

The Zero-Root Property : Permanent vs Temporary Terms of Trade Shocks

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THE ZERO-ROOT PROPERTY: PERMANENT VS TEMPORARY TERMS OF TRADE SHOCKS¹

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Abstract

This contribution shows that the persistence and the time of occurrence of the shock matter in determining the long-run macroeconomic aggregates' responses after permanent and transitory terms of trade shocks. Within a simple two-good small open economy model, we differentiate analytically between the (long-run) effects of four types of negative terms of trade disturbances of different degrees of persistence and occurrence. We distinguish between permanent and transitory shocks, on the one hand, and between anticipated and unanticipated shocks, on the other hand, and finally we consider a permanent perturbation but viewed as temporary by agents. The application of a new analytical two-step procedure to the zero-root property case allows to obtain new conclusions not developed by previous studies: [i] a strong persistent terms of trade worsening may induce a decline in the long-run in the real expense and in the stock of net foreign assets greater than after a permanent disturbance, [ii] real expense may rise or fall in the long-run after an anticipated future permanent negative external shock depending on the time of occurrence of the disturbance, and [iii] fails in expectations may induce large cut in real expense and translate into large losses in welfare.

Keywords: Small open economy; Terms of trade; Savings; Temporary shock.

JEL Classification: F32; F41; E21.

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1 Introduction

In a small open economy model that consists of a large number of identical and infinitely living agents with perfect foresight, the equality between the constant domestic rate of time preference and the exogenous world interest rate must be imposed to insure a finite interior steady-state value for the marginal utility of wealth. Although this strong assumption may seem unreasonable, it has repeatedly been useful in infinite-horizon models with time separable preferences and perfect capital mobility.¹ Beyond the justification of setting such an equality,² the literature has explored its analytical and economic implications. Giavazzi and Wyplosz [1984] and Sen and Turnovsky [1990] emphasize the dependency of the steady-state on initial conditions which implies in turn that temporary shocks have permanent effects. In empirical words, consumption data may follow a difference stationary process and be integrated of order one. Moreover, the constancy of the marginal utility of wealth rules out the possibility of transitional dynamics after a permanent disturbance. As the shock hits the economy, macroeconomic aggregates jump instantaneously to the new steady-state.

The absence of sluggish adjustment has been avoided by previous studies in four ways. Previous studies [i] incorporate heterogeneity among agents like overlapping generations models (see Blanchard [1985], Weil [1989]), [ii] relax the assumption of time separable preferences by endogenizing the psychological time discount rate as in recursive preferences models (see Epstein [1987], Obstfeld [1990]), [iii] introduce status preferences or assume that utility depends on financial wealth (see Fisher [2004], Zou [1997]), or [iv] suppose imperfect international capital markets as international banks require a risk premium (see Fisher [1995], Senhadji [2003]). All these approaches restore transitional dynamics in the small open economy model. Nevertheless, retaining a new framework implies usually other assumptions which have been source of criticism. For example, Blanchard [1985] assumes that each individual faces a constant probability of death, and Epstein [1987] supposes that the psychological time discount rate is an increasing function of consumption. Obstfeld [1990] introduces recursive preferences in a small open economy model. The unappealing result is that permanent and temporary shocks leave unchanged the consumption in the long-run, which may translate in empirical words by consumption series that are stationary in levels (or integrated of order zero). Finally, the investigation of the consequences of imperfections on international capital markets in a deterministic context may not be relevant and it seems preferable to model default risk in a stochastic framework.

Our approach consists to explore the effects of an unanticipated temporary relative price shock and an anticipated future permanent terms of trade perturbation when the knife-edge condition holds. The main result is that long-run changes depend critically upon the degree of the shock persistence, the time of occurrence of the disturbance (i. e. in a distant or near future), and the ability of agents to foresee the persistence of the perturbation. The framework is taken from Obstfeld [1983] who considers a simple small open economy consuming a domestic and a foreign good. Our analysis departs from Obstfeld's study in many respects. First, instead of focusing on initial responses as Obstfeld does, we are interested

in long-term reactions. Second, we apply the *two-step* approach proposed by Schubert and Turnovsky [2002] to the zero-root property and show that the economy no longer jumps instantaneously to the new steady-state. The Schubert and Turnovsky procedure to analyze the effects of temporary and future permanent shocks shows its capacity to deal in a consistent and explicit way with dynamic models in continuous time. Their *two-step* approach has an attractive use since it allows to express the macroeconomic aggregates prevailing at the steady-state in terms of the marginal utility of wealth and calculate in a clear way their long-run changes. Third, we enlarge the scope of shocks studied by Obstfeld by considering an anticipated future permanent disturbance and by investigating the role of fails in expectations in a deterministic context. Fourth, formal solutions for temporary and future permanent disturbances are obtained without retaining a functional form for the instantaneous utility function.

The application of a new analytical *two-step* procedure to the zero-root property case allows to derive new results not developed and/or not obtained by previous studies. [i] We show formally that the emergence of non degenerate dynamics rests critically upon the permanence or temporary nature of the disturbance, and more particularly that a strong persistent terms of trade worsening may induce a greater fall in the real expense and in the foreign bonds holding than after a permanent shock. [ii] As the economic literature is silent on permanent shocks' effects which are perfectly expected to occur in the future, such a formal study is performed. This analysis allows to show that the response of real expense in the long-run is not clear cut and depends on the time of occurrence of the perturbation.³ [iii] The presence of an explosive component in the solution of net foreign assets implies a monotonic adjustment of traded bonds at an increasing rate. In the same time, as Obstfeld has shown, a transitory terms of trade shock may generate a discontinuity in real expense adjustment but the trajectory remains flat over the period $0 \leq t < T$ when the shock is at work, and over the period $t \geq T$ once the shock ends due to the specification of time separable preferences. [iv] More importantly, formal solutions indicate that fails in expectations may exacerbate the negative perturbation effects and may translate into large changes of real expense and into deep losses in welfare. From an economic point of view, our analysis emphasizes the key role of the persistence of the disturbance in determining the long-term reactions of macroeconomic aggregates and the harmful effect of a long-lived temporary relative price shock on welfare. Finally, as Senhadji [2003] has argued, we demonstrate that large cuts in real expense and losses in welfare may be exacerbated by expectation mistakes on shock persistence.

What are the implications of changes in terms of trade on the current account? This question has received a lot of attention in the eighties by abstracting from capital accumulation and by deriving consumption decisions from solutions to an optimization problem. In two-period models, Sachs [1981] and Svensson and Razin [1983] have studied the impacts on current account of an unanticipated and anticipated, permanent and transitory adverse terms of trade shocks. By abstracting from the restrictive two-period framework, Obstfeld [1983] was the first to study the implications of an (exogenous) unanticipated permanent and transitory terms of trade deterioration in an intertemporal optimizing framework with an infinite

horizon. He showed that a temporary terms of trade shock generates both a *smoothing* effect and an *intertemporal speculation* effect. The first effect follows from the differential between the current income and the permanent income. The second is induced by the transitory variation of the consumption-based real interest rate. Following a negative terms of trade perturbation which lasts a short period, the fall of real interest rate in terms of consumption provides an incentive for raising present real expense. Sen and Turnovsky [1989] and Servèn [1999] enlarge Obstfeld's formal setup by allowing for capital accumulation in presence of installation costs. The former applies a procedure which contains an inconsistency (see Schubert and Turnovsky [2002]) and does not investigate the implications of fails in expectations. The latter restricts his study to a permanent and a temporary terms of trade disturbance and emphasize the role of the shock's persistence on short-run dynamics, neglecting to develop the long-run changes in economic aggregates and the hysteresis phenomenon. Our analysis aims to clarify the effects of an unanticipated transitory, a future anticipated permanent, and a permanent, viewed as temporary, terms of trade perturbations, by emphasizing the key role of the persistence of the shock and its time of occurrence, and by comparing the macroeconomic aggregates changes with those prevailing after a permanent disturbance.

The justification for focusing on unanticipated temporary or anticipated future permanent terms of trade disturbances is fourfold. First, a relative price transitory disturbance gives rise to *wealth*, *smoothing*, *intra-temporal substitution* and *inter-temporal substitution* effects, which enlarge the scope of economic aggregates' responses as we shall see below. Second, empirical studies performed by Fischer [1993] and Easterly et al. [1993] highlight the importance of terms of trade changes for economic growth. According to panel data estimations of Loayza et al. [2000], terms of trade play a key role in explaining saving rates variations. Other empirical works show that a great part of external position fluctuations can be attributed to terms of trade shocks, particularly for small open economies (see e. g. Cashin and Mc Dermott [2002], Otto [2003]). Third, terms of trade changes varies widely across countries in their temporary component and their duration.⁴ As Deaton [1999] underlines, the misperception between a long-lived temporary shock and a permanent one may be catastrophic for consumption and external indebtedness in the long-run as the optimal fall in real expense may be not sufficient regarding the permanence of the real income decline. Fourth, forecasts of commodity prices may encourage a country to increase its consumption and accumulate external debt as the the economy expects a windfall in the future viewed as permanent.⁵ But if the time length of the windfall is overestimated, the subsequent fall in real expense to insure intertemporal solvency may be dramatic. As in the real world, perturbations take various forms, say permanent or temporary, unanticipated or anticipated, and agents may misdiagnose a negative disturbance as temporary, a formal study is needed to understand the implications of these different shocks and evaluate the optimal macroeconomic aggregate reactions. Previous studies emphasize the striking difference of aggregate responses in the short-run after permanent and temporary shocks. Our contribution goes further by investigating the reactions in the long-run of real expense and of the stock of foreign assets according to wether: [i] the temporary shock is weakly or strongly persistent ($T \rightarrow \infty$), [ii] the permanent disturbance is expected to occur in the distant or in the near future, and [iii] the permanent perturbation was expected by

agents to be a short-lived or a long-lived temporary shock.

The purpose of this paper is to re-examine and differentiate analytically the long-run effects of terms of trade changes of different degrees of persistence and of different time of occurrence in a framework similar to Obstfeld's. A new consistent procedure is applied to study temporary and future permanent shocks in continuous time. This approach leads to formal solutions that allow to investigate accurately transitional dynamics when the linearized system exhibits a zero-root. We compare analytically transitory and permanent perturbations and anticipated and unanticipated shocks, underscore hysteresis phenomenon, and bring out the determinants of long-term reactions of macroeconomic aggregates. The paper is organized as follows. In section 2, we present the framework of a two-good dynamic optimizing small open economy, facing given terms of trade and world interest rate. In section 3, we analyze the equilibrium dynamics and the viable steady-state of the model. Section 4 explores in detail the effects of unanticipated permanent, unexpected temporary and anticipated future permanent worsening of the terms of trade. The discussion emphasizes the long-run reactions of macroeconomic aggregates. This section ends with the investigation of implications of fails in expectations, that is the effects of a permanent negative external shock, which was expected to be only temporary. Analysis of welfare is provided in section 5. Conclusions and a short outlook on further research are contained in section 6. Almost all formal computations can be retrieved in the Appendix.

2 The Model

The Framework

Consider a small open economy that is populated by a constant number of identical households that have perfect foresight and live forever. At each instant the representative household consumes a domestic good and a foreign good denoted respectively by d and f . The country is small in world good and capital markets and faces given terms of trade (price of the domestic good in terms of the foreign good), p , and world interest rate, r^* .

The measure of utility of consumption at t , $c(t)$, is given by the relation :

$$c(t) = c(d(t), f(t)), \quad (1)$$

where $c(\cdot)$ is a positive, increasing, concave and linearly homogeneous aggregator function.

Since $c(\cdot)$ is homothetic, the household's maximization problem can be decomposed into two stages. At the first stage, the household minimizes the cost, $z_c(t) = p(t)d(t) + f(t)$, for a given level of subutility, $c(t)$, where $p(t)$ is the relative price of the domestic good. For any chosen $c(t)$, the optimal basket $(d(t), f(t))$ is a solution to

$$p_c(p(t)) c(t) = \min_{\{d(t), f(t)\}} \{p(t)d(t) + f(t) : c(d(t), f(t)) \geq c(t)\}. \quad (2)$$

At the second stage, consumers choose their real expense, c , and rate of accumulation of traded bonds to maximize their objective function subject to the flow budget constraint:

$$\max_{c(t)} \int_0^{\infty} u[c(t)] \exp(-\delta t) dt, \quad u_c > 0, u_{cc} < 0, \quad (3a)$$

subject to,

$$\dot{b}(t) = r^* b(t) + p(t)y(t) - p_c(p(t)) c(t), \quad (3b)$$

$$b(0) \quad \text{given}, \quad (3c)$$

$$\lim_{t \rightarrow \infty} \exp(-r^* t) b(t) \geq 0, \quad (3d)$$

where δ is the consumer's discount rate; $p_c(p)$ is the unit cost function dual (or consumption-based price index) to c ; equation (3d) is the *no-ponzi* game condition. Each household is endowed with y units of the domestic exportable good which is assumed to be constant and accumulates net foreign bonds expressed in terms of the foreign good, b , that pay an exogenously given world interest rate, r^* .

Macroeconomic Equilibrium

To obtain the macroeconomic equilibrium, we first derive the optimality conditions for consumers and combine these with the accumulation equation. This leads to the set of equations:

$$u_c(c(t)) = \lambda(t) p_c(p(t)), \quad (4a)$$

$$\dot{\lambda}(t) = \lambda(t) (\delta - r^*), \quad (4b)$$

and the dynamic equation (3b), and the transversality condition

$$\lim_{t \rightarrow \infty} \bar{\lambda} b \exp(-r^* t) = 0, \quad (5)$$

where λ is the co-state variable associated with the dynamic equation (3b).

The first static efficiency condition (4a) requires that along an optimal path the marginal current utility of real expense is equal to the marginal utility of wealth in the form of internationally traded bonds measured in terms of the domestic good, $p_c \lambda$. Equation (4b) is the dynamic efficiency condition requiring the return on consumption, $\delta - \dot{\lambda}/\lambda$, to equal the return on traded bonds, r^* .

With a constant rate of time preference and an exogenous world interest rate, we impose the following equality

$$\delta = r^*, \quad (6)$$

in order to generate an interior solution. This standard assumption made in the literature implies that the marginal utility of wealth, λ , must remain constant over time (see (4b)), and gives rise to the *zero-root property* (see Sen [1994], Turnovsky [1997], chap. 2).

Static Efficiency Condition

The static efficiency condition (4a) can be solved for the short-run macroeconomic equilibrium which of course must hold at any point of time. We obtain :

$$c = c(\bar{\lambda}, p), \quad \text{with} \quad c_{\bar{\lambda}} < 0, \quad c_p < 0, \quad (7)$$

where $c_{\bar{\lambda}} = p_c/u_{cc} < 0$, et $c_p = [p'_c \bar{\lambda}]/u_{cc} < 0$. A rise in the marginal utility of wealth, measured in terms of the foreign good, $\bar{\lambda}$, induces the representative agent to reduce its real expense. Given the marginal utility of wealth, an increase in the terms of trade raises the marginal utility of wealth measured in terms of the domestic good through the rise in the consumption price index and leads to a fall in consumption.

Intertemporal Budget Constraint

As the shadow value of foreign assets λ must remain constant over time, the solution for the consumer's holdings of internationally traded bonds is given for an expected constant path of terms of trade

$$b(t) = e^{r^*t} \left[b_0 + \frac{(py - p_c c(\bar{\lambda}, p))}{r^*} (1 - e^{-r^*t}) \right], \quad (8)$$

where we integrate the accumulation equation (3b) and b_0 is the initially given stock of foreign bonds. By invoking the transversality condition (5), we obtain the intertemporal solvency constraint of the small open economy for $p(t) = p$

$$b_0 = -\frac{(py - p_c c(\bar{\lambda}, p))}{r^*} = \bar{b}, \quad (9)$$

where we denote a steady-state value with a bar.

Stationary-State

From (9), there are no dynamics and the economy is always in steady-state. The stationary-state is reduced to the following three equations

$$u_c(\bar{c}) = \bar{\lambda} p_c(p), \quad (10a)$$

$$r^* \bar{b} + py - p_c \bar{c} = 0, \quad (10b)$$

$$b_0 = \bar{b}. \quad (10c)$$

What happens after an unanticipated terms of trade shock? From (10c), the stock of foreign assets is unaffected and remains to its initial value. For a new level of the home goods' relative price, the current account equilibrium is guaranteed in the long-run through the instantaneous change of real expense (see eq (10b)). Finally, the initial and unique jump of λ allows for equality between the marginal utility of consumption and the marginal utility of wealth expressed in terms of the home good (see eq (10a)).

3 Equilibrium Dynamics and the Zero-Root Property

Equilibrium Dynamics

We now investigate the equilibrium dynamics of the model. Insert (7) into (3b) and (4b) and linearize around the steady-state $(\bar{b}, \bar{\lambda})$:

$$\begin{pmatrix} \dot{b}(t) \\ \dot{\lambda}(t) \end{pmatrix} = \begin{pmatrix} r^* & -p_c c_{\bar{\lambda}} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} b(t) - \bar{b} \\ \lambda(t) - \bar{\lambda} \end{pmatrix}, \quad (11)$$

The characteristic equation writes as follows : $-\mu_i (r^* - \mu_i) = 0$ from which we obtain a null characteristic root, $\mu_1 = 0$, et and a positive characteristic root, $\mu_2 = r^* > 0$. The eigenvector is given by: $\omega_2^i = (r^* - \mu_i) / p_c t_{\bar{\lambda}}$.

The general solution to the linearized system may be written as

$$b(t) = \bar{b} + A_1 + A_2 e^{\mu_2 t}, \quad (12a)$$

$$\lambda(t) = \bar{\lambda} + \frac{r^*}{p_c t_{\bar{\lambda}}} A_1, \quad (12b)$$

where we use the fact that $\omega_2^2 = 0$ and $\mu_1 = 0$.

To ultimately approach the steady-state $(\bar{b}, \bar{\lambda})$ and to satisfy the transversality condition (5), it is necessary to set the arbitrary constants A_1 and A_2 equal to zero. Thus the stable paths are given by

$$b(t) = \bar{b}, \quad (13a)$$

$$\lambda(t) = \bar{\lambda}. \quad (13b)$$

From (13), the adjustment of the small open economy's external position is immediate following an unanticipated terms of trade perturbation if the country is initially in a stationary position. This result no longer obtains when the country faces a temporary price shock since the economy follows transitorily unstable paths.

The Viable Stationary-State

Following Schubert and Turnovsky [2002], we define a viable steady-state i , corresponding to a level of terms of trade p_i starting at time T_i with a stock of traded assets b_{T_i} , to be one that is consistent with long-run solvency, given b_{T_i} . It is described by the following set of equations:

$$u_c(\bar{c}_i) = \bar{\lambda}_i p_c(p_i), \quad (14a)$$

$$r^* \bar{b}_i + p_i \bar{y} - p_c(p_i) \bar{c}_i = 0, \quad (14b)$$

together with the intertemporal solvency condition

$$\bar{b}_i = b_{T_i}, \quad (14c)$$

where the last equality comes from (9) and (14b).

System (14) may be solved for the steady-state values by applying the *two-step* solution method described by Schubert and Turnovsky [2002]. Equations (14a) and (14b) can be first solved for \bar{c}_i and \bar{b}_i in terms of $\bar{\lambda}_i$ and p_i

$$\bar{c}_i = t(\bar{\lambda}_i, p_i), \quad t_{\bar{\lambda}} < 0, \quad t_p < 0, \quad (15a)$$

$$\bar{b}_i = v(\bar{\lambda}_i, p_i), \quad v_{\bar{\lambda}} < 0, \quad v_p < 0, \quad (15b)$$

where the partial derivatives are given in the Appendix.

By substituting (15b) into (14c) and solving, the intertemporal solvency condition can be rewritten as follows :

$$v(\bar{\lambda}_i, p_i) = b_{T_i}. \quad (16)$$

which may be solved for the equilibrium value of $\bar{\lambda}_i$ in the form

$$\bar{\lambda}_i = \lambda(b_{T_i}, p_i), \quad \text{with } \bar{\lambda}_b < 0, \quad \bar{\lambda}_p < 0, \quad (17)$$

where the partial derivatives are given in the Appendix.

Whenever a permanent unanticipated terms of trade shock is considered, the initial condition is given by $b_{T_i} = b_0$. In this case, (17) determines $\bar{\lambda}_i$, and (15a)-(15b) determine \bar{c}_i and \bar{b}_i in terms of the initial stock of traded bonds and the terms of trade. This implies that the steady-state depends on initial conditions and therefore temporary shocks have permanent effects. When the disturbance is temporary, we must take into account that the stock of traded bonds has been accumulated over the unstable period $(0, T)$. This implies a new initial condition for the small open economy, b_T , once the shock ends, which influences in turn the new steady state.

4 Permanent vs Temporary, Anticipated vs Unanticipated Terms of Trade Shocks

This section examines the effects of changes in the terms of trade, p , on the real expense and the stock of foreign bonds in the short-run and more importantly in the long-run. We consider an unanticipated permanent shock (denoted by the subscript *perm*), an unanticipated temporary perturbation denoted by the subscript *temp*, and an anticipated future permanent disturbance (denoted by the subscript *fut*). Finally we analyze the consequences on real expense and current account of a permanent relative price variation that was anticipated transitory by agents (denoted by the subscript *fail*).

4.1 An Unanticipated Permanent Terms of Trade Worsening

We investigate the effects of a permanent decrease in the relative price of the domestic good, p , from p_0 to p_1 , which occurs at time $t = 0$, where the economy is originally in steady state. Like Obstfeld [1983], Servèn [1999], we assume that the small open economy is a net exporter of the domestic good at the steady-state,⁶ that is $(y - \bar{d}) > 0$. All agents perfectly understand the permanence of the terms of trade deterioration, but its occurrence at time $t = 0$ is unanticipated.

Following a permanent terms of trade worsening, $dp < 0$, the real expense reduces initially by the same amount as the fall in the real income, which leaves unchanged the stock of traded

bonds:

$$\left. \frac{d\bar{c}}{dp} \right|_{perm} = \frac{\partial \bar{c}}{d\bar{\lambda}} \frac{\partial \bar{\lambda}}{dp} + \frac{\partial \bar{c}}{\partial p} = \frac{(y - \bar{d})}{p_c} > 0, \quad (18a)$$

$$\left. \frac{d\bar{\lambda}}{dp} \right|_{perm} = \frac{u_{cc}(y - \bar{d})}{p_c^2} - \frac{p'_c \bar{\lambda}}{p_c} < 0, \quad (18b)$$

$$\left. \frac{d\bar{b}}{dp} \right|_{perm} = \frac{\partial \bar{b}}{d\bar{\lambda}} \frac{\partial \bar{\lambda}}{dp} + \frac{\partial \bar{b}}{\partial p} = 0. \quad (18c)$$

From expression (18b), the permanent decrease in p has two influences which work in the same direction on marginal utility of wealth expressed in terms of the foreign good. First, a fall in p lowers domestic real income which induces a drop in consumption. This *wealth* effect is proportional to $(y - \bar{d})$ (see the first term on the right-hand side of eq (18b)). Second, a negative change in p favors the consumption of the domestically produced good, d , and incites households to reduce the level of their consumption in the foreign good. This *relative price* effect is proportional to $\alpha_c \equiv p'_c p / p_c$, the share of domestic goods in consumption expenditure (see the second term on the right-hand side of eq (18b)). The latter effect disappears in a one-good model. These two adjustments lead to an increase in marginal utility of wealth expressed in terms of the foreign good. The change in the steady-state of c is the sum of two effects: a *wealth* effect (the first term on the right-hand side of (18a)) and a *relative price* effect (the second term on the right-hand side of (18a)). A fall in p raises $\bar{\lambda}$ which lowers c . In the same time the diminution in the relative price encourages the agent to raise their real expense as the consumption price index is definitively smaller. This latter effect is cancelled by the *relative price* effect on $\bar{\lambda}$, i. e. $t_{\bar{\lambda}} p'_c \bar{\lambda} / p_c = t_p$.

4.2 An Unanticipated Temporary Terms of Trade Worsening

We now investigate the short-term and the long-term effects of a relative price disturbance which lasts over the period $(0, T)$. We suppose that terms of trade change unexpectedly at time $t = 0$ from the original level p_0 to level p_1 over the period $0 \leq t < T$, and they revert back at time T permanently to their initial level, $p_T = p_2 = p_0 > p_1$.

The initial jump of the marginal utility of wealth following a transitory terms of trade shock is given by (see Appendix B):

$$\left. \frac{d\bar{\lambda}}{dp} \right|_{temp} = -\frac{v_p}{v_{\bar{\lambda}}} (1 - e^{-\mu_2 T}) = \lambda_p (1 - e^{-\mu_2 T}) = \left. \frac{d\bar{\lambda}}{dp} \right|_{perm} (1 - e^{-\mu_2 T}) < 0, \quad (19)$$

where $\mu_2 = r^*$. The positive term $0 < (1 - e^{-r^* T}) < 1$ is a monotonic increasing function of T . Since $\lambda_p < 0$ denotes the steady-state change of λ for a permanent variation in p , we see that the change of λ for a temporary perturbation in p is smaller but of the same direction. This is quite intuitive since the *wealth* effects induced by the transitory relative price change extend over successively shorter periods as the persistence of the perturbation diminishes. Following a negative temporary terms of trade shock, the *wealth* effect incites households to reduce their real expense through the rise of the marginal utility of wealth,

which is nevertheless dampened with respect to a permanent disturbance. Since agents know that the decrease in p is only temporary, the present value of the necessary reduction in real expense to satisfy the intertemporal budget constraint is less than for an equal but permanent decline in p .

Impact Effects and Transitional Dynamics

The impact on real expense follows from (15a):

$$\left. \frac{dc(0)}{dp} \right|_{temp} = \frac{(y - \bar{d})}{p_c} (1 - e^{-r^*T}) - \frac{\sigma \bar{d} e^{-r^*T}}{p_c} \geq 0, \quad (20a)$$

$$\left. \frac{dc(0)}{dp} \right|_{temp}^{T \rightarrow \infty} = \frac{(y - \bar{d})}{p_c} = \left. \frac{d\bar{c}}{dp} \right|_{perm}, \quad (20b)$$

where σ is the intertemporal elasticity of substitution. From equation (20a), real expense may fall or rise following a terms of trade worsening depending on the *wealth* effect (the first term on the right-hand side of (20a)) is greater or smaller than the *intertemporal speculation* effect (the second term on the right-hand side of (20a)).⁷ The equation (20b) reflects that after a strong persistent terms of trade deterioration, real expense initially falls of the same amount that after a permanent perturbation. Once the real expense jumped, its temporal path remains flat. At time T , when terms of trade revert back to their initial level, the marginal utility of wealth measured in terms of the domestic good rises abruptly, $p_c(p_2) \bar{\lambda} > p_c(p_1) \bar{\lambda}$, which induces an immediate and definitive decline in the real expense to its final steady-state value, \bar{c}_2 . On figure 1, we have considered two cases: a short-lived shock of length T_1 and a strongly persistent shock of length T_2 .

Even in the presence of the zero-root property, a transitory shock (and not a permanent one) induces a transitory and monotonic adjustment of foreign assets (see figure 2). The stock of internationally traded bonds declines over the period $(0, T)$

$$ca(t) = \dot{b}(t) = -r^* v_p e^{r^*(t-T)} dp = [(y - \bar{d}) + \bar{d}\sigma] e^{r^*(t-T)} dp < 0, \quad (21)$$

where we denote the current account by ca and we have differentiated the solution (45a) with respect to time by using (51) and substituting (41d). The term on the right-hand side of (21) represents an unstable component of the transitional path due to the term e^{r^*t} . In contrast to a permanent shock, which leaves unchanged the steady-state value of b , the economy decumulates net foreign assets at an increasing rate along an unsustainable path concerning intertemporal solvency.⁸ At time T , net foreign assets reach a lower level, $b(T) < b_0$, decumulation ends, and the current account equilibrium is immediately restored, $b(T) = \bar{b}_2 = -(p_2 y - p_c(p_2) \bar{c}_2) / r^*$.

Long-Term Effects

As the adjustments of real expense and of net foreign assets are characterized by an hysteresis phenomenon and since the transitional dynamics are driven by the agents' expectations of the ultimate steady-state under the perfect foresight assumption, we pursue our analysis with the investigation of the long-run effects of an unanticipated transitory change in p .

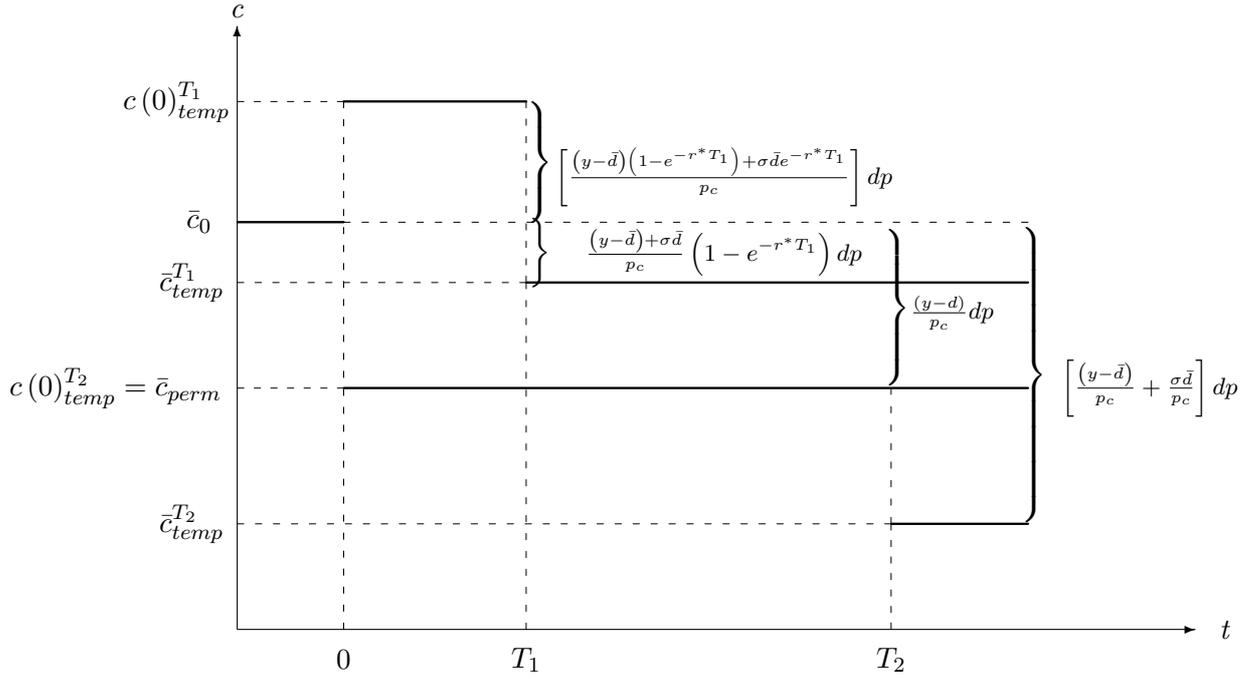


Figure 1: An unanticipated permanent and an unexpected temporary terms of trade worsening: the real expense response

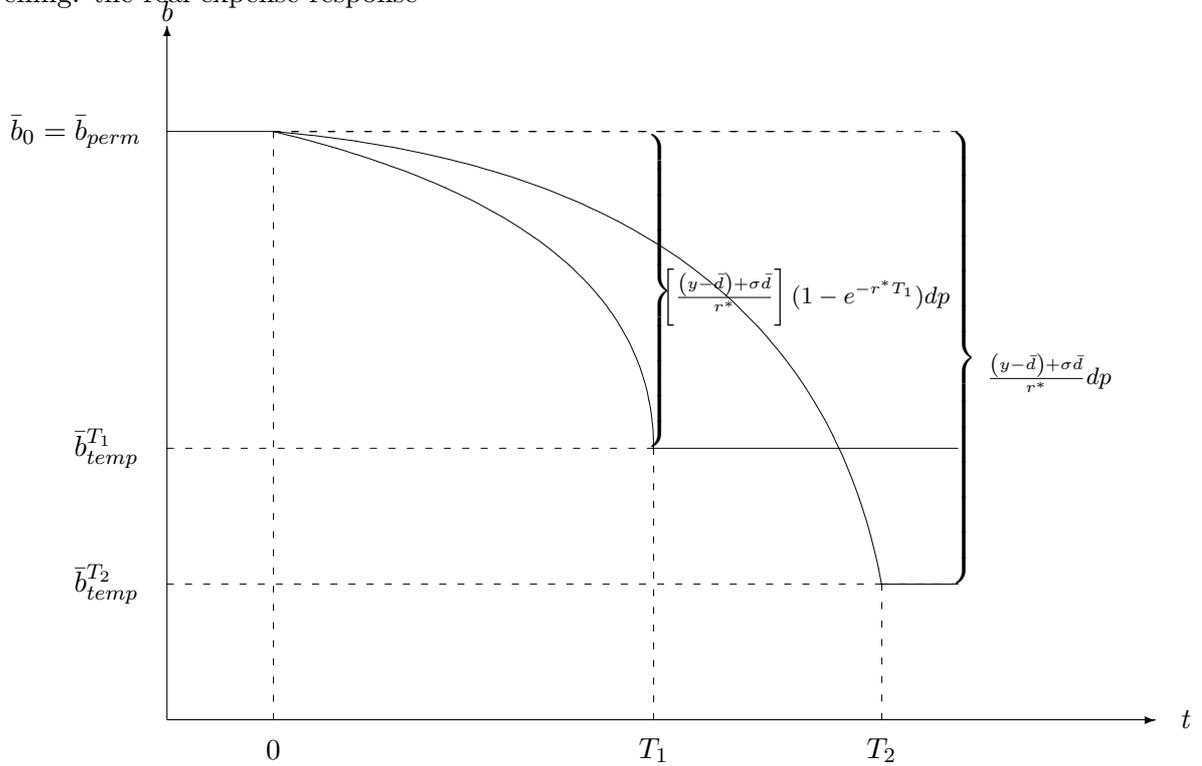


Figure 2: An unanticipated permanent and an unexpected temporary terms of trade worsening: current account dynamics

The ultimate steady-state changes compared to the initial \bar{c}_0, \bar{b}_0 for an unanticipated transitory shock can be calculated as

$$\left. \frac{d\bar{c}}{dp} \right|_{temp} = t_{\bar{\lambda}} \left. \frac{d\bar{\lambda}}{dp} \right|_{temp} \begin{matrix} \geq \\ \leq \end{matrix} \left. \frac{d\bar{c}}{dp} \right|_{perm} = t_{\bar{\lambda}} \lambda_p + t_p > 0, \quad (22a)$$

$$\left. \frac{d\bar{b}}{dp} \right|_{temp} = v_{\bar{\lambda}} \left. \frac{d\bar{\lambda}}{dp} \right|_{temp} > \left. \frac{d\bar{b}}{dp} \right|_{perm} = v_{\bar{\lambda}} \lambda_p + v_p = 0, \quad (22b)$$

where $t_{\bar{\lambda}} < 0, v_{\bar{\lambda}} < 0, (d\bar{\lambda}/dp)_{temp} = \lambda_p (1 - e^{-r^*T})$ (see (19)).

There exists a length of the shock denoted by \hat{T} such that changes in the long-run following a permanent or a transitory shock are equal, that is :

$$\hat{T} = -\frac{1}{r^*} \log \left[-\frac{t_p}{t_{\bar{\lambda}} \lambda_p} \right] = -\frac{1}{r^*} \log \left[\frac{\frac{\sigma \bar{d}}{p_c}}{\frac{(y-\bar{d})}{p_c} + \frac{\sigma \bar{d}}{p_c}} \right]. \quad (23)$$

Let T tend to infinity which implies:

$$\left. \frac{d\bar{c}}{dp} \right|_{temp}^{T \rightarrow \infty} = t_{\bar{\lambda}} \left. \frac{d\bar{\lambda}}{dp} \right|_{temp}^{T \rightarrow \infty} = \left. \frac{d\bar{c}}{dp} \right|_{perm} + \frac{\sigma \bar{d}}{p_c} > \left. \frac{d\bar{c}}{dp} \right|_{perm} > 0, \quad (24a)$$

$$\left. \frac{d\bar{b}}{dp} \right|_{temp}^{T \rightarrow \infty} = v_{\bar{\lambda}} \left. \frac{d\bar{\lambda}}{dp} \right|_{temp}^{T \rightarrow \infty} > 0, \quad (24b)$$

$$\left. \frac{d\bar{\lambda}}{dp} \right|_{temp}^{T \rightarrow \infty} = \left. \frac{d\bar{\lambda}}{dp} \right|_{perm} < 0. \quad (24c)$$

Following a very persistent negative terms of trade shock, the real expense may be definitively reduced at a lower level than following a permanent negative perturbation. Intuitively, since the relative price change is only temporary, ultimately, only the *wealth* effect remains. Therefore, the steady-state value of the real expense is only affected by the variation of the marginal utility of wealth and not by the change in the relative price p since terms of trade are expected to revert back to their initial level whatever the degree of shock's persistence. At the opposite of a permanent perturbation, the stock of foreign assets is definitively reduced. The *smoothing* effect coupled with the *intertemporal speculation* effect during the unstable period $(0, T)$ have encouraged the agents to decumulate net foreign assets until the level $b_T = \bar{b}_2$. This lower long-run level of net foreign assets, i. e. $\bar{b}_2 < \bar{b}_0$, is consistent with the intertemporal solvency constraint and allows the the small open economy to reach the steady-state where the gross national product must be equal to the lower level of absorption, that is

$$r^* \bar{b}_2 + p_2 y = p_c (p_2) \bar{c}_2 \quad (25)$$

where $p_2 = p_0$ and $\bar{c}_2 < \bar{c}_0$.

Permanent and temporary shocks may induce markedly different or opposite effects in the short-run and the long-run only when the knife-edge condition holds, that is when *hysteresis* effects are at work.⁹ On figure 1, we consider a weakly persistent terms of trade worsening. The *intertemporal speculation* effect incites consumers to raise initially their real expense.

Once the shock ends, the marginal utility of wealth measured in terms of the domestic good rises abruptly from the level $p_c(p_1)\bar{\lambda}$ to the level $p(p_2)\bar{\lambda}$ and the real expense falls to a lower level but higher than in the permanent case¹⁰. If the shock is strongly persistent ($T_2 > \hat{T}$), the *wealth* effect outweighs the *intertemporal speculation* effect and real expense falls initially. Once the terms of trade are restored to their initial level, the real expense reaches immediately and definitively a lower level than after a permanent terms of trade worsening. As the real expense is permanently reduced, the current account equilibrium in the long-run calls for a diminution in the stock of internationally traded bonds.

4.3 A Future Anticipated Permanent Terms of Trade Worsening

We investigate the effects of a future anticipated permanent decline in the domestic relative price, $dp \equiv p_2 - p_1 = p_T - p_0 < 0$. We assume that at time 0 agents perfectly anticipate that terms of trade deteriorate at time T , from $p_0 = p_1$ to $p_T = p_2$.¹¹

A permanent terms of trade shock, which is expected to occur in the future, induces an equilibrium change of the marginal utility of wealth given by

$$\left. \frac{d\bar{\lambda}}{dp} \right|_{fut} = -\frac{v_p}{v_\lambda} e^{-\mu_2 T} = \lambda_p e^{-r^* T} = \left. \frac{d\bar{\lambda}}{dp} \right|_{perm} e^{-r^* T} < 0. \quad (26)$$

From the above equation, the shadow value of wealth in the form of internationally traded bonds rises at time $t = 0^+$ once agents perfectly expect that the home goods' relative price will be definitively lower in the future. Compared to an unanticipated permanent negative perturbation, the initial jump of λ is moderated by the term $e^{-r^* T}$. The change is smaller the farther the anticipated decrease in the relative price is expected to occur since the *wealth* effect is more softened.

Impact Effects and Transitional Dynamics

At time 0, the marginal utility of wealth increases abruptly, which in turn induces an immediate fall in real expense and a current account surplus:

$$\left. \frac{dc(0)}{dp} \right|_{fut} = \left[\frac{(y - \bar{d})}{p_c} + \frac{\sigma \bar{d}}{p_c} \right] e^{-r^* T} > 0, \quad (27a)$$

$$ca(0) = \dot{b}(0) = r^* v_p e^{-r^* T} dp = -[(y - \bar{d}) + \bar{d}\sigma] e^{-r^* T} dp > 0, \quad (27b)$$

where we used (15a) to obtain (27a). As the *wealth* and *intertemporal speculation* effects work in the same direction, the real expense decreases initially without ambiguity. On the one hand, the present value of wealth is reduced at time 0, and on the other hand, agents perfectly expect a terms of trade worsening, which induces a transitory rise in the consumption-based real rate of interest above the time preference rate, $r^c = r^* - \alpha_c \dot{p}/p > r^* = \delta$ (over period 1). These effects both lead to a reduction in c . The fall in absorption induces an initial improvement of the current account (see eq (27b)). From (27b), the higher the value of the intertemporal elasticity of substitution, σ , the larger is the incentive to reallocate consumption towards the

future, and the bigger is the initial fall in real expense, which translates into a greater current account surplus.

As in the temporary case, a future anticipated permanent terms of trade worsening calls for a transitory and monotonic adjustment of the external asset position:

$$ca(t) = \dot{b}(t) = r^* v_p e^{r^*(t-T)} dp = - [(y - \bar{d}) + \bar{d}\sigma] e^{r^*(t-T)} dp > 0. \quad (28)$$

Although the shock is permanent, it is expected to occur in the (distant or near) future and the *smoothing* effect incites agents to accumulate foreign bonds ($ca(t) > 0$) at an increasing rate ($\dot{ca}(t) > 0$) during the unstable period (see figure 4).

When time T is reached, the perfectly anticipated decrease in p actually occurs. The current account equilibrium is instantaneously restored and the real expense reaches its long-run equilibrium value as the dynamics degenerate at the time terms of trade deteriorate. The long-term responses of real expense which depend on the length of the lead time T are given by

$$\left. \frac{d\bar{c}}{dp} \right|_{fut} = t_{\bar{\lambda}} \lambda_p e^{-r^*T} + t_p = \frac{(y - \bar{d})}{p_c} e^{-r^*T} - \frac{\sigma \bar{d}}{p_c} (1 - e^{-r^*T}) \geq 0, \quad (29a)$$

$$\left. \frac{d\bar{c}}{dp} \right|_{fut}^{T \rightarrow \infty} = -\frac{\sigma \bar{d}}{p_c} < 0, \quad (29b)$$

where we make use of (41a) and (43a) (with (18b)) to obtain (29a)-(29b). The relative price decline from $p_0 = p_1$ to $p_T = p_2$ leads to a fall in the marginal utility of wealth expressed in terms of the foreign good through the reduction in the consumption price index, that is from $p_c(p_1) \bar{\lambda}$ to $p_c(p_2) \bar{\lambda}$. This in turn encourages individuals to raise their real expense and works in the opposite direction of the *wealth* effect. From equation (29a), the change in real expense in the long-run following an anticipated future negative perturbation reflects two opposite forces: a *wealth* effect, which is a decreasing function of the shock's length, and the *relative price* effect, which is an increasing function of T . The farther the anticipated decline in the terms of trade, that is the longer the lead time T , the lower is the fall in present value of wealth (the smaller the *wealth* effect) and the weaker is the incentive to postpone real expense (the smaller the *intertemporal speculation* effect). There exists a time $T = \tilde{T}$ for which the *wealth* effect and the *relative price* effect cancel each other. It is given by

$$\tilde{T} = -\frac{1}{r^*} \log \left[\frac{\frac{\sigma \bar{d}}{p_c}}{\frac{(y - \bar{d})}{p_c} + \frac{\sigma \bar{d}}{p_c}} \right] = \hat{T}. \quad (30)$$

For a sufficiently high value of T ($T_2 > \tilde{T}$), the *relative price* effect predominates and real expense may reach a higher level at the new steady-state, say $\bar{c}_{fut}^{T_2}$ (see figure 3). A negative future permanent shock, which is expected to occur in a distant future ($T_2 > \tilde{T}$) leads to a rise in the long-term value of real expense without ambiguity since only the *relative price* effect is at work (see (29b)). Intuitively, as the external negative shock occurs at a very distant date, say $T \rightarrow \infty$, the wealth in present value does not change. It can be seen more formally by letting the parameter T tend towards infinity in equation (26), which results in the absence of a change in $\bar{\lambda}$, that is $\bar{\lambda}_{fut}^{T \rightarrow \infty} = \lambda_0$.

Concerning external asset position at the new steady-state, the stock of foreign bonds of the small open economy will be definitively higher whatever is the time occurrence of the perturbation:

$$\left. \frac{d\bar{b}}{dp} \right|_{fut} = v_{\bar{\lambda}} \lambda_p e^{-r^*T} + v_p = v_p (1 - e^{-r^*T}) < 0 \quad (31)$$

Compared to an unanticipated permanent terms of trade worsening beginning at time $t = 0$, transitional dynamics emerge during which agents accumulate some foreign bonds as they wish to smooth their consumption over time (see figure 4). Compared to an unanticipated transitory home goods' relative price decline, even if the negative disturbance occurs in the near future leading to a fall in real expense in the long-run, the permanent nature of the shock and the fall in the real income necessitate a rise in the interest receipts, which in turn calls for a rise in the stock of internationally traded bonds during the unstable period (that is over period 1).

4.4 Fails in Expectations: What are the Implications?

By using a stochastic dynamic general equilibrium, Senhadji [2003] has recently explored the consequence for external borrowing when underestimating the longevity of negative external shocks like terms of trade changes. In a deterministic context, we investigate the implications of agents' failure from anticipating the degree of shock's persistence. As Senhadji [2003] we argue that fails in expectations may exacerbate the effects of a negative external disturbance and rational agents may desaccumulate foreign assets in an optimal way regarding their expectations of shock's length.

We consider that at time $t = 0$, terms of trade deteriorate from level p_0 to level p_1 (with $p_0 < p_1$). Agents are supposed to expect that the negative relative price disturbance will last over the period $(0, T)$. But at time T , agents' expectations are disappointed as terms of trade are not restored back to their initial level. Transitional dynamics over the period $(0, T)$ (period 1) are similar to those discussed in section 4.2 and therefore do not necessitate further comment. The unexpected constancy of the home goods' relative price implies that the negative external shock anticipated temporary by individuals turns to be permanent and therefore calls for an economic adjustment. As the agents initially viewed the terms of trade deterioration as only temporary, the reduction in the present value of wealth is larger than it was expected. New information at time T leads to a jump of the marginal utility of wealth from the level $\bar{\lambda}_{temp}$ to the level $\bar{\lambda}_{fail}$.¹²

Over period 1, as the shock was expected to be only temporary, the small open economy desaccumulates foreign assets and is dotted at time T with a lower stock of internationally traded bonds than initially, that is $b_T < b_0$. At time T , agents are informed on the unexpected permanent nature of the disturbance, and the equilibrium value of the marginal utility of wealth changes as follows (see Appendix C):

$$\left. \frac{d\bar{\lambda}(T)}{dp} \right|_{fail} = \lambda_p < 0. \quad (32)$$

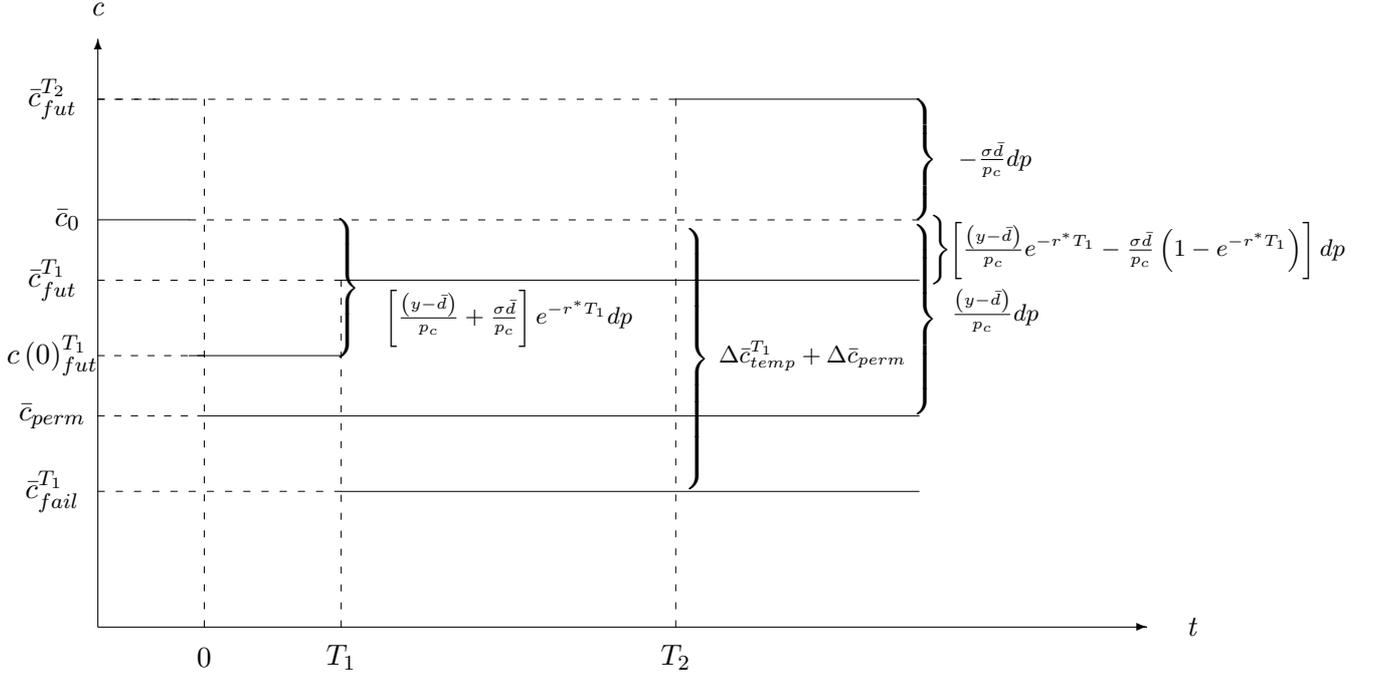


Figure 3: A future anticipated permanent and a permanent viewed as temporary terms of trade worsening: the real expense response

From (32), the marginal utility of wealth increases instantaneously at time T by the same amount that after a permanent shock. Although the shadow value of wealth in the form of internationally traded bonds has already jumped upward by an amount equal to $\lambda_p (1 - e^{-r^* T}) dp > 0$, the excessive decumulation of foreign assets over the unstable period 1 calls for a deeper accumulation than after a permanent shock which translates into a greater jump of $\bar{\lambda}$ over the entire period, that is (see Appendix C)

$$\left. \frac{d\bar{\lambda}}{dp} \right|_{fail} = \lambda_p (1 - e^{-r^* T}) + \lambda_p = \left. \frac{d\bar{\lambda}}{dp} \right|_{temp} + \left. \frac{d\bar{\lambda}}{dp} \right|_{perm} < \left. \frac{d\bar{\lambda}}{dp} \right|_{perm} < 0. \quad (33)$$

From (19), (26) and (33), we can deduce the following inequalities prevailing for an adverse terms of trade disturbance

$$\bar{\lambda}_{fail} > \bar{\lambda}_{perm} > \bar{\lambda}_{temp} > \lambda_0 > \bar{\lambda}_{fut}. \quad (34)$$

By remembering that $\bar{c}_{fail} = v(\bar{\lambda}_{fail}, p_1)$ and $\bar{b}_{fail} = v(\bar{\lambda}_{fail}, p_1)$, the long-run changes in the real expense and in the net foreign assets are given by

$$\left. \frac{d\bar{c}}{dp} \right|_{fail} = t_{\bar{\lambda}} \left. \frac{d\bar{\lambda}}{dp} \right|_{fail} + t_p = t_{\bar{\lambda}} \left[\lambda_p (1 - e^{-r^* T}) + \lambda_p \right] + t_p = \left. \frac{d\bar{c}}{dp} \right|_{temp} + \left. \frac{d\bar{c}}{dp} \right|_{perm} > 0, \quad (35a)$$

$$\left. \frac{d\bar{b}}{dp} \right|_{fail} = v_{\bar{\lambda}} \left. \frac{d\bar{\lambda}}{dp} \right|_{fail} + v_p = \left. \frac{d\bar{b}}{dp} \right|_{temp}, \quad (35b)$$

where we used the fact that $(d\bar{b}/dp)_{perm} = 0$ (see (18c)). Our results may be summarized by

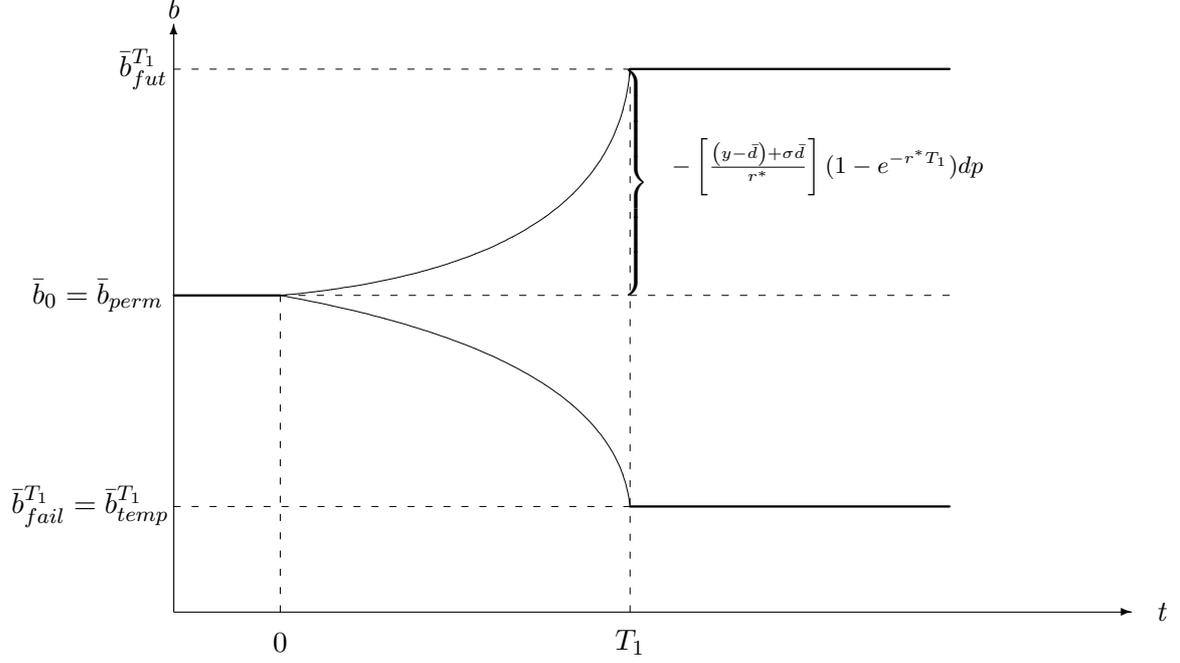


Figure 4: A future anticipated permanent terms of trade worsening and fails in expectations: current account dynamics

the following inequalities

$$\bar{c}_0 > \bar{c}_{fut}^{T_1} > \bar{c}_{temp}^{T_1} > \bar{c}_{perm} > \bar{c}_{temp}^{T_2} > \bar{c}_{fail}, \quad (36a)$$

$$\bar{b}_{fut} > \bar{b}_{perm} = b_0 > \bar{b}_{temp} = \bar{b}_{fail}, \quad (36b)$$

where T_1 is such that $T_1 < \hat{T} = \tilde{T}$ and reflects the case of a short-lived negative terms of trade disturbance. At the opposite, the parameter T_2 refers to a strong persistent relative price perturbation ($T_2 > \hat{T} = \tilde{T}$). As we depict on figures 1 and 3, when the parameter T takes very high values, real expense may definitely reach: [i] a higher level than its initial value after a future anticipated permanent terms of trade deterioration, [ii] a lower level after a strongly persistent negative perturbation than after a permanent disturbance, and [iii] a very low level whenever expectations are disappointed. In the latter case, the deviation is equal to the sum of a permanent and a long-lived adverse external shocks.

5 Welfare Analysis

In this section, we shall investigate the welfare implications of an adverse external shock for the small open economy's individuals. Since there are no transitional dynamics in consumption because of the zero-root property, the instantaneous utility remains constant over each period, i. e. over respectively $(0, T)$ and $t \geq T$. In order to have a correct measure of welfare, we follow Turnovsky [1995] (see chap. 8) by evaluating in present value terms the

instantaneous utility prevailing respectively over periods 1 and 2, that is the accumulated (or desaccumulated) welfare over the infinite planning horizon. From the knife-edge condition (6), the flow of utility is discounted by the means of the world interest rate.

The impact of an adverse temporary terms of trade change on overall welfare $U [C(0)]$ is obtained by differentiating with respect to p the discounted values of the two periods of the instantaneous utility (see Appendix B, eq (57)):

$$\frac{dU [C(0)]}{dp} \Big|_{temp} = \frac{u_c}{r^*} \left[t_{\bar{\lambda}} \frac{d\bar{\lambda}}{dp} \Big|_{temp} + (1 - e^{-r^*T}) t_p \right] = \frac{u_c}{r^*} \frac{d\bar{c}}{dp} \Big|_{perm} (1 - e^{-r^*T}) > 0, \quad (37a)$$

$$\frac{dU [C(0)]}{dp} \Big|_{temp}^{T \rightarrow \infty} = \frac{u_c}{r^*} \left[t_{\bar{\lambda}} \frac{d\bar{\lambda}}{dp} \Big|_{temp}^{T \rightarrow \infty} + t_p \right] = \frac{u_c}{r^*} \frac{d\bar{c}}{dp} \Big|_{perm} = \frac{dU [C(0)]}{dp} \Big|_{perm} > 0. \quad (37b)$$

In obtaining the expression (37a), use has been made of $(d\bar{\lambda}/dp)_{temp} = \lambda_p (1 - e^{-r^*T})$. Although the real expense may be reduced in a greater amount after a long-lived negative disturbance than after a permanent perturbation, (37b) indicates that change in welfare is equal. This comes from the fact that the discounted utility flow obtained from period 2's real expense tends toward zero as the shock tends toward infinity.

Following a perfectly future anticipated permanent shock, the change in welfare may be quite different depending on the perturbation occurs in the close or distant future

$$\frac{dU [C(0)]}{dp} \Big|_{fut} = \frac{u_c}{r^*} \left[t_{\bar{\lambda}} \frac{d\bar{\lambda}}{dp} \Big|_{fut} + t_p \right] e^{-r^*T} = \frac{u_c}{r^*} \frac{d\bar{c}}{dp} \Big|_{perm} e^{-r^*T} > 0, \quad (38a)$$

$$\frac{dU [C(0)]}{dp} \Big|_{fut}^{T \rightarrow \infty} = 0. \quad (38b)$$

When the adverse disturbance is expected to occur in the near future, that is the length of the lead time T is short, the change in welfare is negative (see (38a)); compared with a permanent shock, the welfare loss is dampened because of the constancy of terms of trade at level $p_0 = p_1$ over period $(0, T)$ (period 1) during which the current income is higher than the permanent income. When the lead time T tends towards infinity, the home goods' relative price remains to its initial level over an infinite period and the present value of the instantaneous utility's gain (when the shock occurs) converges towards zero (see (38b)).

What are the consequences of a failure in expectations for welfare? Its change is equal to the sum of variations of U after a permanent and a temporary shock:

$$\frac{dU [C(0)]}{dp} \Big|_{fail} = \frac{u_c}{r^*} \left[t_{\bar{\lambda}} \frac{d\bar{\lambda}}{dp} \Big|_{fail} + t_p \right] = \frac{u_c}{r^*} \left[\frac{d\bar{c}}{dp} \Big|_{temp} + \frac{d\bar{c}}{dp} \Big|_{perm} \right] > 0. \quad (39)$$

The negative loss in welfare is greater when an expectations' failure occurs the farther the date T at which agents become aware of the permanence of the adverse disturbance.

6 Conclusion

In a simple small open economy model where the infinite-living representative agent consumes both a domestic and an imported good, Obstfeld [1983] has shown that a transitory terms of trade worsening induces a transitory fall in the consumption-based real interest rate, a possibly initial rise in the real expense if the *intertemporal speculation* effect is sufficiently strong, and a current account deficit whatever is the persistence of the shock. In a framework close to Obstfeld, we provide new insights about the *hysteresis* effects of an unanticipated transitory terms of trade worsening, and particularly propose a comparison with the long-run changes of macroeconomic aggregates after a permanent disturbance. The scope of Obstfeld's analysis is enlarged by considering two other types of shocks: a future anticipated permanent perturbation and a permanent viewed as temporary negative terms of trade disturbance.

A recent *two-step* analytical procedure proposed by Schubert and Turnovsky [2002] is applied to the zero-root property, which allows to obtain formal solutions in a consistent and explicit way. New results non developed and not obtained by previous studies are shown formally. First, the once-for-all rise in the marginal utility of wealth after a temporary and a future permanent shock is a scaled-down factor of the response to a permanent relative price disturbance. Second, the real expense may reach a lower level than after a permanent shock if the terms of trade perturbation is strongly persistent since in the transitory disturbance case, the final steady-state change is only affected by the variation of the marginal utility of wealth. Third, agents may raise or reduce their real expense after a future anticipated permanent terms of trade deterioration depending on the shock occurs in the distant or in the near future. Fourth, falls in expectations may induce large cut in real expense. Fifth, the overall welfare changes in the same amount after a strongly persistent relative price perturbation or a permanent disturbance. The variation in welfare tends toward zero whenever a permanent negative external shock is expected to occur in a very distant future and is as lower as the expectations upon the shock's persistence are disappointed in the distant future. Sixth, the stock of foreign assets is definitively reduced or raised according to the shock is temporary or future anticipated permanent although an unanticipated permanent negative external shock leaves unchanged the stock of foreign assets. In the latter case, the economy jumps immediately to the new steady-state. This result is no longer obtained and transitional dynamics emerge whenever the perturbation is unanticipated temporary, future anticipated permanent, or permanent viewed as temporary. After a temporary (resp. future permanent) adverse external shock, the representative agent decumulates (resp. accumulates) traded bonds during the unstable period, that is when the terms of trade are unfavorable (resp. constant). Once the shock ends (resp. occurs), the stock of foreign assets reaches its new lower (resp. higher) long-run steady-state value.

Our aim is not to support an extensive use of the time separable utility function in open economy macroeconomics but rather to clarify its implications through the simplest small open economy model. Previous studies refer mostly to the case of time separable preferences as the case of immediate reduction of real expense by the same amount as the income drop,

leaving the net foreign asset position unaffected. This view seems restrictive as current account dynamics emerge whenever the shock is temporary, future permanent, or permanent expected as transitory, and real expense may rise or fall in the short-run or in the long-run depending upon the persistence of the shock, the time it occurs, and the ability of agents not to fail in its time of occurrence.

A two-country model will be an interesting way to extend our model as it would endogenize the world interest rate as well as the relative price. Such a formalization has been performed by Turnovsky [1997] (chap. 7) by abstracting from temporary shocks. A two-country variant of the solution procedure presented here could be performed as the zero-root property of a small open economy model translates in a restriction in the ratio λ^*/λ in a world economy.

A Steady-State Changes: The Two-Step Method

In a first step, we totally differentiate the set of relations (14) without (14c), which gives in a matrix form:

$$\begin{pmatrix} u_{cc} & 0 \\ -p_c(p_i) & r^* \end{pmatrix} \begin{pmatrix} d\bar{c}_i \\ d\bar{b}_i \end{pmatrix} = \begin{pmatrix} p_c(p_i) d\bar{\lambda}_i + p'_c(p_i) \bar{\lambda}_i dp_i \\ -(\bar{y} - \bar{d}_i) dp_i \end{pmatrix}. \quad (40)$$

The steady-state changes are given by

$$\frac{\partial \bar{c}_i}{\partial \bar{\lambda}_i} = \frac{r^* p_c}{G} < 0, \quad (41a)$$

$$\frac{\partial \bar{c}_i}{\partial p_i} = \frac{r^* p'_c \bar{\lambda}_i}{G} < 0, \quad (41b)$$

$$\frac{\partial \bar{b}_i}{\partial \bar{\lambda}_i} = \frac{p_c^2}{G} < 0, \quad (41c)$$

$$\frac{\partial \bar{b}_i}{\partial p_i} = \frac{-u_{cc}(\bar{y} - \bar{d}_i) + p'_c \bar{\lambda}_i p_c}{G} < 0. \quad (41d)$$

where $G \equiv r^* u_{cc} < 0$.

In a second step, we totally differentiate (16) to obtain the equilibrium change of the marginal utility of wealth

$$v_{\bar{\lambda}} d\bar{\lambda}_i = db_{T_i} - v_p dp_i. \quad (42)$$

The partial derivatives of the function $\bar{\lambda}_i = \lambda(b_{T_i}, p_i)$ write as follows

$$\lambda_p \equiv \frac{d\bar{\lambda}_i}{dp_i} = -\frac{v_p}{v_{\bar{\lambda}}} < 0, \quad (43a)$$

$$\lambda_b \equiv \frac{d\bar{\lambda}_i}{db_{T_i}} = \frac{1}{v_{\bar{\lambda}}} < 0. \quad (43b)$$

B Formal Solutions for Temporary Disturbances: The Zero-Root Property Case

Solutions for the Foreign Assets Stock and the Marginal Utility of Wealth for Temporary Terms of Trade Shocks

In this appendix, we apply the *two-step* procedure proposed by Schubert and Turnovsky [2002] to the zero-root property case. We assume that the small open economy is initially in steady-state equilibrium, denoted by the subscript $i = 0$:

$$b_0 = \bar{b}_0 = v(\bar{\lambda}_0, p_0) = v(g(b_0, p_0), p_0), \quad (44a)$$

$$\lambda_0 = \bar{\lambda}_0 = g(b_0, p_0). \quad (44b)$$

We suppose now that the terms of trade change unexpectedly at time $t = 0$ from the original level p_0 to level p_1 over the period $0 \leq t < T$, and they revert back at time T permanently to their initial level $p_T = p_2 = p_0$.

Period 1 ($0 \leq t < T$)

Whereas the terms of trade disturbance is at work, the economy follows unstable transitional paths:

$$b(t) = \bar{b}_1 + A_1 + A_2 e^{\mu_2 t}, \quad (45a)$$

$$\lambda(t) = \bar{\lambda}_1 + \frac{r^*}{p c t_\lambda} A_1. \quad (45b)$$

The steady-state value \bar{b}_1 is given by the following function (determined from (15b) with $i = 1$)

$$\bar{b}_1 = v(\bar{\lambda}, p_1), \quad (46)$$

where the marginal utility of wealth remains constant over periods 1 and 2 at level $\bar{\lambda}_1 = \bar{\lambda}_2 = \bar{\lambda}$ after its initial jump at time $t = 0$.

Period 2 ($t \geq T$)

Once terms of trade revert back to their initial level, the economy jumps instantaneously to the final steady-state:

$$b(t) = \bar{b}_2, \quad (47a)$$

$$\lambda(t) = \bar{\lambda}_2, \quad (47b)$$

where the steady-state value \bar{b}_2 is given by the following function (determined from (15b) with $i = 2$)

$$\bar{b}_2 = v(\bar{\lambda}, p_2). \quad (48)$$

During the transition period 1, the economy accumulates (or decumulates) foreign assets. Since this period is by its very nature unstable, it would lead the nation to violate

its intertemporal budget constraint. By contrast, the adjustment process taking place in period 2 is stable and must satisfy the economy's intertemporal budget constraint. The dynamics prevailing with time separable preferences differ markedly with the adjustment in an habit-forming model since in the former case, the economy jumps immediately to the new steady-state as the relative price shock ends. The zero-root problem requires the equilibrium value of marginal utility of wealth to adjust once-and-for-all when the shock hits the economy. So λ remains constant over the periods 1 and 2 (see Obstfeld [1983]). The aim of the *two-step* method is to calculate the deviation of λ such that the country satisfies one single and overall intertemporal budget constraint, given the new relevant initial condition b_T prevailing when the terms of trade are restored to their initial level. Therefore, for the country to remain intertemporally solvent, we require:

$$b_T = \bar{b}_2 = -\frac{(p_2 y - p_c \bar{c}_2)}{r^*}. \quad (49)$$

In order to determine the two constants A_1 and A_2 , and the change in the equilibrium value of marginal utility of wealth, we impose two conditions:

1. Initial condition $b(0) = b_0$ must be met.
2. Economic aggregate b must remain continuous in the neighborhood of time T . This implies in turn that the intertemporal solvency holds.¹³

To deal conveniently with temporary and future terms of trade disturbances simultaneously, we define the dummy-variable τ as follows

$$\begin{cases} \tau = 1 & \text{for temporary shock in period 1,} \\ \tau = 0 & \text{for future permanent shock in period 2.} \end{cases}$$

Then, we approximate the steady-state changes with the differentials by using the dummy notation:

$$\bar{c}_1 - \bar{c}_0 \equiv t(\bar{\lambda}, p_1) - t(\lambda_0, p_0) = t_{\bar{\lambda}} d\bar{\lambda} + t_p \tau dp, \quad (50a)$$

$$\bar{c}_2 - \bar{c}_1 \equiv t(\bar{\lambda}, p_2) - t(\bar{\lambda}, p_1) = t_p (-1)^\tau dp, \quad (50b)$$

$$\bar{b}_1 - \bar{b}_0 \equiv v(\bar{\lambda}, p_1) - v(\lambda_0, p_0) = v_{\bar{\lambda}} d\bar{\lambda} + v_p \tau dp, \quad (50c)$$

$$\bar{b}_2 - \bar{b}_1 \equiv v(\bar{\lambda}, p_2) - v(\bar{\lambda}, p_1) = v_p (-1)^\tau dp, \quad (50d)$$

where $t_{\bar{\lambda}} < 0$, $v_{\bar{\lambda}} < 0$, $t_p < 0$, $v_p < 0$ (see (41)).

Set $t = 0$ in solution (45a), equate (45a) and (47a), (45b) and (47b) evaluated at time $t = T$, one obtains:

$$A_1 + A_2 = b_0 - \bar{b}_1 = -v_{\bar{\lambda}} d\bar{\lambda} - v_p \tau dp, \quad (51a)$$

$$A_1 + A_2 e^{\mu_2 T} = \bar{b}_2 - \bar{b}_1 = v_p (-1)^\tau dp, \quad (51b)$$

$$\frac{r^*}{p_c t_{\bar{\lambda}}} A_1 = \bar{\lambda}_2 - \bar{\lambda}_1 = 0, \quad (51c)$$

where $d\bar{\lambda} \equiv \bar{\lambda} - \lambda_0$ and $\lambda_2 = \lambda_1 = \bar{\lambda}$ since the shadow value of internationally traded bonds jumps abruptly and permanently to its new level at time $t = 0^+$ after an unanticipated temporary disturbance or a future anticipated permanent perturbation.

The solutions are given by:¹⁴

$$A_1 = 0, \quad (52a)$$

$$A_2 = v_p (-1)^\tau e^{-\mu_2 T} dp, \quad (52b)$$

$$d\bar{\lambda} = -\frac{v_p}{v_{\bar{\lambda}}} (\tau + (-1)^\tau e^{-\mu_2 T}) dp. \quad (52c)$$

By substituting the constants in (45a), the solution for net foreign assets over the unstable period 1 writes as follows¹⁵

$$b(t) = \bar{b}_1 + v_p (-1)^\tau e^{\mu_2(t-T)} dp. \quad (53)$$

C Fails in Expectations: The New Equilibrium Value of the Marginal Utility of Wealth

The change in the equilibrium value of marginal utility of wealth at time T is given by:

$$d\bar{\lambda}(T) \equiv \bar{\lambda}_{fail} - \bar{\lambda}_{temp} = (\bar{\lambda}_{fail} - \lambda_0) - (\bar{\lambda}_{temp} - \lambda_0), \quad (54)$$

where $\bar{\lambda}_{temp} - \lambda_0 = \lambda_p (1 - e^{-r^*T}) dp$ (see eq (19)). By using the function $\bar{\lambda}_i = \lambda(b_{T_i}, p_i)$, we are able to calculate the first term on the right hand side of (54)

$$\bar{\lambda}_{fail} - \lambda_0 = \lambda_b db + \lambda_p dp = \lambda_p (1 - e^{-r^*T}) dp + \lambda_p dp > \bar{\lambda}_{perm} - \lambda_0 = \lambda_p dp > 0, \quad (55)$$

where $\bar{\lambda}_{fail} = \lambda(b_T, p_1)$ (that is $p_2 = p_1 < p_0$); from (53) we have $db \equiv b_T - b_0 = v_{\bar{\lambda}} \lambda_p (1 - e^{-r^*T}) dp$ and by using (43b) we obtain (55).

Combining the change in $\bar{\lambda}$ after a failure in expectation and the change following an unanticipated transitory shock, we get

$$d\bar{\lambda}(T) \equiv \bar{\lambda}_{fail} - \bar{\lambda}_{temp} = \lambda_p dp > 0. \quad (56)$$

D Solutions for Welfare for Terms of Trade Shocks

By denoting the instantaneous welfare $\phi(t) = u(c(t)) = u(t(\bar{\lambda}, p))$, remembering that $\lambda(t) = \bar{\lambda} = \lambda_1 = \lambda_2$, and taking the discounted value of the two periods of ϕ over an infinite horizon,¹⁶ one may evaluate the effect of a relative price disturbance on overall welfare $U[C(0)] = \int_0^\infty u[c(t)] \exp(-\delta t) dt$:

$$\begin{aligned} U[C(0)] &= \int_0^T \phi(t) e^{-r^*t} dt + \int_T^\infty \phi(t) e^{-r^*t} dt, \\ &= \frac{u(t(\bar{\lambda}, p_1)) (1 - e^{-r^*T})}{r^*} + \frac{u(t(\bar{\lambda}, p_2)) e^{-r^*T}}{r^*}, \end{aligned} \quad (57)$$

where we used the imposed equality $\delta = r^*$ (see (6)).

Notes

¹See for example Obstfeld [1983], Sen and Turnovsky [1989], [1990], Karayalçin [1999], Servèn [1999].

²See Turnovsky and Sen [1995] for a justification.

³To our knowledge, only Sen and Turnovsky [1989] examine the effects of a future anticipated permanent terms of trade shock in a continuous time model with infinite horizon. Our study departs from their analysis by our focus on saving dynamics, the analytical method retained for studying temporary and permanent disturbances, the numeraire chosen, and more importantly by differentiating analytically between the effects of transitory relative price changes of different degrees of persistence. When the domestic good is taken as the numeraire, the *wealth* effect and the *relative price* effect go in opposite ways, leaving undetermined the long-run change in the equilibrium value of the marginal utility of wealth. For clarity of discussion, we have retained the foreign good as the numeraire.

⁴Reinhart and Wickham [1994] for developing countries, Cashin and Mc Dermott [2002] for developed countries, evaluate the importance of the temporary component of terms of trade variations. Cashin, Mc Dermott, Patillo [2004] estimate the duration of relative price changes.

⁵Deaton [1999] compares the World Bank's commodity price forecasts with their actual evolution and conclude to an overestimation, more particularly for copper and cotton (see pp. 31-33).

⁶See the exchange between Obstfeld [1996] and Lahiri [1996] who discuss the sufficient conditions for a current account deficit following an adverse terms of trade shock, and particularly the role of the patterns of trade in a framework close to ours.

⁷The expression (20a) is analogue to the equation obtained by Servèn [1999] who provides a remarkable discussion of impact effects after a terms of trade improvement. There exists a length of the shock denoted by $\check{T} = \frac{1}{r^*} \log \left[\frac{(y-\bar{d})+\sigma\bar{d}}{y-\bar{d}} \right]$ such that the *wealth* effect is exactly outweighed by the *smoothing* and *intertemporal speculation* effects, that is the initial change in real expense is null. For $T > \check{T}$ ($T < \check{T}$), real expense falls (rises) following a temporary terms of trade worsening.

⁸In contrast to formal solutions for temporary shocks obtained when recursive preferences or an habit-forming behavior are considered, the formal solution for net foreign assets contains only an explosive component as $\mu_1 = 0$.

⁹Changes of macroeconomic aggregates after a permanent or a transitory terms of trade disturbance differ markedly too if we suppose an habit-forming behavior since the knife-edge condition is imposed. For a particular form of intertemporal preferences (the distant complementarity case), a permanent worsening of terms of trade implies a rise in the stock of foreign assets. In contrast, a transitory worsening implies a fall in the stock of foreign assets (if the shock is not too short) for the same reason we suggest in this paper.

¹⁰The marginal utility of wealth, measured in terms of the foreign goods, remains constant over the unstable period 1, $0 \leq t < T$, and over the stable period 2, $t \geq T$, that is $\bar{\lambda}_1 = \bar{\lambda}_2 = \bar{\lambda}$.

¹¹The dummy variable takes the value zero, that is $\tau = 0$.

¹²We have calculated the change $d\lambda(T)$ in the equilibrium value of the marginal utility of wealth by retaining a similar procedure as Schubert [2002] proposed.

¹³Schubert and Turnovsky [2002] set three conditions. In our model, the second and third conditions are identical.

¹⁴The change in the equilibrium value of the marginal utility of wealth can be calculated from the solution of internationally traded bonds (9) by imposing the condition of continuity of foreign assets at time T . By remembering that the small open economy follows an unstable path during period 1, the solution for net foreign assets during the unstable period is given by (8). Once the shock ends, the economy follows stable paths and jumps immediately to the final steady state at level $b(T) = \bar{b}_2$. Equate (8) (by using the fact that $\bar{b}_1 = -\frac{p_1 y - p_c(p_1)\bar{c}_1}{r^*}$) and (47a) evaluate at time T , we get

$$\bar{b}_1 + (b_0 - \bar{b}_1) e^{r^*T} = \bar{b}_2.$$

Substitute the steady-state value of net foreign assets (15b), approximate by the differentials, one obtains (52c).

¹⁵By applying the general procedure proposed by Buiter [1984], we find for an unanticipated temporary shock:

$$\begin{aligned} b(t) &= b_0 + e^{r^*T} \left(1 - e^{r^*t}\right) \frac{[(y - \bar{d}) - p_c t_p]}{r^*} dp, \\ &= b_0 + v_p e^{-r^*T} dp - v_p e^{r^*(t-T)} dp, \end{aligned}$$

where $b_0 + v_p e^{-r^*T} dp = \bar{b}_1$ since by using (52c) we have $\bar{b}_1 - b_0 = v_{\bar{\lambda}} d\bar{\lambda} + v_p dp = v_p e^{-r^*T}$. For our purpose, we have adopted the *two-step* Schubert and Turnovsky solution procedure as it allows to express the steady-state values of macroeconomic aggregates as functions of the marginal utility of wealth and the relative price. This permits to tackle the hysteresis phenomenon in a consistent and explicit way since the calculations of long-run changes of real expense and net foreign assets are straightforward.

¹⁶See Turnovsky [2000] for a similar procedure (chap. 8).

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