Tax Progression in Imperfect Labour Markets: A Survey*

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Abstract

We look at the effect of tax progression in imperfect labour markets. The models considered are union models, an equiblibrium search model with wage bargaining, an equilibrium search model with wage posting by firms and efficiency wage models. We find that in all basic models, an increase in tax progression leads to lower wages and higher employment. Extensions of the models can however change these results.

Keywords: Tax progression, Redistribution, Labour Market Imperfections

JEL numbers: H42, J41, J42, J51

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1 Introduction

Tax progression has a negative impact on output if labour markets are perfectly competitive. The individual decides on his own about labour supply, i.e. work effort and working time. If the marginal tax rate is high, an additional increase in individual labour supply allows only a small increase in net income. The individual therefore has less incentives to work long hours, and total labour supply diminishes. Since the efficient outcome is achieved in a world without government intervention, progressive taxes move the equilibrium away from the efficient outcome.

However, economists generally agree that labour markets are not perfect. There is however no general agreement about where the labour market imperfections come from. Different theories have been put forward during the last years. In this survey, we look at some of the most prominent modern labour market theories. The impact of tax progression is shown to have a considerably different impact on outcomes than in a perfect labour market. Surprisingly, the basic models lead all to the same conclusion: A higher progression of the tax system decreases wages and increases employment. If labour supply is taken as exogenous, output increases.

The aim of this article is neither to give deep insights into modern labour market theories[1], nor to check the relevance of the models against each other. We expose the models in the most simple way, but such that they still capture the main mechanisms of market failures. We then check the impact of tax progression in each of these models. The analysis is concentrated on the theoretical models[2].

The structure of this article is as follows. Section 2 shows the tax functions that we consider and how we define tax progressivity. Sections 3 to 5 constitute the main part of this paper, where different labour market models are exposed and the impact of tax progressivity on outcomes is explained. We start with simple models of the trade union in section 3. Trade unions are often blamed for distorting the outcome by setting wages too high compared to the competitive level. We show that progressive taxation has a wage moderating effect in various variants of trade union models. Tax progression therefore weakens the distortions that are induced by the presence of unions. In section 4, we assume that workers and firms have to go through a costly search process to find a partner on the labour market. Once they are matched, they enjoy a rent. In a first model, we assume that this rent is shared through wage bargaining between the worker and the firm. A second model assumes that firms post vacancies

[1] The textbook by Cahuc and Zylberberg (2004) gives an in-depth analysis of the models of this article, and confronts them to the empirical data.

[2] Short overviews on empirical findings can be found in Sørensen (1997) and Røed and Strøm (2002).

and choose the wage unilaterally, but competition between firms and on-the-job search of workers induces an equilibrium wage distribution. We show that in both cases, tax progression decreases the average wage and increases employment. Next, section 5 looks at efficiency wage models. A higher wage increases the effort of the worker, which leads firms to set wages above the competitive level. Tax progression reduces the gain for the worker of an additional unit of effort in terms of net income. It becomes more costly for firms to give incentives for high effort levels and they set lower wages. As a consequence, effort, wages and unemployment decrease. In all these models, we assume that labour supply is exogenous. Section 6 lets this assumption fall and considers the case when there is wage bargaining and elastic labour supply as in the competitive model. Finally, section 7 concludes.

2 General notations and definitions

2.1 Tax function

Throughout this article, we assume that individuals are all identical. Their net income x equals the gross wage w minus the tax $T(w)$. We restrict ourselves to linear tax schemes[3] $T(w) = -\tau_0 + \tau w$, such that the workers' net income x can be written

$$x = w(1-\tau) + \tau_0$$

The marginal tax rate $\frac{\partial T(w)}{\partial w}$ equals τ and the average tax rate $\frac{T(w)}{w}$ is denoted as τ^a and equal to $\tau - \frac{\tau_0}{w}$. We impose the natural assumption that the marginal tax rate is below 1. Finally, we define the coefficient of residual income progression (CRIP) ν as the elasticity of the after-tax income with respect to pre-tax income. Therefore, $\nu = \frac{1-\tau}{1-\tau^a}$.

As appendix B shows, it does not matter whether the tax on the wage is paid by the employer or the worker. For simplicity, we therefore assume that only the worker pays the tax.

2.2 Tax progression

Next, we have to define what exactly we mean by progression[4] and how we measure the degree of progression. It is generally agreed that a tax system is progressive if the marginal tax rate is higher than the average tax rate, implying that the average tax rate is increasing in gross income (see Creedy, 1996;

[3]This is without loss of generality in this article, because the basic models of this paper only include one stage of optimisation. Therefore, only the marginal and average tax rates are of importance.

[4]Throughout this paper, we will use "progression" and "progressivity" as synonyms.

Musgrave and Thin, 1948). This allows us to say whether a tax system is progressive or not, but it does not tell anything about whether one tax system is more progressive than another one. Musgrave and Thin (1948) put forward four different local measures for the degree of tax progression:

1. Average rate progression: A tax system is more progressive than another tax system if its derivative of the average tax rate with respect to the pre-tax income is higher.

2. Marginal rate progression: A tax system is more progressive than another tax system if the derivative of the marginal tax rate with respect to the pre-tax income is higher.

3. Liability progression: A tax system is more progressive than another tax system if the elasticity of the tax with respect to the pre-tax income is higher.

4. Residual income progression: A tax system is more progressive than another tax system if the elasticity of the after-tax income with respect to the pre-tax income is lower.

The degree of tax progression is assumed to measure the redistributive power of a tax system. The Lorenz criterion has become dominant for comparing two tax systems. As Jakobsson (1976) showed, the coefficient of residual income progression (CRIP) is the only local measure that is compatible with the Lorenz criterion, i.e. if a tax system has a lower CRIP for all income levels than another tax system, then it is also more redistributive as measured by the Lorenz criterion.

It is then interesting to concentrate on tax reforms that increase the marginal tax rate holding the average tax rate constant, since this makes the tax scheme more progressive in the senses 1, 3 and 4 mentioned above[5]. The effect of such a change can be decomposed. Consider a change that holds the tax to be paid constant, but increases the marginal tax rate. As we will show, this decreases the wage rate. This however implies that the average tax rate increases. A tax change that holds the average tax rate constant has thus to decrease the tax paid by the individual. This income effect has intuitively a negative effect on the wage: A reduction in the tax decreases the gross wage. Unfortunately, this income effect also makes calculations more tedious without adding many new insights. That is why we will in the main part only give the results for a tax change that increases the marginal tax rate at a constant tax. The case of a tax

[5]The second definition of the degree of tax progression is without importance in our models, since there is only one stage of optimisation. An increase in the second derivative of the tax function does not affect the outcomes.

reform that increases the marginal tax rate at a constant average tax rate are relegated to the appendix C.

The tax reform considered is thus the one that increases τ and adjusts τ_0, such that the total amount of the tax paid T stays unchanged. The appendix considers the case of an increase in τ that adjusts τ_0 in a way that τ^a is not affected.

3 Trade union models

There are three popular models of trade union behaviour that can be found in the literature:

- The monopoly model (Dunlop, 1944): In this model, the union unilaterally sets the wage rate, whereas the firm chooses the level of employment. When setting the wage rate, the union takes the effect of the chosen wage on employment into account.
- The right-to-manage model: As in the monopoly model, the firm chooses the level of employment unilaterally. The firm and the union bargain about wages in an asymmetric Nash bargain. It can be shown that the monopoly model is a special case of the right-to-manage model where the union has all the bargaining power.
- The efficient bargain model (McDonald and Solow, 1981): The firm and the union bargain about both the employment level and the wage rate.

Even though it can be shown that these three models can be unified in one framework (see Manning, 1987), we check the effects of tax progression on the outcomes in the three models separately to make the mechanisms at work more intuitive.

Next, we have to describe the objective function of the union. It is clear that unions are interested in numerous things (see Freeman and Medoff, 1984; Booth, 1995), many of them not being directly connected to pay issues (e.g. working conditions). To simplify the analysis, it is however often assumed in the literature that unions only care about total employment N and the workers' net income x. We also follow this approach.

Different functional forms for the union's objective function $U(x, N)$ have been considered in the literature. Dunlop (1944) favoured the maximisation of the total wage income of the union members as the most convincing objective of the union. If subscription costs are negligeable, the objective function becomes $U = xN$. A very similar functional form was put forward by Rosen (1969) where he assumes that the union maximises the rent of its members. The objective function then becomes $U = (x - x^c)\, N$, where x^c denotes the income at the competitive wage level in the non-unionised sector. Another possible objective of the union might be to maximise the sum of individuals' utilities. The objective function then writes $U = Nu(x)$, where $u(.)$ denotes the individual's utility of income. Finally, the union might only be interested in the net income, but not in the level of employment. This applies if the union is dominated by insiders.

In our models, we assume that the union's objective can take any form $U(x, N)$. We only impose that the first derivatives with respect to net income and employment are strictly positive. That is, both income and employment are desirable for the union.

To keep our models as simple as possible, we look at the effect of an increase in the marginal tax rate, holding the amount of taxes paid by individuals constant. An extension to the case where the marginal tax rate increases but the average tax rate stays constant, is intuitively simple since one only has to add an income effect, but implies tedious calculations. Therefore, this extension has been relegated to the appendix C.

3.1 The monopoly model

The union chooses the wage rate to maximise its objective function $U(x, N)$ and the firm sets employment given this wage rate. When the union sets the wage rate, it anticipates the behaviour of the firm. We assume that the output of the firm is strictly concave in labour input, such that the employment level chosen by the firm is decreasing in the wage rate. The first-order condition of the union's problem can then be written

$$U_w \equiv \frac{\partial U}{\partial w}(1-\tau) + \frac{\partial U}{\partial N}\frac{\partial N}{\partial w} = 0 \tag{1}$$

We are interested in the effect of a change in the marginal tax rate τ, holding the total tax T paid by the individual constant. The implicit derivation of (1) gives

$$\left.\frac{\partial w}{\partial \tau}\right|_T = \frac{\frac{\partial U}{\partial w}}{U_{ww}} < 0$$

assuming that the second-order condition $U_{ww} < 0$ of the maximisation problem holds.

An increase in the marginal tax rate therefore decreases the wage rate, and increases the level of employment. The intuition for this result is very standard: An increase in the marginal tax rate is equivalent to say that the price of an increase in net income has become higher relative to the price of an increase in employment. The union therefore substitutes net income towards employment. This is a pure substitution effect, as was first highlighted by Hersoug (1984) and Malcomson and Sartor (1987).

If we were interested by a tax reform that increases the marginal tax holding the average tax constant, an additional income effect would show up. Since the higher marginal tax rate decreases the wage, a constant average tax would imply that the individuals pay less taxes. It is quite intuitive that a lower tax decreases the wage demands of the union, but one has to make specific assumptions on the utility and profit functions. However, Lockwood and Manning (1993) as well as our appendix C show that this effect goes in the expected direction for reasonable assumptions.

3.2 The right-to-manage model

In this model, the firm and the union negotiate the wage level in a asymmetric Nash bargain and the firm sets unilaterally the employment level given the wage rate. This effect is anticipated by the union and the firm during the Nash bargain. If there is no agreement, the union gets the outside utility $\bar{U}$, and the firm gets the profit $\bar{\Pi}$, which are both independent of the wage rate. The maximisation problem then becomes

$$\max_{w} \quad \left(U - \bar{U}\right)^{\beta} \left(\Pi - \bar{\Pi}\right)^{1-\beta} \tag{2}$$

where β denotes the relative bargaining power of the union. To simplify, we take the logarithm of (2)

$$\Omega \equiv \beta \ln \left(U - \bar{U}\right) + (1-\beta) \ln \left(\Pi - \bar{\Pi}\right)$$

The first-order condition can be written as

$$\Omega_w \equiv \frac{\beta}{U - \bar{U}} \left[\frac{\partial U}{\partial w}(1-\tau) + \frac{\partial U}{\partial N}\frac{\partial N}{\partial w}\right] + \frac{1-\beta}{\Pi - \bar{\Pi}}\frac{\partial \Pi}{\partial w} = 0 \tag{3}$$

We are again interested in the effect of a change in the marginal tax rate τ, holding the total tax T paid by the individual constant. The implicit derivation of (3) gives

$$\left.\frac{\partial w}{\partial \tau}\right|_T = \frac{\frac{\beta}{U-\bar{U}}\frac{\partial U}{\partial w}}{\Omega_{ww}} < 0$$

assuming that the second-order condition $\Omega_{ww} < 0$ of the maximisation problem holds.

This effect was first noted by Lockwood and Manning (1993). The intuition is a bit more complex than in the monopoly union model, because there are now two effects that show up. First, the same substitution effect as in the monopoly union plays. A higher marginal tax rate makes increases in the employment level relatively cheaper compared to increases in the net income. The union therefore substitutes employment for income. Second, the higher marginal tax rate decreases the relative bargaining *strength* of the union. In fact, to get an increase in the net income of its members by one unit, the wage has to rise by $(1+\tau)$ units. This increase is however opposed more strongly by the firm if the marginal tax τ is high. The relative bargaining *strength* of the union therefore decreases even though the relative bargaining *power* does not change.

Once more, if we were interested in an increase in the marginal tax rate holding the average tax rate constant, an additional income effect would show up. Since the wage rate decreases with higher marginal tax rates, a constant average tax rate would imply that the tax paid by the union members decreases. Intuitively, a lower tax decreases the wage demands of the unions. Lockwood and Manning (1993), Sørensen (1999) and our appendix C show that this is in fact the case for reasonable assumptions on the union's objective function and the firm's profit function.

3.3 The efficient bargain model

The right-to-manage model has been criticized because it does not lead to a Pareto-efficient outcome. Both the firm and the union could get higher utility levels when they bargain about both employment and the wage rate. However, there is little evidence that unions really do bargain about the employment levels with the firms (see Booth, 1995).

In this framework, the union and the firm bargain about the wage rate and the level of employment in an (asymmetric) Nash bargain

$$\max_{w,N} \quad \left(U - \bar{U}\right)^{\beta} \left(\Pi - \bar{\Pi}\right)^{1-\beta}$$

The derivation of the impact of the marginal tax rate gets more complicated and tedious in this model, but the result is the same as in the other union models (see Koskela and Vilmunen, 1996; Sørensen, 1999): An increase in the marginal tax rate decreases the wage level and increases employment. And also the intuition is the same as above: A higher marginal tax rate makes increases in net income relatively more expensive compared to increases in the level of employment. The firm and the union therefore substitute employment for net income. Moreover, as in the right-to-manage model, an increase in the marginal tax rate decreases the relative bargaining strength of the worker.

4 Matching models

One does not only observe unemployed individuals on the labour market, but also unfilled vacancies. This indicates that it is not always easy for a firm and a worker to find a match. In search models (see Mortensen and Pissarides, 1999; Pissarides, 2000), one assumes that there is heterogeneity on the labour market such that it is not easy for the firm to find a worker for its vacant job with whom it can produce output, and vice versa. Therefore, the firm and the worker have to search for a partner, and this is costly for both parties.

Frictions on the labour market are implicitly modeled by a matching function that gives the number of successful matches M as a function of the number of workers who search for a job and the number of firms that search a worker for an open vacancy. The number of matches can be written

$$M = m(L, V) \tag{4}$$

where L denotes the number of individuals searching for a job and V is the number of vacant jobs. This matching function is assumed to be increasing in both its arguments, concave and homogeneous of degree 1. Empirical studies have shown that a Cobb-Douglas functional form fits the data well (see Petrongolo and Pissarides, 2001). To keep things simple, we assume this to be satisfied.

Let $\theta = \frac{V}{L}$ denote the labour market tightness. A vacant job is then filled at a rate

$$q(\theta) \equiv \frac{M}{V} = m\left(\frac{1}{\theta}, 1\right)$$

and workers find a job with probability

$$\theta q(\theta) = \frac{M}{V}\frac{V}{L}$$

The properties of the matching function imply that $q'(\theta) < 0$. The elasticity of $q(\theta)$ is a number between -1 and 0, and its absolute value is denoted by η. If the matching function has a Cobb-Douglas form, η is a constant.

We normalise the total labour force to 1 and denote the unemployment rate by u. Jobs are destroyed at an exogenous rate λ. The flow into unemployment therefore equals $\lambda(1-u)$. Unemployed individuals find a vacant job with probability $\theta q(\theta)$, such that the flow out of unemployment can be written as $\theta q(\theta) u$. In steady state, the unemployment rate is constant, which implies that

$$u = \frac{\lambda}{\lambda + \theta q(\theta)} \tag{5}$$

This equation gives a relation between vacant jobs and unemployed individuals and is commonly known as the Beveridge curve. In an unemployment-vacancy space, this curve is convex to the origin by the properties of the matching function, as shown in figure 1.

There are then two different assumptions that can be taken to describe how wages are set in the economy. The next sections check the impact of a rise in tax progressivity in both models. We develop the models for infinitely lived individuals and in continuous time[6]. Moreover, we concentrate our analysis on the steady state.

[6] A static version of the bargaining model gives similar results, as explained in appendix D.

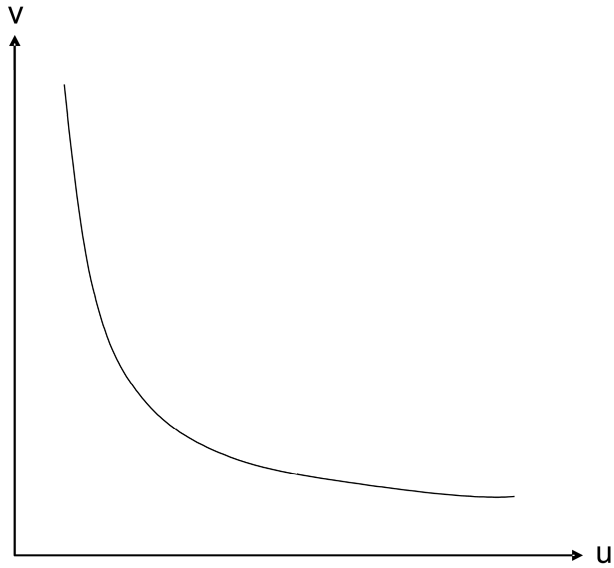

Figure 1: **Beveridge Curve**

4.1 Bargaining

In this model, we assume that only unemployed individuals search for a job such that $L = u$ in the matching function (4).

Denote by J^f and J^o the discounted values of a filled and open vacancy respectively. Once a vacancy is filled, the worker produces an output a. The firm pays a wage w to the worker. The match is destroyed at the exogenous probability λ. This allows us to write the Bellman equation

$$rJ^f = a - w - \lambda \left(J^f - J^o\right) \tag{6}$$

If firms open a vacancy, they have to pay a fixed cost κ per period and per vacancy. This represents the hiring cost for the firm while searching for a worker. The Bellman equation for an open vacancy can be written

$$rJ^o = -\kappa + q\left(\theta\right)\left(J^f - J^o\right) \tag{7}$$

Firms enter the market and open vacancies as long as the expected profit is positive. This free-entry condition therefore implies that $J^o = 0$. Combining these equations, one gets

$$\frac{\kappa}{q\left(\theta\right)} = \frac{a-w}{r+\lambda} \tag{8}$$

This is the so-called vacancy supply (or job creation) curve. By the properties of the matching function, this curve is negatively sloped in the $w - \theta$ space, as depicted in figure 2. Intuitively, if the wage w falls, firms post more vacancies because they get a higher profit once they have found a worker. This implies that the labour market tightness θ increases.

Denote by W and U the discounted value of the expected income for an employed and unemployed worker, respectively. Unemployed workers get an unemployment benefit b and find a job with probability $\theta q(\theta)$ in which case they move to the employed state. The Bellman equation becomes

$$rU = b + \theta q(\theta)(W - U) \tag{9}$$

Employed workers get a wage w and pay a tax $T(w)$. They become unemployed at an exogenous rate λ in which case they move to the unemployed state. The Bellman equation can then be written

$$rW = w - T(w) + \lambda(U - W) \tag{10}$$

Once a firm and a worker have matched on the labour market, they enjoy a rent because both are better off than when they were searching for a partner on the labour market. In this section, we assume that this rent is shared in an asymmetric Nash bargain. The maximisation problem then becomes

$$\max_{w} \quad (W - U)^{\beta} \left(J^{f} - J^{o}\right)^{1-\beta} \tag{11}$$

where β denotes the relative bargaining power of the worker. The first-order condition can be written after rearranging the terms (see appendix A.1)

$$w = \frac{\beta(1-\tau)(r + \lambda + \theta q(\theta))(a + \kappa) + (1-\beta)(r + \lambda + q(\theta))(T + b)}{\beta(1-\tau)(r + \lambda + \theta q(\theta)) + (1-\beta)(r + \lambda + q(\theta))} \tag{12}$$

where τ is the marginal tax. This is the so-called wage-setting curve. As appendix A.1 shows, this curve is upward-sloping in the $w - \theta$ space, as depicted in figure 2. Intuitively, if labour market tightness θ increases, there are more vacancies on the labour market. This implies that it becomes easier for the worker to find a job in case the current match is destroyed. Therefore, his outside option in the bargaining game increases. On the other side, an increase in θ makes it more difficult for a firm to find a worker if the current match is destroyed. This decreases the outside option for the firm in the bargaining game. As a consequence of these changes in the outside options in favour of the worker, the negotiated wage increases.

Together, the wage setting curve (12) and the vacancy supply curve (8) determine a unique equilibrium in the $w - \theta$ space, as depicted in figure 2. The equilibrium value of θ can then be inserted as a straight line through the origin into the figure with the beveridge curve, to get the equilibrium values for the vacancy and the unemployment rates. This is shown in figure 3.

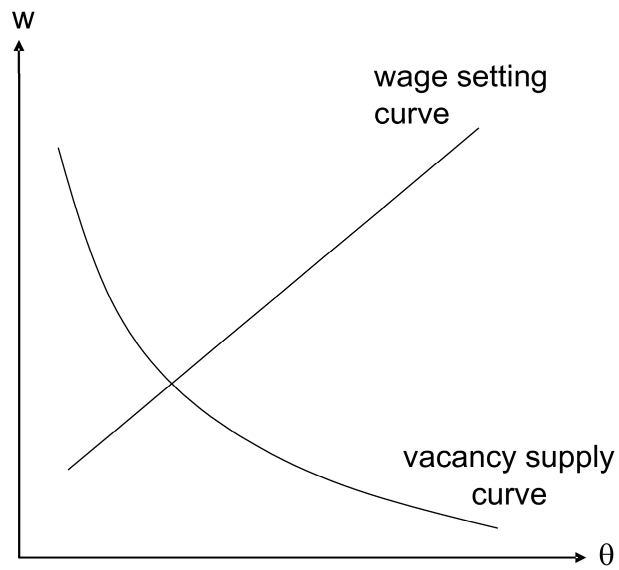

Figure 2: **Equilibrium: wage and labour market tightness**

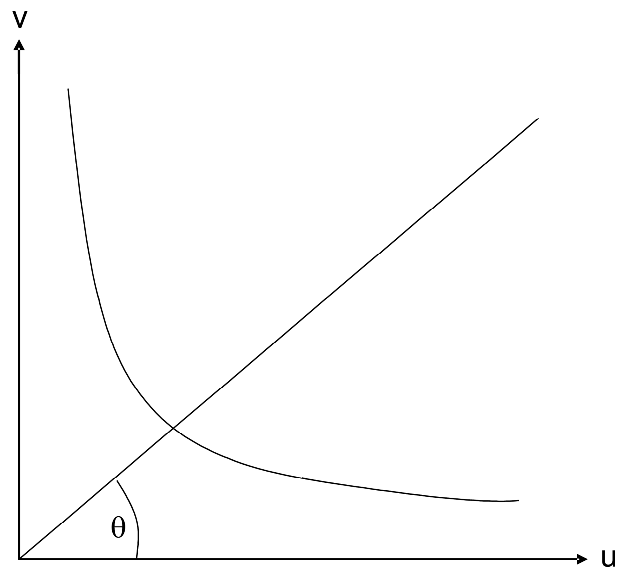

Figure 3: **Equilibrium values for u and v**

What is then the impact of a tax reform that increases the marginal tax rate τ, but holds the total tax paid unchanged? First note that tax policy does affect neither the beveridge curve (5) nor the vacancy-supply curve (8). It only matters for the wage-setting curve (12). As appendix A.1 shows, an increase in the marginal tax rate moves the wage-setting curve downwards. The intuition is exactly the one that we already saw in the right-to-manage union model: An increase in the marginal tax rate makes an increase in the net wage more costly for the firm in terms of increase in the gross wage. Therefore, the firm resists more heavily the wage claims of the worker, and the wage decreases. It can then be seen from figure 2 that a downward shift of the wage-setting curve decreases the equilibrium wage and increases labour market tightness. This in turn decreases unemployment according to figure 3.

As in the union models, a tax change that increases the marginal tax rate and keeps the average tax rate unchanged adds an income effect. Intuitively[7], this income effect is negative. Since the equilibrium wage decreases for a fixed tax, the average tax rate increases. To keep the average tax rate constant, the tax must thus decrease. According to the Nash bargaining solution, this lowers the gross wage even more.

4.2 Wage posting by firms

In this model, employed individuals continue to search for a better-paying job. Firms announce the wage when posting a vacancy. If they announce a low wage, their profit per worker is high, but it is more difficult to find a worker. Moreover, low-wage firms loose their workers rapidly, since the workers might get a better job offer. The firms have thus the choice between a low wage and high labour turnover on the one side, and a high wage and low labour turnover on the other side. It can be shown[8] that at equilibrium, the wage distribution is non-degenerate and workers with the same productivity are paid different wages.

Our setting is based on the paper by Manning (2004) that introduces taxes into the Burdett-Mortensen model of wage posting with homogeneous workers and firms (see Burdett and Mortensen, 1998; Mortensen, 2000).

In this context, both employed and unemployed individuals search for a job such that $L = 1$ in the matching function (4). For simplicity, we assume that both the unemployed and the employed get a job offer with the same probability[9]. Hence, from job-search theory, the reservation wage of the unemployed is simply a constant b. While the unemployed accept every job that gives them

[7] The mathematical developments are given in appendix C.

[8] For the proof, see Burdett and Mortensen (1998).

[9] This simplifies the model without changing the basic mechanism. For the case of different arrival probabilities of job offers, see Burdett and Mortensen (1998).

a net income $w - T(w)$ above their reservation utility b, the employed only accept jobs that pay a higher wage than their actual job. It can then be shown (see Burdett and Mortensen, 1998) that the lowest wage offered by the firms w_0 equals

$$w_0 = b + T(w_0) \tag{13}$$

Intuitively, the firm has no incentive to offer a lower wage because no worker would accept such a job. On the other side, the firm that offers the wage at the lower end of the wage offer distribution has no incentives to pay a higher wage than (13), since this would only decrease its profits without decreasing labour turnover.

Denote by $F(w)$ the wage offer distribution. The unemployed u get a job offer with probability $\theta q(\theta)$, and with probability $F(w)$, this job offer is below the wage level w. The flow of the number of people into jobs with a wage no higher than w is thus equal to $\theta q(\theta) F(w) u$. Denote then by $G(w)$ the fraction of workers employed at a wage w or less. The employed, who constitute a fraction $(1-u)$ of the population, leave employment at an exogenous rate λ, but also leave employment when they receive a better job offer. Job offers arrive at a rate $\theta q(\theta)$ and with probability $1 - F(w)$, this job offer is above their current wage w. The flow of the number of people out of jobs with a wage no higher than w is thus equal to $[\lambda + \theta q(\theta)(1 - F(w))] G(w)(1-u)$. In steady state, one gets

$$G(w) = \frac{\lambda F(w)}{\lambda + \theta q(\theta)(1 - F(w))}$$

Denote by $J^f(w)$ and J^o the discounted value of a filled job for a firm that pays a wage w and the discounted value of a vacant job respectively. The match produces an output of a and the firm pays the wage w. The match gets destroyed when the worker gets a better job offer, which happens with probability $\theta q(\theta)(1 - F(w))$, or when the job is destroyed at the exogenous rate λ. The Bellman equation becomes thus

$$rJ^f(w) = a - w - \theta q(\theta)(1 - F(w))\left(J^f(w) - J^o\right) - \lambda\left(J^f(w) - J^o\right)$$

When posting a new vacancy, the firm chooses the wage that maximises its expected profits. Posting a vacancy has a cost κ per period. The job gets filled with probability $q(\theta)[u + (1-u)G(w)]$, such that the Bellman equation can be written

$$rJ^o = \max_{w \geq w_0} \left\{q(\theta)[u + (1-u)G(w)]\left(J^f(w) - J^o\right) - \kappa\right\}$$

Firms open vacancies as long as the expected profit is positive, such that the free-entry condition implies $J^o = 0$. Since this is also true for the lowest wage offered w_0, the equilibrium labour market tightness solves[10]

$$\frac{\kappa}{q(\theta)} = \frac{\lambda}{\lambda + \theta q(\theta)} \frac{a - w_0}{r + \lambda + \theta q(\theta)} \tag{14}$$

Finally, since firms choose the wages they post to maximise profits, all wages must give the same profit at the equilibrium. Using the fact that $F(w_0) = 0$, one can then derive an equilibrium wage distribution (see Mortensen, 2000). For simplicity, we take the limit as $r \to 0$, such that the wage offer distribution can be written

$$F(w) = \frac{\lambda + \theta q(\theta)}{\theta q(\theta)} \left[1 - \sqrt{\frac{a - w}{a - w_0}} \right]$$

From this equation, one can derive the average wage rate $E(w)$ (see Manning, 2003, 2004)

$$E(w) = a - \frac{\lambda}{\lambda + \theta q(\theta)} (a - w_0) \tag{15}$$

The description of a tax reform that increases the progressivity of the tax system seems at first view more difficult to describe in this context, since we face a distribution of wages and not any more a single wage for the representative agent. An increase in the marginal tax rate that holds the tax paid (or the average tax rate) constant at some income level necessarily changes the total tax paid (or the average tax rate) at all other income levels. However, it is intuitively clear that a tax reform that increases progressivity increases τ and decreases τ_0, such that the low-income earners pay less taxes, whereas the taxes paid by the high-income earners increase. This implies that the individual with wage w_0 has to pay less taxes, and $T(w_0)$ decreases. This then allows the lowest-wage firm to cut wages to bring the net income of the lowest-wage workers back to their outside option b as shown in (13). This decreases the average wage by (15) and (14)[11] and therefore increases profits for firms. As a consequence, firms post more vacancies and unemployment decreases. As Manning (2004) puts it out, "another way to think about this result is that subsidising low-wage labour encourages the payment of low wages and, in a monopsonistic labour market, employers have the market power to take advantage of this."

5 Efficiency wage models

Efficiency wage models can be motivated in different ways. But the basic idea behind these models is always the same: The productivity of the worker depends

[10]This equation has two solutions in θ, one at 0 and the other one at some strictly positive number. As Mortensen (2000) notes, only the strictly positive value is stable.

[11]Since the proof in Manning (2004) is not complete, you can find the proof in appendix A.2.

positively on the wage that the firm pays relative to the market wage. Yellen (1984) notes four possible microfoundations for efficiency wage models:

- The shirking model: Workers can choose whether they shirk at work or not. Monitoring the worker's effort is costly for firms and is therefore imperfect. Workers who shirk have some chance to get caught, in which case they are fired. If the wage is then at the market-clearing level of a completely competitive labour market, the worker who was fired could immediately get a new job at the same wage. The worker has then no incentive not to shirk. However, if the firm offers a wage that is higher than the market-clearing level, the worker looses the high wage in case he is caught shirking. This gives some incentives to the worker not to shirk. At the equilibrium, all firms behave in the same way and set wages above the competitive level. This induces involuntary unemployment. A worker who is caught shirking then looses his job and might not immediately find a new job. He is punished by being unemployed for a while. This gives incentives not to shirk at the equilibrium.

- The labour turnover model: Firms offer high wages to avoid costly labour turnover. The higher the wage rate in a firm and the higher the unemployment rate, the less the individual has incentives to quit the firm. Since all firms are identical, they pay a wage above the market-clearing level and unemployment arises.

- Adverse selection: Workers are assumed to be heterogeneous in ability. Moreover, ability and the workers' reservation wages are assumed to be positively correlated. By offering a high wage, the firm can attract high-ability workers. Since all firms want to attract the high-ability types, the wage increases above the market-clearing level. Firms do not hire workers who are willing to work at a lower wage since the firms fear that they have low ability (i.e. that they are "lemons"). The high wage induces involuntary unemployment.

- Sociological models (see also Akerlof, 1982, 1984): A worker's effort depends on the work norms of his group. By giving a gift to the workers in the form of a higher wage, the firm increases the work norm of the group and therefore their productivity. Since all firms behave identically, the wage is set above the market-clearing level and unemployment arises.

We will use the shirking model of Shapiro and Sitglitz (1984) as basic motivation for our model.

The instantaneous utility $U(x, 1-e)$ of a worker depends positively on the net income x and leisure time $1-e$, where e denotes effort. If unemployed, the worker has all his time for leisure and gets an unemployment benefit b. If the worker is caught shirking, he is fired. The probability q of being fired is then a given function of e, $q = q(e)$. We assume that q is decreasing and convex in effort. There is also an exogenous separation rate λ. The exit rate out of unemployment ϕ depends positively on aggregate employment N.

Denote by V^e and V^u the discounted expected utilities of an employed and unemployed worker respectively. An employed individual gets the instantaneous utility $U(x, 1-e)$ and looses his job at rate $(q(e) + \lambda)$. An unemployed individual gets the instantaneous utility $U(b, 1)$ and finds a job with rate $\phi(N)$. The Bellman equations can then be written as

$$\begin{aligned} rV^e &= U(x, 1-e) - (q(e) + \lambda)(V^e - V^u) \qquad (16) \\ rV^u &= U(b, 1) + \phi(N)(V^e - V^u) \end{aligned}$$

where r denotes the exogenous discount rate.

5.1 Discrete effort choice

In the model of Shapiro and Stiglitz, the worker has the binary choice to shirk or not. Therefore, either $e = 1$ or $e = 0$. A worker does not shirk as long as $V^e(e = 1) \geq V^e(e = 0)$. This is the so-called no-shirking condition.

The firm then sets wages unilaterally. The firm wants the workers to supply effort. There is thus an incentive for the firm to offer a wage that satisfies the no-shirking condition, but the firm has no incentives to offer a higher wage than the minimal wage that satisfies the no-shirking condition.

For simplicity, we assume that the worker's utility function is linear in wages and effort, i.e. $U(x, 1-e) = x - e$. Moreover, a worker that supplies effort is never fired, whereas the shirking worker is fired at probability q.

At the steady state, the flow out of unemployment equals the flow into unemployment, implying that, if no-one shirks, an unemployed gets a job with probability $\frac{\lambda(1-u)}{u}$, where u denotes the unemployment rate. Combining the no-shirking condition with the value equations (16) and solving for the wage rate, one obtains

$$w = \frac{1}{1-\tau^a}\left(b + \frac{r+\lambda+q}{q} + \frac{1-u}{u}\frac{\lambda}{q}\right) \qquad (17)$$

This labour supply curve can then be combined with a labour demand curve to get the equilibrium values of w and u.

As can easily be seen from (17), the marginal tax rate has no impact on the wage levels, as found by Pissarides (1998). However, this finding depends crucially on the assumption that the effort choice is binary, as will be demonstrated in the next section. In fact, the marginal tax rate can influence the substitution between effort and net income in the worker's utility function. If the effort choice is binary, this substitution does not take place, since the firm will always induce the worker to supply the effort[12].

5.2 Continuous effort choice

Here, we assume that the worker's instantaneous utility U is increasing and concave in income and linearly decreasing in effort. Moreover, we impose $\frac{\partial^2 U}{\partial x \partial e} = 0$. Effort e is continuous and can take any value between 0 and 1.

From (16) we can derive the discounted utility for an employed individual

$$V^e = \frac{(r + \phi(N)) U(x, 1-e) + (q(e) + \lambda) U(b, 1)}{r(r + \phi(N) + q(e) + \lambda)}$$

The worker chooses effort to maximise this value. The first order condition can be written as

$$\frac{q'(e)[U(x, 1-e) - U(b, 1)]}{r(r + \phi(N) + q(e) + \lambda)} + \frac{\partial U(x, 1-e)}{\partial (1-e)} = 0$$

This equation implicitly defines a function $e = e(x, N)$. Using the implicit function theorem, one can show that $e_x = \frac{\partial e}{\partial x} > 0$, $e_N = \frac{\partial e}{\partial N} < 0$, $e_{xx} = \frac{\partial^2 e}{\partial x^2} < 0$, and $e_{xN} = \frac{\partial^2 e}{\partial x \partial N} = 0$. Further, we assume that an increase in aggregate employment also leads to an increase in aggregate effective labour input, i.e. $\frac{d(e(x,N)N)}{dN} = e + e_n > 0$.

The firms maximise profits. For the moment, we assume that the number of firms is fixed. Output depends on effective labour, i.e. effort times employment. The firm takes unilaterally the decisions about the wage w and employment level n, but takes into account the worker's effort function. Given a general production function $y = f(en)$ where the production function is assumed to be increasing and concave, the firm maximises

$$\Pi = f(e(x, N) n) - wn$$

The first-order condition can be written as

$$\begin{aligned} \frac{\partial \Pi}{\partial w} &= f' e_x (1-\tau) n - n = 0 \\ \frac{\partial \Pi}{\partial n} &= f' e - w = 0 \end{aligned} \tag{18}$$

Combining these two equations, one gets the Solow condition

$$\frac{e_x (1-\tau) w}{e} = 1$$

[12]Note that another assumption is crucial in this context: If the worker's utility is linear in effort and income, then the worker will always chose a corner solution.

Since aggregate employment equals the employment at the representative firm times the number of firms, the two equations of (18) determine implicitly the wage and employment levels as a function of the tax parameters.

Concerning the tax reform, we are again interested in an increase in the marginal tax rate that holds the tax paid by the individual constant. It can be shown that such an increase in progressivity decreases the wage rate and increases employment (Pisauro, 1991; Hoel, 1990). Intuitively, to increase the effort of the workers, the firm has to increase the worker's net income. The price in term of gross wages is higher if the marginal tax rate is high. Therefore, the firm has less incentives to induce workers to produce high efforts. As a consequence, the firm offers lower wages and unemployment decreases.

Further, if we look at a tax change that increases the marginal tax rate holding the average tax rate constant, an additional income effect shows up. As shown by Pisauro (1991), this effect again goes in the expected direction: A decrease in the tax paid decreases the wage.

5.3 General equilibrium

In the long run, the profits of the firms are equal to zero and the number of firms is endogenous. The zero-profit condition can be written

$$f\left(e\left(x,N\right)n\right)-wn=0$$

Together with the equations of (18), this determines implicitly the wage w, employment at the firm level n and aggregate employment N.

Tax progression has then different effects compared to the case where the number of firms is fixed. In fact, As Rasmussen (1999, 1998) shows, an pure increase in progressivity, i.e. an increase in the marginal tax rate holding the average tax rate constant has then no effect on employment but still decreases wages. This also implies that a budget-balanced increase in progressivity decreases employment, since the tax base decreases.

5.4 Heterogeneity among firms

Andersen and Rasmussen (1997) consider an efficiency wage model similar to the model presented in section 5.1 and inspired by Shapiro and Sitglitz (1984). Workers choose an effort level that is either low (e_L) or high (e_H). The firm then chooses wages and employment to maximise its profits. As in section 5.1, we again assume that the utility of a worker is linear in after-tax income and effort. For simplicity, we assume that the firm can perfectly observe the effort of the workers. This assumption simplifies the model, but does not change the conclusions, as shown by Andersen and Rasmussen (1997). The no-shirking condition then becomes

$$(1 - T(w_L))w_L - e_L \leqslant (1 - T(w_H))w_H - e_H$$

The participation constraint is

$$b \leqslant (1 - T(w_i))w_i - e_i \qquad i = L, H$$

where b is net of tax utility for an unemployed person. Therefore, the minimum wage to induce effort e_L is

$$\underline{w_L} = \frac{b + e_L}{1 - T\left(w_L\right)}$$

Since firms unilaterally set wages, they have no incentive to pay a higher wage than $\underline{w_L}$ for an individual that chooses effort e_L. If there are firms who offer the wage $\underline{w_L}$, the minimum wage for effort e_H has to satisfy the no-shirking condition. If $\underline{w_L}$ is not offered, $\underline{w_H}$ has to satisfy the participation constraint of the individuals. In both cases, $\underline{w_H}$ is given by

$$\underline{w_H} = \frac{b + e_H}{1 - T\left(w_H\right)}$$

Again, no firm has an incentive to offer a higher wage than $\underline{w_H}$ to the workers. A firm i offers the high wage $\underline{w_H}$ if

$$\Pi_i(\underline{w_H}, e_H) \geqslant \Pi_i(\underline{w_L}, e_L)$$

There exists then a critical wage level $\overline{w_i}$, given by

$$\Pi_i(\overline{w_i}, e_H) = \Pi_i(\underline{w_L}, e_L)$$

If the wage needed to induce effort e_H is above the critical level $\overline{w_i}$, the firm does not offer high-wage jobs, but only low-wage jobs.

We assume that the firms have heterogeneous production functions. This implies that $\overline{w_i}$ is not the same for all firms. Some firms therefore offer high-wage jobs, whereas some others offer low-wage jobs. Not all workers earn the same wage.

Let us then consider a tax reform that raises marginal taxes, holding the average tax rate at some mean wage constant. Taxes become more progressive. Due to the higher marginal taxes, the average tax for wage levels above the mean income grows, whereas the average tax for wage levels below the mean income falls.

Since we are in a efficiency wage model where the effort is a binary choice, the marginal tax rate does not directly influence wages and unemployment (see section 5.1). But average taxes do. In our model, the average tax for low-income earners decreases, and the average tax for high-income earners increases. Therefore, $\underline{w_L}$ falls and $\underline{w_H}$ rises.

A firm i can then be in three situations, depending on its critical value $\overline{w_i}$:

- The firm offers the high wage before and after the tax reform. This is true if $\overline{w_i}$ is above $\underline{w_H}$ before and after the tax reform. In such a firm, the wage rises. The effort stays constant. But a higher wage without higher effort leads to lower employment in this firm.
- The firm offers a low wage before and after the tax reform. This is true if $\overline{w_i}$ is lower than $\underline{w_H}$ in both cases. The firm then continues to pay $\underline{w_L}$, which falls due to the decrease in average tax at the income $\underline{w_L}$. The effort is the same as before. This leads to higher employment in the low-wage firm.
- The firm offers a high wage before the tax reform and a low wage after the tax reform: This is true if $\overline{w_i}$ is higher than $\underline{w_H}$ before and lower after the tax reform. In these firms, wage and effort decrease. The impact on employment is ambiguous.

The aggregate effect on employment is ambiguous. However, there will be fewer jobs in high-paying firms and more jobs in low-wage firms. Moreover, gross wage inequality rises since the high wage goes up and the low wage goes down. The conclusions of this model show thus that the results obtained in the models without heterogeneity are not so robust.

Note that Andersen and Rasmussen (1997) extend this analysis also to the cases where there is imperfect and endogenous monitoring of work effort and where a union sets wages in the high-wage sector. This does not change the results of the model qualitatively.

6 Labour supply and wage bargaining

Up to now, we have assumed that labour supply was exogenous. Since the competitive model has put much emphasis on labour supply distortions, it might be interesting to check what happens when labour supply effects are added to the wage moderating effects showed above. We will do this analysis in the context of a bargaining model.

A higher marginal tax rate decreases labour supply in a competitive model with quasi-linear preferences in income. The effect of an increase in the marginal tax rate on wages in a mixed model then seems at first view straightforward: Such a tax reform decreases the pie (i.e. the total surplus) since the worker has less incentives to supply labour. And from the bargaining models that we have seen in sections 3 and 4, we know that a higher marginal tax rate decreases the share of the pie that goes to the worker. For both these reasons, the wage decreases. This is confirmed by Hansen (1999) who shows that in a basic matching model with wage bargaining, an increase in tax progression decreases hourly wages, working hours and unemployment. The size of these effects depends on how working hours are determined. If the worker chooses the working hours unilaterally after the wage bargain, labour supply decreases more and wages less than if the worker and the firm bargain about both working hours and the wage rate[13]. This comes from the fact that the worker does not take into account the effect that his choice has on the surplus of the firm. As a consequence, the firm agrees on a higher hourly wage to give incentives to the worker to work long hours and thus to increase the total surplus, part of which goes to the firm.

Things however become more complicated if labour demand and/or different forms of preferences are taken into account. For example, if the utility function implies that the marginal utility of income is decreasing in work hours, the marginal utility of income increases as a response to the increased tax progression, because of the labour supply effect that decreases working hours. This increase in the marginal utility has then a positive effect on wages, and the impact of tax progression on wages becomes ambiguous (see Fuest and Huber, 2000). Similarly, if the hourly marginal product of the worker is decreasing in working hours, a decrease in working hours due to an increase in tax progression has a positive impact on hourly wages, and the total effect becomes again ambiguous (see Hansen *et al.*, 2000). If the labour supply elasticity is high enough and if the bargaining power of the worker is low, then the labour supply effect might in fact dominate.

7 Conclusion

This paper investigated the impact of tax progression on wages and employment in different popular models of imperfect labour markets. The results of the basic models all lead to the same conclusion: A higher degree of tax progression reduces wages and unemployment. However, we also noted that extensions of these models might change the results. Introducing endogenous labour supply in the bargaining models may make these effects ambiguous. Introducing a zero-profit condition or heterogeneity among firms in an efficiency wage model may also change the results.

The main criticism to our models is that they assume all individuals to be identical. The government has thus no redistributive goals. This simplifies the analysis, since an increase in tax progression can then be modelled as an

[13]Note also that if the worker sets the working hours unilaterally, there are two stages of optimisation in the framework. This implies that the second derivative of the tax function plays a role. The papers considered in this section however all assume implicitely linear tax schemes.

increase in the marginal tax rate holding the average tax rate constant, such that tax progression has mainly a substitution effect. However, an increase in tax progression will also decrease the average tax rate of the low-income earners and increase the average tax rate of high-income earners. Income effects are then added to the substitution effects.

Further, tax progression has also been accused to decrease the incentive for investments in human capital (see the discussion in Sørensen, 1997). In the models presented in this paper, human capital was assumed to be exogenous.

Finally, as Andersen and Rasmussen (1993) note, tax progression decreases incentives for job-to-job changes. Gentry and Hubbard (2004) have shown that this effect seems to be significant in the United States. However, the effects on equilibrium outcomes have not been studied yet. This is left for future research.

Appendix A Proofs and formulas

A.1 The wage-setting equation (12)

First, we show how one gets the wage-setting equation (12). From (6) and (7), one gets

$$J^f - J^o = \frac{a - w - \kappa}{q(\theta) + r + \lambda} \tag{19}$$

Similarly, from (9) and (10), one obtains

$$W - U = \frac{w - T(w) - b}{\theta q(\theta) + r + \lambda} \tag{20}$$

Introducing these equations into the bargaining problem (11), and solving the first-order condition with respect to w, one gets equation (12).

Next, we show that (12) is upward-sloping in the $w - \theta$ space. To simplify notations, let $y(\theta) \equiv \beta [\theta q(\theta) + r + \lambda]$ and $z(\theta) \equiv (1-\beta)[q(\theta) + r + \lambda]$. By the properties of the matching function, $y' > 0$ and $x' < 0$[14]. The wage-setting equation can then be rewritten as

$$w [y(\theta)(1-\tau) + z(\theta)] - y(\theta)(1-\tau)(a+\kappa) - z(\theta)(T(w) + b) = 0$$

Using the implicit function theorem on this equation, one gets

$$\frac{\partial w}{\partial \theta} = \frac{y'(1-\tau)(a+\kappa-w) - z'(w - T(w) - b)}{(1-\tau)(y+z)}$$

Since the rent of both the worker and the firm have to be positive (otherwise they would prefer to not form the match), this derivative is positive and the wage-setting curve thus upward-sloping.

[14]Intuitively, a higher labour market tightness makes it easier for a worker to find a job, and more difficult for a firm to find a worker.

Finally, we want to show that an increase in the marginal tax rate (holding the tax to be paid constant) shifts the wage-setting curve downwards. From (12) and again using the definitions of y and z to simplify the notations, one gets directly

$$\left.\frac{\partial w}{\partial \tau}\right|_{\theta} = -\frac{yz\left(a-\kappa-T-b\right)}{\left[\left(1-\tau\right)y+z\right]^{2}}$$

The term in parenthesis in the denominator is the total surplus to the agents which has to be positive. The derivative is thus negative, and an increase in the marginal tax rate decreases the wage rate.

A.2 The impact of tax progression in the wage posting equilibrium

From (14) and taking the limit as $r \to 0$, one gets

$$\frac{\partial \theta}{\partial w_0} = -\frac{\lambda+\theta q\left(\theta\right)}{\left(a-w_0\right)\left[2q\left(\theta\right)\left(1-\eta\right)+\frac{\eta}{\theta}\left(\lambda+\theta q\left(\theta\right)\right)\right]} < 0$$

Deriving equation (15) with respect to the wage w_0, one gets

$$\frac{\partial E\left(w\right)}{\partial w_0} = \frac{\lambda}{\lambda+\theta q\left(\theta\right)}\left[1+\frac{q\left(\theta\right)\left(1-\eta\right)\left(a-w_0\right)}{\lambda+\theta q\left(\theta\right)}\frac{\partial \theta}{\partial w_0}\right]$$

Putting these two equations together, one obtains finally

$$\frac{\partial E\left(w\right)}{\partial w_0} = \frac{\lambda}{\lambda+\theta q\left(\theta\right)}\frac{\left(1-\eta\right)+\frac{\eta}{\theta q\left(\theta\right)}\left(\lambda+\theta q\left(\theta\right)\right)}{2\left(1-\eta\right)+\frac{\eta}{\theta q\left(\theta\right)}\left(\lambda+\theta q\left(\theta\right)\right)} > 0$$

Appendix B Is it equivalent to tax the firm or the worker?

Conventional wisdom says that it is irrelevant whether the worker or the firm pays the taxes (see e.g. Blinder, 1988). Picard and Toulemonde (2003) have recently put some doubts on this wisdom. With the help of some examples, this appendix tries to clarify the issue. To keep things simple, we consider a simple bargaining framework which should be sufficient to give the necessary intuitions. The results however extend to the union models, matching models with wage bargain and efficiency wage models as shown by Picard and Toulemonde (2003)[15].

[15] For the matching model with wage posting by firms, the equivqlence is straightforward, as it is only the amount of tax at the lowest wage that counts, no matter whether this tax is paid by the firm or by the worker.

The market wage (gross wage) is denoted by w, while w_n and w_c are the net wage and labour cost respectively. As in Picard and Toulemonde (2003), we propose a general tax structure in which the net wage and labour cost are related to the market wage by the following functions $n(w;\alpha)$ and $c(w;\varepsilon)$

$$w_n = n(w;\alpha) \quad \text{and} \quad w_c = c(w;\varepsilon)$$

where α and ε are the vectors of taxation parameters. The tax paid by the worker then equals $w - n(w;\alpha)$, and the tax paid by the firm is equal to $c(w;\varepsilon) - w$.

When the firm and the worker bargain about the wage level, they maximise the Nash product

$$\max_w \quad (w_n)^\beta (a - w_c)^{1-\beta} \tag{21}$$

where a denotes the productivity of the worker and β is the worker's relative bargaining power. To keep things simple, we set the outside options equal to 0. The first-order condition writes

$$\frac{\beta n'_w}{w_n} = \frac{(1-\beta) c'_w}{a - w_c}$$

where n'_w and c'_w are the first derivatives of the respective functions with respect to w.

Consider now the following examples:

Example 1 *Initially, all taxes are paid by the worker, through a function* $n(w;\alpha)$. *The government then decides to shift all taxes to the firm, through the function* $c(w;\varepsilon)$. *It chooses this function such that* $c(w;\varepsilon) = n^{-1}(w_n;\alpha)$. *This tax shift does not affect the outcomes, and there is thus an equivalence of the two tax schemes.*

Example 2 *Consider the functions* $n(w;\alpha) = \alpha_0 + \alpha_1 w$ *and* $c(w;\varepsilon) = \varepsilon_0 + \varepsilon_1 w$. *Tax functions are thus linear. The government then decides to decrease the lump-sum part* α_0 *of the worker, and to compensate it by an increase in the lump-sup part* ε_0 *of the firm that holds the government's budget balanced. As shown by Picard and Toulemonde (2003), this tax change does not affect economic outcomes.*

Example 3 *Consider now some non-linear tax functions implied by the functions* $n(w;\alpha)$ *and* $c(w;\varepsilon)$. *The government then as in the previous example decides to decrease the lump-sum part* α_0 *of the worker, and to compensate it by an increase in the lump-sup part* ε_0 *of the firm that holds the government's budget balanced. As shown by Picard and Toulemonde (2003), this tax change does affect economic outcomes.*

Why does the third example not result in an equivalence, while the first two do? The intuition for this difference is not very difficult. Look at the definition of the Nash bargaining problem given by equation 21. The only values that the firm and the worker care about are the net wage w_n and the labour cost w_c. If two tax systems give the same choices of net wages and labour costs, the outcome will in fact be equivalent, since the maximisation problem is the same. This is however not the case any more in our third example. Intuitively, if the lump-sum part of the firm is shifted to the worker, they will agree on a higher market wage to compensate this shift. However, if the tax system is non-linear, this implies that they will also face a different marginal tax rate at this higher market wage. And as we have seen in the previous sections, a change in the marginal tax rate changes outcomes.

From this discussion, it follows that two tax systems are equivalent, if they relate the same labour cost to the same net wage, i.e. if they have the same function $n\left(c^{-1}\left(w_c;\varepsilon\right);\alpha\right)$. Whenever in this dissertation we refer to the equivalence between taxing the worker and taxing the firm, we assume this to be the definition of the equivalence between two tax systems.

This is however a purely theoretical result. In reality, workers might not always be aware of the tax function to the firm when negotiating the wage rate. Imperfect information and bounded rationality can thus in practice imply that the equivalence does not show up.

Appendix C Including income effects

C.1 The right-to-manage union model

We show the income effect in a right-to-manage union model. This discussion obviously extends to the monopoly union model, since the monopoly union model is just a special case of the right-to-manage union model where the union has all the bargaining power. The exposition follows closely Lockwood and Manning (1993) but simplify their framework for expository reasons[16].

We assume that the product market is imperfect. The demand schedule facing firm i is given by

$$Y_i^d = \left(\frac{P_i}{P}\right)^{-\eta}$$

with the absolute elasticity $\eta > 1$. P_i denotes the price charged by firm i, and P is the economy-wide price level. We normalise $P = 1$. Output by firm i is

[16] Especially, we do not include consumption taxes and taxes on labour paid by the employer. Moreover, we assume that the union maximises the total rent of the workers and that workers are risk-neutral. Our conclusion however also hold for more complicated settings.

given by its Cobb-Douglas production function

$$Y_i^s = AN_i^\alpha$$

where N_i denotes employment in firm i. The profit of the firm then equals

$$\Pi_i = P_i^{1-\eta} - w_i \left(\frac{P_i}{A}\right)^{\frac{1}{\alpha}}$$

The firm chooses P_i to maximise its profits. This gives the following equations for employment and profit at the firm level:

$$N_i = c w_i^{-\frac{1}{1-\alpha'}} \qquad (22)$$

$$\Pi_i = k w_i^{-\frac{\alpha'}{1-\alpha'}} \qquad (23)$$

where $\alpha' = \alpha(\eta - 1)/\eta$ and c and k are constants.

The firm bargains about wages at the firm-level with the firm-specific trade union. The trade union has utilitarian preference, i.e. it tries to maximise the total rent of workers. The union's preferences are thus described by

$$U = N_i (w_i - T(w_i) - U_l)$$

where U_l denotes the worker's utility in case he finds no work at firm i.

The wage w_i is chosen to maximise the asymmetric Nash bargain

$$\left(U_i - \bar{U}\right)^\beta \left(\Pi_i - \bar{\Pi}\right)^{1-\beta}$$

where $\bar{U}$ and $\bar{\Pi}$ are the outside options of the union and the firm respectively. We assume that if there is no agreement, production does not take place and workers have to search for another job. This implies that $\bar{U}$ and $\bar{\Pi}$ are equal to 0. The first-order condition can then be written as[17]

$$\beta \left[\varepsilon_N + \frac{w - T(w)}{w - T(w) - U_l} \frac{1-\tau}{1-\tau^a}\right] + (1-\beta)\varepsilon_\Pi = 0 \qquad (24)$$

where ε_N and ε_Π are the elasticities of the labour demand and profit functions respectively, with respect to the wage. By (22) and (23), these values are constant at $-1/(1-\alpha')$ and $-\alpha'/(1-\alpha')$ respectively. Next, let us assume that the workers utility in case he does not find a job at firm i is a weighted sum of the after-tax wage and unemployment benefits b, where the weights depend on the probability of being reemployed and on the discount rate of the worker,

$$U_l = \phi(u(w))\, b + [1 - \phi(u(w))(w - T(w))]$$

[17] Since all firms are identical, we can drop the firm-specific subscripts at this stage.

where $\phi' > 0$ and $u' > 0$, i.e. the lower the wage rate in the economy, the more jobs are offered and the easier it becomes to find a job. Equation (24) can then be rewritten as

$$\mu = \frac{w(1-\tau)}{\phi[w(1-\tau^a) - b]}$$

where $\mu = -((1-\beta)\varepsilon_\Pi + \beta\varepsilon_N)/\beta$ is a constant. Finally, implicit derivation of this equation gives

$$\frac{\partial w}{\partial \tau_a} = \frac{w\phi\mu}{\frac{(1-\tau)w}{(1-\tau^a)w-b} + \phi' u'[w(1-\tau^a) - b]} > 0$$

An increase in the average tax rate thus increases gross wages.

C.2 The matching model with wage bargain

The case in the matching model with wage bargain is analytically much simpler. In fact the wage-setting curve (12) can directly be rewritten as

$$w = \frac{\beta(1-\tau)(r+\lambda+\theta q(\theta))(a+\kappa) + (1-\beta)(r+\lambda+q(\theta))b}{\beta(1-\tau)(r+\lambda+\theta q(\theta)) + (1-\beta)(r+\lambda+q(\theta))(1-\tau^a)}$$

It is then straightforward to see that an increase in the average tax rate τ^a increases the gross wage.

Appendix D A static matching model with wage bargaining

The aim of this appendix is twofold. First we want to develop a static matching model with wage bargain and intuitively explain the differences to the dynamic model. We show that tax changes have similar effects in both models. Second, we calculate the optimal marginal tax rates for both the static and dynamic model. As we show, these optimal marginal tax rates coincide.

D.1 The static model

The setting is similar to the dynamic model. Workers search for a job and firms post vacancies. When they match, they bargain about the wage rate. Finally, output is produced.

Since the setting is static, all individuals are without job at the beginning, so that $L = 1$ in the matching function (4).

The firms have to pay a cost κ to post a vacancy. If they find a worker, they produce output a and pay a wage w to the worker. Otherwise they produce nothing. Firms enter the market and open vacancies as long as the expected profit is positive,which gives the vacancy supply curve

$$\frac{\kappa}{q(\theta)} = a - w \tag{25}$$

Comparing this equation with the vacancy-supply curve (8) of the dynamic model, one notes that the curve is steeper in the static model in a $w - \theta$ space.

If the individual finds a job and agrees upon a wage w with the firm, he earns the net income $w - T(w)$. If he does not find a job or if there is no agreement with the firm on the wage rate, he gets the utility b of an unemployed individual. This utility may consist of the unemployment benefit, but may also include the utility of leisure or household production. The bargaining problem of the firm and the worker can thus be written

$$\max_{w} \quad (w - T(w) - b)^{\beta} (a - w)^{1-\beta}$$

which leads to the first-order condition

$$w = \frac{\beta(1-\tau)a + (1-\beta)(T+b)}{\beta(1-\tau) + (1-\beta)} \tag{26}$$

This equation again looks very similar to the equation (12) in the dynamic model. The main difference is that the wage rate w is now independent of labour market tightness θ, i.e. the wage-setting equation is now a horizontal line in the $w - \theta$ space.

The impact of tax policy on outcomes are qualitatively the same. An increase in the marginal tax rate decreases the wage by (26), and this lower wage increases labour market tightness by (25). Similarly, an increase in the tax increases wages and decreases labour market tightness.

D.2 Optimal marginal tax rates

The social planner is only concerned about efficiency, and does not care about redistribution. He thus maximises total net output. We further assume that the unemployment benefit b and the tax T are fixed. We derive the optimal marginal tax rates for both the static and dynamic model. We consider the case where the matching function is of the Cobb-Douglas form such that (the absolute value of) the elasticity of $q(\theta)$ is a constant η.

D.2.1 The dynamic model

We assume that the economy is infinitely lived. The total net output can then be written as

$$Y = \int_0^{\infty} e^{-rt} \left[a(1-u) - \kappa\theta u\right] dt$$

where u denotes the unemployment rate. The social planner's choice is also subject to the matching function, such that the evolution of the unemployment rate can be written

$$\dot{u} = \lambda (1 - u) - \theta q(\theta) u \tag{27}$$

Let μ denote the co-state variable. The optimal path of unemployment and labour market tightness then satisfies (27) and the first-order conditions

$$\begin{aligned} -\dot{\mu} &= e^{-rt} (a + \kappa\theta) - (\lambda + \theta q(\theta)) \mu \\ 0 &= e^{-rt} \kappa u + \mu u q(\theta) (1 - \eta) \end{aligned}$$

Putting these two equations together and solving at steady state, one obtains the following equation that implicitly defines the optimal value of θ:

$$(1 - \eta) a - \frac{\kappa}{q(\theta)} [(1 - \eta)(r + \lambda) + \eta (r + \lambda + \theta q(\theta))] = 0 \tag{28}$$

The private outcome is given by equations (8) and (12). We then search for the values of τ such that these two equations imply (28).

First,note that introducing the free-entry condition (8) into (28) gives

$$w = \eta a \tag{29}$$

as optimality condition. Next, putting (8) and (12) together lead to the following equation

$$\begin{aligned} 0 &= (1 - \beta)(a - T - b) \\ &\quad - \frac{\kappa}{q(\theta)} [(1 - \beta)(r + \lambda) + \beta (1 - \tau)(r + \lambda + \theta q(\theta))] \end{aligned} \tag{30}$$

This equation looks similar to (28) and it is straightforward to see that if there is no government intervention (thus $T = b = \tau = 0$), the social optimum is only achieved if $\beta = \eta$. This is the so-called Hosios condition (Hosios, 1990).

Putting together the equations (28) and (30) and using the vacancy supply curve (8), (19) and (20), the private outcome and the social optimum coincide if

$$-\frac{1-\beta}{\beta} \frac{T+b}{w-T-b} \frac{W-U}{J^f - J^o} + \frac{\eta - \beta}{\beta} \frac{w}{w-T-b} \frac{W-U}{J^f - J^o} + \frac{\eta}{\beta} = 1 - \tau \tag{31}$$

Moreover, the first-order condition on the Nash product implies

$$\frac{W-U}{J^f - J^o} = \frac{\beta (1 - \tau)}{1 - \beta}$$

Introducing this into (31), using (29) and solving for τ, one finally obtains

$$\tau = 1 - \frac{\eta}{\beta} \frac{1-\beta}{1-\eta} \left(1 - \frac{T+b}{\eta a}\right)$$

that describes the optimal marginal tax rate.

D.2.2 The static model

Total net output in the static model is given by

$$\theta q(\theta)\, a - \theta\kappa$$

The maximisation with respect to θ gives

$$(1-\eta)\, q(\theta)\, a - \kappa = 0$$

This equation defines implicitly the optimal value of labour market tightness θ.

The free-entry condition in the decentralised economy can be written

$$q(\theta)(a-w) - \kappa = 0$$

Introducing this free-entry condition into the equation that defines the optimal value for θ, one again obtains $\eta a = w$, as in the dynamic model.

Further, from the first-order condition of the Nash bargaining problem,

$$\frac{a-w}{w-T-b} = \frac{1-\beta}{\beta(1-\tau)}$$

Putting all these equations together and solving for τ, one obtains again

$$\tau = 1 - \frac{\eta}{\beta}\frac{1-\beta}{1-\eta}\left(1 - \frac{T+b}{\eta a}\right)$$

which is the same value as in the dynamic model.

The optimal marginal tax rates thus coincide in the dynamic and the static models.

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