# VOLATILITY-REDUCING BIODIVERSITY CONSERVATION UNDER STRATEGIC INTERACTIONS

Emmanuelle Augeraud-Véron, Giorgio Fabbri and Katheline Schubert

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EMMANUELLE AUGERAUD-VÉRON, GIORGIO FABBRI,
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ABSTRACT. We study a model of strategic competition among farmers for land use in an agricultural economy. Each agent can take possession of a part of the collective forest land and convert it to farming. Unconverted forest land helps preserving biodiversity, which contributes to reducing the volatility of agricultural production. Agents' utility is given in terms of a Kreps Porteus stochastic differential utility capable of disentangling risk aversion and aversion to fluctuations.

We characterize the land used by each farmer and her welfare at the Nash equilibrium, we evaluate the over-exploitation of the land and the agents' welfare loss compared to the socially optimal solution and we study the drivers of the inefficiencies of the decentralized equilibrium.

After characterizing the value of biodiversity in the model, we use an appropriate decomposition to study the policy implications of the model by identifying in which cases the allocation of property rights is preferable to the introduction of a land conversion tax.

**KEY WORDS:** Biodiversity, insurance value,land conversion, recursive preferences, stochastic differential games.

**JEL CLASSIFICATION:** Q56, Q58, Q10, Q15, O13, O20, C73, D62.

## 1. INTRODUCTION

According to FAO (2018) projections, agricultural production is expected to increase by 70% between 2005 and 2050 to feed a population that is expected to reach 9.1 billion people by 2050. Whereas part of this increase will come from higher agricultural yields, another part will inevitably require the extension of agricultural land (by 5% per year, FAO, 2018), at the expense of natural ecosystems. This land conversion phenomenon is, and will be in the future, one of the main drivers of biodiversity destruction (IPBES, 2019).

The evaluation of the phenomenon demands to appraise the trade-off between agricultural production and damages due to biodiversity loss. It is part of a broader reflexion on the development of theoretical tools for identifying the economic value of biodiversity. Traditional approaches evaluate direct and indirect use values (see for instance Simpson *et al.*, 1996, for the case of the pharmaceutical industry or Maille and Mendelsohn, 1993, for ecotourism) or, in a context of uncertainty, consider the option and quasi-option values of biodiversity (see e.g. Lasserre and Kassar, 2004). Again looking at the relations between biodiversity and risk a more recent stream of literature introduced and exploited the notion of *insurance value* of biodiversity. The idea comes from the recognition that a series of services offered by biodiversity have an impact on the variability of ecological and then economic conditions. It is the case, for example, with regulating services (control of the local climate, flood control, regulation of soil fertility, pollination...) or biocontrol services (pest control, resistance to plant invasion, disease control...). In a series of contributions Baumgärtner (2007), Quaas *et al.* (2007), Quaas and Baumgärtner (2008), Baumgärtner and Quaas (2010) and Baumgärtner and Strunz (2014) measured the insurance value as the reduction of the risk premium that can be achieved by relying on conservative ecosystem management. Augeraud-Véron *et al.* (2019) [1] analyzed the total value of biodiversity, as the welfare gain that can be attributed to biodiversity conservation in a dynamic framework, distinguishing between the notion of risk and the notion of intertemporal substitutability. That study also highlighted the insurance value of biodiversity in a dynamic set-up.

However, as far as the trade-off between agricultural production and biodiversity conservation is concerned, the study of the decentralized solution is also

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[1] See also Quaas *et al.* (2019) for a similar argument applied to a model with natural capital.

necessary.[2] Indeed, even if in some countries the number of farmers is decreasing, agriculture remains a highly distributed activity, for which farmers' productive choices act as an externality on the profit of others.

In this work we develop and study a decentralized model where two farmers[3] have free access to land, that is a common-property resource. Each farmer chooses the share of land she converts to agriculture, then produces and consumes the harvest from her plot. The evolution of agricultural productivity over time is influenced by the endogenous choices about the size of the plots. More precisely we suppose that the part of land which is not used for farming is devoted to biodiversity conservation and, as already explained, that services from the biodiversity attenuate future uncertainty on agricultural productivity. The two farmers/players coordinate on a Nash equilibrium of the economy.

In order to disentangle aversion to fluctuations and aversion to risk we use a recursive utility set-up, initially introduced in the seminal works of Epstein and Zin (1991) and of Duffie and Epstein (1992). The relevance of recursive utilities in natural resources and environmental economic models is well documented (see for instance Knapp and Olson, 1996, Epaulard and Pommeret, 2003 or Howitt *et al.*, 2005) but this is, as far as we know, the first work where a dynamic environmental economic model with strategic interaction and Epstein–Zin preferences is studied and solved.

The proposed model is completely solved by explicitly finding the Nash equilibrium of the differential game, characterizing the corresponding portion of land used for agricultural activities and for the preservation of biodiversity and, finally, identifying the welfare of the agents.

To understand the impact of decentralization we compare, in the symmetric situation with uniform preference and technology parameters, the equilibrium in the strategic competition set-up and the social optimal choice. As expected, since the agents only partly internalize the negative effects of their actions, an

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[2]The trade-off between biodiversity and agriculture has been mostly studied in the context of agro-biodiversity. Indeed, monoculture favors the emergence of disease and increases production risks. In a survey, Heal *et al.* (2004) look at a game set on the genetic diversity in crops, showing that due to external effect, the diversity is lower than the social optimal level. Baumgärtner and Quaas (2010) study the insurance value of agro-diversity.

[3]The results for the *n*-farmer case are also provided, see Section 6.

over-exploitation of land systematically arises, so that the land available for bio-diversity conservation is always lower in the decentralized case than at the social optimum.

We further proceed in two steps. First we delve further into how the phenomenon of over-exploitation depends on natural and technological conditions and on agents' preferences. Then we identify the policy implications of the model.

After characterizing the dependencies of best responses of the agents in terms of their preferences and other agents' ones, we better quantify the over-exploitation taking into account two measures: the *relative increase of the land devoted to farming* (and then, symmetrically, the relative loss of land devoted to biodiversity conservation) and the per-capita *welfare gain from biodiversity conservation*, and we look at their behavior with respect to model parameters. While aversion to fluctuations has a sharply negative effect on both these variables, they are completely insensitive to risk aversion.

To investigate the policy implications of the model we show how to decompose the per-capita welfare gain from biodiversity conservation in two parts: the welfare gain of moving from the open access equilibrium to the Nash equilibrium, and the welfare gain of moving from the Nash equilibrium to the optimum. Depending on which of the two is prevalent we can identify in which cases the allocation of property rights is preferable to the introduction of a land conversion tax. In particular we show that enforcing property rights is more relevant in case of stagnant technological progress in agricultural activities (especially for low levels of aversion to fluctuations) while policies consisting in pricing the volatility externality (for instance through a tax on land conversion) are more suited for rapidly developing economics.

The paper proceeds as follows. In Section 2 we present the main elements of the model which is completely solved in Section 3. Section 4 is devoted to the comparison between the two-players setting and the social optimum benchmark while in Section 5 we discuss the policy hints that we get from the model. In Section 6 we look at the results for the $n$-player extension of the model and in Section 7 we conclude. The appendices contains technical details and the proofs of all the results.

## 2. The model

We develop and study a decentralized version of the model proposed by Augeraud-Véron *et al.* (2019). We describe here a 2-player setting; the $n$-player extension is discussed in Section 6.

We consider an agricultural economy with a stock $L = 1$ of land. Land is a common property resource. There are two farmers in this economy, who can appropriate land for farming purposes. Land left undeveloped supports biodiversity and provides ecosystem services. The larger the undeveloped land area, the larger the number of species and the higher the level of biodiversity. The two farmers are indexed by $i \in \{1, 2\}$. Farmer $i$ can appropriate some share $f_i(t)$ of total land, although the following constraint needs to be verified:

$$(1) \qquad f_1(t) + f_2(t) \le 1, \qquad t \ge 0.$$

We denote by $f(t) := f_1(t) + f_2(t) \in [0, 1]$ and $1 - f(t) \in [0, 1]$ the shares of land respectively used for farming and left undeveloped, that is devoted to biodiversity conservation.

We assume that the agricultural production output of farmer $i$ at time $t$ is given by:

$$(2) \qquad Y_i(t) = f_i(t) A_i(t)$$

where $A_i(t)$ is the productivity of a unit of land farmed by agent $i$ at time $t$, whose dynamics is described by the following stochastic differential equation:

$$(3) \qquad \begin{cases} \mathrm{d}A_i(t) = \alpha_i A_i(t)\,\mathrm{d}t + \sigma_i f(t)^{1/2} A_i(t)\,\mathrm{d}W(t) \\ A_i(0) = A_i^0 \end{cases}$$

where $W$ is a real standard Brownian motion $W \colon [0, +\infty) \times \Omega \to \mathbb{R}$, adapted to some filtration $\mathcal{F}_t$, defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. In such an expression, $\alpha_i \in \mathbb{R}$ represents some fixed and exogenous rate of technological progress in farming activities (it can be equal to 0), and the term $\sigma_i f(t)^{1/2}$ measures the volatility of agricultural productivity. Total volatility has an exogenous component $\sigma_i$, stemming for instance from weather events, and an endogenous component $f(t)^{1/2}$, common to both farmers, increasing in the total share of land converted to agriculture, that is decreasing in the biodiversity level[4]. It is in

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[4]Augeraud-Véron et al. (2019) write more generally this endogenous component as: $f(t)^{\chi/2}$, $\chi \ge 1$. We restrict ourselves to the case $\chi = 1$ for simplicity, without loss of generality.

this sense that biodiversity appears in the model as insurance against adverse outcomes.

The two farmers play a non-cooperative game with Markovian strategies so that farmer $i$ chooses the size of land to cultivate in the form $f_i = \Psi_i(A_1, A_2)$ where $\Psi_i \colon \mathbb{R}^2_+ \to [0, 1]$ is a measurable function. A couple of Markov strategies $\Psi := (\Psi_1, \Psi_2)$ is admissible if (1) is verified for all positive couple of real numbers $(A_1, A_2)$, i.e. if

$$\Psi_1(A_1, A_2) + \Psi_2(A_1, A_2) \leq 1, \quad \text{for all } (A_1, A_2) \in \mathbb{R}^2_+$$

and if the two equations (3) have a unique solution.[5] We denote by $\mathcal{C}$ the set of admissible strategy couples:

$$\mathcal{C} := \left\{ \Psi := (\Psi_1, \Psi_2) \colon \mathbb{R}^2_+ \to [0, 1]^2 \text{ measurable } : \Psi \text{ is an admissible strategy couple } \right\}$$

Given a strategy $\Psi_1$ of Player 1 (respectively $\Psi_2$ of Player 2), we denote by $\mathcal{C}_2(\Psi_1)$ (respectively $\mathcal{C}_1(\Psi_2)$) the set

$$\mathcal{C}_2(\Psi_1) := \left\{ \Phi \colon \mathbb{R}_+ \to [0, 1] \text{ measurable } : (\Psi_1, \Phi) \in \mathcal{C} \right\}$$

(and similarly for $\mathcal{C}_1(\Psi_2)$).

We suppose that at each time $t \geq 0$ all the production is consumed:

$$(4) \qquad\qquad\qquad C_i(t) = Y_i(t).$$

Farmer $i$'s target is to maximize an infinite horizon, continuous time, Epstein-Zin-Weil utility function with Kreps Porteus utility. This specification of utilities makes it possible to disentangle the notions of aversion to risk and aversion to fluctuations which are identified (and conflated) in the usual time additive expected utility function i.e. in the specifications where the target can be written as the expected value of utility on each trajectory. We denote by $\theta_i > 0$ (and $\theta_i \neq 1$) the (constant) relative risk aversion of Player $i$ and by $\phi_i > 0$ her aversion to fluctuations i.e. the inverse of her intertemporal elasticity of substitution. The case $\theta_i = \phi_i$ corresponds to the usual time additive case. The discount rate of Player $i$ is denoted by $\rho_i > 0$.

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[5]This second requirement is not very strong: the term $f$ is bounded (by 0 and 1) so the system of equations for $A$ has always linear growth, so that, by standard existence and uniqueness results for the solution of a system of SDE (see Chapter V of Protter, 2005), we only need to have weak Lipschitz-type continuity in the diffusion term (w.r.t. of $A_i$) and some mild regularity of $\Psi$ to guarantee the existence and the uniqueness of the solution.

In this framework the problem focuses on the study of a pair of Hamilton-Jacobi-Bellman (HJB) equations of the following form:

$$\rho_i \frac{1 - \theta_i}{1 - \phi_i} V_i(A_i) = \max_{f_i \in [0, 1 - f_j]} \left[ \frac{(f_i A_i)^{1 - \phi_i}}{1 - \phi_i} \frac{1}{((1 - \theta_i) V_i(A_i))^{\frac{1 - \phi_i}{1 - \theta_i} - 1}} + X_i(A_i, f_1 + f_2) \right]$$

where $V_i(A_i)$ is the value function of Player $i$ evaluated on some trajectory $A_i$, and

$$X_i(A_i, f_1 + f_2) = V_i'(A_i) A_i \alpha_i + \frac{1}{2} V_i''(A_i) A_i^2 \sigma_i^2 (f_1 + f_2).$$

More technical details on the formulation we use can be found in Appendix A. The study of the HJB equations is carried out in Appendix B.

## 3. The characterization of the strategic interaction equilibrium

In this section we see how to solve the model we presented above and get an explicit description of the Nash equilibrium. Let's start by defining the quantity

$$(5) \qquad x_i = \frac{\rho_i - \alpha_i(1 - \phi_i)}{\theta_i \frac{\sigma_i^2}{2}}, \qquad \text{for } i = 1, 2.$$

that will be frequently used blow. The strategies of the two players to balance are described in the following theorem.

**Theorem 3.1.** *Suppose that*

$$(6) \qquad \rho_i > (1 - \phi_i)\alpha_i, \qquad \text{for } i = 1, 2$$

*(it implies in particular $x_1$ and $x_2$ positive),*

$$(7) \qquad \phi_1 + \phi_2 > 1$$

*and that the following transversality condition is verified:*

$$(8) \qquad \rho_i > (1 - \theta_i) \left( \alpha_i - \theta_i \frac{\sigma_i^2}{2} \frac{x_1 + x_2}{\phi_1 + \phi_2 - 1} \right), \qquad \text{for } i = 1, 2.$$

*Then, if*

$$(9) \qquad \bar{f}_1 := \frac{\phi_2 x_1 + (1 - \phi_1) x_2}{\phi_1 + \phi_2 - 1} \in (0, 1)$$

*and*

$$(10) \qquad \bar{f}_2 := \frac{\phi_1 x_2 + (1 - \phi_2) x_1}{\phi_1 + \phi_2 - 1} \in (0, 1 - \bar{f}_1),$$

*the couple*

$$\Psi_2(A_1, A_2) \equiv \bar{f}_2, \qquad \Psi_1(A_1, A_2) \equiv \bar{f}_1$$

*is a Nash equilibrium of Markov strategies. Moreover it is the unique Nash equilibrium of Markov strategies where strategies are constant and deterministic.*

*Proof.* See Appendix B.                                                      □

Using the expressions of the land used by each player at the equilibrium we can easily determine the total amount of land converted for agricultural purposes and therefore, in a complementary way, the land available to preserve biodiversity. The result is contained in the corollary below.

**Corollary 3.2.** *Suppose that hypotheses of Theorem 3.1 are satisfied. Then the total land surface devoted to farming is*

$$(11) \qquad \bar{f} := \bar{f}_1 + \bar{f}_2 = \frac{x_1 + x_2}{\phi_1 + \phi_2 - 1}.$$

To complete the analysis of the system at the equilibrium we can finally calculate the utility of each of the players:

**Proposition 3.3.** *Suppose that the hypotheses of Theorem 3.1 are verified. Then, along the described equilibrium, the welfare of the two players is*

$$(12) \qquad V_i\left(A_i\right) = \frac{\beta_i}{1 - \theta_i} A_i^{1-\theta_i}$$

*with*

$$(13) \qquad \beta_i = \left(\theta_i \frac{\sigma_i^2}{2} \bar{f}_i^{\phi_i}\right)^{-\frac{1-\theta_i}{1-\phi_i}}$$

*where the expression of $x_i$ and $\bar{f}_i$ are given respectively in (5) and in (9)-(10).*

*Proof.* It is a corollary of the proof of Theorem 3.1. See Appendix B.     □

A first study of the results of the model is possible using the comparative analysis contained in the following proposition.

**Proposition 3.4.** *Suppose that the hypotheses of Theorem 3.1 are satisfied. Then we have the following dependencies of $\bar{f}_i$ on the parameters characterizing Player i:*

$$\frac{\partial \bar{f}_i}{\partial \rho_i} > 0, \qquad \frac{\partial \bar{f}_i}{\partial \sigma_i} < 0, \qquad \frac{\partial \bar{f}_i}{\partial \theta_i} < 0,$$

*and on the parameters characterizing Player $j$:*

$$
\begin{cases}
\frac{\partial \bar{f}_i}{\partial \rho_j} \geq 0, & \frac{\partial \bar{f}_i}{\partial \sigma_j} \leq 0, & \frac{\partial \bar{f}_i}{\partial \theta_j} \leq 0, & \text{if} \quad \phi_i \leq 1 \\
\frac{\partial \bar{f}_i}{\partial \rho_j} < 0, & \frac{\partial \bar{f}_i}{\partial \sigma_j} > 0, & \frac{\partial \bar{f}_i}{\partial \theta_j} > 0, & \text{otherwise}
\end{cases}
$$

*Proof.* See Appendix B.                                                   □

The trade-off agent $i$ faces when she chooses the amount of land $f$ she converts to agriculture is indeed between consuming more today (higher $f_i$) and having a smaller uncertainty tomorrow (higher $1 - f_i$). Therefore the fact that $\frac{\partial \bar{f}_i}{\partial \rho_i} > 0$ is rather intuitive: the higher is the subjective discount rate, the lower the importance of the future and then the higher the relative weight the player assigns to present consumption and the higher the $f_i$ she will choose. The effects of $\theta_i$ and $\sigma_i$ are pure risk-aversion effects: the higher $\sigma_i$ (respectively $\theta_i$) the higher the future volatility (respectively its perception) and then the higher the effort of the agent to reduce it.

To understand the effects of preference parameters of Player $j$ on the value of $\bar{f}_i$ it is instructive to have a look at the best response of Player $i$ to a generic choice $\bar{f}_j$ of Player $j$. It reads (see equations (45) and (47) in Appendix B):

$$
\bar{f}_i = \frac{x_i + (1 - \phi_i)\bar{f}_j}{\phi_i}
$$

Then:

$$
\frac{d\bar{f}_i}{d\bar{f}_j}
\begin{cases}
> 0 & \text{if} \quad \phi_i < 1 \\
= 0 & \text{if} \quad \phi_i = 1 \\
< 0 & \text{otherwise}
\end{cases}
$$

If Player $i$'s aversion to intertemporal fluctuations $\phi_i$ is smaller than 1, her best response is to increase her share of land devoted to farming when Player $j$ increases her own. In this case, there is strategic complementarity. It is the reverse if $\phi_i$ is larger than 1: Player $i$, who is now strongly averse to intertemporal fluctuations, will decrease her share of land devoted to farming when Player $j$ increases her own. Now the decisions of the two players are strategic substitutes.

The total amount of land, that has to be divided between three uses: farming by Player 1, farming by Player 2, and biodiversity conservation, is fixed. When Player $j$ chooses to convert more forest to farming, Player $i$, expecting more volatility of agricultural productivity in the future, may either increase $f_i$ to increase its average consumption at each date, or, on the contrary, decrease $f_i$ to

counteract the volatility increase due to the behavior of Player $j$. The first effect is stronger than the second one when Player $i$ is not very averse to intertemporal fluctuations, and vice-versa. In case of strategic complementarity, the discount rate and the risk parameters of Player $j$ affect the behavior of Player $i$ in the same way as her owns do. It is the reverse in case of strategic substitutability. When $\phi_i = 1$ the two effect compensate exactly, and the decision of Player $i$ is independent on the choice made by Player $j$.

## 4. THE SYMMETRIC CASE AND THE COMPARISON WITH THE OPTIMAL SOLUTION

To compare now the results with the optimal solution presented by Augeraud-Véron *et al.* (2019) we consider the case where parameters ($\alpha$, $\sigma$, $\rho$, $\theta$ and $\phi$) do not depend on $i$. We also suppose that the initial value of $A$ (and then the value of $A$ all the time, since the two players are identical they will take exactly the same decisions) is the same for both farmers. More precisely we compare the effect of the strategic interaction presented in the previous section with a planner which gives, in the social welfare function, the same weight to each farmer. We start by re-stating the result of Section 3 in this circumstances. In the next two subsections we will compare the results with the social planner benchmark in two perspectives: the biodiversity conservation and the welfare of the agents.

Define

$$x = \frac{\rho - \alpha(1 - \phi)}{\theta \frac{\sigma^2}{2}}.$$

The results of Theorem 3.1 and Proposition 3.3 read as follows.

**Corollary 4.1.** *Suppose that*

(14)
$$\rho > (1 - \phi)\alpha$$

*(it implies in particular that $x$ is positive),*

(15)
$$\phi > 1/2$$

*and that the following transversality condition is verified:*

(16)
$$\rho\left(1 + 2(\phi - \theta)\right) > (1 - \theta)\alpha.$$

*Suppose that*

(17)
$$\bar{f}_h := \frac{x}{2\phi - 1} \in \left[0, \frac{1}{2}\right].$$

*Then the couple*

$$\Psi_2(A_1, A_2) \equiv \bar{f}_h, \qquad \Psi_1(A_1, A_2) \equiv \bar{f}_h$$

*is a Nash equilibrium of Markov strategies. Moreover it is the unique Nash equilibrium of Markov strategies where strategies are constant and deterministic. The total land used for farming is then*

$$(18) \qquad \bar{f} := 2\bar{f}_h = \frac{2x}{2\phi - 1} \leq 1.$$

*The corresponding welfare of each farmer is*

$$(19) \qquad V_h(A) = \frac{\beta_h}{1 - \theta} A^{1-\theta}$$

*with*

$$(20) \qquad \beta_h = \left( \theta \frac{\sigma^2}{2} 2^{-\phi} \bar{f}^{\phi} \right)^{-\frac{1-\theta}{1-\phi}}.$$

The expression of $\bar{f}$ given in (18) is particularly transparent. One can first easily verify that $\bar{f}$ is always a decreasing function of $\theta$. This is the same "risk hedging" reaction we have seen in Proposition 3.4 for the dependence of the values of $\bar{f}_1$ and $\bar{f}_2$ on $\theta_1$ and $\theta_2$. On the other hand we can observe that $\bar{f}$ is a decreasing function of $\phi$ if and only if $\alpha < 2\rho$. This condition is indeed the comparison of the magnitudes of the growth and discount rates. When the trend of agricultural productivity is smaller than twice the discount rate, farmers' future prospects are not good. Then the more averse to fluctuations they are the more willing they are to insure against bad outcomes in the future by conserving more biodiversity. The opposite occurs when $\alpha > 2\rho$.

Notice that when $\alpha > 2\rho$, the two parameters characterizing preferences play in opposite directions. This is a strong argument in favor of disentangling the two parameters. See the discussion in Subsection 4.2.

## 4.1. The effect of strategic competition on land use and biodiversity.
We recall that the part $f^*$ of land used for farming in the social planner model of Augeraud-Véron *et al.* (2019) is equal to

$$(21) \qquad f^* = \frac{x}{\phi}.$$

The restrictions on the parameters which are needed to obtain this result are (14), a transversality condition which reads:

$$\rho\left(1 + (\phi - \theta)\right) > (1 - \theta)\alpha, \tag{22}$$

and the condition $\frac{x}{\phi} \in [0, 1]$. So, all in all, the conditions that allow to work in the intersection of the hypotheses needed for the current results and for the social planner case are the following: (14), (15), and

$$\rho\left(1 + \min\left[\phi - \theta, 2(\phi - \theta)\right]\right) > (1 - \theta)\alpha. \tag{23}$$

When they are satisfied,

$$\begin{cases} f^* = \frac{x}{\phi} & \text{if } x < \phi, \quad 1 \quad \text{otherwise} \\ \bar{f} = \frac{x}{\phi - \frac{1}{2}} & \text{if } x < \phi - \frac{1}{2}, \quad 1 \quad \text{otherwise.} \end{cases} \tag{24}$$

The optimal conversion rate increases with the discount rate $\rho$, decreases when intrinsic volatility $\sigma$ increases, decreases when risk aversion $\theta$ increases, decreases when aversion to fluctuations $\phi$ increases if $\alpha < \rho$, and vice versa. Notice that, for $\rho < \alpha < 2\rho$, $f^*$ increases with $\phi$ whereas $\bar{f}$ decreases with $\phi$.

Define $\Lambda = \frac{\bar{f}}{f^*}$ the ratio of total land converted to farming at the Nash equilibrium and at the optimum. We have:

$$\Lambda = \begin{cases} \frac{\phi}{\phi - \frac{1}{2}}, & \text{if } x < \phi - \frac{1}{2} \\ \frac{\phi}{x}, & \text{if } \phi - \frac{1}{2} \leq x < \phi \\ 1, & \text{if } \phi \geq x. \end{cases} \tag{25}$$

**Proposition 4.2.** *Suppose that (14), (15) and (23) are satisfied. Then $\Lambda \geq 1$ so that the land used for biodiversity conservation is weakly smaller in the strategic competition situation than that chosen by the social planner. $\Lambda$ is strictly greater than 1 as soon as the optimal land devoted to farming is strictly smaller than 1.*

*Proof.* Direct inspection of formulas in (25). □

Proposition 4.2 shows that a *tragedy of the commons* mechanism is at work in the model: the exploitation of the natural resource is larger in the strategic competition set-up than at the social optimum and then the land used for biodiversity conservation is smaller. This is what we expected since the two players do not completely internalize the volatility externality: the conversion decisions of each farmer affect the volatility of agricultural productivity and then the utility of both. In the following proposition we try to dig a little deeper into this result.

**Proposition 4.3.** *Suppose that (14), (15) and (23) are satisfied and that $x \leq \phi - \frac{1}{2}$, so that the solution is interior at the optimum and at the Nash equilibrium. Then $\Lambda$ does not depend on the risk aversion $\theta$ and is decreasing in the aversion to fluctuations $\phi$.*

*Proof.* Direct inspection of the first formula in (25). $\qquad\square$

The previous proposition identifies the parameters that play a role in the gap between the optimal choice of land conversion and the outcome of a strategic competition for land appropriation between two identical farmers. The result is sharp: even if the risk aversion has a role in establishing the share of land devoted to production it has no impact on the gap between the optimal and the decentralized solutions. Conversely the elasticity of intertemporal substitution parameter does play a role: the smaller it is the higher is $\Lambda$, that is the farther is the Nash solution from the optimal one.

4.2. **A technical *excursus*: comments on the role of the Epstein-Zin-Weil utility.** In this paper we have chosen to use the Epstein-Zin-Weil specification of preferences. Of course as a general statement there are several good reasons to disentangle the effect of risk aversion and intertemporal elasticity of substitution: they are related to different characteristics of the individuals, one concerning intra-temporal preferences, the other (as the name suggests) intertemporal behavior.

At the level of our model the relevance of this distinction is immediately clear once we look at the realistic possible values of the parameters (see for instance Barro, 2015). Plausible calibrations of the inverse of the intertemporal elasticity of substitution $\phi$ are smaller than 1 while the relative risk aversion parameter $\theta$ is typically largely higher than 1 (3 and more). This fact, besides pointing out once again the unlikelihood of the expected utility case $\theta = \phi$, has qualitative implications in the model.

Disentangling the effect of risk aversion and intertemporal elasticity of substitution often gives the possibility of more deeply understanding some behaviors of the model. To clearly make the point consider for instance the value of the variable $x$ which appears several times in the paper. Its expression is

$$x = \frac{\rho - \alpha(1 - \phi)}{\theta \frac{\sigma^2}{2}}$$

while its "expected utility counterpart" is given by

$$x_{EU} = \frac{\rho - \alpha(1 - \varphi)}{\varphi \frac{\sigma^2}{2}}$$

where $\varphi$ is the unique parameter appearing in the expected utility case. One can readily see that the derivative $\frac{\partial x_{EU}}{\partial \varphi}$ does not have a univocal sign while the two derivatives $\frac{\partial x}{\partial \theta}$ and $\frac{\partial x}{\partial \phi}$ are respectively always negative and positive (under assumption (6)) so that introducing the two-parameters Epstein-Zin-Weil specification allows to resolve the ambiguity that we observe in the expected utility case. This explanatory effect of the Epstein-Zin-Weil specification on $x$ brings a similar results to more interesting variables. If we look for instance at the expected utility counterpart of (18) (we could comment in a similar way (9) and (10)), it reads as

$$\bar{f}_{EU} = \frac{2x_{EU}}{2\varphi - 1} = \frac{2\frac{\rho - \alpha(1 - \varphi)}{\varphi \frac{\sigma^2}{2}}}{2\varphi - 1}.$$

The dependence of this expression on $\varphi$ relies on various relative magnitudes of the parameters and they are not easy to justify, while the signs of the two derivatives $\frac{\partial x}{\partial \theta}$ and $\frac{\partial x}{\partial \phi}$ is net as we stressed in the discussion below Corollary 4.1: $\bar{f}$ is always a decreasing function of $\theta$ and it is an increasing function of $\phi$ if and only if $\alpha > 2\rho$. As discussed above these results have a neat economic interpretation.

Observe that in particular, as the two variables indicate different notions and characteristics, their effects on the agent's behavior can be very different. We have already mentioned above that, as far as $\alpha > 2\rho$ the role of $\theta$ and $\phi$ in establishing the size of the land devoted to farming (and then the size of the land devoted to biodiversity conservation) at the equilibrium $\bar{f}$ is opposite. This is also true for the size of the land devoted to farming at the social optimum $f^*$.

## 5. THE EFFECT OF STRATEGIC COMPETITION ON WELFARE AND POLICY IMPLICATIONS OF THE MODEL

5.1. **The per-capita welfare gain of moving from the Nash equilibrium to the optimum.** We look now at the welfare loss w.r.t. the optimum, in the most interesting case where parameters are such that the solution at the Nash equilibrium is interior ($\bar{f} < 1$,) which guarantees that the optimal solution is interior as well.

The welfare of each farmer at the Nash equilibrium is given in Proposition 3.3. At the optimum, the planner fixes the level of $f$ at $f^*$ and then she gives half of total production to each farmer[6]. Since the production is linear in productivity, the welfare of each farmer is equal to the total welfare $V_p$ computed in Augeraud-Véron *et al.* (2019), calculated for a productivity level equal to $A/2$, where

$$(26) \qquad V_p(A) = \frac{1}{1-\theta}\beta_p A^{1-\theta}$$

with

$$(27) \qquad \beta_p = \left(\theta\frac{\sigma^2}{2}(f^*)^\phi\right)^{-\frac{1-\theta}{1-\phi}}.$$

We denote $\omega$ the per capita welfare gain of moving from the Nash equilibrium to the optimum. It is defined by:

$$(28) \qquad V_p\left(\frac{A}{2}\right) = V_h((1+\omega)A).$$

Using equations (26), (27), (19) and (20) we obtain:

$$(29) \qquad \omega = \left[\frac{1}{2}\left(\frac{\phi}{\phi-\frac{1}{2}}\right)^\phi\right]^{\frac{1}{1-\phi}} - 1.$$

Similar to what we demonstrated in Propoition 4.3 for the ration $\Lambda$ we have, for the per-capita welfare gain the following result.

**Proposition 5.1.** *Suppose that hypotheses of Corollary 4.1 are verified. $\omega$ does not depend on risk aversion, and is a decreasing function of aversion to fluctuations.*

*Proof.* See Appendix B. $\qquad\qquad\square$

**5.2. Decomposition of the total value of biodiversity and policy implications of the model.** In reference of Lucas' welfare cost of fluctuations (Lucas, 1987), the total value of biodiversity can be defined as the welfare gain from biodiversity conservation at its optimal value compared to no conservation at all. The case where all land is converted to farming and no biodiversity is conserved can be interpreted as the consequence of a land in open access, where property rights are absent or poorly defined or enforced. Then each farmer appropriates

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[6]We suppose implicitly that the planner affects the same weight to each farmer in the social welfare function.

1/2 of total land and all land is converted to farming. Following Augeraud-Véron *et al.* (2019), the value function in this case is:

$$(30) \qquad V_{oa}(A) = \frac{1}{1-\theta} \beta_{oa} A^{1-\theta}$$

with

$$(31) \qquad \beta_{oa} = \left[ \rho - (1-\phi) \left( \alpha - \theta \frac{\sigma^2}{2} \right) \right]^{-\frac{1-\theta}{1-\phi}}.$$

The per capita welfare gain from biodiversity conservation is then $\lambda$ defined by:

$$(32) \qquad V_p \left( \frac{A}{2} \right) = V_{oa} \left( (1+\lambda) \frac{A}{2} \right).$$

Using equations (26), (27), (30) and (31) we obtain:

$$(33) \qquad \lambda = \left( \frac{\phi f^* + (1-\phi)}{f^{*\phi}} \right)^{\frac{1}{1-\phi}} - 1.$$

$\lambda$ decreases when the discount rate $\rho$ increases, increases (resp. decreases) when the trend of agricultural productivity increases if $\phi < 1$ (resp. $> 1$), increases with the intrinsic volatility $\sigma$, increases with the risk aversion $\theta$ and is an ambiguous function of the aversion to fluctuations $\phi$. Augeraud-Véron *et al.* (2019) show that when $\alpha < \rho$, $\lambda$ unambiguously increases with $\phi$. In this case, a higher aversion to fluctuations translates into more biodiversity conservation and a higher value of biodiversity. In the opposite case where $\alpha > \rho$, the effect of $\phi$ on $\lambda$ is ambiguous. Then, increasing aversion to fluctuations decreases the share of land devoted to biodiversity conservation and may at the same time increase the total value of biodiversity.

The total insurance value of biodiversity $\lambda$ can be decomposed in two parts: the welfare gain of moving from the open access equilibrium to the Nash equilibrium, denoted $\bar{\lambda}$, and the welfare gain of moving from the Nash equilibrium to the optimum, $\omega$. Assessing the relative magnitudes of these two components is important for policy. Indeed, if the insurance value of biodiversity is mostly due to the departure from the open access situation, then the appropriate policy consists in dividing total land equally among farmers and giving them secure property rights on their plot; if on the contrary it is mostly due to the lack of internalization of the volatility externality, then the appropriate policy consists in pricing the volatility externality, for instance through a tax on land conversion.

Similarly to the definition of $\lambda$, $\bar{\lambda}$ is defined by:

$$(34) \qquad V_h\left(\frac{A}{2}\right) = V_{oa}\left((1+\bar{\lambda})\frac{A}{2}\right).$$

It is easy to see that the decomposition reads:

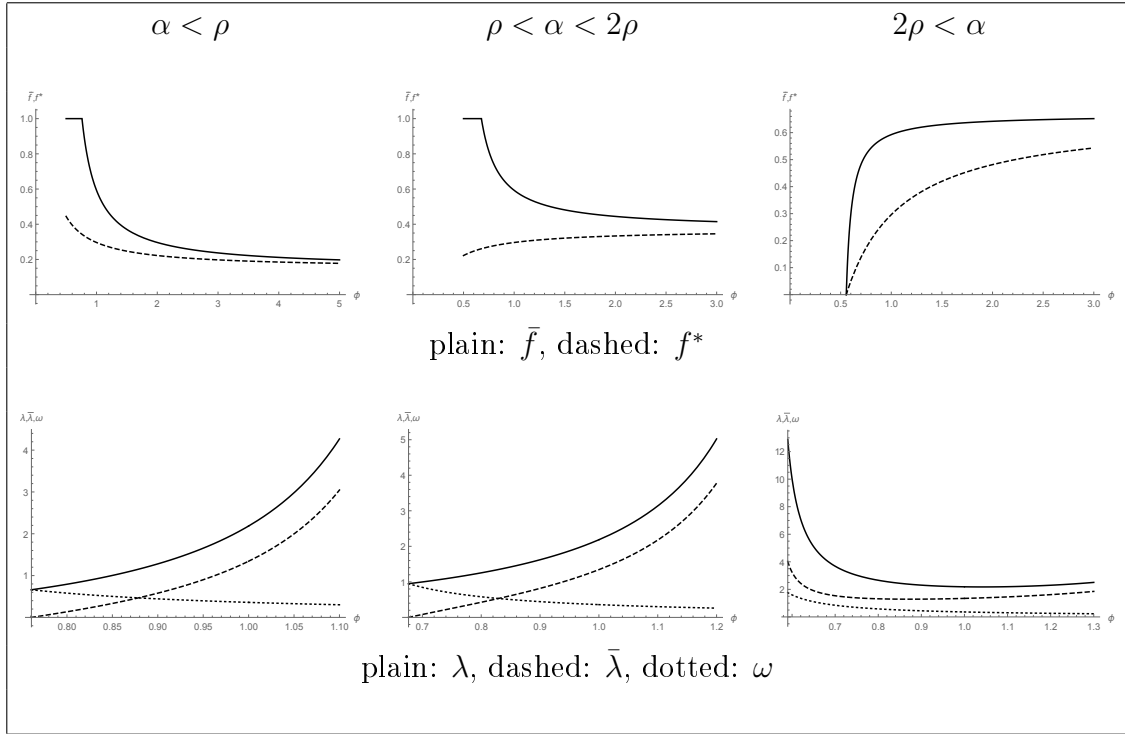$$(35) \qquad 1 + \lambda = (1+\omega)(1+\bar{\lambda}).$$



FIGURE 1. Land conversion (upper panel) and welfare gains (lower panel) as functions of aversion to fluctuations

Figure 1 shows the amount of land converted to farming at the optimum and at the Nash equilibrium (upper panel) and the three welfare gain measures (lower panel) as functions of $\phi$, for $\theta = 3$, $\rho = 0.04$ and $\sigma = 0.3$, in three configurations: $\alpha = 0.02 < \rho$, $\rho < \alpha = 0.05 < 2\rho$, $2\rho < \alpha = 0.09$. The value chosen for $\theta$ is empirically relevant (see for instance Barro, 2015); the value of $\rho$ is in the standard range considered in dynamic models and it is for instance consistent with the data from Lopez (2008); finally, the range of values of $\alpha$ is consistent with the growth rate observed in developing countries (see e.g. World Bank Group, 2018).

We have chosen on purpose a very high volatility of agricultural productivity (30 %) in the case where all land is converted to farming, to reflect firstly the fact that agricultural output is more volatile than global output, and second that without the dampening effect of biodiversity this volatility will become very high. Finally, in the comparison of the welfare gains (lower panel of Figure 1), the value of the aversion to fluctuations is restricted to empirically plausible values, around 1. These values ensure that the restrictions necessary to obtain interior solutions and the transversality conditions are satisfied. All in all, our calibration is related to the case of biodiversity-rich developing countries where agricultural productivity is very volatile, deforestation and land conversion are an issue, and farmers have a preference for the early resolution of uncertainty (Gollier, 2001), in the sense that they are more risk averse than they are concerned about smoothing consumption.[7]

In the first two cases depicted on Figure 1 ($\alpha < \rho$ and $\rho < \alpha < 2\rho$), the trend of agricultural productivity in relatively small compared to the discount rate. Then when aversion to fluctuations is small, the welfare gain of conserving the optimal level of biodiversity is mostly explained by the welfare gain of moving from the open access situation to the Nash equilibrium. It becomes less and less so as $\phi$ increases. In the third case ($\alpha > \rho$), the agricultural productivity trend is high and the effects are completely reversed. These results suggest that for a very low aversion to fluctuations the most relevant policy option in poorly productive agricultural economies is to put in place land property rights, whereas in highly productive economies it is to tax land conversion. For higher degrees of aversion to fluctuations taxing land conversion always dominate.

## 6. EXTENSION TO THE $n-$PLAYER CASE

The results obtained for the two-player case can be extended to the $n$-player case. In this section we briefly sketch the results in the general case.

As proved in Appendix C the best response function of Player $i$ to the decisions of the other players is:

$$(36) \qquad \bar{f}_i = \frac{1}{\phi_i} \left( x_i + (1 - \phi_i) \sum_{j \neq i} \bar{f}_j \right).$$

---

[7]See for instance Lybbert and McPeak (2012) for an empirical highlighting of the preference for an early resolution of uncertainty in the case of an agricultural economy.

As in the two-player case, there is strategic complementarity if $\phi_i < 1$, strategic substitutability if $\phi_i > 1$, and the decision of Player $i$ is independent on the choices of the other players when $\phi_i = 1$. Moreover, the interior Nash equilibrium is characterized by the following set of strategies:

$$(37) \qquad \bar{f}_i = \min \left( \frac{x_i \left( \sum_{\substack{j=1 \\ j \neq i}}^{n} \phi_j - (n-2) \right) + (1 - \phi_i) \sum_{\substack{j=1 \\ j \neq i}}^{n} x_j}{\sum_{j=1}^{n} \phi_j - (n-1)}, 1 - \sum_{\substack{j=1 \\ j \neq i}}^{n} f_j \right).$$

When preferences parameters are the same for all players and $\phi > 1 - \frac{1}{n}$, according to (37), the interior optimal quantity of land devoted to farming for each farmer in the $n-$player game is given by

$$\bar{f}_h = \frac{x}{n\phi + 1 - n} \in \left[ 0, \frac{1}{n} \right]$$

and then the total land devoted to farming is

$$\bar{f} = \frac{x}{\phi - \frac{n-1}{n}} \leq 1.$$

If can be noticed that

$$\frac{d\bar{f}_h}{dn} \begin{cases} > 0 & \text{if} \quad \phi < 1 \\ = 0 & \text{if} \quad \phi = 1 \\ < 0 & \text{otherwise} \end{cases} \quad \text{and} \quad \frac{d\bar{f}}{dn} > 0.$$

Not surprisingly, as the number of farmers increases, the total share of land devoted to biodiversity conservation decreases. The more farmers there are the bigger the volatility externality. The open access situation corresponds to the case of free entry, where new farmers appropriate land until all land is converted to farming.

More interestingly, how individual choices depend on the number of farmers is radically different whether the aversion to intertemporal fluctuations is less or more than 1. We have seen in Section 3 that farmers' decisions are strategic complements when $\phi < 1$; then, when the number of farmers increases every individual share of land devoted to farming increases as well. In this sense, a small aversion to intertemporal fluctuations is detrimental to biodiversity conservation.

Conversely, if farmers have a large aversion to intertemporal fluctuation ($\phi > 1$), farmers' decisions are strategic substitutes and when the number of farmers increases each one reduces its own farming.

In the $n-$player situation, the relationship $1 + \lambda = (1 + \omega)(1 + \bar{\lambda})$ still holds, with $\lambda$ unchanged and (see Appendix C)

$$\omega = \left( \frac{1}{n} \left( \frac{\phi}{\phi + \frac{1}{n} - 1} \right)^{\phi} \right)^{\frac{1}{1-\phi}} - 1$$

We can see that

$$\frac{\partial \omega}{\partial n} = \frac{\left( 1 - \frac{1}{n} \right) \left( \frac{\left( \frac{\phi}{\phi + \frac{1}{n} - 1} \right)^{\phi}}{n} \right)^{\frac{1}{1-\phi}}}{1 + n(\phi - 1)}$$

thus as $\phi > 1 - \frac{1}{n}$, $\omega$ is an increasing function of $n$. The more farmers there are the bigger the gain of putting in place land property rights.

## 7. CONCLUSION

Biodiversity is a multiform and articulated concept as are the ways to calculate its economic value. Biodiversity is, among other things, a flow of services that allows to reduce the variability of ecological and economic conditions and can therefore be thought of, among other things, as a natural form of insurance. What is the loss of biodiversity insurance value that results from the decentralized decision of individuals and volatility externalities? Under which conditions is this loss most pronounced? What policies are best suited to counter this loss?

In this article we propose a framework to answer these questions. For empirically plausible values of the parameters, we argue that, notably, enforcing properties rights is more effective in case of stagnant productivity in agricultural sector while taxes on land conversion are a more suited policy for rapidly developing economics.

A first possible model extensions could be the addition of production storage technologies. In this way the consumption at any given time would not need to be equal to production and the smoothing of the consumption trajectory could go through a new channel compared to the model presented in this paper. Another possible generalization could be taking into account catastrophic events that may

have an impact on production/productivity whose frequency or magnitude could be endogenously influenced by conservation choices as suggested by a great deal of evidence (for example, see Isbell *et al.*, 2015).

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## APPENDIX A. THE UTILITY SPECIFICATION AND THE DEFINITION OF EQUILIBRIUM

We spend some words to recall how this utility is defined. We start by defining the utility for the finite horizon problem with horizon $T > 0$. Chosen a couple of strategies $(\Psi_1, \Psi_2) \in \mathcal{C}$ we have, through (3), the dynamics of $A_1$ and $A_2$ and then, thanks to (2) the processes $Y_i$, and (with (4)) the consumptions paths. The Epstein-Zin-Weil utility (for player 1, the situation for player 2 is completely analogous) is defined (see

Duffie and Epstein, 1992a, 1992b) as the solution at time $t = 0$ of the following integral equation (it is a backward SDE with final condition equal to 0):

$$(38) \qquad U_1^{t,T}(\Psi_1, \Psi_2) = \mathbb{E}\left[\int_t^T F_1(C_1(s), U_1^{s,T}(\Psi_1, \Psi_2))\, ds \;\Big|\; \mathcal{F}_t\right]$$

where the aggregator $F$ can be chosen[8] in the following form (Epstein and Zin, 1991, Svensson, 1989 and and Duffie and Epstein, 1992a) :

$$(39) \qquad F_1(C, V) = \frac{\rho}{1 - \phi_i}(1 - \theta_i)V\left[\left(\frac{C}{\rho\big((1 - \theta_i)V\big)^{\frac{1}{1-\theta_i}}}\right)^{1-\phi_i} - 1\right].$$

In the infinite horizon case the utility is defined as

$$U_1(\Psi_1, \Psi_2) = \lim_{T \to \infty} U_1^{0,T}(\Psi_1, \Psi_2).$$

Proving that this utility is well defined is not in general an easy problem (one has to be sure that the backward equation has a solution and that the limit exists). Results for existence are given for instance by Duffie and Epstein (1992a) in the finite horizon case, and by Duffie and Lions (1992) in the infinite horizon Kreps Porteus case (i.e. the case we are interested in).

Given a strategy $\Psi_2$ of farmer 2 we say that $\Psi_1 \in \mathcal{C}_1(\Psi_2)$ is a best response to $\Psi_2$ if

$$(40) \qquad \Psi_1 \in \arg\max_{\Phi \in \mathcal{C}_1(\Psi_2)} U_1(\Phi, \Psi_2).$$

Similarly, given a strategy $\Psi_1$ of farmer 1, $\Psi_2 \in \mathcal{C}_2(\Psi_1)$ is a best response to $\Psi_1$ if

$$(41) \qquad \Psi_2 \in \arg\max_{\Phi \in \mathcal{C}_2(\Psi_2)} U_1(\Psi_1, \Phi).$$

We say that the couple $(\Psi_1, \Psi_2)$ is a Nash equilibrium of Markov strategies if both (40) and (41) are verified.

## Appendix B. Proofs in the 2−player case

*Proof of Theorem 3.1.* Suppose first that Player 2 plays a constant strategy $\Psi_2(A_1, A_2) \equiv f_2$ for some $f_2 \in [0, 1]$. Then the optimal response of Player 1 can be

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[8]Several ordinally equivalent choices of the aggregator are possible, the one described in (39) has the advantage of eliminating the variance term that in general appears in (38), it is indeed the unique with this property, see Duffie and Epstein (1992b) and Duffie and Lions (1992).

found as the solution (in feedback form) of an optimal control whose Bellman equation may be written as:

$$\rho_1 \frac{1-\theta_1}{1-\phi_1} V_1(A_1) = \max_{f_1 \in [0, 1-f_2]} \left[ \frac{(f_1 A_1)^{1-\phi_1}}{1-\phi_1} \frac{1}{((1-\theta_1)V_1(A_1))^{\frac{1-\phi_1}{1-\theta_1}-1}} + X_1(A_1, f_1 + f_2) \right]$$

where

$$X_1(A_1, f_1 + f_2) = V_1'(A_1)A_1\alpha_1 + \frac{1}{2}V_1''(A_1)A_1^2\sigma_1^2(f_1 + f_2)$$

We guess and check the following solution: $V_1(A_1) = \beta_1 \frac{A_1^{1-\theta_1}}{1-\theta_1}$ for some positive constant $\beta_1$. Then

$$X_1(A_1, f_1 + f_2) = \beta_1 A_1^{1-\theta_1} \left[ \alpha_1 - \theta_1 \frac{\sigma_1^2}{2}(f_1 + f_2) \right]$$

and

$$\rho_1 \frac{1}{1-\phi_1} \beta_1 A_1^{1-\theta_1} = \max_{f_1} \left[ \frac{(f_1 A_1)^{1-\phi_1}}{1-\phi_1} \frac{1}{\left(\beta_1 A_1^{1-\theta_1}\right)^{\frac{1-\phi_1}{1-\theta_1}-1}} + \beta_1 A_1^{1-\theta_1} \left[ \alpha_1 - \theta_1 \frac{\sigma_1^2}{2}(f_1 + f_2) \right] \right]$$

i.e.

$$\frac{\rho_1}{1-\phi_1} = \max_{f_1} \left[ \frac{f_1^{1-\phi_1}}{(1-\phi_1)\beta_1^{\frac{1-\phi_1}{1-\theta_1}}} + \left[ \alpha_1 - \theta_1 \frac{\sigma_1^2}{2}(f_1 + f_2) \right] \right]$$

i.e.

$$(42) \qquad \frac{\rho_1}{1-\phi_1} - \alpha_1 = \max_{f_1} \left[ \frac{f_1^{1-\phi_1}}{(1-\phi_1)\beta_1^{\frac{1-\phi_1}{1-\theta_1}}} - \theta_1 \frac{\sigma_1^2}{2}(f_1 + f_2) \right]$$

The maximizer $f_1$ on $(0, +\infty)$ (we will check later that point of maximum will be in $(0, 1 - f_2]$) is given by the solution of

$$(43) \qquad \frac{f_1^{-\phi_1}}{\beta_1^{\frac{1-\phi_1}{1-\theta_1}}} = \theta_1 \frac{\sigma_1^2}{2}$$

If we replace $\beta_1^{\frac{1-\phi_1}{1-\theta_1}}$ taken from (43) into (42) we get:

$$(44) \qquad \rho_1 - \alpha_1(1 - \phi_1) = \theta_1 \frac{\sigma_1^2}{2} [f_1 \phi_1 - (1-\phi_1)f_2]$$

i.e.

$$(45) \qquad f_1 = \frac{1}{\phi_1}(x_1 + (1-\phi_1)f_2)$$

where $x_i$ is defined as in (5). If this expression belongs in $[0, 1 - f_2]$, using classical results for optimization of problems with stochastic differential utilities and infinite horizon (see for instance Appendix C of Duffie et Epsten, 1992a), the feedback optimal control of Player 1 when she knows that Player 2 plays $\Psi_2(A_1, A_2) \equiv f_2$ (i.e. her optimal response) is $\Psi_1(A_1, A_2) \equiv f_1$ if a transversality condition in the form (see Smith, 1996):

$$(46) \qquad \lim_{t \to \infty} \mathbb{E}_0 \left[ e^{-\rho_1 t} V(A(t)) \right] = 0$$

is verified. We will verify it at the equilibrium.

A similar argument can be used for Farmer 2 responding to a constant strategy of Farmer 1. The equation which corresponds to (45) is then

$$(47) \qquad f_2 = \frac{1}{\phi_2}(x_2 + (1 - \phi_2)f_1)$$

The solution of the system (45)-(47) gives (9)-(10). Thanks to hypotheses of the Theorem the couple $\Psi_2(A_1, A_2) \equiv \bar{f}_2$, $\Psi_1(A_1, A_2) \equiv \bar{f}_1$ is an admissible couple. When the players use this couple the total area devoted to farming is, as described in (11), $\bar{f} := \bar{f}_1 + \bar{f}_2 = \frac{x_1 + x_2}{\phi_1 + \phi_2 - 1}$. To conclude the proof we only need to see that, along the equilibrium path the condition (46) and its counterpart for Player 2 are verified. We can see that

$$\mathbb{E}_0 \left[ e^{-\rho_i t} V_i(A(t)) \right] = \frac{\beta_i}{1 - \theta_i} \lim_{t \to \infty} e^{-\rho_i t} \mathbb{E}_0 \left[ A_i(t)^{1 - \theta_i} \right] = \frac{\beta_i}{1 - \theta_i} A_0 \lim_{t \to \infty} e^{-\rho_i t} e^{(1 - \theta_i)(\alpha_i - \theta_i \frac{\sigma_i^2}{2} \bar{f}) t},$$

and one can easily see that both (for $i = 1, 2$) these expressions goes to 0 under the conditions (8).

Finally we can observe that, since the solution of the two equations (45) and (47) is unique, so it is the equilibrium of constant strategies. $\qquad \square$

*Proof of Proposition 3.4.* By direct computation we have

$$\frac{\partial \bar{f}_1}{\partial \rho_1} = \frac{\partial x_1}{\partial \rho_1} \frac{\phi_2}{\phi_1 + \phi_2 - 1} > 0$$

$$\frac{\partial \bar{f}_1}{\partial \theta_1} = \frac{\partial x_1}{\partial \theta_1} \frac{\phi_2}{\phi_1 + \phi_2 - 1} < 0$$

$$\frac{\partial \bar{f}_1}{\partial \sigma_1} = \frac{\partial x_1}{\partial \sigma_1} \frac{\phi_2}{\phi_1 + \phi_2 - 1} < 0$$

and similarly when we compute $\frac{\partial \bar{f}_2}{\partial \rho_2}$, $\frac{\partial \bar{f}_2}{\partial \theta_2}$ and $\frac{\partial \ln \bar{f}_2}{\partial \sigma_2}$. Conversely, when we compute the derivatives of $\bar{f}_1$ w.r.t. to parameters of player 2 we get

$$\frac{\partial \bar{f}_1}{\partial \rho_2} = \frac{\partial x_2}{\partial \rho_2} \frac{1 - \phi_1}{\phi_1 + \phi_2 - 1}$$

$$\frac{\partial \bar{f}_1}{\partial \theta_2} = \frac{\partial x_2}{\partial \theta_2} \frac{1 - \phi_1}{\phi_1 + \phi_2 - 1}$$

$$\frac{\partial \bar{f}_1}{\partial \sigma_2} = \frac{\partial x_2}{\partial \sigma_2} \frac{1 - \phi_1}{\phi_1 + \phi_2 - 1}$$

whose sign depend on the value of $\phi_1$ compared to 1.

However the sign of $f_i$ w.r.t. $\phi_i$ remains unspecified as clear looking at the following expression:

$$\frac{\partial \bar{f}_1}{\partial \phi_1} = \frac{\phi_2}{\phi_1 + \phi_2 - 1} \left( \frac{\partial x_1}{\partial \phi_1} - \frac{x_1 + x_2}{\phi_1 + \phi_2 - 1} \right)$$

$\square$

*Proof of Proposition 4.2.* Since, by assumption $\phi > 1/2$, the result is straightforward.

$\square$

*Proof of Proposition 4.3.* By direct computation we have

$$\frac{\partial \ln \Lambda}{\partial \phi} = \frac{1}{\phi} - \frac{2}{2\phi - 1} = -\frac{1}{\phi (2\phi - 1)} < 0.$$

$\square$

*Proof of Proposition 5.1.* We have

$$\ln [1 + \omega] = -\frac{1}{1 - \phi} \left( \ln 2 + \phi \ln \left( 1 - \frac{1}{2\phi} \right) \right)$$

By direct computation of the derivative we obtain:

$$\begin{aligned}
\frac{d \ln (1 + \omega)}{d\phi} &= \frac{1}{1 - \phi} \left[ \frac{1}{1 - \phi} \ln \frac{\phi}{2\phi - 1} - \frac{1}{2\phi - 1} \right] \\
&= \frac{1}{1 - \phi} \left[ \frac{1}{1 - \phi} \ln \left( 1 + \frac{1 - \phi}{2\phi - 1} \right) - \frac{1}{2\phi - 1} \right] \\
&< \frac{1}{1 - \phi} \left[ \frac{1}{1 - \phi} \frac{1 - \phi}{2\phi - 1} - \frac{1}{2\phi - 1} \right] = 0
\end{aligned}$$

$\square$

## APPENDIX C. PROOFS IN THE $n-$PLAYER CASE

*Optimal response and Nash equilibrium.* We assume that the parameters are the same for all players. Suppose first that all players $j \neq i$ play a constant strategy $\Psi j(A_1, .., A_n) \equiv f_j$. Then the optimal response of Player $i$ can be found as the solution (in feedback form) of an optimal control whose Bellman equation may be written as:

$$\rho\frac{1-\theta}{1-\phi}V_i(A_i) = \max_{f_1 \in [0, 1-\sum_{j \neq i} f_j]} \left[ \frac{(f_i A_i)^{1-\phi}}{1-\phi} \frac{1}{((1-\theta)V_i(A_i))^{\frac{1-\phi}{1-\theta}-1}} + X_i(A_i, \sum_{j=1}^{n} f_j) \right]$$

where

$$X_i(A_i, \sum_{j=1}^{n} f_j) = V_i'(A_i)A_i\alpha + \frac{1}{2}V_i''(A_i)A_i^2\sigma^2\sum_{j=1}^{n} f_j$$

We guess and check the following solution: $V_i(A_i) = \beta_i\frac{A_i^{1-\theta}}{1-\theta}$ for some positive constant $\beta_i$. Then

$$X_i(A_i, \sum_{j=1}^{n} f_j) = \beta_i A_i^{1-\theta_1} \left[ \alpha - \theta\frac{\sigma^2}{2}\sum_{j=1}^{n} f_j \right]$$

and

$$\frac{\rho}{1-\phi}\beta_i A_i^{1-\theta} = \max_{f_i} \left[ \frac{(f_i A_i)^{1-\phi}}{1-\phi} \frac{1}{\left(\beta_i A_i^{1-\theta}\right)^{\frac{1-\phi}{1-\theta}-1}} + \beta_i A_i^{1-\theta_1} \left[ \alpha - \theta\frac{\sigma^2}{2}\sum_{j=1}^{n} f_j \right] \right]$$

i.e.

$$\frac{\rho}{1-\phi}\beta_i A_i^{1-\theta} = \max_{f_i} \left[ \frac{(f_i A_i)^{1-\phi}}{1-\phi} \frac{1}{\left(\beta_i A_i^{1-\theta}\right)^{\frac{1-\phi}{1-\theta}-1}} + \beta_i A_i^{1-\theta_1} \left[ \alpha - \theta\frac{\sigma^2}{2}\sum_{j=1}^{n} f_j \right] \right]$$

i.e.

$$(48) \qquad \frac{\rho}{1-\phi} = \max_{f_i} \left[ \frac{f_i^{1-\phi}}{1-\phi} \frac{1}{\beta_i^{\frac{1-\phi}{1-\theta}}} + \left[ \alpha - \theta\frac{\sigma^2}{2}\sum_{j=1}^{n} f_j \right] \right]$$

The maximizer $f_i$ on $(0, +\infty)$ (we will check later that point of maximum will be in $(0, 1 - \sum_{j \neq i} f_j])$ is given by the solution of

$$(49) \qquad \frac{f_i^{-\phi}}{\beta_i^{\frac{1-\phi}{1-\theta}}} = \theta\frac{\sigma^2}{2}$$

If we replace $\beta_i^{\frac{1-\phi}{1-\theta}}$ taken from (49) into (48) we get:

$$\rho - \alpha\left(1-\phi\right) = \theta\frac{\sigma^2}{2}\left(\phi f_i - (1-\phi)\sum_{j\neq i} f_j\right)$$

i.e.

(50)
$$f_i = \frac{x + (1-\phi)\sum_{j\neq i} f_j}{\phi}$$

When all the strategies are symmetric, then

$$f_i = \frac{x}{n\phi - n + 1}$$

Thus using (49),

$$\frac{\left(\frac{x}{n\phi-n+1}\right)^{-\phi}}{\theta\frac{\sigma^2}{2}} = \beta_i^{\frac{1-\phi}{1-\theta}}$$

$\square$

*Per-capita welfare gain and gain from biodiversity conservation.* Let us now compute the per-capita welfare gain of moving from Nash equilibrium to optimum, defined as $\omega$ such that

$$V_p\left(\frac{A}{n}\right) = V_h\left((1+\omega)\,A\right)$$

As $V_p(A) = \frac{1}{1-\theta}\beta_p A^{1-\theta}$ with $\beta_p = \left(\theta\frac{\sigma^2}{2}(f^*)^\phi\right)^{-\frac{1-\theta}{1-\phi}}$

$$\omega = \left(\frac{1}{n}\left(\frac{n\phi}{n\phi+1-n}\right)^\phi\right)^{\frac{1}{1-\phi}} - 1.$$

Finally, the per capita welfare gain from biodiversity conservation $\lambda$ is

$$V_p\left(\frac{A}{n}\right) = V_{oa}\left((1+\lambda)\frac{A}{n}\right)$$

The value of $\lambda$ is the same as in the $2-$player case

$$\lambda = \left(\frac{\phi f^* + (1-\phi)}{f^{*\phi}}\right)^{\frac{1}{1-\phi}} - 1.$$

$\square$

## Acknowledgements

Emmanuelle Augeraud-Véron, GREThA, Université de Bordeaux, France
*Email address*: emmanuelle.augeraud@u-bordeaux.fr

Giorgio Fabbri, Univ. Grenoble Alpes, CNRS, INRA, Grenoble INP, GAEL, Grenoble, France.
*Email address*: giorgio.fabbri@univ-grenoble-alpes.fr

Katheline Schubert, Paris School of Economics, Université Paris 1 Panthéon-Sorbonne, France.
*Email address*: schubert@univ-paris1.fr

# INSTITUT DE RECHERCHE ÉCONOMIQUES ET SOCIALES

Place Montesquieu 3
1348 Louvain-la-Neuve

**UCLouvain**

LiDAM

IRES | Louvain Institute of Data Analysis and Modeling in economics and statistics