IPR for Public and Private Innovations, and Growth

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IPR for Public and Private Innovations, and Growth

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Abstract

The empirical analyses show that public and private R&D are strongly intertwined. On the one hand, the existence of large direct spillovers from public R&D to private industry has extensively proven. Yet, both a substitutability and complementarity relationship between private and public R&D investment has been found. From an institutional point of view, to stimulate the technology transfer from public R&D to private industry the U.S. adopted an uniform patent policy for public funded research, such as that guaranteed by the Bayh-Dole Act. This paper contributes to explain this empirical evidence. Within a neo-Schumpeterian endogenous growth model, it is shown that the intellectual appropriation share of a new commercial valuable idea by private firms and the subsidy of private R&D cost are two equivalent ways to stimulate private R&D effort, and they affect in the same way the endogenous per capita output growth rate. The existence of a trade off between the per capita output growth rate and level has found. The main policy implication of these results consists into guarantee two different regimes of IPR for industrial and public innovations. Furthermore, it is shown that the large direct spillovers from public R&D to private industry allows to have better growth performance even if public R&D investment crowds out private innovative effort. Again a trade off between the per capita output growth rate and level has found.

Keywords: Intellectual Property Rights, Private and Public R&D, Growth

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1 Introduction

The role of the technological progress and of the Research and Development (R&D) investments for the growth performance of a country is emphasized by

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both academic and non-academic analyses. In several developed countries both private and public institutions invest large amounts of resources in R&D (see the National Science Foundation reports). In such countries intellectual property rights (IPR) - often in the form of a patents - have granted at the offsprings of the R&D effort. Since the '80s the U.S. adopted several legislative and institutional arrangements to reinforce the ties between public and private R&D, and to spur the technology transfer of discoveries and inventions from public research to private industry. Among the several legislative acts the most influential has been the Bayh-Dole Act of the 1980. This Act instituted an uniform federal patent policy for universities and small businesses under which they obtained the rights to any patents resulting from grants and contracts funded by any federal agency and to licence these patents on an exclusive or non-exclusive basis.[1] Based on the belief that legislative arrangements such as the Bayh-Dole Act enhance the technology transfer and the academic contributions to innovation and growth in the U.S., similar legislation is being considered in other OECD countries (OECD, 2002).

The existence of interrelations between public and private R&D is also proven by the empirical evidence. The empirical analyses show that private firms benefit of large spillovers from public discoveries and innovations (direct spillovers), whatever is their effect on the private innovative effort. Narin *et al.* (1997), and McMillan *et al.* (2000) show that, for the U.S. industry, relying in external sources of knowledge centers on public science. In particular, Narin *et al.* (1997) show that during the 1993-1994, 73% of the scientific paper cited by U.S. industrial patents were firm public science sources. From a theoretical point of view these spillovers are explained both through the intrinsic nature of knowledge as a nonrival input (see Arrow 1962) and the Merton's issue of priority of scientific discoveries.[2]

Furthermore, some empirical analyses study the effect of public R&D investment in stimulating private research effort. Recently, David *et al.* (2000) also find the existence of large and direct spillovers from public R&D to private industry, and the same authors maintain that these spillovers "...often are held to enhance private sector productive capabilities, and, specifically, to encourage applied R&D investments by firms that lead to technological innovations - from which will flow future streams of producer and consumer surpluses."[3]

[1] Others legislative acts in such direction are the Stevenson-Wydler Technology Innovation Act of the (1980), the Small Business Innovation Development Act (1982), the National Cooperative Research Act (1984), the Federal Technology Transfer Act (1986), the National Cooperative Research and Production Act (1993), the Technology Transfer and Commercialization Act (2000).

[2] Merton (1973) argued that - within the non market rewards structure - the goal of scientists is to establish the priority of discovery by being the first to announce an advance in knowledge. Therefore, the rewards to priority are the recognition awarded by the scientific community for being the first.

[3] The same authors maintain: "Several recent econometric studies, for example, document positive, statistically significant 'spillovers' effects via the stimulation of private R&D investment by publicly funded additions to the stock of scientific knowledge." See also Leyden and Link (1991). Other macro empirical works that show the existence of complementarity between publicly funded R&D and private R&D are Levy (1990), Robson (1993), Diamond

Yet, this complementarity relationship between public and private R&D investments is not general, in fact the empirical analysis also shows the existence of a substitutability relationship between private and public R&D investment.[4]

Therefore, the empirical evidence shows that private and public R&D efforts are strictly intertwined. The existence of large direct spillovers from public R&D to private industry has extensively proven. Yet, a clear substitutability-complementarity relationship between public and private research effort is not proven. Moreover, the institutional set-up of a country in the form of IPR can affect the ties between private and public R&D.

This paper considers these interrelations, and it sheds light to explain the empirical evidence mentioned above. To this aim a neo-Schumpeterian growth framework à la Aghion and Howitt (1992, 1998), and Howitt (1999, 2000) is adopted. The neo-Schumpeterian growth literature considers innovation, knowledge accumulation, and technological progress as the engines of the economic growth. In large part of the neo-Schumpeterian literature policy consists into guarantee patent protection at the private innovative firms and to subsidize their R&D cost, without any direct presence of the government in the R&D sector.[5] The patent-design literature, on the other hand, addresses the question of how does patent policy affect incentives for industrial R&D. Recently, O'Donoghue and Zweimuller (2004) extend the patent-design literature to a general equilibrium framework. However, also these authors only pay attention at the effect on the industrial R&D.

This paper distinguishes in the description of the R&D sector, where both the private and public sectors invest in R&D. In this framework the industrial sector can conduct both the basic and development stage of the R&D activity,[6] while the public sector invests in basic research. Because the empirical evidence shows that the development stage is the prominent activity of the private research effort,[7] the drastic, yet realistic, assumption that public R&D only consists in

(1998), Von Tunzelmann and Martin (1998).

[4] David *et al.* (2000) remark that empirical evidence on public R&D as complement or substitute for private R&D is not conclusive; in fact the authors maintain: "...available empirical evidence on the issue remains rather short of being conclusive, to say at least."

[5] The theoretical conclusions are not univocal. Some theoretical and empirical analyses conclude that policy has positive effect on both per capita output growth rate and on per capita output level. The alternative view concludes that policy is ineffective on per-capita output growth rate, even if it can produce positive effects on the per capita output level (see Jones 1995, 1999). Although a recent empirical analysis by Ha and Howitt (2006) seems to consider public policy effective even on the per capita growth rate of countries, a conclusive result, both theoretical and empirical, can not be again obtained.

[6] The Science and Engineering Indicators (SEI, 2006) by the National Science Foundation defines as basic the research aimed "to gain more comprehensive knowledge or understanding of the subject under study without specific application in minds". The development stage is defined as "the systematic use of the knowledge or understanding gained from research directed toward the production of useful materials, devices, systems, or methods, including the design and development of prototypes and processes."

[7] The SEI states: "The federal government, estimated to have found 61,8% of U.S. basic research in 2004, has historically been the primary source of support for basic research...Industry devoted only an estimated 4,8% of its total R&D support to basic research in that year." (SEI 2006, ch.4 p.13). In addition, "The development of new and improved goods, services, and

basic research programs is introduced. Furthermore, the innovations of both public and industrial R&D are granted of patent protection. Public research introduces new basic discoveries a share of which can be usefully developed by the private industry to create a new commercial valuable product. This share can be assumed to be stochastic. The industrial sector pays the licenses to develop the patented public innovations. Therefore, it will only appropriate a share of the final market value of a new product.

This paper studies the macroeconomic implications of the interplay between industrial and public investments in R&D. The results show that the subsidy to private R&D cost and the intellectual appropriation's share of a commercial valuable idea by a private R&D firm are two - in some sense - equivalent ways to finance the industrial R&D effort. In fact, both these ways increase the private innovative effort and the per capita output growth rate, while they reduce the per capita output level. Yet, the subsidy and the intellectual appropriation of a new idea have some fundamental differences The subsidy of the private R&D cost concerns the certainty aspect of a R&D process, while the intellectual appropriation's share of a commercial valuable idea concerns the uncertainty aspect of a R&D process. Furthermore, the subsidy and the intellectual appropriation have deep differences from an institutional point of view. The intellectual appropriation of new ideas concerns both public and private research activity, and the policy 'design' of the IPR involves the political, executive, legal, and jurisprudential powers of a country. The subsidy to private research cost does not deeply shape the institutional set-up of a country as the IPR policy 'design' does, and it can also be used as a fine-tuning policy instrument.

Although the mere introduction of some form of IPR regime for public innovations reveals to be an efficient way to stimulate both the private innovative effort and the per capita output growth rate of a country. A policy implication is that - once IPR are introduced for public innovations - a different regime for private and public innovations should be introduced. Patents to public innovations should be only granted to some radical and very innovative ideas. In fact, the existence of IPR for public innovations generates a further R&D cost for the industrial sector to acquire the licenses from public institutions. This implies that a patent design should guarantee a larger patentability requirement and a lower leading breadth for public basic ideas than for industrial innovations.[8] In fact, a larger patentability requirement and a lower leading breadth both allow the private firm to appropriate of a higher share of a new product's market

processes is dominated by industry, which performed 90.2% of all U.S. development in 2004." (SEI 2006, ch.4 p.13).

[8] The patentability requirement is a minimum innovation size required to receive a patent. A patent breadth's put restrictions on the products other firms can produce without a license. In particular, the leading breadth limits future innovators by specifying superior products that other firms can not produce (see O'Donoghue, 1998, and O'Donoghue and Zweimuller, 2004). Based on these definitions, the leading breadth could only concern a new marketable product, and it could not apply at a basic innovation that - by its own definition - does not have an immediate and specific market application. However, if the leading breadth also applies at a basic innovation, the policy implication of the paper follows.

value. The policy to grant patents at few and very radical basic innovations for the public sector does not restrict the technological transfer from universities to industrial R&D. In fact, the empirical analyses show that the private sector uses many channels to access public and universities discoveries. Cohen *et al.* (2002) find that the most important channels to access publicly funded research are publications, conferences, informal interactions rather than more institutional channels such as patents, licenses, and cooperative ventures. These results refer to all industrial sectors, even the high-tech industries.[9]

Recently, a paper by Cozzi and Galli (2007) focuses on the sequential nature of the innovation process within a dynamic general equilibrium framework. Their paper considers a two stage innovation process, and it evaluates the conditions under which IPR should be extended to basic discoveries that does not have an immediate and specific marketable application and a commercial value. Cozzi and Galli (2007) show that a pro-growth policy consisting into guarantee an intellectual property protection for 'basic half-ideas' was at the ground of the reforms undertaken in the U.S. around the '80s. Moreover, Cozzi and Galli (2007) explicitly determine the market value of a 'basic half-idea'. The focus of this paper is on the strength and width of patents granted to public R&D, once patent protection for public innovation already exists in a country. Therefore, this paper complements Cozzi and Galli's (2007) contribution. O'Donoghue and Zweimuller (2004) study the general equilibrium implications of a larger patent protection against future innovators within a neo-Shumpeterian growth framework. Their results show that both a larger patentability requirement and a larger leading breadth spur the private innovative effort, and therefore the per capita output growth rate. However, their work does not consider neither the role of the public R&D investment nor a patent protection for public innovations.

Furthermore, and differently from Cozzi and Galli (2007), this paper shows how the mixed empirical evidence about the substitutability-complementarity relationship between public and private R&D investment can be explained. It is shown that a higher public investment in R&D can either crowd out or complement private R&D effort. The substitutability-complementarity relationship between public and private R&D investment depends on the structural parameters of the economy, on the population growth rate, and on the intensity through which private and public R&D affect the probability to find a new commercial valuable idea. In particular, it is shown that whenever population growth rate is not high enough, a higher public R&D investment diverts too much resources from the private sector - both manufacturing and R&D - and thus it crowds out private R&D investment. The effect on the per capita output growth rate depends on the direct spillovers public innovations generate on the private industry. As indicated above, the existence of large and positive spillovers from public R&D to private industry has found by the empirical analysis. This paper

[9] Cohen *et al.* (2002) find that the pharmaceutical industry more heavily conveys public research knowledge through patents and licenses. However, the same authors maintain that even in this industry informal channels and open science are still more important in conveying public R&D discoveries.

proves that - because of these large spillovers - a higher public investment in R&D better off the growth performance of a country. These results are reinforced when population growth rate is high enough, so that a higher public R&D investment only crowds out the manufacturing resources, while it complements the private innovative effort. Finally, the existence of a trade-off between the per capita output growth rate and level has found. This trade-off is stronger when a complementarity relationship between private and public R&D exists.

The paper is organized as follows. Section 2 sets up the model, section 3 describes the general equilibrium results, section 4 concludes.

2 The Model

This economy is composed of a final good sector, of an intermediate good sector, and of a R&D sector. As in Aghion and Howitt (1992, 1998), competitive firms produce a homogeneous final consumption good by using a fixed input, and a continuum of intermediate goods with heterogeneous productivity. Intermediate sector produces a continuum of products denoted by N_t at a given time $t \geq 0$. The mass of intermediate goods is continuously enlarging thanks to serendipitous imitation as in Howitt (2000). The manufacturing sector is characterized by free entry and exit, and by a constant returns to scale technology: workers can be hired by a continuum of firms that produce their intermediate goods on a one to one basis from labor. A legally imposed distortion - the Patent System - renders each of them a local monopoly. According to the standard Schumpeterian approach à la Aghion and Howitt (1992 and 1998), each intermediate monopolistic firm is challenged by outsider R&D firms trying to invent and patent a better product and - due to instantaneous price competition - drive the former monopolist out of the market.

The R&D sector is composed of both private and public research. Private R&D consists into upgrade the quality (or the production process) of the intermediate products (vertical innovation). As said above, a perfectly enforceable patent law allows the researcher to gain monopolistic rents for all the effective duration of the patent, because - as usual in neo-Schumpeterian growth models with vertical innovation - the incumbent monopolist can be replaced by the next innovator in the same product line.[10] Therefore, it generates the Schumpeterian creative destruction effect. The existence of a perfect stock market channels consumer savings to firms engaged in R&D. Moreover, the R&D sector is composed of public research: the government employes skilled workers to obtain basic innovations and discoveries. According to the legislative acts mentioned in the introduction, patents are granted to basic innovations. These basic innovations are developed by private R&D firms to find a new commercial valuable innovation and to introduce new intermediate goods.

[10] See Aghion and Howitt (1992, 1998), Segerstrom (1998), Howitt (1999). Cozzi (2007) proves that the standard neo-Schumpeterian growth models are compatible with a positive and finite R&D investment by the incumbent monopolistic firms. The analysis of this paper is also compatible with Cozzi's (2007) findings.

2.1 Basic Framework

Let us assume continuous time and unbounded horizon. In this economy a mass $L_t > 0$ of infinitely lived families exists. Each family has an identical preference for non-negative consumption flows represented by the intertemporal utility function $\int_0^\infty e^{-rt} C_t dt$, where C_t is the non negative consumption flow of each household, $r > 0$ is the common and constant subjective rate of time preference. The linear instantaneous utility implies constant real interest rate always equal to r. Moreover, each family is endowed with a unit mass of flow labor time bearing no disutility. Population growth is constant and equal to $g_L > 0$. The labor market is perfect and the inelastic supply of labor L_t is instantaneously employed by manufacturing firms and by the R&D sector.

Final output is produced by perfectly competitive firms combining the fixed factor with a large variety of intermediate goods, that is:

$$Y_t = M^{1-\alpha} \int_0^{N_t} A_{it} x_{it}^{\alpha} di \tag{1}$$

with $0 < \alpha < 1$. x_{it} is the amount of intermediate good i produced and used as an input at a given time $t \geq 0$, and A_{it} is the productivity parameter of the current version of that good. M is the aggregate mass of fixed factor (such as for example, "land, minerals, oils", etc.). $N_t \in [0, \infty)$ denotes the mass of intermediate goods already invented in the economy at date $t \geq 0$. Since in each sector instantaneous Bertrand competition guarantees that only the most advanced patent holder will be producing, N_t also denotes the mass of active intermediate good industries. The elasticity of substitution between intermediate products is equal to $\varepsilon \equiv \frac{1}{1-\alpha} > 1$.

The perfectly competing R&D firms try to achieve and appropriate the next generation of any intermediate good (vertical innovation process). According to Aghion and Howitt (1998), and Howitt (1999) the leading-edge technology has considered, with an economy-wide leading edge productivity parameter $A_t^{\max}$ that exerts positive R&D spillovers in all intermediate goods. When a new commercial valuable discovery is introduced into an intermediate product line i (a better quality of that intermediate good is introduced) the productivity parameter A_{it} in that line jumps to $A_t^{\max}$. This specification incarnates Aghion and Howitt's (1998 ch. 3) and Howitt's (1999) inter-sector knowledge spillovers.

The technological frontier $A_t^{\max}$ grows deterministically at a rate proportional to the per product line rate of vertical innovations. The Poisson arrival rate of vertical innovations in any product line i is $\lambda_A f(l_{At}, b_t)$. λ_A is a productivity factor, $l_{At} = \frac{L_{At}}{N_t}$ is the per product line research labor time, $b_t = \frac{B_t}{N_t}$ is the per product line stock of basic knowledge that can be usefully developed to generate a new patentable and commercial valuable idea in any product line i. The function $f(l_{At}, b_t)$ captures the effect of the two R&D efforts, private and public, into generate a new marketable product and therefore to increase the technological frontier (see the appendix A, point 1). As the economy develops an increasing number of intermediate goods, an innovation of a given size in any product line will have a smaller impact on the aggregate economy; hence the marginal impact of each innovation on the stock of public knowledge will be $\frac{\sigma}{N_t}$, where $\sigma > 1$ is a proportionality factor. The aggregate flow of vertical innovations is the number of intermediate goods N_t times the expected flow of vertical innovations per industry line. The economy-wide rate of vertical technological progress is described by the following:

$$g_{At} = \frac{\dot{A}_t^{\max}}{A_t^{\max}} = \frac{\sigma}{N_t} \int_0^{N_t} \lambda_A f(l_{At}, b_t) = \sigma \lambda_A f(l_{At}, b_t) \tag{2}$$

The generic specification of $f(\cdot)$ leaves room to many ways through which the stock of basic knowledge affects the productivity of private R&D. As an example, only a share of the stock of basic knowledge can be usefully developed, and/or the duplication argument can be applied both to private innovative effort and to basic discoveries.[11] Therefore, a new better quality version of any intermediate product is the result of private innovation that renders marketable and commercial valuable the offsprings of both public basic and private research effort.

According to this framework, in equilibrium we will observe an ever-evolving intersectoral distribution of the absolute productivity parameters A_{it}, with values ranging from 0 to $A_t^{\max}$. Defining $a \equiv \frac{A_{it}}{A_t^{\max}}$, we can concentrate on the relative intersectoral distribution, that - as shown in Aghion and Howitt (1998, ch. 3) and in Howitt (1999) - converges to the unique stationary distribution of relative productivity parameters - a - characterized by cumulative distribution function $H(a) = a^{\frac{1}{\sigma}}$, with $0 \leq a \leq 1$. Every time a better quality of an intermediate good is introduced into the economy, the absolute distribution will be re-scaled rightward because the technological process rises to $A_t^{\max}$.

The mass of intermediate products grows as a result of serendipitous imitation, not deliberate innovation.[12] Each person has the same propensity to imitate $\beta > 0$, thus the aggregate flow of new products is:

$$\dot{N}_t = \beta L_t \tag{3}$$

Sine population grows at the constant rate g_L, the number of workers per product line $\frac{L_t}{N_t}$ converges monotonically to $\frac{g_L}{\beta}$.

[11] The per product line stock of basic ideas b_t encompasses all discoveries that can be useful to private industry in a wide meaning. It includes all basic ideas that help each private economic agent to have new ideas, insights, developments, etc. Since intellectual property rights for public R&D ideas exist - a share of the stock of basic discoveries b_t has obtained a patent grant - the private firm incurs a cost to use patented basic discoveries. Notice that the spillovers per product line basic discoveries have on the productivity of private research effort concerns the stock of basic knowledge accumulated over time, and not only the flow of new basic discoveries. Hence, the stock of basic ideas can be used for the development of different versions of an intermediate product.

[12] See Howitt (2000). In Howitt (1999), and Cozzi and Spinesi (2005) horizontal innovation is motivated by the same profit seeking objectives as quality improving innovation. The results of this paper are not qualitatively affected by this specification for horizontal innovation.

2.2 Asset Market, Manufacturing, and Vertical R&D

The private and public research effort allows to increase the technological frontier. As remarked in the introduction, a public basic idea does not have an immediate and specific commercial application, and it will be developed by the industrial R&D sector to find a new marketable intermediate product. Therefore, the commercial value of a new intermediate product is given by the firm's expected stock market value that monopolizes the commercialization of the new intermediate product. Let V_t be the expected stock market value of a new intermediate product with maximum productivity $A_t^{\max}$. The private firm pays for the use of patented public basic ideas, and therefore it will appropriate a share of the commercial value of the new intermediate product. The share of the market value respectively appropriated by the private R&D firm and by the public research unit that have contributed to introduce the new intermediate product is described as a Nash-bargaining solution between these two forces. The bargaining process is described as taking place between a representative firm and a public institution from which the rights on the patented basic innovations are acquired. Because of the symmetry in the R&D sector, each R&D firm behaves symmetrically.[13] Let V_t^p be the expected stock market value of a new intermediate product appropriated by the private R&D firm, and let V_t^b be the expected stock market value of a new intermediate product appropriated by the public institution, with $V_t = V_t^p + V_t^b$. The expected stock market value appropriated by private and public innovators is the solution to the following:

$$\begin{aligned} &\max_{V_t^p, V_t^b} \left(V_t^p\right)^{\phi} \left(V_t^b\right)^{1-\phi} \\ s.t.\ V_t &= V_t^p + V_t^b \end{aligned} \tag{4}$$

The solution to this problem gives $V_t^p = \phi V_t$, and $V_t^b = (1-\phi) V_t$. The parameter $\phi \in (0,1)$ represents the institutional set-up in which the bargaining process takes place.[14] The existence of laws such as the Bayh-Dole Act - and of other legislative arrangements - heavily contribute to determine the value of the parameter ϕ in the economy. In fact, ϕ indicates that a private R&D firm

[13] Because of the macroeconomic framework of this paper, the bargaining process is here described with a centralized set-up. A representative firm bargains with a centralized public institution to buy the rights on a stock of basic ideas per unit of time. This stock can be usefully developed by private R&D firms.

[14] As shown by Cozzi and Galli (2007) the mere introduction of IPR for public basic ideas can better off the growth performance of a country. In this paper the tightness and the ease of the IPR regime - as measured by ϕ - has studied. Therefore, $\phi < 1$ is assumed. On the other hand a low value of ϕ can indicate that it is extremely easy for public R&D to obtain patent grants for any basic innovation. This implies that private firms must pay to also use basic discoveries that have a very low innovative power. When $\phi = 0$ any incentive for private R&D effort disappears, i.e. $l_{At} = 0$. Therefore, $\phi > 0$ is assumed. Notice that, in this macroeconomic set-up, the share $(1-\phi)$ appropriated by the public sector can encompass different public patents all useful to develop a new intermediate product, and not just one patented basic public idea. As indicated in the Appendix A, point 1, the per product line stock of basic innovations granted of patent protection can be a random variable $p \in [0, \bar{p}]$ with

pays to use basic ideas which are granted of some form of IPR. In what follows the parameter ϕ is assumed constant within each intermediate product line and between them. Because the break-down of the bargaining process between private and public institutions gives a zero value at both of them, at least one firm will buy the basic patented ideas. In fact, because of the symmetry, all R&D firms will pay the same price for the basic ideas and they will appropriate of a share ϕ of the expected commercial value of an innovation V_t. On the other hand, the government will have the incentive to license the same basic ideas at least at one private R&D firm.

Each vertical R&D firm targeting an intermediate product i chooses its R&D intensity to maximize $\phi V_t \lambda_A \psi\left(l_{Ajt}, b_t\right) - (1-s)\, w_t l_{Ajt}$, where l_{Ajt} is the labor time flow employed by the vertical R&D firm j at time t, s is the subsidy to private research. Rational individuals and firms know they will appropriate a fraction ϕ of the expected stock market value of a patentable and commercial valuable idea. Notice that, the per product line basic stock knowledge is taken as given by each individual and firm. This condition determines the optimal choice $l_{Ajt} = l^*_{Ajt}$ for each R&D firm (see the appendix A, point 2).

Applying Aghion and Howitt's (1992 and 1998) methods, the intermediate good i production level that maximizes the monopolist profits at time t is

$$x_{it} = M\left(\frac{\alpha^2 A_{it}}{w_t}\right)^{\frac{1}{1-\alpha}},$$

because the distribution of relative productivities is unchanging, we do not classify the sectors by their index i but by their relative productivity $a \equiv \frac{A_{it}}{A_t^{\max}}$. Defining the productivity-adjusted real wage as $\omega_t \equiv \frac{w_t}{A_t^{\max}}$ and normalizing the fixed factor to one (that is positing $M = 1$) the instantaneous labor demand function for a sector with relative productivity a at date t is rewritten as:

$$\tilde{x}_{it}\left(\frac{\omega_t}{a}\right) = \left(\frac{\alpha^2 a}{\omega_t}\right)^{\frac{1}{1-\alpha}} \tag{5}$$

where $\tilde{x}_{it}\left(\frac{\omega_t}{a}\right)$ is a labor demand function for the manufacturing firm. The labor force employed in the manufacturing sector negatively depends on the productivity-adjusted real wage.

We will focus on the symmetric steady state, that is $x_{it} = x_t$, $l_{Ait} = l_{At}$, etc., for every intermediate product line i.[15] Because the R&D sector is characterized by free entry and exit, in equilibrium the following condition must hold for a successful R&D firm: $\phi V_t \lambda_A \psi\left(l^*_{Ajt}, b_t\right) - (1-s)\, w_t l^*_{Ajt} = 0$ (see the appendix A, point 2). This condition can be rewritten as:

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a common cdf $F(p)$. Therefore, each product line has the same average value $\tilde{p} = \int_0^{\bar{p}} p dF(p)$. Because of the intersectoral knowledge spillovers and by the Law of large numbers, the share of the per product line patented public ideas acquired by the industrial sector is deterministic.

[15] As proven by Cozzi (2005), Howitt's (1999) model admits a continuum of symmetric balanced growth paths.

$$w_t = \frac{\phi \lambda_A \psi \left(l^*_{Ajt}, b_t\right)}{(1-s)\, l^*_{Ajt}} V_t \tag{6}$$

Therefore, in a the multisector economy a condition very similar to the manufacturing-R&D no-arbitrage equation as in Aghion and Howitt (1998) has obtained. Notice that, in eq. (6) the wage paid to a researcher in the industrial R&D laboratory w_t is proportional to the average Poisson arrival rate of innovation of the industrial R&D laboratory itself, i.e. is proportional to $\frac{\lambda_A \psi\left(l^*_{Ajt}, b_t\right)}{l^*_{Ajt}}$. By following the same steps as in Aghion and Howitt (1998, ch. 3, Appendix) from eq. (6) it follows:

$$\begin{aligned} \frac{(1-s)\, l^*_{Ajt}}{\phi \lambda_A \psi \left(l^*_{Ajt}, b_t\right)} w_t &= V_t = A_t^{\max} \int_0^\infty e^{-(r+g_A/\sigma)\tau} \tilde{\pi}\left(\omega e^{g_A \tau}\right) d\tau = \\ &= A_t^{\max} \int_0^\infty e^{-(r+g_A/\sigma)\tau} \frac{1-\alpha}{\alpha} \omega_t \tilde{x}(\omega)\, e^{-\frac{\alpha}{1-\alpha} g_A \tau} d\tau \end{aligned} \tag{7}$$

On the right hand side of eq. (7) the discount rate $(r + g_A/\sigma)$, and the profit flows $A_t^{\max} \tilde{\pi}\left(\omega e^{g_A \tau}\right)$ accruing to a successful innovator from date t to infinity have been considered.

2.3 Public R&D

Population differs in the basic research ability, while there are no quality differences among workers employed in vertical R&D and in manufacturing. Let us $G(\theta)$ be the cumulative distribution function (cdf) of the 'basic research ability' θ, with θ taking value on $\left[0, \bar{\theta}\right]$ and $\bar{\theta} \leq 1$. The usual properties $G^{'}(\theta) > 0$, $G(0) = 0$, $G\left(\bar{\theta}\right) = 1$ apply. Since each worker must be indifferent between manufacturing and vertical research activity, it will be $w_t = \frac{\phi \lambda_A \psi\left(l^*_{Ajt}, b_t\right)}{(1-s) l^*_{Ajt}} V_t$. The additional no-arbitrage condition between improving/manufacturing and basic research effort can be written as:

$$w_t = \lambda_B \varphi \left[E \left(\frac{A_{it}}{A_t^{\max}} \right)^{-1} \right] \theta_0 w_t = \tilde{\lambda}_B \theta_0 w_t \tag{8}$$

where the function $\varphi \left[E \left(\frac{A_{it}}{A_t^{\max}} \right)^{-1} \right]$ represents the spillovers from vertical innovation to basic research, and $\tilde{\lambda}_B \equiv \varphi(1+\sigma) \lambda_B$.[16] The left hand side of eq. (8) indicates the expected returns of manufacturing and improving the quality of an intermediate product. The right hand side of the last part of eq. (8) indicates the expected flows return to be employed in basic research. Because

[16] $\varphi(\cdot)$ can be any positive function of the average relative productivity $E\left[\left(\frac{A_{it}}{A_t^{\max}}\right)\right] = (1+\sigma)^{-1}$.

$\theta \in [0, \bar{\theta}]$, the research gross salary paid by the public sector is lower than the wage paid by the industrial R&D laboratory.[17] The parameter $\tilde{\lambda}_B$ allows to increase the perceived salary of a public researcher. In this framework the parameter $\tilde{\lambda}_B$ also positively depends on public expenditures.[18]

Let us denote θ_0 the threshold value of the 'basic research ability' that satisfies equality (8): θ_0 ability researchers are indifferent between trying to improve the quality of one of the existing intermediate goods, to be employed in basic research, and to be employed in the manufacturing sector. The higher the basic research talent an individual is endowed with, the higher the gain to be employed in basic research. The no-arbitrage equation (8) determines the threshold ability value

$$\theta_0 = \frac{1}{\tilde{\lambda}_B}, \tag{9}$$

which is constant along the BGP.[19] Each individual endowed of a research ability $\theta > \theta_0$ will find it profitable to be employed in basic research. Hence, in such an economy, for $\theta > \theta_0$, $[1 - G(\theta_0)] L_t$ individuals will choose to be employed in basic research. Instead, the individuals endowed with an ability $\theta \leq \theta_0$, that is $G(\theta_0) L_t$, will decide either to introduce a better quality of the existing intermediate goods, or to work in the manufacturing sector. A policy that affects the productivity of basic research effort also affects the threshold ability parameter θ_0. This in turn changes the population employed in basic research programs, and therefore the per product line stock of basic knowledge. This implies that the institutional set-up can affect in different ways the interplay between public and private research effort.

The government conducts basic research to accumulate basic knowledge B_t according to the following dynamic law

$$\dot{B}_t = \tilde{\lambda}_B \left[\int_{\theta_0}^{\bar{\theta}} \theta G'(\theta) \, d\theta \right] L_t = \tilde{\lambda}_B m(\theta_0) L_t \tag{10}$$

where $m(\theta_0) L_t \equiv \left[\int_{\theta_0}^{\bar{\theta}} \theta G'(\theta) \, d\theta \right] L_t$ is the expected conditioned cumulated basic research effort, $\tilde{\lambda}_B$ is the productivity of each researcher engaged in basic research. Eq. (10) implies that the stock of basic knowledge B_t is accumulated at the same rate as the population growth rate g_L.

[17] See Aghion et. al (2005), and the Science and Engineering Indicators (2004, 2006) for the empirical evidence of an average higher wage gained in the private R&D laboratories than in nonprofit/government R&D laboratories.

[18] To fix ideas, the public expenditures affect the quality of laboratories equipment, because higher public expenditures allow to have higher quality of equipment. This affect the expected and perceived gain of a public researcher. Moreover, public researcher often obtains some financial support from a variety of institutions above their contractual gross salary. These elements contribute to increase the expected gain of a public researcher.

[19] It assumed that $\frac{1}{\tilde{\lambda}_B} < \bar{\theta} \leq 1$.

The government uses proceeds from basic innovations to finance the public research and to subsidize vertical R&D firms. The public balanced budget in each instant of time requires (see Appendix A, point 3):

$$\begin{aligned} m(\theta_0) L_t w_t + s w_t N_t l_{At} + E\left(A_t^{\max}\right) L_t = \\ = N_t d (1-\phi) V_t + \tau \left(A_t^{\max}\right) L_t \end{aligned} \tag{11}$$

The left hand side of eq. (11) represents public outlays to finance both private and public R&D effort. $m(\theta_0) L_t w_t$ are the total outlays for the wages paid at researchers employed in public research, $s w_t N_t l_{At}$ are the total outlays to subsidize private research effort, $E\left(A_t^{\max}\right) L_t$ indicates the public expenditures that are proportional to the leading-edge productivity parameter and to population. The right hand side of eq. (11) indicates the total proceeds of the public sector. The government appropriates a fraction $(1-\phi)$ of the expected stock market value of a marketable patented idea V_t from both successful and unsuccessful R&D firms, d indicates the per product line number of the outsider R&D firms. The parameter τ is a lump-sum tax that is proportional to leading-edge technology parameter $A_t^{\max}$.[20]

2.4 Labor and Asset Market Equilibrium

Each researcher endogenously decides to allocate her research labor time to inventive or to manufacturing activity.

Plugging these results in the manufacturing/vertical R&D arbitrage condition (7), and solving the integral yields:

$$\frac{(1-s)\, l_{Ajt}^{*}}{\phi \lambda_A \psi\left(l_{Ajt}^{*}, b_t\right)} = \frac{\frac{1-\alpha}{\alpha} \tilde{x}(\omega)}{r + \frac{g_A}{\sigma} + \frac{\alpha}{1-\alpha} g_A} \tag{12}$$

Solving the above equation for $\tilde{x}(\omega)$, the labor force employed in the production of the top quality intermediate good is obtained:

$$\tilde{x}(\omega) = \frac{(1-s)\, l_{Ajt}^{*}}{\phi \lambda_A \psi\left(l_{Ajt}^{*}, b_t\right)} \left(r + \frac{g_A}{\sigma} + \frac{\alpha}{1-\alpha} g_A\right) \frac{\alpha}{1-\alpha} \tag{13}$$

from which, by inverting eq. (13), it is possible to determine the productivity-adjusted real wage ω_t.

The labor market clearing condition for manufacturing and vertical innovation is:

[20] In this framework all variables are proportional to the leading-edge productivity parameter $A_t^{\max}$. Along the BGP, the per capita consumption grows over time at the same rate as the per capita final output $\frac{Y_t}{L_t}$. In fact, the final output is also the unique and homogeneous consumption good of the economy. Moreover, the linear preferences imply a constant path of the per capita consumption over time. These two elements imply that - along the BGP - what is constant over time is the productivity-adjusted per capita consumption. Therefore, also a lump-sum tax must be proportional to the leading-edge productivity parameter $A_t^{\max}$.

$$G\left(\theta_0\right) L_t = N_t l_{At} + N_t \int_0^1 \tilde{x}\left(\omega/a\right) h\left(a\right) da = N_t l_{At} + \frac{N_t \tilde{x}\left(\omega\right)}{1+\frac{\sigma}{1-\alpha}} \tag{14}$$

where $\tilde{x}(\omega/a)$ is the labor demand function of a product line with relative productivity parameter a at the date t, and $h(a)$ is the density function of the relative productivities' cumulative distribution function $H(a)$.

The labor market clearing condition for basic research is:

$$\left[1 - G\left(\theta_0\right)\right] L_t = L_{Bt} \tag{15}$$

which - because the threshold ability parameter θ_0 is constant along the BGP - is a constant fraction of the population.

From eq. (1), and reclassifying intermediate goods by their relative productivities, the aggregate GDP can be written as (see Aghion and Howitt 1998, ch. 3, and Howitt 1999):

$$\begin{aligned} Y_t &= A_t^{\max} N_t \textstyle\int_0^1 a\tilde{x}\left(\omega/a\right)^{\alpha} h\left(a\right) da = \\ &= A_t^{\max} N_t \textstyle\int_0^1 \frac{1}{\sigma} a^{\frac{1}{\sigma}} \left(\frac{\alpha^2 a}{\omega_t}\right)^{\frac{\alpha}{1-\alpha}} da = \frac{N_t A_t^{\max}\left(\frac{\alpha^2}{\omega_t}\right)^{\frac{\alpha}{1-\alpha}}}{\left(1+\frac{\sigma}{1-\alpha}\right)} \end{aligned} \tag{16}$$

Notice that, in the light of eq.s (16) and (1), the productivity-adjusted fixed factor rent is:

$$\frac{re}{A_t^{\max}} = \left(1-\alpha\right)\frac{Y_t}{MA_t^{\max}} = \left(1-\alpha\right)\frac{N_t \tilde{x}^{\alpha}\left(\omega\right)}{\left(1+\frac{\sigma}{1-\alpha}\right)} = \left(1-\alpha\right)\frac{N_t\left(\frac{\alpha^2}{\omega_t}\right)^{\frac{\alpha}{1-\alpha}}}{\left(1+\frac{\sigma}{1-\alpha}\right)} \tag{17}$$

Therefore, the fixed factor rent increases in the number of intermediate goods, simply because they complement it in the production of the final good; and it decreases in the productivity-adjusted real wage.

3 General Equilibrium

The economy has a unique rational expectation equilibrium on which rational individuals instantaneously jump on. From now onward the time index is eliminated for the sake of notational simplicity.

Let us consider the law of motion of the basic knowledge (10), along the BGP it is obtained:

$$b \equiv \frac{B}{N} = \frac{m\left(\theta_0\right)\tilde{\lambda}_B}{\beta} \tag{18}$$

Therefore, the labor demand in eq. (13) for the top quality intermediate good becomes:

$$\tilde{x}(\omega) = \frac{(1-s)\, l^{*}_{Ajt}}{\phi \lambda_A \psi\left(l^{*}_{Aj}, \frac{m(\theta_0)\tilde{\lambda}_B}{\beta}\right)} \left(r + \frac{g_A}{\sigma} + \frac{\alpha}{1-\alpha} g_A\right) \frac{\alpha}{1-\alpha} \tag{19}$$

Let us consider both eq.s (19) and (14), along the rational expectation equilibrium, a positive and finite value for the per product line vertical research effort exists such that:

$$l_{At} = \left[\frac{L}{N} G(\theta_0) - \frac{\frac{(1-s) l^{*}_{Aj}}{\phi \lambda_A \psi\left(l^{*}_{Aj}, b\right)} \left(r + \frac{g_A}{\sigma} + \frac{\alpha}{1-\alpha} g_A\right) \frac{\alpha}{1-\alpha}}{\left(1 + \frac{\sigma}{1-\alpha}\right)} \right] \tag{20}$$

where, along the BGP, $b = \frac{m(\theta_0)\tilde{\lambda}_B}{\beta}$. From eq. (20) the per product line private research effort is obtained (see the Appendix B).

The per capita output is:

$$\frac{Y}{L} = \frac{\frac{N}{L} A^{\max} \tilde{x}^{\alpha}(\omega)}{\left(1 + \frac{\sigma}{1-\alpha}\right)} = \frac{\frac{\beta}{g_L} A^{\max} \left(\frac{\alpha^2}{\omega}\right)^{\frac{\alpha}{1-\alpha}}}{\left(1 + \frac{\sigma}{1-\alpha}\right)} \tag{21}$$

where eq. (16) has been used. Therefore, the per capita output growth rate is equal to the technological frontier growth rate:

$$g_{Y/L} = g_A = \sigma \lambda_A f(l_A, b) \tag{22}$$

By considering the eq.s from (18) to (22) the following can be stated:

Proposition 1 *Along the rational expectation BGP, a constant fraction of population is employed in manufacturing, private and public research. Along the BGP, an increase either in the private intellectual appropriation parameter ϕ or in the subsidy s positively affects the per capita output growth rate and negatively affects the per capita output level.*

Proof. *See Appendix B and C* ■

The industrial intellectual appropriation's share of a new commercial valuable idea and the subsidy of the private R&D cost are two alternative ways to finance industrial research effort. Yet, some fundamental differences between these two ways exist. The policy 'design' of the IPR regime of valuable ideas concerns the uncertainty aspect of a R&D process and it involves the political, executive, jurisprudential authorities of a country. Therefore, this policy 'design' strongly shapes the institutional set-up in which both private and public R&D operate. The subsidy of the industrial R&D cost does not shape the institutional set-up of the economy as the policy 'design' of the IPR does, and it only directly affects the private R&D cost, and therefore the certainty aspect of a R&D process. Moreover, the subsidy can be also managed in short time horizon.

Given these fundamental differences between the IPR regime and the support of industrial R&D cost, and the subsidy to R&D, both the industrial intellectual appropriation share ϕ and the subsidy s spur the per product line vertical research effort. However, this effect can be different in magnitude, as the following states:

Proposition 2 *Whenever condition (D1) holds along the BGP with a public balanced budget - i.e. $s \geq 1-\phi$ - a larger subsidy would produce a higher per capita output growth rate and a lower per capita output level than what could be obtained from a higher private intellectual appropriation parameter ϕ.*

Proof. *See Appendix D* ■

Let us consider the public balanced budget along the BGP:

$$\begin{array}{c} \frac{g_L}{\beta} m(\theta_0) + s\bar{l}_A + e = \\ = \frac{(1-\phi)(1-s)l^*_{Ajt}}{\phi \lambda_A \psi\left(l^*_{Ajt}, b_t\right)} d + \tau \frac{g_L}{\beta} \end{array} \tag{23}$$

where $e \equiv \frac{E(A_t^{\max})L_t}{A_t^{\max} N_t} = E\frac{g_L}{\beta}$ is a constant term, and $\bar{l}_A$ is the per product line vertical labor time research effort. Eq. (23) allows to obtain the tax rate τ such that equality $\bar{s} = 1-\phi$ holds. Once this equality holds along the BGP, a marginal increase in the subsidy greatly spurs the industrial research effort and the per capita output growth rate. Yet, a marginal increase in the subsidy magnifies the trade-off between the per capita output growth rate and the per capita output level. Notice that the higher ϕ the lower the threshold subsidy $\bar{s}$ that greatly affects the per capita output growth rate and the per capita output level. This means that the larger the private appropriation share of the expected stock market value of a new commercial valuable idea, the lower the subsidy to spur the industrial investment in R&D and the growth rate of a country.

Because ϕ is a measure of the strength, tightness and ease of IPR granted to public innovations, proposition 2 has a main policy implication. Once some form of IPR are introduced for public ideas, two different regimes of IPR against future innovators should be provided for public and private innovations. In particular, the different regime concerns in this framework a patent protection against future innovators, and therefore it refers to both patentability requirement and to patent breadth.[21] O'Donoghue and Zweimuller (2004) show that both a larger patentability requirement and leading breadth better off the growth performance of a country. This paper shows that because patented basic ideas are a cost for a private R&D firm, a larger patentability requirement for a public basic idea reduces the industrial R&D cost and better off the growth performance of a country. Instead, a larger leading breadth for a public basic innovation generates a cost of licences for both universities and private firms, and this discourages the R&D effort. Therefore, a larger patentability requirement and a

[21] Because the leading breadth limits future innovators by specifying superior products that other firms can not produce, it could not apply at a basic idea that does not have an immediate and specific market application. However, to be as general as possible, also a leading breadth for non marketable basic ideas has considered.

lower leading breadth should be set for public basic ideas than for industrial innovations. In this way, a higher value of the parameter ϕ can be obtained, and therefore also a low subsidy of the industrial R&D cost greatly spurs the innovative effort and the per capita output growth rate of a country.

Let us now suppose that the offsprings of public research also depends on public expenditures e.[22] As proven in the Appendix E, whenever the population grows at a high enough rate, a larger per product line public expenditures e in basic R&D programs spurs the private R&D investment. This in turn implies a higher per capita output growth rate and a lower per capita output level. The results are summarized in the following

Proposition 3 *When condition (E4) holds along the BGP a higher public investment in R&D determines: a) a higher per product line industrial R&D effort; b) a higher per capita output growth rate; c) a lower per capita output level.*

Proof. *See Appendix E* ■

When the public sector invests resources in public R&D to endow researchers of better equipment and instruments, more talented researchers will find it profitable to work in that research programs. A larger proportion of the population will choose to be employed in public research. This in turn can also spur the private research effort and investment to gain from a higher productivity of basic research. The positive effect of a higher public expenditure on the per capita output growth rate is then magnified from both these elements. On the other hand, the workforce employed in the manufacturing sector - as well as the demand for each intermediate monopolistic firm - will be lower along the new BGP. Therefore, also the negative effect on the per capita output level has magnified from a higher public expenditure in basic R&D.

These results also depend on a threshold for the population growth rate. When the population growth rate is not high enough, an increase in the public expenditure to finance public R&D discourages the industrial research effort and reduces the per capita output level (see the Appendix E). In fact, a larger fraction of the population will find it profitable to work in public R&D, and a lower faction of the population will be employed in both manufacturing and private R&D. Yet, in this case the population growth rate is not high enough to compensate the private firm for the reduced market demand of an intermediate product. This in turn implies a lower private innovative effort due to a too low market demand for each intermediate good. In this case, the final effect on the per capita output growth rate is not univocal and it depends on the relative

[22] We can think to e as public investment in material goods used in the public R&D programs, such as more productive machineries, more modern laboratory equipment, libraries, latest computers and software programs, etc. These expenditures can increase the productivity of each R&D worker employed in public research. Therefore $\tilde{\lambda}_B = \tilde{\lambda}_B(e)$, with $\frac{\partial \tilde{\lambda}_B}{\partial e} > 0$. This means that the productivity of each researcher in finding new basic discoveries depends on her personal ability, and it is also positively affected by the equipment, libraries, machineries, etc. she works with. Moreover, a higher public expenditure can concern larger financial support and research funds for each researcher, so that all these elements increase the salary gained by public researchers above the contractual salary w_t.

strength of basic discoveries and of private effort in the function $f\left(\bar{l}_A, b\right)$. In fact, if the spillovers of basic discoveries to the Poisson arrival rate of innovation are strong enough to compensate the reduction in the private innovative effort, the per capita output growth rate rises. Therefore, also the existence of a trade-off between the per capita output growth rate and the per capita output level depends on the spillovers' strength that basic discoveries produce on the innovative capacity of private R&D firms.

Notice that these results depends on a population growth rate threshold that is determined by the structural and institutional parameters of the economy. Moreover, these results contribute to explain the mixed substitutability/complementarity relationship between private and public R&D investment.

4 Conclusions

Since the '80s the U.S. adopted several legislative acts to spur the transfer of knowledge and innovations from public funded research programs to private industry. The most influential legislative act has been the Bayh-Dole Act of the 1980 that created an uniform federal patent policy that allowed universities and small businesses to retain rights to any patents resulting from government and federal agency funded research and to licence these patents on an exclusive or non-exclusive basis. Based on the belief that legislative arrangements such as the Bayh-Dole Act enhance the technology transfer and the academic contributions to innovation and growth in the U.S., similar legislation is being considered in other OECD countries.

Strong intertwined relationships between private and public R&D have documented by the empirical evidence. On the one hand, the existence of large spillovers from public research activity towards private industry is widely recognized. On the other hand, both a complementarity and substitutability relationship between public and private R&D investment has been found.

This paper investigates on the macroeconomic implications of the ties between public and private R&D and it sheds light to explain the empirical evidence just mentioned. To this aim a neo-Schumpeterian growth model à la Aghion and Howitt (1992, 1998) and à la Howitt (1999, 2000) has been adopted. In the R&D sector, public research generate basic ideas that do not have an immediate and specific commercial value and application. According to the institutional set-up of the U.S., IPR are granted at these basic ideas. The private R&D sector pays for licenses on such basic patented innovations and it develops these basic ideas to introduce a better intermediate product in the marketplace. Therefore, the industrial sector appropriates a share of the final commercial value of each product, being another share paid at the public sector for the licenses on the usefully developed patented basic ideas. Moreover, an industrial R&D firm obtains a subsidy for its research cost.

This paper shows that the IPR and the subsidy to private R&D cost are two alternative ways to finance the private research effort. Both the intellectual appropriation parameter and the subsidy to R&D spur the private innovative effort. This in turn generates a trade off between the per capita output growth rate and level, by increasing the former and reducing the second. This result seems remarkable because the policy design of the IPR strongly shapes the institutional set-up of the economy, and it concerns the uncertainty aspect of a R&D process. While the subsidy of the private R&D cost does not have an institutional 'weight' as the IPR has, and it concerns the certainty aspect of a R&D process. An important policy implication concerns the IPR regime. Once the IPR are also granted at public ideas, two different regimes of IPR against future innovators should be provided for public and industrial innovations. In particular, a larger patentability requirement and a lower leading breadth should be granted for basic public ideas than for industrial innovations. This policy allows at a low subsidy to private R&D cost to greatly spur the innovative effort and the per capita output growth rate of the economy.

Moreover, it is shown that public investments in R&D can explain the mixed empirical evidence about the substitutability-complementarity relationship between public and private innovative effort. It is shown that an increase in the public investment in R&D can either crowd out or complement private R&D effort. The substitutability-complementarity relationship between public and private R&D effort depends on the structural parameters of the economy and on the population growth rate. When population grows at a high enough pace, a higher public investment in R&D complements the industrial research effort and it only crowds out the demand for each intermediate product. This in turn generates a trade off between the per capita output growth and level. However, because the existence of large and positive direct spillovers from public R&D to private industry has found by the empirical analyses, the this paper shows that a higher public investment better off the growth performance of a country whatever is the relationship between private and public R&D investments, either substitutability or complementarity.

Appendix A

1. Each vertical R&D firm j has a Poisson arrival rate of innovation at a given time $t>0$ described by $\lambda_A\psi\left(l_{Ajt},b_t\right)$. The function $\psi\left(l_{Ajt},b_t\right)$ has positive first partial derivative, it is concave in l_{Ajt}, and it satisfies the Inada condition, that is $\underset{l_{Ajt}\to 0}{lim}\psi_1\left(l_{Ajt},b_t\right)\to\infty$ and $\underset{l_{Ajt}\to\frac{g_L}{\beta}G(\theta_0)}{lim}\psi_1\left(l_{Ajt},b_t\right)\to 0$, where $\frac{g_L}{\beta}G\left(\theta_0\right)$ denotes the maximum employment level in the private R&D sector. This point will become clear later to the reader. Along the BGP, a symmetry in each product line i between the R&D firms exists, so that $l_{Ajt}=\frac{l_{At}}{d}$, where d is the number of R&D firms in the product line i. The Poisson arrival rate of innovation is independently distributed across firms, across industry lines, and over time. Therefore the Poisson arrival rate of innovation along a product line i is $\sum_{j=1}^{d}\lambda_A\psi\left(l_{Ajt},b_t\right)=\sum_{j=1}^{d}\lambda_A\psi\left(\frac{l_{At}}{d},b_t\right)=\lambda_Af\left(l_{At},b_t\right)$. The function $f\left(l_{At},b_t\right)$ is assumed to have strictly increasing partial derivative and it is concave in l_{At}. Q.E.D.

The per product line stock of basic knowledge b_t encompasses both patented and non-patented basic public ideas. The share of public discoveries granted of patent protection - for which industrial R&D pays for their use - can be described as a random variable. In this case, this share is a continuum random variable $p\in[0,\bar{p}]$ common for all the industry lines and constant over time , with $\bar{p}<b_t$ and with a common cumulative distribution function $F\left(p\right)$.

2. Each vertical R&D firm targeting an intermediate product i chooses its R&D intensity to maximize $\phi V_t\lambda_A\psi\left(l_{Ajt},b_t\right)-\left(1-s\right)w_tl_{Ajt}$. The first order condition for a private R&D firm implies

$\phi V_t\lambda_A\psi_1\left(l_{Ajt},b_t\right)\leq\left(1-s\right)w_t$. Because $\psi\left(l_{Ajt},b_t\right)$ is assumed concave in l_{Ajt} the first order condition is also sufficient for a maximum. The interior solution implies $\phi V_t\lambda_A\psi_1\left(l_{Ajt},b_t\right)=\left(1-s\right)w_t$. The R&D sector is characterized by free entry and exit, therefore the following condition must be satisfied for any R&D firm: $\phi V_t\lambda_A\psi\left(l_{Ajt},b_t\right)-\left(1-s\right)w_tl_{Ajt}\geq 0$. When the marginal firm enters in the R&D race this condition will be binding, at that point the marginal firm will be indifferent between to enter or do not enter in the R&D race. By combining the first order condition and the free entry condition the following inequality $\psi\left(l^*_{Ajt},b_t\right)\geq l^*_{Ajt}\psi_1\left(l^*_{Ajt},b_t\right)$ is obtained. This condition can be rewritten as $\frac{\psi\left(l^*_{Ajt},b_t\right)}{l^*_{Ajt}}\geq\psi_1\left(l^*_{Ajt},b_t\right)$. Because $\psi\left(l^*_{Ajt},b_t\right)$ is concave in l_{Ajt}, the average productivity $\frac{\psi(l_{Ajt},b_t)}{l_{Ajt}}$ is decreasing. Therefore, the condition $\frac{\psi\left(l^*_{Ajt},b_t\right)}{l^*_{Ajt}}\geq\psi_1\left(l^*_{Ajt},b_t\right)$ is always satisfied. Q.E.D.

3. In this point the eq. (11) has obtained. By following the same steps as in Aghion and Howitt (1998), the profit flow of a monopolistic firm that manufactures an intermediate product i with productivity A_{it} is

$$\pi_{it}=A_t^{\max}\frac{1-\alpha}{\alpha}\omega_t\left(\frac{\alpha^2}{\omega_t}\right)^{\frac{1}{1-\alpha}}a^{\frac{1}{1-\alpha}}=A_t^{\max}\tilde{\pi}\left(\omega\right)a^{\frac{1}{1-\alpha}}$$

where $\omega_t \equiv \frac{w_t}{A_t^{\max}}$ is the productivity-adjusted real wage, $\tilde{\pi}(\omega)$ is the profit flow of the intermediate good with the maximum productivity parameter $A_t^{\max}$. The expected stock market value of the last successful R&D firm that has productivity $A_t^{\max}$ is described by the eq. (5) in the text. The expected stock market value of an intermediate product i with absolute productivity A_{it} and relative productivity $\frac{A_{it}}{A_t^{\max}}$ is $V_{it} = V_t a^{\frac{1}{1-\alpha}}$. Therefore, the cumulative expected stock market value of all manufacturing monopolies at a given time $t \geq 0$ is:

$$\int_0^{N_t} V_{it} di = N_t \int_0^1 V_{it} dH(a) = N_t V_t \int_0^1 a^{\frac{1}{1-\alpha}} dH(a) = \frac{N_t A_t^{\max} V_t}{1 + \frac{\sigma}{1-\alpha}} \tag{A1}$$

Let d be the number of the per product line outsider R&D firms. The public sector appropriates a share $(1-\phi)$ of the expected stock market value of a patented idea from each R&D firm, successful and unsuccessful. Therefore a balanced budget requires

$$\begin{aligned} m(\theta_0) L_t w_t + s w_t N_t l_{At} + E(A_t^{\max}) L_t = \\ = d N_t (1-\phi) V_t + \tau(A_t^{\max}) L_t, \end{aligned} \tag{A2}$$

In this setting the Arrow's effect is assumed to be at work. This implies that the incumbent firm does not find it profitable to undertake R&D. However, as proven by Cozzi (2007) this framework can not exclude positive investment in R&D by the incumbent firm. If this argument would be considered, all the analysis remained valid by simply replacing $n = d+1$ to d in the paper. Q.E.D.

Appendix B

In this Appendix the per product line vertical research labor time l_{At} along the BGP is obtained. From now onward the time index is eliminated for the sake of notational simplicity. By eq. (20) the following i obtained:

$$\begin{aligned} \frac{\frac{L_t}{N_t} G(\theta_0)\left(1+\frac{\sigma}{1-\alpha}\right)\phi\lambda_A \psi\left(l_{Aj}^*, b\right)\frac{1-\alpha}{\alpha}}{(1-s) l_{Aj}^*} - r - \frac{\frac{1-\alpha}{\alpha}\left(1+\frac{\sigma}{1-\alpha}\right)\phi\lambda_A \psi\left(l_{Aj}^*, b\right)}{(1-s) l_{Aj}^*} l_A = \\ = \lambda_A \left(1 + \frac{\sigma\alpha}{1-\alpha}\right) f(l_A, b) \end{aligned} \tag{B1}$$

where - along the BGP - $\frac{L}{N} = \frac{g_L}{\beta}$. Let us consider the left hand side of eq. (B1). The research labor time of a R&D firm j is $l_{Aj}^* = \frac{l_A}{d}$. For $l_A \to 0$, so that also $l_{Aj}^* \to 0$, and by applying de l'Hopital rule, the left hand side of eq. (B1) tends to $\frac{\frac{g_L}{\beta} G(\theta_0)\left(1+\frac{\sigma}{1-\alpha}\right)\phi\lambda_A \psi_1\left(l_{Aj}^*, b\right)\frac{1-\alpha}{\alpha}}{\frac{(1-s)}{d}} - r$. The Inada conditions for $\psi\left(l_{Aj}^*, b\right)$ imply $\lim_{l_{Aj}^* \to 0} \psi_1\left(l_{Aj}^*, b\right) = \infty$, and therefore the left hand side of eq. (B1) is always strictly positive for $l_{Aj}^* \to 0$. For $l_A \to \frac{g_L}{\beta} G(\theta_0)$ the left hand side of eq. (B1) tends to $\frac{\left(1+\frac{\sigma}{1-\alpha}\right)\phi\lambda_A \psi\left(\frac{g_L}{d\beta} G(\theta_0), b\right)\frac{1-\alpha}{\alpha}}{(1-s)\frac{g_L}{d\beta} G(\theta_0)} \left[\frac{g_L}{\beta} G(\theta_0) - l_A\right] - r \to -r < 0$.

Let us turn to prove that the left hand side of eq. (B1) is a strictly monotonic decreasing function of l_{At}. Let us consider the eq. (20) rewritten as:

$$
\begin{aligned}
\Lambda = \frac{\frac{L}{N} G(\theta_0) \left(1 + \frac{\sigma}{1-\alpha}\right) \phi \lambda_A \psi \left(l^*_{Aj}, b\right) \frac{1-\alpha}{\alpha}}{(1-s) l^*_{Aj}} - r + \\
- \frac{\frac{1-\alpha}{\alpha} \left(1 + \frac{\sigma}{1-\alpha}\right) \phi \lambda_A \psi \left(l^*_{Aj}, b\right)}{(1-s) l^*_{Aj}} l_A - \lambda_A \left(1 + \frac{\sigma\alpha}{1-\alpha}\right) f\left(l_A, b\right)
\end{aligned}
\tag{B2}
$$

where the research labor time of a R&D firm j is $l^*_{Aj} = \frac{l_A}{d}$. Therefore, it will be

$$
\begin{aligned}
\frac{\partial \Lambda}{\partial l_A} = \frac{\left(1 + \frac{\sigma}{1-\alpha}\right) \phi \lambda_A \frac{1-\alpha}{\alpha} \frac{(1-s)}{d} \left[\frac{g_L}{\beta} G(\theta_0) - l_A\right]}{\left[\frac{(1-s) l_A}{d}\right]^2} \left[\frac{l_{At}}{d} \psi_1 \left(l^*_{Aj}, b\right) - \psi\left(l^*_{Aj}, b\right)\right] + \\
- \frac{\left(1 + \frac{\sigma}{1-\alpha}\right) \phi \lambda_A \frac{1-\alpha}{\alpha} \psi \left(l^*_{Aj}, b\right)}{\frac{(1-s) l_A}{d}} - \lambda_A \left(1 + \frac{\sigma\alpha}{1-\alpha}\right) f_1\left(l_A, b\right) < 0
\end{aligned}
\tag{B3}
$$

The first order condition for a maximum profit and the free entry condition in the R&D race imply that $\psi\left(l^*_{Aj}, b\right) \geq l^*_{Aj}\ \psi_1\left(l^*_{Aj}, b\right)$, and therefore it also is $\psi\left(l^*_{Aj}, b\right) \geq \frac{l_{At}}{d}\ \psi_1\left(l^*_{Aj}, b\right)$. Moreover, the labor market clearing condition implies that $\frac{g_L}{\beta} G(\theta_0) \geq l_A$. Therefore the inequality (B3) is proven. The left hand side of eq. (B1) is a strictly monotonic decreasing function of $l_A \in \left[0, \frac{g_L}{\beta} G(\theta_0)\right]$, with values ranging from $+\infty$ to $-r$.

The right hand side of eq. (B1) is assumed to be an increasing and concave function of l_A, with b taken as given by each R&D firm. These conditions imply the existence of a unique and stable steady state of the per product line vertical research labor time effort $0 < \bar{l}_A < \frac{g_L}{\beta} G(\theta_0)$. Notice that in this proof $f(l_A, b)$ is not necessarily a concave function of both its arguments l_A and b. Q.E.D.

Appendix C

The first part of this Appendix proves the effect of a higher private intellectual appropriation parameter ϕ on the economic performance of a country, $\phi \in (0, 1)$ is assumed. The second part analyses the effect of a higher R&D subsidy.

1. Let us consider the eq. (B2). By using the Implicit Function Theorem it is obtained:

$$
\frac{\partial l_A}{\partial \phi} = -\frac{\frac{\partial \Lambda}{\partial \phi}}{\frac{\partial \Lambda}{\partial l_A}} = -\frac{\frac{\left(1 + \frac{\sigma}{1-\alpha}\right) \lambda_A \psi \left(l^*_{Aj}, b\right) \frac{1-\alpha}{\alpha}}{(1-s) l^*_{Aj}} \left[\frac{g_L}{\beta} G(\theta_0) - \bar{l}_A\right]}{\frac{\partial \Lambda}{\partial l_A}} > 0 \tag{C1}
$$

where $\frac{\partial \Lambda}{\partial l_A} < 0$ from inequality (B3), and $\left[\frac{g_L}{\beta} G(\theta_0) - \bar{l}_A\right] > 0$. Therefore, along the BGP, a higher intellectual appropriation parameter ϕ increases the per product line private R&D effort. Q.E.D.

In order to determine the effects of a higher private intellectual appropriation parameter ϕ on the market demand for any existing intermediate good, we use the labor market clearing condition:

$$
L = G(\theta_0) L + [1 - G(\theta_0)] L = N \bar{l}_A + \frac{N \tilde{x}(\omega)}{1 + \frac{\sigma}{1-\alpha}} + N l_B \tag{C2}
$$

where $l_B = \frac{L_B}{N} = [1 - G(\theta_0)] \frac{L}{N}$ denotes the per product line basic research effort. From eq. (8) a constant threshold ability parameter θ_0 is obtained. Therefore - along the new BGP with a higher private intellectual appropriation parameter ϕ - the per product line basic research effort $[1 - G(\theta_0)] \frac{L}{N}$ is constant and equal to $[1 - G(\theta_0)] \frac{g_L}{\beta}$. Moreover, eq. (C1) proves that, along the new BGP, the per product line vertical research effort is higher. Therefore, eq. (C2) necessarily implies a lower market demand $\tilde{x}\left(\frac{\omega}{a}\right)$ for each existing intermediate good. Finally, from eq. (21), it immediately follows that a higher appropriation parameter ϕ determines a lower per capita output level. Q.E.D.

The positive effect of a change in the appropriation parameter ϕ on the per capita output growth rate is easily proven:

$$\frac{\partial g_{Y/L}}{\partial \phi} = \sigma \lambda_A \frac{\partial f\left(\bar{l}_A, b\right)}{\partial l_A} \frac{\partial \bar{l}_A}{\partial \phi} > 0 \tag{C3}$$

where the inequality follows from eq. (C1). Q.E.D.

2. This part analyses the effect of a change in the subsidy to private research effort s on the economic performance of the economy; $s \in (0,1)$ is assumed. From eq. (B2), and by using the Implicit Function Theorem the following is obtained:

$$\frac{\partial l_A}{\partial s} = -\frac{\frac{\partial \Lambda}{\partial s}}{\frac{\partial \Lambda}{\partial l_A}} = -\frac{\frac{l^*_{Aj}\left(1+\frac{\sigma}{1-\alpha}\right)\phi\lambda_A\psi\left(l^*_{Aj},b\right)\frac{1-\alpha}{\alpha}}{\left[(1-s)l^*_{Aj}\right]^2}\left[\frac{g_L}{\beta}G(\theta_0) - \bar{l}_A\right]}{\frac{\partial \Lambda}{\partial l_A}} > 0 \tag{C4}$$

where $\frac{\partial \Lambda}{\partial l_A} < 0$ from inequality (B3), and $\left[\frac{g_L}{\beta}G(\theta_0) - \bar{l}_A\right] > 0$. Therefore, along the BGP a positive relationship between the subsidy to private research effort s and the per product line private R&D labor time $\bar{l}_A$ is proven. Q.E.D.

In order to determine the effects of a higher subsidy s on the market demand for any existing intermediate good, the labor market clearing condition (C2) is used. As proven above, along the BGP, the per product line basic research effort is constant and equal to $[1 - G(\theta_0)] \frac{g_L}{\beta}$. Moreover, eq. (C4) proves that - along the new BGP with a higher subsidy to private R&D firms - the per product line vertical research effort is higher. Therefore, eq. (C2) necessarily implies a lower market demand $\tilde{x}\left(\frac{\omega}{a}\right)$ for each existing intermediate good. Finally, from eq. (21), it immediately follows that a higher subsidy s determines a lower per capita output level. Q.E.D.

The positive effect of a change in the subsidy to private research effort s on the per capita output growth rate is easily proven:

$$\frac{\partial g_{Y/L}}{\partial s} = \sigma \lambda_A \frac{\partial f\left(\bar{l}_A, b\right)}{\partial l_A} \frac{\partial \bar{l}_A}{\partial s} > 0 \tag{C5}$$

where the inequality follows from eq. (C4). Q.E.D.

Appendix D

This Appendix compares the effect of a marginal change in the appropriation parameter ϕ with the effect of a marginal change in the subsidy s. Along a new BGP with a larger value of either ϕ or s determines a higher per product line private innovation effort and a higher per capita output growth rate. In order to compare the magnitude of these effects it suffices to consider the eq.s (C1) and (C4). Whenever the following condition is satisfied $\frac{\partial \bar{l}_A}{\partial s} \geq \frac{\partial \bar{l}_A}{\partial \phi}$, an increase in the subsidy generates the same economic effects as an increase in the appropriation parameter ϕ, but the former are higher in magnitude. In fact, from eq.s (C1) and (C4), it immediately follows that $\frac{\partial \bar{l}_A}{\partial s} \geq \frac{\partial \bar{l}_A}{\partial \phi}$ if and only if

$$s \geq 1 - \phi \tag{D1}$$

Q.E.D.

Appendix E

In this Appendix the relationship between the per product line vertical R&D effort and the per product line stock of public basic knowledge is derived. From eq. (B2) and by using the Implicit Function Theorem it is obtained:

$$\frac{\partial l_A}{\partial b} = -\frac{\frac{\partial \Lambda}{\partial b}}{\frac{\partial \Lambda}{\partial l_A}} = -\frac{\frac{\left(1+\frac{\sigma}{1-\alpha}\right)\phi\lambda_A\psi_2\left(l_{Aj}^*,b\right)\frac{1-\alpha}{\alpha}\left[\frac{g_L}{\beta}G(\theta_0)-\bar{l}_A\right]}{(1-s)l_{Aj}^*} - \lambda_A\left(1+\frac{\sigma\alpha}{1-\alpha}\right)f_2\left(\bar{l}_A,b\right)}{\frac{\partial \Lambda}{\partial l_A}} \tag{E1}$$

with $\psi_2\left(l_{Aj}^*,b\right) > 0$, $f_2\left(\bar{l}_A,b\right) > 0$, $\frac{\partial \Lambda}{\partial l_A} < 0$. The inequality (E1) will be strictly positive if and only if

$$\begin{aligned} g_L > & \left\{\lambda_A\left(1+\frac{\sigma\alpha}{1-\alpha}\right)f_2\left(\bar{l}_A,\frac{m\left(1/\tilde{\lambda}_B\right)\tilde{\lambda}_B}{\beta}\right)* \right. \\ & \left. *\frac{(1-s)l_{Aj}^*}{\left(1+\frac{\sigma}{1-\alpha}\right)\phi\lambda_A\psi_2\left(l_{Aj}^*,b\right)\frac{1-\alpha}{\alpha}} + \bar{l}_A\right\} * \frac{\beta}{G\left(1/\tilde{\lambda}_B\right)} \end{aligned} \tag{E2}$$

where - along the BGP - the per product line private research effort $\bar{l}_A$ is strictly lower than $\frac{g_L}{\beta}G\left(1/\tilde{\lambda}_B\right)$. When inequality (E2) holds a complementarity relationship between public and private R&D effort exists.

When the productivity of public basic research depends on per product line public expenditures e, that is $\tilde{\lambda}_B(e)$, it is $\frac{\partial \tilde{\lambda}_B(e)}{\partial e} > 0$, and therefore it is $\frac{\partial \theta_0}{\partial e} < 0$. In this case the calculation in eq. (E1) modifies to:

$$\begin{aligned} \frac{\partial l_A}{\partial e} = -&\left\{k\psi_2\left(l_{Aj}^*,b\right)\frac{\partial b}{\partial e}\left[\frac{g_L}{\beta}G\left(\theta_0\right)-\bar{l}_A\right] + k\psi\left(l_{Aj}^*,b\right)\frac{g_L}{\beta}\frac{\partial G(\theta_0)}{\partial e} + \right. \\ & \left. -\lambda_A\left(1+\frac{\sigma\alpha}{1-\alpha}\right)f_2\left(l_A,b\right)\frac{\partial b}{\partial e}\right\} * \\ & *\left\{\frac{\partial \Lambda}{\partial l_A}\right\}^{-1} \end{aligned} \tag{E3}$$

where $k \equiv \frac{\left(1+\frac{\sigma}{1-\alpha}\right)\phi\lambda_A\frac{1-\alpha}{\alpha}}{(1-s)l_{Aj}^*}$, $\frac{\partial b}{\partial e} > 0$ because along the BGP $b = \frac{m\left[\frac{1}{\tilde{\lambda}_B(e)}\right]\tilde{\lambda}_B(e)}{\beta}$, and $m\left[\frac{1}{\tilde{\lambda}_B(e)}\right]$ is an increasing function of $\tilde{\lambda}_B(e)$; $\frac{\partial G(\theta_0)}{\partial e} < 0$. Condition (E3) is strictly positive whenever population growth rate satisfies the following inequality,

$$g_L > \beta \left\{ k\psi_2 \left(l_{Aj}^*, b\right) \bar{l}_A \frac{\partial b}{\partial e} + \lambda_A \left(1 + \frac{\sigma\alpha}{1-\alpha}\right) f_2 \left(l_A, b\right) \frac{\partial b}{\partial e} * \right. \\ \left. * \left[k\psi_2 \left(l_{Aj}^*, b\right) \frac{\partial b}{\partial e} G\left(\theta_0\right) + k\psi \left(l_{Aj}^*, b\right) \frac{\partial G\left(\theta_0\right)}{\partial e} \right]^{-1} \right\} \tag{E4}$$

Finally, whenever $g_L > \max\left\{(E2), (E4)\right\}$ a higher per product line basic knowledge - either exogenous or endogenously obtained through a higher public expenditure - complements the industrial R&D effort, i.e. l_A is higher. Q.E.D.

To determine the effects of a higher productivity of the basic research programs on the per capita output level, we use the labor market clearing condition (C2):

$$L = G\left(\theta_0\left(e\right)\right) L + 1 - G\left(\theta_0\left(e\right)\right) L = N\bar{l}_A + \frac{N\tilde{x}\left(\omega\right)}{1 + \frac{\sigma}{1-\alpha}} + Nl_B$$

where $l_B = \frac{L_B}{N} = \left[1 - G\left(\theta_0\right)\right] \frac{L}{N}$ is higher because $\frac{\partial G(\theta_0)}{\partial \theta_0} \frac{\partial \theta_0}{\partial e} < 0$, and $G\left(\theta_0\right) L$ is lower because $\frac{\partial G(\theta_0)}{\partial \theta_0} \frac{\partial \theta_0}{\partial e} < 0$. Moreover, eq. (E3) proves that, along the new BGP, the per product line vertical research effort is higher. The eq. (C2) necessarily implies a lower market demand $\tilde{x}\left(\frac{\omega}{a}\right)$ for each existing intermediate good. From eq. (21), it immediately follows that a higher productivity of basic research programs reduces the per capita output level. Finally, because the population employed by private firms is lower - $G\left(\theta_0\right) L$ is lower - and the per product line vertical research effort is higher, the negative effect on per capita output level is magnified by a higher public expenditure. Q.E.D.

The effect of a change in e on the per capita output growth rate is:

$$\frac{\partial g_{Y/L}}{\partial e} = \sigma\lambda_A \left[\frac{\partial f\left(\bar{l}_A, b\right)}{\partial l_A} \frac{\partial \bar{l}_A}{\partial b} + \frac{\partial f\left(\bar{l}_A, b\right)}{\partial b} \right] \frac{\partial b}{\partial \tilde{\lambda}_B\left(e\right)} \frac{\partial \tilde{\lambda}_B\left(e\right)}{\partial e} \tag{E5}$$

When condition (E4) holds the eq. (E5) is strictly positive. Q.E.D.

When the population growth rate is not high enough, the inequality (E3) is strictly negative and an increase in the public expenditure to finance basic R&D reduces the private innovation effort. As the eq. (E5) shows, the effect on the per capita output growth rate can not be univocally determined. It depends on the relative strength of partial derivative $\frac{\partial f\left(\bar{l}_A, b\right)}{\partial l_A}$ and $\frac{\partial f\left(\bar{l}_A, b\right)}{\partial b}$. Yet, the per capita output level is lower because of the lower market demand along each intermediate product line. Q.E.D.

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