

# On Dictatorship, Economic Development and Stability\*

Lionel Artige

Universitat Autònoma de Barcelona

October 14, 2004

## Abstract

This paper aims to account for varying economic performances and political stability under dictatorship. We argue that economic welfare and social order are the contemporary relevant factors of political regimes' stability. Societies with low natural level of social order tend to tolerate predatory behavior from dictators in exchange of a provision of civil peace. The fear of anarchy may explain why populations are locked in the worst dictatorships. In contrast, in societies enjoying a relative natural civil peace, dictatorship is less likely to be predatory because low economic welfare may destabilize it.

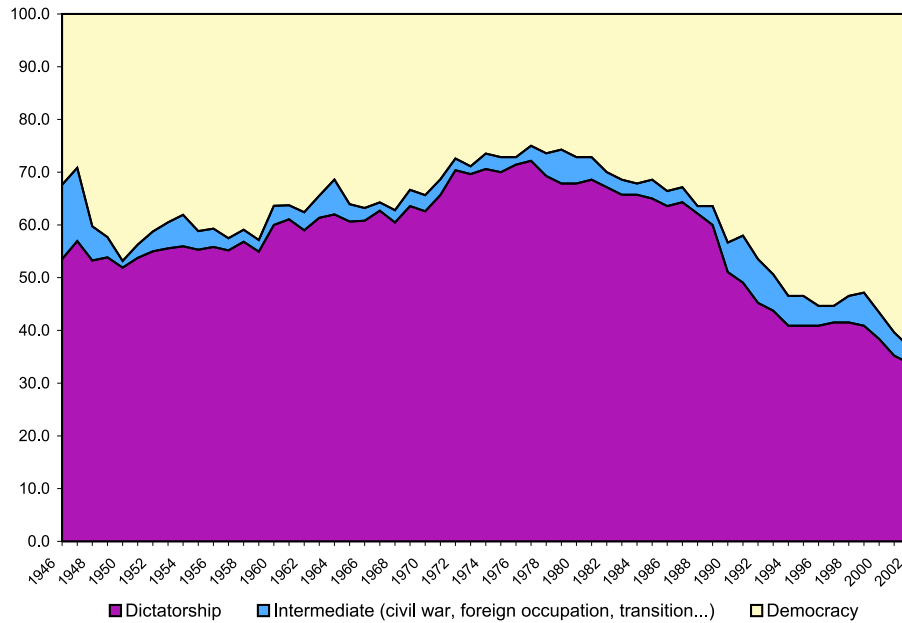
Keywords: anarchy, dictatorship, economic development, predation, social order.  
JEL Classification numbers: H1, H5, O1, P16.

---

\*I have greatly benefitted from comments of David de la Croix, Philippe Michel, as well as of Klaus Desmet, Philippe Monfort, Henri Snessens, participants of the 2003 Public Choice Annual Meeting in Nashville and participants of the Macroeconomic seminar at the Universitat Autònoma de Barcelona. Responsibility for all errors is my own. I acknowledge the financial support of the Belgian French speaking community (Grant ARC 99/04-235 "Growth and incentive design") and of the Belgian Federal Government (Grant PAI P5/10, "Equilibrium theory and optimization for public policy and industry regulation"). Address for correspondance: Universitat Autònoma de Barcelona, Edifici B, Campus de la UAB, 08193 Bellaterra, Barcelona, Spain. Email: lartige@idea.uab.es

# 1 Introduction

Dictatorship - as a catch-all concept for non-democratic societies - is universally considered as an undesirable political regime.<sup>1</sup> Although the number of dictatorships has significantly decreased in the last two decades (Figure 1) and across continents, this regime has been persistent and widespread throughout history, still exists in large parts of the world today and constantly threatens fragile democracies.<sup>2</sup>



Source: Polity IV data set.

Figure 1: Political regimes (% of countries in the world)

If a political regime is assumed to be the result of rational choices rather than a cultural or a random event, what can explain the existence and persistence of a decried political regime in many societies?

We propose to define dictatorship by the concentration of force, the political specialization and the absence of a constraint obliging the government to be continuously responsive

<sup>1</sup>The normative judgement on political regimes has evolved much through history. Before the French Revolution, monarchy was considered as the best form of government in most of the classics of political thought. From the nineteenth century on, democracy has gained the status of the best political regime (Bobbio 1989).

<sup>2</sup>For instance, Russia under the rule of Vladimir Putin is moving away from democracy little by little with, as it seems, the support of its population (Pipes 2004). Pakistan has returned to dictatorship in 1999 after 12 years of democracy while its neighbor, India, remains firmly democratic. In Latin America, a recent UN report shows a large disillusionment regarding democracy. This represents a worrying and partly surprising feeling in a continent with a long experience of tough and bloody dictatorships (UNDP 2004).

to the preferences of its citizens. The first two criteria characterize the existence of a state apparatus that is common to dictatorships and democracies but differentiates them from anarchy. There is political specialization when some individuals or groups play political roles and others not. These politicians enforce their decisions thank to the monopoly of organized force. The third criterion distinguishes dictatorship and democracy by their procedures along the lines of modern political thought (Dahl 1971). Therefore, this definition suggests the existence of two possible forms of government - dictatorship or democracy - and one possible situation characterized by the absence of a government. Since Kelsen's *General Theory of the Law and State* (1945) the dichotomic classification of political regimes has been widely accepted. Kelsen suggested to oppose democracy and autocracy<sup>3</sup> based on the level of political liberty. The procedural approach of modern political science retains this dichotomy and endorses the norm inherent in the use of Kelsen's criterion. Supported by overwhelming empirical evidence, past and present, dictatorship has become a byword for any regime violating freedom and other basic human rights. Hence, its universal dislike.

However, its persistence through history and the defense of some forms of non-democratic regimes by philosophers from Antiquity to the nineteenth century are puzzling. Plato and Aristotle classified political regimes according to the number of rulers, namely one, a few or many. For each type of this tripartite division, they established good and corrupt political regimes depending on whether government is in the common interest or self-interested. Their classifications suggest a large variety of government forms and an issue of stability of the good ones. Nevertheless, the good and the bad types ultimately depend on the classification criteria, often, reflecting the perspective either from the individual or from the society. For instance, Bobbio (1989) argues that, starting with the same conceptions about the state of nature and power, Hobbes favors monarchy while Spinoza leans towards democracy. The former considered peace and order as the supreme objective while the latter focused on liberty. The dispute between Hobbes and Spinoza pleads for a multi-dimensional analysis of political regimes.

During these centuries of philosophical debate, the arguments were limited to the political domain and no mention was made about any economic dimension. This is hardly surprising as income growth per capita was then at best of imperceptible magnitude. From the industrial revolution on, the living standards of ordinary people growing at an unprecedented rate in some countries and the large income disparities that followed between them and those of the countries which did not take off have modified the context and created a new dimension susceptible to differentiate political regimes. After Lipset (1959) formulated the hypothesis that democracy was related to economic development, the analysis of political regimes based on the economic performance's criterion has developed enormously. The increasing availability of data has also encouraged the realization of many empirical studies but has failed so far to lead to clear-cut conclusions.<sup>4</sup>

Using a data set encompassing the experiences of 135 countries between 1950 and 1990, a

---

<sup>3</sup>The term 'autocracy' has been widely replaced in the literature as in the public by the term 'dictatorship' after the Bolshevik and the fascist regimes of the 1930s (Bobbio 1989).

<sup>4</sup>See surveys by Sirowy and Inkeles (1990) and Przeworski and Limongi (1993).

recent work by Przeworski, Alvarez, Cheibub, and Limongi (2000) confirms the existence of a link between political regimes and economic development, as measured by per capita income. Almost 80% of their annual observations of regimes are predicted only by using this indicator. Unfortunately, even though this result is strong, the authors rightly stress that it is inconclusive about the way the relationship goes. If the political regime were the cause of economic development, the result would be useful to define a political theory of economic development. If economic development determined political regime, then it would deliver an economic theory of political transition (endogenous democratization or 'dictatorization'). The contemporary literature favors the first causality on the basis of personal beliefs in default of doubtless empirical evidence.<sup>5</sup>

Three other of their results do not provide much help to pick either causality. First, they confirm a well-known result that, on average, per capita incomes are lower in dictatorships. As various statistical tables can show, the poorest countries in the world are dictatorships. All the famines have happened under autocratic rule (Drèze and Sen 1989). Nevertheless, the relationship still can go every which way. Second, they found that growth rates are either very low or very high in dictatorships and in between in democracies. This observed large variance in economic performance casts doubt on the effect of dictatorship on economic development. Dictatorial regimes can exhibit economic miracles and disasters.<sup>6</sup> For instance, since the economic reforms of 1978, China's GDP per capita in constant US dollars has doubled every 8-9 years. Finally, their empirical results reveal that dictatorships are likely to democratize in middle-income countries and remain stable in low- and high-income countries, thus invalidating endogenous democratization.<sup>7</sup>

Despite their inconclusive results, these empirical studies offer two preliminary findings. First, the economic dimension is relevant for the analysis of political regimes. Second, economic welfare is not neutral for the stability and instability of political regimes.

The present paper attempts to account for the large variance of economic performance under dictatorship and its political stability. This will depend ultimately on the distance between the preferences of the ruled in terms of economic welfare and social order, and the effect of the dictator's own preferences on these two variables. We think that economic welfare and social order are the contemporary relevant factors of legitimacy and, hence, stability of political regimes. By social order it is meant security of persons and their property. As mentioned before, the economic factor is relatively recent and backed by empirical evidence. Social order has been a perpetual preoccupation of political thinkers and one of the central problems of political philosophy. Before any form of government, there are two ways to settle arrangements among individuals whenever they interact: peacefully or conflictingly. Even though the end-goal of most people is to live in peace, conflict is a means that cannot be ruled out to achieve it. Considering these two possibilities for human behavior leads to Thomas Hobbes' state of nature. Hobbes in *Leviathan* (1651) thought that the human search to secure 'felicity' through power, the possibility

---

<sup>5</sup>Barro (1996) and Boix and Stokes (2003), for example, favor the second causality and focus on endogenous democratization.

<sup>6</sup>See also examples in Temple (1999) (table 2).

<sup>7</sup>Boix and Stokes (2003) challenge this econometric result.

that any human being had the capacity of killing any other, and the scarcity of goods would bring all at war with all. In fact, even the abundance of goods can lead to war. If a good is excludable (such as a primary resource for example), a finite large amount of it does not necessarily imply peace. The power that the monopoly of such a good can give to the holder is sufficient to drive everyone into a fight to control the production of this good, either to prevent others from establishing a disproportionate power or to acquire it oneself. Whenever individuals interact, power becomes an issue. In the state of nature there is no guarantee that individuals will refrain voluntarily accumulating and using it to threaten each other, even for legitimate purposes. Thus, social order is uncertain due to the absence of any concentration of force and of any political specialization together. This is precisely the characterization of *anarchy* (Taylor 1982). In this situation, everyone has to face alone all the potential threats with uncertain results. This prospect is fearsome for most people. That is the reason why social order is generally found desirable and anarchy disliked. Nonetheless, the problem remains to find a mechanism performing the maintenance of social order. Such a mechanism is generally called an *institution* and social order a *public good*. Because some individuals may free ride on bearing the provision cost of social order in large societies, a voluntary agreement is likely to fail. This was the main argument on which Hobbes justified in *Leviathan* the establishment of a state. The internal wars of Hobbes' time corresponded to the asserting of the state in England and France through the victories of monarchs over the multitude of local lords. This institutional success provided a peaceful internal environment propitious for the nascent sustained economic growth. Many other nations in Europe and in the rest of the world will soon try to imitate this pattern.

Once the concentration of force is realized and social order successfully maintained, the possibility of abusing the asymmetry of force creates another threat against individuals: the threat of dictatorship. The problem is then how to limit the state's power to avoid dictatorship. Since the state cannot commit itself, individuals swap potential dispersed violence against potential centralized violence. One may wonder whether they are eventually better off. Olson (1993) provides an argument to assert so. In dictatorship, the single ruler has an 'encompassing interest' in refraining from preying on all revenues today to have something to prey on in the future. In other words, his own intertemporal welfare is tied to the individuals' welfare. In anarchy, none has the force to dominate the others. This gives every individual the incentive to prey as much as possible whenever possible, leaving nothing to build the future. As it will be proved in this paper, Olson's economic argument does not constitute a limit to the dictator's power. However it provides an important intuition to understand dictatorship's stability.

In the political economy literature, there are a number of works which consider predation and violence as an economic activity competing with production activity (Usher (1989), Grossman and Noh (1990), Grossman (1991), Hirshleifer (1991), Olson (2000)). The individuals face a tradeoff between the appropriative and production activities creating a threat to both economic development and social order. The resulting need for security gives the Hobbesian justification for the establishment of the state. However, the centralization of violence through the state could also lead to the centralization of predation, also

hampering economic development (North (1990), Grossman and Noh (1990), Grossman (1996), and Wintrobe (1998)). Even in democracies economic development can be halted by corrupt governments (Kanczuk (1998) and Azariadis (2001)). The question is then why inefficient political institutions are not necessarily unstable. Dictatorial regimes can resist democratization because wealth inequality might be such that it is too costly for a governing elite to accept an inevitable redistribution under a democratic regime (Acemoglu and Robinson 2001). Another explanation can be the difficult cooperation that the toppling of a dictator requires, especially when the ruler can use plenty of resources to divide and rule (Acemoglu, Robinson, and Verdier 2004). However, the difficulty of democratization can also be accounted for by the possible existence of a violent transition on the road to democracy. This potential violence may lock populations in a dictatorial *statu quo*. This is the view that our paper takes up by analyzing the economic performance and the stability of dictatorship in relation to anarchy. In many societies, the gain of deposing a dictator must be assessed against the risk of anarchy. This threat may be sufficient for the people to prefer keeping the dictator.

Section 2 characterizes the model of dictatorship. The relationship between dictatorship and economic development is analyzed in section 3. Section 4 studies the political stability of dictatorship. The role of the threat of anarchy is examined in section 5. Finally, section 6 concludes.

## 2 A model of dictatorship: characterization

Consider an economy under dictatorship that is populated by overlapping generations of people living for two periods and ruled by a dictator. We assume that the dictator is infinitely lived (i.e. the dictator and his successors) because we are interested in dictatorship as a regime. The dictator has the monopoly of power over the fiscal and police administrations. He solely determines the amount of taxes to levy. A part of them is allocated to the payroll of his police forces, judiciary and fiscal agents as well as to his own consumption. The other part is used to finance the maintenance of social order.

We study the behavior of this dictator in an overlapping generations model extended from Diamond (1965). Time is discrete and goes from 0 to  $\infty$ . At each period there are  $N$  (normalized to 1) citizens and there is no population growth.

### 2.1 The economy

All agents (including the dictator) in this economy are price-takers and all markets are competitive. At each period  $t$ , there is one physical good produced from capital,  $K$ , and labor,  $L$ . This physical good can be consumed or invested by the citizens to build future capital, and consumed by the dictator or redistributed to the citizens under the form of a public good representing social order. Thus, government consumption is unproductive. We take the good produced at each period  $t$  as the numéraire.

### 2.1.1 Technology

The production technology is represented by a Cobb-Douglas production function,  $F(K_t, L_t) = AK_t^\alpha L_t^{1-\alpha}$ , exhibiting constant returns to scale. This technology is the same for all periods. At the end of each period the capital stock is completely depreciated.

We assume a representative firm producing at period  $t$ . The capital stock is owned by the households who receive the profits of the firm. Since the production function is homogenous of degree one, we can rewrite as:

$$f(k_t) = Ak_t^\alpha, \quad (1)$$

where  $A > 0$  and  $\alpha < 1$  are technological parameters and  $k = \frac{K}{L}$ .

### 2.1.2 Households

**Preferences** The individuals live for two periods. When young, they work, receive a wage and allocate this income between consumption and savings. When old they retire and consume the return of their savings. These consumers also care about social order which is preserved by institutions such as the police and the justice. The objective of the households is to maximize their utility deriving from consumption and social order. The young generation has to make two decisions: one about how much to save and one about whether to overthrow the dictator.

The behavior of the individuals is represented by a logarithmic life-cycle utility function, which is assumed to be additively separable,

$$\ln(c_t) + \beta \ln(d_{t+1}) + \ln(p_t + l) + \beta \ln(p_{t+1} + l). \quad (2)$$

The left part of (2) is the utility derived from consumption of the single private good at time  $t$ ,  $c_t$ , and at time  $t + 1$ ,  $d_{t+1}$ .  $\beta > 0$  is the discount factor denoting a relative impatience.  $l > 0$  is the exogenous natural level of social order associated with a particular society. The budget constraint is

$$c_t + s_t = (1 - \tau_t)w_t \quad (3)$$

$$d_{t+1} = (1 - \tau_{t+1})R_{t+1}s_t, \quad (4)$$

where savings are denoted  $s_t$ , the real wage is  $w_t$  and the interest factor is  $R_{t+1}$ .

The second part of (2) is the utility derived from the public good, the level of social order,  $p$ , defined by :

$$p_t = (1 - q_t)\tau_t f(k_t), \quad (5)$$

where  $\tau \in (0, 1)$  is the income tax rate set by the dictator and  $q_t \in (0, 1)$  is the rate of predation. The public good<sup>8</sup>,  $p$ , provided by the dictator, represents the protection of

---

<sup>8</sup>Social order cannot be a private good because it is a good with externalities. The absence of violence is essentially a social phenomenon. If an individual pays to prevent another individual from using violence against him, both will benefit from the absence of violence. Therefore, both will have an incentive to free ride on bearing the cost of social order.

persons and individual property rights on the consumption good against other individuals, but not against the dictator's predation.<sup>9</sup>

**Optimal behaviors** The representative consumer maximizes (2) subject to (3) and (4). The first order conditions yield:

$$\begin{aligned} c_t &= \frac{1}{1+\beta} (1-\tau_t)w_t, \\ s_t &= \frac{\beta}{1+\beta} (1-\tau_t)w_t. \end{aligned}$$

The representative firm must only choose the labor input and maximizes profits subject to (1). Marginal productivities are thus equal to factors prices:

$$\begin{aligned} R_t &= \alpha Ak_t^{\alpha-1}, \\ w_t &= (1-\alpha)Ak_t^\alpha. \end{aligned}$$

The savings function is therefore:

$$s_t = \frac{\beta}{1+\beta} (1-\tau_t)(1-\alpha)Ak_t^\alpha. \quad (6)$$

Savings increases with the wage and decreases with the income tax.

## 2.2 The dictatorial regime

At the initial date,  $t = 0$ , the dictatorial regime is already in place starting with initial conditions reflecting its history. The objective is to study dictatorship from date  $t = 0$  onwards.

### 2.2.1 The dictator

**Definition** A dictator is an individual who uses his monopoly of force across a territory and through time to impose his own preferences. Therefore, following the criteria proposed by Przeworski and Limongi (1993) to define political regimes, the dictator monopolizes the locus of decision-making and the property right to the fiscal residuum which is the difference between the total output and the cost of government.

---

<sup>9</sup>Alternatively,  $p$  could be interpreted as the provision of public utilities or infrastructure. What is important for the analysis is that the public good, whatever it is, requires an authority to deliver it. If the authority breaks down, there is no spontaneous private substitute for this public good.



**Preferences** The dictator maximizes an additively separable utility function of the form:

$$U = \sum_{t=0}^{\infty} \frac{g_t^{1-\sigma}}{(1-\sigma)(1+\rho)^t}, \quad \sigma \geq 0, \rho > 0 \quad (7)$$

subject to

$$g_t = q_t \tau_t f(k_t), \quad (8)$$

where  $\tau \in (0, 1)$  is the income tax rate,  $q_t \in (0, 1)$  is the rate of predation and  $g_t$  is the dictator's consumption stream at time  $t$ . The consumption  $g_t$  includes all the expenditures required to maintain his power. All of this is financed by an income tax whose rate,  $\tau_t$ , is set by himself. His choice depends on  $\rho$ , the dictator's rate of time preference,  $q$ , the intensity of predation and  $\sigma$ , the degree of concavity of the utility function.

The parameter  $\rho$  describes the dictator's attitude toward the future. A high  $\rho$  means that the dictator has a strong preference for the present and cares little about the future. This will result in level effects on the long-term output.

### 2.2.2 Predation and repression

The dictator imposes his preferences through violence against the population. However, the police forces that are the intermediaries of the exertion of the dictator's violence do not work for free. Violence has its price: the wages of these repression forces. *A priori*, the individuals who oppose the dictator are not willing to buy this violence. As a result, the dictator will seize a part of the individuals' income to finance violence. Violence and predation are thus intertwined: predation involves violence and violence requires predation.

Therefore, the rate of predation, represented by  $q_t$ , also reflects the level of repression in the economy. Formally,  $q_t$  determines the amount of tax revenues that the dictator keeps for himself and  $(1 - q_t)$  is the rate of social order provided to the citizens. The more the dictator wants to consume, the more repression is needed to make it possible and the less resources to maintain social order. The tax rate,  $\tau$ , and the rate of predation,  $q$ , thus characterize all possible types of dictatorships. The higher  $\tau$  and  $q$ , the more predatory the dictator.

### 2.2.3 The insurrection threat

By using repression the dictator has the power to prey on national revenues. There are two possible reactions of the population to repression and predation: obedience and insurrection. Obedience means toleration of the regime and welfare losses. Insurrection must imply a cost, otherwise no dictatorship would ever survive. In Grossman (1991), the cost of insurrection is the foregone production by devoting time to the insurrection. It can also be the mortality risk in a framework with probabilistic survival as in Usher (1989). In our approach, insurrection leads to anarchy and the cost of rebellion is the loss of social order. This loss emphasizes the cost of an *institutional vacuum*. Then, people have to compare their welfare in dictatorship with that in anarchy. If the households

consider that repression is too high and their economic welfare too low, they may plan an insurrection and depose the dictator. If they do so, then  $\tau_t = 0$  and  $p_t = 0$ ; they get rid of the burden of the dictator's consumption but also the authority setting the tax rate and the provision of social order. All the institutions collapse so that the individual rights, as few as they were, are no longer guaranteed and there is no longer an established authority to make any decision. This situation corresponds to anarchy where everyone has to fear everyone.

At date  $t$ , the dictator holds power and imposes a tax rate to the households. At the end of the period  $t$ , the households must make a decision about to rise against the dictator or not and, hence, tolerate or depose the dictator at period  $t + 1$ . The decision is made after comparing the expected welfare in dictatorship and that in anarchy. If the gain of insurrection in terms of consumption and social order is higher than the loss, then people revolt. If they do not overthrow the dictator, their intertemporal utility is as follows:

$$\ln \left( \frac{(1 - \tau_t)w_t}{1 + \beta} \right) + \beta \ln \left( \frac{(1 - \tau_{t+1})R_{t+1}\beta(1 - \tau_t)w_t}{1 + \beta} \right) + \ln(p_t + l) + \beta \ln(p_{t+1} + l)$$

If they topple the dictator from power, their intertemporal utility is:

$$\ln \left( \frac{(1 - \tau_t)w_t}{1 + \beta} \right) + \beta \ln \left( \frac{R_{t+1}\beta(1 - \tau_t)w_t}{1 + \beta} \right) + \ln(p_t + l) + \beta \ln l$$

As a result, the condition for an insurrection is the difference between the two. This difference must be negative:

$$I_{t+1} = \ln[(1 - \tau_{t+1})] + \ln[(1 - q_{t+1})\tau_{t+1}Ak_{t+1}^\alpha + l] - \ln l < 0. \quad (9)$$

Obviously, for the dictator the constraint  $I$  must be positive or null.

## 2.3 Equilibrium

The equilibrium on the goods market at time  $t$  is given by the accounting identity in per capita terms,

$$y_t = f(k_t) = g_t + c_t + d_t + s_t + p_t, \quad (10)$$

where  $y_t$  is output,  $c_t$  consumption,  $s_t$  savings and  $p_t$  resources devoted to the maintenance of social order at time  $t$ .

The total stock of capital is built from the savings of the young generation:

$$k_{t+1} = s_t. \quad (11)$$

### 2.3.1 Problem of the dictator

The objective of the dynastic dictator is to maximize his utility with respect to consumption,  $g_t$ , from time 0 to  $\infty$ . This requires that he must stay in power through the

entire period. In other words, the problem is to find an equilibrium path that guarantees the permanence of dictatorship and a maximum present value of utility for the dictator. Since the dictator has no savings<sup>10</sup>, he must choose a suitable tax rate to have an optimal intertemporal utility. The income tax rate chosen by the dictator,  $\tau_t$ , is the solution to its maximization program:

$$\max_{g_t} U = \sum_{t=0}^{\infty} \frac{g_t^{1-\sigma}}{(1-\sigma)(1+\rho)^t}, \quad \sigma \geq 0,$$

subject to,

$$g_t = q_t \tau_t A k_t^\alpha \quad (12)$$

$$\ln[(1 - \tau_{t+1})] + \ln[(1 - q_{t+1})\tau_{t+1}A k_{t+1}^\alpha + l] - \ln l \geq 0. \quad (13)$$

$$k_{t+1} = \frac{\beta}{1+\beta}(1 - \tau_t)(1 - \alpha)A k_t^\alpha \quad (14)$$

$$0 \leq q_t \leq 1 \quad (15)$$

Equation (12) is the dictator's budget constraint. Equation (13) represents the no-insurrection constraint and equation (14) is an implementation (binding) constraint, describing how households react to the dictator's choice of tax rate. Equation (15) indicates the feasible values for  $q$ .

Thus, the behavior of the dictator depends on two exogenous preference parameters ( $\sigma$  and  $\rho$ ) and three endogenous variables ( $\tau$ ,  $q$  and  $k$ ).

### 2.3.2 First-order Kuhn-Tucker conditions

The first-order Kuhn-Tucker conditions are computed by forming and maximizing the following Lagrangean with respect to  $\tau_t$ ,  $q_t$  and  $k_{t+1}$ :

$$\begin{aligned} \max_{\tau_t, q_t, k_{t+1}} \mathcal{L} &= \sum_{t=0}^{\infty} \frac{(q_t \tau_t A k_t^\alpha)^{1-\sigma}}{(1-\sigma)(1+\rho)^t} - \\ &- \sum_{t=0}^{\infty} \frac{\lambda_t}{(1+\rho)^t} \left\{ k_{t+1} - \frac{\beta}{1+\beta}(1 - \tau_t)(1 - \alpha)A k_t^\alpha \right\} + \\ &- \sum_{t=0}^{\infty} \frac{\mu_t}{(1+\rho)^t} \left\{ \ln[(1 - \tau_{t+1})] + \ln[(1 - q_{t+1})\tau_{t+1}A k_{t+1}^\alpha + l] - \ln l \right\} \\ &- \sum_{t=0}^{\infty} \frac{\delta_t}{(1+\rho)^t} \{ q_t - 1 \} \end{aligned}$$

---

<sup>10</sup>The dictator does not need save as he has the power to prey on national resources in the future. In an open economy framework, he could open saving accounts in foreign countries, as it often happens to be the case. This would diminish the effect of the implementation constraint and hence would increase his predatory appetite, if he were certain to rely on these foreign funds and their returns later on.

The first-order Kuhn-Tucker conditions are :

$$\begin{aligned} \tau_t \frac{\partial \mathcal{L}}{\partial \tau_t} &= (q_t \tau_t A k_t^\alpha)^{1-\sigma} - \lambda_t \tau_t \left\{ \frac{\beta}{1+\beta} (1-\alpha) A k_t^\alpha \right\} - \\ &- \mu_{t-1} (1+\rho) \tau_t \left\{ \frac{-1}{1-\tau_t} + \frac{(1-q_t) A k_t^\alpha}{(1-q_t) \tau_t A k_t^\alpha + l} \right\} = 0 \end{aligned} \quad (16)$$

$$\begin{aligned} q_t \frac{\partial \mathcal{L}}{\partial q_t} &= (q_t \tau_t A k_t^\alpha)^{1-\sigma} + \\ &+ q_t \mu_{t-1} (1+\rho) \left\{ \frac{\tau_t A k_t^\alpha}{(1-q_t) \tau_t A k_t^\alpha + l} \right\} - q_t \delta_t = 0 \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial k_{t+1}} &= \alpha (q_t \tau_{t+1} A)^{1-\sigma} k_{t+1}^{\alpha(1-\sigma)-1} - \lambda_t (1+\rho) + \\ &+ \lambda_{t+1} \left\{ \frac{\beta}{1+\beta} \alpha (1-\alpha) (1-\tau_{t+1}) A k_{t+1}^{\alpha-1} \right\} - \\ &- \mu_t (1+\rho) \left\{ \frac{(1-q_{t+1}) \tau_{t+1} \alpha A k_{t+1}^{\alpha-1}}{(1-q_{t+1}) \tau_{t+1} A k_{t+1}^\alpha + l} \right\} = 0 \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \lambda_t} &\geq 0, & \lambda_t &\geq 0, & \lambda_t \frac{\partial \mathcal{L}}{\partial \lambda_t} &= 0 \\ \frac{\partial \mathcal{L}}{\partial \mu_t} &\geq 0, & \mu_t &\geq 0, & \mu_t \frac{\partial \mathcal{L}}{\partial \mu_t} &= 0 \\ \frac{\partial \mathcal{L}}{\partial \delta_t} &\geq 0, & \delta_t &\geq 0, & \mu_t \frac{\partial \mathcal{L}}{\partial \delta_t} &= 0 \end{aligned}$$

$$\lim_{t \rightarrow \infty} \frac{k_t g_{t-1}^{-\sigma}}{(1+\rho)^t} = 0 \quad (19)$$

Equation (16) is the Kuhn-Tucker first-order condition with respect to  $\tau_t$ , equation (17) is the Kuhn-Tucker first-order condition with respect to  $q_t$  and equation (18) is the first-order condition with respect to  $k_{t+1}$ . Equation (19) is the transversality condition stating that the actual value of the capital in terms of welfare is exhausted.

### 2.3.3 Dictatorship's equilibrium

Given initial condition  $\{k_0\}$ , a dictatorship's equilibrium can be characterized by a path  $\{k_{t+1}, \tau_t, q_t, \lambda_t, \mu_t\}_{t>0}$  such that equations (13)-(18) hold.

### 3 Dictatorship and economic development

In this section, we want to know whether all types of dictatorship regardless of political constraints are economically feasible. Therefore, we do not take the insurrection constraint (13) into account, i.e., we set  $\mu = 0$  in equations (16) to (18). We thus study the existence and stability of steady states for all types of dictatorship and examine their implications on economic development. The dynamic system of dictatorship without insurrection threat is as follows:

$$k_{t+1} = \frac{\beta}{1+\beta}(1-\tau_t)(1-\alpha)Ak_t^\alpha \quad (20)$$

$$q_{t+1} = 1 \quad (21)$$

$$\tau_{t+1} = \frac{\alpha^{\frac{1}{\sigma}}\tau_t k_t^{\alpha(\frac{\sigma-1}{\sigma})(1-\alpha)}}{(1+\rho)^{\frac{1}{\sigma}}(1-\tau_t)^{\frac{1+\alpha(\sigma-1)}{\sigma}} \left[ \frac{\beta}{1+\beta}(1-\alpha)A \right]^{\frac{\alpha(\sigma-1)}{\sigma}}} \quad (22)$$

Equation (20) equalizes savings and investment. Equation (21) indicates that the dynamics of the predation rate,  $q$ , is constant and equal to one. Finally, equation (22) gives the expected income tax rate as a function of the current income tax rate and the current capital stock. It is computed by eliminating the Lagrange multiplier  $\lambda$  from (16)-(18), and plugging (20) in (18).

#### 3.1 Steady state

**Proposition 1** *Any form of dictatorship characterized by the dynamic system (20)-(22) admits a unique interior steady state.*

Proof:

- There exists a unique interior steady state characterized by:

$$\begin{aligned} \bar{k} &= \left( \frac{\beta\alpha(1-\alpha)A}{(1+\beta)(1+\rho)} \right)^{\frac{1}{1-\alpha}} \\ \bar{\tau} &= 1 - \frac{\alpha}{1+\rho} \end{aligned} \quad (23)$$

It is straightforward to compute  $(\bar{k}, \bar{\tau})$  by solving the system (20)-(22) at the steady state.

Hence, the dictator's steady state consumption:

$$\bar{g} = q \left( 1 - \frac{\alpha}{1+\rho} \right) A^{\frac{1}{1-\alpha}} \left( \frac{\beta\alpha(1-\alpha)}{(1+\beta)(1+\rho)} \right)^{\frac{\alpha}{1-\alpha}}$$

- The steady state is "degenerate" if and only if  $\sigma > 1$ ,  $k = 0$  and  $\tau = 1$ .

$(k, \tau) = (0, 1)$  is a solution to the system (20)-(22) if and only if the exponent of  $k$  in equation (22) is positive, i.e., if  $\sigma > 1$ . ■

The "degenerate" steady state will not be studied since it has no interest. The interior steady state describes how the dictator's time preference,  $\rho$ , affects the long run output of the economy. If the dictator strongly prefers to consume in the present instead of smoothing his consumption streams, the income tax rate,  $\bar{\tau}$ , will be high and will hamper capital accumulation. Such a political economic argument may be a cause of poverty traps in developing countries (see Kanczuk (1998) and Azariadis (2001)). In contrast, if the dictator cares about his future consumption or, equivalently, about the long-term development of the country he rules, then he will choose a lower tax rate,  $\bar{\tau}$ .

Note that the steady state does not depend on the choice variable  $q$  since it does not enter the rule of the capital accumulation. The dictator enjoys a total freedom to set the rate of predation. However, this choice variable has distributional effects. It affects the utility of the dictator and the consumers. In total, the rate of predation/repression has a big impact on the consumers' life-time utility without long-term effect on the economy.

The dictatorships can thus be distinguished along two dimensions: an intertemporal dimension represented by  $\rho$  and a distributional dimension represented by  $q$ . The former has a level effect on the long-term aggregate output and the latter has a level effect on the individual utility, which depends on consumption and social order.

These two dimensions may help to account for the large variety of living conditions under dictatorship and the possibility of very different economic performances, as it is observed in the empirical literature.

### 3.2 Local stability of the interior steady state

**Proposition 2** *The interior steady state of any form of dictatorship is a saddle point.*

Proof: see appendix A

Two conclusions can be drawn from propositions 1 and 2. First, in the absence of any political constraints, all possible behaviors of a dictator are compatible, in our model, with a unique and locally saddle-path stable interior steady state. Therefore, the economic constraint (the rule of capital accumulation) does not provide any economic limit to the dictator's power. Second, the dictator's behavior has a significant impact on the long-term level of aggregate output.

### 3.3 Phase diagram: a few graphical examples

To describe the dynamics graphically, we build a phase diagram using the two equations of the dynamical system (20)-(22). Following the method shown by de la Croix and Michel (2002), we characterize the set of points  $(k_t, \tau_t)$  for which there is no change in  $k_t$  in equation (20). Solving the equation leads to:

$$1 - \tau_t = \frac{(1 + \beta)}{\beta(1 - \alpha)A} k_t^{1-\alpha} \quad (24)$$

This equation maps the points of the function  $k_{t+1} = k_t$ . Its derivative with respect to  $k$  is:

$$\frac{d(1 - \tau_t)}{dk_t} = \frac{(1 + \beta)}{\beta A} k_t^{-\alpha} > 0$$

Then we characterize the set of points  $(k_t, \tau_t)$  for which there is no change in  $\tau_t$  in equation (22). Solving for this equation leads to:

$$1 - \tau_t = \frac{\alpha^{\frac{1}{1+\alpha(\sigma-1)}}}{(1 + \rho)^{\frac{1}{1+\alpha(\sigma-1)}} \left[ \frac{\beta}{1+\beta}(1 - \alpha)A \right]^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}}} k_t^{\frac{\alpha(\sigma-1)(1-\alpha)}{1+\alpha(\sigma-1)}} \quad (25)$$

This equation maps the points of the function  $\tau_{t+1} = \tau_t$ . Its derivative with respect to  $k$  is:

$$\frac{d(1 - \tau_t)}{dk_t} = \frac{\alpha(\sigma - 1)(1 - \alpha)}{1 + \alpha(\sigma - 1)} \left( \frac{\alpha^{\frac{1}{1+\alpha(\sigma-1)}}}{(1 + \rho)^{\frac{1}{1+\alpha(\sigma-1)}} \left[ \frac{\beta}{1+\beta}(1 - \alpha)A \right]^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}}} \right) k_t^{-\frac{1+\alpha^2(\sigma-1)}{1+\alpha(\sigma-1)}} \leq 0$$

The sign of this derivative depends on  $\sigma$ . If  $\sigma < 1$ , then the derivative is negative. It is positive otherwise.

Hence, the function (24) is increasing in  $k_t$  and the function (25) is decreasing in  $k_t$  if  $\sigma < 1$  and increasing if  $\sigma > 1$ . Knowing these two functions, we can proceed to a graphical exposition with a few examples. These examples will illustrate the dynamics of the economy depending on different types of dictatorships.

#### 3.3.1 Dynamics when $\sigma < 1$

If  $\sigma < 1$ , then the function (24) is increasing and the function (25) is decreasing. They intersect only once at the interior steady state. This steady state is a saddle point.

#### 3.3.2 Dynamics when $\sigma > 1$

If  $\sigma > 1$ , the functions (24) and (25) are both increasing in  $k$  and intersect twice if the slope of (25) is steeper than the one of (24). We can check it by dividing the slope of (24) by the slope of (25) and take the limits when  $k \rightarrow 0$  and  $k \rightarrow +\infty$ :

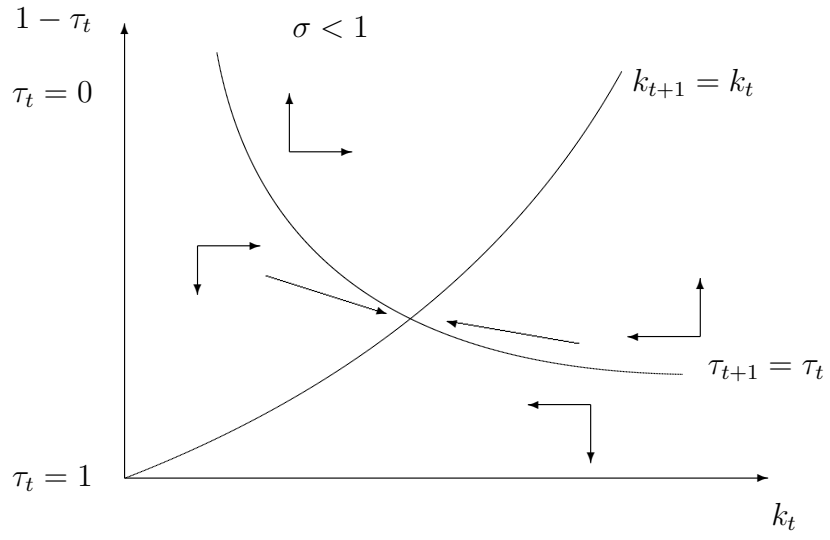


Figure 2: Dynamics of  $\tau$  and  $k$  when  $\sigma < 1$

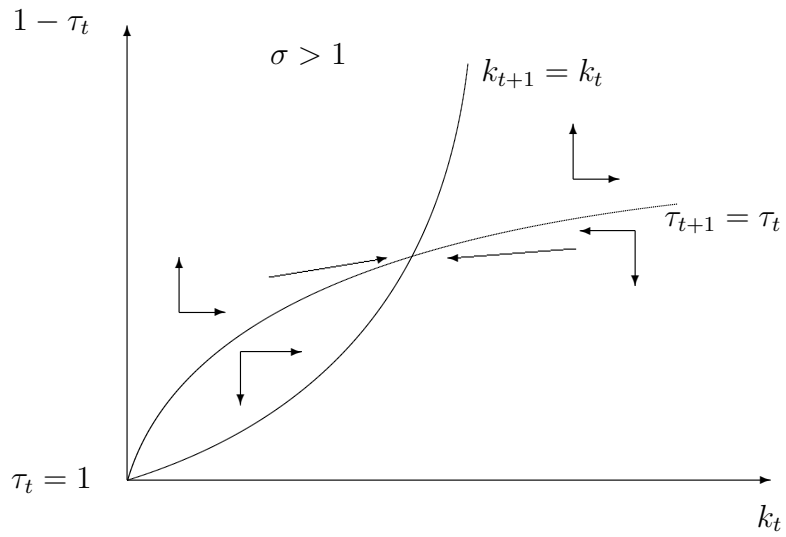


Figure 3: Dynamics of  $\tau$  and  $k$  when  $\sigma > 1$



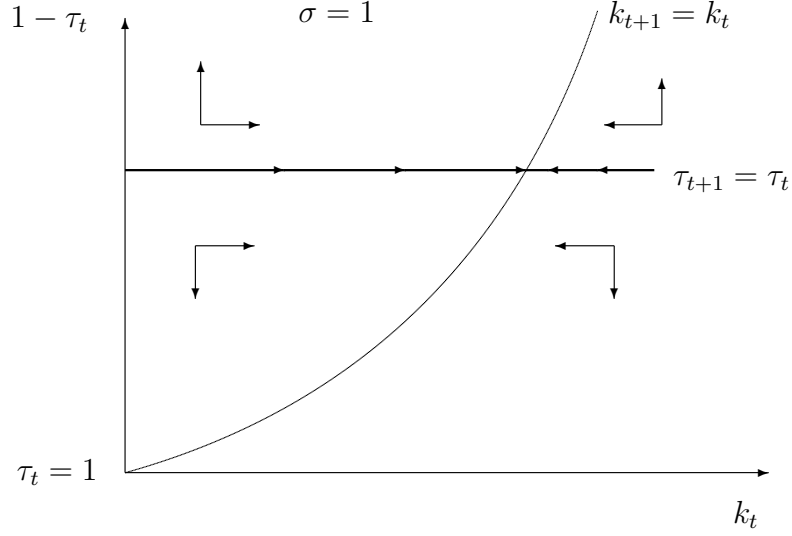


Figure 4: Dynamics of  $\tau$  and  $k$  when  $\sigma = 1$

$$\lim_{k \rightarrow 0} \frac{\frac{(1+\beta)}{\beta A}}{\frac{\alpha(\sigma-1)(1-\alpha)}{1+\alpha(\sigma-1)} \left( \frac{\alpha^{\frac{1}{1+\alpha(\sigma-1)}}}{(1+\rho)^{\frac{1}{1+\alpha(\sigma-1)}} \left[ \frac{\beta}{1+\beta} (1-\alpha) A \right]^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}}} \right)} k^{\frac{1-\alpha}{1+\alpha(\sigma-1)}} = 0$$

since  $\frac{1-\alpha}{1+\alpha(\sigma-1)} > 0$  when  $\sigma > 1$ , and

$$\lim_{k \rightarrow +\infty} \frac{\frac{(1+\beta)}{\beta A}}{\frac{\alpha(\sigma-1)(1-\alpha)}{1+\alpha(\sigma-1)} \left( \frac{\alpha^{\frac{1}{1+\alpha(\sigma-1)}}}{(1+\rho)^{\frac{1}{1+\alpha(\sigma-1)}} \left[ \frac{\beta}{1+\beta} (1-\alpha) A \right]^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}}} \right)} k^{\frac{1-\alpha}{1+\alpha(\sigma-1)}} = +\infty$$

Therefore, the function (25) representing  $\tau_{t+1} = \tau_t$  has a steeper slope than the function (24) representing  $k_{t+1} = k_t$  for small value of  $k$ . This is the inverse for large values of  $k$ . This means that both functions start from 0, cross at the interior steady state and diverge afterwards. We can see from the phase diagram that the interior steady state is a saddle point.

### 3.3.3 Dynamics when $\sigma = 1$

If  $\sigma = 1$  (logarithmic utility function), then the function (24) is increasing and the function (25) is linear. They intersect once at the interior steady state. The steady state is a saddle point.

## 4 Political stability of dictatorship

In the previous section, any dictatorship was shown to admit a unique and stable steady state. The only imposed constraint on the dictator's behavior was a constraint of intertemporal economic feasibility. It turned out that this economic constraint, alone, did not exert a limitation on the dictator's preferences. We want next to take the political constraint (equation 13) into account in the model. This additional constraint represents the potential insurrection threat as a reaction of the people, or at least a group of people, to the behavior of the dictator. This is the Sword of Damocles under which any dictator lives continuously. We will now analyze dictatorship under both constraints.

The dynamical system of dictatorship with insurrection threat depends on whether the insurrection constraint is binding or not. If not, the system is the same as in section 4. If the threat is binding, then the first-order conditions are (16)-(18). The dynamic system of dictatorship with insurrection threat is then the following:

$$k_{t+1} = \frac{\beta}{1+\beta}(1-\tau_t)(1-\alpha)Ak_t^\alpha \quad (26)$$

$$\tau_{t+1} = 1 - \frac{l}{(1-q_{t+1})Ak_{t+1}^\alpha} \quad (27)$$

$$q_{t+1} = \frac{[(1+\rho)(1+\beta)]^{\frac{1}{1-\sigma}} \tau_t k_t^\alpha (1-q_t)^{\frac{2}{1-\sigma}}}{\alpha^{\frac{1}{1-\sigma}} q_t^{\frac{\sigma}{1-\sigma}} \tau_{t+1} k_{t+1}^{\alpha - \frac{1}{1-\sigma}}} \quad (28)$$

Equation (26) is the same as equation (20). Equation (27) gives the expected income tax rate as a function of the current income tax rate and the current capital stock. Equation (28) gives the expected predation rate as a function of the current predation rate, the current income tax rate and the current capital stock. The system of these three equations is computed by eliminating the Lagrange multipliers from the equations (16)-(18), and by solving the new system to find the expressions of  $k_{t+1}$ ,  $\tau_{t+1}$  and  $q_{t+1}$ .

### 4.1 Steady states

**Proposition 3** *Any politically stable dictatorship characterized by the dynamic system (26)-(28) admits a unique steady state.*

Proof:

By solving the system (26)-(28) at the steady state, we obtain the following unique solu-

tion:

$$\begin{aligned}
\bar{k} &= \frac{(1 + \rho)[\beta(1 - \alpha)l]^2 + \alpha\beta(1 - \alpha)l}{\alpha(1 + \beta)} \\
\bar{\tau} &= 1 - \frac{l(\beta(1 + \rho)(1 - \alpha)l + \alpha)^{1-\alpha}}{A(\alpha)^{1-\alpha} \left(\frac{\beta(1-\alpha)l}{1+\beta}\right)^\alpha} \\
\bar{q} &= \frac{1}{1 + \frac{\alpha}{\beta(1+\rho)(1-\alpha)l}}
\end{aligned} \tag{29}$$

Hence, the dictator's steady state consumption is:

$$\bar{g} = \frac{[\beta(1 - \alpha)l]^{2-\alpha}}{[\beta(1 - \alpha)l + A\alpha]^{2(1-\alpha)}} \left( \frac{A}{\alpha} \left( 1 + \left[ \frac{\alpha A}{\beta(1 - \alpha)l} \right]^{1-\alpha} \right) \left( \frac{\beta(1 - \alpha)l}{1 + \beta} \right)^\alpha - \frac{l}{A^2\alpha} \right) \tag{30}$$

If the insurrection constraint is not binding, then the steady state solution is the same as in proposition 1. ■

Proposition 3 gives the steady state of dictatorship that is politically stable. The insurrection constraint makes the rate of predation,  $q$ , one of the variables determining the steady state point of dictatorship. The threat of insurrection has therefore three implications. First, it can improve the utility of the consumers by constraining the distribution of the tax revenues between them and the dictator. Second, it contributes to economic development by influencing the choice of a more economically sustainable tax rate. Finally, it represents the limit to the dictator's power by making unstable some of the (socially) worst possible preferences of the dictator.

## 4.2 Local stability

**Proposition 4** *The local stability of the interior steady state of any form of politically stable dictatorship depends on the dictator's preference parameters (the elasticity of substitution,  $\sigma$ , and the time preference,  $\rho$ ), the initial level of social order,  $l$ , and the productivity level,  $A$ .*

Proof: see appendix B

In contrast with the situation without insurrection threat, Proposition 4 points out that some dictatorships, even though satisfying the political constraint, may not have a locally stable steady state. Depending on the parameters, the steady state may be locally stable, locally saddle-path stable or unstable. This proposition shows the large variety of situations a dictator may face.

## 5 The role of the threat of anarchy

The political stability of dictatorship eventually depends on the insurrection threat but the conditions of this threat's occurrence may vary across dictatorships. Each society

possesses its own structural parameters that may differentiate the perimeter of the dictator's political stability across different societies. Depending on these parameters, some societies may offer more propitious conditions for the establishment and the stability of dictatorship. However, all societies experience internal evolution and, hence, a modification of these structural parameters. Any unexpected change in them may be fatal for dictators. Although there are certainly many structural parameters constituting a society, we will study two of them. The first is the natural level of social order,  $l$ . We consider that this level is different across societies because it depends on factors such as demography, geography, history, culture, etc..., all the factors which make that the world population does not form a single political community but a plurality of them. Therefore, it may be more or less challenging across societies to maintain social order. The second parameter is probably the one that has modified the most all societies in the world in the last two centuries: sustained economic growth, represented in the model by the total factor productivity parameter  $A$ . The objective is then to examine in what way economic growth may affect the stability of dictatorship.

To study the effect of the parameters on the perimeter of the dictatorship's political stability, we need to analyze how they affect the insurrection constraint. Recall that the insurrection constraint,  $I$ , is not violated if and only if

$$I = \ln[(1 - \bar{\tau})] + \ln[(1 - \bar{q})\bar{\tau}A\bar{k}^\alpha + l] - \ln l \geq 0. \quad (31)$$

A condition for its no-violation is given, for example, by the steady state predation rate of (29),

$$\bar{q} \leq \frac{1}{1 + \frac{\alpha A}{\beta(1-\alpha)l}}. \quad (32)$$

If the dictator chooses a predation rate lower or equal to  $\bar{q}$ , dictatorship is politically stable. This is the threshold of the perimeter of the political stability. How does it evolve with the parameters?

## 5.1 The natural level of social order, $l$

$$\frac{\partial I}{\partial l} = \frac{\partial I}{\partial q} \frac{\partial q}{\partial l} < 0$$

The partial derivative of  $I$  with respect to  $l$  is negative. The lower  $l$ , the higher the threshold. This means that when the natural level of social order is low, the population's need for security is large. The fear of anarchy is great and makes the population accept to trade a bit of social order against predation/repression. As already mentioned, societies are structurally different. In some of them, rivalries among ethnic groups, inequality in political rights, income inequality or any strife among a human group can create difficulties to reach a peaceful collective choice resulting in a low natural level of social order. In our analysis, such a level increases the toleration level of dictatorship.

In contrast, if the natural level of social order is high, the population enjoys a relatively peaceful order and is less ready to tolerate the dictator's predatory behavior.

## 5.2 The level of technology, $A$

$$\frac{\partial I}{\partial A} = \frac{\partial I}{\partial q} \frac{\partial q}{\partial A} > 0$$

The partial derivative of  $I$  with respect to  $A$  shows that the insurrection constraint is an increasing function of  $A$ . The higher the level of technology, the higher the threshold. This result means that economic growth, represented by an exogenous increase in the level of technology, augments the perimeter of dictatorship's political stability. The dictator has thus more leeway to satisfy his preferences and increase predation/repression to impose them.<sup>11</sup>

In economies with low level of technology, the dictator is more vulnerable. This is especially the case when the natural level of social order is relatively high. However, a stagnant or declining economy is more likely to experience difficulties to maintain social order and, hence, often turns out to secure the political power of the dictator.

The largest perimeter of dictatorship's political stability occurs when economic growth is high and the natural level of social order is low.

## 6 Conclusion

Everyone agrees that dictatorship is a political regime violating human rights. However, its effect on economic performance remains heatedly debated in the absence of clear-cut empirical results. The poorest countries are indeed dictatorships but the spectacular economic growth of the East-Asian countries in the last half a century occurred under dictatorial rule. The causes of its stability and instability are even more poorly understood. Without this knowledge, how to explain the wave of democratization that happened in the last two decades in numerous parts across the world simultaneously? Will this historical shift be durable?

To answer these questions, it is necessary to focus the research on the dynamics of dictatorship. The objective of this paper is to propose a framework to account for the observed large variance of economic performance under dictatorship and its political stability.

In a simple growth model, we formalize the intertemporal behavior of a dictator illustrating Olson's idea that a dictator has an 'encompassing interest' in the fate of the economy he rules. As a result, the economic performance depends a lot on the dictator's impatience rate. It turns out that all the dictator's preferences represented by a CIES utility function are economically feasible and, for each type of preference, the dynamics of the economy possesses a locally stable steady state. In this framework, there is no economic limit to the dictator's preferences.

The maintenance in power of a dictator does not rely on its popularity but on the reduction of the risks to be deposed. The threat of a popular insurrection or a plot by a group

---

<sup>11</sup>This result seems to be supported by the evolution of the Chinese regime. The Polity IV scoring indicates that the Chinese dictatorship has hardened with economic expansion from 1978 to 2002 (see the Polity IV 2002 database and Marshall and Jaggers (2002)).

of people is the limit to the dictator's power. To remain in power he has to create the conditions dissuading these threats. These conditions depend on the choice of his preferences but also on the economic performance and the risk of anarchy in case of a regime change. Our model emphasizes the conception that between dictatorship and democracy, there may be an anarchic situation working in favor of the stability of dictatorship. Nevertheless, by focusing on social order and economic welfare, our model ignores a possible presence of political liberties in the citizens' preferences, and hence, the attractiveness effect of democracy on the stability of dictatorship. However, as shown in Acemoglu, Robinson, and Verdier (2004), this attractiveness is counterbalanced by the difficult and yet necessary cooperation among various groups in the polities to establish democracy.

## References

- Acemoglu, Daron and James A. Robinson. 2001. "A Theory of Political Transitions." *American Economic Review* 91 (4): 938–963.
- Acemoglu, Daron, James A. Robinson, and Thierry Verdier. 2004. "Kleptocracy and Divide-And-Rule: A Model of Personal Rule." *Journal of the European Economic Association* 2 (2-3): 162–192 (April-May).
- Azariadis, Costas. 2001. "The Theory of Poverty Traps: What Have we Learned?" *UCLA Working Paper*.
- Barro, Robert J. 1996. "Democracy and Growth." *Journal of Economic Growth* 1:1–27.
- Bobbio, Norberto. 1989. *Democracy and Dictatorship. The Nature and Limits of State Power*. Cambridge: Polity Press.
- Boix, Carles and Susan C. Stokes. 2003. "Endogenous Democratization." *World Politics* 55 (July): 517–549.
- Dahl, Robert A. 1971. *Polyarchy: Participation and Opposition*. New Haven: Yale University Press.
- de la Croix, David and Philippe Michel. 2002. *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations*. Cambridge: Cambridge University Press.
- Diamond, Peter A. 1965. "National Debt in a Neoclassical Growth Model." *American Economic Review* 55 (5): 1126–1150 (Dec).
- Drèze, Jean and Amartya Sen. 1989. *Hunger and Public Action*. Oxford University Press.
- Grossman, Herschel I. 1991. "A General Equilibrium Model of Insurrections." *American Economic Review* 81 (4): 912–921 (September).
- 1996. "Predation and Accumulation." *Journal of Economic Growth* 1 (September): 333–351.
- Grossman, Herschel I. and Suk Jae Noh. 1990. "A Theory of Kleptocracy with probabilistic Survival and Reputation." *Economics and Politics* 2 (2): 157–171 (July).
- Hirshleifer, Jack. 1991. "The Technology of Conflict as an Economic Activity." *American Economic Review* 81 (2): 130–134 (May).
- Kanczuk, Fabio. 1998. "The Political Economy of Non-Convergence." Ph.D. diss., UCLA, Los Angeles.
- Lipset, Seymour Martin. 1959. "Some Social Requisites of Democracy: Economic Development and Political Legitimacy." *American Political Science Review* 53 (1): 69–105.
- Marshall, Monty G. and Keith Jaggers. 2002. "Polity IV Project: Dataset User's Manual." [www.cidcm.umd.edu/inscr/polity](http://www.cidcm.umd.edu/inscr/polity).
- North, Douglass C. 1990. *Institutions, Institutional Change, and Economic Performance*. Cambridge: Cambridge University Press.

- Olson, Mancur. 1993. "Dictatorship, Democracy, and Development." *American Political Science Review* 87 (3): 567–576.
- . 2000. *Power and Prosperity. Outgrowing Communist and Capitalist Dictatorships*. New York: Basic Books.
- Pipes, Richard. 2004. "Flight From Freedom. What Russians Think and Want." *Foreign Affairs* 83 (3): 9–15 (May/June).
- Przeworski, Adam, Michael E. Alvarez, José Antonio Cheibub, and Fernando Limongi. 2000. *Democracy and Development: Political Institutions and Well-being in the World, 1950-1990*. New York: Cambridge University Press.
- Przeworski, Adam and Fernando Limongi. 1993. "Political Regimes and Economic Growth." *Journal of Economic Perspectives* 7 (3): 51–69 (Summer).
- Sirowy, Larry and Alex Inkeles. 1990. "The Effects of Democracy on Economic Growth and Inequality: A Review." *Studies in Comparative International Development* 25:126–157.
- Taylor, Michael. 1982. *Community, Anarchy and Liberty*. Cambridge: Cambridge University Press.
- Temple, Jonathan. 1999. "The New Growth Evidence." *Journal of Economic Literature* 37 (1): 112–156 (March).
- UNDP. 2004, April. *Democracy in Latin-America: Towards a Citizens' Democracy*. New York.
- Usher, Dan. 1989. "The Dynastic Cycle and the Stationary State." *American Economic Review* 79 (5): 1031–1044 (Dec).
- Wintrobe, Ronald. 1998. *The Political Economy of Dictatorship*. Cambridge: Cambridge University Press.



## A Proof of proposition 2

The dynamical system of the dictatorship without insurrection threat is characterized by these two equations:

$$k_{t+1} = \frac{\beta}{1+\beta}(1-\tau_t)(1-\alpha)Ak_t^\alpha \quad (33)$$

$$\tau_{t+1} = \frac{\alpha^{\frac{1}{\sigma}}\tau_t k_t^{\alpha(1-\frac{1}{\sigma})(1-\alpha)}}{(1+\rho)^{\frac{1}{\sigma}}(1-\tau_t)^{\frac{1+\alpha(\sigma-1)}{\sigma}} \left[ \frac{\beta}{1+\beta}(1-\alpha)A \right]^{\frac{\alpha(\sigma-1)}{\sigma}}} \quad (34)$$

The linearized system is:

$$\begin{bmatrix} k_{t+1} - \bar{k} \\ \tau_{t+1} - \bar{\tau} \end{bmatrix} = J \cdot \begin{bmatrix} k_t - \bar{k} \\ \tau_t - \bar{\tau} \end{bmatrix} + O(\varepsilon)$$

with

$$J = \begin{bmatrix} \alpha \frac{\beta}{1+\beta}(1-\bar{\tau})(1-\alpha)A\bar{k}^{\alpha-1} & -\frac{\beta}{1+\beta}(1-\alpha)A\bar{k}^\alpha \\ \frac{\alpha^{1+\frac{1}{\sigma}}(1-\alpha)(1-\frac{1}{\sigma})\bar{\tau}\bar{k}^{\alpha(1-\frac{1}{\sigma})(1-\alpha)-1}}{(1+\rho)^{\frac{1}{\sigma}} \left( \frac{\beta}{1+\beta}(1-\alpha)A \right)^{\frac{\alpha(\sigma-1)}{\sigma}} (1-\bar{\tau})^{\frac{1+\alpha(\sigma-1)}{\sigma}}} & \frac{\alpha^{\frac{1}{\sigma}}\bar{k}^{\alpha(1-\frac{1}{\sigma})(1-\alpha)} \left( 1 + \frac{1+\alpha(\sigma-1)}{\sigma} \bar{\tau} \right)}{(1+\rho)^{\frac{1}{\sigma}} \left( \frac{\beta}{1+\beta}(1-\alpha)A \right)^{\frac{\alpha(\sigma-1)}{\sigma}} (1-\bar{\tau})^{\frac{1+\alpha(\sigma-1)}{\sigma}+1}} \end{bmatrix}$$

Replacing  $\bar{k}$  and  $\bar{\tau}$  by their expressions, we can make some simplifications:

$$J = \begin{bmatrix} \alpha & -\frac{\beta}{1+\beta}(1-\alpha)A \left( \frac{\beta\alpha(1-\alpha)A}{(1+\beta)(1+\rho)} \right)^{\frac{\alpha}{1-\alpha}} \\ \alpha(1-\alpha) \left( \frac{\sigma-1}{\sigma} \right) \frac{1-\frac{\alpha}{1+\rho}}{\left( \frac{\beta\alpha(1-\alpha)A}{(1+\beta)(1+\rho)} \right)^{\frac{1}{1-\alpha}}} & \frac{1+\rho}{\alpha} - \frac{(1-\alpha)(\sigma-1)}{\sigma} \left( \frac{1+\rho}{\alpha} - 1 \right) \end{bmatrix}$$

The trace of  $J$  is :

$$trJ = \alpha + \frac{1+\rho}{\alpha} - \frac{(1-\alpha)(\sigma-1)}{\sigma} \left( \frac{1+\rho}{\alpha} - 1 \right)$$

The determinant of  $J$  is:

$$DetJ = 1 + \rho > 1$$

Rearranging the terms of the trace of  $J$  yields:

$$trJ = 2 + \rho + \frac{1}{\sigma} \left( \frac{(1-\alpha)(1+\rho-\alpha)}{\alpha} \right)$$

Since the term in brackets is positive, it is straightforward to conclude that  $|1 + DetJ| < |trJ|$ . As a result, the steady state is a saddle point for any  $\sigma$  and  $\rho$ .

## B Proof of proposition 4

By eliminating the Lagrange multipliers from the equations (16)-(18), we obtain the following system:

$$k_{t+1} = \frac{\beta}{1+\beta}(1-\tau_t)(1-\alpha)Ak_t^\alpha \quad (35)$$

$$\tau_{t+1} = 1 - \frac{l}{(1-q_{t+1})Ak_{t+1}^\alpha} \quad (36)$$

$$q_{t+1} = \frac{[(1+\rho)(1+\beta)]^{\frac{1}{1-\sigma}} \tau_t k_t^\alpha (1-q_t)^{\frac{2}{1-\sigma}}}{\alpha^{\frac{1}{1-\sigma}} q_t^{\frac{\sigma}{1-\sigma}} \tau_{t+1} k_{t+1}^{\alpha - \frac{1}{1-\sigma}}} \quad (37)$$

Plugging in (35) the expression of  $\tau_t$  given by (36), and plugging (36) in (37), we obtain a system of two equations,

$$k_{t+1} = \frac{l\beta(1-\alpha)}{(1+\beta)(1-q_t)} \quad (38)$$

$$q_{t+1} - \frac{lq_{t+1}}{(1-q_{t+1})Ak_{t+1}^\alpha} = \frac{[(1+\rho)(1+\beta)]^{\frac{1}{1-\sigma}} \tau_t k_t^\alpha (1-q_t)^{\frac{2}{1-\sigma}}}{\alpha^{\frac{1}{1-\sigma}} q_t^{\frac{\sigma}{1-\sigma}} k_{t+1}^{\alpha - \frac{1}{1-\sigma}}} \quad (39)$$

Using (36) and (38) to rearrange the expression (39), we obtain a system equivalent to (35)-(37):

$$k_{t+1} = \frac{l\beta(1-\alpha)}{(1+\beta)(1-q_t)} \quad (40)$$

$$q_{t+1}q_t^{\frac{\sigma}{1-\sigma}} - \frac{q_{t+1}q_t^{\frac{\sigma}{1-\sigma}} l[(1+\beta)(1-q_t)]^\alpha}{1-q_{t+1} A[l\beta(1-\alpha)]^\alpha} = \quad (41)$$

$$= \frac{\left( (1-q_t)^{\frac{1}{1-\sigma} + \alpha} k_t^\alpha - \frac{l(1-q_t)^{\frac{\sigma}{1-\sigma} + \alpha}}{A} \right)}{\alpha^{\frac{1}{1-\sigma}} [(1+\rho)(1+\beta)]^{\frac{-1}{1-\sigma}} \left( \frac{l\beta(1-\alpha)}{1+\beta} \right)^{\alpha - \frac{1}{1-\sigma}}}$$

Then we take a first-order Taylor expansion around the steady state. To compute the partial derivatives of  $q_{t+1}$  with respect to  $k_t$  and  $q_t$ , we use the implicit function theorem. Given the function  $F(q_{t+1}, k_t, q_t) = 0$ , with

$$F(q_{t+1}, k_t, q_t) = q_{t+1}q_t^{\frac{\sigma}{1-\sigma}} - \frac{q_{t+1}q_t^{\frac{\sigma}{1-\sigma}} l[(1+\beta)(1-q_t)]^\alpha}{1-q_{t+1} A[l\beta(1-\alpha)]^\alpha} - \frac{\left( (1-q_t)^{\frac{1}{1-\sigma} + \alpha} k_t^\alpha - \frac{l(1-q_t)^{\frac{\sigma}{1-\sigma} + \alpha}}{A} \right)}{\alpha^{\frac{1}{1-\sigma}} [(1+\rho)(1+\beta)]^{\frac{-1}{1-\sigma}} \left( \frac{l\beta(1-\alpha)}{1+\beta} \right)^{\alpha - \frac{1}{1-\sigma}}},$$

there exists an implicit function  $q_{t+1} = f(k_t, q_t)$  if  $F$  has continuous partial derivatives  $F_{q_{t+1}}, F_{k_t}, F_{q_t}$ , and if  $\partial F / \partial q_{t+1} \neq 0$ . Both conditions are verified if  $q_{t+1} \neq 1$  and  $q_t \neq 0$ . As only the values between 0 and 1 are of interest, we can define an interval  $]0, 1[$  for  $q$  to ensure that  $F$  admits the implicit function  $q_{t+1} = f(k_t, q_t)$ . The linearized system of (40)-(41) is the following:

$$\begin{bmatrix} k_{t+1} - \bar{k} \\ q_{t+1} - \bar{q} \end{bmatrix} = J. \begin{bmatrix} k_t - \bar{k} \\ q_t - \bar{q} \end{bmatrix} + O(\varepsilon)$$

with

$$J = \begin{bmatrix} 0 & \frac{l\beta(1-\alpha)}{(1+\beta)(1-q_t)^2} \\ \frac{A(1-q_{t+1})^2(1-q_t)^{\frac{1}{1-\sigma}+\alpha} k_t^{\alpha-1}}{(1+\rho)^{\frac{1}{1-\sigma}} \Upsilon} & \Omega + \frac{A(1-q_{t+1})^2(1-q_t)^{\frac{\sigma}{1-\sigma}+\alpha} \left( \left( \frac{1}{1-\sigma} + \alpha \right) k_t^\alpha - \frac{l \left( \frac{\sigma}{1-\sigma} + \alpha \right)}{A(1-q_t)} \right)}{\alpha(1+\rho)^{\frac{1}{1-\sigma}} \Upsilon} \end{bmatrix}_{(\bar{q}, \bar{k}, \bar{q})}$$

where,

$$\Upsilon = (\alpha q_t)^{\frac{\sigma}{1-\sigma}} [l\beta(1-\alpha)]^{-\frac{1}{1-\sigma}} \left( \frac{(1-q_{t+1})^2 A [l\beta(1-\alpha)]^\alpha}{(1+\beta)^\alpha} - l(1-q_t)^\alpha \right)$$

$$\Omega = \frac{\frac{\sigma q_{t+1}}{(1-\sigma)q_t} - \frac{(1-q_t)^\alpha}{q_t} \left( \frac{\sigma}{(1-\sigma)} - \frac{\alpha q_t}{1-q_t} \right) \frac{q_{t+1} l (1+\beta)^\alpha}{(1-q_{t+1}) A [l\beta(1-\alpha)]^\alpha}}{1 - \frac{l(1+\beta)(1-q_t)^\alpha}{A [l\beta(1-\alpha)]^\alpha (1-q_{t+1})^2}}$$

Therefore, the Jacobian matrix is:

$$J = \begin{bmatrix} 0 & \frac{l\beta(1-\alpha)}{(1+\beta)(1-\bar{q})^2} \\ \frac{A(1-\bar{q})^{\frac{1}{1-\sigma}+\alpha+2} \bar{k}^{\alpha-1}}{(1+\rho)^{\frac{1}{1-\sigma}} \Upsilon} & \Omega + \frac{A(1-\bar{q})^{\frac{\sigma}{1-\sigma}+\alpha+2} \left( \left( \frac{1}{1-\sigma} + \alpha \right) \bar{k}^\alpha - \frac{l \left( \frac{\sigma}{1-\sigma} + \alpha \right)}{A(1-\bar{q})} \right)}{\alpha(1+\rho)^{\frac{1}{1-\sigma}} \Upsilon} \end{bmatrix}$$

where,

$$\Upsilon = (\alpha \bar{q})^{\frac{\sigma}{1-\sigma}} [l\beta(1-\alpha)]^{-\frac{1}{1-\sigma}} \left( \frac{(1-\bar{q})^2 A [l\beta(1-\alpha)]^\alpha}{(1+\beta)^\alpha} - l(1-\bar{q})^\alpha \right)$$

$$\Omega = \frac{\frac{\sigma}{(1-\sigma)} - (1-\bar{q})^{\alpha-1} \left( \frac{\sigma}{(1-\sigma)} - \frac{\alpha \bar{q}}{1-\bar{q}} \right) \frac{l(1+\beta)^\alpha}{A [l\beta(1-\alpha)]^\alpha}}{1 - \frac{l(1+\beta)^\alpha}{A [l\beta(1-\alpha)]^\alpha (1-\bar{q})^{2-\alpha}}}$$

The trace of  $J$  is:

$$tr J = \Omega + \frac{A(1 - \bar{q})^{\frac{\sigma}{1-\sigma} + \alpha + 2} \left( \left( \frac{1}{1-\sigma} + \alpha \right) \bar{k}^\alpha - \frac{l \left( \frac{\sigma}{1-\sigma} + \alpha \right)}{A(1-\bar{q})} \right)}{\alpha(1 + \rho)^{\frac{-1}{1-\sigma}} \Upsilon}, \quad (42)$$

and the determinant of  $J$  is:

$$Det J = - \left( \frac{l\beta(1 - \alpha)}{1 + \beta} \right) \left( \frac{A(1 - \bar{q})^{\frac{1}{1-\sigma} + \alpha} \bar{k}^{\alpha-1}}{(1 + \rho)^{\frac{-1}{1-\sigma}} \Upsilon} \right). \quad (43)$$

Then, we rearrange the determinant and the trace by removing the  $\bar{k}^l$ s and express them in terms of  $\bar{q}$  and  $\bar{\tau}$  to analyze them more easily.

$$Det J = -(1 - \bar{q}) \left( \frac{\frac{l}{1-\bar{\tau}}(1 - \bar{q})^{\frac{\sigma}{1-\sigma} + \alpha}}{(1 + \rho)^{\frac{-1}{1-\sigma}} \Upsilon} \right)$$

Using the expression of  $\Upsilon$ , the determinant becomes:

$$\begin{aligned} Det J &= - \frac{\frac{l}{1-\bar{\tau}}(1 - \bar{q})^{\frac{\sigma}{1-\sigma} + 1 + \alpha} [(1 + \rho)\beta(1 - \alpha)l]^{\frac{1}{1-\sigma}}}{(\alpha\bar{q})^{\frac{\sigma}{1-\sigma}} \left( \frac{(1-\bar{q})^2 A [l\beta(1-\alpha)]^\alpha}{(1+\beta)^\alpha} - l(1 - \bar{q})^\alpha \right)} \\ &= - \frac{\frac{l}{1-\bar{\tau}}(1 - \bar{q})^{\frac{\sigma}{1-\sigma} + 1 + \alpha} \left( \frac{\bar{q}}{1-\bar{q}} \right)^{\frac{1}{1-\sigma}}}{\alpha^{-1} \bar{q}^{\frac{\sigma}{1-\sigma}} l(1 - \bar{q})^\alpha \left( \frac{(1-\bar{q})^2 A [l\beta(1-\alpha)]^\alpha}{(1+\beta)^\alpha l(1-\bar{q})^\alpha} - 1 \right)} \end{aligned}$$

Simplifying and replacing  $(1 - \bar{q})^\alpha$  by its steady state expression yields

$$Det J = - \frac{\alpha\bar{q}}{(1 - \bar{\tau}) \left( \frac{(1-\bar{q})^2 A \alpha^{-\alpha} [l\beta(1-\alpha)]^\alpha [\beta(1+\rho)(1-\alpha)l + \alpha]^\alpha}{l(1+\beta)^\alpha} - 1 \right)}$$

Replacing  $(1 - \bar{q})$  by its steady state expression, the large term in the denominator can be identified as  $(1 - \bar{q})/(1 - \bar{\tau})$ . Finally, after simplification, we obtain:

$$Det J = \frac{-\alpha\bar{q}}{-\bar{q} + \bar{\tau}} \quad (44)$$

- Condition 1: if  $\bar{q} \in ]\frac{\bar{\tau}}{1+\alpha}, \frac{\bar{\tau}}{1-\alpha}[$ , then  $|Det J| > 1$ ;
- Condition 2: if  $\bar{q} \in ]\frac{\bar{\tau}}{1-\alpha}, 1[$ , then  $|Det J| < 1$ ;

where  $\bar{q}$  is a function of the parameters  $\rho$  and  $l$ ;  $\bar{\tau}$  is a function of the parameters  $l$ ,  $\rho$  and  $A$  (see equation (29)); and  $0 < \bar{\tau}, \bar{q} < 1$ .

As for the trace, rearranging (42) yields

$$trJ = \Omega + \frac{\sigma}{1-\sigma} \left( \frac{\alpha}{(1+\rho)\beta(1-\alpha)l + \alpha} \right) \frac{A(1-\bar{q})^{\frac{1}{1-\sigma} + \alpha} \bar{k}^\alpha \left( \frac{1+\alpha(1-\sigma)}{\alpha\sigma} - \frac{(1-\bar{\tau})(\sigma+\alpha(1-\sigma))}{\alpha\sigma} \right)}{(1+\rho)^{\frac{-1}{1-\sigma}} \Upsilon}$$

where  $1 - \bar{\tau} = \frac{l}{A(1-\bar{q})k^\alpha}$  (see equation (36)).

The second additive term of the trace can be simplified in the same way as the determinant.

After some computations, we obtain:

$$\frac{\sigma}{1-\sigma} \frac{-\alpha\bar{q}}{-\bar{q}+\bar{\tau}} \left( \frac{1+\alpha(1-\sigma)}{\alpha\sigma} - \frac{(1-\bar{\tau})(\sigma+\alpha(1-\sigma))}{\alpha\sigma} \right),$$

and after simplifications, the second term of the trace becomes

$$\frac{-\bar{q}}{-\bar{q}+\bar{\tau}} \left( \frac{\bar{\tau}(\sigma+\alpha-\alpha\sigma)}{1-\sigma} + 1 \right).$$

As for the first additive term of the trace,  $\Omega$ , it can be rewritten as follows:

$$\Omega = \frac{\frac{\sigma}{1-\sigma} \left( 1 - \frac{l(1+\beta)^\alpha(1-\bar{q})^{\alpha-2}}{A[l\beta(1-\alpha)]^\alpha} \left( 1 - \bar{q} - \frac{\alpha\bar{q}(1-\sigma)}{\sigma} \right) \right)}{1 - \frac{l(1+\beta)^\alpha(1-\bar{q})^{\alpha-2}}{A[l\beta(1-\alpha)]^\alpha}} \quad (45)$$

The big term that is common to the nominator and the denominator can be identified, by replacing  $1 - \bar{q}$  by its steady state expression, as  $(1 - \bar{\tau})/(1 - \bar{q})$ .

After simplifications,  $\Omega$  becomes:

$$\Omega = \frac{\frac{\sigma}{1-\sigma} [\bar{\tau}(1 - \bar{q})] + \alpha\bar{q}}{-\bar{q} + \bar{\tau}}$$

As a result, we obtain the final expression for the trace  $J$ :

$$trJ = \frac{\frac{\bar{\tau}\sigma}{1-\sigma} [1 - \bar{q}(2 - \alpha + \frac{\alpha}{\sigma})] - \bar{q}(1 - \alpha)}{-\bar{q} + \bar{\tau}} \quad (46)$$

The final part of this proof consists in comparing  $|trJ|$  and  $|1 + DetJ|$ , i.e.,

$$\left| \frac{\frac{\bar{\tau}\sigma}{1-\sigma} [1 - \bar{q}(2 - \alpha + \frac{\alpha}{\sigma})] - \bar{q}(1 - \alpha)}{-\bar{q} + \bar{\tau}} \right| \text{ and } \left| \frac{-\bar{q}(1 - \alpha) + \bar{\tau}}{-\bar{q} + \bar{\tau}} \right|.$$

This depends on  $\sigma$ , the magnitude of  $\bar{q}$ , which itself depends on the value of the parameters  $\rho$  and  $l$ , and on whether  $|DetJ| \leq 1$  (conditions 1 and 2, established above).

When  $\sigma > 1$ , it can be shown that:

- If  $\bar{q} \in \left] 0, \frac{1}{2-\alpha(1-\frac{1}{\sigma})} \left[ \cup \right] \frac{2-\frac{1}{\sigma}}{2-\alpha(1-\frac{1}{\sigma})}, 1 \left[ \right]$ , then either  $\frac{\sigma}{1-\sigma} [1 - \bar{q}(2 - \alpha + \frac{\alpha}{\sigma})] < 0$  or  $\frac{\sigma}{1-\sigma} [1 - \bar{q}(2 - \alpha + \frac{\alpha}{\sigma})] > 1$  and, therefore,  $|trJ| > |1 + DetJ|$ . The steady state is a saddle.
- If  $\bar{q} \in \left] \frac{1}{2-\alpha(1-\frac{1}{\sigma})}, \frac{2-\frac{1}{\sigma}}{2-\alpha(1-\frac{1}{\sigma})} \left[ \right]$ , then  $\frac{\sigma}{1-\sigma} [1 - \bar{q}(2 - \alpha + \frac{\alpha}{\sigma})] > 1$  and, therefore,  $|trJ| < |1 + DetJ|$ . If  $|DetJ| > 1$ , the steady state is unstable. If  $|DetJ| < 1$ , the steady state is a sink.

When  $\sigma < 1$ , there are two cases:

- If  $\sigma < \frac{1}{2}$ , it can be shown that  $0 < \frac{\sigma}{1-\sigma}[1 - \bar{q}(2 - \alpha + \frac{\alpha}{\sigma})] < 1$  and, therefore,  $|trJ| < |1 + DetJ|$ . If  $|DetJ| > 1$ , the steady state is unstable. If  $|DetJ| < 1$ , the steady state is a sink.

- If  $1 > \sigma > \frac{1}{2}$ , there are two subcases depending on  $\bar{q}$ :

a/ If  $\bar{q} < \frac{2-\frac{1}{\sigma}}{2-\alpha(1-\frac{1}{\sigma})}$ , then  $\frac{\sigma}{1-\sigma}[1 - \bar{q}(2 - \alpha + \frac{\alpha}{\sigma})] > 1$  and, therefore,  $|trJ| > |1 + DetJ|$ . The steady state is a saddle.

b/ If  $\bar{q} > \frac{2-\frac{1}{\sigma}}{2-\alpha(1-\frac{1}{\sigma})}$ , then  $\frac{\sigma}{1-\sigma}[1 - \bar{q}(2 - \alpha + \frac{\alpha}{\sigma})] < 1$  and, therefore,  $|trJ| < |1 + DetJ|$ . If  $|DetJ| > 1$ , the steady state is unstable. If  $|DetJ| < 1$ , the steady state is a sink.

■