# Capital Stock and Depreciation: Theory and an Empirical Application

F. J. Escribá-Pérez, M. J. Murgui-García and J. R. Ruiz-Tamarit

Discussion Paper 2019-4

Institut de Recherches Économiques et Sociales
de l'Université catholique de Louvain

IRES

UCLouvain

# Capital Stock and Depreciation: Theory and an Empirical Application*

F. J. Escribá-Pérez† M. J. Murgui-García‡ J. R. Ruiz-Tamarit§

January 3, 2019.

*The authors acknowledge the financial support from the Spanish Ministerio de Economía y Competitividad, Projects ECO2015-65049-C2-1-P and ECO2016-76818-C3-3-P as well as the support of the Belgian research programmes ARC on Sustainability. The authors also acknowledge the financial support from the Generalitat Valenciana PROMETEO/2016/097.

†Department of Economic Analysis, Universitat de València (Spain); francisco.j.escriba@uv.es

‡Department of Economic Analysis, Universitat de València (Spain); maria.j.murgui@uv.es

§Corresponding author. Department of Economic Analysis, Universitat de València (Spain), and Department of Economics IRES, Université Catholique de Louvain (Belgium). Address: Facultat d'Economia; Av. dels Tarongers s/n; E-46022 València (Spain). Phone: +34 963828250; Fax: +34 963828249; ramon.ruiz@uv.es

Capital Stock and Depreciation: Theory and an Empirical Application


**Abstract:**

There are many puzzles and unresolved problems in empirical economics that depend on the reliability of the productive capital series. Some macroeconomic topics and questions cannot be addressed correctly or answered with the available standard statistical measures of capital stock. We make an innovative contribution to the theory of capital together with an exercise that quantifies the depreciation rate and the capital stock for the U.S. economy. An intertemporal optimization model with adjustment and maintenance costs, gives us the algorithm and the corresponding economic estimation of capital deterioration and obsolescence. Our measures are based on profitability and the Tobin's $q$ ratio.



**Keywords**:

Capacity Utilization, Capital, Depreciation, Maintenance, Obsolescence, Tobin's $q$.

**JEL classification:** C61, D21, E22.

# 1 Introduction

In studying the evolution of capital theory from a historical perspective, we find that the neoclassical school has managed to develop a powerful theory of investment,[1] but has failed to create a consensus around a theory of depreciation. Looking at the basics first, we can see that investment decisions are channeled towards the explicit acquisition of new capital goods that involve market observable transactions. Instead, the broad label of capital depreciation is an umbrella under which a wide variety of heterogeneous concepts are sheltered. From the decline in productive capacity, through efficiency losses, to retirements or scrapping. Moreover, decisions on the depreciation of capital are basically associated with market unobservable transactions, which makes available records unreliable. These are related to a collection of implicit costs and benefits having its origin in a composite of two different causes: deterioration and obsolescence. Deterioration, which may be both physical (output decay) and economic (input decay), is an inherent characteristic of capital goods associated with the aging, use, and maintenance of equipment. Obsolescence, which may be both technological and structural, is extrinsic to assets and comes from outside associated with either technical progress (mainly embodied but also disembodied), energy prices, patterns of international trade, regulatory programs, or changes in the output composition that affect the relative prices.

The lack of a unified and widely accepted theory of capital depreciation has been supplemented for years by the results of two main but separate branches of research.[2] The first one uses the vintage capital model as a natural instrument, and has focused on the study of obsolescence. The second one, through the proportionality theorem, has provided the framework for the study of physical deterioration, i.e. wear and tear, which narrowly refers to depreciation caused by aging and the regular and constant use of capital.

The hypothesis of proportionality, from Jorgenson's (1963) pioneering work, states that the depreciation of capital is proportional to the capital stock, and recommends using the exponential form at a constant rate to represent the depreciation pattern of capital goods. In addition, the double-declining balance method establishes a close connection between this pattern and the notion of an exogenous service life of capital assets, which must be taken as fixed in correspondence with the assumed constancy of the depreciation rate. This implies that only output decay is considered relevant, and the role of variations in capacity utilization, decisions on maintenance, and embodied technical progress is ignored in the process of determining the economic useful life of capital.

The popularity of this branch eclipsed the strength of the studies that over the years centered on the endogenous determination of the optimal economic lifetime for different vintages and types of capital. In these studies technically more complex the endogeneity had a double sense because it also implied the endogenous causation of capital depreciation. Furthermore, from this point of view, the depreciation rate was no longer a constant parameter or something characterized by strict stability. Indeed, the old tradition of vintage capital models[3] focussed on the replacement of obsolete capital goods due to the technical progress embodied in new equipment. Nevertheless, they did not pay too much attention to the economic deterioration.

[1] For a review of the investment literature we recommend reading the works of Jorgenson (1967), Fazzari et al. (1988), Abel (1990), Chirinko (1993) and Caballero (1999).

[2] The literature on this subject has been thoroughly reviewed by Bitros (2010a, 2010b).

[3] The first model was proposed early in Johansen (1959) and an extensive literature followed it, specially after Malcomson (1975). The reader will find a summary review in Boucekkine et al. (2008).

That is, the depreciation associated with the variable activity and uncertainty typical of business cycle, the (lower) expenditure devoted to maintenance, and the structural change.[4]

Behind the two previous opposite views there was a more fundamental controversy between those who treat the depreciation of capital as a simple technical requirement that is exogenously determined, and those who consider the depreciation as a more complex phenomenon in which agents' decisions play an important role determining endogenously its economic value. Nonetheless, the debate disappeared from the top positions in the research agenda of the theory of capital, and only persisted what is known as the embodiment problem. That is, the discussion about the quantitative relevance in economic growth of the investment-specific embodied technical progress with respect to the standard neutral disembodied technical progress.[5]

It must be remarked that classical studies about the transmission of technical progress that are based on aggregate production functions, usually assume capital malleability in the sense that old and new capital goods share the same marginal productivity. However, the heterogeneity of capital goods is too important to be ignored and, in particular, the study of investment-specific technical progress requires the use of vintage capital models, which assume that investment goods of different generations differ in its productivity. In consequence, although capital goods that incorporate different technologies coexist at any time, improvements in technology do affect output only by means of the net investment or by the replacement of old equipment. The heterogeneity of the vintage model, from Solow's (1960) seminal work, may be represented with a single measure equivalent to the stock of capital, which is also known as the *jelly* capital. Solow assumes a putty-putty technology where the average useful life of capital goods is constant. Under the aggregation properties of his model it is possible to calculate both deterioration and obsolescence, but the latter is a constant fraction of the value of capital. On the whole, capital stock depreciates at a constant rate and this represents an important shortcoming because a geometric depreciation pattern means that technological change and market forces do not affect the average lifetime of capital goods.

The purpose of this paper is to address the challenge faced by the theory of capital to obtain an economic measure of the true capital stock at the aggregate level. This measure is required because capital stock, or the flow of services it provides, is one of the basic macroeconomic aggregates contributing to describe the main empirical facts of modern economies. To reach this goal we need to correctly measure depreciation in economic terms, because any forgotten component or measurement error can lead researchers to misinterpret reality and make inaccurate economic predictions. The pursued measure of depreciation should include all depreciation, that is, depreciation caused by age, use, maintenance, embodied technical progress and so on.

The traditional solution to the measure of depreciation and capital stock has been a statistical solution implemented according to the perpetual inventory method. This is a statistical measure of capital, not an economic measure in terms of value, which implies that the variability observed in the implicit depreciation rate mainly reflects changes in the composition of the capital stock. Instead of that, we provide an alternative solution consisting in a model that

[4]This point was initially remarked by Feldstein and Rothschild (1974) and Nickell (1975), for which depreciation varies considerably under the influence of conventional economic forces.

[5]Two different strategies are followed to model the investment-specific technological change, usually associated with Solow (1960) and Jorgenson (1966) respectively, which differ in the resource constraint but share the same law of motion for capital. The corresponding implications for growth accounting are analyzed in Greenwood et al. (1997) and Hulten (1992) respectively.

allows for the simultaneous computation and the endogenous measurement of both depreciation and capital.[6]

In parallel with the previous dichotomy, it is well known that the standard economic models hypothesize a capital stock depreciating at a constant exogenous rate and being fully used in the production process. However, firms do not always decide to use all the installed capital and they are able to change the depreciation rate. One of the first attempts to change things was the depreciation-in-use hypothesis, which means that a higher level of economic activity leads to a higher rate of capital utilization and, hence, to a greater rate of depreciation. A second attempt was the causal relationship according to which firms can indirectly affect the depreciation rate by devoting resources to keep the equipment in good working conditions, that is, repairing and maintaining the deteriorated capital stock. Despite these attempts, even if depreciation became an endogenous variable, it continues to play a residual role.

In an integrated theoretical-empirical framework, we solve a dynamic optimization model that endogenizes the depreciation rate by adding it to the set of controls, jointly with the variables gross investment, capacity utilization, and employment. This optimal control problem may be read as a general model explaining simultaneously the behavior of investment and depreciation. Depreciation is no longer a residual variable; together with the rate of capital utilization and the other controls, it is one of the instruments used by firms in setting their optimal plans. The key elements are the costs of capital adjustment (Hayashi, 1982), the maintenance and repair of equipment (Escribá-Pérez and Ruiz-Tamarit, 1996; McGrattan and Schmitz, 1999), the depreciation-in-use mechanism (Epstein and Denny, 1980; Bischoff and Kokkelenberg, 1987; Motahar, 1992; Boucekkine and Ruiz-Tamarit, 2003), and the technical progress (Boucekkine et al., 2009 and 2010). This model allows us to explore an algorithm capable of generating the series of the depreciation rate and the value of the capital stock. Our methodology yields an economic market value estimation based on a profitability indicator: the Tobin's $q$ ratio.

In this paper we differentiate between the model-based marginal $q$, the model-based average $q$ and the Tobin's $q$ ratio. The first and the second are theoretical and endogenous to the model; the third one is empirical and exogenous. According to the first order conditions the marginal $q$ is the determinant of the flows of investment and depreciation, which explain the dynamics of the capital stock. In addition, under some standard technical assumptions that characterize the functions of production, adjustment costs and maintenance expenditures, marginal $q$ equals average $q$. Therefore, we have the connection between the average $q$ ratio and the control variables. Finally, as the cornerstone of our empirical procedure we introduce the distinction between the theoretical average $q$ ratio and the observable Tobin's $q$ ratio. The former includes the market value of the firm in the numerator and the economic value of the capital stock measured in nominal terms at its replacement cost in the denominator. The Tobin's $q$ ratio is an empirical measure taken exogenously from studies that manage book value of capital assets, as well as financial and stock-exchange data of companies.[7]

Our model, our algorithm, and our calculations are coherent with most of the recent theoretical and empirical research about capital depreciation, capital measurement, and its influence

[6] A detailed exposition of these two alternative frameworks for the study of the economic and statistical measurement of capital may be found in Escribá-Pérez et al. (2018) and in the literature referenced there.

[7] It is assumed that in no case the empirical computation of the Tobin's $q$ ratio uses data on capital stock at historical prices or acquisition costs.

on other related problems like the productivity slowdowns and accelerations experienced by growing economies during the last fifty years. We provide a method for the measurement of capital stock that overcomes the shortcomings of the constant depreciation rate hypothesis, and which connects directly with the more general and economically rooted theory of capital based on vintage capital models. It is important to remark that, initially, Solow (1960) contributed showing that the vintage capital model with Cobb-Douglas technology can be rewritten as a standard neoclassical model, provided that the market value of capital is used as the measure of capital. Secondly, Greenwood et al. (1997) came to show, in its disregarded Appendix B, that we can transform the vintage capital model with investment-specific technological change, so that the distinction with a conventional model is mainly found in the rate of economic depreciation as opposed to the rate of physical depreciation, but also in the measure of capital stock and consequently in the measure of total factor productivity. The difference between the two above depreciation rates largely represents the obsolescence effect from technological change. Finally, Boucekkine et al. (2009) in a two-sector vintage capital model with neutral and investment-specific technical progress as well as variable utilization of vintages, raises a study similar to ours about depreciation as an endogenous phenomenon. Their total rate of depreciation is the sum of a physical age-related depreciation rate, plus an economic use-related depreciation rate, and the scrapping or obsolescence rate.

Our contribution has to do with the previous controversy concerning the depreciation rate, but goes beyond the related literature because it provides a measure of capital that corresponds to the market value of capital stock. In fact, it is a measure of aggregate capital at equilibrium assuming that the economy operates under perfect competition and there is no uncertainty. Given this measure, we compare with the series supplied by the agency *U.S. Bureau of Economic Analysis* (BEA). This series is statistical in nature, not an economic measure in terms of value, because statistical depreciation is estimated by adopting a pattern of constant depreciation rates. Consequently, there is a fundamental difference between the statistical depreciation and the economic depreciation. The first one may be considered as a proxy of the physical deterioration alone, while it is supposed that economic depreciation also includes economic deterioration and obsolescence.

We observe that the two series of the U.S. capital stock during the period 1960-2016 differ substantially in their short-run evolution, but it is possible to establish a close correspondence between the long-run profiles. In particular, we find that the economic depreciation rate fluctuates around the statistical depreciation rate, which may be read as if the long-run statistical measurement of depreciation were a good approximation of the long-run economic depreciation.

On the other hand, the market-based measure of economic depreciation and the corresponding economic value of capital stock are suitable for both the economic accounts of income and wealth and the production analysis and productivity measurement (Triplett, 1996). Consequently, our market measure or economic value of capital is a good proxy of either the net stock of capital in terms of wealth and the productive capital stock from which it is immediate to deduce a proportional flow of capital services. Our economic measure of the capital stock can be used as indicator of the productive capacity and as a measure of capital input in studies of multifactor productivity. It can also be related to the added value to calculate capital-output ratios.

The remaining of the paper is organized as follows. Section 2 describes the model that gives theoretical support to the economic measurement of capital stock and depreciation. Section 3 discusses about the content of the economic depreciation rate and its relationship with obsolescence. Section 4 first derives the algorithmic procedure that can be used to obtain quantitative results in an empirical application. Next, we show the series of economic capital stock and depreciation rate for the economy of the United States compared with the standard measures supplied by the statistical agency BEA. Finally, Section 5 summarizes.

# 2 Theory

Let us consider the supply side of an economy with a large number of identical firms. We shall present here a simple theoretical model that shows the optimizing behavior of the individual price-taking firm in a competitive environment. However, given the representative agent assumption and the absence of externalities, the variables of the model might also represent aggregate levels and the problem could be read as if all firms jointly made decisions in a centralized economy. The optimization problem to be solved is an intertemporal maximization problem that generalizes the standard model in which the employment $L(t)$ and the gross investment $I^G(t)$ are controlled in order to maximize the present discounted value of cash-flow. The generalization consists in adding to the set of controls the rate of capital depreciation $\delta^*(t)$ and the rate of capital utilization $u(t)$. These two rates are endogenous variables in the model and are linked to each other due to their relation to maintenance and repair expenditures. The objective functional or cash-flow is determined by revenues that depend on the production function, minus adjustment costs, maintenance costs, the wage bill, and investment spending. Adjustment and maintenance expenditures enter the cash-flow associated with their corresponding cost function because they are internal to the firm. The model has a single state variable, the capital stock $K^*(t)$, so that the optimal control problem must include a dynamic constraint to express the corresponding accumulation process. Putting it all together, we can write

$$\max_{\{K^*,L,I^G,\delta^*,u\}} V(t_0) =$$

$$\int_{t_0}^{+\infty} \left(G\left(p(t), A^*(t), K^*(t), L(t), I^G(t), \delta^*(t), u(t)\right) - W(t) L(t) - p^k(t) I^G(t)\right) e^{-\int_{t_0}^{t} R(s)ds} dt$$

$$s.t. \qquad \overset{\bullet}{K}^*(t) = I^G(t) - \delta^*(t) K^*(t), \tag{1}$$

$$K^*(t_0) = K_0^* > 0.$$

In this economy output is produced according to the production function $Y = A^* F(L, uK^*)$. Here $A^*$ is the current level of technology and $F(.)$ is a function homogeneous of degree one in its two determinants: labor and the portion of the capital stock that is used in the productive activity.[8] This function satisfies Inada conditions.

For the sake of simplicity we normalize the price of output, $p(t) = 1$. The price of labor $W(t)$, the market price of capital goods $p^k(t)$ and the nominal interest rate $R(t)$ are given for the competitive firm. To make things even easier, we define the function $G(.)$ that represents the value of net production after subtracting investment-related adjustment costs $C\left(I^G, K^*\right)$

[8] Actually, the function could be written as $F(L, KU^*)$, where $KU^* = uK^*$, $F_L > 0$, $F_2 > 0$, $F_{LL} < 0$, and $F_{22} < 0$. Our homogeneity assumption involves the variables $L$ and $KU^*$ instead of $L$, $u$, and $K^*$, taken separately.

and maintenance expenditures $M(\delta^* K^*, uK^*)$.These two functions are assumed homogenous of degree one in their corresponding pair of determinants.[9] Under the linear homogeneity assumptions we can write $G\left(A^*, K^*, L, I^G, \delta^*, u\right) = A^* F(L, uK^*) - \phi\left(\frac{I^G}{K^*}\right) K^* - \varpi\left(\frac{\delta^*}{u}\right) uK^*$, and we can also characterize the function by means of the sign of the first and second derivatives with respect to the controls and the state variable.[10] That is, $G_L = A^* F_L > 0$, $G_{LL} = A^* F_{LL} < 0$, $G_{I^G} = -C_{I^G} = -\phi' < 0$, $G_{I^G I^G} = -C_{I^G I^G} = -\frac{\phi''}{K^*} < 0$, $G_{\delta^*} = -\varpi' K^* > 0$, $G_{\delta^*\delta^*} = -\varpi'' \frac{K^*}{u} < 0$, $G_u = A^* F_2 K^* - \left(\varpi - \varpi' \frac{\delta^*}{u}\right) K^* \gtreqless 0$, $G_{uu} = A^* F_{22} K^{*2} - \frac{\varpi'' \delta^{*2} K^*}{u^3} < 0$. Moreover, given that $\phi(.)$ is strictly convex we get $C_{K^*} = \phi - \phi' \frac{I^G}{K^*} < 0$ and $C_{K^* K^*} = \phi'' \frac{\left(I^G\right)^2}{(K^*)^3} > 0$, consequently $G_{K^* K^*} = A^* F_{22} u^2 - C_{K^* K^*} < 0$. Finally, we assume that the net marginal productivity of capital is positive, $G_{K^*} = A^* F_2 u - C_{K^*} - \varpi u > 0$.

When we add the dynamic constraint to the objective functional by introducing the multiplier $\mu$ as expression of the shadow price of capital, we get the following Hamiltonian function written in current value

$$H^c = G\left(A^*(t), K^*(t), L(t), I^G(t), \delta^*(t), u(t)\right) - W(t) L(t) - p^k(t) I^G(t)$$
$$+\mu(t)\left(I^G(t) - \delta^*(t) K^*(t)\right). \tag{2}$$

In addition, we assume that the discount rate for the cash-flow is given exogenously and is perceived as a constant $R$. We avoid then the time-consistency problem associated with a non-constant discount rate that makes preferences intertemporally dependent, and apply the Pontryagin's maximum principle, from which we get the necessary conditions for the control variables[11]

$$H_L^c(.) = 0 = A^*(t) F_L(L(t), u(t) K^*(t)) - W(t), \tag{3}$$

$$H_{I^G}^c(.) = 0 = -\phi'\left(\frac{I^G(t)}{K^*(t)}\right) - p^k(t) + \mu(t), \tag{4}$$

$$H_{\delta^*}^c(.) = 0 = -\varpi'\left(\frac{\delta^*(t)}{u(t)}\right) K^*(t) - \mu(t) K^*(t), \tag{5}$$

[9] The adjustment cost function $C(.)$ has the usual properties: it is increasing in $I^G$ and decreasing in $K^*$. The maintenance cost function originally could be written as $M(D^*, KU^*)$, where $D^* = \delta^* K^*$ represents the volume of total depreciation. It is assumed that maintenance expenditures decrease with depreciation $D^*$, but increase with the quantity used of capital stock $KU^*$. Our homogeneity assumption involves the variables $D^*$ and $KU^*$ instead of $\delta^*$, $u$, and $K^*$, taken separately.

[10] The adjustment unit cost function $\phi(i)$ satisfies the properties $\lim_{i\to 0^+} \phi(i) = 0$, $\lim_{i\to+\infty} \phi(i) = +\infty$, $\phi'(i) > 0$, $\lim_{i\to 0^+} \phi'(i) = 0$, $\lim_{i\to+\infty} \phi'(i) = +\infty$, $\phi''(i) > 0$. The maintenance unit cost function $\varpi\left(\frac{\delta^*}{u}\right)$ satisfies the properties $\lim_{x\to 0^+} \varpi(x) = +\infty$, $\varpi\left(\frac{1}{u}\right) = 0$, $\lim_{x\to+\infty} \varpi(x) = 0$, $\varpi\left(\frac{\delta^*}{1}\right) \geqslant 0$, $\varpi'(x) < 0$, and $\varpi''(x) > 0$. It is easy to deduce the results $\varpi_{\delta^*} = \frac{\varpi'}{u} < 0$, $\varpi_{\delta^*\delta^*} = \frac{\varpi''}{u^2} > 0$, $\varpi_u = -\frac{\varpi'\delta^*}{u^2} > 0$, $\varpi_{uu} = \frac{\delta^*}{u^3}\left(2\varpi' + \varpi'' \frac{\delta^*}{u}\right)$, and $\varpi_{\delta^* u} = -\frac{1}{u^2}\left(\varpi' + \varpi'' \frac{\delta^*}{u}\right)$. Moreover, if we assume that $\varpi_{uu} > 0$, then we get $\varpi_{\delta^* u} < 0$.

[11] Although the model also includes the following control constraints $\forall t$: $L(t) \geq 0$, $I^G(t) \geq 0$, $0 \leq \delta^*(t) \leq 1$, and $0 \leq u(t) \leq 1$, for the sake of simplicity we do not make them explicit in the optimization problem. We are going to consider interior solutions alone. The latter is guaranteed by the characterization we have made of the functions involved in our model throughout the previous three footnotes.

$$H_u^c(.) = 0 = A^*(t)\, F_2(L(t), u(t)\, K^*(t))\, K^*(t) - \left(\varpi\left(\frac{\delta^*(t)}{u(t)}\right) - \varpi'\left(\frac{\delta^*(t)}{u(t)}\right)\frac{\delta^*(t)}{u(t)}\right) K^*(t), \tag{6}$$

the Euler equation

$$\overset{\bullet}{\mu}(t) = R\mu(t) - H_{K^*}^c\left(A^*(t), K^*(t), L(t), I^G(t), \delta^*(t), u(t)\right), \tag{7}$$

where $H_{K^*}^c(.) = A^* F_2 u - \left(\phi - \phi' \frac{I^G}{K^*}\right) - \varpi u - \mu\delta^*$, the dynamic constraint

$$\overset{\bullet}{K^*}(t) = I^G(t) - \delta^*(t)\, K^*(t), \tag{8}$$

and the transversality condition

$$\lim_{t\to+\infty} \mu(t)\, K^*(t)\, e^{-R(t-t_0)} = 0. \tag{9}$$

We observe that the first order conditions (3)-(6) could implicitly define a system of four control functions. However, after total differentiation, we can prove that the implicit function theorem cannot be applied because the assumed homogeneity of degree one on the different functions involved in $G(.)$ makes the determinant of the Jacobian matrix equal to zero. Therefore, except for the investment equation that is given independently of the others, we proceed to solve the sign of the partial effects of the state, co-state and parameters on the two new control variables defined as ratios between the original variables.

$$\frac{L(t)}{u(t)} = N\left(\overset{+}{K^*}(t); \overset{+}{A^*}(t), \overset{-}{W}(t)\right), \tag{10}$$

$$I^G(t) = I^G\left(\overset{+}{K^*}(t), \overset{+}{\mu}(t); \overset{-}{p^k}(t)\right), \tag{11}$$

$$\frac{\delta^*(t)}{u(t)} = x\left(\overset{-}{\mu}(t)\right). \tag{12}$$

Equation (6), in turn, establishes a tight link between the two ratios

$$Z\left(\frac{L(t)}{u(t)}, \frac{\delta^*(t)}{u(t)}, K^*(t), A^*(t)\right) = 0. \tag{13}$$

Coming back to the set of optimality conditions, we find that the differential equation (7) may be integrated forward solving for $\mu(t)$, under the non-explosivity condition $\lim_{t_F\to+\infty} \mu(t_F)$ $\exp\left\{-\int_t^{t_F}(R + \delta^*(\tau))\, d\tau\right\} = 0$. The result we get may be put in terms of the model-based definition of marginal $q$,

$$q^M(t) = \frac{\mu(t)}{p^k(t)} =$$

$$\frac{1}{p^k(t)}\int_t^{+\infty}\left(A^*(s)\, F_2(L(s), u(s)\, K^*(s))\, u(s) - \left(\phi\left(\frac{I^G(s)}{K^*(s)}\right) - \frac{I^G(s)}{K^*(s)}\phi'\left(\frac{I^G(s)}{K^*(s)}\right)\right)\right.$$

$$-\varpi\left(\frac{\delta^*(s)}{u(s)}\right)u(s))e^{-\int_t^s(R+\delta^*(v))dv}ds. \tag{14}$$

That is, the present value of the future stream of the net marginal productivity of capital, discounted by the sum of the constant discount rate plus the variable depreciation rate, and all that divided by the current market price of one unit of capital. In other words, the quotient between the shadow price of one unit of capital (the Hamiltonian multiplier $\mu$) and its replacement cost (the market price of capital goods $p^k$). This is the variable which directly explains the flows of investment and depreciation that determine the dynamics of the capital stock.

On the other hand, the property of homogeneity assumed on the production and cost functions together with the first order conditions of the dynamic optimization problem, allow us to set the following linear ordinary differential equation in $X=\mu K^*$: $\dot{X}=RX-A^*F(L,uK^*)+\phi\left(\frac{I^G}{K^*}\right)K^*+\varpi\left(\frac{\delta^*(s)}{u(s)}\right)uK^*+WL+p^kI^G$. This one may be integrated forward solving for the product $\mu(t)K^*(t)$, under the transversality condition (9). The result we get may be put in terms of the model-based definition of average $q$, the quotient between the market value of (all) the firm(s) and the economic value of the capital stock measured in nominal terms at its replacement cost,

$$q^A(t)=\frac{\mu(t)K^*(t)}{p^k(t)K^*(t)}= \tag{15}$$

$$=\frac{\int_t^{+\infty}\left(G\left(A^*(s),K^*(s),L(s),I^G(s),\delta^*(s),u(s)\right)-W(s)L(s)-p^k(s)I^G(s)\right)e^{-R(s-t)}ds}{p^k(t)K^*(t)}.$$

Given (14) and (15) it is apparent the equality between marginal and average $q$.[12] That is, the two theoretical $q$ ratios are equivalent and, according to the literature, they can be empirically approximated by the observable Tobin's $q_t$, which is the ratio of the stock market value of the firm to the current-cost book value of capital assets.

Our variables in the model refer to economic or market values but, many times, the literature uses other names with the same meaning. For example, the valuation at replacement cost, in nominal terms or at current prices are equivalent, and means that the capital goods are valued at the prices of the current period. This is opposed to valuation at historical prices, which means that the assets are valued at the prices at which they were originally purchased. According to Siegel (2008) most of the empirical measures of Tobin's q use financial data and adjust for inflation to compute the replacement cost of the assets and liabilities. Available measures of Tobin's $q$ ratio are based on financial market valuation of the corporate assets corresponding to the fundamentals of the firm, as well as on data obtained from balance sheets. Consequently we can use them as exogenous proxies for our model-based $q$ ratios.

# 3 Does the endogenous rate of depreciation capture obsolescence?

An important issue related to the previous model is whether the variable depreciation rate $\delta^*(t)$ is a good indicator, representative of all depreciation or not. We want to know if it also includes the obsolescence of capital goods or just deterioration, either physical and economic. Our model endogenizes the depreciation rate, which is no longer a fixed proportion of the

[12] See Hayashi (1982), Blanchard et al. (1993) or Kalyvitis (2006).

capital stock, on the basis that economic agents choose its value according to the costs and benefits of the different activities related to capital goods. The key piece of this building is a maintenance cost function that associates resources devoted to maintenance and repair with the intensity of capital utilization and the rate of depreciation. We have the certainty that by combining the maintenance argument with the depreciation-in-use mechanism, the model accounts for deterioration in all its richness and complexity. However, as regards obsolescence things are a bit different.

Traditionally, obsolescence has been analyzed using the framework of vintage capital models and studying the effect that the embodied technological change causes in the economic useful life of capital goods.[13] More recently, many papers and among them especially Greenwood et al. (1997), pointed out that the observed negative relationship between the series for the investment price index and the quantity of investment should be taken as evidence of technological change in the production of new units of capital. Henceforth, we will use the term $\theta(t)$ to represent the state of the technology, or productivity level, in the production of new capital. Increases in $\theta(t)$ may be read as quality improvements or investment-specific technological change that boost the productivity of the last vintage of capital goods.[14] The embodiment means that to benefit from the advantages of innovation it is necessary to invest in new equipment. But it should not be forgotten that the embodied technical progress has a counterpart inducing the obsolescence of installed capital goods.

It is important to mention here Boucekkine et al. (2009), which in a two-sector vintage capital model with both neutral and embodied technical progress, as well as variable utilization of vintages, derives endogenously the economic rate of depreciation. Their total rate of depreciation is the algebraic sum of a physical age-related depreciation rate, plus an economic use-related depreciation rate, and the scrapping or obsolescence rate. However, our model is based on the more standard neoclassical premise that there is something like an aggregate stock of capital. We do not consider explicitly capital vintages in our framework and, consequently, our model cannot produce an explicit scrapping rate. In addition, our model with an aggregate capital stock does not differentiate between the usual consumption and capital sectors, and we do not make explicit the role of the investment-specific technological change that is the main cause of obsolescence.

Even so, it may be interesting to know whether our strategy for modelling an endogenous rate of economic depreciation is also implicitly or indirectly capturing obsolescence. To address this question we count on the help of the conceptual developments from Solow (1960) and the methodological arguments exhibited in Greenwood et al. (1997), which significantly extends the model of the previous one. Solow was the first to show how to manage the investment-specific technological change in a model with aggregate capital stock. But it was Greenwood

[13] According to Boucekkine et al. (2009), while improvements through neutral technical progress increase the profitability of all vintages, which leads to lengthen their lifetime, capital-embodied technical progress leads to shorter lifetimes.

[14] In empirical studies the hypothesis of the link between the investment price index and capital-embodied technological change is introduced by assuming $\theta(t) = p(t)/p^k(t)$, which is the reciprocal of the relative price index of investment goods with respect to output. The idea is simple, technological change makes new capital goods simultaneously less expensive and more productive than old ones. One unit of new capital is $\frac{\dot{\theta}(t)}{\theta(t)}$ times more productive than another one period older, but $\frac{\dot{\theta}(t)}{\theta(t)}$ is also the rate of declining in prices of new capital goods. Consequently, the declining of the relative price index of investment reveals an increasing level of investment-specific technology.

and his coauthors who proposed two equivalent representations of the model. On one hand, in the main text of the article they provide a version of the model that includes both the level of neutral technical progress and the level of embodied technical progress, but the depreciation rate is constant. The latter is considered an important shortcoming because only physical deterioration would be recorded. Alternatively, in the Appendix B they provide a transformed version of the same model, in which the depreciation rate is variable and intends to represent all the economic depreciation, but now the technological level apparently only takes the form of the standard neutral technical progress. In any case, there is a clear equivalence between the two specifications of the model and, consequently, any change in the formal representation of technology will be compensated in the formal representation of depreciation and, hence, in the measure of capital stock.

In our case, the model shown in the previous section is closer to the transformed model in Greenwood et al. (1997), because $A^*(t)$ apparently takes the form of the standard neutral technological level, and the depreciation rate $\delta^*(t)$ is a variable that covers the whole economic depreciation. Methodologically, we can establish an homomorphic parallelism with its double representation of the model and show, first, that depreciation caused by obsolescence is also included in $\delta^*(t)$ and, second, that the embodied technical progress is also taken into account in $A^*(t)$. Output and the dynamics of capital are determined as follows

$$Y(t) = A^*(t) F(L(t), u(t) K^*(t)), \tag{16}$$

$$\overset{\bullet}{K}^*(t) = I^G(t) - \delta^*(t) K^*(t), \tag{17}$$

where $K^*(t)$ and $\delta^*(t)$ represent the market value of the variables capital stock and depreciation rate. In the production function the variable $A^*(t)$ represents the corresponding level of total factor productivity. In the absence of maintenance costs our model reduces to the conventional neoclassical model, and equations (16) and (17) transform into

$$Y(t) = A(t) F(L(t), K(t)), \tag{18}$$

$$\overset{\bullet}{K}(t) = I^G(t) - \delta K(t), \tag{19}$$

where $\delta$ is the constant depreciation rate usually associated with physical deterioration, $A(t)$ is the exogenous level of neutral technical progress, and $K(t)$ is obtained according to the perpetual inventory method. It is well known that in this model depreciation only takes into account the effects of aging capital and its use at a given constant rate.

According to Solow, Greenwood et al., and identifying their variables with the subscript $SG$, output and the dynamics of capital are determined as follows

$$Y(t) = A(t) F(L(t), K_{SG}(t)), \tag{20}$$

$$\overset{\bullet}{K}_{SG}(t) = \theta(t) I^G(t) - \delta_{SG} K_{SG}(t), \tag{21}$$

where it is implicitly assumed that $A_{SG}(t) = A(t)$. Now, looking for a relationship that allows us to establish the correspondence between the two specifications of the model, we use by analogy the following one:

$$K^*(t) = \frac{K_{SG}(t)}{\theta(t)}. \tag{22}$$

Then, combining (17), (21), and (22) we get the important relationship

$$\delta^*(t) = \delta_{SG} + \frac{\dot{\theta}(t)}{\theta(t)}. \tag{23}$$

A key variable in our model is the economic rate of depreciation $\delta^*(t)$, and equation (23) shows that obsolescence is properly measured by this variable along with physical and economic deterioration. In fact, the above equation only says that our $\delta^*(t)$ is the sum of the depreciation rate corresponding to the Solow-Greenwood framework plus the rate of embodied technical progress. Of course, neither Solow nor Greenwood included maintenance costs and variable capacity utilization in their studies of investment and technological change. Consequently, the above equation should be interpreted as the affirmative answer to the question posed in this subsection, regardless of whether $\delta_{SG}$ also measures economic deterioration or only physical deterioration. This is so because there is no doubt about whether in our model $\delta^*(t)$ is a measure of depreciation that includes economic deterioration.

Moreover, from (16), (20), and (22) we get

$$A^*(t) = A(t)\frac{F(L(t), K_{SG}(t))}{F(L(t), u(t)K^*(t))} = A(t)\frac{F(L(t), \theta(t)K^*(t))}{F(L(t), u(t)K^*(t))}. \tag{24}$$

Given that $F(.)$ is homogeneous of degree one we can write[15]

$$A^*(t) = A(t)\left(1 + \left(\frac{\theta(t)}{u(t)} - 1\right)\left(1 - \frac{F_L L}{F}(t)\right)\right) = A(t)\,h(\theta(t), u(t), \alpha(t)). \tag{25}$$

This expression says that the model-based measure of total factor productivity $A^*(t)$ is something beyond the pure exogenous level of neutral technical progress associated with the measure of capital stock that arises from the perpetual inventory method. The above endogenous measure of total factor productivity actually represents the level of the global technical progress associated with the economic measure of capital stock and depreciation, including obsolescence. It encompasses the standard neutral technical progress, but also depends positively on the level of investment-specific technology. Moreover, we find that $A^*(t)$ depends negatively on the rate of capital utilization and positively on the elasticity of capital in the production function. After these new insights concerning the total factor productivity, there is no doubt that the economic measurement of capital will have important consequences for the empirical exercises of growth accounting, as well as for any attempt to explain output slowdowns and accelerations.

# 4 Algorithm and measurement

In this section we will first simplify notation, calling the market value of the firm along the optimal equilibrium path as $V_t^*$, and we shall rewrite variables in discrete terms to make the relevant expressions computationally operative. Then, from the first order conditions and on the side of the value of the firm we can write the Tobin's $q$ ratio as

$$q_t = \frac{V_t^*}{p_t^k K_t^*}. \tag{26}$$

[15] With a Cobb-Douglas specification for $F(.)$ we get $1 - \frac{F_L L}{F} = \alpha$ and $A^*(t) = A(t)(\theta(t)/u(t))^{\alpha}$.

On the other side, the stock of capital is determined at each moment according to the first-order difference equation

$$K_t^* = I_t^G + (1 - \delta_t^*) K_{t-1}^*. \tag{27}$$

Given the capital stock of the previous period, by adding the flow of gross investment $I_t^G$ and subtracting the depreciation flow $\delta_t^* K_{t-1}^*$, we obtain the stock of capital of the current period. Although the gross investment is a control variable in the model, from the point of view of our computations it will be considered as a forcing variable. Since the transactions related to the acquisition of capital goods are observable transactions in the market, we shall consider these records as the result of optimal decisions and will introduce them in our calculations. In this way, we can try to obtain a simpler algorithm to generate the values of depreciation and capital stock that are not directly observable in the market.

Furthermore, we specify the market value of the firm $V_t^*$ as the discounted present value of the infinite flow of distributed profits, $B_t^*$. According to the financial theory of the firm, there is a clear candidate for discounting profits: the required returns to capital or its average cost. However, from an empirical point of view there is a problem of choice between different alternatives. In our case, we use the long term interest rate because we deduce the measurement algorithm from a set of hypotheses that include an infinite horizon (only distributed profits are relevant) and perfectly competitive capital markets (all measures of the returns to capital are equivalent).[16] Given that variables $V_t^*$ and $B_t^*$ are both expressed in nominal terms, we discount the stream of dividends with the nominal interest rate, $R_t$,

$$V_t^* = \sum_{s=t}^{\infty} \frac{B_s^*}{\prod\limits_{\tau=t+1}^{s} (1 + R_\tau)}. \tag{28}$$

In this intertemporal context without uncertainty, it is still necessary for our computations to specify how agents form their expectations regarding the future value of variables. We assume that, along the equilibrium path, economic agents expect that nominal profits increase with inflation. Moreover, under perfect competition agents are price-takers: they consider the inflation rate and the nominal interest rate as exogenously given at the moment of making decisions. Although these two price variables could change over time in accordance with the market forces of demand and supply, at any moment the firms taken individually will perceive them as constant parameters. Consequently, we assume that in each period they behave as if the current values of the rate of inflation and the nominal interest rate were to be repeatedly observed in the future. When we apply these assumptions to the terms of equation (28) we find that, $\forall s, \tau \in [t, \infty[$, $B_s^* = B_t^* \left(1 + \pi_s^k\right)^{s-t}$ being $\pi_s^k = \pi_t^k$ the inflation rate associated with the price index of capital goods $p^k$, and $\prod\limits_{\tau=t+1}^{s} (1 + R_\tau) = (1 + R_\tau)^{s-t}$ with $R_\tau = R_t$. We define the real interest rate $r_t = R_t - \pi_t^k > 0$ and approximate the term $\frac{1+\pi_t^k}{1+R_t} = 1 + \pi_t^k - R_t$, taking the product $r_t R_t$ as negligible. Then, we can write

$$V_t^* = B_t^* \sum_{s=t}^{\infty} \left( \frac{1 + \pi_t^k}{1 + R_t} \right)^{s-t} = B_t^* \sum_{s=t}^{\infty} (1 - r_t)^{s-t} = \frac{B_t^*}{r_t}. \tag{29}$$

[16] These assumptions result in an extreme position of measurement with theory, but our target in this paper is to obtain the measure of the capital stock according to the purest requirements of the neoclassical theory of capital without the limitations raised by the assumption of a constant depreciation rate.

Substituting this result in (26) we get

$$q_t = \frac{B_t^*}{r_t p_t^k K_t^*}. \tag{30}$$

Equations (30) and (27) give us a clear idea of how the process of capital accumulation is defined in economic terms but they also tell us how the markets evaluate this process. In these equations we are considering the economic or market value of each of the variables. On the quantity-variables side: distributed profits, flows of gross investment and depreciation, and the capital stock; and on the side of the price-variables: interest rate, price of investment goods and the Tobin's $q$ ratio.

Concerning the revenues of productive factors generated by firms and distributed through the market mechanisms, it must be remarked that the sum of the economic value of net distributed profits, $B_t^*$, and the flow of economic depreciation in nominal terms is equal to the distributed gross profits, $B_t^G = B_t^* + \delta_t^* p_t^k K_{t-1}^*$. Substituting in (30) we get

$$q_t r_t p_t^k K_t^* = B_t^G - \delta_t^* p_t^k K_{t-1}^*. \tag{31}$$

Therefore, if we know the value of all the price variables as well as the value of the economic-accounting flows of gross investment and gross distributed profits, we can use sequentially the equations (27) and (31) to obtain the values of the depreciation rate $\delta_t^*$ and the capital stock $K_t^*$. This dynamic system of two first-order difference equations allows us to express the values of the two previous endogenous variables as a function of the variables $q_t$, $r_t$, $p_t^k$, $B_t^G$, and $I_t^G$, given the predetermined value of $K_{t-1}^*$. In other words, from a known initial value $K_0^*$ we can solve forward to obtain the complete series for the capital stock and the depreciation rate. Next, we show in closed-form their explicit solutions:

$$\delta_t^* = \frac{\frac{B_t^G}{q_t r_t p_t^k} - K_{t-1}^* - I_t^G}{\left(\frac{1}{q_t r_t} - 1\right) K_{t-1}^*}, \tag{32}$$

$$K_t^* = \frac{K_{t-1}^* + I_t^G - \left(B_t^G / p_t^k\right)}{1 - q_t r_t}. \tag{33}$$

Hence, the comparative statics of these two variables is symmetric with each other, for all independent variables and prices. Ceteris paribus, we can see that an increase in investment expenditures will increase the capital stock but reduces the depreciation rate. Moreover, the higher the level of distributed profits the higher the depreciation rate and the lower the capital stock. Finally, Tobin's $q$ ratio and the real interest rate are inversely (positively) correlated with the depreciation rate (capital stock).

Next, we apply the algorithmic procedure above to the non-financial business sector data of the U.S. economy during the period 1960-2016. This sector involves most of the activities in the economy excluding the financial intermediation sector, real estate and non-market services. We obtain the economic measures of capital stock and depreciation and we compare them with the corresponding statistical measures. As explained along the previous sections, these standard measures are recorded by agencies like BEA on the basis of the perpetual inventory method. It is therefore assumed that the useful life of capital goods is exogenously determined by technological parameters, and depreciation is then calculated at a constant exponential rate.

Figures 1 and 2 plot the evolution of the two pairs of measures of capital stock and depreciation rate. In Figure 1, the economic depreciation rate is compared to the statistical depreciation rate. In Figure 2 the two time profiles of the economic and statistical capital stocks are shown. Table 1 contains the exact figures for the series of economic and statistical depreciation rates and capital stocks over the period 1960-2016. Here it is assumed that the initial capital stocks are equal, $K^*_{1960} = K_{1960}$. The complete series of the variables required to run our algorithm are available in Table 2. These values and the initial capital stock have been adopted from BEA database.

**Figure 1**
[U.S. Economic and Statistical Depreciation Rates]

**Figure 2**
[U.S. Economic and Statistical Capital Stocks]

**Table 1**
[U.S. Data: Economic and Statistical Depreciation Rates and Capital Stocks]

**Table 2**
[U.S. Data: Non-financial Business Sector Capital-related Variables 1960-2016]

Our finding contrasts sharply with that coming from the empirical application of the theorem of proportionality. However, our results are consistent with the accumulated evidence surrounding the subject of capital and depreciation measurement, the discussion about the relevance of obsolescence in this matter and other issues related to the general process of capital accumulation and substitution.

# 5 Conclusions

In this paper a model of dynamic optimization for the competitive firm is developed. Capital utilization and depreciation are both endogenous variables of control. They are determined by profit maximization together with investment and labor demands. However, this is at variance with standard models where the rate of capital utilization as well as the depreciation rate are treated as exogenous constants. Our changes to the basic model make it more realistic and rely on the introduction of a maintenance cost function that connects the previous variables with each other. Moreover, the solution under the usual homogeneity conditions allows us to establish the decisive equality between different versions of the Tobin's $q$ ratio, which is so useful from an empirical point of view. The equations of the model enable us to endogenously calculate the variables rate of depreciation and capital stock, yielding an economic estimation based on indicators of profitability. This estimation differs from the standard measurement of depreciation and capital stock according to the perpetual inventory method, which is based on the Jorgenson's proportionality principle.

According to the previous statements, we launched an empirical application focussed on the U.S. data. The results show that the endogenous economic depreciation rate fluctuates around the exogenous statistical rate. Moreover, we obtain the corresponding series for both economic and statistical capital stock, whose differences are the result of a greater or lesser destruction of capital in different periods, but which are not recorded in official statistics. Although depreciation may be caused by deterioration and obsolescence, official depreciation records usually appear associated with physical deterioration alone. Accordingly, statistical measurements assume the exponential form at a constant rate to represent the depreciation pattern of capital goods. In this paper, however, we have theoretically modeled and empirically obtained an endogenous variable depreciation rate. As we have shown, this economic depreciation rate covers both causes economic deterioration and obsolescence. Concerning the first one, it is obvious because our model allows for the optimal choice of maintenance expenditures and the rate of capacity utilization. With respect to the second, we have proceed methodologically as in Greenwood et al. (1997), establishing an homomorphic transformation that allows us to conclude that depreciation caused by obsolescence is encompassed in our economic measure of depreciation and the embodied technical progress is included in our measure of total factor productivity.

Related with the latter, there is a consolidated literature proving that any distortion in the measurement of capital stock may cause a substantial bias in the measurement of total factor productivity growth. Musso (2004) and Mukoyama (2008) analyze this point in the context of a vintage capital model. Now, theoretical and quantitative results shown in our paper open an alternative way of studying the old problem of finding a reasonable explanation for labor productivity slowdowns and accelerations in the United States. If the economic value of depreciation and capital stock are poorly measured using the traditional perpetual inventory method, growth accounting will not reflect correctly the role that plays capital deepening and total factor productivity. With the help of our results it is possible to revise these exercises and interpretations, but for the moment this is left as a challenge for future research.

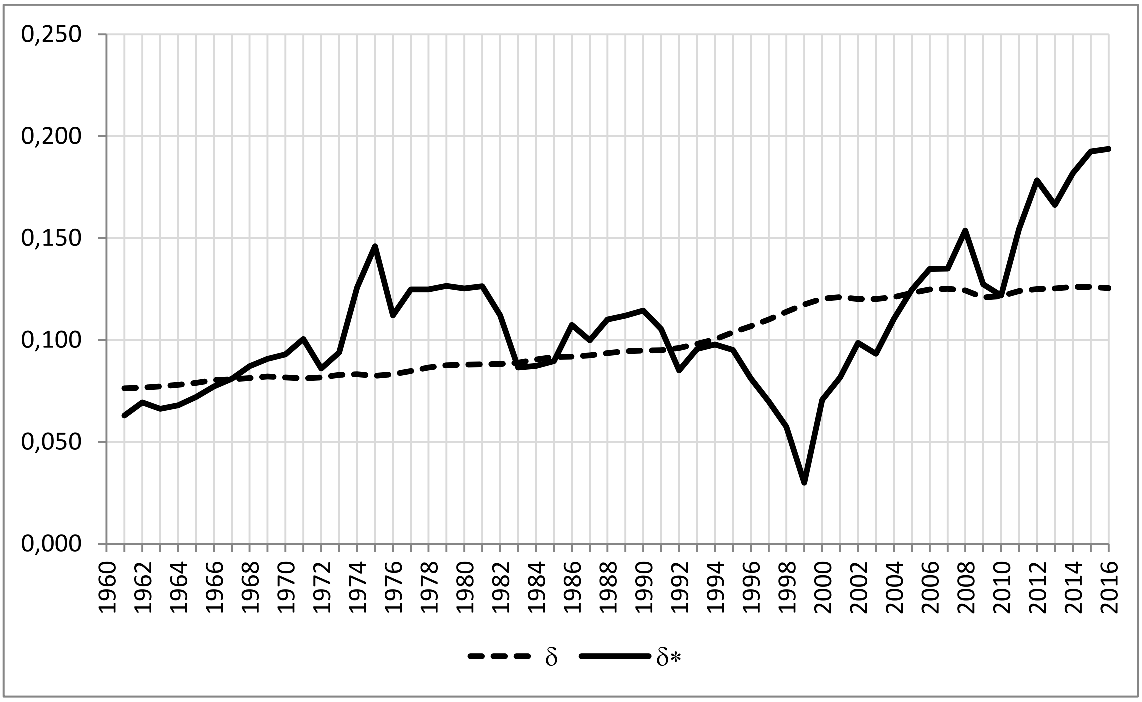


**Figure 1**. Statistical and economic depreciation rates: U.S. Non-Financial Business Sector, 1960-2016. Source: Own elaboration and BEA.

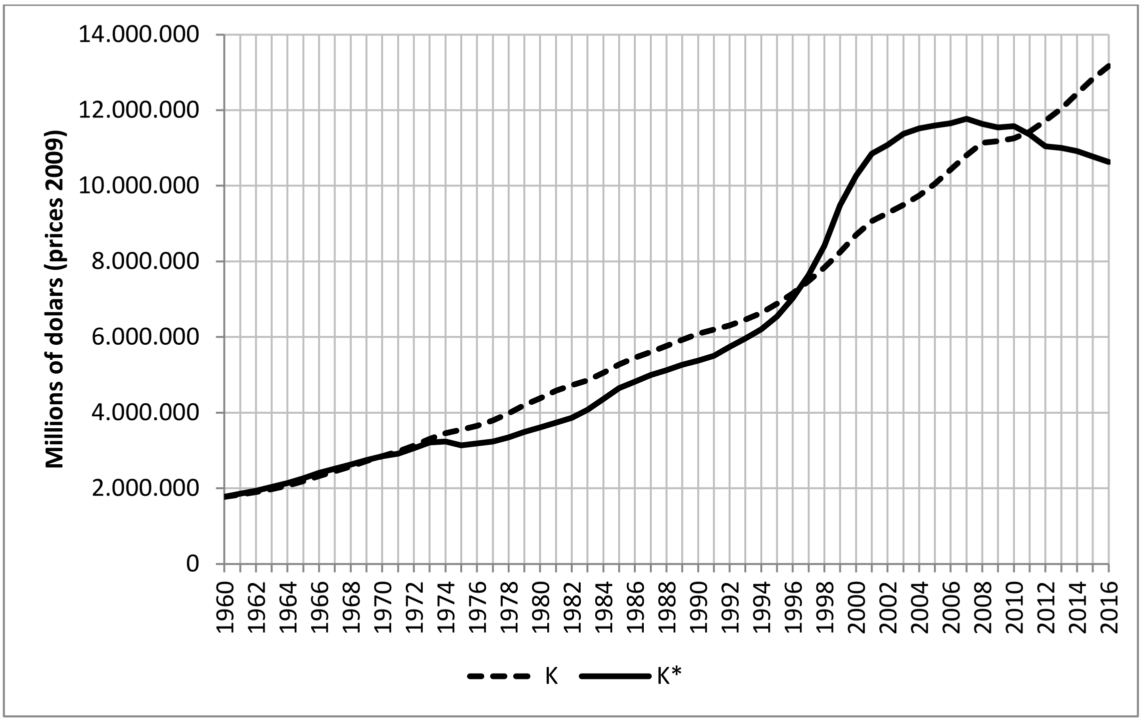


**Figure 2**. Statistical and economic capital stocks: U.S. Non-Financial Business Sector, 1960-2016. Source: Own elaboration and BEA.

**Table 1**. Economic and statistical depreciation rates and capital stocks (millions of dollars, prices 2009) in the United States Non-Financial Business Sector, 1960-2016.

| Year | Depreciation rate | | Capital Stock | | Year | Depreciation rate | | Capital Stock | |
|---|---|---|---|---|---|---|---|---|---|
| | Economic $\delta^*$ | Statistical $\delta$ | Economic $K^*$ | Statistical $K$ | | Economic $\delta^*$ | Statistical $\delta$ | Economic $K^*$ | Statistical $K$ |
| **1960** | | | 1,777,396 | 1,777,396 | **1989** | 0.1119 | 0.0944 | 5,264,742 | 5,931,358 |
| **1961** | 0.0630 | 0.0762 | 1,856,425 | 1,832,817 | **1990** | 0.1144 | 0.0948 | 5,381,330 | 6,088,083 |
| **1962** | 0.0694 | 0.0766 | 1,936,615 | 1,901,354 | **1991** | 0.1052 | 0.0949 | 5,501,912 | 6,197,402 |
| **1963** | 0.0662 | 0.0772 | 2,033,257 | 1,979,326 | **1992** | 0.0851 | 0.0961 | 5,742,120 | 6,310,507 |
| **1964** | 0.0679 | 0.0779 | 2,142,278 | 2,072,220 | **1993** | 0.0956 | 0.0981 | 5,960,438 | 6,458,602 |
| **1965** | 0.0719 | 0.0789 | 2,269,426 | 2,189,960 | **1994** | 0.0977 | 0.1003 | 6,208,748 | 6,641,839 |
| **1966** | 0.0772 | 0.0804 | 2,407,093 | 2,326,821 | **1995** | 0.0952 | 0.1037 | 6,547,209 | 6,882,438 |
| **1967** | 0.0810 | 0.0807 | 2,522,044 | 2,448,940 | **1996** | 0.0811 | 0.1066 | 7,024,398 | 7,156,831 |
| **1968** | 0.0871 | 0.0813 | 2,631,582 | 2,578,862 | **1997** | 0.0698 | 0.1101 | 7,646,737 | 7,481,428 |
| **1969** | 0.0907 | 0.0820 | 2,748,465 | 2,723,027 | **1998** | 0.0574 | 0.1138 | 8,412,878 | 7,834,792 |
| **1970** | 0.0929 | 0.0816 | 2,844,895 | 2,852,714 | **1999** | 0.0300 | 0.1173 | 9,485,802 | 8,240,705 |
| **1971** | 0.1004 | 0.0812 | 2,917,503 | 2,979,286 | **2000** | 0.0706 | 0.1202 | 10,269,508 | 8,702,977 |
| **1972** | 0.0860 | 0.0815 | 3,061,622 | 3,131,358 | **2001** | 0.0815 | 0.1210 | 10,846,158 | 9,063,624 |
| **1973** | 0.0939 | 0.0829 | 3,211,779 | 3,309,321 | **2002** | 0.0985 | 0.1200 | 11,077,461 | 9,275,958 |
| **1974** | 0.1257 | 0.0832 | 3,234,407 | 3,460,335 | **2003** | 0.0932 | 0.1201 | 11,377,765 | 9,494,793 |
| **1975** | 0.1459 | 0.0823 | 3,138,094 | 3,551,243 | **2004** | 0.1103 | 0.1209 | 11,516,426 | 9,740,347 |
| **1976** | 0.1121 | 0.0832 | 3,185,738 | 3,655,362 | **2005** | 0.1246 | 0.1230 | 11,597,194 | 10,058,262 |
| **1977** | 0.1248 | 0.0847 | 3,236,029 | 3,793,862 | **2006** | 0.1348 | 0.1248 | 11,654,840 | 10,423,731 |
| **1978** | 0.1248 | 0.0865 | 3,345,829 | 3,979,271 | **2007** | 0.1350 | 0.1251 | 11,767,757 | 10,804,967 |
| **1979** | 0.1265 | 0.0876 | 3,490,569 | 4,198,444 | **2008** | 0.1537 | 0.1244 | 11,633,851 | 11,135,653 |
| **1980** | 0.1253 | 0.0878 | 3,607,214 | 4,383,697 | **2009** | 0.1273 | 0.1208 | 11,543,431 | 11,181,353 |
| **1981** | 0.1263 | 0.0881 | 3,733,961 | 4,579,735 | **2010** | 0.1217 | 0.1215 | 11,575,538 | 11,259,782 |
| **1982** | 0.1121 | 0.0881 | 3,863,625 | 4,724,484 | **2011** | 0.1545 | 0.1240 | 11,356,972 | 11,433,336 |
| **1983** | 0.0865 | 0.0888 | 4,078,811 | 4,854,392 | **2012** | 0.1783 | 0.1249 | 11,047,042 | 11,720,277 |
| **1984** | 0.0872 | 0.0904 | 4,363,768 | 5,055,849 | **2013** | 0.1662 | 0.1253 | 10,998,597 | 12,039,430 |
| **1985** | 0.0896 | 0.0917 | 4,652,811 | 5,272,320 | **2014** | 0.1818 | 0.1261 | 10,919,466 | 12,442,020 |
| **1986** | 0.1073 | 0.0917 | 4,820,376 | 5,455,335 | **2015** | 0.1924 | 0.1260 | 10,776,555 | 12,833,001 |
| **1987** | 0.0998 | 0.0924 | 4,997,138 | 5,609,385 | **2016** | 0.1937 | 0.1253 | 10,631,612 | 13,167,002 |
| **1988** | 0.1100 | 0.0935 | 5,126,795 | 5,764,047 | | | | | |

Note: The figures of the statistical capital stock represent the measurement of capital according to the PIM, and rely on the Bureau of Economic Analysis (BEA) data. See Table 2 for the gross fixed capital formation and the consumption of fixed capital series. The capital stock in 1960 corresponds to the non-financial corporate and non-corporate business assets (BEA Account Codes: LM102010005 and LM11201005) excluding real estate (LM105035005 and LM115035023), in millions of dollars and deflated using the price index for private nonresidential investment (see Table 2).

**Table 2**. United States Non-Financial Business Sector, 1960-2016.

| Year | $I_t^G$ [1] | $p_t^k$ [2] | $R_t$ [3] | $q_t$ [4] | $B_t^G$ [5] | Year | $I_t^G$ | $p_t^k$ | $R_t$ | $q_t$ | $B_t^G$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1960** | 186793 | 0.297 | 0.041 | 0.75 | | **1989** | 711381 | 0.885 | 0.085 | 0.72 | 714900 |
| **1961** | 190930 | 0.296 | 0.039 | 0.87 | 53600 | **1990** | 719034 | 0.904 | 0.086 | 0.66 | 750500 |
| **1962** | 208987 | 0.296 | 0.039 | 0.83 | 56500 | **1991** | 686960 | 0.921 | 0.079 | 0.81 | 767000 |
| **1963** | 224803 | 0.296 | 0.040 | 0.89 | 59300 | **1992** | 708378 | 0.918 | 0.070 | 0.88 | 772000 |
| **1964** | 247158 | 0.298 | 0.042 | 1.01 | 64000 | **1993** | 767006 | 0.920 | 0.059 | 0.91 | 787000 |
| **1965** | 281251 | 0.302 | 0.043 | 1.09 | 70100 | **1994** | 830716 | 0.927 | 0.071 | 0.83 | 841500 |
| **1966** | 312919 | 0.306 | 0.049 | 0.9 | 76100 | **1995** | 929263 | 0.936 | 0.066 | 1 | 896900 |
| **1967** | 309919 | 0.314 | 0.051 | 1.06 | 83200 | **1996** | 1008293 | 0.930 | 0.064 | 0.98 | 946100 |
| **1968** | 329115 | 0.325 | 0.056 | 1.14 | 91600 | **1997** | 1112457 | 0.925 | 0.064 | 1.16 | 1019600 |
| **1969** | 355687 | 0.339 | 0.067 | 0.9 | 101400 | **1998** | 1204799 | 0.910 | 0.053 | 1.36 | 1109800 |
| **1970** | 351847 | 0.355 | 0.073 | 0.82 | 111800 | **1999** | 1325097 | 0.902 | 0.056 | 1.63 | 1142500 |
| **1971** | 358172 | 0.371 | 0.062 | 0.87 | 119900 | **2000** | 1452995 | 0.907 | 0.060 | 1.25 | 1244700 |
| **1972** | 395021 | 0.384 | 0.062 | 0.98 | 128500 | **2001** | 1413716 | 0.904 | 0.050 | 1.05 | 1303600 |
| **1973** | 437525 | 0.400 | 0.068 | 0.71 | 140400 | **2002** | 1299877 | 0.900 | 0.046 | 0.75 | 1335800 |
| **1974** | 426352 | 0.438 | 0.076 | 0.39 | 165000 | **2003** | 1333148 | 0.899 | 0.040 | 0.98 | 1348800 |
| **1975** | 375723 | 0.496 | 0.080 | 0.54 | 190600 | **2004** | 1393884 | 0.911 | 0.043 | 1.01 | 1449000 |
| **1976** | 399473 | 0.523 | 0.076 | 0.6 | 205200 | **2005** | 1516146 | 0.938 | 0.043 | 0.95 | 1485300 |
| **1977** | 447932 | 0.558 | 0.074 | 0.5 | 229100 | **2006** | 1620634 | 0.966 | 0.048 | 0.98 | 1716700 |
| **1978** | 513570 | 0.595 | 0.084 | 0.48 | 256900 | **2007** | 1685739 | 0.986 | 0.046 | 0.99 | 1842600 |
| **1979** | 567827 | 0.643 | 0.094 | 0.5 | 287900 | **2008** | 1674557 | 1.003 | 0.037 | 0.6 | 1946100 |
| **1980** | 554048 | 0.700 | 0.115 | 0.55 | 342800 | **2009** | 1390700 | 1.000 | 0.033 | 0.74 | 1788000 |
| **1981** | 582243 | 0.767 | 0.139 | 0.46 | 407200 | **2010** | 1437468 | 0.991 | 0.032 | 0.84 | 1791500 |
| **1982** | 548268 | 0.810 | 0.130 | 0.48 | 449000 | **2011** | 1570143 | 1.005 | 0.028 | 0.82 | 1919900 |
| **1983** | 549423 | 0.809 | 0.111 | 0.52 | 463800 | **2012** | 1714507 | 1.022 | 0.018 | 0.92 | 2083900 |

| | | | | | |
|---|---|---|---|---|---|
| **1984** | 640471 | 0.812 | 0.124 | 0.48 | 494600 |
| **1985** | 679903 | 0.820 | 0.106 | 0.56 | 525400 |
| **1986** | 666739 | 0.834 | 0.077 | 0.62 | 564300 |
| **1987** | 657913 | 0.844 | 0.084 | 0.6 | 589300 |
| **1988** | 679104 | 0.865 | 0.088 | 0.62 | 649400 |
| **2013** | 1787368 | 1.030 | 0.024 | 1.17 | 2098300 |
| **2014** | 1920516 | 1.044 | 0.025 | 1.27 | 2262100 |
| **2015** | 1958522 | 1.051 | 0.021 | 1.15 | 2402500 |
| **2016** | 1942552 | 1.048 | 0.018 | 1.2 | 2469200 |

Notes:

1. The variable $B_t^G$ is expressed in millions of current dollars and $I_t^G$ in millions of dollars, prices 2009.
2. The figures in column [1] represent the gross fixed capital formation (non-financial corporate –BEA Account Code: FA105019085, Table s.5.a– and non-corporate –BEA Account Code: FA115019085, Table s.4.a) deflated using the price index for private nonresidential investment (column [2])**.**
3. The figures in column [2] are the price index for private fixed investment by type (nonresidential), BEA Account Code: B008RG, Annual data from 1947 to 2017, Table 5.3.4.
4. The series in column [3] is the long-term interest rate, OECD Stat. Direct source: Federal Reserve Board. U.S. data refers to yields on government securities with outstanding maturities of 10 years.
5. The series in column [4] is the observable Tobin's $q$ supplied in "Returns for Domestic Nonfinancial Business" by S. Osborne and B. A. Retus, Survey of Current Business (December 2017), BEA. We use Q3: the market value of outstanding equity plus market value of outstanding corporate bonds plus net liquid assets divided by the net stock of produced assets valued at current cost.
6. The figures in column [5] represent the distributed gross profits that are computed by adding property income and consumption of fixed capital. The series of property income corresponds to both non-financial corporate [BEA Account Codes: FA106150105 (received) and FA106150005 (paid)] and non-corporate [BEA Account Codes: FA116130101 (received) and FA11615005 (paid)] sectors. The series of consumption of fixed capital also corresponds to non-financial corporate (BEA Account Code: FA106300001) and non-corporate (BEA Account Code: FA116300001) sectors.

# References


[1] Abel, A. B., 1990. Consumption and Investment. In: Benjamin M. Friedman and Frank H. Hahn (eds.), *Handbook of Monetary Economics*, Vol. 2., Ch. 14, 725–778. New York: Elsevier Science.

[2] Bischoff, C. W. and Kokkelenberg, E. C., 1987. Capacity Utilization and Depreciation-in-Use. *Applied Economics* 19 (8), 995-1007.

[3] Bitros, G. C., 2010a. The Theorem of Proportionality in Contemporary Capital Theory: An Assessment of its Conceptual Foundations. *The Review of Austrian Economics* 23 (4), 367-401.

[4] Bitros, G. C., 2010b. The Theorem of Proportionality in Contemporary Capital Theory: An Assessment of its Applicability. *Journal of Economic and Social Measurement* 35, 1-31.

[5] Blanchard, O., Rhee, Ch. and Summers, L., 1993. The Stock Market, Profit and Investment. *Quarterly Journal of Economics* 108, 115-136.

[6] Boucekkine, R., de la Croix, D. and Licandro, O., 2008. Vintage Capital. In S. Durlauf and L. Blume (Eds.): *New Palgrave Dictionary of Economics*, 2nd Edition, 628-631. Palgrave-McMillan.

[7] Boucekkine, R., del Rio, F. and Martinez, B., 2009. Technological Progress, Obsolescence, and Depreciation. *Oxford Economic Papers* 61, 440–466.

[8] Boucekkine, R., Fabbri, G. and Gozzi, F., 2010. Maintenance and Investment: Complements or Substitutes? A reappraisal. *Journal of Economic Dynamics and Control* 34, 2420-2439.

[9] Boucekkine, R. and Ruiz-Tamarit, J. R., 2003. Capital Maintenance and Investment: Complements or Substitutes? *Journal of Economics* 78 (1), 1-28.

[10] Caballero, R., 1999. Aggregate Investment. In: Taylor, J. and M. Woodford (eds.), *Handbook of Macroeconomics*, Vol. 1A, Ch. 12, 813-862. North Holland, Amsterdam.

[11] Chirinko, R. S., 1993. Business Fixed Investment Spending: A Critical Survey of Modelling Strategies, Empirical Results, and Policy Implications. *Journal of Economic Literature* 31 (December), 1875-1911.

[12] Epstein, L. and Denny, M., 1980. Endogenous Capital Utilization in a Short-Run Production Model. Theory and an Empirical Application. *Journal of Econometrics* 12, 189-207.

[13] Escribá-Pérez, F. J., Murgui-García, M. J. and Ruiz-Tamarit, J. R., 2018. Economic and Statistical Measurement of Physical Capital: From Theory to Practice. *Economic Modelling* 75, 246-255.

[14] Escribá-Pérez, F. J. and Ruiz-Tamarit, J. R., 1996. Maintenance Costs and Endogenous Depreciation. *Revista Española de Economía* 13 (2), 261-277.

[15] Fazzari, S. M., Hubbard, R. G. and Petersen, B. C., 1988. Financing Constraints and Corporate Investment. *Brookings Papers on Economic Activity* 1988 (1), 141-195.

[16] Feldstein, M. and Rothschild, M. S., 1974. Towards an Economic Theory of Replacement Investment. *Econometrica* 42 (3), 393-423.

[17] Greenwood, J., Hercowitz, Z. and Krusell, P., 1997. Long-Run Implications of Investment-Specific Technological Change. *American Economic Review* 87 (3), 342-362.

[18] Hayashi, F., 1982. Tobin's Marginal $q$ and Average $q$: A Neoclassical Interpretation. *Econometrica* 50 (1), 213-224.

[19] Hulten, Ch. R., 1992. Growth Accounting When Technical Change is Embodied in Capital. *American Economic Review* 82 (4), 964-980.

[20] Johansen, L., 1959. Substitution versus Fixed Production Coefficients in the Theory of Economic Growth. *Econometrica* 29, 157-176.

[21] Jorgenson, D. W., 1963. Capital Theory and Investment Behavior. *American Economic Review* 53 (2), 247-259.

[22] Jorgenson, D. W., 1966. The Embodiment Hypothesis. *Journal of Political Economy* 74 (1), 1-17.

[23] Jorgenson, D. W., 1967. The Theory of Investment Behavior. *Brookings Papers on Economic Activity* 1967 (1), 129-175.

[24] Kalyvitis, S., 2006. Another Look at the Linear $q$ Model: An Empirical Analysis of Aggregate Business Capital Spending with Maintenance Expenditures. *Canadian Journal of Economics / Revue Canadienne d'Economique* 39 (4), 1282-1315.

[25] Malcomson, J. M., 1975. Replacement and the Rental Value of Capital Equipment subject to Obsolescence. *Journal of Economic Theory* 10 (1), 24-41.

[26] McGrattan, E. and Schmitz, J. A., 1999. Maintenance and Repair: too Big to Ignore. *Federal Reserve Bank of Minneapolis Quarterly Review* 4, 2-13.

[27] Motahar, E., 1992. Endogenous Capital Utilization and the $q$ Theory of Investment. *Economics Letters* 40 (1), 71-75.

[28] Mukoyama, T., 2008. Endogenous Depreciation, Mismeasurement of Aggregate Capital, and the Productivity Slowdown. *Journal of Macroeconomics* 30 (1), 513-522.

[29] Musso, P., 2004. Productivity Slowdown and Resurgence. The Role of Capital Obsolescence. *Revue Économique* 55 (6), 1215-1239.

[30] Nickell, S., 1975. A Closer Look at Replacement Investment. *Journal of Economic Theory* 10 (1), 54-88.

[31] Siegel, J. J., 2008. Stocks for the Long Run, The Definitive Guide to Financial Market Returns and Long-Term Investment Strategies. McGraw-Hill, New York, USA.

[32] Solow, R. M., 1960. Investment and Technological Progress. In Kenneth J. Arrow, Samuel Karlin and Patrick Suppes (Eds.): *Mathematical Methods in the Social Sciences*, 89-104. Stanford University Press, Stanford.

[33] Triplett, J. E., 1996. Depreciation in Production Analysis and in Income and Wealth Accounts: Resolution of an old debate. *Economic Inquiry* 34 (1), 93-115.

Institut de Recherches Économiques et Sociales
Université catholique de Louvain

Place Montesquieu, 3
1348 Louvain-la-Neuve, Belgique

ISSN 1379-244X D/2019/3082/4