Public Education for the Children Left Behind
C. Camacho and I-L. Shen

Discussion Paper 2010-6
Public Education for the Children Left Behind

Carmen Camacho\textsuperscript{a} I-Ling Shen\textsuperscript{b}

March, 2010
(Preliminary Version. Do not quote without permission.)

Abstract

This paper examines the role of public education in the context of parental migration, and it studies the effects of an expansive income tax policy that is adopted to increase public education expenditure per pupil. It is shown that such a policy may exacerbate income inequality in the long run if for the less skilled dynasties, the benefits of more public spending on education does not make up for the negative effects of increased parental absences. However, if the migration-induced tax base erosion is not severe, an expansive income tax policy indeed enhances future human capital for all dynasties, and moreover, it may help the less skilled households escape from the poverty trap, thus reducing long-run income inequality.

JEL Classification: H20, H52, O15, O40.

Keywords: Human Capital; Income Inequality; Parental Migration; Public Education Expenditure; Tax Base Erosion.

\textsuperscript{a}Belgian National Foudation of Scientific Research and IRES, Université catholique de Louvain. E-mail: carmen.camacho@uclouvain.be.

\textsuperscript{b}IRES, Université catholique de Louvain, Department of Econometrics, Université de Genève, and Institute for the Study of Labor (IZA). E-mail: shen.iling@gmail.com.
1 Introduction

Most international migrants from East and South-East Asia go to countries that want temporary workers rather than settlers, so most migrants leave their children behind. ... If correct, a figure of 2 million [overseas Filipino workers] implies that around 5 per cent of Filipino children had one or two parents overseas in 2000.

– Situation Report on International Migration in East and South-East Asia (2008), International Organization for Migration

The continuous rise in the global amount of remittances has triggered heated debates over their economic impacts on the recipient side. Indeed, the money sent home by adults working abroad can be of important help to finance education of the young generation (Kandel and Kao 2001; Cox Edwards and Ureta 2003; Hanson and Woodruff 2003; McKenzie and Rapoport 2006). On the one hand, remittances may boost household income and increase private educational investments. On the other hand, households that are previously liquidity constrained may become able to afford the fix costs of sending children to school (e.g., tuition fees and material expenses). At the same time, however, it is widely acknowledged that parents play an indispensable role in children’s human capital formation.

Despite the potential benefits of remittances entailed by parental migration, empirical studies have suggested that parental absences may imply more housework for the children left behind and that lack of effective supervision may cause these children to develop behavioral problems, all of which adversely affects their school attendance and performances.

Hence, the overall impact of parental migration is rather obscure on children’s human capital.

\[\text{\footnotesize 1\textsuperscript{st} See Rapoport and Docquier (2006) for the survey of related literature.}\]

\[\text{\footnotesize 2\textsuperscript{nd} For example, using the U.S. Army personnel data, Lyle (2006) finds that parental absences due to military deployment have a negative impact on children’s test scores.}\]
accumulation, which is however an extremely important determinant of economic growth.\textsuperscript{3} Moreover, it is considered in the related literature that human capital investment through formal schooling is crucial in generating positive growth effects (Glomm and Ravikumar 1992; Glomm and Ravikumar 2001). At the same time, public schools are the main vehicle to achieve universal primary education, one of the Millennium Development Goals set out by the United Nations to eradicate poverty. Additionally, public resources may also be spent through schools in order to prevent children of migrants from dropping out due to weak parental control.\textsuperscript{4} Last but not least, public education can be used as the second-best policy to correct the negative externality caused by decentralized educational decisions made by parents (Eckstein and Zilcha 1994). While taxation is necessary to finance public education expenditures, the departure of working-age adults may however undermine the domestic tax base.

This paper sets out, in the context of parental migration, a theoretical model where economic growth is human capital driven. Human capital formation requires three inputs, including parental transmission of human capital, private educational investment, as well as public education expenditure per pupil. The government has one policy tool: it chooses the income tax rate in order to adjust the amount of public spending.\textsuperscript{5} The aim of the paper is then to study how an expansive income tax policy, via its direct impacts on public education expenditures and indirect effects on household decisions, may affect economic growth and income inequality in the long run.

\textsuperscript{3}See the pioneering works of Romer (1986) and Lucas (1988). Krueger and Lindahl (2001) provides a comprehensive review of the related literature.

\textsuperscript{4}Several policy reports observe that parental absences are often accompanied by children’s increased absences in school or dropping out. See, for example, Education for some more than others? A regional study on education in Central and Eastern Europe and the Commonwealth of Independent States (2007), prepared by the UNICEF regional office for CEE/CIS.

\textsuperscript{5}In many developing countries, government revenue relies much more heavily on consumption tax than on personal income tax (see, for example, Table 1 in Gordon and Li (2009)). This paper focuses on personal income tax as it directly influences the incentive to work abroad for temporary migrants. In Footnote 11, we will briefly discuss the implications of consumption tax in our baseline model.
The trade-off between higher earnings abroad and parental absences are an especially meaningful issue for temporary migration. Temporary migrants tend to be young adults, located in the age group that is most likely to have early school-age children. For instance, as one of the world’s largest guest worker sending countries, the Philippines have more than 50% of their overseas workers below 35 years old in 2006 and 2007, with one quarter of the total being those aged between 25 and 29. Moreover, several studies hint that temporary migrants seem to be more likely to remit (Merkle and Zimmerman 1992; Glytsos 1997; Duraisamy and Narasimhan 2000). Research on remittances decay also shows that the amount of remittances decline more mildly over time for migrants with strong attachments to the origin (Funkhouser 1995; Amuedo-Dorantes and Pozo 2006).

On top of the pros and cons at the household level, parental migration may also generate externalities on other children’s schooling through public education. First, the difficulty to tax foreign income may thwart the use of income tax policy to raise public spending on education, as Maimbo and Ratha (2005) put it: “Efforts to tax remittances or direct them to specific investments are likely to prove ineffective” since migrants may simply resort to unofficial channels to remit. Hence, due to the possibility of migration, tax base erosion is inevitable. Moreover, data on international migration reveals a strong pattern of positive self-selection into migration (see Figure 1). It suggests that better skilled individuals are more likely to emigrate, which consequently shifts the tax burden toward the less skilled families. At the end, the direction of income redistribution through public education is a priori unclear.

Positive self-selection into migration also implies that the better skilled benefits more from foreign earnings than their less skilled counterparts, which will eventually result in a larger gap in private educational investments for their offspring. This, combined with the negative externality of tax base erosion, may generate serious implications on economic growth and

6See the Survey on Overseas Filipinos released by the Filipino National Statistics Office.
income inequality in the long run. Hence, the government may invoke an expansive income tax policy in the hope of achieving a growth-enhancing and inequality-reducing outcome. However, as an increased tax rate indicates a decline in the net domestic income, an expansive income tax policy gives rise to a stronger incentive to migrate and may further erode the tax base. This complexity is fully analyzed in the paper.

Our work contributes to two streams of literature. In the literature on growth and inequality, Glomm and Ravikumar (1992) and Galor and Tsiddon (1997) both suggest that adopting a redistributive policy in the short-run may hinder economic performances in the long run. However, in the context of parental migration, our paper shows that an expansive income tax policy, by increasing public provision of education, enhances future human capital for all dynasties if it does provoke serious tax base erosion due to a larger flow of out-migration in response to heavier taxation (i.e., with a sufficiently high tax rate elasticity of public education expenditure per pupil); moreover, such a policy may help the less skilled households...
escape from the poverty trap, thus reducing long-run income inequality. In this respect, our findings echo the results in Saint-Paul and Verdier (1993) and Eckstein and Zilcha (1994) who study the role of public intervention in education. Nonetheless, our paper is novel in showing that an expansive income tax policy may exacerbate income inequality in the long run because, for the less skilled dynasties, the benefits of more public spending on education does not make up for the negative effects of increased parental absences.\footnote{In comparison, Glomm and Ravikumar (2003) demonstrate, in a different framework and in a closed economy, that the relationship between the tax rate (used to fund public education) and income inequality is U-shaped over time. In Blankenau and Simpson (2004), it is shown that public education expenditures may crowd out other relevant factors for economic growth, thus producing an overall negative growth impact. In a certain sense, via the income tax policy, public education can be considered to crowd out parental transmission of human capital in our model.} This is in line with the finding that personal income tax is ineffective in terms of reducing inequality in developing countries (Bird and Zolt 2005), and our paper provides an alternative explanation particularly for the labor sending countries.

Our work also supplements the literature on brain drain and economic growth. Haque and Kim (1995) argue that, in an open economy, subsidizing education induces brain drain by imposing heavier taxation, and as emigrants do not repay the subsidies in the form of future taxes, public intervention in education does not contribute to economic growth at origin. In this paper, however, public education provides resources to children from different families alike. Moreover, while migrants are positively self-selected and more would migrate following an expansive income tax policy to increase the funding for public education, they may still indirectly contribute to the future tax base by privately investing more in their children’s education. This feedback effect is particularly important when it comes to temporary parental migration.

The rest of the paper is organized as follows. Section 2 describes the theoretical model. Section 3 begins with analyzing the evolution of human capital distribution, then it scrutinizes the impacts of an expansive income tax policy. Finally, Section 4 concludes.

This is in line with the finding that personal income tax is ineffective in terms of reducing inequality in developing countries (Bird and Zolt 2005), and our paper provides an alternative explanation particularly for the labor sending countries.
2 A Baseline Model

We model an overlapping generations economy inhabited by two-period living individuals, who belong to an infinite number of heterogeneous dynasties indexed by \( i \in [0, \infty) \). For a dynasty \( i \) at period \( t \), the household is composed of parents and children, both of mass 1.\(^8\) The heterogeneity across dynasties lies in the parental human capital \( h^i_t \), which is distributed on a non-negative support. Let us denote \( g(h^i_t) \) as the density function, with the initial human capital \( h^i_0 > 0 \) given for all \( i \). Hence, our assumption of constant population implies

\[
N = \int_0^\infty g(h^i_t) \, dh^i_t \quad \forall \, t, \quad h^i_t \geq 0,
\]

with \( N \) denoting the total size of the population.

Individuals spend their entire childhood going to school at origin. They turn into adults and have children as soon as the second period begins. Parental human capital \( h^i_t \) is determined by education received in the previous period, and by assumption, it perfectly transforms into the efficiency units of labor possessed by each adult belonging to dynasty \( i \) at period \( t \). During adulthood, individuals inelastically supply labor and work either at origin or abroad. At origin, a composite good is produced uniquely with total efficiency units of labor supplied by resident adults:

\[
Y_t = \omega H_t \equiv \omega \int_0^\infty (1 - m^i_t) \, h^i_t \, g(h^i_t) \, dh^i_t, \quad m^i_t \in [0, 1],
\]

where \( \omega > 0 \) denotes the exogenous technology parameter, which can be seen as an indicator of the level of development at origin, and \( m^i_t \) can be regarded as the fraction of household adults who migrate to work abroad, or as the fraction of total adult time spent abroad, at period \( t \).\(^9\) In other words, \( H_t \) is the total human capital remaining in the sending country.

---

\(^8\)The fertility rate is assumed to be one, \( \forall i, \, t \), so that economic growth in our model originates only from human capital accumulation.

\(^9\)Although our model fits better temporary migration, we assume only one period of adult life and omit
The domestic wage rate per unit of human capital equates its marginal productivity $\omega$, which is assumed to be strictly lower than the foreign wage rate $\bar{w}$, i.e., $\omega < \bar{w}$.

The utility maximization problem for a dynasty $i$ at period $t$ is written as

$$\max_{\{0 \leq m_i^t \leq 1, b_i^t \geq 0\}} u_i^t = \ln c_i^t + \gamma \ln h_{t+1}^{i,e}$$

subject to

$$c_i^t = \left[(1 - \tau)(1 - m_i^t) \omega + m_i^t \bar{w}\right] h_i^t - b_i^t - \nu m_i^t, \quad 0 < \tau < 1, \quad \nu > 0$$

$$h_{t+1}^{i,e} = \left[(1 - m_i^t) h_t^{i,e}\right]^{\alpha} b_i^t \epsilon_t^{1-\alpha-\beta}, \quad 0 < \alpha, \beta < 1, \quad (\alpha + \beta) < 1$$

The household utility function (3) is characterized by “warm glow,” where $\gamma$ is the altruistic parameter that shows how much parents care about their children’s expected future earning potential $h_{t+1}^{i,e}$ relative to household consumption $c_i^t$. The latter is required by the household budget balance (4) to equate net household income, whether earned domestically or abroad, less private educational investment $b_i^t$ and total migration costs, with $\nu$ denoting costs per migrant.\textsuperscript{10} Note that the government is able to levy tax only on domestic income.\textsuperscript{11} Thus, remittances are implicitly assumed to be non-taxable.

\textsuperscript{10}It is assumed that migration costs can be financed at a zero interest rate from foreign sources, e.g. foreign recruitment agencies. This assumption is made because we are not interested in credit constraints that may prevent the poorer households from emigrating, and it also exempts us from modeling a domestic credit market. The assumption of zero interest rate could be easily relaxed, so that households below a certain threshold of human capital face higher gross migration costs, i.e. $\nu$ plus interests. It will be observed later that this relaxation only reinforces positive self-selection that already exists in our current setting.

\textsuperscript{11}Alternatively, we can also include a consumption tax $\tau^c$ such that Eq. (4) becomes $(1 + \tau^c) c_i^t = \left[(1 - \tau)(1 - m_i^t) \omega + m_i^t \bar{w}\right] h_i^t - b_i^t - \nu m_i^t$. However, $\tau^c$ does not change the optimal solutions for $b_i^t$ and $m_i^t$ in the log-linear utility setting. Therefore, the effect of raising $\tau^c$ is straightforward: it increases tax revenue, raises public education funding per pupil, and helps to reduce economic disparity.
Parents’ expectation about their children’s future earning potential \((h_{t+1}^{i,e})\) is derived from the human capital formation process specified above. It takes place in childhood, and Eq. (5) captures all effects of parental migration described in Section 1. The first term \((1 - m_t^i) h_t^i\) measures parental transmission of human capital, positively depending on the size of non-migrant parents. By the same token, it captures the negative effect of parental absences due to migration. In addition, children’s future human capital is increasing in the amount of private educational investment, as well as in the tax-financed public education expenditure per pupil, denoted by \(\epsilon_t\).

Parents make the optimal decisions (denoted by \(\ast\)) at the beginning of each period. We focus on the interior solution. It can be easily seen that \(b_{i}^{t*} > 0\) and \(m_{i}^{t*} < 1\) due to the log-linear specification for the household utility function (3). Lemma 1 below shows that \(m_{i}^{t*} > 0\) when the following assumptions hold.

**Assumption 1** Every child is born with a unit of basic human capital. Human capital formation during the childhood period improves but does not diminish the inborn unit:

\[
h_{t+1}^i = \max \left\{ 1, \left[ (1 - m_t^i) h_t^i \right]^{\alpha} b_t^{i^\beta} \epsilon_t^{1-\alpha-\beta} \right\}, \quad h_0^i \geq 1.
\]

**Assumption 2** Migration costs are sufficiently low relative to the foreign wage such that

\[
\nu < \bar{w} - \frac{1 + \gamma(\alpha + \beta)}{1 + \gamma \beta} \cdot \omega.
\]

**Lemma 1** As long as Assumptions 1 and 2 hold, every household has some but not all adults working abroad, i.e., \(0 < m_{i}^{t*} < 1\).

**Proof:** See Appendix A.

The interior solution to the household maximization problem is unique and the optimal
values for $m^*_t$ and $b^*_t$ are described below:

$$m^*_t = \frac{1 + \gamma \beta}{1 + \gamma (\alpha + \beta)} \left( \bar{w} - \frac{1+\gamma(\alpha+\beta)}{1+\gamma\beta} (1 - \tau) w \right) h^*_t - \nu,$$

(6)

$$b^*_t = \frac{\gamma \beta}{1 + \gamma (\alpha + \beta)} (\bar{w} h^*_t - \nu).$$

(7)

It is straightforward to verify that the optimal migration rate increases with the level of human capital (i.e., $\partial m_t^*/\partial h_t^* > 0$). The reason is that, for a better-skilled adult, migration costs $\nu$ are relatively cheaper when compared to the pecuniary benefit of migration, i.e., the international difference of net earnings: $(\bar{w} - (1 - \tau) \omega) h_t^*$. Hence, a positive self-selection pattern emerges that is consistent with the stylized fact. This also provides the rationale behind Lemma 1: when the migration costs are sufficiently low such that the lowest-skilled household has some family members working abroad, then so do all other households.

Furthermore, the optimal migration rate is increasing also with the tax rate (i.e., $\partial m_t^*/\partial \tau > 0$). Thus, when the government attempts at period $t$ to increase public educational spending by augmenting the tax rate, the concurrent tax base (i.e., total domestic wage income $\omega H_t$) may erode further with more adults working abroad. If the tax base shrinks significantly, the government budget for public education may actually decline, as opposed to the initial intention.

3 Evolution of Human Capital Distribution

This section begins by studying the evolution of human capital and its long-run distribution. Then, in the remaining, we scrutinize the effects of an expansive income tax policy. First, we study whether such a policy can indeed boost public education expenditure per pupil and whether it acts to redistribute income from the rich to the poor. In what follows, we analyze how a marginal increase in the tax rate may affect the evolution of human capital
and its implications on long-term income distribution.

To begin with, let us replace \( m_i^* \) and \( b_i^* \) into the human capital formation equation (5). Combined with Assumption 1, we obtain:

\[
h_{t+1}^i = \max \left\{ 1, \frac{(\gamma \alpha \beta)^{\alpha + \beta}}{[1 + \gamma(\alpha + \beta)]^{\alpha + \beta}} \cdot \frac{(\bar{w} h_i^t - \nu)^{\alpha + \beta}}{[(\bar{w} - (1 - \tau) w) h_i^t - \nu]^{\alpha}} \cdot (h_i^t)^{\alpha} \cdot \epsilon_t^{1-\alpha - \beta} \right\}
\]  

(5’)

Since only domestic income is assumed to be taxable, we define public education expenditure per pupil as:

\[
\epsilon_t = \frac{\tau w H_t}{N}.
\]

**Lemma 2** Under Assumptions 1 and 2, children’s human capital \((h_{t+1}^i)\) is non-decreasing in parental human capital \((h_i^t)\) if

\[
\nu \leq \bar{w} \cdot \frac{2(\alpha + \beta)(1 - \omega)}{(2\alpha + \beta) + \sqrt{\beta^2 + 4\alpha(\alpha + \beta) \frac{2}{\bar{w}}}},
\]

(8)

or migration costs \((\nu)\) are low enough relative to the foreign wage \((\bar{w})\). The sufficient condition may or may not be satisfied by Assumption 2. However, both Inequality (8) and Assumption 2 are more likely to be satisfied with a larger international difference in gross wages \((\bar{w} - \omega)\).

**Proof:** See Appendix B. ■

As we are interested in South-North migration, where international wage differences are undoubtedly sizable, we disregard the empirically irrelevant case: \((\partial h_{t+1}^i/\partial h_i^t) \leq 0\), by refining the assumption on migration costs as below:

**Assumption 3** Migration costs per migrant are small enough relative to the foreign wage
such that:

\[
\nu < \min \left\{ \left( \bar{w} - \frac{1 + \gamma (\alpha + \beta)}{1 + \gamma \beta} \cdot \omega \right), \left( \bar{w} \cdot \frac{2(\alpha + \beta)(1 - \frac{\omega}{\bar{w}})}{(2\alpha + \beta) + \sqrt{\beta^2 + 4\alpha(\alpha + \beta)^2}} \right) \right\}
\]

After a careful examination of Eq. (5'), we can analyze the long-term behavior of human capital:

**Proposition 1** Given Assumptions 1 and 3, the distribution of human capital \( \{h_i^t\}, i \in [0, \infty) \) has at least one locally stable steady state: \( \bar{h}^i \geq 1 \) for any initial condition \( \{h_0^i\}, i \in [0, \infty) \). In the case when there exist more than one locally stable steady state, depending on the initial distribution of human capital, a poverty trap may appear which prevents low-skilled dynasties to reach the locally stable steady state with a higher level of human capital.

**Proof:** See Appendix C. ■

We prove in Appendix C that the number of locally stable steady states is one, two, or three (see Figure C.1 for an illustration). Depending on the initial distribution of human capital and the parameter set, some dynasties may fall into a poverty trap, which is defined as “any self-reinforcing mechanism which causes poverty to persist” (Azariadis and Stachurski 2005). Obviously, a poverty trap is more likely to occur with a flatter distribution of initial human capital. In this case, unless there exists some kind of redistributive mechanism, knowledge or technical spill-overs from the rich to the poor, those individuals at the lowest end of distribution will be trapped at a low level of human capital.

In our model, a dynasty’s current human capital depends on its initial value and the dynastic history, and as in Glomm and Ravikumar (2003), public education is the only channel for the poor dynasties to escape from a poverty trap. Note in the literature that the aggregated level of human capital produces positive externality and affects individual human capital.
formation in a monotonic fashion (for example, see Galor and Tsiddon (1997)). In the present model, however, it is the remaining human capital \( (H_t) \) that matters because public education is financed by domestic income. Since migration gives rise to the problem of tax base erosion, a high level of aggregated human capital does not necessarily imply a high remaining level. Furthermore, the size of the externality is determined by the tax rate as public education is tax-financed.

In an economy with highly heterogeneous human capital and where the poor dynasties are haunted by the possibility of a poverty trap, the policy maker can resort to an expansive income tax policy in order to increase the public input \( (\epsilon_t) \) in the human capital formation process. Despite the threat of tax base erosion that might scrape government revenue, the following proposition shows that an expansive income tax policy always increases public education expenditure per pupil under the model assumptions.

**Proposition 2** Under Assumptions 1 and 2, the level of public education expenditure per pupil is strictly increasing and strictly concave in the tax rate, i.e. \( \frac{\partial \epsilon_t}{\partial \tau} > 0 \) and \( \frac{\partial^2 \epsilon_t}{\partial \tau^2} < 0 \). In other words, the tax rate elasticity of public education expenditure per pupil \( (\frac{\epsilon_t}{\epsilon_t} \frac{\partial \epsilon_t}{\partial \tau}) \) is higher at a lower tax rate.

**Proof:** See Appendix D.

The non-linear structure of the elasticity in Proposition 2 comes from the possibility to migrate, which produces the second-order effect of tax base erosion. It negatively affects the government budget to provide public education and renders the income tax policy less effective. In an open economy as specified in this paper, the positively selective pattern of

\(^{12}\)In Galor and Tsiddon (1997) inequality may be a growth engine for the entire economy. In their model, an individual’s human capital depends upon the aggregated human capital. Consequently, if there exists a leading group with very high human capital, it can become high enough to pull upwards the entire economy. Notice that a temporal poverty trap may have been created and/or exacerbated in the medium term, but it is important for the long-term not to slow down the leaders (by means of increasing taxes for instance).
out-migration indicates that the dynasties with higher human capital have a larger share of total income coming from the non-taxable foreign source. Consequently, the direction of redistribution is \textit{a priori} unclear.

\textbf{Lemma 3} \ A household’s tax contribution is non-monotonic in the level of human capital:

\[
\text{If } \ 1 \leq h_t^i < \nu \cdot \frac{1+\sqrt{(1-\tau)\omega}}{\bar{w}-(1-\tau)\omega}, \quad \frac{\partial \tau \omega(1-m_t^*(h_t^i)) h_t^i}{\partial h_t^i} < 0; \quad \text{otherwise,} \quad \frac{\partial \tau \omega(1-m_t^*(h_t^i)) h_t^i}{\partial h_t^i} \geq 0. \tag{9}
\]

\textit{Proof:} \ See Appendix E. \hfill \blacksquare

Hence, if the human capital distribution is such that there exist some households with very low human capital, it is not necessarily the poorest households but those with \(h_t^i\) around the neighborhood of \(\nu(1+\sqrt{(1-\tau)\omega/\bar{w}})/(\bar{w}-(1-\tau)\omega)\) who receive the largest net redistribution through public education. Hence, if there are insufficient number of highly skilled households, the poorest households may actually serve as the main contributors to the financing of public education. Nevertheless, it is straightforward to see that the upper bound of human capital in Condition (9) is decreasing in the tax rate. That is to say, the issue of reverse redistribution can be alleviated by an expansive income tax policy, which also acts to raise public education expenditure per pupil as shown in Proposition 2.

Next, we need to ask the question: does a marginal increase in the tax rate benefit all dynasties in terms of their future human capital? As a matter of fact, the next Proposition warns that an expansive income tax policy does not necessarily promote a more equalized income distribution. Instead, it can possibly lead to deepened inequality in the long-run.

\textbf{Proposition 3} \ A marginal increase in the tax rate, via increased provision of public education, improves (or does not deteriote) the future human capital for the dynasties whose human capital is sufficiently high at the time of policy intervention such that \(h_t^i > \bar{h}_t\), with
\( \dot{h}_t \) satisfying the following equation:

\[
\frac{\tau}{\epsilon_t} \cdot \frac{\partial \epsilon_t}{\partial \tau} = \frac{\alpha}{1 - \alpha - \beta} \cdot \frac{\tau \omega \dot{h}_t}{(\bar{w} - (1 - \tau)\omega) \bar{h}_t - \nu}.
\]

For dynasties with lower human capital \( h^i_t \leq \bar{h}_t \), however, the same policy deteriorates (or does not improve) their future human capital. In short,

\[
\frac{\partial h^i_{t+1}}{\partial \tau} \geq 0 \iff h^i_t > \bar{h}_t; \quad \frac{\partial h^i_{t+1}}{\partial \tau} \leq 0 \iff h^i_t \leq \bar{h}_t.
\]

Proof: See Appendix F.

In an economy with two stable steady states for example (see Figure C.1.b for an illustration), if \( \bar{h}_t \) is located in between, Proposition 3 implies that an expansive income tax policy may push the two equilibrium levels of human capital further away from each other. Therefore, those dynasties with \( h^i_t > \bar{h}_t \) become better off in the long-run, while others will be worse off. In fact, there is a possibility that an expansive income tax policy may create perpetual income inequality in an economy originally characterized by one stable steady state, or long-run equality. Figure 2 illustrates such a case, where the curvature of \( h^i_{t+1}(h^i_t) \) changes for intermediate levels of human capital due to a larger \( \tau \).

**Corollary 1** If \( h^i_{t+1}(h^i_t) \) is originally concave in the neighborhood of \( h^i_t \), a marginal increase in the tax rate may change the local curvature to convex. Ceteris paribus, this change is more likely to occur with a larger public education expenditure per pupil.

Proof: See Appendix G.

The cause behind the income polarization described in Proposition 3 is as follows. First, recall in Eq. (5) that human capital formation requires three inputs: parental transmission,
Before the adoption of an expansive income tax policy, \( h_{t+1}^i(h_t^i) \) has one stable steady state \( A \). After the policy intervention, households with \( h_t^i \in [1, B^*] \) converge to the stable steady state \( A^* \) whereas those with \( h_t^i \in (B^*, \infty) \) converge to \( C^* \) which is also stable.

Figure 2: Perpetual inequality created by a shift of \( h_{t+1}^i(h_t^i) \) due to an expansive income tax policy

Public education expenditure per pupil, and private educational investment. Second, an expansive income tax policy certainly increases public provision of education (\( \epsilon_t \)). Third, notice that the optimal private educational investment (\( b_i^{i*} \)) is not affected by the tax rate nor by changes in public education. Hence, parental absence is the only channel that may produce negative impact on human capital formation when the government marginally raises the tax rate. It is observed that

\[
\frac{\partial^2 m_i}{\partial h_i \partial \tau} = \frac{-\gamma \alpha}{1 + \gamma (\alpha + \beta)} \cdot \nu \omega \left\{ \left[ \bar{w} + (1 - \tau) \omega \right] h_i^i - \nu \right\} \left\{ \left[ \bar{w} - (1 - \tau) \omega \right] h_i^i - \nu \right\}^3 < 0.
\]

That is, although the migration rate is increasing in human capital (i.e., positive self-selection), a marginal increase in the tax rate augments the low-skilled parents’ migration rate faster. Thus, following an expansive income tax policy, the marginal rise in the costs of parental absences is larger for the less skilled. In the meantime, the marginal increase in the benefits from elevated public education expenditure per pupil is lower for the less skilled because public education is complimentary to the combination of other inputs for human capital formation. As poorer parents have fewer of which, they benefit less from the
complementarity.

Other things being equal, Corollary 1 tells us that income polarization following an expansive income tax policy is more likely to happen when the public education expenditure is large, or when the tax rate is already high. If public education plays an important role for the future generations’ human capital, i.e., a large \(1 - \alpha - \beta\), the government then has a great interest in setting a high tax rate. In this case, a progressive income tax scheme may help to better avoid income polarization. With an \textit{ad valorem} tax, however, the following Corollary specifies a sufficient condition under which an expansive income tax policy improves future human capital for \textit{all} dynasties, regardless the distribution of human capital at the time of policy intervention.

**Corollary 2** A marginal increase in the tax rate, via increased provision of public education, improves the future human capital for all dynasties, if the tax rate elasticity of public education expenditure per pupil is sufficiently high such that

\[ \frac{\tau}{\epsilon_t} \cdot \frac{\partial \epsilon_t}{\partial \tau} > \frac{\alpha}{1 - \alpha - \beta} \cdot \frac{\tau \omega}{\bar{w} - (1 - \tau) \omega - \nu}. \]  

(10)

The condition is more likely to be satisfied at a lower tax rate \(\tau\) and with lower migration costs \(\nu\).

\textbf{Proof:} See Appendix H.

The intuition behind the corollary above is illustrated in Figure 3. Moreover, as shown in the lower panel b., if originally the economy would have ended up with perpetual income inequality (i.e., more than one locally stable steady states), then a large enough increase in public education through an expansive income tax policy may help to create equality in the long-run, when all dynasties experience growing human capital and the entire economy joins a unique steady state. Corollary 2 also implies that this new steady state level of
a. The value of the unique stable steady state for $h_{t+1}^i(h_t^i)$ grows larger from $A$ to $A^*$, following an expansive income tax policy.

b. Before the adoption of an expansive income tax policy, $h_{t+1}^i(h_t^i)$ has two locally stable steady states $A$ and $C$. After the policy intervention, there is only one stable steady state: $C^*$, whose value is greater than the previously largest stable steady state value.

Figure 3: Upward shift of $h_{t+1}^i(h_t^i)$ trajectory following an expansive income tax policy
human capital is consequently larger than the original largest steady state. This finding is in contrast to Glomm and Ravikumar (1992) and Galor and Tsiddon (1997). Both suggest that public intervention in the short-run (for equality purposes) could hinder long-run economic performances. In this paper, while public education carries the role of effective income redistribution, it is also an essential stimulus for human capital growth.

Finally, Corollary 2 suggests that, in order for an expansive income tax policy to achieve a growth-enhancing and equality-promoting outcome, it should be adopted when the issues of tax base erosion and of parental absences are still relatively minor (i.e., when the tax rate is sufficiently low), in comparison to the benefits of elevated public education expenditure per pupil. Besides, such a policy is more likely to be successful when migration costs are lower. It implies that the net gain from working abroad rises, and migration becomes more efficient in terms of enlarging the household budget. As a result of increased private educational investment, the complementary inputs of human capital formation (i.e., parental transmission and public provision of education) both grow more productive. As parents find their absences more costly, an expansive income tax policy is less likely to cause serious tax base erosion, which in turn indicates a larger tax rate elasticity of public education expenditure per pupil.

### 3.1 A note on the corner solution

We have just mentioned that, when migration costs ($\nu$) are lower, an expansive income tax policy is more likely to achieve positive outcomes of economic growth and of income inequality. But, what if migration costs are so high such that Assumption 2 does not hold? Then, it is found that there exists a critical value of human capital: $\hat{h} > 1$, below which households do not migrate at all.\textsuperscript{13} This is because, if migrating, they will have

\textsuperscript{13}Notice that, although Assumption 2 is relaxed, the assumption $\hat{w} - \left[\frac{(1 + \gamma(\alpha + \beta))}{(1 + \gamma\beta)}\right] \cdot \omega > 0$ still maintains throughout the discussion of this section; otherwise, the foreign wage is too low for any household to migrate. This is an easily satisfied assumption as the World Development Indicators show that
such low levels of net foreign wage \((\bar{\omega} h_i^t - \nu)\) that the benefit of migration is not worth of the costs caused by parental absences. Depending on the initial distribution of human capital, income polarization may already occur at the very beginning of the migration history. More specifically, the non-migrating parents do not benefit from the high foreign wage rate whereas other dynasties enjoy enlarged household budgets, which enable them to make more private investments in their children’s education.\(^{14}\) In the long-run, the initial human capital disparity will be reinforced unless there is sufficient public intervention in education in order to lift up the poor dynasties (i.e., through \(\epsilon_t\)).

Due to tractability issues, this paper has focused its dynamic analysis on the migrating dynasties; however, it is worth investigating the determinants of the migration threshold \(\hat{h}\), which affects the number of households who do not migrate at each period.

**Corollary 3** If Assumption 2 fails to hold, then given an initial distribution of human capital \(\{h_i^t\}_i \in [0, \infty)\), the number of households who do not migrate at each period

i) increases with migration costs (\(\nu\)), domestic wage (\(\omega\)) and the elasticity of human capital to parental transmission (\(\alpha\));

ii) decreases with tax rate (\(\tau\)), foreign wage (\(\bar{\omega}\)), and the elasticity of human capital to private educational investment (\(\beta\));

**Proof:** See Appendix I.

The number of non-migrating households is influenced by the monetary benefits of migration: less households have adults working abroad when their disposable domestic income

---

the cross-country income differences are huge. The ratio of average GNI per capita (adjusted by purchasing power parity) is around 10 for high to lower middle income countries, and around 25 for high to low income countries.

\(^{14}\)“Children of migrants belong, on average, to wealthier households than children of non-migrants.” accounts the IOM report quoted in the introduction. Moreover, “Previous studies have shown that children of migrants are more likely to attend expensive private schools than children of non-migrants, so children of migrants receive higher quality education, on top of the higher quantity [...]”
\((1 - \tau) \omega h_i^t\) is higher or when the net foreign income \((\bar{w} h_i^t - \nu)\) is lower. Despite monetary benefits, however, parental migration bears the costs of reduced human capital transmission. As \(\alpha\) and \(\beta\) represent respectively the efficiencies of parental transmission and of private educational investment, a rise in the marginal benefit of each input then produces contrasting effects, with the former increases and the latter decreases the number of non-migrating households.

Based on the observations above, the existence of the corner solution offers some interesting dynamic conjectures. First, if there are multiple steady states, more dynasties may fall into the low equilibrium with a higher \(\alpha\), or a more important role of parental transmission of human capital. In this case, even though the low-skilled dynasties do not migrate and suffer minimally from parental absences, they have very limited human capital to transmit to their offspring. At the upper tail, however, higher levels of human capital coupled with elevated private educational investments (aided by foreign income) have the potential to dynamically enlarge the gap and separate two ends of the human capital distribution. Without a strong enough public intervention in education, a perpetual income polarization will be created.

Second, although non-migrating households do not directly benefit from the foreign wage, their offspring may do so through the role of public education. This is because, if tax base erosion is not severe, higher levels of private investment enabled by foreign income will help to increase migrant children’s future human capital; consequently, the aggregate human capital in the following period also rises. Again, if tax base erosion is not severe such that the remaining human capital is also higher, then the access to foreign wage will eventually raise the funding for public education at origin and benefit non-migrating families as well. However, high migration costs will undermine this sort of trick-down mechanism as the net foreign income declines.
4 Conclusion

This paper examines the role of public education in the context of parental migration, and it studies the effects of an expansive income tax policy that is adopted to increase public education expenditure per pupil. The novelty of our work lies in its demonstration that such a policy may exacerbate income inequality in the long run if the income tax policy is not sufficiently effective (i.e., without a sufficiently high tax rate elasticity of public education expenditure per pupil). This is because, for the less skilled dynasties, the benefits of more public spending on education does not make up for the negative effects of increased parental absences. However, if tax base erosion is not severe, an expansive income tax policy, via increased provision of public education, indeed enhances future human capital for all dynasties, and moreover, it may help the less skilled households escape from the poverty trap, thus reducing long-run income inequality.

Furthermore, it is found that an expansive income tax policy is more likely to achieve the growth-enhancing and inequality-reducing outcome when migration costs are lower, as migration is then more efficient in raising the household budget and the marginal costs of parental absences also grow higher. Thus, public education spending enjoys the complementary of more private educational investments and more parental transmission of human capital. This result calls for a better international coordination in regulating the industry of guest worker recruitment agencies. More often than not, migrant workers need to subtract a handsome amount from their paychecks in order to pay off the fees and other expenses charged by these agencies.

In summary, migration is a knife with two edges. On the one hand, lest the government is able to efficiently levy taxes on foreign income, tax base erosion can weaken public provision of education; moreover, a positively self-selective migration pattern shifts the tax burden towards those who are less skilled. On the other hand, however, remittances that are invested in children’s education may produce positive externality in a dynamic perspective, as long
as tax base erosion does not cancel out the benefits. In real life, the development deadlock
is often formed and reinforced by the fact that too many of the migrant children grow up
more educated but still follow their parents’ footsteps to work overseas.\footnote{See, for example, Lisa Wiltse’s 2008 essay for the Time Magazine: “The Motherless Generation,” where she reports the intergenerational cycle of emigration in the Philippines.}

**Acknowledgements** The authors gratefully acknowledge financial support from the Belgian French-speaking Community (convention ARC 09/14-019 on “Geographical Mobility of Factors”) and from the Marie Curie Research Training Network “Transnationality of Migrants” (TOM). We thank Raouf Boucekkine, David de la Croix and Frédéric Docquier for valuable comments. We also thank the audience at the Second TOM Conference and seminar participants at the Université catholique de Louvain. All remaining errors are of course ours.

**Appendices**

A  **Proof of Lemma 1**

By rearranging $m_t^*|_{\tau=0} > 0$, where $m_t^*$ is given by Eq. (6), we obtain:

$$\nu < h_t \cdot (\bar{w} - \frac{1 + \gamma (\alpha + \beta) - 1 + \gamma / \beta}{1} \cdot \omega).$$

By Assumptions 1 and 2, $m_t^*|_{\tau=0} > 0$ holds for every household $i \in [0, \infty)$. Then, it is easily shown that $\partial m_t^*/\partial \tau > 0$, $\forall \tau \in [0, 1]$. ■
**B Proof of Lemma 2**

After examining Eq. (5'), we can rewrite it as

\[
h_{i+1} = \begin{cases} 
\Theta(h_i) & \text{if } \Theta(h_i) > 1 \\
1 & \text{if } \Theta(h_i) \leq 1
\end{cases},
\]

where

\[
\Theta(h_i) = \frac{(\gamma \alpha)^\alpha (\gamma \beta)^\beta}{[1 + \gamma(\alpha + \beta)]^{\alpha + \beta}} \cdot \frac{(\bar{w}h_i - \nu)^{\alpha + \beta}}{[(\bar{w} - (1 - \tau)w) h_i - \nu]^\alpha} \cdot (h_i)^\alpha \cdot \epsilon_t^{1-\alpha - \beta}, \quad \text{with } h_i^\alpha \geq 1. \quad (B.1)
\]

When \( \Theta(h_i) \leq 1 \), \( h_{i+1} \) is constant and therefore non-decreasing in \( h_i \). In order to derive the condition under which \( h_{i+1}(h_i) \) is also non-decreasing when \( \Theta(h_i) > 1 \), it is sufficient to show that \( \Theta'(h_i) \geq 0 \ \forall h_i > 1 \), with

\[
\Theta'(h_i) = \frac{(\gamma \alpha)^\alpha (\gamma \beta)^\beta}{[1 + \gamma(\alpha + \beta)]^{\alpha + \beta}} \cdot \epsilon_t^{1-\alpha - \beta} \cdot h_i^\alpha \cdot \frac{(\bar{w}h_i - \nu)^{(\alpha + \beta - 1)}}{[(\bar{w} - (1 - \tau)w) h_i - \nu]^{\alpha + 1}} \cdot \{ h_i(\alpha + \beta)\bar{w} \left[ (\bar{w} - (1 - \tau)w) h_i - \nu \right] - \alpha \nu (\bar{w}h_i - \nu) \},
\]

which is non-negative if and only if the last factor in the expression above is non-negative. After some computations, it is found that, \( \forall h_i > 1 \)

\[
\Theta'(h_i) \geq 0 \iff h_i \geq \nu \cdot \frac{(2\alpha + \beta) + \sqrt{\beta^2 + 4\alpha(\alpha + \beta)(1 - \tau)\bar{w}}}{2(\alpha + \beta)(\bar{w} - (1 - \tau)w)} \left( > \nu \cdot \frac{1}{\bar{w} - (1 - \tau)w} \right).
\]

A sufficient condition for the inequalities to hold is

\[
(\alpha \beta \geq 1) \geq \left( \nu \cdot \frac{(2\alpha + \beta) + \sqrt{\beta^2 + 4\alpha(\alpha + \beta)(1 - \tau)\bar{w}}}{2(\alpha + \beta)(\bar{w} - (1 - \tau)w)} \right)_{\tau = 0}.
\]

23
After some rearrangements, we obtain Inequality (8) whose satisfaction in fact implies that, if \( \Theta(h^i_t) > 1 \), \( h^i_{t+1}(h^i_t) \) is strictly increasing. Inequality (8) will always hold under Assumption 2 if

\[
\omega \cdot \frac{\gamma \alpha}{1 + \gamma \beta} > (\bar{w} - \omega) \cdot \left(1 - \frac{1}{1 + \frac{\sqrt{\beta^2 + 4 \alpha (\alpha + \beta) \bar{w} - \beta}}{2 (\alpha + \beta)}}\right),
\]

which is however not necessarily true. Nevertheless, it can be easily verified that Inequality (8) and Assumption 2 are more likely to be satisfied with a larger \((\bar{w} - \omega)\).

### C  Proof of Proposition 1

Under Assumptions 1 and 3 and with \( \Theta(h^i_t) \) defined in Eq. (B.1), it is shown that \( h^i_{t+1}(h^i_t) \) has the following properties:

\[
\begin{cases}
\text{If } \Theta(h^i_t) > 1, \quad h^i_{t+1} = \Theta(h^i_t) \quad \text{and} \quad \frac{\partial h^i_{t+1}}{\partial h^i_t} = \Theta'(h^i_t) > 0 \quad \forall \ h^i_t > 1 ; \\
\text{If } \Theta(h^i_t) \leq 1, \quad h^i_{t+1} = 1 \quad \text{and} \quad \frac{\partial h^i_{t+1}}{\partial h^i_t} = 0 \quad \forall \ h^i_t \in ]1, \Theta^{-1}(1)[ .
\end{cases}
\]

We firstly compute the second derivative of \( \Theta(h^i_t) \) for all \( h^i_t > 1 \) and study its sign:

\[
\Theta''(h^i_t) = \frac{(\gamma \alpha) (\gamma \beta)^{\beta}}{[1 + \gamma (\alpha + \beta)]^{\alpha + \beta}} \cdot \epsilon^{1-\alpha-\beta} \cdot h^i_t^{\alpha-2} \cdot \frac{(\bar{w} h^i_t - \nu)^{(\alpha + \beta - 2)}}{[(\bar{w} - (1 - \tau) \omega) h^i_t - \nu]^{\alpha + \beta}} \cdot P(h^i_t)
\]

(+) by Assumptions 1, 3

with \( P(h^i_t) \) denoting a fourth order polynomial. Secondly, it is straightforward to show

\[
\lim_{h^i_t \to \infty} h^i_{t+1}(h^i_t) = \lim_{h^i_t \to \infty} \Theta(h^i_t) = \infty \quad \text{and} \quad \lim_{h^i_t \to \infty} \frac{\partial^2 h^i_{t+1}}{\partial h^i_t^2} = \lim_{h^i_t \to \infty} \Theta''(h^i_t) < 0
\]

Hence, \( h^i_{t+1}(h^i_t) \) is strictly concave when \( h^i_t \) tends to infinity. Next, let us consider \( P(h^i_t), \forall h^i_t \in \mathbb{R} \).

If all roots of \( P(h^i_t) \) are complex with non-zero imaginary parts, then it implies \( \Theta''(h^i_t) \) is always negative for all \( h^i_t > 1 \), i.e., \( \Theta(h^i_t) \) is globally concave and \( \Theta(h^i_t) \) crosses the 45° line at most twice.
a. If all roots of $P(h_i^t)$ are complex, the number of locally stable steady state for $h_{t+1}^i(h_i^t)$ is at least one: $A$ (solid curve) or 1 (dash dotted curve), and at most two: $A'$ and 1 (dashed curve).

b. If $P(h_i^t)$ has two real roots for all $h_i^t > 1$, the number of locally stable steady state for $h_{t+1}^i(h_i^t)$ is at least one: $A$ (solid curve) or 1 (dash dotted curve), and at most three: $A'$, $B'$, and 1 (dashed curve).

c. If $P(h_i^t)$ has four real roots for all $h_i^t > 1$, the number of locally stable steady state for $h_{t+1}^i(h_i^t)$ is at least one: $A$ (solid curve) or 1 (dash dotted curve), and at most four: $A'$, $B'$, $C'$, and 1 (dashed curve).

Figure C.1: Steady State of $h_{t+1}^i(h_i^t)$
(see Figure C.1.a). If $\Theta(h^i_t)$ crosses the 45° line at least once at $\Theta(h^i_t) > 1$, then the number of locally stable steady state for $h_{t+1}^i(h^i_t)$ is at least one: $\bar{h}^i > 1$ (two if $1 < \Theta^{-1}(1)$, with the smaller one being $\tilde{h}^i = 1$). Otherwise, there exists a unique stable steady state $\bar{h} = 1$.

If $P(h^i_t)$ has two complex roots with non-zero imaginary parts, then $\Theta''(h^i_t)$ has at most two real roots for all $h^i_t > 1$ such that $\Theta(h^i_t)$ crosses the 45° line at most four times, with the first and third largest steady states being stable (see Figure C.1.b). Similar to the argument above, if $\Theta(h^i_t)$ crosses the 45° line at least once at $\Theta(h^i_t) > 1$, then the number of locally stable steady state for $h_{t+1}^i(h^i_t)$ is at least one: $\bar{h}^i > 1$ (at most three if $1 < \Theta^{-1}(1)$, with the smallest one being $\tilde{h}^i = 1$, or else at most two). Otherwise, there exists a unique stable steady state $\bar{h} = 1$.

If all roots of $P(h^i_t)$ are real, $\Theta''(h^i_t)$ has at most four real roots for all $h^i_t > 1$ such that $\Theta(h^i_t)$ crosses the 45° line at most six times, with the first, third, and fifth largest steady states being stable (see Figure C.1.c). If $\Theta(h^i_t)$ crosses the 45° line at least once at $\Theta(h^i_t) > 1$, then the number of locally stable steady state for $h_{t+1}^i(h^i_t)$ is at least one: $\bar{h}^i > 1$ (at most four if $1 < \Theta^{-1}(1)$, with the smallest one being $\tilde{h}^i = 1$, or else at most three). Otherwise, there exists a unique stable steady state $\bar{h} = 1$.

\[ \square \]

D Proof of Proposition 2

By taking the derivative of Eq. (3) with respect to $\tau$, we decompose the effects of an marginal increase in the tax rate:

\[
\frac{\partial \epsilon_t}{\partial \tau} = \frac{\omega}{N} \left( \frac{H_t}{(+)} + \tau \cdot \frac{\partial H_t}{\partial \tau} \right). \tag{D.1}
\]

On the one hand, a marginal increase in the tax rate directly implies that the government extracts a greater fraction of domestic income to finance public education expenditure per pupil (hence the positive sign for the first term). On the other hand, however, migration rates are increasing with
the tax rate; therefore, there is less remaining human capital and tax base erosion deepens (hence the negative sign for the second term). Below, we investigate which effect dominates by replacing Eq. (2) into Eq. (D.1). Moreover, under Assumptions 1 and 2, we replace the migration rate by its optimal value expressed in Eq. (6). We obtain:

\[
\frac{\partial \epsilon_t}{\partial \tau} = \frac{\omega}{N} \cdot \frac{\gamma \alpha}{1 + \gamma (\alpha + \beta)} \cdot \int_0^\infty \frac{(\bar{w} h_t - \nu) h_t}{(\bar{w} - (1 - \tau) \omega) h_t - \nu} \cdot \left[ (\bar{w} - \omega) h_t - \nu \right] \cdot g(h_t^i) dh_t^i > 0
\]

(+) by Assumptions 1, 2

Similarly, the second derivative is derived below:

\[
\frac{\partial^2 \epsilon_t}{\partial \tau^2} = \frac{\omega}{N} \cdot \left( 2 \cdot \frac{\partial H_t}{\partial \tau} + \tau \cdot \frac{\partial^2 H_t}{\partial \tau^2} \right)
\]

\[
= \frac{2 \gamma \omega^2}{N [1 + \gamma (\alpha + \beta)]} \cdot \int_0^\infty \frac{(\bar{w} h_t - \nu)(h_t^i)^2}{(\bar{w} - (1 - \tau) \omega) h_t - \nu} \cdot \left[ (\bar{w} - \omega) h_t^i - \nu \right] \cdot g(h_t^i) dh_t^i < 0
\]

\[\blacksquare\]

**E Proof of Lemma 3**

A household’s tax contribution is written as \(\tau \omega (1 - m_t^*(h_t^i)) h_t^i\), where the optimal migration rate is expressed in Eq. (6). Taking the derivative of the tax contribution with respect to human capital, it is found that

\[
\frac{\partial \tau \omega (1 - m_t^*(h_t^i)) h_t^i}{\partial h_t^i} < 0 \iff \nu \cdot \frac{1 - \sqrt{(1 - \tau) \omega}}{\bar{w} - (1 - \tau) \omega} < h_t^i < \nu \cdot \frac{1 + \sqrt{(1 - \tau) \omega}}{\bar{w} - (1 - \tau) \omega}
\]

Under Assumptions 1 and 2, the lower bound is always satisfied. \[\blacksquare\]
F Proof of Proposition 3

In order to study the sign of \( \partial h_{t+1}^i / \partial \tau \), we firstly compute the derivative of \( \Theta(h_t^i) \) with respect to \( \tau \), with \( \Theta(h_t^i) \) defined in Eq. (B.1):

\[
\frac{\partial \Theta(h_t^i)}{\partial \tau} = \frac{(\gamma \alpha)^{\alpha}(\gamma \beta)^{\beta}}{[1 + \gamma(\alpha + \beta)]^{\alpha+\beta}} \cdot \frac{(\bar{w} h_t^i - \nu)^{\alpha+\beta}}{[(\bar{w} - (1 - \tau)\omega) h_t^i - \nu]^{\alpha}} \cdot \epsilon_t^{1-(\alpha+\beta)} \cdot h_t^i \cdot \\
\cdot \left\{ (1 - \alpha - \beta) \cdot \frac{1}{\epsilon_t} \cdot \frac{\partial \epsilon_t}{\partial \tau} - \frac{\alpha \omega h_t^i}{(\bar{w} - (1 - \tau)\omega) h_t^i - \nu} \right\}.
\]

Hence, \( \frac{\partial \Theta(h_t^i)}{\partial \tau} > 0 \) if and only if

\[
(1 - \alpha - \beta) \cdot \frac{1}{\epsilon_t} \cdot \frac{\partial \epsilon_t}{\partial \tau} > \frac{\alpha \omega h_t^i}{(\bar{w} - (1 - \tau)\omega) h_t^i - \nu}.
\]

(F.1)

It suggests that, if the inequality above is satisfied, an increase in tax from \( \tau \) to \( \tau' \) leads to \( (\partial h_{t+1}^i / \partial \tau) > 0 \) unless \( 1 \leq h_t^i \leq \Theta^{-1}(1)|_{\tau'} \), for which range \( (\partial h_{t+1}^i / \partial \tau) = 0 \). Let us define \( \tilde{h}_t^i \) such that the right and left hand sides of Inequality (F.1) equalize. As the right hand side is strictly decreasing in \( h_t^i \), we obtain the proposition.

G Proof of Corollary 1

Since we focus on the neighborhood of \( h_t^i \) where the curve is originally concave, we need to study the values of \( h_t^i \) such that \( h_{t+1}^i = \Theta(h_t^i) > 1 \). By taking the second derivative of \( h_{t+1}^i(h_t^i) \) w.r.t. \( \tau \), we obtain:

\[
\frac{\partial \partial^2 h_{t+1}^i}{\partial \tau^2} = \left( \frac{1 - \alpha - \beta}{\epsilon_t} - \frac{(\alpha + 2) \omega h_t^i}{(\bar{w} - (1 - \tau)\omega) h_t^i - \nu} + \frac{Q(h_t^i)}{P(h_t^i)} \right) \cdot \frac{\partial^2 h_{t+1}^i}{\partial h_t^i^{12}},
\]

with \( Q(h_t^i) < 0 \). Eq. (G.1) tells us how a marginal increase in \( \tau \) changes the curvature around \( h_t^i \).

Given an originally concave curve in the neighborhood of \( h_t^i \), the second derivative and \( P(h_t^i) \) are
both negative. If
\[ \epsilon_t > (1 - \alpha - \beta) \left( \frac{(\alpha + 2)\omega h_t^i}{(\bar{w} - (1 - \tau)\omega) h_t^i - \nu} - \frac{Q(h_t^i)}{P(h_t^i)} \right)^{-1} \]
such that the first factor in Eq. (G.1) is negative, then it implies that the second order derivative of \( h_{t+1}^i(h_t^i) \) increases with \( \tau \). Hence, \( h_{t+1}^i(h_t^i) \) may turn from concave to convex with a sufficiently large \( \tau \), and the distribution of human capital may exhibit multiplicity of steady states.

\[ \blacksquare \]

### H Proof of Corollary 2

By Proposition 3 and Assumption 1, a sufficient condition for \( \frac{\partial h_{t+1}^i}{\partial \tau} > 0 \forall i \) is
\[ \frac{\tau}{\epsilon_t} \cdot \frac{\partial \epsilon_t}{\partial \tau} > \frac{\alpha}{(1 - \alpha - \beta)} \cdot \frac{\tau \omega}{[\bar{w} - (1 - \tau)\omega - \nu]} \cdot \frac{1}{\omega (1 - \tau) \omega - \nu} \cdot \frac{\nu}{(1 + \gamma)} \cdot \frac{1}{1 + \gamma} \]

The left hand side of the inequality (i.e., the tax rate elasticity of public education expenditure per pupil) is decreasing in \( \tau \), as shown in Proposition 2, whereas the right hand side can be easily proven to be increasing in \( \tau \) and \( \nu \).

\[ \blacksquare \]

### I Proof of Corollary 3

First, we derive:
\[ \hat{h} = \frac{\nu}{\bar{w} - (1 + \gamma/\alpha + \gamma) \cdot (1 - \tau)\omega} \]

The corollary is proved by taking the partial derivatives of \( \hat{h} \) with respect to each of the model parameters.

\[ \blacksquare \]
References


