Remittances and inequality : A dynamic migration model

F. Docquier, H. Rapoport and I-Ling Shen

Discussion Paper 2007-3

Département des Sciences Économiques
de l'Université catholique de Louvain

UCL

Remittances and inequality: A dynamic migration model*

Frédéric Docquier[†], Hillel Rapoport[‡] and I-Ling Shen[§]

Revised Version
December 15, 2006

Abstract

We develop a model to study the effects of migration and remittances on inequality in the origin communities. While wealth inequality is shown to be monotonically reduced along the time-span, the short- and the long-run impacts on income inequality may be of opposite signs, suggesting that the dynamic relationship between migration/remittances and inequality may well be characterized by an inverse U-shaped pattern. This is consistent with the findings of the empirical literature, yet offers a different interpretation from the usually assumed migration network effects. With no need to endogenize migration costs through the role of migration networks, we generate the same result via intergenerational wealth accumulation.

JEL Classification: O11, O15, J61, D31.
Keywords: Migration, remittances, inequality.

*We thank an anonymous referee for this journal and participants at the 2003 Meeting of the European Economic Association in Venice, the XVIII Villa Mondragone International Economic Seminar, the NAKE Research Day 2006 in Amsterdam, and various seminar audiences.

[†]Fonds National de la Recherche Scientifique, IRES, Université catholique de Louvain, World Bank and IZA Bonn, Germany.

[‡]Department of Economics, Bar-Ilan University, EQUIPPE-CADRE, University of Lille 2, and CReAM, University College London.

[§]IRES, Université catholique de Louvain. Corresponding author: Place Montesquieu 3, B-1348 Louvain-la-Neuve, Belgium. Email: shen@ires.ucl.ac.be

1 Introduction

Does international migration increase or decrease economic inequality in developing (sending) countries? What are the possible forces that may decide whether there exists a positive or a negative relationship between the two? These are important issues because inequality is an outcome of interest in its own right and because the distribution of income conditions the extent to which liquidity constraints impinge on investment in physical and human capital.[1] Consequently, the growth-enhancing potential of international migration largely depends on its distributional impact. A series of recent studies covering a large sample of developing countries have demonstrated the growth potential of migration in a context of capital market imperfections, with remittances and savings accumulated abroad relaxing credit constraints on farm investment (Lucas, 1987, Rozelle et al., 1999), education (Hanson and Woodruff, 2003, Cox Edwards and Ureta, 2003) and investment in micro-enterprises (e.g., Mesnard and Ravallion, 2001, Dustmann and Kirchkamp, 2002, Woodruff and Zenteno, 2006). However, there are also studies pointing to more negative effects of remittances and migration on investment, be it because remittances are mainly consumed (Rempel and Lobdell, 1978), generate moral hazard problems leading to lower effort or labor force participation (Azam and Gubert, 2005) or because migration may in some circumstances depress educational attainments of children (McKenzie and Rapoport, 2006a).[2]

The empirical literature on the migration-inequality relationship does not offer decisive conclusions as to whether international migration in general, and migrants' remittances in particular, increase or decrease economic inequality at origin. This lack of consensus may be attributed to the diversity of the environments studied as well as to differences in the empirical approaches adopted: static versus dynamic models, with and without endogenous migration costs,

[1] See the pionneeing contributions of Banerjee and Newman, 1993, Galor and Zeira, 1993, Aghion and Bolton, 1997; see also Bardhan, Bowles and Gintis (1999) for a survey including additional channels.

[2] See Rapoport and Docquier (2006) for a survey of this literature.

and different conceptions about whether remittances must be treated as a substitute for domestic earnings (in which case the effect of migration on domestic income sources must also be taken into account). There is however a general sense in the literature that the impact of migration and remittances on economic inequality is likely to vary over time and display an inverse U-shaped pattern. The underlying assumption is that international migration may be viewed as a diffusion process with decreasing information costs thanks to the role of migration networks: since migration costs are initially high in communities lacking migration experience, only households at the middle of the income and wealth distribution have both the incentives and the means to send members abroad; as these relatively affluent households benefit from additional remittance income, inequality at first increases; however, early migrants may supply information and assistance to future migrants from their hometown, thus making migration affordable to households at the lower end of the distribution and allowing for a decrease in inequality.

In this paper, we propose a dynamic theoretical framework that goes part of the way towards reconciling the conflicting results from empirical studies and complements the "networks" view in showing that the same predictions may be obtained with exogenous (i.e., constant) migration costs. While previous literature has examined the relationship between migration and inequality in static, partial equilibrium frameworks, we investigate the impact of migration on income and inequality both via the direct effect of migrants' households increasing their income via higher wages abroad and also the indirect effects of the outbound flow of individuals on the local labor market, and do so in a way that demonstrates the importance of the pre-migration distribution of wealth in determining the impact of migration on the dynamic path and long run levels of income and wealth inequality.

The rest of the paper is organized as follows. In Section 2, we review the related literature on migration, remittances and inequality. In Section 3 we build a model with two classes of agents characterized by different non-liquifiable capital endowments, which result in different levels of productivity. In the rural regions, these endowments generally take the form of a plot of land, the quality and the quantity of which determine a household's agricultural productivity, as well as its income potential and migration incentives. Familial wealth is accumulated over time; it is saved from one generation's disposable income plus wealth and transmitted to the next generation. Obviously, migration incentives are stronger for poor households, but rich households are less constrained; as a result, the exact composition of migration flows in terms of social origin is a priori unclear.[3] In Sections 4 and 5, we first characterize different regimes of high, higher-medium, lower-medium, and low initial inequalities that condition the dynamics of migration and inequality in the migrants' origin communities. Then, we investigate how migration and remittances affect the evolution of wealth and income inequalities both at the beginning of transition and at the steady state. We show that migration and remittances always lower wealth inequality. In contrast, although income inequality is also reduced in the long run, it may either increase or decrease in the short run, depending on the initial distribution of endowments. That is to say, the short- and the long-run effects on the income distribution may be of opposite signs and display an inverse U-shaped relationship. In Section 6, we extend the model by introducing a local labor market into our agricultural economy. We still observe an inverse U-shaped relationship, but only in an originally impoverished economy when both types of households have very low levels of productivity. Moreover, with the presence of this inverse U-shaped curve, we find that migration and remittances are likely to worsen the income inequality for good. Finally, Section 7 presents the conclusions.

[3]Migrations decisions may also be affected by the level of information on foreign opportunities, which may be related to skills and income, or by incentive compatibility constraints (e.g., wealthy households have a stronger enforcement power to secure remittance through inheritance - see for example Hoddinott, 1994). These aspects are not dealt with in this paper.

2 Related Literature

As mentioned in the previous section, the results from empirical studies on the migration-inequality relationship are mixed. For example, Adams (1989) found that international migration tends to worsen economic inequality in rural Egypt, while the same author found a neutral effect in rural Pakistan (Adams, 1992). In the case of rural Mexico, Taylor and Wyatt (1996) showed that remittances are distributed almost evenly across income groups, hence inducing a direct equalizing effect in terms of economic inequality. In addition, they also showed that remittances have the highest shadow value for households at the middle-to-low-end of the income distribution; for such households indeed, remittances ease access to productive assets (land) and/or complementary inputs; a second equalizing effect is thereby obtained. This suggests that the impact of remittances on rural development depends not only on the initial distribution of wealth in the origin community, but also on a host of factors affecting their shadow value (e.g., degree of liquidity of land rights, costs of complementary inputs, availability of local labor, etc.). In their study of remittances to a small coastal city of Nicaragua, Barham and Boucher (1998) also find that remittances decrease income inequality, but only when domestic income sources are treated as exogenous; after constructing different no-migration counterfactuals to control for self-selection into migration and local labor-force participation, they show instead that remittances increase income inequality; this is explained by the fact that the potential home earnings of erstwhile migrants have a more equalizing effect on income distribution than remittances.

The impact of remittances on economic inequality, however, needs not be monotonic. Stark, Taylor and Yitzhaki (1986) suggested that the dynamics of remittances and inequality may be represented by an inverse U-shaped relationship along the lines described above. Their analysis was based on the decomposition of a Gini index of household income by income sources, taking into account the correlations between different income components. The method was applied to household data from two Mexican villages, one with a relatively recent Mexico-to-US migration experience, and one with a longer migration history. The distributional impact of remittances was shown to depend on the village's migration history, which implicitly captures the magnitude of migration costs.[4] They showed that income dispersion was decreased in both villages once migrants' remittances were taken into account, but more so in the village characterized by a longer migration tradition. With a similar approach applied to Yugoslavia, Milanovic (1987) also tested for the possibility of such a trickle-down effect. Using data from the 1973, 1978 and 1983 Yugoslavian household surveys, Milanovic found no empirical support for this hypothesis: remittances were shown to raise income inequality throughout the period, and more so for agricultural households. Taylor's (1992) longitudinal study of a Mexican village also shows that remittances may well have an inequality-enhancing effect in the short-run and yet contribute to decrease income inequality in the long-run as poor rural households gradually transform remittance income into productive assets.

Finally, McKenzie and Rapoport (2006b) examine the overall impact of migration on inequality in a large number of Mexican rural communities. This impact is composed of the direct and the indirect (e.g., multiplier) effects of remittances, and other potential spillover and general equilibrium effects of migration. They confirm that Mexican immigrants to the United States come from the middle of the asset wealth distribution, with the migration probability displaying an inverse-U shaped relationship with wealth. The presence of migration networks, both at the family and at the community level, is found to increase the likelihood of migration, which accords with their ability to raise the expected benefits and lower the costs of migration, and to generate a Kuznets-

[4]Treating migration costs as exogenous may be adapted to situations where they mainly include transportation and border crossing expenditures, but is clearly unsatisfactory when information costs (e.g., search process for a destination, and a job at destination) are substantial; in this case, it is well known that migration costs tend to decrease as the size of the relevant network at destination increases. Such network effects have first been recognized in the sociological literature (e.g, Massey et al., 1994) and, more recently, in the economic literature (Carrington et al., 1996, Munshi, 2003).

type relationship between migration and inequality. Indeed, migration appears to increase inequality at low levels of community migration prevalence and then to reduce inequality at high levels of migration prevalence.

3 The Benchmark Model

In this section, we lay out the basic framework of our model in the closed economy, that is, without access to migration. We consider a rural economy, where each household is engaged in self-sustaining agricultural production (i.e., no exchange of labor or goods). There are two classes of households: low-productivity (LP) and high-productivity (HP), whose difference originates from the quantity and quality of their inherited and non-liquifiable familial land. These characteristics are captured by a technological parameter α, which equals to $\overline{\alpha}$ for HP households and to $\underline{\alpha}$ for LP households, with $\overline{\alpha} > \underline{\alpha} > 1$. Despite its productivity level, every household consists of the same given number of one-period-lived agents. Without loss of generality, this number, or the size of each household, is normalized to unity.

Next, we assume a quadratic production function for each family.[5] We write:

$$q_t = \alpha \cdot l_t - \frac{l_t^2}{2}$$

where $l_t \in [0, 1]$ is the amount of household labor used on own farm. The negative term captures the decreasing marginal productivity of labor. For mathematical convenience, it is assumed that the scale parameter α only affects the linear term.

Since we are primarily interested in the characterization of inter-household inequality and not in the intra-household distribution of income, we assume that the familial income is equally shared between the members of a given

[5]With a quadratic function, the marginal productivity of labor is bounded from above. This avoids unrealistic solutions where a very small proportion of household members stays in the familial farm with a very high marginal productivity.

household. With homogeneous agents, each household maximizes its utility according to the following Cobb-Douglas function:

$$
\begin{aligned}
u_t &= (x_t - x_m)^{1-\sigma} b_{t+1}^{\sigma} \\
s.t. \quad x_t + b_{t+1} &= y_t + b_t
\end{aligned}
$$

where x_t is the level of consumption, x_m is the subsistence level (it is assumed that $x_m < \underline{\alpha} - \frac{1}{2}$), b_t is the wealth inherited from the previous generation and b_{t+1} is the bequest left to the next generation. $\sigma \in]0,1[$ is the parameter indicating preferences between consumption and bequests. Lastly, y_t is the amount of household income. The usual utility maximization leads to

$$
\begin{aligned}
x_t &= (1-\sigma)(y_t + b_t) + \sigma x_m \\
b_{t+1} &= \sigma(y_t + b_t - x_m)
\end{aligned}
$$

In a closed economy, there is no access to migration so that the sole source of familial income is the household agricultural production: $y_t = q_t$.[6] Given our utility function, the maximization of utility is equivalent to maximizing income. Since we assume that productivity levels of both types are greater than 1, we obtain $l_t^* = 1$ for both types of household (i.e., everybody works), and

$$
y_t = \alpha - \frac{1}{2}; \;\; b_{t+1} = \sigma(\alpha - \frac{1}{2} - x_m + b_t)
$$

From our assumptions that (i) $\sigma \in]0,1[$ and (ii) $b_{t+1}(0) > 0$, we know there exists a unique and stable steady state for the linear function $b_{t+1}(b_t)$. At the steady state,

$$
y_{ss} = \alpha - \frac{1}{2}; \;\; b_{ss} = \frac{\sigma}{1-\sigma}(\alpha - \frac{1}{2} - x_m) \tag{1}
$$

[6] In section 6, we will introduce a local labor market, but for the time being, it is assumed that there is no exchange amongst households.

4 Open Economy: with Access to Migration

Let us now assume that after this rural society evolves into its steady state in the closed economy, for some exogenous and unexpected reason, there is a migration possibility to a high-wage foreign destination from period $t = 1$.[7] The foreign wage per migrant, w^*, is given (i.e., the home country or region is small enough to keep wages at destination unaffected by migration).[8] Meanwhile, each migrant incurs a fixed and positive amount of migration costs denoted by c. Due to the absence of credit markets, migration costs must be financed at the beginning of each period with the family's accumulated wealth.

The familial motivation for sending out migrants is to increase total family income, and thus household utility. Migration by some members is an implicit familial arrangement involving: (i) collective financing of migration costs, and (ii) remittances from the migrants to the remaining household members. Note that we rule out the possibility for remittances to be invested in land or in any other productive asset. This has two justifications. First, as detailed above, the empirical literature on the impact of remittances on investment is mixed, with some studies showing that remittances can alleviate liquidity constraints that impede investment in different forms of capital and other studies emphasizing instead their depressing effect on investment. Second and maybe more importantly, migration is assumed to be the only possibility of investment for domestic households. However, the economic impact of the investment in "migrants" is by no means different from the impact of investment in other productive assets. In our dynamic framework, previous migration flows allow households to accumulate wealth, which further increases the number of migrants they can afford. Introducing an alternative and substitutable form of investment would generate similar qualitative predictions on the dynamics of inequality, both in the short-run and in the long-run.

[7]In fact, we can open the frontier at any period without altering our inequality analysis. Our assumption is made to facilitate the proofs of the propositions presented in section 5.

[8]The foreign wage is the same for all migrants because we assume that they are homogeneous agents. The difference in agricultural productivity between the HP and the LP household members comes from their inherited land and has nothing to do with agent-specific characters.

With these understandings, let $\overline{m}_t$ be the number of migrants in HP households and $\underline{m}_t$ for that in LP households. Each household is therefore subjected to a liquidity constraint as follows:

$$c \cdot \overline{m}_t \leq \bar{b}_t \quad \text{and} \quad c \cdot \underline{m}_t \leq \underline{b}_t, \qquad \forall\, t \geq 1$$

4.1 LP Household's Migration Decision

We assume $w^* - c > \underline{\alpha}$, or the net-of-migration-cost foreign wage is greater than the highest level of marginal productivity for the LP household. Hence, if there is no liquidity constraint, the LP household will choose to have all its family members migrating to earn foreign wages, ($\underline{m}_t^{nc} = 1$). In order to avoid such unrealistic situations where a certain type of household fully disappears from the local economy, we impose the following condition:[9]

$$\underline{b}_t < c, \quad \forall\, t \geq 1$$

which implies that, due to a binding liquidity constraint, there are always some members of the LP household who stay behind for the agricultural production. Therefore, their effective number of migrants is

$$\underline{m}_t^* = \frac{\underline{b}_t}{c} \in]0, 1[$$

The income of LP households now comes from agricultural production as well as from received foreign wage:

$$\begin{aligned} \underline{y}_t &= \underline{\alpha}(1 - \underline{m}_t^*) - \frac{(1 - \underline{m}_t^*)^2}{2} + \underline{m}_t^*(w^* - c) \\ &= \underline{\alpha} - \frac{1}{2} + \frac{\underline{b}_t}{c}(w^* - \underline{\alpha} - c + 1 - \frac{\underline{b}_t}{2c}) \end{aligned} \tag{2}$$

[9] Although based on our assumption of one-period-living agent, there seems to be no need to distinguish between permanant and temporary migration, our model still better portrays the picture of temporary migration, such as guest worker programs, where migration usually involves only part of a household and the migrated members send back home considerable amount of their earned wages.

Based on our assumptions, we find that $\underline{y}_t$ is strictly increasing in $\underline{b}_t$. This is an intuitive result which says that, when a LP household is less bound by the liquidity constraint, more family members will be able to work abroad, which generates a higher net reward than working on the familial farm.

Due to the change in sources of income, the decision rule of bequest is no longer linear in the bequeathed wealth, as shown in Section 3. Instead, it becomes a quadratic function:

$$\underline{b}_{t+1} = \sigma \left[(\underline{\alpha} - \frac{1}{2} - x_m) + \frac{\underline{b}_t}{c}(w^* - \underline{\alpha} + 1 - \frac{\underline{b}_t}{2c}) \right] \quad (3)$$

Our assumptions indicate that (i) the function $\underline{b}_{t+1}(\underline{b}_t)$ is strictly increasing and strictly concave, and (ii) $\underline{b}_{t+1}(0)$ is positive. Thereby, there exists a unique positive stable steady state. With proper restriction on the parameters, this steady state satisfies the liquidity constraint $\underline{b}^m_{ss} < c$.

Lemma 1 *If the intergenerational altruistic parameter σ is sufficiently small such that $\sigma < \frac{c}{w^* - x_m}$, then for equation (3) there exists a real-numbered stable steady state $\underline{b}^m_{ss} \in]0, c[$ in the open economy:*

$$\underline{b}^m_{ss} = c \left[1 + (w^* - \underline{\alpha} - \frac{c}{\sigma}) + \sqrt{(w^* - \underline{\alpha} - \frac{c}{\sigma})^2 + 2(w^* - x_m - \frac{c}{\sigma})} \right] \quad (4)$$

Proof. See Appendix A. ■

4.2 HP Household's Migration Decision

With migration possibilities, the HP households face the following maximization problem: $\forall t \geq 1$,

$$\max_{\{\overline{m}_t \geq 0\}} \overline{y}_t = \overline{\alpha}(1-\overline{m}_t) - \frac{(1-\overline{m}_t)^2}{2} + \overline{m}_t(w^* - c)$$

$$s.t. \begin{cases} \overline{m}_t \leq \frac{\overline{b}_t}{c} \\ \overline{m}_t \leq 1 \\ \overline{b}_1 = \frac{\sigma}{1-\sigma}(\overline{\alpha} - \frac{1}{2} - x_m) \end{cases}$$

Using the Kuhn-Tucker formulation and excluding the situation where $\overline{m}_t^* = 1$ (for the same reason as with the LP households), we obtain the migration rates respectively for the following scenarios.

First, when $\overline{\alpha} \geq 1 + w^* - c$, even the lowest level of marginal productivity on family farm ($\overline{\alpha} - 1$) is no less than the net-of-migration-cost foreign wage. Therefore, the HP households do not have any incentive to send abroad their family members: $\overline{m}_t^* = 0$.

Second, when $\overline{\alpha} \leq w^* - c$, the HP households face a similar situation as do the LP households. That is, since the highest level of marginal productivity $\overline{\alpha}$ is no greater than the net-of-migration-cost foreign wage, the HP households have incentives to send out all family members. For the same reason as for the LP households, their liquidity constraint is assumed to be always binding (i.e. $\overline{b}_t < c, \ \forall t \geq 1$). Thus, $\overline{m}_t^* = \frac{\overline{b}_t}{c} \in]0, 1[$.

Lastly, when $w^* - c < \overline{\alpha} < 1 + w^* - c$, the net-of-migration-cost foreign wage lies somewhere between the highest and the lowest levels of marginal productivity for the HP households. Hence, the HP households have an incentive to send out part of their family members, but not all. We find that, without liquidity constraint, the optimal number of migrants is $\overline{m}_t^{nc} = (1 + w^* - c - \overline{\alpha}) \in]0, 1[$. Nevertheless, depending on our choices of parameter values, the HP households could be either liquidity-constrained or not. The following are the three possible settings:

- If $c(1+w^*-c-\overline{\alpha}) \leq \overline{b}_1 \leq \overline{b}_{ss}^m$, the HP households are never liquidity-constrained. Thus, $\overline{m}_t^* = 1+w^*-c-\overline{\alpha}$.

- If $\overline{b}_1 < c(1+w^*-c-\overline{\alpha}) \leq \overline{b}_{ss}^m$, the HP households are liquidity-constrained in the beginning of the open economy, so $\overline{m}_t^* = \frac{\overline{b}_t}{c}$. With the accumulation of family wealth, however, the HP households will be able to overcome the liquidity constraint at a certain period, when $t = \min\left\{t \mid \overline{b}_t \geq c\,(1+w^*-c-\overline{\alpha})\right\}$. From then on, $\overline{m}_t^* = 1+w^*-c-\overline{\alpha}$.

- If $\overline{b}_1 \leq \overline{b}_{ss}^m < c\,(1+w^*-c-\overline{\alpha})$, the liquidity constraint persists, so $\overline{m}_t^* = \frac{\overline{b}_t}{c}$, $\forall t \geq 1$.

In fact, whether a migration decision falls into one of these three settings is decided by the level of $\overline{\alpha}$.

Lemma 2 *For all $\overline{\alpha} \in]w^*-c, 1+w^*-c[$, there are three types of migration decisions depending on $\overline{\alpha}$:*

- $\forall\, \overline{\alpha} \geq \overline{\alpha}_1$, $\overline{m}_t^* = 1+w^*-c-\overline{\alpha}$.

- $\forall\, \overline{\alpha} \in [\overline{\alpha}_0, \overline{\alpha}_1[$, $\overline{m}_t^* = \frac{\overline{b}_t}{c}$ *for all* $t < \min\left\{t \mid \overline{b}_t \geq c\,(1+w^*-c-\overline{\alpha})\right\}$; *otherwise,* $\overline{m}_t^* = 1+w^*-c-\overline{\alpha}$.

- $\forall\, \overline{\alpha} < \overline{\alpha}_0$, $\overline{m}_t^* = \frac{\overline{b}_t}{c}$.

where $\overline{\alpha}_0 = w^*-c+\frac{c(1-\sigma)}{\sigma}\left(\sqrt{1-\frac{2\sigma}{c(1-\sigma)}\left[\frac{\sigma}{c(1-\sigma)}(w^*-c-x_m)-1\right]}-1\right)$ *and* $\overline{\alpha}_1 = \frac{c(1-\sigma)(1+w^*-c)+\sigma(\frac{1}{2}+x_m)}{c(1-\sigma)+\sigma}$.

Proof. See Appendix B. ■

Since migration decisions alter a households' income level, which subsequently changes the dynamics of wealth accumulation, we can now use the above derived optimal rates of migration to categorize different cases of income and wealth dynamics.

(i) High Inequality Case: $\overline{\alpha} \geq 1 + w^* - c$

In this case, $\overline{m}_t^* = 0$, so the income and the wealth levels of the HP households remain at the steady state of the closed economy: $\forall t \geq 1$,

$$\begin{aligned} \overline{y}_t &= \overline{\alpha} - \frac{1}{2} \\ \overline{b}_{t+1} &= \frac{\sigma}{1-\sigma}(\overline{\alpha} - \frac{1}{2} - x_m) \end{aligned}$$

(ii) Higher-Medium Inequality Case: $\overline{\alpha}_1 \leq \overline{\alpha} < 1 + w^* - c$

In this case, $\overline{m}_t^* = 1 + w^* - c - \overline{\alpha}$. Therefore,

$$\begin{aligned} \overline{y}_t &= \frac{(1 + w^* - c - \overline{\alpha})^2}{2} + \overline{\alpha} - \frac{1}{2} \\ \overline{b}_{t+1} &= \sigma \left[\frac{(1 + w^* - c - \overline{\alpha})^2}{2} + \overline{\alpha} - \frac{1}{2} - x_m \right] + \sigma \overline{b}_t \end{aligned}$$

Based on our assumptions that (i) $\sigma \in]0, 1[$ and (ii) $\overline{b}_{t+1}(\overline{b}_t = 0) > 0$, there exists a unique and stable steady state of wealth, which is

$$\overline{b}_{ss}^m = \frac{\sigma}{1-\sigma} \left[\frac{(1 + w^* - c - \overline{\alpha})^2}{2} + \overline{\alpha} - \frac{1}{2} - x_m \right]$$

(iii) Lower-Medium Inequality Case: $\overline{\alpha}_0 \leq \overline{\alpha} < \overline{\alpha}_1$

In the beginning of this case, $\overline{m}_t^* = \frac{\overline{b}_t}{c}$, so both the income and wealth dynamics follow the same patterns as in the low inequality case that will be clarified below. However, as soon as the liquidity constraint is overcome and so $\overline{m}_t^* = 1 + w^* - c - \overline{\alpha}$, the dynamics switch to what have been described in the higher-medium inequality case.

(iv) Low Inequality Case: $\overline{\alpha} < \overline{\alpha}_0$

In this case, $\overline{m}_t^* = \frac{\overline{b}_t}{c}$. Hence,

$$\begin{aligned}
\overline{y}_t &= \overline{\alpha} - \frac{1}{2} + \frac{\overline{b}_t}{c}(w^* - \overline{\alpha} - c + 1 - \frac{\overline{b}_t}{2c}) \\
\overline{b}_{t+1} &= \sigma\left[(\overline{\alpha} - \frac{1}{2} - x_m) + \frac{\overline{b}_t}{c}(w^* - \overline{\alpha} + 1 - \frac{\overline{b}_t}{2c})\right]
\end{aligned}$$

Based on the same reasons as for equation (3), there exists a real-numbered stable steady state of wealth

$$\overline{b}^m_{ss} = c\left[1 + (w^* - \overline{\alpha} - \frac{c}{\sigma}) + \sqrt{(w^* - \overline{\alpha} - \frac{c}{\sigma})^2 + 2(w^* - x_m - \frac{c}{\sigma})}\,\right]$$

and $\overline{b}^m_{ss} \in]0, c[$.

4.3 Remittances

As emphasized above, we focus in this paper on inter-household inequality rather than on intra-household inequality. For this reason, we assume an equal-sharing rule for family income, meaning that remittances sent per migrant are such that they equalize income per-member across migrants and non-migrants in a given household. Hence, the equilibrium amount of remittances is given by the difference between the average income of a household and the domestic income per member in the home region. Since migration costs are equally shared within a household, the amount received by each remaining member can be written as:

$$r_t = m^*_t \cdot \left\{w^* - \frac{1}{1 - m^*_t}\left[\alpha(1 - m^*_t) - \frac{(1 - m^*_t)^2}{2}\right]\right\}$$

This is the product of two terms: the migration rate and the averaged-income gap between migrants and remaining HP household members.

Rewriting the general expression above for each case, we have for the LP households $\underline{r}_t = \frac{\underline{b}_t}{c}(w^* - \underline{\alpha} + \frac{1}{2} - \frac{\underline{b}_t}{2c})$, and for the HP households,[10] in the

[10]In order to have a positive level of remittances in all cases, we need to impose extra conditions on the parameters. However, whether these conditions hold or not does not change our later

(i)

1. *High inequality case:* $\overline{r}_t = 0$

2. *Higher-medium inequality case:* $\overline{r}_t = (1 + w^* - c - \overline{\alpha})(\frac{w^*+c-\overline{\alpha}}{2})$

3. *Lower-medium inequality case:*

$$\begin{aligned} \overline{r}_t = & \quad \frac{\overline{b}_t}{c}(w^* - \overline{\alpha} + \frac{1}{2} - \frac{\overline{b}_t}{2c}), & \forall\, \overline{b}_t < c(1 + w^* - c - \overline{\alpha}) \\ \overline{r}_t = & \quad (1 + w^* - c - \overline{\alpha})(\frac{w^*+c-\overline{\alpha}}{2}), & \forall\, \overline{b}_t \geq c(1 + w^* - c - \overline{\alpha}) \end{aligned}$$

4. *Low inequality case:* $\overline{r}_t = \frac{\overline{b}_t}{c}(w^* - \overline{\alpha} + \frac{1}{2} - \frac{\overline{b}_t}{2c})$

5 The effect of migration and remittances on inter-household inequality

We have seen so far that migration to a high-wage destination enables rural households to raise their average income via (i) a rise in farm productivity, and (ii) remittances from migrated family members. This change in income subsequently alters the inter-household inequalities of income and wealth. Below, we will characterize this alteration in the short and the long runs in each case, but before doing so, let us define the measures that we use to evaluate inter-household inequality:[11]

Definition 1 *Wealth Inequality:* $\Gamma^b_{t+1} = \frac{\overline{b}_{t+1}}{\underline{b}_{t+1}}$

Definition 2 *Income Inequality:* $\Gamma^y_t = \frac{\overline{y}_t}{\underline{y}_t}$

---

analysis on inequality. Thus, even though it only consists of a small set of parameter values (a subset of ($c \leq 1$), or when migration costs are very low), we tolerate this peculiar situation where the remaining household subsidizes the migrated members because migration results in higher average returns on the domestic farm than overseas.

[11]Note that the wealth level at each period is chosen by the previous generation.

From equation (1), we derive the wealth inequality at the steady state of the closed economy:

$$\Gamma_1^b = \frac{\overline{\alpha} - \frac{1}{2} - x_m}{\underline{\alpha} - \frac{1}{2} - x_m};$$

Similarly, we obtain the income inequality at the steady state of the closed economy:

$$\Gamma_0^y = \frac{\overline{\alpha} - \frac{1}{2}}{\underline{\alpha} - \frac{1}{2}};$$

5.1 High Inequality Case

In this case, we can easily conclude that both income and wealth inequalities decrease whether in the short or the long run. This is because the HP households' income and wealth levels remain at the closed-economy steady state while the LP households gain greater income and wealth via remittances from family members working abroad.

Proposition 1 *In the high inequality case, where $\overline{\alpha} \geq 1 + w^* - c$, wealth as well as income inequalities are reduced once migration becomes possible. This result holds true both in the short and the long runs. In fact, inequalities decline along the time-span, i.e. $\forall\, t \geq 1,\ \Gamma_{t+1}^b < \Gamma_1^b$ and $\Gamma_t^y < \Gamma_0^y$.*

Proof. See Appendix C. ■

5.2 Other Cases

In the higher- and lower-medium inequality cases as well as the low inequality one, both types of households benefit from remittances sent back by family members working abroads. Therefore, the effects on inequalities are not as clear-cut as in the high inequality case. However, we find that all the three cases generate the same properties in terms of inequality, despite the fact that the proofs of these properties differ greatly from case to case.

Proposition 2 (Effects on the wealth inequality) *In the higher-medium, the lower-medium, and the low inequality cases, migration and remittances reduce the wealth inequality in both the short and the long runs as in the high inequality case, i.e.* $\Gamma_2^b < \Gamma_1^b$ *and* $\Gamma_{ss}^b < \Gamma_1^b \; \forall \, \overline{\alpha}$.

Proof. See Appendix D. ■

Proposition 3 (Effects on the income inequality) *In the higher-medium, the lower-medium, and the low inequality cases, it is uncertain whether the income inequality rises or reduces in the short run, i.e.* $\Gamma_1^y \gtreqless \Gamma_0^y, \; \forall \, \overline{\alpha} < 1 + w^* - c$. *In the long run, however, the income inequality is reduced as in the high inequality case, i.e.* $\Gamma_{ss}^y < \Gamma_0^y \; \forall \, \overline{\alpha}$.

Proof. See Appendix E. ■

Therefore, in the cases where the income inequality rises in the short run, our model generates the inverse U-shaped pattern between migration/remittances and inequality, the same end result described by the migration network theory. However, we need not endogenize migration costs in order for the disadvantaged households to overcome their liquidity constraints. Instead, they do so simply via intergenerational wealth accumulation. As shown in the proofs for Proposition 3, we find that the closer $\underline{\alpha}$ goes toward the subsistence level of consumption, the more likely we will have this inverse U-shaped relationship.[12] Figure 1 best illustrates the reasons behind.

This figure shows the income and the wealth dynamics in a low-inequality example, with the LP productivity set to a very low level. In the beginning of the open economy, the LP households are severely constrained by their little familial wealth since most of their agricultural production was used to satisfy the subsistence needs. Thus, unlike their wealthier HP counterparts who can already capitalize to a large extent on the access to migration, the LP households are only able to send out few migrants to earn the high foreign wage.

[12] From numerical simulations, we also observe rising income inequality in the short run, when $\overline{\alpha}$ is sufficiently close to $\underline{\alpha}$. That is to say, in a sufficiently "equalized" closed economy, the access to migration could trigger higher income inequality in the short run.

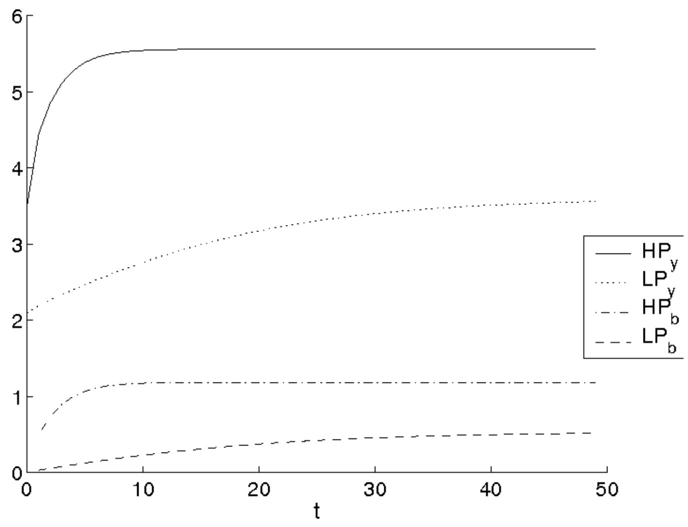

Figure 1: Income and wealth dynamics (low inequality case): from top to bottom, the four curves respectively represent the income dynamics for the HP household, then for the LP household, and the wealth dynamics for the HP household, then for the LP household. ($\overline{\alpha} = 4$, $\underline{\alpha} = 2.6$, $x_m = 2, \sigma 0.25$, $w^* = 7.5$, $c = 1.5$.)

Consequently, the income inequality rises. In more technical terms, the HP households' income quickly converges toward its open-economy steady state whereas the speed of convergence is by far slower for the LP households. This is exactly what causes the bump of income inequality in Figure 2, which corresponds to Figure 1.

Later on, while the increase in income becomes negligible for the HP households, the LP households have accumulated more wealth to relax their liquidity constraint. Hence, they become more and more capable of sending migrants abroad to raise the household income level. Finally, even though wealth accumulation also slows down for the LP households, the catch-up continues due to the difference in their speeds of convergence to the respective steady states. That is why after some point we observe a continuing decline in income inequality.

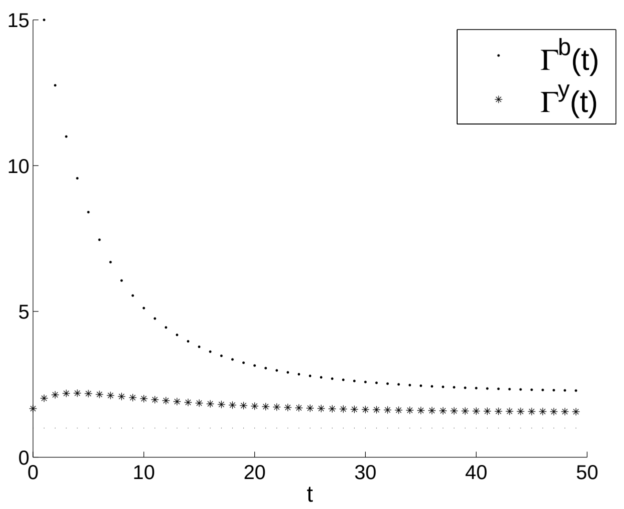

Figure 2: Interhousehold inequalities (low inequality case): the dotted curve represents the dynamics of the wealth inequality while the starred curve depicts that of the income inequality. In the short run, the income inequality may rise when the LP households are severely constrained by little familial wealth.

The same reasoning also applies to the lower- and higher-medium inequality cases for the possible rise in income inequality in the short run. In the lower-medium inequality case, the short-run dynamics is identical to that in the low inequality case, and in the higher-medium inequality case, the HP households' income jumps to the new steady state as soon as the frontier is open whereas the LP households are not yet able to benefit much in the beginning.

Corollary 1 *In an economy without local labor market, migration and remittances are most likely to generate an inverse U-shaped pattern in terms of income inequality when (i) the LP household is sufficiently poor in the closed economy, and (ii) the initial inequality is not too high so that HP households have incentives to send at least some migrants out.*

In contrast, we observe a constant decline of wealth inequality in Figure 2, thanks to migration and remittances. The reason is straightforward. Since the initial wealth of the LP households is only slightly above zero, a small absolute amount of wealth increase is enough to lower the wealth inequality $\Gamma_t^b = \frac{\overline{b}_t}{\underline{b}_t}$.

We could of course also measure the inter-household inequality in terms of utility. Due to our Cobb-Douglas setting, the utility-inequality ratio equals to the wealth-inequality ratio ($\Gamma_t^u = \frac{\overline{u}_t}{\underline{u}_t} = \Gamma_t^b$). Hence, as concluded in Proposition 2, the utility inequality is reduced in all cases, whether in the short or in the long runs.

5.3 The Gini Coefficient

One may wonder whether our theory would still be robust once we replace the inter-household measure of income inequality with the empirically-used Gini coefficient, which takes into account the initial population composition and the subsequent population changes due to migration. Below, we will show that the use of Gini coefficient does not at all exclude the possibility of rising income inequality in the short-run. Let us assume that, at each period of time, there are ρ units of LP households for each unit of HP household. By the definition of the Gini coefficient, we can construct another measure G_t to gauge the income inequality in our rural economy:[13]

$$G_t = \frac{\rho(\Gamma_t^y - 1)}{(\rho + \phi_t)(\frac{\rho}{\phi_t} + \Gamma_t^y)}$$

where $\phi_t = \frac{1-\overline{m}_t^*}{1-\underline{m}_t^*}$, or the ratio of the remaining members of the HP households to that of the LP households. We do not consider the migrated population in this inequality measure because, in most cases, they are no more included in the census data, where the Gini coefficient is derived from. If instead, we include the migrants, then $\phi_t = 1, \forall\, t$ and we would find that the Gini coefficient has the same general behavior as Γ_t^y since $\frac{\partial G_t}{\partial \Gamma_t^y}|_{\phi_t=1} > 0$. Accordingly, the propositions about the income inequality are preserved.

However, ϕ_t does not stay constant over time once we exclude the migrants, and so to study the dynamics of the Gini coefficient in the open economy we have to consider the joint effects of Γ_t^y and ϕ_t. With numerical simulations, we

[13] See Appendix F.

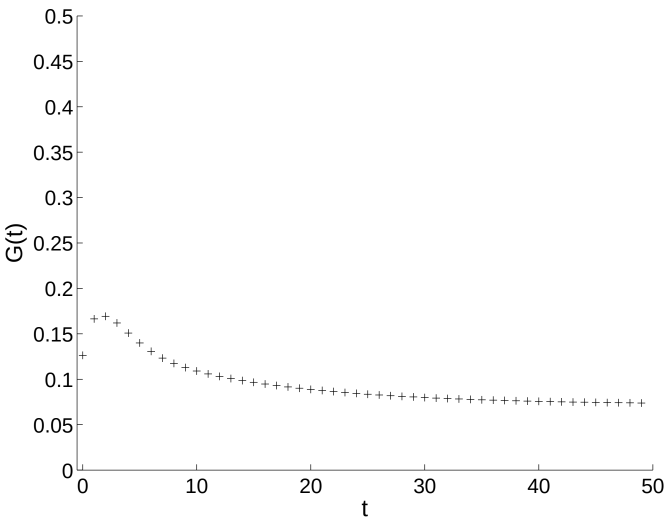

Figure 3: Gini coefficient (low inequality case): using the Gini coefficient as another inequality measure, we may still find the presence of the inverse U-shaped relationship between migration/remittances and income inequality.

find that the inverse U-shaped pattern may be preserved, especially when the difference in agricultural productivity is small. Figure 3, which corresponds to the same scenario as the two figures above and with $\rho = 1.5$, presents such an example of our simulation.

6 Extension: Local Labor Market

In this section, we introduce general equilibrium effects in the form of a local labor market into our agricultural economy. The LP households may choose to work on the HP farm if the wage rate offered (w_t) is higher than their marginal productivity on own farm. As a result, both households enjoy higher income and thus higher wealth when compared to an economy without labor exchange; nevertheless, changes in inequalities are not a priori clear. Hence, we would like to investigate below: i) whether there is still an inverse U-shaped relationship between migration and income inequality over time; ii) whether long-run income inequality always falls under the closed-economy level, as observed in the economy without local labor market.

6.1 Closed Economy

For HP households, the income maximization problem in the closed economy becomes

$$\max_{\{0\leq n_t^d\}} \quad \overline{y}_t = \overline{\alpha}(1+n_t^d) - \frac{(1+n_t^d)^2}{2} - w_t n_t^d$$

which gives the labor demand (n_t^d) condition:

$$n_t^d = \begin{cases} 0 & \text{if} \quad w_t \geq \overline{\alpha} - 1 \\ \overline{\alpha} - 1 - w_t & \text{if} \quad w_t < \overline{\alpha} - 1 \end{cases}$$

Similarly, LP households maximize their income according to

$$\max_{\{0\leq n_s^d \leq 1\}} \quad \underline{y}_t = \underline{\alpha}(1-n_t^s) - \frac{(1-n_t^s)^2}{2} + w_t n_t^s$$

which gives the labor supply (n_t^s) condition:

$$n_t^s = \begin{cases} 0 & \text{if} \quad w_t \leq \underline{\alpha} - 1 \\ 1 - (\underline{\alpha} - w_t) & \text{if} \quad \underline{\alpha} - 1 < w_t \leq \underline{\alpha} \\ 1 & \text{if} \quad w_t > \underline{\alpha} \end{cases}$$

Recall that there are ρ LP households per HP household. Therefore, at the labor market equilibrium, $n_t^* = n_t^s = \frac{n_t^d}{\rho}$. Using the labor demand and supply conditions, we find that, if the HP productivity ($\overline{\alpha}$) is not too much higher than the LP productivity ($\underline{\alpha}$) and/or if the LP population is relatively large, then only *some* of the LP household members work on the HP farm and the equilibrium wage rate is lower than $\underline{\alpha}$. Otherwise, *all* LP members work on the HP farm and receive a wage at least as high as the LP households' highest marginal productivity $\underline{\alpha}$. In short, the two possible labor market equilibria, denoted as *CS* and *CA* respectively, are:

[CS] when $\overline{\alpha} < 1 + \rho + \underline{\alpha}$,

$$n_t^* = \frac{\overline{\alpha} - \underline{\alpha}}{1+\rho} \quad \text{and} \quad w_t = \underline{\alpha} - (1 - n_t^*)$$

[CA] when $\overline{\alpha} \geq 1 + \rho + \underline{\alpha}$,

$$n_t^* = 1 \quad \text{and} \quad w_t = \overline{\alpha} - (1 + \rho)$$

Since the labor market equilibrium in the closed economy is time-invairant, the income and the wealth inequalities at the steady state are

$$\Gamma_0^y = \frac{\overline{\alpha}(1 + \rho n_t^*) - \frac{(1+\rho n_t^*)^2}{2} - w_t \rho n_t^*}{\underline{\alpha}(1 - n_t^*) - \frac{(1-n_t^*)^2}{2} + w_t n_t^*} \tag{5}$$

$$\Gamma_1^b = \frac{\frac{\sigma}{1-\sigma}(\overline{y}_t - x_m)}{\frac{\sigma}{1-\sigma}(\underline{y}_t - x_m)} = \frac{\overline{\alpha}(1 + \rho n_t^*) - \frac{(1+\rho n_t^*)^2}{2} - w_t \rho n_t^* - x_m}{\underline{\alpha}(1 - n_t^*) - \frac{(1-n_t^*)^2}{2} + w_t n_t^* - x_m} \tag{6}$$

with (n_t^*, w_t) being the *CS* or the *CA* labor market equilibrium.

6.2 Open Economy: with Access to Migration

From our discussion on the open economy without labor exchange, we have learned that when the HP productivity is so high that $\overline{\alpha} \geq 1 + w^* - c$, the HP households do not send out any migrants and inequalities must fall whether in the short- or the long-run. Hence, in this section we restrict our attention within $\overline{\alpha} < 1 + w^* - c$ such that $\overline{m}_t \neq 0$. As before, we enforce the condition $\overline{m}_t \neq 1$; that is, the HP household is always constrained if $\overline{\alpha} < w^* - c$.

As soon as the frontier is open at $t = 1$, the HP household's maximization problem becomes

$$\max_{\{\overline{m}_t, n_t^d\}} \overline{y}_t = \overline{\alpha}(1 + n_t^d - \overline{m}_t) - \frac{(1 + n_t^d - \overline{m}_t)^2}{2} - n_t^d w_t + \overline{m}_t(w^* - c)$$

$$s.t. \quad \begin{cases} \overline{m}_t \leq \frac{\overline{b}_t}{c} \\ 0 < \overline{m}_t < 1 \end{cases}$$

which gives the labor demand condition

$$n_t^d = \begin{cases} 0 & \text{if} \quad w_t \geq \overline{\alpha} - (1 - \overline{m}_t) \\ \overline{\alpha} - (1 - \overline{m}_t) - w_t & \text{if} \quad w_t < \overline{\alpha} - (1 - \overline{m}_t) \end{cases}$$

For the LP households, we maintain the assumption that they are always liquidity constrained so that $\underline{m}_t = \frac{b_t}{c} \in \,]\,0, 1\,[$.[14] They maximize income according to

$$\max_{\{n_t^s\}} \underline{y}_t = \underline{\alpha}(1 - n_t^s - \frac{b_t}{c}) \qquad - \frac{(1 - n_t^s - \frac{b_t}{c})^2}{2} + n_t^s w_t + \frac{b_t}{c}(w^* - c)$$
$$s.t. \quad n_t^s + \tfrac{b_t}{c} \leq 1$$

which gives the labor supply condition

$$n_t^s = \begin{cases} 0 & \text{if} \quad w_t \leq \underline{\alpha} - (1 - \frac{b_t}{c}) \\ 1 - \frac{b_t}{c} - (\underline{\alpha} - w_t) & \text{if} \quad \underline{\alpha} - (1 - \frac{b_t}{c}) < w_t \leq \underline{\alpha} \\ 1 - \frac{b_t}{c} & \text{if} \quad w_t > \underline{\alpha} \end{cases}$$

Similar to the closed economy, there are two possible labor market equilibria. In the first one, denoted as *OS*, *some* of the remaining LP family members work for the HP households, and in the second one, denoted as *OA*, *everyone* who remains behind works on the HP farm. Listed below are the equilibrium amounts of exchanged labor and their corresponding wage rates.[15]

For the equilibrium [OS], which applies when $\overline{\alpha} \leq (1 - \overline{m}_t) + \rho(1 - \frac{b_t}{c}) + \underline{\alpha}$:

[14] In fact, when $w_t = w^* - c$, we can have $\underline{m}_t \neq 1$ without assuming that LP households are always liquidity constrained. However, this requires the LP population to be sufficiently small and $\overline{\alpha}$ to be sufficiently large such that they satisfy $\rho < \overline{\alpha} - (w^* - c) < 1$. Moreover, we cannot exclude the possibility that $\underline{m}_t = 1$. Therefore, we forgo this case and simply assume that the LP households are liquidity constrained at all times.

[15] $\overline{m}_t = \arg\max \left[\overline{y}_t = \overline{\alpha}(1 + \rho n_t^* - \overline{m}_t) - \frac{(1+\rho n_t^* - \overline{m}_t)^2}{2} - \rho n_t^* w_t + \overline{m}_t(w^* - c) \right]$ if the HP household is *not* liquidity constrained.

$$n_t^* = \frac{\overline{\alpha}-\underline{\alpha}+\overline{m}_t-\frac{b_t}{c}}{1+\rho} \text{ and } w_t = \underline{\alpha} - (1 - \frac{b_t}{c} - n_t^*)$$

where $\overline{m}_t = \frac{\overline{b}_t}{c}$ if the HP household is liquidity constrained;

$$\overline{m}_t = \frac{1}{1+2\rho}\left[(1+\rho)^2(1+w^*-c) - \rho^2(\underline{\alpha}+\frac{b_t}{c})\right] - \overline{\alpha} \text{ otherwise.}$$

For the equilibrium [OA], which applies when $\overline{\alpha} > (1-\overline{m}_t) + \rho(1-\frac{b_t}{c}) + \underline{\alpha}$:

$$n_t^* = 1 - \frac{b_t}{c} \text{ and } w_t = \overline{\alpha} - 1 + \overline{m}_t - \rho n_t^*$$

where $\overline{m}_t = \frac{\overline{b}_t}{c}$ if the HP household is liquidity constrained;

$$\overline{m}_t = 1 + w^* - c - \overline{\alpha} \text{ otherwise.}$$

How does the local labor market equilibrium change in response to migration? Since $(1-\overline{m}_t)+\rho(1-\frac{b_t}{c})+\underline{\alpha} < 1+\rho+\underline{\alpha}$, if the labor market equilibrium in the closed economy is *CA*, where $\overline{\alpha} \geq 1+\rho+\underline{\alpha}$, then after opening to migration, the equilibrium must change to *OA*. In other words, if everybody works on the HP farm before migration becomes a possibility, then afterwards, all the remaining LP members will still work for the HP households. Otherwise, when the closed-economy equilibrium is *CS*, it may change to either *OS* or *OA* in the short run. With less and less labor supply as the LP household sends out increasing number of migrants, the equilibrium may maintain within the same type, or start at *OS* and end up at *OA* in the long run. Hence, even if productive in the closed economy, the LP farm may be abandoned permanently after migration becomes possible.

The income and the wealth inequalities in the open economy are therefore:

$$\Gamma^y_t = \frac{\overline{\alpha}(1+\rho n^*_t - \overline{m}_t) - \frac{(1+\rho n^*_t - \overline{m}_t)^2}{2} - \rho n^*_t w_t + \overline{m}_t(w^* - c)}{\underline{\alpha}(1 - n^*_t - \frac{b_t}{c}) - \frac{(1-n^*_t - \frac{b_t}{c})^2}{2} + n^*_t w_t + \frac{b_t}{c}(w^* - c)} \tag{7}$$

$$\begin{aligned}\Gamma^b_{t+1} &= \frac{\sigma(\overline{y}_t + \overline{b}_t - x_m)}{\sigma(\underline{y}_t + \underline{b}_t - x_m)} \\ &= \frac{\overline{\alpha}(1+\rho n^*_t - \overline{m}_t) - \frac{(1+\rho n^*_t - \overline{m}_t)^2}{2} - \rho n^*_t w_t + \overline{m}_t(w^* - c) + \overline{b}_t - x_m}{\underline{\alpha}(1 - n^*_t - \frac{b_t}{c}) - \frac{(1-n^*_t - \frac{b_t}{c})^2}{2} + n^*_t w_t + \frac{b_t}{c}(w^* - c) + \underline{b}_t - x_m}\end{aligned} \tag{8}$$

with (n^*_t, w_t) being the *OS* or the *OA* labor market equilibrium.

6.3 Evolution of Inter-household Inequalities

In order to answer the questions that we raised in the beginning of this section: i) is there still an inverse U-shaped relationship between migration and income inequality over time? and ii) does long-run income inequality always fall under the closed-economy level? We resort to numerical simulation using equations (5), (6), (7), and (8). We fix $x_m = 2$, $c = 1.5$, $w^* = 7.5$ with three different values of ρ so that we create three scenarios where the LP population can be smaller ($\rho = 0.5$), equal ($\rho = 1$), or larger ($\rho = 1.5$) than their HP counterpart. In addition, we experiment with different values of $\underline{\alpha}$, and we vary σ and $\overline{\alpha}$ within the ranges that satisfy our assumptions.

We find that, after introducing a local labor market, we may still observe the inverse U-shaped pattern. Recall that without labor market, the condition for observing such a pattern was that $\underline{\alpha}$ must be close enough to the subsistence level. In fact, the existence of a Kuznets curve now requires $\underline{\alpha}$ to be even closer to the subsistence level. Additionally, it also requires that $\overline{\alpha}$ is sufficiently low, or not too much higher than $\underline{\alpha}$. This acts to minimizes the positive effect of labor exchange on the LP household's income: with lower productivity, the HP household offers lower wage and hires less LP workers.[16] Under such circumstances, the LP household is very much constrained in the beginning to benefit

[16]More rigorously speaking, the equilibrium wage (w_t) and/or hired labor (n^*_t) decrease with the HP household's productivity ($\overline{\alpha}$).

from the access to migration.

Corollary 2 *In an economy with a local labor market, migration and remittances are most likely to generate an inverse U-shaped pattern in an originally impoverished community, where both the LP and HP households are living close to the subsistence level before migration becomes possible.*

More importantly, we find that with a Kuznets curve, the income inequality falls but may not fall under the closed-economy level in the long-run (see figure 4(a)). In other words, the access to migration may worsen the income inequality *for good*, which is a very different result from Proposition 3. The reason is that, with labor exchange, the HP households incur lower opportunity costs of migration than without: they are able to, at least partly, compensate for their loss of labor by employing the LP family members for their agricultural production. In contrast, although the LP household also benefits from migration and labor exchange, the gains are limited due to a tight liquidity constraint and low equilibrium wage rates.[17]

Proposition 4 (Increased income inequality in the long run) *With the existence of a local labor market, when $\overline{\alpha}$ is sufficiently close to $\underline{\alpha}$, the income inequality is increased in the long run, i.e.* $\Gamma_0^y < \Gamma_{ss}^y$.

Proof. See Appendix G. ■

We also examine the corresponding Gini coefficients, and as shown in figure 4, they exhibit the same pattern as measured by the inter-household income inequality except for the median case (b) where there is an overshooting of decreased inequality at the beginning. This seems paradoxical if we look at the corresponding income dynamics: we observe that the timing of overshooting corresponds exactly to the overshooting of the HP household's income. This is because, at the beginning of the open economy, the HP household gains by a large amount from sending many members abroad while enjoying a still low

[17] Even though equilibrium wage rates go up with migration due to rising demand and reduction in supply, the increases are small because the HP household also has a very low level of productivity.

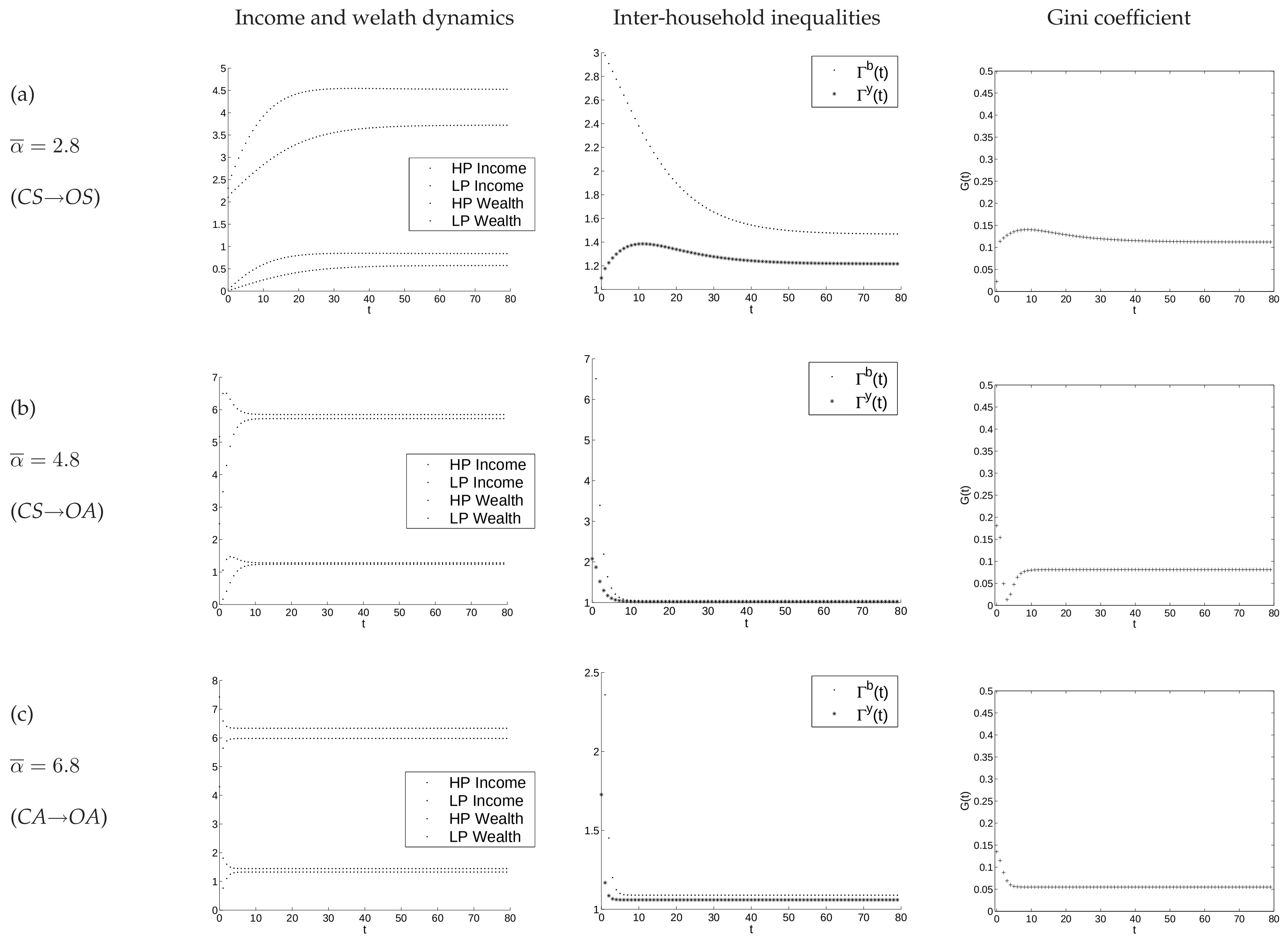

Figure 4: Remittances and inequality with a local labor market ($\underline{\alpha} = 2.6,\ \sigma = 0.25,\ x_m = 2,\ c = 1.5,\ w^* = 7.5,\ \rho = 1.5$).

wage rate at home. Since the Gini coefficient takes into account only the remaining population, it creates the phenomenon that income inequality is sharply reduced right after opening the border. However, in terms of what we are concerned of, the Gini coefficient in this case also demonstrates the same result as does the inter-household inequality measure; that is, the income inequality is decreased both in the short- and the long-runs.

The last interesting note is that migration and remittances are not always Pareto-improving in an economy with a local labor market. When the HP household's productivity is so high that they do not benefit much from migration opportunities, the opening of frontier actually makes them worse off because the equilibrium wage rate hikes up due to decreasing labor supply. These decreases in HP household's absolute income and wealth can be observed for the low-inequality case in figure 4(a).

7 Conclusion

The dynamic famework proposed in this paper demonstrates that the impact of migration and remittances on inequalities in the migrants' origin communities largely depends on the difference in their initial endowments of non-liquifiable capital. These endowments translate into different productivity levels and in turn determine migration incentives and opportunities, and they also affect the exact way in which remittances and labor markets respond to migration. Combined together, the global effects eventually determine the intergenerational transmission of wealth across households.

The main results of the analysis may be summarized as follows. First, whether in the short or the long runs, migration and remittances always reduce wealth inequality, through a proportionally larger increase in wealth for the poor. Second, except when the inequality in productivity is sufficiently high, income inequality may decrease continuously over time or be characterized by a "trickle-down" transition path. In the latter case, income inequality rises in the short-run and then subsides after some period of time. That is to say, an inverse U-shaped relationship may be generated via intergenerational wealth transfers, which is true with either the inter-household inequality measure or the Gini coefficient. We find that this case is most likely to occur when low-productivity households are sufficiently poor in the economy without labor exchange, or when both low- and high-productivity households are rather indigent in a closed economy with local exchange. Third, in the long run, migration and remittances are shown to decrease income inequality in an economy *without* labor exchange. However, when a local labor market exists and the income inequality is increased in the short run, the deteriorated inequality may never recover to its closed-economy level, when migration was not possible.

The first two results imply that migration network effects are not a necessary condition for observing an inverse U-shaped pattern, and they have strong implications for the empirical analysis of the migration-inequality relationship. An immediate implication is that domestic income sources should be treated as endogenous, as advocated for example by Adams (1989), Taylor (1992) or Braham and Boucher (1998). Indeed, our results suggest that studies based on Gini Index decompositions with exogenous distributions of domestic incomes may yield biased estimates of the inequality impact of migration, with the direction of the bias being theoretically uncertain and depending on the initial distribution of wealth. This also suggests that the lack of consensus in the empirical literature on the inequality impact of migration may be partly explained by the omission of labor market responses. Indeed, in a country such as Mexico where inequality is high by international standards, this omission is likely to lead to an underestimation of the inequality-reducing effect of migration, but not to a reversal of the sign of the effect. By contrast, in a country such as Yugoslavia where inequality is much lower, taking labor market responses into account could possibly reverse the conclusions on the inequality-enhancing effect of remittances (Milanovic, 1987).

Finally, our model suggests that the usual hypothesis of migrant network effects must be tested for directly rather than inferred from the observation of lower inequality levels within communities with a longer migration tradition. Indeed, as explained, our framework generates the possibility of an inverse U-shaped relationship between migration/remittances and inequality. This is consistent with the findings of the related empirical literature, but offers a different explanation based on the dynamic accumulation of wealth across generations, with no need to endogenize migration costs through network effects.

References

Adams, R. (1989): Workers remittances and inequality in rural Egypt, Economic Development and Cultural Change, 38, 1: 45-71.

Adams, R. (1992): The impact of migration and remittances on inequality in rural Pakistan, Pakistan Development Review, 31, 4: 1189-203.

Aghion, P. and P. Bolton (1997): A theory of trickle-down growth and development, Review of Economic Studies, 64(2): 151-172.

Azam, J.-P. and F. Gubert (2005): Those in Kayes. The impact of remittances on their recipients in Africa, Revue Economique, 56(6): 1331-58.

Bardhan, P., S. Bowles and H. Gintis (2000): Wealth inequality, wealth constraints and economic performance, in A.B. Atkinson and F. Bourguignon, eds.: Handbook of Income Distribution, Elsevier-North Holland, Vol. 1, Chapter 10, pp. 541-603.

Barham, B. and S. Boucher (1998): Migration, remittances and inequality: estimating the net effects of migration on income distribution, Journal of Development Economics, 55: 307-31.

Banerjee, A. V. and A.F. Newman (1993): Occupational choice and the process of development, Journal of Political Economy, 105, 501-26.

Carrington, W.J., E. Detragiache and T. Vishwanath (1996): Migration with endogenous moving costs, American Economic Review, 86 (4): 909-30.

Cox Edwards, A. and M. Ureta (2003): International migration, remittances and schooling: evidence from El Salvador, Journal of Development Economics, 72, 2: 429-61.

Dustmann, C. and O. Kirchkamp (2002): The optimal migration duration and activity choice after remigration, Journal of Development Economics, 67, 2: 351-72.

Galor, O. and J. Zeira (1993): Income distribution and macroeconomics, Review of Economic Studies, 60(1): 35-52.

Hanson, G.H. and C. Woodruff (2003): Emigration and educational attainment in Mexico, Mimeo., University of California at San Diego.

Hoddinott, J. (1994): A model of migration and remittances applied to Western Kenya, Oxford Economic Papers, 46(8): 459-76.

Lucas, R.E.B. (1987): Emigration to South Africa's mines, American Economic Review, 77,3: 313-30.

Massey, D.S., L. Goldring and J. Durand (1994): Continuities in Transnational Migration: An Analysis of Nineteen Mexican Communities, American Journal of Sociology, 99, 6: 1492-1533.

McKenzie, D. and H. Rapoport (2006a): Can migration reduce educational attainment? Evidence from Mexico, World Bank Policy Research Working Paper No 3952, June.

McKenzie, D. and H. Rapoport (2006b): Network effects and the dynamics of migration and inequality: theory and evidence from Mexico, Journal of Development Economics, forthcoming.

Mesnard, A. and M. Ravallion (2001): Wealth distribution and self-employment in a developing economy, CEPR Discussion Paper No 3026.

Milanovic, B. (1987): Remittances and income distribution, Journal of Economic Studies, 14(5): 24-37.

Munshi, K. (2003): Networks in the modern economy. Mexican migrants in the US labor market, Quarterly Journal of Economics, 118, 2: 549-99.

Rapoport, H. and F. Docquier (2006): The economics of migrants' remittances; in S.-C. Kolm and J. Mercier Ythier, eds.: Handbook of the Economics of Giving, Altruism and Reciprocity, Amsterdam: North-Holland, Vol. 2, Chapter 17, pp. 1135-98.

Rempel, H. and R. Lobdell (1978): The role of urban-rural remittances in rural development, Journal of Development Studies, 14: 324-41.

Rozelle, S., J.E. Taylor and A. deBrauw (1999): Migration, remittances and agricultural productivity in China, American Economic Review, 78, 2: 245-50.

Stark, O., J.E. Taylor and S. Yitzhaki (1986): Remittances and inequality, Economic Journal, 96(383): 722-40.

Taylor, J.E. (1992): Remittances and inequality reconsidered: direct, indirect and intertemporal effects, Journal of Policy Modeling, 14, 2: 187-208.

Taylor J.E. and T.J. Wyatt (1996), "The Shadow Value of Migrant Remittances, Income and Inequality in a Household-farm Economy", Journal of Development Studies, 32, 6: 899-912.

Woodruff, C. and R. Zenteno (2006): Remittances and micro-enterprises in Mexico, Journal of Development Economics, forthcoming.

8 Appendices

A Proof for Lemma 1

We know that at the steady state $\underline{b}_{t+1} = \underline{b}_t$ for equation (3), which has two roots. However, only the larger one

$$\underline{b}_{ss}^m = c\left[1 + (w^* - \underline{\alpha} - \frac{c}{\sigma}) + \sqrt{(w^* - \underline{\alpha} - \frac{c}{\sigma})^2 + 2(w^* - x_m - \frac{c}{\sigma})}\,\right]$$

is stable. From our formulation of the utility function, it is clear that wealth should be always positive, and it is straightforward to show that this stable steady state is indeed positive. In the meantime, it should also satisfy the binding liquidity constraint:

$$c\left[1 + (w^* - \underline{\alpha} - \frac{c}{\sigma}) + \sqrt{(w^* - \underline{\alpha} - \frac{c}{\sigma})^2 + 2(w^* - x_m - \frac{c}{\sigma})}\,\right] < c$$

$$\Leftrightarrow \quad w^* - x_m < \frac{c}{\sigma} \quad (\text{or } \sigma < \tfrac{c}{w^* - x_m})$$

Armed with this condition, now we can prove that this stable steady state is a real solution, that is, $(w^*-\underline{\alpha}-\frac{c}{\sigma})^2+2(w^*-x_m-\frac{c}{\sigma}) \geq 0$. Rewrite the inequality above as $\left[\frac{c}{\sigma}-(w^*-\underline{\alpha})\right]^2 \geq 2\left[\frac{c}{\sigma}-(w^*-x_m)\right]$. With the assumption $x_m < \underline{\alpha}-\frac{1}{2}$, it is sufficient to show that

$$\begin{aligned} & \left[\tfrac{c}{\sigma}-(w^*-(x_m+\tfrac{1}{2}))\right]^2 \geq 2\left[\tfrac{c}{\sigma}-(w^*-x_m)\right] \\ \Leftrightarrow \quad & \left[\tfrac{c}{\sigma}-(w^*-x_m)-\tfrac{1}{2}\right]^2 \geq 0 \end{aligned}$$

Since the last inequality always holds, we conclude that $\underline{b}_{ss}^m$ is indeed real.

B Computations for $\overline{\alpha}_0$ and $\overline{\alpha}_1$ in Lemma 2

We derive $\overline{\alpha}_1$ simply from $c\,(1+w^*-c-\overline{\alpha}) = \overline{b}_1 = \frac{\sigma}{1-\sigma}(\overline{\alpha}-\frac{1}{2}-x_m)$. Similarly but with more complexity, we derive $\overline{\alpha}_0$ from $c\,(1+w^*-c-\overline{\alpha}) = \overline{b}_{ss} = \frac{\sigma}{1-\sigma}\left[\frac{(1+w^*-c-\overline{\alpha})^2}{2}+\overline{\alpha}-\frac{1}{2}-x_m\right]$. This equality leads to

$$w^*-c-\overline{\alpha}_0 = \frac{1-\sqrt{1-\frac{2\sigma}{c(1-\sigma)}\left[\frac{\sigma}{c(1-\sigma)}(w^*-c-x_m)-1\right]}}{\frac{\sigma}{c(1-\sigma)}}$$

We eliminate the other root that is certainly positive because $w^*-c-\overline{\alpha}_0 < 0$ in this scenario, but we still have to check if this root is negative. We find that with $w^*-x_m < \frac{c}{\sigma}$, the condition we derive in Lemma 1, it is indeed negative. After some rearrangements, we have $\overline{\alpha}_0$ as shown in Lemma 2.

C Proof for Proposition 1

In order to prove that the wealth inequality abates in the high inequality case, we need to show that $\Gamma^b_{t+1} - \Gamma^b_1$ has a negative sign. Since

$$\begin{aligned}\Gamma^b_{t+1} - \Gamma^b_1 &= \frac{\frac{\sigma}{1-\sigma}(\overline{\alpha} - \frac{1}{2} - x_m)}{\underline{b}_{t+1}} - \frac{\overline{\alpha} - \frac{1}{2} - x_m}{\underline{\alpha} - \frac{1}{2} - x_m} \\ &= (\frac{\overline{\alpha} - \frac{1}{2} - x_m}{\underline{\alpha} - \frac{1}{2} - x_m}) \cdot (\frac{\underline{b}_1 - \underline{b}_{t+1}}{\underline{b}_{t+1}})\end{aligned}$$

it suffices to demonstrate that $\underline{b}_1 - \underline{b}_{t+1} < 0$. Using equations (1) and (3), we know that $\underline{b}_1 < \underline{b}_2$. From this as well as the functional properties of $\underline{b}_{t+1}(\underline{b}_t)$ and Lemma 1, we conclude that $\underline{b}_1 < \underline{b}_2 < \cdots < \underline{b}_{ss}$. That is to say, $(\underline{b}_1 - \underline{b}_{t+1})$ is indeed negative.

Similarly, to prove that the income inequality also declines, we have

$$\Gamma^y_t - \Gamma^y_0 = (\frac{\overline{\alpha} - \frac{1}{2}}{\underline{y}_t}) - (\frac{\overline{\alpha} - \frac{1}{2}}{\underline{\alpha} - \frac{1}{2}}) = (\frac{\overline{\alpha} - \frac{1}{2}}{\underline{\alpha} - \frac{1}{2}}) \cdot (\frac{\underline{y}_0 - \underline{y}_t}{\underline{y}_t})$$

so it suffices to show that $\underline{y}_0 - \underline{y}_t < 0$, which is evident by comparing equations (1) and (2).

D Proofs for Proposition 2

D.1 Higher-Medium Inequality Case

In order to confirm that the wealth inequality is reduced in the short run, we need to prove that

$$\Gamma^b_2 - \Gamma^b_1 = \frac{\sigma\left[\frac{1}{2}(1 + w^* - c - \overline{\alpha})^2 + \overline{\alpha} - \frac{1}{2} - x_m + \overline{b}_1\right]}{\sigma\left[\underline{\alpha} - \frac{1}{2} - x_m + \frac{\underline{b}_1}{c}(w^* - \underline{\alpha} + 1 - \frac{\underline{b}_1}{2c})\right]} - \frac{\overline{\alpha} - \frac{1}{2} - x_m}{\underline{\alpha} - \frac{1}{2} - x_m} \; < \; 0$$

After some rearrangements, we know that it is equivalent to demonstrate the following to be true.

$$\frac{(1+w^*-c-\overline{\alpha})^2}{2(\overline{\alpha}-\frac{1}{2}-x_m)} - \frac{\sigma}{c(1-\sigma)}\left[1+w^*-c-\underline{\alpha}-\frac{\sigma(\underline{\alpha}-\frac{1}{2}-x_m)}{2c(1-\sigma)}\right] \;<\; 0$$

Denoting the left-hand side of the inequality as $f(\overline{\alpha})$, one finds that $\frac{\partial f(\overline{\alpha})}{\partial \overline{\alpha}} \leq 0, \forall \overline{\alpha}$. In other words, $f(\overline{\alpha})$ reaches its maximum when $\overline{\alpha} = \overline{\alpha}_{min} = \frac{c(1-\sigma)(1+w^*-c)+\sigma(\frac{1}{2}+x_m)}{c(1-\sigma)+\sigma}$, the minimal value of $\overline{\alpha}$. Thereby, it is sufficient to show that $f(\overline{\alpha}_{min}) < 0$. Again after some rearrangements, we find that to determine the sign of $f(\overline{\alpha}_{min})$ is equivalent to know the sign of

$$g(\underline{\alpha}) = (1+w^*-c-\underline{\alpha})(1-2\left[\frac{c(1-\sigma)}{\sigma}+1\right]) + (\underline{\alpha}-\frac{1}{2}-x_m)(2+\frac{\sigma}{c(1-\sigma)})$$

where $g(\underline{\alpha})$ monotonically increases in $\underline{\alpha}$, $\forall\, c$, σ, $\underline{\alpha}$. By assumption, $\frac{1}{2}+x_m < \underline{\alpha} < w^*-c$; thus, as long as we can prove $g(w^*-c) < 0$, it immediately implies that $g(\underline{\alpha}) < 0$, $\forall\, \underline{\alpha}$. With

$$g(w^*-c) = (2+\frac{\sigma}{c(1-\sigma)}) \cdot (w^*-x_m-\frac{c}{\sigma}-\frac{1}{2})$$

and $w^*-x_m < \frac{c}{\sigma}$ by Lemma 1, $g(w^*-c)$ is indeed negative. Hence, we have proven that wealth inequality is reduced in the short run.

The proof for the long-run wealth inequality can be found in the proof for the next case.

D.2 Lower-Medium Inequality Case

The proof for the short-run wealth inequality is identical to that of the low inequality case (see below).

In order to confirm that the wealth inequality is reduced in the long run, we need to prove that

$$\Gamma^b_{ss} - \Gamma^b_1 = \frac{\sigma\left[\frac{1}{2}(1+w^*-c-\overline{\alpha})^2 + \overline{\alpha} - \frac{1}{2} - x_m + \overline{b}^m_{ss}\right]}{\sigma\left[\underline{\alpha} - \frac{1}{2} - x_m + \frac{b^m_{ss}}{c}(w^* - \underline{\alpha} + 1 - \frac{b^m_{ss}}{2c})\right]} - \frac{\overline{\alpha} - \frac{1}{2} - x_m}{\underline{\alpha} - \frac{1}{2} - x_m} < 0$$

After some rearrangements, we know that it is equivalent to demonstrate the following to be true.

$$\frac{(1+w^*-c-\overline{\alpha})^2}{2(\overline{\alpha} - \frac{1}{2} - x_m)} + 1 - \frac{\frac{c(1-\sigma)}{\sigma}(1+w^* - \underline{\alpha} - \frac{c}{\sigma} + A)}{\underline{\alpha} - \frac{1}{2} - x_m} < 0$$

where $A = \sqrt{(w^* - \underline{\alpha} - \frac{c}{\sigma})^2 + 2(w^* - x_m - \frac{c}{\sigma})}$.

Denoting the left-hand side of the inequality as $f(\overline{\alpha})$, one finds that $\frac{\partial f(\overline{\alpha})}{\partial \overline{\alpha}} \leq 0, \forall\, \overline{\alpha} \in]\underline{\alpha},\, 1+w^*-c[$. In other words, $f(\overline{\alpha})$ reaches its maximum when $\overline{\alpha} = \overline{\alpha}_{min} = w^* - c + B$, where $B = \frac{c(1-\sigma)}{\sigma}\left(\sqrt{1 - \frac{2\sigma}{c(1-\sigma)}\left[\frac{\sigma}{c(1-\sigma)}(w^* - c - x_m) - 1\right]} - 1\right)$. Thereby, it is sufficient to demonstrate that $f(\overline{\alpha}_{min}) < 0$. Again after some rearrangements, we find that to determine the sign of $f(\overline{\alpha}_{min})$ is equivalent to know the sign of

$$g(\underline{\alpha}) = (\underline{\alpha} - \frac{1}{2} - x_m)(1-B) - (B + w^* - c - x_m - \frac{1}{2})(1 + w^* - \underline{\alpha} - \frac{c}{\sigma} + A)$$

We find that $\frac{\partial^2 g(\underline{\alpha})}{\partial \underline{\alpha}^2} > 0,\ \forall\, \underline{\alpha} \in \Re$, or $g(\cdot)$ is a convex function in $\underline{\alpha}$. Moreover, $g(x_m + \frac{1}{2}) < g(\overline{\alpha}_{min}) = 0$ while $x_m + \frac{1}{2} < \underline{\alpha} < w^* - c < \overline{\alpha}_{min}$. Thus, we conclude that $g(\underline{\alpha}) < 0$, and so we have proven that wealth inequality is reduced in the long run. Since this result is valid for $w^* - c + B \leq \overline{\alpha} < 1 + w^* - c$, the proof above also applies to the higher-medium inequality case.

D.3 Low Inequality Case

In order to know whether the wealth inequality is reduced in the short run, we look at the sign of $\Gamma^b_2 - \Gamma^b_1$. After some computation, we have

$$\Gamma^b_2 - \Gamma^b_1 = \frac{-\sigma(\overline{\alpha} - \frac{1}{2} - x_m)(\underline{\alpha} - \frac{1}{2} - x_m)}{c(1-\sigma)} \cdot (1 + \frac{\sigma}{2c(1-\sigma)}) \cdot (\overline{\alpha} - \underline{\alpha})$$

which is obviously negative and indicates that the wealth inequality is indeed reduced in the short run.

To demonstrate that the wealth inequality is also reduced in the long run, let us first write

$$b_{t+1} = f(b_t, \alpha) = \sigma\left[(\alpha - \frac{1}{2} - x_m) + \frac{b_t}{c}(w^* - \alpha + 1 - \frac{b_t}{2c})\right]$$

with $b_{ss} = f(b_{ss}, \alpha)$, which implies $b_{ss} = b(\alpha)$. Thus, we can define that at $b = b_{ss}$, $g(b, \alpha) = f(b(\alpha), \alpha) - b(\alpha) = 0$. From the function form of $f(\cdot)$, we know $\frac{\partial g}{\partial b} \neq 0$. This allows us to apply the implicit function theorem and obtain

$$\frac{d\,b(\alpha)}{d\,\alpha} = -\frac{\frac{\partial g}{\partial \alpha}}{\frac{\partial g}{\partial b}} = \frac{\frac{\partial f}{\partial \alpha}}{1 - \frac{\partial f}{\partial b}} \tag{9}$$

.

Given $\overline{\alpha} > \underline{\alpha}$, we need to prove that $\Gamma^b_{ss} < \Gamma^b_1 = \frac{\overline{\alpha} - \frac{1}{2} - x_m}{\underline{\alpha} - \frac{1}{2} - x_m}$. That is,

$$\frac{f\left[b(\overline{\alpha}), \overline{\alpha}\right]}{f\left[b(\underline{\alpha}), \underline{\alpha}\right]} < \frac{f\left[0, \overline{\alpha}\right]}{f\left[0, \underline{\alpha}\right]} \quad or \quad \frac{f\left[b(\overline{\alpha}), \overline{\alpha}\right]}{f\left[0, \overline{\alpha}\right]} < \frac{f\left[b(\underline{\alpha}), \underline{\alpha}\right]}{f\left[0, \underline{\alpha}\right]}$$

Suppose $\overline{\alpha} = \underline{\alpha}$, the right hand side (RHS) of the second inequality is equal to the left hand side (LHS). Hence, in order to prove our speculation to be true, it suffices to show that $\frac{\partial LHS}{\partial \overline{\alpha}} < \frac{\partial RHS}{\partial \overline{\alpha}} = 0$ holds for all $\overline{\alpha} > \underline{\alpha}$. In other words, we need to demonstrate that

$$\begin{aligned}
& \frac{f[0,\overline{\alpha}]}{f[b(\overline{\alpha}),\overline{\alpha}]} \cdot \frac{\partial f[b(\overline{\alpha}),\overline{\alpha}]}{\partial \overline{\alpha}} < \frac{\partial f[0,\overline{\alpha}]}{\partial \overline{\alpha}} \\
\Rightarrow \quad & \frac{\sigma(\overline{\alpha} - \frac{1}{2} - x_m)}{\overline{b}_{ss}} \cdot \frac{\sigma(1 - \frac{\overline{b}_{ss}}{c})}{1 - \frac{\sigma}{c}(w^* - \overline{\alpha} + 1 - \frac{\overline{b}_{ss}}{c})} < \sigma \quad \text{(using equation (9))} \\
\Rightarrow \quad & \sigma\left[(\overline{\alpha} - \tfrac{1}{2} - x_m) + \tfrac{\overline{b}_{ss}}{c}(w^* - \overline{\alpha} + 1 - \tfrac{\overline{b}_{ss}}{c})\right] < \overline{b}_{ss} + \tfrac{\sigma \overline{b}_{ss}}{c}(\overline{\alpha} - \tfrac{1}{2} - x_m) \\
\Rightarrow \quad & f(\overline{b}_{ss}, \overline{\alpha}) - \overline{b}_{ss} - \frac{\sigma \overline{b}^2_{ss}}{2c^2} < \frac{\sigma \overline{b}_{ss}}{c}(\overline{\alpha} - \tfrac{1}{2} - x_m) \\
\Rightarrow \quad & -\frac{\sigma \overline{b}^2_{ss}}{2c^2} < \frac{\sigma \overline{b}_{ss}}{c}(\overline{\alpha} - \tfrac{1}{2} - x_m)
\end{aligned}$$

Since the last inequality always holds, we have proven our case.

E Proof for Proposition 3

E.1 Higher-Medium Inequality Case

In order to know how the income inequality changes in the short run, we write

$$\Gamma_1^y - \Gamma_0^y = \frac{\frac{1}{2}(1+w^*-c-\overline{\alpha})^2+\overline{\alpha}-\frac{1}{2}}{\underline{\alpha}-\frac{1}{2}+\frac{b_1}{c}(w^*-\underline{\alpha}-c+1-\frac{b_1}{2c})} - \frac{\overline{\alpha}-\frac{1}{2}}{\underline{\alpha}-\frac{1}{2}}$$

whose sign is the same as that of the following equation:

$$f(\overline{\alpha}) = \frac{(1+w^*-c-\overline{\alpha})^2}{2(\overline{\alpha}-\frac{1}{2})} - \frac{\frac{b_1}{c}(w^*-\underline{\alpha}-c+1-\frac{b_1}{2c})}{\underline{\alpha}-\frac{1}{2}}$$

Obviously as $\overline{\alpha} \to (1+w^*-c)$, the upper bound of $\overline{\alpha}$, we have $f(\overline{\alpha}) < 0$.

However, $f(\overline{\alpha})$ does not always stay negative. To illustrate, we set $\overline{\alpha} = \overline{\alpha}_{min} = \frac{c(1-\sigma)(1+w^*-c)+\sigma(\frac{1}{2}+x_m)}{c(1-\sigma)+\sigma}$. We know that to determine the sign of $f(\overline{\alpha}_{min})$ is equivalent to look at the sign of the equation below.

$$g(\underline{\alpha}) = \frac{(\underline{\alpha}-\frac{1}{2})(w^*-c-x_m+\frac{1}{2})^2}{1+\frac{c(1-\sigma)}{\sigma}} - (\underline{\alpha}-\frac{1}{2}-x_m)(w^*-c+\frac{1}{2}+\frac{\sigma x_m}{c(1-\sigma)}) \cdot \left[1+w^*-\underline{\alpha}-c-\frac{\sigma(\underline{\alpha}-\frac{1}{2}-x_m)}{2c(1-\sigma)}\right]$$

where $\max(1, x_m+\frac{1}{2}) < \underline{\alpha} < w^*-c$. In the case where $x_m > \frac{1}{2}$, we have $g(\underline{\alpha}) > 0$ as $\underline{\alpha} \to (x_m+\frac{1}{2})$, the lower bound of $\underline{\alpha}$. It therefore implies that $\lim_{\underline{\alpha}\to(x_m+\frac{1}{2})} f(\overline{\alpha}_{min}) > 0$.

Because we have shown above that the sign of $f(\overline{\alpha})$ varies with the parameter values, we conclude that the effect of migration and remittances on income equality is ambiguous in the short run.

The proof for the long-run income inequality can be found in the proof for the next case.

E.2 Lower-Medium Inequality Case

To know whether the income inequality increases or decreases in the short run, we have to investigate the sign of

$$\Gamma_1^y - \Gamma_0^y = \frac{\overline{\alpha} - \frac{1}{2} + \frac{\overline{b}_1}{c}(w^* - \overline{\alpha} - c + 1 - \frac{\overline{b}_1}{2c})}{\underline{\alpha} - \frac{1}{2} + \frac{b_1}{c}(w^* - \underline{\alpha} - c + 1 - \frac{b_1}{2c}} - \frac{\overline{\alpha} - \frac{1}{2}}{\underline{\alpha} - \frac{1}{2}}$$

Since we have derived the wealth levels at $t = 1$, after some rearrangements, we know it is equivalent to look at the sign of

$$\begin{aligned} f(\overline{\alpha}, \underline{\alpha}) \quad = \quad & \frac{(\overline{\alpha} - \frac{1}{2} - x_m)(w^* - \overline{\alpha} - c + 1 - \frac{\sigma(\overline{\alpha} - \frac{1}{2} - x_m)}{2c(1-\sigma)})}{\overline{\alpha} - \frac{1}{2}} \\ & - \frac{(\underline{\alpha} - \frac{1}{2} - x_m)(w^* - \underline{\alpha} - c + 1 - \frac{\sigma(\underline{\alpha} - \frac{1}{2} - x_m)}{2c(1-\sigma)})}{\underline{\alpha} - \frac{1}{2}} \end{aligned}$$

Obviously $\lim_{\underline{\alpha} \to (x_m + \frac{1}{2})} f(\cdot) > 0$. However, $f(\cdot)$ does not always stay positive. For example, when we set $\underline{\alpha} = w^* - c$, we obtain

$$f(\overline{\alpha}) = [(w^* - c) - \overline{\alpha}] \left[1 + \frac{\sigma}{2c(1-\sigma)} - \frac{x_m}{(\overline{\alpha} - \frac{1}{2})(w^* - c - \frac{1}{2})} - \frac{\sigma x_m^2}{2c(1-\sigma)(\overline{\alpha} - \frac{1}{2})(w^* - c - \frac{1}{2})} - \frac{x_m}{\overline{\alpha} - \frac{1}{2}}\right]$$

Studying the properties of $f(\overline{\alpha})$, we first find that it is a strictly concave function, i.e. $\frac{\partial^2 f(\overline{\alpha})}{\partial \, \overline{\alpha}^2} < 0, \ \forall \overline{\alpha} > \frac{1}{2}$. Second, $f(\overline{\alpha}) = 0$ has two solutions: $\overline{\alpha} w^* - c$ and $\overline{\alpha} = \frac{1}{2} + x_m \left[\frac{2c(1-\sigma)(w^* - c + \frac{1}{2}) + \sigma x_m}{[2c(1-\sigma) + \sigma](w^* - c - \frac{1}{2})}\right]$. When we choose a parameter set such that the second root is smaller than the first one, both roots are then smaller than $\overline{\alpha}_0$. So, this strictly concave function $f(\overline{\alpha})$ is negative within the range of $\overline{\alpha}$, which implies that the income inequality is reduced in the short run.

In order to confirm that the income inequality is reduced in the long run, we need to prove that

$$\Gamma_{ss}^y - \Gamma_0^y = \frac{\frac{1}{2}(1 + w^* - c - \overline{\alpha})^2 + \overline{\alpha} - \frac{1}{2}}{\underline{\alpha} - \frac{1}{2} + \frac{b_{ss}^m}{c}(w^* - \underline{\alpha} - c + 1 - \frac{b_{ss}^m}{2c})} - \frac{\overline{\alpha} - \frac{1}{2}}{\underline{\alpha} - \frac{1}{2}} \ < \ 0$$

The rest of the proof is very similar to the proof for the wealth inequality in the long run. We can also rearrange this inequality so that it suffices to demonstrate that the function $f(\overline{\alpha}_{min}) < 0$, which in turn is equivalent to show the following to be true

$$g(\underline{\alpha}) = (\underline{\alpha} - \tfrac{1}{2})(1 - B + \tfrac{\sigma x_m}{c(1-\sigma)}) - (B + w^* - c - \tfrac{1}{2})(\tfrac{\sigma x_m}{c(1-\sigma)} + 1 + w^* - \underline{\alpha} - \tfrac{c}{\sigma} + A) < 0$$

We find that $\frac{\partial^2 g(\underline{\alpha})}{\partial \underline{\alpha}^2} > 0, \ \forall \underline{\alpha} \in \Re$, or $g(\cdot)$ is a convex function in $\underline{\alpha}$. Moreover, $g(\frac{1}{2}) < g(\overline{\alpha}_{min}) = 0$ while $\frac{1}{2} < x_m + \frac{1}{2} < \underline{\alpha} < w^* - c < \overline{\alpha}_{min}$. Thus, we conclude that $g(\underline{\alpha}) < 0$, and so we have proven that income inequality is reduced in the long run. Since this result is valid for $w^* - c + B \leq \overline{\alpha} < 1 + w^* - c$, the proof above also applies to the higher-medium inequality case.

E.3 Low Inequality Case

Since in the short run, the income dynamics is the same for the low and the lower-medium inequality cases, we can follow the same steps in the last case to show that the income inequality may rise in the short run (i.e. $\lim_{\underline{\alpha} \to (x_m + \frac{1}{2})} f(\cdot) > 0$). However, the effect is not certainly positive. For example, when $\overline{\alpha} = w^* - c$, we obtain

$$f(\underline{\alpha}) = [\underline{\alpha} - (w^* - c)] \left[1 + \frac{\sigma}{2c(1-\sigma)} - \frac{x_m}{(\underline{\alpha} - \frac{1}{2})(w^* - c - \frac{1}{2})} - \frac{\sigma x_m^2}{2c(1-\sigma)(\underline{\alpha} - \frac{1}{2})(w^* - c - \frac{1}{2})} - \frac{x_m}{\underline{\alpha} - \frac{1}{2}}\right]$$

Studying the properties of $f(\underline{\alpha})$, we first find that it is a strictly convex function, i.e. $\frac{\partial^2 f(\underline{\alpha})}{\partial \underline{\alpha}^2} > 0, \ \forall \underline{\alpha} > \frac{1}{2}$. Second, $f(\underline{\alpha}) = 0$ has two solutions: $\underline{\alpha} = w^* - c$ and $\underline{\alpha} = \frac{1}{2} + x_m \left[\frac{2c(1-\sigma)(w^* - c + \frac{1}{2}) + \sigma x_m}{[2c(1-\sigma) + \sigma](w^* - c - \frac{1}{2})}\right]$ $(> \frac{1}{2} + x_m)$. We can choose a set of parameter values such that the second root is smaller than the first one. So, we can ensure that the strictly convex function $f(\underline{\alpha})$ is negative, which implies that the income inequality is reduced in the short run, for all $\underline{\alpha}$ located within these two roots.

To prove that income inequality is reduced in the long run, we firstly rewrite y_{ss}^m as $x_m + (\frac{1}{\sigma} - 1)b_{ss}^m$, so it is equivalent to demonstrate that

$$\frac{x_m + (\frac{1}{\sigma} - 1)\overline{b}_{ss}^m}{x_m + (\frac{1}{\sigma} - 1)\underline{b}_{ss}^m} < \frac{\overline{\alpha} - \frac{1}{2}}{\underline{\alpha} - \frac{1}{2}} \quad \text{or} \quad \frac{x_m + (\frac{1}{\sigma} - 1)\overline{b}_{ss}^m}{\overline{\alpha} - \frac{1}{2}} < \frac{x_m + (\frac{1}{\sigma} - 1)\underline{b}_{ss}^m}{\underline{\alpha} - \frac{1}{2}}$$

Suppose $\overline{\alpha} = \underline{\alpha}$, the right hand side (RHS) of the second inequality is equal to the left hand side (LHS). Thus, it suffices to show that $\frac{\partial LHS}{\partial \overline{\alpha}} < \frac{\partial RHS}{\partial \overline{\alpha}} = 0$ holds for all $\overline{\alpha} > \underline{\alpha}$. Indeed, $\frac{\partial LHS}{\partial \overline{\alpha}} =$

$$-\left\{\frac{c(\frac{1}{\sigma} - 1)}{\overline{\alpha} - \frac{1}{2}} \left[1 + \frac{w^* - \overline{\alpha} - \frac{c}{\sigma}}{\sqrt{(w^* - \overline{\alpha} - \frac{c}{\sigma})^2 + 2(w^* - x_m - \frac{c}{\sigma})}}\right] + \frac{x_m + (\frac{1}{\sigma} - 1)\overline{b}_{ss}^m}{(\overline{\alpha} - \frac{1}{2})^2}\right\} < 0$$

F Gini Coefficient

Following the definition of the Gini coefficient that it is the ratio of area between the Lorenz curve and the uniform distribution of income to the whole area under the uniform distribution, we have

$$G_t = 1 - \frac{\frac{1}{2} \cdot \rho^2 \cdot \underline{y}_t(1 - \underline{m}_t^*)^2 + \frac{1}{2} \cdot \overline{y}_t(1 - \overline{m}_t^*)^2 + \rho \cdot \underline{y}_t(1 - \underline{m}_t^*)(1 - \overline{m}_t^*)}{\frac{1}{2}\left[\rho(1 - \underline{m}_t^*) + (1 - \overline{m}_t^*)\right] \cdot \left[\underline{y}_t \cdot \rho(1 - \underline{m}_t^*) + \overline{y}_t^*(1 - \overline{m}_t^*)\right]}$$

Define $\phi_t = \frac{1-\overline{m}_t^*}{1-\underline{m}_t^*}$. After some computations, we obtain the Gini coefficient shown in Section F.

G Proof for Proposition 4

We look at the scenario where the HP household is liquidity constrained (so $\overline{m}_t = \frac{\overline{b}_t}{c}$) and the labor market equilibrium changes from *CS* in the closed economy to *OS* in the open economy, or where there are always some LP family members working on their own farm. We choose to examine this type of transition because it is when $\overline{\alpha}$ and $\underline{\alpha}$ are the closest amongst all the possible transitions of equilibria.

In order to know whether $\Gamma^y_{ss} > \Gamma^y_0$, we examine the sign of

$$\Gamma^y_{ss} - \Gamma^y_0 = \frac{\overline{y}^m_{ss}}{\underline{y}^m_{ss}} - \frac{\overline{y}_0}{\underline{y}_0} = \frac{(\frac{1-\sigma}{\sigma})\overline{b}^m_{ss} + x_m}{(\frac{1-\sigma}{\sigma})\underline{b}^m_{ss} + x_m} - \frac{(\frac{1-\sigma}{\sigma})\overline{b}_1 + x_m}{(\frac{1-\sigma}{\sigma})\underline{b}_1 + x_m}$$

which is equivalent to know the sign of Ψ

$$\Psi = \underline{b}^m_{ss} \cdot \underline{b}_1 \cdot (\frac{\overline{b}^m_{ss}}{\underline{b}^m_{ss}} - \frac{\overline{b}_1}{\underline{b}_1}) + (\frac{x_m\sigma}{1-\sigma})(\overline{b}^m_{ss} - \underline{b}^m_{ss}) - (\frac{x_m\sigma}{1-\sigma})(\overline{b}_1 - \underline{b}_1)$$

with

$$\overline{b}_1 = \tfrac{\sigma}{1-\sigma}\left[\overline{\alpha} - \frac{1}{2} - x_m + \tfrac{\rho^2(\overline{\alpha}-\underline{\alpha})^2}{2(1+\rho)^2}\right] \text{ and } \underline{b}_1 = \tfrac{\sigma}{1-\sigma}\left[\underline{\alpha} - \frac{1}{2} - x_m + \tfrac{(\overline{\alpha}-\underline{\alpha})^2}{2(1+\rho)^2}\right].$$

Next, we derive $\overline{b}^m_{ss}$ as a function of $\underline{b}^m_{ss}$ from the LP household's wealth accumulation at its steady state:

$$\begin{aligned}\underline{b}^m_{ss} = \sigma\Bigg[&\underline{\alpha}(1 - \frac{\overline{\alpha} - \underline{\alpha} + \frac{\overline{b}^m_{ss}}{c} - \frac{\underline{b}^m_{ss}}{c}}{1+\rho} - \frac{\underline{b}^m_{ss}}{c}) - \frac{1}{2}(1 - \frac{\overline{\alpha} - \underline{\alpha} + \frac{\overline{b}^m_{ss}}{c} - \frac{\underline{b}^m_{ss}}{c}}{1+\rho} - \frac{\underline{b}^m_{ss}}{c})^2 \\ &+(\frac{\overline{\alpha} - \underline{\alpha} + \frac{\overline{b}^m_{ss}}{c} - \frac{\underline{b}^m_{ss}}{c}}{1+\rho})(\underline{\alpha} - 1 + \frac{\overline{\alpha} - \underline{\alpha} + \frac{\overline{b}^m_{ss}}{c} - \frac{\underline{b}^m_{ss}}{c}}{1+\rho} + \frac{\underline{b}^m_{ss}}{c}) \\ &+\frac{\underline{b}^m_{ss}}{c}(w^* - c) + \underline{b}^m_{ss} - x_m\Bigg]\end{aligned}$$

$$\Rightarrow \tfrac{\overline{b}^m_{ss}}{c(1+\rho)} = \tfrac{\underline{b}^m_{ss}}{c(1+\rho)} - \tfrac{\overline{\alpha}-\underline{\alpha}}{1+\rho} + \sqrt{(\tfrac{\underline{b}^m_{ss}}{c})^2 + 2\cdot(\tfrac{c}{\sigma} - w^* + \underline{\alpha} - 1)\cdot\tfrac{\underline{b}^m_{ss}}{c} - 2(\underline{\alpha} - x_m - \tfrac{1}{2})}$$

Then, we obtain

$$\begin{aligned}\frac{\overline{b}^m_{ss}}{\underline{b}^m_{ss}} - \frac{\overline{b}_1}{\underline{b}_1} = (1+\rho)&\sqrt{1 + 2\cdot(\tfrac{c}{\sigma} - w^* + \underline{\alpha} - 1)\cdot\tfrac{c}{\underline{b}^m_{ss}} - 2\cdot(\underline{\alpha} - x_m - \tfrac{1}{2})\cdot(\tfrac{c}{\underline{b}^m_{ss}})^2} \\ &-(\overline{\alpha} - \underline{\alpha})\left[\tfrac{c}{\underline{b}^m_{ss}} + \frac{1 - \frac{(\overline{\alpha}-\underline{\alpha})(1-\rho)}{2(1+\rho)}}{\underline{\alpha} - \frac{1}{2} - x_m + \frac{(\overline{\alpha}-\underline{\alpha})^2}{2(1+\rho)^2}}\right]\end{aligned}$$

With the two relationships we just obtained above, we can now rewrite Ψ:

$$\begin{aligned}\Psi = c(1+\rho)(\underline{\alpha} - \tfrac{1}{2} + \tfrac{(\overline{\alpha}-\underline{\alpha})^2}{2(1+\rho)^2})&\sqrt{(\tfrac{\underline{b}^m_{ss}}{c})^2 + 2\cdot(\tfrac{c}{\sigma} - w^* + \underline{\alpha} - 1)\cdot\frac{\underline{b}^m_{ss}}{c} - 2\cdot(\underline{\alpha} - x_m - \tfrac{1}{2})} \\ &-(\overline{\alpha} - \underline{\alpha})\left[c(\underline{\alpha} - \tfrac{1}{2} + \tfrac{(\overline{\alpha}-\underline{\alpha})^2}{2(1+\rho)^2}) + (\underline{b}^m_{ss} + \tfrac{\sigma x_m}{1-\sigma})(1 - \tfrac{(\overline{\alpha}-\underline{\alpha})(1-\rho)}{2(1+\rho)})\right]\end{aligned}$$

Finally,

$$\lim_{\overline{\alpha}\to\underline{\alpha}} \Psi = c(1+\rho)(\underline{\alpha}-\tfrac{1}{2})\sqrt{(\tfrac{b_{ss}^m}{c})^2+2\cdot(\tfrac{c}{\sigma}-w^*+\underline{\alpha}-1)\cdot\frac{b_{ss}^m}{c}-2\cdot(\underline{\alpha}-x_m-\tfrac{1}{2})} \quad > 0$$

Hence, we have proved that when $\overline{\alpha}$ is sufficiently close to $\underline{\alpha}$, the income inequality is increased in the long run, i.e. $\Gamma_0^y < \Gamma_{ss}^y$.

In figure 5, where we choose identical parameter values to those in figure 4, we actually observe that Ψ is positive, or the long-run income inequality is increased, for a good range of $\overline{\alpha}$. In comparison, the wealth inequality is always decreased, i.e. $\Gamma_1^b > \Gamma_{ss}^b$.

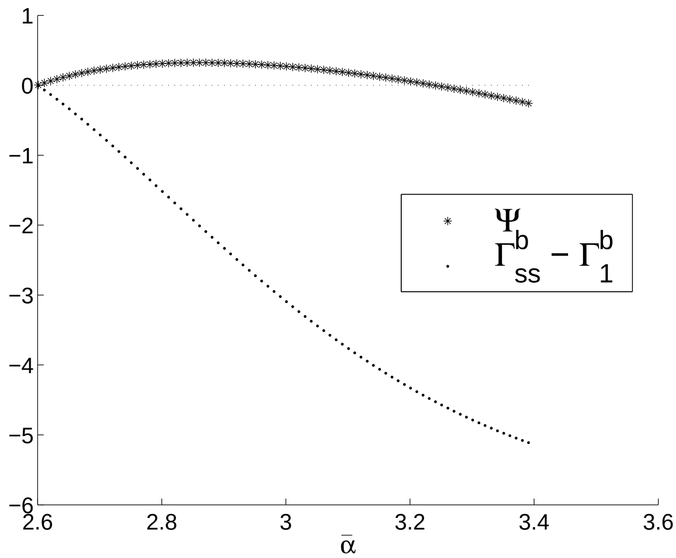

Figure 5: The income inequality is increased in the long run when $\overline{\alpha}$ is sufficiently close to $\underline{\alpha}$ ($\underline{\alpha} = 2.6$, $\sigma = 0.25$, $x_m = 2$, $c = 1.5$, $w^* = 7.5$, $\rho = 1.5$).

Département des Sciences Économiques
de l'Université catholique de Louvain
Institut de Recherches Économiques et Sociales

Place Montesquieu, 3
1348 Louvain-la-Neuve, Belgique

ISSN 1379-244X D/2007/3082/003