Public goods' attractiveness and migrations

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Public goods' attractiveness and migrations

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Abstract

The aim of this paper is to develop a dynamic model of migrations, in which migration is driven by size asymmetries between countries and by the relative preferences of consumers between private consumption and consumption of public goods. The dynamic tra jectories heavily depend on the degree of attractiveness for public goods We show that monotone migrations require sufficiently strong preferences for public goods, and can only be sustained from the small to the large countries. We identify the threshold value of the public goods' intensity of preferences guaranteeing the survival of the small country. For weaker preference intensities, oscillating migrations may arise, but they finally converge to situation where both countries are of equal size.

Keywords: migration, public goods, income tax.

JEL Classification: H

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1 Introduction

This paper is based on the following simple intuition. The provision of most public goods is financed through public spending. Public spending is based on the product of taxes. When a country has many citizens, the basis for tax collection is wider than when the country has only a few ones. Thus larger countries are able to provide a larger amount of public goods than the small ones and, as far as individuals are attracted by public goods' abundance, they should be more satisfied when living in large than in small countries[1]. Accordingly, if the cost of moving is not too large, one should expect a populations' drain from small countries to the big ones through migration.

This explanation is however a little simplistic, and forgets a crucial element. When more and more people leave the small country to enjoy the larger amount of public good provided in the big one, it cuts down progressively the amount of labour force available in the small country, with the likely effect of increasing the local wage rate. Now, while becoming higher and higher in the small country, the wages can at some point make the latter more attractive, compared with the attractiveness of the big country, resulting from its higher amount of public good ! In turn, this increase in attractiveness of the small country may well constitute soon a stronger countervailing force than the original attractiveness of the large country, and lead accordingly to a reversal in the direction of migration!

In order to disentangle the precise effects of these forces on the pattern of migrations, it is useful to embed the problem into a simple general equilibrium model for studying rigorously the dynamics of migrations entailed by these forces. We propose hereafter such a framework, which develops along the following lines. We suppose that there are two countries, *home* and *foreign*, of unequal size in terms of the number of their citizens. Heterogeneity among the citizens in each country is introduced by assuming that they incur varying (subjective) costs when moving abroad. In each country, a private composite commodity is produced by a profit-maximising firm *via* a decreasing returns technology, using labour as sole intput. Individuals are each endowed with a single unit of homogeneous labour supplied inelastically on the local labour market. The local wage rate is determined by the equality of demand and labour supply. Individuals are utility maximizers.

[1] Alesina et al. (2005, page 1503) notice for example that , "...safety is a public good that increases with country size". Moreover Ram (2008) has recently challenged the finding of Alesina and Wacziarg (1998) that there is a negative covariation of country size with the share of public consumption in GDP. The author even finds a significant positive covariation between government size and country size.

In each country, the government levies an income tax which serves to finance the provision of the public good produced under a constant returns technology, and available for consumption to all residents in the country. The net income is then spent by residents as a private consumption. The resulting individual utility thus depends, when staying at home, on the local wage and tax rates. When the home citizen decides to emigrate, it depends now on the foreign wage and tax rates, but the cost of migrating has now to be deducted from the utility. Accordingly, the only decision to be taken by an individual is whether she/he prefers to stay in her/his home country to enjoy private consumption and the amount of local public good, or to go abroad and enjoy there private consumption and foreign public good, but then paying the cost incurred for moving abroad. The only decision to be taken by each government is the national tax rate, which automatically determines the amount of the public good *via* the state budget constraint. In order to focus on labour mobility we assume that firms are (internationally) immobile and that entry costs for new firms are prohibitively high.

After solving this static model, we turn to its dynamics. The above equilibrium analysis, which is essentially static, spontaneously invites to a dynamic extension: migration observed at equilibrium in period 0 as a consequence of the initial populations' shares now generates renewed initial populations' sizes in period 1. More generally, the migration observed at equilibrium in period t as a consequence of the initial populations' shares existing in the period now creates renewed initial populations'sizes for period $t+1$.Accordingly, a sequence of successive competitive equilibria is naturally generated as a consequence of the equilibrium migration arising in each period, which constitutes the initial conditions of the equilibrium for the next one.

The question to be answered is to clarify the trajectories of these dynamics. In particular, we want to know whether the forces described above drive inexorably the small country to become smaller and smaller due to migrations to the larger one, even with the risk of its disappearance, or whether some oscillating behavior should be observed in the direction of migration. Clearly, due to the preferences of individuals which ultimately drive the dynamics of the model, the answer to this question should depend on the relative intensity of individual preferences for the private *versus* public good. The attractiveness of the larger country is tantamount larger, the larger the intensity of preferences for the public good, compared with the intensity of preferences for private spending. Our main result consists first in finding a sufficient condition, bearing on the intensity of preferences for the public good, for obtaining a monotone convergence of migrations from the smaller to the larger country. We also show that this condition is necessary for obtaining monotone convergence: *monotone* convergence can never arise from the larger to the smaller country. Of course, when the intensity of public good's preferences is very strong, one can imagine that this could even endanger the very survival of the small country. Thus, we also identify the upperlimit on the intensity of public good's preferences which guarantees the survival of the small country in the long run. On the contrary, when the intensity of preferences for the public good decreases below a threshold value, we show that two issues are possible. Either, migration takes place only during a single period, and then stops for ever. Or, when the preferences for the public good are really weak, migration's dynamics oscillate from period to period but, finally, populations' sizes tend to become equal in the two countries.

There is a significant literature pointing to the fact that small states suffer from their smallness (for a brief survey, see Easterly and Kray, 2000)). In this context it is often argued that small countries face high per capita costs for providing public goods because large indivisibilities are the rule in the provision of these goods. In this paper we focus on another limitation which results from the standard theory of local public goods and concerns the attractiveness of public goods supplied by small jurisdictions in the context of migration. In the vein of Tiebout (1956), jurisdictions are assumed to provide local public goods so that everyone living in the same community consumes the whole quantity of this good. The production of the local good being financed by a local tax levied on the residents of the jurisdiction, it follows that the volume (intensity or quality) of public goods rises with the size of the local tax base. Accordingly, when people are allowed to choose the jurisdiction where to move, small sized jurisdictions tend to be disadvantaged with respect to the larger ones.

Our paper is also related to the political economics approach to the migration problem which studies, in the tradition of Tiebout, how people migrate on the basis of public good preferences. Factor mobility is not only motivated by differences in income but also by the provision of public goods. In our model size asymmetry between countries is crucial in causing migration. The dynamics of the model then show that migration also explains how this asymmetry may evolve. Accordingly, the size of the involved countries are shown to be determined endogenously. In this respect, our approach may also be related to Alesina and Spolaore (1997) and Alesina, Spolaore and Wacziarg (2005), who are concerned with the endogenous determination of country size.

Our contribution also highlights the fact that some size asymmetries between countries cannot remain sustainable in the long term. Indeed, labor mobility and migration may exacerbate size asymmetry between countries to such an extent that the very survival of the small country could be threatened in the long run. In this context, the model is related to the literature concerned with geographic agglomeration (for an interesting survey, see Ottaviano, and Thisse, 2004). On the contrary, our paper also shows that countries' asymmetries are wiped out when preferences for the public good are weak.

2 The model

We consider two countries, home (h) and foreign (f); the population *types* in h (resp. f) is represented by the $[0,1]$-interval, with density λ_h in country h (resp. λ_f in country f), $\lambda_h + \lambda_f = 1$. Each individual in each country is endowed with one unit of homogenous labour. Wages are denoted by w_h and w_f, respectively. An individual of type $x, x \in [0,1]$, either decides to work in his/her own country, say country i, or he/she migrates to the foreign one, $j, j \neq i, (j, i = h, f)$. In the first case, his/her utility is given by

$$u_i(x) = w_i(1 - t_i) + 2\sqrt{\alpha \lambda_i w_i t_i}$$

and by

$$u_j(x) = w_j(1 - t_j) + 2\sqrt{\alpha \lambda_j w_j t_j} - x$$

when migrating, with t_i denoting the tax rate in country $i, i = h, f$, and x the disutility of emigrating for an individual of type x. Accordingly, heterogeneity among types in each country is generated by the disutility of emigrating, and types are ranked in the $[0,1]$-interval by order of increasing disutility incurred from emigrating. As for the term $2\sqrt{\alpha \lambda_i w_i t_i}$, it measures the utility of the public good made available in country i thanks to the tax proceeds $\lambda_i w_i t_i$, with α denoting the *intensity of preferences* for the public good. For simplicity, we set in the following: $\lambda_f = \lambda_0$ and $\lambda_h = 1 - \lambda_0$, with λ_0 and $1 - \lambda_0$ measuring respectively the foreign and the domestic country initial populations' densities. Populations' densities at some date t will be denoted accordingly by λ_t and $1 - \lambda_t$.

The production function $F(L)$ of the representative firm in each country is identical and given by

$$F(L) = L - \frac{1}{2}L^2,$$

with L denoting labour used in the production process.

3 The competitive equilibrium at time $t = 0$.

Assume that individuals in both countries supply inelastically the unit of labour they are initially endowed with. Aggregate labour supply in country i is then equal to the population's density in this country, namely $L_h = 1 - \lambda_0$ and $L_f = \lambda_0$ in country h and f, respectively. From profit maximisation, the demand function for labour obtains from the condition that the wage rate is equal to its marginal product, namely

$$w_i = F'(L_i) = 1 - L_i.$$

Accordingly, equilibrium wage rates are given by

$$w_h = \ \lambda_0 \text{ and } w_f = 1 - \lambda_0.$$

For simplicity, we assume that the resulting profits of the firm are invested outside the area in which we are interested in[2]. The government in each country is assumed to select a tax rate t_i,$i = h, f$, (and an ensuing amount of public good) in order to maximize the utility of the representative *resident*[3]. This assumption leads to the government's payoff function

$$R_h(t_h, t_f) = (1 - t_h)\lambda + 2\sqrt{\alpha t_h \lambda(1 - \lambda)} \tag{1}$$

in the home country with population's density $1 - \lambda$, and to the payoff

$$R_f(t_h, t_f) = (1 - t_f)(1 - \lambda) + 2\sqrt{\alpha t_f \lambda(1 - \lambda)} \tag{2}$$

in the foreign one with population's density λ. From the first order conditions, we easily get the optimal taxes t_h^* and t_f^* at time $t = 0$:

$$t_h^* = \frac{\alpha(1 - \lambda_0)}{\lambda_0} \text{ and } t_f^* = \frac{\alpha\lambda_0}{1 - \lambda_0} \tag{3}$$

for the initial populations' densities λ_0 and $1 - \lambda_0$.

Let us now state three simple properties observed at the competitive equilibrium.

Proposition 1 *The net wage $w_i = w_i(1 - t_i)$,$i = h, f$, at the competitive equilibrium is the highest in the smallest country.*

[2]Think of tax heavens, like the Channel Islands or Cayman Islands... Our capitalists can be viewed as the *absentee landlords* in Hansen A. and Kessler (2001).

[3]Notice that this policy does not take into account the fact that it can induce some home residents to leave the country for abroad or, conversely, to attract some foreign residents at home. Accordingly, the government's behavior is assumed to be myopic.

Proof. From $w_h = \lambda_0$ and $w_f = 1 - \lambda_0$.we have that $w_f - w_h = (1 - 2\lambda_0)$. It follows that $w_f > w_h$ iff $0 \leq \lambda_0 < \frac{1}{2}$ (f is the small country) and $w_h > w_f$ iff $1 > \lambda_0 > \frac{1}{2}$($h$ is the small country). ■

Similarly, we have that

Proposition 2 *The smallest country charges the smallest tax rate at the competitive equilibrium and provides the smallest amount of public good.*

Proof. The first part of the proposition follows from a direct comparison of the optimal taxes (see (3)). Now, given the optimal taxes obtained in (3), the state budget is equal to $\frac{\alpha(1-\lambda_0)^2}{\lambda_0}$ at home and to $\frac{\alpha\lambda_0^2}{1-\lambda_0}$ in the foreign country. Accordingly, the state budget at home is the largest iff either $0 \leq \lambda_0 < \frac{1}{2}$ (f is the small country), or $1 > \lambda_0 > \frac{1}{2}$($h$ is the small country). Since the amount of public good is equal to the amount of the state budget, the second part of the proposition follows. ■

Denote by Δ the difference $R_h(t_h, t_f) - R_f(t_h, t_f)$ with R_h and R_f as given by (1) and (2). When this difference is positive, country h is more attractive than country f, and the most mobile citizens in country f are willing to migrate to country h. The reverse holds when the difference Δ is negative, entailing a migration from h to f. Assume, without loss of generality, that $\lambda_0 > 1 - \lambda_0$.Then, using the equilibrium values t_h and t_f given by (3), we get

$$\Delta = R_h(t_h, t_f) - R_f(t_h, t_f) = (1 - \alpha)(2\lambda_0 - 1) < 0$$

when $\alpha > 1$,so that migration takes place from home to foreign, and

$$\Delta = R_h(t_h, t_f) - R_f(t_h, t_f) = (1 - \alpha)(2\lambda_0 - 1) > 0$$

when $\alpha < 1$, in which case migration takes place from foreign to home. Without loss of generality, assume that $\alpha > 1$ so that migration takes place from home to foreign.The type x in the home country who is indifferent between migrating or staying at home must satisfy $\Delta = x < 0$. Substituting in the difference $R_h(t_h, t_f) - R_f(t_h, t_f)$, with R_h and R_f as given by (1) and (2), for the equilibrium values t_h and t_f given by (3), we obtain

$$|\Delta(\lambda_0)| = x_0 = (1 - \alpha)(1 - 2\lambda_0), \tag{4}$$

given the initial sizes of the populations λ_0 and $1 - \lambda_0$. Consequently, all individuals in the home country belonging to the types in the interval $[0, x_0]$ migrate to the foreign country, as a consequence of the equilibrium taxes and the resulting public goods' differential between the two countries. Accordingly, equation (4) reveals that the migration pattern at equilibrium crucially depends on the parameters α and λ_0, i.e., the intensity of preferences for the public good and the initial sizes of the countries. The following proposition describes unambiguously the migration pattern at equilibrium.

Proposition 3 *Migration takes place at the competitive equilibrium from the home country to the foreign one* $\Longleftrightarrow \alpha > 1$ *and* $\lambda_0 > \frac{1}{2}$*, or* $\alpha < 1$ *and* $\lambda_0 < \frac{1}{2}$*. In the first alternative, the home country is the small country while the reverse holds in the second.*

4 The dynamics of migration

The above equilibrium analysis, which is essentially static, spontaneously invites to a dynamic extension: the migration observed at equilibrium in period 0 as a consequence of the initial populations' shares λ_0 and $1 - \lambda_0$ now creates renewed initial conditions in period 1, say λ_1 and $1 - \lambda_1$. More generally, the migration observed at equilibrium in period t as a consequence of the initial populations' shares λ_t and $1 - \lambda_t$ now creates renewed initial conditions for period $t + 1$, say λ_{t+1} and $1 - \lambda_{t+1}$. Accordingly, a sequence of successive competitive equilibria is naturally generated as a consequence of the equilibrium migration arising in each period, which constitutes the initial conditions of the equilibrium for the next one.

4.1 Migration dynamics when $\alpha > 1$

In order to study the dynamics of these successive competitive equilibria, we consider first the case when $\alpha > 1$. As we show later, this condition guarantees that these dynamics are monotone through time. Then, assuming that $x_0 = (1 - \alpha)(1 - 2\lambda_0) > 0$, equation (4) can be written more generally as

$$x_t = (1 - \alpha)(1 - 2\lambda_t) > 0, \tag{6}$$

where t refers to any time period. Under the monotonicity assumption, the variable x_t is positive because we have assumed $x_0 > 0$.The population's density in country f at time $t + 1$ writes as

$$\lambda_{t+1} = \lambda_0 + (1 - \lambda_0)\, x_t. \tag{7}$$

Combining the two above equations gives the first order difference equation

$$x_{t+1} = f(x_t) = -2(1-\alpha)(1-\lambda_0)x_t + x_0 \tag{8}$$

which describes the trajectory of migrations corresponding to the successive competitive equilibria in the sequence. Whenever it exists, the fixed point of this trajectory is given by the solution x^* to the equation $-2(1-\alpha)(1-\lambda_0)x + x_0 = x$, namely,

$$x^* = \frac{(\alpha-1)(2\lambda_0-1)}{2(\alpha-1)(\lambda_0-1)+1}.$$

.

In order to identify under which condition the above dynamics are indeed monotone, we analyse equilibrium migrations as a function of the preference intensity for the public good, α.

Proposition 4 *Migrations are monotone if, and only if,* $1 < \alpha < \frac{3-\lambda_0}{2-2\lambda_0}$. *Furthermore, they can only take place from the small to the large country.*

Proof. First remember that the steady state share of migrants (in total initial population of the foreign country) is equal to $x^* = \frac{(\alpha-1)(2\lambda_0-1)}{2(\alpha-1)(\lambda_0-1)+1}$.The condition for having monotone convergence to x^*(migration from f to h with no reversal) is given by $0 < \left|\frac{\partial f(x_t)}{\partial x_t}\right|_{x_t=x^*} < 1$. According to equation (8), we get $-2(1-\alpha)(1-\lambda_0) < 1 \Longleftrightarrow 1 < \alpha < \frac{3-\lambda_0}{2-2\lambda_0}$. The second part of the proposition follows directly from the fact that convergence implies that the preference parameter α has to be larger than 1 and, since $x_0 > 0$, it follows that λ_0 has to exceed $\frac{1}{2}$. ■

The case considered in the above proposition is depicted in figure 1 in the appendix.

We have assumed above that migration takes place from the home to the foreign country. According to the above proposition, we conclude that this can happen under monotone dynamics *only* when the home country is the small one. The above proposition implies that the small country becomes smaller and smaller through time. This raises the question of its long-run survival.

Proposition 5 *Under monotone convergence* $(\alpha > 1)$, *the small country survives in the long run if, and only if,* $\alpha < 2$.

Proof. Survival arises if and only if the unequality $x^* = \frac{(\alpha-1)(2\lambda_0-1)}{2(\alpha-1)(\lambda_0-1)+1} < 1$ holds, namely, iff

$$\alpha < \min\left\{2, \frac{3-\lambda_0}{2(1-\lambda_0)}\right\}$$

This condition reduces to $\alpha < 2$ since $\lambda_0 > \frac{1}{2}$ according to proposition 4. ■

A sufficient condition for migration to flow monotonically from the small country to the big one thus requires that individuals have strong preferences for public goods ($\alpha > 1$). This flow is sustained until a steady state is reached since migration to the country supplying the largest amount of public good increases monotonically this supply through the increase in taxes. The larger the population in the receiving country, the higher the tax rates and, accordingly, the larger the tax income available to finance public goods' supply.

4.2 Migration dynamics when $\alpha < 1$

When individuals value more weakly the public good ($0 < \alpha < 1$), it is conceivable that migration starts with individuals moving from the foreign country to the home one, in which taxation is milder and net wages more attractive. However, as the population of the home country increases as a result of immigration, its tax rate should rise and its wage decrease, making the home country less attractive than in the previous stage. Then two alternatives are possible. Either it can still attract agents from the foreign country which are less mobile than those who were moving in the first period, or its attractiveness has decreased to such an extent that all candidates to migration from country f in the first period are already in country h. Or it can even decrease to such an extent that some past migrants now wish to go back to the foreign country ! In the latter case, migration will reverse and now takes place from country h to f. In turn, this flow can however be reversed again, after that country f has reached a population size inducing a sufficiently low net wage level to justify again a reversal of migration, now from f to h again. We examine now, as a function of the intensity parameter $\alpha, \alpha < 1$, when the migration process stops after one period or, on the contrary, adopts an oscillating behaviour of backward and forward migration.

When $\alpha < 1$ *and* $\lambda_0 > \frac{1}{2}$, migration takes place at the competitive equilibrium from the foreign country to the home one. Since $\Delta(\lambda_0) = x_0 = (1-\alpha)(2\lambda_0 - 1) > 0$, the population in the foreign country decreases to λ_1

with

$$\lambda_1 = (1 - x_0)\lambda_0.$$

According to the sign of $\Delta(\lambda_1) = (1-\alpha)(2\lambda_1 - 1)$

$= (2\alpha\lambda_0 - 2\lambda_0 + 1)(2\lambda_0 - 1)(1-\alpha)$, migration takes place in period 1 from the home to the foreign country when $\Delta(\lambda_1) < 0$, or again from the foreign to the home one when $\Delta(\lambda_1) > 0$. We notice that, when $\alpha < 1$, the above quadratic expression in λ_0 has two roots, namely, $\frac{1}{2}$ and $\frac{1}{2(1-\alpha)}$. Furthermore it is concave so that, when $\lambda_0 \in \left[\frac{1}{2}, \frac{1}{2(1-\alpha)}\right]$, this expression is positive. Consequently, when $\alpha > \frac{1}{2}$, we have also $\frac{1}{2(1-\alpha)} > 1$, and the expression is positive for all acceptable values for α. We conclude that migration takes place again from the foreign to the home one. Furthermore, since a direct comparison shows that $x_0 > x_1$, all types who should migrate from abroad to home in period 1, have already migrated at the end of period 0, so that no migration takes place again in period 1, and never in the future. In other words,

Proposition 6 *When $\alpha \in \left[\frac{1}{2}, 1\right]$ and $\lambda_0 \in \left[\frac{1}{2}, 1\right]$, migration takes place only in period 1 and stops for ever afterwards.*

The case considered in the above proposition is depicted in figure 2 of the appendix.

Now we move to the case $\alpha \in \left(0, \frac{1}{2}\right)$, which implies that $\frac{1}{2(1-\alpha)} < 1$. When $\lambda_0 \in (\frac{1}{2}, \frac{1}{2(1-\alpha)})$, then $\Delta(\lambda_1) > 0$ and migration takes place again from abroad to home, as in the previous case. Applying the above proposition, we conclude that migration stops for ever after period 1. It remains to examine the case $\Delta(\lambda_1) < 0$.

Proposition 7 *When $\alpha \in \left(0, \frac{1}{2}\right)$ and $\lambda_0 \in (\frac{1}{2(1-\alpha)}, 1)$, the populations' dynamics oscillate for ever, but the measure of the set of migrants in each period t is decreasing with t. Furthermore,*

$$\lim_{t\to\infty} \{\lambda_t\} = \frac{1}{2} = \lim_{t\to\infty} \{1 - \lambda_t\}.$$

Proof. Suppose that $\alpha < \frac{1}{2}$ and that $\Delta(\lambda_1) = (2\alpha\lambda_0 - 2\lambda_0 + 1)(2\lambda_0 - 1)(\alpha - 1) < 0$, which happens if and only if $\lambda_0 \in \left[\frac{1}{2(1-\alpha)}, 1\right]$. This implies that, at period 1, migration takes place from home to abroad. Then $x_1 = |\Delta(\lambda_1)| = (1-\alpha)(1-2\lambda_1)$. Notice that migration, which was from abroad to home at the end of period 0, now takes place from home to abroad. Also we observe again that $x_0 > x_1$ so that only the more mobile types of agents are coming back in their home country. Now let us move to period 2. We have

$$\lambda_2 = \lambda_1 + x_1 > \frac{1}{2},$$

since $\lambda_1 + x_1 - \frac{1}{2} = \lambda_1 + (1-\alpha)(1-2\lambda_1) - \frac{1}{2} = (\frac{1}{2} - \alpha)(1-2\lambda_1) > 0$. According to proposition 3, migration in period 2 takes place from foreign to home since the large country is now the foreign one. We get $x_2 = |\Delta(\lambda_2)| = (1-\alpha)(2\lambda_2 - 1)$. We observe again that $x_1 > x_2$[4]. More generally,

$$\begin{aligned} x_{2k} &= |\Delta(\lambda_{2k})| = (1-\alpha)(2\lambda_{2k} - 1) \\ x_{2k+1} &= |\Delta(\lambda_{2k+1})| = (1-\alpha)(1-2\lambda_{2k+1}), \end{aligned} \tag{9}$$

Furthermore, we have

$$\begin{aligned} \lambda_{2k} &= \lambda_{2k-1} + x_{2k-1} > \frac{1}{2} \\ \lambda_{2k+1} &= \lambda_{2k} - x_{2k} < \frac{1}{2}. \end{aligned} \tag{10}$$

Accordingly, the home and foreign countries are alternatively the large and the small country according as t is even or odd. Substituting (9) in equation (10), we obtain

$$\begin{aligned} \lambda_{2k} &= (1-\alpha) + (2\alpha - 1)\lambda_{2k-1} \\ \lambda_{2k+1} &= (1-\alpha) + (2\alpha - 1)\lambda_{2k} \end{aligned}$$

or,

$$\lambda_t = (1-\alpha) + (2\alpha - 1)\lambda_{t-1}.$$

This first order linear difference equation has, as general solution, the following expression

$$\lambda_t = (1-\alpha)\frac{(2\alpha-1)^t - 1}{2(\alpha - 1)} + (2\alpha - 1)^t \lambda_0.$$

Taking into account the fact that $\alpha < \frac{1}{2}$, we obtain

$$\lim_{t\to\infty} \{\lambda_t\} = \frac{1}{2}.$$

QED. ■

The case considered in the above proposition is depicted in figure 3 in the appendix.

[4] $x_1 > x_2 \Longleftrightarrow 1 - 2\lambda_1 > 2\lambda_2 - 1$
$\Leftrightarrow 1 - \lambda_1 - \lambda_2 = 1 - 2\lambda_1 + (1-\alpha)(2\lambda_1 - 1) =$
$= \alpha(1 - 2\lambda_1) > 0.$

5 Conclusion

The aim of this paper is to develop a simple model of migrations, in which migration is fundamentally driven by size asymmetries between countries and by the relative preferences of consumers between private consumption and consumption of public goods. Public goods' provision is generally financed through public funds. Public spending depends itself on the product of taxes. When a country is large, the basis for tax collection is wider than when the country has only few citizens. Thus larger countries are able to provide a larger amount of public goods and, as far as individuals are attracted by public goods' abundance, the attraction of living in large countries should exceed the attraction of living in smaller ones. Accordingly, one expects migrations to move from small countries to the big ones. Since local labour markets are closed and the supply of local public goods depends on local tax income, size asymmetry creates inter-country differentials in (net) wages and in the provision of public goods. In the above model, agents are heterogenous concerning their disutility to move abroad, while their preferences for public goods are uniform. Also there is no inter-country competition due to the myopia of the governments, which do not take into account the effects of their tax decisions on labour mobility. Countries tax their residents and provide them with public goods without trying to alter the conditions faced by the other country. In a one-shot approach, migration flows from the large to the small country if the preferences for public goods are weak, and flows from the small to the big one if public goods' valuation is high. In a dynamic setting, populations' dynamics are driven by the relative preferences of consumers between private consumption and consumption of public goods. The migration observed at equilibrium at any period, as a consequence of the populations' shares in the preceding period, creates renewed populations' shares for the next one. Monotone migrations require sufficiently strong preferences for public goods, and can only be sustained from the small to the large country. Nevertheless, if these preferences are too strong, the small country incurs the risk of disappearance.We identify the threshold value of the public goods' intensity of preferences guaranteeing the survival of the small country. When the intensity of preferences for the public good is weaker than this threshold value ($\alpha < 1$), two alternative dynamic paths can be followed. Either the attractive country in period 0 can still attract in period 1 agents from the other country which are less mobile than those who were moving in the first period, or its attractiveness has decreased to such an extent that all candidates to migration in period 1 have already moved in the other country in period 0. Then no further migration is observed in the future. Or the attractiveness of the receiving country in period 0 has even decreased in period 1 to such an extent that some past migrants now wish to go back to their initially home country ! In the latter case, migration now takes place in the reverse direction. In turn, this flow can however be reversed again, after the receiving country in period 1 has reached a population size inducing a sufficiently low net wage level to justify again a new reversal of migration. We show that this dynamic process finally leads the countries to become of equal size in the long-run.

An interesting extension of our analysis would consist in assuming a more sophisticated behaviour of the governments when deciding about taxes. In the present set-up we assume that governments select taxes so as to maximize the utility of their residents, without accounting for the incidence of these taxes on the migration pattern these taxes generate (see footnote 3, page 6). It would be more appealing to assume that governments anticipate this incidence, and take it into account in their tax decisions. Proceeding in this way introduces an element of interaction between the decisions of the two countries' governments : both the direction of migration and its amplitude depend on the strategies of both of them since the tax selected by each government finally determines these magnitudes. The situation could then be formalized as a tax game in which countries adopt a strategic behaviour and try to be more (or less) attractive in taxes and in the ensuing provision of public goods provided to its residents.

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Appendix

Figure 1:

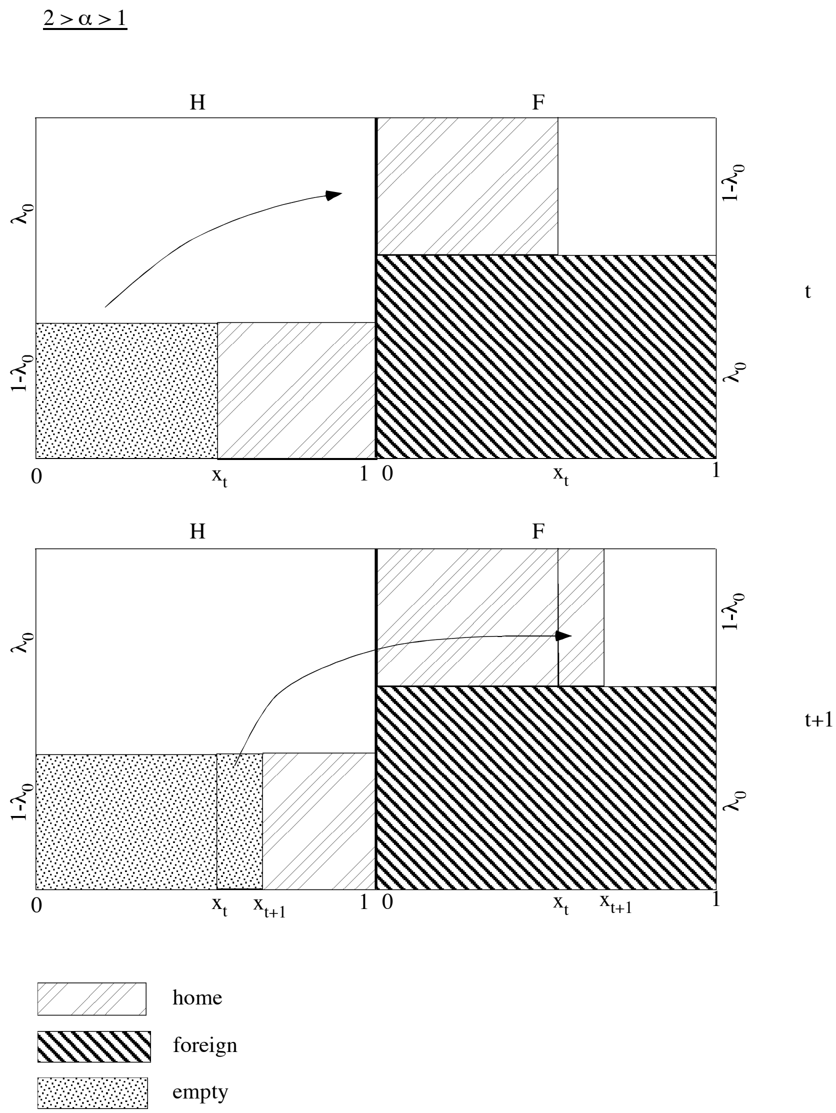
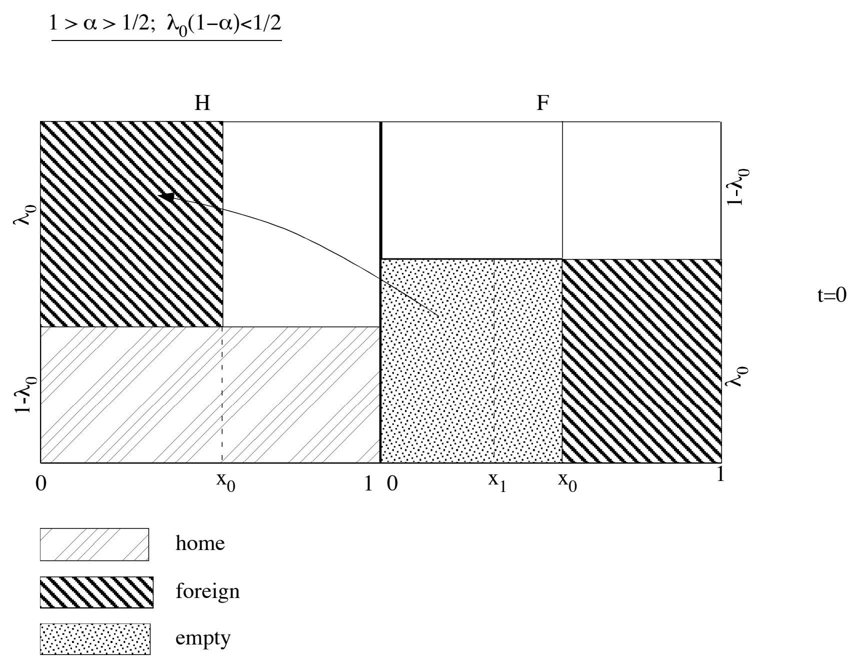
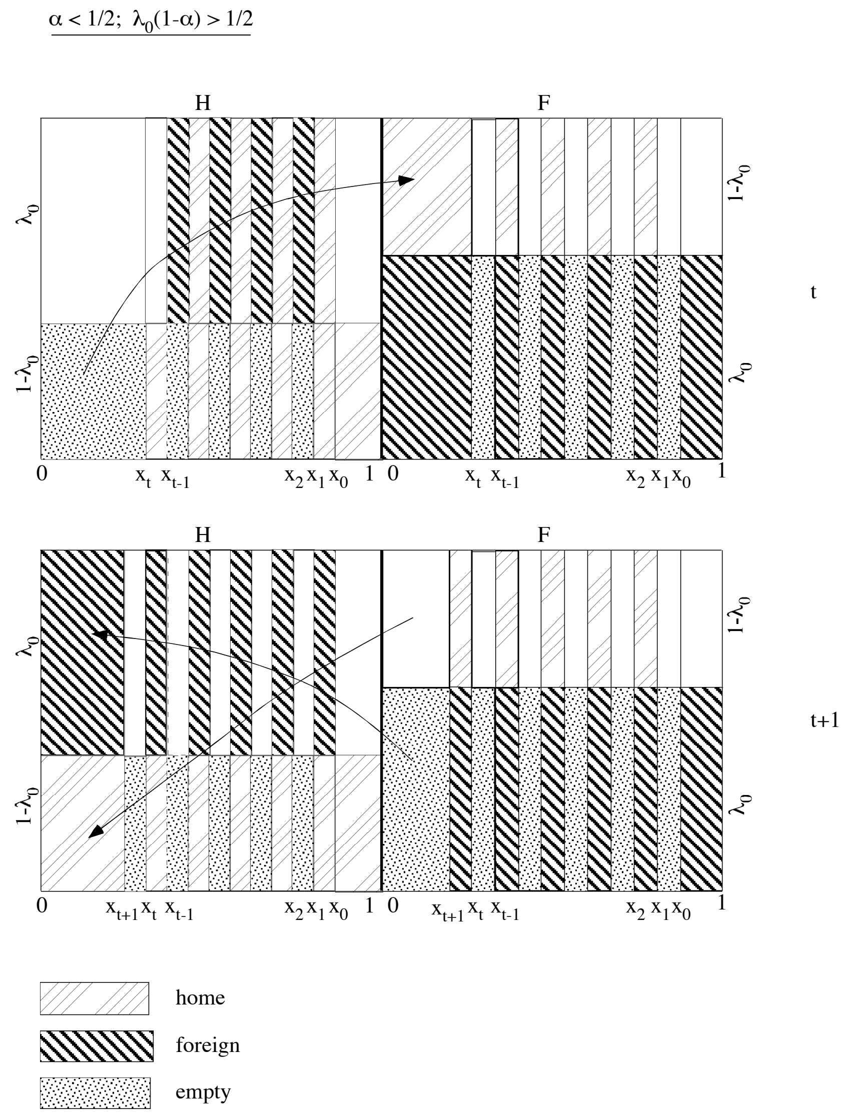


Figure 2:



1 > α > 1/2; λ0(1−α)<1/2
H
F
λ0
1-λ0
1-λ0
λ0
t=0
0
x0
1
0
x1
x0
1
home
foreign
empty

Figure 3:


$\alpha < 1/2;\ \lambda_0(1-\alpha) > 1/2$
H
F
λ_0
$1-\lambda_0$
t
t+1
0
x_{t+1} x_t x_{t-1}
x_2 x_1 x_0
1
home
foreign
empty

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