Moral hazard in financial markets : Inefficient equilibria and monetary policies

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Moral Hazard in Financial Markets: Inefficient Equilibria and Monetary Policies*

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Abstract

This paper presents a moral hazard model of financing in which borrowers adopt two modes of finance, either issuing bonds or applying for bank loans. The bond rate is set by the borrowers, while the loan rate is chosen by a monopolistic bank. Bank finance ameliorates the moral hazard problem by monitoring borrowers. Monetary interventions, which affect real economy through the bank lending channel, are justified on the basis of welfare considerations. When the informational problem is not severe, monitoring is wasteful and welfare is enhanced through a monetary tightening. When the moral hazard problem is severe, monitoring is useful and welfare is increased by a monetary expansion.

Keywords: moral hazard, monitoring, monetary policies, bank lending channel.

JEL Classification: D82, E44, E52.

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1 Introduction

There are two views of the initial impact of monetary policy on financial markets: money and credit view.[1] The first one is the traditional IS/LM textbook description where (i) loans, the bank assets, are considered perfect substitutes for other debt instruments and then lumped together with these in a general bond market, (ii) authorities can directly control the quantity of money by adjusting reserves. Accordingly, when authorities reduce the level of reserves, the availability of money (i.e., bank liability) decreases; banks passively suffer the consequent reduction of assets (bonds) and relative repayment increases: investment and aggregate production are negatively affected. Money (deposits) is then the only mean of the monetary policy transmission.

The credit view refuses the first hypothesis by stressing the importance of asymmetric information in credit markets, on the ground of which loans and bonds are considered to be imperfect substitutes both for borrowers and for banks. We focus our attention on a particular strand of the credit channel literature, called bank lending channel, which assumes that a monetary tightening produces reduction in the supply of loans. Equilibrium of the lending market then plays a central role in how monetary policy affects aggregate production. Empirical works show strong evidence in favor of a high correlation between monetary intervention and supply of loans: Kashyap *et al.* (1993) find that a restrictive policy is followed by a raise in commercial paper issuance and a decline in bank loans. Many theoretical works give a formalization of how the credit channel operates. Two types of them may be found in the literature: papers *à la* Bernanke and Blinder (1988) in which a general equilibrium framework is proposed, but financial choices of agents are not derived from first principles, and models *à la* Repullo and Suarez (2000) or Holmstrom and Tirole (1997), in which choices are micro-founded, but in a partial equilibrium framework where monetary authorities are not explicitly modeled and are assumed to target directly an endogenous interest rate. Nevertheless, no paper seems to consider the problem of why authorities intervene: effects of monetary policies are studied in economies where equilibrium level of welfare is constrained efficient. In contrast, the current paper assumes that the bank lending channel is in action and focuses on constructing a partial equilibrium model in which equilibria arise with inefficient levels of aggregate production: monetary intervention is addressed to reduce such an inefficiency.

[1] See Freixas and Rochet (1997) for a survey.

More exactly, a moral hazard framework of financing (Holmstrom and Tirole, 1997) is proposed, where homogeneous borrowers need money to implement a project. The borrowers raise funds either by issuing bonds or by applying for bank loans. After the financial contract is signed, they can choose between two levels of effort: when choosing the low one they obtain a private benefit, but the project is efficient when effort is high. Private benefits are nonmonetary and nontransferable quantities through which the borrowers embezzle resources from the projects. Lenders cannot verify the choice of the effort. The borrowers' participation constraint is always satisfied in equilibrium, so that the demand for credit is equal to the amount of savings that can be lent.

A household sector is introduced that buys bonds and deposits money in the bank sector. Households' total savings are assumed to be equal to the demand for credit, hence all the borrowers obtain funding. Loans are supplied by a monopolistic bank, which has no initial capital and raises funds by insured deposits. We limit assets and liabilities of the bank respectively to loans and deposits relying on the idea, behind the bank lending view, that loans and bonds are imperfect substitutes for the bank and that insured deposits are the cheapest way of financing loans (Stein, 1998), hence the correlation between availability of deposits and supply of bank loans is high. We assume for simplicity that after a tightening that reduces loans, our homogeneous borrowers can shift to bond financing. In other words, a monetary intervention does not modify the quantity of credit, rather affecting the composition of lending.

We argue that the bank can better monitor the borrowers than the dispersed households. A free-riding argument can enforce the assumption: each household is small, so it is not worth paying the monitoring cost because everybody would like someone else to bear the cost (Allen and Gale, 2000). The dispersion of the households can also generate wasteful multiplication of monitoring costs (Diamond, 1984). Monitoring ameliorates the moral hazard problem, by reducing private benefits to the borrowers. The bank decides to monitor in equilibrium and to induce the borrowers to choose high effort. Moreover, by taking into account the empirical evidence that bank credit is more expensive than direct debt (James, 1987), we assume that borrowers' profits on bonds are higher than profits on loans for any admissible value of the bond rate.

We define as welfare the sum of profits of all agents and we assume that monetary authorities act directly on the bond rate to maximize welfare. The assumption is related to the idea that, following a monetary operation which targets the T-bills interest rate, effects on the bond repayment are less delayed and less ambiguous than effects on the loan rate and it is also due to the difficulties involved in constructing a congruous model of a monetary economy. Changes in the bond rate affect the allocation of savings between bonds and deposits. When remuneration on bonds rises less borrowers are financed by loans because the supply of deposits decreases. When the opposite holds, more borrowers are financed by loans because the supply of deposits increases.

Gertler and Gilchrist (1994) show that after a tightening external financing of large firms increases and that it is the small firms that experience a large decreases in bank loans, which are essentially their only source of financing. Even if no exogenous heterogeneity of the borrowers is taken into account, the model gives a normative interpretation to evidence on the cross-sectional impact of monetary policy by studying how the equilibrium bond repayment changes with the severity of the moral hazard problem. Such a severity is defined as follows: the bigger are private benefits, the higher is the incentive for the borrowers to choose the inefficient project and the harder the moral hazard problem will be. The size of private benefits may depend on how the legal system works: if it works well the borrowers cannot easily manipulate balance-sheets, whereas if it works badly, the borrowers can take significant amounts of money from the project.

Indeed, when moral hazard is weak for private benefits are low, the equilibrium bond repayment is such that the borrowers choose high effort. In this case monitoring is wasteful because the borrowers behave well even if they are not monitored, hence a monetary tightening is justified by the need of reducing deposits and thereby the supply of loans. In other words, low private benefits depict a situation where the pool of borrowers is good: in such a case a tightening is addressed to increase the access to direct debt. We show that it is welfare-enhancing for bonds are more efficient. When moral hazard is severe the equilibrium bond rate induces the borrowers to choose low effort, hence bond finance represents the inferior mode. Nonetheless monitoring is costly, hence authorities reduce the targeted rate to induce the borrowers to behave well under bond finance. In other words, when private benefits are high the pool of borrowers is bad; a reduction of the bond rate augments their stake in the project and induces them to increase the effort.

The reminder of the paper is organized as follows. Section 2 presents the model. Section 3 studies equilibria with weak and severe moral hazard. Section 4 analyses the efficiency of equilibria and explains how monetary authorities can increase welfare. Section 5 contains concluding notes.

2 The model

Borrowers. Consider n risk neutral and homogeneous borrowers, with n arbitrarily large, each of whom needs one unit of capital to implement a project. Each project yields A with probability p_i and zero otherwise. Let $i = s, r$ and $p_s > p_r$: the borrowers may decide to reduce the probability of success (or, say, the effort level) from p_s to p_r in order to enjoy private benefits equal to $B > 0$. Let s be the project when the borrowers choose p_s and r the project when p_r is chosen. The borrowers have two alternatives of financing: they can either issue bonds or apply for bank loans. The choice of the project is made after the financial contract is signed and it is not verifiable by lenders.

Households and Monopolistic Bank. Consider m risk neutral and homogeneous households, with m arbitrarily large, each of whom endowed with an amount of savings equal to $\frac{n}{m}$ and to be allocated between bonds and bank deposits. Bonds and deposits are not perfect substitutes: the households need deposits in order to perform daily transactions; on the contrary, bonds cannot offer this service (Bolton and Freixas, 2000). Let D be the deposit remuneration set by a monopolistic bank and p_iF be the unitary return of each household on bonds, where F is the bond repayment set by the borrowers. The supply of deposits is assumed to have the following functional form:

$$n\lambda\left(D,F\right) = \begin{cases} n\frac{D}{p_iF} \text{ for any } D < p_iF \\ n \text{ for any } D \geq p_iF \end{cases} \tag{1}$$

When $D < p_iF$ deposits are less remunerative, however the households allocate some savings to deposits. When $D \geq p_iF$ deposits have at least the same remuneration as bonds and the households allocate all savings to the former.

The bank has no initial capital and it can raise funds only by deposits. Deposits are insured, so that if the bank fails an outside insurance fund repays households. The cost of deposit insurance is fixed and, without loss of generality, normalized to zero. For the sake of simplicity the bank is assumed to lend all deposits it raises, therefore its loanable funds are given by $n\lambda\left(D,F\right)$.[2]

Monitoring and Contracts. The households are dispersed, so that they cannot monitor the effort level chosen by the borrowers. The bank can instead monitor the effort at a unitary cost $c > 0$. When the bank monitors private benefits of the borrowers diminish to $b > 0$.

[2] The demand for bonds is equal to $n\left(1-\lambda\left(D,F\right)\right)$.

Assumption 1 $c < \frac{p_r p_s}{(p_s - p_r)^2} b$ and $\underline{B} \leq B < \overline{B}$, where $\underline{B} = \frac{p_s}{p_s - p_r} b$ and $\overline{B} = (p_s - p_r) A - c$.[3]

Assumption 1 states that monitoring is both sufficiently cheap and sufficiently efficient and implies $p_s A > p_r A + B$: throughout the paper we refer to project s as the efficient project and to project r as the inefficient project. Both bank loans and bonds take the following contractual form: when the project succeeds the borrowers repay a gross interest rate which must not exceed A, whereas in the case of failure no repayment is made. The upper bound on the repayment derives from the fact that private benefits are assumed not to be transferable: lenders can at most obtain the monetary outcome of the project. Profit of the bank amounts to[4]

$$n\lambda (D, F) (p_i R - c_j - D). \tag{2}$$

where R is the loan repayment set by the bank, $c_j = c$ if monitoring is implemented and $c_j = 0$ otherwise. Remark that $\lambda (D, F)$ and $1 - \lambda (D, F)$ can be interpreted as the ex ante probabilities that each borrower receives money either from the bank or from the households, respectively. Ex ante profit of each borrower can thus be written as

$$\lambda (D, F) \left[U^i_{c_j} (R) \right] + (1 - \lambda (D, F)) \left[U^i (F) \right], \tag{3}$$

where $U^i_c (R) = p_i (A - R) + b_i$, $b_s = 0$ and $b_r = b$, $U^i_0 (R) = p_i (A - R) + B_i$, $B_s = 0$ and $B_r = B$, $U^i (F) = p_i (A - F) + B_i$. Finally, unitary profit of each household is

$$\lambda (D, F) D + (1 - \lambda (D, F)) p_i F. \tag{4}$$

Welfare is defined as the sum of profits of bank, borrowers and households:

$$W = \begin{cases} n \left[\lambda (p_i A + b_i - c) + (1 - \lambda) (p_i A + B_i) \right] & \text{if monitoring is implemented} \\ n \left[\lambda (p_i A + B_i) + (1 - \lambda) (p_i A + B_i) \right] & \text{if monitoring is not implemented} \end{cases} \tag{5}$$

Timing. The timing of the model is as follows:

1. At $t = -2$ the bank decides whether to monitor or not and selects loan and deposit rates; simultaneously, the borrowers set the bond rate.

2. At $t = -1$ the households allocate savings.

[3] Notice that the interval $[\underline{B}, \overline{B})$ is nonempty if $(p_s - p_r) A > \left(\frac{p_s}{p_s - p_r} \right)^2 b$.

[4] Costs of retail banking are assumed to be zero, so that cost of deposits for the bank is given by D. Moreover, we neglect the possibility of the bank to monitoring only a fraction of the projects it finances.

3. At $t = 0$ the borrowers obtain funds either from the households or from the bank and invest: the choice of the project is made between 0 and 1 and is not verifiable by the lenders. The borrowers have no time preference.

4. At $t = 1$ returns of project accrue, the borrowers repay bonds and loans and the bank repays deposits.

In the next section we define the game between bank and borrowers and we study how the Nash equilibrium of the game varies with the parameter B.

3 Equilibria with Weak and Severe Moral Hazard

The bank decides whether to monitor and chooses R and D in order to maximize (2) for any F chosen by the borrowers. The borrowers choose F to maximize (3) for any choice of the bank. The game between bank and borrowers is analysed by restricting the attention to pure strategy equilibria. The set of players is $\{Ba, Bo\}$, where Ba is the bank and Bo are the borrowers. Before defining the set of actions of the bank, notice that the borrowers' choice of the project depends on R if they are financed by the bank. When the bank decides to monitor, $U_c^i(R)$ is the ex post profit of the borrowers under bank finance. If $R = R_c^*$, where $R_c^* = A - \frac{b}{p_s - p_r}$ is the solution to $U_c^s(R) = U_c^r(R)$, then the borrowers are indifferent between the two projects, in which case they are assumed to choose project s. If $R > R_c^*$, the increase in expected repayment $p_i(R - R_c^*)$ is lower if project r is chosen, for $p_r < p_s$. For the same reasoning, if $R < R_c^*$, the decrease in expected repayment $p_i(R_c^* - R)$ is higher if project s is chosen. It follows that the borrowers choose project r for $R > R_c^*$, while project s is chosen for $R \leq R_c^*$. The maximum repayment the bank can set and still induces investment in the project r is $\hat{R}_c = A$. When the bank decides not to monitor, cut-off values are $R_0^* = A - \frac{B}{p_s - p_r}$, where R_0^* is solution to $U_0^s(R) = U_0^r(R)$, and $\hat{R}_0 = A$. By comparing bank finance with and without monitoring, one can verify that $R_c^* > R_0^*$. This stems from the fact that monitoring reduces private benefits to the borrowers, who are thereby more oriented towards project s. Since (2) is linearly increasing in R, the bank sets it as high as possible in equilibrium by anticipating the above effects on borrowers' incentives. We then restrict the bank choice of R to the four cut-off values computed above. Its set of actions is

$$\mathbb{A}^{Ba} = \left\{ \{monitoring, no\ monitoring\}; R = \left\{ R_0^*, R_c^*, \hat{R}_0, \hat{R}_c \right\}; D \in [0, +\infty) \right\}. \tag{6}$$

On the contrary, the borrowers' choice of the project depends on F if they receive money from the households. The ex post profit of the borrowers under bond finance is $U^i(F)$. If $F = F^*$, where $F^* = A - \frac{B}{p_s - p_r}$ is solution to $U^s(F) = U^r(F)$, then the borrowers choose project r for $F > F^*$ and project s for $F \leq F^*$. Again, the maximum repayment which induces investment is $\hat{F} = A$, for which the borrowers select the project r. The set of actions of the borrowers is $\mathbb{A}^{Bo} = F \in [0, +\infty)$.

The timing of the game is simultaneous: the bank selects a strategy from the set $\mathbb{A}^{Ba}$ and the borrowers select a strategy from the set $\mathbb{A}^{Bo}$. In Proposition 1 we compute the best response of the bank. Formal proofs of this and all other results are in the Appendix.

Proposition 1 *The best response of the bank is as follows: for any F the bank decides to monitor the borrowers and to set $R_c^* = A - \frac{b}{p_s - p_r}$ that induces them to choose the efficient project. Moreover,*

$$D(F) = \begin{cases} p_s F \text{ for any } F \leq \frac{D^*}{p_s} \\ D^* \text{ for any } \frac{D^*}{p_s} < F \leq F^* \\ p_r F \text{ for any } F^* < F \leq \frac{D^*}{p_r} \\ D^* \text{ for any } \frac{D^*}{p_r} < F \leq A \end{cases} \tag{7}$$

where $D^* = \frac{p_s R_c^* - c}{2}$.

Proposition 2 defines the reaction function of the borrowers.

Proposition 2 *The best response of the borrowers is*

$$F(D) = \begin{cases} F_s^*(D) \text{ for any } D \leq D_s \\ F_r^*(D) \text{ for any } D_s < D \leq D_r \end{cases} \tag{8}$$

where $F_s^*(D) = \sqrt{\frac{DR_c^*}{p_s}}$, $D_s = \frac{p_s(F^*)^2}{R_c^*}$ is such that $F_s^*(D_s) = F^*$, $F_r^*(D) = \frac{\sqrt{D(B+\zeta)}}{p_r}$, $\zeta = p_r A - \frac{p_s}{p_s - p_r} b$ and $D_r = \frac{(p_r A)^2}{B + \zeta}$ is such that $F_r^*(D_r) = A$.

We then analyze how the Nash equilibrium changes with the parameter B, which indicates private benefits. The borrowers must repay only when the project succeeds because they have limited liability. Moreover, they obtain private benefits from the project r, hence they prefer it for high values of the borrowing rate. Lenders do not obtain any repayment in the case of failure, hence they prefer the project s. The bank sets $R = R_c^*$, which induces the borrowers to choose the efficient project. In contrast, a conflict of interests between households and borrowers arises for high values of the bond rate and such a conflict is increasing in the amount of private benefits. Let $B_0 = (p_s - p_r)[A - F_s^*(D^*)]$ and recall $\underline{B} = \frac{p_s}{p_s - p_r} b$ and $\overline{B} = (p_s - p_r)A - c$, both derived by Assumption 1.

Definition 1 *When $\underline{B} \leq B \leq B_0$ private benefits are low and the moral hazard problem is weak. When $B_0 < B < \overline{B}$ private benefits are high and the moral hazard problem is severe.*

Proposition 3 *When the moral hazard problem is weak the Nash equilibrium is $(monitoring, R_c^*, D^*, F_s^*(D^*))$, for which the borrowers select the efficient project under bond finance. When the moral hazard problem is severe the Nash equilibrium is $(monitoring, R_c^*, D^*, F_r^*(D^*))$, for which the borrowers select the inefficient project under bond finance.*

The borrowers face the following trade-off under bond finance: if they set a low repayment their stake in the project is high, hence they choose the high effort with the aim of increasing the probability of success, but they give up private benefits. When private benefits are little the borrowers decide to give up them. In contrast, when they are large the borrowers set a high repayment, thereby choosing the inefficient project. We denominate the equilibria described in Proposition 3 as the market equilibria.

4 Efficiency of Equilibria and Monetary Policies

When information is symmetric, i.e., the choice of the project is verified by the lenders, p is contractable hence no monitoring occurs. The bank sets p_i, R and D to maximize (2) for any choice of the borrowers. The borrowers choose p_i and F to maximize (3) for any choice of the latter. The market equilibrium with symmetric information is such that the efficient project is chosen by both bank and borrowers, $R = A$, $D = \frac{p_s A}{2}$ and $F = \frac{A}{\sqrt{2}}$. By substituting these values into (5) one gets

$$W^* = np_s A, \tag{9}$$

which we refer to as the efficient level of welfare.

We consider monetary authorities who act directly on the bond rate with the aim of maximizing welfare given the informational constraints. In our model an increase (decrease) of the bond rate is equivalent to a monetary tightening (expansion). Authorities intervene by undoing the borrowers' equilibrium choice before the households allocate savings, i.e., between $t = -2$ and $t = -1$. Recall that with asymmetric information the bank decides to monitor and to induce the borrowers to choose the efficient project for any F. Therefore the problem of authorities is as follows:

$$\max_F n\left[\lambda\left(D,F\right)\left(p_s A - c\right) + \left(1-\lambda\left(D,F\right)\right)\left(p_i A + B_i\right)\right]. \tag{10}$$

The solution to (10) is $F = F^*$, for which the best response of the bank is $D = D^*$ and $\lambda_s^* = \lambda\left(D^*, F^*\right)$ is minimum provided that the borrowers keep on choosing high effort under bond finance: the new equilibrium is $(monitoring, R_c^*, D^*, F^*)$. We refer to the resulting level of welfare

$$W_* = n\left[p_s A - \lambda_s^* c\right] \tag{11}$$

as the constrained efficient level.

Proposition 4 *The market equilibrium levels of welfare are below the constrained efficient level. The inefficiency is eliminated by a monetary tightening when the moral hazard problem is weak and by a monetary expansion when the moral hazard problem is severe. Moreover, the resulting equilibrium represents a Pareto improvement with compensation with respect to the market equilibria.*

Monitoring is wasteful with low private benefits because the borrowers select the efficient project even if they are not monitored. It follows that from the efficiency point of view bonds are superior than loans as a mode of financing because they save on monitoring costs. Authorities then decides to implement a monetary restriction which reduces the availability of loans.

Monitoring is useful with high private benefits because the borrowers select the inefficient project if they are not monitored. Nevertheless, monitoring is costly. Therefore authorities decide to implement a monetary expansion which decreases the bond rate and induces the borrowers to select the efficient project under bond finance.

5 Conclusion

The model presented here tries to fill a gap in the credit channel literature because an efficiency issue is analysed. More precisely, we assume that the bank lending channel is in action and we find that direct debt is more efficient than bank debt if the moral hazard problem of financing is weak. In such a case, a monetary tightening which is addressed to reduce the supply of bank loans, is welfare enhancing.

On the contrary, severe informational problems make monitoring useful: bank credit becomes more efficient. Nonetheless monitoring is costly and welfare is enhanced by a monetary expansion which induces the borrowers to behave well even if they are not monitored. We interpret the severity of the informational problem as negatively related to the quality of the legal system and we deduce that the latter affects positively the quality of the pool of borrowers. We conclude that a monetary restriction (expansion) is likely to be effective in an economy where the legal environment induces entrepreneurs to behave well (badly).

Further research in this area should overcome the main limitation of this analysis, due to the difficulties involved in constructing a consistent model of a monetary economy: the assumption that monetary authorities act directly on the bond rate.

6 Appendix

(Proposition 1). Consider the case in which $F \leq F^*$, then the borrowers choose project s under bond finance. If $F \leq \frac{D}{p_s}$ remuneration on bonds is not higher than remuneration on deposits, then the supply of deposits is n and bank profit is equal to

$$n\left(p_i R - c_j - D\right). \tag{A1}$$

Under Assumption 1, $(A1)$ is maximum when monitoring is implemented, R is set equal to R_c^*, so that the borrowers are induced to choose the high effort, and D equal to $p_s F$. If $\frac{D}{p_s} < F \leq F^*$, then the supply of deposits is $n\frac{D}{p_s F}$ and bank profit can be rewritten as

$$\frac{n}{p_s F}[-D^2 + \left(p_i R - c_j\right) D], \tag{A2}$$

Under Assumption 1, $(A2)$ is maximum when monitoring is implemented, R is set equal to R_c^* and D equal to $\frac{p_s R_c^* - c}{2}$. Let $D^* = \frac{p_s R_c^* - c}{2}$: it is worth noting that D^* does not depend on F. It is easy to check that if $F \leq \frac{D^*}{p_s}$ bank profits, which are represented by $(A2)$ when D is set lower than $p_s F$, are increasing in $D \leq p_s F$: the optimal choice is setting D as high as possible, i.e., equal to $p_s F$. On the contrary, if $\frac{D^*}{p_s} < F \leq F^*$, then $(A2)$ for $D = D^*$ is higher than $(A1)$ for $D = p_s F$.

If $F^* < F \leq A$, then the borrowers choose project r under bond finance. If $F^* < F \leq \frac{D}{p_r}$, then bank profit is equal to $(A1)$. If $\frac{D}{p_r} < F \leq A$, then bank profit is $\frac{n}{p_r F}[-D^2 + \left(p_i R - c_j\right) D]$, which is maximum when monitoring is implemented, $R = R_c^*$ and $D = D^*$. The result in the text follows. As the bank decides to monitor the borrowers and to set $R = R_c^*$ for any F, in the next proofs we focus our attention on the interaction between D and F.

(Proposition 2). Consider the case in which $F \leq F^*$, then remuneration on bonds is $p_s F$. If $D < p_s F$, then the supply of deposits is $n\frac{D}{p_s F}$ and the ex ante profit of each borrower is

$$\mathcal{U}_s = \frac{D}{p_s F}\underline{B} + \left(1 - \frac{D}{p_s F}\right) U^s\left(F\right), \tag{A3}$$

where $\underline{B} = p_s\left(A - R_c^*\right) = \frac{p_s}{p_s - p_r}b$ is the borrowers' ex post profit under bank finance. Each borrower sets F to maximize expression $(A3)$, which can be rewritten as follows: $\mathcal{U}_s = -p_s F + (p_s A + D) - DR_c^*\left(F\right)^{-1}$. By computing first and second derivatives one can obtain $\frac{\partial \mathcal{U}_s}{\partial F} = -p_s + DR_c^*\left(F\right)^{-2}$ and $\frac{\partial^2 \mathcal{U}_s}{\partial^2 F} = -2DR_c^*\left(F\right)^{-3}$. The first derivative is equal to zero for $F_s^*\left(D\right) = \sqrt{\frac{DR_c^*}{p_s}}$. The second derivative is strictly negative, hence $F_s^*\left(D\right)$ is point of maximum. The corresponding value of the profit is $\mathcal{U}_s\left(F_s^*\left(D\right)\right) = p_s A + D - 2\sqrt{p_s R_c^* D}$. On the contrary, when $D \geq p_s F$, all borrowers are financed by loans and profit of each one, $\underline{B}$, does not depend on F. Note that $\frac{\partial \mathcal{U}_s\left(F_s^*(D)\right)}{\partial D} \leq 0 \Leftrightarrow D \leq p_s R_c^*$ and $\mathcal{U}_s\left(F_s^*\left(p_s R_c^*\right)\right) = \underline{B}$. It follows that the borrowers will set $F_s^*\left(D\right)$ for any $D \leq p_s R_c^*$. Note that $F_s^*\left(p_s R_c^*\right) = R_c^* > F^*$. Nevertheless, we are restricting our attention to values of F that do not exceed F^*, i.e., $F_s^*\left(D\right) \leq F^*$. Solving by D the last inequality, we get $D \leq \frac{p_s\left(F^*\right)^2}{R_c^*} = D_s$.

Consider now the case in which $F^* < F \leq A$, hence remuneration on bonds is $p_r F$. If $D < p_r F$, then the supply of deposits is $n\frac{D}{p_r F}$ and the ex ante profit of each borrower is

$$\mathcal{U}_r = \frac{D}{p_r F}\underline{B} + \left(1 - \frac{D}{p_r F}\right) U^r\left(F\right) \tag{A4}$$

Expression $(A4)$, which can be rewritten as $\mathcal{U}_r = -p_r F + (p_r A + B + D) - \frac{(B+\zeta)D}{p_r}\left(F\right)^{-1}$, is maximum for $F_r^*\left(D\right) = \frac{\sqrt{(B+\zeta)D}}{p_r}$, where recall that $B + \zeta = B + p_r A - \frac{p_s}{p_s - p_r}b$. The corresponding value of the profit is $\mathcal{U}_r\left(F_r^*\left(D\right)\right) = p_r A + B + D - 2\sqrt{\left(B + \zeta\right)D}$. When $D \geq p_r F$, all borrowers are financed by loans and profit of each one, $\underline{B}$, does not depend on F. Note that $\frac{\partial \mathcal{U}_r\left(F_r^*(D)\right)}{\partial D} \leq 0 \Leftrightarrow D \leq B + \zeta$ and $\mathcal{U}_r\left(F_r^*\left(B + \zeta\right)\right) = \underline{B}$. It follows that the borrowers will set $F_r^*\left(D\right)$ for any $D \leq B + \zeta$. Note that $F_r^*\left(B + \zeta\right) = \frac{B+\zeta}{p_r} > A$ under Assumption 1. Nevertheless, we are restricting our attention to values of F that belong to $(F^*, A]$. Let $D_r' = \frac{\left(p_r F^*\right)^2}{B+\zeta}$ be such that $F_r^*\left(D_r'\right) = F^*$ and $D_r = \frac{\left(p_r A\right)^2}{B+\zeta}$ be such that $F_r^*\left(D_r\right) = A$. $F_r^*\left(D\right)$ is thus defined in $(D_r', D_r]$ and recall that $F_s^*\left(D\right)$ is defined in $[0, D_s]$. Note that $D_s \geq D_r'$ for any $B \geq B_1$, where $B_1 = \frac{p_r^2}{p_s}R_c^* - \zeta < \underline{B}$. If $D_s < D_r$, then the ranges of $F_s^*\left(D\right)$ and $F_r^*\left(D\right)$ overlap in $(D_r', D_s]$. In such a case the borrowers compare $\mathcal{U}_s\left(F_s^*\left(D\right)\right)$ to $\mathcal{U}_r\left(F_r^*\left(D\right)\right)$: the former is higher for $D < \overline{D} = \frac{\left[(p_s - p_r)A - B\right]^2}{4\left(\sqrt{p_s R_c^*} - \sqrt{B+\zeta}\right)^2}$. If $D_r < \overline{D}$, then the result in the text follows.

(Proposition 3). Consider the following cut-off values of B: $B_2 = D^* - \zeta$ and $B_3 = \frac{(p_r A)^2}{D^*} - \zeta$. Recall $B_0 = (p_s - p_r)[A - F_s^*(D^*)]$, for which $F_s^*(D^*) = F^*(B)$ and $D^* = D_s$ and $B_1 = \frac{p_r^2}{p_s} R_c^* - \zeta$. If $B \geq \underline{B} > B_1$, then $\frac{\partial F_s^*(D)}{\partial D} < \frac{\partial F_r^*(D)}{\partial D}$, i.e., the function $F_r^*(D)$ is steeper than the function $F_s^*(D, B)$, and $D_r(B) < D_s(B)$. If $\frac{D^*}{p_r} < A$, then $B_2 < \underline{B} < B_3$. The inequality $B_3 > \overline{B}$ can be rewritten as $\frac{D^*}{p_r} < \frac{A}{\sqrt{2}}$. It follows that if $\frac{D^*}{p_r} < \frac{A}{\sqrt{2}} (< A)$, then $\max\{B_1, B_2\} < \underline{B} < \overline{B} < B_3$. In such an interval the admissible values of B are higher than B_2 and lower than B_3. Therefore, $\frac{D^*}{p_r} < F_r^*(D^*) \leq A$ and $D_r \geq D^*$, where recall that $D_r = \frac{(p_r A)^2}{B+\zeta}$ is such that $F_r^*(D_r) = A$. We focus on the interval $\frac{D^*}{p_r} < \frac{A}{\sqrt{2}}$ for which three cases must be taken into account: (i) $B_0 < \underline{B}$, (ii) $\underline{B} \leq B_0 < \overline{B}$, (iii) $B_0 \geq \overline{B}$. We provide a numerical example in which $B_0 \in [\underline{B}, \overline{B})$. Let $A = 25$, $b = 2$, $c = 1.9$, $p_s = 0.8$ and $p_r = 0.4$. We get: $\underline{B} = 4$, $\overline{B} = 8.1$, $R_c^* = 20$, $D^* = 7.05$, $\zeta = 6$, $B_1 = -2$, $B_2 = 1.05$, $B_3 = 8.18$ and $B_0 = 4.69$. Moreover we can check that $D_s < D_r < \overline{D}$ for any $B \in [4, 8.1)$ so that results of Proposition 2 hold. Note that for $B \leq B_0$, then $F_s^*(D^*) \leq F^*(B)$ and $D^* \leq D_s(B)$. In Figure 1 we depict both the case of weak moral hazard (when $4 \leq B \leq 4.69$) and severe moral hazard ($4.69 < B < 8.1$): bold lines represent best responses of bank and borrowers in the interval $F \leq F^*$ and grey lines in the interval $F^* < F \leq A$. Condition $\underline{B} \leq B \leq B_0$ is sufficient to state that the Nash equilibrium is $E = (F_s^*(D^*), D^*)$, whereas condition $B_0 < B < \overline{B}$ is sufficient to state that the Nash equilibrium is $E' = (F_r^*(D^*), D^*)$.

(Proposition 4). The market equilibrium is defined by Proposition 3: with weak moral hazard welfare is

$$n[\lambda_s(p_s A - c) + (1 - \lambda_s) p_s A] = n[p_s A - \lambda_s c], \tag{A5}$$

where $\lambda_s = \lambda(D^*, F_s^*(D^*)) > \lambda_s^*$.[5] With severe moral hazard welfare amounts to

$$n[\lambda_r(p_s A - c) + (1 - \lambda_r)(p_r A + B)], \tag{A6}$$

where $\lambda_r = \lambda(D^*, F_r^*(D^*)) > \lambda_s > \lambda_s^*$. Note that conditions $\lambda_r > \lambda_s > \lambda_s^*$ and $p_s A - c > p_r A + B$ imply that $(A6) < (A5) < (11)$. Therefore authorities undo the borrowers' equilibrium choice by increasing the bond rate from $F_s^*(D^*)$ to F^* when moral hazard is weak and by reducing the bond rate from $F_r^*(D^*)$ to F^* when moral hazard is severe. After the restriction loss of the bank is equal to

$$n(\lambda_s - \lambda_s^*) D^*; \tag{A7}$$

[5] When $B = B_0$, then $\lambda_s = \lambda_s^*$ and $(A5) = (11)$. In such a case no monetary intervention is needed.

the borrowers incur a total loss $n\left[\mathcal{U}_s\left(F_s^*\left(D^*\right)\right)-\mathcal{U}_s\left(F^*\right)\right]$ which can be rewritten as

$$n\left[\left(1-\lambda_s\right)p_s\left(A-F_s^*\right)-\left(1-\lambda_s^*\right)p_s\left(A-F^*\right)+\left(\lambda_s-\lambda_s^*\right)\underline{B}\right]; \tag{A8}$$

finally, gain of the households amounts to[6]

$$n\left[\left(1-\lambda_s^*\right)p_sF^*-\left(1-\lambda_s\right)p_sF_s^*-\left(\lambda_s-\lambda_s^*\right)D^*\right]. \tag{A9}$$

Notice that $(A9)-[(A7)+(A8)]\equiv W_*-n\left[p_sA-\lambda_sc\right]$ is equal to

$$n\left(\lambda_s-\lambda_s^*\right)c. \tag{A10}$$

If compensations $(A7)$ and $(A8)$ are given to the bank and to the borrowers, respectively, they are indifferent between the market equilibrium and the equilibrium after the restriction. Moreover, the households are better-off for their profits increase by the amount $(A10)$.

After the expansion the bank incurs loss

$$n\left(\lambda_r-\lambda_s^*\right)D^*; \tag{A11}$$

the borrowers incur loss $n\left[\mathcal{U}_r\left(F_r^*\left(D^*\right)\right)-\mathcal{U}_s\left(F^*\right)\right]$ which can be rewritten as

$$n\left[\left(1-\lambda_r\right)\left(p_r\left(A-F_r^*\right)+B\right)-\left(1-\lambda_s^*\right)p_s\left(A-F^*\right)+\left(\lambda_r-\lambda_s^*\right)\underline{B}\right]; \tag{A12}$$

finally, gain of the households amounts to

$$n\left[\left(1-\lambda_s^*\right)p_sF^*-\left(1-\lambda_r\right)p_rF_r^*-\left(\lambda_r-\lambda_s^*\right)D^*\right]. \tag{A13}$$

Notice that $(A13)>(A9)$ and $(A13)-[(A11)+(A12)]\equiv W_*-n\left[p_sA-\lambda_rc\right]$ is equal to

$$\left(1-\lambda_r\right)\left[\left(p_s-p_r\right)A-B\right]+\left(\lambda_r-\lambda_s^*\right)c. \tag{A14}$$

If compensations $(A11)$ and $(A12)$ are given to the borrowers and to the bank, respectively, they are indifferent between the market equilibrium and the equilibrium after the expansion. Moreover, the households are better-off for their profits increase by the amount $(A14)$. The result in the text follows.

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[6] Note that $(A9)$ is higher than $\left(\lambda_s-\lambda_s^*\right)\left(p_sF^*-D^*\right)$, which is positive for $B\in\left[\underline{B},B_0\right]$.

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