Is there always a trade-off between insurance and incentives? The case of unemployment with subsistence constraints

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Is there always a trade-off between insurance and incentives? 
The case of unemployment with subsistence constraints

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Abstract

This article analyzes the behavioral effects of unemployment benefits (UB) and it characterizes their optimal level when jobless people only survive if they have access to a minimum or subsistence consumption level in each period. To survive when the level of UB is very low, they carry out a subsistence activity. Our model shows that if the level of UB is very low, increasing its level or providing liquidity to the agent can decrease the duration in unemployment; for higher levels of UB we reencounter the standard properties that increasing UB increases duration and that providing liquidity to the agent increases duration (Chetty, 2008). We also show that the optimal level of UB satisfies the Baily-Chetty formula (Baily, 1978, Chetty, 2006), but contrary to Chetty (2008), in our model the gain from insurance cannot be rewritten using sufficient statistics; we show that such decomposition requires specific modeling assumptions.

JEL classification: D91, H21, J64, J65.
Keywords: liquidity effect, scarcity, monetary costs, optimal insurance.

1 Introduction

Unemployment insurance is not present in a number of countries (Vodopivec, 2013, Bosch and Esteban-Pretel, 2015) and, where it is in place, the level of benefits is sometimes low (Kupets, 2006). This raises the question of the subsistence of jobless people. This article analyzes the behavioral effects of unemployment benefits (UB) and it characterizes their optimal level when jobless people only survive if they have access to a minimum or subsistence consumption level in each period. Even though the existence of subsistence constraints has been recognized in the economic literature,¹ to the best of our knowledge, the design of unemployment insurance schemes has not explicitly integrated daily subsistence requirements.² Our model shows that if the level of UB is very low, increasing its level can decrease the duration in unemployment; for higher levels of UB we reencounter the standard property that increasing UB increases duration. Our model also shows that if the level

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¹In the literature of development economics, see for instance: Dercon (1998) and Zimmerman and Carter (2003) about the role of subsistence constraints on assets accumulation for the poor and Bhalotra (2007) about the link between subsistence constraints and child work. In the literature on social insurance it has been mentioned by Chetty (2006) and Chetty and Looney (2006).

²As will soon be clear, this goes beyond the assumption that the marginal utility of consumption becomes huge when the level of consumption tends to zero. Pavoni (2007) analyzes the design of unemployment insurance when the planner must respect a lower bound on the expected discounted utility of the agent. The unemployed agent decides whether to search, or not (binary decision) subject to the scheme proposed by the planner.
of UB is very low, providing some liquidity to the agents (irrespectively of their employment status, and that should not be repaid through specific taxes) can actually decrease the duration in unemployment; for higher levels of UB we reencounter the standard property that liquidity increases duration (Chetty, 2008). Furthermore, compared to a framework where minimum consumption is ignored, the optimal level of UB can be larger. We also show that the optimal level of UB satisfies the Bailey-Chetty formula (Baily, 1978, Chetty, 2006), but contrary to Chetty (2008), in our model the gain from insurance cannot be rewritten using sufficient statistics; we show that such decomposition is not general, it actually requires specific modeling assumptions.

Some authors have assumed that the factors affecting the chance of finding a job are not summarized by a scalar variable (van den Berg and van der Klaauw, 2006 distinguish formal and informal search effort; Caliendo et al., 2015, distinguish search effort and locus of control). We emphasize that exiting to a job requires two inputs: some job-search effort(s) and cognitive resources.

According to Mullainathan, Shafir and co-authors, (Shah et al., 2012, Mullainathan and Shafir, 2013, Mani et al., 2013, Shah et al., 2015, and Schilbach et al., 2016) who develop a number of experiments both in the United States and in developing countries, the cognitive capacity or “bandwidth” of agents is limited. “Bandwidth measures our computational capacity, our ability to pay attention, to make good decisions, to stick with our plans, and to resist temptations” (Mullainathan and Shafir, 2013, p.41).

When the subsistence of the individual needs to be guaranteed and UB are low or absent, the unemployment risk is not covered by private insurers and credit markets are imperfect or absent, the person needs a behavioral margin: a daily life activity (for example, looking for discounts in the supermarket, fixing old clothes etc) and/or a casual informal activity (for example, selling homemade food, subsistence farming, etc) that allow the person to increase her consumption out of the same level of UB.

Performing these activities, “taxes” the cognitive capacity of the agent, and leaves less cognitive resources available for job search. Mullainathan, Shafir and co-authors state that: “We suggest that cognitive load arises because people are more engaged with problems where scarcity is salient. This consumes attentional resources and leaves less for elsewhere.” Shah et al. (2012). “Scarcity taxes our bandwidth, and as a result, inhibits our most fundamental capacities.” (Mullainathan and Shafir, 2013, p. 42). Consequently, in our model the exit probability from unemployment is affected by job search effort and also by the cognitive resources available for this task. Given that cognitive resources are limited, dealing with subsistence reduces the cognitive resources available for job search, and therefore has a negative impact on the probability of exiting unemployment. This is an intuitive, yet neglected consideration, whose consequences are at the heart of our analysis.

Related Literature

As we mentioned, in our model, when UB are low, an increase in its level could decrease the duration in unemployment. For higher values of the UB, we re-encounter the standard effect, i.e. that increasing the level of UB increases duration. Several studies in the literature show that increments in the level of the UB increase duration in unemployment (See Tatsiramos and van Ours, 2014 for a survey). Nevertheless, none of these studies focuses on modifications of the UB for levels that are below or very close to the subsistence requirements. We found two papers which analyze the impact of UB in a low-income sample. LaLumia (2013) estimates a hazard model for a sample of people eligible to the earned income tax credit (EITC) in the United States. She finds that the effect of UB is never significant for women, and for men it is significant and positive in some of her specifications. Kupets (2006), develops a duration analysis for Ukraine. She finds that the receipt of UB does not have a significant effect on duration for agents who carry out subsistence activities. We will comment in more detail these papers in section 4.1.

In our model, when UB are low, providing some liquidity to the agents can actually increase their

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3There is a literature about the lack of plausibility of private unemployment insurance, see for instance Easley et al. (1985), or more recently Hendren (2017).
job finding probability. For higher values of UB, we re-encounter the standard effect, i.e., that providing liquidity or cash transfers to agents increase their duration in unemployment. In fact, several studies (Card et al., 2007, Chetty, 2008, Basten et al., 2014) report that the probability of finding a job decreases when cash-constrained agents receive cash transfers (severance payments or annuities of any kind). On the other hand, several recent studies show that cash transfers to people who live in poverty (conditional cash transfer (CCT)), usually conditional on family structure, and/or maintaining kids on school, or unconditional cash transfers (UCT) do not have a negative effect on employment outcomes or may even have positive ones (Banerjee et al., forthcoming, for an analysis of seven CCT and UCT, Barrientos and Villa, 2015 who analyzes a CCT in Colombia, and Schwartz (2015). The two first papers assume that search requires time and money. Barron and Mellow (1979) does not assume any complementarity between time and money. Tannery (1983) criticizes that assumption but does not develop a theoretical analysis. Schwartz (2015) assumes that looking for a job requires effort and money. He assumes that job search can be influenced through search capital, which requires monetary expenditures to be maintained; in his framework, the monetary expenditure is not a flow per period; he develops a theoretical analysis for a simple two period setting, and he produces a numerical analysis.

Some papers consider that job search requires only monetary expenditures, namely: Ben-Horim and Zuckerman (1987), Decreuse (2002), Mazur (2016). These papers, as ours, highlight the positive effect that UB can have on the duration in unemployment. Nevertheless, if job search only requires money, the cost of search is a decreasing function of cash-on-hand. In this specification providing liquidity to the agents always increases the probability of finding a job, which is at odds with empirical evidence which finds that cash-transfers increase duration (Chetty, 2008, Card et al., 2007, Basten et al., 2014), and with empirical evidence that shows that richer agents experience longer unemployment spells (Algan et al., 2003, Lentz and Tranaes, 2005, Lentz, 2009 and Centeno and Novo, 2014).

Finally, the literature about the design of UB has always put forward the trade-off caused by it. The standard view is that the role of UB is to smooth consumption, the price being a distortion of incentives. See among many others Frederiksson and Holmlund (2006), Tatsiramos and van Ours (2014), Spinnewijn (2015), Schmieder and Watcher (2016). Our model suggests that for some low values of UB, this trade-off may not be present: increasing the level of this benefit increases the possibilities of the agent to smooth consumption and also increases her probability of finding a job. This implies that for low levels of UB there is no behavioral cost, instead, there could be a behavioral gain. If that is the case, the role of UB would not only be to allow the agent to smooth consumption but also to provide her with tools that help her to find a job.

These CCT or UCT usually last for long periods of time, and do not change with the income level, Banerjee et al., forthcoming state: “Once a household becomes eligible for any of the programs that we study, the amount of benefit that one receives is the same regardless of actual income level and lasts at least a period between 2 and 9 years, depending on the program. This differs from many U.S. transfer programs (e.g. EITC, SNAP), where the stipend depends (either positively or negatively) on family income, and is updated frequently”.

The paper is organized as follows: In section 2 we start by presenting two standard results of the literature. Then we introduce our baseline model and do comparative statics. We further introduce
three extensions, and comment the differences and similarities between our baseline model and a model in which job search requires both search and monetary expenditures. In section 3 we solve the planner’s problem and find the Baily-Chetty (Baily, 1978 and Chetty, 2006) formula, we also discuss the conditions that would allow to write the gain of insurance in terms of sufficient statistics. In section 4 we solve the baseline model and its extensions numerically, we show that when UB are low our model questions the two standard properties with which we started the paper and we comment some empirical results found in the literature consistent with these findings. Finally, we analyze the effect of the new variables of the baseline model on the optimal level of UB.

## 2 Positive Analysis

In this section we start by presenting two standard results of the literature: (1) a higher level of UB increases the expected duration unemployment and (2) providing liquidity to the agent increases the expected duration in unemployment. Then, we introduce our baseline model, which incorporates subsistence requirements and a subsistence activity, we analyze the optimal choices of the agent in that context. We continue by introducing three extensions to our baseline model (finite entitlement to flat UB, the presence of incomplete financial markets and stochastic wage offers) and finally we point out the similarities and differences between our baseline model and a model in which finding a job requires both job search effort and flow monetary expenditures.

### 2.1 Standard Job Search Model [SM]

We start by introducing what we call the “standard model” [SM]. It is a partial equilibrium job search model in a stationary discrete-time setting. Infinitely-lived, homogeneous and hand-to-mouth unemployed workers choose their search effort intensity, \( s \). \( \lambda(s) \) denotes the cost of job search effort and it is assumed that \( \lambda(0) = 0, \lambda_s > 0, \lambda_{ss} \geq 0 \). Unemployed workers are entitled to a flat unemployment benefit, \( b \), with no time limit. Hence, there’s no room for an “entitlement effect” (Mortensen, 1977). In each period the consumption of the unemployed agent, \( c_u \), is equal to \( b \). It is further assumed that the agent is risk averse, implying that her utility function is increasing and concave \( u(c) > 0, u_{cc}(c) < 0 \). In each period job offers arrive with a probability \( P(s) \) and it is assumed that \( P(0) = 0, P_s > 0, P_{ss} \leq 0 \). The net wage associated to a job offer is equal to \( w - \tau \), where \( w \) is the gross wage, and \( \tau \) is the level of taxes needed to finance the UB scheme; the consumption when employed, \( c_e \), is equal to \( w - \tau \).\(^6\) The disutility of the in-work effort is normalized to zero and it is assumed that the employed agent loses her job with an exogenous probability \( \phi \).

The agent discounts the future at a rate \( \beta = \frac{1}{1+r} \) where \( r \) is the interest rate. The timing is as follows: the unemployed chooses \( s \) in the current period, if she receives an offer, she starts working in the next period.

In the “Standard Model” [SM], the lifetime value \( V^U \) in unemployment (respectively \( V^E \) in employment) verifies the following Bellman equation:

\[
[SM] = \begin{cases} 
V^U = \max_s \left( u(c^u) - \lambda(s) + \beta[P(s)V^E + (1 - P(s))V^U] \right) \\
V^E = u(c^e) + \beta[\phi V^U + (1 - \phi)V^E] 
\end{cases}
\]

(1)

Where \( u(c^u) = u(b), u(c^e) = u(w - \tau) \).

Subject to: \( V^E - V^U \geq 0, s \geq 0 \).

\(^6\)As in Chetty (2008) or Hopenhayn and Nicolini (1997), we consider a degenerate distribution of wage offers. Moreover, as it is standard, the wage is the only feature that characterizes the job.
We want to highlight two well-known properties of an interior solution to problem [SM]:

1. Increasing \( b \) increases \( D \), the expected duration in unemployment, where \( D = \sum_{t=0}^{\infty} (1 - P)^t = \frac{1}{\tau} \).

2. Providing liquidity to the agent increases the duration in unemployment. This was pointed out by Chetty (2008).

To analyze the effect of liquidity, Chetty (2008) introduces an annuity, \( A \), in a setup essentially equal to the [SM] just presented. This annuity is a lump-sum income (independent from the UB scheme) received by the agent regardless of her employment status, so that \( c^u = b + A \) and \( c^e = w - \tau + A \). In Chetty’s model, providing liquidity to the agent always has a negative effect on the probability of finding a job, and therefore it unambiguously increases expected duration in unemployment.

2.2 Baseline Model [BM]

Our baseline model builds upon the standard model [SM], we incorporate four differences. First, the agent has a subsistence requirement \( c_{\text{min}} \), which means that her total consumption in each period has to be greater than or equal to \( c_{\text{min}} \). Second, we assume that the unemployed agent can carry out a subsistence activity by exerting effort \( a \). Third, we assume that effort \( a \) is costly, meaning that \( \lambda(a, s) > 0 \); we also assume that \( \lambda_{sa} > 0 \). Fourth, the job finding probability \( P \) is a function of \( s \) and \( a \). Following the scarcity literature (Shah et al., 2012, Mullainathan and Shafir, 2013, Mani et al., 2013, Shah et al., 2015, and Schilbach et al., 2016) we assume that cognitive capacity is limited. Dealing with subsistence, which is a pressing activity, taxes this cognitive capacity, meaning that less cognitive capacity is left for job search. Formally, the effort devoted to the subsistence activity has a negative effect on the job finding probability: for the same level of job search effort, the job finding probability is lower the higher the quantity of effort devoted to the subsistence activity, i.e., \( P_a < 0 \). It also has a negative effect on the productivity of \( s \), i.e., \( P_{as} \leq 0 \).

We assume that the agent has a utility function of the Stone-Geary type \( u(c - c_{\text{min}}) \). The consumption when unemployed becomes:

\[
c^u = b + g(a)
\]

where \( g(a) \) is the subsistence activity, with: \( g(0) = 0, g_a > 0, g_{aa} \leq 0 \).

We further assume that \( g(a) \equiv 0 \) when the agent is employed, meaning that the agent does not carry out the subsistence activity when employed.

All along the paper we assume that \( a \) and \( s \) are not observable by the government or UI agency, therefore there are no fines or punishments associated to \( g(a) \) or to the lack of \( s \).

The Bellman equations of the the unemployed, and employed agent in the “Baseline Model” [BM] are defined as:

\[
[BM] = \begin{cases} 
V^U = \max_{s,a} \left[ u(c^u) - \lambda(s,a) + \beta[P(s,a)V^E + (1-P(s,a)]V^U \right] \\
V^E = u(c^e) + \beta[\phi V^U + (1-\phi)V^E] 
\end{cases}
\]  

\[\text{\footnote{van den Berg and van der Klaauw (2006) consider a model with two job search search channels (formal and informal). Contrary to our setting, in their paper the two channels are independent, one leads to a formal job and the other one to an informal job.}}\]

\[\text{\footnote{Otherwise devoting effort to the subsistence activity would have negative effects on the productivity of the employed agent at work, and this should also be analyzed. In such a setup, the probability of losing the job, \( \phi \), would be a function of \( a \). Given our focus on the problem of the unemployed, this analysis is beyond the scope of this paper.}}\]

\[\text{\footnote{On the difficulty of observing job search effort without errors, see for instance Cockx et al. (forthcoming).}}\]
where \( u(c^u) = u(b + g(a) - c_{min}) \), \( u(c^s) = u(w - \tau - c_{min}) \), \( \lambda_a > 0, \lambda_{aa} \geq 0, \lambda_{as} \geq 0, P_a < 0, P_{aa} \leq 0, P_{as} \leq 0, P_s > 0, P_{ss} \leq 0, P(0, a) = 0 \).

Subject to: \( c^u \geq c_{min} \), \( V^E - V^U \geq 0 \) and \( s \geq 0 \).

Our specification pretends to be in line with the scarcity literature, from which we extract four stylized facts: (1) Scarcity captures the mind automatically: in our model if \( b < c_{min} \) the agent must devote some effort (and therefore, cognitive resources) to subsistence, and in that sense, scarcity captures the mind automatically. (2) Scarcity makes us better at solving problems related with scarcity (focus effect); in our model by devoting effort to the subsistence activity the agent is able to increase her consumption. (3) Scarcity leaves less bandwidth (less attention or cognitive resources) to other aspects of our life (tunnel effect); in our model devoting effort to subsistence leaves less cognitive resources available for job search, and therefore affects \( P \) negatively: \( P_a < 0 \) and \( P_{as} \leq 0 \). (4) None of these characteristics are personal traits: any person faced with scarcity would act in this way: we model homogeneous agents, we do not claim that just one type (namely “agents with intrinsic characteristics that predict poverty”) behave in this way.

Note that even when we interpret \( a \) and \( s \) as effort devoted to the subsistence activity and to job search, respectively, they could also be interpreted as time devoted to these activities. In a framework with time, the agent enjoys consumption and leisure. If one assumes that her utility function is separable in these two arguments and that time is not a binding constraint, then such a framework could be encompassed by the baseline model [BM].

**Comparative Statics in the Baseline Model**  The first order conditions of this maximization program, if the solution is interior, are:

\[
G_a \equiv u_a(c^u)g_a - \lambda_a + \beta P_a[V^E - V^U] = 0
\]

\[
G_s \equiv -\lambda_s + \beta P_s[V^E - V^U] = 0
\]

Where \( V^E - V^U = \frac{u(c^s) - u(c^u) + \lambda(s,a)}{1 - \beta(1 - P(s,a) - \phi)} \).

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10 Scarcity captures our mind automatically. And when it does, we do not make trade-offs using a careful cost-benefit calculus. We tunnel on managing scarcity both to our benefit and to our detriment**, pg. 34-35 Mullainathan and Shafir (2013).

11 The very lack of available resources makes each expense more insistent and more pressing. A trip to the grocery store looms larger, and this month’s rent constantly seizes our attention. Because these problems feel bigger and capture our attention, we engage more deeply in solving them. This is our theory’s core mechanism: Having less elicits greater focus.” Shah et al. (2012)

12 One could also capture this effect by having that \( g(a, b) \) with \( g_a > 0 \) and \( g_b < 0 \), that is, the smaller the \( b \) the higher is the ability of the agent to transform effort into consumption. Having \( g(a, b) \) instead of \( g(a) \) reinforces our findings. The development is available from the authors upon request.

13 Focusing on something that matters to you makes you less able to think about other things you care about. Psychologists call this goal inhibition. Goal inhibition is the mechanism underlying tunneling**, pg.31 Mullainathan and Shafir (2013).

14 Being poor means coping not just with a shortfall of money, but also with a concurrent shortfall of cognitive resources. The poor, in this view, are less capable not because of inherent traits, but because the very context of poverty imposes load and impedes cognitive capacity. The findings, in other words, are not about poor people, but about any people who find themselves poor.” Mani et al. (2013).

15 Several studies highlight that unemployed agents do not devote more than a few hours per week (or even less) to look for a job. For instance, Krueger and Muller (2010) report that the average unemployed person searches 4 minutes a day in Nordic countries, 10 minutes in the rest of Europe and 30 minutes in North America (see also Manning, 2011 for a synthesis of five studies, and more recently Aguilar et al., 2013). Accordingly, jobless people have plenty of time left and hence we do not assume that time is a binding constraint for the unemployed.

16 Corner solutions are discussed in Appendix A.1.
We are particularly interested in the reaction of the agent when $b$ changes, i.e., in $\frac{ds}{db}$ and in $\frac{ds}{db}$. In general, an increment in $b$ induces ambiguous effects on $s$ and on $a$, which implies that the standard property of the literature $\frac{dp}{db} < 0$ is not necessarily met. In what comes, we discuss the condition under which the quantity of effort devoted to the subsistence activity decreases when $b$ increases, $\frac{da}{db} < 0$, and we comment on the fact that the quantity on effort devoted to job search when $b$ increases almost always decreases, $\frac{ds}{db} < 0$.17

The marginal effect of the UB level on attention devoted to job search The following expression summarizes the necessary and sufficient conditions under which an increase in $b$ causes a reduction in the effort devoted to the subsistence activity, $a$, i.e., such that $\frac{da}{db} < 0$:18 (see Appendix A.1 for the general formula)

$$-u_c(c^u)_{sa} > \beta \frac{\partial (V^E - V^U)}{\partial b} (P_a - P_s (\frac{\lambda_{as}}{\lambda_{ss}} + \frac{p_{as}}{p_{ss}}))$$

necessary condition

sufficient condition

When $b$ increases, two different forces affect $a$. Consider first the left hand side (LHS) of the inequality: it tells us that when $b$ increases the marginal utility gain of effort devoted to the subsistence activity is smaller. This is because the marginal utility of consumption is decreasing (concave utility). This effect goes in the direction of reducing $a$. We call this an income effect: having additional money ($db > 0$), reduces the marginal utility of further consumption.

Consider now the right hand side (RHS) of the inequality: $\beta \frac{\partial (V^E - V^U)}{\partial b} < 0$ implies that an increment in $b$ distorts the relative value of being employed vs. being unemployed.19 This affects $a$ through two channels, a direct one, $P_a < 0$, and an indirect one through the effect of the change in $b$ on $s$, $-P_s (\frac{\lambda_{as}}{\lambda_{ss}} + \frac{p_{as}}{p_{ss}}) \leq 0$. The direct channel: since employment is less attractive, the negative effect that $a$ has on $P$ is marginally less detrimental for the utility of the agent. The indirect channel: An increase in $b$ has a negative direct impact on $s$ (The partial derivative of the FOC of $s$ with respect to $b$, $G_{sb}$, is negative; see in Appendix A.1), given the interactions between $s$ and $a$ ($\lambda_{as} \geq 0$, $P_{as} \leq 0$) this affects the optimal level of $a$. On the one hand, given the reduction of $s$, the marginal cost of $a$ becomes smaller. On the other hand, given the reduction in $s$, $a$ is now marginally less detrimental to $P(s, a)$. Both the direct and the indirect effects go in the direction of increasing $a$. We call this a substitution effect: increasing $b$ distorts the relative value of being employed vs. being unemployed. In the numerical exercise of section 4 we find, for a broad set of parameters, that the income effect always dominates the substitution effect, i.e., $\frac{da}{db} \leq 0$.

The marginal effect of the UB level on search effort The total expression for $\frac{ds}{db}$ is equation (29) in Appendix A.1. All forces in that equation, except one, push it to be negative. The only force against is due to the interaction between $a$ and $s$. When $\frac{da}{db} > 0$, the quantity of effort devoted to the subsistence activity decreases with $b$. If in addition $P_{as} < 0$ and $\lambda_{as} > 0$, these two effects push $s$ upwards. This is only possible, of course, when $\frac{da}{db} < 0$. Nevertheless, in the numerical exercise for the baseline model [BM], this effect is never big enough to compensate for the fact that a higher $b$ makes employment less attractive as compared with unemployment.

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17 We consider partial changes of $b$, i.e., we leave $\tau$ constant. In the numerical analysis we allow $\tau$ to increase as to compensate the increment in $b$.

18 If $\lambda_{as} = 0$, $\frac{1}{\lambda_{as}}$ must be replaced by $-\frac{1}{\lambda_{as}}$. In the inequality (5). If $P_{as} = 0$, $\frac{p_{as}}{p_{ss}}$ must be replaced by $-\frac{1}{\lambda_{as}}$. In the inequality (5), see Appendix A.1 for the details.

19 This effect is of course reinforced if we consider that financing a higher $b$ requires higher taxes, and thus the net wage $w - \tau$ is smaller.
2.3 Extensions to the Baseline Model [BM] and Monetary Cost of Job Search [MC]

In this section we introduce three extensions to the baseline model [BM] in which we abandon the stylized stationary setup we had so far. The aim of these extensions is to allow for a more realistic framework; in all the extensions time is finite and the agent lives for \( T \) periods (from period 0 to period \( T - 1 \)). In the first extension we assume that the agent is entitled to the UB for \( B < T \) periods, we call it “model with finite entitlement” [FE]. In the second one, we allow for the presence of incomplete financial markets: the agent starts her life with an exogenous level of assets, she can save and get indebted up to a certain limit \( L \), and she has to repay her debt at the end of period \( T - 1 \), we call it “model with incomplete financial markets” [FM]. In the third one, we assume a sequential search model when there is a distribution of wage offers and no recall (McCall, 1970): when the agent receives an offer she has to decide whether to accept it or to reject it. We call this model “stochastic wage offers” [SWO]. In Appendix A.1.2 we develop the three theoretical frameworks, and in section 4 we solve them numerically.

Moreover, our baseline model [BM] is formally similar to a model in which looking for a job requires job search effort and monetary expenditures. In fact, the model would be equivalent if (1) \( c_{min} = 0 \), (2) \( g(a) \) is replaced by \( -m \), the monetary expenditure, with \( P(s,a) \) replaced by \( P(s,m) \) where now \( P_m > 0 \), i.e. spending money in job search increases the probability of finding a job and if (3) \( \lambda(a,s) \) is replaced by \( \lambda(s) \) for all levels of \( a \), that is, if there’s no direct cost of the effort devoted to the subsistence activity. In Appendix A.1.2 we develop this setting theoretically and in section 4 we solve it numerically.

3 Optimal Unemployment Insurance

3.1 The Planner’s Problem

We consider here the baseline model [BM], a setting with a hand-to-mouth and infinitely lived agent. The optimal level of the flat benefit \( b \) maximizes the lifetime utility of the unemployed subject to a budget-balanced condition for the Government (that is, the agent covers all the expected costs of the UB scheme).

To construct the budget-balanced condition we transpose the approach of Shimer and Werning (2007) to a discrete time setup. The idea is that the net actualized cost of the job seeker should be null. Let \( C^U \) be the net actualized cost of the UB scheme for a job seeker, and \( C^E \) be the net actualized cost of a wage earner written in a recursive way. For simplicity, we write \( P \) instead of \( P(s,a) \):

\[
C^U = b + \beta[P C^E + (1 - P)C^U] 
\]

(6)

\[
C^E = -\tau + \beta[\phi C^U + (1 - \phi)C^E] 
\]

(7)

The net actualized cost of the job seeker should be zero. Then, \( C^E = \frac{-\tau}{1 - \beta(1 - \phi)} \). Plugging this last expression and \( C^U = 0 \) in (6), gives us:

\[
\frac{b}{\beta P} = \frac{\tau}{1 - \beta(1 - \phi)} 
\]

The expected duration of one episode of unemployment is equal to \( D = \sum_{t=0}^{\infty} (1 - P)^t = \frac{1}{p} \). Considering
this, \( \tau \) can be written as:

\[
\tau = \frac{1 - \beta (1 - \phi)}{\beta} b D
\]  

(8)

We are now ready to compute the optimal level of \( b \), i.e., the one that maximizes \( V^U \) subject to (8).

\[
\max_b V^U = \quad u(c^u) - \lambda(s,a) + \beta [P V^E + (1 - P) V^U]
\]  

(9)

The problem is stationary, therefore, \( V^U \) can be written as:

\[
V^U = \frac{1 - \beta (1 - \phi)}{(1 - \beta)(1 - \beta + \beta \phi + \beta P)} \left( u(c^u) + \frac{\beta P}{1 - \beta (1 - \phi)} u(c^e) - \lambda(s,a) \right)
\]

We need to look only at the direct impact of a change of \( b \), because the envelope conditions eliminate the first-order effects of the behavioral responses (Chetty, 2006). Deriving the previous expression with respect to \( b \) gives:

\[
\frac{dV^U}{db} = \frac{1 - \beta (1 - \phi)}{(1 - \beta)(1 - \beta + \beta \phi + \beta P)} \left( u'(c^u) - \frac{\beta P}{1 - \beta (1 - \phi)} u'(c^e) \right)
\]

(10)

Take \( \frac{dV^U}{db} = 0 \), and note from (8) that \( \frac{d\tau}{db} = \frac{1 - \beta (1 - \phi)}{\beta} \left( -\frac{1}{P} \frac{dP}{db} b + \frac{1}{b} \right) \). Plugging this in (10) and simplifying gives:

\[
0 = u'(c^u) - u'(c^e) \left( -\frac{1}{P} \frac{dP}{db} b + 1 \right)
\]

(11)

Note that the elasticity of duration with respect to \( b \), i.e \( \varepsilon_{D,b} \) is equal to \( \frac{b \frac{dP}{db}}{P \frac{dP}{db}} \). So, the optimal level of \( b \) verifies the following implicit equation:

\[
\frac{u'(c^u) - u'(c^e)}{u'(c^e)} = \varepsilon_{D,b}
\]

(12)

This is the Baily-Chetty formula (Baily, 1978 and Chetty, 2006). Its interpretation is the standard one, the LHS of the equation is equal to the gain of \( b \): consumption smoothing. The RHS, in turn, captures the moral hazard costs of benefit provision due to behavioral response; it is typically assumed that a higher \( b \) reduces the job search effort of the agent, causing her to remain longer in unemployment, this negative reaction to the distorted incentives is captured by the RHS. \(^{21}\)

Appendix A.2.2 generalizes this analysis in a non-stationary setup. The resulting formula has only slight modifications, the most important being that, since \( c_t \neq c_{t+1} \), then what is relevant is the average consumption when employed and when unemployed.

\(^{20}\)This is the quantity of taxes that has to be paid during one episode of employment to cover the costs of one episode of unemployment. Job destruction, i.e, \( \phi \neq 0 \), means that there will be several employment and unemployment spells. Nevertheless, since time is infinite, this formula remains valid: the total expenditure in unemployment will be paid during the total time the agent is employed.

\(^{21}\)Schmieder and von Watcher (2017) highlight that the RHS can be written as the behavioral cost of changing \( b \) divided by the mechanical cost of changing \( b \), where the mechanical cost is defined as “by how much the policy change actually increases the transfer to the unemployed in the absence of behavioral responses”.

---

9
If the underlying model is the standard model [SM] -or a model like the one of Chetty (2008)-, the previous formula can be rewritten as:

\[
\frac{\text{liquidity effect}}{\text{moral hazard effect}} = -bD(\text{liquidity effect} + \text{moral hazard effect}) \tag{13}
\]

where liquidity effect \( = \frac{\partial P}{\partial A} \) and moral hazard effect \( = \frac{\partial P}{\partial w} \).

The decomposition of the LHS is put forward by Chetty (2008), the decomposition of RHS is evident since \( \epsilon_{D,b} = -bD \frac{\partial P}{\partial b} \), and \( \frac{\partial P}{\partial b} \) = liquidity effect + moral hazard effect. Several comments are in order. First, from this expression it is very clear that conditionally on \( \epsilon_{D,b} \), the higher the liquidity effect, the higher the optimal level of \( b \), and in this sense “if an agent chooses a longer duration primarily because he has more cash on hand (as opposed to distorted incentives), we infer that UI benefits bring the agent closer to the social optimum” Chetty (2008). Note, however, that unconditionally, a higher liquidity effect does not necessarily imply a higher optimal level of \( b \), because the liquidity effect affect both sides of (13) in the same direction. Second, as Chetty, 2008 and Chetty, 2009 highlight, the “sufficient statistics” approach is very useful, but since all the inputs (\( \epsilon_{D,b} \), the moral hazard effect and the liquidity effect) are endogenous to the level of \( b \), an empirical analysis that follows this sufficient approach applies only locally. Third, the “sufficient statistics” approach is less model dependent than other alternatives (structural estimation or numerical simulations), as Chetty (2008) claims. Nevertheless, it should be noticed that the LHS decomposition crucially depends on some modeling assumptions: (1) the probability of finding a job, \( P \), cannot be affected by more than one variable: When \( b \) changes, all variables except for one, should remain constant; (2) The cost of search needs to be separable from consumption. If one of these two assumptions is not verified, it is no longer possible to re-write the LHS of the Baily-Chetty formula using sufficient statistics, i.e, as the ratio of the liquidity effect and the moral hazard effect. In our setting (baseline model [BM] and its extensions) these two conditions are not satisfied: devoting attention to job search has a cost that is not separable from consumption and the probability of finding a job is affected by two choice variables (\( P \) is a function of \( a \) and \( s \)), meaning that the decomposition in sufficient statistics is not possible (See Appendix A.2.1 for the proof).

3.2 Interpretation and the role of \( c_{min} \) and \( g(a) \)

Starting from our baseline model [BM], the Baily-Chetty formula, can be written as:

\[
\frac{u'(b + g(a) - c_{min}) - u'(w - \tau - c_{min})}{u'(w - \tau - c_{min})} = \epsilon_{D(a,s),b} \tag{14}
\]

Even if the formula for the optimal \( b \) is the standard one, our baseline model [BM] incorporates two new elements with respect to the standard model [SM]: \( c_{min} \) and \( g(a) \), which introduce changes both to the LHS and to the RHS of the equation. In the following paragraphs we study the impact of each of them separately.

The role of \( c_{min} \): If we consider the standard model [SM], and we introduce a subsistence requirement, the LHS of (12) will be higher when preferences exhibit decreasing absolute risk aversion, which is a common assumption (See for instance Mas-Colell et al., 1995 p.193). Regarding the RHS, which can be written as: \( \epsilon_{D,b} = -bD \frac{\partial P}{\partial b} = bD \frac{\partial P}{\partial s} (\frac{\partial s}{\partial b}) \), one can show that in the presence of \( c_{min} \), \( s \) is

22 Gerard and Gonzaga (2016) also report that the decomposition is not possible in their setting because, as we do, they consider several choice variables. If one is willing to assume that when \( b \) changes one of the two variables remain constant -as Chetty (2008) does when he incorporates the reservation wage (a second variable) in his setup- then the decomposition remains possible, if the variable that changes has a cost which is separable from consumption.
higher,\textsuperscript{23} and thus \( P(s) \) is higher, this causes the two first terms of this expression to be lower, but it is not clear whether \( \frac{ds}{db} \) is also lower in the presence of \( c_{\min} \), meaning that the effect over the RHS is ambiguous.\textsuperscript{24}

The role of \( g(a) \): If we take the standard model [SM] and (1) add the subsistence activity \( g(a) \), (2) consider a specification for \( P(s,a) \) such that if the agent devotes no effort to the subsistence activity, i.e \( a = 0 \), the function reduces to the same function of the standard model \( P(s) \) (for example \( E s \beta_{1} e^{- \beta_{2} a} \)), then the optimal level of \( b \) will be lower in the resulting model as compared with the [SM], ceteris paribus. This is because the resulting model is more general, in the sense that the agent always has the option of devoting zero effort to the subsistence activity and by doing so to go back to the standard model [SM]. In a sense, the subsistence activity has a self-insurance role, because it allows to increase consumption when the agent is unemployed.\textsuperscript{25}

4 Numerical Exercise

In this section we solve the optimization problem numerically. First, we show that the property of a hump-shaped \( P(s,a) \) is present in the baseline model [BM]; it is robust to a wide number of parametrizations, and to the extensions introduced in section 2.3. Second, we show that in our baseline model [BM] and in its extensions, providing liquidity to the agent when the level of \( b \) is low, can increase the probability of finding a job, instead, when the level of \( b \) is high, we re-encounter the standard property, that is, providing liquidity to the agent decreases the probability of finding a job. Third, we analyze the effect of \( g(a) \) and \( c_{\min} \) on the optimal level of \( b \).

The benchmark parametrization takes the functions and parameters specified in Table 1. We take the time unit to be a week. The values of \( \phi \) and \( r \) are taken from Shimer and Werning (2007). We assume a constant relative risk aversion (CRRA) utility function where \( \sigma \) (the CRRA) is taken from Chetty (2008). Due to lack of evidence, it is hard to pinpoint the values for the other parameters. Nevertheless, the chosen parametrization applied to the baseline model [BM] gives, for the optimal level of \( b \) (which is \( b = 0.62w \), i.e, a gross replacement rate (RR) of 0.62) an expected duration of one episode of unemployment equal to of 18.13 weeks, which is reasonable.\textsuperscript{26} A sensitivity analysis for other 42 specifications is provided in Table 2.

4.1 Impact of \( b \) on the Duration in Unemployment

In this section we numerically solve the baseline model [BM], its extensions ([FE], [FM] and [SWO]) and the model with monetary costs of job search [MC], and we analyze the relationship between the level of \( b \) and the expected duration in unemployment. Solving the baseline model [BM] numerically we find that the relationship between \( P \) and the level of \( b \) is hump-shaped: when \( b \) is low enough, increasing it increases the probability of finding a job; instead, when \( b \) is big enough, increasing it, reduces the probability of finding a job. This property is robust to the presented extensions of the model. In this section we also comment some empirical results found in the literature.

Baseline Model [BM]: In section 2.2 we showed that in the baseline model [BM] an increment in \( b \) has an ambiguous effect on \( P \), the probability of finding a job, contrary to the standard job

\textsuperscript{23}Consider the standard model [SM], The FOC is: \( G_{s} \equiv - \lambda_{s} + \beta P_{s}(V^{E} - Y^{U}) = 0 \), and \( \frac{ds}{d_{c_{\min}}} = - \frac{G_{s,c_{\min}}}{G_{s}} \). We know that \( G_{ss} < 0 \), and \( G_{b,c_{\min}} \equiv \frac{\beta P_{s}}{1 - 1/(1 - P_{s}) - \phi} (-u'(c') + u'(c'_{a})) > 0 \), therefore \( \frac{ds}{d_{c_{\min}}} > 0 \).

\textsuperscript{24}Recall that the standard model with subsistence requirements, without any mechanism to cope with those subsistence requirements, is only defined for levels of \( b > c_{\min} \).

\textsuperscript{25}In a setup with home production Taskin (2011) finds a similar result.

\textsuperscript{26}Chetty (2008) calibrates his model for the US to have an average unemployment duration of 15.8 weeks.
### Table 1: Functional Forms and Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Functional Form</th>
<th>Source</th>
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<tbody>
<tr>
<td>$u(c^u)$</td>
<td>Utility Function</td>
<td>$\frac{1}{1-\sigma}$</td>
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<tr>
<td>$\lambda(s,a)$</td>
<td>Cost of Search effort</td>
<td>$e^{(\mu_1 s + \mu_2 a)} - 1$</td>
</tr>
<tr>
<td>$g(a)$</td>
<td>Subsistence Production</td>
<td>$Ga^\gamma$</td>
</tr>
<tr>
<td>$P(s,a)$</td>
<td>Prob. of finding a job</td>
<td>$Es^{\beta_1}e^{-\beta_2 a}$</td>
</tr>
</tbody>
</table>

### Parameters (baseline)

| $\phi$ | Job Destruction rate | 0.00443 | Shimer and Werning (2007) |
| $r$ | Interest rate | 0.001 | Shimer and Werning (2007) |
| $\beta$ | Discount rate | 0.999 | $1/(1+r)$ |
| $E$ | Coefficient in front of $P(s,a)$ | 0.2 |
| $\beta_1$ | Exponent of $s$ in $P(s,a)$ | 0.5 |
| $\beta_2$ | Exponent of $a$ in $P(s,a)$ | 0.5 |
| $w$ | Wage | 100 |
| $c_{min}$ | Subsistence level | 20 |
| $\sigma$ | RRA | 1.75 | Chetty (2008) |
| $\mu_1$ | Parameter of $s$ in $\lambda(s,a)$ | 0.3 |
| $\mu_2$ | Parameter of $a$ in $\lambda(s,a)$ | 0.3 |
| $G$ | Scale parameter of $g(a)$ | 22 |
| $\gamma$ | Exponent if $g(a)$ is isoelastic | 0.8 |

Changing the values of the parameters changes the levels of $b$ for which $P(s,a)$ is maximal. Nevertheless, the qualitative shape of the graphs remains the same for a broad set of parameters. Table 2 reports the results for 42 different specifications. In this table, the 13 first columns show the values of the parameters, and the last six the results: “argmax $P$”, is the level of $b$ for which $P(s,a)$ reaches the maximum (the equivalent to 19 in the previous graph). In all but two of the specifications of Table 2, when $c_{min} > 0$ the hump-shape of $P(s,a)$ is preserved. It is not preserved when $\beta_2$, the exponent of the isoelastic $a$ in $P(s,a)$ is very low (equal to 0.1) and when $G$, the scale parameter of $g(a)$, is very high (equal to 70). In the first case, the negative slope of $P(s,a)$ for all values of $b$ is explained because for any level of job search effort, the negative effect on the job finding probability of devoting...
Figure 1: Baseline Model [BM]: The three graphs show the optimal level of $a$, $s$ and $P(s,a)$ respectively, in the baseline model [BM] for different values of $b$. The functions and parameters are the ones presented in Table 1.

effort to the subsistence activity is very small. So even if $a$ always decreases (strictly for low values of $b$, weakly for high values of $b$) when $b$ increases, this effect is dominated by $\frac{ds}{db} < 0$. In the second case, the agent has a very big capacity to self-insure, this implies that by devoting a small quantity of effort to the subsistence activity she is able to meet the subsistence requirements, and thus that the negative effects of this activity on the job finding probability are very limited and always dominated by $\frac{ds}{db} < 0$.

Even though there is a large evidence showing that duration increases when $b$ increases, see Tatsiramos and van Ours (2014) for a survey, to the best of our knowledge none of the original studies this paper cites has looked at the effects of $b$ when its level is “low”, i.e below subsistence requirements. The closest one among those is Lalive et al. (2006), which analyzes the effect of a rise in UBs in Austria in 1989 of 4.6 percentage points, starting from a replacement rate of 41%, to people with low wages (below 12 610 ATS, the median was 16 400 ATS), who on top of the UB receive family allowances. They found using a diff-in-diff approach that the increment in the UB increased duration by 0.38 weeks (from 20.60 -treated- to 20.97 -control- weeks), which implies an elasticity of about 0.15.\textsuperscript{27} The analyzed people are probably above the subsistence level, (because the pre-reform level of the UB is already substantial and because on top of that they have family allowances). Nevertheless, the effect of an increase of UB is very small. The rest of the studies, which do not consider low income people, have estimates which are higher, and range between 0.35 and 1.7.

LaLumia (2013) estimates a hazard model for a sample of people eligible to the earned income tax credit (EITC) in the United States. She has a sample of 5881 unemployment spells, 2173 men and 3708 women, by construction this is a low income sample. 23% of the unemployment spells in her sample involve the receipt of UB. On average, individuals in her sample are eligible for about $150 weekly UB measured in 2007 real dollars. She finds that the effect of UB on women is non significant in any specification. For men, in some of her specifications the effect of UB on the hazard rate is positive, and in others it is nonsignificant.

Kupets (2006) makes a duration analysis for Ukraine, using the Ukranian Longitudinal Monitoring Survey for the years 1998-2002. The level of UB is low, of around 25-28% of the official average wage. Only 4.6% of the sample reported UB to be their main source of support, 13.9% of the sample states that casual activities or subsistence farming constitute their main source of subsistence. She finds that: “The estimate of the variable on receipt of unemployment benefits fails to reject our hypothesis of insignificant effect of unemployment benefits on reemployment probability in the case of the total sample of unemployed. However, the effect of unemployment benefits is found to be significant and negative if we take only “standard” unemployed without any income from casual work”. Moreover, she finds a negative effect of the presence of casual work on the finding rate probability, she states: “On the other hand, those usually unemployed persons who are occasionally engaged in unreported activities or subsistence farming tend to search for regular jobs less intensively and, therefore, they are less likely to receive a job offer.”

Moreover, Morris and Wilson (2014) studies the impact of a reduction in the level of unemployment

\textsuperscript{27}In their sensitivity analysis they use a RDD and find that the effect is even smaller, of 0.31 weeks.
assistance “Newstart” in Australia (“The Newstart payment at AUD255.25 per week for a single person (March 2014) is well below the poverty line even when government rent assistance is included.”) using a survey and in-depth interviews, they state: “Insufficient income contributed to stress, and it added to circumstances in which interviewees struggled to maintain their confidence in a job interview (if they even reached that point). Physical appearance particularly suffered; interviewees told of how difficult it was to keep themselves groomed, appropriately attired and motivated”.

All this suggests that the impact of $b$ on the probability of finding a job may vary according to the actual level of the UB, and that if the level of UB is very low the effect of increasing it is not necessarily negative.

For the coming three extensions, unless stated otherwise, we use the functions and parameters specified on Table 1, with one exception: For simplicity (as Hopenhayn and Nicolini, 1997, Chetty, 2006, Shimer and Werning, 2008, Schmieder et al., 2012, Kolsrud et al., 2015, Kroft and Notowidigdo, 2016) we consider that employment is an absorbing state ($\phi = 0$). We take the time unit to be a week and we set $T = 200$ (from $t = 0$ to $t = 199$), as the total quantity of time.

**Finite Entitlement [FE]:** The model with finite entitlement considers the case in which the agent is entitled to a flat benefit $b$ for a number of periods $B$ strictly smaller than $T$. We solve this problem numerically, using the functions and parameters of Table 1, moreover, we set to $B = 100$ (from $t = 0$ to $t = 99$). To show how the behavior of the agent changes when the end of entitlement is approaching, in Fig. 2 we report the optimal choice of the agent in several periods (period 0, 24, 74 and 99). As can be observed, the graph at the beginning of the unemployment spell (period 0) looks very similar to the graph for the case where we had stationarity (Fig.1), in particular the hump-shape of $P(s,a)$ is preserved. In period 99, the last period in which the agent receives $b$, $\frac{d(V^U_t - V^I_t)}{db} = 0$ this causes $s$ to have a weakly positive slope for all $b$ (see equation (31) in the Appendix A.1.2).

**Incomplete Financial Markets [FM]:** The model with incomplete financial markets considers the case in which the agent starts her life with an exogenous level of assets $k_0$, and she can get indebted up to a certain binding limit $L$. We assume that she has to repay the debt at the end of the last period. We solve this problem numerically, using the functions and parameters of Table 1, moreover, we assume that the agent starts the unemployment spell with an exogenous level of assets $k_0 = 0$, and we allow her to get indebted up to 200, that is, up to two times the gross wage. In each period the agent chooses $a_t$ and $s_t$, moreover, she also chooses the optimal level of assets for the next period, $k_{t+1}$ from a grid of different values of assets that goes from -200 to 0. In Fig. 3, we report the optimal choices of $a_t$, $s_t$ and the probability of finding a job $P(s_t,a_t)$ at the beginning of the unemployment spell. As Fig. 3 shows, the hump shape of the exit rate from unemployment is preserved in this context.

**Stochastic Wage Offers [SWO]:** The model with stochastic wage offers considers the case in which the distribution of offers is not degenerate and is known. If an offer is received, the agent follows a stopping rule: she accepts the job offer if the wage is above her reservation wage (an additional choice variable), otherwise she rejects it. We solve this problem numerically, using the functions and parameters of Table 1, moreover we assume that wages are Pareto distributed with minimum possible value $w_{min} = 66.66$ and shape parameter $\alpha = 3$, so that the average wage is equal to 100. We set the coefficient of relative risk aversion $\sigma = 2$ because an integer allows us to find a closed form expression for $V^U_t$, which simplifies the numerical analysis. In Fig. 4 we report the optimal choices of $a_t$, $s_t$ and the exit probability out of unemployment $P(s_t,a_t) \times (1 - H(x_t))$ at the beginning of the

---

28 Both $B$ and $b$ could be part of the optimal unemployment insurance design (see for instance Hopenhayn and Nicolini (1997)), nevertheless in this paper we look at the optimal level of $b$ conditional on $B$, as Baily (1978), Chetty (2006) and Chetty (2008) do.
<table>
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</table>

Table 2: Sensitivity Analysis for the Baseline Model [BM]: This table reports the results for 42 different specifications. The 13 first columns show the values of the parameters, and the last five the results. \("\text{argmax } P\)\) is the level of \(b\) for which \(P(s,a)\) reaches the maximum. \("\text{Gross RR}\)\) gives, for each specification, the optimal gross replacement rate: \(\frac{b}{w}\), respectively \("\text{Net RR}\)\), the optimal net replacement rate \(\frac{b}{w} - \tau\). Moreover, \("\frac{c_{u}^{*}}{c_{e}^{*}}\)\) gives the ratio of the consumption in unemployment \(c_{u}^{*} = b + g(a)\) divided by the consumption in employment \(c_{e}^{*} = w - \tau\), when \(b\) is the optimal one. Finally, \(a^{*}\) and \(s^{*}\) are the levels of \(a\) and \(s\) when \(b\) is optimal.
Figure 2: **Finite Entitlement [FE]**: Each column has three graphs that show the optimal level of $a_t$, $s_t$, and $P(s_t, a_t)$ respectively, in the model finite entitlement [FE] for different values of $b$. We report in each line the results for period 0, 24, 74 and 99, respectively. The functions and parameters are the ones presented in Table 1, except for $\phi$ which is now equal to zero. We set $T = 200$, the total quantity of time, and $B = 100$, the number of periods in which the agent is entitled to the flat benefit $b$.

Figure 3: **Incomplete Financial Markets [FM]**: The three graphs show the optimal level of $a_t$, $s_t$, and $P(s_t, a_t)$ respectively, at the beginning of the unemployment spell, in the model with incomplete financial markets [FM] for different values of $b$. The functions and parameters are the ones presented in Table 1, except for $\phi$ which is now equal to zero, moreover we allow the agent to get indebted up to 200 (two times the wage) and we assume that the agent has to repay her debt at the end of the $T = 200$ periods.
Figure 4: **Stochastic Wage Offers [SWO]**: The three graphs show the optimal level of $a_t$, $s_t$ and $P(s_t, a_t) \cdot (1 - H(x_t))$ respectively, at the beginning of the unemployment spell, in the model with stochastic wage offers for different values of $b$. The functions and parameters are the ones presented in Table 1, except for $\phi$ which is now equal to zero, and $\sigma = 2$. We set $T = 200$, the total quantity of time. We assume that wages are Pareto distributed with parameters $w_{min} = 66.66$ and $\alpha = 3$.

Monetary Costs of Job search [MC]: Even without subsistence requirements, in a framework in which looking for a job requires effort and money, the relationship between $b$ and the probability of finding a job can also be hump-shaped. To show this we solve this stationary problem numerically using the functions and parameters of Table 1, where now instead of $a$ we have $m$: money devoted to job search and we change three things: (1) the probability of finding a job is now $P(s, m) = E \beta_1 (m/10)^{\beta_2}$, (2) Instead of $g(a)$ we have now $-m$, where $m$ is the quantity of money spent in job search (3) $c_{min} = 0$ and finally (4) we set $\beta_1 = \beta_2 = 0.5$.

To have a hump-shaped $P(s, m)$, the monetary expenditure needs to have an important contribution to the job finding probability $P(s, m)$, that is $\beta_2$ cannot be negligible compared with $\beta_1$. In Fig. 5, $\beta_1 = \beta_2 = 0.5$, this is enough to generate the hump-shape. Nevertheless, if $\beta_2 = 0.2$, for instance, then we would not have the hump-shape any more, and $P(s, m)$ would always be decreasing with respect to $b$. Also the functional form of $P(s, m)$ in general is important. If we would have instead $P(s, m) = E \beta_1 (1+m)^{\beta_2}$ for instance, which means that, even when the monetary expenditure increases the probability of finding a job, it’s role is not crucial, then we would not have a hump-shaped curve either.

There is very few data about how much money is spent in job search. One of the few estimates is the one of Stephenson (1976) who finds that white youth use 25% of their income to find work. Schwartz (2015) uses a baseline expenditure of 30% of the level of the UB.
4.2 Liquidity Effect

Providing an annuity to the agent in each period, regardless of her employment status, increases the expected duration in unemployment in the standard model [SM]. In this section we question this property in the baseline model [BM], its extensions ([FE], [FM] and [SWO]) and the model with monetary costs of job search [MC] numerically. Additionally, we comment some empirical results found in the literature.

Let us start by analyzing the effect of an annuity in the baseline model [BM]. We analyze the effect of the annuity for each possible budget-balanced level of \( b \). Consider Fig. 6, where all the functions and the parameters are the ones described in Table 1. In this graph we show which is the effect of providing liquidity to the agent. To do that we compare two cases: (1) the only income of the agent is \( b \) (the continuous line) and (2) on top of \( b \) the agent receives an annuity of 10 (the dashed line). Both curves intersect in a point close to \( b = 15 \). When \( b \) is above 15, providing liquidity to the unemployed decreases her expected probability of finding a job (the dashed line is below the continuous line). Nevertheless, when \( b \) is below 15, providing an annuity to the agent increases her probability of finding a job (the dashed line is above the continuous line). The intuition behind this effect is that for low levels of \( b \) subsistence is not guaranteed: providing money to the agent when \( b \) is low allows her to reduce the effort devoted to the subsistence activity and therefore to increase the cognitive resources devoted to job search. This effect is also present in the extensions to the baseline model, as will be shown below.

There is some recent evidence showing that when people are close to subsistence levels of consumption, providing money may help them to leave unemployment: Franklin (forthcoming) develops an experiment in Ethiopia where he gives money (intended to cover transportation costs) to young job-less people. He finds that four months after the start, people who received the subsidy were seven percentage points more likely to have a permanent work. He states: “The positive impacts on employment seem to be driven directly by increased job search in response to subsidies. Weekly data from the phone surveys show that treatment has a significant impact on job search, and an even larger impact on search in city centre, and search through formal methods.” and “Finally the treatment effects on search and employment are particularly strong among the relatively poor and cash constrained”. Barrientos and Villa (2015) find, using a regression discontinuity design, that a conditional anti-poverty cash transfer in Colombia (conditional on maintaining kids in school) had positive effects not only on labor force participation, but also on the level of employment of adult males. On the same line, Banerjee et al. (forthcoming) analyze the effect of conditional and unconditional cash transfers to low income families of seven different programs in developing countries on work outside the household (which includes casual or permanent employment, and excludes any

\[ \text{The program is called "Familias en Acción", "Familias en Accion was introduced in 2001 by the government of Colombia with the aim of strengthening human capital investment among children in poorest households in rural areas and small towns" Barrientos and Villa (2015).} \]
Figure 7: **Liquidity Effect**: These graphs show the optimal value of $P$ for different values of $b$, for the extensions: [FE], [FM] and [SWO] (For [SWO] instead of reporting $P$, we report $P \times (1 - H(x))$ for the optimal $x$). We also include the results for the model with monetary costs [MC]. The continuous line is generated with the functions and parameters discussed in the previous section for each model, the dashed line is generated with the same parameters except for the fact that the agent receives an annuity of 10 regardless of her employment status.

After pooling the samples of the five comparable programs they do not find evidence of a negative effect, moreover, when treating each program separately in some cases they find a positive effect. This evidence is not in line with the [SM] or a model like the one of Chetty (2008) in which providing an annuity to the agent always has negative effects on the probability of finding a job.

We now check whether the above numerical properties are robust in two senses. First, Fig. 6 has been derived for an annuity of 10. The qualitative properties of Fig. 6 hold true as long as the annuity is at most equal to 35% of the wage. When the annuity is above 35% of the gross wage, the annuity increases expected duration, even if $b$ is low. In this case, we are back to the standard property found in previous papers quoted in the introduction.

Second, we check whether Fig. 6 remains valid for extensions of the model [BM]. We provide the numerical results for the extensions ([FE], [FM] and [SWO]) and for the model with monetary costs of job search [MC]. The parametrization for each model is the one used in the previous subsection, where the effect of $b$ on the expected duration was analyzed. Fig. 7 shows the results. For the extensions to the baseline model [BM], the intuition is the same as before. For low levels of $b$ subsistence is not guaranteed: providing money to the agent when $b$ is low allows her to reduce the effort devoted to the subsistence activity and therefore to increase the cognitive resources devoted to job search. For the model with monetary costs [MC] the intuition is the following: when the level of $b$ is very low, the marginal utility of consumption is very high, which causes that just a small amount of money is devoted to job search; the presence of the annuity relaxes this trade-off and allows the agent to devote more money to job search.

Regarding these programs Banerjee et al. (forthcoming) state: “Once a household becomes eligible for any of the programs that we study, the amount of benefit that one receives is the same regardless of actual income level and lasts at least a period between 2 and 9 years, depending on the program. This differs from many U.S. transfer programs (e.g. EITC, SNAP), where the stipend depends (either positively or negatively) on family income, and is updated frequently.”
4.3 The Optimal level of $b$

In section 3.2 we showed that the new elements of the baseline model [BM] ($c_{min}$ and $g(a)$) have an ambiguous effect on the optimal level of $b$. In this section we first comment Table 2 to see the impact of $c_{min}$ and the shapes of $\lambda(s,.)$ and $g(.)$ on the optimal level of $b$, $b^*$. Then, we compare the $b^*$ obtained in the [BM] with the one of the [SM].

Table 2 displays different indicators when $b$ verifies the optimality condition (12). “Gross RR” is the optimal gross replacement rate: $\frac{b}{w}$ where in the simulations $w = 100$, “Net RR”, is the optimal net replacement rate $\frac{b}{w-\tau}$, “$c^*/c_\tau^*$” is the ratio of the consumption in unemployment $c_\tau = b + g(a)$ divided by the consumption in employment $c^*_\tau = w - \tau$, at $b^*$, and finally “$a^*$” and “$s^*$” are, respectively, the effort devoted to the subsistence activity and to job search when $b = b^*$. Note that, in general, the level of “Net RR” is very similar to the level of “$c^*/c_\tau^*$”. This is because $g(a)$, is very close to zero when $b$ is optimal.

As can be observed, the optimal gross replacement rate is, in most of the cases, between 0.55 and 0.72. Several other studies have computed the optimal value of $b$ in different contexts and with different assumptions, but they tend to find replacement rates close to 0.50-0.60 (see for instance Pavoni, 2007 and Chetty, 2008).

Let us now analyze the impact of $c_{min}$ and the shapes of $\lambda(s,.)$ and $g(.)$ on $b^*$. It can be seen that higher levels of $c_{min}$ imply higher levels of $b^*$. Four parameters of Table 2 are linked to $a$ ($\beta_2$, $\mu_2$, $G$, $\gamma$). As can be observed, the optimal level of $b$ doesn’t change with $\beta_2$, $\mu_2$ or $G$. This is because $a \approx 0$ for levels of $b$ above 60 in these specifications, therefore changing the parameters associated with $a$ has no implications. Instead, when $\gamma$ changes, the optimal level of $b$ changes, because for low levels of $\gamma$, $a$ is not negligible even for high values of $b$. In that case, $b^*$ increases when $\gamma$ increases, which is intuitive, because the value of $g(a)$ for a given $a$ is smaller the higher the $\gamma$ (for values of $a < 1$). Therefore higher values of $\gamma$ reduce the self-insurance capacity of the agent. It can be seen that higher levels of $c_{min}$ imply higher levels of $b^*$.

Our numerical exercise shows that the optimal replacement rate in the standard model [SM] is 0.57. Table 2 shows the optimal replacement rate in the baseline model [BM] for the different parameters linked to $a$ ($\beta_2$, $\mu_2$, $G$, $\gamma$) and for different levels of $c_{min}$. In all cases, except in one, the optimal replacement rate is higher or equal in the [BM] as compared with the [SM]. The only exception is the case in which $\gamma = 0.1$. When $\gamma = 0.1$, low levels of $a$ are able to generate a high $g(a)$, therefore this constitutes an important margin of self-insurance which reduces $b^*$.

5 Conclusion

The available theory of job-search is developed in a very streamlined setting. This has the advantage of providing a number of clear-cut testable predictions. On this basis and given the accumulated empirical evidence, it is commonly accepted that a rise in the level of unemployment compensation lowers the probability of finding a job. It is also well accepted that providing cash irrespectively of the employment status has the same qualitative effect. However, the stylized nature of job-search theory sets aside a number of day-to-day problems encountered during joblessness. This paper has put forward the need to cover a minimal level of consumption in an otherwise standard job-search problem. Under realistic assumptions about the absence of private unemployment insurances and about imperfect capital markets, a minimal consumption level cannot be guaranteed when benefits are very low or absent (a feature shared by many countries). As is now well documented, living in scarcity is extremely stressing. Through some subsistence activities, the jobless individual can tackle scarcity. However, performing those activities limits cognitive capacities available for job-search. Providing a higher level of benefit can then relax the constraints imposed by those limits.

Through this channel, this paper has shown that a rise in benefits (restricted to the unemployed or
also given in case of employment) can raise the exit probability to a job. This property is established numerically in a range of standard job-search settings. Qualitatively, this occurs when benefit levels are low enough. These properties are not counter-factual. For, most empirical evaluations of the impact of benefits do not focus on the low-income jobless population or study the impact of unemployment insurance systems that are fairly generous. The few papers we have found dealing with this population and considering low benefits actually provide evidence that goes in the direction predicted by our model.

This paper has also studied the optimal level of unemployment benefits. From an analytical point of view, we retrieve the standard Baily-Chetty formula. Our theoretical contribution consists in highlighting that writing the gain from insurance as the ratio of the liquidity effect to the moral hazard effect (Chetty, 2008) cannot be done without imposing restrictive and, to the best of our understanding, often unnoticed modeling assumptions. Finally, our numerical exercise indicates that the optimal replacement ratio is typically slightly higher in our setting in comparison with a standard job-search model. The optimal level of benefits is also typically such that the agent is not or not much devoting effort to subsistence activities. From these properties, we conjecture that many actual unemployment insurance systems are suboptimal.

A Appendix

A.1 Positive Analysis

A.1.1 The Baseline Model [BM]

The first order conditions are already stated in the main text (3, 4). The second order derivatives are:

\[ G_{ss} \equiv -\lambda_{ss} + \beta P_{ss}(V^E - V^U) < 0 \] \hspace{1cm} (15)

\[ G_{sa} \equiv -\lambda_{as} + \beta P_{sa}(V^E - V^U) < 0 \] \hspace{1cm} (16)

\[ G_{as} \equiv -\lambda_{as} + \beta P_{sa}(V^E - V^U) < 0 \] \hspace{1cm} (17)

\[ G_{aa} \equiv u_{cc}(c^u)^2 + u_{c}(c^u)g_{aa} - \lambda_{aa} + \beta P_{aa}(V^E - V^U) < 0 \] \hspace{1cm} (18)

\[ G_{sb} \equiv \beta P_{a} \frac{\partial(V^E - V^U)}{\partial b} = \frac{-\beta P_{s}}{1 - \beta[1 - (P(a,s) - \phi)]} u_{c}(c^u) < 0 \] \hspace{1cm} (19)

\[ G_{ab} \equiv u_{cc}(c^u)g_{sa} + \beta P_{a} \frac{\partial(V^E - V^U)}{\partial b} = u_{cc}(c^u)g_{sa} - \frac{\beta P_{a}}{1 - \beta[1 - (P(a,s) - \phi)]} u_{c}(c^u) \geq 0 \] \hspace{1cm} (20)

\[ G_{sw} \equiv \beta P_{s} \frac{\partial(V^E - V^U)}{\partial w} = \frac{\beta P_{s}}{1 - \beta[1 - (P(a,s) - \phi)]} u_{c}(c^e) > 0 \] \hspace{1cm} (21)

\[ G_{aw} \equiv \beta P_{a} \frac{\partial(V^E - V^U)}{\partial w} = \frac{\beta P_{a}}{1 - \beta[1 - (P(a,s) - \phi)]} u_{c}(c^e) < 0 \] \hspace{1cm} (22)

\[ G_{sA} \equiv \beta P_{s} \frac{\partial(V^E - V^U)}{\partial A} = \frac{\beta P_{s}}{1 - \beta[1 - (P(a,s) - \phi)]}[u_{c}(c^e) - u_{c}(c^u)] < 0 \] \hspace{1cm} (23)

\[ G_{aA} \equiv u_{cc}(c^u)g_{sa} + \beta P_{a} \frac{\partial(V^E - V^U)}{\partial A} = u_{cc}(c^u)g_{sa} + \frac{\beta P_{a}}{1 - \beta[1 - (P(a,s) - \phi)]}[u_{c}(c^e) - u_{c}(c^u)] \geq 0 \] \hspace{1cm} (24)
where \( V^E - V^U = \frac{1}{1-\beta[1-\rho(s,a)-\varphi]}(u(c^e) - u(c^u) + \lambda(s,a)). \)

The following conditions are sufficient to guarantee that a solution, if any, to the system (3, 4) is a unique maximum: \( G_{ss} < 0 \) (so that \( P_{ss}, \lambda_{ss} \) cannot simultaneously be zero), \( G_{aa} < 0 \) and \( G_{ss}G_{aa} - G_{as}^2 > 0 \).

Totally differentiating the FOC (3, 4) with respect to \( s \) and \( b \) leads to:

\[
\begin{align*}
G_{ss}ds + G_{sa}da + G_{sb}db &= 0 \\
G_{as}ds + G_{aa}da + G_{ab}db &= 0
\end{align*}
\]

Hence:

\[
\begin{align*}
\frac{da}{db} &= -\frac{G_{ss}G_{ab} + G_{as}G_{sb}}{G_{ss}G_{aa} - G_{as}^2} \\
\frac{ds}{db} &= -\frac{G_{as}G_{ab} + G_{aa}G_{sb}}{G_{ss}G_{aa} - G_{as}^2}
\end{align*}
\]

Where, the denominator of both expressions needs to be positive by the second order conditions.

**The sign of \( \frac{da}{db} \):**

Since the denominator needs to be positive, let’s concentrate on the numerator of \( \frac{da}{db} \).

\[
-[-\lambda_{ss} + \beta P_{ss}(V^E - V^U)]u_{cc}(e^u)g_a + \beta P_a \frac{\partial(V^E - V^U)}{\partial b} + \beta P_s \frac{\partial(V^E - V^U)}{\partial b} [-\lambda_{as} + \beta P_{sa}(V^E - V^U)]
\]

Gss: (-) \hspace{1cm} Gab: (-) \hspace{1cm} Gsb: (-)

Note first that having \( G_{ab} < 0 \) is a necessary condition to have \( \frac{da}{db} < 0 \), nevertheless it is not sufficient.

In what comes, we look for a sufficient condition for \( \frac{da}{db} < 0 \).

The previous expression can be re-written as:

\[
\begin{align*}
\lambda_{ss}u_{cc}(e^u)g_a + \beta \frac{\partial(V^E - V^U)}{\partial b} [P_a \lambda_{ss} - P_s \lambda_{as}] - \beta P_{sa}[V^E - V^U]u_{cc}(e^u)g_a - 2 \beta P_s \frac{\partial(V^E - V^U)}{\partial b} [V^E - V^U][P_a P_{ss} - P_s P_{as}]
\end{align*}
\]

1: (-) \hspace{1cm} 2: (+) \hspace{1cm} 3: (-) \hspace{1cm} 4: (+)

The expression above is negative if the terms 1 + 2 < 0 and 3 + 4 < 0.

**First condition:** 1 + 2 < 0

\[\lambda_{ss}u_{cc}(e^u)g_a + \beta \frac{\partial(V^E - V^U)}{\partial b} [P_a \lambda_{ss} - P_s \lambda_{as}] < 0 \text{ iff} \]

\[-u_{cc}(e^u)g_a > \beta \frac{\partial(V^E - V^U)}{\partial b} [P_a - P_s \frac{\lambda_{ss}}{\lambda_{as}}] \]

**Second condition:** 3 + 4 < 0

\[-\beta P_{sa}[V^E - V^U]u_{cc}(e^u)g_a - 2 \beta \frac{\partial(V^E - V^U)}{\partial b} [V^E - V^U][P_a P_{ss} - P_s P_{as}] < 0 \text{ iff} \]

\[P_{ss}u_{cc}(e^u)g_a + \beta \frac{\partial(V^E - V^U)}{\partial b} [P_a P_{ss} - P_s P_{as}] > 0 \text{ iff} \]

\[u_{cc}(e^u)g_a + \beta \frac{\partial(V^E - V^U)}{\partial b} [P_a - P_s \frac{P_{sa}}{P_{ss}}] < 0 \text{ iff} \]
Putting together the two conditions we get the sufficient condition (5) in the main text. If \( P_{ss} = 0 \), that is, if the probability of finding a job is linear with respect to \( s \), the term 3 of expression (28) above disappears, and also a part of term 4. After some simplifications, and using the first order condition with respect to \( s \), we are then left with this condition:

\[
-u_{cc}(c^u)g_a > \beta \frac{\partial(V^E - V^U)}{\partial b}[P_a - P_{as} \frac{P_a}{P_{as}}]
\]

If \( \lambda_{ss} = 0 \), that is, if the cost of effort is linear with respect to \( s \), the term 1 of expression (28) above disappears, and also a part of term 2. After some simplifications, and using the first order condition with respect to \( s \), we are then left with this condition:

\[
-u_{cc}(c^u)g_a > \beta \frac{\partial(V^E - V^U)}{\partial b}[P_a - \frac{\lambda_s}{\lambda_{ss}} + \frac{P_a}{P_{as}}]
\]

The sign of \( \frac{ds}{db} \):

Since the denominator needs to be positive, let’s concentrate on the numerator of \( \frac{ds}{db} \).

\[
-\beta P_a \frac{\partial(V^E - V^U)}{\partial b} [u_{cc}(c^u)g_a + u_{cc}(c^u)g_{sa} + \beta P_{as}(V^E - V^U)] + [-\lambda_{as} + \beta P_{as}(V^E - V^U)] [u_{cc}(c^u)g_a + \beta P_a \frac{\partial(V^E - V^U)}{\partial b}] (29)
\]

Note that \( ds/db > 0 \) could be possible only when \( G_{ab} \) is negative, which is a necessary condition to have \( da/db < 0 \). But, even in that case, there are several terms pushing in the direction of having \( ds/db < 0 \).

Corner Solutions:

The previous analysis assumed that \( a > 0 \) and \( s > 0 \), i.e., that the optimum is interior. Let’s analyze now the possibility of having corner solutions:

- Note first that choosing \( a = 0 \) when \( b < c_{min} \) is not possible. In that case, the agent needs to generate some subsistence consumption.
- If \( b > c_{min} \) and if the agent chooses \( a = 0 \), then the problem becomes exactly equal the standard model [SM], and hence (4) is the unique FOC (the only difference with respect to the standard model being the presence of \( c_{min} \)).
- We avoid having \( s = 0 \) by imposing \( \lambda_s(0,a) < \beta P_s(0,a)[V^E - V^U] \) for all possible values of \( a \). Except for the presence of \( a \), this inequality is standardly assumed (explicitly or not) in the job search literature.

A.1.2 Extensions to the baseline model [BM]

In this section we start by presenting the three extensions to the baseline model [BM]. In all of them time is finite and equal to \( T \) periods (from \( t = 0 \) to \( t = T - 1 \)). Since time is finite, the analysis is not stationary anymore, which means that \( a_t \neq a_{t+1} \), same for \( s \). The structure of the problem, nevertheless, remains very similar. First, we present the model with finite entitlement [FE], where the agent is entitled to a flat \( b \) during \( B \) periods, \( B < T \). Second, we incorporate incomplete financial markets into the setting with finite horizon and flat \( b \) for \( T \) periods. Finally, we assume that there is a distribution of wage offers (McCall, 1970), in this environment the agent chooses a reservation wage, \( x_t \), below which job offers are rejected. We finish the section by presenting the stationary model with
monetary costs of job search in which looking for a job requires both job search effort and also a flow expenditure.

**Model with Finite entitlement [FE]**

Using the same notations as before, the lifetime values in unemployment and in employment solve respectively the following Bellman equations:

$$[FE] = \begin{cases} 
V_t^U = \max_{s_t, a_t} u(c_t^u) - \lambda(s_t, a_t) + \beta[P(s_t, a_t)V_{t+1}^E + (1 - P(s_t, a_t))V_{t+1}^U] \\
V_t^E = u(c_t^e) + \beta[\phi V_{t+1}^U + (1 - \phi)V_{t+1}^E] 
\end{cases}$$

(30)

where $c_t^u = b + g(a_t)$ if $t \leq B - 1$ and $c_t^u = g(a_t)$ if $B - 1 < t < T$, $c_t^e = w - \tau$.

Subject to: $c_t^u \geq c_{\text{min}}$, $V_{t+1}^E - V_{t+1}^U \geq 0$, $a_t \geq 0$, $s_t \geq 0$ and $V_T^U = V_T^E = 0$

The First Order Conditions:

$$G_a \equiv u_c(c_t^u)g_a(a_t) - \lambda_a + \beta P_a[V_{t+1}^E - V_{t+1}^U] = 0$$

$$G_s \equiv -\lambda_s + \beta P_s[V_{t+1}^E - V_{t+1}^U] = 0$$

Which are the same as in the stationary case (except from the fact that now the timing for $V_U$ and $V_E$ is relevant). The same happens with the second order partial derivatives:

$$G_{ss} \equiv -\lambda_{ss} + \beta P_{ss}(V_{t+1}^E - V_{t+1}^U)$$

$$G_{sa} \equiv -\lambda_{as} + \beta P_{sa}(V_{t+1}^E - V_{t+1}^U)$$

$$G_{as} \equiv -\lambda_{as} + \beta P_{sa}(V_{t+1}^E - V_{t+1}^U)$$

$$G_{aa} \equiv u_{cc}(c_t^u)g_a(a_t)^2 + u_c(c_t^u)g_{aa}(a_t) - \lambda_{aa} + \beta P_{aa}(V_{t+1}^E - V_{t+1}^U)$$

$$G_{sb} \equiv \beta P_b \frac{\partial(V_{t+1}^E - V_{t+1}^U)}{\partial b}$$

$$G_{ab} \equiv u_{cc}(c_t^u)g_a(a_t) + \beta P_a \frac{\partial(V_{t+1}^E - V_{t+1}^U)}{\partial b}$$

And, therefore, a sufficient condition to have $da/db < 0$ is the same as before:

$$-u_{cc}(c_t^u)g_a(a_t) > \beta \frac{\partial(V_{t+1}^E - V_{t+1}^U)}{\partial b} \left( P_a - P_s \left( \frac{\lambda_{as}}{\lambda_{ss}} + \frac{P_{as}}{P_{ss}} \right) \right)$$

Note, nevertheless, that the expression $\frac{\partial(V_{t+1}^E - V_{t+1}^U)}{\partial b}$ is different from the expression that we had before. While the expression is relatively simple when employment is an absorbing state, it gets more complicated when $\phi \neq 0$. 

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Equivalently, $ds/db < 0$ is equal to:

$$\frac{-\beta z}{\partial b} \frac{\partial (V_{t+1} - V_{t+1}^{U})}{\partial b} \left[ u(c_t^{a})g_{a}(a_t) + u(c_t^{e})g_{e}(e_t) - \lambda a + \beta P_{a}(V_{t+1}^{E} - V_{t+1}^{U}) \right] +$$

- $G_{b}: (+)$

$$- \lambda a + \beta P_{a}(V_{t+1}^{E} - V_{t+1}^{U}) \] \left[ u(c_t^{a})g_{a}(a_t) + \beta P_{a} \frac{\partial (V_{t+1}^{E} - V_{t+1}^{U})}{\partial b} \right]$$

- $G_{a}: (-)$

$$- \lambda a + \beta P_{a}(V_{t+1}^{E} - V_{t+1}^{U}) \] \left[ u(c_t^{a})g_{a}(a_t) + \beta P_{a} \frac{\partial (V_{t+1}^{E} - V_{t+1}^{U})}{\partial b} \right]$$

(31)

**Model with Incomplete Financial Markets [FM]**

Consider now the problem where the time horizon is finite and equal to $T$ (from $t = 0$ to $t = T - 1$) and the agent is entitled to a flat benefit for $B = T$ periods. The difference now is that agents have access to an asset traded on an imperfect credit market. For simplicity (as Hopenhayn and Nicolini, 1997, Chetty, 2008, Shimer and Werning, 2008, Schmieder et al., 2012, Kolsrud et al., 2015, Kroft and Notowidigdo, 2016) we consider that employment is an absorbing state ($\phi = 0$). Using the same notations as before, and denoting by $k_t$ the level of assets in each period, the lifetime values in unemployment and in employment solve respectively the following Bellman equations:

$$[FM] = \begin{cases} V_{t}^{U} = \max_{s_t, a_t, k_{t+1}} & u(c_t^{a}) - \lambda(s_t, a_t) + \beta [P(s_t, a_t)V_{t+1}^{E} + (1 - P(s_t, a_t))V_{t+1}^{U}] \\ V_{t}^{E} = \max_{k_{t+1}} & u(c_t^{e}) + \beta V_{t+1} \end{cases}$$

(32)

where $c_t^{a} = b + g(a_t) + (1 + r)k_t - k_{t+1}$ and $c_t^{e} = w - \tau + (1 + r)k_t - k_{t+1}$.

Subject to: $c_t^{a} \geq c_{\min}$, $V_{t+1}^{E} - V_{t+1}^{U} \geq 0$, $a_t \geq 0$, $s_t \geq 0$, $V_{T}^{U} = V_{T}^{E} = k_t = 0$ and $k_t \geq L$. This last condition can be interpreted as a capital market imperfection.\(^{32}\)

Since $\phi = 0$ the setup is deterministic when the agent is employed. The optimal consumption path satisfies the Euler equation:

$$u_c(c_t^{e}) = \beta(1 + r)u_c(c_{t+1}^{e})$$

We assume that $\beta = \frac{1}{1+r}$, this implies that the agent entering in employment in period $t$ keeps the same level of consumption until $T$.

In order to find $c_t^{e}$, let us consider the budget constraint of the employed agent hired in period $t$ with an initial level of assets of $k_t$: $c_t^{e} = w - \tau + (1 + r)k_t - k_{t+1}$. This expression can be rewritten as: $k_t = \frac{c_t^{e} - (w - \tau) + k_{t+1}}{1+r}$. By iterating forward (that is, by replacing $k_{t+1} = \frac{c_{t+1}^{e} - (w - \tau) + k_{t+2}}{1+r}$ on the previous expression, and then replacing $k_{t+2}$, etc...) and since $k_T = 0$, we have that:

$$k_t(1 + r) = c_t^{e} - (w - \tau) + \frac{c_t^{e} - (w - \tau)}{1 + r} + \ldots + \frac{c_t^{e} - (w - \tau)}{(1 + r)(T - 1) - 1} = [c_t^{e} - (w - \tau)] \sum_{j=0}^{(T-1)-1} \frac{1}{1+r}$$

Which implies that:

$$c_t^{e} = k_t \left[ \frac{r}{1 - \left( \frac{1}{1+r} \right)^{(T-1)-1}} \right] + w - \tau$$

\(^{32}\) As highlighted by Chetty (2008), it is easy to show that $V_{t}^{E}$ is concave, because there is no uncertainty following reemployment; however, $V_{t}^{U}$ could be convex. Nevertheless, this is not the case in our simulations - non concavity never arises in Chetty (2008) nor in Lentz and Tranaes (2005) either.
Now, \( c^e_t \) is a function of \( t \) because it depends on the moment in which the agent starts working: If the agent is recruited in \( t+1 \) her constant level of consumption when employed is different from the one she would have if she was hired in \( t \).

Moreover, since consumption is constant from the moment in which the agent is employed, we can write:

\[
V^E_t = \sum_{j=0}^{(T-1)-t} \beta^j u(c^e_t) = u(c^e_t) \frac{1-\beta^{(T-1)-t+1}}{1-\beta}
\]

In unemployment, the First Order Conditions can be written as:

\[
G_{s_t} \equiv u(c^u)g_a(a_t) - \lambda + \beta P_a[V^E_{t+1} - V^U_{t+1}] = 0
\]

\[
G_{s_t} \equiv -\lambda + \beta P_s[V^E_{t+1} - V^U_{t+1}] = 0
\]

\[
G_{k_{t+1}} \equiv -u(c^u) + \beta \left( P(s_t, a_t) \frac{\partial V^E_{t+1}}{\partial k_{t+1}} + (1 - P(s_t, a_t)) \frac{\partial V^U_{t+1}}{\partial k_{t+1}} \right) = 0
\]

where:

\[
\frac{\partial V^E_{t+1}}{\partial k_{t+1}} = u(c^e_{t+1}) \frac{1-\beta^{(T-1)-t}}{1-\beta} \frac{\partial c^e}{\partial k_{t+1}} = u(c^e_{t+1}) \frac{1-\beta^{(T-1)-t}}{1-\beta} \frac{r}{1-(1+\beta)^{(T-1)-t}} = u(c^e_{t+1}) \frac{1}{\beta}
\]

and:

\[
\frac{\partial V^U_{t+1}}{\partial k_{t+1}} = u(c^u_{t+1})(1+r) = u(c^u_{t+1}) \frac{1}{\beta}
\]

Which allows to re-write \( G_{k_{t+1}} \) as:

\[
G_{k_{t+1}} \equiv -u(c^u_t) + P(s_t, a_t)u(c^e_{t+1}) + (1 - P(s_t, a_t))u(c^u_{t+1}) = 0
\]

The comparative statics are now more complex, since we have three choice variables. In section 4, we will solve this problem numerically.

**Model with Stochastic Wage Offers [SWO]**

Consider now the problem where the time horizon is finite and equal to \( T \) (from \( t = 0 \) to \( t = T-1 \)) and the agent is entitled to a flat benefit for \( B = T \) periods. The difference now is that wage offers are stochastic (McCall, 1970). The distribution of wage offers is known, we denote the support of the distribution by \( [w, \overline{w}] \) \( 0 \leq w < \overline{w} \), the cumulative distribution function (CDF) by \( H(w) \) and the probability density function (PDF) by \( h(w) \). If an offer is received, the agent has to decide to accept it or not (no recall). Then, the agent follows a stopping rule: if the wage offer is higher than the reservation wage, \( x \), she accepts the offer, otherwise, she rejects it. Using the same notations as before, the lifetime values in unemployment and in employment solve respectively the following Bellman equations:

\[
[\text{SWO}] \begin{align*}
V^U_t &= \max_{s_t, a_t, x_t} u(c^u_t) - \lambda(s_t, a_t) + \beta [P(s_t, a_t)V^U_{t+1} + (1 - P(s_t, a_t))V^E_{t+1}] \\
V^E_t &= u(c^e_t) + \beta V^E_{t+1}
\end{align*}
\]

(33)
where \( V_{t+1}^U = E_{w} \max \{V_{t+1}^E(w), V_{t+1}^U \} = \int_{x_t}^\infty V_{t+1}^U(w) dH(w) + \int_{x_t}^\infty V_{t+1}^E(w) dH(w) \)

Subject to: \( c_t^u \geq c_{min}, a_t \geq 0, s_t \geq 0, V_T^U = V_T^E = 0. \)

\( V_t^U \) can be rewritten as:

\[
V_t^U = \max_{s_t,a_t,x_t} u(c_t^u) - \lambda(s_t,a_t) + \beta \left[ P(s_t,a_t) \int_{x_t}^\infty (V_{t+1}^E(w) - V_{t+1}^U) dH(w) + V_{t+1}^U \right]
\]

The First Order Conditions can be written as:

\[
G_{a_t} \equiv u_c(c_t^u) g_a(a_t) - \lambda_a + \beta P_a \int_{x_t}^\infty (V_{t+1}^E(w) - V_{t+1}^U) dH(w) = 0
\]

\[
G_{s_t} \equiv -\lambda_s + \beta P_s \int_{x_t}^\infty (V_{t+1}^E(w) - V_{t+1}^U) dH(w) = 0
\]

\[
G_{x_t} \equiv \beta P(s_t,a_t) \left( V_{t+1}^E(x_t) - V_{t+1}^U \right) h(x_t) = 0
\]

The comparative statics are now more complex, since we have three choice variables. In section 4, we will solve this problem numerically.

**Model with Monetary Costs of Job Search [MC]**

Consider now again a stationary setup, with infinite time horizon. In this setup looking for a job requires both job search effort, \( s \), but also a flow expenditure, \( m \). The the lifetime values in unemployment and in employment solve respectively the following Bellman equations:

\[
[M] = \begin{cases} 
V_{t+1}^U = \max_{s,m,a} & u(c_t^u) - \lambda(s) + \beta [P(s,m)V_{t+1}^E + (1 - P(s,m))V_{t+1}^U] \\
V_{t+1}^E = u(c_t^e) + \beta [\phi V_{t+1}^E + (1 - \phi) V_{t+1}^E] 
\end{cases}
\]

(34)

Where \( u(c_t^u) = u(b - m), u(c_t^e) = u(w - \tau) \) and \( m \) is the quantity of money devoted to job search.

Subject to: \( V_{t+1}^E - V_{t+1}^U \geq 0 \) and \( s \geq 0. \)

If the direct cost of the effort devoted to the subsistence activity is zero, problems [BM] and [MC] can be seen as equivalent if one ignores \( c_{min} \). When \( c_{min} \) is taken into account in [BM], the difference between the two setups is deeper than it appears at first glance, because \( b - m \leq b \) and subsistence cannot be guaranteed if \( b < c_{min} \).

### A.2 Optimal Unemployment Insurance

#### A.2.1 Impossibility to write

\[
\frac{u(c_t^e) - u(c_t^e)}{u_t(c^e)} = \frac{\text{liquidity effect}}{\text{moral hazard effect}}
\]

In our baseline model

\[
\frac{\text{liquidity effect}}{\text{moral hazard effect}} = \frac{P_a \frac{da}{dA} + P_s \frac{ds}{dA}}{-(P_a \frac{da}{dw} + P_s \frac{ds}{dw})}
\]

\[
= \frac{P_a [-G_{ss} G_{aa} + G_{as} G_{sA}] + P_s [-G_{aa} G_{sa} + G_{sa} G_{aA}]}{-(P_a [-G_{ss} G_{aw} + G_{as} G_{sw}] + P_s [-G_{aa} G_{sw} + G_{sa} G_{aw}])}
\]

(35)
Which cannot be reduced to \( \frac{u_i(c^u) - u_i(c^e)}{u_i(c^e)} \).

Nevertheless, if we assume that when \( b \) changes only \( s \) changes (\( a \) remains constant), then the previous expression becomes:

\[
\frac{\text{liquidity effect}}{\text{moral hazard effect}} = \frac{P_s \frac{ds}{db}}{-P_s \frac{ds}{dw}} = \frac{-G_{aA}}{G_{sw}} = \frac{-u_i(c^u) - u_i(c^e)}{u_i(c^e)} = \frac{\text{liquidity effect}}{\text{moral hazard effect}} \tag{36}
\]

Instead, if we assume that when \( b \) changes only \( a \) changes (\( s \) remains constant), then (35) becomes:

\[
\frac{\text{liquidity effect}}{\text{moral hazard effect}} = \frac{P_a \frac{da}{db}}{-P_a \frac{da}{dw}} = \frac{-G_{aA}}{G_{sw}} = \frac{-u_i(c^u) - u_i(c^e)}{u_i(c^e)} + \frac{\beta P_i u_i(c^e)}{1 - \beta} \tag{37}
\]

This shows that, \( \frac{u_i(c^u) - u_i(c^e)}{u_i(c^e)} = \frac{\text{liquidity effect}}{\text{moral hazard effect}} \) if one assumes that only one choice variable changes when \( b \) changes and \( a \) is such that the utility function is separable in consumption and the cost of this variable (36). Instead, \( \frac{u_i(c^u) - u_i(c^e)}{u_i(c^e)} \neq \frac{\text{liquidity effect}}{\text{moral hazard effect}} \) if there are two choice variables that change when \( b \) changes (35) or if there is just one variable that changes, but such that the utility function is not separable in consumption and the cost of this variable (37).

### A.2.2 Optimal UI in a non-stationary setting

In this section we assume that time is finite, and equal to \( T \) (from \( t = 0 \) to \( t = T - 1 \)). For simplicity (as Chetty, 2008, Schmieder et al., 2012, Kolsrud et al., 2015, Kroft and Notowidigdo, 2016) we assume that there is no job destruction (\( \phi = 0 \)).

Because of the envelope conditions, the following analysis is independent of the underlying structure of the model behind it (see Chetty, 2006 and Chetty, 2008), in particular on whether or not the agent is allowed to save, or whether or not she chooses a reservation wage in a context of stochastic wage offers. In the development below, we assume that there is finite entitlement, meaning that the UB is paid during \( B \) periods when \( B < T \) (If one wants to assume that \( B = T \), it suffices to replace \( B \) for \( T \) everywhere).

On top of the underlying model, our analysis for the optimal UB differs from the one of Chetty (2008) in two respects: (1) \( \beta = \frac{1}{1 + \tau} \neq 1 \) and more importantly (2) our timing assumption: in our model, if the agent finds a job in \( t \), she starts working in \( t + 1 \).

The optimal level of UB is the level of \( b \) that maximizes the inter-temporal utility of the job seeker subject to the budget balanced condition. Formally:

\[
\max_b V^U_0 = u(c^u_0) - \lambda(s_0, a_0) + \beta \left[ P^E_0 V^U_1 + (1 - P_0) V^U_1 \right]
\]

Subject to: \( D_B b = (T - D) \tau \), where \( D \) is the expected duration in unemployment, \( D = 1 + \sum_{t=0}^{T-2} \prod_{i=0}^{t} (1 - P_i) \) and \( D_B \) is equal to the expected compensated duration in unemployment \( D_B = 1 + \sum_{t=0}^{B-2} \prod_{i=0}^{t} (1 - P_i) \). In these expressions \( P \) is a short notation for \( P(s, a) \).

Applying the envelope theorem, the derivative of the previous expression with respect to \( b \) gives:
Now, let us compute the unconditional average marginal utility while employed:

\[
\frac{dV_{t}^{U}}{db} = u'(c_{t}^{u}) - \beta P_{0} \frac{dV_{t}^{E}}{dw} \frac{d\tau}{db} + \beta (1 - P_{0}) \left[ \frac{dV_{t}^{U}}{db} - \frac{dV_{t+1}^{U}}{db} \right]
\]

where \( \frac{dV_{t}^{J}}{dw} = \sum_{t=1}^{T-1} \frac{dV_{t}^{J}}{dw} \), for \( J \in \{ E, U \} \) and \( \frac{dV_{t}^{U}}{db} = \sum_{t=1}^{B-1} \frac{dV_{t}^{U}}{db} \).

Which could be rewritten as:

\[
\frac{dV_{t}^{U}}{db} = u'(c_{t}^{u}) - \sum_{t=1}^{B-2} \beta^{t} \prod_{j=1}^{T-1} (1 - P_{j}) u'(c_{t+1}^{u}) + \beta (1 - P_{0}) \frac{dV_{t}^{U}}{db}
\]

\( (38) \)

We now need to find the values of \( \frac{dV_{t}^{E}}{dw} \), \( \frac{dV_{t}^{U}}{db} \) and \( \frac{dV_{t}^{U}}{db} \) to plug them in the expression above. We obtain the following derivatives for the value functions (assuming \( T \geq B > 2 \)): \( ^{33} \)

\[
\frac{dV_{t}^{U}}{db} = \frac{dV_{t}^{U}}{db}
\]

\( (39) \)

\[
\frac{dV_{t}^{U}}{db} = \frac{dV_{t}^{U}}{db}
\]

\( (40) \)

Plugging (39) and (40) in (38) gives:

\[
\frac{dV_{t}^{U}}{db} = D_{B} E_{0, B-1} u'(c_{t}^{u}) - (T - D) E_{0, T-1} u'(c_{t}^{u}) \frac{d\tau}{db}
\]

\( (41) \)

\( ^{33} \)Note that since \( \beta = \frac{1}{T} \), and since employment is an absorbing state, the agent will have a constant level of consumption since the moment at which she starts to work.
where \( \frac{d\tau}{db} = D_B T - D (1 + \varepsilon_D b + \varepsilon_D b \frac{D}{T-D}) \), and \( \varepsilon_D b = \frac{dD_b}{db}, \varepsilon_D b = \frac{dD}{db} \).

Plugging \( \frac{d\tau}{db} \) in (41), simplifying and setting \( \frac{dV_U}{db} = 0 \) gives:

\[
E_0, B - E_0, T - 1 u'(c^u_t) - E_0, T - 1 u'(c^e_t) = \varepsilon_D b + \varepsilon_D D - D
\]

(42)

Consider \( \sigma = \frac{T-D}{T} \), then the previous expression can be written as:

\[
\frac{E_0, B - E_0, T - 1 u'(c^u_t) - E_0, T - 1 u'(c^e_t)}{E_0, T - 1 u'(c^e_t)} = \frac{1}{\sigma} (\sigma \varepsilon_D b + (1-\sigma)\varepsilon_D b)
\]

(43)

Having \( B < T \) has two implications: (1) The LHS of the formula takes the average consumption while unemployed and entitled to \( b \) minus the average consumption when employed and (2) the RHS is a weighted average of \( \varepsilon_D b \) and \( \varepsilon_D b \), because the cost of the insurance depends not only on the time the agent is unemployed and compensated \( (D_B) \) but also on the time the agent is not paying taxes \( (D) \). If \( B = T \), that is, without finite entitlement, the previous expression becomes:

\[
\frac{E_0, T - 1 u'(c^u_t) - E_0, T - 1 u'(c^e_t)}{E_0, T - 1 u'(c^e_t)} = \varepsilon_D b \frac{T}{T-D}
\]

(44)

Moreover, without finite entitlement and if \( T = \infty \) we are back to the stationary case analyzed in the main text, where \( E_0, T - 1 u'(c^e_t) = u'(c) \) -because consumption is constant-, and \( \lim_{T \to \infty} \frac{T}{T-D} = 1 \). This reduces the previous expression to the formula (12) in the main text.

References


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