The effects of the marginal tax rate in a matching model with endogenous labor supply

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The effects of the marginal tax rate in a matching model with endogenous labor supply *

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Abstract

This paper analyzes the effects of the marginal tax rate on unemployment and economic efficiency in a matching model with homogenous agents when wages and working hours are bargained over. I show that the theoretical impact of a higher marginal tax rate on unemployment is ambiguous whatever the instantaneous utility in unemployment i.e. for an utility in unemployment that is either fixed or perfectly indexed on net wages. These results are in sharp contrast with the literature. Numerical simulations applied to France suggest that a higher marginal tax rate generally reduces the unemployment rate but at the expense of lower economic efficiency. The simulations point also out that the relation between the optimal marginal tax rate and the elasticity of labor supply is not monotonic.

Keywords: Matching model, Marginal Tax Rate, Labor supply, Utility in unemployment.

JEL Codes: D82, H21, H24, J64.

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1 Introduction

Average and marginal tax rates affecting low-paid jobs have decreased the last twenty years in many countries (e.g. the E.I.T.C. in the US, the W.F.T.C. in UK, reductions in social security contributions in France). Since it is recognized that these policies were beneficial in terms of employment to the low-paid employees, many governments wonder if these policies should be intensified. However, these policies might have generated costs because of the induced increase in the marginal tax rate for average-paid groups\footnote{Their average tax rates might have also decreased. However, for the average-paid groups, marginal tax rates are much more affected than average tax rates (especially in France, see The OECD Tax Database).}. Therefore, it is important to shed light on the effects of a higher marginal tax rate on equilibrium unemployment and social efficiency, two crucial economic dimensions for assessing the effects of taxes. These are the two main objectives of this article.

The theoretical framework is based on the matching model (e.g., Mortensen and Pissarides (1999) and Pissarides (2000)) with endogenous working hours\footnote{In the paper, I use the term “working hours” to refer to the more general notion of “intensive labor supply margin” which encompasses all types of in-work efforts.}. Firms post job vacancies at a fixed cost. Matching frictions create search externalities and generate rents that are shared between workers and firms through Nash-bargaining over wages and hours. Two traditional effects found in the literature analyzing the impact of an increase in the marginal tax rate are present in my model. First, by excluding labor supply income effects (as is usually done in the literature), a higher marginal tax rate distorts working hours through a substitution effect\footnote{Even if the intensity of the reaction remains controversial, the negative impact of the marginal tax rate on the labor supply intensive margin has been widely empirically recognized (see Blundell and MaCurdy (1999), Blundell (2000) and OECD (2002)).}. Second, the wage moderating effect of a higher marginal tax rate (henceforth called the standard wage moderating effect) was put forward in the literature that considers imperfect labor markets with fixed hours\footnote{This effect has been highlighted in theoretical and empirical studies for many countries (see Hersoug (1984), Malcolmson and Sartor (1987), Lockwood and Manning (1993), Holmlund and Kolm (1995), Sorensen (1997), Pissarides (1998) and Røed and Stroem (2002), among others.).}. A higher marginal tax rate (keeping average tax rates fixed) implies that the labor cost becomes more sensitive to an increase in the net wage. Moreover, in the presence of positive taxes, a rise in the marginal tax rate implies that an increase in gross wages has a reduced impact on net wages. Therefore, it becomes less rewarding for workers to bargain aggressively and wages fall. This in turn stimulates labor demand and decreases unemployment.

Three new theoretical findings are put forward in this paper. By taking endogenous working
hours into account, I enrich the *standard wage moderating effect* through two channels emerging from indirect ("feedback") macroeconomic equilibrium effects. First, the decrease in working hours due to a higher marginal tax rate reduces firm’s net profit and thus the possibilities for firms to recover their vacancy costs. Firm’s total profit being lower, fewer firms enter the market and the probability for workers to match a vacancy decreases. This in turn decreases workers’ wage claims. Therefore, this effect, henceforth called the *scale economy effect*, reinforces the *standard wage moderating effect* of the marginal tax rate. The second effect emerges from utility in unemployment. The paper considers two polar cases for the instantaneous utility in unemployment. When this utility is perfectly indexed to net wages (i.e. the Bismarckian component of the unemployment compensation), the wage rate decreases when the marginal tax rate increases. When this utility is fixed (e.g. the Beveridgian component of the unemployment compensation or the utility of leisure), an increase in the marginal tax rate has an ambiguous impact on the wage rate. Indeed, the *scale economy* and the *standard wage moderating effects* coupled with the reduction in working hours increase the net replacement ratio and thus the reservation wage. Employees are then more reluctant to accept a decrease in their wage rate: This is the *net replacement ratio effect*. This effect on the wage rate can dominate the *scale economy* and the *standard wage moderating effects*, especially when the fixed utility level in unemployment and the labor supply elasticity are high. Finally, I show that even if the hourly profit increases through a possible decrease in the wage rate, the effects of a higher marginal tax rate on the profit per worker are ambiguous because the possibilities to recover the vacancy costs decrease. As a consequence, the impact of a higher marginal tax rate on labor demand and unemployment is undetermined.

Numerical simulations applied to France are also provided, an exercise that few papers analyzing this problem propose. The simulations indicate to what extent the effect on unemployment relies on three important dimensions: The level of utility in unemployment, whether the replacement ratio is constant or not and the labor supply elasticity. They show also that the evolution of social efficiency is essentially dictated by working hours. Furthermore, they point out two striking findings. First, except for extreme values of the parameters, increasing the marginal tax rate improves employment but deteriorates social efficiency. Second, the evolution of the optimal marginal tax rate with respect to the labor supply elasticity is not monotonic.

Three branches of the literature are linked to my paper. A first literature focusing on imperfect
labor markets has stressed the importance of distinguishing the unemployment compensation regimes. However, Pissarides (1998), Holmlund (2002), Altenburg and Schaub (2002), Kilponen and Sinko (2005) and Van der Ploeg (2006) do not analyze the effects of an increase in tax progression. In a union model, Holmlund and Kolm (1995) show that the qualitative effects of the marginal tax rate (taking average tax rates as given) on unemployment are the same in each regime. It turns out that this latter property is lost when the labor supply is endogenous in a search model.

Second, other articles have enriched the standard wage moderating effect of the marginal tax rate by incorporating an intensive labor supply margin. The vast majority of these studies have considered union models. Among others, Calmfors (1995), Sørensen (1999) and Hansen et al. (2000) have shown that the effects of the marginal tax rate on the wage rate might be ambiguous. In contrast to my paper, these analyses do not compare the effects of the marginal tax rate when utility in unemployment is fixed or indexed on wages. Furthermore, except Sørensen (1999) and Fuest and Huber (2000), no paper has analyzed the effects on unemployment. However, these two papers show that a higher marginal tax rate always reduces unemployment. This is due to the assumption, made in all these union models, that firms consider men and hours as perfectly substitutable. This implies that firms and unions compensate for a decrease in hours by an increase in employment. The matching model is usually not characterized by such a property. However, Hansen (1999) does not share my scale economy effect since he considers that vacancy costs are proportional to hours, an assumption that is not standard in the matching literature with endogenous hours (see Pissarides (2000)). Moreover, he considers that utility in unemployment is equal to zero. This explains why Hansen obtains positive employment effects of a higher marginal tax rate in a matching model. Cahuc and Zylberberg (2004) consider fixed vacancy costs and a positive utility in unemployment. However, they also show that the unemployment rate always decreases when the marginal tax rate is pushed up. This is due to the worker’s utility function they consider and which is characterized by income and substitution effects that offset each other when the initial tax system is proportional. My study and the analysis provided in Cahuc and Zylberberg can be considered as complementary since empirical studies indicate that for

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5 Tax progression increases when the average tax rate increases with income. This occurs when the marginal tax rate increases more than the average tax rate.

6 The theoretical analysis of Andersen and Rasmussen (1999) has also shown that the marginal tax rate has an ambiguous effect on wages and employment in an efficiency wage setting with two levels of effort.

7 Fuest and Huber (2000) show that the impact on unemployment depends on the existence of labor supply income effects. However, the comparability of their article with my paper and the papers aforementioned is difficult since possible negative effects of a higher marginal tax rate are due to income effects emerging from a balanced government budget.
some categories, the substitution effect dominates significantly the income effect and for other (richer) categories, the two effects offset each other (see Blundell (1995)).

The third literature analyzes the effects of the marginal tax rate on social efficiency. In a static matching framework with fixed hours, Boone and Bovenberg (2002) have shown that when the Hosios condition is not satisfied \(^8\), the optimal marginal tax rate increases with the worker’s bargaining power in order to compensate for its possible negative effects on trading externalities. This property is in line with my results. My simulations are also consistent with the theoretical contribution of Pissarides (1983). Even if the mechanisms are different, the basic idea is the same. A higher utility in unemployment implies a higher reservation wage, whose negative effects are compensated for by a higher marginal tax rate. Finally, the numerical simulations of Sørensen (1999) emphasize that the optimal tax progression decreases sharply with the labor supply elasticity. My simulations reveal a non-monotonic relation between the optimal marginal tax rate and the labor supply elasticity. This important difference is explained by the fact that the scale economy effect induced by search frictions and the net replacement ratio effect induced by a fixed component in the utility in unemployment are absent in Sørensen (1999).

The paper is organized as follows. Section 2 presents the model and the theoretical effects of a higher marginal tax rate on wage rates and unemployment. Section 3 provides numerical simulations. Section 4 concludes.

## 2 The model

### 2.1 General assumptions

The framework is the matching model developed in Mortensen and Pissarides (1999) and Pissarides (2000). For the sake of simplicity, I restrict my analysis to the steady-state equilibrium and time variables are omitted. Agents are homogenous but transaction costs imply that vacant jobs and unemployed workers meet through an imperfect matching process. The matching function \( M(U,V) \) gives the number of matches that are formed in the market at each instant. This number is a function

\(^8\)This condition expresses the fact that without taxation and with exogenous hours, a decentralized wage formation internalizes the trading externalities due to the matching frictions. This condition is satisfied when the worker’s bargaining power is equal to the elasticity of the matching function (see Hosios (1990) and Pissarides (2000)).
of the total number of agents searching for a job, \( U \), and of the (endogenous) number of vacant jobs \( V \) at each instant. The matching function satisfies the standard assumptions i.e. it is increasing in each argument, continuously differentiable, homogenous of degree one and yields no hiring if \( U \) or \( V \) is nil.

Denoting by \( \theta \equiv \frac{V}{U} \) the labor market tightness, a vacant job is filled at the rate

\[
q(\theta) = \frac{M(U,V)}{V} \quad q'(\theta) < 0
\]

and a worker finds a job at a rate

\[
\theta q(\theta) = \frac{M(U,V)}{U} \quad (\theta q(\theta))' > 0
\]

Normalizing the labor force to 1, the unemployment rate in each period is \( U \). The entry flows into employment are therefore equal to \( U \theta q(\theta) \). I assume that negative idiosyncratic shocks arrive at an exogenous rate \( \lambda \). These shocks lead to the destruction of the job and to the entry of the worker into unemployment. Therefore, the flows into unemployment are \( \lambda (1 - U) \). Thus, the unemployment rate at the steady-state equilibrium is given by the following Beveridge relation:

\[
U = \frac{\lambda}{\lambda + \theta q(\theta)}
\]  

(1)

In the following sections, I introduce the characteristics of each participant in the labor market.

2.1.1 Workers

Workers have no aversion to risk, are infinitely living and their discount rate is \( r \). They are either unemployed or employed. An unemployed worker searches for a job and has an unemployment utility of \( B \). I assume that there is no cost of search i.e. that the extensive margin of the labor supply is absent \(^{10}\). The expected value of the stream of income of an unemployed worker at the stationary

\(^9\)Ljundqvist and Sargent (1995) study the effects of taxation with on-the-job-search but they consider an exogenous intensive labor supply margin. I don’t allow for on-the-job-search in order to simplify the analysis.

\(^{10}\)It is however possible to interpret the unemployment utility \( B \) as an utility net of exogenous search costs. Lehmann and Van der Linden (2004) analyze the effects of fiscal progression with an endogeneous search effort but with an exogeneous labour supply intensive margin.
equilibrium is:

\[ rV_u = B + \theta q(\theta)(V_e - V_u) \]  

(2)

where \( V_e \) represents the expected present value of the stream of income of a worker who is employed at the stationary equilibrium. I distinguish two polar cases for utility in unemployment. In the first case, one has \( B = \rho(WH - T(WH)) \) with \( \rho \) the (fixed) net replacement ratio. In the second case, I consider an extreme form of endogenous net replacement ratio by setting \( B \) at \( \underline{B} \). One might wonder if analyzing a fixed utility in unemployment is relevant to a comparative steady state study. In particular, one may argue that in the long run, utility in unemployment is necessarily perfectly indexed on net wages. If it was not the case, unemployment would not be constant along a balanced growth path. However two arguments support the relevance to analyze the effects of a higher marginal tax rate when utility in unemployment is fixed. First, the costs involved by the acceptance of a job are not only financial (e.g. forgone leisure) and these costs are not necessarily proportional to the wage rate obtained in the labor market (e.g. home production or revenues from working in the underground economy). Second, since the transitory dynamics in matching models are quite short, the steady state is attained in the medium run. Now, in the medium run, non human wealth or assistance benefits have not always enough time to adjust (for a discussion, see Pissarides (2000)) 11.

A worker is employed when he has been matched with a firm and has agreed with her on the wage rate \( W \) 12 and the working hours \( H \). Once the worker and the firm have reached an agreement, production takes place, the employed worker earns \( WH \) and pays taxes \( T(WH) \). Working hours are costly for workers. The disutility is denoted \( d(H) \). I consider a separable utility function which is quasi-linear in consumption. Income effects are therefore omitted. This assumption is also generally made in the literature because it seems less important to take the income effects into account when one focuses on working hours rather than participation to the labor market. Moreover, Blundell (1995) shows that the labor supply is an inverse-U shaped function of the net wage rate and that it becomes flatter when the net wage rate increases. However, the level of the wage rate from which the substitution and the income effects offset each other is not precisely known. Finally, the great majority of the literature has considered such a separable utility function so it allows for a better comparability

11 Another argument that might give some support to the case of an endogenous net replacement ratio would be to assume that wage taxes are used to finance unemployment benefits, that the government budget is balanced and that the average tax rate is held constant. However, I do not consider the government budget equilibrium constraint.

12 \( W \) corresponds basically to the hourly cost of labor. Henceforth, I use the term wage rate for simplification.
of my results with the literature. The disutility function is isoelastic and writes $d(H) = a_o \frac{H^\alpha}{\alpha}$, $a_o$ being a scale parameter. As is usual in the literature, this function is increasing and convex, which implies that the elasticity of the disutility of labor $\alpha$ is higher than $1$\footnote{The elasticity of the disutility of labor is related to the elasticity of the labor supply $\eta$ in the following way: $\alpha = 1 + \frac{1}{\eta}$.}. Finally, an employed worker loses his job at a rate $\lambda$. Thus, one has:

$$rV_e = WH - T(WH) - a_o \frac{H^\alpha}{\alpha} + \lambda (V_u - V_e)$$

(3)

### 2.1.2 Firms

There is an endogenous number of firms producing a single good (whose price is normalized to one) with labor as only input. It is sold in a perfectly competitive market. As is standard in the matching literature, I consider small firms (i.e. each firm restricts to one job) characterized by a discount rate equal to $r$. When the job is vacant the firm is searching for a worker. The fixed vacancy cost being $C$, the asset value on a vacant job at the stationary equilibrium verifies:

$$rJ_v = -C + q(\theta)(J_f - J_v)$$

(4)

with $J_f$, the asset value of a filled job. Once a firm has met a worker and has agreed on the wage rate and the working hours, she produces a fixed number of product per hour, $Y$. This assumption is consistent with the medium run period on which I focus. It implies that firms aim at maximizing the total number of hours worked in a job in order to recover their vacancy costs. Since a job is destroyed at a constant rate $\lambda$, the asset value of a filled job writes:

$$rJ_f = (Y - W)H + \lambda (J_v - J_f)$$

(5)

In order to shed light on the intuitions behind the effects of a higher marginal tax rate, I present gradually its underlying mechanisms. Therefore, I analyze the effects of the marginal tax rate on the equilibrium relations before presenting its total effects on hours, wages and unemployment.
2.2 Effects of the marginal tax rate on the equilibrium relations

Before analyzing the effects of an increase in the marginal tax rate on wage formation and labor demand relations, it is necessary to bring up two assumptions used in the paper and usually adopted in the literature. The first assumption consists in leaving out the government’s budget constraint from the analysis. Two reasons explain this assumption. First, it allows to consider the marginal tax rate as a parameter i.e. to study the effects of variations in the marginal tax rate taking the average tax rate as given. Second, a lump-sum tax/subsidy given to all the agents may insure a balanced budget constraint without affecting the results. The second assumption is that the matching function is Cobb Douglas. The elasticity of the matching function with respect to the number of unemployed, \( \gamma \equiv -\frac{\theta q'(\theta)}{q(\theta)} \), is thus constant and lower than 1. This function is frequently used in the literature and is supported by empirical analyses (see e.g. Blanchard and Diamond (1989), Pissarides (2000) and Petrongolo and Pissarides (2001)).

2.2.1 Effects of the marginal tax rate on wage formation

I consider an efficient bargaining process between matched firms and workers i.e. the wage rate and the working hours are simultaneously bargained. Considering that workers choose their working hours would introduce a bias toward a lower decrease in wage rates following an increase in the marginal tax rate. Firms would take into account the negative effects of a higher marginal tax rate on hours and would have to compensate for these effects by a smaller decrease in the wage rate (see Hansen (1999)). Introducing this mechanism would thus yield more complex relations and limit the understanding of my specific mechanisms.

Matched firms and workers share their rents through a Nash bargaining process. The negotiation is decentralized in the sense that the outside options are taken as given by the players. Denoting \( \beta \) the worker’s bargaining power, \((W, H)\) solves the following program:

\[
\max_{W,H} (V_e - V_u)^\beta (J_f - J_v)^{1-\beta}
\]

The F.O.C. yield the following working hours relation:

\[
a_o H^{\alpha-1} = Y (1 - T_m)
\]
where \( T_m \equiv \frac{dT(WH)}{dW} = \frac{dT(WH)}{dH} \) is the marginal tax rate. Relation (6) expresses the fact that the marginal cost of an additional working hour (the marginal disutility of work) has to be equal to its marginal benefit (the hourly productivity minus additional taxes) for the worker-firm pair. The properties of working hours are standard: The number of hours increases with \( Y \) and the reactions to an increase in the marginal tax rate are decreasing in \( \alpha \) (i.e. increasing with the labor supply elasticity).

The free entry condition of firms implies that \( J_v = 0 \). Therefore, defining the average tax rate by \( T_M \equiv \frac{T(WH)}{WH} \), the wage rate is given by:

\[
W = \frac{\beta Y (1 - T_m) + (1 - \beta) \frac{(a_\alpha H^\alpha / \alpha + rV_u)}{H}}{\beta (1 - T_m) + (1 - \beta) (1 - T_M)}
\]  

(7)

The wage rate increases with productivity \( Y \), the worker’s bargaining power \( \beta \) and the worker’s reservation wage \( rV_u \). The effect of working hours on the wage rate is ambiguous. Consider the effects of an increase in the marginal tax rate. On the one hand, by pushing down the disutility of labor \( a_\alpha * H^{\alpha - 1} \), a decrease in hours reduces the necessary wage rate compensation. On the other hand, there are “feedback” effects emerging from the macroeconomic equilibrium since the reservation wage is affected by taxation. In order to capture the total effect of the marginal tax rate on the wage rate at the steady state macroeconomic equilibrium, it is necessary to distinguish the unemployment utility regimes.

**Effects of the marginal tax rate in the fixed net replacement ratio regime.**

Using relations (2), (3), taking into account that \( J_v = 0 \) in relation (5), and replacing the expression of \( B \), the F.O.C. of the Nash bargaining program yields the wage relation \( WS \):

\[
W = \frac{Y \beta (1 - T_m) + (1 - \beta) * a_\alpha \frac{H(T_m)^{\alpha - 1}}{\alpha} \cdot g(\theta)}{\beta (1 - T_m) + (1 - \beta) (1 - \rho) (1 - T_M) \cdot g(\theta)}
\]  

(8)

with \( g(\theta) \equiv \frac{r + \lambda}{r + \lambda + \theta q(\theta)} \) (and thus \( g'(\theta) < 0 \)) and where \( H(T_m) \) is obtained by relation (6). This relation defines an increasing curve in the \((\theta, W)\) plan \(^{14}\) because when the labor market tightness is high, agents who have met a firm exert a stronger pressure on wages.

\(^{14}\) The concavity of this curve is not signed. It is represented on Figure 1 as a concave curve since this form appears in the numerical simulations.
Figure 1: Effects of the marginal tax rate on $W$ and $\theta$. $E$ is the initial equilibrium. When the utility in unemployment is perfectly indexed on net wages, the possible new equilibria are $E_1$ and $E_2$. When the utility in unemployment is fixed, possible new equilibria are $E_1$, $E_2$, $E_{21}$ and $E_{22}$.

An increase in the marginal tax rate shifts the $WS$ curve to the South-East region (i.e. from $WS$ to $WS_1'$ on Figure 1). This is due to two effects. An increase in the gross wage rate is less favorable to the employee which gives him less incentives to be aggressive in the bargaining process and this reduces the wage rate. This is the \textit{standard wage moderating effect} of a higher marginal tax rate and emerges formally when hours are fixed \( \left( \frac{\partial W}{\partial t_m} \right|_{t_m=0,h=0} < 0 \). The impact of an increase in the marginal tax rate on hours amplifies this effect. The increase in the marginal tax rate diminishes working hours, reduces labor disutility and thus the financial compensation asked by the employee.

\[15\text{The small letters correspond to the differentiated variables (e.g. for } H, \text{ one has } h = \frac{\partial H}{\partial H}. \]
Effects of the marginal tax rate in the fixed unemployment utility regime.

In this regime, the wage relation $WS$ writes:

\[
W = \frac{Y\beta (1 - T_m) + (1 - \beta) g(\theta) \left( a_o \frac{H(T_m)^{\alpha-1}}{\alpha} + \frac{B}{H(T_m)} \right)}{\beta (1 - T_m) + (1 - \beta) (1 - T_M) g(\theta)}
\]

(9)

where $H(T_m)$ is determined by relation (6). Since the analysis of this relation is very close to the preceding one, I only comment on the differences. The term $\frac{\bar{B}}{H(T_m)}$ captures this difference and shows that the net replacement ratio rises when the marginal tax rate increases. As a response to this effect, employed workers aim at limiting the decrease in their wage rate. This net replacement ratio effect, that emerges exclusively from the introduction of working hours is new in the literature.

It suggests that when utility in unemployment $\bar{B}$ and the labor supply elasticity are sufficiently high, an increase in the marginal tax rate does not necessarily decrease the wage rate (this case is displayed in Figure 1 by the curve $WS_2'$, the shift $WS_1'$ corresponding to the usual case put forward in the literature). Thus, the effect of working hours on the wage rate is ambiguous.

2.2.2 Effects of the marginal tax rate on aggregate labor demand

The free entry condition implies that firms decide to enter the market as long as their intertemporal discounted expected profit on a filled job is equal to the average cost of a vacant job. Thus, at the equilibrium, the value of the expected profit on a vacant job $J_v$ is nil, which is expressed by the following relation:

\[
J_f = \frac{C}{q(\theta)}
\]

Substituting this relation and relation (6) in relation (5), one obtains the aggregate labor demand (called also the price relation):

\[
W = Y - \frac{C}{H} * \frac{(r + \lambda)}{q(\theta)} = Y - \frac{C (r + \lambda)}{(Y (1 - T_m) / a_o)^{\alpha - 1}} \frac{1}{q(\theta)}
\]

(10)

The curve associated to this relation is decreasing and convex in the $(\theta, W)$ plan (see Figure 1). Ceteris paribus, an increase in the wage rate decreases the intertemporal profit on a filled job and thus the total number of vacant jobs and labor market tightness. The tax system affects this
relation through working hours. An increase in the marginal tax rate pushes down working hours and diminishes the possibilities for firms to recover their sunk vacancy costs. As a consequence, for a given wage rate, the decrease in hours diminishes the expected profits on filled jobs and restricts the number of firms. Thus, an increase in the marginal tax rate shifts the aggregate labor demand curve $PS$ to the South-West region (from $PS$ to $PS'$ on Figure 1).

As is shown in Figure 1, the effect of the marginal tax rate on employment is ambiguous. In the fixed net replacement ratio regime, the wage rate decreases as the marginal tax rate increases but the effect on employment depends on the extent of the shift of the $PS$ curve. For a fixed utility in unemployment, an increase in the marginal tax rate might imply an increase in the wage rate and in the unemployment rate.

Before investigating what are the key variables governing the results, it is important to determine the conditions under which the equilibrium exists. The curves $WS$ and $PS$ defining the equilibrium are respectively increasing and decreasing. When an equilibrium exists, it is therefore unique. In Appendix A, it is shown that an equilibrium exists in the fixed net replacement ratio regime when the condition $(1 - \beta) \left( \frac{1-T_m}{\alpha} - (1 - \rho)(1 - T_M) \right) < 0$ is satisfied. In the fixed utility in unemployment regime, an equilibrium exists when the condition $(1 - \beta) \left( \frac{Y(1-T_m)}{\alpha} - Y(1 - T_M) + \frac{B}{H(T_m)} \right) < 0$ is satisfied. Intuitively, these conditions express the fact that firms’ and workers’ rents are positive. Henceforth, I assume that these conditions hold.

2.3 Effects of an increase in the marginal tax rate on hours, wages and unemployment

The relations describing the total effects of an increase in the marginal tax rate on wages and unemployment are very complex. Thus, in order to simplify the presentation and to capture the intuitions of the mechanisms, my analysis is based on the study of total elasticities $\xi_{I,T_m}$ of the endogenous variables $I$ with respect to the marginal tax rate $T_m$ (the average tax rate remaining fixed). Moreover, the total elasticities $\xi_{I,T_m}$ are expressed as functions of the partial elasticities $\varepsilon_{I,Z}^P$ emerging from relations (6), (8) or (9) and (10) ($Z$ corresponds thus to endogenous or exogenous variables). More intuitively, the total elasticities emerge from the macroeconomic equilibrium and the partial elasticities are related to the relations at the microeconomic level. The proofs of the results are in Appendix B.
Since working hours are solely affected by the marginal tax rate, the elasticity of working hours with respect to $T_m$ is trivially given by the differentiation of relation (6). One obtains:

$$\xi_{H,T_m} = -\frac{T_m}{1-T_m} \frac{1}{\alpha-1} < 0$$

This elasticity increases (in absolute value) with the labor supply elasticity and with the initial marginal tax rate. When the initial marginal tax rate is high, working hours are initially highly distorted. A further increase in the marginal tax rate increases more the hours distortion than when the initial marginal tax rate is low.

Since the effects of the marginal tax rate on the wage relations are different in each unemployment utility regime, it is necessary to distinguish them.

2.3.1 The fixed net replacement ratio regime

The total elasticity of the wage rate with respect to the marginal tax rate writes:

$$\xi_{W,T_m} = \frac{\varepsilon_{p,T_m}^W + \left(\frac{\varepsilon_{p,\theta}}{\gamma}\right) \cdot \xi_{H,T_m}}{1 - \left(\frac{\varepsilon_{p,\theta}}{\gamma}\right) \cdot \varepsilon_{f,T,W}^p} < 0$$ (11)

The impact of an increase in the marginal tax rate on the wage rate is influenced by three effects. When the intensive margin is exogenous (i.e. when $\xi_{H,T_m}$ is equal to zero), the first term in the numerator and the denominator are solely present. The term $\varepsilon_{p,T_m}^W$ represents the standard wage moderating effect. It is weakened by a feedback macroeconomic effect since the decrease in the wage rate pushes up labor demand and labor market tightness. Since the term in the denominator is higher than 1, the final variation of the wage rate following an increase in the marginal tax rate is lower than the variation at the microeconomic level ($\xi_{W,T_m} < \varepsilon_{W,T_m}^p$). The intensity of this feedback effect depends on $\varepsilon_{W,\theta}$ and the matching function elasticity $\gamma$. $\varepsilon_{W,\theta}$ measures the intensity of the wage rate variation when the labor market tightness is modified. $\frac{1}{\gamma}$ gives the effectiveness of vacancies to generate matching. The higher $\varepsilon_{W,\theta}/\gamma$, the higher the increase in the labor market tightness when the wage rate is initially pushed down by the marginal tax rate, and the higher the induced wage claims.

\textsuperscript{16}One has $\left(\frac{\varepsilon_{W,\theta}}{\gamma}\right) \varepsilon_{f,T,W}^p < 0$ with $\varepsilon_{f,T,W}^p$ the elasticity (at fixed hours) of the profit on a filled job with respect to the wage rate.
of the employees. Thus, the higher $\varepsilon_{W,\theta}/\gamma$, the smaller is the final decrease in the wage rate when the marginal tax rate is raised.

Taking an endogenous labor supply into account adds a new effect, the scale economy effect, which is captured by the term $\left(\varepsilon_{W,\theta}/\gamma\right) \times \xi_{H,T_m}$. A higher marginal tax rate pushes working hours down. This reduces firms’ net flow profits and thus the possibilities for firms to recover their vacancy costs. Firms’ expected profit is therefore decreased and from relation (10), this implies that labor demand and labor market tightness diminish. These induced effects push down the wage rate in the negotiations. The intensity of the scale economy effect increases with $\varepsilon_{W,\theta}$ and/or $\frac{1}{\gamma}$.

Finally, since the standard wage moderating effect and the scale economy effect affect similarly the wage rate, the wage rate decreases with the marginal tax rate. However, even if the hourly profit on a filled job is pushed up, total profit and labor demand do not necessarily increase with the marginal tax rate. The effects of a variation in the marginal tax rate on the unemployment rate are captured by the variation in the expected profit on a filled job. Log-differentiating the Beveridge relation (1), one has:

$$\xi_{U,T_m} = -\xi_{J_f,T_m} \times \left(\frac{\theta q(\theta)}{\lambda + \theta q(\theta)} \times \frac{1 - \gamma}{\gamma}\right) \tag{12}$$

with $\xi_{J_f,T_m}$, the elasticity of the profit on a filled job with respect to the marginal tax rate. This elasticity is given by (using the log-differentiation of relation (5) with $J_v = 0$):

$$\xi_{J_f,T_m} = \xi_{H,T_m} + \varepsilon_{J_f,W}^{p} \xi_{W,T_m} \xi_{H,T_m} \tag{13}$$

$\xi_{H,T_m}$ has a negative impact on the numerator of $\xi_{J_f,T_m}$ but $\varepsilon_{J_f,W}^{p} \xi_{W,T_m}$, which measures the transmission of a variation in $T_m$ on firm’s profit $J_f$ (and thus on labor demand) through the wage rate channel, has a positive influence on $\xi_{J_f,T_m}$. Thus, the elasticity of the unemployment rate with respect to the marginal tax rate is more likely positive when the labor supply elasticity (i.e. $\xi_{H,T_m}$) is high. A higher marginal tax rate diminishes the wage rate and working hours. When the labor supply elasticity is sufficiently high, the loss due to the lower scale economy possibilities is not covered by the gain coming from the decrease in the wage rate. In this case, although the hourly profit increases, total profit, labor demand and employment decline.

This result is summarized in the following proposition:
Proposition 1 For a fixed net replacement ratio, an increase in the marginal tax rate diminishes the wage rate and has an indeterminate impact on the unemployment rate. The higher the elasticity of the labor supply, the more likely the unemployment rate increases.

2.3.2 The fixed utility in unemployment regime

The total elasticity of the wage rate with respect to the marginal tax rate writes:

\[ \xi_{W,Tm} = \varepsilon_{pW,Tm} + \frac{\left( \varepsilon_{pW,\theta}/\gamma \right) * \xi_{H,Tm} + \varepsilon_{pW,H} * \xi_{H,Tm}}{1 - \left( \varepsilon_{pW,\theta}/\gamma \right) \varepsilon_{f_{j,W}}} \]  

The analysis of this regime is close to the latter one. Therefore, I only comment the differences. A new term appears in the numerator. The term \( \varepsilon_{pW,H} * \xi_{H,Tm} \) is negative and emerges from the impact of working hours on the wage rate through the net replacement ratio effect. Lower working hours increase the net replacement ratio and render the situation of the unemployed less “unattractive”. This effect tends to increase the wage claim of the employees. Moreover, the intensity of this effect increases with the labor supply elasticity. The total effects of the marginal tax rate on the wage rate is therefore ambiguous.

It is interesting to assess the influence of the level of utility in unemployment to understand how important the net replacement ratio effect can be. To do this, compare two economies (or two different groups in a same economy) differing only by their level of utility in unemployment. As is shown in Appendix C, the higher the level of utility in unemployment, the more likely the wage rate increases when the marginal tax rate increases. Basically, when hours decrease, the increase in the net replacement ratio (and the reservation wage) is pushed up by the level of utility in unemployment. Thus, employees resist more to the wage rate moderating effects of the marginal tax rate. As a consequence, the unemployment rate is more likely to increase when the level of utility in unemployment is high. The next Proposition summarizes these results.

Proposition 2 For a fixed utility in unemployment, an increase in the marginal tax rate has an ambiguous impact on the wage rate and the unemployment rate. The higher the utility in unemployment and/or the higher the labor supply elasticity, the more likely the wage rate and the unemployment rate increase.
3 Numerical simulations

This section is devoted to the study of three questions. The theoretical analysis suggests that the total effect of the marginal tax rate on employment might be different when utility in unemployment is fixed or indexed on net wages. However, real world falls between these two polar cases. The numerical simulations intend therefore to assess which effect dominates. The second objective of the simulations consists in determining what is the effect of the marginal tax rate on economic efficiency, defined as the total net product minus a possible government budget deficit, which writes\footnote{One might wonder why $\Omega$ does not encompass home production. This is because I restrict my analysis to the unemployment compensation. However, I have made other simulations by taking a home production into account and the results were not significantly affected. This is because $\Omega$ is much more affected by gross production than by the government budget -or the home production.}:

$$\Omega = \left[ YH - d(H) + WHT_M (1 - U) - U \left[ \bar{B} + \rho WH (1 - T_M) \right] \right] - CU \theta$$  \hspace{1cm} (15)

The impact on $\Omega$ is \textit{a priori} ambiguous since the effect on unemployment is not signed and the impact on working hours is negative. Moreover, an important objective of this analysis consists in shedding light on an eventual political economy problem. More precisely, a central planner whose objective is not only social economic efficiency but also employment might be confronted with a dilemma. Finally, this section assesses the quantitative effects of the parameters on the optimal marginal tax rate. The optimal marginal tax rate is defined as the marginal tax rate that maximizes economic efficiency when the average tax rate characterizing the French population studied is fixed.

This section is organized as follows. First, I explain how the model is calibrated. Second, I report the results of the numerical simulations.

3.1 Calibration

The model is characterized by two types of parameters: The parameters which value is “known” and the parameters to which I attribute a value in order to reproduce some stylized facts characterizing the French economy in 2000. The reference period is the year because wage negotiations in France take place generally each year since the Auroux laws of 1982.

The baseline parameters summarized in Table 1 are the following. The matching function is a Cobb Douglas, so $M(U,V) = d_o U^\gamma V^{1-\gamma}$ with $d_o$ a scale parameter. The empirical analyses of Blanchard and...
Diamond (1990), Broersma and Van Ours (1999) and Pissarides (2000) show that the Cobb Douglas function is a good approximation and they find that $\gamma = 0.5$. It is difficult to assess empirically the order of magnitude of $\beta$. Thus, I use the conventional assumption of an equal bargaining power for the firm and the worker i.e. $\beta = 0.5$. Thus, the Hosios condition is satisfied. The empirical estimations of the labor supply elasticity conduct to very different levels. The labor supply of male workers is found to be quite inelastic (0.1) while the elasticity of married women and lone parents is found to be much more elastic (from 0.7 to 1). Therefore, I set the elasticity of the labor supply $\eta$ to an average value of 0.4 which seems consistent with Blundell (1995), Blundell and MaCurdy (1999) and Roed and Stroem (2002). The discount rate is set at 0 to obtain figures consistent with the definition of economic efficiency in relation (15) \(\text{18}\). The job destruction rate is given by the evaluations of the OECD (1995) and is equal to 20%. The OECD defines wage categories with respect to the wage of the average product worker (APW). My simulations focus on this category for several reasons. First, the level of the wage rate of the APW is such that it is not directly affected by the minimum wage (see CSERC (1999)). Secondly, this is the category that would be affected by an increase in the marginal tax rate if the subsidies or social security contribution reductions on low-paid jobs (roughly those paid at the minimum wage) were intensified. Finally, the linear tax rate assumed in the paper is quite representative for the APW category (see Laroque (2005)). The tax wedge evaluations are found in the OECD Tax Database. In 2000, the average tax rate of the labor cost is equal to 48.2% and the marginal tax rate of the labor cost is equal to 52.9% \(\text{19}\). I set the initial wage rate at 100 and the initial number of hours at 1. Finally, Martin (1996) provides evaluations of the net replacement rate for different qualification categories. He finds a total net replacement rate of 65% \(\text{20}\) for individuals with an “average” qualification, a good approximation of the APW category. Moreover, the unemployment compensation French system is linear (see OECD (2000)). The Beveridgian and the Bismarckian components of the unemployment compensation are calculated on the basis of the UNEDIC unemployment compensation general regime. $\rho$, the component indexed on net wages, is equal to 0.4. This yields a fixed component of utility in unemployment equal to $\tilde{B} = 12.9$.

The second group of parameters is fixed in order to reproduce some key figures of the French

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\(\text{18}\) The results wouldn’t have been changed if I had taken a positive $r$.

\(\text{19}\) These taxes include: “The combined central and sub-central government income tax plus employee and employer social security contribution taxes, as a percentage of labour costs defined as gross wage earnings plus employer social security contributions” (see The OECD Tax Database).

\(\text{20}\) This figure is in line with the OECD Social Indicators (2005).
Table 1: Baseline parameters for the French labour market.

<table>
<thead>
<tr>
<th>APW</th>
<th>W</th>
<th>H</th>
<th>T_M</th>
<th>T_m</th>
<th>U_o</th>
<th>ρ</th>
<th>B</th>
<th>λ</th>
<th>r</th>
<th>η</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>1</td>
<td>0.482</td>
<td>0.529</td>
<td>0.1</td>
<td>0.4</td>
<td>12.9</td>
<td>0.2</td>
<td>0</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 2: The calibrated parameters in the benchmark.

<table>
<thead>
<tr>
<th>APW</th>
<th>Y</th>
<th>C_o</th>
<th>θ_o</th>
<th>a_o</th>
<th>d_o</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>101</td>
<td>48.22</td>
<td>0.18</td>
<td>47.55</td>
<td>4.24</td>
</tr>
</tbody>
</table>

population. Five variables have to be evaluated: The scale parameter of the matching function $d_o$, the initial labor market tightness $\theta_o$, the workers productivity $Y$, the scale parameter of the disutility of work $a_o$ and the vacancy cost $C_o$. The matching function scale parameter and the initial labor market tightness are evaluated by using the unemployment rate and the rate at which a vacant job is filled. I take a reasonable initial unemployment rate $U_o$ of 10%. The rate at which vacant jobs are filled is given by Maillard (1997) and is equal to 5 weeks approximately. Since the reference period is the year, one obtains $q(\theta) = 10$. The value of $Y$ is then deduced from the wage formation relation (19) (see Appendix D). The parameter $a_o$ is deduced from the working hours relation (6). Finally, I obtain the vacancy cost $C_o$ from the labor demand relation (10). Table 2 summarizes the values of the calibrated parameters in the benchmark economy.

In order to verify the validity of my calibration exercise, I check if some parameters computed take reasonable values. To do this, I compare the data available for some parameters with their value induced by my calibration. First, my calibration yields a vacancy cost of 4.8% of the total yearly labor cost. This figure seems reasonable since it is in the range of the empirical estimations of Hamermesh (1993) and Abowd and Kramarz (2003). Second, the calculations of the elasticity $\xi^{S}_{1-T_m}$ of the total wage $S$ with respect to $1-T_m$ have become very popular because this elasticity encompasses two crucial fiscal distortion channels i.e. the wage bargaining and the in-work effort (working hours and effort at work). I calculate this elasticity (see Appendix D) and find a value equal to 0.417 which is close to the average value of 0.4 reported in Gruber and Saez (2001).

Fiscal distortions emerge therefore essentially through the working hours channel. This is partly explained by the two opposite effects of the marginal tax rate on the wage rate as exposed in the theoretical section.
3.2 Results

Four graphs are provided for each simulation. The horizontal axis corresponds to the marginal tax rate. The vertical axis reports respectively the percentage variations with respect to the initial situation for the wage rate $W$, the working hours $H$, the unemployment rate $U$ and social economic efficiency $\Omega$.

3.2.1 The benchmark

![Graphs showing percentage variations](image)

Figure 2: The effects (in percentage) of a modification in the marginal tax rate (holding constant the average tax rate) in the benchmark case.

The results are displayed in Figure 2. A higher marginal tax rate decreases working hours through the traditional labor supply substitution effect. The wage rate also decreases through the standard wage moderating and the scale economy effects but this variation is smaller than the variation in working hours. However, this small decrease compensates for the decrease in working hours and leads to a lower unemployment rate. This result is due to the fact that the initial wage rate $W$ is high with respect to the worker’s productivity $Y$. Therefore, the increase in the hourly profit compensates for the decrease in per head profit emerging from the lower scale economies. The modifications in the total net product are essentially driven by the working hours variations and the increase in the marginal
tax rate deteriorates the government budget. From a social efficiency point of view, the decrease in working hours dominates therefore the decrease in the unemployment rate. As a consequence, a central planner would be confronted with an objective’s conflict between the reduction in the unemployment rate and the increase in economic efficiency: An increase in tax progression does not appear to be a “free lunch” as was suggested by Pissarides (1998) \textsuperscript{22}.

3.2.2 Sensitivity analysis

In this section, I investigate to what extent the results obtained in the benchmark case are sensitive to the parameters of the model.

The theoretical analysis suggests that the unemployment utility component and the level of utility in unemployment play an important role for the effects of the marginal tax rate on unemployment. I therefore provide a sensitivity analysis concerning these unemployment utility parameters. Finally, I propose a sensitivity analysis with respect to the elasticity of labor supply.

\textbf{The influence of the unemployment compensation.}

The benchmark case shows that from an unemployment point of view, the \textit{standard wage moderating} and the \textit{scale economy effects} dominate the labor supply \textit{substitution effect}. However, the theoretical analysis suggests that when the marginal tax rate increases, the worker’s rent decreases more in the fixed unemployment utility regime. As a consequence, worker’s resistance to a wage rate decrease is more severe. It seems therefore interesting to analyze the effects of the marginal tax rate variations when $\rho$ is lower than 0.4. The form of the curves obtained in the benchmark case is not affected for a large range of $\rho$ values. Thus, I restrict my analysis to the extreme case of $\rho = 0$ which is displayed in Figure 3 \textsuperscript{23}.

For a fixed utility in unemployment, the wage rate is less affected by the modification in the marginal tax rate than in the benchmark case. Thus, the decrease in the wage rate implies a reduction in the unemployment rate as long as the marginal tax rate increases from 53\% (its initial value) to

\textsuperscript{22}Indeed, Pissarides (1998) explains that “a reform of the employment tax structure from regressive to progressive can be one of the very few free lunches that one encounters in the analysis of the economic policy” (p. 177).

\textsuperscript{23}This case should be analyzed only for individuals whose unemployment spell period is sufficiently long i.e. for those whose unemployment rate is higher than 10\%, the benchmark value. However, Figure 3 is not fundamentally modified when the initial unemployment rate is significantly increased (in particular, the graph is very close to the one presented here when the unemployment rate is equal to 30\% and corresponds to an unemployment spell duration of 25 months).
Figure 3: The effects (in percentage) of a modification in the marginal tax rate (holding constant the average tax rate) for a fixed utility in unemployment ($\rho = 0$).

64\%. When the marginal tax rate increases more, the reduction in the wage rate is shrunk since the *net replacement ratio effect* becomes more important. Moreover, since the wage rate does not vary very much for a higher marginal tax rate, the loss due to smaller scale economies on the sunk vacancy costs decreases the labor demand and increases unemployment. However, these results are solely obtained for a quasi-totally fixed utility in unemployment. Our benchmark results seem therefore quite robust to this dimension.

The sensitivity analysis concerning the unemployment utility level seems also interesting. First, the figures provided in Martin (1996) refer only to average evaluations. Some population can have access to conditional revenues due to their family situation and these revenues can strongly affect their replacement ratio. Secondly, the parameter $B$ can be interpreted as a pure utility of leisure or as domestic production. These dimensions are difficult to take into account in a statistic evaluation but they might play an important role in the agents’ decisions. Therefore, I investigate the effects of the marginal tax rate when the total net replacement ratio is increased from 0.65 to 0.7.\(^{24}\) The simulations,

\(^{24}\)It is not possible to set a higher value since the equilibrium existence conditions would no longer hold.
not reported here, show that the results are not fundamentally affected by this modification.

Figure 4: The effects (in percentage) of a modification in the marginal tax rate (holding constant the average tax rate) when the utility in unemployment is fixed ($\rho = 0$) and higher than the benchmark case.

However, the wage rate and the unemployment rate variation curves are affected when one considers simultaneously that utility in unemployment is completely fixed and that the level of the total net replacement ratio is higher than in the benchmark. This case is displayed in Figure 4. The curve describing the evolution of the unemployment rate is inverted with respect to the benchmark case (Figure 2). The importance of this modification is explained by the wage rate variation. When the marginal tax rate is higher than 57%, the decrease in working hours increases strongly the replacement ratio, the workers resistance to a decrease in the wage rate is stronger and the wage rate increases when the marginal tax rate is further increased. Thus, the profit per head is reduced through lower scale economies and through a reduction in the hourly profit. This explains why the unemployment rate increases significantly with the marginal tax rate. Moreover, since the conflict between the effects on the unemployment rate and the total net product disappears, it seems that a decrease in the marginal tax rate would be convenient for these populations.

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25 In the case where the unemployment benefit is not totally fixed, the curves are similar to Figure 4 as long as $\rho < 0.07$. 23
The influence of the labor supply elasticity.

The conflict between an economic efficiency objective and an unemployment rate objective might be driven by the labor supply elasticity $\eta$. However, as explained in the calibration section, empirical evaluations of this elasticity differ with the category of population studied. It seems therefore necessary to provide a sensitivity analysis with respect to this elasticity. I take an elasticity equal to 0.2. In order to assess the influence of the labor supply elasticity on the eventual conflict between an economic efficiency and an unemployment objective, I restrict my analysis to the extreme case of a total net replacement ratio equal to 0.7 and a totally fixed unemployment utility ($\rho = 0$). The results, not reported here, are approximately the same as in the benchmark case: The shapes of the curves are identical and solely the extent of the variation rates is affected. Since the labor supply elasticity is lower, the working hours diminish less. The net replacement ratio is thus less affected and the standard wage moderating and the scale economy effects dominate the negative impact of the labor supply on the possibilities for firms to recover their vacancy costs.

In sum, three conditions have to be simultaneously satisfied for an increase in the marginal tax rate to reduce economic efficiency and increase the unemployment rate: A total wage sufficiently sensitive to the marginal tax rate, a sufficiently high level of utility in unemployment and an utility in unemployment largely disconnected from the net wage. These three conditions seem to be appropriate only for some married women or lonely mothers. When one of these conditions is not satisfied, an increase in the marginal tax rate reduces simultaneously the unemployment rate and economic efficiency. Finally, the sensitivity analysis has suggested that it is particularly important to distinguish the two polar unemployment utility regimes when one studies the effects of the marginal tax rate with endogenous hours.

3.2.3 The optimal marginal tax rate

This section analyzes the main forces driving the optimal marginal tax rate (taking constant the average tax rate). To do this, I extend the range of marginal tax rates on the horizontal axis of the Figures. Since the value of the labor supply elasticity is quite controversial, Figure 5 depicts the effects of marginal tax rates for different labor supply elasticities, the other variables remaining set at their benchmark value. Three different values of the labor supply elasticity are represented: Solid lines
<table>
<thead>
<tr>
<th>Labor Supply Elas.</th>
<th>0.001</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark $\frac{W_Y}{T^*_m}$</td>
<td>96.3</td>
<td>97.2</td>
<td>97.9</td>
<td>98.5</td>
<td>99.0</td>
<td>99.5</td>
</tr>
<tr>
<td>$\beta = 0.3$ $\frac{W_Y}{T^*_m}$</td>
<td>91.8</td>
<td>93.7</td>
<td>95.3</td>
<td>96.7</td>
<td>97.9</td>
<td>98.9</td>
</tr>
<tr>
<td>Low Unempl. Utility (0.4) $\frac{W_Y}{T^*_m}$</td>
<td>93.8</td>
<td>94.7</td>
<td>95.4</td>
<td>96.0</td>
<td>96.5</td>
<td>96.9</td>
</tr>
<tr>
<td>Fixed Unempl. Utility $\frac{W_Y}{T^*_m}$</td>
<td>96.3</td>
<td>97.2</td>
<td>97.9</td>
<td>98.5</td>
<td>99.0</td>
<td>99.5</td>
</tr>
</tbody>
</table>

Table 3: Optimal Marginal Tax rates w.r.t the elasticity of labor supply. All the results are given in percentage.

For the low unemployment utility case, total net replacement ratio decreases from 0.65 to 0.4 but the distribution between the indexed and the fixed utility of the benchmark remains fixed. For the fixed unemployment utility case, total initial utility in unemployment is the same as in the benchmark (i.e. the total net replacement ratio remains equal 0.65).

correspond to an elasticity of 0.4, dotted lines to an elasticity of 0.2 and dashed lines to an elasticity close to zero (0.001). Table 3 considers even more labor supply elasticity values to emphasize more precisely the effects of the exogenous variables on the optimal marginal tax rate. At this stage, it should be pointed out that the values obtained in Table 3 should not be considered as the marginal tax rate that a French government should implement. The values should rather be taken as illustrative.

Several striking properties are put forward in Figure 5 and Table 3. First, Figure 5 indicates that the shapes of the social efficiency curves are very different. In the benchmark case (with a labor supply elasticity of 0.4), the social efficiency curve is an inversed-U shape with respect to the marginal tax rate and the optimal marginal tax rate is equal to 47%. When the labor supply elasticity is equal to 0.2, the shape of the social efficiency curve is the same but its slope is much less important and the optimal marginal tax rate is equal to 39%. When the elasticity is close to 0, the shape of the social efficiency curve is very different: It is increasing in the marginal tax rate (in the range considered in Figure 5) and the conflict between the evolution of the unemployment rate and the total net product disappears.

These properties imply that the relation between the labor supply elasticity and the optimal marginal tax rate $T^*_m$ is not monotonic (see Table 3). The U-shaped curve of the optimal marginal tax rate with respect to the labor supply elasticity can be explained as follows. When the labor supply elasticity is very small, hours are fixed and solely the trading externalities are important for economic efficiency.

---

In Figure 5, I consider that the Hosios condition is satisfied i.e. that $\beta = \gamma = 0.5$. Other simulations (not reported here) indicate that the results are not significantly modified when $\beta$ takes other values in the range $[0.2; 0.8]$ (keeping $\gamma$ equal to 0.5).
Figure 5: Sensivity analysis with respect to the labor supply elasticity $\eta$ (other baseline parameters keeping their benchmark value). Solid, dotted and dashed lines for $\eta$ equal to 0.4, 0.2 and 0, respectively.

In this case, the higher the average tax rate and the level of utility in unemployment, the higher the wage bargained and the lower the labor demand and economic efficiency. As shown in Hungerbühler et al. (2005), when the Hosios condition is satisfied, this implies that the marginal tax rate should compensate for these negative effects. Thus, the optimal marginal tax rate increases with the average tax rate and utility in unemployment. This explains why the optimal marginal tax rate is so high when the labor supply elasticity is nil (see Table 3). When the labor supply elasticity increases but remains lower than 0.2, an increase in the marginal tax rate harms working hours and thus the possibilities for firms to recover their vacancy costs. This explains why the optimal marginal tax rate decreases with the labor supply elasticity. However, when the labor supply elasticity is higher than 0.3, the initial wage rate-productivity $\frac{W}{Y}$ ratio becomes very high (see Table 3). This is due to the increase in the disutility of work for which workers need to be compensated for and because of the net replacement ratio effect. Thus, since the wage rate variations are approximately the same for each elasticity of the labor supply (see Figure 5), a decrease in the wage rate is necessary so that the hourly profit, the labor demand, the employment and the government budget do not decline too much. This is made
possible by fixing a higher marginal tax rate (see Figure 5) and this explains why the optimal marginal tax rate is pushed up. Therefore, the complementarity between the effects on hours, wage rates and unemployment explains why the evolution of the optimal marginal tax rate with the elasticity of the labor supply is not monotonic. This result contrasts with Sørensen (1999) who finds in efficiency wage and union models that the higher the labor supply elasticity, the lower optimal tax progression. The qualitative difference with the results of Sørensen is explained by the existence of the scale economy and net replacement ratio effects in my model.

The second interesting result concerns the sensitivity of the optimal marginal tax rate to the introduction of an endogenous labor supply. The optimal marginal tax rate is very high when the elasticity of the labor supply is close to zero (0.001) but it decreases sharply even for relatively low elasticities (0.1). When the labor supply elasticity is higher than 0.1, the optimal marginal tax rate varies less with the labor supply elasticity. This points out some offsetting effects of the marginal tax rate on hours and the wage rate. These effects are due to the impact of the marginal tax rate on the initial ratio $\frac{W}{Y}$ and on the variation of the wage rate.

Third, as shown in Table 3, the optimal marginal tax rate increases with the worker’s bargaining power (keeping $\gamma$ equal to 0.5). This result is in line with Boone and Bovenberg (2002) and Hungerbühler (2004) who demonstrate that when the labor supply intensive margin is fixed, the optimal marginal tax rate decreases with the difference between the elasticity of the matching function and the worker’s bargaining power. In fact, when the Hosios condition is satisfied, trading externalities are efficiently allocated when the intensive labor supply is exogenous. When the worker’s bargaining power is lower than the matching function elasticity, the initial wage rate-productivity ratio is too low. This induces too many firms to enter the market and implies too high total vacancy costs. Decreasing the marginal tax rate implies a higher wage rate and allows thus to compensate for this negative effect. However, the importance of this property decreases with the labor supply elasticity since the variation in hours becomes more important for economic efficiency.

Fourth, the higher the level of utility in unemployment, the higher the optimal marginal tax rate. This result is in line with Pissarides (1983). In particular, I generalize the result of Pissarides by endogenizing labor demand and working hours. The basic mechanisms is the same as in Pissarides (1983): A higher level of utility in unemployment implies a higher reservation wage and pushes the wage rate up. Higher marginal tax rates compensate for this negative effect.
Finally, the degree of indexation of the utility in unemployment on the net wage has not a monotonic impact on the optimal marginal tax rate. When the elasticity of labor supply is low, the net replacement rate effect is weak and the (negative) differences between the optimal marginal tax rate in the benchmark case and in the case of fixed utility in unemployment is low (3 percentage points, see Table 3). The higher the labor supply elasticity, the stronger is the net replacement rate effect. As shown in Figure 4, this implies some pressure for an increase in the wage rate when the marginal tax rate is pushed up. Thus, the optimal marginal tax rate declines.

4 Conclusion

This article has shown that the theoretical effects of an increase in the marginal tax rate on wages and employment are ambiguous. The model has put forward three theoretical contributions. First, I have shown that working hours affect the wage bargained through a scale economy effect. This effect reinforces the standard wage moderating effect of a higher marginal tax rate previously put forward in the literature considering imperfect labor markets. Second, according to the relation between utility in unemployment and the net wage, an increase in the marginal tax rate can have negative or ambiguous effects on the wage rate. These results have suggested that a higher marginal tax rate might increase the wage rate when utility in unemployment is fixed and initial utility in unemployment and/or the labor supply elasticity are sufficiently high. This case applies to the individuals that only receive an assistance benefit when they lose their job but also to the individuals characterized by a high utility of leisure. The numerical simulations have confirmed these intuitions and have shown that small changes in the marginal tax rate can have important impacts on unemployment and economic efficiency. They have also suggested that for a large part of the population, higher marginal tax rates might decrease the unemployment rate but at the expense of lower economic efficiency. Moreover, the variation of the optimal marginal tax rate with the labor supply elasticity is not monotonic. Thus, since the empirical values of the labor supply elasticity are controversial and might be very different across groups (in particular men and women), it seems very difficult to draw general policy recommendations.

Some research avenues might follow my paper. My theoretical results are different from those obtained by Cahuc and Zylberberg (2004). This is particularly surprising since union models have driven to radically opposite results: When the agent’s utility function is separable (Sørensen (1999)),
a higher marginal tax rate improves employment but when the agent’s utility function is not separable (Fuest and Huber (2002)), the effects on employment are no longer always positive. In this perspective, it would be interesting to conduct the analysis with a general utility function in order to explain these differences. Furthermore, I have shown that the level of utility in unemployment has a major importance for the results. Taking simultaneously the intensive and the extensive labor supply margins would be interesting. However, it would probably be much more complex to stress the theoretical mechanisms of a higher marginal tax rate in such a model.

Appendix

A The existence of the equilibrium

I determine the existence of the equilibrium by comparing the wage rate levels given in relations WS and PS for \( \theta \to 0 \) and \( \theta \to +\infty \). This is done by using the wage formation relation (8) (or (9)) and the labor demand (10) after having substituted the working hours relation (6). For the labor demand relation, I obtain:

\[
\lim_{\theta \to 0} W^{PS} = Y \quad \text{and} \quad \lim_{\theta \to +\infty} W^{PS} = -\infty
\]

In the fixed net replacement ratio regime, one has:

\[
\lim_{\theta \to 0} W^{WS} = \frac{Y\beta (1 - T_m) + (1 - \beta) \frac{(1-T_m)Y}{\alpha}}{\beta (1 - T_m) + (1 - \beta) (1 - \rho) (1 - T_M)} \quad \text{and} \quad \lim_{\theta \to +\infty} W^{WS} = Y
\]

Thus, for \( \theta \to +\infty \), one has \( W^{WS} > W^{PS} \). For \( \theta \to 0 \), one has \( W^{WS} < W^{PS} \) if

\[
Y (1 - \beta) \left( \frac{(1-T_m)}{\alpha} - (1 - \rho) (1 - T_M) \right) < 0. \quad \text{Therefore, a unique equilibrium exists if}
\]

\[
(1 - \beta) \left( \frac{(1-T_m)}{\alpha} - (1 - \rho) (1 - T_M) \right) < 0.
\]

In the fixed unemployment utility regime, one has:

\[
\lim_{\theta \to 0} W^{WS} = \frac{Y\beta (1 - T_m) + (1 - \beta) \left( Y\frac{(1-T_m)}{\alpha} + \frac{B}{H(T_m)} \right)}{\beta (1 - T_m) + (1 - \beta) (1 - T_M)} \quad \text{and} \quad \lim_{\theta \to +\infty} W^{WS} = Y
\]

Thus, for \( \theta \to +\infty \), one has \( W^{WS} > W^{PS} \). For \( \theta \to 0 \), one has \( W^{WS} < W^{PS} \) if

\[
(1 - \beta) \left( \frac{Y(1-T_m)}{\alpha} - Y (1 - T_M) + \frac{B}{H(T_m)} \right) < 0.
\]

One can easily check that theses conditions are equivalent to the conditions ensuring positive rents
for employees and firms.

B Determination of the partial and total elasticities with respect to the marginal tax rate

The model is defined by three endogenous variables: The wage rate, the working hours and the labor market tightness. The equilibrium is determined by the working hours relation (6), the aggregate labor demand (10) and the wage formation relations, (8) or (9) depending on the regime analyzed. In this appendix, I log-differentiate each relation with respect to the endogenous variables and the marginal tax rates.

I start with the relations common to the two regimes. The small letters correspond to the differentiated variables. One has thus $h = \frac{dH}{H}$, an exception being for $\theta = \frac{d\theta}{\theta}$.

The aggregate labor demand relation (10) (without substituting the working hours) gives:

$$\frac{1}{\gamma}(-\varepsilon_{Jf,W}^p w - h) + \theta = 0$$

(16)

with $\varepsilon_{Jf,W}^p = -\frac{W}{Y - W} < 0$. The relation defining the working hours becomes:

$$(\alpha - 1) * h = - \frac{T_m}{1 - T_m} * t_m$$

(17)

I have now to distinguish the wage formation relations in each regime.

B.1 The fixed net replacement ratio regime

The log differentiation of relation (8)\(^{27}\) gives:

$$w = \varepsilon_{W,Tm}^p * t_m + \varepsilon_{W,\theta}^p * \theta$$

(18)

\(^{27}\)In order to simplify the log-differentiations, I replace the working hours relation in the wage formation relations i.e. the term $\frac{H^{\alpha - 1}}{\alpha}$ by $\frac{Y(1 - T_m)}{\alpha}$. 

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The partial elasticities write:

$$
\varepsilon_{W,T_m}^p = \frac{(1 - \beta)(1 - \rho)(1 - T_m) g(\theta)}{(1 - \beta)(1 - \rho)(1 - T_m) g(\theta) + \beta (1 - T_m) \frac{T_m}{1 - T_m}} \begin{array}{c} \text{with respect to the marginal tax rate } T_m. \text{ One has thus} \\ \end{array}
$$

$$
\varepsilon_{W,\theta}^p = \frac{(1 - \beta) \theta g'(\theta) \beta (\frac{1 - T_m}{\alpha} - (1 - \rho)(1 - T_m))}{[(1 - \beta)(1 - \rho)(1 - T_m) g(\theta) + \beta (1 - T_m)] \left(\beta + \frac{1 - \beta}{\alpha} g(\theta)\right)}
$$

It is straightforward to show that $\varepsilon_{W,T_m}^p < 0$ and $0 < \varepsilon_{W,\theta}^p$ since $g'(\theta) < 0$ and the equilibrium existence condition implies $\frac{1 - T_m}{\alpha} - (1 - \rho)(1 - T_m) < 0$.

Denoting by $\xi_{I,T_m} = \frac{dI/I}{dT_m/T_m}$, the total elasticity of the variable $I$ with respect to the marginal tax rate $T_m$, one can retrieve relation (11) by using the relations (16), (17) and (18).

### B.2 The fixed unemployment utility regime

I follow the same method as before to obtain relation (14). The log-differentiation of relation (9) gives $^{28}$:

$$
w = \varepsilon_{W,T_m}^p t_m + \varepsilon_{W,\theta}^p \hat{\theta} + \varepsilon_{W,H}^p h
$$

In order to simplify the analysis, I define $X(H) = \frac{B}{H(T_m)}$. One has thus

$$
\varepsilon_{W,T_m}^p = \frac{T_m (1 - \beta) g(\theta) \left[\beta (X(H) - Y (1 - T_m)) - \frac{(1 - \beta)(1 - T_m) g(\theta) Y}{\alpha}\right]}{[(1 - \beta)(1 - T_m) g(\theta) + \beta (1 - T_m)] \left[Y(1 - T_m) \left(\beta + \frac{(1 - \beta)g(\theta)}{\alpha}\right) + (1 - \beta)g(\theta) X(H)\right]}$$

$$
\varepsilon_{W,\theta}^p = \frac{(1 - \beta) (1 - T_m) \beta \left(Y \frac{(1 - T_m)}{\alpha} + X(H) - Y (1 - T_m)\right) g'(\theta)}{[(1 - \beta)(1 - T_m) g(\theta) + \beta (1 - T_m)] \left[Y(1 - T_m) \left(\beta + \frac{(1 - \beta)g(\theta)}{\alpha}\right) + (1 - \beta)g(\theta) X(H)\right]}$$

$$
\varepsilon_{W,H}^p = -\frac{(1 - \beta) g(\theta) X(H)}{Y (1 - T_m) \left(\beta + \frac{(1 - \beta)g(\theta)}{\alpha}\right) + (1 - \beta) g(\theta) X(H)}
$$

It is straightforward to show that $\varepsilon_{W,T_m}^p < 0$, $0 < \varepsilon_{W,\theta}^p$ (since one has $g'(\theta) < 0$ and

$$
\left(\frac{Y (1 - T_m)}{\alpha} - Y (1 - T_m) + X(H)\right) < 0
$$

< 0$ to get an equilibrium and as long as $T_m \leq 1$, which is a reasonable assumption) and $\varepsilon_{W,H}^p < 0$.

$^{28}$To simplify the log-differentiation and highlight the differences between each regime, the term $a_0 * H(T_m)^{\alpha - 1}$ is replaced by $Y(1 - T_m)$ but $H(T_m)$ in the term $\frac{B}{H(T_m)}$ of relation (9) is not substituted.
C The impact of $\bar{B}$ on the sign of $\xi_{W,Tm}$

One has $\xi_{W,Tm} = \frac{\varepsilon_{W,Tm}^p + \left(\varepsilon_{W,\theta}^p / \gamma \right) \ast \xi_{H,Tm} + \varepsilon_{W,H}^p \ast \xi_{H,Tm}}{(1 - \left(\varepsilon_{W,\theta}^p / \gamma \right) \varepsilon_{J,f,W}^p)}$, its denominator being positive and the sign of its numerator being ambiguous. The sign of $\xi_{W,Tm}$ is therefore dictated by the evolution of its numerator. At the partial equilibrium i.e. taking $\theta$ as given, one can show that:

$$\frac{\partial \varepsilon_{W,Tm}^p}{\partial X} = \Gamma \beta Y (1 - T_m) \left(\beta + \frac{(1-\beta)g(\theta)}{\alpha}\right) + (1 - \beta)g(\theta)(1 - T_M) \left[\beta Y + \frac{(1-\beta)g(\theta)Y}{\alpha}\right] > 0$$

with

$$\Gamma = \frac{T_m(1-\beta)g(\theta)}{[(1-\beta)(1-T_M)g(\theta) + \beta(1-T_m)]} > 0$$

$$\frac{\partial \varepsilon_{W,\theta}^p}{\partial X} = \Xi \frac{Y((1 - T_m) \beta + (1 - \beta)g(\theta)(1 - T_M))}{Y(1 - T_m) \left(\beta + \frac{(1-\beta)g(\theta)}{\alpha}\right) + (1 - \beta)g(\theta)X}^2 < 0$$

with

$$\Xi = \frac{(1-\beta)(1-T_m)\beta\theta g'(\theta)}{[(1-\beta)(1-T_M)g(\theta) + \beta(1-T_m)]} < 0$$

and

$$\frac{\partial \varepsilon_{W,H}^p}{\partial X} = \Lambda \frac{Y(1 - T_m) \left(\beta + \frac{(1-\beta)g(\theta)}{\alpha}\right)}{Y(1 - T_m) \left(\beta + \frac{(1-\beta)g(\theta)}{\alpha}\right) + (1 - \beta)g(\theta)X}^2 < 0$$

with

$$\Lambda = -(1 - \beta)g(\theta) < 0$$

Therefore when $\bar{B}$ (and thus $X(H)$) increases, the term $\varepsilon_{W,Tm}^p$ is less negative, the term $\left(\varepsilon_{W,\theta}^p / \gamma \right) \ast \xi_{H,Tm}$ is less negative and the term $\varepsilon_{W,H}^p \ast \xi_{H,Tm}$ is more positive. Thus, $\xi_{W,Tm}$ is more likely to be positive when $\bar{B}$ is higher.

D The calibration

The aim of this appendix is twofold. First, the wage setting relation is rewritten in order to take into account that the utility in unemployment has a fixed component $\bar{B}$ and a net wage indexed component
\[ \rho (S - T (S)) \text{ with } S = W \ast H, \text{ the total wage. Thus, one obtains:} \]
\[ \beta (1 - T_m) (Y H - S) = (1 - \beta) \left( (S - T (S)) (1 - \rho) - a_o \frac{H^\alpha}{\alpha} - B \right) g (\theta) \] (19)

Second, I point out the expression of the total wage elasticity with respect to the marginal tax rate. The method for determining the elasticity \( \xi^S_{1-T_m} = \frac{dS}{d(1-T_m)} (1-T_m) \) is close to the one used in Appendix B. I differentiate working hours, the labor demand \((PS)\) and the wage formation relation \((WS)\). The difference with the theoretical evaluation in Appendix B is that the differentiations are made with respect to the total wage. The labor demand relation writes now:

\[ S = Y \ast H - \frac{C (r + \lambda)}{q (\theta)} \]

Differentiated, it becomes (denoting \( s = \frac{dS}{S} \))

\[ \frac{Y \ast H \ast h - S \ast s}{Y \ast H - S} = \gamma \theta \] (20)

After the differentiation of the wage formation relation (19), and by taking into account of relation (20), of the differentiated working hours relation \( \left( h = \frac{d(1-T_m)}{1-T_m} \ast \frac{1}{\alpha-1} \right) \) and the fact that

\[ \frac{\theta q' (\theta)}{g (\theta)} = - \frac{\theta q (\theta) (1 - \gamma)}{r + \lambda + \theta q (\theta)} = \frac{\theta q (\theta) (1 - \gamma)}{r + \lambda} g (\theta), \]

one obtains after some basic mathematics:

\[ \xi^S_{1-T_m} = \frac{\beta (Y \ast H - S) + \frac{\beta Y \ast H}{\alpha-1} + (1 - \beta) g (\theta) \frac{Y \ast H}{\alpha-1} \left( 1 + \frac{\beta (1-\gamma)}{(1-\beta) \gamma} \ast \frac{\theta q (\theta)}{r + \lambda} \right)}{S \ast \left( \beta + (1 - \beta) g (\theta) \left( 1 - \rho + \frac{\beta (1-\gamma)}{(1-\beta) \gamma} \ast \frac{\theta q (\theta)}{r + \lambda} \right) \right)} \] (21)

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