Imperfect mobility of labor across sectors and fiscal transmission

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Abstract

Our paper investigates the impact of government spending shocks on relative sector size and contrasts the effects across countries. Using a panel of sixteen OECD countries over the period 1970-2007, our VAR evidence shows that a rise in government consumption i) increases the share of non tradables in labor and real GDP and lowers the share of tradables, and ii) causes a significant increase in non traded wages relative to traded wages. While the first finding reveals that the non traded sector is more intensive in the government spending shock and experiences a labor inflow that increases its relative size, the second finding suggests the presence of labor mobility costs preventing wage equalization across sectors. Turning to cross-country differences, empirically we detect a positive relationship between the magnitude of the impact responses of sectoral output shares and the degree of labor mobility across sectors. To account for our evidence, we develop an open economy version of the neoclassical model with tradables and non tradables. Our quantitative analysis shows that the model is successful in replicating the responses of sectoral output shares to a fiscal shock, as long as we allow for a difficulty in reallocating labor across sectors along with adjustment costs to capital accumulation. Finally, calibrating the model to country-specific data, we are able to generate a cross-country relationship between the degree of labor mobility and the responses of sectoral output shares which is similar to that in the data.

Keywords: Fiscal policy; Labor mobility; Investment; Non tradables; Sectoral wages.

JEL Classification: E22; E62; F11; F41; J31.

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1 Introduction

Does a government spending shock affect the production sectors in an open economy uniformly? If not, how can we explain the heterogeneity in the sectoral effects of a rise in government consumption? Does the magnitude of the sectoral effects vary across countries and what factors cause such differences? Our paper provides an attempt to answer these questions by exploring the sectoral effects of a government spending shock empirically and calibrating an open economy version of the neoclassical model with tradables and non tradables.

The motivation for the analysis of the sectoral effects of a rise in government consumption is based on our panel VAR evidence for 16 OECD countries over the period 1970-2007. First, our evidence reveals that a government spending shock has a strong expansionary effect on hours worked and output in the non traded sector relative to the traded sector. Such a finding suggests that the rise in government consumption is biased toward non traded goods. Second, we also find empirically that a government spending shock leads to a shift of labor toward the non traded sector that increases its relative size. Such a reallocation of labor toward the non traded sector is costly, however, as we detect empirically a significant increase in non traded wages relative to traded wages. Third, when we turn to cross-country differences, these labor mobility costs are found to play a pivotal role in explaining international differences in the sectoral impact of fiscal policy. More precisely, we find that both the increase in the share of non tradables and the decline in the share of tradables are more pronounced in countries where the degree of labor mobility across sectors is higher.

Estimates of the responses of sectoral output shares to a government spending shock allow us to evaluate the contribution of the reallocation of resources to sectoral fiscal multipliers empirically. More specifically, our evidence reveals that a rise in government spending by one percentage point of GDP increases non traded output by 0.7% of GDP on impact while the output share of non tradables rises initially by 0.35% of GDP. Since the latter result indicates that non traded output would increase by 0.35% if GDP remained constant, the reallocation of resources toward the non traded sector thus contributes to 50% of non traded output growth. The rise in the share of non tradables in real GDP also suggests that the non traded sector receives a disproportionate share of the shock to government spending. In this regard, our estimates show that government consumption of non tradables contributes to 90% on average of increases in government spending.

While government spending shocks are biased toward non tradables and generate a substantial reallocation of resources which significantly affects the relative size of sectors, our evidence suggests some difficulty in reallocating labor between sectors. To assess the extent of mobility costs, we estimate the elasticity of labor supply across sectors, which
captures the degree of labor mobility, for each country in our sample. Estimating the panel
VAR model for countries with a low and a high elasticity of labor supply across sectors,
we find that non traded wages increase more relative to traded wages while hours worked
(output) in tradables relative to non tradables fall less in countries where the elasticity is
low. This corroborates our conjecture that in countries where mobility costs are higher, non
traded firms wishing to produce more must pay much higher wages to attract workers. In
order to emphasize the importance of mobility costs for fiscal transmission, we explore the
cross-country relationship between the responses of sectoral output shares and the degree
of labor mobility captured by the elasticity of labor supply across sectors. While the vast
majority of the economies experience a rise (decline) in their output share of non tradables
(tradables), we empirically detect a positive relationship between the size of the responses
of sectoral output shares and the degree of labor mobility.

To account for our evidence on fiscal transmission, we put forward an open economy
version of the neoclassical model with tradables and non tradables. In calibrating the
model to a representative OECD economy, we allow for the fraction of higher government
consumption expenditure on non tradables to be higher that the share of non tradables in
real GDP, in line with our evidence, so that the government shock is biased toward non
tradables. Our quantitative results show that the model is successful in replicating the
sectoral effects of government spending shocks as long as we allow for imperfect mobility
of labor across sectors and capital adjustment costs. With these two features, the model
produces a rise in the share of non tradables by 0.38 percentage point of GDP, close to our
empirical findings. If we remove both or either one of these ingredients, the model fails to
account quantitatively for our evidence on fiscal transmission, in particular the responses of
sectoral output shares which we estimate empirically. Intuitively, if we abstract from capital
adjustment costs, a government spending shock leads to a dramatic fall in investment which
offsets the rise in government consumption. As a result, the excess demand in the non traded
goods market is low or even nil. Due to low incentives to shift resources toward the non
traded sector, the model understates substantially the rise in the share of non tradables.
Conversely, if we allow for capital adjustment costs, the decline in investment is mitigated,
which leads to significant excess demand in the non traded goods market. However, if we
impose perfect mobility of labor across sectors, high incentives to shift resources toward
the non traded sector lead the model to overstate the responses of sectoral output shares
considerably.

The final exercise we perform is to investigate whether the model can account for cross-

\footnote{To generate imperfect mobility of labor, we consider limited substitutability in hours worked across
sectors along the lines of Horvath [2000]. See e.g., Bouakez et al. [2011], Cardi and Restout [2015] who
assume that sectoral hours worked are aggregated by means of a CES function in order to account for the
evidence related to monetary policy shocks or the long-run effects of productivity shocks biased toward the
traded sector.}
country differences in the responses of sectoral output shares to a fiscal shock. We thus calibrate the model to match data from the 16 OECD countries regarding dimensions such as the non tradable content of labor, consumption, investment, government spending, and the elasticity of labor supply across sectors capturing the degree of labor mobility. In line with the evidence, the decline in the output share of tradables and the rise in the output share of non tradables are more pronounced in countries with a higher degree of labor mobility. We find quantitatively that impact responses of sectoral output shares to a government spending shock are sensitive to the degree of labor mobility, as they vary between 0.26% and 0.49% of GDP for non tradables when we move from the lowest to the highest value of elasticity of labor supply across sectors. Although the model tends to understate changes in the relative size of sectors, it is able to generate a cross-country relationship between the responses of sectoral output shares and the degree of labor mobility that is similar to that in the data.

We contribute to the vast literature investigating fiscal transmission both empirically and theoretically by focusing on the reallocation effect of government spending shock. Like Ramey and Shapiro [1998], we emphasize the importance of the composition of government spending for understanding both the aggregate and the sectoral effects of a fiscal shock. In contrast to the authors who consider a rise in defense spending during a military buildup, which is heavily concentrated in the manufacturing sector, we investigate a rise in government consumption in 'normal times' and find that such a government spending shock leads to a sharp increase in non traded output relative to traded output. This finding is in line with estimates documented by Monacelli and Perotti [2008] and Benetrix and Lane [2010] which reveal that an increase in government spending disproportionately benefits the non traded sector. In contrast to the authors who restrict their attention to sectoral output effects, we highlight the changes in sectoral shares in labor and real GDP. In this regard, one major finding is that the share of non tradables in employment and real GDP increases significantly while the share of tradables declines. These findings reveal that government spending shocks are strongly biased toward non tradables and produce a reallocation of labor across sectors that affect their relative size. Like Perotti [2008], the sector which is relatively more intensive in the government spending shock experiences an increase in real wages. In contrast to Perotti who considers Ramey-Shapiro episodes (i.e., Vietnam War and the Carter-Reagan buildup) and finds empirically higher increases in the real wage in industries that are defense related, we detect empirically a significant increase in wages paid by non traded industries which are relatively more intensive in government spending shocks in 'normal times'. One additional key finding with respect to the papers mentioned above is that international differences in workers’ costs of switching sectors can account for

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2 More precisely, all else being equal, for the share of non tradables in real GDP to increase, the fraction of the rise in government consumption spent on non tradables must be higher than the share of non tradables in real GDP.
the cross-country dispersion in the responses of sectoral shares, as we uncover a positive cross-country relationship between the degree of labor mobility and the changes in relative sector size.

Our theoretical approach is at the cross-roads between two strands of the literature investigating the adjustment of open economies to sectoral demand shocks. First, it is closely related to the analysis by Morshed and Turnovsky [2004], Cardi and Restout [2015], Chatterjee and Mursagulov [2016] who develop variants of the neoclassical model and investigate the effects of a rise in government spending on non tradables. All of these works have in common that they impose perfect mobility of labor across sectors and focus mainly on the real exchange rate dynamics by considering either intersectoral capital mobility costs, endogenous markups, or an increase in public investment, respectively. In contrast to these works, we document panel VAR evidence on fiscal transmission and show how a difficulty in reallocating labor across sectors can account for our evidence following a rise in government purchases biased toward non tradables. In this respect, our study can be viewed as complementary to the literature which investigates the quantitative implications of a sector-specific government spending shock in a model with labor market frictions. By developing a multi-sector model with search frictions in the labor market, Phelan and Trejos [2000] show that an adverse sectoral demand shock originating from a cut in military purchases can be greatly magnified as a result of the combined effect of labor shifts across sectors and a slow reallocation. Like the authors, we show that the combined effect of sectoral intensity in the government spending shock and imperfect mobility of labor across sectors matter for fiscal transmission. In contrast, we estimate the degree of labor mobility across sectors, quantify both empirically and numerically the changes in the relative sector size following a rise in government consumption and show that international differences in the degree of labor mobility across sectors can account for cross-country differences in the sectoral impact of fiscal policy. Furthermore, the authors abstract from physical capital while we find that both imperfect mobility of labor along capital adjustment costs are necessary to produce the responses of sectoral shares in real GDP that we document empirically.

The remainder of the paper is organized as follows. In section 2, we establish panel VAR evidence on aggregate and sectoral effects of a government spending shock and then document an empirical relationship between the responses of sectoral output shares and the degree of labor mobility. In section 3, we develop an open economy version of the neoclassical model with a difficulty in reallocating labor across sectors. In section 4, we abstract from physical capital accumulation in order to derive a number of analytical results and to build up intuition on fiscal transmission with imperfect mobility of labor. In section 5, we report the results of our numerical simulations and assess the ability of the model

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3It is worth mentioning that we consider a rise in government consumption which is splits between non tradables and tradables in accordance with their respective share in government spending.
to account for the evidence. In section 6, we summarize our main results and present our conclusions.

2 Stylized Facts on Fiscal Transmission

In this section, we revisit the time-series evidence on fiscal transmission by differentiating the effects of fiscal policy between the traded and non-traded sectors. Because the sectoral impact of an expansionary fiscal shock varies considerably across the countries in our sample, we also contrast the effects of government spending shocks in economies with low and high workers’ mobility costs. We denote below the level of the variable in upper case and the logarithm in lower case.

2.1 VAR Model and Identification

In order to shed some light on fiscal transmission and guide our quantitative analysis, we estimate the VAR model in panel format on annual data. We consider a structural model with \( k = 2 \) lags in the following form:

\[
AZ_{i,t} = \sum_{k=1}^{2} B_k Z_{i,t-k} + \epsilon_{i,t},
\]

where subscripts \( i \) and \( t \) denote the country and the year, respectively, \( Z_{i,t} \) is the vector of endogenous variables, \( A \) is a matrix that describes the contemporaneous relation among the variables collected in vector \( Z_{i,t} \), \( B_k \) is a matrix of lag specific own- and cross-effects of variables on current observations, and the vector \( \epsilon_{i,t} \) contains the structural disturbances which are uncorrelated with each other.

Because the VAR model cannot be estimated in its structural form, we pre-multiply (1) by \( A^{-1} \) which gives the reduced form of the VAR model:

\[
Z_{i,t} = \sum_{k=1}^{2} A^{-1} B_k Z_{i,t-k} + e_{i,t},
\]

where \( A^{-1} B_k \) and \( e_{i,t} = A^{-1} \epsilon_{i,t} \) are estimated by using a panel OLS regression with country fixed effects and country specific linear trends. To identify the VAR model and recover the government spending shocks, we need assumptions on the matrix \( A \) as the reduced form of the VAR model that we estimate contains fewer parameters than the structural VAR model shown in eq. (1).

To identify fiscal shock, we follow Blanchard and Perotti [2002] and assume that government spending is predetermined relative to the other variables in the VAR model. We thus adopt a Cholesky decomposition in which government spending is ordered before the other variables so that the fiscal shock is exogenous. The identifying assumption holds as long as public spending does not react contemporaneously to the state of the economy.
due to delays between current output observation and the implementation of fiscal measures. The potential problem is that we use annual data and some adjustment could be possible within the year. To support our assumption, we estimated the same panel VAR model that includes aggregate variables which are available on a yearly and a quarterly basis. The responses of variables to an exogenous fiscal shock are similar whether we use annual or quarterly series as our estimates using quarterly data lie within the confidence interval of those obtained from yearly data.\(^4\) Our results accord well with the conclusion reached by Born and Müller [2012] whose test reveals that the assumption that government spending is predetermined within the year cannot be rejected. Moreover, as government spending does not include transfers (such as unemployment benefits), it is therefore much less likely to respond automatically to the other variables. An additional obstacle is to identify unexpected fiscal events. We conducted an investigation of the potential presence of anticipation effects by using a dataset constructed by Born, Juessen and Müller [2013] which contains one year-ahead OECD forecasts for government spending.\(^5\) First, we run Granger-causality tests and do not find that fiscal forecasts have any predictive power for our identified government spending shocks. Second, it turns out that differences are rather moderate when we control for the anticipation effects and that our main results are not altered by the inclusion of forecasts for government spending growth.\(^6\)

2.2 Data Construction

Before presenting the VAR model, we briefly discuss the dataset we use. Our sample consists of a panel of 16 OECD countries: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Finland (FIN), France (FRA), Ireland (IRL), Italy (ITA), Japan (JPN), the Netherlands (NLD), Norway (NOR), Spain (ESP), Sweden (SWE), the United Kingdom (GBR), and the United States (USA). Our sample covers the period 1970-2007 and contains annual observations.

As detailed in the next subsection, we consider a number of VAR specifications as we wish to shed some light on the reallocation of resources triggered by a fiscal shock that affects the relative size of sectors. Because their movements are strongly intertwined, we explore both the aggregate and sectoral effects of government spending shocks empirically. The former variables consist of government consumption \((G_{it})\), GDP \((Y_{it})\), private fixed investment \((JE_{it})\), current account \((CA_{it})\), labor \((L_{it})\), and real consumption wage \((WC_{it})\).

\(^4\)The results are included in a Technical Appendix available on request from the authors. An alternative to deal with the potential endogeneity problem is to identify exogenous changes in government spending directly from historical events or official documents. In a robustness exercise, we augment each VAR model with the ’spending-based’ events variable constructed by Guajardo, Leigh, and Pescatori [2014] whose dataset contains 173 fiscal policy changes for 17 OECD countries over the period 1978-2009. Overall, our results show low sensitivity to the identifying assumption.

\(^5\)We use an alternative dataset constructed by Fioramanti et al. [2016] which contains one year-ahead forecasts for the budget balance-GDP ratio performed by the European Commission and do not find any evidence of anticipation effects.

\(^6\)Details about our empirical strategy and data construction can be found in a Technical Appendix.
All data are obtained from the OECD Economic outlook and OECD STAN database together with EU KLEMS database. For government final consumption expenditure, GDP, and private investment (excluding residential investment), we use the volumes reported by the OECD. Aggregates $G_{it}$, $Y_{it}$, $JE_{it}$ are deflated with their own deflators. We use hours worked to measure labor. All quantities are scaled by the working age population and are measured in logs, except for the current account which is expressed as a fraction of GDP. The real consumption wage is the ratio of the nominal aggregate wage, $W_{it}$, to the consumption price index, $P_{C, it}$, and is measured in logs. The nominal wage is obtained by calculating the ratio of labor compensation to the number of hours worked. Details of data construction and the source of variables used in our estimate are given in Appendix A.

Because our primary objective is to investigate the sectoral effects of fiscal transmission, we describe below how we construct time series at a sectoral level. Our sample covers the period 1970-2007 (except for Japan: 1974-2007), for eleven 1-digit ISIC-rev.3 industries. We use the EU KLEMS [2011] and OECD STAN [2011] database which provide domestic currency series of value added in current and constant prices, labor compensation and employment (number of hours worked) at an industry level. To split these eleven industries into traded and non traded sectors, we follow the classification suggested by De Gregorio et al. [1994]. Agriculture, hunting, forestry and fishing; Mining and quarrying; Total manufacturing; Transport, storage and communication are classified as traded industries. Following Jensen and Kletzer [2006], we updated the classification by De Gregorio et al. [1994] by treating Financial intermediation as a traded industry. Electricity, gas and water supply; Construction; Wholesale and retail trade; Hotels and restaurants; Real estate, renting and business services; Community, social and personal services are classified as non traded industries.7

Once industries have been classified as traded or non traded, series for sectoral value added in current (constant) prices are constructed by adding value added in current (constant) prices for all sub-industries in sector $j = T, N$, from which we construct price indices, $P_{it}^j$, which correspond to sectoral value added deflators. The relative price of non tradables, $P_{it}$, is defined as the ratio of the non traded value added deflator to the traded value added deflator (i.e., $P_{it} = P_{N, it}^N / P_{T, it}^T$). The same logic applies to constructing series for hours worked and labor compensation in the traded and the non traded sectors which allow us to construct sectoral wages, $W_{it}^j$. The relative wage, $\Omega_{it}$, is computed as the ratio of the

7In contrast to De Gregorio et al. [1994] who treat “Financial intermediation” as non tradable, we classify this industry as tradable, following Jensen and Kletzer [2006]; our sensitivity analysis reveals that our conclusions hold whether “Financial intermediation” is classified as tradable or non tradable. The classification of the “Wholesale and Retail Trade”, “Hotels and Restaurants”, “Transport, Storage and Communication”, and “Real Estate, Renting and Business Services” items may also display some ambiguity. In order to address this issue, we re-estimated the various VAR specifications for different classifications in which one of the above industries initially marked as tradable (non tradable resp.) is classified as non tradable (tradable resp.), all others industries staying in their original sector. Because results are very similar, to save space we do not present them and they are therefore relegated to the Technical Appendix.
non traded wage to the traded wage (i.e., $\Omega = W^N_t/W^T_t$). The real consumption wage in sector $j$, $W_{C,j,t}$, is defined as the sectoral nominal wage, $W^j_{it}$, divided by the consumption price index, $P_{C,it}$. 

2.3 VAR Specification

In order to investigate the distribution of the aggregate fiscal multiplier across sectors along with the contribution of the reallocation of resources to sectoral fiscal multiplier, we consider four specifications. The choice of variables is motivated in part by the variables discussed in the quantitative analysis.

- To explore the magnitude of the aggregate fiscal multiplier empirically, we consider a VAR model that includes in the baseline specification (log) government consumption, $g_{it}$, GDP, $y_{it}$, total hours worked, $l_{it}$, private fixed investment, $j_{it}$, and the real consumption wage denoted by $w_{C,it}$. Our vector of endogenous variables, is given by: $z_{it} = [g_{it}, y_{it}, l_{it}, j_{it}, w_{C,it}]$. In the second specification we replace private investment with the current account expressed in percentage of GDP, $ca_{it}$.

- To investigate the magnitude of the sectoral fiscal multiplier, we consider a VAR model that includes value added at constant prices in sector $j$, $y_{j,it}$, hours worked in sector $j$, $l_{j,it}$, and the real consumption wage in sector $j$, $w_{j,C,it}$. Our vector of endogenous variables, is given by: $z^j_{it} = [g_{it}, y_{j,it}, l_{j,it}, w_{j,C,it}]$ with $j = T, N$.

- To estimate the change in relative sector size defined as the excess of the sectoral fiscal multiplier over the aggregate fiscal multiplier, we consider a VAR model where we divide sectoral value added at constant prices (sectoral labor) by GDP (total labor) in order to filter the change in sectoral output (sectoral labor) arising from GDP (total labor) growth which allows us to isolate the ‘pure’ reallocation effect and thus gauge the importance of the shift of resources across sectors that affects their relative size. Denoting the output and labor share of sector $j$ by $\nu^Y_{it} = y_{it} - y_{it}$ and $\nu^L_{it} = l_{it} - l_{it}$, respectively, our vector of endogenous variables, is given by: $z^{S,j}_{it} = [g_{it}, \nu^Y_{it}, \nu^L_{it}, w_{C,it}]$ with $j = T, N$.

- Finally, to investigate the relative price ($p$) and relative wage ($\omega$) effects of a fiscal shock, we consider a VAR model where we replace sectoral quantities with the ratio of sectoral quantities for both the product and the labor market. Our vector of endogenous variables, is given by: $z^P_{it} = [g_{it}, y_{it}^T - y_{it}^N, p_{it}]$ and $z^W_{it} = [g_{it}, l_{it}^T - l_{it}^N, \omega_{it}]$, respectively.

2.4 Effects of Government Spending Shocks: VAR Evidence

We generated impulse response functions which summarize the responses of variables to an increase in government spending by 1 percentage point of GDP. Fig. 1-2 displays the
The estimated effects of a fiscal shock for our four alternative sets of specifications. The horizontal axis measures time after the shock in years and the vertical axis measures percentage deviations from trend. GDP together with its demand components, sectoral output and sectoral output share are measured in percentage points of total output relative to trend. Sectoral labor and sectoral labor share are both measured in percentage deviations of total hours worked from trend. The remaining variables are measured in percentage deviations from trend. In each case, the solid line represents the point estimate, while the shaded area indicates the 90% confidence bounds obtained by bootstrap sampling. Point estimates are shown in Panel A of Table 1 at a one year-, two-year and four-year horizon.

2.4.1 Aggregate Effects

We start with the aggregate effects of a government spending shock. Fig. 1 shows results for the first VAR model. The top left panel of Fig. 1 displays the endogenous response of government spending to an exogenous fiscal shock. The response of government consumption is hump-shaped, as it peaks after one year and then gradually declines; it shows a high level of persistence over time as it is about 8 years before the shock dies out. The impact on GDP is fairly moderate as the fiscal multiplier is about 0.5 and averages 0.29 during the first four years after the shock. As shown in the last row, the dynamic adjustment of real GDP seems to mimic the dynamic adjustment of hours worked which increase on impact by 0.53% and declines after one year. In addition, we detect a moderate increase in the real consumption wage followed by a rapid decline. Its cumulative response over a two-year horizon is 0.6% approximately, and subsequently becomes negative.

Turning to the response of investment and the current account as shown in the second column of Fig. 1, the top panel indicates that investment is fairly unresponsive on impact which suggests the presence of installation costs, while the middle panel reveals that the current account moves into deficit in the short-run. The government spending shock leads to a protracted decline in investment which remains below trend while the current account recovers after two years and moves into surplus after about 5 years. As shown in Table 1, after four years, the cumulative decline in investment amounts to -1.29 percent of GDP while the current account deficit is substantial at -3.35 GDP percentage points.

*Like Ilzetzki, Mendoza, and Vegh [2013], we calculate the (aggregate or sectoral) multiplier at a k-year horizon by computing the ratio of the present value of the cumulative change in output to the present value of the cumulative change in government consumption, setting the world interest rate set to 4% to be consistent with the model calibration.

*Overall, our panel VAR evidence for aggregate variables is well in line with that reported in earlier studies, see e.g., Corsetti et al. [2012] who use a panel of 17 OECD countries for the period 1975-2008, and Beetsma, Giuliodori and Klaassen [2008] who consider a panel of 11 Euro Area Members.
2.4.2 Sectoral Effects

We now discuss the sectoral effects of a government spending shock. In Fig. 2, we report results for the second, third and fourth VAR model.\(^{10}\) The first column displays sectoral multiplier results. We find that a rise in government consumption has a strong expansionary effect on non traded output which increases significantly on impact by 0.70 percentage point of GDP, as reported in column 3 of Table 1, while its four-year horizon cumulative response is substantial at 1.88 percentage points of GDP. During the first four years after the shock, the non traded output multiplier of government spending averages at about 0.47 percentage point of GDP. In contrast, the traded sector displays a negative fiscal multiplier for the first four years as the government spending shock gives rise to a contraction in traded output which remains below trend.\(^{11}\) For non traded output to increase relative to traded output, the fraction of the rise in government spending spent on non tradables, \(\omega_{G,N}\), must be higher than that on tradables, \(\omega_{G,T}\).\(^{12}\) Henceforth, our evidence shown in Fig. 2 reveals that the government spending shock is biased toward non tradable goods as it benefits the non traded sector at the expense of the traded sector. Furthermore, as shown in the second row of Fig. 2, higher non traded output is associated with a sharp increase in hours worked on impact, while the traded sector experiences a gradual decline in labor for the first five years.

The second column of Fig. 2 enables us to gauge the contribution of the reallocation of inputs, labor especially, to the expansion of the relative size of the non traded sector. The second and fourth rows show that the labor share of tradables declines by 0.27 percentage point of total labor while the reverse is true for non tradables. Since the response of sectoral labor share filters the change in sectoral labor arising from growth in total hours worked, our

\(^{10}\)For reason of space, weomit the endogenous responses of government spending along with the dynamic adjustment of sectoral real consumption wages which can be found in a Technical Appendix. Because we consider alternative VAR models, one might be concerned by the fact that identified government spending shocks display substantial differences across VAR specifications. Reassuringly, the correlation between structural government spending shocks across VAR specifications averages 0.97. To further address this issue, we ran a number of robustness checks by augmenting each VAR model with the same identified spending shock, ordered first. Because in the quantitative analysis, we take \(z_{it} = [g_{it}, y_{it}, l_{it}, j_{it}, w_{it, C, it}]\) as our benchmark model to calibrate the government spending shock, we augment each VAR model with the spending shock identified for this benchmark specification on annual or quarterly data. Results reveal that the discrepancy in the estimated effects is quite moderate whether the spending shock is identified on a yearly or a quarterly basis.

\(^{11}\)Like us, Monacelli and Perotti [2008] who use U.S. quarterly data from 1954 to 2006 and Benetrix and Lane [2010] who consider of panel of 11 EU countries over 1970-2005, document a significant increase in non traded output following a government spending shock. While we find a protracted decline in traded output, Monacelli and Perotti [2008] detect a fall on impact only while traded output rises above trend after two years. Benetrix and Lane [2010] report an increase in traded output followed by a gradual decline. When we re-estimate the VAR model on U.S. annual data, we obtain similar sectoral output effects, at least qualitatively, to those reported by Monacelli and Perotti [2008]. Second, when we restrict our sample to EU countries over 1970-2005, we find that traded output increases instead of declining, in line with evidence reported by Benetrix and Lane [2010]. While the sample matters for the response of traded output, our main conclusions hold for the U.S. or a restricted set to EU countries: the relative size of the non traded sector increases significantly while the relative wage of non tradables appreciate.

\(^{12}\)More specifically, keeping private sector’s demand components fixed, the sectoral output growth differential in percentage points of GDP is determined by the sectoral intensity differential in the government spending shock, i.e., \(\nu^YN \hat{Y}^N(t) - \nu^YT \hat{Y}^T(t) = (\omega_{G,N} - \omega_{G,T})dG(t)/Y\) where we denote the percentage deviation relative to initial steady-state by a hat.
estimates thus suggest that over the first year, a government spending shock causes 0.27% of workers to shift from the traded to the non traded sector. Since non traded hours worked increase by 0.55% of total employment, 50% of non traded employment growth is the result of labor reallocation. As shown in the first and the third row of the second column, a fiscal shock lowers the output share of tradables significantly and substantially increases that of non tradables. Because changes in output shares indicate how much sectoral output would increase if GDP remained constant, they provide us with valuable information on the shift of inputs across sectors and the resulting changes in their relative size. Quantitatively, since non traded output rises by 0.7 percentage point of GDP while the output share of non tradables rises by 0.35 percentage point of GDP, the shift of resources toward the non traded sector alone contributes to 50% of non traded output growth. Our second set of findings shown in Fig. 2 thus reveals that a government spending shock generates a reallocation of labor that significantly affects the relative size of sectors.

Inequality $\omega_{GN} > \omega_{GT}$ is a necessary but not sufficient condition for the share of non tradables in GDP to increase following a fiscal shock. The reason is that such a condition does not take into account that the share of non tradables in almost twice as large as that of tradables. A sufficient condition for the share of non tradables in GDP to increase is $\omega_{GN} > \nu_{Y,N}$, i.e., the fraction of government spending spent on non traded goods must exceed the share of non tradables in GDP. We may consider inequality $\omega_{GN} > \nu_{Y,N}$ as a stricter definition of a government spending shock biased toward non tradables. Obviously, for the increase in the GDP share of non tradables to materialize, resources must be reallocated away from the traded sector to the non traded sector.\(^{13}\)

In order to check whether the government spending shock is strongly intensive in non tradables, we first split government final consumption expenditure between government consumption on non tradables, $g_N$, and tradables, $g_T$, by using the COFOG database from the OECD which provides a breakdown of government expenditure by function.\(^{14}\) The sample covers 13 OECD countries over the period 1995-2015. We choose this period as time series for government consumption by function are not available before 1995 for most of the countries in our sample while the period 1995-2007 would be too short to obtain consistent estimates.\(^{15}\) Then, we estimate a VAR model in panel format on annual data that includes unanticipated government spending shocks, $\epsilon_t^G$, or-

\(^{13}\)Keeping private sector’s demand components fixed, the growth differential in GDP units between sectoral value added at constant prices and real GDP is positive as long as the intensity of the non traded sector in the government spending shock is higher than its share in real GDP, i.e., $\nu_{Y,N} (\hat{Y}_N - \hat{Y}) = (\omega_{GN} - \nu_{Y,N}) dG(t)/Y$.

\(^{14}\)While there is some degree of arbitrariness in treating certain items as non tradable and the others as tradable, the content of items is such that there is little doubt in the breakdown. See Appendix B for details about the breakdown of $g$ into $g_N$ and $g_T$.

\(^{15}\)Data to construct time series for sectoral government consumption expenditure are available for all the countries in our sample except Canada. In efforts to have a balanced panel and time series of a reasonable length, Australia (1998-2015) and Japan (2005-2015) are removed from the sample, which leaves us with 13 OECD countries over the period 1995-2015.
dered first, government consumption spending and sectoral government consumption on non tradables and tradables in panel format on annual data. To identify exogenous and unanticipated fiscal shocks, \( \epsilon_{G,t} \), we estimate the VAR model that includes aggregate variables, i.e., \( z_{i,t} = [g_{i,t}, y_{i,t}, l_{i,t}, j_{i,t}, w_{C,i,t}] \), and adopt a Cholesky decomposition. Fig. 3 displays the responses of government consumption of non tradables and tradables to an exogenous and unanticipated increase in government spending by 1% of GDP. On impact, government consumption of non tradables increases by 0.88%. Its contribution to the government spending shock averages 90% and is quite stable over time as it varies from 88% up to 91%.\(^{16}\) Moreover, we find that the responses of sectoral government consumption to an exogenous fiscal shock are both hump-shaped and seem to mimic the adjustment of government spending shown in 1(a).\(^ {17}\)

The third column of Fig. 2 enables us to shed some light on fiscal transmission. The first two rows support the conjecture that an aggregate government spending shock triggers a sectoral demand shock in favor of non tradables. More specifically, the relative price of non tradables appreciates significantly in the short-run which signals an excess demand in the non traded goods market while the ratio of traded output relative to non traded output decreases substantially. The last two rows show that the sharp decline in hours worked in the traded sector relative to the non traded sector is associated with a significant increase in non traded wages relative to traded wages. The positive response of the relative wage to a government spending shock suggests the presence of intersectoral labor mobility costs.

< Please insert Figures 2-3 about here >

2.5 Cross-Country Differences in the Sectoral Impact and Imperfect Mobility of Labor

The presence of labor mobility costs preventing from wage equalization after a government spending shock square well with evidence documented by Artuç et al. [2010], Dix-Carneiro [2014], Lee and Wolpin [2006] who find substantial barriers to mobility and observe that wages are not equalized across sectors in the short run following both trade liberalization episodes and sector-biased technological change. Workers’ costs of switching sectors, which can be interpreted as psychological or geographic mobility costs, or can be the result of sector-specific human capital, can rationalize the increase in non traded wages relative to traded wages. Intuitively, following a rise in public purchases that are heavily concentrated in non traded industries, establishments in the non traded sector wish to increase their production to meet this additional demand. To attract workers, non traded firms must pay

\(^{16}\)See Table 3 in Appendix B.2 which displays the mean responses of the two components of government consumption.

\(^{17}\)Interestingly, when we breakdown government consumption on non tradables into collective and individual expenditure, we find empirically that the latter component which includes in particular health and education services, accounts for 77% of increases in \( g^N \).
higher wages in order to cover their mobility costs, and all the more so as the difficulty in reallocating hours worked across sectors is more pronounced. In countries where workers’ mobility costs are higher, we thus expect the positive response of the relative wage to a fiscal shock to be greater as non traded firms must pay much higher wages to increase hours worked. As the labor demand shifts along a steeper labor supply schedule in countries with greater mobility costs, the decline in relative hours worked in tradables is expected to be less pronounced. Since the traded sector experiences a lower labor outflow, the fall in traded output relative to non traded output should also be less (in absolute terms).

To gauge the importance of workers’ mobility costs for fiscal transmission, we thus ask whether the positive response of the relative wage to a fiscal shock is more pronounced whereas the reallocation of labor is lower in countries where mobility costs are higher. To explore our conjecture empirically, we draw on Horvath [2000] and estimate the elasticity of labor supply across sectors for each country.\(^{18}\) This parameter measures the extent to which workers are willing to reallocate their hours worked toward the non traded sector following a 1% increase in the relative wage. When the elasticity of labor supply across sectors is greater, workers’ mobility costs are thus lower which in turn implies a higher degree of labor mobility.

Building on our panel data estimates for the 16 OECD countries over the period 1970-2007, we split our sample into groups of 'high mobility' and 'low mobility' economies and re-estimate the sectoral effects for each of the two groups. The 'low mobility' economies are those for which the switching cost is above average for the sample. In order to provide some support for our measure of workers’ mobility cost, we compute an intersectoral labor reallocation index in year \(t\) for each country \(i\), denoted by \(LR_{i,t}\), by calculating the average change between year \(t\) and \(t - \tau\) in the amount of labor employed in sector \(j\) as a fraction of total employment:\(^{19}\)

\[
LR_{i,t}(\tau) = 0.5 \left[ \frac{\sum_{j=\tau}^{N} \frac{L_{j,t} - L_{j,t-\tau}}{\sum_{j=\tau}^{N} L_{j,t}}}{\sum_{j=\tau}^{N} L_{j,t}} \right].
\]

(3)

We choose \(\tau = 2\) to eschew year-to-year changes because of the low frequency changes in labor at that horizon and consider only differences over 2 years. As the values of the labor reallocation index, \(LR_i\), increase, the fraction of workers who are working in a different sector in year \(t\) than in year \(t - \tau\) is thus larger.\(^{20}\)

In the following, we compare the cumulative responses of the labor reallocation index, hours worked in tradables relative to non tradables, and the relative wage for the 'low mobility' group with those for the 'high mobility' group. The last two columns of Panel B

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\(^{18}\)Details about the empirical strategy can be found in Appendix B while details of derivation of the testable equation are provided in a Technical Appendix.

\(^{19}\)See e.g., Kambourov [2009] who computes the same labor reallocation index (3).

\(^{20}\)When we estimate the response of the intersectoral labor reallocation index to a government spending shock, we replace hours worked in the traded sector in terms of hours worked in the non traded sector, \(l_{it}^T - l_{it}^N\) with \(LR(2)_{it}\) and thus consider the 'labor reallocation' specification that is given by: \(\gamma_i^W = [g_{it}, LR_{it}(2), \omega_{it}]\).
of Table 1 show the point estimates for both sub-samples for selected horizons. Contrasting point estimates reported in columns 5 and 6 of Table 1, we find that the magnitude of the shift of labor in the 'low mobility' group, as captured by the LR index (3), is about five times less in first year. This finding thus lends credence to our measure of mobility costs. Importantly, in accordance with our conjecture, we find that the magnitude of the responses of the relative wage and relative hours worked in tradables are different across the sub-samples. As can be seen in the last two columns of Table 1, non traded wages increase substantially relative to traded wages for the 'low mobility' economies while the relative wage response for 'high mobility' countries is not statistically different from zero. Columns 5 and 6 of Table 1 also show that the 'low mobility' economies experience a fall in relative hours worked of tradables which is less pronounced as labor supply is less elastic to the relative wage. Because the shift of labor toward the non traded sector is less, 'low mobility' economies also experience a smaller decline in output of tradables relative to non tradables as shown by point estimates reported in the last two columns of Table 1.

Overall, our results emphasize the importance of labor mobility for fiscal transmission. We now move a step further and explore the cross-country relationship between changes in the relative size of sectors and the magnitude of workers’ costs of switching sectors. We estimate the same model as in eq. (2) but for a single country at a time. Then in Fig. 4, we plot the impact responses of sectoral shares on the vertical axis against our measure of the degree of labor mobility, denoted $\epsilon$, on the horizontal axis. This exercise may be viewed as tentative as the sectoral effect of a government spending shock varies considerably across countries and there is substantial uncertainty surrounding point estimates given the relatively small number of observations available per country.

< Please insert Figure 4 about here >

Fig. 4 plots sectoral labor and sectoral output shares against the degree of labor mobility across sectors. The cross-country analysis highlights two major findings. First, as shown in the top panels, whether we use labor or output, almost all countries in our sample experience a fall in the relative size of the traded sector as impact responses from the VAR model are below the X-axis. The bottom panels reveal that the reverse is true for the non traded sector which benefits from the reallocation of inputs. This evidence supports our earlier conjecture according to which government spending shock is strongly biased toward non tradables. Second, as can be seen in the top panels of Fig. 4, countries where workers have lower mobility costs experience a larger decline in the share of tradables while the bottom panels show that the relative size of non tradables increases more in these economies. In sum, our findings reveal that the magnitude of the change in relative

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21 When estimating the responses of sectoral labor and sectoral output shares to a government spending shock for each country, we omit $w^j_{C,it}$ in order to economize some degrees of freedom; the vector of endogenous variables is thus $z^S_{it} = [\nu_{it}, \nu^Y_{it}, \nu^L_{it}]$. We also estimated the VAR model by including $w^j_{C,it}$ and find that the results are similar. We allow for two lags (i.e., $k = 2$ in eq. (1)), as we did for the panel data estimate.
sector size following a government spending shock increases with the degree of labor mobility across sectors.

In the following, we develop a dynamic general equilibrium model with imperfect mobility of labor and capital installation costs in order to account for our evidence on fiscal transmission. While these two features along with a high intensity of the government spending shock in non tradables are necessary to replicate the change in relative sector size for a representative OECD economy, we have to let the degree of labor mobility across sectors vary across countries to account for the cross-country dispersion in the sectoral output responses.

3 A Two-Sector Open Economy Model with Imperfect Mobility of Labor across Sectors

We consider a small open economy populated by a constant number of identical households and firms that have perfect foresight and live forever. The country is small in terms of both world goods and capital markets, and faces a given world interest rate, \( r^* \). One sector produces a traded good denoted by the superscript \( T \) which can be exported at no cost, invested and consumed domestically. A second sector produces a non traded good denoted by the superscript \( N \) which can be consumed domestically or invested. The traded good is chosen as the numeraire.\(^{22}\) Time is continuous and indexed by \( t \).

3.1 Households

At each instant the representative household consumes traded and non traded goods denoted by \( C^T \) and \( C^N \), respectively, which are aggregated by means of a CES function:

\[
C(t) = \left[ \varphi \left( C^T(t) \right)^{\phi-1} \varphi \left( C^N(t) \right)^{\phi-1} + (1 - \varphi) \right]^{\phi-1},
\]

where \( 0 < \varphi < 1 \) is the weight of the traded good in the overall consumption bundle and \( \phi \) corresponds to the elasticity of substitution between traded goods and non traded goods.

The representative household supplies labor \( L^T \) and \( L^N \) in the traded and non traded sectors, respectively. To rationalize the rise in the non traded wage relative to the traded wage, we assume limited labor mobility across sectors. A shortcut to produce a difficulty in reallocating hours worked is to assume that workers experience a utility loss when shifting hours worked from one sector to another. We follow Horvath [2000] and consider that hours worked in the traded and the non traded sectors are aggregated by means of a CES function:

\[
L(t) = \left[ \vartheta^{-1/\epsilon} \left( L^T(t) \right)^{\epsilon-1} \vartheta^{-1/\epsilon} \left( L^N(t) \right)^{\epsilon-1} + (1 - \vartheta) \right]^\epsilon \vartheta \epsilon, \tag{5}
\]

\(^{22}\)The price of the traded good is determined on the world market and exogenously given for the small open economy.
and $0 < \vartheta < 1$ parametrizes the weight attached to the supply of hours worked in the traded sector and $\epsilon$ is the degree of substitutability in hours worked across sectors.

The representative agent is endowed with one unit of time, she/he supplies a fraction $L(t)$ as labor, and consumes the remainder $l(t) \equiv 1 - L(t)$ as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Assuming that the felicity function is additively separable in consumption and labor, the representative household maximizes the following objective function:\footnote{In a Technical Appendix, we show that relaxing the assumption of separability in preferences between consumption and labor merely affects the results.}

$$U = \int_0^\infty \left\{ \ln C(t) - \frac{L(t)^{1+\vartheta}}{1 + \frac{1}{\sigma_L}} \right\} e^{-\beta t} \, dt,$$

(6)

where $\beta$ is the discount rate and $\sigma_L > 0$ is the Frisch elasticity of labor supply or intertemporal elasticity of substitution for (aggregate) labor supply.

Factor income is derived by supplying labor $L(t)$ at a wage rate $W(t)$, and capital $K(t)$ at a rental rate $R(t)$. In addition, households accumulate internationally traded bonds, $B(t)$, that yield net interest rate earnings of $r^* B(t)$. Denoting lump-sum taxes by $T(t)$, households’ flow budget constraint states that real disposable income (on the RHS) can be saved by accumulating traded bonds, consumed, $P_C(t) C(t)$, or invested, $P_J(t) J(t)$:

$$\dot{B}(t) + P_C(t) C(t) + P_J(t) J(t) = r^* B(t) + R(t) K(t) + W(t) L(t) - T(t),$$

(7)

where $P_C(P(t))$ and $P_J(P(t))$ are consumption and the investment price index, respectively, which are a function of the relative price of non traded goods, $P(t)$. The aggregate wage index, $W(t) = W \left( W^T(t), W^N(t) \right)$, associated with the labor index (5) is:

$$W(t) = \left[ \vartheta \left( W^T(t) \right)^{\epsilon+1} + (1 - \vartheta) \left( W^N(t) \right)^{\epsilon+1} \right]^{\frac{1}{\epsilon+1}},$$

(8)

where $W^T(t)$ and $W^N(t)$ are wages paid in the traded and the non traded sectors, respectively.

The investment good is produced (costlessly) using traded good and non traded good inputs according to a constant returns to scale function which is assumed to take a Cobb-Douglas form:\footnote{In accordance with the empirical findings documented by Bems [2008] for OECD countries, we choose an elasticity of substitution between $J^N$ and $J^T$ of 1.}

$$J(t) = \left( \frac{J^N(t)}{\alpha_J} \right)^{\alpha_J} \left( \frac{J^T(t)}{1 - \alpha_J} \right)^{1 - \alpha_J},$$

(9)

where $\alpha_J$ and $1 - \alpha_J$ are investment expenditure shares on non tradables and tradables, respectively.

Installation of new investment goods involves increasing and convex costs, assumed quadratic, of net investment. Thus, total investment $J(t)$ differs from effectively installed new capital, $I(t)$:

$$J(t) = I(t) + \frac{\kappa}{2} \left( \frac{I(t)}{K(t)} - \delta_K \right)^2 K(t),$$

(10)
where the parameter $\kappa > 0$ governs the magnitude of adjustment costs to capital accumulation, and $0 \leq \delta_K < 1$ is a fixed depreciation rate. Net investment gives rise to capital accumulation according to the dynamic equation:

$$\dot{K}(t) = I(t) - \delta_K K(t).$$  \hspace{1cm} (11)

Households choose consumption, worked hours and investment in physical capital by maximizing lifetime utility (6) subject to (7) and (11) together with (10). Denoting by $\lambda$ and $Q$’ the co-state variables associated with (7) and (11), the first-order conditions characterizing the representative household’s optimal plans are:\textsuperscript{25}

\begin{align*}
C(t) &= (P_C(t)\lambda(t))^{-1}, \quad \text{(12a)} \\
L(t) &= (W(t)\lambda(t))^{\sigma L}, \quad \text{(12b)} \\
\frac{I(t)}{K(t)} &= \frac{1}{\kappa} \left( \frac{Q(t)}{P_J(t)} - 1 \right) + \delta_K, \quad \text{(12c)} \\
\dot{\lambda}(t) &= \lambda(t) (\beta - r^*), \quad \text{(12d)} \\
\dot{Q}(t) &= (r^* + \delta_K) Q(t) - \left\{ R(t) + P_J(t) \frac{\kappa}{2} \left( \frac{I(t)}{K(t)} - \delta_K \right) \left( \frac{I(t)}{K(t)} + \delta_K \right) \right\}, \quad \text{(12e)}
\end{align*}

and the transversality conditions $\lim_{t \to \infty} \lambda(t) e^{-\beta t} = 0$, $\lim_{t \to \infty} Q(t) K(t) e^{-\beta t} = 0$. In an open economy model with a representative agent who has perfect foresight, a constant rate of time preference and perfect access to world capital markets, we impose $\beta = r^*$ in order to generate an interior solution. Setting $\beta = r^*$ into (12d) yields $\lambda = \bar{\lambda}$.

Eq. (12c) can be solved for investment:

$$\frac{I(t)}{K(t)} = v \left( \frac{Q(t)}{P_J(t)} \right) + \delta_K, \quad v(\cdot) = \frac{1}{\kappa} \left( \frac{Q(t)}{P_J(t)} - 1 \right).$$  \hspace{1cm} (13)

Equation (13) states that investment is an increasing function of Tobin’s $q$, which is defined as the shadow value to the firm of installed capital, $Q(t)$, divided by its replacement cost, $P_J(t)$. For the sake of clarity, we drop the time argument below provided this causes no confusion.

Applying Shephard’s lemma (or the envelope theorem) to consumption expenditure yields the following demand for the traded and non traded good, respectively:

$$C^T = \varphi \left( \frac{1}{P_C} \right)^{-\varphi} C, \quad C^N = (1 - \varphi) \left( \frac{P}{P_C} \right)^{-\varphi} C.$$  \hspace{1cm} (14)

Denoting the share of non traded goods in consumption expenditure by $\alpha_C$, expenditure in non tradables and tradables is given by $P_C N = \alpha_C P_C C$ and $C^T = (1 - \alpha C) P_C C$.\textsuperscript{26}

Applying the same logic for labor, given the aggregate wage index (8), we can derive the allocation of aggregate labor supply to the traded and non traded sectors:

$$L^T = \vartheta \left( \frac{W^T}{W} \right)^\epsilon L, \quad L^N = (1 - \vartheta) \left( \frac{W^N}{W} \right)^\epsilon L.$$  \hspace{1cm} (15)

\textsuperscript{25}To derive (12c), we used the fact that $Q(t) = Q'(t)/\lambda$ which is the shadow value of capital in terms of foreign assets.

\textsuperscript{26}Specifically, the non tradable content of consumption expenditure is given by $\alpha_C = \frac{(1 - \varphi) P^{1 - \varphi}}{\varphi + (1 - \varphi) P^1}$. 

where $\epsilon$ is the elasticity of labor supply across sectors; it measures the extent to which agents are willing to increase their relative hours worked in sector $j$, $L_j/L$, following a 1% rise in the relative wage in sector $j$, $W_j/W$. As $\epsilon$ takes higher values, more labor shifts from one sector to another and thus the degree of labor mobility across sectors increases. When we let $\epsilon$ tend toward infinity, the special case of perfect labor mobility is obtained.

Because workers are willing to devote their whole time to the sector that pays the highest wages, the sectors pay the same wage. Denoting by $\alpha_L$ the share of non tradable labor revenue in labor income, labor income from supplying hours worked in the non traded and the traded sectors are $W^N L^N = \alpha_L W L$ and $W^T L^T = (1 - \alpha_L) W L$.27

### 3.2 Firms

Each sector consists of a large number of identical firms which use labor, $L_j$, and physical capital, $K_j$, according to a constant returns to scale technology:

$$Y_j = Z_j (L_j)^{\theta_j} (K_j)^{1-\theta_j}, \quad (16)$$

where $Z_j$ represents the TFP index which is introduced for calibration purposes only and $\theta_j$ corresponds to the labor income share in the value added of sector $j$. Firms lease capital from households and hire workers. They face two cost components: a capital rental cost equal to $R$, and wage rates in the traded and non traded sectors equal to $W^T$ and $W^N$, respectively. Both sectors are assumed to be perfectly competitive and thus choose capital and labor by taking prices as given. Since capital can move freely between the two sectors, the value of marginal products in the traded and non traded sectors equalizes while costly labor mobility implies a wage differential across sectors:

$$Z^T (1 - \theta^T) (k^T)^{-\theta^T} = P Z^N (1 - \theta^N) (k^N)^{-\theta^N} \equiv R, \quad (17a)$$

$$Z^T \theta^T (k^T)^{1-\theta^T} \equiv W^T, \quad (17b)$$

$$P Z^N \theta^N (k^N)^{1-\theta^N} \equiv W^N, \quad (17c)$$

where $k_j \equiv K_j/L_j$ denotes the capital-labor ratio for sector $j = T, N$.

Aggregating over the two sectors gives us the resource constraint for capital:

$$K^T + K^N = K. \quad (18)$$

### 3.3 Government

The final agent in the economy is the government. Total government spending, $G$, falls on goods, $G^N$, produced by non traded firms and goods, $G^T$, produced by traded firms. Both components of government spending are determined exogenously. The government finances public spending by raising lump-sum taxes, $T$. As a result, Ricardian equivalence obtains

27Specifically, we have $\alpha_L = \frac{(1-\theta)(W^N)^{\epsilon+1}}{[\theta(W^T)^{\epsilon+1} + (1-\theta)(W^N)^{\epsilon+1}]}$.\[19\]
and the time path of taxes is irrelevant for the real allocation. We may thus assume without
loss of generality that government budget is balanced at each instant:\(^{28}\)

\[ G = G^T + PG^N = T. \] (19)

### 3.4 Model Closure and Equilibrium

To fully describe equilibrium, we first impose the market clearing condition for non tradables:

\[ Y^N = C^N + J^N + G^N. \] (20)

Equality between non traded output and its demand counterpart is achieved through ad-
justments in the relative price of non tradables, \( P \), which guarantee that eq. (20) holds at
each point of time.

Regarding the allocation of government consumption in good \( j = T, N \), we consider
a rise in government consumption which is split between non tradables and tradables in
accordance with their respective share in government expenditure which we denote by \( \omega_{GN} \)
and \( \omega_{GT} \equiv 1 - \omega_{GN} \), respectively;\(^{29}\) more specifically, denoting the long-term values with a
tilde, we have in linearized form:

\[
\begin{pmatrix}
\dot{G}(t) - \dot{G} \\
\dot{Q}(t)
\end{pmatrix} = \begin{pmatrix}
\begin{pmatrix}
\Upsilon_K & \Upsilon_Q \\
\Sigma_K & \Sigma_Q
\end{pmatrix}
\begin{pmatrix}
K(t) - \bar{K} \\
Q(t) - \bar{Q}
\end{pmatrix} + \\
\begin{pmatrix}
\Upsilon_G \\
\Sigma_G
\end{pmatrix}
\begin{pmatrix}
G(t) - \bar{G} \\
G(t) - \bar{G}
\end{pmatrix},
\end{pmatrix}
\] (21)

After inserting appropriate first-order conditions into the non traded good market clear-
ing condition (20) and the no arbitrage condition (12e), it can be shown that the adjustment
of the open economy towards the steady-state is described by a dynamic system which com-
prises two equations that form a separate subsystem in \( K \) and \( Q \), i.e., \( \dot{K} \equiv \Upsilon (K, Q, G) \)
and \( \dot{Q} \equiv \Sigma (K, Q, G) \). Linearizing these equations in the neighborhood of the steady-state
and using (21), we get in a matrix form:

\[
\begin{pmatrix}
\dot{K}(t) \\
\dot{Q}(t)
\end{pmatrix} = \begin{pmatrix}
\begin{pmatrix}
\Upsilon_K & \Upsilon_Q \\
\Sigma_K & \Sigma_Q
\end{pmatrix}
\begin{pmatrix}
K(t) - \bar{K} \\
Q(t) - \bar{Q}
\end{pmatrix} + \\
\begin{pmatrix}
\Upsilon_G \\
\Sigma_G
\end{pmatrix}
\begin{pmatrix}
G(t) - \bar{G} \\
G(t) - \bar{G}
\end{pmatrix},
\end{pmatrix}
\] (22)

where the coefficients of the Jacobian matrix are partial derivatives evaluated at the steady-
state, e.g., \( \Upsilon_X = \frac{\partial \Upsilon}{\partial X} \) with \( X = K, Q \), and the direct effects of an exogenous change in
government spending on \( K \) and \( Q \) are described by \( \Upsilon_G = \frac{\partial \Upsilon}{\partial G} \) and \( \Sigma_G = \frac{\partial \Sigma}{\partial G} \), also evaluated
at the steady-state.

To determine the solutions for physical capital and the shadow value of installed capital,
we have to set the endogenous response of government spending to an exogenous fiscal

\(^{28}\)In a Technical Appendix, we allow for distortionary labor taxation and consider a rise in government
spending which is debt-financed. Denoting by \( D(t) \) the stock of (traded) bonds issued by the government,
the flow budget constraint reads as \( \dot{D}(t) = r^*D(t) + G(t) - T(t) \) with \( T(t) = \tau(t)W(t)L(t) \) where \( \tau \) is the
wage tax levied on households’ wage income. Our quantitative results show that the sectoral impact of fiscal policy is similar to that obtained when assuming a balanced-budget government spending shock; as
expected, the rise in the labor tax in the short-run mitigates substantially the positive response of hours
worked and thus the size of the aggregate fiscal multiplier.

\(^{29}\)We provide more details on the non tradable content of the government spending shock in section 5.1
and Appendix B.2.
shock. In order to account for the non-monotonic pattern of the dynamic adjustment of government consumption in line with our evidence (see Figure 1(a)), we assume that the deviation of government spending relative to its initial value as a percentage of initial GDP is:

$$\left( G(t) - \bar{G} \right) / \bar{Y} = e^{-\xi t} - (1 - g) e^{-\chi t},$$  \hspace{1cm} (23)$$

where \( g > 0 \) parametrizes the magnitude of the exogenous fiscal shock, \( \xi > 0 \) and \( \chi > 0 \) parametrize the degree of persistence of the fiscal shock; as \( \xi \) and \( \chi \) take higher values, government spending returns to its initial level more rapidly. More specifically, eq. (23) allows us to generate an inverted \( U \) pattern for the endogenous response of \( G(t) \): if \( \chi > \xi \), we have \( \bar{G}(t) > 0 \) following the exogenous fiscal shock and then government consumption declines after reaching a peak at some time \( t \).

Denoting the negative eigenvalue by \( \nu_1 \) and the positive eigenvalue by \( \nu_2 \), applying the standard method to solve systems of deterministic first-order linear differential equations and making use of (23), the general solutions for \( K \) and \( Q \) can be written in a compact form:  \(^{30}\)

$$K(t) - \bar{K} = X_1(t) + X_2(t), \quad Q(t) - \bar{Q} = \omega_1^1 X_1(t) + \omega_2^2 X_2(t),$$  \hspace{1cm} (24)$$

where \( \omega_2 \) is the element of the eigenvector associated with the eigenvalue \( \nu_i \) (with \( i = 1, 2 \)) and \( X_1(t) \) and \( X_2(t) \) are solutions which characterize the trajectory of physical capital and the shadow value of capital:

$$X_1(t) = e^{\nu_1 t} \left[ \left( K_0 - \bar{K} \right) + \Gamma_2 \left( 1 - \Theta_2 \right) - \Gamma_1 \left( 1 - \Theta_1 \right) \right] + \Gamma_1 \left( e^{-\xi t} - \Theta_1 e^{-\chi t} \right),$$  \hspace{1cm} (25a)$$

$$X_2(t) = -\Gamma_2 \left( e^{-\xi t} - \Theta_2 e^{-\chi t} \right),$$  \hspace{1cm} (25b)$$

where \( K_0 \) is initial stock of physical capital, \( \Gamma_i = -\Phi_i / \nu_i - \nu_2 \), \( \Phi_1 = (\Upsilon_K - \nu_2) \Upsilon_G + \Upsilon_Q \Sigma_G \), \( \Phi_2 = (\Upsilon_K - \nu_1) \Upsilon_G + \Upsilon_Q \Sigma_G \), and \( \Theta_i = (1 - g) \left( \nu_i + \xi \right) / \nu_i + \chi \) (with \( i = 1, 2 \)). When the shock is permanent, \( X_2(t) = 0 \) while \( X_1(t) \) reduces to \( e^{\nu_1 t} \left( K_0 - \bar{K} \right) \). Because our objective is to account for VAR evidence, we restrict our attention to a temporary fiscal shock.

Using the fact that \( RK + WL = Y^T + PY^N \) and inserting the market clearing condition for non tradables (20) into (7) gives the current account equation:

$$\dot{B} = r^* B + Y^T - C^T - G^T - J^T.$$  \hspace{1cm} (26)$$

Substituting appropriate short-run solutions, eq. (26) can be written as a function of state and control variables, i.e., \( \dot{B} = r^* B + \Xi (K, Q, G) \). Linearizing around the steady state, substituting the solutions for \( K(t) \) and \( Q(t) \) given by (24), solving and invoking the transversality condition, yields the solution for traded bonds:

$$B(t) - \bar{B} = \frac{\omega_1^1}{\nu_1 - r^*} e^{\nu_1 t} - \frac{\Xi G}{\xi + r^*} \left( e^{-\xi t} - \Theta_1 e^{-\chi t} \right) - \frac{N_1 \Gamma_1}{\xi + r^*} \left( e^{-\xi t} - \Theta_1 e^{-\chi t} \right) + \frac{N_2 \Gamma_2}{\xi + r^*} \left( e^{-\xi t} - \Theta_2 e^{-\chi t} \right),$$  \hspace{1cm} \text{eq. (27)}$$

\(^{30}\)See e.g., Buiter [1984] who presents the continuous time adaptation of the method of Blanchard and Kahn.
where $\omega_B^1 = [\Xi_K + \Xi_Q \omega_B^2] \left[ (K_0 - \tilde{K}) + \Gamma_2 (1 - \Theta_2) - \Gamma_1 (1 - \Theta_1) \right]$, with $\Xi_K = \frac{\partial \Xi}{\partial K}$, $\Xi_Q = \frac{\partial \Xi}{\partial Q}$, and $\Xi_G = \frac{\partial \Xi}{\partial G}$ evaluated at the steady-state, and $\Theta' = (1 - g) \frac{\xi_T + \xi}{\xi_T + \chi}$, and $\Theta_1' = \Theta_1 \frac{\xi_T + \xi}{\xi_T + \chi}$ (with $i = 1, 2$). To ultimately remain solvent, the open economy must satisfy the following condition:

$$
\tilde{B} - B_0 = -\frac{\omega_B^1}{\nu_1 - r^*} + \frac{\omega_B^2}{\xi + r^*},
$$

(28)

where $B_0$ is the initial stock of traded bonds and $\omega_B^2 = \Xi_G (1 - \Theta') + [\Xi_K + \Xi_Q \omega_B^2] \Gamma_1 (1 - \Theta_1') - [\Xi_K + \Xi_Q \omega_B^2] \Gamma_2 (1 - \Theta_2')$. The assumption $\beta = r^*$ implies that temporary policies have permanent effects. In this regard, eq. (28) determines the steady-state change in the net foreign asset position following a temporary fiscal expansion.

## 4 Imperfect Mobility of Labor and the Transmission of Government Spending

In this section, we solve the model analytically by abstracting from physical capital. This enables us to derive a number of analytical results which show that a model assuming imperfect mobility of labor across sectors can account for the evidence on fiscal transmission documented in section 2, as long as the government spending shock is biased toward non tradables. To avoid unnecessary complications, we solve the model by assuming that the endogenous response to an exogenous fiscal shock is governed by the following dynamic equation:

$$
dG(t)/Y = g e^{-\xi t},
$$

(29)

which amounts to setting $\xi = \chi$ into eq. (23). We consider a rise in $G$ which is split between non tradables and tradables in accordance with their respective share in government spending, $\omega_{Gj}$, as described by (21). Building on our evidence which reveals that a rise in government consumption is biased toward non tradables, we consider that $\omega_{GN}$ is high enough to produce an appreciation in the relative price of non tradables, in line with our empirical findings.

Both sectors use labor as the sole input in a constant returns to scale technology, i.e., $Y^j = L^j$ with $j = T, N$. Because there is a difficulty in reallocating labor, sectoral wages do not equalize, i.e., $1 = W^T$ and $P = W^N$. The key equations characterizing optimal household behavior are given by first-order conditions described by (12a)-(12b) and (14)-(15). The market clearing conditions for non traded and traded goods read as $Y^N = C^N + G^N$ and $\dot{B} = r^* B + Y^T - C^T - G^T$, respectively.

### 4.1 Solving the Model Analytically

Substituting first (12a) into (14), (12b) into (15), using $W^N = P$, totally differentiating the market clearing condition for the non traded good and denoting the percentage deviation relative to initial steady-state by a hat leads to the change in the relative price of non
tradables:
\[ \hat{P}(t) = \frac{\alpha_L \sigma_L + \alpha_C \omega_C}{\Psi} \hat{\lambda} + \frac{\omega_{GN} dG(t)}{Y}, \]  
(30)

where we set \( \Psi = \alpha_L [\epsilon (1 - \alpha_L) + \sigma_L \alpha_L] + \omega_C \alpha_C [1 - (1 - \alpha_C) \phi + \alpha_C] > 0 \). In eq. (30), we denote by \( \omega_C = \frac{P_C C}{Y} \) consumption expenditure as a share of GDP, \( \alpha_L \) and \( \alpha_C \) the non tradable content of consumption expenditure and labor compensation, respectively; in a model without capital, \( \alpha_L \) also measures the share of non tradables in GDP, i.e., \( \alpha_L = \frac{P_Y N}{Y} \).

Inserting first the demand for tradables (14) and labor supply to the traded sector (15), linearizing in the neighborhood of the steady-state, substituting the law of motion of government spending (29), solving and invoking the transversality condition leads to the solution for traded bonds:
\[ B(t) - \tilde{B} = \frac{\Upsilon_Y \tilde{Y}}{\xi + r^* g} e^{-\xi t}, \]  
(31)

consistent with the intertemporal solvency condition
\[ \left( \tilde{B} - B_0 \right) = -\frac{\Upsilon_Y \tilde{Y}}{\xi + r^* g} g, \]  
(32)

where \( \Upsilon_Y = \Upsilon^N_G \omega_{GN} + \omega_{GT} \) with \( \Upsilon^N_G = \left[ \frac{1 - \alpha_L}{1 - \alpha_L} \right] \frac{(1 - \sigma_C) \omega_C (1 - \alpha_C) \phi - 1}{\Psi} \). If the elasticity of labor supply across sectors, \( \epsilon \), is large enough with respect to aggregate labor supply, \( \sigma_L \), then we have \( \Upsilon^N_G > 0 \), so that the current account unambiguously deteriorates following a temporary fiscal expansion, in line with our VAR evidence.\(^{31}\)

To determine the change in the equilibrium value of the marginal utility of wealth, we have to differentiate the market clearing condition for the traded good evaluated at the steady-state (i.e., \( \tilde{B}(t) = 0 \)), using the fact that in the long-run, government spending is restored to its initial level (i.e., \( dG = 0 \)); next, inserting (32) into the resulting expression leads to the change in the equilibrium value of the marginal utility of wealth:
\[ \hat{\lambda} = \frac{\Psi \Upsilon_Y}{\Gamma} \frac{r^* g}{\xi + r^* g} > 0, \]  
(33)

where \( \Gamma = \Psi \left\{ \frac{(1 - \alpha_L) \sigma_L + \omega_C (1 - \alpha_C) \sigma_C}{\Psi} + [\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C] \Upsilon^N_G \right\} > 0 \).

### 4.2 Implications of Imperfect Mobility of Labor

What are the implications of imperfect mobility for fiscal transmission? As in a model that imposes perfect mobility of labor, a rise in government consumption produces an increase in the shadow value of wealth as taxes must be raised to balance the budget which reduces households’ disposable income. The negative wealth effect described by (33) encourages agents to work more and cut real expenditure. Because the decline in real expenditure is spread over the two goods, the rise in \( G^N \) more than offsets the fall in \( C^N \) if \( \omega_{GN} \) is

\(^{31}\)Differentiating (31) with respect to time leads to the current account response in percentage of GDP which is unambiguously negative as long as \( \Upsilon^N_G > 0 \). Intuitively, non traded output must increase to meet higher demand for non tradables. At the same time, households wish to avoid a large reduction in consumption and/or a large increase in labor supply. Because traded goods can be imported, resources are reallocated toward the non traded sector so that a current account deficit shows up.
As long as there is a difficulty in reallocating labor, an excess demand arises in the non traded goods market, which in turn causes the relative price of non tradables to appreciate. To see this formally, we determine the initial response of the relative price of non tradables by evaluating (30) at time \( t = 0 \), inserting (33), and using the fact that \( dG(0)/Y = g \):

\[
\hat{P}(0) = \left\{ \omega_{GN} - [\alpha_L \sigma_L + \alpha_C \omega_C] \frac{\Psi \left[ \frac{\Gamma}{r^*} \omega_{GN} + \omega_{GT} \right]}{\xi + r^*} \right\} \frac{g}{\Psi} > 0. \tag{34}
\]

Eq. (34) shows that both the composition of government spending along with the degree of labor mobility across sectors matter in determining the response of the relative price of non tradables. First, when the rise in government consumption is fully biased toward non tradables (i.e., \( \omega_{GN} = 1 \)), the relative price of non tradables unambiguously appreciates.\(^{32}\) In contrast, if the government spending shock were fully biased toward tradables (i.e., \( \omega_{GT} = 1 \)), the relative price would depreciate, in contradiction with our evidence. Because \( \hat{P}(0) \) is monotonically increasing with \( \omega_{GN} \), there is a critical value \( \bar{\omega}_{GN} \) so that \( \hat{P}(0) > 0 \) for \( \omega_{GN} > \bar{\omega}_{GN} \). Second, as the degree of labor mobility across sectors increases, a government spending shock leads to a lower appreciation in the relative price of non tradables. The reason is that the shadow value of wealth, \( \bar{\lambda} \), increases further, which results in a larger increase in non traded output and a greater decline in \( C_N \).\(^{33}\) In a model imposing perfect mobility of labor across sectors (i.e., \( \epsilon \rightarrow \infty \)), the relative price of non tradables remains unaffected by a fiscal shock. Intuitively, the appropriate amount of labor moves instantaneously toward the non traded sector to eliminate any excess demand in the non traded goods market.

Conversely, as long as \( \epsilon < \infty \) and \( \omega_{GN} > \bar{\omega}_{GN} \), an excess demand shows up in the non traded goods market so that the relative price of non tradables appreciates on impact. Non traded firms are encouraged to produce and thus to hire more workers. To persuade workers who experience mobility costs to increase their hours worked in the non traded sector, non traded firms must pay higher wages, i.e., \( \hat{W}_N(0) = \hat{P}(0) > 0 \). The subsequent shift of labor toward the non traded sector unambiguously raises non traded output. It can be shown analytically that the response of traded output is ambiguous; more precisely, \( Y^T \) may fall if the degree of labor mobility, \( \epsilon \), is higher than \( \sigma_L \).

We now turn to the initial response of the sectoral output share. As documented in section 2, we find that a government spending shock increases the share for non tradables in real GDP and all the more so in countries where the degree of labor mobility across sectors is higher. In the data, the response of the sectoral output share is calculated as the

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\(^{32}\)To see this formally, the sign of the term in braces is unambiguously positive since \( 0 < \left[ \alpha_L \sigma_L + \alpha_C \omega_C \right] \frac{\Gamma^2 \Psi}{\xi + r^*} < 1 \) and \( 0 < \frac{r^*}{\epsilon + r^*} < 1 \).

\(^{33}\)As the degree of labor mobility across sectors increases, more labor shifts toward toward the non traded sector which results in a greater decline in traded labor and thus triggers a larger current account deficit. In the long-run, for the intertemporal solvency condition to hold, the open economy must run a trade surplus and consumption must thus be reduced more though a stronger negative wealth effect.
growth differential in GDP units between sectoral value added at constant prices and real GDP denoted by $Y_R$. Totally differentiating non traded output and real GDP, the latter being equal to overall labor compensation $WL$ with $L = (\bar{\lambda}W)^{\sigma_L}$, and evaluating at time $t = 0$ leads to the impact response of the output share of non tradables in real terms:  

$$\alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) = \alpha_L (1 - \alpha_L) \epsilon \hat{P}(0) > 0, \tag{35}$$

where $\hat{P}(0)$ is given by (34). According to (35), the appreciation in the relative price of non tradables and the subsequent increase in non traded wages leads to a shift of labor toward the non traded sector which increases its share in real GDP. A rise in the parameter $\epsilon$ exerts two opposite effects on the magnitude of the positive response of the output share of non tradables. On the one hand, as the parameter $\epsilon$ on the RHS of (35) takes higher values, more labor shifts toward the non traded sector, thus amplifying the positive response of the output share of non tradables. On the other hand, as mentioned above, the negative wealth effect turns out to be greater as labor becomes more mobile across sectors; as a result, increased labor mobility mitigates the excess demand in the non traded goods market and thus the appreciation in the relative price of non tradables as reflected in smaller values of $\hat{P}(0) > 0$. It can be shown analytically that the former effect predominates so that $\alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right)$ is increasing with $\epsilon$ since the elasticity of the relative price response with respect to the degree of labor mobility is smaller than one, i.e., $\frac{-\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)} < 1$. \[35\]

Letting $\epsilon$ tend toward infinity into eq. (35) and applying l’Hôpital’s rule leads to:  

$$\lim_{\epsilon \to \infty} \alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) = \left[ \sigma_G \left( \alpha_L \sigma_L + \alpha_C \omega_C \right) \frac{r^*}{\xi + r^*} \right] g > 0. \tag{36}$$

The analytical expression of the response of the share of non tradables in real GDP described by (36) in the special case of perfect mobility of labor enables us to shed some light on the relationship between the non tradable content of the government spending shock, $\sigma_G$, and $\alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right)$. Keeping the responses of the private sector’s demand components fixed, it is straightforward to show that the share of non tradables in real GDP increases

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34Real GDP is the sum of value added at constant prices, i.e., $Y_R = Y_T + \hat{P}Y^N$ where $\hat{P}$ corresponds to the initial steady-state value for the relative price of non tradables. Using the fact that $Y_R = WL$, totally differentiating real GDP and inserting $L = \sigma_L \lambda + \sigma_L W$ with $W = \alpha_L \hat{P}$, leads to $\hat{Y}_R = \sigma_L \lambda + \alpha_L \sigma_L \hat{P}$. Using the fact that $\hat{Y}^N = \left[ \epsilon (1 - \alpha_L) + \alpha_L \sigma_L \right] \hat{P} + \sigma_L \lambda$, multiplying the growth differential between non traded output and real GDP (i.e., $\hat{Y}^N - \hat{Y}_R$) by $\alpha_L$ and evaluating at time $t = 0$ leads to (35).

35In a Technical Appendix, we are able to show that $\alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) > 0$ is increasing with the degree of labor mobility across sectors, $\epsilon$, by considering two polar cases: a weakly and a highly persistent fiscal shock. More specifically, we show that $\alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right)$ is increasing with $\epsilon$ as long as $-\frac{\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)} < 1$.

Since the elasticity $-\frac{\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)}$ ranges from a low when the shock is weakly persistent to a high when the shock is highly persistent, i.e.,

$$-\frac{\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)} \in \left\{ \frac{\alpha_L (1 - \alpha_L) \epsilon \sigma_L (1 - \alpha_L) \epsilon}{\Psi} \left\{ 1 + \frac{(\alpha_L \sigma_L + \omega_C \alpha_C)^2}{\xi} \right\}, \right\},$$

where both bounds of interval are smaller than 1, the initial reaction of the share of non tradables in GDP is unambiguously increasing with $\epsilon$.

36First inserting (34) into eq. (35), letting $\epsilon$ tend toward infinity and applying l’Hôpital’s rule that implies that $\lim_{\epsilon \to \infty} \frac{\partial \hat{Y}_R}{\partial \epsilon} = \frac{1}{\sigma_L + \omega_C}$ together with $\lim_{\epsilon \to \infty} \frac{\alpha_L (1 - \alpha_L)}{\psi} = 1$ gives eq. (36).
as long as $\omega_{GN} > \alpha_L$.\textsuperscript{37} In a general equilibrium model, demand components react to the government spending shock and thus slightly modify this condition. Adding and subtracting $\alpha_L$ in the RHS of (36) implies that the response $\alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right)$ is larger than $(\omega_{GN} - \alpha_L) \varrho$ since $\alpha_L > \left( \frac{\alpha_L \sigma_L + \alpha_C \omega_C}{\sigma_L + \omega_C} \xi^* \right)$. Intuitively, households smooth their consumption while non traded output must meet higher demand for non tradables. Because traded goods can be imported, net exports decline on impact which in turn further biases the spending shock toward non tradables. Because $0 < \frac{\alpha_L \sigma_L + \alpha_C \omega_C}{\sigma_L + \omega_C}$ (see the second term on the RHS of eq. (34)), this result also holds when assuming imperfect mobility of labor across sectors. Henceforth, the critical value $\bar{\omega}_{GN}$ above which the relative price appreciates on impact, i.e., $\hat{P}(0) > 0$, and thus the share of non tradables in real GDP increases, is smaller than $\alpha_L$ but would reduce to $\alpha_L$ if the current account were unresponsive to the fiscal shock.

How do hours worked and the real consumption wage react to a fiscal shock? Higher non traded wages increase the aggregate wage $\hat{W}$ in proportion to the non tradable content of labor compensation, i.e., $\hat{W} = \alpha_L \hat{P}$. Differentiating $W_C = W/P_C$, using the fact that $\hat{P}_C = \alpha_C \hat{P}$, the initial response in the real consumption wage is given by:

$$\hat{W}_C(0) = (\alpha_L - \alpha_C) \hat{P}(0) > 0.$$ \textsuperscript{(37)}

As long as the non tradable content of labor compensation $\alpha_L$ is higher than the non tradable content of consumption expenditure $\alpha_C$, the rise in the aggregate wage index more than offsets the increase in the consumption price index so that a fiscal shock initially raises the real consumption wage $W/P_C$, in line with the evidence.

The initial reaction of hours worked to a temporary government spending shock is unambiguously positive as the result of the negative wealth and the rise in the aggregate wage:

$$\hat{L}(0) = \sigma_L \left( \hat{\lambda} + \alpha_L \hat{P}(0) \right) > 0,$$ \textsuperscript{(38)}

where $\hat{\lambda} > 0$ and $\hat{P}(0) = \hat{W}^N(0) > 0$ are given by (33) and (34), respectively. It can be shown analytically that $\hat{L}(0)$ is decreasing with $\epsilon$.\textsuperscript{38} Intuitively, as the degree of labor mobility increases, the non traded wage and thus $W$ increases less. As a result, hours worked rise by a smaller amount as $\epsilon$ takes higher values.\textsuperscript{39}
5 Quantitative Analysis

In this section, we analyze the effects of a temporary and unanticipated rise in government consumption quantitatively. For this purpose we solve the model described in section 3 numerically.\(^{40}\) First we discuss parameter values before turning to the short-term consequences of higher government consumption.

5.1 Calibration

To calibrate our model, we estimated a set of parameters so that the initial steady state is consistent with the key empirical properties of a representative OECD economy. Our sample covers the sixteen OECD economies in our dataset. Our reference period for the calibration corresponds to the period 1990-2007.\(^{41}\) Since we calibrate a two-sector model with tradables and non tradables, we pay particular attention to ensure that the non tradable content of the model matches the data. Table 5 summarizes our estimates of the non tradable content of GDP, employment, consumption, gross fixed capital formation, government spending, labor compensation, and gives the share of government spending on the traded and non traded goods in their respective sectoral output, the shares of labor income in output in both sectors, for all countries in our sample. Moreover, columns 12-14 of Table 5 display investment expenditure and government spending as a percentage of GDP together with the labor income share, respectively, for the whole economy. To capture the key properties of a typical OECD economy, chosen as the baseline scenario, we take unweighted average values, as shown in the last line of Table 5. Some of the parameter values can be taken directly from the data, but others like \(\varphi\), \(\vartheta\), \(\delta K\) together with initial conditions \((B_0, K_0)\) need to be endogenously calibrated to fit a set of aggregate and sectoral ratios.\(^{42}\) We choose the model period to be one year and therefore set the world interest rate, \(r^*\), which is equal to the subjective time discount rate, \(\beta\), to 4%.

In light of our discussion above, \(\epsilon\) plays a key role in fiscal transmission. The degree of labor mobility captured by \(\epsilon\) is set to 0.75, in line with the average of our estimates shown in the last line of Table 5.\(^{43}\) Our estimates display a sharp dispersion across countries and we therefore conduct a sensitivity analysis with respect to this parameter. Excluding

\(^{40}\)Technically, the assumption \(\beta = r^*\) requires the joint determination of the transition and the steady state.

\(^{41}\)The choice of this period was dictated by data availability for all countries in the sample.

\(^{42}\)As detailed in a Technical Appendix, the steady-state can be reduced to four equations which jointly determine \(P\) (and thus \(\alpha C\)), \(Y_T/Y_N\) (and thus \(\omega J\)), \(K/Y\) (and thus \(\omega J = Y_J / Y\)) and \(\nu_B = \frac{\nu_B}{Y}\) (and thus \(\nu_B = \frac{NX}{Y}\)) where we denote net exports by \(NX\). Among the 19 parameters that the model contains, 16 have empirical counterparts while the remaining 3, i.e., \(\varphi\), \(\vartheta\), \(\delta K\) together with initial conditions \((B_0, K_0)\) must be set in order to match \(\alpha C = \frac{Y_C}{Y_N}\), \(\omega J = \frac{Y_J}{Y}\), and \(\nu_B = \frac{NX}{Y}\) with \(NX = Y_T - C_T - G_T - I_T\).

\(^{43}\)Since estimates of \(\epsilon\) for Denmark and Norway are not statistically significant at a standard threshold, the values are left blank and we set \(\varphi = 0.75\) which corresponds to the average value. To estimate \(\epsilon\), we first derive a testable equation by combining first-order conditions for labor supply and labor demand. We next run the regression of the sectoral employment growth arising from labor reallocation across sectors on the percentage change in the relative share of sectoral value added accrued to labor, see Appendix B.
the estimates of $\epsilon$ for Denmark and Norway which are not statistically significant at 10%, estimates of $\epsilon$ range from a low of 0.22 for the Netherlands to a high of 1.39 for the U.S. and 1.64 for Spain. Hence, we allow for $\epsilon$ to vary between 0.22 and 1.64 in the sensitivity analysis.

Building on our panel data estimates, the elasticity of substitution $\phi$ between traded and non traded goods is set to 0.77 in the baseline calibration since this value corresponds to the average of estimates shown in the last line of column 15 of Table 5. The weight of consumption in non tradables $1 - \phi$ is set to 0.51 to target a non-tradable content in total consumption expenditure (i.e., $\alpha C$) of 53%, in line with the average of our estimates shown in the last line of column 2. In our baseline parametrization, we set intertemporal elasticity of substitution for labor supply $\sigma_L$ to 0.4, in line with evidence reported by Fiorito and Zanella [2012], but conduct a sensitivity analysis with respect to this parameter. The weight of labor supply to the non traded sector, $1 - \vartheta$, is set to 0.68 to target a non-tradable content of labor compensation of 66%, in line with the average of our estimates shown in the last line of column 6 of Table 5.

We now describe the calibration of production-side parameters. We assume that physical capital depreciates at a rate $\delta_K$ of 6% to target an investment-to-GDP ratio of 21% (see column 12 of Table 5). Labor income shares in the traded ($\theta_T$) and the non traded sector ($\theta_N$) are set to 0.58 and 0.68, respectively, which correspond roughly to the averages for countries with $k^T > k^N$. Such values, i.e., $\theta_T = 0.58$ and $\theta_N = 0.68$, give an aggregate labor income share of 64%, in line with the average value shown in the last line of column 14 of Table 5. In line with our evidence shown in the last column of Table 5, we assume that traded firms are 28 percent more productive than non traded firms; hence we set $Z_T$ and $Z_N$ to 1.28 and 1 respectively. We set the investment expenditure share on non-tradable goods, $\alpha_J$, to 64%, in accordance with the evidence shown in column 3 of Table 5. We choose the value of parameter $\kappa$ so that the elasticity of $I/K$ with respect to Tobin’s $q$, i.e., $Q/P_J$, is equal to the value implied by estimates in Eberly, Rebelo, and Vincent [2008]. The resulting value of $\kappa$ is equal to 17.

As shown in column 4 of Table 5, the non tradable content of government spending, $\omega_{GN}$, averages 90%. We set government consumption on non traded goods, $G^N$, and traded goods, $G^T$, so as to yield a non tradable share of government spending, $\omega_{GN}$, of 90%, and government spending as a share of GDP to 20%.

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44 The average value is calculated by excluding estimates for Italy which are negative.
45 Table 5 gives the labor share of sector $j$ $\theta^j$ (with $j = T, N$) for the sixteen OECD countries in our sample. While $\theta^T$ and $\theta^N$ are set to 0.58 and 0.68, respectively, in the baseline calibration, we use reverse but symmetric values, i.e., $\theta^T = 0.68$ and $\theta^N = 0.58$, when $k^N > k^T$. For reason of space, we do not show numerical results for this case which can be found in the Technical Appendix of a longer version of the paper. Overall, the quantitative analysis reveal that our results are similar whether $k^T > k^N$ or $k^N > k^T$ as long as we assume imperfect mobility of labor across sectors.
46 Eberly, Rebelo, and Vincent [2008] run the regression $I/K = \alpha + \beta \ln(q)$ and obtain a point estimate for $\beta$ of 0.06. In our model, the steady-state elasticity of $I/K$ with respect to Tobin’s $q$ is $1/\kappa$. Equating $1/\kappa$ to 0.06 gives a value for $\kappa$ of 17.
We choose initial conditions for $B_0$ and $K_0$ so that trade is initially balanced. Since net exports are nil and $P_J I/Y = 21\%$ and $G/Y = 20\%$, the accounting identity according to which GDP is equal to the sum of the final uses of goods and services, leads to a consumption-to-GDP ratio of $P_C C/Y = 59\%$.\(^{47}\) It is worthwhile mentioning that the non tradable content of GDP is endogenously determined by the non tradable content of consumption, $\alpha_C$, investment, $\alpha_J$, and government expenditure, $\omega_{GN}$, along with the consumption-to-GDP ratio, $\omega_C$, and the investment-to-GDP ratio, $\omega_J$. More precisely, dividing the non traded good market clearing condition (20) by $Y$ leads to the non tradable content of GDP:

$$PY^N/Y = \omega_C \alpha_C + \omega_J \alpha_J + \omega_{GN} \omega_G = 63\%,$$

where $\omega_C = 59\%$, $\alpha_C = 53\%$, $\omega_J = 21\%$, $\alpha_J = 64\%$, $\omega_{GN} = 90\%$, and $\omega_G = 20\%$. According to (39), the ratios we target are consistent with a non tradable content of GDP of 63% found in the data, as reported in the last line of column 1 of Table 5.

In order to capture the endogenous response of government spending to exogenous fiscal shock, we assume that the dynamic adjustment of government consumption is governed by eq. (23). In the quantitative analysis, we set $g = 0.01$ so that government consumption increases by 1 percentage point of initial GDP. To calibrate $\xi$ and $\chi$ that parametrize the shape of the dynamic adjustment of government consumption along with its persistence, we proceed as follows.\(^{48}\) Because $G(t)$ peaks after one year, we have $dG(1)/Y = [e^{-\xi} - (1 - g) e^{-\chi}] = g' > g$ with $g' = 0.011265$ and $\dot{G}(1)/Y = -\left[\xi e^{-\xi} - \chi (1 - g) e^{-\chi}\right] = 0$. Solving the system gives us $\xi = 0.408$ and $\chi = 0.415$. Left-multiplying eq. (23) by $\omega_{Gj}$ (with $j = N, T$) gives the dynamic adjustment of sectoral government consumption to an exogenous fiscal shock:

$$\omega_{Gj} \left( G(t) - \hat{G} \right)/Y = \omega_{Gj} \left[ e^{-\xi t} - (1 - g) e^{-\chi t} \right],$$

where $\omega_{Gj}$ is the fraction of government consumption in good $j$.\(^{49}\) To determine (40), we assume that the parameters that govern the persistence and shape of the response of sectoral government consumption are identical across sectors, while the sectoral intensity of the government spending shock is constant over time and thus corresponds to the share of government final consumption expenditure in good $j$. The right panel of Figure 9 in

\(^{47}\)Remember that $J = I$ at the steady-state.

\(^{48}\)Our calibration of the government consumption shock is based on estimates of the first VAR model $z_t = [g_t, y_t, l_t, j_{ev}, w_{C,t}, \cdots]$. Since we consider alternative VAR specifications and the endogenous response of government consumption may thus differ across VAR models, we ran a robustness check in which we compute numerically the responses of variables to an exogenous government spending consistent with the VAR specification used to estimate the empirical IRF of the corresponding variable. Reassuringly, as can be found in the Technical Appendix, we find that whether we consider a baseline or alternative spending shocks, the discrepancy in numerically-computed responses is insignificant.

\(^{49}\)While one may be concerned by the fact that the methodology to break down total government spending into non tradables and tradables can be viewed as somewhat arbitrary, the computation of $\omega_{C,N}$ is consistent with eq. (39), i.e., the non tradable content of expenditure coincides exactly with the non tradable content of GDP.
Appendix B.2 contrasts empirical responses of sectoral government consumption to an exogenous fiscal shock with theoretical responses derived from eq. (40) by setting $\omega_{GN}$ and $\omega_{GT}$ to 0.9 and 0.1, respectively. Overall, the theoretical responses perform well in reproducing the evidence and thus the assumptions underlying the dynamic equation (40) which governs the adjustment of $G^j$ are consistent with data.

As the baseline scenario, we take the model with imperfect mobility of labor across sectors and capital adjustments costs. In our baseline calibration we set $\epsilon = 0.75$, $\sigma_L = 0.4$, $\kappa = 17$, but we also conduct a sensitivity analysis with respect to these three parameters by setting alternatively: $\epsilon$ to 0.22 and 1.64, $\sigma_L$ to 1, and $\kappa$ to 0. In order to contrast our results with those obtained when imposing perfect mobility of labor across sectors, we let $\epsilon$ tend toward infinity.

5.2 Results

In this subsection, we analyze in detail the role of imperfect mobility of labor in shaping the dynamics of the open economy in response to a government spending shock. Our primary objective is to explain in what workers’ costs of switching sectors change the model’s predictions in a way that makes them consistent with our empirical findings on fiscal policy transmission.

Table 2 shows the simulated impact effects of an exogenous and unanticipated increase in government consumption by 1 percentage point of GDP while column 1 shows impact responses from our VAR model for comparison purposes. Column 2 shows results for the baseline model which we contrast with those obtained when we impose perfect mobility of labor (i.e., we set $\epsilon \to \infty$) and abstract from capital installation costs (i.e., we set $\kappa = 0$) as well. Other columns give results for alternative scenarios discussed below. While in Table 2, we restrict our attention to impact responses, in Fig. 5 and 6 we show the dynamic adjustment to an increase in government consumption by 1% of GDP. Figures show the model predictions together with the respective VAR evidence. In each panel, the solid blue line displays the point estimate of the VAR model, with the dotted blue lines indicating the 90% confidence bounds. The solid black line shows the transitional paths obtained in a model with imperfect mobility of labor and capital adjustment costs. To gauge the importance of labor mobility across sectors for fiscal transmission, we contrast our baseline case featuring imperfect mobility with the perfect mobility case shown by the dashed black line. It is worth mentioning that the endogenous response of government spending to an exogenous fiscal shock that we generate theoretically in Figure 5(a) by specifying the law of motion (23) reproduces the dynamic adjustment from the VAR model remarkably well as the black line and the blue line cannot be differentiated.
5.2.1 Aggregate Effects

We begin with the aggregate effects of a government spending shock shown in panels A and B of Table 2. Contrasting the numerical results reported in columns 2 and 7 with the evidence shown in column 1, whether we assume imperfect or perfect mobility of labor, both models tend to understate the responses of real GDP and hours worked. However, the model performance improves with imperfect mobility of labor as the rise in GDP by 0.19% lies within the confidence interval, as shown in Figure 5(c). The reason is that with imperfect mobility of labor, the existence of workers’ costs of switching sectors puts upward pressure on non traded wages and thus on the aggregate wage. This then amplifies the positive response of hours worked which increases on impact by 0.30% instead of 0.11% when the mobility cost is absent. Because agents supply more labor, real GDP rises by a larger amount as long as there is a difficulty in reallocating labor. While the real consumption wage is unaffected on impact when we let \( \epsilon \) tend toward infinity, a government spending shock generates a rise in the wage rate which more than offsets the increase in the consumption price index and thus pushes up the real consumption wage by 0.07% in the baseline model where \( \epsilon = 0.75 \).

Turning to the dynamic adjustment of investment and the current account displayed in Fig. 5(b) and 5(d), a model assuming perfect mobility and abstracting from capital installation costs dramatically overstates the decline in investment and predicts a current account surplus in the short-run, contrary to the evidence. Because capital-labor ratios are fixed, the return on domestic capital remains unchanged as well. The substantial decline in private savings generates such a physical capital decumulation that the current account moves into surplus. In contrast, as long as we relax the assumption of perfect mobility of labor, the neoclassical model is able to produce the crowding out of investment along with the current account deficit in the short-run, as shown in column 5 of Table 2 where we abstract from capital installation costs to isolate the role of limited labor mobility. Intuitively, as long as there is a difficulty in reallocating labor across sectors, the capital-labor ratio falls in the traded sector as the workers’ mobility costs moderate the shift of labor. Thus, the return on domestic capital increases, which in turn mitigates the fall in investment and produces a current account deficit. However, the model tends to overstate the crowding-out of investment and to understate the decline in the current account. In contrast, as shown in column 2, when we allow for capital installation costs along with imperfect mobility of labor, the model predicts a current account deficit of 0.34% of GDP, which accords well with our estimate, by further mitigating the decline in investment.

We then ask whether both capital adjustments costs and imperfect mobility of labor are essential to account for the evidence. To answer this, column 8 considers a scenario where we assume that physical capital accumulation is subject to installation costs while hours
worked are perfect substitutes across sectors. The model predicts a rise in investment instead of a decline and considerably overstates the current account deficit found in the data: while the shadow price of investment, $Q$, increases as in a model assuming imperfect mobility of labor, the rise in the investment price index, $P_J$, is not large enough to drive down Tobin’s $q$. As will become clear below, perfect mobility of labor implies that the relative price of non tradables merely appreciates, thus hampering the increase in $P_J$.

Contrasting the model’s predictions with VAR evidence in Fig. 5, the simulated responses lie within the confidence interval along the transitional adjustment, with the exception of the real consumption wage. Although quite stylized, the model is able to account for the time-series evidence on the aggregate effects of a government spending shock as long as we allow for both capital installation costs and a difficulty in reallocating labor.

\[ \text{Please insert Table 2 and Figures 5-6 about here} \]

### 5.2.2 Reallocation Effects across Sectors

Turning to the sectoral impact of a rise in government consumption, the baseline model can account reasonably well for the dynamic adjustment of the non traded sector and somewhat less well for the traded sector. Panels C and D of Table 2 show impact responses of labor and product market variables, respectively, while in Fig. 6, we report the model predictions together with the VAR evidence of selected sectoral variables.

Focusing first on impact responses, column 7 of Table 2, shows that a model assuming perfect mobility of labor fails to account for the evidence along a number of dimensions. More specifically, comparing the VAR evidence reported in column 1 with simulated impact effects, we find that a model abstracting from workers’ mobility costs understates the expansionary effect of a government spending shock on non traded output, cannot generate an appreciation in the relative price of non tradables or a rise in the non traded wage relative to the traded wage, and substantially understates the changes in sectoral output shares.

In contrast, as displayed in column 2, the performance of the neoclassical model improves as long as we allow for imperfect mobility of labor. To begin with, the baseline model which considers costs of switching sectors can account for the rise in the relative wage. Intuitively, because government spending is biased toward non tradables, non traded firms are encouraged to produce and thus to hire more to meet additional demand. As workers experience intersectoral mobility costs, non traded firms must pay higher wages to attract workers which raises the relative wage, $\Omega$, by 1.44% as shown in the sixth line of panel C.

Because labor shifts toward the non traded sector, the baseline model predicts a rise in hours worked in non tradables by 0.44% which accords well with the evidence shown in column 1. Labor reallocation pushes up non traded output by 0.50%, the response being almost double that obtained with perfect labor mobility (see column 7). The reason is
twofold. First, the capital-labor ratio in the non traded sector increases as workers are reluctant to shift their hours worked across sectors. Second, because the aggregate wage increases when we allow for imperfect mobility of labor, workers supply more labor which further raises output in the non traded sector since it is relatively more labor intensive. While the baseline model is able to account pretty well for impact responses of hours worked and output of non tradables, it tends to somewhat overstate the contraction in hours worked and output of tradables which are fairly muted according to VAR evidence.

As long as there is a difficulty in reallocating labor across sectors, excess demand shows up in the non traded goods market. As a result, the price of non traded goods relative to traded goods appreciates by 0.88%, as shown in the fourth line of panel D. The appreciation in the relative price triggers a reallocation of resources toward the non traded sector, raising its output share by 0.38% of GDP, while that of tradables falls by exactly the same amount. As we move from column 3 to column 4 of Table 2, the utility loss resulting from the shift from one sector to another is reduced. As shown analytically in section 4, a rise in the degree of labor mobility exerts two opposite effects on sectoral output shares: while workers are more willing to shift across sectors, the relative price of non tradables appreciates less which mitigates the incentive for labor reallocation. We find numerically that raising the elasticity of labor supply across sectors, $\epsilon$, from 0.22 to 1.64 amplifies the rise in the output share of non tradables from 0.26% to 0.49% of GDP, in accordance with our evidence documented in section 2.5. Thus, the former effect more than offsets the latter.\(^{50}\)

Turning to the adjustment of sectoral variables following a government spending shock as shown in Fig. 6, the dynamics of the relative price and the relative wage are captured fairly well by the model. As government spending falls and is restored to its initial level, excess demand in the non traded goods market is reduced, which depreciates the relative price of non tradables along the transitional path, as shown in Fig. 6(a). Decreasing prices of non tradables relative to tradables encourage non traded firms to reduce hours worked and thus to lower output, in line with the evidence in Fig. 6(h) and 6(g). Because non traded wages fall relative to traded wages during the transitional adjustment, as shown in Fig. 6(b), labor is reallocated toward the traded sector, which recovers gradually, while both hours worked and output remain below their initial levels for almost ten years. As shown in Fig. 6(e) and 6(d), the model tends to somewhat understate the contraction of labor and the output of tradables in the medium run.\(^{51}\)

In order to further highlight the performance of the baseline model with imperfect mobility of labor and capital installation costs, it is useful to analyze the dynamic adjustment

\(^{50}\)However, the latter influence may predominate if the values of $\epsilon$ are higher because the relative price merely appreciates in this case. In the polar case where $\epsilon$ tends toward infinity, the output share of non tradables increases by only 0.24%, a value that is much smaller than the estimated response of 0.35% of GDP.

\(^{51}\)The explanation is intuitive: the baseline model underpredicts the decumulation of physical capital along the transitional path while the traded sector is more capital intensive.
of sectoral variables when these two features are absent. The dotted line in Fig. 6 displays the model predictions if we let $\epsilon$ tend toward infinity, while the parameter governing the magnitude of adjustment cost, $\kappa$, is set to zero. First, a model assuming $\epsilon \to \infty$ and setting $\kappa = 0$ predicts a flat temporal path for the relative wage and the relative price which conflict with the evidence. Second, it substantially understates the impact responses of sectoral output shares while the simulated responses for the baseline model accord well with the evidence. Intuitively, the relative price of non tradables appreciates when $\epsilon$ takes intermediate values, which in turn amplifies the shift of capital toward the non traded sector. Third, the model imposing perfect mobility of labor considerably overstates the changes in sectoral output shares along the transitional path. The reason is that the capital stock falls sharply in the short-run and then recovers rapidly after two years, resulting in sharp changes in the relative size of sectors due to the Rybczynski effect.

5.2.3 Sensitivity Analysis

To gauge the relative role of limited labor mobility and capital adjustment costs, we also report results from two restricted versions of the model where one of the two features is, respectively, shutdown. Column 8 of Table 2 shows the predictions of a model imposing perfect mobility of labor along with capital installation costs while column 5 reports impact responses from a model assuming imperfect mobility while setting $\kappa = 0$. Both models fail to account for the responses of sectoral output shares to a government spending shock. While introducing capital installation costs restore transitional dynamics for the relative price of non tradables, the restricted model where labor is perfectly mobile across sectors considerably overstates the responses of sectoral output shares. Intuitively, workers no longer experience a mobility cost and are thus willing to shift their whole time to the sector that pays the highest wage. As a result, sectoral labor and thus sectoral output become unrealistically sensitive to a change in relative price, thus leading to a change in the sectoral output share which is about twice what is estimated empirically, as can be seen in column 3. In contrast, as reported in column 7, a model assuming imperfect mobility of labor while abstracting from capital installation costs tends to substantially understate the responses of sectoral output shares. As investment is crowded out by a larger amount than if capital were subject to adjustment costs, the excess demand in the non traded goods market is lower so that the relative price appreciates less, resulting in smaller shifts of labor and capital toward the non traded sector.

Column 6 shows results when the elasticity of labor supply, $\sigma_L$, is set to 1. Raising $\sigma_L$ from 0.4 to 1 amplifies the rise in hours worked triggered by the negative wealth effect and

\[\sigma_L \to 1\]

To save space we develop intuition regarding the implications of imperfect mobility of labor and capital adjustment costs by restricting attention to impact responses. In a Technical Appendix, we contrast the dynamic adjustment from baseline model with the responses from the restricted model where one of the two features is shut down.
the increase in the aggregate wage, which further raises real GDP. Because larger labor supply benefits both sectors, hours worked (and subsequently output) increase more in the non traded sector while employment (and subsequently output) falls less in the traded sector. Since the non traded sector is more labor intensive, the rise in non traded labor is somewhat more pronounced. However, the responses of sectoral output shares are almost unchanged compared with those obtained from the baseline model as the relative price of non tradables appreciates by a smaller amount, thus mitigating the shift of capital toward the non traded sector.

5.3 Cross-Country Differences in Sectoral Impact: Taking the Model to Data

We have shown above that the performance of the neoclassical model in replicating the evidence related to fiscal transmission improves as long as we allow for imperfect mobility of labor and capital adjustment costs. We now move a step further and assess the ability of the model to generate a similar cross-country relationship between the degree of labor mobility and changes in the relative size of sectors to that in the data.

To compute the impact responses of sectoral output shares to a government spending shock numerically, we calibrate our model to match key characteristics of the 16 OECD economies in our sample, including the share of non traded hours worked to total hours worked, the non tradable content of consumption, investment and public expenditure, investment- and government spending-to-GDP ratios, and the degree of labor mobility across sectors. Table 5 summarizes the country-specific data for non tradable and GDP component shares. The elasticity of labor supply across sectors, $\epsilon$, which plays a pivotal role in fiscal transmission, is set in accordance with our estimates shown in the last column of Table 5. As mentioned in section 5.1, $\varphi$, $\varphi_J$, $\vartheta$, $\delta_K$ together with initial conditions ($B_0$, $K_0$) need to be calibrated endogenously to target $\alpha_C$, $\alpha_J$, $L^N/L$, $\omega_J$ along with $\nu_{NX} = NX/Y^T$ where $NX = Y^T - C^T - G^T - J^T$ corresponds to net exports. The remaining parameters are set to their empirical counterparts. Some parameters, such as the elasticity of labor supply, $\sigma_L$, and $\kappa$ governing the magnitude of adjustment costs to physical capital accumulation, along with the world interest rate, are kept constant however for all countries.

While we explore the sectoral effects of a rise in government consumption by 1% of GDP (i.e., $g$ is set to 0.01) for each country in our sample, to be consistent with the calibration to a representative OECD economy described in section 5.1, we assume that the increase in public purchases is split between non tradables and tradables in accordance with their respective shares in government spending, i.e., $\omega_{GN}$ and $1 - \omega_{GN}$, respectively, where $\omega_{GN}$ is set in accordance with its country-specific value shown in column 4 of Table 5, except for Australia and Ireland.\textsuperscript{53}

\textsuperscript{53}For Australia and Ireland, we find empirically that the output share of tradables increases on impact while the relative size of the non traded sector declines. To be consistent with empirical evidence, we
To explore the cross-country relationship quantitatively, we first plot in Fig. 7 the simulated responses of sectoral output shares on the vertical axis against the degree of labor mobility captured by the parameter $\epsilon$ on the horizontal axis. Restricting our attention to countries where the rise in government consumption is biased toward non tradables, impact changes in non traded output relative to real GDP range from 0.26% of GDP for the Netherlands to 0.49% of GDP for Spain. Fig. 7(a) and 7(b) also show that these differences in the responses of sectoral output shares are correlated with the measure of the degree of labor mobility across sectors. As $\epsilon$ takes higher values, countries with a higher degree of labor mobility experience a larger decline in the relative size of the traded sector and a larger increase in the relative size of the non traded sector. These results thus reveal that the sectoral impact of fiscal policy increases with the degree of labor mobility, which accords with our evidence. Quantitatively, as we move along the trend line shown in Fig. 7(a), our model predicts that a country with a low degree of labor mobility as captured by a value of $\epsilon$ of 0.2 will experience a decline in the output share of tradables of 0.2% of GDP, while a country with a higher degree of labor mobility as captured by a value of $\epsilon$ of 1.2 will face a fall by 0.4% of GDP, a decline which is twice as strong. Hence, cross-country differences in the degree of labor mobility generate a substantial dispersion in the sectoral impact of fiscal policy.

In Fig. 8, we contrast the cross-country relationship from the calibrated baseline model shown by the solid blue line with the cross-country relationship from the VAR model shown by the solid black line. When we calibrate our model to cross-country data, we obtain a correlation between the responses of sectoral output shares and the measure of the degree of labor mobility of -0.207 for tradables ($t \text{– } \text{stat} = -2.238$) and 0.207 for non tradables ($t \text{– } \text{stat} = 2.238$). While it tends to understate the changes in the relative size of sectors since the cross-country relationship is higher for tradables and lower for non tradables, the model is able to generate a cross-country relationship between the responses of sectoral output shares and the degree of labor mobility which is quite similar to that in the data.

6 Conclusion

While the literature analyzing fiscal transmission mainly focuses on the aggregate effects of a rise in government consumption, our empirical results reveal that the impact of fiscal policy varies significantly between sectors and across countries. Using a panel of 16 OECD countries over the period 1970-2007, we find empirically for the whole sample that a government purchase which is fully biased toward tradables. It is worthwhile mentioning that at the initial steady-state, we set the non tradable content of government spending, $\omega_GN$, to 88% and 90% for Australia and Ireland, respectively, in accordance with the shares reported in column 4 of Table 5.

54Because our panel data estimates are not statistically significant at 10% for Denmark and Norway, these two countries are removed from the cross-country analysis. If we include them, the conclusions are unaffected.
ernment spending shock has an expansionary effect on hours worked and output of non tradables, whereas it gives rise to contractions in hours worked and output of tradables. Such a finding along with the appreciation in the relative price of non tradables suggests that public purchases are biased toward non traded goods. Importantly, non traded output increases substantially relative to GDP (in real terms) while the reverse is true for the traded sector. This evidence thus highlights the fact that resources are shifted toward the non traded sector, with the reallocation of inputs contributing to 50% of non traded output growth. If labor were freely mobile across sectors, sectoral wages would equalize. However, we find empirically that non traded wages increase substantially relative to traded wages, thus suggesting the presence of labor mobility costs across sectors. Contrasting the sectoral impact across the economies in our sample, the output share of non tradables (in real terms) rises for the vast majority of the economies while its magnitude varies sharply across countries. Estimating the elasticity of labor supply across sectors for each country, we find that impact responses of output shares for tradables and non tradables are more pronounced in countries with lower mobility costs.

To rationalize our panel VAR evidence, we develop a two-sector open economy model with imperfect mobility of labor across sectors and adjustment costs to physical capital accumulation. As in Horvath [2000], agents cannot costlessly reallocate hours worked from one sector to another. Because mobility is costly in utility terms, workers demand higher wages in order to compensate for their cost of switching sectors. Calibrating the model to a representative OECD economy and considering a rise in government consumption biased toward non tradables, we find quantitatively that the open economy version of the neoclassical model with tradables and non tradables can account for the panel VAR evidence, in particular the changes in relative sector size, as long as we allow for adjustment costs to physical capital accumulation along with imperfect mobility of labor across sectors. The first feature mitigates the decline in investment and thus guarantees that the excess demand and thus incentives to shift resources toward the non traded sector are high enough. By reducing the elasticity of labor supply across sectors, the second feature hampers the reallocation of labor and thus allows the model to match the changes in relative sector size quantitatively. In contrast, the restricted version of the model where one of the two features is shut down fails to account for the evidence.

When we calibrate our baseline model to each OECD economy in our sample, our numerical results reveal that international differences in the degree of labor mobility generate a wide dispersion in the responses of sectoral output shares to a government spending shock: changes in the relative size of sectors are twice as strong in the country with the highest degree of labor mobility than in the economy with the lowest labor mobility. Finally, we find quantitatively that the model reproduces pretty well the cross-country relationship between the degree of labor mobility and the responses of sectoral output shares that we
References


Figure 1: Effects of Unanticipated Government Spending Shock on Aggregate Variables.

Notes: Exogenous increase of government consumption by 1% of GDP. Aggregate variables include GDP (constant prices), total hours worked, private fixed investment, the current account and the real consumption wage. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in output units (government spending, GDP, investment, current account), percentage deviation from trend in labor units (total hours worked), percentage deviation from trend (real consumption wage). Solid blue lines: point estimates; shaded areas: bootstrapped 90% confidence intervals; sample: 16 OECD countries, 1970-2007, annual data.
Figure 2: Effects of Unanticipated Government Spending Shock on Sectoral Variables. Notes: Exogenous increase of government consumption by 1% of GDP. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in output units (sectoral output, sectoral output shares), percentage deviation from trend in labor units (sectoral labor, sectoral labor shares), deviations from trend (ratio of traded value added to non traded value added, ratio of hours worked of tradables to hours worked of non tradables), and percentage deviation from trend (relative price, relative wage). Solid blue lines: point estimates; shaded areas: bootstrapped 90% confidence intervals; sample: 16 OECD countries, 1970-2007, annual data.
Table 1: Responses to Government Spending Shock: Point Estimates

<table>
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<th>Variables</th>
<th>Horizon</th>
<th>A. Aggregate and Sectoral Effects</th>
<th>B. Low Vs. High Labor Mobility</th>
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</table>

Notes: Horizon measured in year units. * denote significance at 10% level. Standard errors are bootstrapped with 10000 replications. The last three columns report, for selected horizons and samples, the cumulative responses of relative labor, relative output, the intersectoral labor reallocation index and relative wage to an increase in government spending by 1% of GDP. The response of relative labor (relative output resp.) is estimated from a 3-variable VAR that includes government spending, relative labor (relative output), \( L^T/L^N \) \( (Y^T/Y^N) \), and the relative wage of non tradables (relative price of non tradables), \( W^N/W^T \) \( (P^N/P^T) \). Finally, the response of labor reallocation \( LR \) is estimated from a 3-variable VAR that includes government spending, the intersectoral labor reallocation index, \( LR(2) \), and the relative wage of non tradables, \( W^N/W^T \).
Figure 3: Effects of Unanticipated Government Spending Shock on Government Final Consumption Expenditure on Non Tradables and Tradables. Notes: Exogenous increase of government consumption by 1% of GDP. The government spending shock is identified by estimating a VAR model that includes real government final consumption expenditure, GDP (constant prices), total hours worked, private fixed investment, and the real consumption wage. The responses of government final consumption expenditure on non tradables (i.e., $G^N$) and tradables (i.e., $G^T$) to the identified government spending shock are displayed by solid blue lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling; sample: 13 OECD countries, 1995-2015, annual data. Source: COFOG, OECD.

Figure 4: Effect of Government Spending Shocks on Sectoral Composition against the Degree of Labor Mobility across Sectors. Notes: Figure 4 plots impact responses of sectoral labor and sectoral output shares. Impact responses shown in the vertical axis are obtained by running a VAR model for each country and are expressed in percentage point. Horizontal axis displays the elasticity of labor supply across sectors, $\epsilon$, which captures the degree of labor mobility across sectors; panel data estimates for $\epsilon$ are taken from column 16 of Table 5.
Table 2: Impact Responses of Aggregate and Sectoral Variables to a Rise in Government Consumption (in %)

<table>
<thead>
<tr>
<th>Data</th>
<th>Imperfect Mobility</th>
<th>Perfect Mobility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bench</td>
<td>Lab. supply</td>
</tr>
<tr>
<td></td>
<td>(ψ = 0.75)</td>
<td>(σ_L = 1)</td>
</tr>
<tr>
<td></td>
<td>(ψ = 0.22)</td>
<td>(κ = 0)</td>
</tr>
<tr>
<td></td>
<td>(ψ = 1.64)</td>
<td></td>
</tr>
<tr>
<td>A. Impact: GDP &amp; Components</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real GDP, (dy_R(0))</td>
<td>0.51</td>
<td>0.19</td>
</tr>
<tr>
<td>Investment, (dI(0))</td>
<td>-0.01</td>
<td>-0.13</td>
</tr>
<tr>
<td>Current account, (dCA(0))</td>
<td>-0.30</td>
<td>-0.34</td>
</tr>
<tr>
<td>B. Impact: Labor &amp; Real Wage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor, (dL(0))</td>
<td>0.53</td>
<td>0.30</td>
</tr>
<tr>
<td>Real consumption wage, (d(W/P_C)(0))</td>
<td>0.48</td>
<td>0.07</td>
</tr>
<tr>
<td>C. Impact: Sectoral Labor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traded labor, (dL^T(0))</td>
<td>0.01</td>
<td>-0.14</td>
</tr>
<tr>
<td>Non traded labor, (dL^N(0))</td>
<td>0.54</td>
<td>0.44</td>
</tr>
<tr>
<td>Relative labor, (d(L^T/L^N)(0))</td>
<td>-0.71</td>
<td>-0.52</td>
</tr>
<tr>
<td>Relative wage, (d(W^N/W^T)(0))</td>
<td>0.93</td>
<td>1.44</td>
</tr>
<tr>
<td>Labor share of tradables, (d(L^T/L)(0))</td>
<td>-0.27</td>
<td>-0.24</td>
</tr>
<tr>
<td>Labor share of non tradables, (d(L^N/L)(0))</td>
<td>0.27</td>
<td>0.24</td>
</tr>
<tr>
<td>D. Impact: Sectoral Output</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traded output, (dy^T(0))</td>
<td>-0.03</td>
<td>-0.31</td>
</tr>
<tr>
<td>Non traded output, (dy^N(0))</td>
<td>0.70</td>
<td>0.50</td>
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<tr>
<td>Relative output, (d(Y^T/Y^N)(0))</td>
<td>-1.03</td>
<td>-0.97</td>
</tr>
<tr>
<td>Relative price, (dP(0))</td>
<td>1.06</td>
<td>0.88</td>
</tr>
<tr>
<td>Output share of tradables, (d(Y^T/Y_0)(0))</td>
<td>-0.45</td>
<td>-0.38</td>
</tr>
<tr>
<td>Output share of non tradables, (d(Y^N/Y_0)(0))</td>
<td>0.35</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Notes: Effects of an unanticipated and temporary exogenous rise in government consumption by 1% of GDP. Panels A,B,C,D show the initial deviation in percentage relative to steady-state for aggregate and sectoral variables. Market product (aggregate and sectoral) quantities are expressed in percent of initial GDP while labor market (aggregate and sectoral) quantities are expressed in percent of initial total hours worked; \(\theta_T\) and \(\theta^N\) are the labor income share in the traded sector and non traded sector, respectively; \(\epsilon\) measures the degree of substitutability in hours worked across sectors and captures the degree of labor mobility; \(\sigma_L\) is the Frisch elasticity of labor supply; \(\kappa\) governs the magnitude of adjustment costs to capital accumulation. In our baseline calibration (labelled 'Bench'), we set \(\theta_T = 0.58, \theta^N = 0.68, \epsilon = 0.75, \phi = 0.77, \sigma_L = 0.4, \kappa = 17\).
Figure 5: Dynamic Adjustment of Aggregate Variables to Unanticipated Government Spending Shock. 

Notes: Solid blue line displays point estimate of VAR model with dotted blue lines indicating 90% confidence bounds; the solid black line displays model predictions in the baseline scenario with imperfect mobility of labor across sectors ($\epsilon = 0.75$) and capital installation costs ($\kappa = 17$) while the dotted black line shows predictions of the model imposing perfect mobility of labor ($\epsilon \to \infty$) and abstracting from capital adjustment costs ($\kappa = 0$).
Figure 6: Dynamic Adjustment of Sectoral Variables to Unanticipated Government Spending Shock. Notes: Solid blue line displays point estimate of VAR with dotted blue lines indicating 90% confidence bounds; the solid black line displays model predictions in the baseline scenario with imperfect mobility of labor across sectors ($\epsilon = 0.75$) and capital installation costs ($\kappa = 17$) while the dotted black line shows predictions of the model imposing perfect mobility of labor ($\epsilon \to \infty$) and abstracting from capital adjustment costs ($\kappa = 0$).

Figure 7: Cross-Country Relationship between the Responses of Sectoral Output Shares to Government Spending shock and the Degree of Labor Mobility across Sectors. Notes: Horizontal axes display panel data estimates of the elasticity of labor supply across sectors, $\epsilon$, taken from the last column of Table 5, which captures the degree of labor mobility across sectors. Vertical axes report simulated impact responses from the baseline model with imperfect mobility of labor across sectors and adjustments costs to capital accumulation.
Figure 8: Cross-Country Relationship from Simulated Responses vs. Cross-Country Relationship from VAR Estimates. Notes: Horizontal axes display panel data estimates of the elasticity of labor supply across sectors, $\epsilon$, taken from the last column of Table 5, which captures the degree of labor mobility across sectors. Vertical axes report simulated responses from the baseline model (blue circles) and impact responses from the VAR model (black squares).
A Data Description for Empirical Analysis

Coverage: Our sample consists of a panel of 16 countries: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Finland (FIN), France (FRA), Ireland (IRL), Italy (ITA), Japan (JPN), the Netherlands (NLD), Norway (NOR), Spain (ESP), Sweden (SWE), the United Kingdom (GBR) and the United States (USA). The period is running from 1970 to 2007, except for Japan (1974-2007).

Sources: Our primary sources for sectoral data are the OECD and EU KLEMS databases. We use the EU KLEMS [2011] sectoral database (the March 2011 data release, available at http://www.euklems.net) which provides annual data for eleven 1-digit ISIC-rev.3 industries for all countries of our sample with the exceptions of Canada and Norway. For Canada and Norway, sectoral data are taken from the Structural Analysis (STAN) database provided by the OECD [2011]. In addition, expenditure aggregates are obtained from the Economic Outlook Database provided by the OECD [2017].

The eleven 1-digit ISIC-rev.3 industries are classified as tradables or non tradables. To do so, we adopt the classification proposed by De Gregorio et al. [1994]. Following Jensen and Kletzer [2006], we have updated this classification by treating "Financial Intermediation" as a traded industry. We construct traded and non traded sectors as follows (EU KLEMS codes are given in parentheses):

- **Traded Sector**: "Agriculture, Hunting, Forestry and Fishing" (AtB), "Mining and Quarrying" (C), "Total Manufacturing" (D), "Transport, Storage and Communication" (I) and "Financial Intermediation" (J).

- **Non Traded Sector**: "Electricity, Gas and Water Supply" (E), "Construction" (F), "Wholesale and Retail Trade" (G), "Hotels and Restaurants" (H), "Real Estate, Renting and Business Services" (K) and "Community Social and Personal Services" (LtQ).

Once industries have been classified as traded or non traded, for any macroeconomic variable $X$, its sectoral counterpart $X^j$ for $j = T, N$ is constructed by adding the $X_k$ of all sub-industries $k$ classified in sector $j = T, N$ as follows $X^j = \sum_{k \in j} X_k$.

Relevant to our work, the EU KLEMS and OECD STAN databases provide data, for each industry and year, on value added at current and constant prices, permitting the construction of sectoral deflators of value added, as well as details on labor compensation and employment data, allowing the construction of sectoral wage rates. In the VAR models, with the exception of the current account, all quantity variables are in log levels and scaled by the working age population (15-64 years old), while price deflators and wage rates are in natural logs. Source: OECD ALFS Database for the working age population. We describe below the construction for the sectoral data employed in Section 2 (mnemonics are given in parentheses):

- **Sectoral output**, $Y^j$: sectoral value added at constant prices in sector $j = T, N$ (VA_QI). Sources: EU KLEMS and OECD STAN databases.

- **Relative output**, $Y^T/Y^N$: ratio of traded value added at constant prices to non traded value added at constant prices.

- **Sectoral output share**, $\nu^j$: ratio of value added at constant prices in sector $j$ to GDP at constant prices, i.e., $Y^j/(Y^T + Y^N)$ for $j = T, N$.

- **Relative price of non tradables**, $P$: ratio of the non traded value added deflator to the traded value added deflator, i.e., $P = P^N/P^T$. The sectoral value added deflator $P^k$ for sector $j = T, N$ is calculated by dividing value added at current prices (VA) by value added at constant prices (VA_QI) in sector $j$. Sources: EU KLEMS and OECD STAN databases.

- **Sectoral labor**, $L^j$: total hours worked by persons engaged in sector $j$ (H_EMP). Sources: EU KLEMS and OECD STAN databases.

- **Relative labor**, $L^T/L^N$: ratio of hours worked in the traded sector to hours worked in the non traded sector.

- **Sectoral labor share**, $\nu^j$: ratio of hours worked in sector $j$ to total hours worked, i.e., $L^j/(L^T + L^N)$ for $j = T, N$.

- **Sectoral real consumption wage**, $W^j/CPI$: nominal wage in sector $j$ divided by the consumer price index (CPI). Source: OECD Prices and Purchasing Power Parities for the consumer price index. The sectoral nominal wage $W^j$ for sector $j = T, N$ is calculated by dividing labor compensation in sector $j$ (LAB) by total hours worked by persons engaged (H_EMP) in that sector. Sources: EU KLEMS and OECD STAN databases.

- **Relative wage**, $\Omega$: ratio of the nominal wage in the non traded sector $W^N$ to the nominal wage in the traded sector $W^T$, i.e., $\Omega = W^N/W^T$. 

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Labor reallocation index, $LR$: measures the fraction of workers who are working in year $t$ in a different sector than in year $t - 2$ and is computed as:

$$LR_t(2) = 0.5 \left| \frac{\sum_{j=T}^{N} L_j^t}{\sum_{j=T}^{N} L_j^t} - \frac{\sum_{j=T}^{N} L_j^{t-2}}{\sum_{j=T}^{N} L_j^{t-2}} \right|.$$  

Data for labor (H_EMP), used to compute $LR$, are taken from EU KLEMS and OECD STAN databases.

In the following, we provide details on data construction for aggregate variables (mnemonics are in parentheses):

- **Government spending, $G$**: real government final consumption expenditure (CGV). Source: OECD Economic Outlook Database.
- **Gross domestic product, $Y$**: real gross domestic product (GDPV). Source: OECD Economic Outlook Database.
- **Private investment, $JE$**: real private non-residential gross fixed capital formation (IBV). Source: OECD Economic Outlook Database.
- **Current account, $CA$**: ratio of the current account to the gross domestic product at current prices (CBGDPR). Source: OECD Economic Outlook Database.
- **Labor, $L$**: total hours worked by persons engaged (H_EMP). Sources: EU KLEMS and OECD STAN databases.
- **Real consumption wage, $W/CP1$**: nominal wage divided by the consumer price index (CP1). Source: OECD Prices and Purchasing Power Parities for the consumer price index.

Government spending, investment and GDP variables are deflated with their own deflators.

## B Data for Calibration

### B.1 Non Tradable Content of GDP and its Components

Table 5 shows the non tradable content of GDP, consumption, investment, government spending, labor and labor compensation. In addition, it gives information about the share of government spending on the traded and non traded goods in the corresponding sectoral value added and the sectoral labor income shares. The column 11 shows the ratio of labor productivity of tradables to labor productivity of non tradables as we use labor productivity to approximate technological change. Columns 12 to 14 display the investment-to-GDP ratio and government spending in % of GDP and the labor income share, respectively, for the whole economy. Our sample covers the 16 OECD countries mentioned in Section A. Our reference period for the calibration corresponds to the period 1990-2007. The choice of this period has been dictated by data availability. Columns 15 and 16 report estimates for the elasticity of substitution in consumption between traded and non traded goods, $\phi$, and the elasticity of labor supply across sectors, $\epsilon$. In the following, statistics for the sample as a whole represent (unweighted) averages of the corresponding variables among the group.

To calculate the non tradable share of output, labor and labor compensation, we split the eleven industries into traded and non-traded sectors by adopting the classification proposed by De Gregorio et al. [1994] and updated by Jensen and Kletzer [2006]. Details about data construction for sectoral output and sectoral labor are provided in section A. We calculate the non-tradable share of labor compensation as the ratio of labor compensation in the non traded sector (i.e., $W_N L_N$) to overall labor compensation (i.e., $W L$). Sources: EU KLEMS [2011] and STAN databases. Data coverage: 1990-2007 for all countries. The non tradable content of GDP, labor and labor compensation, shown in columns 1, 5 and 6 of Table 5, average to 63%, 67% and 66% respectively.

To split consumption expenditure (at current prices) into consumption in traded and non traded goods, we made use of the Classification of Individual Consumption by Purpose (COICOP) published by the United Nations (Source: United Nations [2011]). Among the twelve items, the following ones are treated as consumption in traded goods: "Food and Non-Alcoholic Beverages", "Alcoholic Beverages Tobacco and Narcotics", "Clothing and Footwear", "Furnishings, Household Equipment" and "Transport". The remaining items are treated as consumption in non traded goods: "Housing, Water, Electricity, Gas and Fuels", "Health", "Communication", "Recreation and Culture", "Education", "Restaurants and Hotels". Because the item "Miscellaneous Goods and Services" is somewhat problematic, we decided to consider it as both tradable (50%) and non tradable (50%) in...
equal shares. Data coverage: 1990-2007 for AUS, AUT, CAN, DNK, FIN, FRA, GBR, ITA, JPN, NLD, NOR and USA, 1993-2007 for SWE and 1995-2007 for BEL, ESP and IRL. The non-tradable share of consumption shown in column 2 of Table 5 averages to 53%.

To calculate the non tradable share of investment expenditure, we follow the methodology proposed by Burstein et al. [2004] who treat “Housing”, “Other Constructions” and “Other Products” as non-tradable investment and ”Products of Agriculture, Forestry, Fisheries and Aquaculture”, ”Metal Products and Machinery”, ”Transport Equipment” as tradable investment expenditure. Source: OECD Input-Output database [2012]. Data coverage: 1990-2007 for AUT, CAN, ESP, FIN, GBR, IRL, JPN, NLD, and NOR, 1990-2006 for DNK, FRA, ITA and USA, and 1993-2007 for SWE. Data are not available for AUS and BEL. The non tradable share of investment shown in column 3 of Table 5 averages to 64%, in line with estimates provided by Burstein et al. [2004] and Bems [2008].

Sectoral government final consumption expenditure data (at current prices) were obtained from the OECD General Government Accounts database (Source: COFOG, OECD [2017]). ”Economic Affairs” which includes ”Fuel and Energy”, ”Agriculture, Forestry, Fishing, and Hunting”, ”Mining, Manufacturing, and Construction”, ”Transport and Communications” is classified as tradable. Items treated as non tradable are: ”General Public Services”, ”Defense”, ”Public Order and Safety”, ”Environment Protection”, ”Housing and Community Amenities”, ”Health”, ”Recreation, Culture and Religion”, ”Education”, ”Social Protection”. Data coverage: 1995-2007 for AUT, BEL, DNK, ESP, FRA, GBR, IRL, ITA, NLD, NOR and SWE, 1998-2007 for AUS, 1990-2007 for FIN, 2005-2007 for JPN and 1970-2007 for the USA. Data are not available for CAN. The non tradable component of government spending shown in column 4 of Table 5 averages to 90% over the period 1990-2007. Government spending on traded and non traded goods in % of the corresponding sectoral output, i.e., \( G^T/Y^T \) and \( G^N/Y^N \), respectively, is shown in columns 7 and 8 of Table 5. They average 5% and 30%, respectively.

The labor income share for sector \( j \) denoted by \( \theta^j \) is calculated as the ratio of labor compensation of sector \( j \) to value added of sector \( j \) at current prices. Sources: EU KLEMS [2011] and STAN databases. Data coverage: 1990-2007 for all countries. As shown in columns 9 and 10 of Table 5, \( \theta^T \) and \( \theta^N \) average 0.60 and 0.67, respectively. When \( k^T > k^N \), the shares of labor income average 0.58 and 0.67 for the traded and the non traded sector, respectively, while if \( k^N > k^T \), \( \theta^T \) and \( \theta^N \) average 0.70 and 0.64, respectively. In addition, column 14 of Table 5 gives the aggregate labor income share which averages 0.64 in our sample.

We use sectoral labor productivities to approximate technological change. Column 11 of Table 5 displays the ratio of labor productivity in tradables to labor productivity in non tradables \( (Z^T/Z^N) \) averaged over the period 1990-2007. To measure labor productivity in sector \( j = T, N \), we divide value added at constant prices in sector \( j \) (\( VA_Qj \)) by total hours worked by persons engaged (\( H_{EMP} \)) in that sector. Sources: EU KLEMS [2011] and STAN databases. Data coverage: 1990-2007 for all countries. As shown in column 11, the traded sector is 28 percent more productive on average than the non traded sector for the whole sample.

Columns 12 and 13 of Table 5 display gross capital formation and final consumption expenditure of general government as a share of GDP, respectively. Source: OECD National Accounts Database. Data coverage: 1990-2007 for all countries.

**B.2 Non Tradable Intensity of the Government Spending Shock**

We turn to the calibration of the breakdown of the government spending shock between non tradables and tradables. In first approximation, the share of the government spending shock received by the non traded sector could be measured by the non tradable content of government spending calculated above by using the COFOG dataset from the OECD. Denoting by \( \omega_{Gj} \) the content of government spending in good \( j \), we have:

\[
G(t) = \omega_{G^N} G(t) + \omega_{G^T} G(t).
\]

(41)

Assuming that \( \omega_{Gj} \) is fixed over time and differentiating (41) leads to:

\[
dG(t)/Y = \omega_{G^N} (dG(t)/Y) + \omega_{G^T} (dG(t)/Y).
\]

(42)

Thus according to (42), the non tradable intensity of the government spending shock corresponds to the fraction of government consumption spent on non tradables. In order to reproduce the hump-shaped pattern of the endogenous response of government spending to an exogenous fiscal shock, we assume that the deviation of government spending relative to its initial value as a percentage of initial GDP is governed by the dynamic equation (23). Left-multiplying (23) by \( \omega_{Gj} \) (with \( j = N, T \)) gives the dynamic adjustment of sectoral government consumption to an exogenous fiscal shock:

\[
\omega_{Gj} \left( G(t) - \tilde{G} \right)/Y = \omega_{Gj} \left[ e^{-\xi t} - (1 - g) e^{-\chi t} \right].
\]

(43)
We set $g$ to 0.01 as we consider an exogenous increase in government spending by 1% of GDP and choose values of $\xi$ and $\chi$ in order to reproduce the hump-shaped pattern of the endogenous response of government spending to the exogenous fiscal shock. To the extent that $\omega_{Gj}$ is considered as fixed over time, we set $\omega_{Gj}$ to the share of government consumption on good $j$ in government final consumption expenditure, i.e., we set $\omega_{Gj}^\infty$ to 90% and $\omega_{Gj}^T$ to 10%.

The derivation of the dynamic equation (43) that governs the adjustment of sectoral government consumption following an exogenous fiscal shock relies on a number of assumptions. We assume that the parameters that govern the persistence and shape of the response of sectoral government consumption are identical across sectors, while the sectoral intensity of the government spending shock is constant over time and thus corresponds to the share of government final consumption expenditure in good $j$. To investigate whether our assumptions are consistent with the data, we contrast empirical with theoretical impulse response functions of sectoral government consumption. To estimate the dynamic effects on sectoral government consumption of an exogenous fiscal shock, we have to identify the government spending shock. We thus estimate the first VAR model that includes government final consumption expenditure, real GDP, total hours worked, private investment, and the real consumption wage. Then, we estimate a VAR model in panel format on annual data that includes unanticipated government spending shocks, $\epsilon^G_{it}$, ordered first, government spending, $g_{it}$, government consumption on non tradables, $g^N_{it}$, and government consumption on tradables, $g^T_{it}$, i.e., $\epsilon^G_{it} = [\epsilon^G_{it}, g_{it}, g^N_{it}, g^T_{it}]$. All quantities are logged, expressed in real terms and scaled by the working age population.

Since time series for government consumption by function taken from the COFOG dataset are not available before 1995 for most of the countries in our sample, and because our objective is to estimate the non tradable content of the aggregate government spending shock, we consider a period running from 1995 to 2015 in order to have time series of a reasonable length. Data to construct time series for sectoral government consumption expenditure are available for all the countries in our sample except Canada. In efforts to have a balanced panel and time series of a reasonable length, Australia (1998-2015) and Japan (2005-2015) are removed from the sample, which leaves us with 13 OECD countries over the period 1995-2015.

Table 3 reports, for various horizons, the responses of government consumption expenditure on non tradables and tradables to the identified government spending shock. We normalize the impulse responses so that government spending rises by one percentage point of GDP on impact. As can be seen in the first two columns of Table 3, a government spending shock leads to an increase in government consumption expenditure on non tradables by 0.88% on impact while the rise in public purchases of tradables accounts for the remaining share, i.e., 12%. The contribution of government consumption on non tradables to the government spending shock is displayed in the last column of Table 3. Its contribution is quite stable over time and varies between 88% and 91%. The contribution of government expenditure on non tradables averages 90% as can be seen in the last line of the table.

Empirical and theoretical impulse response functions are contrasted and displayed by solid blue lines in the right panel of Figure 9. Before discussing the results, we first focus on the response of government final consumption expenditure to the exogenous fiscal shock shown in the left panel of Figure 9. The endogenous response of government spending to an exogenous fiscal shock displayed in the solid blue line corresponds to the baseline government spending shock in the main text (see Figure 1(a)) we obtain from estimates of the first VAR model. The dynamic response of government final consumption expenditure which has been computed by summing mean responses of government consumption on non tradables and tradables is displayed by the solid red line. While the solid blue line displays point estimate from a sample of 13 OECD countries over 1970-2007, the solid red line displays point estimate from a sample of 15 OECD countries over 1970-2007. The solid red line displays point estimate from a sample of 13 OECD countries over 1995-2015. Whereas the samples are different, the discrepancy is quite moderate. Since theoretical responses of sectoral government consumption are based on the response of government spending shown in the solid blue line in the left panel while the sum of mean responses of government consumption expenditure on non tradables and tradables gives a slightly different response of government spending as shown in the solid red line, we have to rescale empirical responses for $G_j$ so that the sum of mean responses corresponds exactly to the point estimate displayed in the solid blue line. The rescaled empirical responses of sectoral government consumption are displayed by solid blue lines in the right panel of Figure 9 with dotted blue lines indicating the 90 percent confidence bounds obtained by bootstrap sampling. We contrast empirical with theoretical responses displayed by the dotted black lines. It turns out that differences are rather moderate. We may notice that whereas the theoretical response of government consumption of non tradables (tradables) slightly overstates (understates) the estimated response in the short-run, it lies within the confidence bounds for both goods. To conclude, the assumptions underlying the dynamic equation (43) which governs theoretical responses of $G_j$ are reasonable and consistent with data.
Table 3: Responses of $G^N$ and $G^T$ to Identified Government Spending Shock: Point Estimates

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Responses $G^N$</th>
<th>Responses $G^T$</th>
<th>Non tradable intensity of gov. spending shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.876</td>
<td>0.119</td>
<td>88%</td>
</tr>
<tr>
<td>1</td>
<td>1.045</td>
<td>0.150</td>
<td>87%</td>
</tr>
<tr>
<td>2</td>
<td>0.892</td>
<td>0.125</td>
<td>88%</td>
</tr>
<tr>
<td>3</td>
<td>0.753</td>
<td>0.098</td>
<td>88%</td>
</tr>
<tr>
<td>4</td>
<td>0.623</td>
<td>0.076</td>
<td>89%</td>
</tr>
<tr>
<td>5</td>
<td>0.493</td>
<td>0.057</td>
<td>90%</td>
</tr>
<tr>
<td>6</td>
<td>0.381</td>
<td>0.041</td>
<td>90%</td>
</tr>
<tr>
<td>7</td>
<td>0.294</td>
<td>0.030</td>
<td>91%</td>
</tr>
<tr>
<td>8</td>
<td>0.226</td>
<td>0.022</td>
<td>91%</td>
</tr>
<tr>
<td>9</td>
<td>0.175</td>
<td>0.017</td>
<td>91%</td>
</tr>
<tr>
<td>10</td>
<td>0.136</td>
<td>0.013</td>
<td>91%</td>
</tr>
<tr>
<td>Mean</td>
<td>-</td>
<td>-</td>
<td>90%</td>
</tr>
</tbody>
</table>

Notes: Horizon measured in year units. We generate impulse response functions by using a simple VAR, i.e., $z_{G,i,t} = [\epsilon_{G,i,t}, g_{i,t}, g^N_{i,t}, g^T_{i,t}]$, with 2 lags. To identify the government spending shock $\epsilon_{G,i,t}$ we estimate the VAR model that includes aggregate variables, i.e., $z_{i,t} = [g_{i,t}, y_{i,t}, l_{i,t}, j_e_{i,t}, w_{C,i,t}]$, and adopt a Choleski decomposition. The last column of the table displays, for all horizons, the contribution of the response of non tradable component to the government spending shock. Data coverage: 1995-2015 for AUT, BEL, DNK, ESP, FIN, FRA, GBR, IRL, ITA, NLD, NOR, SWE and the USA. All variables are real and scaled by the working age population.

Figure 9: Effects of Unanticipated Government Spending Shock on Government Final Consumption Expenditure on Non Tradables and Tradables: Empirical vs. Theoretical Impulse Response Functions. Notes: The baseline response of government final consumption expenditure is displayed by the solid blue line in the left panel with shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; sample: 16 OECD countries, 1970-2007, annual data. The responses of government final consumption expenditure on non tradables (i.e., $g^N$) and tradables (i.e., $g^T$) to the identified government spending shock (in the baseline VAR model) are displayed by solid blue lines in the right panel with dotted blue lines indicating the 90 percent confidence bounds; sample: 13 OECD countries, 1995-2015, annual data. The red line in the left panel displays the dynamic response of government final consumption expenditure which has been computed by summing mean responses of government consumption expenditure on non tradables and tradables. Theoretical responses of $g^N$ and $g^T$ are displayed by dotted black lines in the right panel.
B.3 Estimates of $\epsilon$ and $\phi$: Empirical strategy

Column 1 of Table 4 shows our estimates of the elasticity of labor supply across sectors, $\epsilon$, while columns 2-3 show our estimates of the elasticity of substitution in consumption between traded and non traded goods, $\phi$. We detail our empirical strategy to estimate these two parameters.

Along the lines of Horvath [2000], we derive a testable equation by combining optimal rules for labor supply and labor demand and estimate $\epsilon$ by running the regression of the worker inflow in sector $j = T$, $N$ of country $i$ at time $t$ arising from labor reallocation across sectors computed as $\hat{l}_{t,i} - \hat{L}_{t,i}$ on the relative labor’s share percentage changes in sector $j$, $\beta_{j,t}^l$,

$$\hat{l}_{t,i} - \hat{L}_{t,i} = f_t + f_i + \gamma_i \beta_{j,t}^l + \nu_{i,t},$$

(44)

where we denote logarithm in lower case and the deviation from initial steady-state by a hat; $\nu_{i,t}$ is an i.i.d. error term; country fixed effects are captured by country dummies, $f_i$, and common macroeconomic shocks by year dummies, $f_t$. The LHS term of (44) is calculated as the difference between changes (in percentage) in hours worked in sector $j$, $\hat{l}_{t,i}$, and in total hours worked, $\hat{L}_{t,i}$. The RHS term $\beta^l$ corresponds to the fraction of labor’s share of output accumulating to labor in sector $j$. Denoting by $\hat{P}_t^j Q_{i,t}^j$ output at current prices in sector $j = T$, $N$ at time $t$, $\beta^l_t$ is computed as $\sum_{j=N}^{\hat{P}_t^j Q_{i,t}^j} \xi_j$, where $\xi_j$ is labor’s share in output in sector $j = T$, $N$ defined as the ratio of the compensation of employees to output in the $j$th sector, averaged over the period 1971-2007. Because hours worked are aggregated by means of a CES function, total hours percentage change $\hat{L}_{t,i}$ is calculated as a weighted average of sectoral employment percentage changes, i.e., $\hat{L}_i = \sum_{j=N}^{T} \beta_{j,-1}^l \hat{l}_{j,t}^l$. The parameter we are interested in, the degree of substitutability of hours worked across sectors, is given by $\epsilon_i = \gamma_i / (1 - \gamma_i)$. In the regressions that follow, the parameter $\gamma_i$ is alternatively assumed to be identical across countries when estimating for the whole sample ($\gamma_i = \gamma_i$, $\gamma_i$ for $i \neq i'$) or to be different across countries when estimating $\epsilon$ for each economy ($\gamma_i \neq \gamma_i$ for $i \neq i'$). Data are taken from the EU KLEMS [2011] and STAN databases, and the sample includes the 16 OECD countries mentioned above over the period 1971-2007 (except for Japan: 1975-2007). Table 4 reports empirical estimates that are consistent with $\epsilon > 0$. All values are statistically significant at 10%, except for Denmark and Norway.

To estimate the elasticity of substitution in consumption, $\phi$, between traded and non traded goods, we first derive a testable equation by inserting the first-order condition equating the marginal revenue of labor and the sectoral wage, i.e., $W_{t,j} = W_t^j$, into the goods market clearing condition. Eliminating $Y^j$, denoting by $Y^T = (W^T L^T - \beta^T P^T N X)$ and $Y^N = W^N L^N$, and taking logarithm yields

$$\ln (Y^T - \gamma N) = f_t + f_i + \alpha i + \phi \ln P_{i,t} + \mu_{i,t},$$

(45)

where $\alpha$ and $\gamma$ are the country fixed effects and time dummies, respectively. Because the term $\alpha = \ln \left( \frac{1 - \nu_{G,N} - \nu_{P,N}}{1 - \nu_{G,T} - \nu_{P,T}} \right)$ is composed of ratios, denoted by $\nu_{G,j}$ and $\nu_{P,j}$, of $G^T$ ($G^N$) and $I^T$ ($I^N$) to $Y^T - NX$ ($Y^N$) and hence may display a trend over time, we add country-specific linear trends, as captured by $\alpha_{i,t}$.58

Instead of using time series for sectoral value added, we can alternatively make use of series for sectoral labor compensation by inserting the first-order condition equating the marginal revenue of labor and the sectoral wage, i.e., $W^j = W_t^j$, into the goods market clearing condition. Eliminating $Y^j$, denoting by $Y^T = (W^T L^T - \beta^T P^T N X)$ and $Y^N = W^N L^N$, and taking logarithm yields

55Details of derivation of the equation we explore empirically can be found in a Technical Appendix.

56As Horvath [2000], we use time series for output instead of value added so that our estimates can be compared with those documented by the author.

57In a Technical Appendix, we address one potential econometric issue. While $\beta^l_t$ (i.e., the RHS term in eq. (44)) is constructed independently from the dependent variable (i.e., the LHS term in eq. (44)), if the labor’s share is (almost) constant over time and thus is close from the average $\xi^j$, an endogeneity problem may potentially show up. Our empirical results reveal that for the majority of the countries in our sample, the dependent variable does not Granger-cause the explanatory variable.

58Because an endogeneity problem of relative prices may potentially affect our econometric results, we ran Granger causality tests. Our empirical results reveal that for the majority of the countries in our sample, the dependent variable does not Granger-cause the explanatory variable. Our results show that one can consider the regressor in eq. (45) as exogenous with respect to the dependent variable.
Table 4: Estimates of the Elasticity of Labor Supply across Sectors (\(\phi\)) and the Elasticity of Substitution in Consumption between Tradables and Non Tradables (\(\phi^*\))

<table>
<thead>
<tr>
<th>Country</th>
<th>Labor Mobility ((\epsilon))</th>
<th>Elasticity of Substitution ((\phi^*))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\text{eq. (44)})</td>
<td>(\text{eq. (45)})</td>
</tr>
<tr>
<td>AUS</td>
<td>0.635a (3.55)</td>
<td>0.268a (2.99)</td>
</tr>
<tr>
<td>AUT</td>
<td>0.548a (2.66)</td>
<td>0.986a (3.09)</td>
</tr>
<tr>
<td>BEL</td>
<td>0.326b (2.51)</td>
<td>0.070 (0.41)</td>
</tr>
<tr>
<td>CAN</td>
<td>0.454a (3.41)</td>
<td>0.391a (3.74)</td>
</tr>
<tr>
<td>DNK</td>
<td>0.150 (1.46)</td>
<td>2.071b (2.95)</td>
</tr>
<tr>
<td>ESP</td>
<td>1.642a (3.92)</td>
<td>0.783a (4.96)</td>
</tr>
<tr>
<td>FIN</td>
<td>0.544a (3.62)</td>
<td>1.072a (8.57)</td>
</tr>
<tr>
<td>FRA</td>
<td>1.287b (2.44)</td>
<td>0.937a (9.04)</td>
</tr>
<tr>
<td>GBR</td>
<td>1.008a (3.79)</td>
<td>0.477a (9.64)</td>
</tr>
<tr>
<td>IRL</td>
<td>0.264a (3.18)</td>
<td>0.374a (4.71)</td>
</tr>
<tr>
<td>ITA</td>
<td>0.686a (2.84)</td>
<td>-0.308 (1.40)</td>
</tr>
<tr>
<td>JPN</td>
<td>0.993a (2.87)</td>
<td>0.654a (4.98)</td>
</tr>
<tr>
<td>NLD</td>
<td>0.224b (1.97)</td>
<td>0.709b (2.33)</td>
</tr>
<tr>
<td>NOR</td>
<td>0.097 (1.49)</td>
<td>0.979b (9.72)</td>
</tr>
<tr>
<td>SWE</td>
<td>0.443a (3.61)</td>
<td>0.356a (4.92)</td>
</tr>
<tr>
<td>USA</td>
<td>1.387a (2.59)</td>
<td>0.668a (8.21)</td>
</tr>
<tr>
<td><strong>Whole Sample</strong></td>
<td>0.479a (12.16)</td>
<td>0.656a (16.13)</td>
</tr>
</tbody>
</table>

| Countries | 16 | 16 | 16 |
| Observations | 1178 | 605 | 605 |
| Country fixed effects | yes | yes | yes |
| Time dummies | yes | yes | yes |
| Time trend | no | yes | yes |

Notes: \(a\), \(b\), and \(c\) denote significance at 1%, 5% and 10% levels; t-statistics are reported in parentheses.

\[
\ln \left( \frac{T}{\gamma} \right) = \eta + \phi \ln P \quad \text{where } \eta \text{ is a term composed of both preference (i.e., } \varphi \text{) and production (i.e.,} \theta^p \text{) parameters, and (logged) ratios of } G^T \left( G^N \right) \text{ and } I^T \left( I^N \right) \text{ to } W^T L^T - \theta^P P^T N X \left( W^N L^N \right). \text{ We estimate } \phi \text{ by exploring alternatively the following empirical relationship:}
\]

\[
\ln (\gamma^T / \gamma^N)_{t,t} = g_t + g_t + \sigma_t + \phi_t \ln P_{t,t} + \zeta_{t,t},
\]

where \(g_t\) and \(g_t\) are the country fixed effects and time dummies, respectively, and we add country-specific trends, as captured by \(\sigma_t\), because \(\eta\) is composed of ratios that may display a trend over time.

Time series for sectoral value added at constant prices, labor compensation, and the relative price of non tradables are taken from the EU KLEMS [2011] and STAN databases (see Section A). Net exports correspond to the external balance of goods and services at current prices taken from OECD Economic Outlook Database. To construct time series for net exports at constant prices, NX, data are deflated by the value added deflator of traded goods \(P_t^L\).

Since LHS terms of (45) and (46) and the relative price of non tradables display trends, we ran unit root and then cointegration tests. Having verified that these two assumptions are empirically supported, we estimate the cointegrating relationships by using fully modified OLS (FMOLS) procedure for cointegrated panel proposed by Pedroni [2000], [2001]. FMOLS estimates of (45) and (46) are reported in the second and the third column of Table 4 respectively. As a reference model, we consider eq. (45) which gives an estimate for the whole sample of \(\phi = 0.66\). This value is roughly halfway between estimates documented by cross-section studies, notably Stockman and Tesar [1995] who find a value for \(\phi\) of 0.44 and Mendoza [1995] who reports an estimate of 0.74.
Table 5: Data to Calibrate the Two-Sector Model (1990-2007)

<table>
<thead>
<tr>
<th>Countries</th>
<th>Non tradable Share</th>
<th>$G_j/Y_j$</th>
<th>Labor Share</th>
<th>Product.</th>
<th>Aggregate ratios</th>
<th>Elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Output (1)</td>
<td>Consump. (2)</td>
<td>Inv. (3)</td>
<td>Gov. Spending (4)</td>
<td>Labor (5)</td>
<td>Lab. comp. (6)</td>
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<td>AUS</td>
<td>0.63</td>
<td>0.56</td>
<td>n.a.</td>
<td>0.88</td>
<td>0.68</td>
<td>0.67</td>
</tr>
<tr>
<td>AUT</td>
<td>0.64</td>
<td>0.52</td>
<td>0.62</td>
<td>0.89</td>
<td>0.64</td>
<td>0.64</td>
</tr>
<tr>
<td>BEL</td>
<td>0.65</td>
<td>0.53</td>
<td>n.a.</td>
<td>0.89</td>
<td>0.68</td>
<td>0.66</td>
</tr>
<tr>
<td>CAN</td>
<td>0.63</td>
<td>0.54</td>
<td>0.67</td>
<td>n.a.</td>
<td>0.69</td>
<td>0.67</td>
</tr>
<tr>
<td>DNK</td>
<td>0.66</td>
<td>0.54</td>
<td>0.60</td>
<td>0.93</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>ESP</td>
<td>0.64</td>
<td>0.54</td>
<td>0.72</td>
<td>0.91</td>
<td>0.66</td>
<td>0.67</td>
</tr>
<tr>
<td>FIN</td>
<td>0.58</td>
<td>0.53</td>
<td>0.68</td>
<td>0.89</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>FRA</td>
<td>0.70</td>
<td>0.51</td>
<td>0.69</td>
<td>0.93</td>
<td>0.69</td>
<td>0.68</td>
</tr>
<tr>
<td>GBR</td>
<td>0.64</td>
<td>0.52</td>
<td>0.58</td>
<td>0.94</td>
<td>0.70</td>
<td>0.65</td>
</tr>
<tr>
<td>IRL</td>
<td>0.52</td>
<td>0.52</td>
<td>0.69</td>
<td>0.90</td>
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<td>ITA</td>
<td>0.64</td>
<td>0.46</td>
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<td>0.92</td>
<td>0.63</td>
<td>0.62</td>
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<tr>
<td>JPN</td>
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<td>0.57</td>
<td>0.63</td>
<td>0.86</td>
<td>0.64</td>
<td>0.65</td>
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<td>NLD</td>
<td>0.65</td>
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<td>0.90</td>
<td>0.70</td>
<td>0.69</td>
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<tr>
<td>NOR</td>
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<td>0.49</td>
<td>0.67</td>
<td>0.91</td>
<td>0.66</td>
<td>0.67</td>
</tr>
<tr>
<td>SWE</td>
<td>0.64</td>
<td>0.56</td>
<td>0.55</td>
<td>0.94</td>
<td>0.68</td>
<td>0.67</td>
</tr>
<tr>
<td>USA</td>
<td>0.69</td>
<td>0.63</td>
<td>0.64</td>
<td>0.88</td>
<td>0.73</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Mean: 0.63 0.53 0.64 0.90 0.67 0.66 0.05 0.30 0.60 0.67 1.28 0.21 0.20 0.64 0.77 0.75

Notes: $G^T/Y^T$ is the share of government spending on good $j$ in output of sector $j$; $\theta^T$ is the share of labor income in value added at current prices of sector $j = T, N$; $Z^T/Z^N$ corresponds to the ratio of labor productivity of tradables to labor productivity of non tradables. $I/Y$ is the investment-to-GDP ratio and $G/Y$ is government spending as a share of GDP.
References


IMPERFECT MOBILITY OF LABOR ACROSS SECTORS AND FISCAL TRANSMISSION

TECHNICAL APPENDIX
NOT MEANT FOR PUBLICATION

Olivier CARDI, Peter CLAEYS, and Romain RESTOUT

- Section A presents the source and construction of the data used in the empirical and quantitative analysis, and empirical strategies to estimate the elasticity of substitution between traded and non traded goods and the elasticity of labor supply across sectors.
- Section B provides more VAR results and conduct a robustness check with respect to the classification of industries as tradables or non tradables, the exclusion of the public sector from aggregate and sectoral variables, the identifying assumption of government spending shocks.
- Section C provides more details about the interpretation of our empirical results and determines the conditions under which a government spending shock is biased toward non traded goods.
- Section D provides an elaborate investigation of the non tradable content of government spending shocks and the reactions of components of government consumption expenditure to a fiscal shock; this section also contrasts our results with those documented by earlier studies, conducts an investigation of the potential presence of anticipation effects, addresses a potential concern related to the fact that various VAR models could identify different structural government spending shocks, and deals with the potential endogeneity problem by using narratively identified government spending shocks from the dataset constructed by Guajardo, Leigh, and Pescatori [2014].
- Section E give more details on the model without physical capital accumulation, sets out the approach taken to solve the model, provides formal solutions for temporary fiscal shocks, investigates the effects of a rise in government consumption on non tradables and tradables as well, analyzes the role of the degree of labor mobility across sectors, and provides the main steps leading to equations in the main text of section 4.
- Section F gives more details on the model with physical capital accumulation, determines first-order conditions and sets out the approach taken to solve the model.
- In section G, we characterize graphically the initial steady-state and analyze the long-run effects of a temporary increase in government consumption.
- Section H provides the main steps leading to formal solutions following a temporary rise in government consumption in a continuous time setup.
- Section I considers a more general form for preferences by relaxing the assumption of separability in preferences in consumption and leisure.
• Section J introduces public debt in the setup.
• Section K lays out the same model except that we allow for the non traded sector to be imperfectly competitive and assume endogenous markups.
• Section L gives more details about the calibration of the model to data.
• Section M gives more numerical results. In this section, we explore the case of imperfect mobility of capital across sectors and we compare the theoretical responses from the baseline model with limited labor mobility and capital installation costs with those when one of these two features is shut down, together with the results from the VAR model. We also explore the case of endogenous markups and the implications of a rise in government spending which is debt-financed.
A Data Description

In this section, we present a complete description of our data set. First, we provide details on the data sources and variables construction used in the empirical analysis and to calibrate the model. Then, we describe empirical strategies to estimate two parameters involved in our quantitative analysis: the elasticity of substitution in consumption between traded and non traded goods, $\phi$, and the degree of substitutability of hours worked across sectors, $\epsilon$.

A.1 Data Description for Empirical Analysis

Coverage: Our sample consists of a panel of 16 countries: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Finland (FIN), France (FRA), Ireland (IRL), Italy (ITA), Japan (JPN), the Netherlands (NLD), Norway (NOR), Spain (ESP), Sweden (SWE), the United Kingdom (GBR), and the United States (USA). The period is running from 1970 to 2007, with the exception of Japan (1974-2007) for which the starting date differs due to sectoral data availability. The choice of countries is restricted by the availability of sufficiently detailed data on sectoral variables over a long time horizon.

A.1.1 Data for Aggregate Variables: Source and Construction

Sources:
- All expenditure aggregates are obtained from the Economic Outlook Database provided by the Organisation for Economic Cooperation and Development [2017].
- Series for aggregate variables are government final consumption expenditure ($G$), GDP ($Y$), total hours worked ($L$), the real consumption wage ($W/CP I$), private non-residential investment ($I$), and the current account-to-GDP ratio ($CA$). The database contains annual observations for the period running from 1970 to 2007 for the 16 OECD countries mentioned above. In the following, we provide details on data construction for aggregate variables (mnemonics are in parentheses):
  - **Government spending, $G$**: real government final consumption expenditure (CGV). Source: OECD Economic Outlook Database.
  - **Gross domestic product, $Y$**: real gross domestic product (GDPV). Source: OECD Economic Outlook Database.
  - **Private investment, $I E$**: real private non-residential gross fixed capital formation (IBV). Source: OECD Economic Outlook Database.
  - **Current account, $CA$**: ratio of the current account to the gross domestic product at current prices (CBGDPR). Source: OECD Economic Outlook Database.
  - **Labor, $L$**: total hours worked by persons engaged (H_EMP). Sources: EU KLEMS and OECD STAN databases.
  - **Real Consumption wage, $W/CP I$**: nominal wage divided by the consumer price index (CPI). Source: OECD Prices and Purchasing Power Parities for the consumer price index. The nominal wage is calculated by dividing labor compensation (LAB) by total hours worked by persons engaged (H_EMP). Sources: EU KLEMS and OECD STAN databases.

For government spending, GDP and investment, we directly use the volumes as reported by the OECD (the series are deflated with their own deflators). All quantity variables, with the exception of the current account, enter in the VAR models in log levels and scaled by the working age population (15-64 years old), while the real consumption wage rate is in natural log. The data source for the working age population is the OECD ALFS database.

A.1.2 Data for Sectoral Variables: Source and Construction

Sources: Our primary data sources are the OECD and EU KLEMS databases. We use the EU KLEMS [2011] sectoral database (the March 2011 data release, available at http://www.euklems.net) which provides for all countries of our sample with the exception of Canada and Norway annual data for eleven 1-digit ISIC-rev.3 industries. For Canada and Norway, sectoral data are taken from the Structural Analysis (STAN) database provided by the OECD [2011].

The eleven 1-digit ISIC-rev.3 industries are classified as tradables or non tradables. To do so, we adopt the classification proposed by De Gregorio et al. [1994] who treat an industry as traded when it exports at least 10% of its output. Following Jensen and Kletzer [2006], we have updated the classification suggested by De Gregorio et al. [1994] by treating “Financial Intermediation” as a traded industry. Jensen and Kletzer [2006] use the geographic concentration of service activities within the United States to identify which service activities are traded domestically. The authors classify activities that are traded domestically as potentially traded internationally. The idea is that
when a good or a service is traded, the production of the activity is concentrated in a particular region to take advantage of economies of scale in production.

Jensen and Kletzer [2006] use the two-digit NAICS (North American Industrial Classification System) to identify tradable and non tradable sectors. We map their classification into the NACE-ISIC-rev.3 used by the EU KLEMS and STAN databases. The mapping was clear for all sectors except for "Real Estate, Renting and Business Services". According to the EU KLEMS/STAN classification, the industry labelled "Real Estate, Renting and Business Services" is an aggregate of five sub-industries: "Real estate activities" (NACE code: 70), "Renting of Machinery and Equipment" (71), "Computer and Related Activities" (72), "Research and Development" (73) and "Other Business Activities" (74). While Jensen and Kletzer [2006] find that industries 70 and 71 can be classified as tradable, they do not provide information for industries 72, 73 and 74. We decided to classify "Real Estate, Renting and Business Services" as non tradable but conduct a robustness check by contrasting our empirical findings when "Real Estate, Renting and Business Services" is non traded with those when "Real Estate, Renting and Business Services" is traded. As shown in section B.2, our conclusions hold and remain unsensitive to the classification. We construct traded and non traded sectors as follows (EU KLEMS codes are given in parentheses):

- **Traded Sector**: "Agriculture, Hunting, Forestry and Fishing" (AtB), "Mining and Quarrying" (C), "Total Manufacturing" (D), "Transport, Storage and Communication" (I) and "Financial Intermediation" (J).
- **Non Traded Sector**: "Electricity, Gas and Water Supply" (E), "Construction" (F), "Wholesale and Retail Trade" (G), "Hotels and Restaurants" (H), "Real Estate, Renting and Business Services" (K) and "Community Social and Personal Services" (LtQ).

Once industries have been classified as tradables or non tradables, for any macroeconomic variable \( X \), its sectoral counterpart \( X_j \) for \( j = T, N \) is constructed by adding the \( X_k \) of all sub-industries \( k \) classified in sector \( j = T, N \) as follows \( X_j = \sum_{k \in j} X_k \).

Relevant to our work, EU KLEMS and OECD STAN database provide data, for each industry and year, on value added at current and constant prices, thus allowing us to construct series for sectoral value added deflators; the database also provide details on labor compensation and employment data, allowing the construction for sectoral wage rates. In the VAR models, with the exception of the current account, all quantity variables are in log levels and scaled by the working age population (15-64 years old), while price deflators and wage rates are in natural logs. Source: OECD ALFS Database for the working age population. We detail below the construction of sectoral data employed in section 2 (mnemonics are given in parentheses):

- **Sectoral output**, \( Y^T_j \): sectoral value added at constant prices in sector \( j = T, N \) (VA_QI). Sources: EU KLEMS and OECD STAN databases.
- **Relative output**, \( Y^T_j / Y^N_j \): ratio of traded value added at constant prices to non traded value added at constant prices.
- **Sectoral output share**, \( \nu^T_j \): ratio of value added at constant prices in sector \( j \) to GDP at constant prices, i.e., \( Y^j_j / (Y^T + Y^N) \) for \( j = T, N \).
- **Relative price of non tradables**, \( P \): ratio of the non traded value added deflator to the traded value added deflator, i.e., \( P = P^N / P^T \). The sectoral value added deflator \( P^j \) for sector \( j = T, N \) is calculated by dividing value added at current prices (VA) by value added at constant prices (VA_QI) in sector \( j \). Sources: EU KLEMS and OECD STAN databases.
- **Sectoral labor**, \( L^j \): total hours worked by persons engaged in sector \( j \) (H_EMP). Sources: EU KLEMS and OECD STAN databases.
- **Relative labor**, \( L^T_j / L^N_j \): ratio of hours worked in the traded sector to hours worked in the non traded sector.
- **Sectoral labor share**, \( \nu^T_j \): ratio of hours worked in sector \( j \) to total hours worked, i.e., \( L^j_j / (L^T + L^N) \) for \( j = T, N \).
- **Sectoral real consumption wage**, \( W^T_j / CPI \): nominal wage in sector \( j \) divided by the consumer price index (CPI). Source: OECD Prices and Purchasing Power Parities for the consumer price index. The sectoral nominal wage \( W^j \) for sector \( j = T, N \) is calculated by dividing labor compensation in sector \( j \) (LAB) by total hours worked by persons engaged (H_EMP) in that sector. Sources: EU KLEMS and OECD STAN databases.
- **Relative wage**, \( \Omega \): ratio of the nominal wage in the non traded sector \( W^N \) to the nominal wage in the traded sector \( W^T \), i.e., \( \Omega = W^N / W^T \).
• **Labor reallocation index,** $LR$: measures the fraction of workers who are working in year $t$ in a different sector than in year $t-2$ and is computed as:

$$LR_t(2) = 0.5 \sum_{j=T}^{N} \frac{L^j_t}{\sum_{j=T}^{N} L^j_t} - \frac{L^j_{t-2}}{\sum_{j=T}^{N} L^j_{t-2}}.$$ 

Data for labor (H\_EMP) are taken from EU KLEMS and STAN databases.

### A.2 Data Description for Calibration

In the numerical analysis, we calibrate a set of parameters by choosing them so that the initial steady-state of the model matches key empirical properties of a representative OECD economy. In particular, we pay attention to the adequacy of the non tradable content of the model to the data. This section gives information on our estimates of the non tradable content of GDP, consumption, investment, government spending, labor and labor compensation. In addition, it gives information about the share of government spending on traded and non traded goods in the corresponding sectoral value added and the labor income shares in sector $j = T, N$.

Our sample covers the 16 OECD countries mentioned in section A.1. In the following, statistics for the sample as a whole represent (unweighted) averages of the corresponding variables among the group. Our reference period for the calibration corresponds to the period 1990-2007. The choice of this period has been dictated by data availability. In the following, we provide details on data construction for non tradable shares:

- **Output, labor and labor compensation:** we split the eleven industries into traded and non traded sectors by adopting the classification proposed by De Gregorio et al. [1994] and updated by Jusen and Kletzer [2006]. Details about data construction for output and labor are provided in Section A.1.2. We calculate the non tradable share of labor compensation as the ratio of labor compensation of non tradables, i.e., $W^N L^N$, to overall labor compensation, i.e., $W L$. Sources: EU KLEMS [2011] and STAN databases. Data coverage: 1990-2007 for all countries.

- **Consumption:** to split consumption expenditure (at current prices) into consumption in traded and non traded goods, we make use of the Classification of Individual Consumption by Purpose (COICOP) published by the United Nations (Source: United Nations [2011]). Among the twelve items, the following ones are treated as consumption in traded goods: "Food and Non-Alcoholic Beverages", "Alcoholic Beverages Tobacco and Narcotics", "Clothing and Footwear", "Furnishings, Household Equipment" and "Transport". The remaining items are treated as consumption in non traded goods: "Housing, Water, Electricity, Gas and Fuels", "Health", "Communication", "Recreation and Culture", "Education", "Restaurants and Hotels". Because the item "Miscellaneous Goods and Services" is somewhat problematic, we decided to consider it as both tradable (50%) and non tradable (50%) with equal shares. Data coverage: 1990-2007 for AUS, AUT, CAN, DNK, FIN, FRA, GBR, ITA, JPN, NLD, NOR and USA, 1993-2007 for SWE and 1995-2007 for BEL, ESP and IRL.

- **Investment:** to map investment expenditure (at current prices) into expenditures on tradable and non tradables, we follow the classification proposed by Burstein et al. [2004], we consider "Housing", "Other Constructions" and "Other Products" as non tradable investment and "Products of Agriculture, Forestry, Fisheries and Aquaculture", "Metal Products and Machinery", "Transport Equipment" as tradable investment expenditure. Source: OECD Input-Output database [2012]. Data coverage: 1990-2007 for AUT, CAN, ESP, FIN, GBR, IRL, JPN, NLD, and NOR, 1990-2006 for DNK, FRA, ITA and USA, and 1993-2007 for SWE. Data are not available for AUS and BEL. Thus, for these two countries, when we calibrate the model to each OECD country, we target a non tradable content of investment expenditure that is given by the unweighed average, i.e., 0.64.

- **Government spending:** Sectoral government final consumption expenditure data (at current prices) were obtained from the OECD General Government Accounts database (Source: COFOG, OECD [2017]). "Economic Affairs" which includes "Fuel and Energy", "Agriculture, Forestry, Fishing, and Hunting", "Mining, Manufacturing, and Construction", "Transport and Communications" is classified as tradable. Items treated as non tradable are: "General Public Services", "Defense", "Public Order and Safety", "Environment Protection", "Housing and Community Amenities", "Health", "Recreation, Culture and Religion", "Education", "Social Protection". Data coverage: 1995-2007 for AUT, BEL, DNK, ESP, FRA, GBR, IRL, ITA, NLD, NOR and SWE, 1998-2007 for AUS, 1990-2007 for FIN, 2005-2007 for JPN and 1970-2007 for the USA. Data are not available for CAN. Thus, for this country, when we calibrate the model to each OECD country, we choose a non tradable content of government expenditure that is given by the unweighed average, i.e., 0.90.
Next, the labor income share for sector \( j = T, N \), denoted by \( \theta^j \), is calculated as the ratio of labor compensation in sector \( j \) (LAB) to value added at current prices (VA,QI) in that sector, i.e., \( \theta^j = (W^j L^j)/(P^j Y^j) \). Sources: EU KLEMS [2011] and STAN databases. Data coverage: 1990-2007 for all countries.

Finally, we approximate technological change in sector \( j \) with labor productivity in this sector which we measure by dividing the value added by constant prices in sector \( j \) (VA,QI) by total hours worked by persons engaged (H_EMP) in this sector, i.e., \( Z^j = Y^j / L^j \). The relative productivity, \( Z^T / Z^N \), is calculated as the ratio of labor productivity of tradables, \( Z^T \), to labor productivity of non tradables, \( Z^N \). Sources: EU KLEMS [2011] and STAN databases. Data coverage: 1990-2007 for all countries.

Because data source and construction are heterogeneous across variables as a result of different nomenclatures, Table 6 provides a summary of the classification adopted to split value added and its demand components as well into traded and non traded goods.

### A.3 Estimates of \( \phi \): Empirical Strategy

In this section, we detail our empirical strategy to estimate the elasticity of substitution between traded and non traded goods \( \phi \). Estimates of the elasticity of substitution \( \phi \) by the existing literature are rather diverse. The cross-section studies report an estimate of \( \phi \) ranging from 0.44 to 0.74, see e.g., Stockman and Tesar [1995] and Mendoza [1995], respectively. The literature adopting the Generalized Method of Moments and the cointegration methods, see e.g. Ostry and Reinhart [1992] and Cashin and Mc Dermott [2003], respectively, reports a value in the range \([0.75, 1.50]\) for developing countries and in the range \([0.63, 3.50]\) for developed countries. Since estimates for \( \phi \) display a sharp dispersion across empirical studies, we conduct an empirical analysis in order to estimate this parameter for each country in our sample.

#### A.3.1 Empirical Strategy

**Using Time Series by Industry Taken from EU KLEMS and STAN**

To estimate \( \phi \), we adopt the following strategy. To determine an empirical relationship, we combine the optimal rule for intra-temporal allocation of consumption (14) (that we repeat for clarity purposes)

\[
\frac{C^T}{C^N} = \left( \frac{\phi}{1 - \phi} \right) P^\phi. \tag{47}
\]

with the goods market equilibrium

\[
\frac{C^T}{C^N} = \frac{Y^T - NX - G^T - IT^T}{Y^N - GN - IN^N}, \tag{48}
\]

where we used the fact that \( B - r^* B = Y^T - C^T - G^T - IT^T \equiv NX \). Inserting (47) into (48) leads to

\[
\frac{Y^T - NX - G^T - IT^T}{Y^N - GN - IN^N} = \left( \frac{\phi}{1 - \phi} \right) P^\phi. \tag{49}
\]

According to the market clearing condition, we could alternatively use data for consumption or for sectoral value added along with times series for its demand components to estimate \( \phi \). Unfortunately, nomenclatures for valued added by industry and for consumption by items are different and thus it is most likely that \( C^T \) differs from \( Y^T - NX - G^T - IT^T \), and \( C^N \) from \( Y^N - GN - IN^N \) as well. Because time series for traded and non traded consumption display a short time horizon for half countries of our sample while data for sectoral value added and net exports are available for the 16 OECD countries of our sample over the period running from 1970 to 2007 (except for Japan: 1974-2007), we find appropriate to estimate \( \phi \) by computing \( Y^T - NX - G^T - IT^T \) and \( Y^N - GN - IN^N \). Yet, an additional difficulty shows up because the classification adopted to split government spending and investment expenditure into traded and non traded items is different from that adopted to break down

---

59While the sample used by Stockman and Tesar [1995] covers 30 countries (including 17 developing and 13 industrialized), Mendoza [1995] uses exactly the same data set in his estimation but includes only the 13 industrialized countries. Note that the estimate of \( \phi \) has been obtained by using the cross sectional dataset by Kravis, Heston and Summers for the year 1975.
### Table 6: Construction of Variables and Data Sources

<table>
<thead>
<tr>
<th>Variable</th>
<th>Countries covered</th>
<th>Period</th>
<th>Construction and aggregation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value added $Y^T$ &amp; $Y^N$</td>
<td>AUS, AUT, BEL, CAN, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA</td>
<td>1970-2007</td>
<td>$T$: Agriculture, Mining, Manufacturing, Transport, Finance intermediation&lt;br&gt;$N$: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services</td>
</tr>
<tr>
<td>(constant prices)</td>
<td></td>
<td></td>
<td>$N$: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services</td>
</tr>
<tr>
<td>Value added $P^T Y^T$ &amp; $P^N Y^N$ (current prices)</td>
<td>AUS, AUT, BEL, CAN, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA</td>
<td>1970-2007</td>
<td>$T$: Agriculture, Mining, Manufacturing, Transport, Finance intermediation&lt;br&gt;$N$: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services</td>
</tr>
<tr>
<td>Labor $L^T$ &amp; $L^N$ (total hours worked by persons engaged)</td>
<td>AUS, AUT, BEL, CAN, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA</td>
<td>1970-2007</td>
<td>$T$: Agriculture, Mining, Manufacturing, Transport, Finance intermediation&lt;br&gt;$N$: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services</td>
</tr>
<tr>
<td>Labor compensation $LAB^T$ &amp; $LAB^N$ (current price)</td>
<td>AUS, AUT, BEL, CAN, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA</td>
<td>1970-2007</td>
<td>$T$: Agriculture, Mining, Manufacturing, Transport, Finance intermediation&lt;br&gt;$N$: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services</td>
</tr>
</tbody>
</table>

**Notes:**
- Times series for $P^T Y^T$ & $P^N Y^N$ are not available for AUS and BEL together with $P^T G^T$ & $P^N G^N$ for CAN.
- **Lab Reallocation Index** $L^R$ (index 1995 = 100)
- **Consumer Price Index** $CPI$ (index 1995 = 100)
- **Government spending** $G$ (constant prices)
- **Gross domestic product** $GDP$ (in % of GDP)
- **Private investment** $I$ (constant prices)
- **Current account** $CA$ (in % of GDP)

**Construction and aggregation**
- Labor compensation ($LAB^T$) over total hours worked by persons engaged ($L^T$)
- Labor compensation ($LAB^N$) over total hours worked by persons engaged ($L^N$)
- Nominal wage ($W^T$) divided by the consumer price index ($CPI^T$)
- Nominal wage ($W^N$) divided by the consumer price index ($CPI^N$)
- Log of real per capita government final consumption expenditure ($CGV$)
- Log of real per capita non-residential gross fixed capital formation ($IBV$)
- Relative Wage $Ω = W^T / W^N$
- Relative Productivity $Z^T / Z^N$
- Real labor productivity in tradables ($Z^T$) over real labor productivity in non tradables ($Z^N$)

**Countries covered**
- OECD, EU KLEMS, AUS, AUT, BEL, CAN, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA

**Value added**
- Agriculture, Mining, Manufacturing, Transport, Finance intermediation
- Electricity, Construction, Trade, Hotels, Real Estate, Personal Services
value added into traded and non traded components. Moreover, the time horizon is short at a disaggregated level (for \(I^j\) and \(G^j\)) for most of the countries, especially for time series of \(G^j\). To overcome these difficulties, we proceed as follows. Denoting by

\[
v_{GT} = \frac{P^T Y^T}{P^T Y^T - P^T NX}
\]

and

\[
v_{IT} = \frac{P^T Y^T - P^TNX}{P^T Y^T - P^T NX}
\]

the ratio of government and investment expenditure on tradables to traded value added adjusted with net exports at current prices, respectively, and by

\[
v_{GN} = \frac{P^NY^N}{P^TY^N} \text{ and } v_{IN} = \frac{P^NY^N}{P^TY^N}
\]

the ratio of government and investment expenditure on non tradable to non traded value added at current prices, the goods market equilibrium can be rewritten as follows:

\[
\frac{(P^TY^T - P^TNX)(1 - v_{GT} - v_{IT})}{P^NY^N(1 - v_{GN} - v_{IN})} = \left(\frac{\phi}{1 - \phi}\right) P^\phi - 1,
\]

or alternatively

\[
\frac{(Y^T - NX)(1 - v_{GT} - v_{IT})}{Y^N(1 - v_{GN} - v_{IN})} = \left(\frac{\phi}{1 - \phi}\right) P^\phi.
\]

(50)

Setting

\[
\alpha \equiv \ln \left(\frac{1 - v_{GN} - v_{IN}}{1 - v_{GT} - v_{IT}}\right) + \ln \left(\frac{\phi}{1 - \phi}\right),
\]

(51)

and taking logarithm, eq. (50) can be rewritten as follows:

\[
\ln \left(\frac{Y^T - NX}{Y^N}\right) = \alpha + \phi \ln P.
\]

(52)

Indexing time by \(t\) and countries by \(i\), and adding an error term \(\mu\), we estimate \(\phi\) by exploring the following empirical relationship:

\[
\ln \left(\frac{Y^T - NX}{Y^N}\right)_{it} = f_i + f_t + \alpha_{it} + \phi \ln P_{it} + \mu_{it}.
\]

(53)

\(f_i\) captures the country fixed effects, \(f_t\) are time dummies, and \(\mu_{it}\) are the i.i.d. error terms. Because the term (51) may display a trend over time, we add country-specific trends, as captured by \(\alpha_{it}\).

Because data to construct time series for traded \((I^T)\) and non traded investment \((I^N)\) are available for twelve countries over the sixteen in our sample over a time horizon varying between 37 years (1970-2007) and 27 years (1980-2007), we computed time series \(Y^T - NX - I^T\) and \(Y^N - I^N\). In this case, eq. (50) can be rewritten as follows:

\[
\frac{(Y^T - NX - I^T)(1 - v_{GT})}{(Y^N - I^N)(1 - v_{GN})} = \left(\frac{\phi}{1 - \phi}\right) P^\phi.
\]

(54)

Denoting by

\[
\kappa \equiv \ln \left(\frac{1 - v_{GN}}{1 - v_{GT}}\right) + \ln \left(\frac{\phi}{1 - \phi}\right),
\]

(55)

where \(v_{GT} = \frac{P^T Y^T}{P^T Y^T - P^T NX}\) and \(v_{GN} = \frac{P^NY^N}{P^TY^N}\) and taking logarithm, we explore alternatively the following relationship to estimate \(\phi\):

\[
\ln \left(\frac{\beta^T}{\beta^N}\right)_{it} = f_i + f_t + \alpha_{it} + \phi \ln P_{it} + \nu_{it}.
\]

(56)

where \(\beta^T = (Y^T - NX - I^T)\) and \(\beta^N = (Y^N - I^N)\).

When determining (52), we can alternatively make use of first-order conditions equating the marginal revenue of labor and the sectoral wage:

\[
\frac{\theta^j P^j Y^j}{L^j} = W^j,
\]

(57)

where \(\theta^j\) is labor’s share in value added in sector \(j = T, N\). Using (57) to eliminate the nominal sectoral value added, \(P^j Y^j\), the goods market clearing condition can be rewritten as follows:

\[
\frac{(W^T L^T - \theta^T P^T NX) \theta^N}{\theta^T} (1 - v_{GT} - v_{IT})}{W^N L^N (1 - v_{GN} - v_{IN})} = \left(\frac{\phi}{1 - \phi}\right) P^\phi - 1.
\]

(58)
We first set
\[
\eta = \ln \left( 1 - \varphi G^N - \theta I^N \right) + \ln \left( \theta^T \gamma^N \right) + \ln \left( \frac{\varphi}{1 - \varphi} \right),
\]
(59)
where \( \varphi G^T = \frac{P^T G^T}{P^T R^T} \) and \( \varphi G^N = \frac{P^N G^N}{P^N R^N} \). Denoting by \( \gamma^T = (W^T L^T - \theta^T P^T N X) \) and \( \gamma^N = W^N L^N \), and taking logarithm, eq. (59) can be rewritten as follows:
\[
\ln \left( \frac{\gamma^T}{\gamma^N} \right) = \eta + (\phi - 1) \ln P.
\]
(60)
Indexing time by \( t \) and countries by \( i \), and adding an error term \( \zeta \), we estimate \( \phi \) by exploring the following empirical relationship:
\[
\ln \left( \frac{\gamma^T}{\gamma^N} \right)_{it} = g_i + g_t + \sigma_i t + \rho p_{it} + \zeta_{it}.
\]
(61)
Because \( \eta_i \) (see eq. (59)) is composed of both preference (i.e., \( \varphi \)) and production (i.e., \( \theta^i \)) parameters, and (logged) ratios which may display trend over time, we introduce country fixed effects \( g_i \) and add country-specific trends, as captured by \( \sigma_i t \). Once we have estimated \( \rho \), we can compute \( \hat{\phi} = \hat{\phi} + 1 \) where a hat refers to point estimate in this context.

Using Time Series for Consumption by Purpose Taken from COICOP

The cross-section studies by Stockman and Tesar [1995] and Mendoza [1995] estimate \( \phi \) by running a regression of the (logged) ratio of consumption in non tradables to consumption in tradables on the (logged) relative price of non tradables:
\[
\ln \left( \frac{C^N}{C^T} \right) = \ln \left( \frac{1 - \varphi}{\varphi} \right) - \phi \ln P.
\]
(62)
Note that when exploring the relationship (62) empirically, we abstract from the goods market clearing condition. Indexing time by \( t \) and countries by \( i \), and adding an error term \( \zeta \), we explore the following relationship empirically by using panel data:
\[
\ln \left( \frac{C^N}{C^T} \right)_{it} = d_i + d_t + \zeta_{it} - \phi \ln P_{C,it} + \zeta_{it},
\]
where \( P_{C,it} = P^N_{C,it}/P^T_{C,it} \) is the ratio of the price deflator for consumption in non traded goods \( (P^N_{C,it}) \) to the price deflator for consumption in traded goods \( (P^T_{C,it}) \): \( d_i \) are country fixed effects while \( d_t \) are time dummies; \( \zeta_{it} \) are the i.i.d. error terms. Because preferences may not be homothetic, there might be income effects in the relative demand for tradable and non tradable goods. Cross-section studies by Stockman and Tesar [1995] and Mendoza [1995] include GDP per capita in the regression to capture the wealth effect. Because it is likely that GDP per capita is correlated with the relative price of non tradables, we capture the wealth effect by time trend, i.e., \( \zeta_{it} \).

A.3.2 Data Construction and Source

Using Time Series by Industry Taken from EU KLEMS and STAN

We provide more details below on the construction of data employed to estimate equations (53), (56) and (61) (codes in EU KLEMS/STAN are reported in parentheses):

- Sectoral value added price deflator \( P^j_t \) \((j = T, N)\): value added at current prices (VA) over value added at constant prices (VA_QI) in sector \( j \). The relative price of non tradables, \( P_t \), corresponds to the ratio of the non traded value added deflator to the traded value added deflator: \( P_t = P^N_t/P^T_t \). Sources: EU KLEMS and OECD STAN databases. Data coverage: 1970-2007 except for JPN 1974-2007.


We also use the time series described above to construct time series for $\frac{Y_t^T - NX_t}{Y_t^N}$, $\frac{\beta_T}{\beta_N}$, $\frac{\gamma_T}{\gamma_N}$, and $P_t$. When estimating equations (53), (56) and (61), all variables are converted into index 1995=100 and are expressed in log levels.

**Using Time Series for Consumption By Purpose Taken from COICOP**

Panel data estimations of $\phi$ are based upon a data set provided by the COICOP database. To split aggregate consumption expenditure into tradables and non tradables, we use the methodology detailed in Appendix A.2 where we provide detailed information about the construction of the non tradable share of consumption expenditure. The COICOP database provides annual data for the sixteen OECD countries of our sample but it has the disadvantage to be unbalanced. Only a few countries have long time series. For example, the US enters the panel with 38 observations, whereas the UK has merely 18. We therefore eschew countries providing no data for periods that extend before 1988 (i.e., countries with more than 20 years). We made this choice in order to ensure the consistency of the estimates of cointegrating vectors. Accordingly, the sample is restricted to eleven countries: AUS (1970-2007), AUT (1976-2007), CAN (1971-2007), DNK (1970-2007), FIN (1975-2007), FRA (1970-2007), ITA (1970-2007), JPN (1980-2007), NLD (1980-2007), NOR (1970-2007) and USA (1970-2007). The following countries: BEL (1995-2007), ESP (1995-2007), GBR (1990-2007), IRL (1995-2007) and SWE (1993-2007) are excluded from the sample due to data limitation.

We now provide information about the construction for the data used to estimate equation (63):

- sectoral price deflator for consumption good $j$ ($P_{C,t}^j$): consumption expenditure in good $j$ at current prices over consumption expenditure in good $j$ at constant prices. Source: COICOP database. The consumption relative price of non tradables, $P_{C,t}$, corresponds to the price deflator for consumption in non tradable goods over the price deflator for consumption in tradable goods: $P_t = P_{C,t}^N / P_{C,t}^T$.

- sectoral consumption expenditure $C_t^j$ ($j = T, N$): final consumption expenditure of households in good $j$ at constant prices (name in COICOP: P31DC). Source: COICOP database.

In equation (63), time series for $(C_t^N / C_t^T)$ and $P_t^C$ are converted into index 1995=100 and are expressed in log levels.
A.3.3 Empirical Results

Since the set of variables of interest in regressions (53), (56), (61) and (63) display trends, we first run panel unit root tests, see Table 7. By and large, all tests, with the exceptions of Breitung and MW(PP) for the variable \( \ln(Y^T - NX/Y^N) \), show that non stationarity is pervasive, making it clear that pursuing a cointegration analysis is appropriate. We thus implement the seven Pedroni’s [2004] tests of the null hypothesis of no cointegration, see Table 8. Across almost all cases the null hypothesis of no cointegration is rejected but only at the 10% level. In small samples, Pedroni’s [2004] simulations reveal that the group-mean parametric t-stat is the most powerful. Based on this result, in the three specifications, the null hypothesis of no cointegration is strongly rejected at the 5% level.

Table 7: Panel Unit Root Tests (p-values)

<table>
<thead>
<tr>
<th>Dep. variable</th>
<th>LLC (t-stat)</th>
<th>Breitung (t-stat)</th>
<th>IPS (W-stat)</th>
<th>MW (ADF)</th>
<th>MW (PP)</th>
<th>Hadri (Zµ-stat)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ln(P^C,N/P^C,T) )</td>
<td>0.206</td>
<td>0.879</td>
<td>0.998</td>
<td>0.441</td>
<td>0.137</td>
<td>0.000</td>
</tr>
<tr>
<td>( \ln(C^N/C^T) )</td>
<td>0.156</td>
<td>0.844</td>
<td>0.255</td>
<td>0.132</td>
<td>0.293</td>
<td>0.000</td>
</tr>
<tr>
<td>( \ln(P^{VA,N}/P^{VA,T}) )</td>
<td>0.670</td>
<td>0.370</td>
<td>1.000</td>
<td>0.976</td>
<td>0.889</td>
<td>0.000</td>
</tr>
<tr>
<td>( \ln(Y^T - NX/Y^N) )</td>
<td>0.322</td>
<td>0.000</td>
<td>0.164</td>
<td>0.061</td>
<td>0.028</td>
<td>0.000</td>
</tr>
<tr>
<td>( \ln(W^T L^T - \theta^T P^T NX)/(W^N L^N) )</td>
<td>0.843</td>
<td>0.854</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: For all tests, except for Hadri [2000], the null of a unit root is not rejected if p-value ≥ 0.05 at a 5% significance level. For Hadri [2000], the null of stationarity is rejected if p-value ≤ 0.05 at a 5% significance level.

Table 8: Panel Cointegration Tests (p-values)

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>( C^N/C^T )</th>
<th>( Y^T - NX/Y^N )</th>
<th>( Y^T - NX - I^T/Y^N - I^N )</th>
<th>( W^T I^T - \theta^T P^T NX/W^N L^N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanatory variable</td>
<td>( P^C,N/P^C,T )</td>
<td>( P^{VA,N}/P^{VA,T} )</td>
<td>( P^{VA,N}/P^{VA,T} )</td>
<td>( P^{VA,N}/P^{VA,T} )</td>
</tr>
<tr>
<td><strong>Panel tests</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-parametric ( \nu )</td>
<td>0.274</td>
<td>0.065</td>
<td>0.000</td>
<td>0.009</td>
</tr>
<tr>
<td>Non-parametric ( \rho )</td>
<td>0.441</td>
<td>0.001</td>
<td>0.006</td>
<td>0.011</td>
</tr>
<tr>
<td>Non-parametric ( t )</td>
<td>0.347</td>
<td>0.000</td>
<td>0.001</td>
<td>0.004</td>
</tr>
<tr>
<td>Parametric ( t )</td>
<td>0.006</td>
<td>0.048</td>
<td>0.000</td>
<td>0.040</td>
</tr>
<tr>
<td><strong>Group-mean tests</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-parametric ( \nu )</td>
<td>0.059</td>
<td>0.047</td>
<td>0.383</td>
<td>0.232</td>
</tr>
<tr>
<td>Non-parametric ( t )</td>
<td>0.245</td>
<td>0.000</td>
<td>0.311</td>
<td>0.021</td>
</tr>
<tr>
<td>Parametric ( t )</td>
<td>0.000</td>
<td>0.068</td>
<td>0.001</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Notes: the null hypothesis of no cointegration is rejected if the p-value is below 0.05 (0.10 resp.) at 5% (10% resp.) significance level.

To estimate the cointegrating vector, we use the group-mean fully modified OLS and the group-mean dynamic OLS estimators of Pedroni [2001]. Table 9 reports panel estimations of the coefficient \( \phi \), when running the regression (53), (56), (61) and (63) respectively; the three former empirical relationships are derived by taking into account the goods market equilibrium. Moreover, exploring alternatively the relationship (53) or (61) empirically has the advantage of allowing us to use time series for sectoral value added or labor compensation which are available over the period 1970-2007 for all countries of our sample (except JPN: 1974-2007).

The first column of Table 9 presents the results corresponding to eq. (63). The dependent variable in both cases is the log of consumption in non tradables in terms of tradables, i.e. \( \ln(C^N/C^T) \). The regressor is the log of the ratio of the price deflator for consumption in non tradables to the price deflator for consumption in tradables. The estimated coefficient for \( \phi \) of 0.579 (DOLS) and 0.615 (FMOLS) are highly significant with a t-statistic of 8.72 and 11.85 respectively. However, there is substantial evidence of parameter heterogeneity across countries of the sample. One drawback of this approach is that when determining the testable equation (63), we abstract from the goods market equilibrium.
Panel data estimates of $\phi$ when running the regression (53) where the dependent variable is $(Y^T - N X)/Y^N$, are shown in column 2 of Table 9. The regressor in this case (and for the rest of the analysis) is the log of the non traded value added deflator to the traded value added deflator. The sample covers all countries we are interested in. For the whole sample, the DOLS and FMOLS estimates give a significant value of $\phi$ of 0.680 and 0.656 respectively. The two estimated coefficients are statistically significant. The vast majority (14 out of 16) of the individual FMOLS estimated coefficients are statistically significant. They vary from a low of 0.070 for BEL to a high of 2.071 for DNK. In addition, we find that $\phi$ is larger than one in only two countries (DNK and FIN). Column 3 of Table 9 shows panel data estimations of $\phi$ when running the regression (56) which explicitly takes into account investment expenditures. This, however, reduces the size of the sample: the series for investment are not available for AUS and BEL, and, SWE and IRL are excluded from the sample due to data limitation. We find that both estimators provide positive and statistically significant $\phi$ coefficients about 0.590. Among the 12 countries, we find that 8 have positive and statistically significant $\phi$ coefficients according to the FMOLS estimator, ranging from a low of 0.252 (CAN) to a high of 1.758 (NLD). Three estimated coefficients are negative (DNK, ESP and ITA), although none of them are statistically significant. Due to data limitations and inconsistent estimates (i.e., negative or statistically insignificant at conventional level for several countries), we find that including investment expenditure does not improve the precision of our estimates, likely due to the classification of investment items which is different to that we used to classify value added and labor as tradables or non tradables.

The last column of Table 9 gives panel data estimates of $\phi$ when running the regression (61); the dependent variable is the (logged) ratio of the labor income in tradables adjusted with net exports at current prices to labor income in non tradables, i.e., $(W^T L^T - \theta^T P^T N X)/W^N L^N$. By and large, estimates are somewhat higher than those shown in columns 1-3 of Table 9: the DOLS and FMOLS estimates give a significant value of $\phi$ of 0.817 and 0.837, respectively. Focusing only on FMOLS estimates which are positive and statistically significant, we find large differences in estimated coefficients across countries. They vary from a low of 0.409 for AUS to a high of 2.056 for NOR.

To calibrate the model, we take FMOLS estimates shown in column 2 as they are in line with earlier studies and values of $\phi$ are consistent for almost all countries in sample, except for Belgium and Italy. Estimate of $\phi$ for Belgium is not statistically significant at a standard threshold while estimates of $\phi$ for Italy are negative. Running the regression (61) allows us to obtain a consistent estimate for $\phi$ for Belgium, i.e., 0.795. Thus, we use this value to calibrate the model to each country. In contrast, estimates of $\phi$ are all inconsistent for Italy. When we calibrate the model to each country, we set $\phi$ to the unweighed average, i.e., 0.77.

A.4 Estimates of $\epsilon$: Empirical Strategy

In this section, we detail our empirical strategy to estimate the elasticity of labor supply across sectors, $\epsilon$, which captures the degree of labor mobility across sectors.

A.4.1 Limited Substitutability of Hours Worked across Sectors and the Derivation of the Testable Equation

To determine the equation we explore empirically, we follow closely Horvath [2000]. The representative agent is endowed with one unit of time, supplies a fraction $L(t)$ as labor, and consumes the remainder $1 - L(t)$ as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Assuming that the felicity function is additively separable in consumption and labor, the representative household maximizes the following objective function:

$$U = \int_0^\infty (1 - \gamma) \ln C(t) + \gamma \ln (1 - L(t)) \, e^{-\rho t} \, dt,$$

(64)
Table 9: DOLS and FMOLS Estimates of $\phi$

<table>
<thead>
<tr>
<th>Sectoral prices</th>
<th>$C^N$</th>
<th>$Y^T - NX$</th>
<th>$Y^T - NX - I^T$</th>
<th>$W^NL^N - \theta^T P^NNX$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td>DOLS</td>
<td>FMOLS</td>
<td>DOLS</td>
<td>FMOLS</td>
</tr>
<tr>
<td>AUS</td>
<td>1.015$^a$</td>
<td>1.041$^a$</td>
<td>0.290$^a$</td>
<td>0.268$^a$</td>
</tr>
<tr>
<td></td>
<td>(7.53)</td>
<td>(9.08)</td>
<td>(2.93)</td>
<td>(2.99)</td>
</tr>
<tr>
<td>AUT</td>
<td>0.008</td>
<td>0.309</td>
<td>0.927$^a$</td>
<td>0.986$^a$</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.57)</td>
<td>(2.07)</td>
<td>(3.95)</td>
</tr>
<tr>
<td>BEL</td>
<td>0.073</td>
<td>0.070</td>
<td>0.073</td>
<td>0.070</td>
</tr>
<tr>
<td></td>
<td>(0.40)</td>
<td>(0.44)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAN</td>
<td>0.183$^b$</td>
<td>0.212$^b$</td>
<td>0.437$^a$</td>
<td>0.391$^a$</td>
</tr>
<tr>
<td></td>
<td>(2.77)</td>
<td>(4.43)</td>
<td>(2.72)</td>
<td>(2.95)</td>
</tr>
<tr>
<td>DNK</td>
<td>0.515$^a$</td>
<td>0.740$^a$</td>
<td>2.234$^a$</td>
<td>2.071$^a$</td>
</tr>
<tr>
<td></td>
<td>(3.71)</td>
<td>(4.96)</td>
<td>(1.56)</td>
<td>(1.83)</td>
</tr>
<tr>
<td>ESP</td>
<td>0.745$^a$</td>
<td>0.783$^a$</td>
<td>0.745$^a$</td>
<td>0.783$^a$</td>
</tr>
<tr>
<td></td>
<td>(9.71)</td>
<td>(9.96)</td>
<td>(9.71)</td>
<td>(9.96)</td>
</tr>
<tr>
<td>FIN</td>
<td>0.461</td>
<td>0.047</td>
<td>1.213$^a$</td>
<td>1.072$^a$</td>
</tr>
<tr>
<td></td>
<td>(0.43)</td>
<td>(0.66)</td>
<td>(9.88)</td>
<td>(8.57)</td>
</tr>
<tr>
<td>FRA</td>
<td>1.292$^a$</td>
<td>0.925$^a$</td>
<td>0.955$^a$</td>
<td>0.937$^a$</td>
</tr>
<tr>
<td></td>
<td>(7.86)</td>
<td>(8.24)</td>
<td>(5.75)</td>
<td>(5.32)</td>
</tr>
<tr>
<td>GBR</td>
<td>0.517$^a$</td>
<td>0.477$^a$</td>
<td>0.517$^a$</td>
<td>0.477$^a$</td>
</tr>
<tr>
<td></td>
<td>(11.30)</td>
<td>(9.64)</td>
<td>(5.64)</td>
<td>(5.29)</td>
</tr>
<tr>
<td>IRL</td>
<td>0.184</td>
<td>0.374$^b$</td>
<td>0.184</td>
<td>0.374$^b$</td>
</tr>
<tr>
<td></td>
<td>(0.63)</td>
<td>(1.71)</td>
<td>(0.63)</td>
<td>(1.71)</td>
</tr>
<tr>
<td>ITA</td>
<td>0.341</td>
<td>0.153</td>
<td>0.436$^a$</td>
<td>0.308$^a$</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
<td>(0.40)</td>
<td>(2.92)</td>
<td>(1.60)</td>
</tr>
<tr>
<td>JPN</td>
<td>0.768$^b$</td>
<td>0.856$^a$</td>
<td>1.012$^b$</td>
<td>0.654$^a$</td>
</tr>
<tr>
<td></td>
<td>(2.15)</td>
<td>(2.81)</td>
<td>(4.38)</td>
<td>(2.98)</td>
</tr>
<tr>
<td>NLD</td>
<td>0.194</td>
<td>0.841$^a$</td>
<td>0.826$^b$</td>
<td>0.709$^b$</td>
</tr>
<tr>
<td></td>
<td>(1.38)</td>
<td>(3.52)</td>
<td>(1.99)</td>
<td>(2.33)</td>
</tr>
<tr>
<td>NOR</td>
<td>0.308$^c$</td>
<td>0.328$^b$</td>
<td>0.992$^a$</td>
<td>0.979$^a$</td>
</tr>
<tr>
<td></td>
<td>(1.72)</td>
<td>(2.50)</td>
<td>(8.38)</td>
<td>(7.92)</td>
</tr>
<tr>
<td>SWE</td>
<td>0.330$^a$</td>
<td>0.356$^a$</td>
<td>0.330$^a$</td>
<td>0.356$^a$</td>
</tr>
<tr>
<td></td>
<td>(3.69)</td>
<td>(4.02)</td>
<td>(3.69)</td>
<td>(4.02)</td>
</tr>
<tr>
<td>USA</td>
<td>3.396$^a$</td>
<td>3.269$^a$</td>
<td>0.586</td>
<td>0.668$^a$</td>
</tr>
<tr>
<td></td>
<td>(5.45)</td>
<td>(6.41)</td>
<td>(1.57)</td>
<td>(2.81)</td>
</tr>
<tr>
<td><strong>Whole Sample</strong></td>
<td>0.579$^a$</td>
<td>0.615$^a$</td>
<td>0.680$^a$</td>
<td>0.656$^a$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Countries</th>
<th>11</th>
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<td>Observations</td>
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<td>605</td>
<td>412</td>
<td>605</td>
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<tr>
<td>Country fixed effects</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Time dummies</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Time trend</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Notes: all variables enter in regression in logarithms. $^a$, $^b$ and $^c$ denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses.
subject to

$$\dot{A}(t) = r^* A(t) + W(t) L(t) - P_C (P(t)) C(t). \quad (65)$$

For the sake of clarity, we drop the time argument below when this causes no confusion.

First-order conditions are:

$$\frac{1 - \gamma}{C} = (P_C \lambda), \quad (66a)$$

$$\frac{\gamma}{1 - L} = W \lambda, \quad (66b)$$

$$\dot{\lambda} = \lambda (\beta - r^*). \quad (66c)$$

The economic system consists of $M$ distinct sectors, indexed by $j = 0, 1, \ldots, M$ each producing a different good. Along the lines of Horvath [2000], the aggregate leisure index is assumed to take the form:

$$1 - L(.) = 1 - \left[ \sum_{j=1}^{M} \left( L^j \right)^{\frac{\xi_j + 1}{\xi}} \right]^{\frac{\xi}{\xi + 1}}. \quad (67)$$

The agent maximizes (67) subject to

$$\sum_{j=1}^{M} W^j L^j = X, \quad (68)$$

where $L^j$ is labor supply in sector $j$, $W^j$ the wage rate in sector $j$ and $X$ total labor income.

Applying standard methods, we obtain labor supply $L^j$ in sector $j$:

$$L^j = \left( \frac{W^j}{W} \right)^\xi L. \quad (69)$$

where we used the fact that $X = WL$.

Combining (66a) and (66b), the aggregate wage index is:

$$W = \frac{\gamma}{1 - \gamma} \frac{P_C C}{1 - L} \quad (70)$$

which allows us to rewrite (69) as follows:

$$L^j = (W^j)^\xi L \left( \frac{\gamma}{1 - \gamma} \frac{P_C C}{1 - L} \right)^{-\xi}. \quad (71)$$

A quantity $Q^j$ of good $j$ is produced by combining capital, $K^j$, labor devoted to the sector, $L^j$, and intermediate inputs, $IM^j$, in a production process described by:

$$Q^j = Z^j \left( L^j \right)^{\xi_j} \left( K^j \right)^{\gamma_j} \left( IM^j \right)^{1 - \xi_j - \gamma_j}, \quad (72)$$

where $\xi^j (\gamma^j)$ is the share of labor (capital) income in gross output of sector $j$.

We assume that labor is imperfectly mobile across sectors, while capital can move freely across sectors. Perfectly competitive firms in sector $j$ seek to maximize the profit function given by:

$$\Pi^j = P^j Q^j - W^j L^j - RK^j - P_{IM} IM^j, \quad (73)$$

where $P^j$ is the price of gross output, $R$ is the user capital cost, $W^j$ the wage rate in sector $j$, and $P_{IM}$ the price of intermediate inputs. Firms take the wage rate (capital rental cost) as given and equate marginal product of labor (capital) to the wage (capital rental rate) to determine demand. First-order conditions are:

$$P^j \frac{\xi^j Q^j}{L^j} = W^j, \quad P^j \frac{\gamma^j Q^j}{K^j} = R, \quad P^j \frac{(1 - \xi^j - \gamma^j) Q^j}{IM^j} = P_{IM}. \quad (74)$$
Eliminating the sectoral wage $W^j_i$ into (71) by using labor demand given by (74), the equilibrium condition for labor is given by:

$$L^j = (\xi^j P^j Q^j)^{\frac{\gamma}{\gamma - 1}} L^{\frac{1}{1 - \gamma}} \left( \frac{\gamma}{1 - \gamma + \gamma} \right)^{\frac{1}{1 - \gamma}}. \quad (75)$$

Summing over the $M$ sectors and using (67), we get:

$$\left( \frac{\gamma}{1 - \gamma + \gamma} \right)^{\frac{1}{1 - \gamma}} = \sum_{j=1}^{M} \theta^j P^j Q^j \frac{L}{L}$$

Plugging this equation into (75) yields:

$$L^j = \left( \frac{\xi^j P^j Q^j}{\sum_{j=1}^{M} \xi^j P^j Q^j} \right)^{\frac{\gamma}{\gamma - 1}} L. \quad (76)$$

As in Horvath [2000], we denote by $\beta^j$ the fraction of labor’s share of aggregate output accumulating to labor in sector $j$:

$$\beta^j = \frac{\xi^j P^j Q^j}{\sum_{j=1}^{M} \xi^j P^j Q^j}. \quad (77)$$

We introduce the time subscript to avoid confusion. Expressing (76) in percentage changes and adding an estimation error term $\nu$ results in the $M$ estimation equations:

$$\hat{l}^j_t - \hat{l}_t = \frac{\epsilon}{\epsilon + 1} \hat{\beta}^j_t + \nu^j_t, \quad j = 1, ..., M, \quad (80)$$

where

$$\hat{l}_t = \sum_{j=1}^{M} \beta^j_{t-1} \hat{l}^j_t. \quad (79)$$

To derive (79), we proceed as follows. Because we consider a traded and a non traded sectors, the labor index (67) can be rewritten as follows:

$$L \left( L^T_t, L^N_t \right) = \left( L^T_t + L^N_t \right)^{\frac{\gamma}{\gamma - 1}}. \quad (80)$$

Approximate changes in aggregate labor with differentials, we get:

$$dL_t \equiv L_t - L_{t-1} = \left( L^T_{t-1} \right)^{\frac{1}{2}} \left( L^N_{t-1} \right)^{\frac{1}{2}} dL^T_t + \left( L^N_{t-1} \right)^{\frac{1}{2}} dL^N_t. \quad (81)$$

Expressing (81) in percentage changes and inserting (76), i.e., $\left( \frac{L^T_t}{L^N_t} \right)^{\frac{\gamma}{\gamma - 1}} = \beta^j$, we have:

$$\hat{l}_t = \frac{L_t - L_{t-1}}{L_{t-1}} = \left( \frac{L^T_{t-1}}{L^N_{t-1}} \right)^{\frac{\gamma}{\gamma - 1}} \hat{l}^T_t + \left( \frac{L^N_{t-1}}{L^N_{t-1}} \right)^{\frac{\gamma}{\gamma - 1}} \hat{l}^N_t,$$

$$= \beta^j_{t-1} \hat{l}^T_t + \beta^j_{t-1} \hat{l}^N_t. \quad (82)$$

According to eq. (82), the percentage change in total hours worked, $\hat{l}_t$, can be approximated by a weighted average of changes in sectoral hours worked $\hat{l}^j_t$ (in percentage), the weight being equal to $\beta^j_{t-1}$.

Combining optimal rules for labor supply and labor demand, we find that the change in employment in sector $j$ is driven by the change in the fraction $\beta^j$ of the labor’s share of aggregate output accumulating to labor in sector $j$. We use panel data to estimate (78). Including country fixed effects captured by country dummies, $f_i$, and common macroeconomic shocks by year dummies, $f_t$, (78) can be rewritten as follows:

$$\hat{l}_{it} - \hat{l}_{it} = f_i + f_t + \gamma_i \beta^j_{it} + \nu_{it}, \quad (83)$$

where $\gamma_i = \frac{\epsilon}{\epsilon + 1}$ and $\beta^j_{it}$ is given by (77); $j$ indexes the sector, $i$ the country, and $t$ indexes time. When exploring empirically (83), the coefficient $\gamma$ is alternatively assumed to be identical, i.e., $\gamma_i = \gamma$, or to vary across countries. The LHS term of (83), i.e., $\hat{l}_{it} - \hat{l}_{it}$, gives the percentage change in hours worked in sector $j$ driven by the pure reallocation of labor across sectors.
A.4.2 Data Description

Data are taken from EU KLEMS and STAN databases. EU KLEMS data provide yearly information for the period 1970-2007 (except for JPN: 1974-2007) for 16 countries of our sample (AUS, AUT, BEL, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, SWE and USA). For CAN and NOR, annual sectoral data stems from the STAN database. To classify employment and gross output as traded or non traded, we adopt the classification described in subsection A.1.2. We provide more details below about the data used to estimate equation (83):

- Sectoral labor $L^j_t$ ($j = T, N$): total hours worked by persons engaged in sector $j$ (H_EMP). Sources: EU KLEMS and STAN databases.
- Sectoral nominal gross output $P^j_t Q^j_t$ ($j = T, N$): gross output at current prices in millions of national currency in sector $j$ (GO). Sources: EU KLEMS and STAN databases.

By combining $\xi^j_t$ and $P^j_t Q^j_t$, we can construct time series $\beta^j_t$ defined by (77).

A.4.3 Exogeneity of the Regressor

By using optimal rules for both labor supply (69) and labor demand (74), we avoid any endogeneity problem. To see it more clearly, when restricting our attention to the optimal labor supply schedule without using firms’ first order conditions, eq. (69) in percentage changes is:

\[
\hat{\beta}_t - \hat{l}_t = \epsilon \left( \hat{w}^j_t - \hat{w}_t \right). \tag{84}
\]

where $\hat{l}_t$ is given by (82). An endogeneity problem may arise because to construct time series of sectoral wages $W^j_t$, we have to divide the labor compensation $W_t L_t$ by sectoral hours worked $L^j_t$; likewise, we have to divide the overall labor compensation $W_t L_t$ by total hours worked $L_t$ to construct time series for the aggregate wage index $W_t$. A way to circumvent any endogeneity problem is to use labor demand $\frac{\xi^j_t P^j_t Q^j_t}{L^j_t} = W^j_t$ to eliminate the sectoral wage from eq. (84), and $W_t = \frac{\sum_j \xi^j_t P^j_t Q^j_t}{L^j_t}$ to eliminate the aggregate wage index; we get $L^j_t / L_t = \left( \frac{\xi^j_t P^j_t Q^j_t}{\sum_j \xi^j_t P^j_t Q^j_t} \right)^{\frac{1}{t}}$. Isolating $L^j_t / L_t$ and differentiating yields (78). Because wages do not show up in eq. (78) as we use the labor income share which is constant over time and gross output (at current prices), we avoid any endogeneity problem. More precisely, the labor’s share in gross output $\xi^j_t$ in sector $j$ is defined as the ratio of the compensation of employees to gross output in the $j$th sector, averaged over the period 1970-2007 so that the explanatory variable (*i.e.*, the RHS term in eq. (83)) is constructed independently from the dependent variable (*i.e.*, the LHS term in eq. (83)).

To check that endogeneity is not a major issue in eq. (83), we test for strict exogeneity of the regressor with respect to the dependent variable. Engle et al. [1983] refer to a variable $x_t$ as strongly exogenous with respect to the variable $y_t$ if $y_t$ does not Granger-cause $x_t$ (see Granger [1969]). Formally, $y_t$ Granger causes $x_t$ if its past value can help to predict the future value of $x_t$ beyond what could have been done with the past value of $x_t$ only. To implement the test of whether $(\hat{\beta}^j_t - \hat{l}_{it})$ (i.e., the LHS term in eq. (83)) Granger-causes $\hat{\beta}^j_{it}$ (i.e., the RHS term in eq. (83)) we run the following regression:

\[
\hat{\beta}^j_{it} = \alpha^j_t + \sum_{k=1}^{p} a^j_{i,t-k} \hat{\beta}^j_{i,t-k} + \sum_{k=1}^{p} b^j_{i,k} (\hat{l}_{i,t-k} - \hat{l}_{i,t-k}) + u^j_{it}, \tag{85}
\]

where $p$ is the autoregressive lag length and $u^j_{it}$ the error term. With respect to (85), in country $i$ and sector $j$, the test of the null hypothesis that $(\hat{\beta}^j_t - \hat{l}_{it})$ does not Granger cause
\[ \hat{\beta}_{it}^{p} \text{ is a } F \text{ test of the form: } H_0 : b_{i1}^{p} = b_{i2}^{p} = \cdots = b_{ip}^{p} = 0. \] By not rejecting the null, one may conclude that the regressor in (83) is strictly exogenous to the dependent variable \((\hat{\ell}_{it}^{p} - \hat{\ell}_{it})\).

Table 10: Granger Causality Test (p-values)

<table>
<thead>
<tr>
<th>Country</th>
<th>Sector</th>
<th>( p = 1 )</th>
<th>( p = 2 )</th>
<th>( p = 3 )</th>
<th>( p = 1 )</th>
<th>( p = 2 )</th>
<th>( p = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>T</td>
<td>0.946</td>
<td>0.835</td>
<td>0.935</td>
<td>GBR</td>
<td>T</td>
<td>0.216</td>
</tr>
<tr>
<td>AUS</td>
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<td>0.215</td>
<td>0.132</td>
<td>0.088</td>
<td>GBR</td>
<td>N</td>
<td>0.087</td>
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<tr>
<td>AUT</td>
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<td>0.665</td>
<td>0.091</td>
<td>IRL</td>
<td>T</td>
<td>0.470</td>
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<tr>
<td>AUT</td>
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<td>0.040</td>
<td>0.014</td>
<td>IRL</td>
<td>N</td>
<td>0.252</td>
</tr>
<tr>
<td>BEL</td>
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<td>0.263</td>
<td>0.934</td>
<td>0.206</td>
<td>ITA</td>
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<td>0.481</td>
</tr>
<tr>
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<td>N</td>
<td>0.362</td>
</tr>
<tr>
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<td>0.006</td>
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<td>T</td>
<td>0.239</td>
</tr>
<tr>
<td>DNK</td>
<td>N</td>
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<td>0.491</td>
<td>0.015</td>
<td>NLD</td>
<td>N</td>
<td>0.285</td>
</tr>
<tr>
<td>ESP</td>
<td>T</td>
<td>0.015</td>
<td>0.024</td>
<td>0.022</td>
<td>NOR</td>
<td>T</td>
<td>0.359</td>
</tr>
<tr>
<td>ESP</td>
<td>N</td>
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<td>0.020</td>
<td>0.021</td>
<td>NOR</td>
<td>N</td>
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<tr>
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<td>0.120</td>
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<td>SWE</td>
<td>T</td>
<td>0.344</td>
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<tr>
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<td>SWE</td>
<td>N</td>
<td>0.133</td>
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<td>0.796</td>
<td>USA</td>
<td>T</td>
<td>0.958</td>
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<tr>
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<td>0.535</td>
<td>0.362</td>
<td>USA</td>
<td>N</td>
<td>0.832</td>
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</tbody>
</table>

Notes: The null hypothesis that \((\hat{\ell}_{it}^{p} - \hat{\ell}_{it})\) does not Granger-cause \((\hat{\beta}_{it}^{p})\) is rejected if p-value \(\leq 0.05\) at a 5% significance level.

The results of causality tests for \(p = 1, 2, 3\) from the change in hours worked in sector \(j\) driven by the pure reallocation of labor across sectors \((\hat{\ell}_{it}^{p} - \hat{\ell}_{it})\) to the fraction of labor’s share of aggregate output accumulating to labor in sector \(j\) \((\hat{\beta}_{it}^{p})\) are displayed in Table 10. The results for \(p = 1\) show that, with the exception of JPN (sector \(T\)) and ESP (both sectors), there is no causality running from \((\hat{\ell}_{it}^{p} - \hat{\ell}_{it})\) to \(\hat{\beta}_{it}^{p}\) at the 5% level of significance. Setting \(p = 2\) and \(p = 3\) leads to similar qualitative results (with the exceptions of the sector \(N\) in AUT for \(p = 2, 3\) and in DNK and ITA for \(p = 3\)). By and large, these results show that one can consider the regressor in eq. (83) as exogenous with respect to the dependent variable.

A.4.4 Panel Data Estimates of \(\epsilon\)

The parameter we are interested in, the degree of substitutability of hours worked across sectors, is given by \(\epsilon_i = \gamma_i/(1 - \gamma_i)\). In the regressions that follow, the coefficient \(\gamma_i\) is alternatively assumed to be identical across countries when estimating for the whole sample \((\hat{\gamma}_i = \hat{\gamma}_i \equiv \gamma \text{ for } i \neq i')\) or to be different across countries when estimating for each economy \((\hat{\gamma}_i \neq \hat{\gamma}_i' \text{ for } i \neq i')\). The sample is running from 1971 to 2007 but we run regression (83) over two sub-periods 1971-1989 and 1990-2007 as well in order to investigate whether our estimates of the degree of labor mobility are relatively stable across sub-periods.

Empirical results reported in Table 11 are consistent with \(\epsilon > 0\). For the whole sample, we find \(\hat{\epsilon} = 0.324\) over the period 1971-2007. Using the fact that \(\hat{\epsilon} = \frac{1}{1 - \hat{\gamma}}\), we find empirically that an increase by 1 percentage point of the labor’s share of aggregate output accumulating to labor in sector \(j\) shifts employment by 0.479 percentage point of total employment toward that sector. When estimating \(\epsilon\) for each economy of our sample over the period 1971-2007, all coefficients are statistically significant, as shown in Table 11, except for DNK and NOR. Excluding these countries, we find that the degree of substitutability of hours worked across sectors ranges from a low of 0.224 for NLD to a high of 1.642 for ESP, with a mean value (across countries) of 0.746. Moreover, the panel data estimations of \(\epsilon\) for the whole sample are quite similar whether the sample is running from 1971 to 2007 or is split into two sub-periods.
Table 11: Panel Data Estimate of $\epsilon$ (eq. (83))

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<td></td>
<td>$\gamma_i$ $\xi_i$ $\hat{\gamma}$ $\hat{\xi}$</td>
<td>$\gamma_i$ $\xi_i$ $\hat{\gamma}$ $\hat{\xi}$</td>
<td>$\gamma_i$ $\xi_i$ $\hat{\gamma}$ $\hat{\xi}$</td>
<td>$\gamma_i$ $\xi_i$ $\hat{\gamma}$ $\hat{\xi}$</td>
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<td></td>
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<td>0.247a 0.327</td>
<td>0.457a 0.841a</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
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<td>0.263a 0.357a</td>
<td>0.225b 0.290</td>
<td>0.457a 0.841a</td>
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</tr>
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<td>CAN</td>
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<td>0.258a 0.348b</td>
<td>0.363a 0.571a</td>
<td>0.457a 0.841a</td>
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<td></td>
</tr>
<tr>
<td>DNK</td>
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<td>0.514a 1.059a</td>
<td>0.457a 0.841a</td>
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<td>FIN</td>
<td>0.352a 0.544a</td>
<td>0.502a 1.007b</td>
<td>0.264b 0.358b</td>
<td>0.457a 0.841a</td>
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</tr>
<tr>
<td>FRA</td>
<td>0.563a 1.287b</td>
<td>0.568a 1.314b</td>
<td>0.556a 1.252c</td>
<td>0.457a 0.841a</td>
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<tr>
<td>GBR</td>
<td>0.502a 1.008a</td>
<td>0.400b 0.667a</td>
<td>0.714a 2.496c</td>
<td>0.457a 0.841a</td>
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</tr>
<tr>
<td>IRL</td>
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<td>0.066 0.073</td>
<td>0.294a 0.477a</td>
<td>0.457a 0.841a</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>ITA</td>
<td>0.407a 0.686a</td>
<td>0.423a 0.734b</td>
<td>0.383a 0.620c</td>
<td>0.457a 0.841a</td>
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<td></td>
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<tr>
<td>JPN</td>
<td>0.498a 0.993a</td>
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<td>0.449b 0.815c</td>
<td>0.457a 0.841a</td>
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<td>NLD</td>
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<td>0.107 0.120</td>
<td>0.354a 0.547c</td>
<td>0.457a 0.841a</td>
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<td></td>
</tr>
<tr>
<td>NOR</td>
<td>0.088 0.087</td>
<td>0.170b 0.217b</td>
<td>0.407a 0.513a</td>
<td>0.457a 0.841a</td>
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</tr>
<tr>
<td>SWE</td>
<td>0.307a 0.443a</td>
<td>0.280a 0.388a</td>
<td>0.339a 0.513a</td>
<td>0.457a 0.841a</td>
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<td></td>
</tr>
<tr>
<td>USA</td>
<td>0.381a 1.387b</td>
<td>0.376a 1.371b</td>
<td>0.588a 1.430</td>
<td>0.457a 0.841a</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Whole Sample | 0.324a (17.99) | 0.479a (12.06) | 0.332a (11.27) | 0.496a (8.87) | 0.314a (11.60) | 0.457a (7.97) |

R-squared | 0.276 | 0.226 | 0.307 | 0.238 | 0.272 | 0.208 |
Observations | 1178 | 1178 | 602 | 602 | 576 | 576 |
Countries | 16 | 16 | 16 | 16 | 16 | 16 |
Sectors | 2 | 2 | 2 | 2 | 2 | 2 |
Country fixed effects | yes | yes | yes | yes | yes | yes |
Time dummies | yes | yes | yes | yes | yes | yes |
Time trend | no | no | no | no | no | no |

Notes: a, b, and c denote significance at 1%, 5% and 10% levels; t-statistics are reported in parentheses.
B More VAR Results and Robustness Check

In this section, we provide more VAR results and conduct a robustness check. In particular, for reason of space, in the main text, we report results of selected variables. Subsection B.1 below reports results for all variables and all VAR models. Due to data availability, we use annual data for eleven 1-digit ISIC-rev.3 industries that we classify as tradables or non tradables. Because at this level of disaggregation, the classification is somewhat ambiguous as some sub-industries could be classified as tradables while other sub-industries are treated as non tradables, subsection B.2 investigates the sensitivity of our empirical results to the classification of industries as tradables or non tradables. In subsection B.3, we estimate the same VAR models as in the main text and investigate empirically the effects of government spending shocks on the business sector by excluding the public sector from aggregate and sectoral variables. Finally, in subsection B.4, since we are constrained to employ annual data as we wish to estimate the sectoral effects of a government spending shock, we investigate the extent to which our empirical results could be altered by our assumption that government spending is predetermined within the year.

B.1 Additional VAR Evidence for the Whole and the Split-Sample Analysis

In section 2, we present VAR evidence on the fiscal transmission. For reason of space and clarity purposes, when we consider the second, third and fourth VAR model that we estimate for the whole sample, we do not show the responses of government spending and the responses of sectoral real consumption wages. Figure 11 report the responses of government spending along with the adjustment of real consumption wages for the second VAR specification. Results are almost identical for the third VAR specification. Panels A and B of Table 1 report the endogenous cumulative response of government spending for the 'labor market' (i.e., \( z_{it}^W = [g_{it}, l_{it}^T - l_{it}^N, \omega_{it}] \)) and the 'product market' specifications (i.e, \( z_{it}^P = [g_{it}, y_{it}^T - y_{it}^N, p_{it}] \)). Contrasting the endogenous cumulative response of government consumption displayed in column 1 of Table 1 with that reported in column 1 of Table 13, we can see immediately that the difference is very small while it shows somewhat higher degree of persistence in the latter case, so that the cumulative response is merely higher. In section 2.4, we split the sample into two sub-samples: a sample of 'low mobility' economies and a sample of 'high mobility' economies. For each country in our sample, we estimate the elasticity of labor supply across sectors, denoted by \( \epsilon \), that captures the extent of workers’ mobility costs across sectors: as \( \epsilon \) takes higher values, workers support relatively less mobility costs and thus are more willing to shift their hours worked from one sector to another. The 'low mobility' economies comprise Australia, Austria, Belgium, Canada, Denmark, Finland, Italy, Ireland, Netherlands, Norway, Sweden, while 'high mobility' economies consist of France, Japan, Spain, United-Kingdom, United States.

In order to give some support for our measure of workers’ mobility cost, we compute an intersectoral labor reallocation index for each country \( i \), which we denote by \( LR_{it}(t) \); we expect the labor reallocation index to increase less in countries where the elasticity of labor supply across sectors \( \epsilon \) takes lower values. To estimate the labor reallocation effect of a government spending shock, we replace the (log) ratio of hours worked in the traded sector to hours worked in the non traded sector, i.e., \( l_{it}^T - l_{it}^N \), with the labor reallocation index \( LR_{it}(2) \), in the 'labor market' specification; this index measures the fraction of workers that shift from one sector to another between year \( t \) and year \( t - 2 \). Our vector of endogenous variables for the 'labor reallocation' specification is thus given by: \( z_{it}^W = [g_{it}, LR_{it}(2), \omega_{it}] \). In Table 1, we do not show the cumulative responses for neither government spending nor the relative wage to an exogenous fiscal shock by 1 percentage point of GDP. Panel C of Table 13 shows cumulates responses for these two variables and the labor reallocation index as well for selected horizons, i.e., at a first-, two-, four-year horizon. First, for the whole sample, we find that a government spending shock increases the labor reallocation across sectors above trend. As emphasized in the main text, contrasting the cumulative responses reported in columns 2 and 3 of Table 13, we find that countries with a smaller elasticity of labor supply across sectors experiences a lower increase in the fraction of workers that shift
Table 12: Cumulative Responses to Spending Shock

<table>
<thead>
<tr>
<th>Variables</th>
<th>Horizon</th>
<th>All sample</th>
<th>Low Mobility</th>
<th>High Mobility</th>
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<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
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<tr>
<td><strong>A. Labor Market</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Gov. spending</td>
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<td>1.000*</td>
<td>1.000*</td>
<td>1.000*</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.190*</td>
<td>2.214*</td>
<td>2.213*</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.294*</td>
<td>4.439*</td>
<td>3.874*</td>
</tr>
<tr>
<td>Relative Labor</td>
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<td>−0.705†</td>
<td>−0.362†</td>
<td>−1.657†</td>
</tr>
<tr>
<td>( (L^T/L^N) )</td>
<td>2</td>
<td>−2.007†</td>
<td>−1.366†</td>
<td>−3.719†</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>−4.968†</td>
<td>−4.141†</td>
<td>−6.835†</td>
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<td>Relative Wage</td>
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<tr>
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<td>2.500†</td>
<td>3.311†</td>
<td>−0.087</td>
</tr>
<tr>
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<td>4</td>
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<td>7.483†</td>
<td>−1.785</td>
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<td><strong>B. Product Market</strong></td>
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</tr>
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<td>1.000*</td>
<td>1.000*</td>
<td>1.000*</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.186*</td>
<td>2.201*</td>
<td>2.2113*</td>
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<td></td>
<td>4</td>
<td>4.195*</td>
<td>4.284*</td>
<td>3.939*</td>
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<td>Relative Output</td>
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<td>−1.025†</td>
<td>−0.674†</td>
<td>−1.936†</td>
</tr>
<tr>
<td>( (Y^T/Y^N) )</td>
<td>2</td>
<td>−2.240†</td>
<td>−1.764†</td>
<td>−3.405†</td>
</tr>
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<td></td>
<td>4</td>
<td>−4.547†</td>
<td>−4.293†</td>
<td>−5.389†</td>
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<td>4</td>
<td>7.984†</td>
<td>8.340†</td>
<td>4.023</td>
</tr>
<tr>
<td><strong>C. Labor Reallocation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gov. spending</td>
<td>1</td>
<td>1.000*</td>
<td>1.000*</td>
<td>1.000*</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.207†</td>
<td>2.199†</td>
<td>2.198†</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.337†</td>
<td>4.304†</td>
<td>4.417†</td>
</tr>
<tr>
<td>Mobility Indicator</td>
<td>1</td>
<td>0.304†</td>
<td>0.163†</td>
<td>0.851†</td>
</tr>
<tr>
<td>( (LR) )</td>
<td>2</td>
<td>0.754†</td>
<td>0.482†</td>
<td>1.772†</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.110†</td>
<td>0.824†</td>
<td>2.191†</td>
</tr>
<tr>
<td>Relative Wage</td>
<td>1</td>
<td>0.939†</td>
<td>1.320†</td>
<td>−0.687</td>
</tr>
<tr>
<td>( (W^N/W^T) )</td>
<td>2</td>
<td>2.667†</td>
<td>3.603†</td>
<td>−1.307</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5.222†</td>
<td>7.683†</td>
<td>−5.248</td>
</tr>
</tbody>
</table>

Notes: Horizon measured in year units. * denote significance at 10% level. Standard errors are bootstrapped with 10000 replications.

For reasons of space, figures in section 2 restrict attention to the responses of selected variables which are included in the VAR models. In this section, we document the effects of an exogenous fiscal shock on all variables which are included in the four specifications of VAR models:

- \( z_{it} = [g_{it}, y_{it}, l_{it}, j_{c, it}, w_{C, it}] \) and \( z_{it} = [g_{it}, y_{it}, l_{it}, c_{it}, w_{C, it}] \) (see Fig. 10, columns 1 and 2 resp.);
- \( z_{it}^T = [g_{it}, y_{it}^T, t_{it}^T, w_{C, it}^T] \), \( z_{it}^N = [g_{it}, y_{it}^N, t_{it}^N, w_{C, it}^N] \), \( z_{it}^{S,T} = [g_{it}, \nu_{it}^{Y,T}, \nu_{it}^{L,T}, w_{C, it}^T] \), and \( z_{it}^{S,N} = [g_{it}, \nu_{it}^{Y,N}, \nu_{it}^{L,N}, w_{C, it}^N] \) (see Fig. 12, columns 1, 2, 3 and 4 resp.);
- \( z_{it}^W = [g_{it}, t_{it}^T - t_{it}^N, \omega_{it}] \) and \( z_{it}^P = [g_{it}, y_{it}^T - y_{it}^N, p_{it}] \) (see Fig. 13, columns 1 and 2 resp.);
- \( z_{it}^{W} = [g_{it}, t_{it}^T - t_{it}^N, \omega_{it}] \) and \( z_{it}^{W} = [g_{it}, LR_{it}(2), \omega_{it}] \) for the 'high' and 'low mobility' sub-samples (see Fig. 14, columns 1 and 2 resp.).
Figure 10: Effects of Unanticipated Government Spending Shock on Aggregate Variables. Notes: Exogenous increase of government consumption by 1% of GDP. Aggregate variables include GDP (constant prices), total hours worked, private fixed investment, the current account and the real consumption wage. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in output units (government spending, GDP, investment, current account), percentage deviation from trend in labor units (total hours worked), percentage deviations from trend (real consumption wage). Results for baseline specification are displayed by solid lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling; sample: 16 OECD countries, 1970-2007, annual data.
Figure 11: Effects of Unanticipated Government Spending Shock on Sectoral Variables. Notes: Exogenous increase of government consumption by 1% of GDP. Sectoral variables include sectoral valued added at constant prices, sectoral hours worked, and real consumption sectoral wages. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in output units (sectoral output), percentage deviation from trend in labor units (sectoral labor), percentage deviation from trend (real consumption sectoral wages). Results for baseline specification are displayed by solid lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling; sample: 16 OECD countries, 1970-2007, annual data.
Figure 12: Effects of Unanticipated Government Spending Shock on Sectoral Variables.

Notes: Exogenous increase of government consumption by 1% of GDP. Sectoral variables include sectoral valued added at constant prices, sectoral hours worked, sectoral labor and sectoral output shares, and real consumption sectoral wages. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in output units (sectoral output, sectoral output share), percentage deviation from trend in labor units (sectoral labor, sectoral labor share), percentage deviations from trend (real consumption sectoral wages). Results for baseline specification are displayed by solid lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling; sample: 16 OECD countries, 1970-2007, annual data.
Figure 13: Effects of Unanticipated Government Spending Shock on Relative Price and Relative Wage. Notes: Exogenous increase of government consumption by 1% of GDP. Sectoral variables include hours worked of tradables in terms of non tradables, the relative wage, output of tradables in terms of non tradables, the relative price of non tradables. Horizontal axes indicate years. Vertical axes measure deviations from trend (ratio of traded value added to non traded value added, ratio of hours worked of tradables to hours worked of non tradables), and percentage deviations from trend (relative price, relative wage). Results for baseline specification are displayed by solid lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling; sample: 16 OECD countries, 1970-2007, annual data.
Figure 14: Effects of Unanticipated Government Spending Shock on Labor Reallocation across Sectors. Notes: Exogenous increase of government consumption by 1% of GDP. Sectoral variables include hours worked of tradables in terms of non tradables, the relative wage, the intersectoral labor reallocation index. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in labor units (intersectoral labor reallocation index), deviations from trend (ratio of hours worked of tradables to hours worked of non tradables), and percentage deviations from trend (relative wage). Panels report cumulative responses for the ‘high mobility’ and the ‘low mobility’countries’ group in the black solid line and the blue dashed line, respectively, with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling; sample: 16 OECD countries, 1970-2007, annual data.
B.2 Robustness Check: Sectoral Classification

This section explores the robustness of our findings to the classification of the eleven 1-digit ISIC-rev.3 industries as tradables or non tradables. When we conduct the robustness analysis, we modify the baseline classification in a number of ways to ensure that some industries with specific characteristics are not driving the results. In particular, the classification of items "Wholesale and Retail Trade", "Hotels and Restaurants", "Transport, Storage and Communication", "Financial Intermediation" and "Real Estate, Renting and Business Services" may display some ambiguity. In order to address this issue, we re-estimate the various VAR specifications for different classifications in which one of the five above industries initially marked as tradable (non tradable resp.) is classified as non tradable (tradable resp.), all other industries staying in their original sector. In doing so, the classification of only one industry is altered, allowing us to see if the results are sensitive to the inclusion of a particular industry in the traded or the non traded sector. The baseline and the five alternative classifications considered in this exercise are shown in Table 13.

Table 13: Robustness check: Classification of Industries as Tradables or Non Tradables

<table>
<thead>
<tr>
<th>KLEMS code</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, Hunting, Forestry and Fishing</td>
<td>AtB</td>
</tr>
<tr>
<td>Mining and Quarrying</td>
<td>C</td>
</tr>
<tr>
<td>Total Manufacturing</td>
<td>D</td>
</tr>
<tr>
<td>Electricity, Gas and Water Supply</td>
<td>E</td>
</tr>
<tr>
<td>Construction</td>
<td>F</td>
</tr>
<tr>
<td>Wholesale and Retail Trade</td>
<td>G</td>
</tr>
<tr>
<td>Hotels and Restaurants</td>
<td>H</td>
</tr>
<tr>
<td>Transport, Storage and Communication</td>
<td>I</td>
</tr>
<tr>
<td>Financial Intermediation</td>
<td>J</td>
</tr>
<tr>
<td>Real Estate, Renting and Business Services</td>
<td>K</td>
</tr>
<tr>
<td>Community Social and Personal Services</td>
<td>LiQ</td>
</tr>
</tbody>
</table>

Notes: T stands for the Traded sector and N for the Non traded sector.

Figures 15 and 16 report the responses of variables of interest to an exogenous increase in government spending by one percent of GDP. The solid blue line shows results for the baseline classification while the responses for alternative classifications are shown in the five colored lines. The last line of Table 13 provides the matching between the color line and the classification between tradables and non tradables. In each panel, the shaded area corresponds to the 90% confidence bounds. Figure 15 contrasts the responses of sectoral output \( (Y^T) \), sectoral labor \( (L^T) \), sectoral output shares \( (Y^j/Y) \), sectoral labor shares \( (L^j/L) \), real consumption sectoral wages \( (W^j/P_C) \) for the baseline classification with those obtained for alternative classifications of industries as tradables or non tradables. Alternative responses are fairly close to those for the baseline classification as they lie within the confidence interval (for the baseline classification) for almost all the selected horizons (8 years). Figure 16 reports the effects of an exogenous increase in government consumption by 1% of GDP on the ratio of sectoral output \( (Y^T/Y^N) \), sectoral labor \( (L^T/L^N) \), the intersectoral labor reallocation index \( (LR) \), the relative price \( (P) \) and the relative wage \( (\Omega) \). For \( LR, P \) and \( \Omega \), the responses are remarkably similar across the baseline and alternative classifications. While the pattern of the dynamic adjustment of \( Y^T/Y^N \) is similar across all classifications, the decline in output of tradables relative to non tradables is more pronounced when the industry "Wholesale and Retail Trade" is treated as tradables (classification #1). We can also notice some differences in responses of \( L^T/L^N \) across the baseline and the five alternative classifications. For specifications #1 and #5, the response of \( L^T/L^N \) does not lie within the confidence interval of the baseline. Yet, across all classifications, \( L^T/L^N \) declines significantly on impact, and stay below trend for a number of periods. By and large, our main

\[60\] We do not report the responses for aggregate variables included since these variables, by construction, are unsensitive to the definition of traded and non traded sectors.
findings hold and remain unsensitive to the classification of one specific industry as tradable or non tradable; in sum, the specific treatment of "Wholesale and Retail Trade", "Hotels and Restaurants", "Transport, Storage and Communication", "Financial Intermediation" and "Real Estate, Renting and Business Services" does not drive the results.
Figure 15: Sensitivity of the Effects of Unanticipated Government Spending Shock on Sectoral Variables to the Classification of Industries as Tradable or Non Tradable. Notes: The blue line shows results for the baseline classification. The red line and the yellow line show results when ‘Whole and retail traded’ and ‘Hotels and restaurants’ are treated as tradables, respectively. The green line and the black line show results when ‘Transport, storage and communication’ and ‘Financial intermediation’ are classified as tradables, respectively. The cyan line reports results when ‘Real Estate, renting and business services’ is treated as tradables.
Figure 16: Sensitivity of the Effects of Unanticipated Government Spending Shock on Relative Price and Relative Wage to the Classification of Industries as Tradable or Non Tradable. Notes: The blue line shows results for the baseline specification. The red line and the yellow line show results when 'Whole and retail traded' and 'Hotels and restaurants' are treated as tradables, respectively. The green line and the black line show results when 'Transport, storage and communication' and 'Financial intermediation' are classified as tradables, respectively. The cyan line reports results when 'Real Estate, renting and business services' is treated as tradables.
B.3 Robustness Check: Excluding the Public Sector

As an additional robustness check, we exclude the industry “Community Social and Personal Services” from the non tradable industries’ set. This robustness analysis is based on the presumption that among the eleven industries provided by the EU KLEMS and STAN databases, this industry is government-dominated. While this exercise is interesting on its own as it allows us to explore the size of the impact of a government spending shock on the business sector, we also purge for the potential and automatic link between non traded output and public spending because government purchases (to the extent that the government is the primary purchaser of goods from this industry) account for a significant part of non traded value added.

Figures 17-19 report the effects of an exogenous increase in government consumption by 1% of GDP for the whole economy (baseline) together with the responses on the business sector (i.e., the public sector is excluded). In each case, the blue line reports the point estimate for the whole economy (with its 90% confidence interval) while the black line shows the point estimate for the business sector (i.e., the industry “Community Social and Personal Services” is excluded). Figure 17 shows the results of a rise in government consumption on GDP, hours worked, investment, the current account and the real consumption wage. We can notice that the dynamic adjustment of hours worked to an exogenous increase in government consumption is somewhat sensitive to the exclusion of the public sector. More precisely, when the public sector is excluded, hours worked increase less in Fig. 17(e) or even may decline on impact as displayed in Fig. 17(f). Whether we consider the whole economy or the business sector, the dynamic adjustment of hours worked displays a similar pattern whether “Community Social and Personal Services” is included or omitted: hours worked decline gradually before starting to recover after 5 years.

Figure 18 shows the results of a rise in government consumption by 1% of GDP on sectoral quantities, on sectoral labor and sectoral output shares, along with real consumption sectoral wages. In each panel, the blue solid line shows the results for the whole economy while the black solid line reports responses for the business sector. When excluding the public sector, we can notice that the contraction in hours worked and output of tradables is somewhat mitigated while the expansionary effect on non tradables is moderated. By and large, the shape of the dynamic adjustment of sectoral variables are similar and mostly lie within the confidence bounds of the baseline specification (i.e., for the whole economy). The third and fourth columns report the dynamic adjustment of output and labor shares of tradables and non tradables. As for sectoral output and sectoral labor in levels, the responses of sectoral output relative to GDP (in real terms) are mitigated when excluding the public sector. However, the conclusions we reach in the main text remain valid. In all instances, whether we use labor or output, the share of tradables falls while the share of non tradables rises. It is worthwhile mentioning that the differences in quantitative adjustments for output shares can be mostly attributed to the modifications of the initial share of each sector in terms of labor or total output. Technically, the responses of sectoral shares are measured as percentage deviation from trend in total output units for sectoral output shares or alternatively as percentage deviation from trend in total hours worked units for sectoral labor shares. Thus, percentage deviations from trend are multiplied by the corresponding share of sector $j$ in the whole economy (for the baseline scenario) or alternatively in the business sector (for the alternative scenario where the industry “Community Social and Personal Services” is excluded). Since the initial share of non tradables is reduced when “Community Social and Personal Services” is excluded, the magnitude of the responses of labor and output share of non tradables are mitigated as well. In the baseline, non traded output and traded output as a share of GDP are 0.60 and 0.40 respectively, while in the alternative scenario where “Community Social and Personal Services” is excluded, the corresponding shares are 0.30 and 0.70. Results without these corrections (not shown) reveal that the differences in the responses of $Y^T/Y$ and $Y^N/Y$ across the two scenarios turn out to be substantially smaller. In the light of this result, it is unlikely that the omitted...
industry plays a major role in explaining the responses of the output shares of tradables and non tradables to an increase in government spending.

Finally, Figure 19 compares the responses with and without the industry "Community Social and Personal Services" for the ratio of sectoral quantities (i.e., $Y_T/Y_N$, $L_T/L_N$), labor reallocation ($LR$), the relative price ($P$) and the relative wage ($\Omega$). When excluding "Community Social and Personal Services", we find that the positive responses of the relative price of non tradables and the relative wage to an exogenous increase in government consumption are more pronounced and display more persistence over time. Because prices and wages are not really determined by the interplay of supply and demand in the public sector, it is not surprising that excluding this sector tends to magnify the responses of the relative wage and the relative price to a fiscal shock. While the ratio of hours worked of tradables relative to non tradables displays a similar magnitude, we may notice that the shift in the ratio of sectoral output is much more pronounced on impact and along the adjustment when we exclude "Community Social and Personal Services". As mentioned above, this behavior is mostly driven by the initial share of tradables which increases sharply. Otherwise, the difference between the two instances would have been much smaller.

To conclude, the results presented in Figures 17-19 show that the conclusions which are drawn in the main text on the basis of the responses to an exogenous fiscal shock when we consider the whole economy remain valid when the industry "Community Social and Personal Services" is excluded.

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62Recall that the percentage deviation from trend of $Y^T/Y^N$ is multiplied by $\frac{P_TX_T}{P_NX_N}$ in order to express the result in percentage points and in the same units. For the baseline scenario (whole economy), the ratio $Y^T/Y^N$ averages to 0.68. When "Community Social and Personal Services" is excluded, the ratio goes up to 2.40.
Figure 17: Sensitivity of the Effects of Unanticipated Government Spending Shock on Aggregate Variables to Exclusion of the Public Sector. Notes: Impulse response functions to an exogenous increase in real government spending by one percent of GDP. Blue line: all sectors; shaded areas: 90 percent confidence intervals; black line: without "Community Social and Personal Services".
Figure 18: Sensitivity of the Effects of Unanticipated Government Spending Shock on Sectoral Variables to Exclusion of the Public Sector. Notes: Impulse response functions to an exogenous increase in real government spending by one percent of GDP. Blue line: all sectors; shaded areas: 90 percent confidence intervals; black line: without "Community Social and Personal Services".
Figure 19: Sensitivity of the Effects of Unanticipated Government Spending Shock on Relative Price and Relative Wage to Exclusion of the Public Sector. Notes: Impulse response functions to an exogenous increase in real government spending by one percent of GDP. Blue line: all sectors; shaded areas: 90 percent confidence intervals; black line: without "Community Social and Personal Services".
B.4 Robustness Check: Identifying Assumption of Government Spending Shocks

Like earlier studies, we adopt the identifying assumption of government spending shocks proposed by Blanchard and Perotti [2002] who assume that there is no contemporaneous response of government spending to macroeconomic aggregates, i.e. that government spending is predetermined. As summarized by Born and Müller [2012]: 'This requires that government spending does i) neither respond automatically to the economy, ii) nor that it is adjusted in a discretionary manner within the period. The first requirement is likely to be satisfied if government spending does not include transfers, but only government consumption and investment (a commonly used definition of government spending). Whether the second requirement is satisfied depends on the extent of decision lags in the policy process and thus on the data frequency'. While the identifying assumption is expected to hold for quarterly data, its fulfilment is less compelling when imposed at annual frequency. Recently, Beetsma, Giuliodori and Klaassen [2008] and Born and Muller [2012] provide evidence that imposing a zero within-year response of government spending to output to identify an annual SVAR is a reasonable identifying restriction for a panel of seven OECD countries and the US, respectively. While these conclusions are reassuring, we provide below additional support for our identifying assumption in annual data. We thus ask whether the assumption that government spending is predetermined within the year by using the largest available subset of the countries in our dataset for which we have sufficient quarterly data. For this purpose, we compare the annualized impulse responses from the quarterly VAR model in panel format with those obtained from a VAR model estimated in panel format on annual data. Because sectoral data are only available at an annual frequency, we restrict the exercise to the VAR models including aggregate variables such as government spending, aggregate GDP, total hours worked, investment, the current account and the real consumption wage. We proceed below in two stages. First, we briefly discuss our data. Second, we compare results obtained on the basis of annual with those obtained with quarterly data.

Data are taken from the OECD Economic Outlook database. The country sample is Australia (AUS), Austria (AUT), Canada (CAN), France (FRA), Italy (ITA), Japan (JPN), the Netherlands (NLD), Sweden (SWE), the United Kingdom (GBR), and the United States (USA), for which quarterly and annual macroeconomic data are available. Given OECD quarterly statistics data, the country and period coverage (identical for the quarterly and annual data sets) is: AUS (1979Q1-2007Q4), AUT (1990Q1-2007Q4), CAN (1981Q1-2007Q4), FRA (1973Q1-2007Q4), ITA (1970Q1-2007Q4), JPN (1970Q1-2007Q4), NLD (1970Q1-2007Q4), SWE (1975Q1-2007Q4), GBR (1972Q1-2007Q4) and USA (1970Q1-2007Q4). Sources and data construction at a quarterly frequency are as follows:

- **Government spending**: real government final consumption expenditure (CGV). Source: OECD Economic Outlook Database.
- **Gross domestic product**: real gross domestic product (GDPV). Source: OECD Economic Outlook Database.
- **Labor**: hours worked per employee, total economy. Source: OECD Economic Outlook Database.
- **Private fixed investment**: real private non-residential gross fixed capital formation (IBV). Source: OECD Economic Outlook Database.
- **Current account**: current account balance (in % of GDP). Source: OECD Economic Outlook Database.
- **Real Consumption wage**: nominal wage rate (total economy) divided by the consumer price index (CPI). Sources: OECD Economic Outlook Database for the nominal wage and OECD Prices and Purchasing Power Parities for the consumer price index.
All data are seasonally adjusted and divided by the population, except for the current account balance and the real consumption wage. We consider per capita variables, and thus divide quantities by the working age population (15-64 years old) provided by OECD Economic Outlook Database (data for the population at quarterly frequency were interpolated from annual data). For government spending, GDP and investment, we directly use the volumes as reported by the OECD (the series are deflated with their own deflators).

In estimating the VAR models on quarterly data, we allow for four lags while the number of lags is set to two when data are at an annual frequency. In order to investigate consistently whether the assumption that government spending is predetermined within the year, we impose the restriction that government spending is predetermined with the year (the quarter) to identify government spending shocks when the model is estimated in panel format on annual (quarterly) data. Figure 20 reports the responses for the variables of interest from the VAR model estimated on annual data shown in the blue solid line and on quarterly data shown in the black solid line. The blue and the black solid lines display the point estimate with shaded areas indicating 90% confidence bounds obtained when the VAR model is estimated on annual data. We take the panel VAR model on annual data as the baseline model. For purposes of comparability, we annualize the responses of the quarterly baseline models. While some differences can be observed, the annualized responses obtained from the quarterly model are fairly close to those obtained from the baseline model as responses lie within the confidence interval of the baseline model for almost all time horizons. It is worthwhile mentioning that following an exogenous increase in government consumption, total hours worked displays much more persistence when the panel VAR model is estimated on quarterly data than on annual data. Note that hours worked revert to its initial level after several decades. We can notice that investment and the current account do not respond to the fiscal shock on impact with quarterly data while they both gradually decline and stay below trend for several years. While the responses somewhat display some minor quantitative differences, the panel VAR evidence is similar whether we assume that government spending does not respond to the other variables included in the VAR model within the year or alternatively within the quarter. In sum, we can conclude that the assumption according to which the fiscal shock is exogenous within the year is not as restrictive as one might think.
Figure 20: Impulse Response Functions from the Panel VAR Model on Annual Data vs. Quarterly Data. Notes: Exogenous increase of government consumption by 1% of GDP. Aggregate variables include GDP (constant prices), total hours worked, private fixed investment, the current account and the real consumption wage. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in output units (government spending, GDP, investment, current account), percentage deviation from trend in labor units (total hours worked), percentage deviations from trend (real consumption wage). Results for baseline specification are displayed by blue lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling; sample: 16 OECD countries, 1970-2007, annual data. Blue line: response from the panel VAR model on annual data; black line: annualized impulse responses from the panel VAR model on quarterly data.
C Condition for the Government Spending Shock to be Biased toward Non Tradables

In this section, we provide more details about the interpretation of our empirical results.

C.1 Standard Definition of Government Spending Shocks Biased toward Non Tradables

In subsection 2.3, we interpret the rise in non traded output relative to traded output that we document empirically as the result of government spending shocks biased toward non tradables. In this subsection, we provide more details about our interpretation and the assumptions underlying this conjecture. The traded good is the numeraire and its price, $P^T$, is normalized to 1.

To begin with, we write down the market clearing conditions:

\[ Y^N(t) = C^N(t) + J^N(t) + G^N(t), \]  
\[ Y^T(t) = C^T(t) + J^T(t) + G^T(t) + NX(t), \]  

where $Y^j$ is value added at constant prices in sector $j = N, T$, $C^j$ and $G^j$ stand for private and public consumption of good $j = N, T$, respectively, $J^j$ corresponds to private investment in sector $j = N, T$, and $NX$ is net exports. Summing value added at constant prices across sectors gives real GDP which we denote by $Y_R$:

\[ Y_R(t) = Y^T(t) + PY^N(t), \]

where $P$ is the price of non traded goods in terms of traded goods that is kept fixed in order to evaluate non traded value added at constant prices. Note that at the initial steady-state, real GDP and nominal GDP coincide, i.e., $Y_R = Y$.

Government spending, $G$, is the sum of public purchases on non traded goods, $G^N$, and traded goods, $G^T$:

\[ G(t) = PG^N(t) + G^T(t). \]

We denote by $\omega_{GN} = \frac{PG^N}{G}$ and $\omega_{GT} = \frac{G^T}{G} = 1 - \omega_{GN}$ the non tradable and tradable content of government spending. Assuming that the share $\omega_{Gj}$ is constant over time, eq. (88) can be rewritten as follows:

\[ G(t) = \omega_{GN}G(t) + \omega_{GT}G(t), \]

with $\omega_{GN} + \omega_{GT} = 1$.

Below, we note by a hat the deviation of variable $X(t)$ relative to its initial level, $X$, in percentage:

\[ \hat{X}(t) = \frac{X(t) - X}{X}. \]

Totally differentiating (90) and dividing by initial GDP, a rise in government spending is split into non tradables and tradables in accordance with their respective shares:

\[ \frac{dG(t)}{Y} = \omega_{GN} \frac{dG(t)}{Y} + \omega_{GT} \frac{dG(t)}{Y}. \]

A government spending shock is said to be biased toward non tradables if

\[ \omega_{GN} > \omega_{GT}. \]

As will be useful later, we totally differentiate (88) and divide the resulting expression by initial GDP:

\[ \frac{dG(t)}{Y} = \omega_G \omega_{GN} \hat{G}^N(t) + \omega_G \omega_{GT} \hat{G}^T(t). \]

where $\omega_G = \frac{G}{Y}$ stands for government consumption-to-GDP ratio. Combining eq. (91) and (93), we thus have:

\[ \omega_G \omega_{Gj} \hat{G}^j(t) = \omega_{Gj} \frac{dG(t)}{Y}, \quad j = N, T. \]
Before investigating the impact of higher government spending on sectoral value added, it is convenient to denote by $\nu^N_t = \frac{\dot{Y}^N_t}{Y}$ the share of non tradables in GDP and $\nu^{Y:T}_t = \frac{\dot{Y}^T_t}{Y}$ the share of tradables in GDP. Keeping private consumption, $C^j$, private investment, $J^j$, and net exports, $NX$, constant, totally differentiating market clearing conditions for non tradables and tradables described by (86a) and (86b), respectively, leads to:

$$\nu^{Y:N} \dot{Y}^N(t) = \omega_G \omega_N G^N(t), \quad (95a)$$

$$\nu^{Y:T} \dot{Y}^T(t) = \omega_G \omega_T G^T(t), \quad (95b)$$

where $\nu^{Y:T} = 1 - \nu^{Y:N}$. The LHS of eqs. (95), $\nu^{Y:j} \dot{Y}^j(t)$, corresponds to the deviation of value added in sector $j$ relative to its initial steady-state value in percentage of initial GDP. Subtracting (95b) from (95a) allows us to relate the change in output of non tradables relative to tradables, both expressed in percentage points of GDP, to changes in sectoral government consumption:

$$\nu^{Y:N} \dot{Y}^N(t) - \nu^{Y:T} \dot{Y}^T(t) = \omega_G \omega_N G^N(t) - \omega_G \omega_T G^T(t),$$

$$= (\omega_G^N - \omega_G^T) \frac{dG(t)}{Y}, \quad (96)$$

where use has been made of (94) to obtain the second line of eq. (96). In accordance with the definition (92), eq. (96) implies that non traded output increases relative to traded output when government spending is biased toward non tradables, i.e.,

$$\nu^{Y:N} \dot{Y}^N(t) - \nu^{Y:T} \dot{Y}^T(t) > 0, \quad \text{if} \quad \omega_G^N > \omega_G^T. \quad (97)$$

### C.2 Stricter Definition of Government Spending Shocks Biased toward Non Tradables

In subsection 2.3 we document a second empirical fact which reveals that the share of non tradables in GDP (in real terms) increases while the share of tradables in GDP (in real terms) falls following a rise in government consumption. In this subsection, we relate the responses of sectoral output shares to changes in sectoral government spending.

To begin with, summing value added at constant prices across sectors, i.e., (86a) and (86a), and using the definition (87) leads to the standard accounting identity according to which GDP is equal to final expenditure:

$$Y_R(t) = P_C C(t) + P_J J(t) + G(t) + NX(t), \quad (98)$$

where $P_C$ and $P_J$ are the consumption and investment price index, respectively, $C$ and $J$ stand for private consumption and private investment in volume.

Keeping consumption and investment expenditure along with net exports fixed, totally differentiating (98) leads to:

$$\dot{Y}_R(t) = \omega_G \dot{G}(t) = \frac{dG(t)}{Y}. \quad (99)$$

Totally differentiating (87) leads to:

$$\dot{Y}_R(t) = \nu^N \dot{Y}^N(t) + \nu^T \dot{Y}^T(t), \quad (100)$$

where $\nu^{Y:j} = \frac{\dot{Y}^j_j}{Y}$, remembering that $P^T = 1$. Subtracting $\dot{Y}_R(t)$ from both sides of eq. (100) and using the fact that $\nu^{Y:T} = 1 - \nu^N$ leads to:

$$\nu^{Y:N} \left( \dot{Y}^N(t) - \dot{Y}_R(t) \right) + \nu^{Y:T} \left( \dot{Y}^T(t) - \dot{Y}_R(t) \right) = 0. \quad (101)$$

The term $\nu^{Y:j} \left( \dot{Y}^j_j(t) - \dot{Y}_R(t) \right)$ corresponds to the deviation of sectoral output share in GDP relative to its initial value:

$$\nu^{Y:j} \left( \dot{Y}^j_j(t) - \dot{Y}_R(t) \right) = \nu^{Y:j} \nu^j_j(t). \quad (102)$$
To relate the response of the share of non tradables in GDP to changes in sectoral government consumption, we make use of eq. (95a) and eq. (99) which relate the change in output to changes in government spending:

\[
\nu^{Y,N} \left( \hat{Y}^N - \hat{Y}_R \right) = \omega_G \omega_G N G^N (t) - \nu^{Y,N} \omega_G \hat{G}(t),
\]

\[
= \omega_G N \frac{dG(t)}{Y} - \nu^{Y,N} \frac{dG(t)}{Y},
\]

\[
= (\omega_G N - \nu^{Y,N}) \frac{dG(t)}{Y},
\]

(103)

where we use eq. (94) to obtain the second line of eq. (103). According to (103), the share of non tradables in GDP increases following a fiscal shock as long as the fraction of government expenditure spent on non traded goods, \(\omega_G N\), is higher than that the share of non tradables in GDP, \(\nu^N\). Thus, a government spending shock is biased toward non tradables if:

\[
\omega_G N > \nu^{Y,N}.
\]

(104)

Building on (92) and (104), there exists two definitions of government spending biased toward non traded goods. The first definition establishes that government spending is biased toward non traded goods if a larger fraction of public spending is spent on non traded traded goods than on traded goods. Such a definition implies that non traded output increases relative to traded output, as documented in Figure 2(c), as long as \(\omega_G N > \omega_G T\). However, this definition does not take into account that non traded output accounts for a larger fraction of GDP. Thus, for the share of non tradables in GDP to increase, inequality (92) is a necessary but not sufficient condition. For the share of non tradables in GDP to increase, as documented in Figure 2(b), the fraction of government spending spent on non traded goods must exceed the share of non tradables in GDP, in line with the stricter inequality (104).

It is worth noting that for the increase in the GDP share of non tradables to materialize, resources must be reallocated away from the traded sector to the non traded sector. In other words, the fact that government spending is biased toward non tradables in accordance with the stricter definition (104) is sufficient as long as labor and/or capital can shift toward the non traded sector.

So far, we have investigated the response of the share of non tradables in real GDP to a rise in government spending by keeping the private sector’s demand components fixed. We now investigate how much the responses of the private sector’s demand components influence our results. To avoid unnecessary complications, we assume that the elasticity of substitution between tradables and non tradables is equal to one for both consumption and investment. Thus, consumption (investment) in non tradables is a fixed fraction \(\alpha_C (\alpha_J)\) of consumption (investment) expenditure, i.e., \(P_C C (P_J J)\). First, log-linearizing (98) leads to the deviation from initial steady-state in percentage for real GDP, \(Y_R\):

\[
\hat{Y}_R(t) = \omega_C \hat{C}(t) + \omega_J \hat{J}(t) + \frac{dG(t)}{Y} + \frac{dNX(t)}{Y},
\]

(105)

where \(Y\) is initial real GDP. Totally differentiating (86a) leads to the deviation from initial steady-state in percentage for real GDP, \(Y^N\):

\[
\nu^{Y,N} \hat{Y}_N(t) = \alpha_C \omega_C \hat{C}(t) + \alpha_J \omega_J \hat{J}(t) + \omega_G N \frac{dG(t)}{Y}.
\]

(106)

Pre-multiplying (105) by \(\nu^{Y,N}\), the initial response of the share of non tradables in real GDP to a government spending shock is:

\[
\nu^{Y,N} \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) = - (\nu^{Y,N} - \alpha_C) \omega_C \hat{C}(t) - (\nu^{Y,N} - \alpha_J) \omega_J \hat{J}(t)
\]

\[
+ (\omega_G N - \nu^{Y,N}) \frac{dG(0)}{Y} - \nu^{Y,N} \frac{dNX(0)}{Y}.
\]

(107)

Since \(\nu^{Y,N}\) takes values which are close to both \(\alpha_J\) and \(\alpha_C\), the response of the share of non tradables in real GDP is mostly affected by \((\omega_G N - \nu^{Y,N})\) along with the deficit in the
balance of trade on impact. In sum, when taking into account the reaction of the private sector’s demand components, the current account deficit triggered by the rise in government spending tends to reinforce the fact that the government spending shock is biased toward non tradables.

D More VAR Results and Robustness Check

In this section, we provide more details about the empirical and calibration strategy in the main text and we conduct several robustness checks:

- In subsection D.1, we provide evidence on the composition of government consumption and quantify the contribution shocks to government consumption on non traded goods to unanticipated changes in total government consumption. We also detail the source and construction of time series for sectoral government consumption. The conclusion that emerges is that shocks to public purchases of non traded goods account on average for about 90 percent of the forecast error variance of total government spending for horizons of 1 to 8 years for the whole sample and the US as well. This finding thus corroborates our conjecture that government spending shocks are strongly biased toward non traded goods. To calibrate our model, we estimate the first VAR model that includes government final consumption expenditure, real GDP, total hours worked, private investment, the real consumption wage, in order to identify unanticipated government spending shocks. Then, we estimate a VAR model in panel format on annual data that includes unanticipated government spending shocks ordered first, government final consumption expenditure, government consumption on non tradables, and government consumption on tradables. Impulse response functions for the two components of government final consumption expenditure we generate following a rise in government spending by 1% of GDP reveal that government consumption expenditure on non tradables accounts on average for 90% of increases in government consumption. This evidence thus indicates that the sectoral components of government final consumption expenditure increase roughly by the same amount than their share in government spending. Because data for government consumption by function are available over 1970-2007 for the U.S., we also estimate the same VAR models as in the main text and contrast dynamic effects on sectoral variables after an aggregate spending shock with those following a shock to government consumption of non tradables. We find empirically that the dynamic responses are very similar in terms of shape and magnitude.

- In subsection D.2, we conduct an elaborate investigation of the responses of components of government final consumption expenditure. Because a large part of unanticipated changes in government final consumption expenditure are due to changes in public purchases of non tradables, we investigate the responses of its components as well. In the first part, following the existing literature, we consider two components in total government expenditure: the purchase of goods and services from the private sector, and the purchases of goods and services from the government sector which accounts for 60% of government final consumption expenditure. To investigate how these two components react to a government spending shock, we identify the unanticipated shock to total government expenditure and then estimate a VAR model that includes the identified shock ordered first and its two components. Estimated responses of the sub-components of government final consumption expenditure to our identified government spending shock reveal that government purchases of goods and services from the private sector is a major part of the variation in government spending over the first five years. More precisely, while the two components react positively to the fiscal shock, the increase in government purchases from the private sector accounts for 80% of the spending shock on impact. Conversely, the contribution of government purchases of government output to the spending shock increases over time and averages 55%. Then, we identify the shocks to the two components of government spending using Blanchard and Perotti’s [2002] approach and estimate the
sectoral effects of each identified shock. Whether we consider a wage or a non-wage government consumption shock, our main conclusions hold. Both shocks appreciate the relative price of non tradables and reallocate resources toward the non traded sector which increases its relative size. We may nevertheless note some differences quantitatively. The magnitude of the reallocation of labor across sectors along with changes in relative sector size are more pronounced following a wage government consumption shock. When we turn to components of government consumption non non tradables, we find that the identified government spending shock increases substantially individual government final consumption expenditure. The contribution of this component to the increase in government consumption of non tradables averages 77%. When we distinguish between defense and non-defense spending, we find that the former accounts for a small fraction (6% on average) of the increase in government consumption of non tradables.

- In subsection D.3, we contrast our results related to the sectoral effects of a government spending shock when differentiating between a traded and a non traded sector with those documented by earlier empirical studies. In particular, we conduct an elaborate analysis of the causes of the discrepancy in our results and those documented by Benetrix and Lane [2010] related to the response of traded output. Our empirical study reveals that the causes are twofold. First, when we restrict the set of countries to those we have in common with the authors’ sample, we find that traded output increases on impact. Second, our analysis also indicates that when quantities are not scaled by the population, as in Benetrix and Lane [2010], then the rise in traded output becomes more persistent over time. The classification of industries between tradables or non tradables along with the VAR specification does not play any noticeable role in driving the discrepancy.

- In subsection D.4, we contrast our empirical results on aggregate effects of government spending shocks with those documented in the empirical literature.

- In subsection D.5, we conduct an investigation of the potential presence of anticipation effects, using alternative measures of forecasts for government spending. The former measure was provided by Born, Juessen and Müller [2013] and stems from the OECD, while the latter measure is taken from a dataset constructed by Fioramanti et al. [2016] where forecasts are performed by the European Commission. We use two alternative datasets as the former contains observations from 1986 to 2007 for all countries, while the latter provides a longer time horizon for a restricted set of countries. As is common in the literature, we alternatively include a forward-looking variable such as stock prices into the VAR model in order to control for potential fiscal foresight. First, we run Granger-causality tests and do not find that fiscal forecasts have any predictive power for our identified government spending shocks. Second, our main results are not altered by the inclusion of forecasts for government spending. We detect some differences quantitatively, however, when we include the OECD forecast for spending growth in the VAR model. More precisely, when we control for anticipation effects, the responses of sectoral output shares are more pronounced, while the appreciation in the relative price and the relative wage of non tradables are somewhat more muted.

- In subsection D.6, we address a potential concern related to the fact that the government spending shock may display noticeable differences across alternative VAR specifications. Such differences could potentially make the comparison of the effects of a government shock across sectors difficult. Because in the quantitative analysis we base our calibration on one unique government spending shock, such differences could potentially undermine the comparison of theoretical with empirical responses. Before summarizing the main conclusions of robustness exercises, it is worth mentioning that, in line with the current practice, to facilitate the interpretation of our results, we normalize the impulse responses so that government consumption rises by one percentage point of GDP on impact. Such a normalization thus makes the responses
of economic variables directly comparable quantitatively across VAR models. Since we base the greatest part of our analysis and discussion on impact effects, potential problems caused by differences in the government spending shock could be mitigated. A straightforward check of the extent of differences of the government spending shock across VAR specifications is performed by contrasting impulse response functions for $G$. All of the empirical impulse response functions for $G$, and to a lesser extent the response of $G$ we generate from estimates of the VAR model that includes the current account, lie within the 90% confidence bounds of the first IRF for $G$ for all horizons. Moreover, the test we perform indicates that assumption that the point estimate for the response of $G$ in the first VAR model is significantly different from that for alternative VAR models is strongly rejected. However, even if the magnitude and the shape of the government spending shock is similar across VAR specifications, different VAR models could pickup different structural government spending shocks. In order to investigate the extent of the discrepancy in the estimated government spending shock across VAR specifications, we perform several robustness exercises. In the first robustness exercise, we augment each VAR specification with the government spending shock identified in the first VAR model which is taken as the baseline in our quantitative analysis. Because the identification scheme is based on the assumption of delays between current output observation and the implementation of fiscal measures and we consider annual rather than quarterly data, we also identify the 'baseline' government spending shock on a quarterly basis. In the latter case, the set of countries is restricted to eight. It turns out that differences with baseline results are rather moderate when the shock is identified on annual data. Some differences are nevertheless noticeable. To some extent, the relative wage increases less in the short-run while total hours worked rise more when anticipation effects are controlled for. The differences are also moderate when our baseline results with a set of countries restricted to eight are contrasted with those for the VAR models augmented with the shock identified on quarterly data. Yet, the rise in government spending following a fiscal shock identified on quarterly data tends to be more pronounced and displays more persistence over time than in the baseline case. As a result, the response of sectoral shares are more pronounced than those in the baseline case. In the last robustness exercise, we provide an attempt to answer the following question numerically: to what extent the dynamic responses of economic variables are affected quantitatively by the differences in the government spending shocks. Reassuringly, we find that the differences are quantitatively small, if not insignificant, when we contrast theoretical IRF that we generate following the baseline government spending shock with theoretical IRF that we generate following a government spending shock that is allowed to vary across VAR specifications.

- The main obstacle in empirical fiscal policy analysis is to identify exogenous and unexpected fiscal events. In subsection D.5, we have addressed the potential effects of fiscal foresight. We now deal with the potential endogeneity problem. We tackle this issue in Appendix B.4 by identifying the spending shock on quarterly instead of annual data; empirical results show that the dynamic effects are rather similar whether the fiscal shock is identified on a quarterly or yearly basis. In order to investigate the extent of the potential endogeneity problem, in subsection D.7, we allow government expenditure series to react to all VAR variables contemporaneously and contrast the IRF for $G$ in the baseline case in which $G$ is ordered first with that when $G$ is ordered last. Results show that differences are rather small and thus our results should not be affected by an endogeneity problem. We nevertheless conduct an empirical investigation by adopting an alternative identification scheme that would enable us to identify ‘truly’ exogenous government spending shocks. The solution suggested by the empirical literature to identify exogenous fiscal shocks is to adopt a narrative approach. In contrast to Blanchard and Perotti’s [2002] identification scheme, the methodology is based on identifying changes in government spending directly from historical events or official documents. Ramey and Shapiro [1998] consider a small number of events which led to large military buildups. While such an analysis is
not feasible for a large panel of countries, Guajardo, Leigh, and Pescatori [2014] use historical documents to construct a dataset that contains 173 fiscal policy changes for 17 OECD countries over the period 1978-2009. Following Ramey [2011], we augment each VAR model with the ‘spending-based’ events variable constructed by the authors, ordered first, and uses shocks to the ‘spending-based’ events variable (identified with the Cholesky decomposition) as the shock. Estimates show that whether changes in government spending are identified by using a narrative approach or by applying Blanchard and Perotti’s [2002] assumption, the main conclusions reached in this paper hold, except for investment which is found to be significantly increasing instead of decreasing in the short-run. We may also note some interesting differences for the sectoral effects which suggest that the fiscal shock events identified by Guajardo, Leigh, and Pescatori [2014] are somewhat less biased toward non traded goods than those identified in this paper. More precisely, the responses of sectoral output shares are somewhat less pronounced in the ‘event’ study while the relative wage of non tradables increases less. We believe that more work needs to be done in order to understand the cause(s) of the quantitative differences between the two approaches.

D.1 Government Spending Shocks Biased Toward Non Traded Goods

Before discussing in details our calibration strategy, it is useful to explain how our panel VAR evidence can be related to the sector intensity in the aggregate government spending shock. In the main text, we run three alternative VAR specifications:

- The first VAR specification aims at exploring empirically the size the aggregate fiscal multiplier by using annual data. Like Corsetti et al. [2012], our panel VAR evidence indicates that the aggregate fiscal multiplier is smaller than 1. All else equal, if the fraction of the rise in government consumption spent on non tradables and tradables are equal, i.e., if \( \omega_{GN} = \omega_{GT} \), then increases in value added expressed in percentage points of GDP are identical across sectors.

- In the second VAR specification, we explore empirically the size of the sectoral fiscal multiplier and estimate effects of an aggregate spending shock on non traded and traded value added at constant prices. Like Benetrix and Lane [2010], our panel VAR evidence shows that the rise in government spending increase non traded output relative to traded output. All else equal, a fall in \( Y_T / Y_N \) indicates that government spending shocks are biased toward the non traded sector. More precisely, as shown in section C.2, we have \( \nu^{Y,N} \tilde{Y}^N(t) - \nu^{Y,T} \tilde{Y}^T(t) > 0 \) as long as \( \omega_{GN} > \omega_{GT} \) (see eq. (97)). This result reveals that the rise in government consumption is concentrated on non traded goods. However, it does not tell us anything about the reallocation of resources across sectors since it does not take into account that the share of non tradables is approximately two-third over 1990-2007. Thus, if \( 1/2 < \omega_{GN} < 2/3 \), non traded output increases more than traded output in percentage points of GDP but the share of non tradables in real GDP declines as the fraction of the rise in government spending spent on non tradables is smaller than the share of non tradables in GDP.

- The third VAR specification explores empirically the responses of the share of tradables and non tradables to a government spending shock. Such a response tells us how much sectoral output would increase if real GDP remained constant. Hence, for the sectoral output share in real GDP to increase, resources must be reallocated toward this sector. And the incentives to reallocate resources toward this sector depend on the extent of the rise in demand for non tradables. More precisely, as shown in section C.2, the share of non tradables in real GDP increases as long as the fraction of the rise in government spending spent on non tradables is higher than the share of non tradables in GDP. More precisely, keeping private sector’s demand components fixed, we have \( \nu^{Y,N} \left( \tilde{Y}^N(0) - \tilde{Y}_R(0) \right) = (\omega_{GN} - \nu^{Y,N}) \frac{4G(0)}{Y_T}. \) Since our panel VAR evidence indicates that the share of non tradables in real GDP, i.e., \( \nu^{Y,N} \left( \tilde{Y}^N(0) - \tilde{Y}_R(0) \right) \), rises by 0.35 percentage points of GDP, using the fact that
the non tradable content of GDP averages 60% over 70-07, the above formula gives us a non tradable content of the government spending shock $dG(0)/Y = 1\%$, i.e.,

$$\omega_{GN} = \nu^{Y,N} (\hat{Y}^N(0) - \hat{Y}_R(0)) + \nu^{Y,N} = 0.35\% + 0.6\% = 0.95\%.$$  

This calculus does not take into account that the private sector’s demand components respond endogenously to the government spending shock. Using eq. (107) and taking into account the reactions of consumption and investment (which merely influence the response of the share of non tradables) along with the decline in net exports, we have (in percentage points of GDP):

$$\omega_{GN} = \nu^{Y,N} (\hat{Y}^N(0) - \hat{Y}_R(0)) + \nu^{Y,N} + \text{Changes in demand components} \simeq 0.78\%.$$  

Thus, our VAR evidence suggest that the non tradable content of the government spending shock is substantial but smaller than 95% due to the current account deficit which further shifts demand toward non traded goods as traded goods can be imported.

Obviously, the discussion above is only informative and enables us to give a sense of the magnitude of the non tradable content of the aggregate spending shock. We detail below our calibration strategy.

Our calibration strategy amounts to calculating the allocation of the rise in government spending between non tradables and tradables. To accomplish this task, we first determine the non tradable content of government spending. Denoting by $\omega_{Gj}$ the content of government spending in good $j$, we have:

$$G(t) = \omega_{GN}G(t) + \omega_{GT}G(t).$$  

To split government spending into expenditure in non traded and traded goods, i.e., to choose a value $\omega_{GN}$, we use time series from COFOG (Classification of the Functions of Government) provided by the OECD. This database ‘classifies government expenditure data from the System of National Accounts by the purpose for which the funds are used’ (more details can be found in the Manual on sources and methods for the compilation of COFOG statistics). COFOG has three levels of detail: Divisions, Groups, and Classes. ‘The ten Divisions could be seen as the broad objectives of government, while the Groups and Classes detail the means by which these broad objectives are achieved’.

Data are available over the period 1995-2007 for AUT, BEL, DNK, ESP, FRA, GBR, IRL, ITA, NLD, NOR and SWE, 1998-2007 for AUS, 1990-2007 for FIN, 2005-2007 for JPN and 1970-2007 for USA. Data are not available for CAN. The advantage of this database is twofold. First, this dataset gives time series for government expenditure net of transfers and makes the distinction between final consumption expenditure and public investment. Thus, there is an exact correspondence between the sum of government consumption on tradables and non tradables on the one hand and time series for total government consumption expenditure used to estimate the effects of a government spending shock in the main text. Second, the first-level COFOG splits expenditure data into ten divisions. While there is some degree of arbitrariness in treating certain items as non tradables and the remaining as tradables, the content of items is such that there is little doubt in treating them as tradable or non tradable. Among the ten items, ”04-Economic Affairs” is treated as tradable while the remaining nine items are classified as non traded: ”01-General Public Services”, ”02-Defense”, ”03-Public Order and Safety”, ”05-Environment Protection”, ”06-Housing and Community Amenities”, ”07-Health”, ”08-Recreation, Culture and Religion”, ”09-Education”, ”10-Social Protection”. It is worth mentioning that ”Economic Affairs” is subdivided in six categories including ”Fuel and Energy”, ”Agriculture, Forestry, Fishing, and Hunting”, ”Mining, Manufacturing, and Construction”, ”Transport and Communications”. Among the nine items treated as non tradables, only one item, namely ”Defense” may display some ambiguity. While in the main text, we treat this item as non tradables, we nevertheless conduct a robustness check in order to explore the extent to which our conclusion for the non tradable content of government spending shocks is altered when we classify ”Defense” as a traded item.
Heading of each item are displayed in the first row of Table 14. Capital letters ‘N’ and ‘T’ indicate whether the item is classified as non tradables or tradables. The first column of Table 14 gives the time horizon over which data are available for each economy in our sample. The second column of Table 14 reports the total government consumption expenditure in percentage point of GDP by summing expenditure on non traded and traded goods.

In the main text, we calibrate the model over the period 1990-2007 as data are not available before 1990 and to be consistent with our empirical analysis in the main text. Column 4 of Table 5 in the Appendix intended for publication gives the non tradable content of government spending over 1990-2007 which averages 90%; thus, when we calibrate the model, we set $\omega_{GN}$ (see eq. (108)) to 0.90. Since time series for government consumption by function are not available before 1995 for most of the countries in our sample, and because our objective in this subsection is to estimate the non tradable content of the aggregate government spending shock, in an effort to have time series of a reasonable length, we consider a period running from 1995 to 2015 except for Australia (1998-2015), Japan (2005-2015). As reported in column 4, the non tradable content of government spending averages 91% and displays a low cross-country dispersion as it varies from a low in Japan (87%) to a high in DNK, FRA, GBR and SWE (94%). Together, "Education" and "Health" account for almost half (48%) of government consumption, except for the US (34%). While there is low cross-country dispersion, it is worth mentioning that the U.S. has distinct features as the share of "Health" is 5.6% of total government consumption while "Defense" accounts for nearly one quarter of $G$.

We also investigate the causes of the cross-country dispersion in the non tradable content of government consumption expenditure. According to "Wagner’s Law”, richer countries choose bigger governments. In spirit of "Wagner’s Law”, we investigate whether richer countries also have a greater non tradable content of government expenditure, given its components includes health and education expenditure. We thus run the regression of $\omega_{GN}$ on GDP per capita (GDP Per head, constant prices, constant PPPs, OECD base year) in panel data (with country fixed effects). As can be seen in the first row of Table 15, there is a positive relationship between these two variables. However, the coefficient in front of GDP per capita is not statistically significant. We believe that the non tradable content of government consumption expenditure is rather explained by the government spending-GDP ratio. Figure 21 plots the non tradable content of government consumption expenditure against the relative size of the public sector measured by the ratio of government consumption expenditure to GDP. The trend line shows that countries where the relative size of the public sector is higher have a greater non tradable content of government consumption expenditure. This results is corroborated since $G/Y$ exerts a statistically significant positive impact on $\omega_{GN}$, as can be seen in the second row of Table 15. Because Rodrik [1998] finds a positive correlation between an economy’s exposure to international trade and the size of its government, we believe that the non tradable content of government consumption expenditure is higher in countries which are more open to international trade. This finding would not be surprising since expenditure classified as non tradables includes spending for an allocative and redistributive motive.

In order to investigate whether government consumption of non tradables is a major part of unanticipated changes in government spending, we estimate a variance decomposition of government final consumption expenditure using a simple VAR including the log of real government consumption on non tradables, $g_{Nt}$, and the log of real government spending, $g_{It}$. The sample covers 13 OECD countries over the period 1995-2015. We choose this period as time series for government consumption by function (COFOG dataset) provided by the OECD are not available before 1995 for most of the countries in our sample while the period 1995-2007 would be too short to obtain consistent estimates. Table 16 reports the share of the forecast error variance of total government

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63One or two lags and quantities are scaled by the working age population
64Data to construct time series for sectoral government consumption expenditure are available for all countries in our sample except Canada. In an effort to have a balanced panel and time series of a reasonable length, Australia (1998-2015) and Japan (2005-2015) are removed from the sample which leaves us with 13 OECD countries over the period 1995-2015.

<table>
<thead>
<tr>
<th>Period</th>
<th>G/Y</th>
<th>G'/G</th>
<th>G''/G</th>
<th>Public Services N</th>
<th>Public Services N/T</th>
<th>Defense N</th>
<th>Economic Affairs T</th>
<th>Environment Protection N</th>
<th>Housing N</th>
<th>Health N</th>
<th>Recreation N</th>
<th>Education N</th>
<th>Social Protection N</th>
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<tbody>
<tr>
<td>AUS</td>
<td>1998-2015</td>
<td>0.172</td>
<td>0.110</td>
<td>0.890</td>
<td>0.088</td>
<td>0.081</td>
<td>0.082</td>
<td>0.110</td>
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<td>0.032</td>
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<tr>
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<td>0.120</td>
<td>0.880</td>
<td>0.109</td>
<td>0.038</td>
<td>0.068</td>
<td>0.120</td>
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<td>0.005</td>
<td>0.306</td>
<td>0.034</td>
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</tr>
<tr>
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</tr>
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<td>n.a.</td>
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</tr>
<tr>
<td>DNK</td>
<td>1995-2015</td>
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<td>0.938</td>
<td>0.084</td>
<td>0.059</td>
<td>0.033</td>
<td>0.062</td>
<td>0.014</td>
<td>0.006</td>
<td>0.274</td>
<td>0.045</td>
<td>0.188</td>
</tr>
<tr>
<td>ESP</td>
<td>1995-2015</td>
<td>0.181</td>
<td>0.087</td>
<td>0.913</td>
<td>0.098</td>
<td>0.057</td>
<td>0.099</td>
<td>0.087</td>
<td>0.022</td>
<td>0.026</td>
<td>0.291</td>
<td>0.047</td>
<td>0.208</td>
</tr>
<tr>
<td>FIN</td>
<td>1995-2015</td>
<td>0.221</td>
<td>0.098</td>
<td>0.902</td>
<td>0.108</td>
<td>0.060</td>
<td>0.050</td>
<td>0.098</td>
<td>0.007</td>
<td>0.008</td>
<td>0.250</td>
<td>0.032</td>
<td>0.207</td>
</tr>
<tr>
<td>FRA</td>
<td>1995-2015</td>
<td>0.230</td>
<td>0.064</td>
<td>0.936</td>
<td>0.114</td>
<td>0.070</td>
<td>0.062</td>
<td>0.064</td>
<td>0.022</td>
<td>0.021</td>
<td>0.283</td>
<td>0.031</td>
<td>0.198</td>
</tr>
<tr>
<td>GBR</td>
<td>1995-2015</td>
<td>0.188</td>
<td>0.057</td>
<td>0.943</td>
<td>0.051</td>
<td>0.114</td>
<td>0.098</td>
<td>0.057</td>
<td>0.024</td>
<td>0.023</td>
<td>0.315</td>
<td>0.033</td>
<td>0.178</td>
</tr>
<tr>
<td>IRL</td>
<td>1995-2015</td>
<td>0.165</td>
<td>0.094</td>
<td>0.906</td>
<td>0.037</td>
<td>0.029</td>
<td>0.087</td>
<td>0.094</td>
<td>0.030</td>
<td>0.030</td>
<td>0.336</td>
<td>0.025</td>
<td>0.225</td>
</tr>
<tr>
<td>ITA</td>
<td>1995-2015</td>
<td>0.189</td>
<td>0.076</td>
<td>0.924</td>
<td>0.129</td>
<td>0.067</td>
<td>0.098</td>
<td>0.076</td>
<td>0.014</td>
<td>0.024</td>
<td>0.324</td>
<td>0.017</td>
<td>0.206</td>
</tr>
<tr>
<td>JPN</td>
<td>2005-2015</td>
<td>0.193</td>
<td>0.131</td>
<td>0.869</td>
<td>0.097</td>
<td>0.043</td>
<td>0.061</td>
<td>0.131</td>
<td>0.032</td>
<td>0.019</td>
<td>0.346</td>
<td>0.014</td>
<td>0.143</td>
</tr>
<tr>
<td>NLD</td>
<td>1995-2015</td>
<td>0.233</td>
<td>0.095</td>
<td>0.905</td>
<td>0.068</td>
<td>0.060</td>
<td>0.067</td>
<td>0.095</td>
<td>0.033</td>
<td>0.018</td>
<td>0.245</td>
<td>0.036</td>
<td>0.182</td>
</tr>
<tr>
<td>NOR</td>
<td>1995-2015</td>
<td>0.207</td>
<td>0.074</td>
<td>0.926</td>
<td>0.103</td>
<td>0.081</td>
<td>0.049</td>
<td>0.074</td>
<td>0.007</td>
<td>0.005</td>
<td>0.293</td>
<td>0.033</td>
<td>0.207</td>
</tr>
<tr>
<td>SWE</td>
<td>1995-2015</td>
<td>0.253</td>
<td>0.063</td>
<td>0.937</td>
<td>0.089</td>
<td>0.062</td>
<td>0.048</td>
<td>0.063</td>
<td>0.003</td>
<td>0.006</td>
<td>0.240</td>
<td>0.034</td>
<td>0.238</td>
</tr>
<tr>
<td>USA</td>
<td>1995-2015</td>
<td>0.151</td>
<td>0.118</td>
<td>0.882</td>
<td>0.093</td>
<td>0.238</td>
<td>0.134</td>
<td>0.118</td>
<td>n.a.</td>
<td>0.005</td>
<td>0.056</td>
<td>0.014</td>
<td>0.303</td>
</tr>
<tr>
<td>Mean</td>
<td>1970-2015</td>
<td>0.156</td>
<td>0.124</td>
<td>0.876</td>
<td>0.090</td>
<td>0.286</td>
<td>0.107</td>
<td>0.124</td>
<td>n.a.</td>
<td>0.006</td>
<td>0.056</td>
<td>0.012</td>
<td>0.284</td>
</tr>
</tbody>
</table>

Notes: Column 1 gives the period of coverage for each country. Column 2 shows the ratio of government final consumption expenditure to GDP. Column 3 and 4 report the tradable and non tradable content of government final consumption expenditure. Column 5 through 14 disaggregates by function and provides a detailed analysis of the share of each component in government expenditure. Capital letters 'N' and 'T' from column 5 to 14 indicates whether the component is classified as tradables or non tradables. Data coverage: 1995-2015 for AUT, BEL, DNK, ESP, FRA, GBR, IRL, ITA, NLD, NOR and SWE, 1998-2015 for AUS, 1990-2015 for FIN, 2005-2015 for JPN, 1970-2015 for USA. Data are not available for CAN.
Table 15: Potential Determinants of the Non Tradable Content of Government Consumption Expenditure

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{it}^{PPP}$</td>
<td>0.011</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(1.503)</td>
<td>(1.839)</td>
</tr>
<tr>
<td>$(G/Y)_{it}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.861</td>
<td>0.860</td>
</tr>
<tr>
<td>Countries</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Observations</td>
<td>301</td>
<td>301</td>
</tr>
<tr>
<td>Country fixed effects</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Notes: in all regressions the dependent variable is $G^N/G$. All variables enter in regression in logarithms. $^a$, $^b$ and $^c$ denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses.

Figure 21: Non Tradable Content of Government Consumption Expenditure against the Relative Size of the Public Sector. Sample: 16 OECD countries 1995-2015; Source: OECD-COFOG database for $G^N$ and OECD Economic Outlook for $G/Y$
Table 16: Variance Decomposition for Government Final Consumption Expenditure on Annual Data (1995-2015)

<table>
<thead>
<tr>
<th>Step</th>
<th>Defense classified in $G_N$</th>
<th>Defense classified in $G_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p = 1$</td>
<td>$p = 2$</td>
</tr>
<tr>
<td>1</td>
<td>0.902</td>
<td>0.874</td>
</tr>
<tr>
<td>2</td>
<td>0.903</td>
<td>0.831</td>
</tr>
<tr>
<td>3</td>
<td>0.905</td>
<td>0.831</td>
</tr>
<tr>
<td>4</td>
<td>0.906</td>
<td>0.844</td>
</tr>
<tr>
<td>5</td>
<td>0.907</td>
<td>0.858</td>
</tr>
<tr>
<td>6</td>
<td>0.907</td>
<td>0.867</td>
</tr>
<tr>
<td>7</td>
<td>0.908</td>
<td>0.872</td>
</tr>
<tr>
<td>8</td>
<td>0.908</td>
<td>0.874</td>
</tr>
</tbody>
</table>

Notes: To estimate a variance decomposition for total government consumption expenditure $g_{i,t}$, we use a simple VAR model that includes government consumption of non tradables and government final consumption expenditure, i.e., $z_{i,t} = [g_{i,t}^N, g_{i,t}]$. We allow for either $p = 1$ or $p = 2$ lags. Data coverage: 1995-2015 for AUT, BEL, DNK, ESP, FIN, FRA, GBR, IRL, ITA, NLD, NOR, SWE and the USA (source: OECD-COFOG database).

spending attributable to shocks to $g^N$ at various horizons. Government spending shocks on non tradables accounts for almost all of the unforeseen changes in total government spending. More precisely, irrespective of whether the item 'Defense' is classified as tradables or as non tradables, shocks to $g^N$ account for 79%-95% of the variance of total government spending for horizons of one to eight years. It is worth mentioning that for each of the four specifications, the contribution of shocks to $g^N$ is rather stable over time, in particular when 'Defense' is classified as non tradables. In this case, the contribution of shocks to $g^N$ to unanticipated changes in $G$ averages 91% with one lag and 86% with two lags.

**Theoretical impulse response functions of sectoral government consumption.**

In order to reproduce the hump-shaped pattern of the endogenous response of government spending to an exogenous fiscal shock, we assume that the deviation of government spending relative to its initial value as a percentage of initial GDP is governed by the dynamic equation (23). Left-multiplying (23) by $\omega_{Gj}$ (with $j = N, T$) gives the dynamic adjustment of sectoral government consumption to an exogenous fiscal shock:

$$\omega_{Gj} \frac{G(t) - \tilde{G}}{Y} = \omega_{Gj} \left[ e^{-\xi t} - (1 - g) e^{-\chi t} \right].$$

(109)

We set $g$ to 0.01 as we consider an exogenous increase in government spending by 1% of GDP and choose values of $\xi$ and $\chi$ in order to reproduce the hump-shaped pattern of the endogenous response of government spending to the exogenous fiscal shock. To the extent that $\omega_{Gj}$ is considered as fixed over time, we set $\omega_{Gj}$ to the share of government final consumption expenditure in good $j$. Thus, we set $\omega_{GN}$ to 90% and $\omega_{GT} = 10\%$ which corresponds to the non tradable and the tradable content of government final consumption expenditure, respectively. The derivation of the dynamic equation (43) that governs the adjustment of sectoral government consumption following an exogenous fiscal shock relies on a number of assumptions. We assume that parameters that govern the persistence and the shape of the response of sectoral government consumption are identical across sectors while sectoral intensity of the government spending shock is constant over time and thus corresponds to the share of government final consumption expenditure in good $j$. We investigate below the extent to which these assumptions are consistent with empirical impulse response functions we generate following a rise in government consumption by 1% of GDP.

**Empirical vs. theoretical impulse response functions of sectoral government consumption.** To generate impulse response functions of sectoral government consumption, we first estimate the first VAR model that includes government final consumption expenditure, real GDP, total hours worked, private investment, the real consumption wage, in order to identify unanticipated government spending shocks. Then, we estimate a VAR model in panel format on annual data that includes unanticipated government spending
shocks, $\epsilon_i^G$, ordered first, government spending, $y_{it}$, government consumption on non-tradables, $g_{it}^N$, and government consumption on tradables, $g_{it}^T$, i.e., $z_{it}^G = [\epsilon_i^G, y_{it}, g_{it}^N, g_{it}^T]$. All quantities are logged, expressed in real terms and scaled by the working age population.

As mentioned above, data to construct time series for sectoral government consumption expenditure are available for all countries in our sample except Canada. In an effort to have a balanced panel and time series of a reasonable length, Australia (1998-2015) and Japan (2005-2015) are removed from the sample which leaves us with 13 OECD countries over the period 1995-2015. To be consistent, we estimate the first VAR model that includes aggregate variables for these 13 OECD countries only. Table 17 reports, for various horizons, the mean responses of government consumption expenditure on non tradables and tradables to the identified government spending shock. We normalize the impulse responses so that government spending rises by one percentage point of GDP on impact. The table show that no matter what the order of the variables, a government spending shock leads to an increase in government consumption expenditure on non tradables by 0.88% on impact while the rise in public purchases of tradables accounts for the remaining share, i.e., 12%. The average contribution of the response of the government spending shock to the government spending shock is displayed in the last line of Table 17. The contribution of government expenditure on non tradables averages 90%. We also find that its contribution is quite stable over time as it varies between 88% and 91%.

Empirical impulse response functions for the two components of government final consumption expenditure we generate following a rise in government spending by 1% of GDP are displayed in solid blue lines in Figure 22. The first and the second row show results for $z_{it}^G = [\epsilon_i^G, y_{it}, g_{it}^N, g_{it}^T]$ and $z_{it}^G = [\epsilon_i^G, y_{it}, g_{it}^N, g_{it}^T]$, respectively. No matter the ordering of variables, impulse response functions for both sectoral components of government spending display an hump-shaped pattern, like the endogenous response of total government spending, and peak after 1 year.

Empirical and theoretical impulse response functions are contrasted and displayed by solid blue lines in the right panel of Figure 23. Before discussing the results, we first focus on the response of government final consumption expenditure to the exogenous fiscal shock shown in the left panel of Figure 23. The endogenous response of government spending to an exogenous fiscal shock displayed in the solid blue line corresponds to the baseline government spending shock in the main text (see Figure 1(a)) obtained from estimates of the first VAR model. The dynamic response of government final consumption expenditure which has been computed by summing mean responses of government consumption consumption on non tradables and tradables is displayed by the solid red line. While the solid blue line displays the point estimates from a sample of 15 OECD countries over 1970-2007, the solid red line displays the point estimates from a sample of 13 OECD countries over 1995-2015. Whereas the samples are different, the discrepancy is quite moderate. Since theoretical responses of sectoral government consumption are based on the response of government spending shown in the solid blue line in the left panel while the sum of mean responses of government consumption expenditure on non tradables and tradables gives a slightly different response of government spending as shown in the solid red line, we have to rescale empirical responses for $G^j$ so that the sum of mean responses corresponds exactly to the point estimate displayed in the solid blue line. The rescaled empirical responses of sectoral government consumption are displayed by solid blue lines in the right panel of Figure 23 with dotted blue lines indicating the 90 percent confidence bounds obtained by bootstrap sampling. We contrast empirical with theoretical responses displayed by dotted black lines. It turns out that differences are quite moderate. We may notice that while the theoretical response of government consumption on non tradables (tradables) slightly overstates (understates) the estimated response, it lies within the confidence bounds for both goods. To conclude, the assumptions underlying the dynamic equation (109) which governs theoretical responses of $G^j$ are reasonable and consistent with data.

**Variance decomposition for government spending: U.S. (1970-2007).** So far, we have quantified the contribution of shocks to $G^N$ to unforeseen changes in government spending. We now move a step further and investigate whether identified shocks to $G^N$ produce similar effects to those triggered by shocks to $G$. Such an analysis is feasible
Figure 22: Effects of an Unanticipated Government Spending Shock on Government Final Consumption Expenditure on Non Tradables and Tradables. Notes: Exogenous increase in government consumption by 1% of GDP. The government spending shock is identified by estimating a VAR model that includes real government final consumption expenditure, GDP (constant prices), total hours worked, private fixed investment, and the real consumption wage. The responses of government final consumption expenditure on non tradables (i.e., $G^N$) and tradables (i.e., $G^T$) to the identified government spending shock are displayed by solid blue lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; sample: 13 OECD countries, 1995-2015, annual data.
Table 17: Responses of $G^N$ and $G^T$ to Identified Government Spending Shock: Point Estimates

<table>
<thead>
<tr>
<th>Horizon</th>
<th>$z_{i,t}^{G} = [\epsilon_{i,t}^{G}, g_{i,t}^{N}, g_{i,t}^{T}]$</th>
<th>$z_{i,t}^{G} = [\epsilon_{i,t}^{G}, g_{i,t}^{N}, g_{i,t}^{T}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.876 0.119</td>
<td>0.877 0.118</td>
</tr>
<tr>
<td>1</td>
<td>1.045 0.150</td>
<td>1.045 0.150</td>
</tr>
<tr>
<td>2</td>
<td>0.892 0.125</td>
<td>0.893 0.124</td>
</tr>
<tr>
<td>3</td>
<td>0.753 0.098</td>
<td>0.753 0.098</td>
</tr>
<tr>
<td>4</td>
<td>0.623 0.076</td>
<td>0.623 0.076</td>
</tr>
<tr>
<td>5</td>
<td>0.493 0.057</td>
<td>0.493 0.057</td>
</tr>
<tr>
<td>6</td>
<td>0.381 0.041</td>
<td>0.382 0.042</td>
</tr>
<tr>
<td>7</td>
<td>0.294 0.030</td>
<td>0.294 0.030</td>
</tr>
<tr>
<td>8</td>
<td>0.226 0.022</td>
<td>0.227 0.022</td>
</tr>
<tr>
<td>9</td>
<td>0.175 0.017</td>
<td>0.176 0.017</td>
</tr>
<tr>
<td>10</td>
<td>0.136 0.013</td>
<td>0.138 0.013</td>
</tr>
<tr>
<td>Contribution</td>
<td>0.895 0.104</td>
<td>0.895 0.104</td>
</tr>
</tbody>
</table>

Notes: Horizon measured in year units. We generate impulse response functions by using a simple VAR, i.e., $z_{i,t}^{G} = [\epsilon_{i,t}^{G}, g_{i,t}^{N}, g_{i,t}^{T}]$ or $z_{i,t}^{G} = [\epsilon_{i,t}^{G}, g_{i,t}^{N}, g_{i,t}^{T}, g_{i,t}^{N}]$ with 2 lags. To identify the government spending shock $\epsilon_{i,t}^{G}$, we estimate the VAR model that includes aggregate variables, i.e., $z_{i,t} = [g_{i,t}, y_{i,t}, l_{i,t}, j_{e_{i,t}}, w_{C,i,t}]$, and adopt a Cholesky decomposition. The last line of the table displays the average contribution of the response of each component to the government spending shock. Data coverage: 1995-2015 for AUT, BEL, DNK, ESP, FIN, FRA, GBR, IRL, ITA, NLD, NOR, SWE and the USA. All variables are real and scaled by the working age population.

Figure 23: Effects of an Unanticipated Government Spending Shock on Government Final Consumption Expenditure on Non Tradables and Tradables: Empirical vs. Theoretical Impulse Response Functions. Notes: The baseline response of government final consumption expenditure is displayed by the solid blue line with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; sample: 16 OECD countries, 1970-2007, annual data. The responses of government final consumption expenditure on non tradables (i.e., $G^N$) and tradables (i.e., $G^T$) to the identified government spending shock (in the baseline VAR model) are displayed by solid blue lines with dotted blue lines indicating 90 percent confidence bounds obtained by bootstrap sampling; sample: 13 OECD countries, 1995-2015, annual data. The red line in the left panel displays the dynamic response of government final consumption expenditure which has been computed by summing mean responses of government consumption expenditure on non tradables and tradables.
Table 18: Variance Decomposition for Government Final Consumption Expenditure on U.S. annual data (1970-2007)

<table>
<thead>
<tr>
<th>Step</th>
<th>( \Phi_t = [g_t, g_t] )</th>
<th>( \Phi_t = [g_t, g_t, y_t] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p = 1 )</td>
<td>( p = 2 )</td>
<td>( p = 1 )</td>
</tr>
<tr>
<td>1</td>
<td>0.940</td>
<td>0.878</td>
</tr>
<tr>
<td>2</td>
<td>0.916</td>
<td>0.961</td>
</tr>
<tr>
<td>3</td>
<td>0.897</td>
<td>0.959</td>
</tr>
<tr>
<td>4</td>
<td>0.882</td>
<td>0.952</td>
</tr>
<tr>
<td>5</td>
<td>0.872</td>
<td>0.950</td>
</tr>
<tr>
<td>6</td>
<td>0.865</td>
<td>0.951</td>
</tr>
<tr>
<td>7</td>
<td>0.859</td>
<td>0.951</td>
</tr>
<tr>
<td>8</td>
<td>0.856</td>
<td>0.952</td>
</tr>
</tbody>
</table>

Notes: Decomposition of variance for government spending \( g_t \) from estimates of alternative VAR specifications \( \Phi_t = [g_t^N, g_t] \) and \( \Phi_t = [g_t^N, g_t, y_t] \), respectively, with either \( p = 1 \) or \( p = 2 \) lags. Data coverage: 1970-2007.

for the US as time series for \( G^N \) are available from 1970 to 2007. Before discussing the dynamic effects, we first estimate a variance decomposition of government spending by using a simple VAR \( \Phi_t = [g_t^N, g_t] \). We alternatively augment the VAR model with real GDP and thus consider the following specification \( \Phi_t = [g_t^N, g_t, y_t] \). The results reported in Table 18 reveal that, regardless of the VAR specification or the number of lags, shocks to \( G^N \) account for 86 to 96 percent of the forecast error variance of total government spending for horizons of 1 to 8 years.

We then estimate the baseline VAR models described in section 2 by using U.S. annual data over 1970-2007. To contrast the effects of a rise in \( G^N \) with those following an increase in \( G \), we re-estimate VAR models in which \( g_t \) is replaced with \( g_t^N \). The latter variable is constructed in accordance with the classification discussed above.\(^{65}\) Figure 24 shows that the share of non tradables in total government consumption is rather stable after 1988 and averages 88%. We generated impulse response functions which are normalized so that government consumption increases by 1 percentage point of GDP. The solid blue line in Figures 25 and 26 shows the results following a rise in total government consumption while the solid black line shows the results following a rise in government consumption of non tradables.\(^{66}\) The results are quite clear:

- First, as displayed in the first row of Figures 25-26, both the shape and the magnitude of the endogenous responses of government consumption are quite similar.

- Second, all the conclusions reached in the main text hold whether we consider a rise in \( G \) or in \( G^N \). More precisely, we find empirically that a government spending shock gives rise to a contraction in hours worked and output in the traded sector while it has an expansionary effect on non traded output. Moreover, a rise in public purchases lowers the share of tradables and increases the relative size of the non traded sector. Finally, both the relative price and the relative wage of non tradables increase.

- Third, and most importantly, the sectoral effects are of the same magnitude whether we consider a rise in total government spending or an increase in government consumption of non traded goods. The evidence also shows that the shape of impulse response functions is quite similar. Across all VAR specifications, differences between the dynamics effects of both shocks are rather moderate. More specifically, the impulse response functions we generated after a shock to \( g_t^N \) lie within the confidence bounds of the IRF we generated after a shock to \( g_t \).

\(^{65}\)\( g_t \) is real government final consumption expenditure (source: OECD Economic Outlook Database) while \( g_t^N \) is real government final consumption expenditure on non tradables (source: OECD COFOG). Government spending on non tradables is deflated by the price of final consumption expenditure of general government (source: OECD Economic Outlook Database).

\(^{66}\)Given the small number of observations (\( T = 38 \)), VAR models are estimated by restricting the number of lags \( p \) to one in order to economize some degrees of freedom.
To conclude, our evidence reveal that the shocks to $G^N$ account on average for about 88% of shocks to $G$ for the whole sample over 1995-2015 while the variance decomposition on U.S. annual data over 1970-2007 suggests that shocks to government consumption of non traded goods accounts on average for 92% of the unforeseen changes in total government spending. Using a panel of 13 OECD countries over 1995-2015, When we estimate mean responses of government consumption expenditure on non tradables and tradables to our identified government spending shock, we find that the former contributes on average to 90% of the change in government spending while the remaining is attributed to government purchases of tradables. When we calibrate the model, we thus consider a rise in government spending which is split between non tradables and tradables in accordance with their respective contribution to the government spending shock, at 90% and 10% respectively.

D.2 An Elaborate Investigation of Responses of Government Expenditure Components to a Government Spending Shock

In this subsection, we conduct an elaborate investigation of the effects on sub-components of government final consumption expenditure and government consumption expenditure on non tradables. First, there are two main components in government consumption expenditure: the purchase of goods and services from the private sector, and the purchases of goods and services from the government sector, the latter corresponding to compensation of government employees. Second, because government consumption expenditure classified as non tradables includes nine divisions which differ from each other along a number of dimensions, such as the purpose (public versus private goods) and the type of expenditure (defense versus non-defense expenditure), in the following, we explore empirically the contribution of each broad category to shocks to government consumption of non traded goods.\footnote{Because the time horizon is too short and the number of observations are not large enough to estimate a VAR model that would enable to compute the contribution of each division of expenditure to increases in aggregate government consumption or government consumption on non traded goods for each country, we estimate a VAR model in panel format and subdivide $G^N$ into broad categories as detailed below.}

\textbf{Wage vs. non-wage government consumption expenditure.} Before going into more details in our empirical investigation, as will be useful below, we introduce a number of definitions. Since in our analysis we abstract from government investment, we denote by $G$ government aggregate consumption expenditure. Government consumption expenditure can be subdivided into \textit{compensation of employees} ($Y_{publ}$) and \textit{non-wage government con-}
Figure 25: Sectoral Effects of Shocks to Aggregate Government Consumption and Government Consumption of Non Tradables. Notes: Exogenous increase in government consumption by 1% of GDP. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend. Results for a shock to government final consumption expenditure are displayed by solid lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the black line displays the responses following a shock to government consumption of non tradables; sample: U.S., 1970-2007, annual data.
Figure 26: Sectoral Composition Effects of Shocks to Aggregate Government Consumption and Government Consumption of Non Tradables. Notes: Exogenous increase in government consumption by 1% of GDP. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend. Results for a shock to government final consumption expenditure are displayed by solid lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the black line displays the responses following a shock to government consumption of non tradables; sample: U.S., 1970-2007, annual data.
sumption \((G_{\text{priv}})\). The former component covers total remuneration paid by government to its employees and relates to the services provided by the government (value added of government), whereas the latter covers public purchases of consumption goods and services from the private sector. While we could restrict our attention to \(G^N\) and differentiate between compensation of employees and non-wage government consumption within this component of \(G\), data availability (since data start from 1995 for most of the countries) would prevent from conducting a VAR analysis.

**Framework.** Government purchases \(G\) consists of the value added of government, \(Y^N_G\), which is part of the non traded sector, and government purchases of goods and services from the private sector \((G_P = G^T + PG^N_P)\). Denoting the value added of the private sector by \(Y^N_P\) and the government value added by \(Y^N_G\), the market clearing condition for non traded goods reads as:

\[
Y^N = Y^N_P + Y^N_G = C^N + J^N + G^N, \tag{110}
\]

where \(G^N\) consists of i) the value added which the government itself produces and sells to itself, i.e., \(G^N_G = Y^N_G\), and ii) government purchases of non tradable goods and services from the private sector, \(G^N_P\). In our study, the private and the public sector are aggregated and thus we explore the effects on non traded value added, \(Y^N\). Conversely, if we restrict attention to the effects on value added of the private sector, using the fact that \(Y^N_G = G^N_G\), eq. \(110\) can be rewritten as follows:

\[
Y^N_P = C^N + J^N + G^N_P, \tag{111}
\]

Since \(L^N = L^N_G + L^N_P\) where \(L^N_G\) and \(L^N_P\) corresponds to hours worked used in the private and the public sector, in our study, we implicitly assume that hours worked can be moved costlessly between the private and the public sector. While it may be viewed as restrictive, most of the literature sets this assumption, see e.g., Cavallo [2005]. The government hires labor, \(L^N_G\), and rents capital, \(K^N_G\), from households, to produce government value added according to the following production function:

\[
Y^N_G = (L^N_G)^{\theta^N} (K^N_G)^{1-\theta^N}. \tag{112}
\]

Due to the assumption of perfect mobility of labor between the private non traded sector and the government (non traded) sector, compensation of government employees is \(W^N L^N_G\). To finance compensation of government employees and rental services from capital, \(R K^N_G\), along with purchases of goods and services from the private sector, the government levies lump-sum taxes in accordance with the following budget constraint:

\[
T = W^N L^N_G + R K^N_G + PG^N_P + G^T, \\
= PY^N_G + PG^N_P + G^T, \\
= PG^N + G^T = PY^N_G + G_P, \tag{113}
\]

where \(G^T\) is government purchases of goods and services from the traded (private) sector. The exposition of a framework that makes the distinction between the government and the private (non traded) sector has the merit to shed some light on the assumptions we set to aggregate these two sectors. First, we assume perfect mobility of labor between the government and the private (non traded) sector. Second, the traded sector is assumed to consist exclusively of private-sector firms. In other words, we assume that the government purchases goods and services from the private traded sector but does not hire traded labor.

**Average government spending share of compensation of government employees.** The second column of Table 19 reports the share of the compensation of employees in total government consumption expenditure for each country and for the whole sample as well. The first column indicates the time period over which data are available to re-estimate the VAR model in panel format on annual data. The time series for compensation of government employees are not available for AUS while data are available from 1970 to 2007 for all remaining countries, except BEL (1976-2007). As can be seen in the second column of Table 19, from 1970 to 2007, the average government spending share of purchases from the private sector is about 40% and the average share of the other component (i.e.,
Table 19: Subcomponents of Total Government Consumption Expenditure and Government Consumption Expenditure on Non Traded Goods

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>AUS</td>
<td>0.594</td>
<td></td>
</tr>
<tr>
<td>AUT</td>
<td>0.588</td>
<td></td>
</tr>
<tr>
<td>BEL</td>
<td>0.640</td>
<td></td>
</tr>
<tr>
<td>CAN</td>
<td>0.674</td>
<td></td>
</tr>
<tr>
<td>ESP</td>
<td>0.671</td>
<td></td>
</tr>
<tr>
<td>FIN</td>
<td>0.579</td>
<td></td>
</tr>
<tr>
<td>FRA</td>
<td>0.575</td>
<td></td>
</tr>
<tr>
<td>IRL</td>
<td>0.544</td>
<td></td>
</tr>
<tr>
<td>ITA</td>
<td>0.604</td>
<td></td>
</tr>
<tr>
<td>JPN</td>
<td>0.437</td>
<td></td>
</tr>
<tr>
<td>NLD</td>
<td>0.502</td>
<td></td>
</tr>
<tr>
<td>NOR</td>
<td>0.641</td>
<td></td>
</tr>
<tr>
<td>SWE</td>
<td>0.641</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>0.637</td>
<td></td>
</tr>
<tr>
<td>Whole</td>
<td>0.597</td>
<td></td>
</tr>
</tbody>
</table>

Notes: $G_{publ}$ is government final wage consumption expenditure (source: OECD Economic Outlook Database); $G_{Npubl}$ is government final wage consumption expenditure on non tradables, $G_{Ncoll}$ collective government final consumption expenditure on non tradables, $G_{Ndef}$ government final consumption expenditure related to 'Defense' (source: OECD, COFOG database).

$G_{publ}/G$) is 60%. Hence, the purchases of goods and services from the government sector are a significant component in government spending. As displayed in column 4 of Table 19, the average government consumption expenditure on non tradables share of purchases from the public sector, i.e., $G_{publ}^{N}/G^{N}$, is a little bit lower at 56%. While the Table does not show it, the average government consumption expenditure on tradables share of purchases from the public sector, i.e., $G_{publ}^{T}/G^{T}$, is lower than that for non tradables, at 41%. Since $G_{publ}^{j}/G^{j}$ is calculated over 1995-2015 and is lower than that over 1970-2007, the average government spending share of purchases from the private sector has increased significantly over time.

**Wage and non-wage government consumption shocks.** We re-estimate the VAR models specified in section 2 in the main text in panel format on annual data by separating the goods and services provided by the government sector (i.e., $G_{publ}$) from the purchase of goods and services from the private sector (i.e., $G_{priv}$):

- $G_{publ}$: Government final wage consumption expenditure. Source: OECD Economic Outlook Database. To express the variable in real terms, we deflate time series by the deflator of government final consumption expenditure (source: OECD Economic Outlook Database).

- $G_{priv} = G - G_{publ}$: Public purchase of goods and services from the private sector. To construct time series for variable $G_{priv}$, we subtract government final wage consumption expenditure from government final consumption expenditure (source: OECD, current prices, in millions of national currency). To express the variable in real terms, we deflate time series by the deflator of government final consumption expenditure (source: OECD Economic Outlook Database).

Because data are not available for AUS, the dataset covers 15 OECD countries over the period running from 1970 to 2007, except for BEL (1976-2007). In order to make our results for a wage government consumption shock comparable with those obtained for a government consumption shock, we re-estimate the VAR models for the restricted set of countries (i.e., 15). The baseline VAR model includes $G = G_{publ} + G_{priv}$ ordered first; when
exploring a shock to wage or non-wage government consumption, $G$ is replaced with $G^{publ}$ or $G^{priv}$ (ordered first), respectively.

**How do compensation of government employees, $G_{publ}$, and the purchases of goods and services from the private sector, $G_{priv}$, react to our identified government shock?** We begin by analyzing how government purchases from the public and private sectors react to a government spending shock. We first estimate the first VAR model that includes government final consumption expenditure, real GDP, total hours worked, private investment, and the real consumption wage, in order to identify unanticipated government spending shocks. Then, we estimate a VAR model in panel format on annual data that includes unanticipated government spending shocks, $\epsilon^G$, ordered first, government spending, $G_{it}$, labor compensation of government employees, $G_{publ}$, and government purchases of goods and services from the private sector, $G_{priv}$, i.e., $z^{G}_{i,t} = [\epsilon^G_{i,t}, G_{it}, G_{publ,i,t}, G_{priv,i,t}]$. To be consistent, we estimate the first VAR model that includes aggregate variables for 15 OECD countries only since time series for the two components are not available for AUS. Table 20 shows, at various horizons for different orderings of the variables, the mean responses of government consumption expenditure from the public and the private sector to the identified government spending shock. We normalize the impulse responses so that government spending rises by one percentage point of GDP on impact. The contribution of the response of labor compensation of government employees to the government spending shock is displayed in the third and the six column of Table 20. The table shows that, no matter what the ordering of the variables, the contribution of government purchases of goods and services from the private sector is large on impact and low after five years while we get the opposite result for compensation of government employees. In other words, a government spending shock seems to be associated first with higher purchases from the private sector and then with an increase in labor compensation. The average contribution of government purchases from the public sector shown in the last line of Table 20 is 55-63%. The contribution increases strongly over time, varying between 25% on impact and 100% after 10 years.

Impulse response functions for the two components of government final consumption expenditure that we generate following a rise in government spending by 1% of GDP are displayed in solid blue lines in Figure 27. The first and the second row show results for $z^{G}_{i,t} = [\epsilon^G_{i,t}, G_{it}, G_{publ,i,t}, G_{priv,i,t}]$ and $z^{G}_{i,t} = [\epsilon^G_{i,t}, G_{it}, G_{priv,i,t}, G_{publ,i,t}]$, respectively. No matter what the ordering of the variables, impulse response functions for the components of government spending are quite distinct. In particular, the endogenous response of government purchases from the private sector is much less persistent while the response of labor compensation is hump-shaped and persistent over time.

The results for a **wage government consumption shock** (solid black lines) are shown and contrasted with those for the baseline (solid blue lines) VAR model that includes total government consumption expenditure in Figures 28 and 29. To facilitate the interpretation of our results, we normalize the impulse responses so that government final (wage) consumption expenditure increases by one percentage point of GDP on impact. When we investigate the effects of a rise in government final wage consumption expenditure, we analyze the impacts on the whole economy (i.e., the private plus the public sector). The reason is that by construction, we have $Y_{publ} = G_{publ}$ where $Y_{publ}$ is government value added and total GDP is the sum of value added of the public and the private sector, i.e., $Y = Y_{priv} + Y_{publ}$. Overall, it turns out that the effects of a wage government consumption shock are more pronounced. All our conclusions in the main text hold though. As can be seen in Figure 28, a wage government consumption shock has a strong expansionary effect on output and hours worked and leads to a greater current account deficit. Importantly, Figure 29 shows that the share of tradables declines and the relative size of the non traded sector increases, while both the relative price and the relative wage of non tradables appreciate. Nevertheless, we may notice a discrepancy in the estimated responses of sectoral real consumption wages, sectoral shares and the relative price, which are more pronounced in both the traded and the non traded sector following a wage government consumption shock. Bermperoglou, Pappa, and Vella [2016] document evidence showing that public employment and public wage (at the state and local level) shocks have expansionary ef-
Figure 27: Effects of an Unanticipated Government Spending Shock on Purchase of Goods and Services from the Government and the Private Sector. Notes: Exogenous increase in government consumption by 1% of GDP. The government spending shock is identified by estimating a VAR model that includes real government final consumption expenditure, GDP (constant prices), total hours worked, private fixed investment, and the real consumption wage. We differentiate between government final consumption expenditure from the public sector and the private sector. The responses of the components of government final consumption expenditure to the identified government spending shock are displayed by solid blue lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; sample: 15 OECD countries, 1970-2007 (except for BEL: 1976-2007), annual data.
Table 20: Responses of Components of Government Consumption Expenditure to Identified Government Spending Shock: Point Estimates

<table>
<thead>
<tr>
<th>Horizon</th>
<th>$z_{i,t}^{G_{publ}}$</th>
<th>$z_{i,t}^{G_{priv}}$</th>
<th>Contribution of $G_{publ}$ in %</th>
<th>$z_{i,t}^{G_{publ}}$</th>
<th>$z_{i,t}^{G_{priv}}$</th>
<th>Contribution of $G_{publ}$ in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.267</td>
<td>0.804</td>
<td>25%</td>
<td>0.661</td>
<td>0.325</td>
<td>33%</td>
</tr>
<tr>
<td>1</td>
<td>0.381</td>
<td>0.788</td>
<td>33%</td>
<td>0.648</td>
<td>0.463</td>
<td>42%</td>
</tr>
<tr>
<td>2</td>
<td>0.386</td>
<td>0.766</td>
<td>34%</td>
<td>0.630</td>
<td>0.469</td>
<td>43%</td>
</tr>
<tr>
<td>3</td>
<td>0.360</td>
<td>0.579</td>
<td>38%</td>
<td>0.476</td>
<td>0.437</td>
<td>48%</td>
</tr>
<tr>
<td>4</td>
<td>0.308</td>
<td>0.416</td>
<td>43%</td>
<td>0.342</td>
<td>0.374</td>
<td>52%</td>
</tr>
<tr>
<td>5</td>
<td>0.249</td>
<td>0.271</td>
<td>48%</td>
<td>0.223</td>
<td>0.303</td>
<td>58%</td>
</tr>
<tr>
<td>6</td>
<td>0.192</td>
<td>0.165</td>
<td>54%</td>
<td>0.135</td>
<td>0.234</td>
<td>63%</td>
</tr>
<tr>
<td>7</td>
<td>0.143</td>
<td>0.089</td>
<td>62%</td>
<td>0.073</td>
<td>0.174</td>
<td>70%</td>
</tr>
<tr>
<td>8</td>
<td>0.104</td>
<td>0.040</td>
<td>72%</td>
<td>0.033</td>
<td>0.126</td>
<td>79%</td>
</tr>
<tr>
<td>9</td>
<td>0.073</td>
<td>0.010</td>
<td>88%</td>
<td>0.009</td>
<td>0.089</td>
<td>91%</td>
</tr>
<tr>
<td>10</td>
<td>0.050</td>
<td>-0.006</td>
<td>114%</td>
<td>-0.005</td>
<td>0.061</td>
<td>100%</td>
</tr>
<tr>
<td>Mean</td>
<td>-</td>
<td>-</td>
<td>55%</td>
<td>-</td>
<td>-</td>
<td>63%</td>
</tr>
</tbody>
</table>

Notes: Horizon measured in year units. We differentiate between compensation of government employees, $G_{publ}$ and the purchase of goods and services from the private sector, $G_{priv}$. We generate impulse response functions by using a simple VAR, $z_{i,t}^{G_{publ}} = [\epsilon_{G_{publ},i,t}, g_{it}, g_{priv,i,t}, g_{publ,i,t}]$, and $z_{i,t}^{G_{priv}} = [\epsilon_{G_{priv},i,t}, g_{it}, g_{priv,i,t}, g_{publ,i,t}]$, with 2 lags. To identify the government spending shock $\epsilon_{G_{i,t}}$, we estimate the VAR model that includes aggregate variables, $z_{i,t} = [g_{i,t}, y_{i,t}, l_{i,t}, je_{i,t}, w_{C,i,t}]$, and adopt a Cholesky decomposition. The third and the sixth column of the table displays the contribution of the response of labor compensation of government employees to the change in government final consumption expenditure while the last line shows the average contribution of this component. Data coverage: 1970-2007 for AUT, CAN, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA, except BEL (1976-2007). In all specifications, all variables are real and scaled by the working age population.

Effects by crowding-in consumption and private-sector employment. The expansionary effect triggered by public employment shocks can be rationalized by assuming a complementarity of the public good with private consumption in the aggregate consumption bundle of the household. This complementarity overturns the negative wealth effect of the shock and leads to an increase in consumption. Public wage shocks stimulate the production of the public good, which thus raises consumption given the complementarity of the latter with the public good. Because a wage government consumption shock produces an increase in government value added, which is biased toward non traded goods, along with an increase in private consumption that leads to a current account deficit, which further biases the government spending shock toward non tradables (since traded goods can be imported while non tradables must be produced by the home country), the non traded sector should be highly intensive in a shock to compensation of government employees. The relative price of non tradables thus appreciates significantly, which provides strong incentives to shift resources toward the non traded sector. Consequently, the share of non tradables increases sharply while the relative size of the traded sector declines substantially.

The results for a non-wage government consumption shock (solid black lines) are shown and contrasted with those for the baseline (solid blue lines) VAR model that includes total government consumption expenditure in Figures 30 and 31. To facilitate the interpretation of our results, we normalize the impulse responses so that government purchases of goods and services from the private sector or government consumption expenditure increases by one percentage point of GDP on impact. When we investigate the effects of a rise in government purchases of goods and services from the private sector, we estimate the effects on private activity. As can be seen in Figure 30, we find that a non-wage government consumption shock has a contractionary effect on economic activity as real GDP and total hours worked decline. To further understand the underlying mechanism leading to a contraction in private activity, we estimate a VAR model that includes non-wage government consumption expenditure, compensation of government employees, real GDP, hours worked, private investment and the real consumption wage; we find that...
an exogenous non-wage government consumption shock lowers compensation of government employees significantly. Thus the production of public goods falls. As long as private consumption and public goods are complements, the decline in the production of public goods along with the negative wealth effect imply that a non-wage government spending shock has a contractionary effect on economy activity. Because government purchases of goods and services from the private sector are biased toward non traded goods, the relative price of non tradables appreciates and the share of non tradables increases. However, as shown in Figure 31, the share of non tradables increases less following a non-wage government consumption shock than after a rise in compensation of government employees. One potential interpretation of this finding is that the non traded sector is relatively less intensive in non-wage government consumption shocks. As can be seen in Figure 31, it might explain the smaller appreciation in the relative wage of non tradables. An additional explanation is that the combined effect of the decline in the production of public goods and the complementarity between private and public goods crowds-out consumption which mitigates the current account deficit and thus makes the government spending shock less biased toward non tradables.

There is a growing literature exploring the impact of a government spending shock on private activity and contrasting the effects of a rise in government final wage consumption expenditure with those caused by an increase in government purchases of goods and services from the private sector. In particular, Cavallo [2005] and Li [2014] study the effects of shocks to different components in government spending, such as more money spent in the private sector or more expenditure in the government sector, by using a neoclassical and a new-keynesian model, respectively. In our paper, we are interested rather in the reallocation effects between a traded and non traded sector of a government spending shock in an open economy and merge the (non traded) private and government (non traded) sectors which form the non traded sector. A government spending shock can be viewed as the result of shocks to government consumption of non tradables and tradables or alternatively as the result of shocks to wage and non-wage government consumption. In our paper, we adopt the first view and shed some light on the role of imperfect mobility of labor between the traded and non traded sectors. As exemplified by the paper by Bermperoglou, Pappa, and Vella [2016], the objective of the literature which investigates the effects of a rise in compensation of government employees is very different from ours, as the authors aim at exploring the impacts on private activity, while we are interested in the reallocation effects across sectors.

Using data from COFOG (source: OECD), government consumption expenditure can be split between expenditure non tradeds goods ($G_N$) and expenditure on traded goods ($G_T$). In subsection D.1, we provide some evidence which reveal that government consumption on non traded goods contributes substantially to unforeseen changes in government consumption expenditure. We now investigate the contribution of sub-components of government consumption expenditure on non tradables following a government spending shock.

Collective vs. individual consumption expenditure on non tradables by the government. Government final consumption expenditure on non tradables can be divided into individual consumption expenditure and collective consumption expenditure. The split between individual and collective consumption is straightforward. In accordance with the COFOG classification, 'Health', 'Recreation and culture', 'Education', and 'Social protection' are provided for allocative and/or distributive motive and can be aggregated under the general heading Individual consumption expenditure of the Government which we denote by $G_{ind}$. As pointed out by the manual of COFOG, Individual consumption expenditure of the Government is close to the definition of private and semi-public goods. The remaining functions are classified as Collective consumption expenditure of the Government which we denote by $G_{coll}$. In terms of the economic theory, collective consumption expenditure by the government includes expenditure on National Defense, Public Order, R&D, ... and thus approximates the definition of pure public goods. As can be seen in column 5 of Table 19, the average government consumption expenditure on non tradables share of collective consumption expenditure by the government, i.e., $G_{coll}^N/G_N^N$, is almost 30%. It varies from a low of 21% in Denmark to a high of 53% in the US.
Figure 28: Aggregate and Sectoral Effects of an Unanticipated Government Final Wage Consumption Shock. Notes: Exogenous increase in government final (wage) consumption expenditure by 1% of GDP. VAR models include government final consumption expenditure (baseline) or government final wage consumption expenditure ordered first. Results for the baseline specification are displayed by solid blue lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the solid black line displays the results following a wage government consumption shock; sample: 15 OECD countries, 1970-2007 (except for BEL: 1976-2007), annual data.
Figure 29: Effects of an Unanticipated Government Final Wage Consumption Shock on Sectoral Composition. Notes: Exogenous increase in government final (wage) consumption expenditure by 1% of GDP. VAR models include government final consumption expenditure (baseline) or government final wage consumption expenditure ordered first. Results for the baseline specification are displayed by solid blue lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the solid black line displays the results following a wage government consumption shock; sample: 15 OECD countries, 1970-2007 (except for BEL: 1976-2007), annual data.
Figure 30:Aggregate and Sectoral Effects of an Unanticipated Shock to Government Purchases on Goods and Services from the Private Sector. Notes: Exogenous increase in government final (non-wage) consumption expenditure by 1% of GDP. VAR models include government final consumption expenditure (baseline) or government final consumption expenditure from the private sector ordered first. Results for the baseline specification are displayed by solid blue lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the solid black line displays the results following a non-wage government consumption shock; sample: 15 OECD countries, 1970-2007 (except for BEL: 1976-2007), annual data.
Figure 31: Effects of an Unanticipated Shock to Government Purchases on Goods and Services from the Private Sector on Sectoral Composition. Notes: Exogenous increase in government final (non-wage) consumption expenditure by 1% of GDP. VAR models include government final consumption expenditure (baseline) or government final consumption expenditure from the private sector ordered first. Results for the baseline specification are displayed by solid blue lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the solid black line displays the results following a non-wage government consumption shock; sample: 15 OECD countries, 1970-2007 (except for BEL: 1976-2007), annual data.
Defense vs. non-defense consumption expenditure on non tradables by the government. A potential alternative breakdown can be performed on the basis of evidence provided by the empirical literature adopting a (Ramey-Shapiro) narrative approach that considers major political events leading to large military buildups, see e.g., Ramey and Shapiro [1998], Ramey [2011]. One key finding documented by Ramey [2011] in an older version (2007) of the paper published in QJE is that shocks to defense spending account for a substantial share of the forecast error variance of total government spending. We thus conduct a robustness check by differentiating between defense spending, $G_{\text{def}}$, and non-defense spending, $G_{\text{nondef}}$. As can be seen in column 6 of Table 19, the average government consumption expenditure on non tradables share of defense expenditure by the government, i.e., $G_{\text{def}}^N/G^N$, is 8% only. While the share of defense expenditure in $G^N$ is lower than 10% for most of the countries in our sample, it averages 12% and 27% for the UK and the US, respectively.

How components of government final consumption expenditure on non tradables react to our identified government shock? We first estimate the first VAR model that includes government final consumption expenditure, real GDP, total hours worked, private investment, the real consumption wage, in order to identify unanticipated government spending shocks. Then, we estimate a VAR model in panel format on annual data that includes unanticipated government spending shocks, $\epsilon^G$, ordered first, government spending, $g$, individual, $g_{\text{ind}}^N$ (defense, $g_{\text{def}}^N$), and collective (non-defense, $g_{\text{nondef}}^N$), $g_{\text{coll}}^N$, government final consumption expenditure on non tradables, i.e., $z_{it}^N = [\epsilon_{it}^G, g_{it}, g_{k,i,t}^N, g_{-k,i,t}^N]$ where $k = \text{ind, def}$ and $-k = \text{col, nondef}$. Table 21 reports at various horizons the mean responses of the two components to the identified government spending shock for two alternative breakdowns. We normalize the impulse responses so that government spending rises by one percentage point of GDP on impact. Since the ordering of variables does not matter, we do not present the results for different orderings. Focusing first on the first two columns of Table 21, a government spending shock by 1 percentage point of GDP increases individual government final consumption expenditure on non tradables by 0.59 percentage point of GDP on impact (i.e., ‘Health’, ‘Recreation and culture’, ‘Education’, and ‘Social protection’). The spending shock also raises collective expenditure by 0.26 percentage point of GDP. The contribution of the response of individual expenditure to the rise in government expenditure on non tradables is displayed in the third column of Table 21 while the average contribution of this component is shown in the last line. The average contribution of individual expenditure to the rise in government final consumption expenditure on non tradables is 77% approximately and thus the contribution of collective expenditure is 23% only. Hence, while the rise in government consumption of non tradables is a major part of the government spending shock, the increase in individual expenditure accounts for more than three-quarters of increases in government consumption of non tradables. Impulse response functions for the two components of government final consumption expenditure on non tradables we generate following a rise in government spending by 1% of GDP are displayed in solid blue lines in the first row of Figure 32. The responses of both components are hump-shaped. We may notice that the rise in individual expenditure is more persistent.

Turning to the last two columns of Table 21, a government spending shock by 1 percentage point of GDP increases spending related to “Defense” by 0.07 percentage point of GDP, in line with its average share in government expenditure on non tradables shown in Table 19. As can be seen in the last line of Table 21, the average contribution of military spending to the rise in government final consumption expenditure on non tradables is 6% while the contribution of other expenditure is 94%. Impulse response functions for the two components of government final consumption expenditure on non tradables we generate following a rise in government spending by 1% of GDP are displayed in solid blue lines in the second row of Figure 32. The responses of both components are hump-shaped. We may notice that the increase in defense spending is much less persistent.
Figure 32: Effects of an Unanticipated Government Spending Shock on Components of Government Final Consumption Expenditure on Non Tradables and Tradables. Notes: Exogenous increase in government consumption by 1% of GDP. The government spending shock is identified by estimating a VAR model that includes real government final consumption expenditure, GDP (constant prices), total hours worked, private fixed investment, and the real consumption wage. We differentiate between collective and individual government final consumption expenditure on non tradables in the first row, and we distinguish between government final consumption expenditure on defense and non-defense in the second row (both classified as non tradables). The responses of components of government final consumption expenditure on non tradables to the identified government spending shock are displayed by solid blue lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; sample: 13 OECD countries, 1995-2015, annual data.
Table 21: Responses of Components of Government Consumption Expenditure on Non Tradables to Identified Government Spending Shock: Point Estimates

<table>
<thead>
<tr>
<th>Horizon</th>
<th>$z_{\text{ind}}^G = [\epsilon_{\text{ind}}, g_{\text{ind}}, g_{\text{col}}]$</th>
<th>Contribution of $G_{\text{ind}}$</th>
<th>$z_{\text{def}}^G = [\epsilon_{\text{def}}, g_{\text{def}}, g_{\text{nondef}}]$</th>
<th>Contribution of $G_{\text{def}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.590 0.260 69%</td>
<td>0.072 0.780 8%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.710 0.315 69%</td>
<td>0.090 0.927 9%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.564 0.314 64%</td>
<td>0.085 0.763 10%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.488 0.216 69%</td>
<td>0.047 0.623 7%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.454 0.140 76%</td>
<td>0.025 0.506 5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.410 0.099 81%</td>
<td>0.017 0.404 4%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.354 0.076 82%</td>
<td>0.014 0.324 4%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.301 0.059 84%</td>
<td>0.011 0.264 4%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.253 0.047 84%</td>
<td>0.010 0.217 4%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.212 0.037 85%</td>
<td>0.008 0.179 4%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.178 0.030 80%</td>
<td>0.007 0.149 4%</td>
<td></td>
<td></td>
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<tr>
<td>Mean</td>
<td>- - 77%</td>
<td>- - 6%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Horizon measured in year units. We generate impulse response functions by using a simple VAR with 2 lags, $z_{i,t}^G = [\epsilon_{i,t}^G, g_{i,t}, g_{\text{col},i,t}, g_{\text{noncol},i,t}]$ where $k = \text{ind, def}$ and $-k = \text{col, nondef}$. To identify the government spending shock $\epsilon_{i,t}^G$ we estimate the VAR model that includes aggregate variables, i.e., $z_{i,t} = [g_{i,t}, y_{i,t}, l_{i,t}, j_{i,t}, w_{C,i,t}]$, and adopt a Cholesky decomposition. The third and the sixth column of the table displays the contribution of the responses of individual government final consumption expenditure on non tradables and defense expenditure to the change in government consumption of non tradables while the last line displays the average contribution of the response of each component to the change in government consumption expenditure on non tradables. Data coverage: 1995-2015 for AUT, BEL, DNK, ESP, FIN, FRA, GBR, IRL, ITA, NLD, NOR, SWE and the USA. In all specifications, all variables are real and scaled by the working age population, and a linear trend is included.

D.3 Comparison of our Estimated Sectoral Effects with those Documented by Earlier Empirical Studies

Monacelli and Perotti (MP henceforth) [2008] and Benetrix and Lane [2010] (BL henceforth) conduct an empirical investigation of the sectoral output effect of a government spending shock by differentiating the traded from the non traded sector. These earlier studies give us the opportunity to address any potential discrepancy between their results and our own VAR evidence. Both MP [2008] and BL [2010] find empirically that a government spending shock produces a much larger increase in non traded output than in traded output. Our estimates corroborate this finding. Nevertheless, we find that traded output falls slightly on impact and remains below trend while the fiscal shock is in effect. MP [2008] also reports a slight decline in traded output on impact like us whereas the authors find that the traded output adjustment is bell-shaped and thus increases significantly above its trend after about two years. BL [2010] detects a positive response of traded output on impact followed by a gradual decline.

We emphasize below the main differences with MP’s [2008] and BL’s [2010] analysis. Monacelli and Perotti [2008] take a different approach from ours in two respects. First, they restrict attention to the US, using quarterly data from 1954 to 2006 while we consider a sample of 16 OECD economies over 1970-2007 and estimate a VAR model in panel format on annual data. Second, Monacelli and Perotti [2008] refer to industries producing services as non tradables and thus take a different approach from ours since we treat industries ‘Financial intermediation’ and ‘Transport and Communication’ as tradables instead of non tradables. In order to investigate whether this discrepancy is attributable to the sample, we re-estimate the VAR specification that includes government consumption, sectoral value added at constant prices, sectoral hours worked and the sectoral real consumption wage, on U.S. annual data over 1970-2007, keeping our own classification for tradables and non tradables. We generate impulse response functions which are normalized so that government consumption increases by 1 percentage point of GDP. The solid blue line in Figure 25 reports the results following a rise in total government consumption while the solid black line reports results following a rise in government consumption of non tradables. The first column shows results for the traded sector while the second column shows results for the
non traded sector. Like MP [2008], our evidence shows that traded output falls on impact and then increases. The adjustment in traded output displays an inverted U-shaped pattern while traded output remains above trend.

A close empirical analysis to ours is that performed by BL [2010]. Like BL [2010], we estimate a panel VAR on annual data and investigate the effects of a government spending shock (identified by adopting Blanchard and Perotti’s [2002] method) on traded and non traded output. Yet, our empirical analysis differs in four respects:

- **Sample.** First, regarding the sample, we use a panel of 16 OECD economies over 1970-2007 while BL [2010] consider a sample of 11 EMU countries over 1970-2005. Since we have eight countries in common with BL [2010]: Austria (AUT), Belgium (BEL), Finland (FIN), France (FRA), Ireland (IRL), Italy (ITA), the Netherlands (NLD) and Spain (ESP), we estimate VAR models by using this sample restricted to 8 EU countries over the period 1970-2005.\(^{68}\)

- **Classification T/N.** Second, BL [2010] treat ‘Transport, Storage and Communication’ and ‘Financial Intermediation’ as non traded rather than traded industries, and classify ‘Electricity, Gas and Water Supply’ in the traded sector while we treat this industry as non tradable.

- **VAR specification.** Third, our VAR specifications are different from those considered by BL [2010]. More specifically, to explore the size of the sectoral fiscal multiplier empirically, we consider a VAR specification \(z_{it}^j = \begin{bmatrix} g_{it}, y_{it}^T, l_{it}^J, w_{it}^C \end{bmatrix} \) with \(j = T, N\), while BL’s [2010] VAR model includes government consumption, traded value added, non traded value added. All variables are in real terms and logged. In terms of our own notations, the VAR specification considered by BL [2010] is: \(z_{it}^{BL} = \begin{bmatrix} g_{it}, y_{it}^T, y_{it}^N \end{bmatrix} \).

- **Construction of variables.** Finally, when we estimate the VAR model, all variables are measured in log, real terms and per capita (except for the current account), while prices and wages are logged, in line with the current practice. In contrast, quantities are not scaled by the working age population in BL [2010].

In the following, ‘CCR’ is a contraction of Cardi, Claeys and Restout while ‘BL’ is a contraction of Benetrix and Lane. Because BL [2010] find that traded output increases in the short-term while our VAR evidence indicates that traded output is slightly negative on impact, then declines and remains below trend while the fiscal shock is in effect, we investigate below the cause of this discrepancy in the estimated contraction in traded output. Since our empirical analysis differs along of four dimensions, namely the sample, the classification T/N, the VAR specification, and data construction, we run several experiments. While we contrast the dynamic effects of a government spending shock by 1 percentage point of GDP across the four alternative experiments in Figure 33-35, for clarity purposes, we first report the impact response of traded output in Table 22. Inspection of the first line of Table 22 shows immediately that the sample is the cause of the discrepancy, i.e., restricting the sample to 8 countries changes the response of traded output from negative to positive, regardless of the classification, VAR specification and the construction of variables. As shown below, the comparison of IRF across alternative scenarios also shows that the construction of variables plays a substantial role as the rise in traded output is more persistent when variables are not scaled by the population.

In each experiment, we focus on the responses of traded and non traded output and thus report neither IRF for sectoral hours worked nor real consumption wages to save space and for clarity purposes as BL [2010] do not consider these variables in their study.

- **Classification T/N and VAR specification.** First, we investigate whether the classification of industries as traded or non traded and/or the VAR specification is responsible for the differences in the short-run response of traded output to a fiscal shock. In columns 1-4 of Table 22 we consider our own sample and compare our baseline results (column 1) with those obtained when adopting BL’s classification

\(^{68}\) We excluded Germany, Greece, Portugal as these countries lack data for a number of aggregate variables.
Table 22: Impact Response of Traded Output to a Fiscal Shock

<table>
<thead>
<tr>
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<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(dY^T(0)/dG)</td>
<td>-0.032</td>
<td>-0.135</td>
<td>-0.109</td>
<td>-0.146</td>
<td>0.579*</td>
<td>0.231</td>
<td>0.340</td>
<td>0.135</td>
<td>0.020</td>
<td>-0.126</td>
<td>-0.102</td>
<td>-0.148</td>
<td>0.625*</td>
<td>0.299</td>
<td>0.380</td>
</tr>
<tr>
<td>Nb of countries</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
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<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Classification</td>
<td>CCR</td>
<td>BL</td>
<td>CCR</td>
<td>BL</td>
<td>CCR</td>
<td>BL</td>
<td>CCR</td>
<td>BL</td>
<td>CCR</td>
<td>BL</td>
<td>CCR</td>
<td>BL</td>
<td>CCR</td>
<td>BL</td>
<td>CCR</td>
</tr>
<tr>
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<td>CCR</td>
<td>BL</td>
<td>BL</td>
<td>CCR</td>
<td>CCR</td>
<td>BL</td>
<td>BL</td>
<td>CCR</td>
<td>BL</td>
<td>CCR</td>
<td>BL</td>
<td>CCR</td>
<td>BL</td>
<td>CCR</td>
<td>BL</td>
</tr>
<tr>
<td>Population scaling</td>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

Notes: Last period refers to the last observation of the period used to estimate the VAR (both samples start in 1970). The sectoral classification CCR (BL resp.) refers to the sectoral classification used in the present (BL’s resp.) paper. The CCR specification is the model with \(z^C_{it} = [g_{it}, y^T_{it}, l^T_{it}, w^T_{C, it}]\) while the BL specification indicates the VAR model with \(z^B_{it} = [g_{it}, y^T_{it}, y^N_{it}]\). In all estimations, we include a country-fixed effect and a country-specific linear trend and two lags are specified in the VAR model. Population scaling indicates if quantities variables included in the VAR models are scaled or not by the working age population. * denotes significance at 10% level.
Figure 33: Effects of an Unanticipated Government Spending Shock on Sectoral Variables. Notes: Exogenous increase in government consumption by 1% of GDP. Sample: 16 OECD Economies (1970-2007). The first column shows results for our classification T/N along with our VAR specification. The second, third and four columns show results when considering BL’s [2010] classification, or VAR specification, or both, respectively. All quantities variables included in the VAR models are scaled by the working age population. Baseline: $N = 16$, classification CCR, VAR model $z_{it} = [g_{it}, Y_{it}, y_{it}^{T}, y_{it}^{N}]$. Results for the baseline specification are displayed by solid blue lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling. Results for an alternative T/N classification or/and VAR specification are displayed by solid black lines.

T/N (column 2), BL’s VAR specification [2010] (column 3), BL’s construction of variables (column 4). The solid blue line in column 1 of Figure 33 shows our baseline results. In columns 2-4 of Figure 33, we contrast baseline results (displayed in the solid blue line) with those obtained when adopting BL’s classification for T/N or their VAR specification (solid black line). Impact responses reported in columns 1-4 along with the dynamic responses show that traded output declines in the short-run and the discrepancy between the baseline scenario and alternatives is not statistically different. Thus neither the classification T/N nor the VAR specification seem to be responsible for the discrepancy in the short-term response of traded output to a fiscal shock.

- **Sample.** Second, we restrict the set of countries to those included in BL’s [2010] sample. Columns 5-8 of Table 22 show the impact response of traded output when we restrict the set of countries to eight and run a number of experiments with respect to the classification T/N and VAR specification. The solid blue line in the first column of Figure 34 shows the IRF in the baseline scenario, i.e., when the sample covers 16 OECD countries, while the solid black line in columns 2-4 displays the results when the sample covers 8 countries. Columns 5-8 of Table 22 show that traded output increases in the short-run when we consider a similar set of countries as BL. As can be seen in the solid blue line in Figure 34, traded output increases on impact whether we consider our own or BL’s VAR specification, or whether we consider our own or BL’s classification for T/N. Hence, alternative scenarios do not lead to substantially different results. While traded output increases in all cases, the expansionary effect
on traded output is not too persistent as it declines below trend after two years approximately.

- **Construction of variables.** Third, as it is common in the literature, all quantities are scaled by the working age population in order to remove trend. Since BL [2010] do not indicate whether they express quantities per capita, we run alternative experiments for both samples: our panel of 16 OECD economies over 1970-2007 and the 8 EMU countries we have in common with BL over the period 1970-2005. In all experiments, we contrast previous results when quantities are divided by the working age population with those obtained without expressing variables per capita. We consider our and BL’s VAR specification, then our and BL’s classification for T/N. The results for alternative scenarios are reported in columns 9-16 of Table 22. The conclusion that emerges is that when the set of countries is restricted to eight (see the last four columns), traded output increases substantially more when quantities are not scaled by the population. The dynamic effects of a government spending shock in Figure 35 reveal that the decline in traded output tends to be much less pronounced with a sample of 16 countries while the rise in traded output tends to be much more persistent when quantities are not scaled by the population and we restrict the set of countries to eight.

Since the set of countries matters for the response of traded output, we investigate the extent to which the responses of sectoral shares are modified when we restrict our sample to eight EU countries. Responses at various horizons are reported in Table 23. Reassuringly, our main conclusion holds: a government spending shock lowers significantly the share of...
<table>
<thead>
<tr>
<th>VAR model: $z_{iit}^j = \left[ g_{iit}, y_{iit}^j, l_{iit}, w_{C,iit} \right]$</th>
<th>VAR model: $z_{iit}^{BL} = \left[ g_{iit}, y_{iit}^T, y_{iit}^N \right]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classification: CCR</td>
<td>Classification: BL</td>
</tr>
<tr>
<td>Nb of countries: 16</td>
<td>Nb of countries: 16</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) $Y^T$</td>
<td>(b) $Y^T$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) $Y^T$</td>
<td>(d) $Y^T$</td>
</tr>
<tr>
<td>(e) $Y^N$</td>
<td>(f) $Y^N$</td>
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<tr>
<td></td>
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<tr>
<td>(g) $Y^N$</td>
<td>(h) $Y^N$</td>
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<td>(j) $Y^T$</td>
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<td>(k) $Y^T$</td>
<td>(l) $Y^T$</td>
</tr>
<tr>
<td>(m) $Y^N$</td>
<td>(n) $Y^N$</td>
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<td></td>
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</tr>
<tr>
<td>(o) $Y^N$</td>
<td>(p) $Y^N$</td>
</tr>
</tbody>
</table>

Figure 35: Effects of an Unanticipated Government Spending Shock on Sectoral Variables. Notes: Exogenous increase in government consumption by 1% of GDP. The first two rows show the results for a panel of 16 OECD countries over 1970-2007 while the last second rows show results for a panel of 8 OECD countries over 1970-2005. The first column shows the results for our T/N classification along with our VAR specification. The second, third and four columns show results when considering BL’s [2010] classification, or VAR specification, or both, respectively. Blue line: the quantity variables included in the VAR models are scaled by the working age population; shaded areas: 90 percent confidence intervals; black line: quantity variables included in the VAR models are not scaled by the working age population.
Table 23: Short-run Responses of Sectoral Variables to a Fiscal Shock (1970-2007, 8 countries)

<table>
<thead>
<tr>
<th>Horizon</th>
<th>$Y^T/Y$</th>
<th>$Y^N/Y$</th>
<th>$L^T/L$</th>
<th>$L^N/L$</th>
<th>$Y^T/Y^N$</th>
<th>$L^T/L^N$</th>
<th>$P$</th>
<th>$\Omega$</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.206</td>
<td>0.195</td>
<td>-0.150</td>
<td>0.151</td>
<td>-0.584</td>
<td>-0.481</td>
<td>1.672</td>
<td>1.247</td>
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<td>1</td>
<td>-0.511</td>
<td>0.370</td>
<td>-0.426</td>
<td>0.407</td>
<td>-1.253</td>
<td>-1.301</td>
<td>2.764</td>
<td>1.853</td>
</tr>
<tr>
<td>2</td>
<td>-0.639</td>
<td>0.557</td>
<td>-0.553</td>
<td>0.527</td>
<td>-1.513</td>
<td>-1.689</td>
<td>2.987</td>
<td>1.909</td>
</tr>
<tr>
<td>3</td>
<td>-0.676</td>
<td>0.656</td>
<td>-0.588</td>
<td>0.565</td>
<td>-1.553</td>
<td>-1.805</td>
<td>2.697</td>
<td>1.485</td>
</tr>
</tbody>
</table>

Classification: CCR CCR CCR CCR CCR CCR CCR CCR

Notes: entries report, for selected horizons, the instantaneous response of sectoral variables to an increase in government spending by 1% of GDP. Responses of $Y^T/Y$ and $L^T/L$, $j = T, N$, stem from the VAR that includes $z_{it} = [g_{it}, \nu_{Y,j}^{it}, \nu_{L,j}^{it}, w_j^{C,it}]$. The response of relative labor $L^T/L^N$ (relative output $Y^T/Y^N$ resp.) is estimated from a 3-variable VAR that includes government spending, relative labor (relative output resp.) and the relative wage of non tradables $\Omega$ (relative price of non tradables $P$ resp.). Sample: 1970-2005 and 8 countries (AUT, BEL, ESP, FRA, FIN, IRL, ITA and NLD). In all estimations, two lags are included in the VAR model. All quantities are scaled by the working age population.

tradables and increases the relative size of the non traded sector. Figure 36 shows the dynamic effects for a sample of eight countries. Again, all our results hold. The share of non tradables increases significantly over the first four years while the share of tradables declines substantially. We also find empirically that both the relative price and the relative wage of non tradables appreciate. We may nevertheless note some differences. In particular, the magnitude of the change in relative sector size is mitigated compared with the baseline case where we consider 16 countries.

To conclude, both the dataset and the construction of variables seem responsible for the discrepancy in the response of traded output in our empirical analysis and that documented by BL [2010]. When the set of countries is restricted to eight, traded output increases on impact, then declines rapidly below trend after two years. Keeping the same set of eight countries, when quantities are not scaled by the working age population, the increase in traded output is more pronounced and more persistent, as it takes about four years before traded output falls below trend. Whether the sample is restricted to eight or sixteen countries, non traded output increases substantially relative to traded output and thus our conclusion according to which government spending shocks are biased toward non traded goods holds. Importantly, in all scenarios, the share of non tradables in employment and real GDP rise which implies that non traded industries which are more intensive in the government spending shock experience a labor inflow.

D.4 Comparison of Aggregate Effects of Government Spending Shocks with those Documented in the Existing Literature

In the previous subsection, we address the main differences between the sectoral effects we document in our paper with those documented by past studies, in particular Benetrix and Lane [2010]. In this subsection, we contrast our empirical results on aggregate effects of government spending shocks with those documented in the empirical literature.

Overall, our panel VAR evidence for aggregate variables is well in line with that reported in earlier studies. In particular, our estimate of an aggregate output multiplier of government spending being lower than one on impact accords well with earlier findings. For example, Corsetti et al. [2012], who use a panel of 17 OECD countries for the period 1975-2008, report an increase in aggregate output by about 0.7 percentage points on impact. As documented in Corsetti et al., an increase in government spending leads to a protracted decline in private investment. The fall in the current account following a rise in public purchases is also in line with earlier findings. Although the empirical literature commonly uses net exports, replacing the current account with the trade balance leads to similar results. Beetsma, Giuliodori and Klaassen [2008] report a fall in the trade balance by 0.5% of GDP for a panel of 11 Euro Area Members while Corsetti et al. [2012] document a decline in net exports on impact which is very similar to ours.

Regarding labor market variables, our evidence reveal that a government spending shock increases hours worked, a finding that again squares well with conventional wisdom and
Figure 36: Effects of an Unanticipated Government Spending Shock on Sectoral Composition. **Sample:** 8 EU Economies over 1970-2005. **Notes:** Exogenous increase in government consumption by 1% of GDP. Results for the baseline specification are displayed by solid lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling.
earlier empirical studies, see e.g., Pappa [2009], Ramey [2011]. While there is no debate in the literature about the empirical facts mentioned above, the response of the real wage to a government spending shock is not a clear-cut result. As summarized by Nekarda and Ramey [2011], the literature adopting Blanchard and Perotti’s [2002] approach to identifying fiscal shock reports an increase in the real consumption wage while application of the ‘narrative’ approach reveals that real consumption wages tend to fall in response to military spending shocks, see e.g., Ramey [2011]. While we find a significant rise in the real consumption wage on impact, our panel VAR evidence indicates that it is followed by a rapid decline. In this regard, our result can be viewed as halfway between these two strands of literature applying different identification schemes to U.S. data.

D.5 Anticipation Effects

In this subsection, we address a major concern regarding the evidence on fiscal transmission we document in the main text due to anticipation effects. As argued by Ramey [2011], Blanchard and Perotti’s [2002] approach to identifying government spending shocks in VAR models may lead to incorrect timing of the identified fiscal shocks. If the fiscal shock is anticipated in advance, agents may have modified their decisions before the rise in government spending actually materializes. Consequently, when the fiscal shock is anticipated, and thus VAR approach captures the shocks too late, it misses the initial changes in variables that occur as soon as the news is learned. As a robustness check, we conduct below an investigation of the potential presence of anticipation effects which draws heavily on previous analysis performed by Beetsma and Giuliodori [2011] and Born, Juessen and Müller [2013]. It is worth mentioning that, as argued by Beetsma and Giuliodori [2011], the effects of anticipation of fiscal policy changes by market participants should be less relevant when using annual data since the fiscal shock is less likely to be anticipated one year before the rise in government spending is implemented than one quarter before it actually takes place.

Drawing on previous studies, we conduct three robustness exercises to explore the potential implications of anticipation effects:

- Like Beetsma and Giuliodori [2011], we run Granger-causality tests in order to investigate whether fiscal forecasts have any predictive power for the identified government consumption shocks.

- A second way to deal with the complications of possibly anticipated government spending shocks is to augment the VAR specification with the forecasts for government spending, see Beetsma and Giuliodori [2011], Born, Juessen and Müller [2013].

- A third route followed by Beetsma and Giuliodori [2011], Brückner and Pappa [2012] is to augment the baseline VAR specification with forward-looking variables such as short-term interest rates, the log of the GDP deflator, stock prices, or oil prices in order to control for fiscal-foresight effects.

In the following, we conduct an investigation of the potential presence of anticipation effects by performing the three robustness exercises mentioned above. To perform the first robustness exercise, we use a dataset constructed by Born, Juessen and Müller [2013] that contains time series for forecasts for government spending from the OECD. Since the OECD prepares forecasts in June and December for each year, the dataset contains semi-annual observations for the period running from 1986 to 2011. To investigate the extent to which evidence on fiscal transmission can be affected by anticipation effects, we use the December forecasts. Data are available over the period 1986-2007 for CAN, FRA, GBR, ITA, JPN and USA, 1997-2007 for AUS, BEL, DNK, ESP, FIN, IRL, NLD, NOR, SWE and 1997-2004 for AUT. We denote \( f_{t+1} \) the period \(-t\) forecast of the growth rate of government spending for the next year \(t+1\). This variable is constructed as the logarithm of real government consumption forecast minus the logarithm of real government consumption.

Before exploring empirically the VAR model augmented with forecasts for government spending, we first test whether the anticipation problem is relevant in our annual VAR.

\footnote{We thank Born, Juessen and Müller [2013] for providing this dataset to us.}
Table 24: Granger Causality Tests (p-values)

<table>
<thead>
<tr>
<th>Country</th>
<th>p-value</th>
<th>Country</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>0.545</td>
<td>GBR</td>
<td>0.042</td>
</tr>
<tr>
<td>AUT</td>
<td>0.090</td>
<td>IRL</td>
<td>0.752</td>
</tr>
<tr>
<td>BEL</td>
<td>0.884</td>
<td>ITA</td>
<td>0.017</td>
</tr>
<tr>
<td>CAN</td>
<td>0.218</td>
<td>JPN</td>
<td>0.000</td>
</tr>
<tr>
<td>DNK</td>
<td>0.617</td>
<td>NLD</td>
<td>0.723</td>
</tr>
<tr>
<td>ESP</td>
<td>0.532</td>
<td>NOR</td>
<td>0.905</td>
</tr>
<tr>
<td>FIN</td>
<td>0.817</td>
<td>SWE</td>
<td>0.761</td>
</tr>
<tr>
<td>FRA</td>
<td>0.073</td>
<td>USA</td>
<td>0.884</td>
</tr>
<tr>
<td>Whole Sample</td>
<td>0.258</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: the null hypothesis that $f^{t+1}_{c_{it}}$ does not Granger-cause $e_{it}^{G}$ is rejected if p-value ≤ 0.05 at a 5% significance level.

We run a Granger-causality test. In particular, we test whether fiscal forecasts $f^{t+1}_{c_{it}}$ have any predictive power for the government spending shocks estimated from our benchmark model $z_{it} = [g_{it}, y_{it}, l_{it}, j_{it}, wc_{it}, w_{C,it}]$. To implement the test of whether $f^{t+1}_{c_{it}}$ Granger-causes the VAR-based government spending shocks $e_{it}^{G}$, we run the following regression:

$$
\begin{align*}
\epsilon_{it}^{G} &= \alpha_i + \sum_{k=1}^{p} a_k \epsilon_{it-k}^{G} + \sum_{k=1}^{p} b_k f^{t+1}_{c_{it-k}} + u_{it},
\end{align*}
$$

(114)

where $p$ is the autoregressive lag length and $u_{it}$, the error term. With respect to (114), in country $i$, the test of the null hypothesis that $f^{t+1}_{c_{it}}$ does not Granger cause $e_{it}^{G}$ is a $F$ test of the form: $H_0 : b_1 = b_2 = \cdots = b_p = 0$. By not rejecting the null, one may conclude that VAR shocks $e_{it}^{G}$ are strictly exogenous to the dependent variable $f^{t+1}_{c_{it}}$. Table 24 reports results.\(^{70}\) The results for individual countries show that, with the exception of GBR, ITA and JPN, there is no causality running from GBR, ITA and JPN, there is no causality running from the VAR-based government spending shocks $z_{it} = [g_{it}, y_{it}, l_{it}, j_{it}, wc_{it}, w_{C,it}]$.

Following Born et al. [2013], we include the forecasts for real government spending growth within the VAR model (ordered after $g_{it}$). More specifically, we augment the four VAR specifications as follows:

- Estimating the magnitude of the aggregate fiscal multiplier: $z_{it} = [g_{it}, f^{t+1}_{c_{it}}, y_{it}, l_{it}, j_{it}, wc_{it}, w_{C,it}]$.
  In the second specification we replace private investment with the current account expressed in percentage of GDP, $ca_{it}$.
- Estimating the magnitude of the sectoral fiscal multiplier: $z_{it}^{j} = [g_{it}, f^{t+1}_{c_{it}}, y_{it}, l_{it}, j_{it}, wc_{it}, w_{C,it}]$ with $j = T, N$.
- Estimating the change in relative sector size: $z_{it}^{Sj} = [g_{it}, f^{t+1}_{c_{it}}, y_{it}, l_{it}, j_{it}, wc_{it}, w_{C,it}]$ with $j = T, N$.
- Estimating the relative price and relative wage effects: $z_{it}^{P} = [g_{it}, f^{t+1}_{c_{it}}, y_{it}^T - y_{it}^N, p_{it}]$ and $z_{it}^{W} = [g_{it}, f^{t+1}_{c_{it}}, y_{it}^T - y_{it}^N, \omega_{it}]$, respectively.

In order to make our evidence comparable with that without government spending forecasts, we estimate the four VAR specifications over the same period, i.e., from 1986 to 2007. Overall, Figures 37 and 38 show that all our results hold regardless of the period considered. We nevertheless may notice that real GDP, total hours worked and the real consumption wage increase less over the period 1986-2007 than over the period 1970-2007. Turning to sectoral variables, we find that the rise in non traded wages relative to traded

\(^{70}\)Given the relatively short time horizon for the variable $f^{t+1}_{c_{it}}$, regression (114) is estimated over the period 1997-2007. Therefore, we choose $p = 1$. Data limitation prevents the use of larger values for $p$. 

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Figure 37: Aggregate and Sectoral Effects of an Unanticipated Government Spending Shock over 1986-2007. Notes: Exogenous increase in government consumption by 1% of GDP. Baseline sample: 16 OECD countries, 1970-2007; restricted sample: 17 OECD countries, 1986-2007. Results for baseline sample are displayed by solid blue lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the solid black line reports results for the restricted sample.

wages is slightly less pronounced than that over the baseline period 1970-2007. This result could suggest that workers’ mobility costs have been reduced in 1986-2007 compared with those prevailing over 1970-1985.
Figure 38: Sectoral Composition Effects of an Unanticipated Government Spending Shock over 1986-2007. Notes: Exogenous increase in government consumption by 1% of GDP. Baseline sample: 16 OECD countries, 1970-2007; restricted sample: 17 OECD countries, 1986-2007. Results for baseline sample are displayed by solid blue lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the solid black line reports results for the restricted sample.
Figures 39 and 40 show IRF when augmenting each baseline VAR specification with forecasts for government spending. The solid blue line reports the results for the baseline case without forecasts for government spending growth, while the solid black line displays the results for the VAR model augmented with forecasts. In both cases, the VAR model is estimated over the period running from 1986 to 2007. Overall, it turns out that differences are rather moderate and anticipation effects thus play a limited quantitative role in the dynamic adjustment to a government spending shock. Yet, some differences can be noticed. Once anticipation effects are controlled for, the rise in hours worked displays more persistence and is more in line with what we obtained numerically. In this regard, this conclusion squares well with estimates documented by Born et al. [2013], except that the authors detect a smaller increase in real GDP when anticipation effects are not controlled for, while this finding applies to hours worked in our case as differences in the dynamic effects for real GDP are almost insignificant quantitatively. Likewise, impact responses of sectoral outputs are also very similar to impact effects computed numerically once the VAR model is augmented with forecasts for government spending. More specifically, the decline in empirically-estimated traded output is found to be more pronounced while the increase in non traded output is slightly smaller. We may also note that the reallocation effects are slightly greater as changes in sectoral shares in real GDP are somewhat more pronounced while the relative wage of non tradables increases less than in the baseline case. However, the responses of the sectoral shares in labor, \( L_j^s / L \), are merely affected once the anticipation effects are controlled for.

As emphasized above, to ensure that our results are not subject to the fiscal foresight problem, we carry out a number of robustness exercises by augmenting the baseline VAR model with forward-looking variables along the lines of Beetsma and Giuliodori [2011], and Brückner and Pappa [2012]. Since one major contribution in this paper is to shed some light on the reallocation effects of a government spending shock, we restrict attention to the dynamic responses of sectoral shares. Columns 1 and 2 of Figure 41 show results when augmenting the baseline VAR model with the forecast for the budget balance-GDP ratio which we denote by \( bbr_t^{t+1} \). The year-ahead forecasts are taken from the Commission’s autumn forecasts, which are published in November.\(^7\) In terms of VAR specifications shown above, we replace \( f_c^{t+1} \) with \( bbr_t^{t+1} \). It is worth mentioning that Beetsma and Giuliodori [2011] find empirically that the balance-ratio forecast has strong predictive power for \( g_t \). Time series for the general government balance to GDP ratio forecast (one year ahead) are available for BEL (1971-2007), DNK (1977-2007), ESP (1987-2007), FRA (1970-2007), GBR (1974-2007), IRL (1974-2007), ITA (1970-2007), NLD (1970-2007). The impulse response functions shown in the solid black line, reported in columns 1-2 of Figure 41, are similar, if not identical, to those under the baseline shown in the solid blue line and thus do not deserve more comments. As an additional, and final, test we augment the baseline model with the log of nominal stock prices denoted by \( sp_t \). Time series for stock prices are taken from OECD Main Economic Indicators. Data are available over 1970-2007 for AUS, AUT, CAN, FIN, FRA, GBR, IRL, ITA, JPN, NLD, SWE and USA, 1983-2007 for DNK, 1985-2007 for ESP and 1986-2007 for BEL and NOR. Again, the discrepancy in the estimated responses is quite moderate.

D.6 Government Spending Shock across VAR Specifications

An additional concern is related to the government spending shock which may vary across alternative VAR specifications. First, panel VAR evidence can be misleading if the spending shock significantly varies across VAR specifications which would make their interpretation difficult; for example, the sectors would not respond to the same shock and thus the comparison of sectoral output responses would become less relevant. In other words, while the responses are qualitatively the same, their magnitude might quantitatively be different with respect to an ideal configuration where we would have considered the same shock across all VAR specifications. Second, when we calibrate and simulate the model, we consider

\(^7\)We thank Fioramanti et al. [2016] for providing this dataset to us.
Figure 39: Aggregate and Sectoral Effects of an Unanticipated Government Spending Shock: Anticipation Effects. Notes: Exogenous increase in government consumption by 1% of GDP. Baseline sample: 16 OECD countries, 1986-2007. Solid blue line: Results for baseline case without controlling for anticipation effects; the solid black line reports the results from estimates of VAR models with forecast of government spending growth (ordered after $g_t$).
Figure 40: Sectoral Composition Effects of an Unanticipated Government Spending Shock: Anticipation Effects. Notes: Exogenous increase in government consumption by 1% of GDP. Baseline sample: 16 OECD countries, 1986-2007. Solid blue line: Results for baseline case without controlling for anticipation effects; the shaded area indicates the 90 percent confidence bounds obtained by bootstrap sampling; the solid black line reports the results from estimates of VAR models with forecast of government spending growth (ordered after \( g_t \)).
Figure 41: Sectoral Composition Effects of an Unanticipated Government Spending Shock: Alternative Forecasts Measures and Anticipation Effects. Notes: Exogenous increase in government consumption by 1% of GDP. Baseline sample: 8 OECD countries (first two columns) and 16 OECD countries (last two columns), 1970-2007. Solid blue line: Results for the baseline case without controlling for anticipation effects; the shaded area indicates the 90 percent confidence bounds obtained by bootstrap sampling; the solid black line reports the results from estimates of VAR models with balance-ratio forecast (first two columns) or stock prices (last two columns).
Table 25: Correlation Matrix between Structural Government Spending Shocks across VAR models

<table>
<thead>
<tr>
<th>VAR models</th>
<th>1-VAR</th>
<th>2-VAR</th>
<th>3-VAR</th>
<th>4-VAR</th>
<th>5-VAR</th>
<th>6-VAR</th>
<th>7-VAR</th>
<th>8-VAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-z_{it} = \begin{bmatrix} g_{it}, y_{it}, l_{it}, j_{it}, w_{it}, \end{bmatrix}</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-z_{it} = \begin{bmatrix} g_{it}, y_{it}, l_{it}, c_{it}, w_{it}, \end{bmatrix}</td>
<td>0.976</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-z_{it} = \begin{bmatrix} g_{it}, y_{it}, l_{it}, w_{it}, \end{bmatrix}</td>
<td>0.989</td>
<td>0.968</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-z_{it} = \begin{bmatrix} g_{it}, y_{it}, l_{it}, w_{it}, \end{bmatrix}</td>
<td>0.986</td>
<td>0.970</td>
<td>0.978</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-z_{it}^{S,T} = \begin{bmatrix} g_{it}, v_{it}^{Y,T}, v_{it}^{L,F}, w_{it}, \end{bmatrix}</td>
<td>0.951</td>
<td>0.938</td>
<td>0.970</td>
<td>0.954</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6-z_{it}^{S,N} = \begin{bmatrix} g_{it}, v_{it}^{Y,N}, v_{it}^{L,N}, w_{it}, \end{bmatrix}</td>
<td>0.956</td>
<td>0.944</td>
<td>0.975</td>
<td>0.958</td>
<td>0.996</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7-z_{it}^{P} = \begin{bmatrix} g_{it}, y_{it}^{P}, \end{bmatrix}</td>
<td>0.958</td>
<td>0.948</td>
<td>0.974</td>
<td>0.955</td>
<td>0.982</td>
<td>0.982</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>8-z_{it}^{W} = \begin{bmatrix} g_{it}, y_{it}^{W} - y_{it}^{N}, \omega_{it}, \end{bmatrix}</td>
<td>0.947</td>
<td>0.935</td>
<td>0.967</td>
<td>0.954</td>
<td>0.995</td>
<td>0.994</td>
<td>0.979</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Notes: The first column shows the VAR model while column 2 through 9 shows the correlation between structural government spending shocks across VAR models.

the same government spending shock to compute the dynamic responses of aggregate and sectoral variables. Thus, the comparison of theoretical and empirical impulse response functions can be misleading as well. In this subsection, we assess the legitimacy of investigating aggregate and sectoral effects by identifying government spending shocks on the basis of alternative VAR specifications. A first and simple check of the discrepancy between estimated structural government spending shocks across the VAR models can be performed by estimating the correlation between structural government spending shocks. The first column of Table 25 is the most interesting as it shows the correlation between structural government spending shocks whose identification is based on the first VAR model and those identified on the basis of alternative VAR models. The correlation varies from a low of 0.947 for the VAR model $z_{it}^{W}$ that includes the relative wage of non tradables to a high of 0.986 and 0.989 for the VAR model $z_{it}^{P}$ that includes sectoral output. Overall, given the high value of correlation between structural government spending shocks across VAR models, one may reasonably expect the discrepancy in the estimated responses caused by slight differences in estimated structural government spending shocks to be small.

Before investigating the extent of the discrepancy in the estimated government spending shock across VAR specifications, we find it useful to begin with a quick refresher on the computation of IRF. For convenience, we repeat the structural VAR model which can be written (abstracting from the constant term) as:

$$AZ_{it} = \sum_{k=1}^{p} B_k Z_{it-k} + \epsilon_{it}. \quad (115)$$

In order to uniquely recover the structural form, we have to impose assumptions on matrix $A$. Following Blanchard and Perotti [2002], we assume that government consumption does not react on impact to other shocks in the system. We thus adopt a Cholesky decomposition in which government spending is ordered before the other variables. Technically, matrix $A$ is thus lower-triangular. Once the restriction is imposed and $A$ has been recovered, the structural form (115) can be written as follows:

$$Z_{it} = \sum_{k=1}^{p} A^{-1} B_k Z_{it-k} + A^{-1} \epsilon_{it}. \quad (116)$$

To avoid unnecessary complications, let us assume one lag so that $p = 1$. Iterating (116) backward leads to:

$$Z_{it} = \sum_{h=0}^{H} B_1^h A^{-1} \epsilon_{it-h} \quad (117)$$

Setting $\phi_h = B_1^h A^{-1}$ with elements $\phi_{lm}(h)$, the vector moving average representation of the
reduced VAR form (116) now reads:

$$Z_{i,t} = \sum_{h=0}^{H} \phi_h \epsilon_{i,t-h}. \tag{118}$$

Letting $e_{i,t}$ be the vector of residuals in the reduced form, we have $e_{i,t} = A^{-1} \epsilon_{i,t}$ and $\epsilon_{1,i,t}$ is the structural government spending shock. VAR model can be estimated in its reduced form by using OLS. OLS provide estimates of elements of $A^{-1} B_t$ along with the variance-covariance matrix which allows the computation of impulse response functions. Since estimates should depend on the VAR specification, the structural government spending shock should be different across VAR specifications. However, this potential problem is mitigated as we normalize the spending shock across all VAR specifications to a rise in government spending by one percentage point of GDP. Such a normalization thus makes impact responses of economic variables directly comparable quantitatively across VAR models. In this regard, as we base the greatest part of our analysis and discussion of fiscal transmission on impact effects, potential problems caused by differences in shape and the magnitude of the fiscal shock could be mitigated. However, even if the magnitude and the shape of the government spending shock is similar across VAR specifications, different VAR models could pickup different structural government spending shocks. Moreover, the endogenous response of government spending to an exogenous fiscal shock normalized to 1% of GDP may vary across VAR specifications while in the quantitative analysis, we compute the theoretical impulse response functions by considering the same exogenous dynamic adjustment of government spending. In order to investigate the extent of the discrepancy in the estimated responses caused by potentially different government spending shocks across VAR specifications, we proceed in four stages:

- **First**, we compare endogenous responses of government spending across all VAR specification in Figure 42. The shaded area corresponds to the 90% confidence bound for the original VAR specification $z_{it} = [g_{it}, y_{it}, l_{it}, j_{it}, w_{C,it}]$. Then we test whether the point estimate for the response of government spending in the first VAR model is significantly different from the point estimate for other VAR models.

- **Second**, once we identified the government spending shock in the first VAR model that includes private investment, $z_{it} = [g_{it}, y_{it}, l_{it}, j_{it}, w_{C,it}]$, we augment each VAR model with the identified government spending shocks, ordered first, and use shocks to government spending (identified with the Cholesky decomposition) as the fiscal shock, see e.g., Ramey [2011] who adopts a similar procedure to identify military spending shocks. Then, we contrast the responses for the baseline model with those for augmented VAR models.

- **Third**, we identify the government spending shock by estimating the first VAR model that includes private investment, i.e., $z_{it} = [g_{it}, y_{it}, l_{it}, j_{it}, w_{C,it}]$ by using quarterly data, and we annualize the estimated shock. Then, we estimate each VAR model augmented with the same annualized identified government spending shock on annual data. Quarterly data are taken from the OECD Economic Outlook database. We apply our method for the largest available subset of the countries in our current sample for which we have sufficient quarterly fiscal data. The sample includes AUS, CAN, FRA, JPN, NLD, SWE, GBR and USA, for which quarterly and annual macroeconomic data are available: AUS (1979Q1-2007Q4), CAN (1981Q1-2007Q4), FRA (1973Q1-2007Q4), JPN (1970Q1-2007Q4), NLD (1970Q1-2007Q4), SWE (1975Q1-2007Q4), GBR (1972Q1-2007Q4) and USA (1970Q1-2007Q4). We provide more details below about the data used in this analysis:

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72Lack of data and/or short time period prevent the inclusion of others countries in the sample. In details: AUT (no data for investment), BEL (no data for government spending, output and the real wage), DNK (no data for the real wage), ESP (no data for investment and the real wage), FIN (no data for hours worked), IRL (no data for investment and quarterly series for government spending and output start in 1990Q1), ITA (no data for investment) and NOR (no data for hours worked and the real wage).
- **Government spending**: real government final consumption expenditure (CGV). Source: OECD Economic Outlook Database.

- **Gross domestic product**: real gross domestic product (GDPV). Source: OECD Economic Outlook Database.

- **Labor**: hours worked per employee, total economy. Source: OECD Economic Outlook Database.

- **Private investment**: real private non-residential gross fixed capital formation (IBV). Source: OECD Economic Outlook Database.

- **Real wage**: nominal wage rate (total economy) divided by the consumer price index (CPI). Sources: OECD Economic Outlook Database for the nominal wage and OECD Prices and Purchasing Power Parities for the consumer price index.

All data are seasonally adjusted and, except for the real wage, are divided by the population. To obtain variables in per capita terms we use the working age population (15-64 years old) provided by OECD Economic Outlook Database (data for the population at quarterly frequency were interpolated from annual data). For government spending, GDP and investment, we directly use the volumes as reported by the OECD (the series are deflated with their own deflators).

- Fourth, instead of taking the identified government spending shock from the estimation of the first VAR model as the baseline spending shock, we allow the government spending shock to vary in the quantitative analysis. More specifically, we compute numerically the responses of variables by calibrating the model so as to replicate the endogenous response of government spending obtained in each VAR specification. In other words, we compute the dynamic responses of economic variables to the government spending shock corresponding to the same VAR model in which this variable is included.

Figure 42 compares the dynamic response of government consumption to an exogenous fiscal shock for the first VAR model with the endogenous response of $G$ for alternative VAR models. The solid blue line reports the point estimates for the first VAR model while the solid black line in each panel reports the point estimates for an alternative VAR model. The evidence is very clear: all responses of government spending are fairly close to the one based on the benchmark classification and remain within the confidence interval of the baseline for all of the selected horizons (10 years). We can notice only one slight difference for the specification $z_{it} = [g_{it}, y_{it}, l_{it}, c_{it}, w_{C,i}]$. When we calculate the correlations between the IRF on government spending from the original VAR specification and the ones obtained from others VAR specifications, we find a strong homogeneity in the estimated value of the correlation coefficient, which varies from a low of 0.992 for the specification $z_{it} = [g_{it}, y_{it}, l_{it}, c_{it}, w_{C,i}]$ to a high of 0.999 for the specification $z_{it}^N = [g_{it}, y_{it}^N, l_{it}^N, w_{C,i}^N]$, with a mean value (across specifications) of 0.997. To further explore empirically the discrepancy between government spending shocks across VAR specifications, Table 26 reports the test (p-value) of the statistical significance of the difference between the point estimate for the baseline model $z_{it} = [g_{it}, y_{it}, l_{it}, e_{it}, w_{C,i}]$ and that for an alternative VAR specification for each horizon $k = 1 \ldots 10$. As can be seen in Table 26, we fail to detect a significant difference in the responses of government spending for all horizons. Overall, the assumption that the government spending shock is similar across VAR specifications appears to be reasonable.

Figures 43 and 44 compare the results in the main text displayed in the solid blue line with those for the same VAR model augmented with the identified government spending shock in the first VAR model estimated on annual data. As shown in the first row of Figures 43 and 44, across all VAR specifications, the endogenous response of government spending is quite similar, if not identical, whether the fiscal shock is identified in the first VAR model (solid black line) or is identified in the corresponding VAR model (solid blue line). The dynamic effects displayed in Figure 43 do not deserve comments as impulse response functions are very similar. Turning to the sectoral composition effects displayed in Figure 44, while we may notice some slight differences for the responses of sectoral shares,
The real consumption wage in non tradables, and the relative wage of non tradables, the discrepancy is not statistically significant, except for the relative wage in the short-run. Overall, reassuringly, this robustness exercise shows that the government spending shock is very similar across VAR specifications and it turns out that differences are quantitatively rather moderate.

The identification scheme proposed by Blanchard and Perotti [2002] is based on the assumption that government spending does not respond contemporaneously to current output developments due to delays between current output observation and the implementation of fiscal measures. The potential problem is that Blanchard and Perotti’s argument is not necessarily true when using annual data as some adjustment could be possible within the year. We use quarterly data and assume that government spending does not respond within the quarter to the other variables included in the VAR model. This assumption is in the spirit of Blanchard and Perotti [2002]. We take the identified government spending shock in the first VAR model as the baseline spending shock. Then we augment each VAR model with
the baseline spending shock identified on a quarterly basis. Such a procedure should ensure that the fiscal shock is exogenous and variables respond to the same identified government spending shock. The disadvantage is that the largest available subset of the countries in our current sample for which we have sufficient quarterly fiscal data is 8 instead of 16. In order to make our baseline results comparable with those when we augment each VAR model with the spending shock identified in the first VAR model, i.e. \( z_{it} = [g_{it}, y_{it}, l_{it}, j_{eit}, w_{C,it}] \), we re-estimate each VAR model on annual data for the set of countries restricted to eight. In the latter case, we do not augment VAR models with the identified government spending shock in the first VAR model. The solid blue line in Figures 45 and 46 displays the baseline results for a government spending shock identified on annual data, while the solid black line displays the results when each VAR model is augmented with government spending shock identified in the first VAR model on quarterly data. As shown in the first row of Figures 45 and 46, the response of government spending to the exogenous fiscal shock identified on quarterly data displayed in the solid black line is somewhat more pronounced in the short term and the IRF of \( G \) remains above that in the baseline. In sum, the government spending shock displays more persistence and is somewhat more pronounced when identified on quarterly data. Because the government spending shock is larger in this configuration, as can be seen in Figure 45, hours worked and output fall more in the traded sector while they increase more in the non traded sector. It is thus not surprising that the responses of sectoral shares are more pronounced, as can be seen in Figure 46. Regarding the responses for the relative price and the relative wage, differences are quantitatively rather moderate.

In the last robustness exercise we perform, we normalize the exogenous fiscal shock to a rise in \( G \) by 1 percentage point of GDP in the quantitative analysis but let the endogenous response of government consumption vary across VAR specifications. Thus, when we compute the theoretical impulse response functions of one variable, we calibrate the government spending shock so as to reproduce the endogenous response of \( G \) we generate from estimates of the VAR model in which the variable is included. The objective of this exercise is to compute quantitatively the discrepancy in the estimated response of each variable caused by an identified government spending shock that potentially varies across VAR models while in the baseline quantitative analysis, we consider one unique spending shock identified in the first VAR model \( z_{it} = [g_{it}, y_{it}, l_{it}, j_{eit}, w_{C,it}] \). In the first row of Figures 47 and 48, the solid blue line displays the adjustment of government spending we generate from estimates of the first VAR model \( z_{it} = [g_{it}, y_{it}, l_{it}, j_{eit}, w_{C,it}] \) while the solid red line displays the endogenous response of \( G \) we generate from estimates of the VAR model in which the variable shown in the second row is included.

As discussed above, the differences between the government spending shock identified in the first VAR model and the government spending shock identified in alternative VAR models are quite moderate, and in most of the cases insignificant, except for the specification that includes the current account shown the first column of Figure 47. The solid blue line in the second row of Figures 47 and 48 shows empirical impulse response functions, while the solid black line displays theoretical responses when the government spending shock is calibrated on the basis of the first VAR model. The dotted black line displays the theoretical responses when the government spending shock is identified in the VAR model in which the economic variable (shown in the second row) is included. Overall, the differences between the solid and the dotted black line are quantitatively small or hardly noticeable for all variables. Thus, our quantitative analysis should not be altered by the small differences in the government spending shock across VAR specifications.

**D.7 Exogeneity of Government Spending Shocks and the Narrative Approach**

Following Blanchard and Perotti [2002], we have based identification on the assumption that government spending does not react contemporaneously to other variables included in the VAR model. Since there are some delays inherent to the legislative system, this is a natural assumption when using quarterly data. However, this argument may not be necessarily true when using annual data since some adjustment could be possible. We thus conduct in this subsection a robustness check by using narratively identified government
Figure 43: Aggregate and Sectoral Effects of an Unanticipated Government Spending Shock: Assessing Differences Caused by Identifying Different Government Spending Shocks on Annual Data. Notes: Exogenous increase in government consumption by 1% of GDP. Results for the baseline specification are displayed by solid lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the solid black line reports the results for the same VAR model which is augmented with the identified government spending shock in the first VAR model estimated on annual data; sample: 16 OECD countries, 1970-2007, annual data.
Figure 44: Sectoral Composition Effects of an Unanticipated Government Spending Shock: Assessing Differences Caused by Identifying Different Government Spending Shocks on Annual Data. Notes: Exogenous increase in government consumption by 1% of GDP. Results for the baseline specification are displayed by solid lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the solid black line reports the results for the same VAR model which is augmented with the identified government spending shock in the first VAR model estimated on annual data; sample: 16 OECD countries, 1970-2007, annual data.
Figure 45: Aggregate and Sectoral Effects of an Unanticipated Government Spending Shock: Assessing Differences Caused by Identifying Different Government Spending Shocks. Notes: Exogenous increase in government consumption by 1% of GDP. Results for the baseline specification are displayed by solid lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the solid black line reports the results for the same VAR model which is augmented with the identified government spending shock in the first VAR model estimated on quarterly data; sample: 8 OECD countries, 1970-2007, annual data.
Figure 46: Sectoral Composition Effects of an Unanticipated Government Spending Shock: Assessing Differences Caused by Identifying Different Government Spending Shocks. Notes: Exogenous increase in government consumption by 1% of GDP. Results for the baseline specification are displayed by solid lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the solid black line reports the results for the same VAR model which is augmented with the identified government spending shock in the first VAR model estimated on quarterly data; sample: 8 OECD countries, 1970-2007, annual data.
spending shocks from the dataset constructed by Guajardo, Leigh, and Pescatori [2014].

Before discussing the results from the narrative approach, it is worth mentioning that we conducted alternative robustness checks. In subsection B.4, following Beetsma and Giuliodori [2011], for the largest available subset of the countries in our current sample, we estimate the same VAR model on a quarterly basis, assuming that government purchases take at least one quarter to react to an output shock. We find that the dynamic responses of government spending along with the remaining aggregate variables included in the VAR model are similar to those when estimating the same VAR model on annual data. To deal further with the potential endogeneity problem, like Beetsma and Giuliodori [2011] and Brückner and Pappa [2012], we also order government purchases last in the VAR model and thus allow for $G$ to respond to all variables included in the VAR model. If the endogenous response of $G$ to an exogenous fiscal shock is similar to that when the VAR is estimated by ordering $G$ first, then we can be confident that the endogeneity problem is mitigated as $G$ is not or at least little responsive to output shocks. As displayed in Figure 49, the endogenous response of $G$ lies within the confidence bounds of the primary VAR model where $G$ is ordered first across all VAR specifications. This finding could be explained by the fact that automatic stabilizers which operate through unemployment benefits or transfers should not pose a problem since we consider government spending net of transfers.

Finally, Born and Müller [2012] test whether annual government spending is predetermined conditional on being predetermined at the quarterly frequency for four countries, i.e., United States, United Kingdom, Canada, and Australia. Their main result is that the restriction that government spending does not respond to other variables in the VAR within a year cannot be rejected. That being said, an alternative estimation strategy is suggested by Ramey and Shapiro [1998]. Their narrative approach amounts to considering major political events which led to large military buildups associated with significant increases in government spending. The advantage of the narrative approach over alternatives
Figure 48: Contrasting Theoretical Impulse Response Functions for Potentially Varying Government Spending Shock. Notes: Impulse response functions to an exogenous increase in real government spending by one percent of GDP. The solid blue line displays the point estimate of VAR with the dotted blue lines indicating 90% confidence bounds; the solid black line displays model predictions in the baseline scenario, i.e., when we calibrate the model on the basis of the first VAR model, $z_{it} = [g_{it}, y_{it}, l_{it}, j_{it}, w_{C, it}]$; the dotted black line displays model predictions when we calibrate the model for a government spending shock specific to the VAR specification. In the first row, the red line and the dotted black line with squares display the empirical and theoretical response of government spending specific to the VAR specification, respectively.
Figure 49: Impulse Response Functions for Government Spending across VAR specifications when $G$ is Ordered Last in the VAR Model. Notes: Exogenous increase in government consumption by 1% of GDP. Results for baseline specification, i.e., $G$ is ordered first in the VAR model, are displayed by solid blue lines with the shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the black line reports results for a VAR model in which $G$ is ordered last; sample: 16 OECD countries, 1970-2007, annual data.

is that political events are arguably exogenous (with respect to economic conditions) and thus identified government spending shocks are not subject to the potential endogeneity problem.

To further explore the potential endogeneity problem empirically, we use a dataset constructed by Gujardo, Leigh, and Pescatori [2014] in order to address the potential problem of endogeneity of government spending. Our objective is to investigate whether our main conclusions hold when adopting an alternative identification method. Using a narrative approach. Guajardo, Leigh, and Pescatori [2014] identify changes in fiscal policy directly from historical documents. More precisely, the dataset contains 173 fiscal policy changes for 17 OECD countries over the period 1978-2009. In order to make our results when adopting the 'event' approach comparable with our evidence obtained by adopting Blanchard and Perotti’s [2002] identifying assumption, we restrict the set of countries to 15 and the period to 1978-2007. We then estimate the same VAR models described in section 2 in the main text. We augment each baseline VAR model with identified fiscal events (ordered first in the VAR specification) corresponding to changes in government spending, i.e. ‘spending-based’ events. Figure 50 reports aggregate and sectoral effects while Figure 51 shows the sectoral composition effects.

While our results obtained by adopting Blanchard and Perotti’s [2002] identifying assumption are broadly in line with those obtained with the narrative approach, we may nevertheless note some differences. Before discussing these differences, it is worth mentioning that Gujardo, Leigh, and Pescatori [2014] identify cases of fiscal consolidation motivated by a desire to reduce the budget deficit. To be consistent with the objective of our paper, we concentrate on episodes of fiscal consolidation implemented through changes in government spending rather than changes in tax rates. The authors find that fiscal consolidation through spending cuts gives rise to a contraction in GDP and investment. In order to be able to compare the effects of a government spending shock identified by applying Blanchard and Perotti’s [2002] identification scheme with those following fiscal shocks identified by Guajardo, Leigh, and Pescatori [2014], we normalize the impulse response functions

\textsuperscript{73}We exclude DEU and PRT which are not included in our empirical study, while NOR is removed from our sample.
Figure 50: Contrasting Aggregate and Sectoral Effects of an Unanticipated Government Spending Shock between Alternative Identification Schemes of Government Spending Shock. Notes: Exogenous increase in government consumption by 1% of GDP. In the baseline case, government spending shocks are identified by assuming that government spending is predetermined relative to the other variables in the VAR model. Results for the baseline case are displayed by solid blue lines with the shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling; the solid black line reports results for narratively identified government spending shocks from the dataset constructed by Guajardo, Leigh, and Pescatori [2014]; sample: 15 OECD countries, 1978-2007, annual data.
Figure 51: Contrasting the Effects of an Unanticipated Government Spending Shock on Sectoral Composition between Alternative Identification Schemes of Government Spending Shock. Notes: Exogenous increase in government consumption by 1% of GDP. In the baseline case, government spending shocks are identified by assuming that government spending is predetermined relative to the other variables in the VAR model. Results for the baseline case are displayed by solid blue lines with the shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling; the solid black line reports results for narratively-identified government spending shocks from the dataset constructed by Gujardo, Leigh, and Pescatori [2014]; sample: 15 OECD countries, 1978-2007, annual data.
so that government consumption rises by 1 percentage point of GDP. In Figures 50 and 51, we report results for our four VAR specifications, augmenting each VAR model with the narrative shocks, ordered first. Results for the ‘event’ study are displayed in the solid black line while the solid blue line reports results in the baseline case in which government spending shocks are identified by applying Blanchard and Perotti’s [2002] method.

- **Differences in responses of aggregate variables.** As displayed in the first row of Figure 50, while the shapes of the endogenous response of government spending are similar, the fiscal shock displays less persistence over time in the ‘event’ approach and is greater in the short-run; more precisely, we observe that increases in government spending over the first two years are more pronounced, while $G$ is restored back toward its initial level more rapidly. In addition, in most of the cases, we observe a spending reversal which echoes Corsetti et al. [2012]; the magnitude of the cut in government spending during the reversal period (after 5 years approximately) is moderate though. Inspection of the dynamic effects of a government spending shock shows that the responses of aggregate variables are qualitatively similar whether the fiscal shock is identified by applying Blanchard and Perotti’s [2002] method or by using an ‘event’ approach, except for investment. More precisely, while a rise in government spending increases both real GDP and hours worked and leads to a decline in the current account in either cases, we detect a significant increase in investment on impact in the ‘event’ approach, while investment declines gradually in the baseline case. We may also note some differences quantitatively, as real GDP and hours worked increase more in the ‘event’ approach than in the baseline case while the current account deficit is more pronounced. This result is not surprising since the government spending shock is more pronounced in the short-run.

- **Differences in responses of sectoral variables.** The last two columns of Figure 50 show the responses of sectoral labor and output, while Figure 51 displays the dynamic adjustment of sectoral shares along with the responses of the relative price and relative wage of non tradables. As can be seen in the last two columns of Figure 50, hours worked in the traded sector increase in the ‘event’ study while traded output rises in the short-run. Because the rise in government spending is more pronounced in the ‘event’ study, we also find empirically that labor and output in the non traded sector increase by a larger amount. While we detect a positive impact on traded output in the short run, the first column of Figure 51 shows that the share of tradables, whether measured in total employment or real GDP, declines after a fiscal shock. As can be seen in the first two columns of Figure 51, the responses of sectoral output shares are somewhat less pronounced in the ‘event’ study but lie within the confidence bounds of the baseline case. As displayed in the last two columns of Figure 51, hours worked and output in the traded sector falls relative to the non traded sector. Yet, the decline is much less pronounced than that in the baseline case. Moreover, both the relative price and the relative wage of non tradables appreciate by a smaller amount.

In sum, whether we consider a narrative approach or Blanchard and Perotti’s [2002] identification scheme, all our results hold qualitatively, except for investment which is found to be crowded-in in the former approach. As mentioned above, we may notice some quantitative differences though. In particular, we find empirically that the rise in government spending has an expansionary effect on non traded output relative to traded output which is somewhat less pronounced in the ‘event’ study. In addition, the government spending shock gives rise to a positive response of traded output on impact and a contraction in the medium-run only.

How can the discrepancy between the two approaches be rationalized? Because the appreciation in the relative price of non tradables along with the responses of sectoral shares are less pronounced in the ‘event’ study, it seems reasonable to conjecture that changes in government spending identified by Gujardo, Leigh, and Pescatori [2014] are somewhat less biased toward non tradables than those identified by applying Blanchard and Perotti’s [2002] method. The decline in hours worked in the traded sector relative to the non traded sector along with the increase in the relative wage of non tradables
which are less pronounced tend to corroborate this conjecture. Furthermore, a relatively less intensive non traded sector in the government spending shock could rationalize the increase in investment expenditure in the short-run in the event approach. More precisely, the response of investment depends on the movement in $Q/P$ where $Q$ is the shadow value of capital and $P$ the investment price index (which is an increasing function of the relative price of non tradables, $P$). Because the capital-labor ratio falls in the traded sector, the return on domestic capital and thus $Q$ increases. Hence, the response of investment depends on the magnitude of the appreciation in the relative price of non tradables. If the government spending shock were not strongly biased toward non traded goods, the increase in $Q$ could thus offset the rise in $P$ so that investment is crowded-in. The second cause of the discrepancy in results obtained in the narrative approach with those obtained by applying Blanchard and Perotti’s [2002] identification scheme could be based on the change in public investment. It may be possible that narratively identified fiscal shocks are associated with an increase in public investment.

To conclude, whether changes in government spending are identified by using a narrative approach or by applying Blanchard and Perotti’s [2002] assumption, our main conclusions hold. This result is reassuring as the Blanchard and Perotti’s [2002] argument is not necessarily true when using annual data and such a robustness check tends to corroborate that government spending shocks we identify in our paper are exogenous. Such an empirical study also suggests that more work needs to be done in order to understand the cause(s) of the quantitative differences between the two approaches.

### E Solving the Two-Sector Model without Physical Capital

In this section, we provide the main steps to solve the two-sector model without capital accumulation. This enables us to shed some light on the implications of a difficulty in real-locating labor across sectors for the fiscal transmission. The small open economy produces a traded and a non traded good by means of a production technology described by linearly homogenous production functions that use labor only. As previously, the output of the non traded good ($Y^N$) can be used for private ($C^N$) and public consumption ($G^N$). The output of the traded good ($Y^T$) can be consumed by households ($C^T$) and the government ($G^T$), or can be exported with $Y^T - C^T - G^T$ corresponding to net exports. To avoid technical details, the reader can jump to subsection E.13 that solves the model in a friendly way by assuming that the endogenous response of government spending to an exogenous fiscal shock decreases monotonically.

Furthermore, to ease the interpretation of analytical results, we set the following assumption:

**Assumption 1** The elasticity of labor supply across sectors, $\epsilon$, is higher than the intertemporal elasticity of substitution for labor, $\sigma_L$.

First, our panel data estimates for $\epsilon$ average 0.75 while empirical studies usually report estimates for the Frisch elasticity of labor supply ranging from 0.4 to 0.6. Second, as will be clear below, such an assumption guarantees that an open economy without physical capital runs a current account deficit, in line with our VAR evidence.

#### E.1 Households

At each instant the representative agent consumes traded goods and non-traded goods denoted by $C^T$ and $C^N$, respectively, which are aggregated by a constant elasticity of substitution function:

$$C (C^T, C^N) = \left[ \varphi^{\frac{1}{\varphi}} (C^T)^{\frac{\varphi+1}{\varphi}} + \left(1 - \varphi\right)^{\frac{1}{\varphi}} (C^N)^{\frac{\varphi+1}{\varphi}} \right]^{\frac{\varphi}{\varphi+1}}. \quad (119)$$

The representative agent must also decide on worked hours in the traded and the non traded sector denoted by $L^T$ and $L^N$ at each instant of time which are aggregated by a
constant elasticity of substitution function:

\[ L(L^T, L^N) = \left[ \vartheta^{-\frac{1}{\epsilon}} (L^T)^{\frac{1}{1+\epsilon}} + (1 - \vartheta)^{-\frac{1}{\epsilon}} (L^N)^{\frac{1}{1+\epsilon}} \right]^{1+\epsilon}. \]  

(120)

The agent is endowed with a unit of time and supplies a fraction \( L(t) \) of this unit as labor, while the remainder, \( 1 - L \), is consumed as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Households decide on consumption and worked hours by maximizing lifetime utility:

\[ U = \int_0^\infty \left\{ \frac{1}{1 - \sigma_C} C(t)^{1-\frac{1}{\sigma_C}} - \frac{1}{1 + \frac{1}{\sigma_L}} L(t)^{1+\frac{1}{\sigma_L}} \right\} e^{-\beta t} dt, \]  

(121)

where \( \beta \) is the consumer’s discount rate, \( \sigma_C > 0 \) is the intertemporal elasticity of substitution for consumption, and \( \sigma_L > 0 \) is the Frisch elasticity of labor supply.

Households decide on consumption and worked hours by maximizing lifetime utility (121) subject to the flow budget constraint which states that the real disposable consisting of interest receipts from traded bonds holding plus labor income less lump sum taxes, \( T \), can be consumed or saved by accumulating traded bonds:

\[ \dot{B}(t) + P_C(P(t)) C(t) = r^* B(t) + W(W^T(t), W^N(t)) L(t) - T(t), \]  

(122)

where the RHS term of (122) corresponds to household’s real disposable income.

Denoting the co-state variable associated with eq. (122) by \( \lambda \), the first-order conditions characterizing the representative household’s optimal plans are:

\begin{align*}
C &= (P_C \lambda)^{-\sigma_C}, \quad \text{(123a)} \\
L &= (\lambda W)^{\sigma_L}, \quad \text{(123b)} \\
\dot{\lambda} &= \lambda (\beta - r^*), \quad \text{(123c)}
\end{align*}

and the transversality condition \( \lim_{t \to \infty} \lambda B(t) e^{-\beta t} = 0 \). In an open economy model with a representative agent having perfect foresight, a constant rate of time preference and perfect access to world capital markets, we impose \( \beta = r^* \) in order to generate an interior solution. This standard assumption made in the literature implies that the marginal utility of wealth, \( \lambda \), will undergo a discrete jump when individuals receive new information and must remain constant over time from thereon, i.e. \( \lambda = \bar{\lambda} \).

The homogeneity of \( C(\cdot) \) and \( L(\cdot) \) allows a two-stage decision: in the first stage, consumption and total hours worked are determined, and the intratemporal allocation between tradables and non tradables is decided at the second stage. Households split consumption between tradables and non tradables according to the following optimal rule:

\[ \left( \frac{1-\varphi}{\varphi} \right) C_T = P^\varphi. \]  

(124)

The allocation of total hours worked between the traded and the non traded sector follows from the following optimal rule:

\[ \left( \frac{\vartheta}{1 - \vartheta} \right) \frac{L^N}{L^T} = \Omega^\epsilon, \]  

(125)

where \( \Omega \equiv W^N/W^T \).

Plugging (124) into total consumption expenditure, i.e., \( E_C = C^T + PC^N \), one obtains the optimal demand for tradables and non tradables:

\begin{align*}
C_T &= \frac{\varphi \cdot E_C}{\varphi + (1 - \varphi) \cdot (P)^{1-\varphi}}, \quad \text{(126a)} \\
C_N &= \frac{(1 - \varphi) \cdot E_C}{(P)^{\varphi} \varphi + (1 - \varphi) \cdot (P)^{1-\varphi}}, \quad \text{(126b)}
\end{align*}
Substituting (126a) and (126b) into the subutility function (119) while setting \( C = 1 \) leads to the consumption price index:

\[
P_C = \left[ \varphi + (1 - \varphi) (P)^{1-\phi} \right]^{\frac{1}{1-\varphi}}, \tag{127}
\]

where \( P'_C = \frac{\partial P_C}{\partial P} > 0 \). Having defined the consumption price index, total consumption expenditure, \( E_C \), can be rewritten as \( P_C C \). Applying the Shephard’s Lemma gives the optimal demand for non tradables:

\[
C^N = \frac{\partial P_C}{\partial P} C. \tag{128}
\]

Using the fact that \( C^T = P_C C - P_C^N \), on obtains the optimal demand for tradables:

\[
C^T = (P_C - PP'_C) .C. \tag{129}
\]

Denoting by \( \alpha_C \) the non tradable content of consumption expenditure defined by:

\[
\alpha_C = \frac{(1 - \varphi) (P)^{1-\phi}}{\varphi + (1 - \varphi)(P)^{1-\phi}} = (1 - \varphi) \left( \frac{P}{P_C} \right)^{1-\phi}, \tag{130a}
\]

\[
1 - \alpha_C = \frac{\varphi}{\varphi + (1 - \varphi) (P)^{1-\phi}} = \varphi P_C^{\phi-1}, \tag{130b}
\]

one can express consumption in non tradables as a share \( \alpha_C \) of total consumption expenditure:

\[
P_C^N = \frac{\partial P_C}{\partial P} \frac{P}{P_C} P_C C = \alpha_C P_C C. \tag{131}
\]

The same logic applies to consumption in tradables:

\[
C^T = \left( 1 - \frac{\partial P_C}{\partial P} \frac{P}{P_C} \right) P_C C = (1 - \alpha_C) P_C C. \tag{132}
\]

The representative household maximizes \( 1 - L(.) \) where \( L(.) \) is a CES function given by (120) with \( \epsilon > 0 \) the intratemporal elasticity of substitution between labor in the traded and non traded sector, given total labor income denoted by \( R_L \) measured in terms of the traded good:

\[
R_L \equiv W^T L^T + W^N L^N, \tag{133}
\]

where \( W^T \) is the wage rate in the traded sector and \( W^N \) is the wage rate in the non traded sector. The linear homogeneity of the subutility function \( L(.) \) implies that total labor income can be expressed as \( R_L = W (W^T, W^N) L \), with \( W \ (W^T, W^N) \) is the unit cost function dual (or aggregate wage index) to \( L \). The unit cost dual function, \( W(.) \), is defined as the minimum total labor income, \( R_L \), such that \( L = L \left( L^T, L^N \right) = 1 \), for a given level of the wage rates \( W^T \) and \( W^N \). We derive below its expression.

Combining (125) together with total labor income denoted by \( R_L \) measured in terms of the traded good, i.e. \( R_L \equiv W^T L^T + W^N L^N \), we are able to express labor supply to the traded and non traded sector, respectively, as functions of total labor income:

\[
L^T = (1 - \vartheta) (W^T)^{-1} \left[ (1 - \vartheta) + \vartheta \left( \frac{W^N}{W^T} \right)^{\epsilon+1} \right]^{-1} R_L, \tag{133a}
\]

\[
L^N = \vartheta (W^T)^{-1} \left( \frac{W^N}{W^T} \right)^{\epsilon} \left[ (1 - \vartheta) + \vartheta \left( \frac{W^N}{W^T} \right)^{\epsilon+1} \right]^{-1} R_L. \tag{133b}
\]

Plugging these equations into (120), setting \( L = 1 \) and \( R_L = W \), yields the aggregate wage index:

\[
W = \left[ \vartheta (W^T)^{\epsilon+1} + (1 - \vartheta) (W^N)^{\epsilon+1} \right]^{\frac{1}{\epsilon+1}}. \tag{134}
\]

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Intratemporal allocation of hours worked between the traded and the non traded sector follows from Shephard’s Lemma (or the envelope theorem):

\[
L^T = \frac{\partial W}{\partial W^T} L = W_T L, \quad \text{and} \quad \frac{W^T L^T}{W_L} = 1 - \alpha_L, \tag{135a}
\]

\[
L^N = \frac{\partial W}{\partial W^N} L = W_N L, \quad \text{and} \quad \frac{W^N L^N}{W_L} = \alpha_L, \tag{135b}
\]

where the non tradable and tradable content of total labor income, respectively, are:

\[
\alpha_L = \frac{(1 - \vartheta) (W^N)^{\epsilon+1}}{\vartheta (W^T)^{\epsilon+1} + (1 - \vartheta) (W^N)^{\epsilon+1}} = (1 - \vartheta) \left( \frac{W^N}{W} \right)^{\epsilon+1}, \tag{136a}
\]

\[
1 - \alpha_L = \frac{\vartheta (W^T)^{\epsilon+1}}{\vartheta (W^T)^{\epsilon+1} + (1 - \vartheta) (W^N)^{\epsilon+1}} = \vartheta \left( \frac{W^T}{W} \right)^{\epsilon+1}. \tag{136b}
\]

We write out some useful properties:

\[
\frac{\partial W}{\partial W^T} \frac{W^T}{W} = (1 - \alpha_L), \quad \frac{\partial W}{\partial W^N} \frac{W^N}{W} = \alpha_L, \tag{137a}
\]

\[
\frac{\partial W}{\partial W^T} \frac{W^T}{W_T} = \frac{\partial^2 W}{\partial (W^T)^2} = \vartheta \epsilon (W^T)^{\epsilon-1} W^{-\epsilon} \alpha_L, \tag{137b}
\]

\[
\frac{\partial W_T}{\partial W^T} \frac{W^T}{W_T} = \epsilon \alpha_L > 0, \tag{137c}
\]

\[
\frac{\partial W_T}{\partial W^N} \frac{W^N}{W_T} = -\epsilon \alpha_L < 0, \tag{137d}
\]

\[
\frac{\partial W_N}{\partial W^N} \frac{W^N}{W} = \epsilon (1 - \alpha_L) > 0, \tag{137e}
\]

\[
\frac{\partial W_N}{\partial W^T} \frac{W^T}{W} = -\epsilon (1 - \alpha_L) < 0. \tag{137f}
\]

where \( W_j = \frac{\partial W}{\partial W^j} \) (with \( j = T, N \)).

**E.2 Firms**

There are two sectors in the economy: a sector which produces a traded good denoted by the superscript \( T \) and a sector which produces a non traded good denoted by the superscript \( N \). Both the traded and non traded sectors use labor, \( L^T \) and \( L^N \), according to linearly homogenous production functions:

\[
Y^T = L^T, \quad \text{and} \quad Y^N = L^N. \tag{138}
\]

Both sectors face a labor cost equal to the wage rate, i.e., \( W^T \) and \( W^N \), respectively. The traded sector and non traded sector are assumed to be perfectly competitive. The first order conditions derived from profit-maximization state that factors are paid to their respective marginal products:

\[
1 = W^T, \quad \text{and} \quad P = W^N. \tag{139}
\]

Dividing the second equality by the first equality leads to a relationship between the relative price of non tradables, \( P \), and the relative wage, \( \Omega = \frac{W^N}{W^T} \):

\[
P = \Omega. \tag{140}
\]

**E.3 Short-Run Static Solutions for Consumption and Labor**

In this subsection, we compute "short-run static solutions". This terminology refers to solutions of static optimality conditions which are inserted in dynamic optimality conditions in order to analyze the equilibrium dynamics. The term "short-run" refers to first-order
conditions, and the term "static" indicates that the solution holds at each instant of time, and thus in the long-run.

**Short-Run Static Solutions for Consumption and Labor**

We begin with those for consumption and labor supply. Static efficiency conditions (123a) and (123b) can be solved for consumption and labor which of course must hold at any point of time:

\[ C = C (\bar{\lambda}, P), \quad L = L (\bar{\lambda}, W^T, W^N), \quad \tag{141} \]

with

\[
\begin{align*}
C_\lambda &= \frac{\partial C}{\partial \lambda} = -\sigma_C \frac{C}{\lambda} < 0, \quad \tag{142a} \\
C_P &= \frac{\partial C}{\partial P} = -\alpha C \sigma_C \frac{C}{P} < 0, \quad \tag{142b} \\
L_\lambda &= \frac{\partial L}{\partial \lambda} = \sigma^L \frac{L}{\lambda} > 0, \quad \tag{142c} \\
L_{W^T} &= \frac{\partial L}{\partial W^T} = \sigma_L \frac{L (1 - \alpha L) }{W^T} > 0, \quad \tag{142d} \\
L_{W^N} &= \frac{\partial L}{\partial W^N} = \sigma_L \frac{\alpha L}{W^N} > 0, \quad \tag{142e}
\end{align*}
\]

where we used the fact that \( \frac{\partial W}{\partial W^T} \frac{W^T}{W} = (1 - \alpha L) \) and \( \frac{\partial W}{\partial W^N} \frac{W^N}{W} = \alpha_L; \sigma_C \) and \( \sigma_L \) correspond to the intertemporal elasticity of substitution for consumption and labor, respectively.

Inserting first the short-run solution for consumption (141), (128) and (129) can be solved for \( C^T \) and \( C^N \):

\[
\begin{align*}
C^T &= C^T (\bar{\lambda}, P), \quad C^N = C^N (\bar{\lambda}, P), \quad \tag{143}
\end{align*}
\]

where the partial derivatives are

\[
\begin{align*}
C^T_\lambda &= -\sigma_C \frac{C^T}{\lambda} < 0, \quad \tag{144a} \\
C^T_P &= \alpha C \frac{C^T}{P} (\phi - \sigma_C) \leq 0, \quad \tag{144b} \\
C^N_\lambda &= -\sigma_C \frac{C^N}{\lambda} < 0, \quad \tag{144c} \\
C^N_P &= -\frac{C^N}{P} \left[(1 - \alpha_C) \phi + \alpha C \sigma_C \right] < 0, \quad \tag{144d}
\end{align*}
\]

where we used the fact that \( \frac{P^P_C}{P} = \phi (1 - \alpha_C) > 0 \) and \( P^C_C = C^N \).

Inserting first the short-run solution for labor (141), into \( L^T = \frac{\partial W(W^T, W^N)}{\partial W^T} L \) and \( L^N = \frac{\partial W(W^T, W^N)}{\partial W^N} L \), we are able to solve for \( L^T \) and \( L^N \):

\[
\begin{align*}
L^T &= L^T (\bar{\lambda}, W^T, W^N), \quad L^N = L^N (\bar{\lambda}, W^T, W^N), \quad \tag{145}
\end{align*}
\]

where the partial derivatives are

\[
\begin{align*}
L^T_\lambda &= \frac{\partial L^T}{\partial \lambda} = \sigma^L \frac{L^T}{\lambda} > 0, \quad \tag{146a} \\
L^T_{W^T} &= \frac{\partial L^T}{\partial W^T} = \frac{L^T}{W^T} \left[ \alpha L + \sigma_L (1 - \alpha_L) \right] > 0, \quad \tag{146b} \\
L^T_{W^N} &= \frac{\partial L^T}{\partial W^N} = \frac{L^T}{W^N} \sigma_L (\sigma_L - \epsilon) \geq 0, \quad \tag{146c} \\
L^N_\lambda &= \frac{\partial L^N}{\partial \lambda} = \sigma^L \frac{L^N}{\lambda} > 0, \quad \tag{146d} \\
L^N_{W^N} &= \frac{\partial L^N}{\partial W^N} = \frac{L^N}{W^N} \left[ \epsilon (1 - \alpha_L) + \sigma_L \alpha_L \right] > 0, \quad \tag{146e} \\
L^N_{W^T} &= \frac{\partial L^N}{\partial W^T} = \frac{L^N}{W^T} (1 - \alpha_L) (\sigma_L - \epsilon) \geq 0, \quad \tag{146f}
\end{align*}
\]
where we used the fact that $\frac{W_{TT}}{W_T} = \epsilon \alpha_L$, $\frac{W_{TN} W_N}{W_T} = \epsilon \alpha_L$, $\frac{W_{NN} W_N}{W_N} = \epsilon (1 - \alpha_L)$, $\frac{W_{NT} W_N}{W_T} = -\epsilon (1 - \alpha_L)$.

**Short-Run Static Solutions for Sectoral Wages**

First order conditions (139) can be solved for the sectoral wages:

\[ W^T = \text{constant}, \quad W^N = W^N (P), \quad (147) \]

where the partial derivative is:

\[ W^N_P = \frac{\partial W^N}{\partial P} = 1 = \frac{W^N}{P} > 0, \quad (148) \]

Inserting (147) into (145) yields:

\[ L^T = L^T (\bar{\lambda}, P), \quad L^N = L^N (\bar{\lambda}, P), \quad (149) \]

where the partial derivatives are

\[ L^T_P = \frac{\partial L^T}{\partial P} = \left( L^T_{W^N W_N} \right) = L^T \frac{\alpha L (\sigma L - \epsilon)}{P} \geq 0, \quad (150a) \]

\[ L^N_P = \frac{\partial L^N}{\partial P} = \left( L^N_{W^N W_N} \right) = L^N \frac{[\epsilon (1 - \alpha L) + \sigma L \alpha L]}{P} > 0, \quad (150b) \]

and $L^T_\bar{\lambda}$ and $L^N_\bar{\lambda}$ are given by (146a) and (146d), respectively.

**E.4 Market Clearing Conditions**

To fully describe the equilibrium, we impose goods market clearing conditions. The non traded good market clearing condition requires that non traded output is equalized with demand for non tradables:

\[ Y^N = C^N + G^N. \quad (151) \]

Plugging this condition into the flow budget constraint (122) and using firms’ optimal conditions (139) yields the market clearing condition for tradables or the current account equation:

\[ \dot{B} = r^* B + Y^T - C^T - G^T, \quad (152) \]

where the sum of the last three terms on the RHS, i.e., $Y^T - C^T - G^T \equiv N X$, corresponds to net exports denoted by $N X$.

Inserting short-run static solutions for $C^N$ for $L^N$ given by (143) and (149), respectively, into the non traded good market clearing condition (151) gives us:

\[ L^N (\bar{\lambda}, P) = C^N (\bar{\lambda}, P) + G^N. \quad (153) \]

The non traded good market clearing condition can be solved for the relative price of non tradables by totally differentiating (153):

\[ \alpha L \dot{L}^N = \omega_C \alpha C \dot{C}^N + \frac{P d G^N}{Y}, \quad (154) \]

where we denote the ratio of consumption expenditure to GDP by $\omega_C = \frac{P d C}{C Y}$; to determine the LHS of (154), we used the fact that $Y = Y^T + P Y^N = W^T L^T + W^N L^N = WL$ because $Y^T = L^T$ and $Y^N = L^N$, since $W^N = P$ together with the definition of $\alpha_L$ given by eq. (135b), we have

\[ \frac{P Y^N}{Y} = \frac{W^N L^N}{WL} = \alpha_L. \quad (155) \]

Inserting short-run static solutions and collecting terms yield:

\[ \dot{P} = \frac{-\dot{\lambda} [\alpha_L \sigma L + \omega_C \alpha C \sigma C] + \frac{P d G^N}{Y}}{\Psi}, \quad (156) \]
where we set
\[
\Psi = \alpha_L [\epsilon (1 - \alpha_L) + \sigma_L \alpha_L] + \omega_C \alpha_C [(1 - \alpha_C) \phi + \alpha_C \sigma_C] > 0.
\] (157)

Invoking the implicit functions theorem, eq. (156) leads to the short-run static solution for the relative price of non tradables:
\[
P = P(\lambda, G^N),
\] (158)
where \(P_\lambda < 0\) and \(P_{G^N} > 0\).

### E.5 Solutions for Sectoral Labor

Totally differentiating the short-run static solution for traded labor \(L^T = L^T(\bar{\lambda}, P)\) given by (149) yields:
\[
\dot{L}^T = \sigma_L \dot{\lambda} + \alpha_L (\sigma_L - \epsilon) \dot{P}.
\]
Inserting the short-run static solution for the relative price \(P\) given by (158) allows us to solve for traded labor:
\[
\dot{L}^T = \left[ \frac{\sigma_L \Psi + \alpha_L (\epsilon - \sigma_L) (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C)}{\Psi} \right] \dot{\lambda} + \frac{\alpha_L (\sigma_L - \epsilon)}{\Psi} \frac{P dG^N}{Y}.
\] (159)

Eq. (159) solves for traded labor:
\[
\dot{L}^T = L^T(\bar{\lambda}, G^N),
\] (160)
where \(L^T_\lambda > 0\), and \(L^T_{G^N} < 0\).

Totally differentiating the short-run static solution for traded labor \(L^N = L^N(\bar{\lambda}, P)\) given by (149) leads to:
\[
\dot{L}^N = \sigma_L \dot{\lambda} + [\epsilon (1 - \alpha_L) + \sigma_L \alpha_L] \dot{P}.
\]
Inserting the short-run static solution for the relative price \(P\) given by (158) allows us to solve for non traded labor:
\[
\dot{L}^N = \left[ \frac{\sigma_L \Psi - (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C) [\epsilon (1 - \alpha_L) + \sigma_L \alpha_L]}{\Psi} \right] \dot{\lambda}
+ \left[ \frac{\epsilon (1 - \alpha_L) + \sigma_L \alpha_L}{\Psi} \right] \frac{P dG^N}{Y},
\] (161)
where
\[
\sigma_L \Psi - (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C) [\epsilon (1 - \alpha_L) + \sigma_L \alpha_L]
= \omega_C \alpha_C \{\sigma_L [(1 - \alpha_C) \phi + \alpha_C \sigma_C] - \sigma_C [\epsilon (1 - \alpha_L) + \sigma_L \alpha_L]\} \geq 0.
\]

Eq. (161) solves for non traded labor:
\[
\dot{L}^N = L^N(\bar{\lambda}, G^N),
\] (162)
where \(L^N_\lambda \geq 0\), and \(L^N_{G^N} > 0\).

### E.6 Equilibrium Dynamics and Formal Solutions

Inserting the short-run static solution for \(L^T(160)\) and for \(C^T(143)\) into the current account equation (152) yields:
\[
\dot{B}(t) = r^* B(t) + L^T(\bar{\lambda}, G^N) - C^T(\bar{\lambda}, P) - G^T.
\] (163)

Remembering that \(P\) is fixed while the shadow value of wealth, \(\lambda\), may jump when new information arrives but remains fixed over time, i.e., \(\lambda = \bar{\lambda}\), and linearizing (163) in the neighborhood of the steady-state leads to:
\[
\dot{B}(t) = r^* \left( B(t) - \bar{B} \right).
\] (164)
The general solution is:

\[ B(t) = \hat{B} + D_2 e^{r_\text{tr}t}, \quad (165) \]

where \( D_2 \) is an arbitrary constant determined by initial conditions. Invoking the transversality condition, i.e., \( \lim_{t \to \infty} \hat{\lambda}B(t)e^{-r_\text{tr}t} = 0 \), the stable solution is:

\[ B(t) = \hat{B}, \quad (166) \]

and the intertemporal solvency condition (ISC) reads:

\[ \hat{B} = B_0. \quad (167) \]

While a permanent fiscal shock does not affect the net foreign asset position, a temporary fiscal shock, by modifying initial conditions, permanently modifies the stock of foreign assets.

### E.7 Steady-State

Inserting the ISC (167) and appropriate short-run static solutions which obviously hold in the long-run, the steady-state can be reduced to one equation

\[ r^*B_0 + L^T (\hat{\lambda}, G^N) - C^T [\hat{\lambda}, P (\hat{\lambda}, G^N)] - G^T = 0. \quad (168) \]

Equation (168) can be solved for the marginal utility of wealth:

\[ \hat{\lambda} = \lambda \left( G^N, G^T \right). \quad (169) \]

Note that we concentrate below on a rise in government spending on non tradables \( G^N \) because empirical evidence indicate that the non-tradable content of public spending averages to 90\% for OECD countries. At the end of the section, we investigate the effects of a temporary increase in \( G^T \) and show that the predictions of the model, in this configuration, are at odds with the panel VAR evidence.

Using the fact that the stock of traded bonds is initially predetermined and totally differentiating (168) yields:

\[ (1 - \alpha_L) \hat{L}^T = \omega_C (1 - \alpha_C) \hat{C}^T + \frac{dG^T}{Y}, \quad (170) \]

where we used the definition of \( \alpha_L \) given by eq. (155).

We first solve for consumption in tradables by totally differentiating \( C^T [\hat{\lambda}, P (\hat{\lambda}, G^N)] \):

\[ \hat{C}^T = -\sigma_C \hat{\lambda} + \alpha_C (\phi - \sigma_C) \hat{P}. \quad (171) \]

Inserting (156) allows us to solve for consumption in tradables:

\[ \hat{C}^T = C^T (\hat{\lambda}, G^N) \]

where partial derivatives are given by:

\[ \hat{\lambda} [(1 - \alpha_L) \sigma_L + \omega_C (1 - \alpha_C) \sigma_C] + \hat{P} [(1 - \alpha_L) \alpha_L (\sigma_L - \epsilon) - \omega_C (1 - \alpha_C) \alpha_C (\phi - \sigma_C)] = \frac{dG^T}{Y}. \quad (172) \]

Inserting (156) into the above equation and collecting terms, the change in the marginal utility of wealth is given by:

\[ \hat{\lambda} = \frac{PdG^N}{Y} \left[ \omega_C (1 - \alpha_C) \alpha_C (\phi - \sigma_C) + (1 - \alpha_L) \alpha_L (\epsilon - \sigma_L) \right] + \frac{dG^T}{Y} \frac{\Psi}{\Gamma}, \quad (173) \]

where \( \Psi \) is given by (156) and we set

\[ \Gamma = [\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C] \left[ \omega_C (1 - \alpha_C) \alpha_C (\phi - \sigma_C) + (1 - \alpha_L) \alpha_L (\epsilon - \sigma_L) \right] \]

\[ + [(1 - \alpha_L) \sigma_L + \omega_C (1 - \alpha_C) \sigma_C] \Psi > 0. \quad (174) \]
E.8 Derivation of Steady-State Solutions

In this subsection, we derive steady-state solutions. The steady-state reduces to two equations:

\[ r^\star \tilde{B} + L^T - \tilde{C}^T - G^T = 0, \]  

(175a)  

together with the intertemporal solvency condition  

\[ \tilde{B} = B_0, \]  

(175b)  

which jointly solve for the stock of traded bonds \( \tilde{B} \) and the marginal utility of wealth \( \tilde{\lambda} \).

We first solve the system (175a) for \( \tilde{B} \) as a function of the marginal utility of wealth, \( \tilde{\lambda} \) and government spending on non tradables \( G^N \) and tradables \( G^T \). To do so, substitute solutions for traded labor (159) and for consumption in tradables (171), into the traded good market clearing condition (175a):

\[ r^\star \tilde{B} + L^T (\tilde{\lambda}, G^N) - C^T (\tilde{\lambda}, G^N) - G^T = 0. \]  

(176)

Solving (176) for the steady-state value of \( B \), we are able to express \( B \) as a function of the shadow value of wealth and government spending on non tradables, \( G^N \), and tradables, \( G^T \):

\[ \tilde{B} = B (\tilde{\lambda}, G^N, G^T), \]  

(177)  

with partial derivatives given by:

\[ B_\tilde{\lambda} \equiv \frac{\partial \tilde{B}}{\partial \tilde{\lambda}} = \frac{(L^T - C^T)}{r^\star}, \]

\[ = -\frac{Y}{r^\star} \left[ (1 - \alpha_L) \frac{\dot{L}^T}{\tilde{\lambda}} - \omega_C (1 - \alpha_C) \frac{\dot{C}^T}{\tilde{\lambda}} \right] \tilde{\lambda}, \]

\[ = -\frac{Y}{r^\star} \Gamma < 0, \]  

(178a)

\[ B_{G^N} \equiv \frac{\partial \tilde{B}}{\partial G^N} = \frac{(L^T_{G^N} - C^T_{G^N})}{r^\star}, \]

\[ = \frac{Y}{r^\star} \left[ (1 - \alpha_L) \frac{L^T_{G^N}}{L^T} - \omega_C (1 - \alpha_C) \frac{C^T_{G^N}}{C^T} \right], \]

\[ = \frac{Y}{r^\star} \left[ \frac{\omega_C (1 - \alpha_C) \alpha_C (\phi - \sigma_C) + (1 - \alpha_L) \alpha_L (\epsilon - \sigma_L)}{\Psi} \right] \frac{P}{Y}, \]  

(178b)

where \( \Psi > 0 \) is given by (156) and \( \Gamma > 0 \) is given by (174) and we used the fact that \( (1 - \alpha_L) \frac{L^T}{\tilde{\lambda}} - \omega_C (1 - \alpha_C) \frac{C^T}{\tilde{\lambda}} = \Gamma \).

Inserting (177) into the ISC (175b) yields:

\[ B (\tilde{\lambda}, G^N, G^T) = B_0. \]  

(179)

Totally differentiating the above equation and collecting terms gives the change in the equilibrium value of the marginal utility of wealth:

\[ \frac{d\tilde{\lambda}}{dG^N}_{\text{perm}} = -\frac{B_{G^N}}{B_\tilde{\lambda}}, \]

\[ = \frac{\tilde{\lambda} \left[ \omega_C (1 - \alpha_C) \alpha_C (\phi - \sigma_C) + (1 - \alpha_L) \alpha_L (\epsilon - \sigma_L) \right] P}{\Gamma \Psi}, \]  

(180)  

where the subscript perm refers to the effect of a permanent increase in government consumption.

E.9 Derivation of Steady-State Changes Following a Permanent Government Spending Shock

We now derive the steady-state changes of key macroeconomic variables following an unanticipated and exogenous permanent government spending shock. Inserting the change in
the equilibrium value of the marginal utility of wealth given by (173) into (155) gives the steady-state change of the relative price of non tradables:

\[ \hat{P} = \frac{[(1 - \alpha_L) \sigma_L + \omega_C (1 - \alpha_C) \sigma_C] PdG^N}{\Gamma} - \frac{[\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C] dG^T}{Y}. \] (181)

Hence, a permanent increase in \( G^N \) unambiguously appreciates the relative price of non tradables in the long-run while a permanent rise in \( G^T \) depreciates it.

Totally differentiating (159), inserting the change in the equilibrium value of the marginal utility of wealth given by (173) gives the steady-state change of traded labor:

\[ \hat{L}^T = \frac{\omega_C (1 - \alpha_C) [\alpha_C (\phi - \sigma_C) \sigma_L + \sigma_C \alpha_L (\sigma_L - \epsilon)] PdG^N}{\Gamma} \]

\[ + \left\{ \frac{\sigma_L \alpha_L \epsilon + \omega_C \alpha_C [\sigma_L [(1 - \alpha_C) \phi + \alpha_C \sigma_C] + \sigma_C \alpha_L (\epsilon - \sigma_L)]}{\Gamma} \right\} \frac{dG^T}{Y}. \] (182)

In contrast to a model imposing perfect mobility of labor across sectors, traded labor does not necessarily fall. Yet, as shown later, the ratio \( L^T / L^N \) unambiguously declines.

Totally differentiating (160), inserting the change in the equilibrium value of the marginal utility of wealth given by (173) gives the steady-state change of non traded labor:

\[ \hat{L}^N = \left\{ \frac{(1 - \alpha_L) \sigma_L \epsilon + \omega_C (1 - \alpha_C) \{ \sigma_L \alpha_C (\phi - \sigma_C) + \sigma_C [\epsilon (1 - \alpha_L) + \sigma_C \alpha_L] \}}{\Gamma} \right\} PdG^N \]

\[ - \frac{\omega_C \alpha_C}{\Gamma} \{ \sigma_C [\epsilon (1 - \alpha_L) + \sigma_C \alpha_L] - \sigma_L [(1 - \alpha_C) \phi + \alpha_C \sigma_C] \} \frac{dG^T}{Y}. \] (183)

According to (183), a permanent rise in \( G^N \) unambiguously raises \( L^N \) in the long-run while a permanent increase in \( G^T \) may raise or lower \( L^N \) depending on whether the cost of shifting hours worked from one sector to another is high or low.

We now derive the steady-state in the consumption wage \( W / P_C \). To do so, remembering that \( W = W \{ W^T, W^N (P) \} \), using the fact that \( \frac{\partial W}{\partial W^T} \frac{W^N}{W} = \alpha_L \) and \( \frac{\partial W^N}{\partial P} \frac{P}{W} = 1 \), the steady-state change in the aggregate wage index is:

\[ \tilde{W} = \alpha_L \hat{P} > 0, \] (184)

where \( \hat{P} \) is given by (181). Moreover, the change in the consumption price index is given by \( \hat{P}_C = \alpha_C \hat{P} \). Hence, using (184), the change in the consumption aggregate wage is given by:

\[ d \left( \frac{W}{P_C} \right) = \frac{W}{P_C} (\alpha_L - \alpha_C) \hat{P} > 0, \] (185)

where the sign follows from the fact that data indicate that \( \alpha_L > \alpha_C \), i.e., the non tradable content of labor income is larger than the non tradable content of consumption expenditure.

### E.10 Derivation of Formal Solutions after Temporary Fiscal Shocks

In this subsection, we determine the solutions following a temporary fiscal expansion. In order to produce a hump-shaped response in line with the evidence, the endogenous response of government spending to an exogenous fiscal shocks is assumed to be governed by the following dynamic equation:

\[ \frac{dG(t)}{Y} = \frac{G(t) - \tilde{G}}{Y} = \left[ e^{-\xi t} - (1 - g) e^{-\chi t} \right], \] (186)

where \( Y \) is initial steady-state GDP, \( \xi \) and \( \chi \) are positive parameters which satisfy the following inequality

\[ \chi (1 - g) > \xi > 0. \] (187)

Inequality (187) guarantees that government spending rises after its initial upward jump. Because the non tradable content of government spending averages 90% for the 15 OECD countries in our sample and thus changes in public expenditure are mostly reflected by
changes in purchases of non tradables by the public sector, we further assume that the rise in government consumption is fully biased toward non tradables; in linearized form, we have:

\[ \dot{P} (G^N(t) - \tilde{G}^N) = G(t) - \tilde{G}, \]  

where we denote the long-term values with a tilde. In the quantitative analysis, we relax this assumption and consider a rise in government spending by 1 percentage point of GDP which is split between non tradables and tradables in accordance with their respective shares, at 90% and 10%, respectively.

**Solution for the Net Foreign Asset Position \( B(t) \)**

To begin with, we linearize the current account equation (163) in the neighborhood of the steady-state:

\[ \dot{B}(t) = r^* \left( B(t) - \tilde{B} \right) + \left( L_T^P - C_P^T \right) P_{GN} \left( G^N(t) - \tilde{G}^N \right). \]  

Inserting \( L_T^P = \frac{L_T}{P} \alpha_L (\sigma_L - \epsilon) \) (see eq. (150a)) and \( C_P^T \alpha_C (\phi - \sigma_C) \) (see eq. (144b)), eq. (189) can be rewritten as follows:

\[ \dot{B}(t) = r^* \left( B(t) - \tilde{B} \right) + \left[ \frac{L_T}{P} \alpha_L (\sigma_L - \epsilon) - \tilde{C} P_{\alpha_C} (\phi - \sigma_C) \right] \frac{P_N}{P} \left( G^N(t) - \tilde{G}^N \right), \]

where \( 1 - \alpha_L = \frac{W^T L^T}{W} = \frac{L_T}{P} \) (with \( Y = Y^T + P Y^N = W^T L^T + W^N L^N = WL \) and \( W^T = 1 \)), we substituted \( \frac{P_N}{P} = \frac{\partial P_N}{\partial \gamma} \) to obtain the second line and we used the fact that \( dG^N(t) = \frac{dG(t)}{P} \) to get the third line. As long as \( \epsilon > \sigma_L \) and \( \phi \simeq \sigma_C \), a rise in government spending above trend tends to affect negatively the net foreign asset position.

Eq. (190) can be rewritten in a more compact form

\[ \dot{B}(t) = r^* \left( B(t) - \tilde{B} \right) - \Upsilon_G Y \left[ e^{-\xi t} - (1 - g) e^{-\gamma t} \right], \]  

where we have inserted (186) and set

\[ \Upsilon_G = - \frac{\partial \tilde{B}(t)}{\partial G(t)} = \frac{[(1 - \alpha_L) \alpha_L (\epsilon - \sigma_L) + (1 - \alpha_C) \omega_C \alpha_C (\phi - \sigma_C)]}{\Psi} \geq 0. \]

Pre-multiplying by \( e^{-r^* \tau} \) and integrating over \((0, t)\) allow us to obtain the general solution for \( B(t) \):

\[ B(t) - \tilde{B} = \left[ (B_0 - \tilde{B}) - \frac{\Upsilon_G Y}{\xi + r^*} (1 - \Theta) \right] e^{r^* t} + \frac{\Upsilon_G Y}{\xi + r^*} \left( e^{-\xi t} - \Theta' e^{-\gamma t} \right), \]

where we used the fact that \( \int_0^t e^{-(\xi + r^*) \tau} d\tau = \frac{1 - e^{-(\xi + r^*) t}}{\xi + r^*} \) and we set:

\[ \Theta' = (1 - g) \frac{\xi + r^*}{\chi + r^*} > 0. \]

Invoking the transversality condition, one obtains the 'stable' solution for the stock of foreign assets so that \( B(t) \) converges toward its steady-state value \( \tilde{B} \):

\[ B(t) - \tilde{B} = \frac{\Upsilon_G Y}{\xi + r^*} \left( e^{-\xi t} - \Theta' e^{-\gamma t} \right). \]

Eq. (193) gives the trajectory for for \( B(t) \) consistent with the intertemporal solvency condition:

\[ (\tilde{B} - B_0) = - \frac{\Upsilon_G Y}{\xi + r^*} (1 - \Theta'), \]
where \( 1 - \Theta' > 0 \) due to inequality (187). While the sign of \( \Upsilon_N^G \) is ambiguous, we expect \( \Upsilon_N^G > 0 \) so that a temporary rise in government spending deteriorates the net foreign asset position, i.e., \( dB < 0 \). More specifically, invoking assumption 1, we have \( \Upsilon_N^G > 0 \) (see eq. (192)) as long as \( \phi \simeq \sigma_C \); in other words, a rise in government consumption produces a decline in hours worked in the traded sector while consumption in tradables is merely affected.

Eq. (195) can be rewritten as follows:

\[
B(t) - \dot{B} = \Upsilon_N^G N_t \int_t^\infty dG(\tau) e^{-r^*(\tau-t)} d\tau, \tag{197}
\]

where \( \int_t^\infty dG(\tau) e^{-r^*(\tau-t)} d\tau \) corresponds to the temporal path for government spending expressed in present value terms:

\[
\int_t^\infty dG(\tau) e^{-r^*(\tau-t)} d\tau = \frac{Ye^{-r't}}{\xi + r^*} \left[ e^{-(\xi+r^*)t} - \Theta' e^{-(\xi+r^*)t} \right],
\]

\[
= \frac{Y}{\xi + r^*} \left( e^{-\xi t} - \Theta' e^{-\xi t} \right). \tag{198}
\]

Differentiating (195) w.r.t. time gives the trajectory for the current account along the transitional path when government spending follows the temporal path given by eq. (186):

\[
\dot{B}(t) = -\frac{\Upsilon_N^G Y}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta' e^{-\chi t} \right). \tag{199}
\]

As long as we impose assumption 1 along with \( \phi \simeq \sigma_C \), we have \( \Upsilon_N^G > 0 \), so that the current account deteriorates monotonically since \( (\xi - \chi \Theta') > 0 \) for \( t > 0 \).

Evaluating (199) at time \( t = 0 \) leads to the initial current account response, expressed as a percentage of initial GDP, following a temporary rise in government spending:

\[
\dot{B}(0) \bigg|_{\text{temp}} = -\frac{\Upsilon_N^G Y}{\xi + r^*} (\xi - \chi \Theta') < 0, \tag{200}
\]

where \( \Upsilon_N^G > 0 \) and \( (\xi - \chi \Theta') > 0 \). Note that \(-[\xi - (1 - g) \chi] > 0 \) guarantees that government spending increases after initial rise \( dG(0) \), i.e., \( \dot{G}(0) > 0 \), inequality \( (\xi - \chi \Theta') > 0 \) implies that the cumulative endogenous response of government spending to an exogenous fiscal shock is decreasing in present discounted value terms.

**The Change in the Equilibrium Value of the Marginal Utility of Wealth**

Eq. (196) gives the steady-state change in the foreign asset position following a temporary (denoted by the subscript \( \text{temp} \)) rise in government spending:

\[
\frac{\dot{B}}{N} \bigg|_{\text{temp}} = -\frac{\Upsilon_N^G Y}{\xi + r^*} (1 - \Theta') < 0. \tag{201}
\]

To determine the change in the equilibrium value of the marginal utility of wealth, we have to differentiate the market clearing condition for the traded good (176):

\[
r^* \frac{d\dot{B}}{Y} \bigg|_{\text{temp}} + (L^T - C^T) \frac{d\lambda}{\lambda} \bigg|_{\text{temp}} = 0.
\]

Expressing the equation above in rate of change and dividing by initial GDP leads to:

\[
\frac{r^* \frac{d\dot{B}}{Y} \bigg|_{\text{temp}}}{\lambda} + \left( 1 - \alpha_L \right) \frac{L^T}{\lambda} - \left( 1 - \alpha_C \right) \frac{C^T}{\lambda} \frac{d\lambda}{\lambda} \bigg|_{\text{temp}} = 0, \tag{202}
\]

where

\[
\left( 1 - \alpha_L \right) \frac{L^T}{\lambda} - \frac{C^T}{\lambda} = \frac{1 - \alpha_L}{\Psi} \left\{ \sigma_L \Psi + \alpha_L (\epsilon - \sigma_L) [\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C] \right\} + \frac{1 - \alpha_C}{\Psi} \left\{ \sigma_C \Psi + \alpha_C (\phi - \sigma_C) [\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C] \right\} = \frac{\Gamma}{\Psi} > 0, \tag{203}
\]

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with \( \Psi > 0 \) given by \((156)\) and \( \Gamma > 0 \) by \((174)\). Using \((203)\), eq. \((202)\) can be rewritten as follows:

\[
\frac{d\lambda}{\lambda}_{\text{temp}} = \frac{-\Psi r^*d\hat{B}}{\Gamma \cdot Y_0} \bigg|_{\text{temp}},
\]

\[
= \frac{\Psi}{\Gamma \xi + r^*} \frac{\nabla_N}{\hat{G}} (1 - \Theta'),
\]

\[
= \frac{\left[(1 - \alpha_C) \omega_C \alpha_C (\phi - \sigma_C) + (1 - \alpha_L) \alpha_L (\epsilon - \sigma_L)\right]}{\Gamma} r^* (1 - \Theta'), \tag{204}
\]

where we have substituted the steady-state change \( d\hat{B} \bigg|_{\text{temp}} \) given by \((201)\) and \( \nabla_N^N \hat{G} \) given by \((192)\). Since the marginal utility of wealth increases across all scenarios, we impose from now on the following condition:

\[
(1 - \alpha_C) \omega_C \alpha_C (\phi - \sigma_C) + (1 - \alpha_L) \alpha_L (\epsilon - \sigma_L) > 0. \tag{205}
\]

Importantly, following a temporary fiscal shock, the marginal utility of wealth increases less than after a permanent rise in \( G^N \).

**Steady-State Effects**

To determine the long-run effects of a temporary fiscal expansion, we approximate the steady-state changes for variable \( X = L, C, P, L^T, L^N, B \) with the differentials:

\[
\dot{X} - \dot{X}_0 \equiv X (\lambda, G^N) - X (\lambda_0, G^N) = X d\lambda \bigg|_{\text{temp}}, \tag{206}
\]

where \( d\lambda \bigg|_{\text{temp}} \equiv \lambda - \lambda_0 \) given by eq. \((204)\), and \( dG^N = 0 \) since government spending is restored to its initial level; note that \( \lambda_0 \) is the initial steady-state value for the shadow value of wealth.

Using the fact that \( P = P (\lambda, G^N) \) and because government spending is restored to its initial level, the relative price of non tradables must depreciate in the long-run:

\[
\dot{P} \bigg|_{\text{temp}} = -\frac{[\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C]}{\Psi} \chi \bigg|_{\text{temp}} < 0, \tag{207}
\]

where we made use of \((155)\) and \( \chi \bigg|_{\text{temp}} \) is given by \((204)\).

Totally differentiating \( L^N = L^N (\lambda, G^N) \) described by eq. \((162)\), using the fact that \( dG^N = 0 \), and inserting \((373)\) leads to the long-run adjustment of non traded employment following a temporary fiscal expansion:

\[
\dot{L}^N \bigg|_{\text{temp}} = \frac{\omega_C \alpha_C \left\{ \sigma_L [1 - \alpha_C] \phi + \alpha_C \sigma_C] - \sigma_C [\epsilon (1 - \alpha_L) + \sigma_L \alpha_L]\right\}}{\Psi} \lambda \bigg|_{\text{temp}}. \tag{208}
\]

On the one hand, the rise in the marginal utility of wealth has an expansionary effect on labor supply and thus on employment in the non traded sector. On the other hand, by driving down consumption in non tradables, the wealth effect depreciates the relative price of non tradables which lowers the non traded wage and thus exerts a negative impact on \( L^N \).

Totally differentiating \( L^T = L^T (\lambda, G^N) \) described by eq. \((160)\), using the fact that \( dG^N = 0 \), and inserting \((373)\) leads to the long-run adjustment of traded employment following a temporary fiscal expansion:

\[
\dot{L}^T \bigg|_{\text{temp}} = \frac{\sigma_L \lambda \epsilon + \omega_C \alpha_C \left\{ \sigma_L [1 - \alpha_C] \phi + \alpha_C \sigma_C] + \sigma_C \alpha_L (\epsilon - \sigma_L)\right\}}{\Psi} \lambda \bigg|_{\text{temp}}. \tag{209}
\]

The combined effect of the rise in the marginal utility of wealth and the depreciation in the relative price of non tradables raises employment in the traded sector. It is worthwhile noticing that \((375)\) is unambiguously positive.
Denoting by $NX$ net exports, differentiating the market clearing condition for the traded good in the long-run, i.e., $r^* B + NX = 0$ and inserting (201) leads to the steady-state change in net exports expressed in percentage of initial GDP:

$$
\frac{dNX|_{\text{temp}}}{Y} = - \frac{r^* d\tilde{B}|_{\text{temp}}}{Y} = \frac{r^* \Theta^N}{\xi + r^*} (1 - \Theta') > 0,
$$

(210)

where $\Theta^N > 0$. In the long-run, a temporary fiscal expansion raises net exports. The reason is that the open economy decumulates traded bonds along the transitional path. To repay its debt, the economy must run a trade surplus.

Consumption unambiguously falls in the long-run:

$$
\tilde{C}|_{\text{temp}} = -\sigma_C \tilde{\lambda}|_{\text{temp}} - \sigma_C \alpha_C \tilde{P}|_{\text{temp}},
$$

$$
= -\sigma_C \tilde{\lambda}|_{\text{temp}} \{\alpha_L [(1 - \alpha_L) \epsilon + \sigma_L (\alpha_L - \alpha_C)] + \omega_C \alpha_C (1 - \alpha_C) \phi\} < 0\ (211)
$$

where the non tradable content of labor, $\alpha_L$, is higher than the non tradable content of consumption expenditure, $\alpha_C$, according to our evidence.

Using the fact that $\tilde{W} = \alpha_L \tilde{W} N$ and $\tilde{P}_C = \alpha_C \tilde{P}$, a temporary fiscal expansion raises employment in the long-run:

$$
\tilde{L}|_{\text{temp}} = \frac{\alpha_L \tilde{\lambda}|_{\text{temp}} + \sigma_L \alpha_L \tilde{P}_C}{\tilde{\lambda}|_{\text{temp}}},
$$

$$
= \frac{\alpha_L \{\alpha_L [(1 - \alpha_L) \epsilon + \sigma_L (\alpha_L - \alpha_C)] + \omega_C \alpha_C (1 - \alpha_C) \phi - \sigma_C (\alpha_L - \alpha_C)\}}{\Psi} > 0. \ (212)
$$

A temporary fiscal expansion unambiguously lowers the real consumption wage in the long-run:

$$
\frac{d}{dt} \left(\frac{W}{\tilde{P}_C}\right)|_{\text{temp}} = \frac{W}{\tilde{P}_C} (\alpha_L - \alpha_C) \tilde{P}_C|_{\text{temp}} < 0. \ (213)
$$

Since data indicate that $\alpha_L > \alpha_C$, the long-run depreciation in the relative price of non tradables drives down the real consumption wage.

**Initial Responses of Sectoral Variables**

To determine the initial reaction of selected variables, we linearize the short-run static solution of variable $X(t)$, i.e., $X(t) = X (\tilde{\lambda}, G^N(t))$, in the neighborhood of the steady-state:

$$
X(t) - \tilde{X} = X_{GN} \left(G^N(t) - \tilde{G}^N\right), \ (214)
$$

and evaluate its initial reaction relative to its initial steady-state value:

$$
dX(0) \equiv X(0) - \tilde{X}_0 = \tilde{X} - \tilde{X}_0 + X_{GN} dG^N(0). \ (215)
$$

Because a temporary fiscal expansion has long-run effects, variables are affected by (indirectly) the change in the shadow value of wealth $\tilde{\lambda}$, as captured by $\tilde{X} - \tilde{X}_0$, and directly by the change in government spending $G^N$, as captured by $dG^N(0)$.

Since we are interested in responses of key macroeconomic variables in the short-run, we analyze the reactions of macroeconomic variables on impact. We first explore the response of the price of non traded goods in terms of traded goods. Evaluating (155) at time $t = 0$ yields the initial response of the relative price of non tradables:

$$
\tilde{P}(0)|_{\text{temp}} = \left[ -\frac{\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C}{\Psi} \tilde{\lambda}|_{\text{temp}} + \frac{1}{\Psi} \frac{PdG^N(0)}{Y}\right],
$$

$$
= \left[ -\frac{\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C}{\Psi} \tilde{\lambda}|_{\text{temp}} + \frac{g}{\Psi} \tilde{P}(0)|_{\text{per}}\right] > 0, \ (216)
$$

where $\Psi > 0$ and we used the fact that:

$$
\frac{PdG^N(0)}{Y} = \frac{dG(0)}{Y} = 1 - (1 - g) = g > 0. \ (217)
$$
Because the rise in the marginal utility of wealth is smaller after a temporary fiscal shock than after a permanent rise in \( G^N \), i.e., \( 0 < \hat{\lambda} \big|_{\text{temp}} < \hat{\lambda} \big|_{\text{perm}} \). \( P \) increases more on impact after a temporary shock than after a permanent shock. Intuitively, as the wealth effect is smaller when the fiscal shock is temporary, consumption in non tradables falls less which in turn triggers a larger excess demand in the non traded goods market, thus causing the relative price of non tradables to appreciate more.

Using the fact that \( dL^N(t) = L^N(t) - \dot{L}^N = L^N(\hat{\lambda}, G^N(t)) - L^N(\alpha_0, G^N_0) \) with \( dG(t) = G(t) - G^N_0 = 0 \) and totally differentiating the short-run solution for non traded labor described by eq. (162), one obtains the initial response of non traded labor following an exogenous increase in government consumption:

\[
\dot{L}^N(0) \big|_{\text{temp}} = \frac{\partial L^N}{\partial \tilde{\lambda}} \text{ } \hat{\lambda} \big|_{\text{temp}} + L^N_{GN} dG^N(0) > 0. \tag{218}
\]

While \( L^N_{GN} > 0 \), the sign of \( L^N_\lambda \) can be positive or negative. If \( L^N_\lambda < 0 \), because the marginal utility of wealth increases less after a temporary rise in \( G^N \) than after a permanent increase in \( G^N \), the negative impact on \( L^N \) produced by the wealth effect (which reduces \( C^N \)) is smaller. Remembering that \( L^N \) rises after a permanent fiscal shock, we can infer from this that non traded labor increases more following a temporary fiscal shock. If \( L^N_\lambda > 0 \), non traded labor increases less after a temporary shock than after a permanent shock.

Using (161), the change in non traded labor in the short-run following a temporary fiscal shock can be written as follows:

\[
\dot{L}^N(0) \big|_{\text{temp}} = \left( \dot{\lambda} \right)_{\text{temp}} \{ \omega_C \alpha_C \{ (1 - \alpha_C) \phi + \alpha_C \sigma_C \} - \sigma_C \{ \epsilon (1 - \alpha_L) + \sigma_L \alpha_L \} \} \}
+ \left( \epsilon (1 - \alpha_L) + \sigma_L \alpha_L \right) \frac{PdG^N}{Y}. \tag{219}
\]

Because \( L^N = \sigma_L \dot{\lambda} \big|_{\text{temp}} + \{ \epsilon (1 - \alpha_L) + \alpha_L \sigma_L \} \dot{P} \big|_{\text{temp}} \) where \( \dot{\lambda} \big|_{\text{temp}} > 0 \) (see eq. (193)) together with condition (205)) and \( \dot{P} \big|_{\text{temp}} > 0 \) (see eq. (216)), non traded labor unambiguously increases on impact after a temporary rise in \( G^N \). Intuitively, the negative wealth effect induces households to supply more labor while the appreciation in the relative price of non tradables pushes up the non traded wage \( W^N \) which encourages workers to shift hours worked toward the non traded sector.

Totally differentiating \( L^T = L^T(\hat{\lambda}, P) \) and inserting the solution for the relative price given by (158), the initial reaction of \( L^T \) following a temporary fiscal expansion can be written as follows:

\[
\dot{L}^T(0) \big|_{\text{temp}} = \sigma_L \dot{\lambda} \big|_{\text{temp}} + \alpha_L \left( \sigma_L - \epsilon \right) \dot{P} \big|_{\text{temp}},
\]

\[
= \frac{\Psi \sigma_L + \alpha_L (\epsilon - \sigma_L) \{ \alpha_L \sigma_L + \psi_C \alpha_C \sigma_C \}}{\Psi} \dot{\lambda} \big|_{\text{temp}} - \frac{\alpha_L (\epsilon - \sigma_L) g}{\Psi} \leq 0, \tag{220}
\]

where \( \Psi > 0 \) (see eq. (156)), \( \dot{\lambda} \big|_{\text{temp}} > 0 \) is given by (204), and we used the fact that \( \frac{PdG^N}{Y} = g \) (see eq. (217)) to determine (197). Using the fact that \( \dot{L}^T = \sigma_L \dot{\lambda} \big|_{\text{temp}} + \alpha_L \left( \sigma_L - \epsilon \right) \dot{P} \big|_{\text{temp}} \), because both the shadow value of wealth \( \hat{\lambda} \) and the relative price of non tradables \( P \) increase, we find that a rise in \( G^N \) raises \( L^T \) if \( \sigma_L > \epsilon \), i.e., if labor is weakly mobile across sectors. Conversely, setting assumption 1, i.e., \( \sigma_L < \epsilon \), traded labor falls because the cost of shifting hours worked from one sector to another is low enough.

Differentiating the short-run change in the real consumption wage, and using the fact that \( \hat{W} = \alpha_L \hat{W}^N \) and \( \dot{P}_C = \alpha_C \dot{P} \), yields:

\[
\frac{d}{dt} \left( \frac{W}{P_C} \right) (0) \big|_{\text{temp}} = \frac{W}{P_C} \left( \alpha_L - \alpha_C \right) \dot{P}(0) \big|_{\text{temp}} > 0. \tag{221}
\]

Because \( P \) appreciates more after a temporary fiscal shock, the real consumption aggregate wage will increase by a larger amount than after a permanent fiscal shock.
E.11 Steady-State Effects of a Temporary Government Spending Shock: Graphical Apparatus

We characterize the equilibrium graphically which allows us to build up intuition on the long-run effects of a temporary rise in $G^N$. Because we focus on steady-state, we omit the tilde below for simplicity purposes when it does not cause confusion.

E.11.1 The Initial Steady-State

We denote by $NX = YT - CT - GT$ net exports. Hence, in the long-run, we have $r^*B = -NX$. Dividing both sides by $YT$, we have: $v_B = -v_{NX}$. The initial equilibrium is thus defined by the following set of equations:

\[
\left( \frac{1 - \varphi}{\varphi} \right) \frac{CT}{CN} = P^\varphi, \tag{222a}
\]

\[
\left( \frac{1 - \vartheta}{\vartheta} \right) \frac{LT}{LN} = \Omega^{-\epsilon}, \tag{222b}
\]

\[
\frac{YT (1 - v_{NX} - v_{GT})}{YN (1 - v_{GN})} = \frac{CT}{CN}, \tag{222c}
\]

where $YT = LT$, $YN = LN$, $\Omega \equiv W^N/W^T$ is the ratio of the non traded wage to the traded wage ratio or the relative wage, and we denote by $v_{NX} \equiv NX/YT$ the ratio of net exports to traded output, and $v_{Gj} \equiv G^j/Y^j$ the ratio of government spending on good $j = T, N$ to output of sector $j = T, N$.

E.11.2 Graphical Apparatus

To build up intuition, we characterize the equilibrium graphically. We denote the logarithm of variables with lower-case letters. The steady state can be described by considering alternatively the goods market or the labor market.

**Goods Market Equilibrium- and Labor Market Equilibrium-Schedules**

The steady-state (222) can be summarized graphically in Figure 52 that traces out two schedules in the $(y^T - y^N, p)$-space. System (222a)-(222d) described above can be reduced to two equations. Substituting (222a) into eq. (222d) yields the goods market equilibrium (henceforth labelled GME) schedule:

\[
y^T (1 - v_{NX} - v_{GT}) \bigg|_{GME} = \phi p + \ln \left( \frac{1 - v_{GN}}{1 - v_{NX} - v_{GT}} \right) + x, \tag{223}
\]

where $x = \ln \left( \frac{\varphi}{1 - \varphi} \right)$. Since a rise in the relative price $p$ raises consumption in tradables, the goods market equilibrium requires a rise in the traded output relative to non traded output. Hence the goods market equilibrium is upward-sloping in the $(y^T - y^N, p)$-space where the slope is equal to $1/\varphi$.

Substituting (222b) into (222c) to eliminate $\omega$ yields the labor market equilibrium (henceforth LME) schedule:

\[
y^T (1 - v_{NX} - v_{GT}) \bigg|_{LME} = -\epsilon p + z, \tag{224}
\]

where $z = \ln \left( \frac{\vartheta}{1 - \vartheta} \right)$. A rise in the relative price $p$ increases the relative wage $\omega$ which encourages agents to supply more labor in the non traded sector, and all the more so as the values of $\epsilon$ are higher. Hence the labor market equilibrium is downward-sloping in the $(y^T - y^N, p)$-space where the slope is equal to $-1/\epsilon$. Assuming that the shift of labor across sectors is costless, i.e., if we let $\epsilon$ tend toward infinity, wages are equalized across sectors. Graphically, the LME-schedule becomes a horizontal line. Conversely, as long as switching hours worked from one sector to another is costly, i.e., if $\epsilon$ takes finite values, the LME-schedule is negatively related to the relative price of non tradables in the $(y^T - y^N, p)$-space.
Labor Demand- and Labor Supply-Schedules

The steady-state (222) can be summarized graphically by focusing alternatively on the labor market. Eq. (222b) describes the labor supply-schedule \((LS\) henceforth) in the \((l^T - l^N, \omega)\)-space. Taking logarithm yields:

\[
(l^T - l^N)_{LS} = -\epsilon_\omega + z, \quad (225)
\]

where \(z = \ln \left(\frac{\phi}{1 - \phi}\right)\). A rise in the non traded wage traded wage ratio \(\omega\) provides an incentive to shift labor supply from the traded sector towards the non traded sector. Hence the \(LS\)-schedule is downward-sloping in the \((l^T - l^N, \omega)\)-space where the slope is equal to \(-1/\epsilon\).

Inserting demand for traded goods in terms of non traded goods (222a) into the market clearing condition given by (222d) yields:

\[
\left(\frac{\phi}{1 - \phi}\right) P^\phi \left(\frac{1 - \nu_{GN}}{1 - \nu_{NX} - \nu_{GT}}\right). \quad (226)
\]

Substituting first-order conditions from the firms’ maximization problem and using production functions, i.e. \(L^T = Y^T\) and \(L^N = Y^N\), we get:

\[
\frac{L^T}{L^N} = \left(\frac{\phi}{1 - \phi}\right) \Omega^\phi \left(\frac{1 - \nu_{GN}}{1 - \nu_{NX} - \nu_{GT}}\right). \quad (227)
\]

Taking logarithm yields the labor demand-schedule \((LD\) henceforth) in the \((l^T - l^N, \omega)\)-space is given by

\[
(l^T - l^N)_{LD} = \phi_\omega + \ln \left(\frac{1 - \nu_{GN}}{1 - \nu_{NX} - \nu_{GT}}\right) + x, \quad (228)
\]

where \(x = \ln \left(\frac{\phi}{1 - \phi}\right)\). A rise in the relative wage \(\omega\) raises the cost of labor in the non traded sector relative to the traded sector. To compensate for the increased labor cost, non traded firms charge prices which encourage agents to substitute traded for non traded goods and therefore produces an expansionary effect on labor demand in the traded sector. Hence the \(LD\)-schedule is upward-sloping in the \((l^T - l^N, \omega)\)-space where the slope is equal to \(1/\phi\).

In order to facilitate the interpretation of analytical results, it is useful to rewrite \(\ln \left(\frac{1 - \nu_{GN}}{1 + v_B - \nu_{GT}}\right)\) by using a first-order Taylor approximation which implies:

\[
\ln (1 - \nu_{NX} - \nu_{GT}) - \ln (1 - \nu_{GN}) \simeq -\nu_{NX} - \nu_{GT} + \nu_{GN}. \quad (229)
\]

E.11.3 Long-Run Adjustments in the Relative Price and Relative Wage

We now analyze graphically and analytically the consequences on the relative price and the relative wage of a temporary increase in \(G^N\). The initial long-run equilibrium is represented at \(E_0\) in Figure 52. The long-run equilibrium is defined by the the system of equations (222).

Equating (233) and (244), differentiating and denoting by a hat the deviation in percentage from initial steady state, one obtains the long-run adjustment in the relative price of non tradables to an exogenous temporary rise in government consumption on non tradables:

\[
\hat{p} = \frac{-d\nu_{NX}}{\phi + \epsilon} < 0, \quad (229)
\]

where we made use of the approximation given by (228) and \(d\nu_{NX} = -\frac{\nu_{NX}}{\nu_{GT}} > 0\) and \(d\nu_{GN} = 0\). By raising net exports and thus the demand for tradables in the long-run, a temporary increase in government spending depreciates the relative price of non tradables.

Equating (225) and (227), differentiating and denoting by a hat the deviation from initial steady state in percentage terms, one obtains the long-run adjustment in the relative wage to an exogenous temporary rise in government consumption on non tradables:

\[
\hat{\omega} = \frac{-d\nu_{NX}}{\phi + \epsilon} < 0, \quad (230)
\]

where we made use of the approximation given by (228); by raising net exports in the long-run, a rise in \(G^N\) shifts the \(LD\)-schedule to the right in the \((l^T - l^N, \omega)\)-space and thus a temporary rise in \(G^N\) permanently lowers the non traded wage relative to the traded wage.
E.12 Solving the Model with Perfect Mobility of Labor across Sectors

In this subsection, we provide analytical results when assuming perfect mobility of labor across sectors. If we let \( \epsilon \) tend toward infinity into eq. (120), hours worked across sectors become perfect substitutes:

\[
L = L^T + L^N.
\]  

(231)

Because workers no longer experience a cost when shifting from one sector to another, hours worked in the traded and the non traded sector are perfect substitutes. Since workers are willing to devote their whole time to the sector that pays the highest wages, firms in both sectors must pay the same wage. Hence, \( 1 = W^T = W^N \). The wage equalization across sectors implies that \( P = 1 \). As a result, the relative price of non tradables remains unaffected by a government spending shock.

Inserting short-run static solutions for \( C^N \) given by (143) into the non-traded good market clearing condition gives us:

\[
L^N = C^N \left( \tilde{\lambda}, P \right) + G^N.
\]

(232)

The non-traded good market clearing condition can be solved for non traded labor

\[
L^N = L^N \left( \tilde{\lambda}, G^N \right),
\]

(233)

where partial derivatives are obtained by totally differentiating (232):

\[
\hat{L}^N = -\frac{\omega CC \sigma_C}{\alpha_L} \hat{\lambda} + \frac{1}{\alpha_L} \frac{P dG^N}{Y},
\]

(234)

with the ratio of consumption expenditure to GDP denoted by \( \omega_C = \frac{P CC}{Y} \), and the non tradable content of GDP denoted by \( \alpha_L = \frac{PY^N}{L} \).

Inserting the short-run static solution for non traded labor (233) and the short-run static solution for aggregate labor supply given by

\[
L = L \left( \tilde{\lambda} \right), \quad \dot{L} = \sigma_L \dot{\lambda},
\]

(235)

the resource constraint for labor given by (231) can be solved for traded labor:

\[
L^T = L^T \left( \tilde{\lambda}, G^N \right),
\]

(236)

where partial derivatives are obtained by totally differentiating the resource constraint for labor given by (231):

\[
(1 - \alpha_L) \dot{L}^T = \sigma_L \dot{\lambda} - \alpha_L L^N.
\]

Inserting the solution for non traded labor expressed in rate of change (234) allows us to solve for traded labor:

\[
(1 - \alpha_L) \dot{L}^T = \left( \sigma_L + \omega CC \sigma_C \right) \dot{\lambda} - \frac{P dG^N}{Y}.
\]

(237)

Effects of a Permanent Rise in Government Spending

Inserting (236) into the current account equation, linearizing and solving yields the intertemporal solvency condition (ISC):

\[
\tilde{B} = B_0.
\]

(238)

Inserting the ISC (238) and appropriate short-run static solutions which obviously hold in the long-run, the steady-state can be reduced to one equation:

\[
\tau^* B_0 + L^T \left( \tilde{\lambda}, G^N \right) - C^T \left( \tilde{\lambda}, P \right) - G^T = 0,
\]

(239)

where \( P \) remains constant. Equation (239) can be solved for the marginal utility of wealth:

\[
\tilde{\lambda} = \lambda \left( G^N, G^T \right).
\]

(240)
Note that we concentrate below on a rise in government spending on non tradables $G^N$ because empirical evidence indicate that the non-tradable content of public spending averages to 90% for OECD countries. Using the fact that the stock of traded bonds is initially predetermined and totally differentiating (239) yields:

$$(1 - \alpha_L) \dot{L}^T = \omega_C (1 - \alpha_C) \dot{C}^T + \frac{dG^T}{Y}.$$  

Inserting (237) and using the fact that $P$ remains unaffected by a fiscal expansion, the change in the equilibrium value of the marginal utility of wealth is:

$$\dot{\lambda} = \frac{\rho dG^N}{Y} + \frac{dG^T}{\sigma_L + \omega_C \sigma_C}.$$  

(241)

Inserting (241) into (234) yields the change in non traded labor following a permanent fiscal expansion:

$$\dot{L}^N = \frac{\sigma_L + \omega_C \sigma_C (1 - \alpha_C) \rho dG^N}{\alpha_L (\sigma_L + \omega_C \sigma_C)} - \frac{\omega_C \alpha_C \sigma_C G^T (t) - \omega_C G^T (t)}{\alpha_L (\sigma_L + \omega_C \sigma_C)}.$$  

(242)

Inserting (241) into (237) yields the change in traded labor following a permanent fiscal expansion:

$$\dot{L}^T = - \frac{\omega_C (1 - \alpha_C) \sigma_C}{(1 - \alpha_L) (\sigma_L + \omega_C \sigma_C)} \frac{P dG^N}{Y} + \frac{\sigma_L + \omega_C \sigma_C \alpha_C}{(1 - \alpha_L) (\sigma_L + \omega_C \sigma_C)} \frac{dG^T}{Y}.$$  

(243)

According to (242) and (243), a permanent fiscal expansion raises non traded labor and lowers traded labor, while wages, the relative price, and the net foreign asset position remain unchanged.

**Effects of a Temporary Rise in Government Spending**

Inserting first the short-run static solutions for traded labor (236) and consumption in tradables (143) into the market clearing condition for the traded good (152) yields:

$$\dot{B} (t) = r^* B (t) + L^T (\hat{\lambda}, G^N (t)) - C^T (\hat{\lambda}, P) - G^T.$$  

(244)

Linearizing the current account equation above around the steady-state gives us:

$$\dot{B} (t) = r^* \left( B (t) - \hat{B} \right) + L^T_{G^N} (G^N (t) - \hat{G}^N).$$  

(245)

Inserting $L^T_{G^N} = - \frac{\dot{L}^T}{1 - \alpha_L} \frac{\rho}{Y} = - \hat{P}$ (see eq. (237)), eq. (245) can be rewritten as follows:

$$\dot{B} (t) = r^* \left( B (t) - \hat{B} \right) - \left( G(t) - \hat{G} \right),$$  

(246)

where we used the fact that $dG^N = \frac{dG(t)}{G} \frac{G(t)}{Y}$ since the relative price of non tradables remains constant over time as $P$ must stick to the marginal product of labor (that reduces to 1).

Inserting the law of motion of government spending given by (186), eq. (246) can be rewritten as follows:

$$\dot{B} (t) = r^* \left( B (t) - \hat{B} \right) - Y \left[ e^{-\xi t} - (1 - g) e^{-r t} \right].$$  

(247)

Pre-multiplying by $e^{-r^* t}$ and integrating over $(0, t)$ allow us to obtain the general solution for $B(t)$:

$$B(t) - \hat{B} = \left[ \left( B_0 - \hat{B} \right) - \frac{Y}{\xi + r^*} (1 - \Theta') \right] e^{-r^* t} + \frac{Y}{\xi + r^*} \left( e^{-\xi t - \Theta' e^{-\xi t}} \right),$$  

(248)

where we used the fact that $\int_0^t e^{-(\xi + r^*) r} d\tau = \frac{(1 - e^{-\xi r})}{\xi + r^*}$ and we set:

$$\Theta' = (1 - g) \frac{\xi + r^*}{\chi + r^*} > 0.$$  

(249)
Invoking the transversality condition, one obtains the 'stable' solution for the stock of foreign assets so that \( B(t) \) converges toward its steady-state value \( \tilde{B} \):

\[
B(t) - \tilde{B} = \frac{Y}{\xi + r^*} \left( e^{-\xi t} - \Theta e^{-\chi t} \right). 
\] (250)

Eq. (250) gives the trajectory for \( B(t) \) consistent with the intertemporal solvency condition:

\[
\left( \tilde{B} - B_0 \right) = -\frac{Y}{\xi + r^*} \left( 1 - \Theta' \right), 
\] (251)

where \( 1 - \Theta' > 0 \) due to inequality (187). According to (251), a temporary rise in government spending deteriorates the net foreign asset position, i.e., \( d\tilde{B} < 0 \).

Differentiating (250) w.r.t. time leads to the trajectory for the current account along the transitional path when government spending follows the temporal path given by eq. (186):

\[
\dot{B}(t) = -\frac{Y}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta' e^{-\chi t} \right). 
\] (252)

According to (252), the net foreign asset position deteriorates monotonically since \( (\xi e^{-\xi t} - \chi \Theta' e^{-\chi t}) > 0 \) for \( t \geq 0 \).

Evaluating (252) at time \( t = 0 \) leads to the initial current account response, expressed as a percentage of initial GDP, following a temporary rise in government spending:

\[
\frac{\dot{B}(0)}{Y} = -\left( \frac{\xi - \chi \Theta'}{\xi + r^*} \right) < 0, 
\] (253)

where \( (\xi - \chi \Theta') > 0 \).

**The Change in the Equilibrium Value of the Marginal Utility of Wealth**

Eq. (251) allows us to calculate the steady-state change in the foreign asset position following a temporary rise in government spending:

\[
d\tilde{B} \bigg|_{\text{temp}} = -\frac{Y}{\xi + r^*} \left( 1 - \Theta' \right). 
\] (254)

To determine the change in the equilibrium value of the marginal utility of wealth, we have to differentiate the market clearing condition for traded goods:

\[
r^* d\tilde{B} \bigg|_{\text{temp}} + (L_T^T - C_T^T) \frac{d\bar{\lambda}}{\bar{\lambda}} \bigg|_{\text{temp}} = 0.
\]

Expressing the equation above in rate of change and dividing by initial GDP leads to:

\[
r^* \frac{d\tilde{B}}{Y} \bigg|_{\text{temp}} + \left( (1 - \alpha_L) \frac{\hat{L}^T}{\lambda} - (1 - \alpha_C) \omega_C \frac{\hat{C}^T}{\lambda} \right) \frac{d\bar{\lambda}}{\bar{\lambda}} \bigg|_{\text{temp}} = 0, 
\] (255)

where

\[
(1 - \alpha_L) \frac{\hat{L}^T}{\lambda} - (1 - \alpha_C) \omega_C \frac{\hat{C}^T}{\lambda} = \sigma_L + \omega_C \sigma_C. 
\] (256)

Inserting (254) and (256), eq. (255) can be solved for the change in the equilibrium value of the marginal utility of wealth:

\[
\frac{d\bar{\lambda}}{\bar{\lambda}} \bigg|_{\text{temp}} = -\frac{1}{\sigma_L + \omega_C \sigma_C} \frac{r^* d\tilde{B}}{Y_0} \bigg|_{\text{temp}}, 
\]

\[
= \frac{1}{\sigma_L + \omega_C \sigma_C} \frac{r^*}{\xi + r^*} (1 - \Theta'). 
\] (257)

Following a temporary fiscal shock, the marginal utility of wealth increases less than after a permanent rise in \( G^N \).
E.13 A Friendly Way to Solve the Model with Imperfect Mobility of Labor

In this subsection, we solve analytically the model with imperfect mobility of labor across sectors by keeping our assumption according to which the government spending shock is fully biased toward non tradables. We relax this assumption in subsection E.19 where we consider a government spending shock which is split between non tradables and tradables. We simplify the government spending shock by assuming that the endogenous response of government spending to an exogenous fiscal shock is governed by the following dynamic equation:

\[ dG(t) = \dot{Y} ge^{-\xi t}. \tag{258} \]

According to (258), government spending rises initially by \( g > 0 \) percentage points of GDP and declines monotonically at rate \( \xi > 0 \). The latter feature simplifies substantially analytical expressions.

The short-run equilibrium can be rewritten as follows:

\[ C = \left( P C \lambda \right)^{-\sigma C}, \tag{259a} \]
\[ L = \left( \lambda W \right)^{\sigma L}, \tag{259b} \]
\[ L^N = \alpha L \frac{W}{W^N} L, \tag{259c} \]
\[ L^T = (1 - \alpha L) \frac{W}{W^T} L, \tag{259d} \]
\[ C^N = \alpha C \frac{P C}{P} C, \tag{259e} \]
\[ C^T = (1 - \alpha C) P C, \tag{259f} \]
\[ W^T = 1, \tag{259g} \]
\[ W^N = P, \tag{259h} \]
\[ Y^N = C^N + G^N, \tag{259i} \]
\[ \dot{B} = r^* B + Y^T - C^T - G^T, \tag{259j} \]

where \( Y^N = L^N, Y^T = L^T, \alpha_C \) is given by eq. (130a) and \( \alpha_L \) is given by eq. (136a).

**Short-Run Solutions**

Substituting first (259a) into (259e), (259b) and (259h) into (259c), the market clearing condition (259i) for the non traded good can be rewritten as follows:

\[ \frac{\alpha L \lambda^{\sigma L} W^{1 + \sigma L}}{P} = \frac{\alpha C P^{1 - \sigma C} \lambda^{1 - \sigma C}}{P} + G^N. \tag{260} \]

As will be useful later, we compute the change in percentage of the shares of non tradables and tradables into consumption and labor. Totally differentiating (130a)-(131), and (136a)-(136b) yields:

\[ \dot{\alpha}_C = (1 - \phi) (1 - \alpha_C) \dot{P}, \tag{261a} \]
\[ (1 - \alpha_C) = (\phi - 1) \alpha_C \dot{P}, \tag{261b} \]
\[ \dot{\alpha}_L = (\epsilon + 1) (1 - \alpha_L) \dot{P}, \tag{261c} \]
\[ (1 - \alpha_L) = - (\epsilon + 1) \alpha_L \dot{P}, \tag{261d} \]

where we used the fact that \( \dot{P} = \alpha C \dot{P} \) (since \( P^T = 1 \)), and \( \dot{W} = \alpha L \dot{P} \) (since \( W^T = P^T = 1 \) and \( W^N = P \)).

Totally differentiating (260), using (261a) et (261c), leads to:

\[ \dot{\alpha}_L + \alpha L \ddot{\lambda} + (1 + \alpha L) \dot{W} - \dot{P} = \frac{C^N}{L^N} \left[ \dot{\alpha}_C + (1 - \sigma C) \dot{P} - \sigma C \dot{\lambda} - \ddot{P} \right] + \frac{dG^N}{L^N}, \]
\[ \frac{PL^N}{Y} \left[ \dot{\alpha}_L + \alpha L \ddot{\lambda} + (1 + \sigma L) \dot{W} - \dot{P} \right] = \frac{PC^N}{Y} \left[ \dot{\alpha}_C + (1 - \sigma C) \dot{P} - \sigma C \ddot{\lambda} - \ddot{P} \right] + \frac{P dG^N}{Y}, \]
\[ \alpha_L \left\{ (1 + \epsilon) (1 - \alpha_L) \dot{P} + [(1 + \sigma L) \alpha L - 1] \dot{P} + \sigma L \ddot{\lambda} \right\} = \alpha C \omega C \left\{ (1 - \phi) (1 - \alpha C) \dot{P} + [(1 - \sigma C) \alpha C - 1] \dot{P} - \sigma C \ddot{\lambda} \right\} + \frac{P dG^N}{Y}, \]

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where $\omega_C = \frac{PC}{Y}$, $\frac{PC^N}{P_C} = \alpha_C$, $\frac{PL_N}{Y} = \frac{W^{NL}}{W} = \alpha_L$. Collecting terms, the deviation in percentage from the initial steady-state for the relative price of non tradables is described by:

$$\hat{P} = -\frac{[\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C]}{\Psi} \hat{\lambda} + \frac{1}{\Psi} \frac{PdG^N}{Y}, \quad (262)$$

where $\Psi$ is given (157).

Totally differentiating (259b) and using the fact that $\hat{W} = \alpha_L \hat{W}^N + (1 - \alpha_L) \hat{W}^T$ with $\hat{W}^N = \hat{P}$ and $\hat{W}^T = 0$ leads to the response of employment in percentage deviation from initial steady-state:

$$\hat{L} = \sigma_L \hat{\lambda} + \sigma_L \alpha_L \hat{P}. \quad (263)$$

Substituting (259c) and (259h) into (259c) leads to $L^T = (1 - \alpha_L)(W)^{1+\sigma_L} (\hat{\lambda})^{\sigma_L}$. Totally differentiating and using the fact that $\hat{1} - \alpha_L = -(1 + \epsilon) \hat{W}$ with $\hat{W} = \alpha_L \hat{P}$, one obtains:

$$\hat{L}^T = - (\epsilon - \sigma_L) \alpha_L \hat{P} + \sigma_L \hat{\lambda},$$

$$= \left\{ \frac{\sigma_L \Psi - \epsilon (1 - \alpha_L) + \alpha_L \sigma_L}{\Psi} \right\} \hat{\lambda} - \frac{\alpha_L (\epsilon - \sigma_L) PdG^N}{Y}. \quad (264)$$

Substituting (259b) and (259h) into (259d) leads to $L^N = \frac{\alpha_L}{Y} (W)^{1+\sigma_L} \hat{\lambda}^{\sigma_L}$. Totally differentiating yields:

$$\hat{L}^N = [(1 + \epsilon) (1 - \alpha_L) + (1 + \sigma_L) \alpha_L - 1] \hat{P} + \sigma_L \hat{\lambda},$$

$$= [\epsilon (1 - \alpha_L) + \alpha_L \sigma_L] \hat{P} + \sigma_L \hat{\lambda},$$

$$= \left\{ \frac{\sigma_L \Psi - \epsilon (1 - \alpha_L) + \alpha_L \sigma_L}{\Psi} \right\} \hat{\lambda} + \frac{\epsilon (1 - \alpha_L) + \alpha_L \sigma_L}{\Psi} \frac{PdG^N}{Y}. \quad (265)$$

**Solution for the Net Foreign Asset Position**

Substituting $L^T = (1 - \alpha_L)(W)^{1+\sigma_L} (\hat{\lambda})^{\sigma_L}$ and $C^T = (1 - \alpha_C) P_C^{1-\sigma_C} \hat{\lambda}^{1-\sigma_C}$ into (259j) leads to:

$$\hat{B}(t) = r^* B(t) + (1 - \alpha_L(t)) W(t)^{1+\sigma_L} \hat{\lambda}^{\sigma_L} - (1 - \alpha_C(t)) P_C(t)^{1-\sigma_C} \hat{\lambda}^{1-\sigma_C} - G^T. \quad (266)$$

Using the fact that both $\hat{\lambda}$ and $G^T$ are constant over time, linearizing (266) in the neighborhood of the steady-state yields:

$$\hat{B}(t) = r^* dB(t) - L^T \frac{\alpha_L (\epsilon - \sigma_L) \hat{P} PdG^N(t)}{Y} - \frac{C^T \alpha_C (\phi - \sigma_C) \hat{P} PdG^N(t)}{Y},$$

$$= r^* dB(t) - \hat{Y} \hat{T}_G \hat{N} e^{-\xi t}. \quad (267)$$

where $\frac{\partial \hat{B}(t)}{\partial \hat{G}(t)} = - \hat{T}_G^N = - \left[ (1 - \alpha_L) \alpha_L \sigma_L + (1 - \alpha_C) \omega_C \phi \alpha_C (\phi - \sigma_C) \right] \frac{\Psi}{\Psi} < 0$ is given by eq. (192).

Substituting the law of motion of government spending (258) and solving leads to the general solution for the net foreign asset position:

$$B(t) - \hat{B} = \left[ \left( B_0 - \hat{B} \right) - \left( \frac{\hat{T}_G \hat{Y}}{\xi + r^*} \right) g \right] e^{r^* t} + \left( \frac{\hat{T}_G \hat{Y}}{\xi + r^*} \right) g e^{-\xi t}. \quad (268)$$

Invoking the transversality condition gives the solution for $B(t)$:

$$B(t) - \hat{B} = \left( \frac{\hat{T}_G \hat{Y}}{\xi + r^*} \right) g e^{-\xi t}, \quad (268)$$

consistent with the intertemporal solvency condition

$$\left( \hat{B} - B_0 \right) = - \left( \frac{\hat{T}_G \hat{Y}}{\xi + r^*} \right) g. \quad (269)$$
To determine the change in the equilibrium value of the marginal utility of wealth, we have to differentiate the market clearing condition (266) for the traded good evaluated at the steady-state (i.e., \( \dot{B}(t) = 0 \)), using the fact that in the long-run government spending reverts to its initial level (i.e., \( dG^N = 0 \)):

\[
\frac{r^* \dot{d}B}{Y} + (1 - \alpha_L) \hat{L}^T = (1 - \alpha_C) \omega_C \hat{C}^T,
\]

where \( \Gamma > 0 \) is given by eq. (174). Substituting (269) into the above equation leads to the change in the equilibrium value of the marginal utility of wealth:

\[
\frac{\dot{\lambda}}{\Gamma} = -\frac{\Psi \; r^* \dot{d}B}{Y},
\]

where \( \Gamma > 0, \Psi > 0, \) and \( r^* > 0 \).

Before evaluating the short-run effects of the fiscal shock, it is useful to rewrite \( \Gamma \) given by eq. (174) as follows:

\[
\Gamma = \frac{\Psi \{[(1 - \alpha_L) \sigma_L + \omega_C (1 - \alpha_C) \sigma_C] + [\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C] \gamma_G^N \Psi\}}{\xi + r^*} + 1.
\]

where we used the fact that \( \gamma_G^N \Psi = [(1 - \alpha_L) \alpha_L (\xi - \sigma_L) + (1 - \alpha_C) \omega_C \alpha_C (\phi - \sigma_C)] \). Eq. (271) implies that the following inequality holds:

\[
0 < \frac{\Psi \gamma_G^N}{\Gamma} (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C) < 1,
\]

where \( \Gamma > 0, \Psi > 0, \) and \( \gamma_G^N > 0 \).

**Impact Effects of a Temporary Fiscal Expansion**

Evaluating (262) at time \( t = 0 \), inserting (270), and using the fact that \( \frac{\dot{d}G^N(0)}{Y} = g > 0 \), leads to the initial response of the relative price of non tradables:

\[
\hat{P}(0) = \frac{- [\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C] r^*}{\Psi} + 1 + \frac{\dot{P}dG^N(0)}{Y} g > 0,
\]

where the term in braces is unambiguously positive due to inequality (272) and \( 0 < \frac{r^*}{\xi + r^*} < 1 \).

Substituting the change in the equilibrium value of the marginal utility of wealth given by eq. (270) into (265), and multiplying both sides by \( \alpha_L \) leads to the initial reaction of non traded labor from initial steady-state in total labor units:

\[
\alpha_L \hat{L}^N(0) = \frac{\alpha_L [\xi (1 - \alpha_L) + \alpha_L \sigma_L]}{\Psi} \left[ 1 - (\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C) \gamma_G^N \Psi \right] \frac{r^*}{\xi + r^*} g > 0,
\]

where the term in brackets \( 1 - (\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C) \gamma_G^N \Psi \frac{r^*}{\xi + r^*} \) is unambiguously positive due to inequality (272) and \( 0 < \frac{r^*}{\xi + r^*} < 1 \); hence, labor in the non traded sector unambiguously increases.

Substituting the change in the equilibrium value of the marginal utility of wealth given by eq. (270) into (264) and multiplying both sides by \( 1 - \alpha_L \) leads to the initial reaction of traded labor from initial steady-state in total labor units:

\[
(1 - \alpha_L) \hat{L}^T(0) = \frac{-(1 - \alpha_L) \alpha_L (\xi - \sigma_L)}{\Psi} \left[ 1 - (\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C) \gamma_G^N \Psi \right] \frac{r^*}{\xi + r^*} g > 0,
\]

where the term in brackets \( 1 - (\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C) \gamma_G^N \Psi \frac{r^*}{\xi + r^*} \) is unambiguously positive due to inequality (272) and \( 0 < \frac{r^*}{\xi + r^*} < 0 \).
where inequality (272) together with \(0 < \frac{r^*}{\xi + r^*} < 1\) imply that the first term on the RHS is unambiguously negative as long as we set assumption 1.

Differentiating (268) with respect to time leads to the response of the current account as a percentage of GDP:

\[
\frac{\dot{B}(t)}{Y} = -\gamma_N \frac{\xi}{\xi + r^*} g e^{-\xi t} < 0,
\]

where \(\gamma_N > 0\).

We now investigate the impact of a government spending shock on sectoral output (or alternatively labor since \(Y^j = L^j\)) shares. To begin with, real GDP which we denote by \(Y_R\) is equal to the sum of value added at constant prices:

\[
Y_R = 
\]

\[
\hat{Y}_R = \sigma_L \hat{\lambda} + \alpha_L \sigma_L \hat{P}.
\]

According to (279), a government spending shock impinges on real GDP through two channels; first, by inducing agents to supply more labor, the negative wealth effect pushes up output; second, since the relative price of non tradables appreciates, non traded firms are induced to produce and thus to hire more; as workers’ experience mobility costs, non traded firms have to pay higher wages which increase the aggregate wage, \(W\), in proportion to the non tradable content of labor compensation, \(\alpha_L\); consequently, agents are encouraged to increase hours worked more which pushes up further real GDP.

To compute the change in the sectoral output share calculated as the growth differential between sectoral output and real GDP in total output units, we divide both sides of eq. (277) by \(Y_R\) and totally differentiate:

\[
0 = (1 - \alpha_L) \left( \hat{Y}_T - \hat{Y}_R \right) + \alpha_L \left( \hat{Y}_N - \hat{Y}_R \right).
\]

The first and the second term on the RHS of eq. (280) corresponds to the response of output share in sector \(j = T, N\) in total output units. More precisely, the change in the sectoral output share is measured by the product of the growth differential between output of sector \(j\) and real GDP and the share of sector \(j\) in GDP.

Using the fact that \(\hat{L}^T = \sigma_L \hat{\lambda} - \alpha_L (\epsilon - \sigma_L) \hat{P}\) and \(\hat{L}^N = \sigma_L \hat{\lambda} + [\epsilon (1 - \alpha_L) + \sigma_L \alpha_L] \hat{P}\), eq. (278) can be rewritten as follows:

\[
\hat{Y}_R = \sigma_L \hat{\lambda} + \alpha_L \sigma_L \hat{P}.
\]

Using the fact that \(\hat{L}^N = \sigma_L \hat{\lambda} + [\epsilon (1 - \alpha_L) + \sigma_L \alpha_L] \hat{P}\), inserting (278), and evaluating at time \(t = 0\), the response of the output share of non tradables is given by:

\[
\alpha_L \left( \hat{Y}_N(0) - \hat{Y}_R(0) \right) = \alpha_L (1 - \alpha_L) \epsilon \hat{P}(0),
\]

where \(\hat{P}(0)\) corresponds to the initial response of the relative price of non tradables in percentage deviation from trend (see eq. (273)).

Using the fact that \(\hat{L}^T = \sigma_L \hat{\lambda} - \alpha_L (\epsilon - \sigma_L) \hat{P}\), inserting (278), and evaluating at time \(t = 0\), the response of the output share of tradables is given by:

\[
(1 - \alpha_L) \left( \hat{Y}_T(0) - \hat{Y}_R(0) \right) = -\alpha_L (1 - \alpha_L) \epsilon \hat{P}(0),
\]

where the initial change in the relative price of non tradables relative to initial steady state in percent, \(\hat{P}(0)\), is described by eq. (273). It is straightforward to see that (282) is exactly the opposite of eq. (281).
When assuming perfect mobility of labor, the short-run equilibrium reduces to:

\[ C = (P \tilde{\lambda})^{-\sigma_C}, \quad \text{(283a)} \]
\[ L = (\tilde{\lambda} W)^{\sigma_L}, \quad \text{(283b)} \]
\[ C^N = \alpha C \frac{P_C}{P} C, \quad \text{(283c)} \]
\[ C^T = (1 - \alpha C) P_C C, \quad \text{(283d)} \]
\[ W^T = 1, \quad \text{(283e)} \]
\[ W^N = P, \quad \text{(283f)} \]
\[ W^N = W^T = W, \quad \text{(283g)} \]
\[ L = L^T + L^N, \quad \text{(283h)} \]
\[ Y^N = C^N + G^N, \quad \text{(283i)} \]
\[ \dot{B} = r^{\star} B + Y^T - C^T - G^T, \quad \text{(283j)} \]

where \( Y^N = L^N, Y^T = L^T \), and \( \alpha_C \) is given by eq. (130a).

**Short-Run Solutions**

Substituting (283e) and (283f) into (283g) leads to:

\[ P = 1. \quad \text{(284)} \]

Because sectoral wages must equalize while the marginal product of labor in the traded sector is fixed, the relative price of non tradables remains unaffected by a government spending shock, both in the short-run and the long-run.

Substituting first (283a) into (283c), the market clearing condition (151) for the non traded good can be rewritten as follows:

\[ L^N = \frac{\alpha C P_C^{1-\sigma_C} \tilde{\lambda}^{-\sigma_C}}{P} + G^N. \quad \text{(285)} \]

Totally differentiating (285), using (284), leads to:

\[ \alpha L \dot{L}^N = -\alpha C \omega_C \sigma C \tilde{\lambda} + \frac{P d G^N}{Y}. \quad \text{(286)} \]

Inserting \( L = \tilde{\lambda}^{\sigma_L} \) (since \( W = 1 \)) into (283h), differentiating and using (286) leads to:

\[ (1 - \alpha L) \dot{L}^T = [\sigma_L + \alpha C \omega_C \sigma C] \tilde{\lambda} - \frac{P d G^N}{Y}. \quad \text{(287)} \]

Inserting \( L^T = L - L^N \) together with \( L^N = C^N + G^N \) and \( L = \tilde{\lambda}^{\sigma_L} \) (since \( W = 1 \)) into (283j), the market clearing condition for the traded good can be written as follows:

\[ \dot{B}(t) = r^{\star} B(t) + L - P_C C - G^T - P G^N(t), \]
\[ = r^{\star} B(t) + \tilde{\lambda}^{\sigma_L} - (1 - \alpha C) P_C^{1-\sigma_C} \tilde{\lambda}^{-\sigma_C} - P G^N(t) - G^T. \quad \text{(288)} \]

Using the fact that both \( \tilde{\lambda}, G^T, \) and \( P \) are constant over time, linearizing (288) in the neighborhood of the steady-state leads to:

\[ \dot{B}(t) = r^{\star} dB(t) - \tilde{P} d G^N(t). \]

Substituting the law of motion of government spending (258) and solving leads to the general solution for the net foreign asset position:

\[ B(t) - \tilde{B} = \left( B_0 - \tilde{B} \right) + \frac{\tilde{Y}}{\xi + r^{\star} g} e^{r^{\star} t} - \frac{\tilde{Y}}{\xi + r^{\star} g} e^{-\xi t}. \quad \text{(289)} \]
Invoking the transversality condition gives the solution for $B(t)$:

$$B(t) - \bar{B} = \frac{\bar{Y}}{\xi + r^*} ge^{-\xi t},$$  \hspace{1cm} (290)

consistent with the intertemporal solvency condition

$$\left( \frac{\bar{B} - B_0}{\xi + r^*} \right) = -\frac{\bar{Y}}{\xi + r^*} g.$$  \hspace{1cm} (291)

To determine the change in the equilibrium value of the marginal utility of wealth, we have to differentiate the market clearing condition (283j) for the traded good evaluated at the steady-state (i.e., $\dot{\bar{B}}(t) = 0$), using the fact that government spending reverts to its initial level in the long-run (i.e., $dG = 0$):

$$r^* d\hat{B} \frac{\tilde{Y}}{\sigma L} + (1 - \alpha_L) \hat{L}^T = (1 - \alpha_C) \omega_C \hat{C}^T,$$

$$\hat{\lambda} = -r^* d\hat{B} \frac{\tilde{Y}}{\sigma L},$$

where we used (287) (setting $dG = 0$) and $\hat{C}^T = -\sigma_C \hat{\lambda}$. Substituting (291) into the above equation leads to the change in the equilibrium value of the marginal utility of wealth:

$$\hat{\lambda} = \frac{1}{\sigma L + \omega_C \sigma_C} \frac{r^*}{\xi + r^*} g > 0,$$  \hspace{1cm} (292)

where $\xi > 0$ and $g > 0$. According to (292), a temporary rise in government consumption generates a negative wealth effect reflected by an increase in the shadow value of wealth.

E.15 Perfect Mobility of Labor as a Special Case of a Model with Limited Substitutability in Hours Worked across Sectors

In order to generate barriers to mobility, we assume limited substitutability in hours worked across sectors along the lines of Horvath [2000]. The degree of substitutability of hours worked across sectors captures the extent of workers’ mobility costs. As the elasticity of labor supply across sectors takes higher values, workers experience lower mobility costs and thus the degree of labor mobility increases. The advantage of this modelling strategy is that it allows us to consider the range of all degrees of labor mobility across sectors. Specifically, if we let $\epsilon$ be zero or tend toward infinity, total immobility ($\epsilon = 0$) and perfect mobility ($\epsilon \rightarrow \infty$), respectively, emerges as a special case. In this subsection, we investigate how the degree of labor mobility affects the magnitude of initial responses of sectoral variables to a government spending shock.

As will be useful later, we compute several expressions. Inserting the expression for $\Psi$ given by (157) into the expression of $\gamma^N_G$ described by (192), letting $\epsilon$ tend toward infinity and applying l'Hôpital’s rule leads to:

$$\lim_{\epsilon \rightarrow \infty} \gamma^N_G = \lim_{\epsilon \rightarrow \infty} \frac{[1 - \alpha_L \sigma L \sigma C \alpha C (\phi - \sigma C)]}{\alpha L ([1 - \alpha_L] + \sigma L \alpha_L) + \omega_C \alpha C ([1 - \alpha_C] \phi + \alpha_C \sigma C)},$$

$$= \frac{\alpha L (1 - \alpha_L)}{\alpha L (1 - \alpha_L)} = 1.$$  \hspace{1cm} (293)

Using the expression for $\Gamma$ given by eq. (271), letting $\epsilon$ tend toward infinity and applying l'Hôpital’s rule leads to:

$$\lim_{\epsilon \rightarrow \infty} \frac{\Psi \gamma^N_G}{\Gamma} = \lim_{\epsilon \rightarrow \infty} \left[ \frac{[1 - \alpha_L \sigma L + \omega_C \sigma C \alpha C] + [\alpha L \sigma L + \omega_C \alpha C \sigma C]}{\sigma L + \omega_C \sigma C} \right],$$

$$= \frac{1}{\sigma L + \omega_C \sigma C},$$  \hspace{1cm} (294)
where we used the fact that \( \lim_{\epsilon \to \infty} \frac{Y_N}{G} = 1 \) (see eq. (293)). Finally, we compute two additional expressions by inserting the expression for \( \Psi \) given by (157), letting \( \epsilon \) tend toward infinity and applying l’Hôpital’s rule:

\[
\lim_{\epsilon \to \infty} \frac{\alpha_L [\epsilon (1 - \alpha_L) + \alpha L \sigma_L]}{\Psi} = \lim_{\epsilon \to \infty} \frac{\alpha_L [\epsilon (1 - \alpha_L) + \sigma_L \alpha L] + \omega_C \alpha C [(1 - \alpha_C) \phi + \alpha_C \sigma_C]}{\alpha_L (1 - \alpha_L)} = 1,
\]

(295a)

\[
\lim_{\epsilon \to \infty} \frac{(1 - \alpha_L) \alpha_L (\epsilon - \sigma_L)}{\Psi} = \lim_{\epsilon \to \infty} \frac{\alpha_L [\epsilon (1 - \alpha_L) + \alpha L \sigma_L]}{\alpha_L (1 - \alpha_L)} = 1.
\]

(295b)

Letting \( \epsilon \) tend toward infinity into eq. (274) and using (293) together with (295a), the initial response of hours worked in the non traded sector relative to the initial steady-state in total labor units can be rewritten as follows:

\[
\lim_{\epsilon \to \infty} \alpha_L \hat{L}^N(0) = \lim_{\epsilon \to \infty} \alpha_L [\epsilon (1 - \alpha_L) + \alpha L \sigma_L] \left[ 1 - (\alpha_L \sigma_L + \alpha_C \omega C \sigma_C) \frac{\Psi Y_N^N}{\Gamma} \frac{r^*}{\xi + r^*} \right] g
\]

\[
+ \lim_{\epsilon \to \infty} \alpha_L \sigma_L \Psi \frac{Y_N^N}{\Gamma} \frac{r^*}{\xi + r^*} g > 0,
\]

\[
= \left[ 1 - \frac{\alpha_L \sigma_L + \alpha_C \omega C \sigma_C}{\sigma_L + \omega C \sigma_C} \frac{r^*}{\xi + r^*} \right] g + \frac{\alpha_L \sigma_L}{\sigma_L + \omega C \sigma_C} \frac{r^*}{\xi + r^*} g,
\]

\[
= 1 - \frac{\alpha C \omega C \sigma_C}{\sigma_L + \omega C \sigma_C} \frac{r^*}{\xi + r^*} g > 0.
\]

(296)

Eq. (296) gives the initial response of hours worked in the non traded sector to an exogenous temporary increase in \( G^N \) when labor can freely move from one sector to another. As discussed below, the magnitude of the rise in non traded labor on impact, i.e., \( \alpha_L \hat{L}^N(0) \), can be larger or lower than that with a difficulty in reallocating labor across sectors. Intuitively, in the latter case, the relative price of non tradables appreciates which exerts a strong positive impact on the reallocation of labor toward the non traded sector.

Letting \( \epsilon \) tend toward infinity into eq. (275) and using (293) together with (295a), the initial response of hours worked in the traded sector relative to the initial steady-state in total labor units can be rewritten as follows:

\[
\lim_{\epsilon \to \infty} (1 - \alpha_L) \hat{L}^T(0) = \lim_{\epsilon \to \infty} \frac{(1 - \alpha_L) \alpha_L (\epsilon - \sigma_L)}{\Psi} \left[ 1 - (\alpha_L \sigma_L + \alpha_C \omega C \sigma_C) \frac{\Psi Y_N^N}{\Gamma} \frac{r^*}{\xi + r^*} \right] g
\]

\[
+ \lim_{\epsilon \to \infty} (1 - \alpha_L) \sigma_L \Psi \frac{Y_N^N}{\Gamma} \frac{r^*}{\xi + r^*} g \leq 0,
\]

\[
= - \left[ 1 - \frac{\alpha L \sigma L + \alpha C \omega C \sigma C}{\sigma L + \omega C \sigma C} \frac{r^*}{\xi + r^*} \right] g + \frac{(1 - \alpha_L) \sigma_L}{\sigma_L + \omega C \sigma_C} \frac{r^*}{\xi + r^*} g,
\]

\[
= - \left[ 1 - \frac{\sigma L + \alpha C \omega C \sigma C}{\sigma L + \omega C \sigma C} \frac{r^*}{\xi + r^*} \right] g < 0.
\]

(297)

Summing (296) and (297) leads to:

\[
\lim_{\epsilon \to \infty} \alpha_L \hat{L}^N(0) + \lim_{\epsilon \to \infty} (1 - \alpha_L) \hat{L}^T(0) = \frac{\sigma L}{\sigma L + \omega C \sigma_C} \frac{r^*}{\xi + r^*} g,
\]

\[
= \lim_{\epsilon \to \infty} \hat{L}(0),
\]

(298)

where the last equality is derived by letting \( \epsilon \) tend toward infinity into eq. (263).

We now investigate the magnitude of the response of the output share of tradables when imposing perfect mobility of labor across sectors. Letting \( \epsilon \) tend toward infinity into eq. (281), using (294) together with the fact that \( \lim_{\epsilon \to \infty} \frac{\alpha_L (1 - \alpha_L)}{\Psi} = 1 \), and applying l’Hôpital’s rule, the initial response of the output share of non tradables can be rewritten as follows:

\[
\lim_{\epsilon \to \infty} \alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) = \lim_{\epsilon \to \infty} \alpha_L (1 - \alpha_L) \epsilon \hat{P}(0),
\]

\[
= \left[ 1 - \left( \frac{\alpha_L \sigma L + \alpha C \omega C \sigma C}{\sigma L + \omega C \sigma_C} \right) \frac{r^*}{\xi + r^*} \right] g > 0,
\]

(299)
where $0 < \left( \frac{\alpha L \sigma L + \alpha C \omega C \sigma C}{\sigma L + \omega C \sigma C} \right) < 1$.

Applying the same logic to the output share of tradables described by eq. (282), the response of the traded output relative to GDP in percent of output when assuming perfect mobility of labor across sectors is:

$$\lim_{\epsilon \to -\infty} (1 - \alpha L) \left( \hat{Y}^T(0) - \hat{Y}_R(0) \right) = -\alpha L \left( 1 - \alpha L \right) \epsilon \hat{P}(0),$$

$$= -\lim_{\epsilon \to -\infty} \alpha L \left( 1 - \alpha L \right) \epsilon \hat{P}(0),$$

$$= -\left[ 1 - \left( \frac{\alpha L \sigma L + \alpha C \omega C \sigma C}{\sigma L + \omega C \sigma C} \right) \frac{r^*}{\xi + r^*} \right] g < 0. (300)$$

E.16 Relationship between the magnitude of impact responses and the degree of labor mobility across sectors: Proofs of Results in Section 4.2

We now investigate the relationship between the magnitude of responses of sectoral labor and the degree of labor mobility across sectors captured by $\epsilon$. To do so, we have to first rewrite $\frac{\Psi \Upsilon_N}{\Gamma}$ (see the first line of eq. (294)) as follows

$$\frac{\Psi \Upsilon_N}{\Gamma} = \frac{\Upsilon_N \Gamma}{[(1 - \alpha L) \sigma L + \omega C (1 - \alpha C) \sigma C] + [\alpha L \sigma L + \omega C \alpha C \sigma C] \Upsilon_N}, \quad (301)$$

and to determine whether $\frac{\Psi \Upsilon_N}{\Gamma}$ increases or decreases as the degree of labor mobility rises. To do so, we have to determine the relationship between $\Upsilon_N$ described by (192) and $\epsilon$:

$$\frac{\partial \Upsilon_N}{\partial \epsilon} = (1 - \alpha L) \alpha L (\alpha L \sigma L + \omega C \alpha C \sigma C) > 0, \quad (302)$$

where $\Psi$ is given by eq. (157). When $\epsilon = 0$, $\Upsilon_N$ described by (192) becomes:

$$\Upsilon_N |_{\epsilon = 0} = 1 - \frac{(\alpha L \sigma L + \omega C \alpha C \sigma C)}{(\alpha L)^2 \sigma L + \omega C \alpha C [(1 - \alpha C) \phi + \alpha C \sigma C] \Upsilon_N} \leq 0. \quad (303)$$

In sum, $\Upsilon_N$ can take negative values when $\epsilon$ is close to 0, is increasing with $\epsilon$ and takes a maximum value of 1 when we let $\epsilon \to -\infty$. Differentiating (301) with respect to $\epsilon$ leads to:

$$\frac{\partial \frac{\Psi \Upsilon_N}{\Gamma}}{\partial \epsilon} = \frac{\frac{\partial \Upsilon_N}{\partial \epsilon}}{(1 - \alpha L) \sigma L + \omega C (1 - \alpha C) \sigma C} \frac{\left[ [1 - (1 - \alpha L) \sigma L + \omega C (1 - \alpha C) \sigma C] \Upsilon_N \right]^2}{\Upsilon_N} > 0,$n$$

$$= \frac{(1 - \alpha L) \alpha L [\alpha L \sigma L + \omega C \alpha C \sigma C] [(1 - \alpha L) \sigma L + \omega C (1 - \alpha C) \sigma C]}{\Gamma^2} > 0. (304)$$

Because $\frac{\Psi \Upsilon_N}{\Gamma}$ and $\Psi$ are both positive and increasing with $\epsilon$ while $\lim_{\epsilon \to -\infty} \frac{\Psi \Upsilon_N}{\Upsilon_N} = \frac{1}{\sigma L + \omega C \sigma C}$ (see eq. 294) and $\lim_{\epsilon \to -\infty} \Psi = \infty$, the initial reaction of the relative price to a government spending shock is unambiguously decreasing with $\epsilon$; differentiating (273) with respect to $\epsilon$ leads to:

$$\frac{\partial \hat{P}(0)}{\partial \epsilon} = -\frac{g}{\Psi} [\alpha L \sigma L + \omega C \alpha C \sigma C] \frac{r^*}{\xi + r^*} \frac{\partial \frac{\Psi \Upsilon_N}{\Gamma}}{\partial \epsilon}$$

$$- \left\{ 1 - [\alpha L \sigma L + \omega C \alpha C \sigma C] \frac{r^*}{\xi + r^*} \frac{\Psi \Upsilon_N}{\Gamma} \right\} g \frac{\partial \Psi}{\Psi^2} < 0. \quad (305)$$

A rise in $\epsilon$ mitigates the appreciation in the relative price of non tradables by amplifying the increase in the supply of non tradables and by reducing the excess of demand for non tradables. First, in countries where labor is more mobile across sectors, a government spending shock biased toward non tradables leads to a larger increase in non traded output which mitigates the appreciation in the relative price of non tradables. Second, as $\epsilon$ takes higher values, the wealth effect becomes larger so that private consumption is crowded out by a larger amount which results in a lower excess demand of non tradables.
Totally differentiating the response of output share of non tradables to a government spending shock described by eq. (281) with respect to \( \epsilon \) leads to:

\[
\frac{\partial \alpha_L(\hat{Y}^N(0) - \hat{Y}_R(0))}{\partial \epsilon} = \alpha_L \left( 1 - \alpha_L \right) \epsilon \hat{P}(0) \left[ 1 + \frac{\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)} \right].
\] (306)

According to (306), the relationship between the positive response of output share of non tradables and the degree of labor mobility across sectors is ambiguous. On the one hand, as shown by the first term in brackets on the RHS of (306), a rise in the parameter \( \epsilon \) amplifies the reallocation of labor toward the non traded sector and thus increases further the output share of non tradables. On the other hand, the rise in the degree of labor mobility also mitigates the rise in the output share as higher mobility increases further the shadow value of wealth which amplifies the crowding out of private consumption by public spending and thus moderates the excess demand in the non traded goods market. Consequently, the relative price of non tradables appreciates by a lower amount which reduces the incentive to increase non traded output. While we address this ambiguity numerically in the main text when we simulate the full model with physical capital accumulation subject to adjustment costs, we provide a formal proof below that \( \frac{\partial \alpha_L(\hat{Y}^N(0) - \hat{Y}_R(0))}{\partial \epsilon} > 0 \) in a model with labor only.

For the RHS of eq. (306) to be positive, we must have:

\[
-\frac{\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)} < 1.
\] (307)

To show that inequality (307) holds, we have to make assumptions. We are able to sign expressions in two polar cases: \( \xi \to \infty \) and \( \xi \to 0 \). The former and the latter case correspond to situations where the government spending shock is weakly persistent (i.e., \( G \) increases initially and is restored back toward its initial level) and highly persistent (i.e., the fiscal shock is permanent).

**A Weakly Persistent Fiscal Shock**

Letting \( \xi \to \infty \) into eq. (273), the initial appreciation in the relative price of non tradables reduces to:

\[
\hat{P}(0)\big|_{\xi \to \infty} = \frac{g}{\Psi} > 0,
\] (308)

where \( \Psi \) is given by eq. (157). Differentiating (308) w.r.t. \( \epsilon \) leads to:

\[
-\frac{\partial \hat{P}(0)}{\partial \epsilon} \big|_{\xi \to \infty} = -\frac{g \alpha_L (1 - \alpha_L)}{\Psi^2} < 0.
\] (309)

Combining (308) and (309), the elasticity (in absolute terms) of the impact response of the relative price to the degree of labor mobility is thus given by:

\[
0 < -\frac{\partial \hat{P}(0)}{\partial \epsilon} \big|_{\xi \to \infty} \frac{\epsilon}{\hat{P}(0)\big|_{\xi \to \infty}} = \frac{\alpha_L (1 - \alpha_L) \epsilon}{\Psi} < 1.
\] (310)

Since \( \Psi > \alpha_L \epsilon (1 - \alpha_L) \), the elasticity is strictly smaller than one and thus the RHS of eq. (306) is unambiguously positive.

**A Highly Persistent Fiscal Shock**

Letting \( \xi \to 0 \) into eq. (273), the initial appreciation in the relative price of non tradables reduces to:

\[
\hat{P}(0)\big|_{\xi \to 0} = \frac{g}{\Psi} \left[ 1 - (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C) \frac{Y^*_G \Psi}{\Gamma} \right].
\] (311)

Differentiating (308) w.r.t. \( \epsilon \) leads to:

\[
-\frac{\partial \hat{P}(0)}{\partial \epsilon} \big|_{\xi \to 0} = -\frac{g \alpha_L (1 - \alpha_L)}{\Psi^2} \left[ 1 - (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C) \frac{Y^*_G \Psi}{\Gamma} \right] - \frac{g}{\Psi} \left( \alpha_L \sigma_L + \omega_C \alpha_C \sigma_C \right) \frac{\partial \frac{Y^*_G \Psi}{\Gamma}}{\partial \epsilon},
\] (312)
where $\frac{\partial \hat{P}_N}{\partial \epsilon}$ is given by eq. (304). Computing the following term:

$$
\left[ 1 - (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C) \frac{\nabla^N \Psi}{\Gamma} \right] = \Psi \left[ (1 - \alpha_L) \sigma_L + \omega_C (1 - \alpha_C) \sigma_C \right],
$$

(313)

the initial reaction of the relative price (311) can be rewritten as follows:

$$
\hat{P}(0) |_{\xi \rightarrow 0} = g \left[ (1 - \alpha_L) \sigma_L + \omega_C (1 - \alpha_C) \sigma_C \right] > 0.
$$

(314)

Making use of (313) and inserting (304), eq. (312) reads as:

$$
\frac{\partial \hat{P}(0)}{\partial \epsilon} |_{\xi \rightarrow 0} = -\frac{g \alpha_L (1 - \alpha_L) [1 - (1 - \alpha_L) \sigma_L + \omega_C (1 - \alpha_C) \sigma_C]}{\Psi^{1/2}} \left[ \Gamma + (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C)^2 \right].
$$

(315)

Calculating the elasticity in absolute terms of $\hat{P}(0)$ w.r.t. $\epsilon$ by combining (314) and (315), we have to show that the following inequality holds for the RHS of eq. (306) to be positive:

$$
-\frac{\partial \hat{P}(0)}{\partial \epsilon} |_{\xi \rightarrow 0} \frac{\epsilon}{\hat{P}(0) |_{\xi \rightarrow 0}} = \alpha_L (1 - \alpha_L) \epsilon \left[ 1 + (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C)^2 \right] < 1.
$$

(316)

To show (316), it is useful to write down the following properties:

$$
\Psi = \alpha_L (1 - \alpha_L) (\epsilon - \sigma_L) + \omega_C C (1 - \alpha_C) (\phi - \sigma_C) + \alpha_L \sigma_L + \omega_C \alpha_C \sigma_C,
$$

(317a)

$$
\nabla^N \Psi = \Psi - (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C),
$$

(317b)

$$
\Gamma = \Psi (\sigma_L + \omega_C \sigma_C) - (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C)^2,
$$

(317c)

where we made use of (317b) to obtain (317c), i.e.,

$$
\Gamma = \Psi \left\{ [(1 - \alpha_L) \sigma_L + \omega_C (1 - \alpha_C) \sigma_C] + [\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C] \nabla^N \Psi \right\},
$$

$$
= \Psi \left\{ [(1 - \alpha_L) \sigma_L + \omega_C (1 - \alpha_C) \sigma_C] + \alpha_L \sigma_L + \omega_C \alpha_C \sigma_C \right\}.
$$

(318)

Using (317c), eq. (316) can be rewritten as follows:

$$
-\frac{\partial \hat{P}(0)}{\partial \epsilon} |_{\xi \rightarrow 0} \frac{\epsilon}{\hat{P}(0) |_{\xi \rightarrow 0}} = \alpha_L (1 - \alpha_L) \epsilon \frac{(\sigma_L + \omega_C \sigma_C)}{\Psi} < 1.
$$

(319)

Thus, making use of eq. (317c), the elasticity (319) is smaller than one if the following inequality holds:

$$
\Gamma > \alpha_L (1 - \alpha_L) \epsilon (\sigma_L + \omega_C \sigma_C),
$$

$$
\Psi (\sigma_L + \omega_C \sigma_C) - (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C)^2 > \alpha_L (1 - \alpha_L) \epsilon (\sigma_L + \omega_C \sigma_C),
$$

$$
\omega_C \alpha_L (1 - \alpha_C) \epsilon (\sigma_L + \omega_C \sigma_C)^2 + (\sigma_L + \omega_C \sigma_C) \omega_C \alpha_C (1 - \alpha_C) \phi > 0.
$$

(320)

Since (320) holds for all range of values of parameters, elasticity (307) is strictly smaller than one and thus the RHS of eq. (306) holds.

**Relationship between $\alpha_L \left( \hat{Y}^N(0) - \hat{Y}^R(0) \right)$ and $\epsilon$**

As shown by eqs. (306) and (307), $\alpha_L \left( \hat{Y}^N(0) - \hat{Y}^R(0) \right)$ is increasing with $\epsilon$ as long as

$$
-\frac{\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)} < 1.
$$

Combining (310) et (319), we find that the elasticity $-\frac{\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)}$ varies from a low of $\frac{\alpha_L (1 - \alpha_L)}{\Psi}$ when the shock is weakly persistent to a high of $\frac{\alpha_L (1 - \alpha_L)}{\Psi} \left[ 1 + \frac{(\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C)^2}{\Gamma} \right]$. Hence, when $\xi$ takes intermediate values, the elasticity falls into the following range of values:

$$
-\frac{\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)} \in \left\{ \frac{\alpha_L (1 - \alpha_L)}{\Psi}, \frac{\alpha_L (1 - \alpha_L)}{\Psi} \left[ 1 + \frac{(\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C)^2}{\Gamma} \right] \right\}.
$$

(321)
E.16.2 Relationship between and \( \alpha_L \hat{Y}^N(0) \) and \( \epsilon \)

In the special case where \( \sigma_L = 0 \), analytical expression of \( \alpha_L \hat{Y}^N(0) \) described by eq. (274) reduces to:

\[
\alpha_L \hat{L}^N(0) \bigg|_{\sigma_L=0} = \frac{\alpha_L \epsilon (1 - \alpha_L)}{\Psi} \left[ 1 - \alpha_C \omega_C \sigma_C \frac{\Psi T^N_C G}{\Gamma} \left( \frac{r^*}{\xi + r^*} \right) \right] g > 0. \tag{322}
\]

Eq. (322) corresponds to eq. (281) when setting \( \sigma_L = 0 \), i.e.,

\[
\alpha_L \hat{L}^N(0) \bigg|_{\sigma_L=0} = \alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) \bigg|_{\sigma_L=0}. \tag{323}
\]

Since our proof for \( \frac{\partial \alpha_L}{\partial \epsilon} \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) \) summarized by inequality (321) also holds for \( \sigma_L = 0 \), we have

\[
\frac{\partial \alpha_L \hat{L}^N(0)}{\partial \epsilon} \bigg|_{\sigma_L=0} < 0. \tag{324}
\]

E.16.3 Relationship between \( \hat{L}(0) \) and \( \epsilon \)

The initial reaction of hours worked to a fiscal shock is:

\[
\hat{L}(0) = \sigma_L \left[ \hat{\lambda} + \alpha_L \hat{P}(0) \right] \tag{325}
\]

where \( \hat{\lambda} \) and \( \hat{P}(0) \) are given by (270) and (273), respectively. **Eq. (325) corresponds to eq. (38) in the main text.**

As for the share of non tradables in GDP, we investigate the relationship between the initial reaction of hours worked to a fiscal shock, \( \hat{L}(0) \), and the degree of labor mobility across sectors, \( \epsilon \), by considering two polar cases.

**A Weakly Persistent Fiscal Shock**

Letting \( \xi \to \infty \) into eq. (273) and (270), the initial appreciation in the relative price of non tradables reduces to:

\[
\hat{P}(0) \bigg|_{\xi \to \infty} = \frac{g}{\Psi} > 0, \tag{326}
\]

where \( \Psi \) is given by eq. (157) and the marginal utility of wealth remains unaffected:

\[
\hat{\lambda} \bigg|_{\xi \to \infty} = 0. \tag{327}
\]

Differentiating (325) w.r.t. \( \epsilon \) and using (309) leads to:

\[
\frac{\partial \hat{L}(0)}{\partial \epsilon} \bigg|_{\xi \to \infty} = \sigma_L \alpha_L \frac{\partial \hat{P}(0)}{\partial \epsilon} \bigg|_{\xi \to \infty},
\]

\[
= -\sigma_L \alpha_L \frac{g \alpha_L (1 - \alpha_L)}{\Psi^2} < 0. \tag{328}
\]

**A Highly Persistent Fiscal Shock**

Letting \( \xi \to 0 \) into eq. (325) and using eqs. (270) and (273), the initial reaction of hours worked can be rewritten as follows:

\[
\hat{L}(0) \bigg|_{\xi \to 0} = \sigma_L \left[ \hat{\lambda}_{\xi \to 0} + \alpha_L \hat{P}(0) \bigg|_{\xi \to 0} \right],
\]

\[
= g \sigma_L \left\{ \frac{T^N_C G}{\Gamma} + \frac{\alpha_L}{\Psi} \left[ 1 - (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C) \frac{T^N_C G}{\Gamma} \right] \right\}. \tag{329}
\]
Differentiating (329) w.r.t. \( \epsilon \), making use of (304) and (316) leads to:

\[
\frac{\partial L(0)}{\partial \epsilon} = \sigma_L \left[ \frac{\partial \Psi \Upsilon}{\partial \epsilon} g + \alpha_L \frac{\partial \hat{P}(0)}{\partial \epsilon} \right] + \frac{\partial \hat{P}(0)}{\partial \epsilon} \]

\[= g \sigma_L \left( 1 - \alpha_L \right) \left[ (1 - \alpha_L) \left( \sigma_L + \omega_C (1 - \alpha_C) \sigma_C \right) + \Psi \left( \alpha_L \sigma_L + \omega_C \alpha_C \sigma_C \right) \right] \]

\[= \frac{g \sigma_L}{\Psi \Gamma^2} \left( 1 - \alpha_L \right) \left[ (1 - \alpha_L) \left( \sigma_L + \omega_C (1 - \alpha_C) \sigma_C \right) + \Psi \left( \alpha_L \sigma_L + \omega_C \alpha_C \sigma_C \right) \right] \]

\[\times \omega_C \sigma_C \left( \alpha_L - \omega_C \alpha_C \right) < 0, \quad (330)\]

where we used (317c) to obtain the last line of eq. (330), i.e., we computed the following term:

\[
\Psi \left( \alpha_L \sigma_L + \omega_C \alpha_C \sigma_C \right) - \alpha_L \Gamma - \alpha_L \left( \alpha_L \sigma_L + \omega_C \alpha_C \sigma_C \right)^2, \]

\[= \Psi \omega_C \sigma_C \left( \alpha_L - \omega_C \alpha_C \right) > 0. \quad (331)\]

The RHS of eq. (330) is positive since according to the market clearing condition for non tradables expressed in percentage point of GDP, we have:

\[
\alpha_L = \omega_C \alpha_C + \omega_G \omega_G N, \quad (332)\]

and thus as long as \( \omega_G N > 0 \), we have:

\[
\alpha_L - \omega_C \alpha_C > 0. \quad (333)\]

In conclusion, the initial reaction of hours worked is decreasing with the degree of labor mobility across sectors, \( \epsilon \).

### E.17 Elasticity of Labor Supply and the Share of Non Tradables: Sensitivity Analysis

In this subsection, we investigate how the elasticity of labor supply, \( \sigma_L \), and the non tradable content of consumption expenditure, \( \alpha_C \), influence the magnitude of the sectoral impact of a government spending shock.

#### E.17.1 Sensitivity to the Intertemporal Elasticity for Labor Supply

We first investigate the implications of the Frisch elasticity of labor supply for the responses of the marginal utility of wealth and the relative price of non tradables which are described by (270) and (273), respectively. To do so, we have to explore the relationship between \( \Psi \Upsilon \frac{\partial N}{\partial \sigma_L} \) and \( \sigma_L \). Differentiating \( \Upsilon \frac{\partial N}{\partial \sigma_L} \) w.r.t. \( \sigma_L \) leads to:

\[
\frac{\partial \Upsilon \frac{\partial N}{\partial \sigma_L}}{\partial \sigma_L} = -\frac{\alpha_L}{\Psi} \left( \alpha_L \left( 1 - \alpha_L \right) \epsilon + \omega_C \sigma_C \left[ (1 - \alpha_C) \phi - (\alpha_L - \alpha_C) \sigma_C \right] \right) \leq 0. \quad (334)\]

While the sign of \( \frac{\partial \Upsilon \frac{\partial N}{\partial \sigma_L}}{\partial \sigma_L} \) is ambiguous, when \( \phi \) is close to \( \sigma_C \), we find that \( \frac{\partial \Upsilon \frac{\partial N}{\partial \sigma_L}}{\partial \sigma_L} < 0\). Eq. (301) can be rewritten as follows:

\[
\frac{\Psi \Upsilon \frac{\partial N}{\partial \sigma_L}}{\Gamma} = \frac{1}{\Upsilon \frac{\partial N}{\partial \sigma_L} + [\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C]} \cdot (335)\]

Because the denominator is higher as the values of \( \sigma_L \) increase, the marginal utility of wealth rises by a smaller amount (see eq. (270)). Intuitively, because agents supply more labor following a rise in government consumption, private savings falls by a smaller amount which results in a lower current account deficit. Thus, the marginal utility of wealth must increase less for the intertemporal solvency condition to hold. According to (273), increasing
σ_L exerts opposite effects on \( \hat{P}(0) \). First, because the marginal utility of wealth increases less, consumption in non tradables falls less while hours worked rises more in the non traded sector as agents supply more labor. If both effects offset each other, excess demand in the non traded goods market is unchanged. On the other hand, raising \( \sigma_L \) makes the relative price more responsive to the excess demand in the non traded goods market following a rise in \( G^N \) as reflected by larger values in \( \Psi \) (see eq. (157)). Overall, one may expect that the last effect predominates so that the relative price of non tradables appreciates less when the elasticity \( \sigma_L \) is high. As a result, the responses of sectoral labor and thus sectoral output shares described by (281) for non tradables and (282) for tradables, respectively, should be less pronounced as the relative price appreciates less.

E.17.2 Sensitivity to Non Tradable Share

We first investigate the implications of increasing \( \alpha_C \) for the responses of the marginal utility of wealth and the relative price of non tradables which are described by (270) and (273), respectively. To do so, we first evaluate \( \Upsilon^N_G \) when we let \( \alpha_C \) tend toward zero and one, respectively:

\[
\lim_{\alpha_C \to 0} \Upsilon^N_G = \frac{(1 - \alpha_L) \alpha_L (\epsilon - \sigma_L)}{\alpha_L [\epsilon (1 - \alpha_L) + \alpha_L \sigma_L]},
\]

\[
\lim_{\alpha_C \to 1} \Upsilon^N_G = \frac{(1 - \alpha_L) \alpha_L (\epsilon - \sigma_L)}{\alpha_L [\epsilon (1 - \alpha_L) + \alpha_L \sigma_L + \omega_C \sigma_C]}.
\]

It is straightforward to see that the following inequality holds:

\[
\lim_{\alpha_C \to 0} \Upsilon^N_G > \lim_{\alpha_C \to 1} \Upsilon^N_G > 0.
\]

Then, we evaluate \( \frac{\Psi \Upsilon^N_G}{\Gamma} \) when we let \( \alpha_C \) tend toward zero and one, respectively:

\[
\lim_{\alpha_C \to 0} \frac{\Psi \Upsilon^N_G}{\Gamma} = \frac{1}{\lim_{\alpha_C \to 0} \Upsilon^N_G},
\]

\[
\lim_{\alpha_C \to 1} \frac{\Psi \Upsilon^N_G}{\Gamma} = \frac{1}{\lim_{\alpha_C \to 1} \Upsilon^N_G + \alpha_L \sigma_L + \omega_C \sigma_C}.
\]

Substituting (336a) into (338a) and (336b) into (338b), after tedious computations, it can be shown analytically that:

\[
\left\{ \frac{(1 - \alpha_L) \sigma_L + \omega_C \sigma_C}{\lim_{\alpha_C \to 0} \Upsilon^N_G} + \alpha_L \sigma_L \right\} - \left\{ \frac{(1 - \alpha_L) \sigma_L}{\lim_{\alpha_C \to 1} \Upsilon^N_G} + \alpha_L \sigma_L + \omega_C \sigma_C \right\} > 0,
\]

and thus

\[
\lim_{\alpha_C \to 1} \frac{\Psi \Upsilon^N_G}{\Gamma} > \lim_{\alpha_C \to 0} \frac{\Psi \Upsilon^N_G}{\Gamma} > 0.
\]

Intuitively, as \( \alpha_C \) takes higher values, the share of tradables falls. At the final steady-state, net exports must be larger for the open economy to be solvent. To improve the balance of trade in the long-run, output of tradables must be higher while consumption in tradables must be lower. Because the share of tradables in the economy is lower, the marginal utility of wealth must increase by a larger amount to lower consumption in tradables and thus to increase net exports.

As the negative wealth effect is stronger, excess demand for non tradables and thus the subsequent appreciation in the relative price \( P \) are smaller which mitigates the shift of resources toward the non traded sector and thus moderates the fall in traded output and the rise in non traded output.
E.18 Effects of a Rise in Government Consumption on Tradables, $G^T$

In this subsection, we explore the effects of a rise in government consumption on tradables, $G^T$, while keeping fixed public purchases of non tradables, $G^N$. Since $G^N$ is unchanged, eq. (262) reduces to:

$$\dot{P} = -\left[\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C\right] \frac{\dot{\lambda}}{\Psi},$$  \hspace{1cm} (341)

where $\Psi = \alpha_L [\epsilon (1 - \alpha_L) + \sigma_L \alpha_L] + \omega_C \alpha_C [(1 - \alpha_C) \phi + \alpha_C \sigma_C] > 0$ (see eq. (157)).

Inserting (341) into $\dot{L}^T = -\alpha_L (\epsilon - \sigma_L) \dot{P} + \sigma_L \dot{\lambda}$ leads to the change in traded labor relative to initial steady-state:

$$\dot{L}^T = \left\{\sigma_L + \frac{\alpha_L (\epsilon - \sigma_L) [\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C]}{\Psi}\right\} \dot{\lambda},$$  \hspace{1cm} (342)

Inserting (341) into $\dot{L}^N = \left[\alpha_L (1 - \alpha_C) \phi + \alpha_C \sigma_C\right] \dot{P} + \sigma_L \dot{\lambda}$ leads to the change in non traded labor from initial steady-state:

$$\dot{L}^N = \left\{\sigma_L - \frac{\alpha_L (1 - \alpha_L) \left[\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C\right]}{\Psi}\right\} \dot{\lambda},$$  \hspace{1cm} (343)

where the term in braces is positive if and only if:

$$\sigma_L [(1 - \alpha_C) \phi + \alpha_C \sigma_C] > \sigma_C [\epsilon (1 - \alpha_L) + \alpha_L \sigma_L].$$

One interesting case is that where $\sigma_C = \phi = 1$. Eq. (343) can be rewritten as follows:

$$\dot{L}^N = -\left\{\frac{\alpha_C \omega_C (1 - \alpha_L) (\epsilon - \sigma_L)}{\Psi}\right\} \dot{\lambda} < 0,$$  \hspace{1cm} (344)

where the sign follows from assumption 1 and we used the fact that $\Psi' = \Psi |_{\phi=\sigma_C=1} = \alpha_L [\epsilon (1 - \alpha_L) + \sigma_L \alpha_L] + \omega_C \alpha_C > 0$ to derive (344).

Applying the theorem of implicit functions, eqs. (341)-(343) lead to the short-run static solutions for the relative price, both traded and non traded labor which depend exclusively on the shadow value of wealth:

$$P = P (\lambda), \hspace{0.5cm} L^T = L^T (\lambda), \hspace{0.5cm} L^N = L^N (\lambda).$$  \hspace{1cm} (345)

Inserting (341) into $\dot{C}^T = \alpha_C (\phi - \sigma_C) \dot{P} - \sigma_C \dot{\lambda}$ leads to the response of consumption in tradables in percentage relative to initial steady-state:

$$\dot{C}^T = \left\{\sigma_C + \frac{\alpha_C (\phi - \sigma_C) [\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C]}{\Psi}\right\} \dot{\lambda},$$  \hspace{1cm} (346)

Inserting (341) into $\dot{C}^N = [\phi (1 - \alpha_C) + \alpha_C \sigma_C] \dot{P} - \sigma_C \dot{\lambda}$ leads to the response of consumption in non tradables in percentage relative to initial steady-state:

$$\dot{C}^N = -\left\{\sigma_C - \frac{\phi (1 - \alpha_C) + \alpha_C \sigma_C [\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C]}{\Psi}\right\} \dot{\lambda},$$  \hspace{1cm} (347)

If $\phi = \sigma_C = 1$, consumption in non tradables falls under assumption 1. Applying the theorem of implicit functions, eqs. (346)-(347) lead to the short-run static solution for consumption in tradables and consumption in non tradables which depend exclusively on the shadow value of wealth:

$$C^T = C^T (\lambda), \hspace{0.5cm} C^N = C^N (\lambda).$$  \hspace{1cm} (348)

Substituting short-run static solutions for traded labor, i.e., $L^T = L^T (\lambda)$ and $C^T = C^T (\lambda)$, into (259j) leads to:

$$\dot{B}(t) = r^* B(t) + L^T (\lambda) - C^T (\lambda) - G^T (t).$$  \hspace{1cm} (349)
Using the fact that both $\lambda$ and $G^N$ are both constant over time, linearizing (349) in the neighborhood of the steady-state yields:

$$
\dot{B}(t) = r^* \left( B(t) - \tilde{B} \right) - \left( G(t) - \tilde{G} \right)
= r^* \left( B(t) - \tilde{B} \right) - \tilde{Y} ge^{-\xi t}.
$$

where we used the fact that $G^T(t) - \tilde{G}^T = G(t) - \tilde{G} = \tilde{Y} ge^{-\xi t}$. Solving leads to the general solution for the net foreign asset position:

$$
B(t) - \tilde{B} = \left( B_0 - \tilde{B} \right) + \tilde{Y} g e^{-\xi t} + r^* \xi + r^* e^{-\xi t}.
$$

(350)

Invoking the transversality condition gives the solution for $B(t)$:

$$
B(t) - \tilde{B} = \tilde{Y} g e^{-\xi t},
$$

(351)

consistent with the intertemporal solvency condition

$$
\left( \dot{B} - B_0 \right) = -\frac{\tilde{Y} g}{\xi + r^*}.
$$

(352)

Setting $\dot{B}(t) = 0$ into (349), totally differentiating and inserting (343) together with (346) allow us to determine the change in the equilibrium value of the marginal utility of wealth:

$$
r^* \frac{d\tilde{B}}{Y} + (1 - \alpha_L) \tilde{L}^T = (1 - \alpha_C) \omega_C \tilde{C}^T,
$$

$$
\hat{\lambda} = -\frac{\Psi}{\Gamma} r^* \frac{d\tilde{B}}{Y},
$$

where $\Gamma > 0$ is given by eq. (174). Substituting (352) into the above equation leads to the change in the equilibrium value of the marginal utility of wealth:

$$
\hat{\lambda} = \frac{\Psi}{\Gamma} r^* g > 0,
$$

(353)

where $\Gamma > 0$, $\Psi > 0$, $Y_G^N > 0$, $\xi > 0$, and $g > 0$. According to (353), an unanticipated temporary rise in $G^T$ increases the shadow value of wealth and thus produces a negative wealth effect.

Inserting (353) into eq. (341) leads to the once-and-for-all decline in the relative price of non tradables:

$$
\tilde{P}(0) = \hat{\tilde{P}} = -\frac{[\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C]}{\Psi} \frac{r^*}{\Gamma} g < 0.
$$

(354)

The depreciation in the relative price of non tradables along with the negative wealth effect which induces agents to supply more labor increases unambiguously traded hours worked:

$$
\tilde{L}^T(0) = \tilde{L}^T = \left\{ \sigma_L + \frac{\alpha_L (\epsilon - \sigma_L)}{\Psi} [\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C] \right\} \frac{\Psi}{\Gamma} r^* g > 0,
$$

(355)

where we substituted (353) into (343). While the negative wealth effect exerts a positive impact on non traded hours worked, the depreciation in the relative price of non tradables has a negative effect on non traded hours worked:

$$
\tilde{L}^N = \left\{ \sigma_L - \frac{[\epsilon (1 - \alpha_L) + \alpha_L \sigma_L]}{\Psi} [\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C] \right\} \frac{\Psi}{\Gamma} r^* g \leq 0.
$$

(356)

In the special case where $\phi = \sigma_C = 1$, non traded labor unambiguously falls under assumption 1 according to which $\epsilon > \sigma_L$. 

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Differentiating (351) with respect to time leads to the response of the current account:

$$\frac{\dot{B}(t)}{Y} = -\frac{\xi}{\xi + \sigma^*} ge^{-\xi t} < 0. \quad (357)$$

According to (357), a rise in $G^T$ triggers a current account deficit and thus permanently lowers the net foreign asset position in the long-run.

Turning to the responses of sectoral output shares, the depreciation in the relative price of non tradables unambiguously lowers the output share of non tradables and increases the output share of tradables. Inserting (354) into (281) leads to the response of the output share of non tradables to a rise in $G^T$:

$$\alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) = \alpha_L (1 - \alpha_L) \epsilon \hat{P}(0),$$

$$= -\alpha_L (1 - \alpha_L) \epsilon \left[ \frac{\alpha_L + \alpha_C \omega_C \sigma_C}{\Psi} \right] \frac{r^*}{\xi + r^*} \sigma < 0. \quad (358)$$

Inserting (354) into (282) leads to the response of the output share of tradables to a rise in $G^T$:

$$(1 - \alpha_L) \left( \hat{Y}^T(0) - \hat{Y}_R(0) \right) = -\alpha_L (1 - \alpha_L) \epsilon \hat{P}(0),$$

$$= \alpha_L (1 - \alpha_L) \epsilon \left[ \frac{\alpha_L + \alpha_C \omega_C \sigma_C}{\Psi} \right] \frac{r^*}{\xi + r^*} \sigma > 0. \quad (359)$$

Finally, inserting (341) into (279) yields the one-for-and-all change in real GDP:

$$\hat{Y}_R = \frac{\sigma_L}{\Psi} \left[ \Psi - \alpha_L \left\{ \alpha_L + \alpha_C \omega_C \sigma_C \right\} \right],$$

$$= \frac{\sigma_L}{\Psi} \left\{ \alpha_L (1 - \epsilon_L) + \omega_C \sigma_C \left[ (1 - \alpha_C) \phi - (\alpha_L - \alpha_C) \sigma_C \right] \right\} \geq 0, \quad (360)$$

where we have inserted $\Psi$ given by eq. (157).

Pre-multiplying eqs. (355) and (356) by $1 - \alpha_L$ and $\alpha_L$, respectively, and letting $\epsilon$ tend toward infinity lead to:

$$\lim_{\epsilon \to \infty} \alpha_L \hat{L}^N(0) = -\frac{\alpha_L \omega_C \sigma_C}{\sigma_L + \omega_C \sigma_C} \frac{r^*}{\xi + r^*} \sigma < 0, \quad (361a)$$

$$\lim_{\epsilon \to \infty} (1 - \alpha_L) \hat{L}^T(0) = \frac{\sigma_L + \alpha_C \omega_C \sigma_C}{\sigma_L + \omega_C \sigma_C} \frac{r^*}{\xi + r^*} \sigma > 0. \quad (361b)$$

Letting $\epsilon$ tend toward infinity into eqs. (358) and (359) leads to the responses of output shares of non tradables and tradables, respectively, when we impose perfect mobility of labor across sectors:

$$\lim_{\epsilon \to \infty} \alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) = -\frac{\alpha_L \omega_C \sigma_C}{\sigma_L + \omega_C \sigma_C} \frac{r^*}{\xi + r^*} \sigma < 0, \quad (362a)$$

$$\lim_{\epsilon \to \infty} (1 - \alpha_L) \left( \hat{Y}^T(0) - \hat{Y}_R(0) \right) = \frac{\alpha_L \omega_C \sigma_C}{\sigma_L + \omega_C \sigma_C} \frac{r^*}{\xi + r^*} \sigma > 0. \quad (362b)$$

**E.19  Government Spending Shock Split between Non Tradables and Tradables and Imperfect Mobility of Labor across Sectors: Proofs of Results in Section 4.2**

So far, we have assumed that the rise in government spending is fully biased toward non tradables. In this subsection, we relax this assumption and assume that the rise in government spending is split between non tradables and tradables in accordance with their share in government spending:

$$\frac{dG(t)}{Y} = \omega_{GN} \frac{dG(t)}{Y} + \omega_{GT} \frac{dG(t)}{Y}. \quad (363)$$

The endogenous response of government spending to an exogenous fiscal shock is governed by eq. (258). We emphasize below the main changes with respect to those obtained when setting $\omega_{GT} = 0$. 

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The short-run equilibrium is identical to (259). Adopting the same steps as in subsection E.13, the deviation in percentage from the initial steady-state for the relative price of non-tradables now reads as follows:

\[
\hat{P}(t) = -\frac{[\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C]}{\Psi} \hat{\lambda} + \frac{1}{\Psi} \hat{\lambda}^+ G(t),
\]

where \(\Psi\) is given by (157). Eq. (364) corresponds to eq. (30) in the main text.

**Solution for the Net Foreign Asset Position**

Substituting \(T(t) = (1 - \alpha_L(t)) \left( W(t) \right)^{1+\alpha_L} \hat{\lambda}^{\alpha_L} \) and \(C(t) = (1 - \alpha_C(t)) \left( P_C(t) \right)^{1-\sigma_C} \hat{\lambda}^{-\sigma_C} G(t) \) into (259j) leads to:

\[
\dot{B}(t) = r^* B(t) + (1 - \alpha_L(t)) W(t)^{1+\alpha_L} \hat{\lambda}^{\alpha_L} - (1 - \alpha_C(t)) P_C(t)^{1-\sigma_C} \hat{\lambda}^{-\sigma_C} - G(t).
\]

Using the fact that \(\hat{\lambda}\) is constant over time, linearizing (365) in the neighborhood of the steady-state yields:

\[
\dot{B}(t) = r^* B(t) - \dot{\bar{L}} T \frac{\alpha_L}{\Psi}(\epsilon - \sigma_L) \omega G^N dG(t) + \dot{C}^T \frac{\alpha_C}{\Psi}(\phi - \sigma_C) \omega G^N dG(t) - \omega G^T dG(t),
\]

where

\[
\frac{\partial \dot{B}(t)}{\partial G(t)} = -r_G \equiv -[r_G N \omega G^N + \omega G^T],
\]

with

\[
\Psi^N_G = \frac{[\alpha_L(1 - \alpha_L) \epsilon - \sigma_L] (1 - \alpha_C) \omega C \omega C (\phi - \sigma_C)}{\Psi} > 0.
\]

It is worth mentioning that it is straightforward to show that

\[
\Psi^N_G \leq 1, \quad \text{and} \quad \Psi^N_G < 1 \quad \text{if} \quad \epsilon < \infty.
\]

Substituting the law of motion of government spending (258) and solving leads to the general solution for the net foreign asset position:

\[
B(t) - \bar{B} = \left[ (B_0 - \bar{B}) - \frac{\Psi_G \bar{Y}}{\epsilon + r^* g} \right] e^{r^* t} + \frac{\Psi_G \bar{Y}}{\epsilon + r^* g} e^{-\hat{\lambda} t}.
\]

Invoking the transversality condition gives the solution for \(B(t)\):

\[
B(t) - \bar{B} = \frac{\Psi_G \bar{Y}}{\epsilon + r^* g} e^{-\hat{\lambda} t},
\]

consistent with the intertemporal solvency condition

\[
\left( \bar{B} - B_0 \right) = -\frac{\Psi_G \bar{Y}}{\epsilon + r^* g}.
\]

**Eq. (370) corresponds to eq. (31) while eq. (371) corresponds to eq. (32) in the main text.**

To determine the change in the equilibrium value of the marginal utility of wealth, we have to differentiate the market clearing condition (266) for the traded good evaluated at the steady-state (i.e., \(B(t) = 0\)), using the fact that in the long-run government spending reverts to its initial level (i.e., \(dG = 0\)):

\[
\frac{r^* \hat{B}}{Y} + (1 - \alpha_L) \hat{L} = (1 - \alpha_C) \omega C \hat{C}^T,
\]

\[
\hat{\lambda} = -\frac{1}{\Gamma} \frac{r^* \hat{B}}{Y},
\]

where \(\Gamma > 0\) is given by eq. (174). Substituting (371) into the above equation leads to the change in the equilibrium value of the marginal utility of wealth:

\[
\hat{\lambda} = \frac{\Psi \Gamma G}{\xi + r^* g} > 0,
\]

(372)
where $\Gamma > 0$, $\Psi > 0$, $\Upsilon_G > 0$, $\xi > 0$, and $g > 0$. Eq. (372) corresponds to eq. (33) in the main text.

Impact Effects of a Temporary Fiscal Expansion

Evaluating (364) at time $t = 0$, inserting (372), and using the fact that $\frac{dG(0)}{G} = g > 0$, leads to the initial response of the relative price of non tradables:

$$
\hat{P}(0) = \left\{ \omega_\mathcal{G}N - [\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C] \frac{\hat{\lambda}}{\Psi} \right\} g \frac{\Psi}{\Upsilon_G \omega_\mathcal{G}N + \omega_\mathcal{G}T},
$$

$$
= \left\{ \omega_\mathcal{G}N - [\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C] \frac{\Psi [\Upsilon_G \omega_\mathcal{G}N + \omega_\mathcal{G}T]}{\Gamma} \frac{r^*}{\xi + r^*} \right\} g \frac{\Psi}{1},
$$

$$
= \omega_\mathcal{G}N \left\{ 1 - [\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C] \frac{\Psi \gamma}{\Gamma} \frac{r^*}{\xi + r^*} \right\} g \frac{\Psi}{\Upsilon_G \omega_\mathcal{G}N + \omega_\mathcal{G}T} \frac{g}{\Psi} \geq 0.
$$

(373)

The second line of eq. (373) corresponds to eq. (34) in the main text. When the rise in government spending, $G(t)$, is split between tradables and non tradables, the relative price of non tradables may appreciate or depreciate. For clarity purposes, let us consider two polar cases. If the government spending shock is fully biased toward non tradables, i.e., $\omega_\mathcal{G}N = 1$, then the second term on the RHS of eq. (373) vanishes. Due to inequality (272), i.e., $0 < \Psi \Upsilon_G (\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C) < 1$, along with and $0 < \frac{r^*}{\xi + r^*} < 1$, a rise in $G(t)$ unambiguously appreciates the relative price of non tradables. Conversely, if the government spending shock is fully biased toward tradables, i.e., $\omega_\mathcal{G}T = 1$, then the first term on the RHS of eq. (373) vanishes. Thus, the relative price of non tradables depreciates. Intuitively, a rise in $G^T$ while keeping $G^N$ fixed, leads to a negative wealth effect which lowers $C^N$. As a result, an excess supply shows up in the non traded goods market which leads to a depreciation in the relative price of non tradables. Because $\hat{P}(0)$ is monotonically increasing with $\omega_\mathcal{G}N$:

$$
\frac{\partial \hat{P}(0)}{\partial \omega_\mathcal{G}N} = \hat{P}(0)|_{\omega_\mathcal{G}N=1} > 0,
$$

(374)

where $\hat{P}(0)|_{\omega_\mathcal{G}N=1}$ is given by (273), there is a critical value $\bar{\omega}_\mathcal{G}N$ so that $\hat{P}(0) > 0$ for $\omega_\mathcal{G}N > \bar{\omega}_\mathcal{G}N$.

Substituting the change in the equilibrium value of the marginal utility of wealth given by eq. (372) into (265), and multiplying both sides by $\alpha_L$ leads to the initial reaction of non traded labor from initial steady-state in total labor units:

$$
\alpha_L \hat{L}^N(0) = \alpha_L [\epsilon (1 - \alpha_L) + \alpha_L \sigma_L] \hat{P}(0) + \alpha_L \sigma_L \frac{\Psi \Upsilon_G}{\Gamma} \frac{r^*}{\xi + r^*} g > 0,
$$

(375)

where $\hat{P}(0) \geq 0$.

Substituting the change in the equilibrium value of the marginal utility of wealth given by eq. (372) into (264) and multiplying both sides by $1 - \alpha_L$ leads to the initial reaction of traded labor from initial steady-state in total labor units:

$$
(1 - \alpha_L) \hat{L}^T(0) = -(1 - \alpha_L) \alpha_L (\epsilon - \sigma_L) \hat{P}(0) + (1 - \alpha_L) \sigma_L \frac{\Psi \Upsilon_G}{\Gamma} \frac{r^*}{\xi + r^*} g \leq 0,
$$

(376)

where $\hat{P}(0) \geq 0$.

Inserting (373) into (281), the response of the share of non tradables in real GDP to a government spending shock is given by:

$$
\alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) = \alpha_L (1 - \alpha_L) \epsilon \hat{P}(0),
$$

$$
= \frac{\alpha_L (1 - \alpha_L) \epsilon}{\Psi} \left\{ \omega_\mathcal{G}N - [\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C] \frac{\Upsilon_G \Psi}{\Gamma} \frac{r^*}{\xi + r^*} \right\} g \hat{P}(0),
$$

where $\Psi = \Upsilon_G \omega_\mathcal{G}N + \omega_\mathcal{G}T > 0$. Eq. (377) corresponds to eq. (36) in the main text.
Differentiating (370) with respect to time leads to the response of the current account as a percentage of GDP:

\[ \frac{B(t)}{Y} = -Y_G \frac{\xi + r^*}{\zeta + r^*} g e^{-\xi t} < 0, \]  

(378)

where \( Y_G > 0 \).

**Relationship between and \( \alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) \)**

In subsection E.20, we have proved that \( \partial_{\alpha_L} (\hat{Y}^N(0) - \hat{Y}_R(0)) > 0 \). We now provide a formal proof that this positive relationship holds with \( 0 < \omega_{GN} < 1 \). As shown in eq. (307), we must have \( 0 < -\frac{\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)} < 1 \). We thus totally differentiate (373) with respect to \( \epsilon \) and multiply the result by \( \epsilon / \hat{P}(0) \):

\[ -\frac{\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)} = \frac{\omega_{GN} \hat{P}(0)|_{\omega_{GN}=1}}{\hat{P}(0)} \left( \frac{-\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)} \right)|_{\omega_{GN}=1} - \frac{1}{\hat{P}(0)} \left[ \alpha_L \sigma_L + \omega_C \alpha_C \sigma_C \right] \frac{\omega_{GN} g \rho}{\xi + r^*} \frac{\partial \Gamma}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)}, \]  

(379)


Thus, whether we assume that the government shock is fully biased toward non tradables or is split between non tradables and tradables, the elasticity of the response of the relative price w.r.t the degree of labor mobility across sectors is identical because the wealth effect vanishes and thus a rise in \( G^T \) has no effect on the relative price.

**A Weakly Persistent Fiscal Shock: \( \xi \rightarrow \infty \)**

Letting \( \xi \rightarrow \infty \) into eq. (379) leads to:

\[ 0 < -\frac{\partial \hat{P}(0)}{\partial \epsilon}|_{\xi \rightarrow \infty} \frac{\epsilon}{\hat{P}(0)|_{\xi \rightarrow \infty}} = \frac{\alpha_L (1 - \alpha_L) \epsilon}{\Psi} < 1. \]  

(380)

A **Highly Persistent Fiscal Shock: \( \xi \rightarrow 0 \)**

As shall be useful, we compute the following term:

\[ \frac{\partial \Gamma}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)} = \alpha_L (1 - \alpha_L) \epsilon \frac{(\sigma_L + \omega_C \alpha_C \sigma_C)}{\Gamma} < 1, \]  

(381)

where \( \Gamma \) is given by eq. (174). Making use of (319), eq. (379) can be rewritten as follows:

\[ -\frac{\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)} = \frac{\omega_{GN} \hat{P}(0)|_{\omega_{GN}=1}}{\hat{P}(0)} - \left[ \alpha_L \sigma_L + \omega_C \alpha_C \sigma_C \right] \frac{\omega_{GN} g \rho}{\Gamma} \frac{\alpha_L (1 - \alpha_L) \epsilon (\sigma_L + \omega_C \alpha_C \sigma_C)}{\Gamma}, \]  

\[ = \alpha_L (1 - \alpha_L) \epsilon \frac{(\sigma_L + \omega_C \alpha_C \sigma_C)}{\Gamma}. \]  

(382)

In sum, whether we assume \( \omega_{GN} = 1 \) or \( 0 < \omega_{GN} < 1 \), when \( \xi \) takes intermediate values, the elasticity falls into the following range of values:

\[ -\frac{\partial \hat{P}(0)}{\partial \epsilon} \frac{\epsilon}{\hat{P}(0)} \in \left\{ \frac{\alpha_L (1 - \alpha_L) \epsilon}{\Psi}, \frac{\alpha_L (1 - \alpha_L) \epsilon}{\Psi} \left[ 1 + \frac{(\alpha_L \sigma_L + \omega_C \alpha_C \sigma_C)^2}{\Gamma} \right] \right\}. \]  

(383)

Finally, we show in subsection E.20 that \( \alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) > (\omega_{GN} - \alpha_L) \epsilon > 0 \) because we need analytical results when imposing perfect mobility of labor across sectors to demonstrate this result when allowing for imperfect mobility of labor.

**E.20 Government Spending Shock Split Between Non Tradables and Tradables and Perfect Mobility of Labor across Sectors**

We now consider a rise in government spending split between non tradables and tradables in the special case of perfect mobility of labor across sectors. We emphasize the main changes. Totally differentiating (285), using (284), leads to:

\[ \alpha_L \hat{L}^N(t) = -\alpha_0 \omega_C \sigma_C \hat{\lambda} + \omega_{GN} \frac{dG(t)}{Y}. \]  

(384)
Inserting $L = \tilde{\lambda}\nu$ (since $W = 1$) into (283h), differentiating and using (384) leads to:
\[(1 - \alpha L) \dot{L}^T(t) = [\sigma L + \alpha_C \omega C \sigma C] \dot{\lambda} - \omega G N \frac{dG(t)}{Y}. \tag{385}\]

Inserting $L^T = L - L^N$ together with $L^N = C^N + G^N$ and $L = \tilde{\lambda}\nu$ (since $W = 1$) into (283j), the market clearing condition for the traded good can be written as follows:
\[
\dot{B}(t) = r^* B(t) + L - P_C C - G^T(t) - P G^N(t),
\]
\[= r^* B(t) + \tilde{\lambda}\nu - (1 - \alpha C) P_C^1 - \sigma C \tilde{\lambda} - \sigma C - P G^N(t) - G^T(t). \tag{386}\]

Using the fact that both $\dot{\lambda}$ and $P$ are constant over time, linearizing (386) in the neighborhood of the steady-state leads to:
\[
\dot{B}(t) = r^* dB(t) - dG(t).
\]

Substituting the law of motion of government spending (258) and solving leads to the general solution for the net foreign asset position as described by eq. (289). The stable solution for the net foreign asset position and the intertemporal solvency condition are identical to (290) and (291), respectively. Thus, the change in the equilibrium value of the marginal utility of wealth is identical as well to eq. (292).

Real GDP is equal to the sum of sectoral value added evaluated at constant prices:
\[Y_R = L^T + P_0 L^N. \tag{387}\]

Totally differentiating (387) and substituting both (384) and (385) leads to:
\[
\dot{Y}_R = \alpha L \dot{L}^N + (1 - \alpha L) \dot{L}^T,
\]
\[= \sigma L \dot{\lambda} > 0. \tag{388}\]

Evaluating (384) at time $t = 0$ and inserting (388), and then substituting (292), the response of the output share of non tradables is given by:
\[
\alpha L \left( \dot{Y}_N^N(0) - \dot{Y}_R(0) \right) = \left[ \omega G N g - \left( \alpha L \sigma L + \omega C \alpha_C \sigma C \right) \dot{\lambda} \right],
\]
\[= \left[ \omega G N - \left( \frac{\alpha L \sigma L + \omega C \alpha_C \sigma C}{\sigma L + \omega C \sigma C} \right) \frac{r^*}{\xi + r^*} \right] g \geq 0. \tag{389}\]

Letting $\epsilon$ tend toward infinity into eq. (377), using (294), i.e., $\lim_{\epsilon \to -\infty} \frac{\epsilon Y_N^N}{\epsilon} = \frac{1}{\sigma L + \omega C \sigma C}$, and using the fact that $\lim_{\epsilon \to -\infty} \epsilon Y_N^N = 1$ (see eq. (293)), and applying l'Hôpital’s rule leads to eq. (389) since $\lim_{\epsilon \to -\infty} \frac{\alpha L [1 - \alpha_L]}{\xi} = 1$. Eq. (389) corresponds to eq. (36) in the main text.

As shown in section C.2, keeping the private demand components fixed, the response of the share of non tradables in real GDP is given by $\alpha_L \left( \dot{Y}_N(0) - \dot{Y}_R(0) \right) = \left( \omega G_N - \alpha L \right) \frac{dG(t)}{Y}$ (see eq. (103)), where in terms of section C.2, we have $\alpha L = \nu Y_N$. In a general equilibrium model, the private sector demand components respond to a fiscal shock and thus the response of the share of non tradables in real GDP deviates from $\left( \omega G_N - \alpha L \right) \frac{dG(t)}{Y}$. To calculate the extent of the discrepancy between the response of the share of non tradables in real GDP described by eq. (389) and that when keeping private sector’s demand components fixed as described by eq. (103), we add and subtract $\alpha L$ in the RHS of eq. (389):
\[
\alpha L \left( \dot{Y}_N^N(0) - \dot{Y}_R(0) \right) = \left\{ \left( \omega G_N - \alpha L \right) + \left[ \alpha L - \left( \frac{\alpha L \sigma L + \omega C \alpha_C \sigma C}{\sigma L + \omega C \sigma C} \right) \frac{r^*}{\xi + r^*} \right] g \right\}. \tag{390}\]

where
\[
\left[ \alpha L - \left( \frac{\alpha L \sigma L + \omega C \alpha_C \sigma C}{\sigma L + \omega C \sigma C} \right) \frac{r^*}{\xi + r^*} \right] > 0. \tag{391}\]

The RHS of eq. (391) is unambiguously positive since $\alpha L \geq \alpha_C$ as our evidence suggest and $0 < \frac{r^*}{\xi + r^*} < 1$. According to (390), the share of non tradables in real GDP unambiguously rises as long as $\omega_G N > \alpha L$. As captured by the positive term (391), the share of
non tradables in real GDP increases more in a general equilibrium model where demand components adjust in response to a fiscal shock because the rise in government spending triggers a fall in net exports on impact which further biases the sectoral demand shock toward non tradables.

While \( \alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) \) is larger than \( (\omega_{GN} - \alpha_L) g \) when \( \epsilon \to \infty \), due to inequality (391), we now show that \( \alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) > (\omega_{GN} - \alpha_L) g > 0 \) holds as well with imperfect mobility of labor across sectors. First, since \( \lim_{\epsilon \to \infty} \frac{\Psi \Gamma^N}{\Gamma} = \frac{1}{\sigma_L + \omega_C \sigma_C} \) (294) and \( \frac{\partial \Psi \Gamma^N}{\partial \epsilon} > 0 \) (see eq. (304)), we thus have:

\[
\frac{\Psi \Gamma^N}{\Gamma} \leq \frac{1}{\sigma_L + \omega_C \sigma_C},
\]

\[
\frac{\Psi \Gamma^N}{\Gamma} (\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C) \leq \frac{\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C}{\sigma_L + \omega_C \sigma_C},
\]

\[
\frac{\Psi \Gamma^N}{\Gamma} (\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C) \leq \frac{\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C}{\sigma_L + \omega_C \sigma_C} g_N. \tag{392}
\]

Second, since \( \lim_{\epsilon \to \infty} \frac{\Psi \Gamma^N}{\Gamma} = 1 \) (see eq. (293)) and \( \frac{\partial \Psi \Gamma^N}{\partial \epsilon} > 0 \) (see eq. (302)), the first line of (392) implies the following inequalities:

\[
\frac{\Psi}{\Gamma} \leq \frac{1}{\sigma_L + \omega_C \sigma_C},
\]

\[
\frac{\Psi}{\Gamma} (\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C) \leq \frac{\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C}{\sigma_L + \omega_C \sigma_C},
\]

\[
\frac{\Psi}{\Gamma} (\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C) \leq \frac{\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C}{\sigma_L + \omega_C \sigma_C} \omega_{GT}. \tag{393}
\]

Combining (392) and (393) leads to:

\[
\frac{\Psi \left[ \frac{\Gamma^N}{\Gamma} \omega_{GN} + \omega_{GT} \right]}{\Gamma} [\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C] \leq \frac{\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C}{\sigma_L + \omega_C \sigma_C} (\omega_{GN} + \omega_{GT}),
\]

\[
\frac{\Psi \left[ \frac{\Gamma^N}{\Gamma} \omega_{GN} + \omega_{GT} \right]}{\Gamma} [\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C] \leq \frac{\alpha_L \sigma_L + \alpha_C \omega_C \sigma_C}{\sigma_L + \omega_C \sigma_C}. \tag{394}
\]

where we used the fact that \( \omega_{GN} + \omega_{GT} = 1 \). From inequalities (391) and (394), we thus have:

\[
\alpha_L \left( \hat{Y}^N(0) - \hat{Y}_R(0) \right) > (\omega_{GN} - \alpha_L) g, \tag{395}
\]

whether \( \epsilon \to \infty \) or \( 0 < \epsilon < \infty \).

F Solving the Model with Physical Capital

This section extends the two-sector model with imperfect mobility of labor to physical capital accumulation which is subject to installation costs.

F.1 Household’s Maximization Problem

At each instant of time, the representative household consumes traded and non traded goods denoted by \( C^T \) and \( C^N \), respectively, which are aggregated by means of a CES function:

\[
C = \left[ \varphi \frac{1}{\sigma} \left( C^T \right)^{\varphi - 1} + (1 - \varphi) \frac{1}{\sigma} \left( C^N \right)^{\varphi - 1} \right]^{\frac{1}{\varphi - 1}}, \tag{396}
\]

where \( 0 < \varphi < 1 \) is the weight of the traded good in the overall consumption bundle and \( \phi \) corresponds to the elasticity of substitution between traded goods and non traded goods.
As in De Cordoba and Kehoe [2000], the investment good is produced using inputs of the traded good and the non traded good according to a constant-returns-to-scale function which is assumed to take a CES form:

\[
J = \left[ \frac{1}{\varphi_J} \left( J^T \right)^{\frac{\varphi_J-1}{\varphi_J}} + \left( 1 - \varphi_J \right) \frac{1}{\varphi_J} \left( J^N \right)^{\frac{\varphi_J-1}{\varphi_J}} \right]^{\frac{\varphi_J}{\varphi_J-1}},
\]

where \( \varphi_J \) is the weight of the investment traded input \((0 < \varphi_J < 1)\) and \( \phi_J \) corresponds to the intratemporal elasticity of substitution in investment between traded and non traded inputs.

Following Horvath [2000], we assume that hours worked in the traded and the non traded sectors are aggregated by means of a CES function:

\[
L = \left[ \vartheta^{-1/\epsilon} \left( L^H \right)^{\frac{\vartheta+1}{1+\vartheta}} + (1 - \vartheta)^{-1/\epsilon} \left( L^N \right)^{\frac{\vartheta+1}{1+\vartheta}} \right]^{\frac{1}{\epsilon+1}},
\]

where \( 0 < \vartheta < 1 \) is the weight of labor supply to the traded sector in the labor index \( L(.) \) and \( \epsilon \) measures the ease with which hours worked can be substituted for each other and thereby captures the degree of labor mobility across sectors.

The representative household chooses consumption, decides on labor supply, and investment that maximizes his/her lifetime utility:

\[
U = \int_0^\infty \left\{ \frac{1}{1 - \sigma_C} C(t)^{1-\sigma_C} - \frac{1}{1 + \sigma_L} L(t)^{1+\sigma_L} \right\} e^{-\beta t} dt,
\]

subject to the flow budget constraint:

\[
\dot{B}(t) = r^* B(t) + R(t) K(t) + W(t) L(t) - T(t) - P_C \left( P(t) \right) C(t) - P_J \left( P(t) \right) J(t),
\]

and capital accumulation which evolves as follows:

\[
\dot{K}(t) = I(t) - \delta_K K(t),
\]

where \( I \) is investment and \( 0 \leq \delta_K < 1 \) is a fixed depreciation rate. The first term on the RHS of (400) \( r^* B(t) + R(t) K(t) + W(t) W^T(t), W^N(t) L(t) - T(t) \) is the representative household’s real disposable income while the second term on the RHS \( P_C \left( P(t) \right) C(t) + P_J \left( P(t) \right) J(t) \) corresponds to consumption and investment expenditure including capital installation costs. More specifically, we assume that capital accumulation is subject to increasing and convex cost of net investment, \( I(t) - \delta_K K(t) \):

\[
J(t) = I(t) + \Psi \left( I(t), K(t) \right) K(t),
\]

where \( \Psi \left( \right) \) is increasing (i.e., \( \Psi' \left( \right) > 0 \)), convex (i.e., \( \Psi'' \left( \right) > 0 \)), is equal to zero at \( \delta_K \) (i.e., \( \Psi \left( \delta_K \right) = 0 \)), and has first partial derivative equal to zero as well at \( \delta_K \) (i.e., \( \Psi' \left( \delta_K \right) = 0 \)). We suppose the following functional form for the adjustment cost function:

\[
\Psi \left( I(t), K(t) \right) = \frac{\kappa}{2} \left( \frac{I(t)}{K(t)} - \delta_K \right)^2.
\]

Using (404), partial derivatives of total investment expenditure are:

\[
\frac{\partial J(t)}{\partial I(t)} = 1 + \kappa \left( \frac{I(t)}{K(t)} - \delta_K \right), \quad (404a)
\]

\[
\frac{\partial J(t)}{\partial K(t)} = -\frac{\kappa}{2} \left( \frac{I(t)}{K(t)} - \delta_K \right) \left( \frac{I(t)}{K(t)} + \delta_K \right). \quad (404b)
\]

Denoting the co-state variables associated with (404a) and (404b) by \( \lambda \) and \( Q' \), respectively, the first-order conditions characterizing the representative household’s optimal plans
are:

\[ C(t) = (P_C(t)\lambda)^{-\sigma_C}, \]  
\[ L(t) = (W(t)\lambda)^{-\sigma_L}, \]  
\[ Q(t) = P_J(t)\left[1 + \kappa \left(\frac{I(t)}{K(t)} - \delta_K\right)\right], \]  
\[ \dot{\lambda}(t) = \lambda (\beta - r^*), \]  
\[ \dot{Q}(t) = (r^* + \delta_K) Q(t) - \left\{R(t) + P_J(t)\frac{\kappa}{2} \left(\frac{I(t)}{K(t)} - \delta_K\right) \left(\frac{I(t)}{K(t)} + \delta_K\right)\right\}, \]

and the transversality conditions \( \lim_{t\to\infty} \bar{\lambda}B(t)e^{-\beta t} = 0 \) and \( \lim_{t\to\infty} Q(t)K(t)e^{-\beta t} = 0; \) to derive (405c) and (405e), we used the fact that \( Q(t) = \dot{Q}(t)/\lambda(t). \)

Since preferences over both goods embodied in the consumption index (396) are homothetic, an exact consumption price index can be derived which we denote by \( P_C: \)

\[ P_C = \left[\varphi + (1 - \varphi) \left(P^{1-\phi}\right)\right]^{1/1-\varphi}, \]  

Given the consumption-based price index (406) and applying Shephard’s lemma (or the envelope theorem), i.e., \( C^N = \frac{\partial P_C}{\partial P} C = P_C C \) and \( C^T = (P_C - PP'_C) C, \) yields the following demand of traded and non traded goods:

\[ C^T = \varphi \left(\frac{1}{P_C}\right)^{-\phi} C, \]  
\[ C^N = (1 - \varphi) \left(\frac{P}{P_C}\right)^{-\phi} C, \]

Combining (407a) and (407b) leads to the optimal intratemporal allocation of expenditure between traded and non traded goods:

\[ \left(\frac{\varphi}{1 - \varphi}\right) \frac{C^N}{C^T} = P^{-\phi}, \]  

where \( P \) is the relative price of non-tradables and \( \phi \) captures the extent to which consumers are willing to raise \( C^T/C^N \) when \( P \) appreciates by 1%. Denoting by \( \alpha_C \) the non tradable content of consumption expenditure defined as follows:

\[ \alpha_C = (1 - \varphi) \left(\frac{P}{P_C}\right)^{1-\phi}, \]  
\[ 1 - \alpha_C = \varphi \left(\frac{1}{P_C}\right)^{1-\phi}, \]

consumption expenditure in non tradables and tradables can be rewritten as \( PC^N = \alpha_C P_C C \) and \( C^T = (1 - \alpha_C) P_C C, \) respectively.

The same logic applies to intratemporal decisions for investment inputs. Given the CES aggregator function (397), we can derive the appropriate price index for investment which we denote by \( P_J: \)

\[ P_J = \left[\varphi_J + (1 - \varphi_J) \left(P^{1-\phi_J}\right)\right]^{1/1-\phi_J}. \]  

Applying the envelope theorem, i.e., \( J^N = \frac{\partial P_J}{\partial P} J = P'_J J, \) we can derive the demand for inputs of the traded good and the non traded good:

\[ J^T = \varphi_J \left(\frac{1}{P_J}\right)^{-\phi_J} J, \]  
\[ J^N = (1 - \varphi_J) \left(\frac{1}{P_J}\right)^{-\phi_J} J. \]
Combining (411a) and (411b) leads to the optimal intratemporal allocation of expenditure between traded and non traded goods:

\[
\left( \frac{\varphi J}{1 - \varphi J} \right)^{J_N / J_T} = P - \phi J,
\]

(412)

where \( \phi J \) is the elasticity of substitution between tradables and non tradables for investment. Denoting by \( \alpha J \) the non tradable content of expenditure expenditure defined as follows:

\[
\alpha J = (1 - \varphi J) \left( \frac{P}{P J} \right)^{1 - \varphi J},
\]

(413a)

\[
1 - \alpha J = \varphi J \left( \frac{1}{P J} \right)^{1 - \varphi J},
\]

(413b)

investment expenditure in non tradables and tradables can be rewritten as \( P J_N = \alpha J P J \) and \( J_T = (1 - \alpha J) P J J \), respectively.

As will be useful later, the percentage change in the consumption and investment price index is proportional to the appreciation in the relative price of non tradables since terms of trade are assumed exogenous:

\[
\hat{P}_C = \alpha C \hat{P}, \quad \hat{P}_J = \alpha J \hat{P}.
\]

(414a) \hspace{1cm} (414b)

The aggregate wage index, \( W(t) \), associated with the labor index defined above (398) is:

\[
W = \left[ \vartheta \left( W_T \right)^{\epsilon + 1} + (1 - \vartheta) \left( W_N \right)^{\epsilon + 1} \right]^{\frac{1}{\epsilon + 1}},
\]

(415)

where \( W_T \) and \( W_N \) are wages paid in the traded and the non traded sectors, respectively. Given the aggregate wage index (415) and applying the envelope theorem, i.e., \( L_T = \frac{\partial W(W_T, W_N)}{\partial W_T} L \) and \( L_N = \frac{\partial W(W_T, W_N)}{\partial W_N} L \), we can derive the allocation of aggregate labor supply to the traded and the non traded sector:

\[
L_T = \vartheta \left( \frac{W_H}{W} \right)^{\epsilon} L, \quad (416a)
\]

\[
L_N = (1 - \vartheta) \left( \frac{W_N}{W} \right)^{\epsilon} L. \quad (416b)
\]

As will be useful later, the percentage change in the aggregate wage index is a weighted average of percentage changes in sectoral wages:

\[
\hat{W} = (1 - \alpha_L) \hat{W}^H + \alpha_L \hat{W}^N, \quad (417)
\]

where \( \alpha_L \) is the non tradable content of aggregate labor compensation:

\[
\alpha_L = (1 - \vartheta) \left( \frac{W_N}{W} \right)^{1 + \epsilon}, \quad (418a)
\]

\[
1 - \alpha_L = \vartheta \left( \frac{W_T}{W} \right)^{1 + \epsilon}. \quad (418b)
\]

Combining (416a) and (416b) leads to the optimal allocation of hours worked to the traded and the non traded sector:

\[
\left( \frac{\vartheta}{1 - \vartheta} \right) \frac{L_N}{L_T} = \Omega^\epsilon, \quad (419)
\]

where \( \Omega \equiv W_N / W_T \) is the relative wage and \( \epsilon \) captures the extent to which workers are willing to shift hours worked toward the non traded sector when \( \Omega \) rises by 1%.
F.2 Firm’s Maximization Problem

Each sector \(j = T, N\) uses physical capital, \(K^j\), and labor, \(L^j\), according to constant returns to scale production functions:

\[
Y^j = Z^j (L^j)^{\theta^j} (K^j)^{1-\theta^j},
\]

where \(\theta^j\) is the labor income share in sector \(j\) and \(Z^j\) corresponds to the total factor productivity index which is introduced for calibration purposes. Both sectors face two cost components: a capital rental cost equal to \(R\), and a labor cost equal to the wage rate, i.e., \(W^T\) in the traded sector and \(W^N\) in the non traded sector. Both sectors are assumed to be perfectly competitive.

Since capital can move freely between the two sectors while the shift of labor across sectors is costly, only marginal products of capital in the traded and the non traded sector equalize:

\[
Z^T (1 - \theta^T) (k^T)^{-\theta^T} = PZ^N (1 - \theta^N) (k^N)^{-\theta^N} \equiv R,
\]

\[
Z^T \theta^T (k^T)^{1-\theta^T} \equiv W^T,
\]

\[
PZ^N \theta^N (k^N)^{1-\theta^N} \equiv W^N,
\]

where the capital-labor ratio for sector \(j = T, N\) is denoted by \(k^j \equiv K^j/L^j\). These static efficiency conditions state that the value of the marginal product of labor in sector \(j\) is equal to the labor cost \(W^j\) while the value of the marginal product of capital in the traded and the non traded sector must be equal to the capital rental cost, \(R\).

Aggregating over the two sectors gives us the resource constraint for capital:

\[
K^T + K^N = K.
\]

F.3 Solving the Model

Before linearizing, we have to determine short-run static solutions. Static efficiency conditions (405a) and (405b) can be solved for consumption and labor which of course must hold at any point of time:

\[
C = C (\tilde{\lambda}, P), \quad L = L (\tilde{\lambda}, W^T, W^N),
\]

with

\[
\dot{C} = -\sigma_C \tilde{\lambda} - \alpha_C \sigma_C C \tilde{P}, \tag{424a}
\]

\[
\dot{L} = \sigma_L \tilde{\lambda} + \sigma_L (1 - \alpha_L) \tilde{W}^T + \sigma_L \alpha_L \tilde{W}^N, \tag{424b}
\]

where we made use of (414a) and (417); \(\sigma_C\) and \(\sigma_L\) correspond to the intertemporal elasticity of substitution for consumption and labor, respectively. A rise in the shadow value of wealth induces agents to cut their real expenditure and to supply more labor. By raising the consumption price index, an appreciation in the relative price of non tradables drives down consumption. A rise in sectoral wage rates increases the aggregate wage index which provides an incentive to increase hours worked.

Inserting first the optimal decision for consumption (405a) into demand for traded and non traded goods described by (407a) and (407b) gives the short-run static solutions for \(C^T\) and \(C^N\):

\[
C^T = C^T (\tilde{\lambda}, P), \quad C^N = C^N (\tilde{\lambda}, P),
\]

with partial derivatives given by

\[
\dot{C}^T = -\sigma_C \tilde{\lambda} + \alpha_C (\phi - \sigma_C) \tilde{P}, \tag{426a}
\]

\[
\dot{C}^N = -\sigma_C \tilde{\lambda} - [(1 - \alpha_C) \phi + \alpha_C \sigma_C] \tilde{P}, \tag{426b}
\]
where we set: $\frac{P^{C}}{P^{G}} = \phi (1 - \alpha_C) > 0$ and $P^{C}C = C^{N}$. A rise in the shadow value of wealth lowers both $C^{T}$ and $C^{N}$. An appreciation in $P$ lowers unambiguously $C^{N}$ and increases $C^{T}$ if $\phi > \sigma_C$.

Inserting first the optimal decision for labor supply (405b) into optimal supply of hours worked to the traded (416a) and the non traded sector (416b), enables us to solve for $L^{T}$ and $L^{N}$:

$$L^{T} = L^{T} (\lambda, W^{T}, W^{N}), \quad L^{N} = L^{N} (\lambda, W^{T}, W^{N}), \quad (427)$$

with partial derivatives given by:

$$\dot{L}^{T} = \sigma_{L} \dot{\lambda} + [\epsilon \alpha_{L} + \sigma_{L} (1 - \alpha_{L})] \dot{W}^{T} - \alpha_{L} (\epsilon - \sigma_{L}) \dot{W}^{N}, \quad (428a)$$

$$\dot{L}^{N} = \sigma_{L} \dot{\lambda} + [\epsilon (1 - \alpha_{L}) + \sigma_{L} \alpha_{L}] \dot{W}^{N} - (1 - \alpha_{L})(\epsilon - \sigma_{L}) \dot{W}^{T}, \quad (428b)$$

Plugging the short-run static solutions for $L^{T}$ and $L^{N}$ given by (427) into the resource constraint for capital (422), the system of four equations consisting of (421a)-(421c) together with (422) can be solved for the sectoral wage rates $W^{T}$ and sectoral capital-labor ratios $k^{j}$. Keeping TFPs unchanged, and log-differentiating (421a)-(421c) together with (422) yields in matrix form:

$$
\begin{pmatrix}
-\theta^{T} & \theta^{N} & 0 & 0 \\
(1 - \theta^{T}) & 0 & -1 & 0 \\
0 & (1 - \theta^{N}) & 0 & -1 \\
(1 - \xi) & \xi & \Psi_{W^{T}} & \Psi_{W^{N}}
\end{pmatrix}
\begin{pmatrix}
\dot{k}^{T} \\
\dot{k}^{N} \\
\dot{W}^{T} \\
\dot{W}^{N}
\end{pmatrix}
= 
\begin{pmatrix}
\dot{\tilde{\rho}} \\
0 \\
-\tilde{\rho} \\
\tilde{K} - \Psi_{\lambda} \dot{\lambda}
\end{pmatrix}, \quad (429)
$$

where we set:

$$
\Psi_{W^{T}} = (1 - \xi^{N}) \frac{L_{W^{T}}W^{T}}{L^{T}} + \xi^{N} \frac{L_{W^{T}}W^{T}}{L^{N}}, \quad (430a)$$

$$
\Psi_{W^{N}} = (1 - \xi^{N}) \frac{L_{W^{N}}W^{N}}{L^{T}} + \xi^{N} \frac{L_{W^{N}}W^{N}}{L^{N}}, \quad (430b)$$

$$
\xi^{N} \equiv \frac{k^{N}L^{N}}{K}, \quad (430c)$$

$$
\Psi_{\lambda} = (1 - \xi^{N}) \sigma_{L} + \xi^{N} \sigma_{L} = \sigma_{L}. \quad (430d)
$$

The determinant of (429) is:

$$G \equiv - \{ \theta^{T} [(1 - \theta^{N}) \Psi_{W^{N}} + \xi^{N}] + \theta^{N} [(1 - \theta^{T}) \Psi_{W^{T}} + (1 - \xi^{N})] \} \leq 0, \quad (431)$$

where

$$\Psi_{W^{T}} = (1 - \xi^{N}) \epsilon + (1 - \alpha_{L})(\sigma_{L} - \epsilon), \quad (432a)$$

$$\Psi_{W^{N}} = \xi^{N} \epsilon + \alpha_{L}(\sigma_{L} - \epsilon), \quad (432b)$$

$$\Psi_{W^{T}} + \Psi_{W^{N}} = \sigma_{L}. \quad (432c)$$
The sign of $G$ depends on $\epsilon \geq \sigma_L$; for the baseline calibration, we have $\epsilon > \sigma_L$; because the discrepancy is small, we find it convenient to assume $\sigma_L \simeq \epsilon$ so that a rise in $W^T$ ($W^N$) does not affect $L^N$ ($L^T$). Hence, we have $G < 0$. In the following, for clarity purposes, when discussing the results, we assume that $\sigma_L \simeq \epsilon$ so that determinant $G$ given by eq. (431) is negative. Note that all our statements below also hold when $\epsilon > \sigma_L$.

The short-run static solutions for sectoral wages are:

$$W^T = W^T (\bar{\lambda}, K, P), \quad W^N = W^N (\bar{\lambda}, K, P),$$

with

$$\frac{\dot{W}^T}{K} = \frac{-(1 - \theta^T) \theta^N}{G} > 0, \quad (434a)$$
$$\frac{\dot{W}^N}{K} = \frac{-(1 - \theta^N) \theta^T}{G} > 0, \quad (434b)$$
$$\frac{\dot{W}^T}{P} = \frac{(1 - \theta^T) (\Psi_{WN} + \xi^N)}{G} < 0, \quad (434c)$$
$$\frac{\dot{W}^N}{P} = \frac{-\{\theta^T \xi^N + [(1 - \xi^N) + (1 - \theta^T) \Psi_{WT}]\}}{G} > 0, \quad (434d)$$
$$\frac{\dot{W}^T}{\bar{\lambda}} = \frac{\sigma_L (1 - \theta^T) \theta^N}{G} < 0, \quad (434e)$$
$$\frac{\dot{W}^N}{\bar{\lambda}} = \frac{\sigma_L (1 - \theta^N) \theta^T}{G} < 0, \quad (434f)$$

The short-run static solutions for capital-labor ratios are:

$$k^T = k^T (\lambda, K, P), \quad k^N = k^N (\bar{\lambda}, K, P),$$

with

$$\frac{\dot{k}^T}{K} = \frac{-\theta^N}{G} > 0, \quad (436a)$$
$$\frac{\dot{k}^N}{K} = \frac{-\theta^T}{G} > 0, \quad (436b)$$
$$\frac{\dot{k}^T}{P} = \frac{\Psi_{WN} + \xi^N}{G} < 0, \quad (436c)$$
$$\frac{\dot{k}^N}{P} = \frac{\theta^T \Psi_{WN} - [(1 - \theta^T) \Psi_{WT} + (1 - \xi^N)]}{G} > 0, \quad (436d)$$
$$\frac{\dot{k}^T}{\bar{\lambda}} = \frac{\sigma_L \theta^N}{G} < 0, \quad (436e)$$
$$\frac{\dot{k}^N}{\bar{\lambda}} = \frac{\sigma_L \theta^T}{G} < 0. \quad (436f)$$

An increase in the capital stock $K$ raises capital-labor ratios and thereby wage rates in both sectors. A rise in $\lambda$ encourages agents to supply more labor which reduces sectoral capital-labor ratios and thereby wage rates in both sectors. In the standard model assuming perfect mobility of labor across sectors, an appreciation in the relative price of non tradables shifts resources toward the non traded sector and increases (lowers) $k^N$ and $k^T$ if the traded sector is more (less) capital intensive than the non-traded sector. In a model with limited labor mobility, $k^N$ increases as $P$ appreciates irrespective of whether the traded sector is more or less capital intensive than the non traded sector.

Inserting first sectoral wages (433), sectoral employment (427) can be solved as functions of the shadow value of wealth, the capital stock and the relative price of non tradables:

$$L^T = L^T (\bar{\lambda}, K, P), \quad L^N = L^N (\bar{\lambda}, K, P),$$

(437)
where the partial derivatives are not shown as we cannot determine the sign of analytical expressions in the general case. Yet, when assuming $\sigma_L \simeq \epsilon$ and using the fact that $\hat{W}^T = (1 - \theta^T) \hat{k}^T$ we have

$$\hat{L}^T = \sigma_L \hat{\lambda} + [\epsilon \alpha_L + \sigma_L (1 - \alpha_L)] (1 - \theta^T) \hat{k}^T.$$

Using (436), we find that traded labor is increasing with the capital stock $K$ and decreasing with the relative price of non tradables. Adopting a similar reasoning for non traded labor, we have:

$$\hat{L}^N = \sigma_L \hat{\lambda} + [\epsilon (1 - \alpha_L) + \sigma_L \alpha_L] \hat{W}^N.$$

Using (436), we find that non traded labor is increasing with both the capital stock $K$ and the relative price of non tradables.

Production functions (420) can be rewritten ad follows:

$$Y_j = Z_j L_j (k_j)^{1 - \theta_j}, \quad j = T, N. \quad (438)$$

Inserting first short-run static solutions for sectoral capital-labor ratios (435) and sectoral labor (437) into the production functions of the traded and non traded sectors yields:

$$Y^T = Y^T (\bar{\lambda}, K, P), \quad Y^N = Y^N (\bar{\lambda}, K, P), \quad (439)$$

where the partial derivatives are not shown as we cannot determine the sign of expressions.

In the standard two-sector model imposing perfect mobility of labor across sectors, the Rybczynski effect implies that a rise in $K$ raises the output of the sector which is relatively more capital intensive. With a difficulty in reallocating labor across sectors, the Rybczynski effect does no hold as a rise in $K$ now increases both traded and non traded outputs. The reason is that due to imperfect mobility of labor, increasing the capital stock raises capital-labor labor ratios in both sectors so that both $Y^T$ and $Y^N$ rise. As in the standard model assuming perfect mobility of labor, an appreciation in the relative price of non tradables shifts resources toward the non traded sector, but all the less so as labor is less mobile across sectors.

**The Return on Domestic Capital, $R$**

The return on domestic capital is:

$$R = Z^T (1 - \theta^T) (k^T)^{-\theta^T}. \quad (440)$$

Inserting first the short-run static solution for the capital-labor ratio $k^T$ given by (435), eq. (440) can be solved for the return on domestic capital:

$$R = R (\bar{\lambda}, K, P), \quad (441)$$

where partial derivatives are given by:

$$R_K = \frac{\partial R}{\partial K} = -\theta^T \frac{R}{k^T} k^T K < 0, \quad (442a)$$

$$R_P = \frac{\partial R}{\partial P} = -\theta^T \frac{R}{k^T} k^T P > 0. \quad (442b)$$

**Optimal Investment Decision, $I/K$**

Eq. (405c) can be solved for the investment rate:

$$\frac{I}{K} = v \left( \frac{Q}{P_j(P)} \right) + \delta_K, \quad (443)$$

where

$$v(.) = \frac{1}{\kappa} \left( \frac{Q}{P_j} - 1 \right), \quad (444)$$

with

$$v_Q = \frac{\partial v(.)}{\partial Q} = \frac{1}{\kappa} \frac{1}{P_j} > 0, \quad v_P = \frac{\partial v(.)}{\partial P} = -\frac{Q}{\kappa} \frac{\alpha_j}{P_j P} < 0. \quad (445)$$
Inserting (443) into (403), investment including capital installation costs can be rewritten as follows:

\[
J = K \left[ \frac{I}{K} + \frac{\kappa}{2} \left( \frac{I}{K} - \delta K \right) \right], \\
= K \left[ v(.) + \delta K + \frac{\kappa}{2} (v(.)^2) \right]. \tag{446}
\]

**The Relative Price of Non Tradables, \( P \)**

Finally, we have to solve for the relative price of non tradables by using the non traded goods market clearing condition:

\[
Y^N = C^N + G^N + J^N. \tag{447}
\]

Remembering that the non traded input \( J^N \) used to produce investment goods is equal to \( P'_J J \), inserting short-run static solutions for \( C^N \) and \( Y^N \) given by (425) and (439), respectively, and substituting (446), the non traded goods market clearing condition (447) can be rewritten as follows:

\[
Y^N (\bar{\lambda}, K, P) = C^N (\bar{\lambda}, P) + G^N + P'_J K \left[ v(.) + \delta K + \frac{\kappa}{2} (v(.)^2) \right]. \tag{448}
\]

Eq. (448) can be solved for the relative price of non tradables:

\[
P = P (\bar{\lambda}, K, Q, G^N), \tag{449}
\]

with partial derivatives given by:

\[
P_K = \frac{\partial P}{\partial K} = -\frac{Y_P^N}{P'_J} + \frac{J}{\Psi^P} \leq 0, \tag{450a}
\]

\[
P_Q = \frac{\partial P}{\partial Q} = \frac{Kv_Q [1 + \kappa v(.)]}{\Psi^P} > 0, \tag{450b}
\]

\[
P_{G^N} = \frac{1}{P'_J \Psi^P} > 0, \tag{450c}
\]

where we set

\[
\Psi^P = \left[ (Y_P^N - C_P^N) + \frac{J^N \phi_J (1 - \alpha_J)}{P} \right] \frac{1}{P'_J} - K v_P [1 + \kappa v(.)] > 0. \tag{451}
\]

**F.4 Equilibrium Dynamics**

Remembering that the non traded input \( J^N \) used to produce the capital good is equal to \( P'_J J \), using the fact that \( J^N = Y^N - C^N - G^N \) and inserting \( I = \dot{K} + \delta K \), the capital accumulation equation can be rewritten as follows:

\[
\dot{K} = \frac{Y^N - C^N - G^N}{P'_J} - \delta K K - \frac{\kappa}{2} \left( \frac{I}{K} - \delta K \right)^2 K. \tag{452}
\]

Inserting short-run static solutions for non traded output (439), consumption in non tradables (425), and optimal investment decision (443) into the physical capital accumulation equation (452) and the dynamic equation for the shadow value of capital stock (404b), the dynamic system is:

\[
\dot{Y} \equiv Y (K, P, Q, G^N) = \frac{Y^N (K, P(\cdot), \bar{\lambda}) - C^N (\bar{\lambda}, P(\cdot)) - G^N}{P'_J (P(\cdot))} - \delta K K - \frac{\kappa}{2K} \left[ \frac{Q}{P'_J (P(\cdot))} - 1 \right]^2, \tag{453a}
\]

\[
\dot{Q} \equiv Q (K, P, Q, G^N) = (\tau^* + \delta K) Q - \left[ R (K, P(\cdot)) + P'_J \frac{K}{2} v(.) (v(.) + 2 \delta K) \right]. \tag{453b}
\]
As will be useful, let us denote by $\Upsilon_K$, $\Upsilon_Q$, and $\Upsilon_P$ the partial derivatives evaluated at the steady-state of the capital accumulation equation w.r.t. K and Q (for given $P$), respectively, and $P$:

$$\Upsilon_K\bigg|_{P \text{ fixed}} \equiv \frac{\partial \tilde{K}}{\partial K} \bigg|_{P \text{ fixed}} = \left(\frac{Y_N}{P_j} - \delta_K\right) > 0, \tag{454a}$$

$$\Upsilon_P \equiv \frac{\partial \tilde{K}}{\partial P} = \left(\tilde{y}_N - C_P\right) + \tilde{I}_N \phi_j (1 - \alpha_j) \frac{1}{P} > 0, \tag{454b}$$

$$\Upsilon_Q\bigg|_{P \text{ fixed}} \equiv \frac{\partial \tilde{K}}{\partial Q} \bigg|_{P \text{ fixed}} = 0, \tag{454c}$$

where we used the fact that in the long-run, $\tilde{I}^N = \tilde{I}^N$ and $\tilde{Q} = P_j\left(\tilde{P}\right)$.

Let us denote by $\Sigma_K$, $\Sigma_Q$, and $\Sigma_P$ the partial derivatives evaluated at the steady-state of the dynamic equation for the marginal value of an additional unit of capital w.r.t. K and Q (for given $P$), respectively, and $P$:

$$\Sigma_K\bigg|_{P \text{ fixed}} \equiv \frac{\partial \tilde{Q}}{\partial K} \bigg|_{P \text{ fixed}} = -R_K > 0, \tag{455a}$$

$$\Sigma_P \equiv \frac{\partial \tilde{Q}}{\partial P} = -R_P - P_j k v P j k v P j > 0, \tag{455b}$$

$$\Sigma_Q\bigg|_{P \text{ fixed}} \equiv \frac{\partial \tilde{Q}}{\partial Q} \bigg|_{P \text{ fixed}} = (r^* + \delta_K) - P_j k v Q \delta_K = r^* > 0, \tag{455c}$$

where $R_K$ given by (442a) is evaluated at the steady-state, i.e., $-P_j (r^* + \delta_K) \theta^T k T \theta^T k T < 0$, and $R_P = -P_j (r^* + \delta_K) \theta^T k T \theta^T k T > 0$ (see eq. (442b)); to derive (455c), we inserted $v_Q = \frac{1}{r P_j}$ given by (445).

Denoting steady-state values with a tilde, linearizing (453a)-(453b) in the neighborhood of the steady-state yields in matrix form:

$$\begin{pmatrix}
\tilde{K}(t) \\
\tilde{Q}(t)
\end{pmatrix}
= \begin{pmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{pmatrix}
\begin{pmatrix}
K(t) - \tilde{K} \\
Q(t) - \tilde{Q}
\end{pmatrix}, \tag{456}
$$

where the coefficients of the Jacobian matrix $J$ are given by:

$$a_{11} = \Upsilon_K\bigg|_{P \text{ fixed}} + \Upsilon_P P_K > 0, \tag{457a}$$

$$a_{12} = \Upsilon_P P_Q > 0, \tag{457b}$$

$$a_{21} = \Sigma_K\bigg|_{P \text{ fixed}} + \Sigma_P P_K > 0, \tag{457c}$$

$$a_{22} = r^* + \Sigma_P P_Q > 0, \tag{457d}$$

with partial derivatives being evaluated at the steady-state; we used the fact that at the steady-state $\tilde{J}/\tilde{K} = \tilde{I}/\tilde{K} = \delta_K$ and $\tilde{R} = Z^T (1 - \theta^T) \left(k T^\theta\right)^{-\theta T} = P_j (r^* + \delta_K)$.

Saddle path stability requires the determinant of the Jacobian matrix $\text{Det}J$ given by $a_{11} a_{22} - a_{21} a_{12}$ to be negative. While the term $a_{21} a_{12}$ is always positive, regardless of sectoral capital intensities, the term $a_{11} a_{22}$ can be positive or negative. Because both the elasticity $k^N$ with respect to $P$ and the tradable content of investment expenditure $(1 - \alpha_j)$ are smaller than one and exert opposite effects on the marginal product of capital, the term $a_{11} a_{22}$ is small so that the saddle-path stability condition is fulfilled regardless of sectoral capital intensities. When investment expenditure are traded only, i.e., $\alpha_j = 0$, we have $a_{22} < 0$ while $a_{11} > 0$; as a result, the determinant of the Jacobian matrix given by $a_{11} a_{22} - a_{21} a_{12}$ is always negative in this case so that the equilibrium is saddle-path. When $0 < \alpha_j < 1$, the sign of the Jacobian matrix is ambiguous; for all plausible sets of parametrization, we find that the long-run equilibrium is saddle path.

Assuming that the saddle-path stability condition is fulfilled, and denoting the negative eigenvalue by $\nu_1$ and the positive eigenvalue by $\nu_2$, the general solutions for $K$ and $Q$ are:

$$K(t) - \tilde{K} = D_1 e^{\nu_1 t} + D_2 e^{\nu_2 t}, \quad Q(t) - \tilde{Q} = \omega_1^2 D_1 e^{\nu_1 t} + \omega_2^2 D_2 e^{\nu_2 t}, \tag{458}$$
where $K_0$ is the initial capital stock and $(1, \omega_2' \nu')$ is the eigenvector associated with eigenvalue $\nu_1$:

$$\omega_2' = \frac{\nu_1 - \alpha_{11}}{\alpha_{12}}.$$  

(459)

Because $\nu_1 < 0$, $a_{11} > 0$ and $a_{12} > 0$, we have $\omega_2' < 0$, regardless of sectoral capital intensities, which implies that the shadow value of investment and the stock physical capital move in opposite direction along a stable path.

Remembering that $J^T = (1 - \alpha_j) P_j J$, the current account equation is given by:

$$\dot{B} = \Xi(B, K, Q, G) = r^* B + Y^T - C^T - G^T - (1 - \alpha_j) P_j J,$$

$$= r^* B + Y^T - C^T - G^T - \left(\frac{1 - \alpha_j}{\alpha_j}\right) P (Y^N - C^N - G^N).$$  

(460)

where we used the fact that $P_j J = Y^N - C^N - G^N$. As will be useful later, let us denote by $\Xi_K$ and $\Xi_P$ the partial derivatives of the accumulation equation for traded bonds w.r.t. $K$ (for given $P$) and $P$:

$$\Xi_K\bigg|_{P_{\text{fixed}}} = \frac{\partial \dot{B}}{\partial K}\bigg|_{P_{\text{fixed}}} = Y^T_K - \left(\frac{1 - \alpha_j}{\alpha_j}\right) \bar{P} Y^N_K \geq 0,$$

$$\Xi_P = \frac{\partial \dot{B}}{\partial P} = (Y^T_P - C^T) - \left(\frac{1 - \alpha_j}{\alpha_j}\right) \bar{P} (Y^N_P - C^N) - \phi_j \left(\frac{1 - \alpha_j}{\alpha_j}\right) \bar{I}^N < \Psi 161b$$

(461a)

(461b)

where we used the fact that $\frac{\partial (1 - \alpha_j)}{\partial P} = - \frac{1}{\bar{P}} \left(\frac{1 - \alpha_j}{\alpha_j}\right) \phi_j \left(\frac{1 - \alpha_j}{\alpha_j}\right)$ and at the steady-state, we have $\bar{J}^N = \bar{I}^N$ since capital installation costs are absent in the long run.

Inserting first the short-run static solutions for traded output (439) and consumption in tradables (425) into the accumulation equation of foreign bonds (460), linearizing, substituting the solutions for $K(t)$ and $Q(t)$ given by (458) yields the general solution for traded bonds:

$$B(t) = \bar{B} + \left[\left(B_0 - \bar{B}\right) - \Psi_1 D_1 - \Psi_2 D_2\right] e^{\nu_1 t} + \Psi_1 D_1 e^{\nu_1 t} + \Psi_2 D_2 e^{\nu_2 t},$$

(462)

where $B_0$ is the initial stock of traded bonds and we set

$$\Xi_K = \Xi_K\bigg|_{P_{\text{fixed}}} + \Xi_P P_K,$$

$$\Xi_P = \Xi_P P_Q,$$

$$N_i = \Xi_K + \Xi_Q \omega_2 \nu_1,$$

$$\Psi_i = \frac{N_i}{\nu_1 - r^*}.$$  

(463a)

(463b)

(463c)

(463d)

Invoking the transversality condition leads to the linearized version of the nations’s intertemporal solvency condition:

$$\bar{B} - B_0 = \Psi_1 \left(\bar{K} - K_0\right),$$

(464)

where $K_0$ is the initial stock of physical capital.

**F.5 Derivation of the Accumulation Equation of Financial Wealth**

Remembering that the stock of financial wealth $A(t)$ is equal to $B(t) + Q(t)K(t)$, differentiating w.r.t. time, i.e., $\dot{A}(t) = \dot{B}(t) + \dot{Q}(t)K(t) + Q(t)\dot{K}(t)$, plugging the dynamic equation for the marginal value of capital (405e), inserting the accumulation equations for physical capital (404b) and traded bonds (404a), yields the accumulation equation for the stock of financial wealth or the dynamic equation for private savings:

$$\dot{A}(t) = r^* A(t) + W(t)L(t) - T(t) - P_C (P(t)) C(t).$$

(465)
We first determine short-run static solutions for aggregate labor supply and aggregate wage index. Inserting short-run static solutions for sectoral wages (433) into the short-run static solution for aggregate labor supply (421), we can solve for total hours worked:

\[ L = L(\bar{\lambda}, K, P), \quad (466) \]

where partial derivatives are given by

\[ L_K = \frac{\partial L}{\partial K} = L_{WT} W_K^T + L_{WN} W_K^N > 0, \quad (467a) \]

\[ L_P = \frac{\partial L}{\partial P} = L_{WT} W_P^T + L_{WN} W_P^N \geq 0. \quad (467b) \]

Substituting short-run static solutions for sectoral wages (433) into the aggregate wage index \( W \equiv W(W^T, W^N) \), we can solve for the aggregate wage index:

\[ W = W(\bar{\lambda}, K, P), \quad (468) \]

where partial derivatives are given by

\[ W_K = \frac{\partial W}{\partial K} = W_{WT} W_K^T + W_{WN} W_K^N, \quad (469a) \]

\[ W_P = \frac{\partial W}{\partial P} = W_{WT} W_P^T + W_{WN} W_P^N, \quad (469b) \]

with \( W_{WT} = (W/W^T)(1 - \alpha_L) \) and \( W_{WN} = (W/W^N) \alpha_L \).

As will be useful, let us denote by \( \Lambda_K \) and \( \Lambda_P \) the partial derivatives of the accumulation equation for financial wealth w.r.t. \( K \) (for given \( P \)) and \( P \):

\[ \Lambda_K = \frac{\partial A}{\partial K} \bigg|_{P \text{ fixed}} = \left( W_K \dot{L} + \dot{W} L_K \right) > 0, \quad (470a) \]

\[ \Lambda_P = \frac{\partial A}{\partial P} = \left( W_P \dot{L} + \dot{W} L_P \right) - \left( \dot{C}^N + P^C C_P + G^N \right) \lesssim 0, \quad (470b) \]

where all partial derivatives are evaluated at the steady-state.

Inserting short-run static solutions for aggregate labor supply (466), for the aggregate wage index (468) and consumption (423) into the accumulation equation of financial wealth (465), linearizing around the steady-state, and solving yields the general solution for the stock of financial wealth:

\[ A(t) = \tilde{A} + \left[ (A_0 - \tilde{A}) - \Delta_1 D_1 - \Delta_2 D_2 \right] e^{r^* t} + \Delta_1 D_4 e^{r^*_1 t} + \Delta_2 D_2 e^{r^*_2 t}, \quad (471) \]

where \( A_0 \) is the initial stock of financial wealth and we set

\[ \Lambda_K = \Lambda_K \bigg|_{P \text{ fixed}} + \Lambda_P P_K, \quad (472a) \]

\[ A_Q = \Lambda_P P_Q, \quad (472b) \]

\[ M_i = \Lambda_K + \Lambda_Q \omega^i, \quad (472c) \]

\[ \Delta_i = \frac{M_i}{\nu_i - r^*}. \quad (472d) \]

The linearized version of the representative household’s intertemporal solvency condition is:

\[ \tilde{A} - A_0 = \Delta_1 \left( \tilde{K} - K_0 \right), \quad (473) \]

where \( K_0 \) is the initial stock of physical capital.
F.6 The Steady-State

In the next section, we use a specific procedure to solve for the steady-state which allows us to summarize graphically the long-run equilibrium. Below, we characterize the whole steady-state and use tilde to denote long-run values. Setting $\tilde{K} = \tilde{P} = \tilde{B} = 0$ into (453a), (453b) and (457a), and inserting short-run static solutions for $k^N$, $Y^N$ and $Y^T$, $C^N$ and $C^T$ derived above, the steady-state can be summarized by four equations:

$$Z^T (1 - \theta^T) \left[ k^T \left( \tilde{K}, \tilde{P}, \lambda \right) \right]^{-\theta^T} = P_j \left( \tilde{P} \right) (r^* + \delta), \quad (474a)$$

$$Y^N \left( \tilde{K}, \tilde{P}, \lambda \right) = C^N \left( \tilde{P}, \lambda \right) + P_j^T \left( \tilde{P} \right) \delta \tilde{K} + G^N, \quad (474b)$$

$$r^* \tilde{B} + Y^T \left( \tilde{K}, \tilde{P}, \lambda \right) = C^T \left( \tilde{P}, \lambda \right) + (1 - \alpha_J) P_j \left( \tilde{P} \right) \delta \tilde{K} + G^T, \quad (474c)$$

$$\tilde{B} - B_0 = \Psi_1 \left( \tilde{K} - K_0 \right). \quad (474d)$$

These four equations jointly determine $\tilde{P}, \tilde{K}, \tilde{B}$ and $\lambda$.

G Solving for the Steady-State

In this section, we characterize the long-run equilibrium graphically.

G.1 Rewriting the Steady-State

In order to summarize graphically the long-run equilibrium and to build up intuition on the long-run effects of fiscal shocks, it is convenient to rewrite the steady-state as follows:

$$\frac{\tilde{C}^T}{C^N} = \frac{\varphi}{1 - \varphi} \frac{\tilde{P}}{\bar{P}}, \quad (475a)$$

$$\frac{\tilde{L}^T}{L^N} = \frac{\vartheta}{1 - \vartheta} \frac{\tilde{\omega}}{\bar{\omega}}, \quad (475b)$$

$$\frac{\tilde{Y}^T (1 + \nu_B - \nu_{JT} + \nu_{GT})}{Y^N (1 - \nu_{JN} - \nu_{GN})} = \frac{\tilde{C}^T}{C^N}, \quad (475c)$$

$$\tilde{P} \left( 1 - \theta^N \right) \left( \tilde{k}^N \right)^{-\theta^N} = P_j \left( \tilde{P} \right) (r^* + \delta), \quad (475d)$$

$$Z^T (1 - \theta^T) \left( \tilde{k}^T \right)^{-\theta^T} = \tilde{P} Z^N \left( 1 - \theta^N \right) \left( \tilde{k}^N \right)^{-\theta^N} = \tilde{R}, \quad (475e)$$

$$\theta_T \left( \tilde{k}_T \right)^{1 - \theta_T} = \tilde{W}^T, \quad (475f)$$

$$P \theta_N \left( \tilde{k}^N \right)^{1 - \theta_N} = \tilde{W}^N, \quad (475g)$$

where $\tilde{\Omega} = \tilde{W}^N / \tilde{W}^T$ is the steady-state relative wage and $\tilde{R} = P_j (r^* + \delta)$ is the steady-state value of the capital rental cost. We denote by $\nu_{JN} \equiv \frac{J^T}{Y^N} (J^T \equiv \frac{J^T}{Y^T})$ the ratio of non traded (traded) investment to non traded (traded) output, by $\nu_B \equiv \frac{B^T}{Y^T}$ the ratio of interest receipts to traded output, by $\nu_{Gj} \equiv \frac{G^j}{Y^j}$ the ratio of government spending in good $j = T, N$ to output of sector $j = T, N$.

Before analyzing the long-run effects of a rise in $G^N$, we characterize the steady state graphically. We denote the logarithm of variables with lower-case letters. Because we restrict ourselves to the analysis of the long-run run equilibrium, the tilde is suppressed for the purposes of clarity. The steady state can be described by considering alternatively the goods or the labor market.

G.2 Goods Market Equilibrium

To begin with, we characterize the goods market equilibrium. The steady state can be summarized graphically in Figure 52. The figure traces out two schedules in the $(y^T - y^N, p)$-space which are derived below. To avoid unnecessary complications, we normalize sectoral TFPs, i.e., $Z^T$ and $Z^N$, to 1.
Combining (475a) and the market clearing condition (475c) yields:

\[
\begin{align*}
\frac{C^T}{C^N} & = \frac{\varphi}{1 - \varphi} P^\phi = \frac{Y^T (1 + v_B - v_{JT} - v_{GT})}{Y^N (1 - v_{JN} - v_{GN})}. \\
\end{align*}
\]  

(476)

The ratio of traded output to non traded output is:

\[
\begin{align*}
\frac{Y^T}{Y^N} & = \frac{(1 - v_{JN} - v_{GN})}{(1 + v_B - v_{JT} + v_{GT})} \frac{\varphi}{1 - \varphi} P^\phi.
\end{align*}
\]  

(477)

Taking logarithm yields the GME-equilibrium:

\[
\left( y^T - y^N \right)^{GME} = \phi p + x',
\]  

(478)

where \(x' = \ln \left( \frac{T}{\varphi} \right) + \ln \left( \frac{1 - v_{JN} - v_{GN}}{1 + v_B - v_{JT} + v_{GT}} \right) \). According to (478), the goods market equilibrium is upward-sloping in the \((y^T - y^N, p)\)-space and the slope of the GME-schedule is equal to \(1/\phi\).

In order to facilitate the interpretation of analytical results, it is useful to rewrite the market clearing condition described by eq. (475c). To do so, take logarithm to \(\left( \frac{1 - v_{JN} - v_{GN}}{1 + v_B - v_{JT} - v_{GT}} \right)\) which gives \(\ln (1 + v_B - v_{JT} - v_{GT}) - \ln (1 - v_{JN} - v_{GN})\), use a Taylor approximation at a first order which implies \(\ln (1 + v_B - v_{JT} - v_{GT}) - \ln (1 - v_{JN} - v_{GN}) \simeq v_B - v_{JT} - v_{GT} + v_{JN} + v_{GN}\). Remembering that at the steady state the traded good market clearing condition is \(r^B + Y^T - J^T - C^T - G^T = 0\), denoting net exports by \(NX\) with \(NX = Y^T - J^T - C^T - G^T\) or alternatively \(-NX = r^B\). Dividing the LHS and the RHS by \(Y^T\) leads to the ratio of net exports to traded output, \(v_B = -v_{NX}\). Totally differentiating eq. (478) and remembering that government spending in non tradables is restored to its initial level so that \(dv_{GN} = 0\), leads to:

\[
\left( y^T - y^N \right)^{GME} = \phi \hat{p} + (dv_{NX} - dv_{JN} + dv_{JT}).
\]  

(479)

In the long-run, investment expenditure are higher and thus, \(dv_{Ji} > 0\) since government spending has returned to its initial level while consumption expenditure are lowered due to the negative wealth effect. In the long-run, the ratio of net exports to traded output increases, i.e., \(dv_{NX} > 0\). Furthermore, the improvement in the trade balance must exceed the investment boom in the non traded sector because along the transitional path, the current account deficit caused by reduced savings more than offsets the fall in investment. The deterioration in the net foreign position in the long-run must be offset by a rise in net exports for the intertemporal solvency condition to hold. Hence, a temporary rise in government spending biased toward non tradables, shifts to the right the GME-schedule in the long-run.

To obtain closed-form solutions, we assume that the aggregator function for inputs of the investment good is Cobb-Douglas since data suggest that the elasticity of substitution \(\phi_J\) is equal to one.

Combining (475b) with the steady-state relative wage given by (475f)-(475g), and using the production functions for the traded sector and non traded sectors which imply \(L^T = \frac{Y^T}{(k^T)^{1-\sigma^T}}\) and \(L^N = \frac{Y^N}{(k^N)^{1-\sigma^N}}\), yields:

\[
\begin{align*}
\frac{Y^T}{Y^N} & = P^{-\epsilon} \left( \frac{\theta^T}{\theta^N} \right)^{\epsilon} \left[ \left( \frac{k^T}{k^N} \right)^{1-\sigma^T} \right]^{1+\epsilon}.
\end{align*}
\]

Combining (475d) and (475e) yields:

\[
\begin{align*}
\left( \frac{k^N}{k^T} \right)^{1-\sigma^T} & = \frac{P^{1-\sigma^N}}{P^{1-\sigma^T}} \left[ P_J (r^* + \delta_K) \right]^{1-\sigma^T-1-\sigma^N} \left[ (1 - \theta_N) \right]^{1-\gamma_N} \left[ (1 - \theta_T) \right]^{1-\gamma_T}.
\end{align*}
\]  

(480)
Inserting (480) to eliminate sectoral capital-labor ratios yields the $LME$-schedule:

$$\frac{Y^T}{Y^N} = P^* \left[ \epsilon + \left( \frac{1 - \theta_N}{\theta_N} \right) (1 + \epsilon) \right] P^* \left( \frac{\varphi_T}{\varphi_T + \delta} \right)^{1/(1+\epsilon)} \Pi, \tag{481}$$

where we set

$$\Pi = \frac{\vartheta}{1 - \vartheta} (\sigma_T + \delta) \left[ \frac{(\theta_T)^{\vartheta_T} (1 - \theta_T)^{(1-\vartheta_T)(1+\epsilon)}}{(\theta_N)^{\vartheta_N} (1 - \theta_N)^{(1-\vartheta_N)(1+\epsilon)}} \right]^{1/\vartheta_T} > 0. \tag{482}$$

Taking logarithm, (481) can be rewritten as follows:

$$\left( y^T - y^N \right) \Big|_{LME} = - \left\{ \epsilon + (1 + \epsilon) \left\{ \left( \frac{1 - \theta_N}{\theta_N} \right) - (1 - \varphi_J) \left( \frac{\theta_T - \theta_N}{\theta_T \theta_N} \right) \right\} \right\} p + \ln \Pi, \tag{483}$$

where $\Pi$ is given by (482).

In a model abstracting from physical capital, we have $\theta_T = 1$, so that the $LME$-schedule described by eq. (483) reduces to:

$$\left( y^T - y^N \right) \Big|_{\theta_T=1} = - \epsilon p + \ln \Pi. \tag{484}$$

In a model with physical capital (i.e., $0 < \theta_T < 1$) but abstracting from traded investment (i.e., $\varphi_J = 0$), the $GME$-schedule described by eq. (481) reduces to:

$$\left( y^T - y^N \right) \Big|_{\varphi_J=0} = - \left\{ \epsilon + \left( \frac{1 - \theta_T}{\theta_T} \right) (1 + \epsilon) \right\} p + \ln \Pi. \tag{485}$$

If $\theta_T < 1$, the $LME$-schedule becomes flatter than that in a model abstracting from physical capital in the $(y^T - y^N, p)$-space. The $LME$-schedule is downward-sloping in the $(y^T - y^N, p)$-space with a slope equal to $-1/ \left\{ \epsilon + \left( \frac{1 - \theta_T}{\theta_T} \right) (1 + \epsilon) \right\}$. A rise in the relative price of non tradables $p$ allows the non traded sector to pay higher wages. Because the relative wage $\omega$ rises, workers are encouraged to shift hours worked from the traded to the non traded sector. As a result, the ratio of sectoral outputs $Y^T/Y^N$ declines. Introducing capital rotates to the left the $LME$-schedule due to the shift of capital across sectors triggered by a change in $P$. Following an appreciation in $P$, the non traded sector experiences a capital inflow which amplifies the expansionary effect on non traded output triggered by the reallocation of labor, which results in a flatter $LME$-schedule.

G.3 The Labor Market

The steady-steady can be characterized alternatively by focusing on the labor market in the $(l^T - l^N, \omega)$-space. Taking logarithm to (475b) yields the labor supply-schedule (henceforth $LS$-schedule):

$$\left( l^T - l^N \right) \Big|_{LS} = - \epsilon \ln \omega + d, \tag{486}$$

where $d = \ln \left( \frac{\vartheta}{1 - \vartheta} \right)$. According to (486), as in the model without capital, a rise in the non traded wage-traded wage ratio $\omega$ provides an incentive to shift labor supply from the traded sector towards the non traded sector. Hence the $LS$-schedule is downward-sloping in the $(l^T - l^N, \omega)$-space where the slope is equal to $-1/\epsilon$.

We turn to the derivation of the labor demand-schedule (henceforth $LD$-schedule). Dividing (475g) by (475f) yields:

$$\frac{P \theta_N \left( \tilde{k}^N \right)^{1-\theta_N}}{\theta_T \left( \tilde{k}^T \right)^{1-\theta_T}} = \Omega. \tag{487}$$

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To eliminate the sectoral capital-labor ratios, we use eqs. (475d)-(475e), i.e.

\[
\frac{(k^N)^{1-\theta^N}}{(k^T)^{1-\theta^T}} = P^{\frac{1-\theta^T}{\theta^T}} (r^* + \delta)^{\frac{1-\theta^T}{\theta^T}} \frac{1-\theta^N}{\theta^N} \left[ (1 - \theta^N) \right]^{\frac{1-\theta^N}{\theta^N}} \left[ (1 - \theta^T) \right]^{\frac{1-\theta^T}{\theta^T}}.
\]

To eliminate the relative price of non tradables, we combine the market-clearing condition (475c) and the demand for tradables in terms of non traded goods (475a) together with production functions (420):

\[
P = \left[ 1 - \frac{\varphi}{\varphi_T} \frac{1 + \nu_B - \nu_{JT} + \nu_{GT}}{1 - \nu_{JT} - \nu_{GN}} \frac{L^T (k^T)^{1-\theta^T}}{L^N (k^N)^{1-\theta^N}} \right]^{\frac{1}{\theta}}.
\]

Substituting (489) into (488) yields:

\[
\frac{(k^N)^{1-\theta^N}}{(k^T)^{1-\theta^T}} = (r^* + \delta)^{\theta^N \frac{[1+\theta^T (\phi-1)]}{\theta^N}} \frac{1-\theta^N}{\theta^N} \left[ (1 - \theta^N) \right]^{\frac{1-\theta^N}{\theta^N}} \left[ (1 - \theta^T) \right]^{\frac{1-\theta^T}{\theta^T}} \left[ \frac{(1 - \theta^N)^{\theta^N \frac{[1+\theta^T (\phi-1)]}{\theta^N}}}{(1 - \theta^T)^{\theta^N \frac{[1+\theta^T (\phi-1)]}{\theta^N}}} \right]^{\frac{1}{\theta^T}}.
\]

Plugging (490) into (487) allows us to relate the relative labor demand to the relative wage:

\[
\frac{L^T}{L^N} \Theta \left( \frac{1 + \nu_B - \nu_{JT} + \nu_{GT}}{1 - \nu_{JT} - \nu_{GN}} \right) = \Omega \left[ 1 + \theta^T (\phi-1) \right],
\]

where we set

\[
\Theta = (r^* + \delta)^{\frac{\theta^N \frac{[1+\theta^T (\phi-1)]}{\theta^N}}{\theta^N}} \left[ 1 - \frac{\varphi}{\varphi_T} \right] \left( \frac{\theta^N}{\theta^T} \right)^{\frac{1}{\theta^T} \frac{[1+\theta^T (\phi-1)]}{\theta^N}} \left[ (1 - \theta^N)^{\frac{1-\theta^N}{\theta^N}} \right]^{\frac{1}{\theta^T} \frac{[1+\theta^T (\phi-1)]}{\theta^N}}.
\]

Taking logarithm to (491) yields the LD-schedule:

\[
(t^T - t^N)^{\text{LD}} = \left[ 1 + \theta^T (\phi-1) \right] \omega + \ln \left( \frac{1 - \nu_{JT} - \nu_{GN}}{1 + \nu_B - \nu_{JT} + \nu_{GT}} \right) - \ln \Theta.
\]
Eq. (493) states that the LD-schedule is upward-sloping in the \((l^T - l^N, \omega)\)-space since an increase in \(\omega\) induces non traded producers to set higher prices, increasing the demand for traded goods and therefore labor demand in that sector relative to the non traded sector. When \(\theta^T < 1\), the LD-schedule is steeper or flatter than that in a model abstracting from physical capital (i.e., when \(\theta^T = 1\)) depending on whether \(\phi\) is larger or smaller than one. In both cases, following an increased non tradable labor cost, the non traded sector is induced to use more capital which raises non traded output and thereby produces a decline in \(p\). Depending on whether \(\phi\) is larger or smaller than one, the share of non tradables in total expenditure increases or decreases, as a result of the shift of capital towards the non traded sector. Hence, a given rise in \(\omega\) produces a smaller or a larger expansionary effect on labor demand in the traded sector depending on whether \(\phi\) exceeds or falls below unity.

Adopting the same methodology described above, the LD-schedule given by eq. (493) can be rewritten in percentage deviation from the initial steady-state in order to facilitate the discussion of the effects of a fiscal shock:

\[
\left(\hat{l}^T - \hat{l}^N\right)^{LD} = \left[1 + \theta^T (\phi - 1)\right] \hat{\omega} + (\hat{dv}_{NX} + \hat{dv}_{JT} - \hat{dv}_{JN}).
\] (494)

As mentioned previously, a fiscal shock deteriorates the current account in the short-run. The short-run current account deficit must be matched in the long-run by a rise in net exports which shifts the LD-schedule to the right, as captured by an increase in \(v_{NX}\) by such an amount that \(\hat{dv}_{NX} + \hat{dv}_{JT} - \hat{dv}_{JN} > 0\). At the final steady-state, the relative wage \(\Omega\) is lower while the ratio \(L^T/L^N\) is higher.

**H Solving for Temporary Fiscal Shocks**

In this section, we provide the main steps for the derivation of formal solutions following a temporary fiscal shock.

**H.1 The Government Spending Shock**

Because the endogenous response of government spending to an exogenous fiscal shock is hump-shaped, we assume that government consumption as a percentage of GDP evolves according to the following dynamic equation:

\[
\frac{dG(t)}{Y} \equiv \frac{G(t) - \bar{G}}{\tilde{Y}} = \left[e^{-\xi t} - (1 - g) e^{-\chi t}\right],
\] (495)

where \(\xi > 0\) and \(\chi > 0\) are (positive) parameters which are set in order to capture the endogenous response of \(G(t)\). Setting \(t = 0\) into (495) yields:

\[
\frac{dG(0)}{Y} \equiv \frac{G(0) - \bar{G}}{\tilde{Y}} = g.
\] (496)

In the quantitative analysis, we set \(g = 0.01\) so that government consumption increases initially by 1 percentage point of initial GDP, \(\tilde{Y}\).

In the quantitative analysis we assume that the rise in government consumption is split between non traded and traded goods in accordance with their respective shares, \(\omega_{GN} = \frac{G_N}{G} \) and \(\omega_{GT} = \frac{G_T}{G}\), respectively. Formally, we thus have:

\[
\frac{dG(t)}{Y} = \omega_{GN} \frac{dG}{Y} + \omega_{GT} \frac{dG}{Y}.
\]

Totally differentiating the balanced budget condition, government expenditure in good \(j = T, N\) can be solved for overall government consumption as follows:

\[
G^N(t) = G^N(G(t)), \quad G^T = G^T(G(t)),
\] (497)

where \(\frac{dG^N}{dG} = \frac{\omega_{GN}}{P}\) and \(\frac{dG^T}{dG} = \omega_{GT}\) with \(\omega_{Gj}\) corresponding to the share of expenditure on good \(j\) in total government spending.
H.2 Formal Solutions for $K(t)$ and $Q(t)$

The adjustment of the open economy towards the steady-state is described by a dynamic system which comprises two equations. Inserting first the short-term static solution for the relative price of non tradables (449) together with (497) into (452), the accumulation equation for physical capital that clears the non-traded goods market along the transitional path can be rewritten as follows:

$$
\dot{K} = Y(K, Q, G) = \frac{Y^N(\lambda, K, P(.)) - C^N(\lambda, P(.)) - G^N(G)}{P^f_j(P(.))} - \delta_K K
$$

$$
- \frac{K(t)}{2\kappa} \left[ \frac{Q}{P^f_j(P(.))} - 1 \right]^2,
$$

(498)

where $P = P(\lambda, K, Q, G)$. Inserting first the optimal choice for the investment rate (443) and the short-term static solution for the relative price of non tradables (449) together with (497) into (405e), the dynamic equation for the shadow price of investment that equalizes the return on domestic capital and traded bonds $r^*$ can be rewritten as follows:

$$
\dot{Q} = \Sigma(K, Q, G) = (r^* + \delta_K) Q - \left[ R(K, P(.)) + P_j(.) \frac{K}{2} v(.) (v(.) + 2\delta_K) \right],
$$

(499)

where $P = P(\lambda, K, Q, G)$ and $v(.) = \frac{1}{\kappa} \left( \frac{Q}{P^f_j(P(.))} - 1 \right)$ (see eq. (444)).

The linearized system can be written in a matrix form:

$$
\begin{pmatrix}
\dot{K}(t) \\
\dot{Q}(t)
\end{pmatrix}
= \begin{pmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{pmatrix}
\begin{pmatrix}
K(t) - \bar{K} \\
Q(t) - \bar{Q}
\end{pmatrix}
+ \begin{pmatrix}
\varepsilon_K \left( G(t) - \bar{G} \right) \\
\varepsilon_Q \left( G(t) - \bar{G} \right)
\end{pmatrix},
$$

(500)

where the coefficients of the Jacobian matrix are given by (457) which we repeat for convenience:

$$
a_{11} = \Upsilon_K = \left( \frac{Y^N_K}{P^f_j} - \delta_K \right) + \left[ \left( \frac{Y^N_P}{P^f_P} - C^N_P \right) + \frac{\bar{I}^N_P (1 - \alpha_j)}{P} \right] \frac{P_K}{P^f_j} \lesssim 0,
$$

(501a)

$$
a_{12} = \Upsilon_Q = \left[ \left( \frac{Y^N_P}{P^f_P} - C^N_P \right) + \frac{\bar{I}^N_P (1 - \alpha_j)}{P} \right] \frac{P_Q}{P^f_j} > 0,
$$

(501b)

$$
a_{21} = \Sigma_K = -R_K - \left( R_P + P_j k v_p \delta_K \right) P_K > 0,
$$

(501c)

$$
a_{22} = \Sigma_Q = r^* - \left( R_P + P_j k v_p \delta_K \right) P_Q > 0,
$$

(501d)

and the direct effects of an exogenous change in government consumption on $K$ and $Q$ are described by:

$$
\varepsilon_K = \left\{ \left( \frac{Y^N_P}{P^f_P} - C^N_P \right) + \frac{\bar{I}^N_P (1 - \alpha_j)}{P} \right\} \frac{P_G}{P^f_j} - \frac{1}{P} \frac{\omega_{GN}}{P},
$$

(502a)

$$
\varepsilon_Q = -R_P \left( P^f_j + P_J k v_p \delta_K \right) \frac{P_G}{P^f_j} \frac{\omega_{GN}}{P},
$$

(502b)

where we used the fact that $\frac{\partial G_N}{\partial G} = \frac{\omega_{GN}}{P}$. Eq. (500) corresponds to eq. (22) in the main text.

We denote by $V = \left( V^1, V^2 \right)$ the matrix of eigenvectors (given by (459)) with $V^{i,j} = (1, \omega_2^j)$ and we denote by $V^{-1}$ the inverse matrix of $V$. Let us define:

$$
\begin{pmatrix}
X_1(t) \\
X_2(t)
\end{pmatrix}
= V^{-1} \begin{pmatrix}
K(t) - \bar{K} \\
Q(t) - \bar{Q}
\end{pmatrix},
$$

(503)

Differentiating w.r.t. time, one obtains:

$$
\begin{pmatrix}
\dot{X}_1(t) \\
\dot{X}_2(t)
\end{pmatrix}
= \begin{pmatrix}
\nu_1 & 0 \\
0 & \nu_2
\end{pmatrix}
\begin{pmatrix}
X_1(t) \\
X_2(t)
\end{pmatrix}
+ V^{-1} \begin{pmatrix}
\varepsilon_K dG(t) \\
\varepsilon_Q dG(t)
\end{pmatrix},
$$

(504)

$$
= \begin{pmatrix}
\nu_1 X_1(t) \\
\nu_2 X_2(t)
\end{pmatrix}
+ \frac{1}{\nu_1 - \nu_2} \begin{pmatrix}
\Phi_1 dG(t) \\
-\Phi_2 dG(t)
\end{pmatrix},
$$

(504)
where \( d \overline{G}(t) = G(t) - \dot{G} \) and we set

\[
\Phi_1 = \left[(a_{11} - \nu_2) \varepsilon K + a_{12} \varepsilon Q\right], \\
\Phi_2 = \left[(a_{11} - \nu_1) \varepsilon K + a_{12} \varepsilon Q\right].
\]

As will be useful later, in order to express solutions in a compact form, we set:

\[
\Gamma_1 = -\frac{\Phi_1 \dot{Y}}{\nu_1 - \nu_2 \nu_1 + \xi}, \\
\Gamma_2 = -\frac{\Phi_2 \dot{Y}}{\nu_1 - \nu_2 \nu_2 + \xi}, \\
\Theta_1 = (1 - g) \frac{\nu_1 + \xi}{\nu_1 + \chi}, \\
\Theta_2 = (1 - g) \frac{\nu_2 + \xi}{\nu_2 + \chi}.
\]

Solving for \( X_1(t) \) gives:

\[
X_1(t) = e^{\nu_1 t} \left\{ X_1(0) + \frac{\Phi_1 \dot{Y}}{\nu_1 - \nu_2} \int_0^t d \overline{G} e^{-\nu_1 \tau} d\tau \right\},
\]

\[
= e^{\nu_1 t} \left\{ X_1(0) + \frac{\Phi_1 \dot{Y}}{\nu_1 - \nu_2} \int_0^t \left[ e^{-\xi (\nu_2 + \xi) \tau} - (1 - g) e^{-\xi (\nu_2 + \chi) \tau}\right] d\tau \right\},
\]

\[
= e^{\nu_1 t} X_1(0) + \frac{\Phi_1 \dot{Y}}{\nu_1 - \nu_2} \left[ \frac{e^{\nu_1 t} - e^{-\xi t}}{\nu_1 + \xi} - (1 - g) \left( \frac{e^{\nu_1 t} - e^{-\xi t}}{\nu_1 + \chi}\right) \right],
\]

\[
= e^{\nu_1 t} \left[ X_1(0) - \Gamma_1 (1 - \Theta_1) + \Gamma_1 \left( e^{-\xi t} - \Theta_1 e^{-\chi t}\right) \right],
\]

where \( \Gamma_1 \) and \( \Theta_1 \) are given by (506a) and (506c), respectively.

Solving for \( X_2(t) \) gives:

\[
X_2(t) = e^{\nu_2 t} \left\{ X_2(0) - \frac{\Phi_2 \dot{Y}}{\nu_1 - \nu_2} \int_0^t d \overline{G} e^{-\nu_2 \tau} d\tau \right\}.
\]

Because \( \nu_2 > 0 \), for the solution to converge to the steady-state, the term in brackets must be nil when we let \( t \) tend toward infinity:

\[
X_2(0) = \frac{\Phi_2 \dot{Y}}{\nu_1 - \nu_2} \int_0^\infty \left[ e^{-\xi (\nu_2 + \xi) \tau} - (1 - g) e^{-\xi (\nu_2 + \chi) \tau}\right] d\tau,
\]

\[
= \frac{\Phi_2 \dot{Y}}{\nu_1 - \nu_2} \left[ \frac{1}{\xi + \nu_2} - (1 - g) \frac{1}{\chi + \nu_2}\right],
\]

\[
= -\Gamma_2 (1 - \Theta_2),
\]

where \( \Gamma_2 \) and \( \Theta_2 \) are given by (506b) and (506d), respectively.

Inserting first \( X_2(0) \), the 'stable' solution for \( X_2(t) \), i.e., consistent with convergence toward the steady-state when \( t \) tends toward infinity, is thus given by:

\[
X_2(t) = e^{\nu_2 t} \frac{\Phi_2 \dot{Y}}{\nu_1 - \nu_2} \int_0^\infty \left[ e^{-\xi (\nu_2 + \xi) \tau} - (1 - g) e^{-\xi (\nu_2 + \chi) \tau}\right] d\tau,
\]

\[
= e^{\nu_2 t} \frac{\Phi_2 \dot{Y}}{\nu_1 - \nu_2} \left[ \frac{e^{-\xi (\nu_2 + \xi) t}}{\xi + \nu_2} - (1 - g) \frac{e^{-\xi (\nu_2 + \chi) t}}{\chi + \nu_2}\right],
\]

\[
= -\Gamma_2 \left( e^{-\xi t} - \Theta_2 e^{-\chi t}\right).
\]

\[\text{Eq. (510) corresponds to eq. (25b) in the main text.}\]

Using the definition of \( X_i(t) \) (with \( i = 1, 2 \)) given by (503), we can recover the solutions for \( K(t) \) and \( Q(t) \):

\[
K(t) - \overline{K} = X_1(t) + X_2(t),
\]

\[
Q(t) - \overline{Q} = \omega_1^2 X_1(t) + \omega_2^2 X_2(t).
\]
Eqs. (511) correspond to eqs. (24) in the main text.

Setting $t = 0$ into (511a) gives $X_1(0) = \left(K(0) - \bar{K}\right) - X_2(0)$; inserting (509) leads to:

$$X_1(t) = e^{\nu t} \left[ \left(K(0) - \bar{K}\right) + \Gamma_2 (1 - \Theta_2) - \Gamma_1 (1 - \Theta_1) \right] + \Gamma_1 \left(e^{-\xi t} - \Theta_1 e^{-\chi t} \right). \quad (512)$$

Eq. (512) corresponds to eq. (25a) in the main text.

**H.3 Formal Solution for the Net Foreign Asset Position, $B(t)$**

To determine the formal solution for the net foreign asset position, we first linearize the current account equation (460) in the neighborhood of the steady-state and substitute the solutions for $K(t)$ and $Q(t)$:

$$\dot{B}(t) = r^* \left(B(t) - \bar{B}\right) + N_1 X_1(t) + N_2 X_2(t) + \Xi_G dG(t), \quad (513)$$

where $N_i$ (with $i = 1, 2$) is given by (463b), and $\Xi_G$ is given by:

$$\Xi_G = \left\{ \left[ \Xi_P P_{GN} + \left( 1 - \alpha_J \right) \frac{\omega_{GN}}{\alpha_J} \right] \frac{\omega_{GT}}{\alpha_J} - \omega_{GT} \right\}, \quad (514)$$

where $\Xi_P < 0$ and $P_{GN} > 0$ are given by (461b) and (450c), respectively.

Substituting $X_1(t)$ given by eq. (512) and $X_2(t)$ given by eq. (510) into (513) leads to:

$$\dot{B}(t) = r^* \left(B(t) - \bar{B}\right) + \omega_B e^{\nu t} + N_1 \Gamma_1 \left(e^{-\xi t} - \Theta_1 e^{-\chi t} \right) - N_2 \Gamma_2 \left(e^{-\xi t} - \Theta_2 e^{-\chi t} \right) + \Xi_G \dot{Y} \left(e^{-\xi t} - (1 - g) e^{-\chi t} \right), \quad (515)$$

where $\Gamma_1$ and $\Gamma_2$ are given by (506a) and (506b), respectively, and we set:

$$\omega_B^1 = N_1 \left[ \left(K(0) - \bar{K}\right) + \Gamma_2 (1 - \Theta_2) - \Gamma_1 (1 - \Theta_1) \right]. \quad (516)$$

Pre-multiplying by $e^{-r^* t}$ and integrating over $(0, t)$ allow us to obtain the general solution for $B(t)$:

$$B(t) - \bar{B} = \left\{ \left( B_0 - \bar{B} \right) - \frac{\omega_B^1}{\nu_1 - r^*} + \frac{\Xi_G \dot{Y}}{\xi + r^*} \left[ (1 - \Theta) + \frac{N_1 \Gamma_1}{\xi + r^*} (1 - \Theta_1) - \frac{N_2 \Gamma_2}{\xi + r^*} (1 - \Theta_2) \right] \right\} e^{r^* t} + \frac{\omega_B^1}{\nu_1 - r^*} e^{\nu t} - \frac{\Xi_G \dot{Y}}{\xi + r^*} \left( e^{-\xi t} - \Theta_1 e^{-\chi t} \right) - \frac{N_1 \Gamma_1}{\xi + r^*} \left( e^{-\xi t} - \Theta_1 e^{-\chi t} \right) + \frac{N_2 \Gamma_2}{\xi + r^*} \left( e^{-\xi t} - \Theta_2 e^{-\chi t} \right), \quad (517)$$

where we set:

$$\Theta' = (1 - g) \frac{\xi + r^*}{\chi + r^*}, \quad (518a)$$
$$\Theta'_1 = \frac{\xi + r^*}{\chi + r^*}, \quad (518b)$$
$$\Theta'_2 = \frac{\xi + r^*}{\chi + r^*}. \quad (518c)$$

Invoking the transversality condition, one obtains the 'stable' solution for the stock of foreign assets so that $B(t)$ converges toward its steady-state value $\bar{B}$:

$$B(t) - \bar{B} = \frac{\omega_B^1}{\nu_1 - r^*} e^{\nu t} - \frac{\Xi_G \dot{Y}}{\xi + r^*} \left( e^{-\xi t} - \Theta' e^{-\chi t} \right) - \frac{N_1 \Gamma_1}{\xi + r^*} \left( e^{-\xi t} - \Theta'_1 e^{-\chi t} \right) + \frac{N_2 \Gamma_2}{\xi + r^*} \left( e^{-\xi t} - \Theta'_2 e^{-\chi t} \right). \quad (519)$$

Eq. (518) corresponds to eq. (27) in the main text.
Eq. (519) gives the trajectory for \( B(t) \) consistent with the intertemporal solvency condition:

\[
\left( \dot{B} - B_0 \right) = -\frac{\omega^1_B}{\nu_1 - r^*} + \frac{\omega^2_B}{\xi + r^*} \tag{520}
\]

where we set

\[
\omega^2_B = \Xi_G \ddot{Y} (1 - \Theta') + N_1 \Gamma_1 (1 - \Theta'_1) - N_2 \Gamma_2 (1 - \Theta'_2). \tag{521}
\]

**Eq. (520) corresponds to eq. (28) in the main text.**

Differentiating (511a) w.r.t. time gives the trajectory for the current account along the transitional path when government spending follows the temporal path given by eq. (495):

\[
\dot{B}(t) = \nu_1 \left( \frac{\omega^1_B}{\nu_1 - r^*} + \frac{\Xi_G \ddot{Y}}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta' e^{-\chi t} \right) \right) + \frac{N_1 \Gamma_1}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta' e^{-\chi t} \right)
- \frac{N_2 \Gamma_2}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta' e^{-\chi t} \right). \tag{522}
\]

**H.4 Formal Solution for the Stock of Financial Wealth, \( A(t) \)**

To determine the formal solution for the stock of financial wealth, we first linearize the private savings equation (465) in the neighborhood of the steady-state and substitute the solutions for \( K(t) \) and \( Q(t) \):

\[
\dot{A}(t) = r^* \left( A(t) - \bar{A} \right) + M_1 X_1(t) + M_2 X_2(t) + A_G dG(t), \tag{523}
\]

where \( M_i \) (with \( i = 1, 2 \)) is given by (472c), and \( A_G \) is given by:

\[
A_G = \Lambda_P \frac{P_G N \omega_G N}{P} - 1, \tag{524}
\]

where \( \Lambda_P \) is given by eq. (470b).

Substituting \( X_1(t) \) given by eq. (512) and \( X_2 \) given by eq. (510) into (523) leads to:

\[
\dot{A}(t) = r^* \left( A(t) - \bar{A} \right) + M_1 A e^{\nu_1 t} + M_1 \Gamma_1 \left( e^{-\xi t} - \Theta_1 e^{-\chi t} \right)
- M_2 \Gamma_2 \left( e^{-\xi t} - \Theta_2 e^{-\chi t} \right) + A_G \ddot{Y} \left( e^{-\xi t} - (1 - g) e^{-\chi t} \right), \tag{525}
\]

where \( \Gamma_1 \) and \( \Gamma_2 \) are given by (501c) and (501d), respectively, and we set:

\[
\omega^1_A = M_1 \left[ \left( K(0) - \bar{K} \right) + \Gamma_2 (1 - \Theta_2) - \Gamma_1 (1 - \Theta_1) \right]. \tag{526}
\]

Pre-multiplying by \( e^{-r^* t} \) and integrating over \((0, t)\) allow us to obtain the general solution for \( A(t) \):

\[
A(t) - \bar{A} = \left\{ \left( A_0 - \bar{A} \right) - \frac{\omega^1_A}{\nu_1 - r^*} + \frac{A_G \ddot{Y}}{\xi + r^*} \left( 1 - \Theta' \right) + \frac{M_1 \Gamma_1}{\xi + r^*} (1 - \Theta'_1) - \frac{M_2 \Gamma_2}{\xi + r^*} (1 - \Theta'_2) \right\} e^{r^* t}
+ \frac{\omega^1_A}{\nu_1 - r^*} e^{\nu_1 t} - \frac{A_G \ddot{Y}}{\xi + r^*} \left( e^{-\xi t} - \Theta' e^{-\chi t} \right) - \frac{M_1 \Gamma_1}{\xi + r^*} \left( e^{-\xi t} - \Theta'_1 e^{-\chi t} \right)
+ \frac{M_2 \Gamma_2}{\xi + r^*} \left( e^{-\xi t} - \Theta'_2 e^{-\chi t} \right), \tag{527}
\]

where \( \Theta', \Theta'_1, \Theta'_2 \) are given by (518)-(518).

Invoking the transversality condition, one obtains the 'stable' solution for the stock of financial wealth so that \( A(t) \) converges toward its steady-state value \( \bar{A} \):

\[
A(t) - \bar{A} = \frac{\omega^1_A}{\nu_1 - r^*} e^{\nu_1 t} - \frac{A_G \ddot{Y}}{\xi + r^*} \left( e^{-\xi t} - \Theta' e^{-\chi t} \right) - \frac{M_1 \Gamma_1}{\xi + r^*} \left( e^{-\xi t} - \Theta'_1 e^{-\chi t} \right)
+ \frac{M_2 \Gamma_2}{\xi + r^*} \left( e^{-\xi t} - \Theta'_2 e^{-\chi t} \right). \tag{528}
\]
Eq. (528) gives the trajectory for $A(t)$ consistent with the intertemporal solvency condition:

$$\left( \dot{A} - A_0 \right) = -\frac{\omega_A^1}{\nu_1 - r^*} + \frac{\omega_A^2}{\xi + r^*}$$

(529)

where we set

$$\omega_A^3 = A_G \bar{Y} (1 - \Theta') + M_1 \Gamma_1 (1 - \Theta'_1) - M_2 \Gamma_2 (1 - \Theta'_2).$$

(530)

Differentiating (528) w.r.t. time gives the trajectory for the current account along the transitional path when government spending follows the temporal path given by eq. (495):

$$\dot{A} (t) = \nu_1 \frac{\omega_A^1}{\nu_1 - r^*} e^{r t} + A_G \bar{Y} \left( \xi e^{-\xi t} - \chi \Theta' e^{-\chi t} \right) + \frac{M_1 \Gamma_1}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta'e^{-\chi t} \right) - \frac{M_2 \Gamma_2}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta'_2 e^{-\chi t} \right).$$

(531)

I Introducing Non-Separability between Consumption and Labor

In this section, we consider a more general form for preferences taken from Shimer [2011]. Since such preferences do not affect the first-order conditions from profit maximization, we do not repeat them and indicate major changes when solving the model.

In the baseline model, we assume that preferences are separable in consumption and leisure. We relax this assumption which implies that consumption and leisure can be substitutes. In particular, this more general specification implies that consumption can be affected by the wage rate while labor supply can be influenced by the change in the relative price of non tradables. As previously, the household’s period utility function is increasing in its consumption $C$ and decreasing in its labor supply $L$, with functional form:

$$\frac{C^{1-\sigma} V(L)^{\sigma} - 1}{1 - \sigma}, \quad \text{if } \sigma \neq 1, \quad V(L) \equiv \left( 1 + (\sigma - 1) \gamma \frac{\sigma_L}{1 + \sigma_L} L^\frac{1 + \sigma_L}{\sigma_L} \right),$$

(532)

and

$$\log C - \gamma \frac{\sigma_L}{1 + \sigma_L} L^\frac{1 + \sigma_L}{\sigma_L}, \quad \text{if } \sigma = 1.$$  

(533)

These preferences are characterized by two pivotal parameters: $\sigma_L$ which is the Frisch elasticity of labor supply, and $\sigma > 0$ that determines the substitutability between consumption and leisure; it is worth noticing that if $\sigma > 1$, the marginal utility of consumption is increasing in hours worked.

The representative household maximizes lifetime utility subject to the flow budget constraint (396) and the accumulation of physical capital (397). Denoting the co-state variables associated with (396) and (397) by $\lambda$ and $Q'$, respectively, the first-order conditions characterizing the representative household’s optimal plans are:

$$C^{-\sigma} V(L)^{\sigma} = P_C \lambda,$$

(534a)

$$C^{1-\sigma} \gamma L^{1/\sigma_L} V(L)^{-1} = W \lambda,$$

(534b)

along with (405c)-(405e) and transversality conditions.

First-order conditions (534a) and (534b) can be solved for consumption and labor as follows:

$$C = C \left( \hat{\lambda}, P, W \right), \quad L = L \left( \hat{\lambda}, P, W \right).$$

(535)

To derive the partial derivatives, we take logarithm and totally differentiate the system which yields in matrix form:

$$\begin{pmatrix} -\sigma & \sigma \left( \frac{1 + \sigma_L}{\sigma_L} \right) \left[ \frac{V(L)-1}{V(L)} \right] \\ (1-\sigma) \left( \frac{1}{\sigma_L} + (\sigma - 1) \left( \frac{1 + \sigma_L}{\sigma_L} \right) \left[ \frac{V(L)-1}{V(L)} \right] \right) & \dot{\lambda} + \alpha_C \hat{P} \end{pmatrix},$$

(536)
where we denote by a hat the deviation in percentage.

Partial derivatives are:

\[
\begin{align*}
\frac{\dot{C}}{\dot{x}} &= \frac{(1 + \sigma_L) \left[ V(L) - 1 \right]}{\sigma} \left[ V(L) - 1 \right] - \frac{1}{\sigma} < 0, \\
\frac{\dot{L}}{\dot{x}} &= \frac{\sigma_L}{\sigma} > 0, \\
\frac{\dot{C}}{\dot{W}} &= (1 + \sigma_L) \left[ \frac{V(L) - 1}{V(L)} \right] > 0, \\
\frac{\dot{L}}{\dot{W}} &= \sigma_L > 0, \\
\frac{\dot{C}}{\dot{P}} &= -\frac{\alpha_C}{\sigma} \left( 1 + (\sigma - 1) (1 + \sigma_L) \left[ \frac{V(L) - 1}{V(L)} \right] \right) < 0, \\
\frac{\dot{L}}{\dot{P}} &= -\frac{\alpha_C}{\sigma} (\sigma - 1) \sigma_L < 0.
\end{align*}
\]

Using the fact that \( W = W(T, W^N) \) with \( \frac{\partial W}{\partial W^T} W^T = (1 - \alpha_L) \) and \( \frac{\partial W}{\partial W^N} W^N = \alpha_L \), we get:

\[
L = L(\lambda, P, W^T, W^N),
\]

where

\[
\begin{align*}
\frac{\dot{L}}{W^T} &= (1 - \alpha_L) \sigma_L > 0, \\
\frac{\dot{L}}{W^N} &= \sigma_L \alpha_L > 0.
\end{align*}
\]

Inserting first the short-run static solution for consumption given by (535) into \( C^N = P_C C \) and \( C^T = [P_C - P_P] C \), one can solve for \( C^T \) and \( C^N \) as follows:

\[
C^T = C^T(\lambda, P, W^T, W^N), \quad C^N = C^N(\lambda, P, W^T, W^N),
\]

where partial derivatives are given by:

\[
\begin{align*}
C^T_P &= C^T_P \left( \alpha_C \phi + \frac{C_P P}{C} \right) \leq 0, \\
C^N_P &= -\frac{C^N}{P} \left[ (1 - \alpha_C) \phi - \frac{C_P P}{C} \right] < 0, \\
C^T_W^T &= C^T_W^T (1 - \alpha_L) \frac{C_W W}{C} > 0, \\
C^N_W^T &= C^N_W^T (1 - \alpha_L) \frac{C_W W}{C} > 0, \\
C^T_W^N &= C^T_W^N \alpha_L \frac{C_W W}{C} > 0, \\
C^N_W^N &= C^N_W^N \alpha_L \frac{C_W W}{C} > 0.
\end{align*}
\]

Inserting first the short-run solution for labor (538), into \( L^T = \frac{\partial W(W^T, W^N)}{\partial W^T} L \) and \( L^N = \frac{\partial W(W^T, W^N)}{\partial W^N} L \), allows us to solve for \( L^T \) and \( L^N \):

\[
L^T = L^T(\lambda, W^T, W^N, P), \quad L^N = L^N(\lambda, W^T, W^N, P),
\]

where partial derivatives w.r.t. \( W^T \) and \( W^N \) are given by (169) and partial derivatives w.r.t. \( P \) are:

\[
\begin{align*}
\frac{\dot{L}^T}{P} &= L^T P \left( \alpha_C (1 - \sigma) \frac{\sigma_L}{\sigma} \right) > 0, \\
\frac{\dot{L}^N}{P} &= L^N P \left( \alpha_C (1 - \sigma) \frac{\sigma_L}{\sigma} \right) > 0.
\end{align*}
\]
I.1 Solving the Model

Plugging the short-run static solutions for $L^T$ and $L^N$ given by (542) into the resource constraint for capital (422), the system of four equations which comprises (421a)-(421c) together with (422) can be solved for sectoral wages and sectoral capital-labor ratios. Taking logarithm and differentiating (421a)-(421c) and (422) yields in matrix form:

$$
\begin{pmatrix}
-\theta^T & \theta^N & 0 & 0 \\
(1-\theta^T) & 0 & -1 & 0 \\
0 & (1-\theta^N) & 0 & -1 \\
(1-\xi) & \Psi_{WT} & \Psi_{WN} & 0
\end{pmatrix}
\begin{pmatrix}
\hat{k}^T \\
\hat{k}^N \\
\hat{W}^T \\
\hat{W}^N
\end{pmatrix}
= \begin{pmatrix}
\hat{P} \\
0 \\
\hat{K} - \Psi_{WT} \bar{\lambda} - \Psi_{WN} \hat{P}
\end{pmatrix}
$$

(544)

where $\Psi_{WT}$ and $\Psi_{WN}$ are given by (430a) and (430b), respectively, $\xi \equiv \frac{k_N}{k_L}$ and we set:

$$
\Psi_P = (1-\xi) \frac{L_P P}{L^T P} + \xi = \frac{L_P P}{L_T} - \alpha C (\sigma - 1) < 0.
$$

(545)

Only the partial derivatives w.r.t. $P$ are modified when preferences are non separable in consumption and leisure. Hence, we thus restrict attention to these partial derivatives. Short-run static solutions for sectoral wages are:

$$
W^T = W^T (\bar{\lambda}, K, P, Z_T, Z^N), \quad W^N = W^N (\bar{\lambda}, K, P, Z_T, Z^N),
$$

(546)

with

$$
\frac{\hat{W}^T}{\hat{P}} = -\frac{(1-\theta^T) (\Psi_{WN} + \theta^N \Psi_P + \xi)}{G} < 0,
$$

(547a)

$$
\frac{\hat{W}^N}{\hat{P}} = -\frac{\left\{1 + (1-\theta^T) \Psi_{WT} - (1-\theta^T) \xi - \theta^T (1-\theta^N) \Psi_P \right\}}{G} > 0,
$$

(547b)

and sectoral capital-labor ratios:

$$
k^T = k^T (\lambda, K, P, Z_T, Z^N), \quad k^N = k^N (\bar{\lambda}, K, P, Z_T, Z^N),
$$

(548)

with

$$
\frac{\hat{k}^T}{\hat{P}} = \frac{\Psi_{WN} + \xi + \theta^N \Psi_P}{G} < 0,
$$

(549a)

$$
\frac{\hat{k}^N}{\hat{P}} = \frac{\theta^T (\Psi_{WN} + \Psi_P) - \left[(1-\theta^T) \Psi_{WT} + (1-\xi)\right]}{G} > 0,
$$

(549b)

To solve the model, insert first short-run static solutions for sectoral wages (546) into sectoral labor (542), then substitute the resulting solutions for sectoral labor and capital-labor ratios (549), production functions can be solved for sectoral outputs.

J Solving the Model with Public Debt

This section extends the two-sector model with imperfect mobility of labor to public debt. In order to avoid confusion, we denote by:

- $K$ is the stock of physical capital;
- $QK$ is the shadow value of the stock of physical capital;
- $D$ is the stock of (traded) bonds issued by the government;
- $B$ is the stock of traded bonds;
- $N = B - D$ is the net foreign asset position;
- $A = QK + N$ is the national non human wealth equal to the shadow value of the stock of physical capital plus the net foreign asset position which gives national savings $\dot{A};$
- $\dot{A} = A + D = QK + N + D$ is non human wealth held by households which gives private savings $\dot{A}.$
J.1 Government

The government issues traded bonds, \( D \), in order to finance the excess of interest payments, \( r^*D \), government spending, and transfers, \( Z(t) \), over taxes, \( T(t) \):

\[
\dot{D}(t) = r^*D(t) + G(t) + Z(t) - T(t)
\]

where we assume that the government raises taxes on labor:

\[
T(t) = \tau(t)W(t)L(t).
\]

with \( \tau \) the wage tax levied on households’ wage income.

J.2 Households

At each instant of time, the representative household consumes traded and non traded goods denoted by \( C^T \) and \( C^N \), respectively, which are aggregated by means of a CES function:

\[
C = \left[ \varphi \left( \frac{C^T}{\phi} \right)^{\varphi - 1} + (1 - \varphi) \left( \frac{C^N}{\phi} \right)^{\varphi - 1} \right]^{\frac{\phi}{\varphi - 1}},
\]

where \( 0 < \varphi < 1 \) is the weight of the traded good in the overall consumption bundle and \( \phi \) corresponds to the elasticity of substitution between traded goods and non traded goods.

As in De Cordoba and Kehoe [2000], the investment good is produced using inputs of the traded good and the non traded good according to a constant-returns-to-scale function which is assumed to take a CES form:

\[
J \equiv J (J^T, J^N) = \left[ \varphi_J \left( J^T \right)^{\varphi_J - 1} + (1 - \varphi_J) \left( J^N \right)^{\varphi_J - 1} \right]^{\frac{1}{\varphi_J - 1}},
\]

where \( \varphi_J \) is the weight of the investment traded input (\( 0 < \varphi_J < 1 \)) and \( \phi_J \) corresponds to the intratemporal elasticity of substitution in investment between traded and non traded inputs.

Following Horvath [2000], we assume that hours worked in the traded and the non traded sectors are aggregated by means of a CES function:

\[
L = \left[ \theta^{-1/\epsilon} \left( L^T \right)^{\epsilon + 1} + (1 - \theta)^{-1/\epsilon} \left( L^N \right)^{\epsilon + 1} \right]^{\frac{1}{\epsilon}},
\]

and \( 0 < \theta < 1 \) is the weight of labor supply to the traded sector in the labor index \( L(\cdot) \) and \( \epsilon \) measures the ease with which hours worked can be substituted for each other and thereby captures the degree of labor mobility across sectors.

The representative household chooses consumption, decides on labor supply, and investment that maximizes his/her lifetime utility:

\[
U = \int_0^\infty \left\{ \frac{1}{1 - \sigma_C} C(t)^{1 - \frac{1}{\sigma_C}} - \frac{1}{1 + \frac{1}{\sigma_L}} L(t)^{1 + \frac{1}{\sigma_L}} \right\} e^{-\beta t} dt,
\]

subject to the flow budget constraint:

\[
\dot{B}(t) = r^*B(t) + R(t)K(t) + W(t) (1 - \tau) L(t) + Z(t) - P_C (P(t)) C(t) - P_J (P(t)) J(t),
\]

and capital accumulation which evolves as follows:

\[
\dot{K}(t) = I(t) - \delta_K K(t),
\]

where \( I \) is investment and \( 0 \leq \delta_K < 1 \) is a fixed depreciation rate. The first term on the RHS of (556) \( r^*B(t) + R(t)K(t) + W(t) (1 - \tau) L(t) + Z(t) \) is the representative household’s real disposable income while the second term on the RHS \( P_C (P(t)) C(t) + P_J (P(t)) J(t) \) corresponds to consumption and investment expenditure including capital installation costs.
More specifically, we assume that capital accumulation is subject to increasing and convex cost of net investment (see (399)-(400)):

\[
J(t) = I(t) + \frac{\kappa}{2} \left( \frac{I(t)}{K(t)} - \delta_K \right)^2 K(t),
\]

(558)

Denoting the co-state variables associated with (556) and (557) by \( \lambda \) and \( Q' \), respectively, the first-order conditions characterizing the representative household’s optimal plans are:

\[
C(t) = (P_C(t)\lambda)^{-\sigma_C},
\]

(559a)

\[
L(t) = (W(t) (1 - \tau) \lambda)^{\sigma_L},
\]

(559b)

\[
Q(t) = P_J(t) \left[ 1 + \kappa \left( \frac{I(t)}{K(t)} - \delta_K \right) \right],
\]

(559c)

\[
\dot{\lambda}(t) = \lambda (\beta - r^\star),
\]

(559d)

\[
\dot{Q}(t) = (r^\star + \delta_K) Q(t) - \left\{ R(t) + P_J(t) \frac{\kappa}{2} \left( \frac{I(t)}{K(t)} - \delta_K \right) \left( \frac{I(t)}{K(t)} + \delta_K \right) \right\},
\]

(559e)

and the transversality conditions \( \lim_{t \to \infty} \bar{\lambda}B(t)e^{-\beta t} = 0 \) and \( \lim_{t \to \infty} Q(t)K(t)e^{-\beta t} = 0 \); to derive (559c) and (559e), we used the fact that \( Q(t) = Q'(t)/\lambda(t) \).

Once households decided on aggregate consumption, they decide on the allocation of expenditure between traded and non traded goods:

\[
C^N = (1 - \varphi) \left( \frac{P_N}{P_C} \right)^{-\phi} C,
\]

(560a)

\[
C^T = \varphi \left( \frac{1}{P_C} \right)^{-\phi} C,
\]

(560b)

where the consumption price index is:

\[
P_C = \left[ \varphi + (1 - \varphi) P^{1-\phi} \right]^{\frac{1}{1-\phi}}.
\]

(561)

As will be useful later, the percentage change in the consumption price index is proportional to the appreciation in the relative price of non tradables:

\[
\hat{P}_C = \alpha_C \hat{P},
\]

(562)

where \( \alpha_C \) is the non tradable content of consumption expenditure.

Once households decided on aggregate investment expenditure, they decide on the allocation between traded and non traded inputs:

\[
J^N = (1 - \varphi_J) \left( \frac{P_N}{P_J} \right)^{-\phi_J} J,
\]

(563a)

\[
J^T = \varphi \left( \frac{1}{P_J} \right)^{-\phi_J} J,
\]

(563b)

where the investment price index is:

\[
P_J = \left[ \varphi_J + (1 - \varphi_J) P^{1-\phi_J} \right]^{\frac{1}{1-\phi_J}}.
\]

(564)

As will be useful later, the percentage change in the investment price index is proportional to the appreciation in the relative price of non tradables:

\[
\hat{P}_J = \alpha_J \hat{P},
\]

(565)

where \( \alpha_J \) is the non tradable content of investment expenditure.
Once households decided on aggregate labor supply, they allocate hours worked to the traded and the non traded sector

\[ L^N = (1 - \vartheta) \left( \frac{W^N}{W} \right)^{\epsilon} L, \quad (566a) \]

\[ L^T = \vartheta \left( \frac{W^T}{W} \right)^{\epsilon} L, \quad (566b) \]

where the aggregate wage index is:

\[ W = \left[ \vartheta \left( \frac{W^T}{W} \right)^{\epsilon+1} + (1 - \vartheta) \left( \frac{W^N}{W} \right)^{\epsilon+1} \right]^{\frac{1}{\epsilon+1}}. \quad (567) \]

As will be useful later, the percentage change in the aggregate wage index is a weighted average of percentage changes in sectoral wages:

\[ \hat{W} = \alpha_L \hat{W}^N + (1 - \alpha_L) \hat{W}^T, \quad (568) \]

where \( \alpha_L \) is the non tradable content of aggregate labor compensation.

**J.3 Firms**

Both the traded and non-traded sectors use physical capital, \( K^j \), and labor, \( L^j \), according to constant returns to scale production functions \( Y^j = Z^j F(K^j, L^j) \) which are assumed to take a Cobb-Douglas form:

\[ Y^j = Z^j (L^j)^{\theta^j} (K^j)^{1-\theta^j}, \quad (569) \]

where \( \theta^j \) is the labor income share in sector \( j \) and \( Z^j \) corresponds to the total factor productivity index which is introduced for calibration purposes. Both sectors face two cost components: a capital rental cost equal to \( R \), and a labor cost equal to the wage rate, i.e., \( W^T \) in the traded sector and \( W^N \) in the non traded sector.

Both sectors are assumed to be perfectly competitive and thus choose capital and labor by taking prices as given:

\[ \max_{K^j, L^j} \Pi^j = \max_{K^j, L^j} \left\{ P^j Y^j - W^j L^j - RK^j \right\}. \quad (570) \]

Since capital can move freely between the two sectors, the value of marginal products in the traded and non traded sectors equalizes while costly labor mobility implies a wage differential across sectors:

\[ Z^T (1 - \theta^T) (k^T)^{-\theta^T} = PZ^N (1 - \theta^N) (k^N)^{-\theta^N} \equiv R, \quad (571a) \]

\[ Z^T \theta^T (k^T)^{1-\theta^T} \equiv W^T, \quad (571b) \]

\[ PZ^N \theta^N (k^N)^{1-\theta^N} \equiv W^N, \quad (571c) \]

where \( k^j \equiv K^j / L^j \) denotes the capital-labor ratio for sector \( j = T, N \).

The resource constraint for capital is:

\[ K^T + K^N = K. \quad (572) \]

**J.4 Solving the Model**

Before linearizing, we have to determine short-run static solutions. First-order conditions (559a) and (559b) can be solved for consumption and aggregate labor supply which of course must hold at any point of time:

\[ C = C(\hat{\lambda}, P), \quad L = L(\hat{\lambda}, W^T, W^N, \tau), \quad (573) \]

with partial derivatives given by

\[ \hat{C} = -\sigma_C \hat{\lambda} - \sigma_C \alpha_C \hat{P}, \quad (574a) \]

\[ \hat{L} = \sigma_L \hat{\lambda} + \sigma_L \alpha_L \hat{W}^N + \sigma_L (1 - \alpha_L) \hat{W}^T - \sigma_L \frac{d\tau}{1 - \tau}. \quad (574b) \]
Inserting first the solution for consumption (575) into (560) allows us to solve for $C^T$ and $C^N$:

$$C^T = C^T (\tilde{\lambda}, P), \quad C^N = C^N (\tilde{\lambda}, P), \quad (575)$$

with partial derivatives given by

$$\begin{align*}
\dot{C}^N &= -[(1 - \alpha_C) \phi + \alpha_C \sigma_C] \dot{P} - \sigma_C \dot{\lambda}, \quad (576a) \\
\dot{C}^N &= \alpha_C (\phi - \sigma_C) \dot{P} - \sigma_C \dot{\lambda}. \quad (576b)
\end{align*}$$

Inserting first the solution for sectoral labor (575) into (566) allows us to solve for $L^T$ and $L^N$:

$$L^T = L^T (\tilde{\lambda}, W^T, W^N, \tau), \quad L^N = L^N (\tilde{\lambda}, W^T, W^N, \tau), \quad (577)$$

with partial derivatives given by:

$$\begin{align*}
\dot{L}^T &= [\epsilon_0 + \sigma_L (1 - \alpha_L)] \dot{W}^T + \sigma_L (\sigma_L - \epsilon) \dot{W}^N + \sigma_L \dot{\lambda} - \sigma_L \frac{dt}{1 - \tau}, \quad (578a) \\
\dot{L}^N &= [\epsilon (1 - \alpha_L) + \sigma_L \alpha_L] \dot{W}^N + (1 - \alpha_L) (\sigma_L - \epsilon) \dot{W}^T + \sigma_L \dot{\lambda} - \sigma_L \frac{dt}{1 - \tau}. \quad (578b)
\end{align*}$$

Plugging the short-run static solutions for $L^T$ and $L^N$ given by (577) into the resource constraint for capital (574), the system of four equations consisting of (571a)-(571c) together with (574) can be solved for sectoral wages $W_j$ and sectoral capital-labor ratios $k^j$. Keeping TFPs unchanged, denoting by $\xi^N \equiv K^N / K$ the share of non traded capital in the aggregate stock of physical capital and log-differentiating (421a)-(421c) and (422) yields in matrix form:

$$\begin{pmatrix}
-\theta^T \\
(1 - \theta^T)
\end{pmatrix} + \begin{pmatrix}
\theta^N & 0 & 0 \\
0 & -1 & 0
\end{pmatrix} \begin{pmatrix}
\dot{L}^T \\
\dot{L}^N
\end{pmatrix} = \begin{pmatrix}
\dot{P} \\
\dot{\lambda} - \dot{\lambda} \frac{dt}{1 - \tau}
\end{pmatrix}, \quad (579)$$

where we set:

$$\begin{align*}
\Psi_{WT} &= (1 - \xi^N) \frac{L^T W^T}{L^T} + \xi^N \frac{L^N W^T}{L^N}, \quad (580a) \\
\Psi_{WN} &= (1 - \xi^N) \frac{L^T W^N}{L^T} + \xi^N \frac{L^N W^N}{L^N}, \quad (580b) \\
\xi^N &= \frac{k^N L^N}{K}, \quad (580c) \\
\Psi_{\lambda} &= (1 - \xi^N) \sigma_L + \xi^N \sigma_L = \sigma_L. \quad (580d)
\end{align*}$$

The short-run static solutions for sectoral wages and capital-labor ratios are:

$$W_j = W_j (\tilde{\lambda}, K, P, \tau), \quad k^j = k^j (\tilde{\lambda}, K, P, \tau). \quad (581)$$

Inserting first sectoral wages (581), sectoral employment (577) can be solved as functions of the shadow value of wealth, the capital stock and the relative price of non tradables:

$$L^j = L^j (\tilde{\lambda}, K, P, \tau). \quad (582)$$

Finally, plugging solutions for sectoral labor (582) and sector capital-labor ratios (581), the production functions (569) can be solved for sectoral output:

$$Y^j = Y^j (\tilde{\lambda}, K, P, \tau). \quad (583)$$
The Return on Domestic Capital, \( R \)

The return on domestic capital is:

\[
R = Z^T (1 - \theta^T) (k^T)^{-\theta^T}. \tag{584}
\]

Inserting first the short-run static solution for the capital-labor ratio \( k^T \) given by (581), eq. (584) can be solved for the return on domestic capital:

\[
R = R (\tilde{\lambda}, K, P, \tau). \tag{585}
\]

The Relative Price of Non Tradables, \( P \)

Finally, we have to solve for the relative price of non tradables by using the non traded goods market clearing condition:

\[
Y^N = C^N + G^N + J^N. \tag{586}
\]

Remembering that the non traded input \( J^N \) used to produce investment goods is equal to \( P'_J J \), inserting solutions for \( C^N \) and \( Y^N \) given by (575) and (583), respectively, and substituting (446), the non traded goods market clearing condition (586) can be rewritten as follows:

\[
Y^N (\tilde{\lambda}, K, P, \tau) = C^N (\tilde{\lambda}, P) + G^N + P'_J K \left[ v(.) + \delta_K + \frac{K}{2} (v(.))^2 \right]. \tag{587}
\]

Eq. (587) can be solved for the relative price of non tradables:

\[
P = P (\tilde{\lambda}, K, Q, G^N, \tau), \tag{588}
\]

with partial derivatives given by:

\[
P_K = \frac{\partial P}{\partial K} = -\frac{Y^N_P + J}{\Psi^P} \leq 0, \tag{589a}
\]

\[
P_Q = \frac{\partial P}{\partial Q} = \frac{K v_Q [1 + \kappa v(.)]}{\Psi^P} > 0, \tag{589b}
\]

\[
P_{GN} = \frac{1}{P'_J \Psi^P} > 0, \tag{589c}
\]

\[
P_{\tau} = -\frac{Y^N_{\tau}}{P'_J \Psi^P} > 0, \tag{589d}
\]

where we set

\[
\Psi^P = \left[ (Y^N_P - C^N_P) + \frac{J^N \phi_J (1 - \alpha_J)}{P} \right] \frac{1}{P'_J} - K v_P [1 + \kappa v(.)] > 0. \tag{590}
\]

J.5 Formal Solutions for public debt \( D(t) \)

Like Gali, Lopez-Salido and Vallès [2007], we assume a fiscal policy rule of the (linearized) form:

\[
dT(t) = \phi_D dD(t) + \phi_G dG(t), \tag{591}
\]

where \( dT(t) = T(t) - \tilde{T}, dD(t) = D(t) - \tilde{D}, \) and \( dG(t) = G(t) - \tilde{G} \). Linearizing first the government budget constraint (550), inserting the fiscal rule (591) and collecting terms yields:

\[
\dot{T} = r^* \left( D(t) - \tilde{D} \right) + \left( G(t) - \tilde{G} \right) - \left( T(t) - \tilde{T} \right),
\]

\[
= \left( r^* - \phi_D \right) \left( D(t) - \tilde{D} \right) + (1 - \phi_G) \left( G(t) - \tilde{G} \right). \tag{592}
\]

Inserting the dynamic equation for \( \frac{dG(t)}{dt} \) given by eq. (495) into (592) and solving the differential equation leads to:

\[
\frac{\left( D(t) - \tilde{D} \right)}{Y} = \left[ \frac{\left( D_0 - \tilde{D} \right)}{Y} + \Theta_D \right] e^{-\delta t} - \left[ \Theta_1 e^{-\xi t} - \Theta_2 e^{-\chi t} \right], \tag{593}
\]

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where we set
\[
\Theta_D = (1 - \phi_G) \left[ \frac{1}{\xi + r^* - \phi_D} - \frac{(1 - g)}{\chi + r^* - \phi_D} \right], \quad (594a)
\]
\[
\Theta_1 = \frac{(1 - \phi_G)}{\xi + r^* - \phi_D}, \quad (594b)
\]
\[
\Theta_2 = \frac{(1 - \phi_G)(1 - g)}{\chi + r^* - \phi_D}, \quad (594c)
\]
\[
\delta = r^* - \phi_D. \quad (594d)
\]

We assume that initial public debt is nil, i.e., \( D_0 = 0 \). Since public debt is back to its initial level in the long-run following a temporary rise in government spending, we have \( \dot{D} = 0 \). Inserting (593) into (591) along with the dynamic equation for government spending (495) leads to the temporal path for taxes in percentage point of GDP:

\[
\frac{dT(t)}{Y} = \phi_D \Theta_D e^{-\delta t} - \phi_D \left[ \Theta_1 e^{-\xi t} - \Theta_2 e^{-\chi t} \right] + \phi_G \left[ e^{-\xi t} - (1 - g) e^{-\chi t} \right],
\]

\[
= \phi_D \Theta_D e^{-\delta t} - \left\{ \left[ \phi_D \Theta_1 - \phi_G \right] e^{-\xi t} - \left[ \phi_D \Theta_2 - \phi_G (1 - g) \right] e^{-\chi t} \right\}. \quad (595)
\]

Dividing (551) by GDP and denoting by \( \theta_L \) the aggregate labor income share, we have:

\[
\frac{T(t)}{Y} = \theta_L(t) \tau(t). \quad (596)
\]

Because the aggregate labor income share is a weighted average of sectoral labor income shares, i.e., \( \theta_L = \frac{Y^N(t)\omega^N(t)\theta^N + Y^{N^T}(t)\theta^{N^T}}{Y(t)} \), \( \theta_L \) varies over time. In order to avoid unnecessary complications, we assume that \( \theta_L \) is fixed, i.e., \( \theta_L(t) = \theta_L \), and thus the adjustment in tax receipts is achieved through changes in the labor tax rate only. Linearizing first (596) and substituting (595), the deviation of the labor tax relative to its initial value is:

\[
d\tau(t) = \frac{1}{\theta_L} \frac{dT(t)}{Y},
\]

\[
= \Omega_D e^{-\delta t} - \left( \Omega_1 e^{-\xi t} - \Omega_2 e^{-\chi t} \right), \quad (597)
\]

where we set

\[
\Omega_D = \frac{\phi_D \Theta_D}{\theta_L}, \quad (598a)
\]

\[
\Omega_1 = \frac{\phi_D \Theta_1 - \phi_G}{\theta_L}, \quad (598b)
\]

\[
\Omega_2 = \frac{\phi_D \Theta_2 - \phi_G (1 - g)}{\theta_L}. \quad (598c)
\]

### J.6 Formal Solutions for \( K(t) \) and \( Q(t) \)

Remembering that the non traded input \( J^N \) used to produce the capital good is equal to \( P^N J \), using the fact that \( J^N = Y^N - C^N - G^N \) and inserting \( I = \dot{K} + \delta K \), the capital accumulation equation can be rewritten as follows:

\[
\dot{K} = \frac{Y^N - C^N - G^N}{P^N J} - \delta K - \frac{\kappa}{2} \left( \frac{I}{\dot{K} - \delta K} \right)^2 K. \quad (599)
\]

Using the fact that \( \frac{\partial G^N}{\partial C^N} = \frac{\omega^N}{\rho} \), inserting short-run static solutions for non traded output (583), consumption in non tradables (575), and optimal investment decision (443) into the physical capital accumulation equation (591) and the dynamic equation for the shadow value (559e), the dynamic system is:

\[
\dot{K} \equiv \Upsilon(K, P(\cdot), Q, G, \tau) = \frac{Y^N(K, P(\cdot), \bar{\lambda}) - C^N(\bar{\lambda}, P(\cdot)) - G^N}{P^N_J(P(\cdot))}
\]

\[
- \delta K - K \frac{\kappa}{2} \left[ \frac{Q}{P^N_J(P(\cdot))} - 1 \right]^2, \quad (600a)
\]

\[
\dot{Q} \equiv \Sigma(K, P, Q, G, \tau) = (r^* + \delta K) Q - \left[ R(K, P(\cdot)) + P^N_J \frac{K}{2} v(\cdot) (v(\cdot) + 2\delta K) \right]. \quad (600b)
\]
The linearized system can be written in a matrix form:

\[
\begin{pmatrix}
\dot{K}(t) \\
\dot{Q}(t)
\end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} K(t) - \tilde{K} \\ Q(t) - \tilde{Q} \end{pmatrix} + \begin{pmatrix} \varepsilon^G_K dG(t) + \varepsilon^r_K d \tau(t) \\ \varepsilon^G_Q dG(t) + \varepsilon^r_Q d \tau(t) \end{pmatrix},
\]

where the coefficients of the Jacobian matrix are given by (501) and the direct effects of changes in government consumption captured by the terms \( \varepsilon^G_K \) and \( \varepsilon^G_Q \) are described by (502). The dynamic effects of changes in the labor tax rate on \( K \) and \( Q \) are described by:

\[
\varepsilon^r_K = \frac{Y^N_P}{P_f} + \Upsilon_P \nu_r,
\]

\[
\varepsilon^r_Q = -R_r + \Sigma_P \nu_r,
\]

where \( \Upsilon_P = \Upsilon_P \left( \tilde{K}, \tilde{Q}, \tilde{G}, \tilde{\tau} \right) \) and \( \Sigma_P = \Sigma_P \left( \tilde{K}, \tilde{Q}, \tilde{G}, \tilde{\tau} \right) \).

Denoting the negative eigenvalue by \( \nu_1 \) and the positive eigenvalue by \( \nu_2 \), the general solutions for \( K \) and \( Q \) can be written as follows:

\[
K(t) - \tilde{K} = X_1(t) + X_2(t), \quad Q(t) - \tilde{Q} = \omega_1^1 X_1(t) + \omega_2^2 X_2(t).
\]

where \( X_1(t) \) and \( X_2(t) \) characterize the trajectory of physical capital and the shadow value of capital. To express these terms in compact form, we set:

\[
\Phi^1_i = \left( a_{11} - \nu_2 \right) \varepsilon^r_K + a_{12} \varepsilon^r_Q,
\]

\[
\Phi^2_i = \left( a_{11} - \nu_1 \right) \varepsilon^r_K + a_{12} \varepsilon^r_Q.
\]

where \( i = G, \tau \), along with

\[
\Gamma^{G}_i = -\frac{\Phi^G_i \tilde{Y}}{(\nu_1 - \nu_2) (\nu_1 + \xi)},
\]

\[
\Theta^{G}_i = (1 - g) \left( \frac{\xi + \nu_1}{\chi + \nu_1} \right),
\]

\[
\Gamma^{D}_i = -\frac{\Phi^D_i \Omega_D}{(\nu_1 - \nu_2) (\nu_1 + \delta)},
\]

\[
\Gamma^{\tau}_i = -\frac{\Phi^{\tau}_i \Omega_{\tau}}{(\nu_1 - \nu_2) (\nu_1 + \xi)},
\]

\[
\Theta^{\tau}_i = \frac{\Omega_2}{\Omega_1} \left( \frac{\xi + \nu_1}{\chi + \nu_1} \right),
\]

where \( i = 1, 2 \).

Adopting the same procedure as in section H.2, solutions for \( X_1(t) \) and \( X_2(t) \) are given by:

\[
X_1(t) = e^{\nu_1 t} \{ X_1(0) - \Gamma^{G}_1 (1 - \Theta^{G}_1) - \Gamma^{D}_1 (1 - \Theta^{D}_1) \}
\]

\[
+ \Gamma^{G}_1 \left( e^{-\xi t} - \Theta^{G}_1 e^{-\chi t} \right) + \Gamma^{D}_1 e^{-\delta t} - \Gamma^{\tau}_1 \left( e^{-\xi t} - \Theta^{\tau}_1 e^{-\chi t} \right),
\]

\[
X_2(t) = -\Gamma^{G}_2 \left( e^{-\xi t} - \Theta^{G}_2 e^{-\chi t} \right) - \Gamma^{D}_2 e^{-\delta t} + \Gamma^{\tau}_2 \left( e^{-\xi t} - \Theta^{\tau}_2 e^{-\chi t} \right),
\]

where

\[
X_1(0) = K_0 - \tilde{K} - X_2(0).
\]

**J.7 Formal Solutions for the Net Foreign Asset Position \( N(t) \)**

To determine the dynamic equation for the net foreign asset position, \( N(t) \), we differentiate \( N(t) = B(t) - D(t) \) w.r.t. time and substitute the dynamic equations for the stock of traded bonds (556) and for the public debt (550):

\[
\dot{N}(t) = \dot{B}(t) - \dot{D}(t),
\]

\[
= r^* N(t) + R(t) K(t) + W(t) L(t) - P_C(P(t)) C(t) - P_F(P(t)) J(t) - G(t).
\]

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Inserting the market clearing condition for non tradables (586) and remembering that  $J^T = (1 - \alpha_J) P_J J$, the current account equation is given by:

$$
\dot{N} = \Xi (N, K, Q, G),
$$

$$
= r^* N + Y^T - C^T - G^T - (1 - \alpha_J) P_J J,
$$

$$
= r^* B + Y^T - C^T - G^T - \left(1 - \frac{\alpha_J}{\alpha_J}\right) P (Y^N - C^N - G^N),
$$

where we used the fact that $P_J J = Y^N - C^N - G^N$.

Linearizing first the current account equation (609) in the neighborhood of the steady-state and substitute the solutions for $K(t)$ and $Q(t)$ leads to:

$$
\dot{N}(t) = r^* dN(t) + N_1 X_1(t) + N_2 X_2(t) + \Xi_G dG(t) + \Xi_d d\tau(t),
$$

where $N_i = \Xi_K + \Xi_Q \omega^i_\tau$ (with $i = 1, 2$). $\Xi_G$ is given by (514) and $\Xi_{\tau}$ reads as follows:

$$
\Xi_{\tau} = \Xi_P P_{\tau} + Y^T_{\tau} - \left(1 - \frac{\alpha_J}{\alpha_J}\right) \tilde{P} Y^N_{\tau},
$$

where $\Xi_P < 0$ is given by (461b).

Substituting $X_1(t)$ given by eq. (512) and $X_2(t)$ given by eq. (606a) into (606b) leads to:

$$
\dot{N}(t) = r^* dN(t) + \omega^N_N e^{\nu t} + N_1 \Gamma^D_G \left(e^{-\Xi t} - \Theta^G_G e^{-\nu t}\right) + N_1 \Gamma^D_1 e^{-\delta t}
$$

$$
= \Gamma^G_G \left(e^{-\Xi t} - \Theta^G_G e^{-\nu t}\right) - N_2 \Gamma^D_2 \left(e^{-\Xi t} - \Theta^G_G e^{-\nu t}\right) - N_2 \Gamma^D_2 e^{-\delta t} + N_2 \Gamma^D_2 \left(e^{-\Xi t} - \Theta^G_G e^{-\nu t}\right)
$$

$$
+ \Xi_G \dot{Y} \left(e^{-\Xi t} - (1 - g) e^{-\nu t}\right) + \Xi_{\tau} \Omega_D e^{-\delta t} - \Xi_{\tau} \left(\Omega_{\tau} e^{-\Xi t} - \Omega_2 e^{-\nu t}\right),
$$

where $\Gamma^l_i$ (with $l = G, \tau, D$, and $i = 1, 2$) is given by (605a), (605b), (605c) and we set:

$$
\omega^N_N = N_1 \left[\left(K(0) - \tilde{K}\right) + \Gamma^G_G (1 - \Theta^G_G) + \Gamma^D_2 - \Gamma^D_2 (1 - \Theta^D_2)\right]
$$

$$
- \Gamma^G_G (1 - \Theta^G_G) - \Gamma^D_2 + \Gamma^D_1 (1 - \Theta^D_2).\tag{613}
$$

Solving the differential equation (612) yields the general solution for the net foreign asset position:

$$
N(t) - \tilde{N} = \left\{\begin{array}{l}
N_0 - \tilde{N} - \frac{\omega^N_N}{\nu_1 - r^*} + \frac{\Xi_G \dot{Y}}{\xi + r^*} (1 - \Theta^G_G) + \Xi_{\tau} \Omega_D \frac{\xi + r^*}{\delta + r^*} (1 - \Theta^\tau_t) \\
+ \frac{N_1 \Gamma^G_1}{\xi + r^*} (1 - \Theta^G_G) + \frac{N_1 \Gamma^D_1}{\delta + r^*} (1 - \Theta^D_G) \\
- \frac{N_1 \Gamma^G_1}{\xi + r^*} (1 - \Theta^G_G) + \frac{N_1 \Gamma^D_1}{\delta + r^*} (1 - \Theta^D_G) \end{array}\right\}
$$

$$
- \frac{\omega^N_N}{\nu_1 - r^*} e^{\nu t} - \frac{\Xi_G \dot{Y}}{\xi + r^*} \left(e^{-\Xi t} - \Theta^G_G e^{-\nu t}\right) - \Xi_{\tau} \Omega_D \frac{\xi + r^*}{\delta + r^*} e^{-\delta t} + \Xi_{\tau} \Omega_1 \frac{\xi + r^*}{\delta + r^*} \left(e^{-\Xi t} - \Theta^\tau_t e^{-\nu t}\right)
$$

$$
- \frac{N_1 \Gamma^G_1}{\xi + r^*} \left(e^{-\Xi t} - \Theta^G_G e^{-\nu t}\right) - \frac{N_1 \Gamma^D_1}{\delta + r^*} + \frac{N_1 \Gamma^D_1}{\xi + r^*} \left(e^{-\Xi t} - \Theta^D_G e^{-\nu t}\right)
$$

$$
+ \frac{N_2 \Gamma^G_2}{\xi + r^*} \left(1 - \Theta^G_G\right) + \frac{N_2 \Gamma^D_2}{\delta + r^*} - \frac{N_2 \Gamma^D_2}{\xi + r^*} (1 - \Theta^D_G),\tag{614}
$$

171
where we set:

\[ \Theta^{G,t} = (1 - g) \frac{\xi + r^*}{\chi + r^*}, \]  
\[ \Theta_1^{G,t} = \Theta_1^{G} \frac{\xi + r^*}{\chi + r^*}, \]  
\[ \Theta_2^{G,t} = \Theta_2^{G} \frac{\xi + r^*}{\chi + r^*}, \]  
\[ \Theta^r = \frac{\Omega_2 \xi + r^*}{\Omega_1 \chi + r^*}, \]  
\[ \Theta_1^r = \Theta_1^r \frac{\xi + r^*}{\chi + r^*}, \]  
\[ \Theta_2^r = \Theta_2^r \frac{\xi + r^*}{\chi + r^*}. \]  

(615a) (615b) (615c) (615d) (615e) (615f)

Invoking the transversality condition, one obtains the 'stable' solution for the stock of net foreign assets so that \( N(t) \) converges toward its steady-state value \( \tilde{N} \):

\[
N(t) - \tilde{N} = \frac{\omega_N^1}{\nu_1 - r^*} e^{\nu_1 t} - \frac{\Xi_G \tilde{Y}}{\xi + r^*} \left( e^{-\xi t} - \Theta^{G,t} e^{-\chi t} \right) - \frac{\Xi_2 \Omega_2 e^{-\delta t}}{\delta + r^*} \left( e^{-\xi t} - \Theta^r e^{-\chi t} \right) - \frac{N_1 \Gamma_1^G}{\xi + r^*} \left( e^{-\xi t} - \Theta^{G,t} e^{-\chi t} \right) - \frac{N_1 \Gamma_1^D}{\delta + r^*} e^{-\delta t} + \frac{N_2 \Gamma_2^G}{\chi + r^*} \left( e^{-\xi t} - \Theta_2^{G,t} e^{-\chi t} \right) + \frac{N_2 \Gamma_2^D}{\delta + r^*} e^{-\delta t} - \frac{N_2 \Gamma_2^G}{\xi + r^*} \left( e^{-\xi t} - \Theta_2^{G,t} e^{-\chi t} \right).
\]

(616)

Eq. (616) gives the trajectory for \( N(t) \) consistent with the intertemporal solvency condition:

\[
\left( \tilde{N} - N_0 \right) = \frac{\omega_N^1}{\nu_1 - r^*} + \frac{\omega_N^G}{\xi + r^*} + \frac{\omega_N^D}{\delta + r^*} + \frac{\omega_N^r}{\chi + r^*},
\]

(617)

where \( \omega_N^1 \) is given by (613) and we set

\[
\omega_N^G = \Xi_G \tilde{Y} \left( 1 - \Theta^{G,t} \right) + N_1 \Gamma_1^G \left( 1 - \Theta_1^{G,t} \right) - N_2 \Gamma_2^G \left( 1 - \Theta_2^{G,t} \right),
\]

(618a)

\[
\omega_N^D = \Xi_2 \Omega_2 - N_1 \Gamma_2^D + N_2 \Gamma_2^G,
\]

(618b)

\[
\omega_N^r = -\Xi_2 \Omega_2 \left( 1 - \Theta^r \right) - N_1 \Gamma_2^r + N_2 \Gamma_2^G \left( 1 - \Theta_2^r \right).
\]

(618c)

Differentiating (616) w.r.t. time gives the trajectory for the current account along the transitional path when government spending follows the temporal path given by eq. (495) and the labor tax rate is governed by the dynamic equation (597)

\[
\dot{N}(t) = \frac{\nu_1}{\nu_1 - r^*} e^{\nu_1 t} + \frac{\Xi_G \tilde{Y}}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta^{G,t} e^{-\chi t} \right) + \frac{\Xi_2 \Omega_2}{\delta + r^*} e^{-\delta t} - \frac{\Xi_2 \Omega_1}{\chi + r^*} \left( \xi e^{-\xi t} - \chi \Theta^r e^{-\chi t} \right) + \frac{N_1 \Gamma_1^G}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta_1^{G,t} e^{-\chi t} \right) + \frac{N_1 \Gamma_1^D}{\delta + r^*} e^{-\delta t} - \frac{N_1 \Gamma_1^r}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta_1^r e^{-\chi t} \right) - \frac{N_2 \Gamma_2^G}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta_2^{G,t} e^{-\chi t} \right) + \frac{N_2 \Gamma_2^D}{\delta + r^*} e^{-\delta t} + \frac{N_2 \Gamma_2^r}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta_2^r e^{-\chi t} \right).
\]

(619)

J.8 Formal Solution for the Stock of Non Human Wealth, \( A(t) \)

Remembering that the stock of national non human wealth \( A(t) \) is equal to \( N(t) + Q(t) K(t) \), differentiating w.r.t. time, i.e., \( \dot{A}(t) = B(t) - \dot{D}(t) + \dot{Q}(t) K(t) + Q(t) \dot{K}(t) \), plugging the dynamic equation for the shadow value of capital (559e), inserting the accumulation equations for physical capital (557), traded bonds (556), traded bonds issued by the government (550) and using the specification of capita adjustment costs (558) along with the first order condition (559d) yields the accumulation equation for the stock of national non human wealth or the dynamic equation for national savings:

\[
\dot{A}(t) = r^* A(t) + W(t) L(t) - P_C \left( P(t) \right) C(t) - G(t).
\]

(620)
We first determine short-run static solutions for aggregate labor supply and aggregate wage index. Inserting solutions for sectoral wages (581) into the solution for aggregate labor supply (575), one can solve for total hours worked:

\[ L = L(\bar{\lambda}, K, P, \tau), \]  

(621)

where partial derivatives are given by

\[ L_K \equiv \frac{\partial L}{\partial K} = L_W T_K^T + L_N W_K^N, \]  

(622a)

\[ L_P \equiv \frac{\partial L}{\partial P} = L_W T_P^T + L_N W_P^N, \]  

(622b)

\[ L_\tau \equiv \frac{\partial L}{\partial \tau} = L_W T_\tau^T + L_N W_\tau^N. \]  

(622c)

Substituting solutions for sectoral wages (433) into the aggregate wage index \( W \equiv W(T, N) \), we can solve for the aggregate wage index:

\[ W = W(\bar{\lambda}, K, P, \tau), \]  

(623)

where partial derivatives are given by

\[ W_K \equiv \frac{\partial W}{\partial K} = W_T T_K^T + W_N W_K^N, \]  

(624a)

\[ W_P \equiv \frac{\partial W}{\partial P} = W_T T_P^T + W_N W_P^N, \]  

(624b)

\[ W_\tau \equiv \frac{\partial W}{\partial \tau} = W_T T_\tau^T + W_N W_\tau^N, \]  

(624c)

with \( W_T = (W/W)^{(1-\alpha_L)} \) and \( W_N = (W/W)^{\alpha_L} \).

Inserting solutions for aggregate labor supply (621), plugging the solutions for aggregate wage index (623) and for consumption (575) into the accumulation equation of financial wealth (620), linearizing around the steady-state leads to:

\[ \dot{A}(t) = r^* dA(t) + \Lambda_K dK(t) + \Lambda_Q dQ(t) + \Lambda_G dG(t) + \Lambda_\tau d\tau(t), \]  

(625)

where partial derivatives evaluated at the steady-state are given by

\[ \Lambda_P = \left( W_T \tilde{L} + \tilde{W} L_P \right) - \left( \tilde{C}^N + P C P \right), \]  

(626a)

\[ \Lambda_K \bigg|_{P \text{ fixed}} = W_K \tilde{L} + \tilde{W} L_K, \]  

(626b)

\[ \Lambda_G \bigg|_{P \text{ fixed}} = -1, \]  

(626c)

\[ \Lambda_\tau \bigg|_{P \text{ fixed}} = W_\tau \tilde{L} + \tilde{W} L_\tau. \]  

(626d)

Substituting the solutions for \( K(t) \) and \( Q(t) \) into (625) leads to:

\[ \dot{A}(t) = r^* dA(t) + M_1 X_1(t) + M_2 X_2(t) + \Lambda_G dG(t) + \Lambda_\tau d\tau(t), \]  

(627)

where \( M_i = \Lambda_K + \Lambda_Q \omega_i^2 \) (with \( i = 1, 2 \)).

Substituting the equation that governs the endogenous response of government consumption to an exogenous fiscal shock (495) along with the equation that governs the endogenous response of the labor tax rate (597) into (627) and solving yields the general
solution for the stock of national non human wealth:

\[ A(t) - \tilde{A} = \left\{ \begin{array}{l}
\left( A_0 - \tilde{A} \right) - \frac{\omega_A^1}{\nu_1 - r^*} + \frac{\Lambda_G \hat{Y}}{\xi + r^*} (1 - \Theta^{G,r}) + \frac{\Lambda_r \Omega_D}{\delta + r^*} - \frac{\Lambda_r \Omega_1}{\xi + r^*} (1 - \Theta^{r'}) \\
+ \frac{M_1 \Gamma^G}{\xi + r^*} \left( 1 - \Theta^{G,r}_1 \right) + \frac{M_1 \Gamma^D}{\delta + r^*} - \frac{M_1 \Gamma^r}{\xi + r^*} \left( 1 - \Theta^{r'}_1 \right) \\
- \frac{M_2 \Gamma^G}{\xi + r^*} \left( 1 - \Theta^{G,r}_2 \right) - \frac{M_2 \Gamma^D}{\delta + r^*} + \frac{M_2 \Gamma^r}{\xi + r^*} \left( 1 - \Theta^{r'}_2 \right) \right\} e^{r^* t} \\
+ \frac{\omega_A^1}{\nu_1 - r^*} e^{\tau t} - \frac{\Lambda_G \hat{Y}}{\xi + r^*} \left( e^{-\xi t} - \Theta^{G,r} e^{-\chi t} \right) - \frac{\Lambda_r \Omega_D}{\delta + r^*} e^{-\delta t} + \frac{\Lambda_r \Omega_1}{\xi + r^*} \left( e^{-\xi t} - \Theta^{r'} e^{-\chi t} \right) \\
- \frac{M_1 \Gamma^G}{\xi + r^*} \left( e^{-\xi t} - \Theta^{G,r}_1 e^{-\chi t} \right) - \frac{M_1 \Gamma^D}{\delta + r^*} + \frac{M_1 \Gamma^r}{\xi + r^*} \left( e^{-\xi t} - \Theta^{r'}_1 e^{-\chi t} \right) \\
+ \frac{M_2 \Gamma^G}{\xi + r^*} \left( 1 - \Theta^{G,r}_2 \right) + \frac{M_2 \Gamma^D}{\delta + r^*} - \frac{M_2 \Gamma^r}{\xi + r^*} \left( 1 - \Theta^{r'}_2 \right),
\end{array} \right. \quad (628)\]

where we set:

\[ \omega_A^1 = M_1 \left[ \left( K_0 - \tilde{K} \right) - X_2(0) - \Gamma^G_1 (1 - \Theta^{G,r}_1) - \Gamma^D_1 \left( 1 - \Theta^{r'}_1 \right) \right]. \quad (629) \]

Invoking the transversality condition, one obtains the 'stable' solution for the stock of national non human wealth so that \( A(t) \) converges toward its steady-state value \( \tilde{A} \):

\[ A(t) - \tilde{A} = \frac{\omega_A^1}{\nu_1 - r^*} e^{\tau t} + \frac{\Lambda_G \hat{Y}}{\xi + r^*} \left( e^{-\xi t} - \Theta^{G,r} e^{-\chi t} \right) - \frac{\Lambda_r \Omega_D}{\delta + r^*} e^{-\delta t} + \frac{\Lambda_r \Omega_1}{\xi + r^*} \left( e^{-\xi t} - \Theta^{r'} e^{-\chi t} \right) \\
- \frac{M_1 \Gamma^G}{\xi + r^*} \left( e^{-\xi t} - \Theta^{G,r}_1 e^{-\chi t} \right) - \frac{M_1 \Gamma^D}{\delta + r^*} e^{-\delta t} + \frac{M_1 \Gamma^r}{\xi + r^*} \left( e^{-\xi t} - \Theta^{r'}_1 e^{-\chi t} \right) \\
+ \frac{M_2 \Gamma^G}{\xi + r^*} \left( e^{-\xi t} - \Theta^{G,r}_2 e^{-\chi t} \right) + \frac{M_2 \Gamma^D}{\delta + r^*} e^{-\delta t} - \frac{M_2 \Gamma^r}{\xi + r^*} \left( e^{-\xi t} - \Theta^{r'}_2 e^{-\chi t} \right). \quad (630)\]

Eq. (630) gives the trajectory for for \( N(t) \) consistent with the intertemporal solvency condition:

\[ \left( \tilde{A} - A_0 \right) = - \frac{\omega_A^1}{\nu_1 - r^*} + \frac{\omega_A^G}{\xi + r^*} + \frac{\omega_A^D}{\delta + r^*} + \frac{\omega_A^r}{\xi + r^*}, \quad (631)\]

where \( \omega_A^1 \) is given by (629) and we set

\[ \omega_A^G = \Lambda_G \hat{Y} (1 - \Theta^{G,r}) + M_1 \Gamma^G_1 \left( 1 - \Theta^{G,r}_1 \right) - M_2 \Gamma^G_2 \left( 1 - \Theta^{G,r}_2 \right), \quad (632a) \]

\[ \omega_A^D = \Lambda_r \Omega_D - M_1 \Gamma^D_1 + M_2 \Gamma^D_2, \quad (632b) \]

\[ \omega_A^r = - \Lambda_r \Omega_1 \left( 1 - \Theta^{r'} \right) - M_1 \Gamma^r_1 \left( 1 - \Theta^{r'}_1 \right) + M_2 \Gamma^r_2 \left( 1 - \Theta^{r'}_2 \right). \quad (632c) \]

Differentiating (630) w.r.t. time gives the trajectory for the national non human wealth along the transitional path when government spending follows the temporal path given by eq. (495) and the labor tax rate is governed by the dynamic equation (597)

\[ \dot{A}(t) = \nu_1 \frac{\omega_A^1}{\nu_1 - r^*} e^{\tau t} + \frac{\Lambda_G \hat{Y}}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta^{G,r} e^{-\chi t} \right) + \frac{\Lambda_r \Omega_D}{\delta + r^*} e^{-\delta t} - \frac{\Lambda_r \Omega_1}{\xi + r^*} \left( \xi e^{-\xi t} - \chi \Theta^{r'} e^{-\chi t} \right) \\
+ \frac{M_1 \Gamma^G}{\xi + r^*} \left( \xi e^{-\xi t} - \Theta^{G,r}_1 e^{-\chi t} \right) + \frac{M_1 \Gamma^D}{\delta + r^*} e^{-\delta t} - \frac{M_1 \Gamma^r}{\xi + r^*} \left( \xi e^{-\xi t} - \Theta^{r'}_1 e^{-\chi t} \right) \\
- \frac{M_2 \Gamma^G}{\xi + r^*} \left( \xi e^{-\xi t} - \Theta^{G,r}_2 e^{-\chi t} \right) + \frac{M_2 \Gamma^D}{\delta + r^*} e^{-\delta t} + \frac{M_2 \Gamma^r}{\xi + r^*} \left( \xi e^{-\xi t} - \Theta^{r'}_2 e^{-\chi t} \right). \quad (633)\]

### K Solving the Model with Endogenous Markups

This section extends the two-sector model with imperfect mobility of labor to an imperfectly competitive non traded sector with endogenous markups. We maintain the assumption capital installation costs. There are two sectors. A traded sector produced a good that can
be consumed by the private, \( C^T \), and the public sector, \( G^T \), invested, \( J^T \), or exported. A non traded sector produces a good for domestic absorption only. The non traded good can be consumed by the private, \( C^N \), and the public sector, \( G^N \), invested, \( J^N \). The final non-traded output, \( Y^N \), is produced in a competitive retail sector with constant-returns-to-scale production which aggregates a continuum measure one of sectoral non-traded goods. We denote the elasticity of substitution between any two different sectoral non traded goods by \( \eta > 0 \). In each industry, there are \( N > 1 \) firms producing differentiated goods that are aggregated into a sectoral non-traded good. The elasticity of substitution between any two varieties within an industry is denoted by \( \rho > 0 \), and we assume that this is higher than the elasticity of substitution across non traded industries, i.e. \( \rho > \eta \) (see Jaimovich and Floetotto [2008]). The number of firms is large enough so that we can ignore the strategic effects but not so large that the effect of entry on the firm’s demand curve is minuscule. Consequently, the price elasticity of demand faced by a single firm is no longer constant and equal to the elasticity of substitution between any two varieties, but rather a function of the number of firms \( N \). We further assume instantaneous entry, which implies that the producers make zero profits. Since the household’s maximization problem is identical to that described in section 2, we restrict our attention to the firms’ maximization problem. More technical details can be found in the working paper version of the paper by Cardi and Restout [2004], [2015].

K.1 Introducing Endogenous Markups

Within each industry, there is monopolistic competition; each firm that produces one variety is a price setter. Output \( X_{i,j} \) of firm \( i \) in industry \( j \) is produced using capital and labor, i.e. \( X_{i,j} = H(K_{i,j}, L_{i,j}) \). Each firm chooses capital and labor by equalizing markup-adjusted marginal products to the marginal cost of inputs, i.e. \( PH_K/\mu = R \), and \( PH_L/\mu = W^N \), where \( \mu \) is the markup over the marginal costs. At a symmetric equilibrium, non-traded output is equal to \( Y^N = NX = H(K^N, L^N) \) where \( L^N = N L_N \) and \( K^N = N K_N \).

Taking into account the fact that output of one variety does not affect the price of final non-traded output, but influences the sectoral price level, in a symmetric equilibrium, the resulting price elasticity of demand is:

\[
e(N) = \rho - \frac{(\rho - \eta)}{N}, \quad N \in (1, \infty).
\] (634)

Assuming that \( \rho > \eta \), the price elasticity of demand faced by one single firm is an increasing function of the number of firms \( N \) within a sector:

\[
\frac{\partial e}{\partial N} = \frac{\rho - \eta}{N^2} > 0,
\] (635)

where the positive sign of the partial derivative follows from the assumption \( \rho > \eta \). Henceforth, the markup defined as follows

\[
\mu(N) = \frac{e}{e - 1}
\] (636)
decreases as the number of competitors increases, i.e. \( \mu_N < 0 \).

We assume instantaneous entry, which implies that the zero-profit condition holds at each instant of time:

\[
\pi^N = H(K^N, L^N) - RK^N - W^N L^N - P\psi,
\]

\[
= P\left[ \frac{Y^N}{N} \left( 1 - \frac{1}{\mu} \right) - \psi \right] = 0,
\] (637)

where we denote fixed costs by \( \psi \); we used the fact that \( P \frac{\partial X}{\partial K_N} = R \) and \( P \frac{\partial X}{\partial L_N} = W^N \) and \( \frac{\partial X}{\partial K_N} K^N + \frac{\partial X}{\partial L_N} L^N = X \). The zero-profit condition \( \pi^N = 0 \) can be solved for the number of firms.

Since capital can move freely between the two sectors while the shift of labor across sectors is costly, only marginal products of capital in the traded and the non traded sector
equalize:

\[
Z^T (1 - \theta^T) (k^T)^{-\theta^T} = \frac{P}{\mu} Z^N (1 - \theta^N) (k^N)^{-\theta^N} \equiv R, \\
Z^T \theta^T (k^T)^{1-\theta^T} \equiv W^T, \\
\frac{P}{\mu} Z^N \theta^N (k^N)^{1-\theta^N} \equiv W^N,
\]

where the capital-labor ratio for sector \( j = T, N \) is denoted by \( k^j \equiv K^j/L^j \). These static efficiency conditions state that the value of the marginal product of labor in sector \( j \) is equal to the labor cost \( W^j \) while the value of the marginal product of capital in the traded and the non traded sector must be equal to the capital rental cost, \( R \).

Aggregating over the two sectors gives us the resource constraint for capital:

\[
K^T + K^N = K. 
\]  (639)

### K.2 Solving the Model with Endogenous Markups

Plugging the short-run static solutions for \( L^T \) and \( L^N \) given by (427) into the resource constraint for capital (639), the system of four equations consisting of (638a)-(638c) together with (639) can be solved for the sectoral wage rates \( W^j \) and sectoral capital-labor ratios \( k^j \). Keeping TFPs unchanged, and differentiating (638a)-(638c) together with (639) yields in matrix form:

\[
\begin{pmatrix}
-\frac{\theta^T}{k^T} & \frac{\theta^N}{k^N} & 0 & 0 \\
0 & -\frac{1}{W^T} & 0 & 0 \\
L^T & L^N & \Psi_{W^T} & \Psi_{W^N} \\
\end{pmatrix}
\begin{pmatrix}
dk^T \\
dk^N \\
dW^T \\
dW^N \\
\end{pmatrix}
= \begin{pmatrix}
dP - \frac{d\mu}{\mu} \\
-\frac{dP}{\mu} + \frac{d\mu}{\mu} \\
-\Psi_{\bar{\lambda}} d\bar{\lambda} \\
\end{pmatrix},
\]  (640)

where we set:

\[
\Psi_{W^T} = k^T L^T_{W^T} + k^N L^N_{W^T}, \\
\Psi_{W^N} = k^T L^T_{W^N} + k^N L^N_{W^N}, \\
\Psi_{\bar{\lambda}} = k^T L^T_{\bar{\lambda}} + k^N L^N_{\bar{\lambda}}.
\]  (641a, 641b, 641c)

The short-run static for sectoral wages and sectoral capital-labor ratios are:

\[
W^j = W^j (\bar{\lambda}, K, P, \mu), \quad k^j = k^j (\bar{\lambda}, K, P, \mu).
\]  (642)

Inserting first solutions for sectoral wages (642) into (427), sectoral hours worked can be solved as functions of the shadow value of wealth, the capital stock, the relative price of non tradables and the markup

\[
L^j = L^j (\bar{\lambda}, K, P, \mu).
\]  (643)

Production functions (16) can be rewritten as follows:

\[
Y^j = Z^j L^j (k^j)^{1-\theta^j}, \quad j = T, N.
\]  (644)

Inserting first short-run static solutions for sectoral capital-labor ratios (642) and sectoral labor (643) into the production functions yields:

\[
Y^j = Y^j (\bar{\lambda}, K, P, \mu).
\]  (645)
It is worth noticing that a rise in the markup $\mu$ produces opposite effects to those induced by an appreciation in $P$.

**The Return on Domestic Capital, $R$**

The return on domestic capital is:

$$R = Z^T (1 - \theta^T) (k^T)^{-\theta^T}.$$  \hspace{1cm} (646)

Inserting first the short-run static solution for the capital-labor ratio $k^T$ given by (642), eq. (646) can be solved for the return on domestic capital:

$$R = R (\bar{\lambda}, K, P, \mu).$$  \hspace{1cm} (647)

**Optimal Investment Decision, $I/K$**

Eq. (405c) can be solved for the investment rate:

$$\frac{I}{K} = v \left( \frac{Q}{P_j(P)} \right) + \delta_K,$$  \hspace{1cm} (648)

where

$$v(.) = \frac{1}{k} \left( \frac{Q}{P_j} - 1 \right).$$  \hspace{1cm} (649)

**The Number of Firms within each Non Traded Industry, $N$**

Substituting the short-run solution for non traded output (144a) and using the fact that $\mu = \mu (N)$ (see (634)-(636)), the zero-profit condition (637) can be rewritten as:

$$Y^N (\bar{\lambda}, K, P, \mu (N)) \left( 1 - \frac{1}{\mu (N)} \right) = N \psi.$$  \hspace{1cm} (650)

Solving yields the short-run solution for the number of firms:

$$N = N (\bar{\lambda}, K, P),$$  \hspace{1cm} (651)

where partial derivatives are given by:

$$N_x = \frac{\partial N}{\partial x} = - \frac{Y^N_x \left( 1 - \frac{1}{\mu} \right)}{\chi} \geq 0,$$  \hspace{1cm} (652)

where $x = K, P, \bar{\lambda}$ and we set

$$\chi = \frac{\mu N}{\mu N} \left\{ Y^N (\mu - 1) + \frac{Y^N}{\mu} \right\} - \psi.$$  \hspace{1cm} (653)

Inspection of (653) shows that $\chi < 0$ since $\mu_N = \frac{\partial \mu}{\partial N} < 0$ if $Y^N_{\mu} = \frac{\partial Y^N}{\partial \mu} < 0$ is not too large. This implies that an input inflow in the non-traded sector that raises $Y^N$ and thereby leads to profit opportunities results in firm entry (i.e, $N$ increases) which lowers the markup, $\mu$.

**Solutions for Sectoral Production Variables**

Since sectoral wages, sectoral capital labor-ratios, sectoral labor and sectoral output depend on the markup and thus on the number of firms, we have to plug back (651) into (642), (643), and (645) in order get the solutions for sectoral wages, sectoral capital labor-ratios, sectoral labor and sectoral output:

$$W^j = W^j \{ \bar{\lambda}, K, P, \mu [N(.)] \} \equiv W^j (\bar{\lambda}, K, P),$$  \hspace{1cm} (654a)

$$k^j = k^j \{ \bar{\lambda}, K, P, \mu [N(.)] \} \equiv k^j (\bar{\lambda}, K, P),$$  \hspace{1cm} (654b)

$$L^j = L^j \{ \bar{\lambda}, K, P, \mu [N(.)] \} \equiv L^j (\bar{\lambda}, K, P),$$  \hspace{1cm} (654c)

$$Y^j = Y^j \{ \bar{\lambda}, K, P, \mu [N(.)] \} \equiv Y^j (\bar{\lambda}, K, P),$$  \hspace{1cm} (654d)

where $Y^j_{\lambda} = \frac{\partial Y^j}{\partial \lambda} + \frac{\partial Y^j}{\partial \mu} \mu_N N_X$ with $X = \lambda, K, P$. The same logic applies to $W^j, k^j, L^j$.

**Sectoral Government Spending, $G^j$**
Making use of (H.1), the budget constraint can be solved for government expenditure in good \( j = T, N \) can be solved for overall government consumption as follows:

\[
G^N(t) = G^N(G(t)), \quad G^T = G^T(G(t)),
\]

(655)

where \( \frac{\partial G^N}{\partial \sigma} = \frac{\omega_G N}{P} \) and \( \frac{\partial G^T}{\partial \sigma} = \omega_{GT} \) with \( \omega_G \) corresponding to the share of expenditure on good \( j \) in total government spending.

**The Relative Price of Non Tradables, \( P \)**

Finally, we have to solve for the relative price of non tradables by using the non traded goods market clearing condition:

\[
Y^N = C^N + G^N + J^N.
\]

(656)

Remembering that the non traded input \( J^N \) used to produce investment goods is equal to \( P_j' J \), inserting short-run static solutions for \( C^N \) and \( Y^N \) given by (425) and (654d), respectively, substituting (446), i.e., \( J = K \left[ v(\cdot) + \delta_K + \frac{\kappa}{2} (v(\cdot))^2 \right] \), and inserting the solution for the number of firms described by (651) into the markup, i.e., \( \mu = \mu \left[ N \left( \lambda, K, P \right) \right] \), the non traded goods market clearing condition (656) can be rewritten as follows:

\[
Y^N \left( \lambda, K, P, \mu \left( N(\cdot) \right) \right) = C^N \left( \lambda, P \right) + G^N + P_j' K \left[ v(\cdot) + \delta_K + \frac{\kappa}{2} (v(\cdot))^2 \right].
\]

(657)

Using eq. (655), eq. (657) can be solved for the relative price of non tradables:

\[
P = P \left( \lambda, K, Q, G \right),
\]

(658)

with partial derivatives given by:

\[
P_K = \frac{\partial P}{\partial K} = \left( \frac{-Y^N}{\mu} + \frac{Y^N \mu N}{P^N N_K} \right) \frac{1}{\Psi P_j} + \frac{J}{\Phi} \leq 0,
\]

(659a)

\[
P_Q = \frac{\partial P}{\partial Q} = \frac{KvQ \left[ 1 + \kappa v(\cdot) \right]}{\Psi P_j} > 0,
\]

(659b)

\[
P_G = \frac{\omega_G N}{PP_j P_j \Psi} > 0,
\]

(659c)

where we set

\[
\Psi P_j = \left[ \left( \frac{Y^N}{\mu} - C^N \right) \frac{J}{P} + \frac{J \phi J (1 - \alpha J)}{P} - \frac{Y^N \mu N}{\mu N} \right] \frac{1}{P_j} - KvP \left[ 1 + \kappa v(\cdot) \right] > 0.
\]

(660)

with \( \mu_N < 0 \) and \( N_P > 0 \).

**K.3 Equilibrium Dynamics**

Remembering that the non traded input \( J^N \) used to produce the capital good is equal to \( P_j' J \), using the fact that \( J^N = Y^N - C^N - G^N \) and inserting \( I = \hat{K} + \delta_K K \), the capital accumulation equation can be rewritten as follows:

\[
\dot{K} = \frac{Y^N}{\mu} - \frac{C^N - G^N}{P_j} - \delta_K K - \frac{\kappa}{2} \left( \frac{I}{\hat{K}} - \delta_K \right)^2 K.
\]

(661)

Inserting short-run solutions for non traded output (654d), consumption in non tradables (425), optimal investment decision (649), and the number of firms (651) into the physical capital accumulation equation (661), and inserting the solution for the return on domestic capital (647) into the dynamic equation for the shadow value of capital stock (405e), the dynamic system reads as follows:

\[
\dot{K} \equiv T \left( K, P, Q, G \right) = \frac{Y^N [K, P(\cdot), \lambda]}{\mu [N, \cdot]} - C^N \left( \lambda, P(\cdot) \right) - G^N \left( G \right) P_j' \left( P(\cdot) \right)
\]

\[-\delta_K K - \frac{K}{2 \kappa} \left[ \frac{Q}{P_j' \left( P(\cdot) \right)} - 1 \right]^2,
\]

(662a)

\[
\dot{Q} \equiv \Sigma \left( K, P, Q, G \right) = \left( r^* + \delta_K \right) Q - \left[ R \left( K, P(\cdot) \right) + P_j \frac{\kappa}{2} v(\cdot) \left( v(\cdot) + 2 \delta_K \right) \right].
\]

(662b)
where we have inserted the solution for the number of firms (651) into (647) in order to solve for the domestic return of physical capital, i.e., \( R = R \{ \hat{\lambda}, K, P, \mu [N()] \} \equiv R (\hat{\lambda}, K, P) \).

As will be useful, let us denote by \( \Upsilon_K, \Upsilon_Q, \) and \( \Upsilon_P \) the partial derivatives evaluated at the steady-state of the capital accumulation equation w.r.t. \( K \) and \( Q \) (for given \( P \)), respectively, and \( P \):

\[
\left. \Upsilon_K \right|_{P \text{ fixed}} = \frac{\partial K}{\partial P} \bigg|_{P \text{ fixed}} = \left( \frac{Y^N}{\mu} - \frac{Y^N}{\mu} \frac{\mu_N}{\mu} N_K \right) \frac{1}{P^T} - \delta_K > 0, \quad (663a)
\]

\[
\left. \Upsilon_P \right|_{P \text{ fixed}} = \frac{\partial K}{\partial P} \bigg|_{P \text{ fixed}} = \left[ \left( \frac{Y^N}{\mu} - \frac{Y^N}{\mu} \frac{\mu_N}{\mu} N_P - C_P^N \right) + \frac{i^N \phi_J (1 - \alpha_J)}{P} \right] \frac{1}{P^T} > 0 (663b)
\]

where we used the fact that in the long-run, \( \dot{J}^N = \tilde{i}^N \) and \( \hat{Q} = P_J \left( \hat{P} \right) \). Partial derivatives evaluated at the steady-state for the marginal value of an additional unit of capital w.r.t. \( K \) and \( Q \) (for given \( P \)), respectively, and \( P \) which we denoted by \( \Sigma_K, \Sigma_Q, \) and \( \Sigma_P \) are identical to (455).

Remembering that \( J^T = (1 - \alpha_J) P_J J \), the current account equation is given by:

\[
\dot{B} \equiv \Xi (B, K, Q, G) = \star^* B + Y^T - C^T - G^T - (1 - \alpha_J) P_J J,
\]

\[
= \star^* B + Y^T - C^T - G^T - \left( \frac{1}{\alpha_J} \right) P \left( \frac{Y^N}{\mu} - C^N - G^N \right) \quad (664)
\]

where we used the fact that \( P_J J = Y^N - C^N - G^N \). As will be useful, let us denote by \( \Xi_K \) and \( \Xi_P \) the partial derivatives of the accumulation equation for traded bonds w.r.t. \( K \) (for given \( P \)) and \( P \):

\[
\left. \Xi_K \right|_{P \text{ fixed}} = \frac{\partial \dot{B}}{\partial K} \bigg|_{P \text{ fixed}} = Y^K - \left( \frac{1 - \alpha_J}{\alpha_J} \right) \hat{P} \left( \frac{Y^K}{\mu} - \frac{Y^K}{\mu} \frac{\mu_N}{\mu} N_K \right) \geq 0, \quad (665a)
\]

\[
\left. \Xi_P \right|_{P \text{ fixed}} = \frac{\partial \dot{B}}{\partial P} = \left( Y_K^T - C_P^T \right) - \left( \frac{1 - \alpha_J}{\alpha_J} \right) \hat{P} \left( \frac{Y^K}{\mu} - \frac{Y^K}{\mu} \frac{\mu_N}{\mu} N_P - C_P^N \right) - \phi_J \left( \frac{1 - \alpha_J}{\alpha_J} \right) \tilde{i}^N < 0, \quad (665b)
\]

where we used the fact that \( \frac{\partial (1 - \alpha_J)}{\partial P} = -1 \left[ \frac{1 - \alpha_J}{\alpha_J} - \phi_J \left( \frac{1 - \alpha_J}{\alpha_J} \right) \right] \) and at the steady-state, we have \( \dot{J}^N = \tilde{i}^N \) since capital installation costs are absent in the long run. The steps for the derivation of solutions for a temporary government shock are identical to those detailed in section H.

### L Calibration Procedure

In this section, we provide more details about the calibration to a representative OECD economy and to data from 16 OECD countries. Section A presents the source and construction of data.

#### L.1 Initial Steady-State

Normalizing total factor productivity (TFP henceforth) for the non traded sector \( Z^N \) to 1, the calibration reduces to 19 parameters: \( \star^*, \beta, \sigma_c, \epsilon, \alpha, \phi, \varphi, \phi_j, \varphi_j, \kappa, \delta_K, \theta_T, \theta_N, \)

\( Z^T, \omega_G \left( = \tilde{\omega}_G \right), \omega_{GN} \left( = \frac{\theta_N}{\theta_N} \right), \xi \), \( \chi \), and initial conditions \( B_0, K_0 \).

Since we focus on the long-run equilibrium, the tilde is suppressed for the purposes of
clarity. The steady-state of the open economy comprises 18 equations:

\[
C = (P_C \lambda)^{-\sigma_C},
\]
\[
L = (W \lambda)^{\sigma_L},
\]
\[
C^N = (1 - \varphi) \left( \frac{P}{P_C} \right)^{-\phi} C,
\]
\[
C^T = (1 - \varphi) \left( \frac{1}{P_C} \right)^{-\phi} C,
\]
\[
L^N = (1 - \vartheta) \left( \frac{W^N}{W} \right)^{\epsilon} L,
\]
\[
I^N = (1 - \varphi_J) \left( \frac{P_J}{P} \right)^{-\phi_J} I,
\]
\[
I^T = (1 - \varphi_J) \left( \frac{1}{P_J} \right)^{-\phi_J} I,
\]
\[
I = \delta_K K,
\]
\[
\frac{G}{Y} = \omega_G,
\]
\[
Z^T (1 - \theta^T) = P_J (r^* + \delta_K),
\]
\[
Z^T (1 - \theta^T) \left( k^T \right)^{-\theta^T} = PZ^N (1 - \theta^N) \left( k^N \right)^{-\theta^N},
\]
\[
Z^T \theta^T \left( k^T \right)^{1 - \theta^T} = W^T,
\]
\[
PZ^N \theta^N \left( k^N \right)^{1 - \theta^N} = W^N,
\]
\[
k^T L^T + k^N L^N = K,
\]
\[
Z^N L^N \left( k^N \right)^{1 - \theta^N} = C^N + G^N + I^N,
\]
\[
r^* B + Z^T L^T \left( k^T \right)^{1 - \theta^T} - C^T - G^T = 0,
\]
\[
and the intertemporal solvency condition
\[
B - B_0 = \Psi_1 (K - K_0),
\]
where we used the fact that at the steady-state \( I^j = J^j \) (with \( j = T, N \)), and we also have

\[
G^N = (\omega_G^N / P) G,
\]
\[
G^T = (1 - \omega_G^N) G,
\]
\[
P_C = \left[ \varphi + (1 - \varphi) (P)^{1-\phi} \right]^{\frac{1}{1-\phi}},
\]
\[
P_J = \left[ \varphi_J + (1 - \varphi_J) P^{1-\phi_J} \right]^{\frac{1}{1-\phi_J}},
\]
\[
W = \left[ \vartheta \ (W^T)^{\epsilon+1} + (1 - \vartheta) \ (W^N)^{\epsilon+1} \right]^{\frac{1}{\epsilon+1}},
\]
\[
Y = Y^T + PY^N = Z^T L^T \left( k^T \right)^{1 - \theta^T} + PZ^N L^N \left( k^N \right)^{1 - \theta^N}.
\]

Using (667), the system (666) jointly determines the following 18 variables \( C, L, C^N, C^T, L^N, L^T, I^N, I^T, I, G, k^T, k^N, W^T, W^N, K, P, B, \lambda \).

Some of the values of parameters can be taken directly from data, but others need to be endogenously calibrated to fit a set of an average OECD economy features. Among the 19 parameters, 4 parameters, i.e., \( \varphi, \varphi_J, \vartheta, \delta_K \) together with initial conditions \( (B_0, K_0) \) must be set in order to match key properties of a typical OECD economy. More precisely, the parameters \( \varphi, \varphi_J, \vartheta, \delta_K \) together with the set of initial conditions are set to target \( \alpha_C, \alpha_J, \alpha_L, \alpha_N, I/Y \). We denote by \( v_{Gj} = G^j/Y^j \) and \( v_{Jj} = J^j/Y^j \) the ratio of government spending and investment expenditure on good \( j \) to output in sector \( j \), respectively, and
$v_B = \frac{r^* \pi^T}{\psi_T}$ the ratio of interest receipts from traded bonds holding to traded output. The steady-state can be reduced to the following four equations:

\begin{align}
\frac{Y^T}{Y^N} (1 + v_B - v_{JT} - v_{GT}) &= \frac{\varphi_T}{1 - \varphi_T} P^\phi, \quad (668a)
\frac{Y^T}{Y^N} &= \frac{P}{Y} \left\{ (1 - (1 + \epsilon) \left( \frac{(1 - \phi_T)}{\frac{\varphi_T \theta_T}{\varphi_T \theta_N}} \right)) \right\} \Pi, \quad (668b)
(1 - \theta^T) \frac{Y^T}{Y} + (1 - \theta^N) \frac{P Y^N}{Y} &= P(1 - \varphi_T) (r^* + \delta_K) \frac{K}{Y}, \quad (668c)
v_B &= v_{B_0} + r^* \frac{Y}{Y^T} \Psi_1 \left( \frac{K}{Y} - v_K \right), \quad (668d)
\end{align}

where $v_{K_0} = \frac{K_0}{Y}$ and $\Pi$ is a term composed of parameters described by:

\begin{equation}
\Pi = \frac{(Z^T)^{1 + \epsilon}}{(N^T)^{1 + \epsilon}} \frac{\vartheta}{1 - \vartheta} (r^* + \delta_K) \left( \frac{\vartheta^T - \vartheta_N}{\vartheta_T - \vartheta_N} \right) (1 + \epsilon)
\end{equation}

The system (668) consisting of four equations determine $P$, $Y^T/Y^N$, $K/Y$ and $v_B$. The four equations (668a)-(668d) described the goods market equilibrium, the labor market equilibrium, the resource constraint for capital, and the intertemporal solvency condition.

Dividing the market clearing condition for the traded good (666q) by the market clearing condition for the non traded good (666p) and equating the resulting expression with the demand of tradables in terms of non tradables obtained by calculating the ratio of (666d) to (666c), i.e., \( \frac{C_T}{C_N} = \frac{P}{\psi_T} P\phi \), leads to the goods market equilibrium (668a). The derivation of the labor market equilibrium requires more steps. As mentioned below, we assume that the aggregator function for inputs of the investment good is Cobb-Douglas since data suggest that $\phi_J = 1$. In this case, the investment price index simplifies, i.e., $P_J = (P)^{1 - \varphi_J}$. First, combining (666k) and (666l) leads to:

\begin{equation}
\frac{(k^N)^{1 - \theta^N}}{(k^T)^{1 - \theta^T}} = P(\frac{1 - \theta^N}{\varphi_T^N}) \left[ P_l (r^* + \delta_K) \right]^{1 - \theta^T} \frac{Z^N (1 - \theta_N)}{\frac{\varphi_T^N}{\varphi_T}} \frac{1 - \theta^N}{\varphi_T \theta_N} \frac{Z^N (1 - \theta_N)}{\frac{\varphi_T^N}{\varphi_T}} \frac{1 - \theta^N}{\varphi_T}.
\end{equation}

Dividing (666f) by (666c) leads to the supply of hours worked in the traded sector relative to the non traded sector, i.e., \( \frac{L^T}{L^N} = \frac{\vartheta_T}{1 - \vartheta_T} \Omega - \epsilon \). Dividing (666n) by (666m) leads to the relative wage, i.e., $\Omega = \frac{\mu P^T (k^N)^{1 - \theta^N}}{Z^T (k^T)^{1 - \theta^T}}$. Inserting the latter expression into the former and using the production functions for the traded sector and non traded sectors which imply $L^T = \frac{Y^T}{Z^T (k^T)^{1 - \theta^T}}$ and $L^N = \frac{Y^N}{Z^N (k^N)^{1 - \theta^T}}$, one obtains:

\begin{equation}
\frac{Y^T}{Y^N} = \frac{\vartheta_T}{1 - \vartheta_T} \left( \frac{Z^T}{Z^N} \right)^{1 + \epsilon} P^{-\epsilon} \left( \frac{\theta^T}{\theta^N} \right) \epsilon \left[ \left( \frac{k^T}{k^N} \right)^{1 - \theta^T} \left( k^N \right)^{1 - \theta^T} \right].
\end{equation}

Inserting (670) into the above expression leads to the labor market equilibrium (668b) while we set $\Pi$ to eq. (669) in order to write the equation in compact form. To determine (668c), use the fact that $K^J = k^J L^J$, multiply both sides of (666a) by $\frac{R}{\psi_T}$ where $R = P_J (r^* + \delta_K)$ is the capital rental cost; we get:

\begin{equation}
\frac{R K^T Y^T}{Y^T} + \frac{R K^N P Y^N}{Y} = \frac{R K}{Y}.
\end{equation}
Using the fact that the capital income share \( \frac{RK^J}{Y^T} \) in sector \( j \) is equal to \((1 - \theta^j)\) and remembering that the investment price index reduces to \( P_j = (P)^{1-\varphi^j} \), one obtains the resource constraint for capital described by eq. (668c). Finally, to get (668d), multiply both sides of (666r) by \( \frac{\epsilon^j}{\epsilon^T} \), denote the ratio of interest receipts from the initial stock of traded bonds to traded output by \( \upsilon_{B_0} = \frac{\epsilon^j B_0}{\epsilon^T Y^T} \) and the ratio of the initial capital stock to GDP by \( \upsilon_K = \frac{K_0}{Y} \) leads to eq. (668d) that describes the intertemporal solvency condition.

Because the ratios we wish to target are different from the macroeconomic aggregates, i.e., \( P, Y^T/Y^N, K/Y \) and \( u_B \), that are jointly determined by the system of equations (668), we have to relate the latter ratios with the former. First, the relative price of non tradables \( P \) determines the non tradable content of consumption expenditure by setting \( \varphi \):

\[
\alpha_C = \frac{(1 - \varphi) P^{1-\varphi}}{\varphi + (1 - \varphi) P^{1-\varphi}}.
\]

The ratio \( K/Y \) along with the relative price of non tradables, \( P \), determines the investment-to-GDP ratio \( P_j I/Y \) by setting \( \delta_K \) (see eq. (666i)):

\[
P_j I/Y = P_j \delta_K K/Y.
\]

The ratio of net interest receipts from traded bonds holding to traded output, i.e., \( \upsilon_B \), determines the ratio of net exports to traded output, i.e. \( \upsilon_{NX} = \frac{NX}{Y^T} \) with \( NX = Y^T - C^T - G^T - J^T \); dividing both sides of the traded goods market clearing condition (666q) leads to:

\[
\upsilon_{NX} = -\upsilon_B.
\]

Finally, we show that \( Y^T/Y^N \) (together with \( P \)) determines \( L^N/L \) by setting \( \theta \). To do so, using the definition of the aggregate wage index (415), the ratio of the aggregate wage to the non traded wage can be rewritten as follows:

\[
\left( \frac{W}{W^N} \right)^{\epsilon+1} = \frac{\vartheta (W^T)^{\epsilon+1} + (1 - \vartheta) (W^N)^{\epsilon+1}}{(W^N)^{\epsilon+1}},
\]

and by solving, we get

\[
\frac{W}{W^N} = \left[ \frac{\vartheta (W^T)}{W^N} + (1 - \vartheta) \right]^{\frac{1}{\epsilon+1}}.
\]

Since \( \theta^j \) is the labor income share in sector \( j \), the ratio of the traded wage to the non traded wage can be written as follows:

\[
\frac{W^T}{W^N} = \frac{\theta^T}{\theta^N} \frac{1}{Y^T} \frac{L^N}{P Y^N L^T}.
\]

Dividing (666f) by (666e) leads to a positive relationship between the supply of hours worked to the traded sector relative to the non traded sector and the traded wage relative to the non traded wage, i.e., \( \frac{L^T}{L^N} = \frac{\theta^T}{1 - \vartheta} \left( \frac{W^T}{W^N} \right)^\epsilon \). Substituting the latter equation, eq. (675) can be solved for \( W^T/W^N \), i.e.,

\[
\frac{W^T}{W^N} = \left[ 1 - \vartheta \left( \frac{\theta^T}{\theta^N} \frac{1}{P Y^N} \right) \right]^{\frac{1}{\epsilon+1}}.
\]

Additionally, since \( \alpha_L = \frac{W^N L^N}{W L} = (1 - \vartheta) \left( \frac{W^N}{W} \right)^{\epsilon+1} \), the share of hours worked in total hours worked is governed by the following optimal rule:

\[
\frac{L^N}{L} = (1 - \vartheta) \left( \frac{W^N}{W} \right)^\epsilon,
\]

\[
= (1 - \vartheta) \left( \frac{W}{W^N} \right)^{-\epsilon}.
\]
Inserting (676) into (674) and plugging the resulting expression into (677) gives us a relationship between the non tradable content of labor and the ratio $Y^T/Y^N$ (together with $P$):

$$\frac{L^N}{L} = (1 - \vartheta) \left[ \vartheta \left( \frac{\theta^T}{\theta^N} \frac{1}{P} \frac{Y^T}{Y^N} \right) + (1 - \vartheta) \right]^{-\frac{1}{\varphi + 1}},$$

$$= (1 - \vartheta)^{\frac{1}{\varphi + 1}} \left[ \frac{\theta^T}{\theta^N} \frac{1}{P} \frac{Y^T}{Y^N} + 1 \right]^{-\frac{1}{\varphi + 1}}. \quad (678)$$

According to (678), given $Y^T/Y^N$ and $P$, setting $\vartheta$ allows us to target the ratio $L^N/L$ found in the data.

L.2 Calibration to a Representative OECD Economy

To calibrate our model, we estimated a set of parameters so that the initial steady state is consistent with the key empirical properties of a representative OECD economy. This section provides more details about how we calibrate the model to match the key empirical properties of a representative OECD economy. Because we consider an open economy setup with traded and non traded goods, we calculate the non tradable content of GDP, employment, consumption, gross fixed capital formation, government spending, labor compensation and the productivity in tradables in terms of non tradables, for all countries in our sample, as summarized in Table 5. To capture the key properties a typical OECD economy which is chosen as the baseline scenario, we take unweighted average values shown in the last line of Table 5. Columns 12-14 of Table 5 also report government spending and investment as a share of GDP along with the aggregate labor income share.

We first describe the parameters that are taken directly from the data; we start with the preference parameters shown in panel A of Table 27:

- One period in the model is a year.
- The world interest rate, $r^\star$, equal to the subjective time discount rate, $\beta$, is set to 4%.
- We assume that utility for consumption is logarithmic and thus set the intertemporal elasticity of substitution for consumption, $\sigma_C$, to 1.
- Next, we turn to the Frisch elasticity of labor supply. We set the intertemporal elasticity of substitution for labor supply $\sigma_L$ to 0.4, in line with the evidence reported by Fiorito and Zanella [2012], but conduct a sensitivity analysis with respect to this parameter.
- The elasticity of labor supply across sectors, $\epsilon$, which captures the degree of labor mobility is set to 0.75 in line with the average of our estimates shown in the last column of Table 5. Our estimates display a wide dispersion across countries and we therefore conduct a sensitivity analysis with respect to this parameter. Excluding estimates of $\epsilon$ for Denmark and Norway which are not statistically significant at 10%, estimates of $\epsilon$ range from a low of 0.22 for the Netherlands to a high of 1.39 for the United States and 1.64 for Spain. Hence, we allow for $\epsilon$ to vary between 0.22 and 1.64 in the sensitivity analysis.
- Building on our panel data estimations (see section A.3), we set the elasticity of substitution (in consumption) between traded and non traded goods to 0.77 in the baseline calibration, in line with the unweighted average value shown in the last line of column 15 of Table 5.

Section A.4 presents the empirical strategy and contains the details of derivation of the relationship we explore empirically.

We derive a testable equation by combining first-order conditions for relative demand and relative supply for tradables in terms of non tradables. Details of derivation of the equation we explore empirically can be found in section A.3. We explore empirically two variants of the testable equation, considering alternatively the ratio of sectoral value added or the ratio of sectoral labor compensation. Estimates of $\phi$
We set the elasticity of substitution, $\phi_J$, in investment between traded and non traded inputs to 1, in line with the empirical findings documented by Bems [2008] for OECD countries.

We also consider a more general specification for preferences which are assumed to be non separable in consumption and labor. The functional form is taken from Shimer [2011]:

$$\frac{C^{1-\sigma}V(L)^{\sigma}-1}{1-\sigma}, \quad \text{if} \quad \sigma \neq 1, \quad V(L) \equiv \left(1 + (\sigma - 1) \frac{\sigma_L}{1 + \sigma_L} L^{\frac{1+\sigma_T}{\sigma_L}}\right).$$

Setting $\sigma = 1$, preferences are separable in consumption and labor, as in (399). When investigating the implications of non separability in preferences, we set $\sigma = 2$ while we keep other parameters unchanged.

We pursue with the non-tradable content of consumption, investment and government expenditure, employment, along with sectoral labor income shares and relative productivity of tradables shown in the last line of Table 5 that reports the average of our estimates while panel B of Table 27 displays the value of parameters we choose to calibrate the model:

- The weight of consumption in non tradables $1-\varphi$ is set to 0.51 to target a non-tradable content in total consumption expenditure (i.e. $\alpha_C$) of 53%.
- In order to target a non tradable content of labor of 67% which corresponds to the 16 OECD countries’ unweighted average shown in the last line of Table 5, we set the weight of labor supply to the traded sector in the labor index $L(,)$, $1-\vartheta$, to 0.68.
- We choose a value for the weight of non traded inputs in the investment aggregator function $J(,)$, $1-\varphi_J$, of 0.64 which allows us to obtain a non tradable content of investment expenditure of 64%.
- In accordance with our estimate shown in the last line of Table 5, we choose a non tradable content of government spending, $\omega_{GT} = \frac{PGN}{G}$, of 90%; by construction, we have a share of government consumption on tradables in total government spending, $\omega_GT = 1 - \omega_{GN}$, of 10%.
- Columns 9 and 10 of Table 5 give the labor income share of the traded and the non traded sector for the sixteen OECD countries in our sample. Labor income shares $\theta^T$ and $\theta^N$ average respectively to 0.60 and 0.67. Because average values suggest that the non traded sector is relatively more labor intensive than the traded sector, in the baseline calibration, we choose values for $\theta^T$ and $\theta^N$ so that $\theta^T < \theta^N$. The figures also show substantial dispersion across countries as the labor income share in the traded sector varies from a low of 0.38 in Norway to a high of 0.71 for Italy. Moreover, the labor income share in the traded sector, $\theta^T$, is higher than that in the non traded sector, $\theta^N$, for two countries, namely France and Italy. Thus, we also conduct a sensitivity analysis by considering a situation where the traded sector is more labor intensive than the non traded sector. When excluding France and Italy, the values of $\theta^T$ and $\theta^N$ average 0.58 and 0.67, respectively. In the baseline calibration, we set $\theta^T$ and $\theta^N$ to 0.58 and 0.68 which correspond roughly to the average for countries with $k^T > k^N$ and are consistent with an aggregate labor income share of 64%, as shown in column 14 of Table 5. Formally, the aggregate labor income share, denoted by $\theta$, is a value-weighted average of the sectoral labor income shares, i.e., $\theta = \frac{\theta^TY^T}{Y} + \frac{\theta^NY^N}{Y}$. When we consider a traded sector that is relatively more labor intensive than the non traded sector, i.e., $k^N > k^T$, we use reverse but symmetric values and thus set $\theta^T = 0.68$ and $\theta^N = 0.58$.
- We assume that traded firms are 28 percent more productive than non traded firms in line with our estimates; we thus normalize $Z^N$ to 1 and set $Z^T$ to 1.28.

for Italy are negative for both variants while for Belgium, only the estimate of the elasticity of substitution in consumption between tradables and non tradables when exploring empirically the second variant of the testable equation is statistically significant (see Table 4). Excluding estimates of $\phi$ for Italy which are negative and considering a value of 0.795 for Belgium, the elasticity of substitution $\phi$ averages to 0.77.
We describe below the choice of parameters displayed in panel C of Table 27 characterizing macroeconomic variables such as investment, government spending and the balance of trade of a typical OECD economy:

- As shown in the last line of column 13 of Table 5, government spending as a percentage of GDP averages 20% and thus we set \( \omega_G = \frac{G}{Y} \) to 0.2.

- In order to target an investment-to-GDP ratio, \( \omega_J = \frac{P_J I}{Y} \), of 21% as shown in the last line of column 12 of Table 5, we set the rate of physical capital depreciation, \( \delta_K \), to 6%.

- We choose the value of parameter \( \kappa \) so that the elasticity of \( \frac{I}{K} \) with respect to Tobin’s \( q \), i.e., \( \frac{Q}{P_J} \), is equal to the value implied by estimates in Eberly, Rebelo, and Vincent [2008]. The resulting value of \( \kappa \) is equal to 17.\textsuperscript{76}

- Finally, we choose initial values for \( B_0 \) and \( K_0 \) for the ratio of net exports to traded output to be nil at the initial steady-state, i.e., \( v_{NX} \simeq 0 \).

Investment- and government spending-to-GDP ratios along with balanced trade endogenously determine the consumption-to-GDP ratio. More precisely, since GDP is equal to the sum of its demand components, remembering that at the steady-state \( I = J \), we thus have the following accounting identity, \( Y = P_C C + P_J I + G + NX \). Dividing both sides by \( Y \) and remembering that net exports are nil, i.e., \( NX = 0 \), the consumption-to-GDP ratio denoted by \( \omega_C = \frac{P_C C}{Y} \) is thus equal to 59%:

\[
\omega_C = \frac{P_C C}{Y} = 1 - \left( \omega_J + \omega_G + \frac{NX}{Y} \right) = 59%,
\]

where \( \omega_J = \frac{P_J I}{Y} = 21\% \), \( \omega_G = \frac{G}{Y} = 20\% \), and \( NX = 0 \).

It is worthwhile mentioning that the non tradable content of GDP is endogenously determined by the non tradable content of consumption, \( \alpha_C \), of investment, \( \alpha_J \), and of government expenditure, \( \omega_G N \), along with the consumption-to-GDP ratio, \( \omega_C \), the investment-to-GDP ratio, \( \omega_J \), and government spending as a share of GDP, \( \omega_G \). More precisely, dividing the non traded good market clearing condition (666p) by GDP, \( Y \), leads to an expression that allows us to calculate the non tradable content of GDP:

\[
\frac{Y^N}{Y} = \omega_C \alpha_C + \omega_J \alpha_J + \omega_G N \omega_G = 63%,
\]

where \( \omega_C = 59\% \), \( \alpha_C = 53\% \), \( \omega_J = 21\% \), \( \alpha_J = 64\% \), \( \omega_G N = 90\% \), and \( \omega_G = 20\% \). According to (681), the values we target for the non tradable content of consumption, investment and government spending along with the consumption-, investment- and government spending-to-GDP ratios are consistent with a non tradable content of GDP of 63% found in the data, as reported in the last line of column 1 of Table 5.

In order to capture the dynamic adjustment of government consumption, we assume that the response of government consumption in percent of GDP is governed by the following dynamic equation:\textsuperscript{77}

\[
\frac{dG(t)}{Y} = \frac{G(t) - \hat{G}}{Y} = \left[ e^{-\xi t} - (1 - g) e^{-\chi t} \right],
\]

where \( g \) parametrizes the exogenous fiscal shock while \( \xi > 0 \) and \( \chi > 0 \) parametrize the persistence of the response of government consumption along with the pattern of its dynamic adjustment. We present below the parameters related to the endogenous response of government spending to an exogenous fiscal shock which are summarized in panel D of Table 27:

\textsuperscript{76}Eberly, Rebelo, and Vincent [2008] run the regression \( \frac{I}{K} = \alpha + \beta \cdot \ln(q) \) and obtain a point estimate for \( \beta \) of 0.06. In our model, the steady-state elasticity of \( \frac{I}{K} \) with respect to Tobin’s \( q \) is \( \frac{1}{\kappa} \). Equating \( \frac{1}{\kappa} \) to 0.06 gives a value for \( \kappa \) of 17.

\textsuperscript{77}More technical details can be found in section H.1.
• We investigate the effects of a rise in government consumption by 1 percentage point of GDP and thus set $g$ to 0.01.

• We choose values of $\xi$ and $\chi$ in order to account for the dynamic adjustment of government consumption. Data indicate that the endogenous response of government spending to an exogenous fiscal shock reaches a maximum at time $t = 1$:

$$dG(1) = G(1) - \tilde{G} = g' = \left[ e^{-\xi} - (1 - g) e^{-\chi} \right].$$

(683)

Differentiating (682) w.r.t. time leads to:

$$\dot{G}(t) = -\left[ \xi e^{-\xi t} - \chi (1 - g) e^{-\chi t} \right].$$

(684)

When government spending reaches its maximum value, we have $\dot{G}(1) = 0$. Setting $t = 1$ into (684) gives:

$$\dot{G}(1) = -\left[ \xi e^{-\xi} - \chi (1 - g) e^{-\chi} \right] = 0$$

(685)

Using the fact that $g = 0.01$ and $g' = 0.01126548$, the system consisting of eq. (683) and eq. (685) jointly determine the values of $\xi$ and $\chi$ which allow us to capture the endogenous response of government spending to an exogenous fiscal shock by $g \times 100$ percentage points of GDP; we set $\xi = 0.408675$ and $\chi = 0.415722$.

• While government purchases both non traded goods, $G^N$, and traded goods, $G^T$, our VAR evidence suggest that the rise in government consumption is strongly biased toward non traded goods as the relative size the non traded sector rises significantly. When we simulate the model, we thus consider a rise in government consumption by 1 percentage point of GDP which is split between non tradables and tradables in accordance with their respective share in government expenditure at 90% and 10%, respectively. Formally, we have:

$$\frac{dG(t)}{Y} = \frac{dG^N(t)}{Y} + (1 - \omega_{GN}) \frac{dG^T(t)}{Y},$$

(686)

where $dG(t) = G(t) - \tilde{G}$ and $dG^i(t) = G^i(t) - \tilde{G}^i$.

M More Numerical Results

In this section, we provide more numerical results:

• First, while in the main text, we restrict attention to impact responses to a government spending shock when we conduct the sensitivity analysis for reasons of space, we provide below more numerical results. In particular, in subsection M.1, we report the cumulative responses over a two-year and a four-year horizon. In order to assess to what extent our results depend on the assumption of separability in preferences between consumption and labor, we also consider a more general specification for preferences. Additionally, while in the main text, we assume that capital can move freely across sectors along with workers’ costs of switching across sectors, in subsection M.1, we investigate the implications of imperfect mobility of capital across sectors. Since this feature merely affects quantitatively the responses, we relegate these results in the Technical Appendix as we believe they are secondary.

• Second, in the main text, we contrast the predictions of our baseline model allowing for imperfect mobility of labor across sectors along with adjustment costs to capital accumulation with those obtained in a model imposing perfect mobility of labor and abstracting from capital installation costs. Because both features play a pivotal role,
### Table 27: Baseline Parameters (Representative OECD Economy)

<table>
<thead>
<tr>
<th>Definition</th>
<th>Value</th>
<th>Sensitivity</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Period of time</strong></td>
<td>year</td>
<td>year</td>
<td>data frequency</td>
</tr>
<tr>
<td><strong>A. Preferences</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subjective time discount rate, $\beta$</td>
<td>4%</td>
<td>4%</td>
<td>equal to the world interest rate</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution for consumption, $\sigma_C$</td>
<td>1</td>
<td>1</td>
<td>logarithmic utility function for consumption</td>
</tr>
<tr>
<td>Elasticity of labor supply at the extensive margin, $\sigma_L$</td>
<td>0.4</td>
<td>0.2-1</td>
<td>Fiorito and Zanella [2012]</td>
</tr>
<tr>
<td>Separability vs. non separability in preferences, $\sigma$</td>
<td>1</td>
<td>2</td>
<td>Shimer [2011]</td>
</tr>
<tr>
<td>Elasticity of substitution between $C^T$ and $C^N$, $\phi$</td>
<td>0.77</td>
<td>0.77</td>
<td>our estimates (KLEMS [2011], OECD Economic Outlook)</td>
</tr>
<tr>
<td>Elasticity of substitution between $J^T$ and $J^N$, $\phi_J$</td>
<td>1</td>
<td>1</td>
<td>Bems [2008]</td>
</tr>
<tr>
<td><strong>B. Non tradable share</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight of consumption in non traded goods, $1 - \varphi$</td>
<td>0.51</td>
<td>0.51</td>
<td>set to target $\alpha_C = 53%$ (United Nations [2011])</td>
</tr>
<tr>
<td>Weight of labor supply to the non traded sector, $1 - \vartheta$</td>
<td>0.15</td>
<td>0.15</td>
<td>set to target $L^N/L = 67%$ (KLEMS [2011])</td>
</tr>
<tr>
<td>Weight of non traded investment, $1 - \varphi_J$</td>
<td>0.64</td>
<td>0.64</td>
<td>set to target $\alpha_J = 64%$ (OECD Input-Output database [2012])</td>
</tr>
<tr>
<td>Non Tradable content of government expenditure, $\omega_G^N$</td>
<td>0.90</td>
<td>0.90</td>
<td>our estimates (COFOG, OECD [2017])</td>
</tr>
<tr>
<td>Labor income share in the non traded sector, $\theta^N$</td>
<td>0.68</td>
<td>0.58</td>
<td>our estimates (EU KLEMS [2011] and OECD STAN databases)</td>
</tr>
<tr>
<td>Labor income share in the traded sector, $\theta^T$</td>
<td>0.58</td>
<td>0.68</td>
<td>our estimates (EU KLEMS [2011] and OECD STAN databases)</td>
</tr>
<tr>
<td>Labor productivity index for the traded sector, $Z^T$</td>
<td>1.28</td>
<td>1.28</td>
<td>our estimates (KLEMS [2011])</td>
</tr>
<tr>
<td><strong>C. GDP demand components</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Physical capital depreciation rate, $\delta_K$</td>
<td>6%</td>
<td>6%</td>
<td>set to target $\omega_J = 21%$ (Source: OECD Economic Outlook Database)</td>
</tr>
<tr>
<td>Parameter governing capital adjustment cost, $\kappa$</td>
<td>17</td>
<td>0</td>
<td>set to match the elasticity $I/K$ to Tobin’s q (Eberly et al. [2008])</td>
</tr>
<tr>
<td>Government spending as a ratio of GDP, $\omega_G$</td>
<td>20%</td>
<td>20%</td>
<td>our estimates (Source: OECD Economic Outlook Database)</td>
</tr>
<tr>
<td><strong>D. Government Spending Shock</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exogenous fiscal shock, $g$</td>
<td>1%</td>
<td>1%</td>
<td>To generate $dG(0)/Y = 1%$</td>
</tr>
<tr>
<td>Persistence and shape of endogenous response of $G$, $\xi$</td>
<td>0.408675</td>
<td>0.408675</td>
<td>set to target $dG(1) = g'$ and $G(1) = 0$</td>
</tr>
<tr>
<td>Persistence and shape of endogenous response of $G$, $\chi$</td>
<td>0.415722</td>
<td>0.415722</td>
<td>set to target $dG(1) = g'$ and $G(1) = 0$</td>
</tr>
</tbody>
</table>
we report in the main text the impact responses to a government spending shock in a model either imposing perfect mobility of labor across sectors or abstracting from capital installation costs. We report below, in subsection M.2, the dynamic adjustment of a model either imposing perfect mobility of labor across sectors or abstracting from capital installation costs and contrast the predictions with those obtained in the baseline model. The model imposing perfect mobility of labor while assuming capital installation costs or the other way around both fail to account for the evidence at an aggregate and a sectoral level while the latter performs better than the former in reproducing the evidence, in particular for sectoral variables.

- Third, in section 5.3 of the main text, we plot the simulated responses of output shares of tradables and non tradables against the degree of labor mobility across sectors and contrast model’s predictions with estimated cross-country relationships for both tradables and non tradables. To look at the cross-country differences in the sectoral impact of a government spending shock empirically, we estimate the same VAR model, i.e., $x_{it}^{S,j}$, as for the whole sample, but for a single a country at a time. To look at the cross-country differences in the sectoral impact of a government spending shock numerically, we calibrate the baseline model to each OECD country in our sample. While in the main text, we only show the scatter-plots, in subsection M.3, we report both estimated and simulated impact responses to a government spending shock of output shares of tradables and non tradables.

**M.1 Numerical Results for a Representative OECD Economy**

Table 28 reports impact effects while Table 29 shows cumulative responses over a two- and four-year horizon following a rise in government consumption by 1 percentage point of GDP. Column 1 of Tables 28 and 29 shows the effects of a government spending shock from our VAR model for comparison purposes while columns 2-14 report simulated responses. We conduct a sensitivity analysis with respect to a number of parameters, including the labor income share of sector $j$, $\theta_j$, the elasticity of labor supply across sectors, $\epsilon$, the parameter $\kappa$ that governs the magnitude of adjustment costs to capital accumulation, the Frisch elasticity of labor supply, $\sigma_L$, and the parameter $\sigma > 0$ that determines the substitutability between consumption and leisure. We provide more details below:

- In columns 2 and 3, we impose perfect mobility of labor across sectors, i.e., $\epsilon \to \infty$. In column 2, we abstract from capital installation costs and thus set $\kappa = 0$ while in column 3, we consider adjustment costs to physical capital accumulation and thus set $\kappa = 17$.
- Column 4 reports results from our baseline model with imperfect mobility of labor across sectors, setting $\epsilon$ to 0.75, while capital accumulation is assumed to be subject to adjustments costs with $\kappa = 17$.
- In columns 5 and 6, we keep unchanged $\kappa$ and investigate the effects of a government spending shock when the degree of labor mobility across sectors is low, i.e., $\epsilon$ is set to 0.22, and when the elasticity of labor supply across sectors is high, i.e., $\epsilon$ is set to 1.64.
- In column 7, we investigate the sensitivity of our results to the Frisch elasticity of labor supply which is raised from 0.4 to 1.
- Column 8 shows results when we allow for imperfect mobility of labor across sectors, setting $\epsilon$ to 0.75, while we abstract from adjustment costs to capital accumulation, and thus set $\kappa$ to 0.
- Column 9 reports results when we relax the assumption of separability in preferences between consumption and labor, setting $\sigma$ to 2.
- In column 10 (IMK), we keep unchanged $\epsilon = 0.75$, $\sigma_L = 0.4$, $\sigma = 1$, $\kappa = 17$ and we allow for imperfect mobility of capital across sectors, setting the elasticity of capital
supply across sectors, \( \eta \), to 0.75, and the weight \( 1 - \zeta \) of capital supply to the non traded sector in the aggregate capital index \( K(\cdot) \) to 0.68 in order to target a non tradable content of capital income of 58%, in line with our estimates.

- While from column 2 to column 10, we assume that the non traded sector is relatively more labor intensive than the traded sector, and thus set \( \theta^N \) to 0.68 and \( \theta^T \) to 0.58, from column 11 to column 13, we explore the case where the non traded sector is relatively more capital intensive and thus choose reverse and symmetric values for the sectoral labor income shares, i.e., we set \( \theta^N \) to 0.58 and \( \theta^T \) to 0.68.

- While column 12 reports our baseline model’s predictions when \( \theta^T > \theta^N \), in column 11, we set \( \kappa = 0 \) and let \( \epsilon \) tend toward infinity, and in column 13 (IMK), we allow for both imperfect mobility of labor and capital across sectors, and thus set \( \eta \) to 0.75.

In column 8, we relax the assumption of separability in preferences between consumption and labor by considering a functional form which is taken from Shimer [2011]:

\[
\frac{C^{1-\sigma}V(L)^{\sigma}-1}{1-\sigma}, \quad \text{if} \quad \sigma \neq 1, \quad V(L) \equiv \left(1 + (\sigma - 1) \frac{\sigma L}{1 + \sigma L} \right)^{1+\sigma L}. \quad (687)
\]

These preferences are characterized by two crucial parameters: \( \sigma_L \) is the Frisch elasticity of labor supply, and \( \sigma > 0 \) determines the substitutability between consumption and leisure; if \( \sigma > 1 \), the marginal utility of consumption increases in hours worked. In contrast, setting \( \sigma = 1 \) implies that preferences are separable in consumption and labor, as in (6). When we investigate the implications of non separability in preferences, we set \( \sigma = 2 \) while keeping other parameters unchanged.

Column 9 shows results when \( \sigma \) is set to 2. As can be seen in column 9, non separability in preferences between consumption and labor amplifies the rise in the real consumption wage while hours and real GDP increase less. Additionally, the open economy runs a larger current account deficit. Intuitively, because non separability in preferences between consumption and labor increases the disutility from working, agents are less willing to supply labor while demanding higher wages. Because consumption increases with the aggregate wage, agents lower their expenditure less. Thus, private savings decline further, which in turn amplifies the decline in the current account. As the crowding out of private consumption is less, the relative price of non tradables appreciates by a larger amount, thus amplifying the responses of sectoral output shares. While the extension of the baseline model to non separability in preferences somewhat improves its performance in reproducing the responses of several sectoral variables, the extended model tends to substantially overstate the contraction in the traded sector and to overpredict the rise in the relative wage. In contrast, all simulated impact responses from the baseline model assuming separability in preferences lie within the confidence interval.

In the last three columns of Table 2, we investigate whether our conclusions hold if we assume a non traded sector that is more capital intensive than the traded sector. While the predictions of the model are very sensitive to sectoral labor income shares if we let \( \epsilon \) tend toward infinity, results are almost unaffected for the baseline model whether \( \theta^T < \theta^N \) or \( \theta^T > \theta^N \). As shown in column 11, the model imposing perfect mobility of labor fails to account for the evidence along a number of dimensions. In particular, the simulated responses of sectoral output shares are more than four times greater than those reported from the VAR model. The reason is that imposing perfect mobility makes labor and thus sectoral output highly sensitive to a change in relative price. Because investment is crowded in, the subsequent excess demand in the non traded goods market causes the relative price of non tradables to appreciate, thus leading to dramatic changes in the relative size of sectors. Since the model’s predictions reported in column 12 are similar to those shown in column 4, they do not merit further comment.

In columns 10 and 13 of Table 28, we extend the baseline model with imperfect mobility of labor along with capital installation costs to imperfect mobility of capital. A shortcut to generate imperfect capital mobility is to assume limited substitutability in capital across sectors. Along the lines of Horvath [2000] who introduce limited substitutability of hours...
worked, we assume that capital in the traded and the non traded sectors are aggregated by means of a CES function:

\[ K(K^T, K^N) = \left[ \zeta^{-\frac{1}{\eta}} (K^T)^{\frac{\eta+1}{\eta}} + (1 - \zeta)^{-\frac{1}{\eta}} (K^N)^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}, \]  

(688)

where \( 0 < \zeta < 1 \) is the weight of capital supply to the traded sector in the aggregate capital index \( K(.) \) and \( \eta \) measures the ease with which capital in the traded and the non traded sector can be substituted for each other and thereby captures the degree of capital mobility across sectors. The case of perfect capital mobility is nested under the assumption that \( \eta \) tends towards infinity; in this case, (688) reduces to \( K = K^T + K^N \) which implies that capital is perfectly substitutable across sectors. When \( \eta < \infty \), sectoral capital goods are no longer perfect substitutes. More specifically, as \( \eta \) becomes smaller, capital mobility across sectors becomes lower as investors perceive a higher cost of shifting capital and therefore become more reluctant to reallocate capital across sectors.

Panels A and B of Table 28 show impact effects of a government spending shock for GDP, investment and the current account along with labor market variables such as total hours worked and the real consumption wage. Panels C and D of Table 28 summarize the theoretical responses of sectoral variables for the labor and product markets. Because the results shown in column 10 when we allow for imperfect mobility of capital across sectors do not improve the performance of the model with imperfect mobility of labor in replicating the evidence, or provide major additional information on the fiscal transmission as the conclusions are similar whether we allow or not for imperfect mobility of capital across sectors, to save space we do not present them in the main text and relegate these results in the Technical Appendix. Panels E and F of Table 29 report cumulative responses over a two- and a fourth-year horizon for aggregate and selected sectoral variables.

### M.2 Numerical Results for a Representative OECD Economy

In the main text, see section 5.2, we show that the model is successful in replicating both aggregate and sectoral effects of a government spending shock as long as we allow for both imperfect mobility of labor across sectors, captured by \( \epsilon \), along with adjustment costs to capital accumulation, captured by the parameter \( \kappa \). Table 2 contrasts impact effects in the baseline scenario with a number of alternative scenarios where we impose perfect mobility of labor across sectors and abstract from capital installation costs (column 2), we consider capital installation costs along with perfect mobility of labor across sectors (column 3), and we allow for imperfect mobility of labor across sectors but abstract from capital installation costs (column 8). Figures 5 and 6 in the main text display the model predictions for the aggregate and sectoral effects, respectively, of a government spending shock under imperfect (solid black line) and perfect mobility of labor across sectors (dotted black line) together with the respective VAR evidence (solid blue line). For reason of space, we do not contrast the dynamic adjustment of the baseline model with that obtained from a model with perfect mobility of labor while assuming capital installation costs or alternatively from a model assuming imperfect mobility of labor but abstracting from adjustment costs to physical capital accumulation. The results are relegated in this subsection. We emphasize very briefly in what a model either abstracting from capital installation costs or imposing perfect mobility of labor across sectors fails to account for our panel VAR evidence.

The solid black line in Figures 53 and 54 show the predictions of the baseline model while the dotted black line displays the predictions of a model with a difficulty in reallocating labor across sectors but abstracting from capital installation costs. As emphasized in the main text, the conclusion that emerges is that the model without capital adjustment costs tend to overstate the crowding out of investment in the short-run and to underestimate substantially the current account deficit. Because investment declines more, excess demand in the non traded goods market and thus the appreciation in the relative price of non tradables is much smaller than that found in the data. Because the model without capital installation costs underpredicts the short-run rise in \( P \), it tends to underestimate the responses of sectoral output shares. The solid black line in Figures 55 and 56 show the predictions of the baseline model while the dotted black line displays the predictions of a model imposing
Table 28: Impact Responses of Aggregate and Sectoral variables to a Rise in Government Consumption (in %)

<table>
<thead>
<tr>
<th>Data</th>
<th>1 - ( \theta^T ) &lt; 1 - ( \theta^N )</th>
<th>1 - ( \theta^T ) &gt; 1 - ( \theta^N )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(( \kappa = 0 ))</td>
<td>(( \kappa = 17 ))</td>
</tr>
<tr>
<td>A. Impact: GDP &amp; Components</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real GDP, ( dY(0) )</td>
<td>0.51</td>
<td>0.07</td>
</tr>
<tr>
<td>Investment, ( dI(0) )</td>
<td>-0.01</td>
<td>-0.84</td>
</tr>
<tr>
<td>Current Account, ( dCA(0) )</td>
<td>-0.30</td>
<td>0.06</td>
</tr>
<tr>
<td>B. Impact: Labor &amp; Real Wage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor, ( dL(0) )</td>
<td>0.53</td>
<td>0.11</td>
</tr>
<tr>
<td>Real wage, ( d(W/P_C)(0) )</td>
<td>0.48</td>
<td>-0.00</td>
</tr>
<tr>
<td>C. Impact: Sectoral Labor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traded labor, ( dL^T(0) )</td>
<td>0.01</td>
<td>-0.20</td>
</tr>
<tr>
<td>Non traded labor, ( dL^N(0) )</td>
<td>0.54</td>
<td>0.30</td>
</tr>
<tr>
<td>Traded wage, ( d(W^T/P_C)(0) )</td>
<td>0.22</td>
<td>0.00</td>
</tr>
<tr>
<td>Non traded wage, ( d(W^N/P_C)(0) )</td>
<td>0.83</td>
<td>0.00</td>
</tr>
<tr>
<td>Relative labor, ( d(L^T/L^N)(0) )</td>
<td>-0.71</td>
<td>-0.53</td>
</tr>
<tr>
<td>Relative wage, ( d(W^T/W^N)(0) )</td>
<td>0.93</td>
<td>-0.00</td>
</tr>
<tr>
<td>Labor share of ( T ), ( d(L^T/L)(0) )</td>
<td>-0.27</td>
<td>-0.23</td>
</tr>
<tr>
<td>Labor share of ( N ), ( d(L^N/L)(0) )</td>
<td>0.27</td>
<td>0.23</td>
</tr>
<tr>
<td>D. Impact: Sectoral Output</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traded output, ( dY^T(0) )</td>
<td>-0.03</td>
<td>-0.22</td>
</tr>
<tr>
<td>Non traded output, ( dY^N(0) )</td>
<td>0.70</td>
<td>0.28</td>
</tr>
<tr>
<td>Relative output, ( d(Y^T/Y^N)(0) )</td>
<td>-1.03</td>
<td>-0.62</td>
</tr>
<tr>
<td>Relative price, ( dP(0) )</td>
<td>1.06</td>
<td>-0.00</td>
</tr>
<tr>
<td>Output share of ( T ), ( d(Y^T/Y_T)(0) )</td>
<td>-0.45</td>
<td>-0.24</td>
</tr>
<tr>
<td>Output share of ( N ), ( d(Y^N/Y_T)(0) )</td>
<td>0.35</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Notes: Effects of an unanticipated and temporary exogenous rise in government consumption by 1% of GDP. Panels A,B,C,D show the initial deviation in percentage relative to steady-state for aggregate and sectoral variables. Market product (aggregate and sectoral) quantities are expressed in percent of initial GDP while labor market (aggregate and sectoral) quantities are expressed in percent of initial total hours worked; \( \theta^T \) and \( \theta^N \) are the labor income share in the traded sector and non traded sector, respectively; \( \epsilon \) measures the degree of substitutability in hours worked across sectors and captures the degree of labor mobility; \( \sigma_L \) is the Frisch elasticity of labor supply; \( \kappa \) governs the magnitude of adjustment costs to capital accumulation; \( \sigma \) determines the substitutability between consumption and leisure when preferences are non separable; \( \eta \) measures the degree of substitutability in capital across sectors and captures the degree of capital mobility. In our baseline calibration we set \( \theta^T = 0.58, \theta^N = 0.68, \epsilon = 0.75, \eta \to \infty, \phi = 0.77, \sigma_L = 0.4, \kappa = 17, \sigma = 1 \).
Table 29: Cumulative Responses of Aggregate and Sectoral Variables to of a Rise in Government Consumption (in %)

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(k = 0)</td>
<td>(k = 17)</td>
<td>(ε = 0.75)</td>
<td>(ε = 0.22)</td>
<td>(ε = 1.64)</td>
<td>(σL = 1)</td>
<td>(k = 0)</td>
<td>(σ = 2)</td>
<td>(η = 0.75)</td>
<td>(ε = ∞)</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
<td>(10)</td>
</tr>
<tr>
<td><strong>E. Cumulative: 2 year</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real GDP, dY_t</td>
<td>1.03</td>
<td>0.07</td>
<td>0.19</td>
<td>0.39</td>
<td>0.45</td>
<td>0.33</td>
<td>0.70</td>
<td>0.24</td>
<td>0.34</td>
<td>0.47</td>
</tr>
<tr>
<td>Investment, dl</td>
<td>-0.33</td>
<td>-1.23</td>
<td>0.10</td>
<td>-0.28</td>
<td>-0.39</td>
<td>-0.18</td>
<td>-0.31</td>
<td>-1.03</td>
<td>-0.31</td>
<td>-0.43</td>
</tr>
<tr>
<td>Current account, dCA</td>
<td>-1.45</td>
<td>-0.47</td>
<td>-1.65</td>
<td>-0.75</td>
<td>-0.48</td>
<td>-1.00</td>
<td>-0.64</td>
<td>-0.20</td>
<td>-1.08</td>
<td>-0.39</td>
</tr>
<tr>
<td>Labor, dl</td>
<td>1.26</td>
<td>0.21</td>
<td>0.30</td>
<td>0.62</td>
<td>0.71</td>
<td>0.53</td>
<td>1.11</td>
<td>0.45</td>
<td>0.55</td>
<td>0.74</td>
</tr>
<tr>
<td>Real wage, dW/P_C</td>
<td>0.59</td>
<td>0.00</td>
<td>0.15</td>
<td>0.15</td>
<td>0.17</td>
<td>0.13</td>
<td>-0.08</td>
<td>0.05</td>
<td>0.25</td>
<td>0.20</td>
</tr>
<tr>
<td>Relative price, dP</td>
<td>3.42</td>
<td>0.00</td>
<td>0.04</td>
<td>1.85</td>
<td>2.39</td>
<td>1.35</td>
<td>1.67</td>
<td>1.22</td>
<td>2.16</td>
<td>2.58</td>
</tr>
<tr>
<td>Relative wage, dW/All</td>
<td>2.50</td>
<td>0.00</td>
<td>0.00</td>
<td>3.04</td>
<td>3.94</td>
<td>2.18</td>
<td>2.82</td>
<td>2.04</td>
<td>3.51</td>
<td>2.64</td>
</tr>
<tr>
<td>Traded output, dY^T</td>
<td>-0.10</td>
<td>-0.85</td>
<td>-1.54</td>
<td>-0.67</td>
<td>-0.40</td>
<td>-0.92</td>
<td>-0.50</td>
<td>-0.45</td>
<td>-0.80</td>
<td>-0.30</td>
</tr>
<tr>
<td>Non traded output, dY^N</td>
<td>1.27</td>
<td>0.92</td>
<td>1.73</td>
<td>1.06</td>
<td>0.85</td>
<td>1.25</td>
<td>1.21</td>
<td>0.69</td>
<td>1.15</td>
<td>0.77</td>
</tr>
<tr>
<td>Output share of T, d(Y^T/Y_N)</td>
<td>-0.99</td>
<td>-0.87</td>
<td>-1.61</td>
<td>-0.81</td>
<td>-0.56</td>
<td>-1.04</td>
<td>-0.77</td>
<td>-0.54</td>
<td>-0.92</td>
<td>-0.46</td>
</tr>
<tr>
<td>Output share of N, d(Y^N/Y_N)</td>
<td>0.76</td>
<td>0.87</td>
<td>1.61</td>
<td>0.81</td>
<td>0.56</td>
<td>1.04</td>
<td>0.77</td>
<td>0.54</td>
<td>0.92</td>
<td>0.46</td>
</tr>
<tr>
<td><strong>F. Cumulative: 4 year</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real GDP, dY_t</td>
<td>1.10</td>
<td>0.05</td>
<td>0.38</td>
<td>0.71</td>
<td>0.81</td>
<td>0.61</td>
<td>1.31</td>
<td>0.21</td>
<td>0.62</td>
<td>0.84</td>
</tr>
<tr>
<td>Investment, dl</td>
<td>-1.29</td>
<td>-1.19</td>
<td>0.20</td>
<td>-0.57</td>
<td>-0.79</td>
<td>-0.35</td>
<td>-0.63</td>
<td>-2.11</td>
<td>-0.62</td>
<td>-0.87</td>
</tr>
<tr>
<td>Current account, dCA</td>
<td>-3.35</td>
<td>-2.15</td>
<td>-3.24</td>
<td>-1.48</td>
<td>-0.95</td>
<td>-1.98</td>
<td>-1.25</td>
<td>-0.46</td>
<td>-2.14</td>
<td>-0.76</td>
</tr>
<tr>
<td>Labor, dl</td>
<td>1.99</td>
<td>0.42</td>
<td>0.58</td>
<td>1.17</td>
<td>1.35</td>
<td>1.01</td>
<td>2.10</td>
<td>0.81</td>
<td>1.04</td>
<td>1.40</td>
</tr>
<tr>
<td>Real wage, dW/P_C</td>
<td>-0.70</td>
<td>0.00</td>
<td>0.28</td>
<td>0.24</td>
<td>0.29</td>
<td>0.23</td>
<td>-0.17</td>
<td>-0.09</td>
<td>0.43</td>
<td>0.34</td>
</tr>
<tr>
<td>Relative price, dP</td>
<td>7.98</td>
<td>-0.00</td>
<td>0.08</td>
<td>3.51</td>
<td>4.53</td>
<td>2.55</td>
<td>3.16</td>
<td>2.34</td>
<td>4.08</td>
<td>4.89</td>
</tr>
<tr>
<td>Relative Wage, dW/All</td>
<td>5.17</td>
<td>0.00</td>
<td>0.00</td>
<td>5.76</td>
<td>7.49</td>
<td>4.14</td>
<td>5.34</td>
<td>3.99</td>
<td>6.66</td>
<td>5.01</td>
</tr>
<tr>
<td>Traded output, dY^T</td>
<td>-0.79</td>
<td>-2.39</td>
<td>-2.91</td>
<td>-1.28</td>
<td>-0.77</td>
<td>-1.75</td>
<td>-0.96</td>
<td>-0.99</td>
<td>-1.53</td>
<td>-0.59</td>
</tr>
<tr>
<td>Non traded output, dY^N</td>
<td>1.88</td>
<td>2.44</td>
<td>3.28</td>
<td>1.98</td>
<td>1.58</td>
<td>2.36</td>
<td>2.28</td>
<td>2.10</td>
<td>2.15</td>
<td>1.43</td>
</tr>
<tr>
<td>Output share of T, d(Y^T/Y_N)</td>
<td>-2.02</td>
<td>-2.40</td>
<td>-3.05</td>
<td>-1.54</td>
<td>-1.06</td>
<td>-1.98</td>
<td>-1.47</td>
<td>-1.06</td>
<td>-1.75</td>
<td>-0.89</td>
</tr>
<tr>
<td>Output share of N, d(Y^N/Y_N)</td>
<td>1.77</td>
<td>2.40</td>
<td>3.05</td>
<td>1.54</td>
<td>1.06</td>
<td>1.98</td>
<td>1.47</td>
<td>1.06</td>
<td>1.75</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Notes: Effects of an unanticipated and temporary exogenous rise in government consumption by 1% of GDP. Panels E and F show the cumulative responses over a two- and four-year horizon, respectively. Market product (aggregate and sectoral) quantities are expressed in percent of initial GDP while labor market (aggregate and sectoral) quantities are expressed in percent of initial total hours worked; θ^F and θ^N are the labor income share in the traded sector and non traded sector, respectively; ε measures the degree of substitutability in hours worked across sectors and captures the degree of labor mobility; σL is the Frisch elasticity of labor supply; κ governs the magnitude of adjustment costs to capital accumulation; σ determines the substitutability between consumption and leisure when preferences are non separable, η measures the degree of substitutability in capital across sectors and captures the degree of capital mobility. In our baseline calibration we set θ^F = 0.58, θ^N = 0.68, ε = 0.75, η → ∞, φ = 0.77, σL = 0.4, κ = 17, σ = 1.
Figure 53: Dynamic Adjustment of Aggregate Variables to Unanticipated Government Spending Shock: The Role of Capital Adjustment Costs. Notes: solid blue line display point estimate of VAR with dotted blue lines indicating 90% confidence bounds; the solid black line displays model predictions in the baseline scenario with imperfect mobility of labor across sectors ($\epsilon = 0.75$) and capital installation costs ($\kappa = 17$) while the dotted black line shows results when abstracting from capital adjustment costs ($\kappa = 0$).

perfect mobility of labor across sectors while assuming that capital accumulation is subject to installation costs. First, the model predicts a rise in investment instead of decline, in contradiction with the evidence, and tends to overstate the current account deficit. Turning to the sectoral effects, while assuming capital installation costs restore transitional dynamics for the relative price of non tradables, the model imposing perfect mobility considerably understates the appreciation in the relative price and cannot account for the rise in non traded wages relative to traded wages as sectoral wages equalize. Moreover, while the relative price of non tradables merely appreciates, because labor is extremely sensitive to relative price changes, the consecutive changes in sectoral output shares conflict with the evidence since their magnitude are about twice what is estimated empirically,
Figure 54: Dynamic Adjustment of Sectoral Variables to Unanticipated Government SpENDING Shock: The Role of Capital Adjustment Costs. Notes: Solid blue line display point estimate of VAR with dotted blue lines indicating 90% confidence bounds; the solid black line displays model predictions in the baseline scenario with imperfect mobility of labor across sectors ($\epsilon = 0.75$) and capital installation costs ($\kappa = 17$) while the dotted black line shows results when abstracting from capital adjustment costs ($\kappa = 0$).
Figure 55: Dynamic Adjustment of Aggregate Variables to Unanticipated Government Spending Shock: The Role of Limited Mobility across Sectors. Notes: solid blue line display point estimate of VAR with dotted blue lines indicating 90% confidence bounds; the solid black line displays model predictions in the baseline scenario with imperfect mobility of labor across sectors ($\epsilon = 0.75$) and capital installation costs ($\kappa = 17$) while the dotted black line shows results when imposing perfect mobility of labor across sectors ($\epsilon \to \infty$).
Figure 56: Dynamic Adjustment of Sectoral Variables to Unanticipated Government Spending Shock: The Role of Limited Labor Mobility across Sectors. Notes: Solid blue line display point estimate of VAR with dotted blue lines indicating 90% confidence bounds; the solid black line displays model predictions in the baseline scenario with imperfect mobility of labor across sectors ($\epsilon = 0.75$) and capital installation costs ($\kappa = 17$) while the dotted black line shows results when imposing perfect mobility of labor across sectors ($\epsilon \to \infty$).
M.3 Simulated Responses of Sectoral Output Shares across Countries

We denote by $\nu_{ij}^Y(t)$ the output ($Y$) share of sector $j$, in country $i$ at year $t$. In terms of our model’s notation, the response of the output share of sector $j$ to a government spending shock is measured in total output units and thus is calculated as the product between the growth differential between sectoral output and GDP (both at constant prices) and the content of production of good $j$ in total output. Formally, the response at year $t$ of the sectoral output share to a government spending shock reads as:

$$\hat{\nu}_{ij}^Y(t) = \frac{P_j^Y}{P_i^Y} \left( \hat{Y}_j^i - \hat{Y}_{R,t} \right).$$

To assess the ability of our model to account for our evidence, we calibrate the model to the data of each country in our sample, except for the world interest rate, elasticity of labor supply, and $\kappa$ that governs the magnitude of capital adjustment costs which are kept unchanged, i.e., $r^* = 4\%$, $\sigma_L = 0.4$, and $\kappa = 17$. When numerically computing $\hat{\nu}_{ij}^Y(0)$ for each country $i$, we set $\phi_i \epsilon_i$ in accordance with their empirical estimates shown the two last columns of Table 5. When we calibrate the model to the whole sample (i.e., a representative OECD economy), we set $\epsilon$ to 0.75 and $\phi$ to 0.77 which correspond to their unweighted average values.

Columns 2 and 4 of Table 30 report the simulated impact responses of the output share of tradables, $\hat{\nu}_{i,T}^Y(0)$, and non tradables, $\hat{\nu}_{i,N}^Y(0)$, respectively, to an exogenous rise in government consumption by 1 percentage point of GDP. Columns 3 and 5 report point estimates from the VAR model for $\hat{\nu}_{ij}^Y(0)$ for each country and the whole sample as well. In line with our model’s predictions, an increase in government consumption gives rise to a contraction in the traded sector and has an expansionary effect on the non traded sector, except for Australia and Ireland. Because in these two economies, the traded sector expands while the non traded sector shrinks, we consider a rise in government consumption by 1 percentage point of GDP triggered by an increase in public purchases on tradables while keeping $G^N$ fixed.

Because the time horizon of the sample is small for each country due to the annual frequency of data, the VAR estimates have to be taken with a grain of salt. More precisely, VAR estimates for $\hat{\nu}_{ij}^Y(0)$ are significant at 10% for only five countries in our sample. As shown in the last line of Table 30, our model predicts remarkably well the contraction in the traded sector and the expansionary effect in the non traded sector. While our results tends to underestimate the changes in output shares of both sectors, the predicted values lie within the 90% confidence interval for most of the economies of our sample. More precisely, when we restrict our attention to statistically significant estimates, the model’s predictions fall in the range of empirical estimates except for Canada. While we find that the model tends to underestimate the responses of sectoral output shares for most of the countries, in particular for Japan, Sweden, and the USA, the correlation between predicted and observed series is 0.65 for tradables and 0.69 for non tradables, as shown in the last line of Table 30, which suggest that the model can account reasonably well for cross-country differences in impact responses of sectoral output shares to a government spending shock.

To investigate the relationship between the magnitude of the sectoral impact of a fiscal shock and the degree of labor mobility across sectors, we regress the estimated sectoral output responses, $\hat{\nu}_{ij}^Y(0)$, on the elasticity of labor supply across sectors, $\epsilon_i$:

$$\hat{\nu}_{ij}^Y(0) = \beta_0 + \beta_1 \cdot \epsilon_i + \epsilon_i.$$  

(689)

According to our estimates reported in Table 31, the regression coefficient, $\beta_1$, is negative for tradables and positive for non tradables which suggests that following a rise in government consumption, the output share of tradables falls more while the output share of non tradables rises by a larger amount in countries with a higher labor mobility across sectors. Importantly, the regression coefficients from simulated and estimated values are roughly similar.

M.4 Robustness Check: Additional Numerical Results

In this subsection, we present results from two extensions of the baseline model:
Table 30: Comparison of Simulated with Estimated Values for Changes in Sectoral Output Shares

<table>
<thead>
<tr>
<th>Country</th>
<th>Parameter</th>
<th>Impact responses: sectoral output shares</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mobility $\epsilon$</td>
<td>$\hat{\nu}_{j}(0)$ simul</td>
</tr>
<tr>
<td>AUS</td>
<td>0.635</td>
<td>0.09†</td>
</tr>
<tr>
<td>AUT</td>
<td>0.548</td>
<td>$-0.32^{†}$</td>
</tr>
<tr>
<td>BEL</td>
<td>0.326</td>
<td>$-0.28^{†}$</td>
</tr>
<tr>
<td>CAN</td>
<td>0.454</td>
<td>$-0.37$</td>
</tr>
<tr>
<td>DNK</td>
<td>-</td>
<td>$-0.31$</td>
</tr>
<tr>
<td>ESP</td>
<td>1.642</td>
<td>$-0.49^{†}$</td>
</tr>
<tr>
<td>FIN</td>
<td>0.544</td>
<td>$-0.34^{*}$</td>
</tr>
<tr>
<td>FRA</td>
<td>1.287</td>
<td>$-0.40^{†}$</td>
</tr>
<tr>
<td>GBR</td>
<td>1.008</td>
<td>$-0.42^{†}$</td>
</tr>
<tr>
<td>IRL</td>
<td>0.264</td>
<td>$0.05^{†}$</td>
</tr>
<tr>
<td>ITA</td>
<td>0.686</td>
<td>$-0.38^{†}$</td>
</tr>
<tr>
<td>JPN</td>
<td>0.993</td>
<td>$-0.41^{†}$</td>
</tr>
<tr>
<td>NLD</td>
<td>0.224</td>
<td>$-0.26^{†}$</td>
</tr>
<tr>
<td>NOR</td>
<td>-</td>
<td>$-0.35^{†}$</td>
</tr>
<tr>
<td>SWE</td>
<td>0.443</td>
<td>$-0.35^{*}$</td>
</tr>
<tr>
<td>USA</td>
<td>1.387</td>
<td>$-0.40^{*}$</td>
</tr>
<tr>
<td>Whole</td>
<td>0.746</td>
<td>$-0.38^{*}$</td>
</tr>
<tr>
<td>Corr.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Table provides simulated (simul) and estimated (estim) responses on impact for $\hat{\nu}_{j}(0)$ (with $j = T, N$); responses correspond to the change in sectoral value added at constant prices relative to real GDP measured in total output units; when computing the change in the share of valued added of sector $j$, we keep relative prices constant so that its change is only triggered by variations in quantities; $\epsilon$ is the elasticity of labor supply across sectors; because estimates of $\epsilon$ for Denmark and Norway are not statistically significant, their values are left blank. Predicted values for Denmark are obtained when setting $\epsilon$ to its value for the whole sample. We denote by superscripts 'simul' and 'estim' the numerically computed values and VAR estimates, respectively; † and * indicate that the predicted value lies within the estimated confidence interval while * indicates that the estimated value is significant at 10%; we calculate 90% confidence intervals based on estimated standard deviations of $\hat{\nu}_{j}(0)$ obtained when the VAR model is estimated, for each country and the whole sample as well; 'Corr.' refers to the correlation coefficient between simulated and estimated values.
Table 31: Relationship between Impact Responses of Sectoral Output Shares to a Rise in Government Consumption and the Degree of Labor Mobility across Sectors (OLS estimates)

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$R^2$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y^T/Y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>−0.272</td>
<td>−0.249</td>
<td>0.058</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>(−1.090)</td>
<td>(−0.860)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>−0.156c</td>
<td>−0.207b</td>
<td>0.295</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>(−1.904)</td>
<td>(−2.240)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y^N/Y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>0.274</td>
<td>0.234</td>
<td>0.087</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>(1.452)</td>
<td>(1.072)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>0.156c</td>
<td>0.207b</td>
<td>0.295</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>(1.904)</td>
<td>(2.240)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: $^a$, $^b$ and $^c$ denote significance at 1%, 5% and 10% levels; t-statistics are reported in parentheses.

- While we assume that both the traded and the non traded sector are perfectly competitive, this assumption can be considered as restrictive for the non traded sector which is sheltered from foreign competition and thus consists of industries with higher markups than the traded sector. The fact that markups are high in non traded industries implies that the elasticity of the markup to entry is high as well; as a result, by producing profit profit opportunities and thus encouraging firm entry, a rise in $G^N$ produce a fall in the markup that may modifies quantitatively the size of the change in the share of non tradables in real GDP. In section K, we lay out the model with an imperfectly competitive non traded sector where the markups are endogenous in order to investigate the extent to which firm entry modifies our baseline results.

- The second extension is related to our assumption that the government budget is balanced at each instant so that the rise in government spending is financed by an increase in lump-sum taxes. As long as taxes are lump-sum, Ricardian equivalence obtains and the time path of taxes is irrelevant for the real allocation. Thus, whether the government budget is balanced or not affects our results neither qualitatively nor quantitatively. If taxes are distorsionary, then the manner of financing higher spending can influence the results. In section J, we lay out a model with public debt which enables us to analyze the differences between the effects of a government spending shock whether it is either debt-financed or budget-balanced.

**Calibration of the Model with Endogenous Markups and Results**

To calibrate our model to a representative OECD economy, we keep the same values for all parameters shown in Table 27 in order to make our results comparable with those obtained for the baseline model. Since we consider an imperfect competitive non traded sector, we have to choose values for the elasticity of substitution between intermediate goods produced non traded industries, $\eta$, and the value for the elasticity of substitution between varieties within one non traded industry, $\rho$. Setting $\rho$ to 4 and $\eta$ to 1 yields a markup $\mu$ charged by the non-traded sector of 1.35, which is close to OECD countries’ unweighted average (1970-2004) documented by Cardi and Restout [2015] for 13 OECD countries that includes all countries in our sample except for Australia, Canada, Finland. We choose a value of fixed costs $\psi$ so that the number of competitors is 20 within each non traded industry which is consistent with our assumption according to which the number of firms is large enough so that we can ignore the strategic effects but not so large that the effect of entry on the firm’s demand curve is minuscule.

**Calibration of the Model with Public Debt and Results**

To calibrate the model with public debt to a representative OECD economy, we estimate a VAR model that includes public debt, $D$, in order to determine the dynamic response of public debt to an exogenous government spending shock. More specifically, the VAR specification includes government consumption, public debt, real GDP, hours worked, non-residential investment, and the real consumption wage. Time series for public debt as
Figure 57: Dynamic Adjustment of Aggregate Variables to Unanticipated Government Spending Shock: The Role of Endogenous Markups. Notes: solid blue line display point estimate of VAR with dotted blue lines indicating 90% confidence bounds; the solid black line displays model predictions in the baseline scenario with imperfect mobility of labor across sectors ($\epsilon = 0.75$) and capital installation costs ($\kappa = 17$) while the dotted black line shows results for the case of endogenous markups.
Figure 58: Dynamic Adjustment of Sectoral Variables to Unanticipated Government Spending Shock: The Role of Endogenous Markups. Notes: Solid blue line display point estimate of VAR with dotted blue lines indicating 90% confidence bounds; the solid black line displays model predictions in the baseline scenario with imperfect mobility of labor across sectors ($\epsilon = 0.75$) and capital installation costs ($\kappa = 17$) while the dotted black line shows results for the case of endogenous markups.
a percentage of GDP are taken from the OECD. As displayed in 59(b), the endogenous response of public to an exogenous government spending shock is hump-shaped and displays high persistence. More precisely, public debt reaches a peak at time $t = 6$ and then is restored back toward its initial level after 30 years. Before discussing the calibration of the model, it is convenient to repeat the equations which govern the adjustment of government spending, public debt, and taxes we derive in section J (see eq. (593), (596), (597)):

$$\frac{dG(t)}{Y} = e^{-\xi t} - (1 - g) e^{-\chi t}, \quad (690a)$$

$$\frac{dD(t)}{Y} = \Theta_D e^{-\delta t} - \Theta_1 e^{-\xi t} + \Theta_2 e^{-\chi t}, \quad (690b)$$

$$\frac{dT(t)}{Y} = \theta_L d\tau(t), \quad (690c)$$

$$d\tau(t) = \Omega_D e^{-\delta t} - \left( \Omega_1 e^{-\xi t} - \Omega_2 e^{-\chi t} \right), \quad (690d)$$

where $\tau$ is a distortive labor tax we impose $D_0 = \tilde{D}$ in line with our VAR evidence, i.e., public debt is restored to its initial level. To calibrate the model with public debt, we have to choose values for three new parameters, $\phi_D, \phi_G, \delta$. Since $\delta = \phi_D - r^*$, it leaves us with only two parameters. These two parameters are chosen so as to reproduce the hump-shaped response of the public debt in percentage of GDP. Using the fact that public debt peaks at $\hat{t} = 6$, we solve the system of equations below to determine the values for $\phi_D$ and $\phi_G$:

$$\dot{D}(\hat{t}) = -\delta \Theta_D e^{-\delta \hat{t}} + \xi \Theta_1 e^{-\xi \hat{t}} - \chi \Theta_2 e^{-\chi \hat{t}} = 0, \quad (691a)$$

$$\frac{dD(\hat{t})}{Y} = \Theta_D e^{-\delta \hat{t}} - \Theta_1 e^{-\xi \hat{t}} + \Theta_2 e^{-\chi \hat{t}}. \quad (691b)$$

Using the fact that at $\hat{t} = 6$, we have $\frac{dD(\hat{t})}{Y} = 1.68738$, solving yields $\phi_G = 0.485728$ and $\phi_D = 0.169004$. 

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Figure 59: Dynamic Adjustment to an Unanticipated Deficit-Financed Government Spending Shock. Notes: The solid blue line displays point estimates of the VAR model with public debt, with dotted blue lines indicating the 90% confidence bounds; the solid black line displays model predictions in the baseline scenario with imperfect mobility of labor across sectors ($\epsilon = 0.75$) and capital installation costs ($\kappa = 17$).
References


