starting from a situation with a unique retirement age, he introduces a first differentiation, for example by average lifetime income. However, there is no justification why a social planner should only consider income if he wants to be *truly* egalitarian. He should then, for example, further differentiate the retirement age according to the lifetime income of the parents (as we know that childhood circumstances affect later morbidity and mortality differences). But there is no reason why he should stop at that step and he should then move to further differentiation. The social planner's agenda will only be fulfilled when all factors influencing longevity have been taken into account. However, at that moment, assuming full and perfect knowledge of the whole set of factors affecting longevity, our social planner will end up with an individualized retirement age, reflecting the true longevity of the individual.[7] Therefore, starting from the point of a social planner who wanted to differentiate the retirement age based on differences of life expectancy, we end up at the point of using differences of longevity. Fleurbaey et al. (2016, p. 201) already stated that point, slightly differently: "ex ante egalitarianism that is based on average mortality statistics is not really ex ante

[7] Fishkin (2014) used a similar argumentation when he explained the problem of achieving equality of opportunity. He argued that we have to withdraw every differences between individuals if we want to achieve equality of opportunity between them. Equality of opportunity would not be achieved if we still leave only one slightly difference (e.g. in some genes). The same argumentation applies in our case. The social planner has to take into account every characteristics affecting longevity if he wants to be truly egalitarian.

egalitarian if it fails to track individual's true life expectancy. In a deterministic world, the true ex ante perspective coincides with the ex post perspective". Therefore, either one concludes that differences of longevity should be taken into account (i.e. by differentiating the retirement age) and uses an ex post framework, or one concludes that differences of longevity should not be taken into account (i.e. an unique retirement age) and uses an ex ante framework. But, it is not possible to use an ex ante framework (i.e. life expectancy) and, at the same time, wanting to differentiate the retirement age.

The argumentation above ends up with a conclusion that ex ante and differentiating the retirement age is incoherent. However, it does not say which framework was the most relevant. The first reason is that there is (currently?) a problem of availability of data to perfectly predict the longevity of an individual. The second reason is that we think that it is a subjective choice that should be left to the population. Problematically, economists have shown that people do not behave consistently with respect to ex ante or ex post choice. Andreoni et al. (2016) made an experiment demonstrating that people reverse their *own* choices when the framework of the question changes from ex ante to ex post. In a nutshell, they did an experiment where people have to allocate 10 lottery cards between two individuals, one of them having already 10 cards. People tend to give the cards to the individual with any card. Then the lottery is drawn and people know that the winning card is one of the cards they have given. When faced with the choice of reallocating their cards, people often want to reallocate 5 cards to each of individual. This contradicts the classical microeconomic theory and shows that people modify their *own* choices if the setting moves from ex ante to ex post. In the end, Andreoni et al. (2016) concluded that people are "deontologically naive" and use the fairness related to the framework they are dealing with. This is troubling if one wants to know how to shape policy. Others experiments have reached similar conclusions (see e.g. Brock et al., 2013). To conclude, both ex ante and ex post theories of fairness appeal to people and there is no clear preferences over one of the other.

6.3. Across individuals or groups?

The second choice to make is to assess the difference across individuals or across groups. The empirical literature has focused on differences across groups whereas this paper concentrates on tagging individuals. The reason comes from the impossibility of putting someone *uncontroversially* in a particular group due to the non-existence of "natural" groups. By consequence, the groups created by statisticians will always be somehow arbitrary, undermining the legitimacy of using them in public policy. One might raise the point that a percentile is not created by statisticians, but exists naturally in the reality (as it comes from the distribution). Although this remark is true, the statistician still chooses to use percentile (instead of e.g. quartile). A individual can always object that the level of precision is too low or too high and that it places him in the wrong group. Moreover, for an individual what matters is his situation and not what may have caused it (Murray, 2001). As such, it makes no difference for an individual if his longevity is smaller due to some between- or within-group factors; what matters for him is his longevity and how it compares with other individuals. Nevertheless, some (e.g. Gakidou et al., 2000) argued that differences in health across groups are more informative than differences across individuals because the former removes the component due to luck. This is true. Differences in average health between socioeconomic status show social health inequalities. But, the question is to know if the differences are sufficiently informative for pension policies. If the normative criterion is that an individual dying earlier should retire earlier, the heterogeneity within group makes undesirable to use groups instead of individuals. Focusing on average differences of longevity, with such a huge dispersion inside each groups, will lead to what Cornia and Stewart (1993) have called F-mistake (failure of coverage) and E-mistake (excessive coverage). F-mistake indicates error when someone is wrongly not included in a policy and E-mistake indicates error when someone is wrongly included. In our context, F-mistake relates to the case of a short-lived rich individuals who would be tag as long lived and E-mistake relates to the case of a long-lived poor individuals who would be tag as short-lived. This could easily be related to the economics of statistical discrimination. As explained in the literature review, Phelps (1972) explained in his seminal works that a decision maker, with time constraint, could based his decision on average characteristics and, by doing so, some high-performing members belonging to an under-performing group are discriminated against. The same arises when the social planner focuses only on life expectancy, the short-lived rich individuals are penalized because the social planner does not take the time to look into each group.

The goal of this section was to explain the underlying philosophical principles behind our choices. Although, we have discussed the problem of ex post vs ex ante and individuals vs groups, we did not state a particular choice as this is, for us, subjective. At the end of the day, fairness is a personal concept. Nevertheless, the issue of the pension of the short-lived individuals deserves attention whatever the philosophical choice being made. We have shown that differentiating the retirement age based on socioeconomic group will not help much the short-lived, due to the high heterogeneity within group. Therefore, other policies should be proposed to deal with this problem. Recently, Ponthière (2018) has proposed an original policy of "reverse retirement". In his paper, people should first retire and then work such that the short-lived individuals enjoy retirement. Other policies could be proposed and there is a need for further research about them.

7. Conclusion

The pension literature has established that the mortality differential between socioeconomic categories reduces or completely eliminates the progressivity of the retirement system. To address this problem, the policy recommendation has often been to differentiate the retirement age by socioeconomic status. However, a lot of different factors (genes, income,...) are causing variation in longevity. In this paper, we have considered tagging individuals according to their income, state and sex as commonly done in the existing literature. We have shown that the distribution of longevity within the different subgroups is such that differentiating the retirement age between US state of residence and/or sex and/or income rank does not lead to a substantial improvement of the gap index. The conclusion is robust to several specifications: gap aversion of the social planner, different weights for positive and negative deviations or truncation below an age threshold.

The take-home message is that pension policy is relatively unable to differentiate optimally the retirement age. At individual level, many factors are causing enormous variation in longevity. Although state, gender and socioeconomic categories are important predictors of longevity, using them to systematically differentiate retirement would still largely fail to account for a large portion of distribution of longevity in the population. Equalising the time spent in retirement requires adopting a lifetime perspective. This implies that the social planner needs to be able to know *in advance* the longevity of people. This paper shows that the gains that may be achieved by abandoning a unique retirement age policy are limited, when the social planner's information is imperfect in the sense that it consists of estimates of group-specific life expectancies. Differentiating the age of retirement by gender and (a limited number of) income categories would account for some of the inequalities of longevity. However, it would not be sufficient to solve the overall problem of longevity differences in the population.

Nevertheless, we expect the question of unfairness of the pension system due to (in some cases growing) longevity heterogeneity to remain a very hot topic in the future. There is therefore a real need for more research in this field to better understand the determinants of that heterogeneity and how it can be addressed within the pension system or via other policies.

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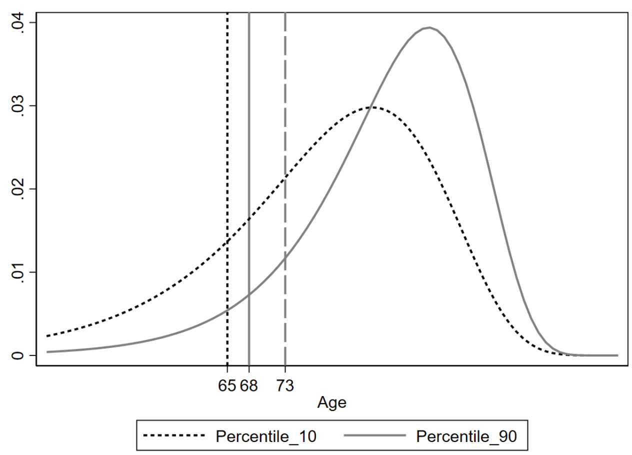
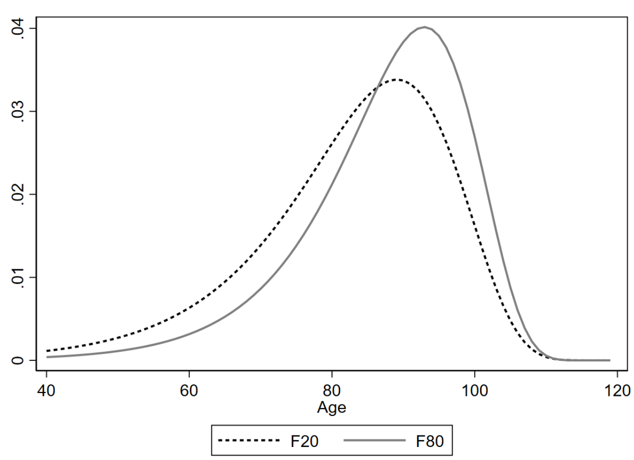


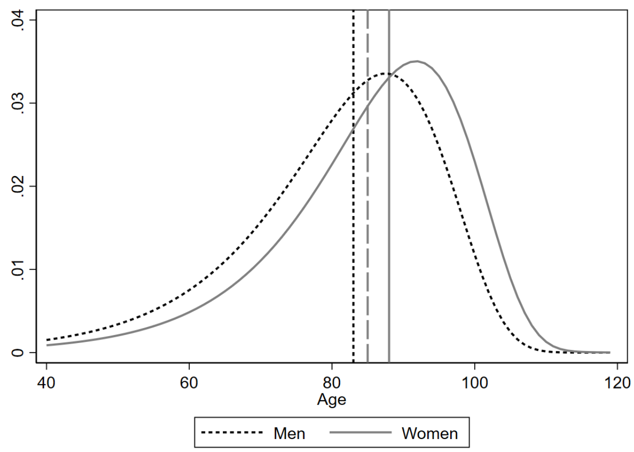
The figure shows the simulation of US longevity distribution for individuals in the 10^{th} and 90^{th} income percentile. Simulations are done using the mortality rates provided by Chetty et al. (2016). The retirement age is set at 68 in case of a unique retirement age, while it is set at 65 for the 10^{th} income percentile and at 73 for the 90^{th}.

Figure 1: Distribution of longevity for the 10^{th} and 90^{th} income percentile



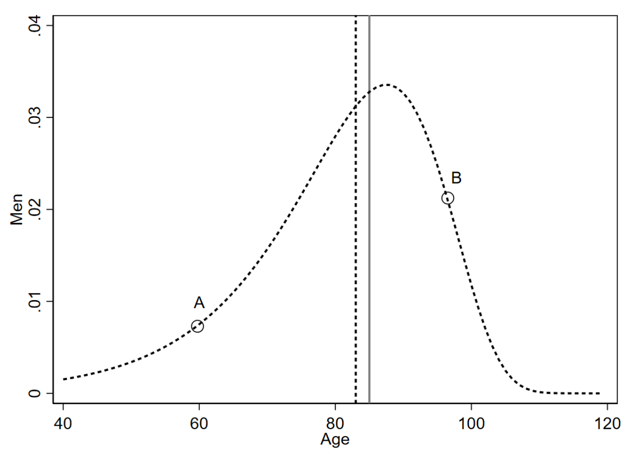
The figure shows the simulation of US longevity distribution for women in the 20^{th} and 80^{th} income percentile. Simulations are done using the mortality rates provided by Chetty et al. (2016). The longevity distribution of the rich women is more negatively skewed which reflects their higher life expectancy. The longevity of the poor women is more dispersed which is a stylized demographic fact.

Figure 2: Distribution of longevity for women in the 20^{th} and the 80^{th} percentile



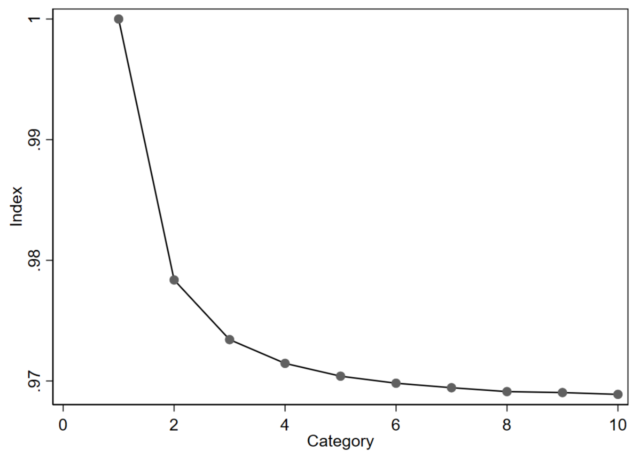
The figure shows the simulation of US longevity distribution for men and women. Simulations are done using the mortality rates provided by Chetty et al. (2016) . The retirement age (with α and β equal to one) for men is the small dashed line and the grey solid line for women. The central dashed line is the unique retirement age.

Figure 3: Distribution of longevity for men and women



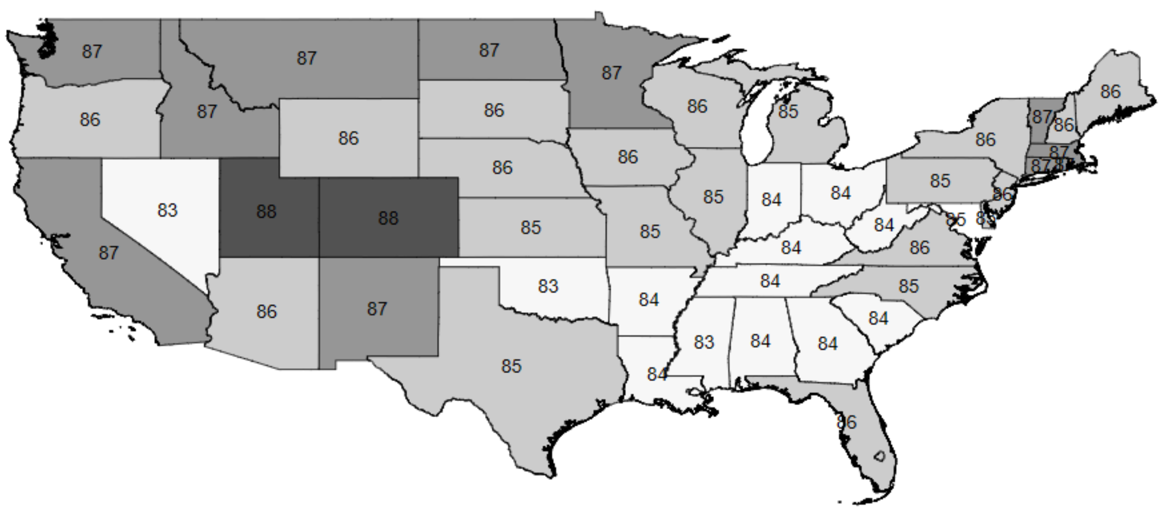
The figure shows the simulation of US longevity distribution for men. Simulations are done using the mortality rates provided by Chetty et al. (2016) . The unique retirement age (with α and β equal to one) is the solid line and the differentiated one (for men) is the dashed one.

Figure 4: Impact of differentiating the retirement age by sex for men



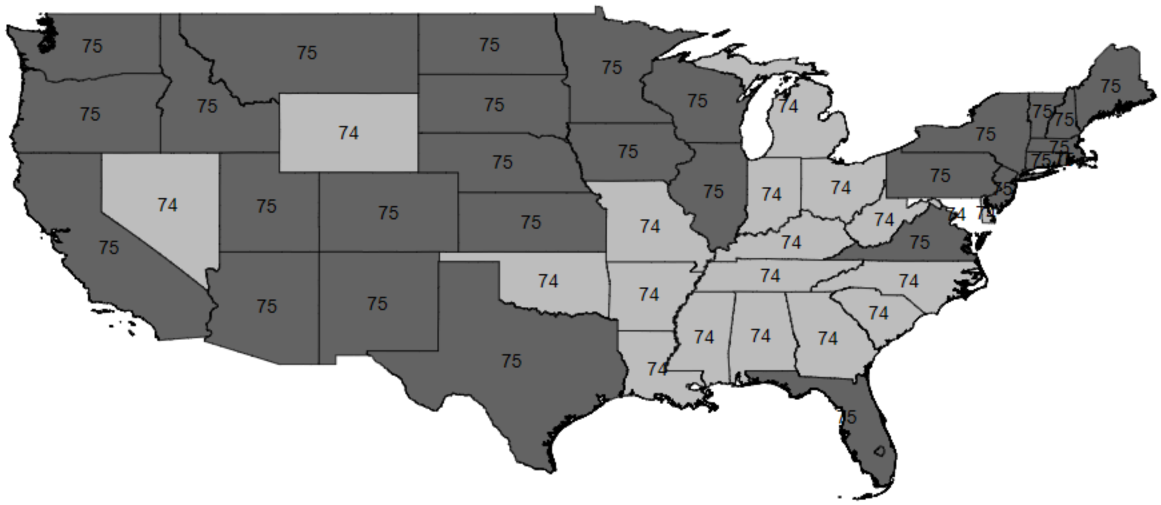
The graph shows the value of the index with the increase in the number of income brackets. One category means that there is only one retirement age, two categories means that the retirement age is different for the individuals having their income in the 1-50$^{\text{th}}$ percentile and in the 51-100$^{\text{th}}$, three categories means that it is differentiated between the individuals in the 1-33$^{\text{th}}$, 34-66$^{\text{th}}$ and 67-100$^{\text{th}}$ income percentile; and so on.

Figure 5: Decrease of the index with the number of category ($\beta = 1$)



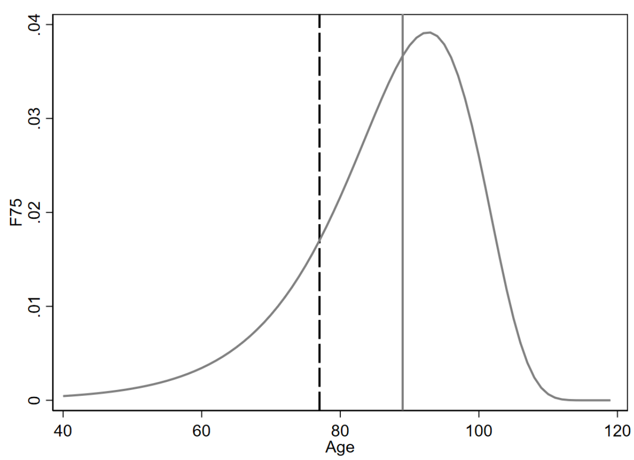
The figure shows the retirement age by state with β and α equal to 1. Simulations are done using the mortality rates provided by Chetty et al. (2016) .

Figure 6: Impact of differentiating the retirement age by state (β and $\alpha = 1$)



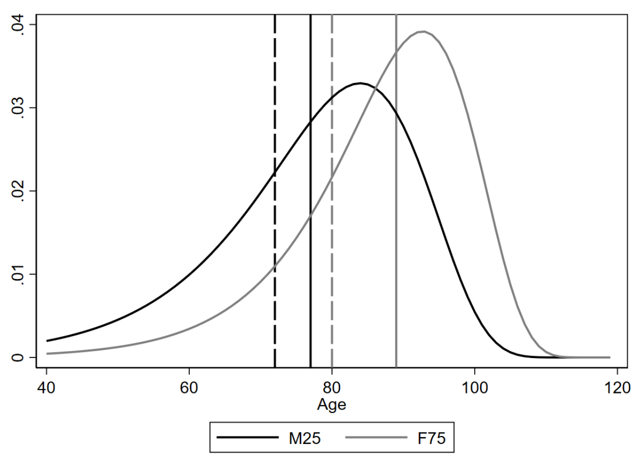
The figure shows the retirement age by state with β equal to 10 (and α equal to 1). Simulations are done using the mortality rates provided by Chetty et al. (2016) .

Figure 7: Impact of differentiating the retirement age by state ($\beta = 10$, $\alpha = 1$)



The figure shows the decrease in the retirement age when β increases. Simulations are done using the mortality rates provided by Chetty et al. (2016). The retirement age (with α equal to 1) is the solid line when β equal 1 and the dashed one when β equal 10.

Figure 8: Explanation of the decrease of the retirement age (Women, 75^{th} percentile)



The figure shows the decrease in the gap between the retirement ages when β increases. Simulations are done using the mortality rates provided by Chetty et al. (2016). The retirement age for the men in the 25^{th} income percentile (with α equal to one) is the solid black line when β equal 1 and the dashed black one when β equal 10. The retirement age for the women in the 75^{th} income percentile (with α equal to 1) is the solid grey line when β equal 1 and the dashed grey one when β equal 10.

Figure 9: Explanation of the narrowing gap

List of Tables

Unique retirement age	85
Income percentile	
10^{th}	-4
20^{th}	-3
30^{th}	-2
40^{th}	0
50^{th}	0
60^{th}	+1
70^{th}	+2
80^{th}	+3
90^{th}	+4
100^{th}	+6

The table shows the difference in retirement age (with β and α equal to 1) between a unique retirement age and ones differentiated based on income percentiles. For example, people in the 20^{th} could retire at $\alpha \times 82$ if the retirement is differentiated.

Table 1: Retirement age by percentile (β and $\alpha = 1$)

Index					
By sex	By percentile	By sex & percentile	By state	By state & sex	By state, sex & quartile
98.32 %	96.80 %	95.04 %	99.59 %	97.92 %	95.08 %

Retirement age ($\alpha = 1$)												
Unique	Male	Female	25^{th}	75^{th}	M, 25^{th}	F, 75^{th}	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1^{st}	Nev, F, 4^{th}
85	83	88	83	87	80	89	87	83	85	85	79	88

The table provides the value of the index when β equal 1. The index is done for tagging by sex; by income percentile; by sex and percentile; by state; by state and sex; by state, sex and income quartile. The line below shows different retirement ages depending on the policy (with α equal to 1). The retirement ages provided are the unique, the ones for each gender, for individuals in the 25^{th} income percentile, for individuals in the 75^{th} income percentile, for men in the 25^{th} income percentile, for women in the 75^{th} income percentile, for residents in Minesotta, for residents in Nevada, for men in Minesotta, for women in Nevada, for men in Minesotta and in the 1^{st} income quartile, for women in Nevada in the 4^{th} income quartile.

Table 2: Index and retirement age for several policies ($\beta = 1$)

β	Index					
	By sex	By percentile	By sex & percentile	By state	By state & sex	By state, sex & quartile
1	98.32 %	96.80 %	95.04 %	99.59 %	97.92 %	95.08 %
2	97.64 %	94.97 %	92.35 %	99.53 %	97.15 %	92.57 %
3	96.89 %	93.38 %	90.16 %	99.38 %	96.40 %	90.54 %
4	96.72 %	92.54 %	88.83 %	99.47 %	96.03 %	89.30 %
5	95.88 %	91.69 %	87.56 %	99.41 %	95.41 %	88.18 %
10	94.39 %	90.18 %	84.30 %	99.07 %	93.17 %	85.14 %

β	Retirement age ($\alpha = 1$)												
	Unique	Male	Female	25^{th}	75^{th}	M, 25^{th}	F, 75^{th}	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1^{st}	Nev, F, 4^{th}
1	85	83	88	83	87	80	89	87	83	85	85	79	88
2	83	81	85	80	85	78	86	84	81	82	83	77	86
3	81	79	82	79	83	76	84	82	79	80	81	75	84
4	79	77	80	77	81	75	82	80	78	79	79	74	82
5	78	76	79	76	79	74	81	79	77	77	78	74	80
10	75	73	76	74	76	72	77	75	74	74	75	72	76

The table provides the value of the index with different values of β. The index is done for tagging by sex; by income percentile; by sex and percentile; by state; by state and sex; by state, sex and income quartile. The line below shows different retirement ages depending on the policy (with α equal to 1). The retirement ages provided are the unique, the ones for each gender, for individuals in the 25^{th} income percentile, for individuals in the 75^{th} income percentile, for men in the 25^{th} income percentile, for women in the 75^{th} income percentile, for residents in Minesotta, for residents in Nevada, for men in Minesotta, for women in Nevada, for men in Minesotta and in the 1^{st} income quartile, for women in Nevada in the 4^{th} income quartile.

Table 3: Index and retirement age for various value of β

σ	Unique	By sex		By percentile		By sex & percentile		By state		By state & sex		By state, sex & quartile	
		Male	Female	25^{th}	75^{th}	Male, 25^{th}	Female, 75^{th}	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1^{st}	Nev, F, 4^{th}
1	85	83	88	83	87	80	89	87	83	85	85	79	88
0.75	83	81	85	80	85	77	87	85	81	83	83	76	86
0.5	80	77	82	77	82	74	84	77	81	79	80	72	83
0.25	73	71	76	70	76	68	78	71	75	73	74	65	81
0	40	40	40	40	40	40	40	40	40	40	40	40	40

The table provides the retirement ages with different values of σ. The retirement ages provided are the unique, the ones for each gender, for individuals in the 25^{th} income percentile, for individuals in the 75^{th} income percentile, for men in the 25^{th} income percentile, for women in the 75^{th} income percentile, for residents in Minesotta, for residents in Nevada, for men in Minesotta, for women in Nevada, for men in Minesotta and in the 1^{st} income quartile, for women in Nevada in the 4^{th} income quartile. σ equals to 1 is our benchmark from previous table.

Table 4: Retirement age by σ (β and $\alpha = 1$)

σ/β	By sex			By percentile			By sex & percentile		
	1	2	5	1	2	5	1	2	5
1	98.32 %	97.64 %	95.88 %	96.80 %	94.97 %	91.68 %	95.04 %	92.35 %	87.56 %
0.75	98.48 %	97.65 %	96.33 %	96.70 %	94.71 %	91.73 %	95.06 %	92.18 %	87.67 %
0.5	98.55 %	97.72 %	96.22 %	96.42 %	94.41 %	91.50 %	94.96 %	92.00 %	87.54 %
0.25	98.78 %	97.86 %	96.50 %	96.09 %	93.91 %	91.29 %	94.89 %	91.74 %	87.50 %
0	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %
σ/β	By state			By state & sex			By state, sex & quartile		
	1	2	5	1	2	5	1	2	5
1	99.59 %	99.53 %	99.41 %	97.92 %	97.15 %	95.41 %	95.08 %	92.57 %	88.18 %
0.75	99.65 %	99.48 %	99.48 %	98.11 %	97.13 %	95.61 %	95.15 %	92.38 %	88.26 %
0.5	99.65 %	99.51 %	99.46 %	98.25 %	97.24 %	95.61 %	95.14 %	92.28 %	88.18 %
0.25	99.67 %	99.48 %	99.45 %	98.48 %	97.36 %	95.77 %	95.16 %	92.10 %	88.16 %
0	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %

The table provides the index with different values of σ and β. σ varies from 0 to 1 and β takes the value of 1, 2 and 5. σ equal to 1 is our benchmark from previous table.

Table 5: Index by σ and by β

Truncation	By sex	By percentile	By sex & percentile	By state	By state & sex	By state, sex & percentile
40	98.32 %	96.80 %	95.04 %	99.59 %	97.92 %	95.08 %
55	98.32 %	97.13 %	95.33 %	99.66 %	97.95 %	95.32 %
60	98.30 %	97.22 %	95.42 %	99.57 %	97.83 %	95.36 %
70	98.28 %	97.67 %	95.88 %	99.64 %	97.87 %	95.85 %

The table provides the index with different level of truncation. Truncation at 40 is our benchmark from previous table, truncation at 55, 60, 70 are arbitrary values of truncation.

Table 6: Index with different level of truncation ($\beta = 1$)

		By sex		By percentile		By sex & percentile		By state		By state & sex		By state, sex & quartile	
Truncation	Unique	Male	Female	25	75	M, 25	F, 75	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1	Nev, F, 4
40	85	83	88	83	87	80	89	87	83	85	85	79	88
55	86	84	88	83	88	81	89	88	84	86	86	80	89
60	86	84	88	84	88	81	89	88	84	86	86	81	89
70	88	86	89	86	89	84	90	89	86	87	87	84	90

The table provides the retirement ages with different level of truncation. The retirement ages provided are the unique, the ones for each gender, for individuals in the 25^{th} income percentile, for individuals in the 75^{th} income percentile, for men in the 25^{th} income percentile, for women in the 75^{th} income percentile, for residents in Minesotta, for residents in Nevada, for men in Minesotta, for women in Nevada, for men in Minesotta and in the 1^{st} income quartile, for women in Nevada in the 4^{th} income quartile. Truncation at 40 is our benchmark from previous table, truncation at 55, 60, 70 are arbitrary values of truncation.

Table 7: Retirement ages with different level of truncation (β and $\alpha = 1$)

Appendix A. The parameter $(\boldsymbol{\mu}(\mathbf{m}), \boldsymbol{\mu_j}(\mathbf{m_j}))$

The social planner desires to minimize the sum of the deviations between the retirement age and the longevity of the observations in each subset/the entire set. Let us study $\mu(\mathbf{m})$:

$$\sum_{i=1}^{n} |\alpha m_i - \alpha\mu(\mathbf{m})|^{\beta} \tag{A.1}$$

The derivative of (A.1) with respect to $\mu(\mathbf{m})$ is:

$$\sum_{i=1}^{n} -\alpha\beta|\alpha m_i - \alpha\mu(\mathbf{m})|^{\beta-2}(\alpha m_i - \alpha\mu(\mathbf{m})) \tag{A.2}$$

If β is odd, then the root is located at the following condition:

$$\sum_{i=1}^{n} |\alpha m_i - \alpha\mu(\mathbf{m})|^{\beta-1}\frac{(\alpha m_i - \alpha\mu(\mathbf{m}))}{|\alpha m_i - \alpha\mu(\mathbf{m})|} = 0 \tag{A.3}$$

Condition (A.3) states that μ is the median when β equals 1. If β is odd and above 1, there exists no statistical name to the condition (A.3) and the social planner finds the root by computation and by applying condition (A.3). For an even value of β, the root is located at the following condition:

$$\sum_{i=1}^{n} (\alpha m_i - \alpha\mu(\mathbf{m}))^{\beta-1} = 0 \tag{A.4}$$

Condition (A.4) states that μ is the mean when β equals 2. There exists no statistical name to the condition (A.4) when β is even and above 2 and the social planner finds the root by computation and by applying condition (A.4).

The same principles apply for $\mu_j(\mathbf{m_j}), \forall j$.

Appendix B. Data construction

Our data have been constructed based on the life table technique detailed in Chiang (1984) and on the mortality rate provided by Chetty et al. (2016). The life-table method starts with a normalized population (called the "radix") and, at each interval of time, a fraction of the population "died" based on the empirically observed mortality rate. The division of the total years lived beyond age x by the population alive at that age gives the life expectancy of the population at age x. Our interest lies in the number of people dying at each age, which provides us our distribution of longevity. We used the mortality rate provided by Chetty et al. (2016)[8] which they computed based on a sample of

[8] They provided adjusted and non-adjusted race mortality rate, we use the non-adjusted to approach more the real distribution.

1,4 billion observations from deidentified tax records between 1999 and 2014. Their mortality rates are the empirical one between 40 and 75 years old, then they computed an interpolation using the Gompertz curve for the ages between 76 and 90 and finally, used the income-independent mortality rates based on NCHS and SSA data for the ages between 91 and 120. Figure B.1 shows us the longevity distribution for women in the 20^{th} and in the 80^{th} percentile. One can notice the presence of a spike at 91, which is the result of the change from the Gompertz curve to the NCHS-SSA mortality rate. It is more important for the 80^{th} percentile because there are more person alive at 91 in it than in the 20^{th} and the change to the NCHS-SSA mortality rate is therefore more reflected in it.

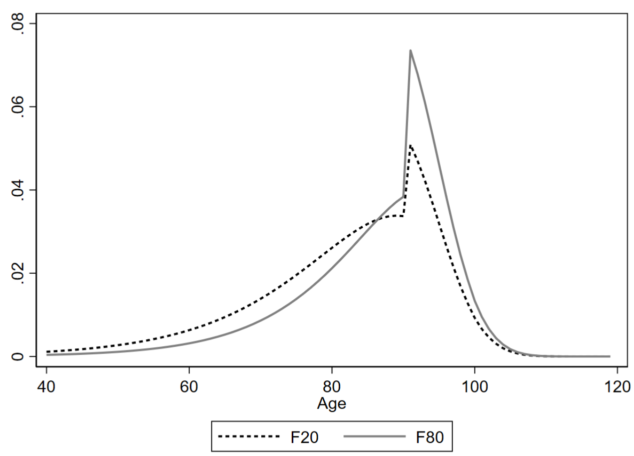


Figure B.1: Distribution of longevity for women in the 20^{th} and the 80^{th} percentile

The presence of this spike could raises the controversy of its importance for the results that we obtain. As a robustness test, we have also computed the distribution using ony the Gompertz curve. There is some empirical debate about the limit age until which it can be used; for example, Gavrilov and Gavrilova (2011) found that it could be extended until 105 without any major issues. Above 105, its relevance is difficult to test due to the small number of observations and the quality of the data.[9] As there is no much individuals living older than 105, extending the Gompertz curve until the end is not a strong assumption. In the main part of the paper, we use graphs based on the Gompertz curve for reading easiness (no spike), but the results reported are those using the assumptions of Chetty. Robustness of results using only the Gompertz curve could be found in Appendix C. A little limitation of our database is that we

[9]An individual older than 105 has his birth date at the beginning of the 1900's and the record are often of poor quality. For example, Gavrilov and Gavrilova (2011) showed that the ratio male-female observed in their data above 105 could not be the true one.

could not decompose longevity by months. We think that it would only change slightly our results as the main part is already captured by using years. There remains some seasonality of death,[10] but this should not impact dramatically our results. Another limitation of our study is that we have to take as exogeneous the longevity distribution; although some papers (see e.g. Dave et al., 2006) have shown that retirement has an effect on the health of individuals.

Appendix C. Table with the data using only the Gompertz curve

Appendix C.1. Index and retirement age for various value of β

β	Index					
	By sex	By percentile	By sex & percentile	By state	By state & sex	By state, sex & quartile
1	98.44 %	97.02 %	95.36 %	99.62 %	98.07 %	95.43 %
2	97.22 %	94.53 %	91.58 %	99.29 %	96.51 %	91.75 %
3	96.27 %	92.71 %	88.69 %	99.15 %	95.36 %	89.03 %
4	95.24 %	91.46 %	86.46 %	98.81 %	94.08 %	86.70 %
5	94.40 %	90.69 %	84.78 %	98.67 %	93.06 %	85.00 %
10	89.79 %	89.20 %	78.85 %	97.55 %	87.95 %	79.19 %

β	Retirement age ($\alpha = 1$)												
	Unique	Male	Female	25^{th}	75^{th}	M, 25^{th}	F, 75^{th}	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1^{st}	Nev, F, 4^{th}
1	85	83	88	83	87	80	89	87	83	85	85	79	88
2	83	81	86	81	85	78	87	85	81	83	83	77	87
3	82	80	84	79	84	77	85	83	80	81	82	76	85
4	80	78	82	78	82	76	84	82	78	80	80	75	83
5	79	77	81	77	81	75	82	80	77	79	79	74	82
10	77	75	78	75	77	73	78	77	75	76	76	73	78

Table C.1: Index and retirement age for various value of β

[10] The seasonality of death is not felt uniformly across the population; old people and those at a low socioeconomic level tend to be more impacted by it (Rau, 2006).

Appendix C.2. Retirement age by σ (β and $\alpha = 1$)

σ	Unique	By sex		By percentile		By sex & percentile		By state		By state & sex		By state, sex & quartile	
		Male	Female	25^{th}	75^{th}	Male, 25^{th}	Female, 75^{th}	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1^{st}	Nev, F, 4^{th}
1	85	83	88	83	87	80	89	87	83	85	85	79	88
0.75	83	81	85	80	85	77	87	85	81	83	83	76	86
0.5	80	77	82	77	82	74	84	81	77	79	80	72	83
0.25	73	71	76	70	76	68	78	75	71	73	74	65	78
0	40	40	40	40	40	40	40	40	40	40	40	40	40

Table C.2: Retirement age by σ (β and $\alpha = 1$)

Appendix C.3. Index by σ and by β

σ/β	By sex			By percentile			By sex & percentile		
	1	2	5	1	2	5	1	2	5
1	98.44 %	97.22 %	94.40 %	97.02 %	94.53 %	90.69 %	95.36 %	91.58 %	84.78 %
0.75	98.57 %	97.37 %	94.46 %	96.89 %	94.43 %	90.45 %	95.35 %	91.62 %	84.69 %
0.5	98.62 %	97.47 %	94.70 %	96.59 %	94.13 %	90.48 %	95.20 %	91.50 %	84.95 %
0.25	98.83 %	97.67 %	95.01 %	96.23 %	93.76 %	90.33 %	95.07 %	91.44 %	85.17 %
0	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %

σ/β	By state			By state & sex			By state, sex & quartile		
	1	2	5	1	2	5	1	2	5
1	99.62 %	99.29 %	98.67 %	98.07 %	96.51 %	93.06 %	95.43 %	91.75 %	85.00 %
0.75	99.68 %	99.43 %	98.47 %	98.22 %	96.78 %	93.03 %	95.45 %	91.86 %	84.94 %
0.5	99.67 %	99.42 %	98.76 %	98.33 %	96.89 %	93.43 %	95.38 %	91.83 %	85.25 %
0.25	99.70 %	99.44 %	98.86 %	98.54 %	97.14 %	93.82 %	95.33 %	91.86 %	85.58 %
0	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %

Table C.3: Index by σ and by β

Appendix C.4. Index with different level of truncation ($\beta = 1$)

Truncation	By sex	By percentile	By sex & percentile	By state	By state & sex	By state, sex & percentile
40	98.44 %	97.02 %	95.36 %	99.62 %	98.07 %	95.43 %
55	98.45 %	97.35 %	95.67 %	99.69 %	98.12 %	95.70 %
60	98.44 %	97.45 %	95.77 %	99.61 %	98.02 %	95.76 %
70	98.47 %	97.89 %	96.23 %	99.68 %	98.12 %	96.28 %

Table C.4: Index with different level of truncation ($\beta = 1$)

Appendix C.5. Retirement ages with different level of truncation (β and $\alpha = 1$)

		By sex		By percentile		By sex & percentile		By state		By state & sex		By state, sex & quartile	
Truncation	Unique	Male	Female	25	75	M, 25	F, 75	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1	Nev, F, 4
40	85	83	88	83	87	80	89	87	83	85	85	79	88
55	86	84	88	83	88	81	89	88	84	86	86	80	89
60	86	84	88	84	88	81	89	88	84	86	86	81	89
70	88	86	89	86	89	84	90	89	86	87	87	84	90

Table C.5: Retirement ages with different level of truncation (β and $\alpha = 1$)

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