Economic Development and the Family Structure: from the *Pater Familias* to the Nuclear Family

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Economic Development and the Family Structure: from the *Pater Familias* to the Nuclear Family*

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Abstract

We provide a theory that is able to account for the observed co-movement between the shift in intergenerational living arrangements from coresidence to non-coresidence and economic development. Our theory is consistent with the diminution in the status of the elderly documented by some sociologists. The results from our analysis show that, when technical progress is fast enough, the economy experiences a shift from stagnation to growth, there is a transition from coresidence to non-coresidence, and the social status of the elderly tends to deteriorate.

Keywords: Unified Growth Theory, Intergenerational Living Arrangements, Bargaining Power, Family Economics

JEL Classification: O40, O11, O33, J10

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1 Introduction

In this article, we propose a theory of the interaction between intergenerational living arrangements and economic growth. This theory is able to rationalize in a neoclassical growth model the observed co-movement between economic growth and the shift from the patriarchal to the nuclear family.

The family structure in Western societies has undergone major changes since the nineteenth century. In the United States, according to the data provided by Ruggles (2007), the percentage of elderly whites residing with their adult children plummeted from almost 70% in 1850, to 13% in 1990. These findings are consistent with earlier works like those by Costa (1997), Michael, Fuchs, and Scott (1980) and Schoeni (1998). Kertzer (1995) discusses similar findings for European countries. A recent study from the United Nations concludes that there is a global trend, across countries and over time, towards independent forms of living arrangements among older persons (United Nations (2005)).

Different theories have been advanced by sociologists and demographers to explain this phenomenon. The results of the debate so far has been that intergenerational coresidence fades away as economic development kicks in. The role of cultural factors is still disputed. The accepted wisdom is that they might play a role in influencing the rapidity and the intensity of the process, but do not seem crucial determinants of the process itself (Ruggles (2009)).[1]

The recorded shift in the living arrangements from coresidence to non-coresidence has often been considered by sociologists and demographers as going together with a change in the social status of the elderly in the society (Kertzer (1995)).

From the economist's point of view, social status is an elusive concept, difficult to define, and even more difficult to quantify. Several variables such as patriarchal control, relative income, education, health and psychological conditions can be thought of as proxy for the social status.

According to Ruggles (2007), the decline of intergenerational coresidence reflects a decline of patriarchal control. Patriarchal control was widely diffused in pre-modern societies. For instance, senators in the ancient Rome were the old heads of the noble families of the city, the so called *pater familias*. The very word 'senator' has the same root as '*senex*', which means 'old' in Latin. In facts, patriarchal control endured and was still strong in the modern age. By comparing the legal entitlements of the parents to their children's income in France, England and the United States since the sev-

[1] See Kramarov (1995) for a different perspective.

enteenth century, Schoonbroodt and Tertilt (2010) make a convincing case for the presence of a *de jure* high degree of patriarchal control in the West until as late as the nineteenth century. Thereafter there was a progressive emancipation of the young generations. Such a feature is a common trait between the West, China and South Asia, although it has been traditionally much stronger in the former (Thornton and Fricke (1987)).

More generally, it has been argued by sociologists that the relative status of the aged, measured as relative income, education, health and psychological conditions, has diminished as modernization has been taking place (see, for instance, Cowgill (1974), Palmore and Whittington (1971)).

So, the literature mentioned above points to a change in the family structure that occurred during, and because of the process of economic development, and maintains that such a change went along with, and depended upon a diminished social status of the elderly.

A peculiarity of the present work is that it is able to account not only for the observed co-movement between the intergenerational living arrangements and economic growth, but also also for its connection with the diminished status of the elderly.

In this article, we build a dynamic general equilibrium model, where the pattern for coresidence is endogenously determined. Drawing inspiration from Kotlikoff and Morris (1988), in Section 2 we build a set up where agents can decide to either live alone, or coreside with their parents/siblings. The household is modeled as having a 'pluralistic' decision-making (Bergstrom (1997)) à la Browning, Bourguignon, Chiappori, and Lechene (1994). In this context, the coresidence decision turns out to be a function of the bargaining power of the young and the old, and is influenced by the utility levels that the young and the old can get by living alone. This implies that living arrangements are ultimately determined by the relative income of the two generations. The higher is the relative income of the young with respect to the old, the less likely coresidence will be.

In this paper, we assume that the human capital of the young is positively affected by exogenous technological progress. The implicit hypothesis here is that technical change is age-biased towards the young generation. Such an hypothesis is justified on the ground of a widespread literature. There are significant adoption costs linked to a new technology (Jovanovic and Nyarko (1996)). The young have more incentive to adopt the new technology, as their lifespan is longer (Ahituv and Zeira (2001)). Moreover, the old are typically more skilled at the existent technology, which makes the opportunity costs of learning the new technology possibly higher for them than for the young (Chari and Hopenhayn (1991)).

On the other hand, the human capital of the old generation will benefit from an experience premium of the learning-by-doing type.

These two assumptions imply that the relative income of the young with respect to the old will depend from the ratio between technological progress and the experience premium.

According to the theory we develop, coresidence was more frequent in ancient societies because of the slow technical progress that characterised those societies. If technical progress is slow, the experience premium of the learning-by-doing type is relatively more important. The relative income of the young will be low, and therefore they will gain from coresiding with the old.

With the industrial revolution first, and with the extraordinary fast path of technical progress in the twentieth century, things have changed. The relative importance of the experience premium has shrunk, while the relative income of the young has increased. In this new context, the incentives for intergenerational coresidence may eventually disappear.

One implication of our theory is that the historical shift in living arrangements depended upon the rate of growth of income rather than its level. While to the best of our knowledge there is not enough direct evidence to test this hypothesis, the micro data for the United States in Ruggles (2007) do provide some indirect evidence. In particular, before World War II, coresidence was more widespread among the white and the wealthier than among the black and the poorer. This casts a serious doubt over the idea that higher income *per sé* should always go together with more independent living.

Our theory has also an additional implication concerning the social status of the elderly. Sticking to a narrow definition of social status, defined as the fraction of resources allocated to each generation, the model can also predict the progressive deterioration of the social status of the elderly documented in the sociological literature.[2]

The rest of the paper is structured as follows. In Section 3 we first illustrate the main mechanisms of the theory in the context of a general model with exogenous growth.

Under some restrictions on the parameters, in Section 4 we go one step further by endogenizing the human capital formation. This allows us to study the feedback effect of living arrangements on the economic performance of a given society.[3] We assume that only the young cares for the human capital of the child, and devotes a fraction of his time to educate him. The results confirm the main findings of Section 3.

[2]In this article we abstract from the presence of public transfers.

[3]Duranton, Rodríguez-Pose, and Sandall (2008) give empirical evidence on the importance of the family structure for determining educational attainment, social capital, labor participation, sectoral structure, wealth, and inequality.

An additional advantage of this endogenous growth model is that it allows us to say something about the respective role of economic and cultural factors in causing the shift from coresidence to non-coresidence. In particular, when the shift in living arrangements is explained by technical change (economic factors), the model economy experiences an increase in the growth rate along a balanced growth path. On the contrary, when the shift in living arrangements is explained by changes in the direct taste for coresidence (cultural factors), the model economy experiences a reduction of the growth rate along a balanced growth path.

Section 5 draws together the thread of the arguments and concludes.

2 The family structure: benchmark model

The economy is an OLG economy populated by two generations of individuals living for two periods, the young ('y') and the old ('o').[4] The economy produces one good using the human capital of the young (H^y) and the human capital of the old (H^o) as inputs. We assume that the two types of human capital are perfect substitute in the production function:

$$Y_t = H_t^y + H_t^o. \tag{1}$$

The human capital of the two generations evolves exogenously according to a law of motion that we shall specify later.[5] The labour supply of each individual is fixed and normalised to 1. The growth rate of the population is 0.

The final good is the numeraire.

There is no physical capital and no saving: in each period the young and the old consume all the available income. As a consequence, the choice of an agent in one period only depends on the variables which affects the utility function in that period.[6] In what follows we omit time subscript for the sake of notation.

The young and the old can either live apart or coreside.

[4]The model presented in this section is a slightly modified version of Kotlikoff and Morris (1988).

[5]This exogeneity assumption will be removed in Section 4.

[6]This assumption is made for the sake of simplicity, as in Blackburn and Cipriani (2005).

2.1 The utility function

The utility function of an agent of type i is:

$$U(c^i, x^i; \delta) = \alpha \log c^i + (1-\alpha) \log x^i + \delta \log \kappa^i, \tag{2}$$

where $i = y, o$ and c^i is consumption. The variable x^i is a good that can be either bought by both the young and the old on the market, or, in case of coresidence, can be shared among the members of the family (for instance, housing services). The price of x is denoted by p. We assume that x is produced using a linear technology $x = ZY^x$, where Y^x are the units of the final good Y used in the production of x. In equilibrium, $Z = \frac{1}{p}$.

The parameter δ is dummy variable. It takes values $\delta = 0$, if agent i lives alone, and $\delta = 1$, if the agents coreside.

The variable κ^i measures the taste for living together. For $\kappa^i = 1$, the individual i draws no direct utility from living together; for $\kappa^i < 1$ (> 1) agents get a negative (positive) direct utility from living in a family.

Therefore, living together has two effects on the utility of the agents: first, there is a direct effect (positive or negative) through the taste for family κ^i; second, there is an indirect (positive) effect through the possibility of sharing the good x.

2.2 Optimal choices

2.2.1 Optimal choices in the case of non-coresidence

If the young and the old live apart, each of them maximizes $\hat{U}(c^i, x^i) \equiv U(c^i, x^i; 0)$ subject to

$$px^i + c^i = wH^i. \tag{3}$$

The variable w is the wage in efficiency unit. This is equal in equilibrium to the marginal productivity of labour in efficiency unit, which in turn is constant and equal to 1.

The optimal choices are characterized by the following equations:

$$\hat{c}^{i*} = \alpha w H^i \tag{4}$$

$$\hat{x}^{i*} = (1-\alpha)\frac{w}{p} H^i \tag{5}$$

Thus the indirect utility function is:

$$\hat{V} \equiv \hat{U}(\hat{c}^{i*}, \hat{x}^{i*}) = \alpha \log \alpha w H^i + (1-\alpha) \log \left((1-\alpha)\frac{w}{p} H^i \right) \tag{6}$$

2.2.2 Optimal choices in the case of coresidence

We assume that, in case of coresidence, the young and the old bargain over the distribution of the resources within the family. We model such a bargaining using a collective model (Browning, Bourguignon, Chiappori, and Lechene (1994)), that only imposes the requirement that the bargaining process lead to an efficient solution.

The household maximizes the sum of the utility functions of the young and the old, weighted by their respective bargaining power:

$$\max \theta \tilde{U}(c^y, x) + (1-\theta)\tilde{U}(c^o, x),$$

s.t.

$$px + c^y + c^o = wH^y + wH^o, \tag{7}$$

where $\tilde{U}(c^i, x) \equiv U(c^i, x; 1)$. Notice that x appears here as a pure public good.

The optimal choices are:

$$\tilde{c}^{y*} = \theta\alpha(wH^y + wH^o) \tag{8}$$

$$\tilde{c}^{o*} = (1-\theta)\alpha(wH^y + wH^o) \tag{9}$$

$$\tilde{x}^* = (1-\alpha)\frac{(wH^y + wH^o)}{p} \tag{10}$$

and the indirect utility functions are:

$$\begin{aligned} \tilde{V}^y(\theta, \kappa^y) \equiv \tilde{U}(\tilde{c}^{y*}, \tilde{x}^*) &= \alpha \log \theta\alpha(wH^y + wH^o) + \\ &+ (1-\alpha)\log(1-\alpha)\frac{(wH^y + wH^o)}{p} + \log \kappa^y \end{aligned} \tag{11}$$

$$\begin{aligned} \tilde{V}^o(\theta, \kappa^o) \equiv \tilde{U}(\tilde{c}^{y*}, \tilde{x}^*) &= \alpha \log(1-\theta)\alpha(wH^y + wH^o) + \\ &(1-\alpha)\log(1-\alpha)\frac{(wH^y + wH^o)}{p} + \log \kappa^o \end{aligned} \tag{12}$$

2.2.3 Optimal choice between coresidence and living alone

The solution to the maximization problem here above gives the allocation of resources within the family for any value of the bargaining power of the young, θ. However, not all the values of θ are compatible with the choice of coresidence. In particular, coresidence can only occur when there exists at least one value of θ such that coresidence is attractive to one of the agents, and indifferent to the other. In what follows we are going to characterize the interval of values of θ that are compatible with the choice of coresidence.[7]

We define θ_{min} as the value of θ such that: $\tilde{V}^y(\theta, \kappa^y) = \hat{V}^y$. By solving the equation, we get

$$\theta_{min} = \left(\frac{H^y}{H^y + H^o} \frac{1}{\kappa^y} \right)^{\frac{1}{\alpha}}. \tag{13}$$

According to this definition, θ_{min} is the value of the bargaining power of the young such that the young is indifferent between living alone or with the old. Notice that, according to equation (11), $\tilde{V}^y(\theta, \kappa^y)$ is increasing in θ. It follows that for any $\theta > \theta_{min}$, the young prefers coresidence. Notice also that a value $0 \leq \theta_{min} \leq 1$ such that $\tilde{V}^y(\theta, \kappa^y) = \hat{V}^y$ exists if and only if

$$\frac{H^y}{H^y + H^o} \leq \kappa^y. \tag{14}$$

If not, the young always decide to live alone. Finally, notice that the condition (14) always holds for any $\kappa^y > 1$.

Similarly, we define θ_{max} as the value of θ such that: $\tilde{V}^o(\theta, \kappa^o) = \hat{V}^o$. By solving the equation we get

$$\theta_{max} = 1 - \left(\frac{H^o}{H^y + H^o} \frac{1}{\kappa^o} \right)^{\frac{1}{\alpha}}, \tag{15}$$

Hence, θ_{max} is the value of the bargaining power of the young such that the old is indifferent between living alone or forming a family with the young. Since $\tilde{V}^o(\theta, \kappa^o)$ is decreasing in θ (equation (12)), the old prefers coresidence for any $\theta < \theta_{max}$. Notice that the condition $0 \leq \theta_{max} \leq 1$ is verified if and only if

$$\frac{H^o}{H^y + H^o} \leq \kappa^o. \tag{16}$$

If not, the old always decide to live alone. Notice also that the condition (16) always holds for any $\kappa^o > 1$.

According to these definitions, we can parametrize the possibility of coresidence to the ratio $\frac{\theta_{min}}{\theta_{max}}$:

- if $\frac{\theta_{min}}{\theta_{max}} < 1$ there exists θ such that coresidence is advantageous for both the young and the old. Living alone is inefficient: the utility levels that can be achieved from living alone are inside the utility possibility set that can be derived in case of coresidence.

[7]The same procedure is used by Iyigun and Walsh (2007) in the analysis of the bargaining between husbands and wives.

- if $\frac{\theta_{min}}{\theta_{max}} = 1$, both the young and the old are indifferent between coresidence and living alone. The utility levels that can be achieved from living alone are on the frontier of the utility possibility set that can be derived in case of coresidence.

- if $\frac{\theta_{min}}{\theta_{max}} > 1$ there is no θ such that coresidence is attractive to both the young and the old. The utility levels that can be achieved from living alone are outside the utility possibility set that can be derived in case of coresidence.

Computing $\frac{\theta_{min}}{\theta_{max}}$ we find:

$$\frac{\theta_{min}}{\theta_{max}} = \frac{\left(\frac{H^y}{H^y+H^o}\frac{1}{\kappa^y}\right)^{\frac{1}{\alpha}}}{1-\left(\frac{H^o}{H^y+H^o}\frac{1}{\kappa^o}\right)^{\frac{1}{\alpha}}} = \frac{\left(\frac{h}{(1+h)\kappa^y}\right)^{\frac{1}{\alpha}}}{1-\left(\frac{1}{(1+h)\kappa^o}\right)^{\frac{1}{\alpha}}}, \qquad (17)$$

where $h \equiv \frac{H^y}{H^o}$.

As a consequence, living arrangements will depend on the taste for coresidence κ^i, and on the relative income of the young and the old. We can distinguish four cases.

1. $\kappa^i = 1$ for $i = y, o$. We have $\frac{\theta_{min}}{\theta_{max}} < 1$ provided that $\alpha < 1$. If living together brings no variation to the utility functions of both the young and the old, then living together is always Pareto improving, because of the possibility of sharing the good x. On the other hand if $\alpha = 1$ then $\frac{\theta_{min}}{\theta_{max}} = 1$ and both the young and the old are indifferent between coresidence and living alone.

2. $\kappa^i > 1$ for $i = y, o$. In this case $\frac{\theta_{min}}{\theta_{max}} < 1$ always. This case is trivial, as both the individuals draw utility from living together.

3. $\kappa^i < 1$ for $i = y, o$. We have that $\frac{\theta_{min}}{\theta_{max}}$ can be $\lesseqqgtr 1$ if $\alpha < 1$. When both the individuals dislike coresidence, the decision to coreside or not depends on the distaste for coresidence and on the relative income. In the limit for $\kappa^i \longrightarrow 0$, $\frac{\theta_{min}}{\theta_{max}} > 1$. If the distaste is maximum for both of them, then coresidence is obviously suboptimal. Living alone is Pareto improving if $\alpha = 1$.

4. $\kappa^y < 1$ and $\kappa^o \geq 1$, or $\kappa^o < 1$ and $\kappa^y \geq 1$. Here the young (the old) dislikes coresidence, but the old (the young) likes it. In this case, we have that $\frac{\theta_{min}}{\theta_{max}}$ can be $\lesseqqgtr 1$.

3 Economic development and the family structure: exogenous growth

In this section, we shall study how technical change affects the family structure in an exogenous growth model.

We assume that the human capital of the two generations evolves according to the following laws of motion

$$H_t^y = (1 + g_t)H_{t-1}^y, \tag{18}$$

$$H_t^o = (1 + e_t)H_{t-1}^y, \tag{19}$$

where g is the exogenous growth rate of the young's human capital, and e is an exogenous experience premium of the learning-by-doing type. Our working hypothesis is that technological developments determine g while leaving e unaffected. The idea is that technical progress is age-biased towards the young. This is so for a number of reasons, put to the fore by a recent widespread literature on vintage capital, technology adoption and diffusion.[8] In a nutshell, the young has more incentive to forego current production to learn the new technology, as his time span is longer, and therefore the discounted reward higher. Moreover, if the technical innovation is a major one, the experience premium of the old gets eroded soon, and the young, who will reasonably learn directly the new technology, will end up being more skilled.[9]

The growth rate of this economy, γ, reads

$$1 + \gamma_{t+1} \equiv \frac{Y_{t+1}}{Y_t} = \frac{(1 + g_{t+1}) + (1 + e_{t+1})}{(1 + g_t) + (1 + e_t)}(1 + g_t). \tag{20}$$

Along a balanced growth path, g and e ought to be constant. This implies

$$1 + \gamma = 1 + g, \tag{21}$$

that is, the growth rate of the economy coincides with the growth rate of the human capital of the young.

Using Equations (18) and (19) we obtain

$$h = \frac{1 + g}{1 + e}. \tag{22}$$

[8]See Ahituv and Zeira (2001), Chari and Hopenhayn (1991), Jovanovic and Nyarko (1996).

[9]This will be particularly true if the young has higher general education than the old as opposed to the vocational education of the skill-specific or the learning-by-doing type (Krueger and Kumar (2004)).

Such a formulation makes the relative skills between generations an increasing function of the rate of technical progress and a decreasing function of the experience premium.

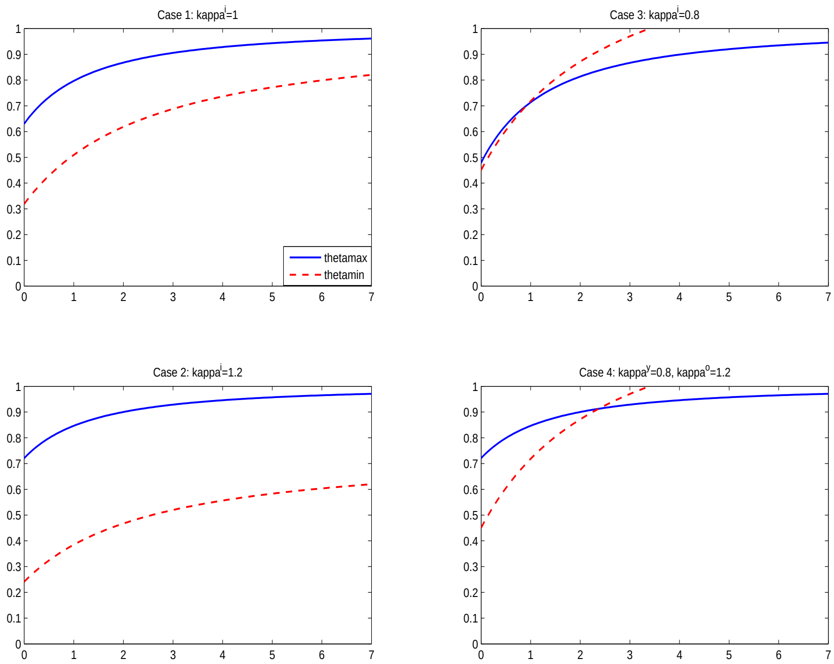

Figure 1: Numerical example: patterns of θ_{min} and θ_{max} as a function of g for different values of κ^i and with $\alpha = 0.7$. Legend: X axis: g; Y axis: θ.

To grasp the final effect of technical change on the coresidence pattern, we need to study how g affects the ratio $\frac{\theta_{min}}{\theta_{max}}$. Studying the derivative, we obtain

$$\frac{\partial \frac{\theta_{min}}{\theta_{max}}}{\partial g} = \frac{1}{1+e} \frac{\partial \frac{\theta_{min}}{\theta_{max}}}{\partial h}, \tag{23}$$

where

$$\frac{\partial \frac{\theta_{min}}{\theta_{max}}}{\partial h} = \frac{\left[1-(1+h)\left(\frac{1}{\kappa^o(1+h)}\right)^{\frac{1}{\alpha}}\right]\left(\frac{h}{\kappa^y(1+h)}\right)^{\frac{1}{\alpha}}}{\alpha h(1+h)\left[\left(\frac{1}{\kappa^o(1+h)}\right)^{\frac{1}{\alpha}}-1\right]^2}. \tag{24}$$

The sign of this derivative is positive if and only if $\kappa^o > \left(\frac{1}{1+h}\right)^{1-\alpha}$. This condition is always verified for any $\kappa^o \geq 1$.

This simple model is able to predict the observed shift in intergenerational living arrangements. In the model, such a shift may happen for several reasons, a change in the attitude towards coresidence embedded in the parameters κ^i, a change in the experience premium e, a change in technology embedded in the parameter g. If for instance κ^i and e are constant and compatible with the cases 3 and 4 in section 2.2.3, a suitable change in g is sufficient to explain the change in living arrangements, provided that the world started in coresidence. Notice that in this case, we do not need to refer to any change in preferences to account for the change in living arrangements: economic development suffices to the scope.

To illustrate the point, in Figure 1, we plot the values of θ_{min} and θ_{max} against g. In this numerical example, for the cases 3 and 4 in section 2.2.3, when g increases enough, living arrangements shift from coresidence to non-coresidence. In the other cases, coresidence is always optimal.

4 Economic development and the family structure: endogenous growth

In the previous section, we studied how economic development affects the family structure. There, we assumed exogenous growth, thereby ignoring the feedback effects that the family structure might hold on economic development.

In this section, we shall remove this assumption and study an extended model with endogenous human capital accumulation. This will allow us to explore the bidirectional link between economic development and the family structure.

4.1 The model

Agents live for three periods. In the first period, the agent is a child and makes no active choice. He lives with the young and accumulates human capital which is assumed to depend on his parent's choices only.

In the second period, the agent is young and has one child.[10] The utility function is almost identical to that of section 2. The only difference is that here we assume that parents have a warm glow motivation for investing in their child's human capital:

[10] We assume that the growth rate of the population is 0.

$$U^y(c_t^y, x_t^y, H_{t+1}^y; \delta) = \alpha \log c_t^y + \zeta \log x_t^y + (1-\alpha-\zeta) \log H_{t+1}^y + \delta \log \kappa^y \quad (25)$$

The young has a time endowment equal to 1. He can either work or invest in the human capital of his child. We call s_t the amount of time the young dedicates to increase the human capital of his child, according to the following production function:

$$H_{t+1}^y = \xi s_t H_t^y, \quad (26)$$

with $\xi > 1$.

In the third period, the old has the same preferences as in section 2, but for a slight change of notation:

$$U^o(c_t^o, x_t^o; \delta) = (1-\zeta) \log c_t^o + \zeta \log x_t^o + \delta \log \kappa^o. \quad (27)$$

Here we denote the weight given to the public good in the utility function as ζ.

As in section 3, the human capital of the old evolves in time according to equation (19).

4.2 Optimal choices in the case of non-coresidence

The young maximizes $\hat{U}^y(c_t^y, x_t^y, H_{t+1}^y) \equiv U^y(c_t^y, x_t^y, H_{t+1}^y; 0)$ subject to the following constraints:

$$H_{t+1}^y = \xi s_t H_t^y \quad (28)$$

$$c_t^y + p_t x_t^y + s_t w H_t^y = w H_t^y \quad (29)$$

The optimal choices are:

$$\hat{s}_t = (1-\alpha-\zeta) \quad (30)$$

$$\hat{c}_t^y = \alpha w H_t^y \quad (31)$$

$$\hat{x}_t^y = \zeta \frac{w H_t^y}{p_t} \quad (32)$$

Notice that the investment in schooling is constant in the case of non-coresidence.

For what concerns the old, nothing changes compared to section 2.

4.3 Optimal choices in the case of coresidence

As before, in the case of coresidence, the household maximizes the sum of the utility functions of the young and the old, weighted by their respective bargaining power:

$$\max_{c_t^y, c_t^o, s_t, x_t} \theta \tilde{U}^y(c_t^y, x_t, H_{t+1}^y) + (1-\theta)\tilde{U}^o(c_t^o, x_t) \tag{33}$$

subject to:

$$p_t x_t + c_t^y + c_t^o + s_t w H_t^y = w(H_t^y + H_t^o), \tag{34}$$

$$H_{t+1}^y = \xi s_t H_t^y, \tag{35}$$

where $\tilde{U}^y(c_t^y, x_t, H_{t+1}^y) \equiv U^y(c_t^y, x_t, H_{t+1}^y; 1)$, and $\tilde{U}^o(c_t^o, x_t) \equiv U^o(c_t^o, x_t; 1)$.

The optimal choices are:

$$\tilde{s}_t = \theta(1-\alpha-\zeta)\frac{(H_t^y + H_t^o)}{H_t^y} \tag{36}$$

$$\tilde{c}_t^y = \theta \alpha w(H_t^y + H_t^o) \tag{37}$$

$$\tilde{c}_t^o = (1-\theta)(1-\zeta) w(H_t^y + H_t^o) \tag{38}$$

$$\tilde{x}_t = \zeta \frac{w(H_t^y + H_t^o)}{p_t} \tag{39}$$

4.4 Optimal choice between coresidence and living alone

Using the same procedure as in section 2, we compute the threshold values θ_{min}, θ_{max}, such that, for $\theta_{min} < \theta < \theta_{max}$, coresidence is always Pareto improving:

$$\theta_{min} = \left(\frac{h_t}{1+h_t}\frac{1}{\kappa^y}\right)^{\frac{1}{1-\zeta}}, \tag{40}$$

$$\theta_{max} = 1 - \left(\frac{1}{1+h_t}\frac{1}{\kappa^o}\right)^{\frac{1}{1-\zeta}}. \tag{41}$$

This allows us to parametrize again the possibility of coresidence to the ratio $\frac{\theta_{min}}{\theta_{max}}$.

4.5 Dynamics

In order to have an analytical solution to the dynamics of the model, we have to assume $\zeta = 0$. This means that we disregard the role of the public good.

Furthermore, we shall restrict the analysis to the case $\kappa^y < 1$ and $\kappa^o > 1$.[11]

From equations (19) and (26), we know that

$$h_t = \frac{\xi s_{t-1}}{1+e}. \tag{42}$$

The relative human capital of the young is now an endogenous variable, and is determined by the amount of time that the young invests in schooling. As s_{t-1} differs depending on whether agents lived together in $t-1$ or not, we have two cases. If in $t-1$ the agents did not live together, then the relevant equation for s is (30). It follows that the value

$$h_t = \frac{\xi(1-\alpha)}{1+e} \equiv h^{nc} \tag{43}$$

does not depend on time. That is, in case of non-coresidence, the relative human capital is always constant and equal to its balanced growth path level h^{nc}.

If instead agents did live together in $t-1$, then the relevant equation for s is (36). It follows that equation (42) now reads:

$$h_t = \theta_{t-1} \frac{\xi(1-\alpha)}{1+e} \frac{1+h_{t-1}}{h_{t-1}}. \tag{44}$$

In this paper, we assume that agents do not leave the possibility of Pareto improvements unexploited. This means that whenever $\frac{\theta_{min}}{\theta_{max}} < 1$, the actual θ always falls within the interval $[\theta_{min}, \theta_{max}]$, and coresidence is chosen. To be more specific, in the following we shall assume that

$$\theta = \lambda\theta_{min} + (1-\lambda)\theta_{max}, \tag{45}$$

[11]As shown in the Appendix, the assumption $\kappa^o > 1$ is necessary to have a sustainable intertemporal equilibrium. Consequently, we assume also $\kappa^y < 1$ so as to have the possibility of a shift between coresidence and non-coresidence, as explained in section (2.2.3). Notice that these assumptions are compatible with the available evidence. Studies of modern family settings by Aquilino (1990), Ward, Logan, and Spitze (1992) and Whittington and Peters (1996) show that there is a negative correlation between coresidence and the children's income, which is compatible with $\kappa^y < 1$ in our model. On the other hand, a recent work by Manacorda and Moretti (2006) shows that there is a positive correlation between coresidence and the parents' income, which is compatible with $\kappa^o > 1$ in our model.

with $0 < \lambda < 1$ standing for exogenous factors affecting the outcome of the bargaining process (for instance, the individual ability to bargain).

Substituting for θ in equation (44), we have

$$h_t = \frac{\xi(1-\alpha)}{1+e}\left(\frac{\lambda}{\kappa^y} + \frac{1-\lambda}{\kappa^o}\frac{\kappa^o(1-h_{t-1})-1}{h_{t-1}}\right). \tag{46}$$

The balanced-growth-path solution of equation (46) reads

$$h^c = \frac{1}{2(1+e)\kappa^y\kappa^o}\{(\alpha-1)\kappa^o\left(\kappa^y(\lambda-1)\right)\xi+$$
$$\sqrt{(\alpha-1)\kappa^o\xi[4(1+e)(\kappa^o-1)\kappa^{y2}(\lambda-1)+(\xi(\alpha-1)\kappa^o(\kappa^y+\lambda-\kappa^y\lambda)^2)]}\} > 0 \tag{47}$$

A comparison between h^{nc} and h^c - the value of human capital along a balanced growth path in the case of non-coresidence and coresidence, respectively - reveals that, *ceteris paribus*, $h^c > h^{nc}$. The rationale of this result lays in the way in which we have formulated the problem. By assuming $\zeta = 0$, we have eliminated the public good from the utility function. This implies that the only rationale for coresidence in this model is a greater availability of resources. To say it differently, young people in this model do coreside if by doing so they get more resources. As a constant share of those resources are dedicated to the education of the offspring, it follows that the relative human capital of the young ought to be higher in case of coresidence.

The overall dynamics of this model is non-trivial. We have two dynamic rules and two balanced growth paths, depending on whether the agents do or do not coreside. Moreover, there is in principle room for switching from one equilibrium to the other. A switching occurs whenever the ratio $\frac{\theta_{min}}{\theta_{max}}$ goes from less than one to more than one, and vice versa. As such ratio depends on h, we can find the threshold level $\hat{h}$ such that $\frac{\theta_{min}}{\theta_{max}} = 1$. That is,

$$\frac{\theta_{min}}{\theta_{max}} = \frac{\left(\frac{h}{(1+h)\kappa^y}\right)}{1-\left(\frac{1}{(1+h)\kappa^o}\right)} = 1 \ \Rightarrow \hat{h} = \frac{\kappa^y(\kappa^o-1)}{\kappa^o(1-\kappa^y)}, \tag{48}$$

For any $h_t > \hat{h}$, people do not coreside, and the relevant dynamics is that of equation (43). For any $h_t < \hat{h}$, people coreside, and the relevant dynamics is that of equation (46). As we shall make clear in a fortnight, the dynamics of the model depends on the relative magnitude of $\hat{h}$, h^{nc} and h^c.

The following proposition characterizes completely the dynamics.

Proposition 1.
Given

$$\xi_1 \equiv \frac{\kappa^y(\kappa^o - 1)}{\kappa^o(1-\kappa^y)} \frac{(1+e)}{1-\alpha}, \tag{49}$$

$$\xi_2 \equiv \kappa^y \xi_1. \tag{50}$$

(i) $\forall\ \xi > \xi_1 \Rightarrow \hat{h} < h^{nc} < h^c$*, and the economy converges towards* h^{nc}*.* Non-coresidence is the chosen living arrangement along the balanced growth path.

(ii) $\forall\ \xi < \xi_2 \Rightarrow h^{nc} < h^c < \hat{h}$*, and the economy converges towards* h^c*.* Coresidence is the chosen living arrangement along the balanced growth path.

(iii) $\forall\ \xi_2 < \xi < \xi_1 \Rightarrow h^{nc} < \hat{h} < h^c$*, and there is no stable balanced growth path.* The economy oscillates perpetually between coresidence and non-coresidence.

Proof
See Appendix. ■

The dynamics of the relative human capital determines the dynamics of output. The growth rate of output, γ, is

$$1 + \gamma_t \equiv \frac{Y_t}{Y_{t-1}} = \frac{H_t^y + H_t^o}{H_{t-1}^y + H_{t-1}^o} = \frac{(1+h_t)(1+e)h_{t-1}}{1+h_{t-1}} \tag{51}$$

Along a balanced growth path, equation (51) reduces to

$$1 + \gamma = (1+e)h, \tag{52}$$

where h can take two values, h^c or h^{nc}, depending on the living arrangements. As a consequence, the value of γ depends on the coresidence status.

4.6 Comparative statics

In this section, we discuss the effects of variations in the parameters of the model economy on living arrangements, on economic growth, and on (a suitable definition of) the social status of the two generations.[12]

[12]Notice that in the model there is also room for endogenous changes in the living arrangements. Take for instance point *(i)* in Proposition 1. For any initial level of $h_0 < \hat{h}$, the economy starts in coresidence, but the dynamics of the model is such that the economy converges towards the non-coresidence steady state h^{nc} (see Figure 3 in the Appendix).

4.6.1 Family structure and economic growth

We start by studying the effects of a change in ξ on the family structure. First notice that $\xi > 1$ plays the same role as $(1 + g)$ in section 3. Accordingly, it can be interpreted as age-biased technical change.

The main effects that changes in ξ have on the family structure have already been spelt out in Proposition 1. An increase in ξ makes coresidence less likely to appear as a stable equilibrium. It follows that in this model, a suitable increase in the technical progress is enough to account for a shift in the living arrangements, from coresidence to non-coresidence.

On the other hand, the following Proposition 2 shows that also suitable changes in the cultural factors κ^y and κ^o lead to the same result.

Proposition 2.

(i) $\frac{\partial \xi_2}{\partial \kappa^y} > \frac{\partial \xi_1}{\partial \kappa^y} > 0.$

(ii) $\frac{\partial \xi_1}{\partial \kappa^o} > \frac{\partial \xi_2}{\partial \kappa^o} > 0.$

(iii) $\frac{\partial \xi_1}{\partial \lambda} = \frac{\partial \xi_2}{\partial \lambda} = 0$

Proof
(i), *(ii)* and *(iii)* follow immediately from the computation of the derivatives of (49), (50). ■

When κ^y decreases, the threshold values ξ_2 and ξ_1 decrease: thus for a given value of ξ, non-coresidence is more likely to appear as a stable living arrangement. Moreover, since ξ_2 decreases more than ξ_1, the region of values of ξ for which a balanced growth path does non exist becomes larger.

By the same token, when κ^o decreases, non-coresidence is more likely to appear as a stable living arrangement. At the same time, the region of values of ξ for which a balanced growth path does non exist shrinks.

Changes in λ stands for changes in unspecified idiosyncratic factors affecting the actual bargaining power of the young, θ. The higher λ, the lower θ. Changes in λ does not produce any shift in living arrangements.

So, according to our model there are two possible explanations for the change in the living arrangement from coresidence to non-coresidence that is observed across countries and through times. One relies on economic factors - technical change. One relies on cultural factors - change in the direct taste for coresidence. As we shall show in following Proposition, only one of the two explanations, specifically that relying on technical progress, is compatible with the observed pattern of economic growth.

Define $\gamma^{nc}(\xi, \kappa^y, \kappa^o, \lambda)$ the growth rate of output along a balanced growth path when non-coresidence is the stable living arrangement. Analogously, define $\gamma^c(\xi, \kappa^y, \kappa^o, \lambda)$ the growth rate of output along a balanced growth path when coresidence is the stable living arrangement. The following Proposition 3 shows how changes in the structural parameters of the model affect the growth rate of the economy. We shall distinguish two cases. First, in point *(i)* and *(ii)* we study the effect of marginal changes in the parameters, such that there is no shift in the chosen living arrangement. Second, in point *(iii)* we study the case when the change in the parameters determines a shift in the living arrangement.

Proposition 3.
(i) $\forall \xi < \xi_2$ *(coresidence is the stable equilibrium):*

1. $\frac{\partial \gamma^c(\xi,\kappa^y,\kappa^o,\lambda)}{\partial \xi} > 0$. *The growth rate of the economy increases with technical change.*
2. $\frac{\partial \gamma^c(\xi,\kappa^y,\kappa^o,\lambda)}{\partial \kappa^y} < 0$. *The growth rate of the economy decreases if the young likes coresidence more.*
3. $\frac{\partial \gamma^c(\xi,\kappa^y,\kappa^o,\lambda)}{\partial \kappa^o} > 0$. *The growth rate of the economy increases if the old likes coresidence more.*
4. $\frac{\partial \gamma^c(\xi,\kappa^y,\kappa^o,\lambda)}{\partial \lambda} < 0$. *The growth rate of the economy decreases if the actual bargaining power of the young decreases.*

(ii) $\forall \xi > \xi_1$ *(non-coresidence is the stable equilibrium):*

1. $\frac{\partial \gamma^{nc}(\xi,\kappa^y,\kappa^o,\lambda)}{\partial \xi} > 0$. *The growth rate of the economy increases with technical change.*
2. $\frac{\partial \gamma^{nc}(\xi,\kappa^y,\kappa^o,\lambda)}{\partial \kappa^y} = 0$. *The growth rate of the economy is not affected by changes in the young's taste for coresidence.*
3. $\frac{\partial \gamma^{nc}(\xi,\kappa^y,\kappa^o,\lambda)}{\partial \kappa^o} = 0$. *The growth rate of the economy is not affected by changes in the old's taste for coresidence.*
4. $\frac{\partial \gamma^{nc}(\xi,\kappa^y,\kappa^o,\lambda)}{\partial \lambda} = 0$. *The growth rate of the economy is not affected by changes in the young's actual bargaining power.*

(iii) Shift from coresidence to non-coresidence.

1. *Following a change in* ξ *from* $\xi < \xi_2$ *to* $\xi' > \xi_1$, $\Rightarrow \gamma^{nc}(\xi', \kappa^y, \kappa^o, \lambda) > \gamma^c(\xi, \kappa^y, \kappa^o, \lambda)$. *When the shift in living arrangements is induced by an increase in technical change, the resulting growth rate of the economy is higher.*

2. *Following a change in* κ^y *from* κ^y *to* $\kappa^{y\prime} < \kappa^y$ *such the economy shifts from* $\xi < \xi_2 < \xi_1$ *to* $\xi > \xi_1' > \xi_2'$, $\Rightarrow \gamma^{nc}(\xi, \kappa^{y\prime}, \kappa^o, \lambda) < \gamma^c(\xi, \kappa^y, \kappa^o, \lambda)$. When the shift in living arrangements is induced by a decrease in the young's taste for coresidence, the resulting growth rate of the economy is lower.

3. *Following a change in* κ^o *from* κ^o *to* $\kappa^{o\prime} < \kappa^o$ *such the economy shifts from* $\xi < \xi_2 < \xi_1$ *to* $\xi > \xi_1' > \xi_2'$, $\Rightarrow \gamma^{nc}(\xi, \kappa^y, \kappa^{o\prime}, \lambda) < \gamma^c(\xi, \kappa^y, \kappa^o, \lambda)$. When the shift in living arrangements is induced by a decrease in the old's taste for coresidence, the resulting growth rate of the economy is lower.

Proof
Equation (52) shows that gamma positively depends on h. So, we shall look at the effects of variations in ξ and κ^i on h. Computing the derivatives of (48), (43) and (47), we get the following results: $\frac{\partial \hat{h}}{\partial \xi} = 0$, $\frac{\partial h^{nc}}{\partial \xi} > 0$, $\frac{\partial h^c}{\partial \xi} > 0$, $\frac{\partial \hat{h}}{\partial \kappa^y} > 0$, $\frac{\partial h^{nc}}{\partial \kappa^y} = 0$, $\frac{\partial h^c}{\partial \kappa^y} < 0$, $\frac{\partial \hat{h}}{\partial \kappa^o} > 0$, $\frac{\partial h^{nc}}{\partial \kappa^o} = 0$, $\frac{\partial h^c}{\partial \kappa^o} > 0$. The points *(i)* and *(ii)* follow immediately. To prove point (*iii.1*), notice that if the return from human capital investment increases from a value ξ below ξ_2 to a value ξ' above ξ_1, living arrangements move from coresidence (with $h^{nc} < h^c < \hat{h}$) to independent living (with $\hat{h} < h^{nc\prime} < h^{c\prime}$). Since $h^c < \hat{h} < h^{nc\prime}$, h increases and so does the balanced growth rate. To prove points (*iii.2*) and (*iii.3*), notice that if κ^i shrinks in such a way that it determines a shift from coresidence (with $h^{nc} < h^c < \hat{h}$) to independent living (with $\hat{h}' < h^{nc} < h^{c\prime}$) ,we are sure that the growth rate decreases, since h^{nc} is lower than h^c. ■

The analysis developed so far leads to conclude that, following an increase in ξ (the economic explanation), economic growth and independent living can be positively correlated. In such a scenario coresidence fades away as economic development kicks in. On the contrary, a decrease in κ^i (the cultural explanation) can account for the observed shift from coresidence to non-coresidence only at the price of reducing economic growth.[13]

[13] It is straightforward to verify that increases in the experience premium e raise the threshold values ξ_1 and ξ_2, and reduce the balanced-growth-path values h^{nc} and h^c. It follows that coresidence is more likely to appear for a given ξ. As to the rate of growth rate of the economy, the effects on γ of variations in e pass through the bargaining power. Consequently, variations in the experience premium have no effect on rate of growth of the economy, if the young and the old live apart. If instead they live together, higher values of e are associated with lower values of h^c: as the old has more bargaining power, the young will invest less in schooling.

4.6.2 The decline of the *pater familias*

So far, we have discussed how the structural parameters of the model affect the coresidence pattern and the growth of the economy. We now turn to the variations in the social status of the two generations. As we have recalled in the Introduction, there is evidence in Sociology of a co-movement between the social status of the elderly and the change in the living arrangements of the family. From the economists's point of view, however, the concept of 'social status' is rather elusive. In the following, we use a narrow definition of social status. We define as social status of each generation i the share of total resources allocated to it, $\frac{H^i}{H^y+H^o}$. In case of coresidence, that is for $\xi < \xi_2$, the social status of the young so defined coincides with the bargaining power θ, while $(1-\theta)$ is the social status of the old. When instead non-coresidence is the equilibrium, that is for $\xi > \xi_1$, along a balanced growth path the social status of the young rewritten in terms of h is $\frac{h^{nc}}{1+h^{nc}}$, while that of the old is $1 - \frac{h^{nc}}{1+h^{nc}}$.

We are now going to study under what conditions the social status of the old as defined here above deteriorates when the economy shifts from coresidence to non-coresidence. The aim is to verify whether our model is compatible with the sociological literature mentioned above.[14]

Proposition 4.
Assume that the economy shifts from coresidence to non-coresidence, following a change in ξ from $\xi < \xi_2$ to $\xi' > \xi_1$.
$\forall \theta \in [\theta_{min}, \theta_{max}]$, $\exists\ \epsilon \in \left[1, \frac{\kappa^y + \sqrt{(\kappa^y)^2 + 4(1-\kappa^y)^2}}{2}\right]$ *such that, if* $\xi' > \xi_1 \epsilon$, *then* $1 - \theta > 1 - \frac{h^{nc}}{1+h^{nc}}$.

Proof

We are comparing two equilibria, one with coresidence and one with non-coresidence. We assumed that the shift from one equilibrium to the other is caused by a change from $\xi < \xi_2$ to $\xi' > \xi_1$. In balanced growth path, we can write the social status of the young living alone as

$$\frac{h^{nc}}{1+h^{nc}} = \frac{\xi'(1-\alpha)}{(1+e) + \xi'(1-\alpha)} \tag{53}$$

(see equation (43)). Notice that $\frac{h^{nc}}{1+h^{nc}}$is increasing in ξ'.

[14]We limit ourselves to the case in which the shift from coresidence to non-coresidence is due to technical change.

To prove Proposition 4 it is enough to prove that:

(i) $1 - \theta_{min} > 1 - \frac{h^{nc}}{1+h^{nc}}$ with $\epsilon_1 = 1$.

(ii) $1 - \theta_{max} > 1 - \frac{h^{nc}}{1+h^{nc}}$ with $\epsilon_1 = \frac{\kappa^y + \sqrt{(\kappa^y)^2 + 4(1-\kappa^y)^2}}{2}$.

The meaning of *(i)* is that, for $\theta = \theta_{min}$, if ξ is only slightly above ξ_1, the social status of the elderly was higher before the shift from coresidence to non-coresidence. The meaning of *(ii)* is the same, but in this case $\theta = \theta_{max}$, and the required change in ξ is more significant.

(i) Using equation (40) for $h = h^c$ and (47) for $\lambda = 1$, we got

$$\theta_{min} = \frac{\xi(1-\alpha)}{(1+e)\kappa^y + \xi(1-\alpha)} \tag{54}$$

The expression for θ_{min} is increasing in ξ. We evaluated equation (54) for $\xi = \xi_2$, which is the highest value of ξ compatible with coresidence. By the same token we evaluated equation (53) for $\xi = \xi_1\epsilon$. Then, we compared the two expressions. This allows us to single out the minimum value of ϵ such that $\theta_{min} < \frac{h^{nc}}{1+h^{nc}}$. It turns out that the latter inequality holds for any $\epsilon > 1$.

(ii) The argument is the same as in point (i), but the relevant expressions are now those for θ_{max}. It turns out that $\theta_{max} < \frac{h^{nc}}{1+h^{nc}}$ $\forall \epsilon > \frac{\kappa^y + \sqrt{(\kappa^y)^2 + 4(1-\kappa^y)^2}}{2}$.
■

This proposition shows that, if technical change is big enough, shifts from coresidence to non-coresidence go along with a deterioration of the social status of the elderly. We can conclude by saying that, on top of being able to discriminate between the economic and the cultural explanations of the observed change in living arrangements, the model presented in this article is also consistent with the progressive diminution of the elderly stressed by the sociological literature mentioned in the Introduction.

5 Conclusions

In this article, we provided a theory that is able to account simultaneously for the observed co-movement between the shift in intergenerational living arrangements from coresidence to non-coresidence, economic growth and the diminution in the status of the elderly documented by the literature.

We used a dynamic general equilibrium model of coresidence, where the decisional set-up of the household is modeled using a collective model of bargaining.

The results from our analysis show that, when technical progress is fast enough, the economy experiences a shift from stagnation to growth, there is a transition from coresidence to non-coresidence, and the social status of the elderly tends to deteriorate.

A significant implication of our theory is that while explanations of the shift in the living arrangements that are based on pure economic factors are compatible with a positive co-movement between coresidence and the rate of economic growth, those relying on cultural factors are not. In particular, when the shift in living arrangements is explained by technical change (economic factors), the model economy experiences an increase in the growth rate along a balanced growth path. On the contrary, when the shift in living arrangements is explained by changes in the direct taste for coresidence (cultural factors), the model economy experiences a reduction of the growth rate along a balanced growth path.

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Appendix

Balanced-growth-path solution to equation (46)

In this section we prove that equation (47) is the balanced-growth-path solution to equation (46). We also prove that such a solution is locally stable.

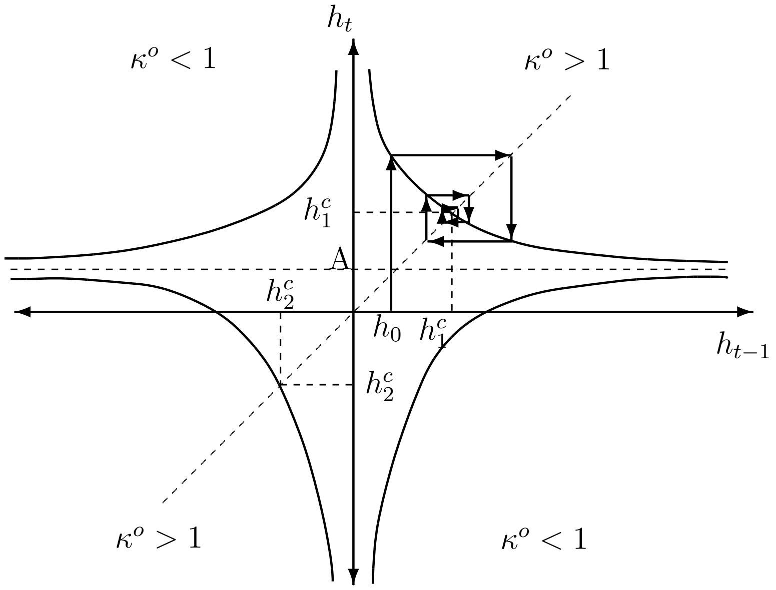

Figure 2: Dynamics in case of coresidence. $A = \frac{\xi(\alpha-1)(\kappa^y(\lambda-1)-\lambda)}{(1+e)\kappa^y}$

Figure 2 shows the geometrical representation of equation (46). We restrict the parameters values to $\kappa^o > 1$, which limits the domain of the function to the first and third quadrant. We do so because for $\kappa^o < 1$, the dynamics delivers negative values of h, as can be easily deduced from the graph. This means that for $\kappa^o < 1$ there is no sustainable intertemporal equilibrium.

Equation (46) is a first order difference equation, that gives us two values for the balanced growth path, that we will label h_1^c and h_2^c respectively. As evident from the graph, $h_2^c < 0$, meaning that only one of the two steady states, namely $h_1^c > 0$, is economically significant. The positive root of equation (46) is equation (47).

For h^c to be a locally stable equilibrium, it must be that the slope of the hyperbola crossing the 45° line in h^c is less than one in absolute value. Doing the computations, it turns out that $0 > \frac{\partial h_t^c}{\partial h_{t-1}^c} > -1$, for $h_{t-1}^c = h^c$. It follows that h^c is a locally stable equilibrium. Convergence occurs along a cobweb pattern. Starting in $t = 0$ from a very low (high) level of h, h_0, the relative human capital in period 1, h_1, will be very high (low). In period 2, h_2 will be again low (high), yet greater (lower) than h_0. By the same token, h_3 will be high (low) but lower (higher) than h_1 and so on and so forth. This cyclical dynamics converges to h^c.

The rationale behind the cyclical behaviour of the dynamics is as follows. For $h = h_0$, the relative human capital of the young is low. Hence, the opportunity cost $w_0 H_0^y$ of investing in schooling is low compared to the income enjoyed by the young in coresidence, $\theta_0 w_0 (H_0^y + H_0^o)$ (see equation (36)). As a consequence, investment in schooling will be high, implying $h_1 > h_0$ (see equation(42)). By the same token, for $h = h_1$, the relative human capital of the young is high, resulting in lower investment in schooling. Notice that in period 2 the economy does not go back to h_0 but it stops at $h_2 > h_0$. The reason is that the actual bargaining power of the young θ is increasing in h, meaning that $\theta_1 > \theta_0$. This means that in period 1 the young is able to capture a larger share of the family income. As schooling is a normal good, this income effect partially offset the increase in the opportunity cost.

Proof that $h^c > h^{nc}$

Using equation (43) and equation (47),

$$h^c > h^{nc} \Leftrightarrow \frac{(1-\lambda)(\kappa^o - 1)}{\kappa^o} > -\frac{\xi(1-\alpha)}{1+e}\frac{\lambda(1-\kappa^y)}{\kappa^y}. \tag{55}$$

Since we assumed $\kappa^y < 1$ and $\kappa^o > 1$, the inequality (55) holds $\forall \lambda \in [0, 1]$.

Proof of Proposition 1

Proof of (i)

The value $\xi_1 \equiv \frac{\kappa^y(\kappa^o-1)}{\kappa^o(1-\kappa^y)}\frac{(1+e)}{1-\alpha}$ is the value of ξ such that $h^{nc} = \hat{h}$. We know that $h^c > h^{nc}$. From equations (43) and (47) and (48), we can deduce that both h^c and h^{nc} increase with ξ, while $\hat{h}$ is invariant with respect to ξ. Therefore, $\forall\ \xi > \xi_1 \Rightarrow \hat{h} < h^{nc} < h^c$.

Figure 3 depicts the dynamics of this case. It represents the first quadrant of Figure 2, to which the line for equation (43) was added to represent the dynamics in the case of non-coresidence. The arrows shows the direction of the global dynamics.

Let's assume that in period 0, $h_0 < \hat{h}$. The relevant dynamics is that of the coresidence case, i.e. the hyperbola. The relative human capital in period 1 will therefore be h_1. Now, as $h_1 > \hat{h}$, the relevant dynamics becomes that of the non-coresidence case, i.e. the parallel line to the axis of the abscissas going through h^{nc}. The relative human capital in period 2 will be h_2. Now, as $h_2 = h^{nc} > \hat{h}$, the economy will remain in h^{nc} for the following periods.

It is straightforward to show that, if $h_0 > \hat{h}$, the economy will converge to h^{nc} already in the first period.

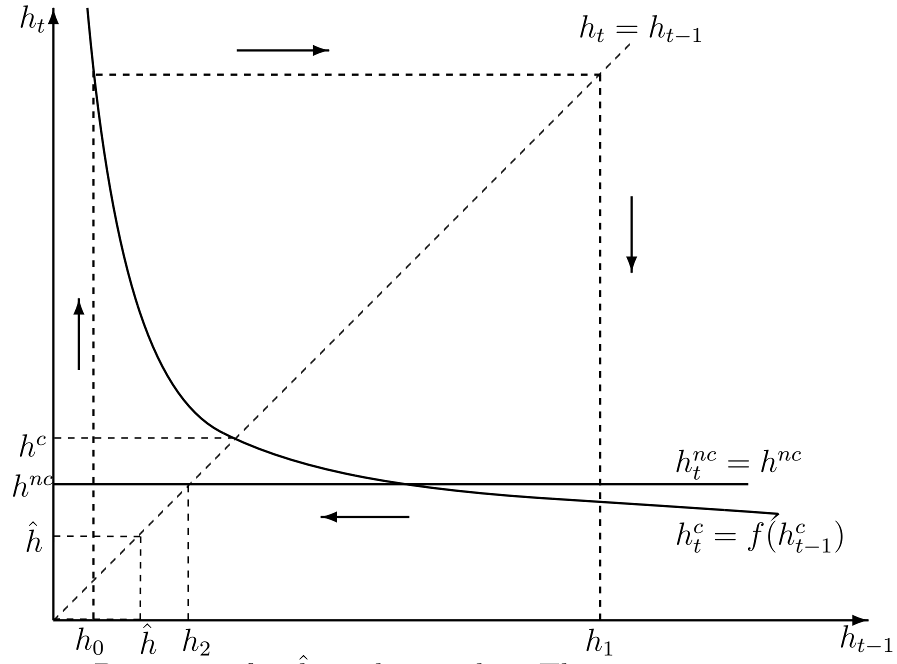

Figure 3: Dynamics for $\hat{h} < h^{nc} < h^c$. The economy converges to h^{nc}: non-coresidence is a globally stable equilibrium.

Proof of (ii)

The value of $\xi_2 \equiv \frac{(1+e)(\kappa^o-1)(\kappa^y)^2}{(\alpha-1)\kappa^o(\kappa^y-1)} = \kappa^y \xi_1$ is the value of ξ such that $h^c = \hat{h}$. By the same argument used in the Proof of (i), $\forall\ \xi < \xi_2 \Rightarrow h^{nc} < h^c < \hat{h}$.

Figure 4 shows the overall dynamics for this case. Starting from $h_0 < \hat{h}$, the relevant dynamics is initially that of coresidence (the hyperbola). The relative human capital in period 1 will be h_1. As $h_1 > \hat{h}$, the relevant dynamics for the second period becomes that of non-coresidence (the line). The relative human capital in period 2 will be h_2. Now, as $h_2 < \hat{h}$, the relevant dynamics will be again that of coresidence. The relative human capital in period 3 will be $h_3 < \hat{h}$. Henceforth, coresidence will always be chosen, until the stable equilibrium h^c is reached.

The result would be unaltered if we started from $h_0 > \hat{h}$. To verify this claim, it is sufficient to start the reasoning from h_1 in Figure 4.

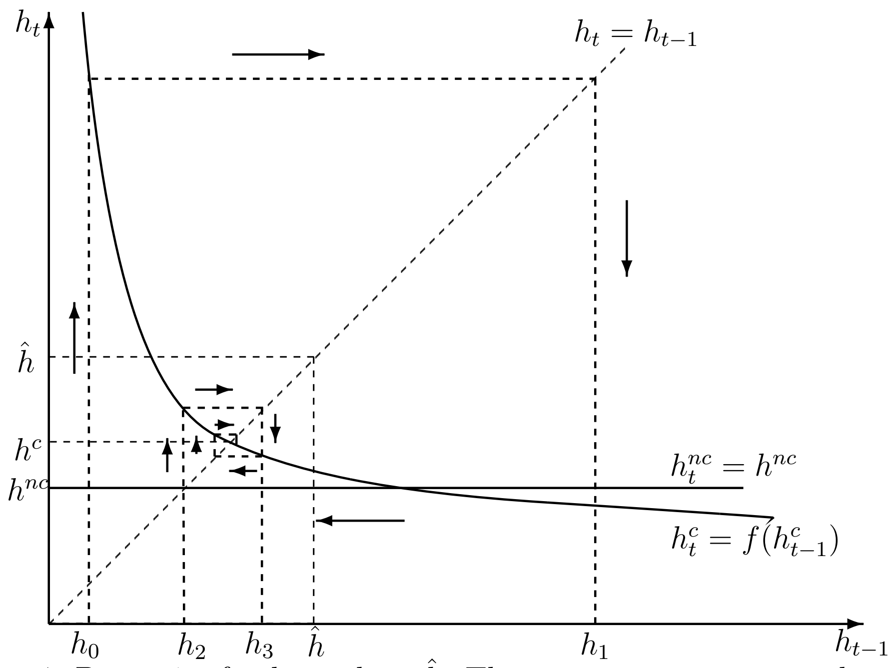

Figure 4: Dynamics for $h^{nc} < h^c < \hat{h}$. The economy converges to h^c: coresidence is a globally stable equilibrium.

Proof of (iii)

From the Proof of (i) and (ii), it follows that $\forall\ \xi_2 < \xi < \xi_1 \Rightarrow h^{nc} < \hat{h} < h^c$

Figure 5 shows the dynamics for this case. Starting from $h_0 < \hat{h}$, the relevant dynamics is initially the hyperbola (coresidence). The relative human capital in period 1 will be h_1. Because $h_1 > \hat{h}$, the relevant dynamics becomes now the line (non-coresidence). The relative human capital in period 2 will be h_2. As $h_2 < \hat{h}$, the relevant dynamics in 2 is again the hyperbola (coresidence), meaning that the economy will move from h_2 to h_3. As $h_3 > \hat{h}$, the relevant dynamics is now the line (non-coresidence), meaning that the economy will move to $h_4 = h_2$. But then, by the same argument, $h_5 = h_3$, and so on and so forth. This means that the economy will oscillate perpetually between h_2 and h_3, switching one period from coresidence to non-coresidence and vice versa the following period. As before, the result does not depend on the value of h_0.

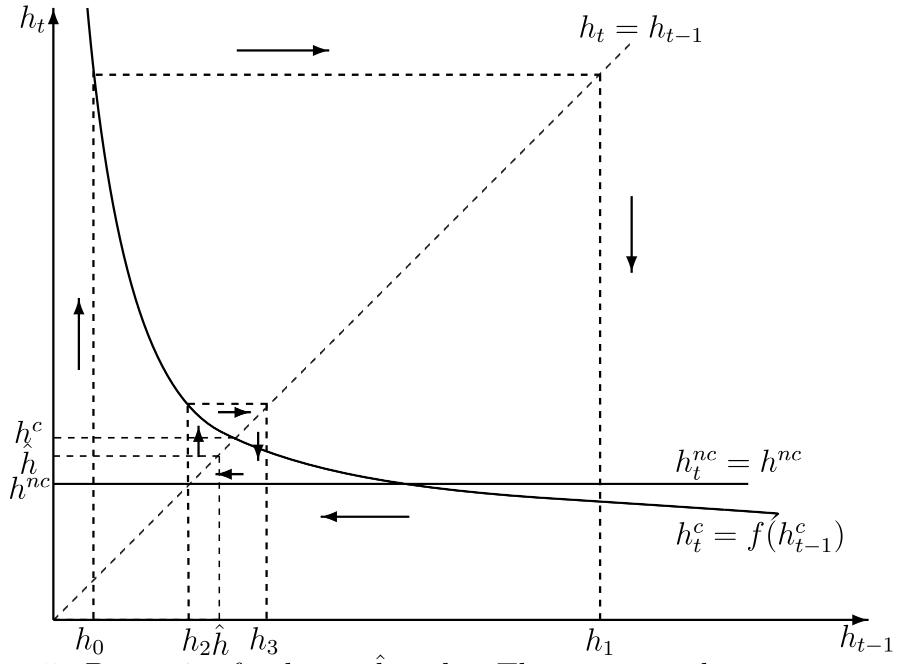

Figure 5: Dynamics for $h^{nc} < \hat{h} < h^c$. The economy does not converge to any stable equilibrium and shows coresidence cycles.

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