Uncertainty, product variety and price competition

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Uncertain quality, product variety and price competition

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Abstract

This paper analyses price competition under product differentiation when goods are defined in a two dimensional characteristic space, and consumers do not know which firm sells which quality. Equilibrium prices consist of two additive terms, which balance consumers' relative valuation of goods' expected quality and consumers' preferences for variety. However the relative importance of these terms differ under vertical and horizontal dominance.

Keywords: product differentiation, variety, quality, uncertainty.

JEL Classification: D43, D80, L15

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1. Introduction

This paper analyses price competition under product differentiation when goods are defined in a two dimensional characteristic space, and consumers do not know which firm sells which quality. Real life economic contexts are often too complex to be analyzed with the simple toolboxes proposed by textbooks. For example, and with some significant exceptions (see Neven and Thisse (1990) and Irmen and Thisse (1996)), competition with differentiated products is basically analysed using alternatively the two models of *horizontal* and *vertical* product differentiation. These models describe goods as points in a one-dimensional characteristic space, like the Hotelling's Main Street paradigm or the Mussa-Rosen's quality ladder. Nonetheless, real life reveals every day plethoric situations which escape to these abstractions, simply because competition among firms often develops along more than one dimension.

Think of restaurants in a town. When consumers are planning to dine out and compare the merits of two restaurants, they consider the quality of the food, but also their location and their respective ambiance and surroundings. Similarly, physicians are not only evaluated with respect to their professional ability, but also with respect to their empathy with patients. Daily newspapers or weekly magazines attract readers not only by the specificity of their content, would it be entertainment or culture, or a mix of both, but also by their look and the quality of their journalists. When students have to decide which university to attend (or professors in which university to work), they generally make a trade off between the quality of the professors (or the potential colleagues), and the length of the road connecting the universities to their own residence. The preferences for clothes and shoes do not depend only on their aesthetic aspects, which are highly subjective, but also on the intrinsic quality of the material used in their production. In all these examples, preferences are not one-dimensional since utility comparisons deal with more than one characteristic: they reflect the interplay between both horizontal and vertical components. As alluded in the title of the Neven and Thisse's paper (see Neven and Thisse (1990)), these utilities combine the preferences for *quality* and the preferences for *variety.* Food quality, professional ability, journalists' or professors' talents, quality of material, all these terms used in the above examples refer to the vertical differentiation of the goods or services. On the contrary, location, empathy, specificity of contents, aesthetic aspects, refer to their horizontal differentiation component.

But often, even moving to models embedding multidimensional characteristics, as Neven and Thisse (1990) do, is not sufficient enough to capture various significant real life ingredients of product differentiation. Among these, one of the most important is consumers' *uncertainty* about quality. Indeed, in several economic contexts, consumers are unable to assert unequivocally either the qualities of the variants offered in the market, or the firms who sell the high quality brands. Going back to the above examples, when consumers are planning to dine out and compare the merits of different restaurants, they consider the quality of the food served in each of them, but most of the time they ignore, or do not know with certainty, which restaurant serves the better food. Similarly, the professional ability of physicians is generally difficult to be unambiguously settled, even if everybody has some idea about their relative competence. When students have to decide which university to attend, or professors in which university to work, the outcomes of the trade off are often difficult to identify, due to the uncertainty about the respective merits of the professors (or the potential colleagues) in each university. Finally, the preferences for clothes and shoes are also submitted to uncertainty, due to the existence of fakes and the resulting difficulty of asserting unequivocally the intrinsic quality of the material used in the production of each brand. When analyzing market competition, imperfect information of consumers about product characteristics, and product quality in particular, is a recurrent problem. Several contributions have been devoted to market behavior under uncertainty (see, e.g. Shapiro (1983), Wolinsky (1986), Ungern-Sternberg and Weizacker (1985), Salop and Stiglitz (1977), Stahl (1982)). More recently, Gabszewicz and Grilo (1992) have studied price competition in a one dimensional model of vertical product differentiation, when consumers are uncertain about which firm sells which quality.

The present paper tends to extend both the contributions by Neven and Thisse (1990) and by Gabszewicz and Grilo (1992). From the first, it borrows the idea of constructing a model which accounts for *two-dimensional* competition, one related to a horizontally differentiated characteristic of the good, and the other to a vertically differentiated component. Such a model captures most ingredients of the examples provided above, but one of them : the consumers' uncertainty about which firm sells which quality. From the paper by Gabszewicz and Grilo (1992), the present approach borrows the idea of relating vertical product differentiation to uncertainty about the quality of the goods. But the analysis is now embedded into a bi-dimensional competition, one along the horizontally differentiated and perfectly observable characteristic, and the other related to the quality of the product, consumers being uncertain about which firm sells which quality.

Uncertainty introduces a new element in the picture. Even if everybody knows that one variant is of a higher quality than the other, it is no longer true that all consumers now necessarily rank the firms selling these products in the same manner, as it is the case under vertical differentiation when quality is perfectly observable. Population's beliefs about which firm sells which quality can make some consumers to view one of the firms, say firm 1, as being more than likely the seller of the high quality variant, while other consumers may well believe the reverse, inverting thereby their preferences for firm 1 at the advantage of firm 2. Thus, and in contrast with the Thisse and Neven's construction, the model of this paper not only couples the two traditional paradigms of product differentiation, but also captures new situations in which firms may be differentiated horizontally in *two* characteristics, due to the presence of uncertainty.

This paper can also be viewed as an extension of Gabszewicz and Grilo (1992). Even if uncertainty is present in both papers concerning the vertical component of the goods, the addition in the present one of a second horizontally differentiated characteristic allows to capture many situations intrinsically represented by two-dimensional competition, as in the examples considered above. Thanks to this added component, consumers may now trade-off a higher willingness to pay for a variant against a smaller likelihood that this variant is the high quality one.

The model used in this paper can be succinctly described as follows. As in Thisse and Neven (1990), we consider that goods are differentiated along two characteristics, being vertically differentiated with respect to the first, but horizontally differentiated with respect to the second. Thus, consumers have different evaluations of goods' merits in relation to the second characteristic while, in a context of perfect information, consumers should be able to unequivocally and unanimously rank variants in terms of the first characteristic. However, we assume that this first characteristic is not perfectly observable (in opposition with the second one, which is considered to be perfectly observable). More precisely, as in Gabszewicz and Grilo (1992), consumers do not know which firm sells which quality and have heterogeneous beliefs about this event. We assume that two rival firms sell each one of the variants and behave non cooperatively when choosing their price strategies.

We focus on interior equilibria, providing sufficient conditions for its existence and uniqueness and showing that the characteristics of the equilibrium are different depending on whether we observe *"vertical dominance"* or *"horizontal dominance"*. Furthermore, equilibrium prices consist of two additive terms, which balance consumers' relative valuation of goods' expected quality and consumers' preferences for variety. However the relative importance of these terms differ under vertical and horizontal dominance. We end up by comparing the equilibrium under imperfect information with the prices obtained when all consumers are perfectly informed about which firm sells which quality. Also, we compare our solutions with the equilibria corresponding to a fully deterministic vertical differentiation model with variants' qualities defined by the average qualities resulting from the consumers' beliefs in the population (certainty equivalent case).

The paper is organized as follows. Section 2 presents the model and introduces the notions of horizontal and vertical dominance. For the two alternative cases, section 3 provides the corresponding price equilibrium analysis. Then, section 4 compares our results with the benchmarks of perfect information and certainty equivalent cases. Finally, section 5 concludes.

2. The model

We consider two profit maximizing firms (firm 1 and firm 2) that produce two goods (good 1 and good 2), differentiated along two characteristics. Goods are produced at a constant marginal cost assumed to be equal to zero, without loss of generality. With respect to the first characteristic, products 1 and 2 differ by their *quality*. However, as in Gabszewicz and Grilo (1992), consumers are uncertain about which firm sells which quality and, furthermore, consumers differ in their beliefs about this uncertain event. The goods are supposed to be differentiated horizontally with respect to the second characteristic which is perfectly observable. Thereby, our model accounts for *two* distinct sources of heterogeneity among consumers: *(i)* consumers formulate different subjective probabilities (beliefs) concerning which good has a higher-quality in terms of the unobservable characteristic; and *(ii)* they differ on their evaluation of goods' merit in relation to the observable characteristic.

Each consumer is identified by a vector (m, θ). The first component (m) represents the subjective probability that consumer of type m assigns to the event: {*product* 1 *corresponds to the high-quality product*}. We assume that m is uniformly distributed in the interval $[\underline{m}, \overline{m}]$, with

$$0 \leq \underline{m} < \frac{1}{2} < \overline{m} \leq 1.$$

We call the interval $[\underline{m}, \overline{m}]$ the *domain of beliefs*. The second component (θ) deals with the heterogeneity of consumers with respect to the horizontal differentiation characteristic, measuring the differential in utilities consumer θ gets when he/she consumes good 1 *versus* good 2. We assume that θ is uniformly distributed in $[\theta_{\min}, \theta_{\max}]$, with $\theta_{\min} < 0$, representing the consumer who, concerning this characteristic, prefers the most good 2 to good 1; and $\theta_{\max} > 0$, representing the consumer who prefers the least good 2 to good 1 regarding this same characteristic.

Hence, consumer (m, θ) can be viewed as a point in the rectangle $[\underline{m}, \overline{m}] \times [\theta_{\min}, \theta_{\max}]$. When consumer (m, θ) buys good 1, at price p_1, she/he expects to obtain an expected utility $E_{U_1}(m, \theta, p_1)$ defined by

$$E_{U_1}(m, \theta, p_1) = V + m(u_h) + (1-m)(u_l) + \frac{\theta}{2} - p_1, \tag{1}$$

where V denotes a positive constant[1], and u_h (resp. u_l) is the utility provided by the variant which corresponds to the high-quality (resp. low-quality) product with respect to the unobservable characteristic,with $u_h > u_l \geq 0$. When consumer (θ, m) buys good 2 (instead of good 1), she/he expects to get an expected utility $E_{U_2}(m, p_2)$ defined by

$$E_{U_2}(m, p_2) = V + m(u_l) + (1-m)(u_h) - \frac{\theta}{2} - p_2. \tag{2}$$

Comparing (1) and (2) ,it becomes clear that the parameter θ indeed represents the differential in utilities (with respect to the horizontal differentiation component) consumer θ gets when she/he consumes good 1 *versus* good 2 .

Conditional on prices (p_1, p_2) and for a given belief $m \in [\underline{m}, \overline{m}]$,the consumer $\left(\tilde{\theta}(m), m\right)$ satisfying the equality

$$E_{U_1}\left(m, \tilde{\theta}, p_1\right) = E_{U_2}(m, p_2) \Leftrightarrow \tag{3}$$

$$\Leftrightarrow \tilde{\theta}(m) = (p_1 - p_2) + (1-2m)(u_h - u_l) \tag{4}$$

[1] The constant V is considered to be large enough for all consumers to find a product for which their utilities are positive at equilibrium (covered market).

is indifferent between buying good 1 or good 2.

From expression (4) it follows that, for a given vector of prices $(p_{1,}p_2)$, the marginal consumer $\tilde{\theta}(m)$ evolves linearly and negatively with m :

$$\frac{\partial\tilde{\theta}}{\partial m} = -2\,(\Delta u) < 0,$$

with $\Delta u = u_h - u_l$. Not surprisingly, for those types of consumers that assign a greater probability m to the event {*product* 1 *corresponds to the high-quality product}*, the critical value of $\tilde{\theta}(m)$ is lower. Thus, the higher the $m-$value, the greater the mass of consumers θ who are willing to buy good 1 instead of good 2.

In order to derive demands as a function of prices, we distinguish between two cases according to the relative importance of the vertical *vsus* the horizontal component. We say that *horizontal (resp. vertical) dominance* is observed whenever the inequality (resp. the reverse inequality)

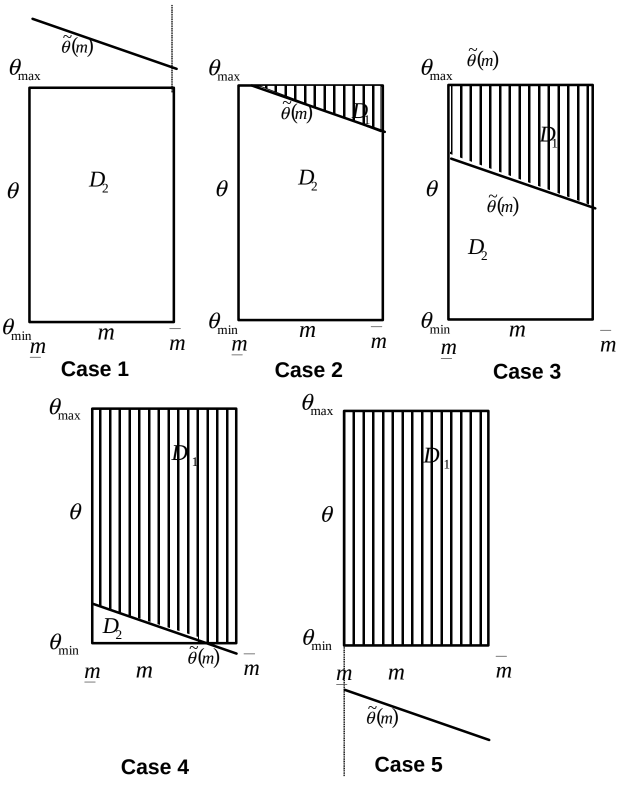
$$2\Delta u\,(\overline{m} - \underline{m}) < \theta_{\max} - \theta_{\min} \tag{5}$$

holds[2].

According to (4), at a given pair of prices (p_1, p_2), the set of consumers can be partitioned into two subsets, each describing those consumers who buy good 1 and good 2, respectively. For a given value of $m \in [\underline{m}, \overline{m}]$, the subset of consumers buying good 1 is given by $\left[\tilde{\theta}(m), \theta_{\max}\right]$, while the subset of consumers buying good 2 is given by $\left[\theta_{\min}, \tilde{\theta}(m)\right]$.

The following figures illustrate the structure of demands for different pairs of prices (p_1, p_2). The first figure deals with the case of horizontal dominance:

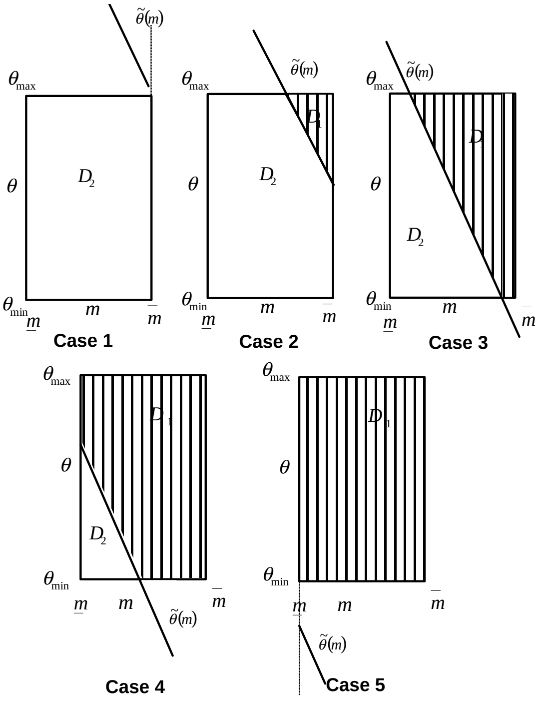
[2] This terminology has been introduced by Neven and Thisse (1990).



Horizontal dominance

Each rectangle in the figure above identifies a specific partition of the set of consumers corresponding to different pairs of prices (p_1, p_2). Under horizontal dominance, case 1 corresponds to a value of p_1 which, given p_2, is so high that there is no consumer willing to buy variant 1 at that price. In cases 2, 3 and 4, both firms are active in the market. In case 2, the market share of firm 1 corresponds to the area of the shadow triangle. When p_1 further decreases, we move to case 3, where all types m are served by both firms and the market share of firm 1 now consists of an area which is the sum of a triangle and a rectangle. When p_1 even further decreases, we move to case 4 and now demand corresponds to an area which is the sum of a triangle and two rectangles. Finally, in case 5, firm 1 becomes a monopolist and the market share of firm 2 is equal to zero.

Under vertical dominance, the same comments apply, *mutatis mutandis* and the structure of demand is represented in the following figure.



Vertical dominance

Addressing the case of horizontal dominance, we define by R_i^H the set $R_i^H = \{(p_1, p_2) :$ (p_1, p_2) *leads to a market share corresponding to case* $i, i = 1, ..., 5\}$. Similarly, under vertical dominance, we define by R_i^V the set $R_i^V = \{(p_1, p_2) : (p_1, p_2)$ *leads to a market share corresponding to case* $i, i = 1, ..., 5\}$.

In this setting, we obtain the analytical expressions of firms' market shares as:

$$D_1^H(p_1, p_2) = \begin{cases} 0 & if \quad (p_1, p_2) \in R_1^H \\ \frac{1}{4}\frac{(\theta_{\max} - p_1 + p_2 + (2\overline{m} - 1)\Delta u)^2}{(\overline{m} - \underline{m})(\theta_{\max} - \theta_{\min})\Delta u} & if \quad (p_1, p_2) \in R_2^H \\ \frac{\theta_{\max} - p_1 + p_2 + (\underline{m} + \overline{m} - 1)\Delta u}{\theta_{\max} - \theta_{\min}} & if \quad (p_1, p_2) \in R_3^H \\ 1 - \frac{1}{4}\frac{(\theta_{\min} - p_1 + p_2 - (1 - 2\underline{m})\Delta u)^2}{(\overline{m} - \underline{m})(\theta_{\max} - \theta_{\min})\Delta u} & if \quad (p_1, p_2) \in R_4^H \\ 1 & if \quad (p_1, p_2) \in R_5^H, \end{cases}$$

$$D_2^H(p_1, p_2) = 1 - D_1^H(p_1, p_2),$$

in the case of horizontal dominance, and

$$D_1^V(p_1,p_2)=\begin{cases} 0 & if \quad (p_1,p_2)\in R_1^V \\ \frac{1}{4}\frac{(\theta_{\max}-p_1+p_2+(2\overline{m}-1)\Delta u)^2}{(\overline{m}-\underline{m})(\theta_{\max}-\theta_{\min})\Delta u} & if \quad (p_1,p_2)\in R_2^V \\ \frac{1}{4}\frac{\theta_{\max}+\theta_{\min}-2p_1+2p_2-2(1-2\overline{m})\Delta u}{\Delta u(\overline{m}-\underline{m})} & if \quad (p_1,p_2)\in R_3^V \\ 1-\frac{1}{4}\frac{(\theta_{\min}-p_1+p_2-(1-2\underline{m})\Delta u)^2}{(\overline{m}-\underline{m})(\theta_{\max}-\theta_{\min})\Delta u} & if \quad (p_1,p_2)\in R_4^V \\ 1 & if \quad (p_1,p_2)\in R_5^V, \end{cases}$$

$$D_2^V(p_1,p_2)=1-D_1^V(p_1,p_2),$$

in the case of vertical dominance. The exact definition of the domains R_i^H and R_i^V is provided in appendix A.

It is easily seen that, in both cases, demands are continuous and decreasing in firms' own prices. However, demands are not everywhere concave functions. Namely, the demand of firm 1 is convex in $(p_1,p_2)\in R_2^j, j=H,V$ (under both horizontal or vertical dominance)[3]. Analogously, demand of firm 2 is convex in $(p_1,p_2)\in R_4^j, j=H,V$[4].

Considering the characteristics of demand, we introduce hereafter sufficient conditions guaranteeing that, in each of these domains (where demands are locally convex), for a given rival's price, individual profits are decreasing in firms' own prices. Thus, these conditions are sufficient to prevent the occurrence of unilateral advantageous deviations leading a pair of prices to fall in these convex domains. This is sufficient to guarantee that any price equilibrium, if it exists, should be in R_3^H, in the case of horizontal dominance; and in R_3^V, in the case of vertical dominance.

The following lemmata identify sufficient conditions for this to occur.

LEMMA 1. ***Horizontal dominance***
Under the case of horizontal dominance, profits of firm 1 *are decreasing in* p_1 *when* $(p_1,p_2)\in R_2^H$ *if*

$$\theta_{\max}\geq(1+\overline{m}-3\underline{m})\,\Delta u \tag{6}$$

Similarly profits of firm 2 *are decreasing in* p_2 *when* $(p_1,p_2)\in R_4^H$ *if*

$$-\theta_{\min}\geq(3\overline{m}-\underline{m}-1)\,\Delta u \tag{7}$$

Proof. In appendix. ∎

[3] Considering that demand of firm 1 is continuous, decreasing and locally concave in p_1 for (p_1,p_2) in R_3^j; R_4^j; and R_5^j, it can be easily shown that the demand of firm 1 is globally concave for $(p_1,p_2)\in\left(R_3^j\cup R_4^j\cup R_5^j\right), j=H,V.$

For $(p_1,p_2)\in R_1^j, j=H,V$, the demand for good 1 is linear and, consequently, locally concave.

[4] The demand of firm 2 is globaly concave for $(p_1,p_2)\in\left(R_1^j\cup R_2^j\cup R_3^j\right), j=H,V.$ and locally concave for $(p_1,p_2)\in R_5^j, j=H,V.$

Lemma 2. ***Vertical dominance***
Under the case of vertical dominance, profits of firm 1 *are decreasing in* p_1 *when* $(p_1, p_2) \in R_2^V$ *if*

$$\Delta u \geq \frac{\theta_{\max} - 3\theta_{\min}}{2\left(2\overline{m} - 1\right)}$$

and profits of firm 2 *are decreasing in* p_2 *when* $(p_1, p_2) \in R_4^V$ *if*

$$\Delta u \geq \frac{3\theta_{\max} - \theta_{\min}}{2\left(1 - 2\underline{m}\right)}.$$

Proof. In appendix. ∎

In the following section we characterize equilibrium prices. Firstly, in subsection 3.1 we analyze equilibrium prices when the conditions in lemma 1 hold (horizontal dominance setting). Then, in subsection 3.2, we restrict our analysis to the values of the parameters accomplishing the conditions in lemma 2 (vertical dominance setting).

3. Equilibrium analysis

3.1. Horizontal dominance

When lemma 1 holds, firms' profits are quasi-concave functions of prices and they reach only one maximum value in region R_3^H, where profit functions write as

$$\pi_1^H\left(p_1, p_2\right) = \left(\frac{\theta_{\max} - p_1 + p_2 + \left(\underline{m} + \overline{m} - 1\right)\Delta u}{\theta_{\max} - \theta_{\min}}\right) p_1,$$

for firm 1, and

$$\pi_2^H\left(p_1, p_2\right) = \left(1 - \frac{\theta_{\max} - p_1 + p_2 + \left(\underline{m} + \overline{m} - 1\right)\Delta u}{\theta_{\max} - \theta_{\min}}\right) p_2,$$

for firm 2.

Accordingly, the unique price equilibrium obtains as the interior solution of the profit maximization problem, yielding:

$$\begin{aligned} p_1^{H*} &= \tfrac{1}{3}\left(\theta_{\max} + \left(\theta_{\max} - \theta_{\min}\right) + 2\left(\tfrac{\underline{m}+\overline{m}}{2} - \tfrac{1}{2}\right)\Delta u\right), \\ p_2^{H*} &= \tfrac{1}{3}\left(\left(\theta_{\max} - \theta_{\min}\right) - \theta_{\min} - 2\left(\tfrac{\underline{m}+\overline{m}}{2} - \tfrac{1}{2}\right)\left(\Delta u\right)\right), \end{aligned} \tag{8}$$

which are both positive as long as the conditions provided in lemma 1 are met. As a consequence, we obtain the following

Proposition 1. *Under horizontal dominance (lemma 1):*

(i) given $\theta_{\min}$ *and* $\theta_{\max}$*, firms' equilibrium prices are determined by the average beliefs*

$\frac{\underline{m}+\overline{m}}{2}$ *: when beliefs are biased towards good* 1 *(resp. good* 2*), with* $\frac{\underline{m}+\overline{m}}{2} > \frac{1}{2}$ *(resp.* $\frac{\underline{m}+\overline{m}}{2} < \frac{1}{2}$*), firm* 1 *(resp. firm* 2*) charges a positive "quality premium" to consumers, while the rival firm offers a "quality discount";*

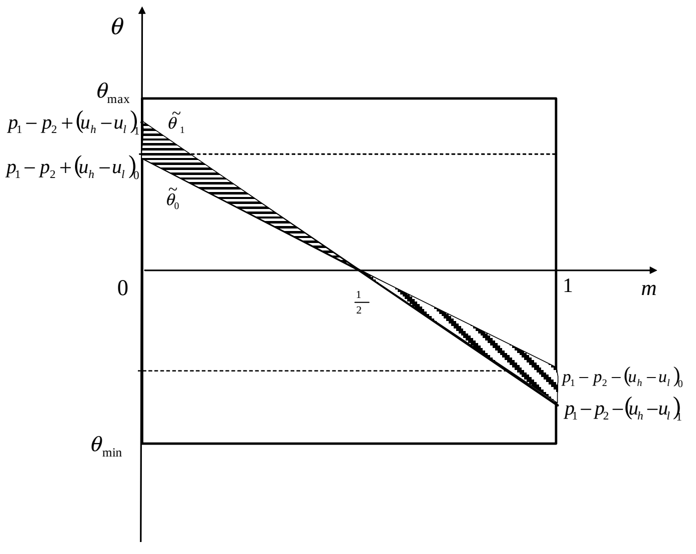
(ii) given the domain of beliefs $[\underline{m}, \overline{m}]$ *and the dispersion of preferences* $\theta_{\max} - \theta_{\min}$*, the larger* $\theta_{\max}$ *(resp. the smaller* $\theta_{\min}$*), the larger the preferences' bias towards variant* 1 *(resp. variant* 2*), and the higher the "variety premium" charged by firm* 1 *(resp. firm* 2*) to the consumers.*

According to proposition 1, the difference between equilibrium prices depends on two components. The first derives from consumers' heterogeneity concerning the horizontal component ($\theta_{\min}$,$\theta_{\max}$) and the second derives from consumers' heterogeneity in terms of their beliefs ($\underline{m}, \overline{m}$) and the quality differential between the variants (Δu).

In order to get a better insight on firms' price behaviour under horizontal dominance, it is convenient to consider the particular case when the domain of beliefs is centered around $m = \frac{1}{2}$ (symmetric beliefs). In this case, the price equilibrium presents some specific properties: regardless of the dispersion of beliefs, $\underline{m} + \overline{m}$ is always equal to 1 and the beliefs of the average consumer are now given by $m = \frac{1}{2}$. Hence, following from proposition 1, only the heterogeneity of consumers concerning the horizontal component determines equilibrium prices. In fact, under a symmetric distribution of beliefs around $m = \frac{1}{2}$, a positive (resp. negative) variation in the differential Δu affects the locus $\tilde{\theta}(m)$ in the following way. It increases (resp. decreases) $\tilde{\theta}(m)$ for consumers of type $m < \frac{1}{2}$ but it decreases (resp. increases) $\tilde{\theta}(m)$ for the remaining consumers (whose beliefs correspond to $m > \frac{1}{2}$). Thus, a positive (resp. negative) variation in the differential Δu entails two effects of opposite sign on firms' demands[5]. It turns out that, when beliefs are symmetric and the conditions imposed by lemma 1 are met, these effects are perfectly symmetric and, consequently, firms' demand do not depend on consumers' valuation of goods in terms of quality (i.e. the quantity demanded of each good is independent of the dispersion of beliefs $\overline{m} - \underline{m}$ and the quality differential Δu).

The following figure graphically illustrates this argument:

[5] On the one hand, this variation leads to an increase (resp decrease) of the demand for good 2 among those consumers who trust more in good 2 (or equivalently, among those consumers whose beliefs are represented by $m < \frac{1}{2}$). On the other hand, this variation leads to a decrease (resp increase) of the demand for good 2 among those consumers who trust more in good 1 (i.e. those consumers whose beliefs are represented by $m > \frac{1}{2}$).



Symmetric beliefs

Thus, as long as the differential in goods' quality is not too large (i.e. lemma 1 is met), when consumers' beliefs are centered around $m = \frac{1}{2}$, equilibrium prices do not depend on goods' qualities and the high-quality firm is not able to charge a quality premium. This is no longer the case when beliefs are not perfectly symmetric. In that case, for a given $[\theta_{\min}, \theta_{\max}]$, the more consumers trust a certain good, the larger the quality premium charged by the trusted firm and the larger the quality discount charged by the less-trusted firm.

3.2. Vertical dominance

When lemma 2 holds, firms' profits are quasi-concave functions of prices and they reach only one maximum value in region R_3^V, where profit functions write as

$$\pi_1^V(p_1, p_2) = \left(\frac{1}{4} \frac{\theta_{\max} + \theta_{\min} - 2p_1 + 2p_2 - 2(1 - 2\overline{m})\Delta u}{\Delta u (\overline{m} - \underline{m})} \right) p_1,$$

for firm 1, and

$$\pi_2^V(p_1, p_2) = \left(1 - \frac{1}{4} \frac{\theta_{\max} + \theta_{\min} - 2p_1 + 2p_2 - 2(1 - 2\overline{m})\Delta u}{\Delta u (\overline{m} - \underline{m})} \right) p_2,$$

for firm 2.

Accordingly, the unique price equilibrium obtains as the interior solution of the profit maximization problem:

$$\begin{aligned} p_1^{V*} &= \tfrac{1}{3}\left(\left(2\left(\overline{m} + (\overline{m} - \underline{m})\right) - 1\right)\Delta u + \tfrac{\theta_{\max}+\theta_{\min}}{2} \right), \\ p_2^{V*} &= \tfrac{1}{3}\left(\left(2\left((\overline{m} - \underline{m}) - \underline{m}\right) + 1\right)\Delta u - \tfrac{\theta_{\max}+\theta_{\min}}{2} \right), \end{aligned} \tag{9}$$

which are both positive, under the conditions pointed out in lemma 2.

PROPOSITION 2. *Under vertical dominance (lemma 2) :*

(i) given the domain of beliefs $[\underline{m}, \overline{m}]$ *, firms' equilibrium prices are determined by the preferences of the average consumer regarding the horizontally differentiated component* $\left(\frac{\theta_{\max}+\theta_{\min}}{2}\right)$ *: when preferences for variety are biased towards good* 1 *(resp. good* 2*), with* $\theta_{\max} > -\theta_{\min}$ *(resp.* $\theta_{\max} < -\theta_{\min}$*), firm* 1 *(resp. firm* 2*) charges a positive "variety premium" to consumers, while the rival firm sets a "variety discount".*

(ii) given $[\theta_{\min}, \theta_{\max}]$ *and* $(\overline{m} - \underline{m})$ *, the larger* $\overline{m}$ *(resp. the smaller* $\underline{m}$*), the greater the trustworthiness of good* 1 *(resp. good* 2*), and the larger the "quality premium" charged by firm* 1 *(resp. firm* 2*).*

As before, in the case of vertical dominance, the magnitude of the difference between the equilibrium prices depends on two components: the horizontal component ($\theta_{\min}$,$\theta_{\max}$) and the vertical component (which is determined by the quality gap, Δu, and the degree of uncertainty concerning products' characteristics($\underline{m}, \overline{m}$)).

However, it is worthy to notice that, differently from before, even when consumers' beliefs are centered around $m = \frac{1}{2}$, firms' demands are no longer independent of the quality gap and, accordingly, even under a symmetric distribution of beliefs ($\overline{m} + \underline{m} = 1$), the quality gap influences firms' equilibrium prices. The larger the quality gap, the more expansive both goods become:

$$\begin{aligned} p_1^{V*} &= (\overline{m} - \underline{m})\, \Delta u + \tfrac{\theta_{\max}+\theta_{\min}}{6}, \\ p_2^{V*} &= (\overline{m} - \underline{m})\, \Delta u - \tfrac{\theta_{\max}+\theta_{\min}}{6}. \end{aligned} \tag{10}$$

When beliefs are symmetric, both goods can be seen as evenly trusted within the population of consumers. Consequently, in a scenario of imperfect information, the low-quality firm takes advantage of uncertainty to charge a quality premium, which coincides with the quality premium charged by the truly high-quality product.

When consumers' beliefs are not symmetric, *ceteris paribus* the larger the trustworthiness of good i, the greater the quality premium charged by the trusted good as well as the quality discount offered by the less-trusted good. Goods' trustworthiness is determined by the interval of beliefs: trustworthiness in good 1 (resp. good 2) positively (negatively) depends on $\overline{m}$ and $\underline{m}$ (for a given value of $\underline{m}$ or $\overline{m}$, respectively).

To sum up, when goods are defined in a two dimensional characteristic space, and consumers do not know which firm sells which quality, equilibrium prices consist of two additive terms, which balance consumers' relative valuation of goods' expected quality and consumers' preferences for variety. However the relative importance of these terms differ under vertical and horizontal dominance.

Furthermore, assuming beliefs are symmetric around $m = \frac{1}{2}$, we have shown that, under horizontal dominance, the relative valuation of goods' expected quality plays no role at equilibrium. On the contrary, under vertical dominance, equilibrium prices depend on this valuation. Nevertheless, in the last case, given the symmetry of beliefs, both firms charge the same quality premium, independently of their true quality. If, instead, the symmetry lies in the preferences for the horizontally differentiated characteristic $(-\theta_{\min} = \theta_{\max})$, under vertical dominance, equilibrium prices do not depend on consumers' preferences for variety. When consumers' preferences towards the horizontal dimension are asymmetric, $(\theta_{\max} \neq -\theta_{\min})$, firm 1 (resp. firm 2) charges a "variety premium" whenever $\theta_{\max} > -\theta_{\min}$ (resp. $\theta_{\max} < -\theta_{\min}$).

Comparing our results with Gabszewicz and Grilo (1992), we observe that the equilibrium prices in the latter coincide with ours when we "neutralize" the heterogeneity of consumers' preferences with respect to variety $(\theta_{\max} = -\theta_{\min} = 0)$. In that case, we are necessarily in the vertical dominance setting. When $\theta_{\max} > -\theta_{\min}$ (resp. $\theta_{\max} < -\theta_{\min}$), the equilibrium price of firm 1 (resp. firm 2) exceeds the corresponding equilibrium price of this firm in Gabszewicz and Grilo (1992), while the other firm charges a lower price in our setting. In the case of horizontal dominance, "neutralizing" the heterogeneity of consumers' preferences with respect to variety $(\theta_{\max} = -\theta_{\min} = 0)$ necessarily requires Δu to be equal to zero[6]. In that case, both firms quote prices equal to zero at equilibrium: since there is no more any source of differentiation between goods, we end up with pure competition "à la Bertrand".

4. Perfect information

In this section, we assume that quality is perfectly observable and, without loss of generality, good 1 is the high-quality good. Then $\overline{m} = \underline{m} = 1$ and the only source of heterogeneity among consumers comes from the horizontal differentiation component (θ).

Accordingly, the demand for good 1 obtains as:

$$D_1(p_1, p_2) = \begin{cases} 0 & if \quad p_1 - p_2 \geq \theta_{\max} + \Delta u \\ \frac{\theta_{\max} - (p_1 - p_2 - \Delta u)}{\theta_{\max} - \theta_{\min}} & if \quad \theta_{\min} + \Delta u < p_1 - p_2 < \theta_{\max} + \Delta u \\ 1 & if \quad p_1 - p_2 \leq \theta_{\min} + \Delta u \end{cases} \tag{11}$$

and

$$D_2(p_1, p_2) = 1 - D_1(p_1, p_2) \tag{12}$$

It is easily seen that firms' demands are continuous, decreasing and concave functions of firms' own prices. Firms' best reply functions are then given by[7]:

$$p_1(p_2) = \begin{cases} \frac{1}{2}(\theta_{\max} + p_2 + \Delta u) & if \quad 0 \leq p_2 < (\theta_{\max} - 2\theta_{\min} - \Delta u) \\ p_2 + \theta_{\min} + \Delta u & if \quad p_2 \geq (\theta_{\max} - 2\theta_{\min} - \Delta u) \end{cases},$$

[6] This corresponds to a degenerate case of lemma 1. When $\theta_{\max} = -\theta_{\min} = 0$, given that $\underline{m} < \frac{1}{2} < \overline{m}$, the conditions in lemma 1 are only valid for $\Delta u = 0$.

[7] We do not consider price policies leading to the eviction of firm 2, since these would require $p_2 < -\theta_1 - \Delta u$, which is inconsistent with the non-negativity constraint.

in the case of firm 1, and

$$p_2(p_1) = \begin{cases} p_1 - \theta_{\max} - \Delta u & if \quad p_1 \geq 2\theta_{\max} - \theta_{\min} + \Delta u \\ -\frac{1}{2}(\theta_{\min} - p_1 + \Delta u) & if \quad \theta_{\min} + \Delta u < p_1 < 2\theta_{\max} - \theta_{\min} + \Delta u \\ p_1 - \theta_{\min} - \Delta u & if \quad p_1 \leq \theta_{\min} + \Delta u \end{cases},$$

in the case of firm 2.

Thus, in a scenario of perfect information, equilibrium prices with both firms active in the market obtain as:

$$p_1^{PI*} = \frac{1}{3}(2\theta_{\max} - \theta_{\min} + \Delta u),$$
$$p_2^{PI*} = \frac{1}{3}(\theta_{\max} - 2\theta_{\min} - \Delta u),$$

which occurs as long as:

$$\theta_{\max} - 2\theta_{\min} \geq \Delta u.$$

PROPOSITION 3. *Under perfect information, when the conditions of lemma 1 (horizontal dominance) hold, both firms are active at equilibrium. Furthermore, equilibrium prices coincide with those obtained for imperfect information when the domain of beliefs is degenerate and reduces to the singleton* $m = 1$.

Accordingly, the introduction of a "perfect label" leads to an increase in the price of the high quality good and a concomitant decrease of the price of the low-quality one.[8]

PROPOSITION 4. *Under perfect information, when the conditions of lemma 2 (vertical dominance) hold, both firms are active at equilibrium if and only if:*

$$\theta_{\max} - 2\theta_{\min} \geq \Delta u.$$

Otherwise only the firm selling the high-quality good is active in the market.

Indeed, in the case of vertical dominance[9] as stated in lemma 2, an interior solution can only arise if consumers' preferences with respect to the horizontal characteristic are biased towards good 2, i.e. $\theta_{\max} - 2\theta_{\min} > \Delta u$. If this condition is not satisfied, the equilibrium price of good 2 would be negative. Therefore, there is only space for the high-quality good in this industry. At equilibrium, the monopolist (firm 1) would charge a price p_1^{*PI} equal to

$$p_1^{*PI} = \theta_{\min} + \Delta u,$$

which prevents entry of firm 2 even when $p_2^{*PI} = 0$ (limit price).

Accordingly, there are situations in which the low quality firm can survive only due to consumers' imperfect information. Informing consumers would lead to the exclusion of the low quality firm, thus entailing a more concentrated equilibrium

[8] See Bonroy and Constantatos (2008) on the effects of perfect labels on equilibrium.

[9] In the case of perfect information ($\underline{m} = \overline{m} = 1$), the vertical dominance as stated in lemma 2 establishes that vertical differentiation must be sufficiently strong: $\Delta u > \frac{\theta_{\max} - 3\theta_{\min}}{2}$.

market structure. In spite of this, the loss of profits by the low quality firm is more than compensated by the welfare improvement for the other agents.

Another benchmark to compare our solutions would consist in comparing the equilibria obtained in our analysis with the one corresponding to a fully deterministic vertical differentiation model with variants' qualities defined by

$$\begin{aligned} u_1 &= \frac{\underline{m}+\overline{m}}{2}u_h + \left(1 - \frac{\underline{m}+\overline{m}}{2}\right)u_l \\ u_2 &= \frac{\underline{m}+\overline{m}}{2}u_l + \left(1 - \frac{\underline{m}+\overline{m}}{2}\right)u_h. \end{aligned}$$

This corresponds to replace the uncertainty bearing on the identity of the firms by the certainty equivalent obtained by the average consumer, say consumer $\hat{m}$, whose beliefs coincide with the average belief[10] $\hat{m} = \frac{\underline{m}+\overline{m}}{2}$.

Notice that, by focusing on consumer $\hat{m}$, we neutralize consumers' dispersion of beliefs since all consumers are now assumed to have the same beliefs as consumer $\hat{m}$. Consequently, as in the case of perfect information, we end up with a pure horizontal differentiation model, with consumers' heterogeneity bearing exclusively on the interval $[\theta_{\min}, \theta_{\max}]$. In this case, when both firms are active at equilibrium, prices are equal to:

$$\begin{aligned} p_1^{CE*} &= \tfrac{1}{3}\left(2\theta_{\max} - \theta_{\min} + (2\hat{m}-1)\Delta u\right), \\ p_2^{CE*} &= \tfrac{1}{3}\left(\theta_{\max} - 2\theta_{\min} - (2\hat{m}-1)(\Delta u)\right), \end{aligned}$$

which coincide with the prices obtained under horizontal dominance with beliefs uniformly distributed on the interval $[\underline{m}, \overline{m}]$, i.e., under imperfect information with horizontal dominance (see (8)).

5. Conclusion

In this paper, we have considered price competition when variants are defined along two dimensions (horizontal and vertical) and consumers are uncertain about which firm sells which quality.

Despite the complexity of this problem, we have succeeded in identifying sufficient conditions for existence and uniqueness. These conditions correspond to a wide domain of the parameters, bearing on consumers' heterogeneity both with respect to their preferences for variety and with respect to their beliefs. This domain comprises two distinct sub-domains, corresponding, respectively, to horizontal and vertical dominance. The first (resp. second) domain is relevant when the quality gap is sufficiently small (resp. large) in comparison with the dispersion of consumers' preferences for variety.

Under these conditions, we were able to explicitly derive the equilibrium values for prices, showing that these equilibrium prices differ under vertical versus horizontal dominance. In both cases, equilibrium prices consist of two additive terms, which

[10] In particular, when $\hat{m} = 1$, the case of certainty equivalent coincides with the case of perfect information when good 1 is the high-quality good.

balance consumers' relative valuation of goods' expected quality and consumers' preferences for variety. The relative importance of each terms will be different depending on whether we have horizontal or vertical dominance.

We have compared our results with a perfect information setting. We concluded that perfect information, by eliminating the dispersion of consumers' beliefs engenders a pure horizontally differentiated model. Furthermore, when the quality differential is sufficiently large and consumers are perfectly informed about it, we have shown that informing consumers would lead to the exclusion of the low quality firm (which would survive in a setting of imperfect information).

Finally, we have considered the certainty equivalent outcome, in which the dispersion of consumers' beliefs is eliminated and replaced by the beliefs of the average consumer. In this benchmark case, equilibrium prices coincide with those obtained under horizontal dominance in a setting of imperfect information. Moreover, when the average consumer is perfectly informed, the equilibrium prices corresponding to the certainty equivalent outcome coincide with those obtained under perfect information.

Appendix

Appendix A - Demand regions

A.1) Demand regions under horizontal dominance

Let us denote $u_h - u_l$ by Δu.

Under horizontal dominance, we have defined by R_i^H the set $R_i^H = \{(p_1, p_2) : (p_1, p_2)$ *leads to a market share corresponding to case* $i, i = 1, ..., 5\}$. Observing figure 1, it becomes evident that R_I^H is observed whenever $\theta(\overline{m}) > \theta_{\max}$, or equivalently:

$$R_1^H = \{(p_1, p_2) : p_1 - p_2 \geq \theta_{\max} + (2\overline{m} - 1)\Delta u\}.$$

From figure 1, it follows as well that R_2^H is observed whenever: $\theta(\underline{m}) > \theta_{\max}$ and, simultaneously $\theta_{\min} < \theta(\overline{m}) < \theta_{\max}$. Under horizontal dominance (condition (5) holds), these conditions imply:

$$R_2^H = \{(p_1, p_2) : (2\underline{m} - 1)\Delta u + \theta_{\max} \leq p_1 - p_2 < \theta_{\max} + (2\overline{m} - 1)\Delta u\}.$$

Similarly, R_3^H is observed when $\theta(\overline{m}) > \theta_{\min}$ and $\theta(\underline{m}) < \theta_{\max}$, which, under horizontal dominance are equivalent to:

$$R_3^H = \{(p_1, p_2) : \theta_{\min} + (2\overline{m} - 1)\Delta u \leq p_1 - p_2 < \theta_{\max} + (2\underline{m} - 1)\Delta u\}.$$

R_4^H is observed when $\theta_{\min} < \theta(\underline{m}) < \theta_{\max}$ and, simultaneously, $\theta(\overline{m}) < \theta_{\min}$. This is equivalent to

$$R_4^H = \{(p_1, p_2) : \theta_{\min} + (2\underline{m} - 1)\Delta u \leq p_1 - p_2 < \theta_{\min} + (2\overline{m} - 1)\Delta u\}.$$

Finally, under R_5^H firm 1 is a monopolist, which is observed whenever $\theta(\underline{m}) < \theta_{\min}$, or equivalently:

$$R_5^H = \{(p_1, p_2) : p_1 - p_2 < \theta_{\min} + (2\underline{m} - 1)\Delta u\}.$$

A.2) Demand regions under vertical dominance

Under vertical dominance, the same comments apply, *mutatis mutandis* and, in this case, demand regions are given by:

$$\begin{aligned}
R_1^V &= \{(p_1, p_2) : p_1 - p_2 \geq \theta_{\max} + (2\overline{m} - 1)\Delta u\}, \\
R_2^V &= \{(p_1, p_2) : \theta_{\min} + (2\overline{m} - 1)\Delta u \leq p_1 - p_2 < \theta_{\max} + (2\overline{m} - 1)\Delta u\}, \\
R_3^V &= \{(p_1, p_2) : \theta_{\max} + (2\underline{m} - 1)\Delta u \leq p_1 - p_2 < \theta_{\min} + (2\overline{m} - 1)\Delta u\}, \\
R_4^V &= \{(p_1, p_2) : \theta_{\min} + (2\underline{m} - 1)\Delta u \leq p_1 - p_2 < \theta_{\max} + (2\underline{m} - 1)\Delta u\}, \\
R_5^V &= \{(p_1, p_2) : p_1 - p_2 < \theta_{\min} + (2\underline{m} - 1)\Delta u\}.
\end{aligned}$$

Appendix B - Proofs

B.1) Proof of Lemma 1

In $(p_1, p_2) \in R_2^H$, firm $1's$ profits are given by a third-degree polynomial:

$$\pi_1 (p_1, p_2)_{\rceil R_2^H} = \left(\frac{1}{4} \frac{(\theta_{\max} - p_1 + p_2 + (2\overline{m} - 1)(\Delta u))^2}{(\overline{m} - \underline{m})(\theta_{\max} - \theta_{\min})(\Delta u)} \right) p_1,$$

with $\lim_{p_1 \to \infty} \left(\left(\frac{1}{4} \frac{(\theta_{\max} - p_1 + p_2 + (2\overline{m} - 1)(\Delta u))^2}{(\overline{m} - \underline{m})(\theta_{\max} - \theta_{\min})(\Delta u)} \right) p_1 \right) = +\infty.$

The extremes of $\pi_1 (p_1, p_2)_{\rceil R_2^H}$ are obtained as $\frac{\partial \left(\pi_1 (p_1, p_2)_{\rceil R_2^H} \right)}{\partial p_1} = 0$, yielding:

$$\begin{aligned} \hat{p}_1 (p_2) &= \theta_{\max} + p_2 + (2\overline{m} - 1)(\Delta u) & (13) \\ \check{p}_1 (p_2) &= \frac{1}{3} (\theta_{\max} + p_2 + (2\overline{m} - 1)(\Delta u)) & (14) \end{aligned}$$

Notice that, for a given p_2, the price level $\hat{p}_1 (p_2)$ corresponds to the switching price between R_2^H and R_I^H (where firm 1 is evicted from the market, obtaining nil profits). Accordingly, firm 1 will never have any incentives to make an unilateral deviation towards $\hat{p}_1 (p_2)$.

Furthermore, it is easy to see that:

$$\begin{aligned} \frac{\partial \pi_1 (p_1, p_2)_{\rceil R_2^H}}{\partial p_1} \rceil^{+}_{\check{p}_1 (p_2)} &< 0 & (15) \\ \frac{\partial \pi_1 (p_1, p_2)_{\rceil R_2^H}}{\partial p_1} \rceil^{-}_{\check{p}_1 (p_2)} &> 0. & (16) \end{aligned}$$

Accordingly, if $\check{p}_1 (p_2)$ occurs for $(p_1, p_2) : \check{p}_1 (p_2) - p_2 < (2\underline{m} - 1)(\Delta u) + \theta_{\max}$, we observe that $\check{p}_1 (p_2)$ would be outside (at the left) of R_2^H . Thus, considering (13)-(16), for any $(p_1, p_2) \in R_2^H$, we observe that $\frac{\partial \pi_1 (p_1, p_2)_{\rceil R_2^H}}{\partial p_1} < 0$, preventing any incentives for advantageous unilateral deviations by firm 1. To end the proof, plug (14) in the condition

$$\check{p}_1 (p_2) - p_2 < (2\underline{m} - 1)(\Delta u) + \theta_{\max}, \tag{17}$$

obtaining:

$$p_2 > -(\theta_{\max} + (3\underline{m} - \overline{m} - 1)(\Delta u)) \tag{18}$$

Given that prices must be non-negative, a sufficient condition to guarantee that (18) holds is:

$$\begin{gathered} -(\theta_{\max} + (3\underline{m} - \overline{m} - 1)(\Delta u)) \leq 0 \Leftrightarrow \\ \Leftrightarrow \theta_{\max} \geq (1 + \overline{m} - 3\underline{m})(\Delta u), \end{gathered}$$

which corresponds to condition (6) in Lemma 1.

Concerning the second condition in lemma 2, one must analyze profits of firm 2 under the case of horizontal dominance. In $(p_1, p_2) \in R_4^H$, firm $2's$ profits are given by a third-degree polynomial:

$$\pi_2(p_1,p_2)_{\rceil R_4^H} = \left(\frac{1}{4} \frac{(\theta_{\min} - p_1 + p_2 - (1 - 2\underline{m})(\Delta u))^2}{(\overline{m} - \underline{m})(\theta_{\max} - \theta_{\min})(\Delta u)} \right) p_2,$$

with $\lim_{p_2 \to \infty} \left(\left(\frac{1}{4} \frac{(\theta_{\min} - p_1 + p_2 - (1 - 2\underline{m})(\Delta u))^2}{(\overline{m} - \underline{m})(\theta_{\max} - \theta_{\min})(\Delta u)} \right) p_2 \right) = +\infty$.

The extremes of $\pi_2(p_1,p_2)_{\rceil R_4^H}$ are obtained as $\frac{\partial\left(\pi_2(p_1,p_2)_{\rceil R_4^H}\right)}{\partial p_2} = 0$, yielding:

$$\begin{aligned} \hat{p}_2(p_1) &= -\theta_{\min} + p_1 - (2\underline{m} - 1)(\Delta u) & (19) \\ \check{p}_2(p_1) &= \frac{1}{3}(-\theta_{\min} + p_1 - (2\underline{m} - 1)(\Delta u)) & (20) \end{aligned}$$

Notice that, for a given p_1, the price level $\hat{p}_2(p_1)$ corresponds to the switching price between R_4^H and R_5^H (where firm 2 is evicted from the market, obtaining nil profits). Accordingly, firm 2 will never have incentives to make an unilateral deviation towards $\hat{p}_2(p_1)$.

Furthermore, it is easy to see that

$$\begin{aligned} \frac{\partial \pi_2(p_1,p_2)_{\rceil R_4^H}}{\partial p_2} \rceil_{\check{p}_2(p_1)}^{-} &> 0 & (21) \\ \frac{\partial \pi_2(p_1,p_2)_{\rceil R_4^H}}{\partial p_2} \rceil_{\check{p}_2(p1)}^{+} &< 0. & (22) \end{aligned}$$

Accordingly, if $\check{p}_2(p_1)$ occurs for $(p_1,p_2) : p_1 - \check{p}_2(p_1) > \theta_{\min} + (2\overline{m} - 1)(\Delta u)$, we observe that $\check{p}_2(p_1)$ would be outside (at the left) of R_4^H . Thus, considering (21)-(22), for any $(p_1, p_2) \in R_4^H$, we observe that $\frac{\partial \pi_2(p_1,p_2)_{\rceil R_4^H}}{\partial p_2} < 0$, preventing any incentives for advantageous unilateral deviations by firm 1. To end the proof, plug (20) in the condition

$$p_1 - \check{p}_2(p_1) > \theta_{\min} + (2\overline{m} - 1)(\Delta u), \tag{23}$$

obtaining

$$p_1 > \theta_{\min} - (\underline{m} - 3\overline{m} + 1)(\Delta u) \tag{24}$$

Given that prices must be non-negative, a sufficient condition to guarantee that the previous condition holds is:

$$\begin{aligned} &\theta_{\min} - (\underline{m} - 3\overline{m} + 1)(\Delta u) \leq 0 \Leftrightarrow \\ &\Leftrightarrow -\theta_{\min} \geq (3\overline{m} - 1 - \underline{m})(\Delta u), \end{aligned}$$

which corresponds to condition (7) in Lemma 1.■

B.2) Proof of Lemma 2

Notice that all the comments concerning the profits of firm 1 up to expression (16) in the proof of lemma 1 also hold in the case of vertical dominance. However, considering the price domains, in the case of vertical dominance, condition (17) becomes

$$\check{p}_1(p_2) - p_2 < \theta_{\min} + (2\overline{m} - 1)(\Delta u),$$

which is equivalent to:

$$p_2 > \frac{1}{2}(\theta_{\max} - 3\theta_{\min} - 2(2\overline{m} - 1)(\Delta u)).$$

Given non-negativity of prices a sufficient condition to guarantee that the above inequality holds for all feasible p_2 is:

$$\frac{1}{2}(\theta_{\max} - 3\theta_{\min} - 2(2\overline{m} - 1)(\Delta u)) \leq 0 \Leftrightarrow$$
$$\Leftrightarrow \Delta u \geq \frac{\theta_{\max} - 3\theta_{\min}}{2(2\overline{m} - 1)},$$

which corresponds to the first condition in lemma 2.

Similarly, all the comments concerning the profits of firm 2 until expression (22) also hold in the case of vertical dominance. However, considering the price domains, in the case of vertical dominance, condition (23) becomes

$$p_1 - \check{p}_2(p_1) > \theta_{\max} + (2\underline{m} - 1)(\Delta u),$$

which is equivalent to:

$$p_1 > \frac{1}{2}(3\theta_{\max} - \theta_{\min} + 2(2\underline{m} - 1)(\Delta u)).$$

Given non-negativity of prices a sufficient condition to guarantee that the above inequality holds for all feasible p_2 is:

$$\frac{1}{2}(3\theta_{\max} - \theta_{\min} + 2(2\underline{m} - 1)(\Delta u)) \leq 0 \Leftrightarrow$$
$$\Leftrightarrow \Delta u \geq \frac{3\theta_{\max} - \theta_{\min}}{2(1 - 2\underline{m})},$$

which corresponds to the second condition in lemma 2.■

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