The impact of Union Power on the Optimal Income Tax Schedule*

Mathias HUNGERBÜHLER †
ERMES, Université Panthéon-Assas Paris II
and
IRES, Université Catholique de Louvain
hungerbuehler@ires.ucl.ac.be

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Abstract

We explain the positive correlation between union power and tax progessivity from a normative point of view by integrating labour market frictions and union power in an optimal taxation framework. We find that unions and redistributive taxation are complementary in the sense that they both create inefficiencies that weaken each other. We find that strong unions increase welfare and efficiency when the government faces an adverse selection problem when redistributing income.

Keywords: Optimal Income Taxation, Unions, Matching
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†ERMES - Université Paris II, 12, place du Panthéon, 75230 Paris, France.

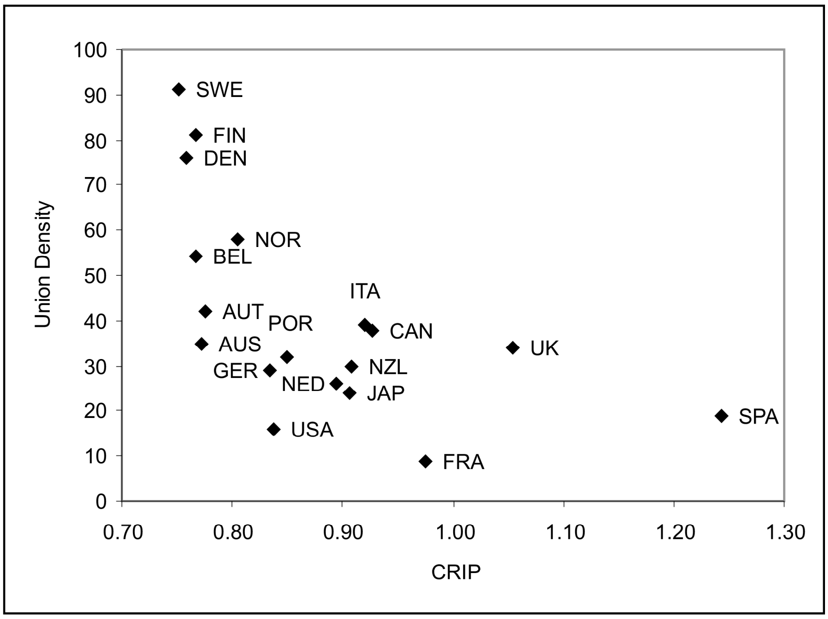

Figure 1: **Union density and tax progressivity**

1 Introduction

In the optimal taxation model of Mirrlees (1971), labour markets are perfectly competitive. Every worker gets his marginal product. This leaves no room for labour market institutions to affect the optimal tax schedule. However, a brief look at raw data (see figure 1[1]) suggests that there exists a positive correlation between union membership and the progressivity of the tax schedule across countries. The present paper tries to explain this correlation from a normative point of view by integrating labour market frictions and union power in an optimal taxation framework.

Unions are often blamed for inefficiencies by setting wages above the competitive level. At the same time, redistributive policies are often blamed for creating inefficiencies by altering the incentives of the workers. Our paper tries to connect these two sources of inefficiency. In a second-best world, it might well be the case that these two sources of inefficiency do not simply add to each other. Especially, as we will show, inefficiencies created by unions might be weakened by high marginal tax rates.

Tax policy affects economic outcomes in several ways and through different

[1] The coefficient of residual income progression (CRIP) measures local tax progressivity, here at 167% of income of an average production worker. Similar results are obtained at other income levels. A high CRIP means low tax progressivity. Union density is union members as part of the working population.
Source: OECD (1997,1998)

channels. Following the article of Mirrlees (1971), much emphasis has been put on the hours-of-work reaction of individuals to tax changes. The results are however rather disappointing, the elasticity of hours worked with respect to the marginal tax rate is estimated to be rather low and often close to zero (see Pencavel, 1986, for a survey). On the other side, the elasticity of income with respect to the marginal tax rate has been shown to be significantly positive (see Blundell and MaCurdy, 1999, for a survey). One way to explain the differences in elasticities is to assume that labour supply consists not only in the choice of hours worked, but also in effort, human capital and occupational choices. Based on the elasticity of income with respect to the marginal tax rate and still assuming perfect labour markets, one can then still use the canonical Mirrlees model to derive the optimal tax schedule, without specifying through which of these channels the tax rate effectively affects outcomes. This might people make believe that the channel has no importance, only the elasticity matters. This is however not true. In a previous paper (Hungerbühler *et al.*, 2003), we have shown that if labour markets are imperfect and tax choices affect wage formation, the optimal tax schedule differs considerably compared to the canonical model of optimal income taxation. It is therefore important to specify exactly how tax changes affect economic outcomes.

For our purpose, we follow the model developed in Hungerbühler *et al.* (2003). Matching frictions in the labour market create rents that are shared between the firm and the workers through a Nash bargain. Tax policy affects the outcomes in two fundamental ways. First, a higher level of taxes decreases the rent and therefore income of the workers. It also increases gross wages and therefore decreases employment. Second, higher marginal tax rates reduce the incentives of the union to claim higher wages. This leads to wage moderation. Unions are assumed to have the same impact as in the canonical right-to-manage union models: They increase wages. Tax policy can then be used to change the effective bargaining strength of unions, as was shown by Hersoug (1984); Lockwood and Manning (1993). More technically, Hungerbühler *et al.* (2003) assume the Hosios condition to be satisfied. While there exist good estimates for the elasticity of the matching function, there are no reliable estimates for the workers' bargaining power. The Hosios condition, that states that the workers' bargaining power equals the elasticity of the matching function, is thus an ad-hoc assumption that has not to be true in the real world. It therefore becomes interesting to extend the framework developed in Hungerbühler *et al.* (2003) to the case where the Hosios condition is not satisfied.

The literature about taxation in matching models has mainly focussed on models with a representative agent. It has been shown (see e.g. Pissarides, 1998; Sørensen, 1999) that tax policy can be used to restore efficiency. These studies however neglect issues of equity between individuals with heterogeneous productivities. On the other side, Hungerbühler *et al.* (2003) examine issues of equity, but assume that the outcome in the *laissez-faire* economy is efficient. Departing form Hungerbühler *et al.* (2003), we do not make this assumption. Taxation can therefore be used to restore efficiency and to increase equity.

Our paper leads to one main result. If unions are strong, then the optimal tax schedule is more progressive than if unions are weak. Labour market institutions can therefore have a considerable effect on the optimal tax schedule. Moreover, we find that the inefficiencies created by unions are partly offset by the adverse selection problem of the government. Welfare is increasing in union power.

Even though labour income is the main income for most households, there are few studies in optimal income taxation that take labour market frictions into account. Our model takes the framework developed by Hungerbühler *et al.* (2003). This study is however also close to the setting of Aronsson and Sjörgen (2001) where unions choose the wage rate. Their model is richer than ours, since the government not only chooses an income tax schedule but also unemployment benefits, the provision of a public good and linear commodity taxation. Moreover, they introduce the intensive hours-of-work margin. This complexity however makes it difficult to derive qualitative results. Our model is different in the sense that we rely on a matching model, which allows us to have an equilibrium model where the behaviour of firms is integrated. Engström (2002) also uses a matching model, but assumes that hourly wages are fixed and income is only affected by the choice of working hours. He finds negative marginal tax rates at the top of the distribution. Another interesting paper that combines imperfect labour markets to the optimal taxation literature has been written by Hariton and Piaser (2004). In their context, there is one monopsonistic firm on the labour market. This firm is however uninformed about the productivity of the worker and offers therefore a menu of wage-working-time contracts to the workers. The model thus includes a two-stage adverse selection problem. They find that the optimal tax schedule has taxes that are decreasing with the income of the individual. These examples show that imperfect labour markets can change dramatically the results obtained in the standard Mirrlees framework. Labour market frictions should thus not be neglected when talking about optimal income taxation. Our paper adds new arguments by integrating union behaviour into this framework.

The structure of the paper is as follows. Section 2 presents the model. In section 3, we derive some analytical results. Section 4 shows numerical simulations and section 5 finally concludes.

2 The Model

2.1 Notations and time setting

The economy lasts for one period.[2] There is a mass 1 of risk-neutral workers. Labour income is their only source of revenue. Workers are heterogeneous in their exogenous ability $a \in [a_0, a_1]$. The workers' abilities are distributed according to the continuous density function $f(.)$ and the corresponding cumulative distribution function $F(.)$. The distribution of abilities is common knowledge, but the ability is private information of the worker. The firm however detects

[2]Our static model simplifies the dynamic version of the matching model but still captures its major mechanisms (see Boone and Bovenberg, 2002).

the ability of a worker through costly screening during the hiring process. Firms are profit-maximising and produce a single good. The number of firms is endogenous. To enter the labour market, a firm has to invest capital κ_a to create a workstation for workers of type a. The complexity of the workstation determines the ability that is needed by the worker to run this workstation. We assume directed search, that is, vacant jobs are advertised with the required ability level and workers only apply for jobs of their ability level. There is perfect competition among firms, such that they enter the labour market as long as expected profits are positive.

Individuals' preferences on consumption and leisure are assumed quasi-linear in consumption. The individual has the binary choice to use all his time endowment as leisure or to search for a job in which case his leisure time is 0.[3] We denote the type-independent utility of leisure by d. A worker in a type-a job creates an output equal to a and receives a gross wage w_a. The government sets a continuously differentiable income tax function $T(.)$ and a welfare benefit b that is given to all non-working individuals. We define the rent x_a that an employed worker gets by $x_a = w_a - T(w_a) - b$. Individuals who enter the labour market but do not find a job get an income equal to the type-independent welfare benefit b. Inactive individuals get the welfare benefit, but also enjoy leisure that they value d.

The timing of events is as follows. First, the government commits to an income taxation scheme and a welfare benefit that is given to all individuals who do not work. Next, workers and firms decide whether to enter the labour market or not. Participation is costly to both the individual and the firm. An individual has to give up his leisure time to search for work. On the other side, firms that want to participate in the labour market have to open a vacancy, install the required equipment and advertise the job opening. This investment is irreversible. Once firms and individuals are on the market, they search for a partner to engage in production. A worker with ability a has to find an open vacancy that requires his productivity. In the next step, the skill-specific union and the firms negotiate the wages. Finally, output is produced and transfers occur. We solve backwards.

2.2 Unions and the wage bargain

Unions are concerned about a wide range of issues. Most of them are not directly related to wage policies of the firm. However, since we are here mainly interested in the determination of wages and employment, we restrain ourselves to these variables as the arguments of the union's objective function. This is in line with most of the literature (see Booth, 1995).

The union bargains with all the firms of one sector about the wage level only. Employment is then determined by the behaviour of firms. A bargain about wage levels and employment would be more efficient, but there is little empirical

[3] This is in line with empirical evidence showing that the intensive hours-of-work margin is small and possibly not significantly different from 0 (see Pencavel, 1986).

evidence that unions negotiate about total employment. In the bargain, the union and the firms are in a bilateral monopoly. To avoid indeterminacy, we assume that the negotiated wage satisfies the solution of the generalized Nash bargain. This can be micro-founded by a game-theoretic approach (Binmore *et al.*, 1986).

In the canonical right-to-manage model of unions, the union has preferences about employment and the wage rate and bargains about the wage with the firm. Due to the free-entry condition for firms and the static nature of our model, we are not able to introduce the canonical model in our framework.[4] In fact, if the bargain takes place before the entry of firms, then firms do not care about wages and employment, since the free-entry condition drives expected profits down to zero anyway. If however the bargain takes place after the entry of firms, then, at the time of the wage bargain, employment is already determined by the free-entry condition and the matching function, and the negotiated wages have no impact on employment any more at this stage. A union that has preferences for employment would then act in the same way as a union that has no preferences for high employment levels, since employment is already fixed at the stage of the wage bargain. We assume that bargaining takes place after workers and firms have matched and the union's preferences are only about the wage rate. The union therefore only cares about insiders and maximises their rents.

Workers with productivity a are represented by the same union. This union is assumed skill-specific, that is, the type-a union only cares of workers with productivity a. The members of union a are therefore all identical. This also implies that the union is only interested in equity among workers with productivity a, but not among workers of different productivities.

Note that the size of the firms is indeterminate in our model. There might be many small firms or one single large firm. It is however important in our context that a small firm can always enter the market, which implies a zero-profit condition for expected profits of the firm. Consequently, the union setting can be viewed in different ways. Our model is compatible with one large firm that negotiates about wages with its workers who are organised in a union. The bargain then takes place at the firm level and the union is firm-specific. Our model is however also consistent with a more corporatist interpretation (Teulings and Hartog, 1998) where workers are employed in small firms but organised in a unique union for a given sector. The union then negotiates wages with the employer's federation and this agreement is applied to all wage contracts in this sector by mandatory extension.

At this stage of the game, the entry costs are sunk. If there is an agreement between the firms and the union on the wage rate, the output is produced and the firm pays the workers the gross wage w_a. In the absence of an agreement, nothing is produced, and the workers only get the welfare benefit b. The rent of the union member is therefore equal to $w_a - T(w_a) - b$, whereas the profit of the firm equals $a - w_a$. As in the canonical union model, we assume that the

[4]The only exception is the case of a monopoly union. As shown in appendix A, this alternative assumption on time setting and union behaviour does not change our results.

wage negotiation takes the form of a Nash bargain

$$\max_{w_a} \left[w_a - T\left(w_a\right) - b\right]^{\beta}\left[a - w_a\right]^{1-\beta}$$

where $\beta \in (0,1)$ denotes the union's relative bargaining power that is assumed to be the same for all types. To simplify notations in what follows, we define the *maximised* Nash product N_a as

$$N_a \equiv \max_{w_a} A^{\frac{1}{\gamma}}\left[w_a - T\left(w_a\right) - b\right]\left[\frac{a - w_a}{\kappa_a}\right]^{\frac{1-\beta}{\beta}} \tag{1}$$

where A and γ are constants of the matching function defined in the next section. This monotonous transformation of the Nash bargain does not change the first-order condition of the maximisation problem and therefore it does not affect our results.

The first-order condition of this maximisation problem leads to the following expression of the gross wage rate:

$$w_a = \frac{\beta\left(1 - T_a'\right)a + (1-\beta)\left(T_a + b\right)}{1 - \beta T_a'} \tag{2}$$

where $T_a \equiv T\left(w_a\right)$ denotes the level of taxes and $T_a' \equiv \partial T(w_a)/\partial w_a$ denotes the marginal tax rate paid by a worker with gross income w_a. This equation can be rewritten as

$$w_a - T_a = b + \frac{\beta\left(1 - T_a'\right)}{1 - \beta T_a'}\left(a - T_a - b\right)$$

This shows the usual intuition in a Nash bargain that the workers get their outside option plus a part of the total surplus created by the match. In this view, the factor $\frac{\beta\left(1-T_a'\right)}{1-\beta T_a'}$ can be seen as the effective relative bargaining *strength* of the union. In the absence of taxation, this bargaining strength is equal to the bargaining power.

The wage equation also gives the intuitive result that an increase in the union's bargaining power leads to higher wages.

The effects of tax policy on wages and unemployment have first been studied by Hersoug (1984) for the case of a monopoly union and by Lockwood and Manning (1993) for the more general case where unions and firms bargain about wages. A higher level of taxes leads to gross wage pressure. An increase in T_a reduces the surplus that can be shared between the firm and the workers. This reduction is however shared between the firm and the worker. This implies that the gross wage w_a increases, by less than the tax increase. On the other side, an increase in the marginal tax rate T_a' decreases the gross wage w_a. The reason of this result lies in the assumption on Nash bargaining. If the marginal tax rate is high, an additional increase in the gross wages results only in a small increase of the workers' net wages. Therefore, the union has less incentives to fight for wage increases and the bargaining *strength* of the union decreases.

2.3 The matching process, participation decisions and employment

We assume that there is a matching function that gives the number of employed individuals as a function of the number of workers searching for a job and the number of firms searching for a worker. This matching function is assumed to represent heterogeneities and frictions that we do not model explicitly. Let U_a denote the number of searching workers of type a and V_a the number of open vacancies for type-a workers. It is usually assumed that the matching function $H(U_a, V_a)$ is increasing in both its arguments, concave and homogeneous of degree 1. Empirical studies have found that a Cobb-Douglas approximation of the matching function fits the data well (see Pissarides, 2000; Petrongolo and Pissarides, 2001). We therefore assume that the number of filled jobs H_a is given by

$$H_a = A\,(U_a)^{\gamma}\,(V_a)^{1-\gamma} \qquad \text{with} \qquad \gamma \in (0,1) \tag{3}$$

where $A > 0$ is a scale parameter of the matching function. If individuals of type a search for work, then $U_a = f\,(a)$, otherwise $U_a = 0$.

To produce a units of labour, the firm has to invest κ_a units of capital, to open a vacancy and find one worker of type a. Since the investment takes place before the matching to the worker, some firms do not find a worker. In that case, the loss of the firm is equal to the investment κ_a. If the firm finds a worker for its vacancy, then they have to bargain about the wage rate and the firm's profit writes $a - w_a - \kappa_a$. Firms enter the market as long as the expected profits are positive. Taking the matching function into account, the expected profit can be written as

$$\frac{H_a}{V_a}\,(a - w_a) - \kappa_a$$

where H_a/V_a denotes the probability that a firm finds a worker of type a. At equilibrium, this expected profit is nil, which implies that the number of vacancies posted by firms equals

$$V_a = H_a \frac{a - w_a}{\kappa_a}$$

Introducing this result into the matching function (3) allows us to write the level of employment L_a for workers of type a as

$$L_a = \frac{H_a}{U_a} = A^{\frac{1}{\gamma}} \left(\frac{a - w_a}{\kappa_a}\right)^{\frac{1-\gamma}{\gamma}} \tag{4}$$

if individuals of type a search for work and 0 otherwise.

An additional firm that enters the market increases employment and therefore *gross* output. But it also increases the resources spent for capital investments. The impact of a new open vacancy on *net* output (net of investment costs) is then ambiguous and depends on the number or vacant jobs that are already on the labour market. If the wage is sufficiently low, the firm has incentives to enter the market, even though this is not optimal from a social point of view.

The gross output generated by workers of type a is equal to $aL_a f(a)$. Let

$$Y_a \equiv aL_a - \frac{V_a \kappa_a}{f(a)}$$

Output net of investment costs on the type-a labour market can then be written as $Y \cdot f(a)$. Using the free-entry condition, one gets $Y_a = w_a L_a$, and can then derive the efficient levels of the wage rate and of employment that maximise net output

$$w_a^* = \gamma a \qquad L_a^* = (1-\gamma)^{\frac{1-\gamma}{\gamma}} A^{\frac{1}{\gamma}} \left(\frac{a}{\kappa_a}\right)^{\frac{1-\gamma}{\gamma}} \tag{5}$$

A worker who is searching for work has the expected utility $L_a [w_a - T(w_a) - b] + b$ which can be rewritten using the definition of the maximised Nash product as

$$N_a \left(\frac{a - w_a}{\kappa_a}\right)^{\frac{1}{\gamma} - \frac{1}{\beta}} + b \tag{6}$$

We assume that κ_a evolves in such a way that this expression is increasing in worker's type a in the second-best solution[5]. A worker decides to search for a job if his expected utility when searching for work is higher than what he gets when he does not search for work, i.e. $b + d$. Our assumption on κ_a implies that there exists a single threshold a_d that separates inactive from searching individuals. All types $a \geq a_d$ search for work, whereas types with $a < a_d$ stay inactive.

2.4 The government's problem

2.4.1 Incentive constraints

Since the government only observes the income of individuals but not their productivity, it faces an adverse selection problem. Therefore, the government has to write a direct taxation contract that leads agents to reveal their ability. The particularity of this problem in our context is that the information revealed through the wage rate is an information that is jointly determined by the workers and the firms. This does however not change the technical tools that can be used to solve the incentive problem and standard techniques apply (see Fudenberg and Tirole, 1991, chapter 7). Using the taxation principle, it is equivalent to design a tax function $T(w)$ or to let the firm-worker pair choose among a menu of allocations (w_a, T_a). To be optimal, the allocations must induce the individual matches to truthfully reveal their type, which is the case for a worker of type a if and only if for all $a' \neq a$

$$\left(\frac{a - w_a}{\kappa_a}\right)^{\frac{1-\beta}{\beta}} (w_a - T_a - b) \geq \left(\frac{a - w_{a'}}{\kappa_a}\right)^{\frac{1-\beta}{\beta}} (w_{a'} - T_{a'} - b)$$

[5] This turns out to be true for our simulation results.

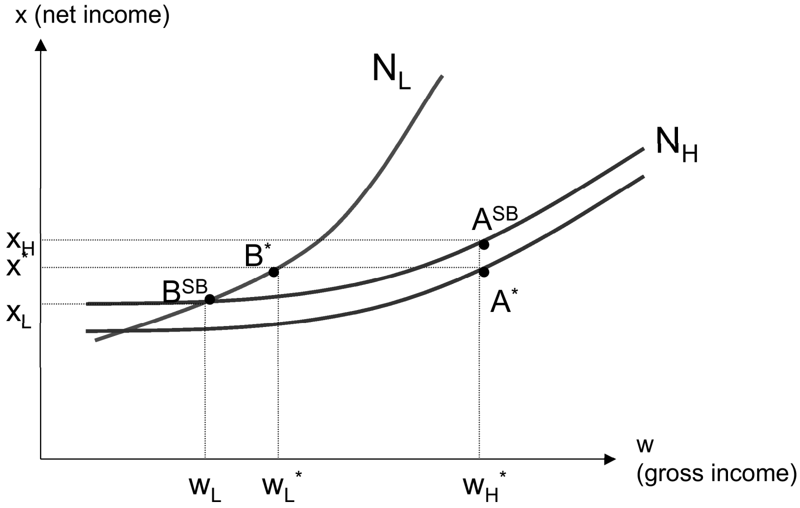

Figure 2: **The incentive problem in the case of two types**

Since at a constant Nash product $[w - T(w) - b]^{\beta} [a - w]^{1-\beta}$, we have that

$$\frac{\partial x}{\partial w} = \frac{1-\beta}{\beta} \frac{x}{a-w}$$

is decreasing in a, the single-crossing property is fulfilled, which allows us to replace the incentive constraint by the first-order condition

$$\dot{N}_a = \frac{1-\beta}{\beta} \left(\frac{1}{a - w_a} - \frac{\dot{\kappa}_a}{\kappa_a} \right) N_a \tag{7}$$

and the second-order condition $\dot{w}_a > 0$[6].

The incentive problem is depicted in figure 2 for the case of two types with $a_H > a_L$. N_H and N_L are the iso-Nash curves for the high-skilled and the low-skilled respectively. Agents prefer bundles that give higher net income and lower gross income. This is because lower gross income implies a lower labour cost supported by firms. Since the government cares about equity, it wants to set a marginal tax rate equal to 1 and give the same net income x^* to all agents. At the same time, the government is concerned about efficiency and therefore optimally sets the respective wage rates at the efficient levels w_L^* and w_H^* by offering contracts A^* to the high-skilled individuals and B^* to the low skilled individuals. However, since the government does not know the ability types, the agents have to self-select the bundle. Having the choice between bundles A^* and B^*, the high-skilled agents prefer the contract B^* designed for the low-skilled since it gives the same net income for a lower gross income. This first-best contract is therefore not incentive compatible. Now consider the

[6]Appendix B shows that this condition is always satisfied given our assumption on κ_a.

contracts A^{SB} and B^{SB}. Both contracts give the same Nash product to the high-skilled agents, such that they are indifferent between these two contracts. The low-skilled still prefer the contract designed for them, in this case B^{SB} to the contract designed for the high-skilled. This contract is thus incentive compatible. Comparing this contract to the first-best contract, there are two major changes. First, the high-skilled agents get an informational rent $(x_H - x^*)$. This has an equity cost for the government because it implies that the high-skilled get higher net incomes than the low-skilled. And second, the wage rate of the low-skilled is distorted downwards from its efficient level. Net output is therefore reduced. The distortion goes downwards, since this reduces the rent that has to be given to the high-skilled agent. An upward distortion of the wage level would even increase the rent of the high-skilled agent. This distortion represents the efficiency cost induced by the incentive problem. Since the wage rate is lower than the optimal level, employment of the low-skilled becomes distorted upwards by equation (4). The reduction in the wage level can be achieved through a high marginal tax rate for the low-skilled by equation (2).

Now consider the impact of union power on this efficiency-equity trade-off. If unions are strong, the wage rate in the *laissez-faire* economy is above its efficient level given by equation (5). An increase in the marginal tax rate of the low-skilled then not only increases the tax that has to be paid by high-skilled individuals and therefore equity, but it also increases efficiency as long as the wage is above its efficient level. The efficiency-equity trade-off does therefore not exist for high union power as long as the marginal tax rate is not too high. On the contrary, if union power is low, efficiency and equity considerations go in opposite directions. The wage rate in the *laissez-faire* economy is below its efficient level. Consider first the case where the *laissez-faire* wage level is between the efficient level w_L^* and the incentive-compatible level w_L. The government then has to decrease the wage rate of the low-skilled to make the taxation scheme incentive compatible. This decrease has however an unambiguous efficiency cost, since it moves the wage rate further away from its efficient level. Consider then the second case, in which the *laissez-faire* wage rate is even below the incentive-compatible level w_L. The government then tries to increase efficiency by increasing the wage rate. To do this, it imposes a negative marginal tax rate. This however decreases the equity of the taxation scheme since it lowers the tax that has to be paid by the high-skilled individuals.

2.4.2 The government's objective and budget constraint

We assume that the government cares only about the distribution of expected utilities. This leaves out the concern about unemployment insurance. In a more realistic dynamic setting, workers would move between unemployment and employment. Capital markets and public insurance could then be used to redistribute among these individuals. Since our setting is static, we consider our model as too poor to deal with these issues. However, we expect that concentrating on expected utilities comes closer to this dynamic idea than differentiating between unemployed and employed individuals. The government therefore only compensates for differences in productivities in our framework and not for differences in labour status. We assume the following objective to the government:

$$\Omega = F(a_d)\,\Phi(b+d) + \int_{a_d}^{a_1} \Phi\left[L_a\left(w_a - T(w_a)\right) + (1-L_a)\,b\right] f(a)\,da$$

where Φ is an increasing and strictly concave function of its argument. Using the definition of the maximised Nash product (1), one can rewrite this objective as

$$\Omega = F(a_d)\,\Phi(b+d) + \int_{a_d}^{a_1} \Phi\left[N_a\left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}} + b\right] f(a)\,da$$

The budget constraint can be written as

$$\int_{a_d}^{a_1} T(w_a)\,L_a f(a)\,da = \left[F(a_d) + \int_{a_d}^{a_1} (1-L_a)\,f(a)\,da\right] b + E$$

where E denotes some exogenous public expenditures. This constraint is equivalent to the resource constraint

$$\int_{a_d}^{a_1} Y_a f(a)\,da = \int_{a_d}^{a_1} N_a\left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}} f(a)\,da + b + E$$

where the left-hand side denotes production and the right-hand side the distribution of the resources.

The government faces therefore the following optimisation problem:

$$\max_{a_d,w_a,N_a,b} F(a_d)\,\Phi(b+d) + \int_{a_d}^{a_1} \Phi\left[N_a\left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}} + b\right] f(a)\,da \quad (8)$$

$$s.t.: \int_{a_d}^{a_1} Y_a f(a)\,da = \int_{a_d}^{a_1} N_a\left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}} f(a)\,da + b + E$$

$$\dot{N}_a = \frac{1-\beta}{\beta}\left(\frac{1}{a-w_a} - \frac{\dot{\kappa}_a}{\kappa_a}\right)N_a$$

$$\begin{cases} N_{a_d}\left(\frac{a_d-w_{a_d}}{\kappa_{a_d}}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}} = d \\ N_{a_d}\left(\frac{a_d-w_{a_d}}{\kappa_{a_d}}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}} \geq d \end{cases} \quad \text{if} \quad \begin{array}{l} a_d > a_0 \\ a_d = a_0 \end{array}$$

3 Properties of the second-best optimum

From the first-order conditions, we get the following expression for the efficiency-equity trade-off (see appendix C.1):

$$\lambda \frac{\partial Y_a}{\partial w_a} f(a) = (\lambda - \Phi'_a) \left(\frac{1}{\beta} - \frac{1}{\gamma} \right) \frac{1}{a - w_a} \cdot N_a \left(\frac{a - w_a}{\kappa_a} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} f(a) + \frac{1-\beta}{\beta} \frac{1}{(a - w_a)^2} \int_a^{a_1} (\lambda - \Phi'_t) \left(\frac{t - w_t}{\kappa_t} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} N_t f(t)\, dt \tag{9}$$

where λ denotes the shadow cost of public funds and Φ'_a equals $\Phi' \left[N_a \left(\frac{a - w_a}{\kappa_a} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} + b \right]$ for $a \geq a_d$ and $\Phi'(b + d)$ for $a < a_d$. Φ'_a is therefore weakly decreasing in a.

Consider the optimisation problem for agents of type a. The Nash product N_a for this type is fixed by the Nash product of types $a' < a$ and the first-order incentive compatibility constraint (7).

An increase in the wage rate w_a implies that less firms enter the market, which has two consequences. First, it decreases employment and therefore gross output. But it also decreases the resources used for investments in capital to build workstations. The effect on output net of investment costs is therefore ambiguous. If $w_a < w_a^*$ (resp. $>$), the total effect is positive (resp. negative). This efficiency effect is represented by the term on the left-hand side of (9).

Consider next the first term on the right-hand side. In our model, the expected surplus that the government focuses on does not coincide with the Nash product that the firms and the union maximise. The relation between the two is given in equation (6). For a given maximised Nash product N_a, a change in the wage w_a has therefore also an impact on the expected surplus. If $\beta > \gamma$, an increase in the wage rate decreases the expected surplus given to agents of type a. The intuition is depicted in figure 3. Agents prefer bundles that give high net income and low gross income (and therefore high employment). Since the bargaining power of the union is high, the Nash product puts much importance on net income compared to gross income. This implies that the iso-Nash curves are relatively flat. Especially, the iso-Nash curve is flatter than the corresponding iso-expected-surplus curve. When the employment level decreases by an increase in gross wages from w_1 to w_2, the net income has to rise by a small amount only, from x_1 to x_2 to give the same Nash product as before to this firm-worker match. However, in terms of expected surplus, this relatively small increase in net income does not compensate for the employment loss due to the relatively strong increase in gross wages. The expected surplus therefore decreases. This decrease is valued at the marginal social utility of type a, namely Φ'_a. Since the government has less resources to give to agents of type a, more resources can be affected to redistribution toward other individuals. This

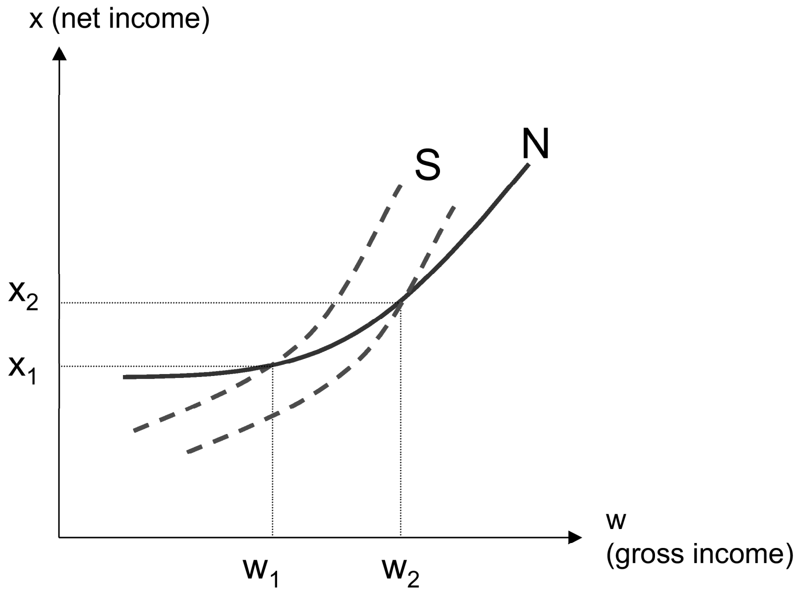

Figure 3: **The relation between the expected surplus (S) and the Nash product (N) for the case $\beta > \gamma$**

increase in budgetary funds is valued at the marginal cost of public funds λ. An increase in the wage rate is therefore desirable if type a is a high-skilled type that gets a higher expected surplus than the low-skilled type because of the first-order incentive constraint. Taking resources away from this high-skilled type is socially valuable because it allows to redistribute more towards other individuals. The contrary is true if type a is a low-skilled type towards whom the government wants to redistribute. Taking away resources from him by increasing his wage is socially not desirable.

The inverse reasoning applies when $\gamma > \beta$. Then, for a given maximised Nash product N_a, an increase in the wage rate increases the expected surplus given to agents of type a. In terms of figure 3, the iso-Nash curve is now steeper than the iso-expected-surplus curve. The increase in expected surplus is desirable if type a is a low-skilled agent to whom the government wants to redistribute resources. It is however not desirable for high-skilled agents from whom the government wants to take resources away to redistribute them to other individuals.

Finally, the second term on the right-hand side represents the impact on informational rents of a higher gross wage for type-a workers. When firm-worker matches endowed with productivity a earn higher gross wages (while keeping the Nash product N_a fixed), more productive firm-worker matches find it more attractive to mimic them. To prevent this, the maximised Nash product accruing to more productive matches has to grow. The term in front of the integral measures the rate at which the growth rate of the worker's maximised Nash product has to increase to prevent slightly more productive matches from mimicking the type a match. Neglecting second-order effects, the incentive compatibility constraints will remain satisfied above a if all matches with a productivity higher than a benefit from an equivalent relative increase in their Nash product. The integral corresponds to the shadow cost of a relative increase in the Nash product of more productive workers. For any type-t above a, this marginal relative increase leads to an absolute rise in the expected surplus equal to $N_t \left(\frac{t-w_t}{\kappa_t}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}}$ times the relative increase in the Nash product. This additional expected surplus generates an increase in the social welfare measured by Φ'_t, but implies a budgetary cost equal to λ.

Proposition 1 *If $\beta > \gamma$ (resp. $\beta < \gamma$), employment is below (resp. above) its efficient level at the top of the productivity distribution.*

Proof. See appendix C.2. ∎

At the top of the distribution, a change in the wage rate has no impact on the informational rents. Since there is too much importance of the wage rate in the wage negotiations between the firm and the union if $\beta > \gamma$, increasing the wage rate above its efficient level decreases the expected utility of the workers with ability a_1. At a given Nash product, this implies that less resources have to be given to individuals with the highest productivity. These resources have a budgetary value of λ which is higher than the marginal utility Φ'_a. It is therefore optimal to decrease employment below its efficient level. The inverse reasoning applies for $\beta < \gamma$.

This proposition is in strong contrast to the usual optimal income tax literature (and even more generally in contract theory) where one gets a no-distortion-at-the-top result. This difference comes from the fact that the government is interested in redistribution of expected surplus, which is not what the agents maximise. In the traditional literature, the individual maximises utility, and the government wants to redistribute to equalize utilities among agents as much as possible. There is therefore no difference between what the agents maximise and what the government wants to redistribute. The same reasoning also applies to our model in the case when $\beta = \gamma$. In fact, if $\beta = \gamma$, the maximised Nash product equals expected utility, and one gets the usual no-distortion-at-the-top result (see also Hungerbühler *et al.*, 2003).

Proposition 2 *If $\beta > \gamma$, employment is above its efficient level for all individuals whose marginal social utility is above the marginal cost of public funds.*

Proof. See appendix C.3. ∎

An increase in the wage rate increases the distributional cost since all the workers with higher ability than a get more informational rents. In terms of equation (9), this implies that the second term on the right-hand side is positive. Moreover, since there is too much importance of the wage rate in the wage negotiations between the firm and the union if $\beta > \gamma$, increasing the wage rate above its efficient level decreases the expected utility of the workers with ability a. At a given Nash product, this implies that less resources have to be given to individuals with productivity a. These resources have a budgetary value of λ which is lower for these individuals than the marginal utility Φ'_a. In terms of equation (9) this is equivalent to saying that the first term on the right-hand side is positive when $\beta > \gamma$ and $\Phi'_a > \lambda$. It is therefore optimal to incur some efficiency cost and to increase employment above its efficient level for skill levels with $\Phi'_a > \lambda$.

We cannot derive analytical results for employment levels of agents with $\Phi'_a < \lambda$. We however know from proposition 1 that employment is below its efficient level at the top of the ability distribution. Proposition 2 tells us that employment is above its efficient level at the bottom part of the ability distribution. For the special case where vacancy costs κ_a are assumed linear in productivity a, the efficient levels of employment given by (5) are independent of a, and we intuitively expect in the case of linear vacancy costs to have employment levels that are decreasing with agent's productivity.

Proposition 3 *If $\gamma > \beta$, employment is above its efficient level for all individuals whose marginal social utility is below the marginal cost of public funds.*

Proof. See appendix C.3. ■

An increase in the wage rate increases the distributional cost since all the workers with higher ability than a get more informational rents. In terms of equation (9), this implies that the second term on the right-hand side is positive. Moreover, since there is too low importance of the wage rate in the wage negotiations between the firm and the union if $\beta > \gamma$, increasing the wage rate above its efficient level increases the expected utility of the workers with ability a. At a given Nash product, this implies that more resources have to be given to individuals with productivity a. These resources have a budgetary cost of λ which is higher for these individuals than the marginal utility Φ'_a. In terms of equation (9) this is equivalent to say that the first term on the right-hand side is positive when $\beta < \gamma$ and $\Phi'_a < \lambda$. It is therefore optimal to incur some efficiency cost and to increase employment above its efficient level for skill levels with $\Phi'_a < \lambda$.

We do not find analytical results for employment levels of agents with $\Phi'_a > \lambda$. On the one hand, the second term on the right-hand side of equation (9) is still positive, but the first term becomes negative.

Proposition 4 *In-work benefits (if any) are lower than assistance benefits.*

Proof. See appendix C.4. ■

Individuals of type a_d are indifferent between participating in the labour market or not. From the social point of view, their participation however induces a cost, since it allows agents with a productivity above a_d to mimic them.

As a consequence, the government has to give informational rents to these individuals. Participation is therefore only optimal, if the government can save some resources by giving them lower in-work benefits than assistance benefits.

4 Simulations

4.1 Calibration

The individuals' abilities are distributed on the support $[a_0 = 1000\ , a_1 = 10\ 000]$. This might be a rather realistic approximation of workers' productivities per month, measured in Euros. We use a truncated log-normal density function of the form:

$$f(a) = \frac{K}{a} \exp\left(\frac{\log a - \log(\mu \cdot a_1 + (1-\mu) a_0)}{2 \cdot \xi^2} \right)$$

where K is the appropriate scale parameter. This form is typical in the literature (Mirrlees, 1971; Tuomala, 1990; Boadway *et al.*, 2000). The parameters of the distribution function are chosen equal to $\mu = 0.5$ and $\xi = 0.7$.

The elasticity of the matching function γ is set at 0.5. This corresponds to the average estimates in empirical models (see Petrongolo and Pissarides, 2001). Furthermore, we assume that vacancy costs are proportional to productivity. This assumption is usual in equilibrium search models (see Pissarides, 2000), even though empirical support is missing. The vacancy cost κ_a and the scale parameter A are adjusted to get an efficient employment level of 0.65 for all types. In the benchmark case, we assume that the value of leisure is set at the income that the lowest-skilled worker would get in the *laissez-faire* economy when $\beta = \gamma = 0.5$. This value is equal to 327. The government's expenditures E are set equal to 0. Finally, the government's utility function is assumed to be a CES function of the expected surplus. We have, $\Phi(\Omega) = \ln \Omega$. This corresponds to the basic parameterisation in Saez (2002). For the union power, we choose levels of 0.4, 0.5 and 0.6 for β to check its impact on the optimal tax schedules. On the following graphs, we use the dotted line for $\beta = 0.4$, the solid line for $\beta = 0.5$ and the dot-dashed line for $\beta = 0.6$. This also implies that with $\beta = 0.5$, the Hosios condition is satisfied and we are back in the framework of the previous Hungerbühler *et al.* (2003).

4.2 Simulation results

Figure 4 shows the employment levels as a function of the individuals' abilities. The efficient employment level is given by the horizontal dashed line. Employment is distorted upwards above its efficient level for most types as we expected it from the incentive constraints. Even the region at the top of the distribution where proposition 1 predicts employment below its efficient level for high union power, is small and the downward distortions are relatively small at the second-best equilibrium. Employment is higher when union power is low. This

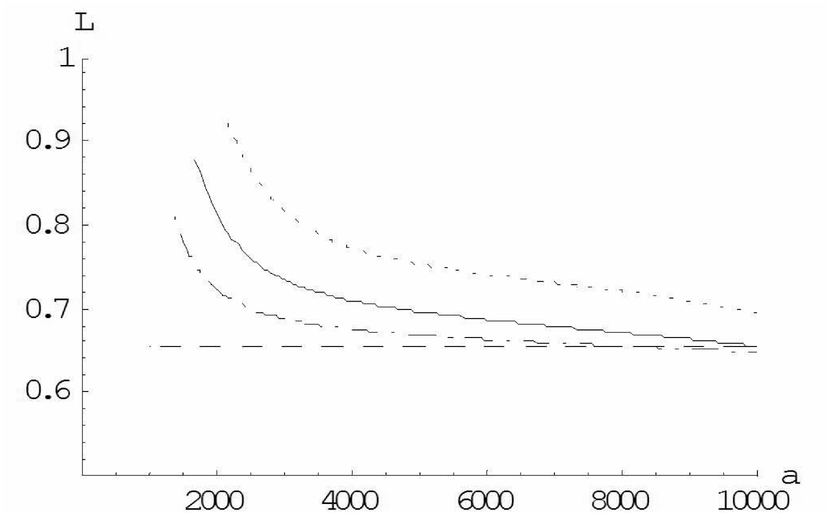

Figure 4: **Employment levels as a function of ability**. Dotted line for $\beta = 0.4$, solid line for $\beta = 0.5$, dot-dashed line for $\beta = 0.6$ and dashed line for efficient employment levels.

is intuitive, since low union power implies that the wage rate in the *laissez-faire* economy is too low and employment therefore too high at the second-best equilibrium. Employment is much closer to its efficient level when union power is high, implying that the efficiency costs are smaller when unions are strong. In fact, figure 2 shows that the incentive constraint pushes wages downwards below their efficient level. An increase in union power β, diminishes this effect. For all levels of union power, employment is a strictly decreasing function of ability. This is first due to our assumption that vacancy costs are linear in ability. If we assumed vacancy costs that are concave in abilities, i.e. $\frac{\dot{\kappa}_a}{\kappa_a} < 1$, the efficient employment level would be increasing in ability (whereas it is constant for linear vacancy costs). The second reason for higher distortions at the bottom of the productivity distribution comes from the fact that distortions are less costly for individuals with low productivities who anyway do not contribute much to net production.

Figures 5 and 6 show the optimal tax schedule and the optimal marginal tax rates. Marginal tax rates are increasing when the bargaining power of the union is above its efficient level, almost linear if it equals the efficient level and decreasing if union power is lower than the efficient bargaining power.

Marginal tax rates are higher when unions are strong, except at the bottom of the ability distribution. This confirms our intuition that the equity-efficiency trade-off is more severe if union power is low. If unions are strong, setting a positive marginal tax rate increases both efficiency and equity, whereas it decreases efficiency when unions are weak.

However, at the bottom of the distribution, marginal tax rates seem to be

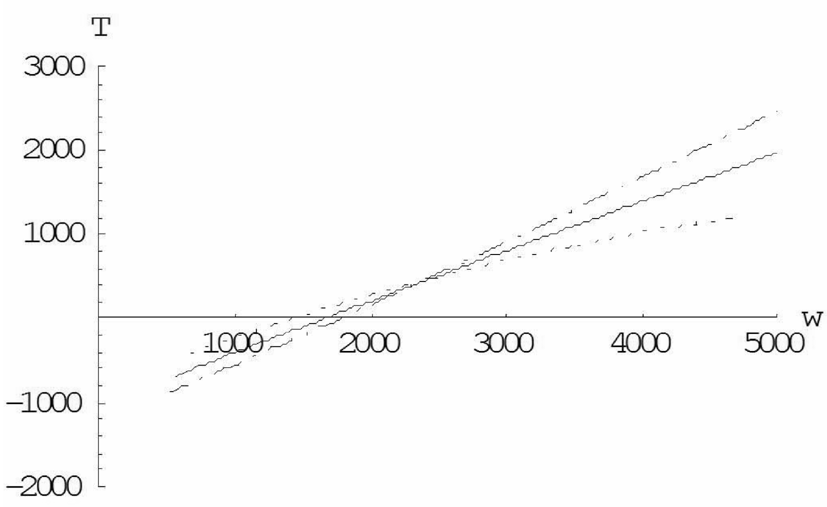

Figure 5: **Optimal tax schedules**. Dotted line for $\beta = 0.4$, solid line for $\beta = 0.5$ and dot-dashed line for $\beta = 0.6$.

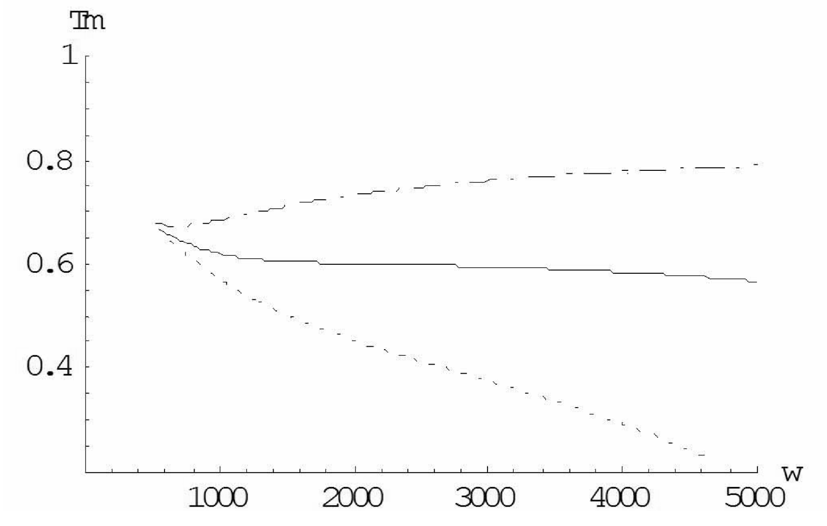

Figure 6: **Optimal marginal tax rates**. Dotted line for $\beta = 0.4$,solid line for $\beta = 0.5$ and dot-dashed line for $\beta = 0.6$.

Table 1: **Numerical results for different union power**

β	L[a]	Y[b]	$\Phi^{-1}(\Omega)$[c]	b	Participation
0.4	0.691	1529	1506	724	90.2%
0.5	0.677	1590	1578	863	95.5%
0.6	0.664	1611	1606	985	98.1%

[a] Employment as a fraction of total population
[b] Net output
[c] Certainty-equivalent of welfare

almost independent of union power. This is mainly due to two reasons. First, efficiency distortions at the bottom of the ability distribution are not very costly since these individuals do not contribute much to net output. The implications on equity are however strongest at the bottom of the distribution. Therefore, even for low union power, the government sets high marginal tax rates. Second, a negative tax decreases the wage levels through the bargaining process. Taxes at the bottom are however lower (i.e. more negative) when the union is strong. This gives a downward distortion on wages, and a higher marginal tax rate would distort this even more. Since taxes are lower when unions are strong, this mechanism is more important when union power is high and the government avoids to set too high marginal tax rates when unions are strong.

Finally, table 1 shows some aggregate values for the economy.

Participation at the second-best equilibrium is increasing with union power. As the optimal tax schemes in figure 5 show, in-work benefits are higher when unions are strong. The optimal tax schedule subisdises low-skilled jobs therefore more when union power is high. This also increases the incentives of the low-skilled individuals to participate in the labour market. Put in another way, high union power implies higher redistribution and therefore a lower informational rent given to high-skilled individuals. When agents of type a_d then enter the market, this implies a lower equity cost. The participation effect however does not dominate the upward distortions for employment shown in figure 4, such that total employment at the second-best equilibrium in the economy is slightly decreasing in union power.

Next, net output at the second-best equilibrium is increasing in union power. This confirms our intuition, that the efficiency-equity trade-off is more severe when unions are weak. High marginal tax rates create high inefficiencies when unions are weak, whereas they restore efficiency up to a certain degree when unions are strong. This is in line with figure 4, where it is shown that employment distortions are higher when unions are weak. Moreover, the increased participation for high union power has a positive impact on net output.

Social welfare (measured by its certainty equivalent $\Phi^{-1}(\Omega)$) at the second-best equilibrium is increasing in union power. This is due to the fact that both net output and redistribution increase with union power.

The welfare benefit b is increasing in union power. Since net output is increasing in union power, there are more resources available for redistribution.

Finally, one can ask who profits from strong unions. Figure 7 shows the

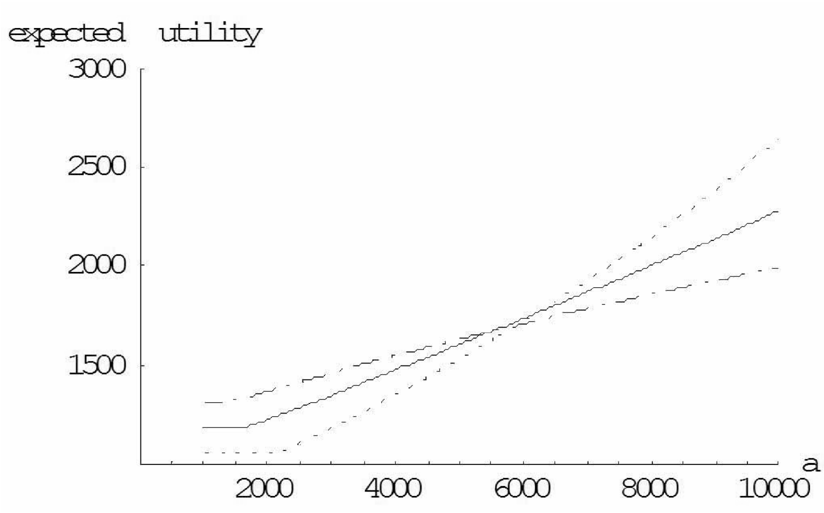

Figure 7: **Expected utilities as a function of ability**. Dotted line for $\beta = 0.4$, solid line for $\beta = 0.5$ and dot-dashed line for $\beta = 0.6$.

expected utilities as a function of ability. Since the tax system is more redistributive when unions are strong, it is intuitive that high-skilled individuals loose when unions become stronger, whereas low-skilled win. Our simulations show a majority support for high union power.

5 Conclusion

Unions are often blamed for setting wages too high and therefore creating inefficiencies. This is also true in our model. However, when an imperfectly informed government is concerned with redistribution, it faces an adverse selection problem. Dealing with that problem creates other inefficiencies. If unions are strong, these inefficiencies partly outweigh each other. By setting high marginal tax rates, the government increases redistribution and therefore equity but also decreases wages. It is then useful that unions set wages too high in a *laissez-faire* economy. In other words, unions create "good" inefficiencies in our model. This mechanism does not work if unions are weak. Redistribution then necessarily increases the already existent inefficiencies. Our model can thus explain the correlation between union power and marginal tax rates.

Our normative analysis might however not be the only way to explain this correlation. If we assume that low-skilled individuals are more likely to join unions than the high-skilled, political economy can give another explanation. In fact, if the income distribution is more concentrated on the bottom of the ability distribution, more individuals join unions and this makes unions more powerful. On the other hand, the concentration of individuals at the bottom of the income distribution implies that the median income is low, and a usual Meltzer and Richard (1981) argument can be used to explain why redistribution is high.

The main conclusion of this paper is that labour market frictions and labour market institutions matter when one designs the optimal taxation of labour incomes. Since for most individuals, labour income is the main income in their household, a theory of optimal income taxation should not neglect labour market theories. It would however be premature to conclude that the tax schedules depicted by our simulations should be implemented in policy. Too many questions are still open. Especially, even if the Nash bargaining solution is often used for wage negotiations, there is no general agreement among labour economists about how wages are set in the economy. Labour market institutions may also affect the wage setting mechanism. These issues are left for further research.

Appendix A Alternative timing and monopoly unions

In this model, we assume that the wage is set by monopoly unions before the entry of firms to the market. It has become common to assume that the union's objective function about employment and wages are represented by a Stone-Geary functional form

$$G\left(L_a, w_a\right) = \left[w_a - T(w_a) - b\right]^{\alpha} \left[L_a\right]^{1-\alpha}$$

where $\alpha \in [0.5, 1)$ denotes the union's preferences for wages. There are two polar cases. If $\alpha = 0.5$, the union is maximising the total rent of all individuals of type a that participate in the labour market. If $\alpha = 1$, the union only cares about wages and not about the employment level. This second polar case is however not realistic in our environment of a monopoly union, since this would lead unions to increase wages to infinity.

Since employment is determined by the matching function and the entry of firms, employment is given as before by the equation

$$L_a = A^{\frac{1}{\gamma}} \left(\frac{a - w_a}{\kappa_a}\right)^{\frac{1-\gamma}{\gamma}}$$

The problem of the union can then be rewritten as

$$\max_{w_a} \; \left[w_a - T(w_a) - b\right]^{\alpha} \left[A^{\frac{1}{\gamma}} \left(\frac{a - w_a}{\kappa_a}\right)^{\frac{1-\gamma}{\gamma}}\right]^{1-\alpha}$$

This is a monotonous transformation of the optimisation problem in the benchmark model and it therefore leads to the same first-order conditions.

Appendix B The second-order IC constraint

We made the assumption that κ_a evolves such that the expected surplus $N_a\left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}}$ is increasing in a. Then, if a given type a participates in the labour market, all individuals with productivity $a' > a$ participate as well. That especially implies that all types $a > a_d$ participate.

We show by contradiction that bunching is not possible in our framework.

Assume that for participating individuals on the interval $I \subset [a_d, a_1]$ there is bunching and one has $\dot{w}_a = 0$, that is, the second order incentive compatibility constraint is binding.

Differentiating the wage equation (2), one gets

$$\dot{w}_a = \frac{1 - T'(w_a)}{\frac{1}{\beta}[1 - T(w_a)] + T''(w_a)(a - w_a)}$$

where $T''(w_a)$ denotes the second derivative of the tax function with respect to the wage w_a. Therefore, $\dot{w}_a = 0$ if and only if $T'(w_a) = 1$. However, putting a marginal tax rate of 1 back into the wage equation (2) gives

$$w_a = T(w_a) + b$$

Introducing this result into the Nash product (1) gives $N = 0$, and therefore the expected surplus $N_a\left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}}$ equals 0. However, type a only participates if his expected surplus is higher than the value of leisure $d > 0$. Therefore, the types on the bunching interval I do not participate in the labour market, which contradicts our assumption that $I \subset [a_d, a_1]$.

Appendix C Proofs and formulas

C.1 First-order conditions of the optimisation problem

We solve problem (8) in two steps. First we solve it for given b and a_d. Second we choose the optimal values of b and a_d. Given b and a_d, we define the Hamiltonian as:

$$\begin{aligned} \mathbb{H}_a &= \Phi\left[N_a\left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}} + b\right] f(a) \\ &+ \left[\lambda Y_a(w_a) - \lambda N_a\left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}}\right] f(a) \\ &+ q_a \frac{1-\beta}{\beta}\left(\frac{1}{a-w_a} - \frac{\dot{\kappa}_a}{\kappa_a}\right) N_a \end{aligned} \qquad (10)$$

where λ is the Lagrange multiplier of the budget constraint and q is the co-state variable. The necessary conditions are:

$$\begin{aligned} 0 &= \lambda \frac{\partial Y_a}{\partial w_a} f(a) + q_a N_a \frac{1-\beta}{\beta} \frac{1}{(a-w_a)^2} \\ &\quad + (\lambda - \Phi'_a) \left(\frac{1}{\gamma} - \frac{1}{\beta}\right) \frac{1}{a-w_a} N_a \left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}} \end{aligned} \qquad (w_a)$$

The co-state variable evolves according to

$$-\dot{q}_a = \{\Phi'_a - \lambda\} \left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}} f(a) + q_a \frac{\dot{N}_a}{N_a} \qquad (N_a)$$

and the transversality conditions are:

$$q_{a_d} \left[N_{a_d} \left(\frac{a-w_{a_d}}{\kappa_{a_d}}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}} - d \right] = 0 \qquad q_{a_1} = 0$$

As usual, q_a is the shadow cost in terms of the social welfare of a marginal increase of N_a. Let $Z_a = q_a N_a$. The condition over N_a implies:

$$-\dot{Z}_a = \{\Phi'_a - \lambda\} N_a \left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}} f(a)$$

So, together with the transversality condition:

$$Z_a = \int_a^{a_1} \{\Phi'_t - \lambda\} N_a \left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}} f(t)\, dt \qquad (11)$$

Since $Z_a \cdot \frac{dN_a}{N_a} = q_a \cdot dN_a$, Z_a stands for the shadow cost of a relative marginal increase of N_a.

The first order condition with respect to w_a can be written as

$$\begin{aligned} \lambda \frac{\partial Y_a}{\partial w_a} f(a) &= (\lambda - \Phi'_a) \left(\frac{1}{\beta} - \frac{1}{\gamma}\right) \frac{1}{a-w_a} N_a \left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1}{\gamma}-\frac{1}{\beta}} \\ &\quad - Z_a \frac{1-\beta}{\beta} \frac{1}{(a-w_a)^2} \end{aligned}$$

which, together with the expression for Z_a gives (9). Furthermore, from the definition of Y_a, we get

$$\frac{\partial Y_a}{\partial w_a} = A^{\frac{1}{\gamma}} \frac{\gamma a - w_a}{\gamma} (a-w_a)^{\frac{1}{\gamma}-2} \kappa_a^{\frac{\gamma-1}{y}} \qquad (12)$$

Furthermore, the conditions with respect to b and a_d write

$$\int_{a_0}^{a_1} (\Phi'_a - \lambda) f(a)\, da = 0 \qquad (13)$$

$$\Phi(b+d) f(a_d) - \mathbb{H}_{a_d} \leq 0 \qquad \text{with} = \text{if } a_d > a_0 \qquad (14)$$

C.2 Proof of proposition 1

The transversality condition $q_{a_1} = 0$ implies that equation (9) can be simplified to

$$\lambda \frac{\partial Y_a}{\partial w_a} f(a) = (\lambda - \Phi'_a) \left(\frac{1}{\beta} - \frac{1}{\gamma} \right) \frac{1}{a - w_a} N_a \left(\frac{a - w_a}{\kappa_a} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} f(a) \tag{15}$$

From our assumption that κ_a evolves such that $N_a \left(\frac{a-w_a}{\kappa_a} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}}$ is increasing in a, we obtain that Φ'_a is decreasing in a. Equation (13) implies that there exists a unique $\hat{a}$ such that $\Phi'_{\hat{a}} = \lambda$. For $a_1 > \hat{a}$, we get $\lambda - \Phi'_{a_1} > 0$. The right-hand side of (15) is therefore positive if $\gamma > \beta$. Therefore, $\frac{\partial Y_a}{\partial w_a} > 0$ and from (12) we have overemployment at a_1. If $\beta > \gamma$, the right-hand side of (15) is negative, which implies $\frac{\partial Y_a}{\partial w_a} < 0$ and employment is below its efficient level at a_1 by (12).

C.3 Proof of propositions 2 and 3

From our assumption that κ_a evolves such that $N_a \left(\frac{a-w_a}{\kappa_a} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}}$ is increasing in a, we obtain that Φ'_a is decreasing in a. Equation (13) implies that there exists a unique $\hat{a}$ such that $\Phi'_{\hat{a}} = \lambda$. For $t < \hat{a}$, we get $\Phi'_t - \lambda > 0$ and $N_t \left(\frac{a-w_t}{\kappa_t} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} < N_{\hat{a}} \left(\frac{a-w_{\hat{a}}}{\kappa_{\hat{a}}} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}}$ and for $t > \hat{a}$, we get $\Phi'_t - \lambda < 0$ and $N_t \left(\frac{a-w_t}{\kappa_t} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} > N_{\hat{a}} \left(\frac{a-w_{\hat{a}}}{\kappa_{\hat{a}}} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}}$. Therefore, for any $t \neq \hat{a}$, we have $(\Phi'_t - \lambda) N_t \left(\frac{a-w_t}{\kappa_t} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} < (\Phi'_t - \lambda) N_{\hat{a}} \left(\frac{a-w_{\hat{a}}}{\kappa_{\hat{a}}} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}}$. Using this inequality and equations (13) and (11), we obtain

$$\begin{aligned}
Z_a &= \int_a^{a_1} (\Phi'_t - \lambda) N_t \left(\frac{a - w_t}{\kappa_t} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} f(t) dt \\
&< \int_a^{a_1} (\Phi'_t - \lambda) N_{\hat{a}} \left(\frac{a - w_{\hat{a}}}{\kappa_{\hat{a}}} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} f(t) dt \\
&< N_{\hat{a}} \left(\frac{a - w_{\hat{a}}}{\kappa_{\hat{a}}} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} \left[\int_a^{a_1} \Phi'_t f(t) dt - \lambda(1 - F(a)) \right] \\
&< N_{\hat{a}} \left(\frac{a - w_{\hat{a}}}{\kappa_{\hat{a}}} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} (1 - F(a)) \left\{ \mathbb{E}_f \left[\lneqq'_t | t \geq a \right] - \lambda \right\} \\
&< N_{\hat{a}} \left(\frac{a - w_{\hat{a}}}{\kappa_{\hat{a}}} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} (1 - F(a)) \left\{ \mathbb{E}_f \left[\lneqq'_t | t \geq a \right] - \mathbb{E}_f \left[\lneqq'_t | t \geq a_0 \right] \right\}
\end{aligned}$$

by equation (13). Therefore, Z_a is negative for all $a < a_1$ because $\lneqq'_t$ is decreasing with respect to the ability. This implies that the second term on the right-hand side of (9) is positive.

If $\beta > \gamma$ and $a < \hat{a}$ such that $\Phi'_a - \lambda > 0$, the first term on the right-hand side of (9) is positive and we get $\frac{\partial Y_a}{\partial w_a} > 0$ from (9) which implies overemployment for all $a_d \leq a \leq \hat{a}$ by (12).

If $\beta < \gamma$ and $a > \hat{a}$ such that $\Phi'_t - \lambda < 0$, the first term on the right-hand side of (9) is positive and we get $\frac{\partial Y_a}{\partial w_a} > 0$ from (9) which implies overemployment for all $\hat{a} \leq a < a_1$ by (12).

C.4 Proof of proposition 4

The first-order condition on a_d (14) can be written

$$
\begin{aligned}
0 \quad \geq \quad & \Phi\left(b+d\right) f\left(a_d\right) - q_{a_d} \frac{1-\beta}{\beta} \left(\frac{1}{a_d - w_{a_d}} - \frac{\dot{\kappa}_{a_d}}{\kappa_{a_d}} \right) N_{a_d} \\
& -\Phi \left[N_{a_d} \left(\frac{a_d - w_{a_d}}{\kappa_{a_d}} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} + b \right] f\left(a_d\right) \\
& + \left[\lambda Y_{a_d}\left(w_{a_d}\right) - \lambda N_{a_d} \left(\frac{a_d - w_{a_d}}{\kappa_{a_d}} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} \right] f\left(a_d\right)
\end{aligned}
$$

Since Z_a is always negative for $a < a_1$, the transversality condition on a_d implies that $N_{a_d} \left(\frac{a_d - w_{a_d}}{\kappa_{a_d}} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} = d$. The previous equation then simplifies to

$$
Y_{a_d}\left(w_{a_d}\right) - N_{a_d} \left(\frac{a_d - w_{a_d}}{\kappa_{a_d}} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} \geq -Z_{a_d} \frac{\dot{N}_{a_d}}{N_{a_d}} \frac{1}{\lambda} \frac{1}{f\left(a_d\right)} > 0
$$

Since $Y_{a_d} = L_{a_d} w_{a_d}$ and $N_{a_d} \left(\frac{a_d - w_{a_d}}{\kappa_{a_d}} \right)^{\frac{1}{\gamma} - \frac{1}{\beta}} = L_{a_d}\left(w_{a_d} - T_{a_d} - b\right)$, this equation implies $T_{a_d} + b > 0$.

References

Aronsson, T. and T. Sjörgen, 2001. "Income Taxation, Commodity Taxation and Provision of Public Goods under Labor Market Distortions." Umeå economic studies 568, Umeå Univerity, Umeå.

Binmore, K., A. Rubinstein and A. Wollinsky, 1986. "The Nash Bargaining Solution in Economic Modelling." *RAND Journal of Economics* 17:176–188.

Blundell, R. and T. MaCurdy, 1999. "Labor Supply: A Review of Alternative Approaches." In O. Ashenfelter and D. Card (eds.), "Handbook of Labor Economics," North-Holland, Amsterdam.

Boadway, R., K. Cuff and M. Marchand, 2000. "Optimal Income Taxation with Quasi-Linear Preferences Revisited." *Journal of Public Economic Theory* 2:435–460.

Boone, J. and L. A. Bovenberg, 2002. "Optimal Labour Taxation and Search." *Journal of Public Economics* 85:53–97.

Booth, A. L., 1995. *The Economics of the Trade Union.* Cambridge University Press, Cambridge.

Engström, P., 2002. "Optimal Non-Linear Taxation in Search Equilibrium." Discussion paper 2002:18, Uppsala University, Uppsala.

Fudenberg, D. and J. Tirole, 1991. *Game Theory.* MIT Press, Cambridge.

Hariton, C. and G. Piaser, 2004. "When Redistribution Leads to Regressive Taxation." Core discussion paper 2004/20, CORE, Louvain-la-Neuve.

Hersoug, T., 1984. "Union Wage Responses to Tax Changes." *Oxford Economic Papers* 36:37–51.

Hungerbühler, M., E. Lehmann, A. Parmentier and B. Van der Linden, 2003. "Optimal Income Taxation in an Equilibrium Unemployment Model: Mirrlees meets Pissarides." Ires discussion paper 2003-24, IRES, Louvain-la-Neuve.

Lockwood, B. and A. Manning, 1993. "Wage Setting and the Tax System." *Journal of Public Economics* 52:1–29.

Meltzer, A. and S. Richard, 1981. "A Rational Theory of the Size of Government." *Journal of Political Economy* 89:914–927.

Mirrlees, J., 1971. "An Exploration in the Theory of Optimum Income Taxation." *Review of Economic Studies* 38:175–208.

Pencavel, J., 1986. "Labor Supply of Men: A Survey." In O. Ashenfelter and R. Layard (eds.), "Handbook of Labor Economics," North-Holland, Amsterdam.

Petrongolo, B. and C. A. Pissarides, 2001. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature* 38:390–431.

Pissarides, C. A., 1998. "The Impact of Employment Tax Cuts on Unemployment and Wages. The Role of Unemployment Benefits and Tax Structure." *European Economic Review* 42:155–183.

———, 2000. *Equilibrium Unemployment Theory.* MIT Press, Cambridge, second edn.

Saez, E., 2002. "Optimal Income Transfer Programs: Intensive versus Extensive Margin Labor Supply." *Quarterly Journal of Economics* 117:1039–1073.

Sørensen, P. B., 1999. "Optimal Tax Progressivity in Imperfect Labour Markets." *Labour Economics* 6:435–452.

Teulings, C. and J. Hartog, 1998. *Corporatism or Competition?* Cambridge University Press, Cambridge.

Tuomala, M., 1990. *Optimal Income Tax and Redistribution.* Clarendon Press, Oxford.