Business Cycle Turning Points:
Mixed-Frequency Data with Structural Breaks

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September 2004

Abstract

This paper develops a dynamic factor models with regime switching to account for the decreasing volatility of the U.S. economy observed since the mid-1980s. Apart from the Markov switching capturing the cyclical fluctuations, an additional type of regime switching is introduced to allow variances to switch between distinct regimes. The resulting four-regime models extend univariate analysis currently used in the literature on the structural break in conditional volatility to the multivariate time series. Besides the dynamic factor model using the data with a single (monthly) frequency, we employ the additional information incorporating the mixed-frequency data, which include not only the monthly component series but also such an important quarterly series as the real GDP. The evaluation of six different nonlinear models suggests that the probabilities derived from all the models comply with NBER business cycle dating and detect a one-time shifting from high variance to low-variance states in February 1984. In addition, we find that: mixed-frequency models outperform single-frequency models; restricted models outperform unrestricted models; four-regime switching models outperform two-regime switching models.

Keywords: Volatility; Structural break; Composite coincident indicator; Dynamic factor model; Markov switching; Mixed-frequency data

JEL classification: E32; C10

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[1] The authors gratefully acknowledge the financial support provided by the Belgian French Community's program Action de recherches concertée 99/04-235 (IRES, Université catholique de Louvain)

1. Introduction

Recent studies suggest that the U.S. economy has become considerably stabilized since the mid-1980s compared to the rest of the postwar era. McConnell and Pérez Quirós[2] (2000) find that the volatility of quarterly GDP growth experienced a one-time drop around the first quarter of 1984: the variance of output fluctuation over the period from 1953 ending in 1983 is at least four times as large as that of the period since then. This view is independently shared by Kim and Nelson (1999a), who distinguish, within the context of a Markov-switching (MS) model, two important sources of stabilization in real GDP growth: a decline in the variance of shocks and a narrowing gap between growth rates during booms and recessions. Blanchard and Simon (1999) estimate that standard error of the autoregressions of GDP has declined from 1% a quarter on average from the 1950s through the mid-1980s to roughly 0.5% a quarter since then. Kim *et al.* (2004), using Bayesian tests, document one or multiple structural breaks in U.S. aggregate and disaggregate real GDP, then volatility and persistence of inflation and interest rates. These new evidences have induced numerous studies to explain the decline in U.S. output volatility. Among them, "better policy", "good luck", and technological improvement such as those used in inventory control strategy are most prominent. See Herrera and Pesavento (2003) and Sensier and Van Dijk (2003) for more details.

[2] From now on we will refer to this paper as MPQ (2000) to save space, since we will cite it on many occasions.

Indeed, the reduction in volatility is not confined to aggregate output or durable goods sector. It is rather widespread in most of economic processes and sectors: employment (Warnock and Warnock, 2001), consumption and income (Chauvet and Potter, 2001), wages and prices (Sensier and Van Dijk, 2003), and a more comprehensive characterization by Stock and Watson (2002).[3] Most of studies so far are based on quarterly series. Yao and Kholodilin (2004) find that reduction in volatility is significant only in two of the coincident indicators. Incorporating structural breaks appears to account for one-time structural breaks in volatility evident in series, but does not remove the break in the estimated co-movement completely. Yao and Kholodilin (2004) also incorporate structural breaks in growth rates and volatility into the dynamic factor model with Markov switching. Likelihood ratio tests suggest that the added feature has significantly improved the statistical property of a plain model based on either Stock and Watson (1989) or Kim and Nelson (1998). It has exaggerated the weak signals for recessions when the state of economy is in low-variance regimes, similar to what the U.S. economy has experienced since the mid-1980s.

The advantage of using dynamic factor model in contrast to the univariate models on reduction in volatility is that it estimates an unobserved underlying state variable or co-movement between the various macroeconomic series, which is one of two key features of business cycles specified by Burns and Mitchell (1946). This paper identifies the business cycle turning points using dynamic factor model. It makes two specific contributions to the literature: using mixed-frequency data and incorporating structural breaks. The first feature enables us to include real GDP (low frequency) together with four coincident indicators (high frequency) when estimating composite coincident index (CCI) or co-movement. GDP, by its very nature, is one of the most important economic indicators, and ignoring information in real GDP certainly would make less efficient the estimation of the conventional dynamic factor models for business cycle studies. The constructed model in this paper should be able to be applied to more

[3] However, studies have shown that the bulk of variations in most macroeconomic series take place at seasonal frequencies and only a much smaller proportion within business cycles. Yao (2004) examines the changes in the output fluctuations at various frequencies. Using univariate spectra of a number of important macroeconomic series, it finds that although variation at frequencies of business cycles in all the series and total variation in most series have been moderated, the variation at business cycles and trading-day-effect frequencies have increased. As firms improve their inventory control and supply-chain management, variations in investment of various inventories have shifted from lower frequencies to higher frequencies, which corresponds to a shift of adjustments to shorter cycles.

general examples. As stated in MPQ (2000), the structural break in volatility affects the implementation of a range of econometric techniques and macroeconomic policies. In a typical dynamic factor model with regime switching, the presence of one-time reduction in volatility of coincident indicators clearly affects the time horizon over which the second or higher moment of the resulting CCI and the estimated probabilities that the overall economy is in recession should be computed. On the empirical side, the volatility break implies that dynamics factor model such as Stock and Watson (1989) model used to estimate CCI could have been mis-specified when the variance is modeled as constant. Generally, the practical realization of the dynamic factor model is impeded by the structural breaks, which introduce discontinuities in the time series. Therefore, considerations of possible structural breaks in coincident indicators are both theoretically and practically interesting. Moreover, the probabilistic approach to the volatility shift adopted in this paper does not impose any predefined breakpoint rather estimating the moment when the structural break occurred which is reflected in the conditional regime probabilities.

The remainder of the paper is organized as follows: Section 2 sets up a basic dynamic factor model with Markov switching and discusses its possible extensions to allow structural breaks and use mixed-frequency data. In section 3 the model is estimated using the U.S. Post-World War II monthly and quarterly macroeconomic time series. Section 4 summarizes the main findings of the paper.

2. Model

2.1. Dynamic Factor Model with Four Regime Switching

Given a set of coincident indicators y_t ($n \times 1$ vector; $t = 1,\ldots,T$), their growth rates are explained by a latent common factor and n idiosyncratic factors, which are specific to each component series. Formally, the measurement equation for Δy_t is defined as:

$$\Delta y_t = \Gamma \Delta c_t + u_t \tag{1}$$

where Δc_t is interpreted as the growth rates of estimated CCI, u_t is the $n \times 1$ vector of specific factors, and Γ is the $n \times 1$ vector of loadings. In the transition equations describing the dynamics of common and specific factors, both the common dynamic factor Δc_t and idiosyncratic factors u_t are assumed to be independent of each other, but serially correlated with their own lags and driven by noise terms w_t and ε_t respectively.

$$\Delta c_t = \mu(s_t^{\mu}) + \Phi(L)\, \Delta c_{t-1} + w_t \tag{2}$$

$$u_t = \Psi(L)\, u_{t-1} + \varepsilon_t \tag{3}$$

where $\mu(s_t^{\mu})$ is regime-dependent mean, s_t^{μ} is the unobserved binary regime variable (taking values of 1 or 2), and $\Phi(L)$ and $\Psi(L)$ are autoregressive (AR) polynomials of common and specific factors with orders p and q ($q = max\{q_1,\ldots,q_n\}$), respectively. w_t are mutually and serially uncorrelated, following normal distribution with zero means. Both the mean and variance of growth in CCI vary depending on the regimes: the mean is usually assumed to be low in recessions and high in expansions, whereas the variance often happens to be high in recessions and low in expansions. Therefore, ε_t is the serially uncorrelated but with regime-dependent variance:

$$\varepsilon_t \sim NID(0, \sigma^2(s_t^{\sigma})). \tag{4}$$

Regimes embedded in variance, s_t^{σ} (high variance or 1 vs. low variance or 2) are assumed to be independent from those in mean s_t^{μ} (high growth or 1 vs. low growth or 2). This specification yields four possible states for the means of CCI, and two state-dependent residual variances, σ_j^2, where j={high, low}. Namely,

	Regime 1	Regime 2	Regime 3	Regime 4
Combined state variable	$s_t=1$	$s_t=2$	$s_t=3$	$s_t=4$
State variable for intercept	$s_t^{\mu}=1$	$s_t^{\mu}=2$	$s_t^{\mu}=1$	$s_t^{\mu}=2$
State variable for variance	$s_t^{\sigma}=1$	$s_t^{\sigma}=1$	$s_t^{\sigma}=2$	$s_t^{\sigma}=2$

The MS 4-regime framework used in our dynamic factor model dealing with multivariate data extends the model used by MPQ (2000) to examine the reduction in volatility of the univariate real GDP series.

The transitions of both mean and variance under different regimes (s_t^{μ} and s_t^{σ}), incorporated into (2) and (4), are governed by two separate hidden Markov processes. They are summarized by the transition probabilities matrix Π^k ($k=\{\mu, \sigma\}$) with a characteristic element $p_{ij} = prob(s_t=j\,|\,s_{t-1}=i)$. Provided that two state variables s_t^{μ} and s_t^{σ} are independent, the 4×4 transition probabilities matrix, Π, governing the behavior of the "combined" state variable, s_t, would look as:

$$\begin{pmatrix} p_{11}^{\mu}p_{11}^{\sigma} & (1\text{-}p_{11}^{\mu})p_{11}^{\sigma} & p_{11}^{\mu}(1-p_{11}^{\sigma}) & (1\text{-}p_{11}^{\mu})(1-p_{11}^{\sigma}) \\ (1\text{-}p_{22}^{\mu})p_{11}^{\sigma} & p_{22}^{\mu}p_{11}^{\sigma} & (1\text{-}p_{22}^{\mu})p_{11}^{\sigma} & p_{22}^{\mu}(1-p_{11}^{\sigma}) \\ p_{11}^{\mu}(1-p_{22}^{\sigma}) & (1\text{-}p_{11}^{\mu})(1-p_{22}^{\sigma}) & p_{11}^{\mu}p_{22}^{\sigma} & (1\text{-}p_{11}^{\mu})p_{22}^{\sigma} \\ (1\text{-}p_{22}^{\mu})(1-p_{22}^{\sigma}) & p_{22}^{\mu}(1-p_{22}^{\sigma}) & p_{22}^{\mu}(1-p_{22}^{\sigma}) & p_{22}^{\mu}p_{22}^{\sigma} \end{pmatrix} \tag{5}$$

In fact, $\Pi^k = \Pi^{\mu} \otimes \Pi^{\sigma}$, where Π^{μ} and Π^{σ} are the transition probabilities matrices for the state variables s_t^{μ} and s_t^{σ}, and $\otimes$ is the Kronecker product. When there is only regime switching governing mean growth rates, the above model gets back to dynamic factor model without (Stock and Watson, 1989) or with regime switching (Chauvet, 1998; Kim and Nelson, 1998).

A restricted model can also be specified following Kim and Nelson (1999a) who also estimate their model based on the univariate GDP data. They treat low-volatility regime as an absorbing state, implying that whenever the system attains this state, it remains there forever. This assumption translates into the following constraint imposed on the transition probabilities matrix Π^{σ}:

$$\Pi^{\sigma} = \begin{pmatrix} p_{11}^{\sigma} & 1 - p_{11}^{\sigma} \\ 0 & 1 \end{pmatrix} \tag{6}$$

Like duration measures in one regime-switching model (for instance, Kim and Nelson, 1998), the expected duration of the high-volatility regime can be calculated as $\frac{1}{1 - p_{11}^{\sigma}}$, which actually suggests the approximate location of the breakpoint. These two models, *i.e.*, unrestricted and restricted, can be compared using the standard likelihood ratio (LR) test.

2.2. Modeling Mixed-Frequency Data

The dynamic factor model with double regime switching can be extended to include mixed-frequency coincident indicators. The conventional models only utilize four monthly coincident indicators that National Bureau of Economic Research (NBER) uses to determine the turning points: non-farm employment (EMP), personal income less transfer payments (INC), industrial production (IP) and manufacturing and trade sales (SAL). However, the most important coincident indicator, real GDP, is usually ignored because it is only available on a quarterly basis. Mariano and Murasawa (2003) explain two benefits of including GDP into dynamic factor model: improving the efficiency due to additional information and being able to interpret the common factor as the growth rates of latent monthly real GDP. The interpretation leads to natural identification of the mean and variance of the common factor as those of quarterly real GDP by assuming the loading factor of the latter to be 1. Stock and Watson (1989) normalize the variance of the common factor to be 1 and identify the mean growth rates of common factor as a weighted average of monthly coincident indicators. As a result, interpretation of common factor was unclear.

The central idea of mixed-frequency data in dynamic factor model is to replace the missing observations of quarterly GDP with artificial observations from the standard distribution independent of the model parameters and rewrite the state-space model accordingly. Assume that we have $n=n_1+n_2$ observable component series. The first n_1 component series, y_1, are observed at lower frequency (each $f>1$ periods), while the remaining n_2 series, y_2, are measured at a higher frequency which we may normalize to 1. Thus, if we have quarterly and monthly data, then $f=3$ and we observe $y_1=\{y_{13}, y_{16}, \ldots, y_{1.T-3}, y_{1.T}\}$ and $y_2=\{y_{21}, y_{22}, \ldots, y_{2.T-1}, y_{2.T}\}$. Denote by y^*_{1t} the values of the first n_1 component series that we might have observed if these series were measured at the same frequency as y_2, that is, $y^*_{1t}=\{y^*_{11}, y^*_{12}, \ldots, y^*_{1.T-1}, y^*_{1.T}\}$. The observed lower-frequency series can be expressed as an arithmetic average of these unobserved values:

$$y_{1t} = \frac{1}{f}\sum_{i=0}^{f-1} L_i y^*_{1t} \tag{7}$$

where the left-hand-side variable is reported every three months at periods f, $3f, \ldots, T$ and the right-hand-side variable is observed in each month, *i.e.*, $1, 2, \ldots, T$. The growth rates of these series[4] are eventually formulated as:

$$(1-L^f)y_{1t} = \frac{1}{f}(\sum_{i=0}^{f-1} L_i)^2(1-L)y^*_{1t} \tag{8}$$

and

$$(\sum_{i=0}^{f-1} L_i)^2 = \sum_{i=0}^{2f-1}(f+1-|i-f|)L^i$$

for a more general case of low frequencies. For instance, (8) for quarterly data becomes

$$\Delta y_{1t} = \frac{1}{3}\left(\Delta y^*_{1t} + 2\Delta y^*_{1t-1} + 3\Delta y^*_{1t-2} + 2\Delta y^*_{1t-3} + \Delta y^*_{1t-4}\right) \tag{9}$$

To estimate the model at the higher frequency, the unobserved values of the lower-frequency time series are treated as missing. As Mariano and Murasawa (2003) have shown, they can be replaced by any random variable that is independent of the parameters of the model. In particular, the missing observations may be substituted by

[4] Let, for example, y_{1t} be the quarterly series. Thence their first difference is the quarterly growth rate. But since our model is expressed in terms of the higher (monthly) frequency, to designate this first-order difference we need operator $\Delta^3=1-L^3$.

zeros. Thus, the growth rates of the first n_1 variables expressed at the higher frequency can be constructed as:

$$(1-L)\hat{y}_{1t}^{*} = \begin{cases} (1-L^{f})y_{1t} & if \quad t = f, 2f, ..., T \\ 0 & otherwise \end{cases} \tag{10}$$

Like in the typical measurement equation, growth rates of mixed-frequency observables, Δy_t, can be explained by a common factor and their idiosyncratic dynamics. Therefore, equation (1) has become:

$$\begin{pmatrix} (1-L^{f})\hat{y}_{1t}^{*} \\ (1-L)y_{2t} \end{pmatrix} = \Gamma \begin{pmatrix} \frac{1}{f}(\sum_{i=0}^{f-1} L^{i})^{2} I_t \\ 1 \end{pmatrix} (1-L)c_t + \begin{pmatrix} \frac{1}{f}(\sum_{i=0}^{f-1} L^{i})^{2} I_t \\ 1 \end{pmatrix} u_t \tag{11}$$

where I_t is the indicator function:

$$I_t = \begin{cases} 1, & when \quad t = f, 3f, ..., T \\ 0, & otherwise \end{cases} \tag{12}$$

and Γ is the $n \times 2$ factor loadings matrix:

$$\Gamma = \begin{pmatrix} \Gamma_1 & O_{n_1} \\ O_{n_2} & \Gamma_2 \end{pmatrix} \tag{13}$$

where Γ_1 and Γ_2 are the vectors of loading factors for the lower- and high-frequency series, respectively, and O_n is $n \times 1$ vector of zeros. Equations (8), (10) and (11) are more general expression of mixed-frequency model in Mariano and Murasawa (2000). Except that equation (1) is replaced by (11), equations (2) – (6) remain the same for our dynamic factor model when mixed-frequency time series are used for estimation. The model is expressed in the state-space form and estimated by the maximum likelihood method as in Kim and Nelson (1999b).

3. Real Example

The data employed in this study are quarterly real GDP published by U.S. Bureau of Economic Analysis and four monthly coincident indicators characterizing the general state of the U.S. economy: EMP, INC, IP, and SAL as defined earlier. These seasonally adjusted historical series are readily available on the NBER website and are plotted in Figure 1, where the shaded areas represent the U.S. recessions defined by NBER. The sample begins in January 1959 (or first quarter of 1959) and goes through March 2004 (or first quarter of 2004).

EMP and INC are broad measures on the labor input and real income of the overall economy. IP and SAL represent real output of two important sectors, manufacturing and trade, respectively. Real GDP measures aggregate output. As with Stock and Watson (1989) and Kim and Nelson (1998), the first differences of the logs of all the series are demeaned and standardized.

3.1. Evidences of Structural Break in Volatility

The existence and exact timing of structural breaks in these five coincident indicators can be tested either by single series or by multiple series. The Wald or likelihood ratio (LM) test[5] developed by Andrews (1993) and Andrews and Ploberger (1994) is used by MPQ (2000) for the significance of reduction in volatility in real GDP data and by Yao and Kholodilin (2004) for that in four monthly coincident indicators. A rejection of the null hypothesis of stability simply implies the existence of structural break. In the former study, a single break is detected around the first quarter of 1984 and in the latter study, a single break is only significant in EMP and IP around February 1984. There is a less significant break point in SAL around February 1984 and no significant break in INC at all.

Yao and Kholodilin (2004) have also found that the structural break that has taken place in monthly coincident indicators since mid-1980s is not one-time and thus adapt a test developed by Bai and Perron (1998) for the presence of multiple breaks. Bai-Perron test eventually calculates a corresponding confidence interval by minimizing the sum of square residuals among all possible sub-samples:

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[5] Wald statistic is based on the comparison of mean estimators before and after each time point of $\sqrt{\frac{\pi}{2}}|\hat{\varepsilon}_{it}|$, which is unbiased estimator of the standard deviation of error term in a AR(2) regression of the indicator.

$$\hat{\tau}_i = \arg\min_{1<\tau_i<T} S_T(\tau_i) \tag{14}$$

where $S_T(\tau_i)$ is the sum of squared residuals from the estimated model in both sub-samples when the break is at time τ_i. The mean and 90% confidence interval of break points suggested by Bai-Perron test are September 1983 [April 1983, November 1986] for EMP, February 1984 [June 1982, August 1989] for IP and February 1987 [August 1982, July 1996] for SAL. The means are almost identical to the single break point detected in the corresponding series using Wald test.

Another alternative test, widely used in the literature of volatility (for instance, Stock and Watson, 2002; Irvine and Schuh, 2004) when the break point is known, is to directly test the equality of means and variances of sub-samples before and after 1984, for which MPQ (2000) detect a one-time break in GDP. These results are reported in Table 1. The columns two and four represent *Z-statistic* following a normal distribution and *F-statistic* following $F(n_1,n_2)$ distribution for means and variances, respectively. n_1 and n_2 represent the sizes of each sub-sample. The *p*-values suggest that variances of all five series are significantly different over two periods and the reduction in volatility is most evident in GDP, EMP, and IP. The means of these series are not so different over two periods except that for EMP.

In addition to univariate tests, we also test and construct valid confidence intervals for the date of a single break in multivariate time series following methods developed by Bai, Lumsdaine and Stock (hereafter BLS, 1998). The BLS test is based on a system equation

$$y_t = \mu + \sum_{j=1}^{p} A_j y_{t-j} + \Gamma X_{t-1} + d_t(k)(\lambda + \sum_{j=1}^{p} B_j y_{t-j} + \Pi X_{t-1}) + \varepsilon_t \tag{15}$$

where y_t, μ, λ, and ε_t are $n\times 1$ and $\{A_j\}$ and $\{B_j\}$ are $n\times n$; $d_t(k) = 0$ for $t \le k$ and $d_t(k) = 1$ for $t > k$; and X_t is a matrix of stationary variables. Equation (15) can be written in matrix form

$$y_t = (V_t^{'} \otimes I)\theta + d_t(k)(V_t^{'} \otimes I)\delta + \varepsilon_t \tag{16}$$

where $V_t^{'} = (1, y_{t-1}^{'}, \ldots, y_{t-p}^{'}, X_{t-1}^{'})$, $\theta = Vec(\mu, A_1, \ldots, A_p, \Gamma)$, $\delta = Vec(\lambda, B_1, \ldots, B_p, \Pi)$, and I is a $n\times n$ identify matrix. The above equation specifies a full structural change in that it allows all coefficients to change. From MPQ (2000) and Yao and Kholodilin (2004), we know that only a subset of coefficients such as the intercept, if we are talking about the series of conditional variance, has a structural break. Therefore, a partial structural change is more appropriate. BLS (1998) also point out that tests for partial structural changes will have better power than those for full structural changes. This consideration would generate a partial structural change model

$$y_t = (V_t^{'} \otimes I)\theta + d_t(k)(V_t^{'} \otimes I)S^{'}S\delta + \varepsilon_t \tag{17}$$

where S is a selection matrix containing 0's and 1's and having full row rank. Its rank is equal to the number of coefficients that are allowed to change. For a break in the intercept only, $S = (s \otimes I)$ with $s = (1, 0, \ldots, 0)$, we have

$$\begin{aligned} y_t &= (V_t^{'} \otimes I)\theta + \lambda d_t(k) + \varepsilon_t \\ &= Z_t^{'}(k)\beta + \varepsilon_t \end{aligned} \tag{18}$$

where $Z_t^{'}(k) = ((V_t^{'} \otimes I), d_t(k)(V_t^{'} \otimes I)S^{'})$ and $\beta = (\theta^{'}, (S\delta)^{'})^{'}$. BLS test for a break in the coefficients are based on the sequence of F-statistics (F_T) testing $S\delta = 0$, for $k = k_*+1, \ldots, T-k_*$, where k_* is a trimming value. From this point forward, procedures of BLS test are similar to those in univariate analysis. Both tests detect the possible break point by maximizing Wald statistics or the logarithm of the Andrews-Ploberger (1994) exponential statistic

$$Sup-W: \sup_{\tau \in (\tau_*, 1-\tau_*)} F_T(\tau) \quad , \tag{19}$$

$$Exp-W: \ln\{\int_{\tau_*}^{1-\tau_*} \exp\{\frac{1}{2}F_T(\tau)\}d\tau\} \quad . \tag{20}$$

where τ_* refers to an initial fraction of the sample which is trimmed. The break-point estimator is defined as the month that maximizes the likelihood function of (17) or (18)

$$\hat{k} = \arg\max_{1 \le k \le T} L(k, \hat{\beta}(k), \hat{\Sigma}(k)), \tag{21}$$

$\hat{\Sigma}(k)$ is the variance-covariance matrix of the error terms.

Table 2 reports test results for univariate through quatrivariate time series analysis among four monthly coincident indicators. The break points of univariate BLS test are similar to those from Bai-Perron test for multiple break points and those from Andrews-Ploberger test for single point. The confidence intervals from the BLS test are wider than those from Bai-Perron test, and have relatively earlier lower and upper limiting dates. Like Yao and Kholodilin (2004), structural break is most significant in EMP and then IP, less significant in SAL and insignificant in INC. This order rules the results in bivariate through quatrivariate time series: the break point and confidence interval of EMP dominate those of combinations of EMP with other three indicators; those of INC would dominate its combination with SAL and INC; SAL would dominate its combination with INC only. As a result, break date and confidence interval from all four monthly coincident indicators are very close to those of EMP. Therefore, BLS test has detected a significant structural break on November 1983 in volatility of four current coincident indicators with 90% confidence interval around [June 1982, April 1985]. We then apply the dynamic factor model with structural breaks to the mixed-frequency data.

3.2. Estimation

We have estimated six types of dynamic factor models with regime switching. These MS models are defined by equations (1) or (11) and (2) – (6) and sorted as:

(a) Dynamic factor model with no structural break based on single-frequency data, which is identical to models in Chauvet (1998) and Kim and Nelson (1998) (2-regime switching model or Base (2));

(b) Dynamic factor model with no structural break based on single-frequency data based on mixed-frequency data (MF (2));

(c) Restricted dynamic factor model with structural break based on single-frequency data (ResSF (4));

(d) Restricted dynamic factor model with structural break based on mixed-frequency data (ResMF (4));

(e) Unrestricted dynamic factor model with structural break based on single-frequency data (UnSF (4));

(f) Unrestricted dynamic factor model with structural break based on mixed-frequency data (UnMF (4)).

The orders of AR polynomial, (p,q), can be determined by minimizing Akaike information criterion (AIC) defined as $-2(l/T) + 2(k/T)$, where k is the number of parameters in the model, T is the sample size and l is the value of the log of the likelihood function using the estimated parameters. In our case, the optimal lag number is 2 for both common and specific factors, which is consistent with Stock-Watson (1989) and Kim-Nelson (1998). For the purpose of identification, the first loading factor in all the models is normalized to 1, *i.e.*, the loading factor for EMP in single frequency models and that for GDP in mixed frequency models.

The parameter estimates and their standard errors of models (a) and (b) are reported in Table 3, where all the parameters are statistically significant. The estimates of Base (2) model is similar to those in Kim and Nelson (1998), where the sum of the AR coefficients (0.519) for the state variable is significantly lower, implying less state-dependence in the resulting CCI. Both these two MS models clearly distinguish between two clear-cut regimes of positive and negative growth rates. They have also estimated negative AR dynamics for specific factors of INC, IP and SAL, but positive for that of EMP. Since Kim and Nelson (1998) only covers the data from January 1960 to January 1995, slowdown of January 1995 to January 1996 and the latest recession of March to November 2001 are not included. The additional period in our study could have impacted the estimated duration and mean growth rates: compared with Kim and Nelson (1998), p_{11} is higher suggesting longer duration of recessions; μ_2 is lower because of more recessions in this sample period. The higher μ_1 is more likely to being attributable to the reduction in volatility. The loading factors (γ_i) of CCI entering each measurement equation are roughly the same, ranging from 0.709 to 1.290 with larger weights on EMP and IP than the other two. Including real GDP data into Base (2) forms MF (2) model. Put in another way, Base (2) is a restricted version of MF (2) where all coefficients related to GDP are set to be 0's. The estimates are largely the same, except a few minor changes. μ_2 has become less negative due to the diffusion effect from real GDP. Variances of common factor and most specific factors are smaller, except for IP, which is complemented by increased weights of their AR dynamics. As a by-product, MS models estimate both filtered and smoothed probabilities that U.S. economy is in recession or low-growth state. The latter of both Base (2) and MF (2) are plotted in Figure 2, where the shaded areas represent NBER business cycle chronology as benchmarks. Both models appear to have missed the latest two recessions after the break point in mid-1980s, while the duration of the earlier recessions is grossly underestimated. It is also not surprising to see that ignoring reduction in volatility since mid-1980s can suppress the signals for the phases of low growth in conjunction with low variance when the model is overviewed by a complete sample from 1959 up to the latest[6].

We next proceed with four different four-regime switching models. The parameter estimates and their standard errors of all four models are reported in Table 4.

[6] This is not the case when a shorter sample is considered. We have also estimated the model for the samples ending in June 2002, December 2002, June 2003, etc. Our observation was that as soon as the data of the second half of 2002 were included, the predicting performance of the conditional regime probabilities drastically went down.

Likewise, both Base (2) and MF (2) can be considered restricted version of their corresponding four-regime switching models, where coefficients related to second regime switching for variances are suppressed to 0's. All the values of log-likelihood function are reported in the last row of Table 4. Neither of the restricted four-regime models alter the coefficient estimates from previous two-regime models. Both restricted models impose $p_{22}^{\sigma}=1$ on the corresponding restricted models. The purpose of this restriction is to constrain the transition probabilities of regime switching for variances to have a one-time permanent shift from regime 1 to 2. Between restricted and unrestricted models, the restriction imposed on the transition probabilities matrix of the state variable for variance can be rejected at 5% significance level using LR test. The test statistic for the single-frequency model is equal 24.8 and that for the mixed-frequency model is 22.4 against a critical value of $\chi^2_{0.95}(1)$ =3.84 in both cases.

All four-regime switching models make clear distinction between states of low growth and high growth or between those of low variance and high variance. The coefficient estimates of μ_1 and μ_3 (states of high growth) are always positive in contrast to the negative estimates of μ_2 and μ_4 (states of low growth). Unrestricted models, compared with restricted models, have expanded the gap between growth rates of low-growth regime and high-growth regime in conjunction with same regime of variances, *i.e.*, μ_1 - μ_2 and μ_3 - μ_4 are increased. Likewise, standard errors of μ_1 and μ_2 (states of high variance) are about 3-4 times larger than those of μ_3 and μ_4 (states of low variance). This implies that the shift in the volatility was accompanied by a "stabilization" of the growth rates. Both recessions and expansions became milder. The gap between standard errors of low-variance regime and high-variance regime under same regime for growth rates has also deepened in unrestricted models. Therefore, the restriction imposed on transitional probabilities has re-arranged the distribution of estimates in different regimes. Loading factors (γ_i — the elements of matrix Γ, where $i=1,\ldots,n$) on EMP and IP in all models are still the heaviest among all, suggesting their larger influence in forming the common factor. The AR coefficients of common factor (ϕ_1 and ϕ_2) in restricted models are similar to those in two base MS models reported in Table 3. However, when the restriction is lifted in unrestricted models, those dynamics have increased from 0.4 to 0.5, implying more state dependence in estimated CCI from unrestricted models. Variances (σ_1 and σ_2) of common factor and those (for instance σ_{EMP}) of specific factors have also increased in unrestricted models. All these increases are complemented by a negative increase in AR coefficients ($\psi_{1,i}$ and $\psi_{2,i}$) of specific factors.

Based on $\dfrac{1}{1-p_{11}^{\mu}}$ and $\dfrac{1}{1-p_{11}^{\sigma}}$ respectively, we can calculate the expected duration of low-growth regimes (recessions) and high-variance regimes. In both restricted models, the expected duration of the high-volatility state is equal to 333 months. Since transition from high to low variances is just one-time, we can figure out that the expected break point corresponds to October 1986. This date is somewhat late compared to the early 1984 or late 1983 suggested by most of tests in previous section, but it is still within the 90% confidence interval reported in Table 2. Both unrestricted models, on the other hand, estimate a much shorter duration. The expected duration of recessions is calculated ranging from 27 to 40 months out of four models.

Figure 3 plots the estimated CCI (or common factor) from six different models, which is derived from the estimated Δc_t in various models:

$$c_t = c_{t-1} + \mu(s_t) + \Delta c_t . \tag{22}$$

All CCIs appear to be identical to one another and they match up well with NBER recessions marked by shaded areas. The common factors from the mixed-frequency models have a far smaller variance than those from the single-frequency model. Figure 4 depicts the smoothed conditional probabilities of low-growth regime (regimes 2 and 4) from four different four-regime switching models. There are relatively good correspondences between the model-derived probabilistic dates and the NBER's chronology, and the former has captured all NBER recessions with matching timing. However, the model-derived probabilities suggest different dynamics of two latest recessions as NBER Dating Committee perceives: both recessions feature "double-dip" as described in Lahiri and Yao (2004). That is, some coincident indicators such as sectoral output measures IP and SAL are more sensitive to shocks than others, such as broader measures EMP and INC. When a monetary policy or supply shock begins, certain industries or processes get hit first, which alter price, output and people's expectation. As the shock dies away, the contaminated industries, regions or processes are back to the track of recovery. But meanwhile as Fed observes change in price and output, a stabilization policy will be implemented, which can well interrupt the recovery process and transmit the initial effect of shocks into broader scope. A fully-fledged recession will come into being. The phase before this point is usually negligible and limited to certain industries or regions and the phase since this point is the recession that NBER defines. As a result of economic stabilization, recessions since mid-1980s have become shorter and milder (Moore (1987)), but they are usually followed or preceded by fairly long slowdowns. The slowdown before recession of July 1990 to March 1991 has started as early as February 1988 and ended around July 1988. The latest recession is followed by a fairly long weak recovery or slowdown till as late as late 2003.

In Figure 4, the smoothed probabilities from both restricted models began to signal recession in February 1989 and drop in August 1989. This period corresponds to a mild slowdown preceding the 1990 recession. The model-derived recession dates are from April 1990 to March 1991. The probabilities signaled the start of the latest recession in June 2000, which is the date when both IP and SAL declined. For the controversy on dating the latest peak, see Lahiri and Yao (2004). Both models also capture a slowdown of July 2002 to April 2003 following the latest recession. The two unrestricted models have the similar probabilities as the corresponding restricted model. But they did not make distinction between two separate recessions in early 1980s. They also picked up an additional down state in 1967.

Figure 5 depicts the smoothed probabilities of low variance state (regimes 3 and 4) from four-regime switching models. The restricted models generate a similar one-time break point from high variance to low variance in February 1984, which is consistent with that in Kim and Nelson (1999), MPQ (2000), and Yao and Kholodilin (2004). Both restricted models have similar probabilities of low variance after February 1984, but they also signaled many episodes of low variances during the period of 1962 to 1983. The four-regime models as depicted in Figure 5 have fully captured two latest recessions since mid-1980s that the previous two two-regime switching models have missed as plotted in Figure 3. We thus plot the smoothed probabilities of low growth and low variance (regime 4) in Figure 6 to show the additional gains in implied probabilities from modeling regime switching for variances. Since the entire period since February 1984 is sorted as low-variance states, the two latest recessions are identified due to the addition of regime switching for variances.

3.3. Evaluation

The above comparison of implied probabilities against official business cycle chronology can be formalized into in-sample evaluation and out-of-sample evaluation when the predicted reference chronology is known using different criteria. We first calculate the Quadratic Probability Score (QPS) (Brier, 1950) based on probabilities implied from each model. Let P_t be the probability that the economy is in recession estimated from the model, R_t be the NBER-defined chronology (1 if recession, 0 otherwise), the QPS is given by:

$$QPS = \frac{1}{T}\sum_{t=1}^{T}(P_t - R_t)^2 \tag{23}$$

which ranges from 0 to 1, with a score of 0 corresponding to perfect accuracy. This is the unique proper scoring rule that is only a function of the discrepancy between realizations and assessed probabilities (see Diebold and Rudebusch (1989) for more discussion). In order to compare the forecasting accuracy of different models, that is, to test whether the differences in QPS of each model are significant, we employ a test statistic developed by Diebold and Mariano (1994). The null hypothesis states no difference between the predicted accuracy of the pair of models being compared. Given a sample path $\{d_t\}_{t=1}^{T}$ of a loss differential series (P_t-R_t in the above definition), we have

$$\sqrt{T}(\bar{d} - \mu) \xrightarrow{d} N(0, 2\pi f_d(0) \tag{24}$$

where $\bar{d}$ is the sample mean loss differential, $f_d(0) = \frac{1}{2\pi}\sum_{\tau=-\infty}^{\infty}\gamma_d(\tau)$ is the spectral density of the loss differential at frequency zero, and $\gamma_d(\tau) = E[(d_t - \mu)(d_{t-\tau} - \mu)]$. Mariano (DM) statistic for testing the H_0 of equal forecast accuracy is defined as

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi \hat{f}_d(0)}{T}}} \tag{25}$$

which is standardized and hence asymptotically distributed as $N(0, 1)$. We calculate the DM statistics with rectangular winder of length 21.

The QPS and DM test statistics are reported in Table 5 for all six models: Base (2), ReSF (4), UnSF (4), MF (2), ResMF (4) and UnMF (4). Overall, a few conclusions can be drawn from the comparison: 1) the smoothed probabilities seem to outperform the filtered ones; 2) mixed-frequency models outperform single-frequency models; 3) restricted models outperform unrestricted models; 4) four-regime switching models outperform two-regime switching models. 1) is true due to the fact that smoothing eliminates the smaller spikes which are very typical for the filtered probabilities and which reflect nothing but the noise. The rest of the findings is actually confirmed by our LR tests and graphs in the previous section. DM test results suggest different conclusions between filtered and smoothed probabilities of different models. In Table 5, rejection of H_0, marked by * or **, suggests that two models are significantly different from each other in terms of signaling recessions. From filtered probabilities, there are nine pairs that can reject the H_0 of equality and six pairs that cannot. That is, Base (2) is significantly different from all but MF (2); MF (2) model is not significantly different from Base (2) and ResMF (4); ResSF (4) is not significantly different from UnSF (4), ResMF (4) and UnSF (4); UnSF (4) is not significantly different from only Base (2) and ResMF (4); ResMF (4) is not significantly different from ResSF (2), UnSF (2) and UnsMF (4); UnMF (4) is not significantly different from ResSF (4) and ResMF (4). Based on smoothed probabilities, two out of six similar pairs can reject H_0 now: MF (2) and Base (2), ResMF (4) and UnSF (4). Since both Base (2) and MF (2) are significantly different from other five models, all four-regime switching models have improved the probabilities from two-regime switching models. Both unrestricted models are not significantly different from their corresponding restricted models in terms of implied probabilities. Although restricted MF model does not improve restricted SF model, unrestricted MF model does relative to unrestricted SF model.

4. Concluding Remarks

This paper models the declining volatility of the U.S. economy using the dynamic factor models with Markov regime switching. Apart from the regime switching accounting for the fluctuations at the business cycle frequencies, an additional type of regime switching is created to account for two distinct states: low variance vs. high variance. The resulting four-regime models thus extend univariate analysis in MPQ (2000) and in Kim and Nelson (1999) to multivariate time series. We examined both the unrestricted model of the former and the constrained model of the latter imposing a restriction on transition probability of the variance state variable ($p_{22}^{\sigma}=1$), which makes the low-variance regime an absorbing one.

Moreover, we consider the models with mixed-frequency data: besides the monthly component series we employ such an important quarterly series as that of the real GDP. Thus, four monthly coincident indicators together with GDP are used to estimate underlying states and model-derived probabilities for different regimes. This technique would enable us to interpret the estimated common factor as monthly GDP, which is not appropriate otherwise.

Prior to the estimation step we test for the significance of structural breaks in five monthly and quarterly coincident indicators. Four different test statistics are used: Wald test for single break point in univariate time series adapted by MPQ (2000); traditional *F*-statistic for the equality of two sub-samples divided by the known break point identified by MPQ (2000), which is widely used in this literature; Bai-Perron test for the signficance of multiple break points in a single time series and the corresponding confidence intervals; BLS test for the significance of a single break point in multivariate time series. In particular, BLS test fits best into the specification of dynamic factor models. The results and exact timing of breaks are largely the same from different tests: November 1983 or February or March of 1984 is the mostly likely break point in U.S. economy. In addition, our study shows that the series with more significant break in univariate analysis usually dominates in biviriate or multivariate time series analysis in conjunction with others. As a result, the break point found in EMP governs the test results when all four coincident indicators are used. Either by comparing with NBER official recession dates or among one another, we find that: mixed-frequency models outperform single-frequency models; restricted models outperform unrestricted models; four-regime switching models outperform two-regime switching models.

Six dynamic factors models with Markov switching were eventually examined: three single-frequency models and three mixed-frequency models. Among each type, a basic MS model with two-regime switching, one restricted and one unrestricted four-regime switching models are applied to the data. LR test reject the validity the restriction imposed on transition probabilities p_{22}^{σ}, and thus suggests the superiority of unrestricted models over restricted ones. The similar test also suggests the improved statistical properties of mixed-frequency models over single-frequency models. Although the estimated CCI from six different models appear to be similar, the filtered and smoothed probabilities of low-growth regimes make distinction among them. Four-regime models have correctly signaled two latest recessions which was missed by two-regime models. Because the former has detected a one-time switching from high variance to low variance in February 1984, the ruling low variance since mid-1980s could have prevented the detection of low-growth states in two-regime models. By incorporating regime-switching in variances, four-regime switching models successfully picked up the recessions. The result shows that additional recessions detected by four-regime switching models are identified as the state of low growth and low variance. Various models are evaluated based on filtered and smoothed probabilities using QPS and DM statistics. A strong link between our models' recession probabilities and the NBER chronology is evident. As the formal forecasting accuracy tests show, the four-regime models do not contribute new information in forecasting the NBER dates but allow detecting the secular structural break in the volatility of U.S. economy.

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Appendix

Table 1. Testing the Equality of Means and Variances Before and After January 1984

Variables	Mean		Variance	
	Z-statistics	p-value	F-statistics	p-value
EMP	2.56	0.005	3.47	0.0
INC	1.27	0.103	0.401	0.0
IP	0.95	0.171	3.98	0.0
SAL	0.080	0.468	1.40	0.004
GDP	0.661	0.254	4.46	0.0

Table 2. Means and Confidence Intervals for a Single Break in Multivariate Series

Variables	Model type	*Sup-W*	*Exp-W*	Break point	90% confidence interval
EMP	univariate	48.91	19.62	1983:11	[1982:5, 1985:5]
INC	univariate	3.99	0.76	1972:7	[1959:1, 1988:5]
IP	univariate	17.69	5.77	1984:3	[1979:10, 1988:8]
SAL	univariate	10.24	3.08	1987:4	[1979:10, 1994:10]
EMP/INC	bivariate	52.30	21.23	1983:11	[1982:6, 1985:4]
EMP/IP	bivariate	51.70	21.01	1983:11	[1982:6, 1985:4]
EMP/SAL	bivariate	48.88	19.63	1983:11	[1982:5, 1985:5]
INC/IP	bivariate	18.88	6.41	1984:3	[1980:1, 1988:5]
INC/SA	bivariate	11.31	3.70	1992:11	[1987:5, 1998:5]
IP/SAL	bivariate	18.44	6.28	1984:3	[1980:1, 1988:5]
EMP/INC/IP	trivariate	54.29	22.25	1983:11	[1982:6, 1985:4]
EMP/INC/SAL	trivariate	51.87	21.05	1983:11	[1982:6, 1985:4]
EMP/IP/SAL	trivariate	50.04	20.22	1983:11	[1982:5, 1985:5]
INC/IP/SAL	trivariate	18.87	6.61	1984:3	[1980:2, 1988:4]
All series	quatrivariate	52.71	21.5	1983:11	[1982:6, 1985:4]

Table 3. Estimates of the Two-Regime Switching Models

Parameter	Single-frequency: LL=-2584.91		Mixed-frequency: LL=-3080.36	
	Estimates	s.e.	Estimates	s.e.
p_{11}	0.991	0.005	0.991	0.005
p_{12}	0.621	0.170	0.639	0.177
μ_1	0.031	0.025	0.017	0.013
μ_2	-2.070	0.277	-1.110	0.155
γ_{EMP}	–	–	1.840	0.144
γ_{INC}	0.709	0.063	1.320	0.112
γ_{IIP}	1.290	0.089	2.310	0.136
γ_{SLS}	0.799	0.062	1.480	0.100
Φ_1	0.399	0.056	0.396	0.056
Φ_2	0.120	0.053	0.125	0.054
$\Psi_{1.EMP}$	0.142	0.041	0.680	0.197
$\Psi_{2.EMP}$	0.512	0.046	-0.577	0.079
$\Psi_{1.INC}$	-0.275	0.047	0.144	0.041
$\Psi_{2.INC}$	-0.015	0.077	0.517	0.046
$\Psi_{1.IP}$	-0.184	0.082	-0.289	0.045
$\Psi_{2.IP}$	-0.233	0.072	-0.027	0.043
$\Psi_{1.SAL}$	-0.385	0.047	-0.115	0.077
$\Psi_{2.SAL}$	-0.180	0.046	-0.177	0.066
$\Psi_{1.GDP}$	–	–	-0.404	0.047
$\Psi_{2.GDP}$	–	–	-0.196	0.046
σ_1	0.258	0.032	0.079	0.011
σ_{EMP}	0.291	0.031	0.067	0.029
σ_{INC}	0.732	0.047	0.283	0.031
σ_{IIP}	0.247	0.036	0.720	0.046

Parameter	Single-frequency: LL=-2584.91		Mixed-frequency: LL=-3080.36	
σ_{SLS}	0.593	0.040	0.279	0.034
σ_{GDP}	–	–	0.577	0.039

LL = value of log-likelihood function

Table 4. Estimates of Four-Regime Switching Models

Parameter	Single Frequency Models				Mixed Frequency Models			
	Restricted		Unrestricted		Restricted		Unrestricted	
	Estimates	s.e.	Estimates	s.e.	Estimates	s.e.	Estimates	s.e.
p_{11}^{μ}	0.964	0.014	0.963	0.023	0.962	0.016	0.975	0.018
p_{22}^{μ}	0.850	0.064	0.854	0.074	0.853	0.059	0.897	0.069
p_{11}^{σ}	0.997	0.003	0.931	0.042	0.997	0.003	0.917	0.060
p_{22}^{σ}	1	–	0.971	0.015	1	–	0.967	0.018
μ_1	0.185	0.094	0.103	0.138	0.095	0.045	0.054	0.124
μ_2	-0.683	0.201	-0.310	0.442	-0.310	0.092	-0.090	0.205
μ_3	0.060	0.023	0.064	0.026	0.029	0.012	0.023	0.013
μ_4	-0.354	0.068	-0.284	0.067	-0.173	0.033	-0.146	0.042
γ_{EMP}	–	–	–	–	2.030	0.144	2.030	0.147
γ_{INC}	0.621	0.052	0.629	0.053	1.290	0.112	1.300	0.113
γ_{IIP}	1.120	0.070	1.150	0.071	2.260	0.131	2.300	0.134
γ_{SLS}	0.705	0.051	0.711	0.051	1.450	0.098	1.460	0.100
Φ_1	0.332	0.090	0.434	0.086	0.325	0.087	0.436	0.095
Φ_2	0.120	0.069	0.147	0.063	0.127	0.070	0.162	0.063
$\Psi_{1.EMP}$	0.164	0.044	0.194	0.046	0.709	0.174	0.695	0.186
$\Psi_{2.EMP}$	0.530	0.048	0.487	0.050	-0.613	0.073	-0.595	0.075
$\Psi_{1.INC}$	-0.279	0.044	-0.281	0.046	0.170	0.045	0.198	0.046
$\Psi_{2.INC}$	-0.019	0.035	-0.020	0.052	0.536	0.048	0.492	0.050
$\Psi_{1.IP}$	-0.193	0.066	-0.241	0.072	-0.290	0.052	-0.291	0.048
$\Psi_{2.IP}$	-0.166	0.059	-0.192	0.064	-0.027	0.077	-0.029	0.056
$\Psi_{1.SAL}$	-0.395	0.046	-0.401	0.046	-0.152	0.063	-0.191	0.066
$\Psi_{2.SAL}$	-0.181	0.045	-0.185	0.045	-0.135	0.059	-0.153	0.061
$\Psi_{1.GDP}$	–	–	–	–	-0.407	0.046	-0.416	0.046
$\Psi_{2.GDP}$	–	–	–	–	-0.191	0.046	-0.197	0.046

Parameter	Single Frequency Models				Mixed Frequency Models			
	Restricted		Unrestricted		Restricted		Unrestricted	
	Estimates	s.e.	Estimates	s.e.	Estimates	s.e.	Estimates	s.e.
σ_1	0.551	0.076	1.040	0.204	0.135	0.022	0.269	0.064
σ_2	0.022	0.012	0.035	0.015	0.006	0.003	0.011	0.005
σ_{EMP}	0.220	0.027	0.236	0.026	0.062	0.025	0.067	0.027
σ_{INC}	0.749	0.047	0.749	0.047	0.217	0.025	0.231	0.026
σ_{IIP}	0.315	0.034	0.294	0.033	0.741	0.046	0.740	0.046
σ_{SLS}	0.609	0.040	0.613	0.040	0.332	0.031	0.312	0.031
σ_{GDP}	–	–	–	–	0.600	0.039	0.599	0.040
LL	-2522.84		-2513.95		-3015.77		-3009.26	

Table 5. In-sample Evaluation of MS Models

Filtered probabilities						
Model	QPS	DM statistics				
		ResSF (4)	UnSF (4)	MF (2)	ResMF (4)	UnMF (4)
Base	0.130	1.493*	1.401*	1.191	1.494*	1.753**
ResSF (4)	0.082		0.857	1.481*	0.901	0.239
UnSF (4)	0.088			1.384*	1.047	1.981**
MF (2)	0.129				1.482*	1.737**
ResMF (4)	0.080					0.073
UnMF (4)	0.080					
Smoothed probabilities						
Model	QPS	DM statistics				
		ResSF (4)	UnSF (4)	MF (2)	ResMF (4)	UnMF (4)
Base	0.131	1.534*	1.368*	1.320*	1.531*	1.729**
ResSF (4)	0.068		1.154	1.521*	0.472	0.053
UnSF (4)	0.081			1.350*	1.290*	1.571*
MF (2)	0.130				1.518*	1.711**
ResMF (4)	0.067					0.124
UnMF (4)	0.069					

Note: * and ** denote the rejection of H_0 at 10% and 5% levels; Base represents the 2-regime MS model based on single-frequency data; "Res" represent restricted models; "Un" represent unrestricted models; SF = single frequency data; MF = mixed-frequency data.

Figure 1.

Mixed Frequency Data: Four Monthly Coincident Indicators and Quarterly Real GDP

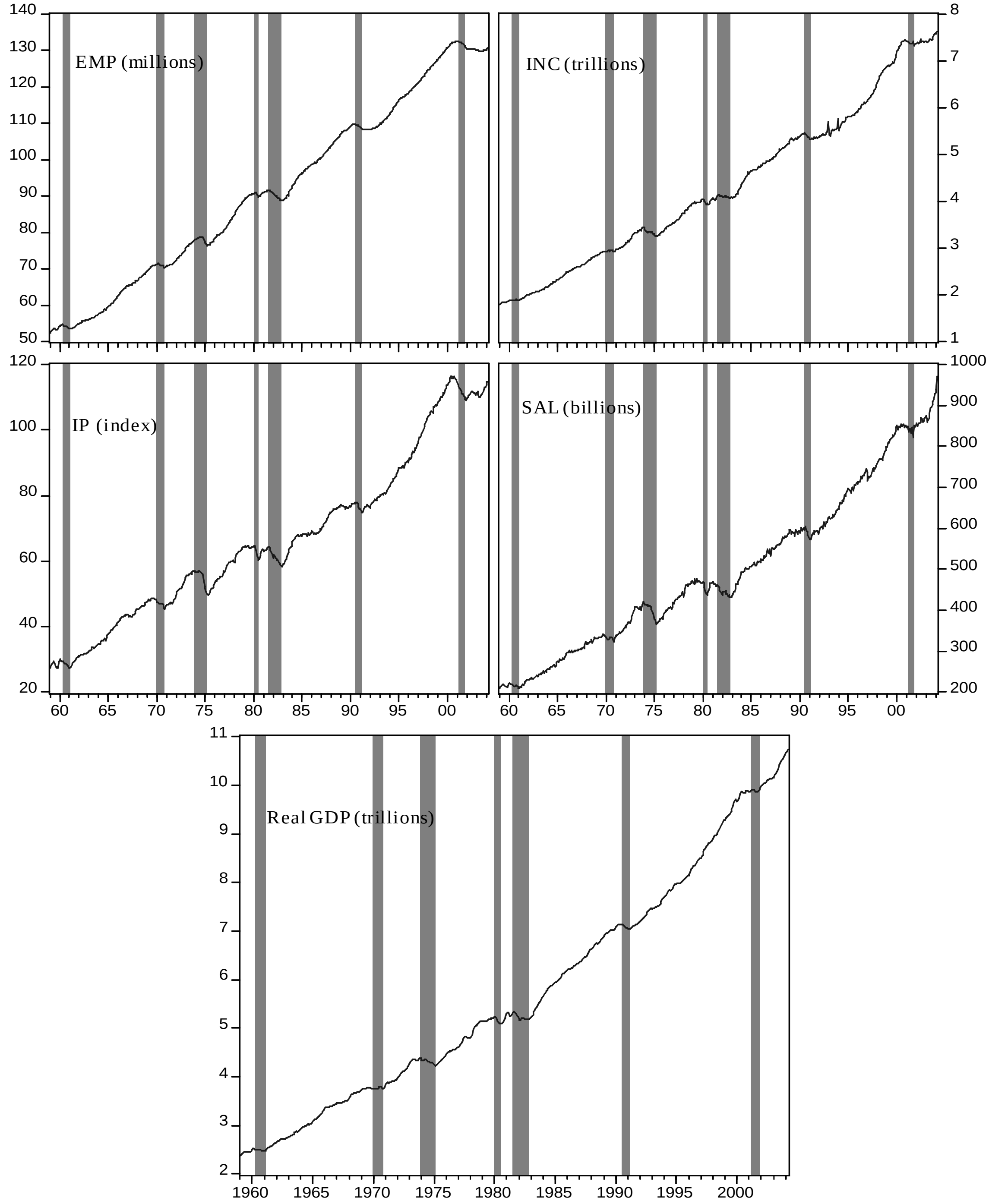


Figure 2.

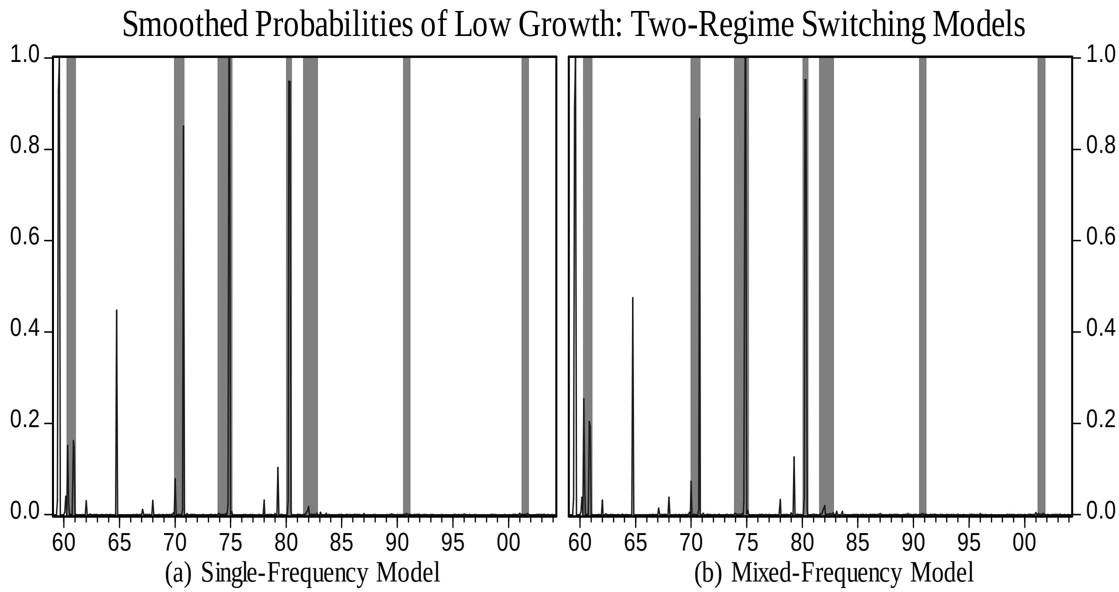

Smoothed Probabilities of Low Growth: Two-Regime Switching Models
1.0
0.8
0.6
0.4
0.2
0.0
60 65 70 75 80 85 90 95 00
(a) Single-Frequency Model
(b) Mixed-Frequency Model

Figure 3.

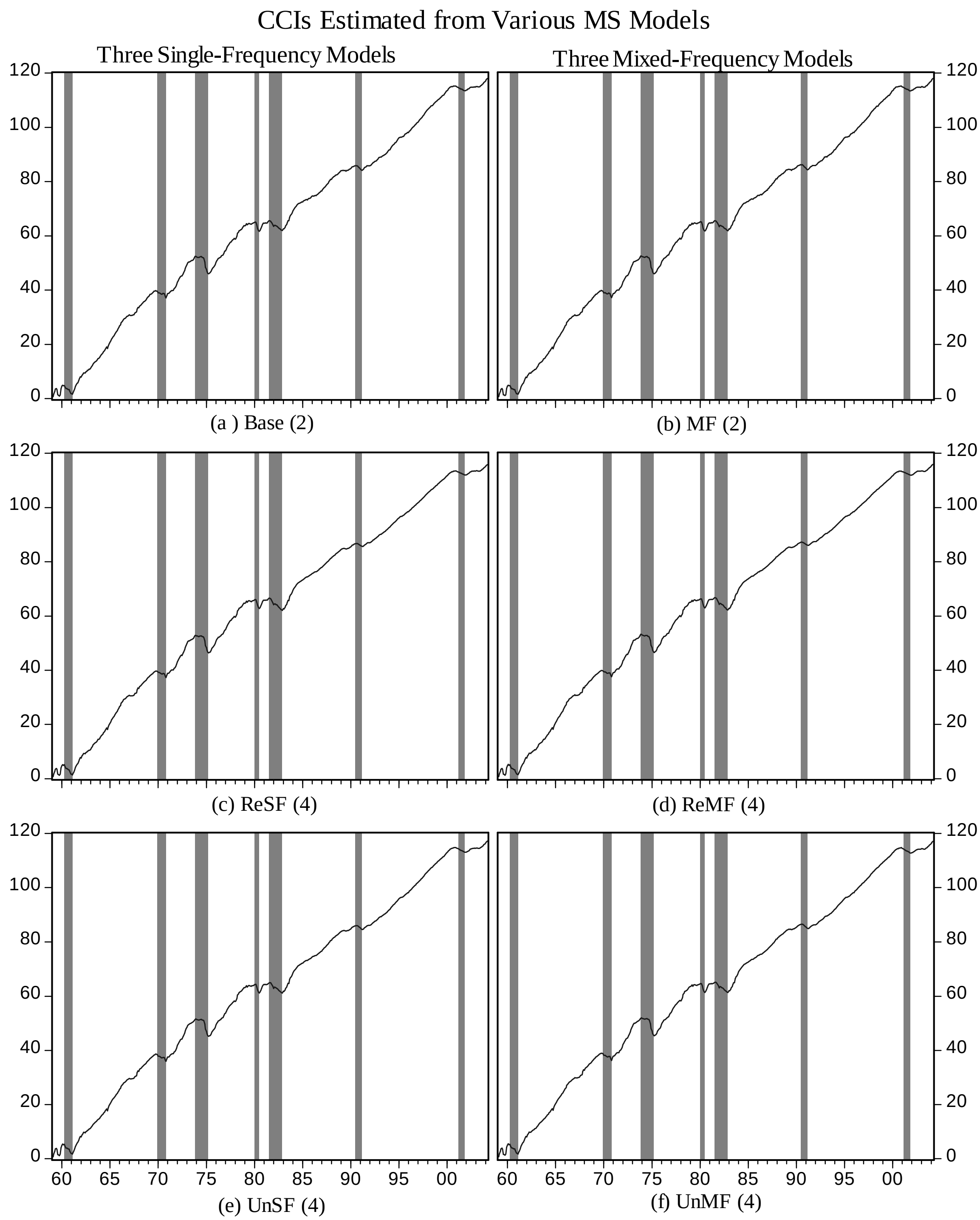
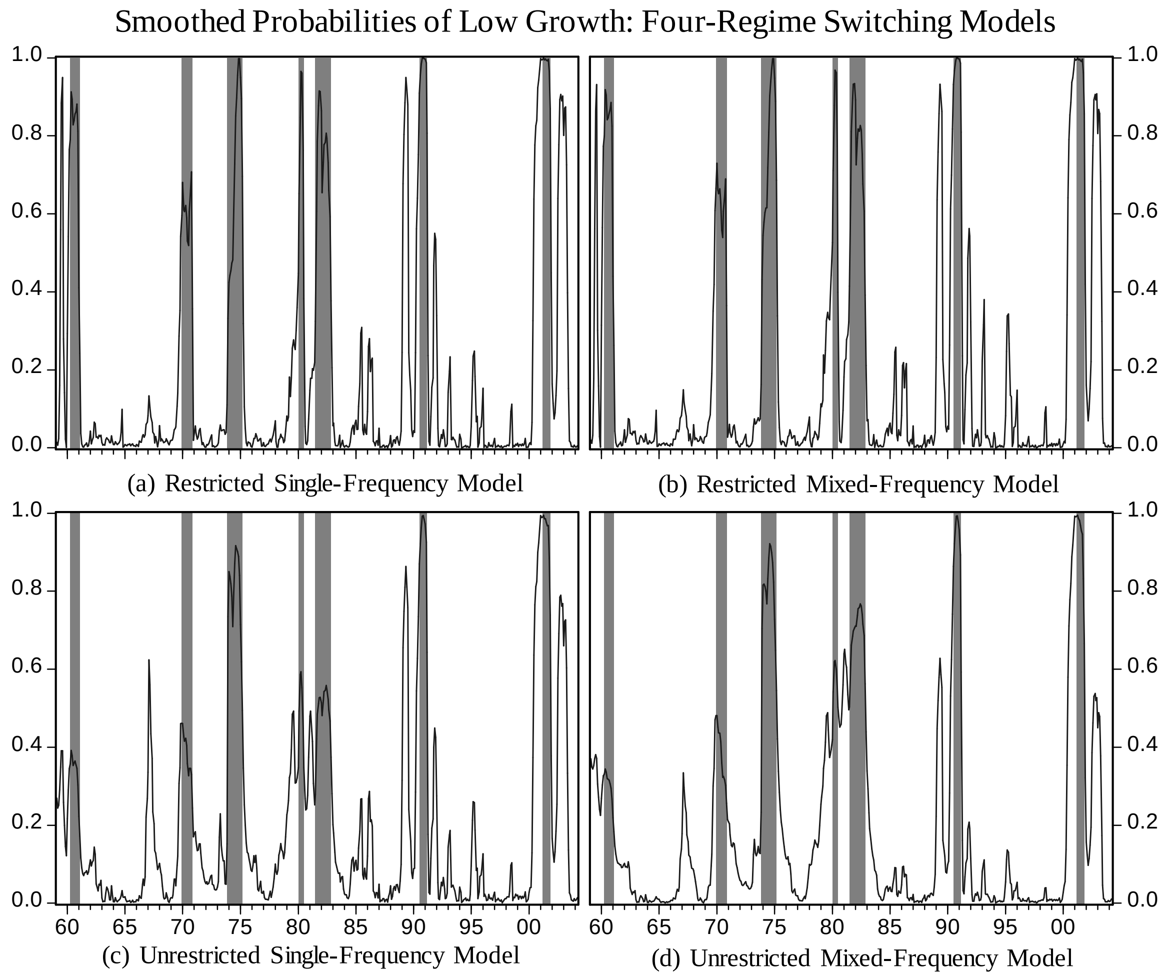

CCIs Estimated from Various MS Models
Three Single-Frequency Models
Three Mixed-Frequency Models
0
20
40
60
80
100
120
60
65
70
75
80
85
90
95
00
(a) Base (2)
(b) MF (2)
(c) ReSF (4)
(d) ReMF (4)
(e) UnSF (4)
(f) UnMF (4)

Figure 4.



Smoothed Probabilities of Low Growth: Four-Regime Switching Models
1.0
0.8
0.6
0.4
0.2
0.0
(a) Restricted Single-Frequency Model
(b) Restricted Mixed-Frequency Model
(c) Unrestricted Single-Frequency Model
(d) Unrestricted Mixed-Frequency Model
60
65
70
75
80
85
90
95
00

Figure 5.

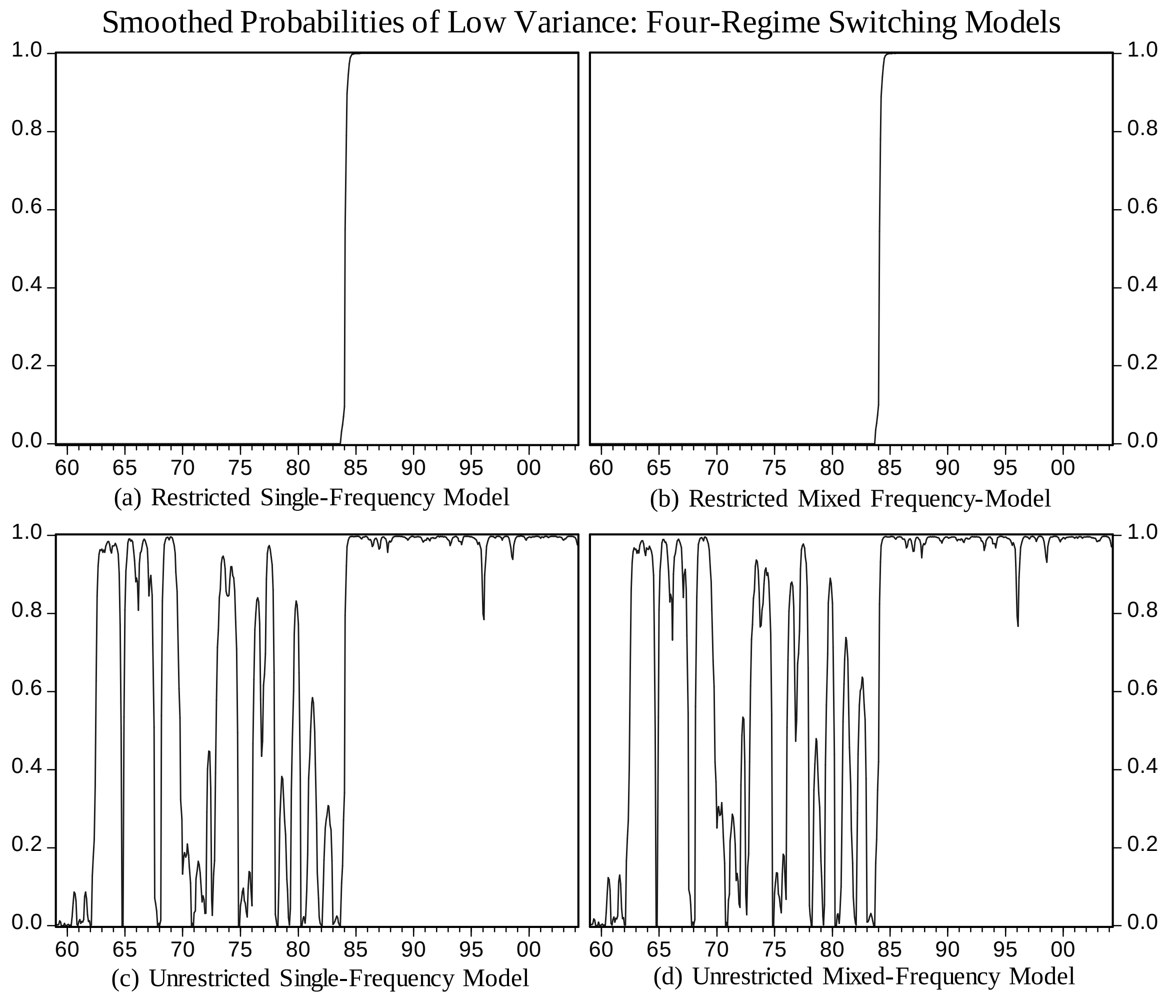
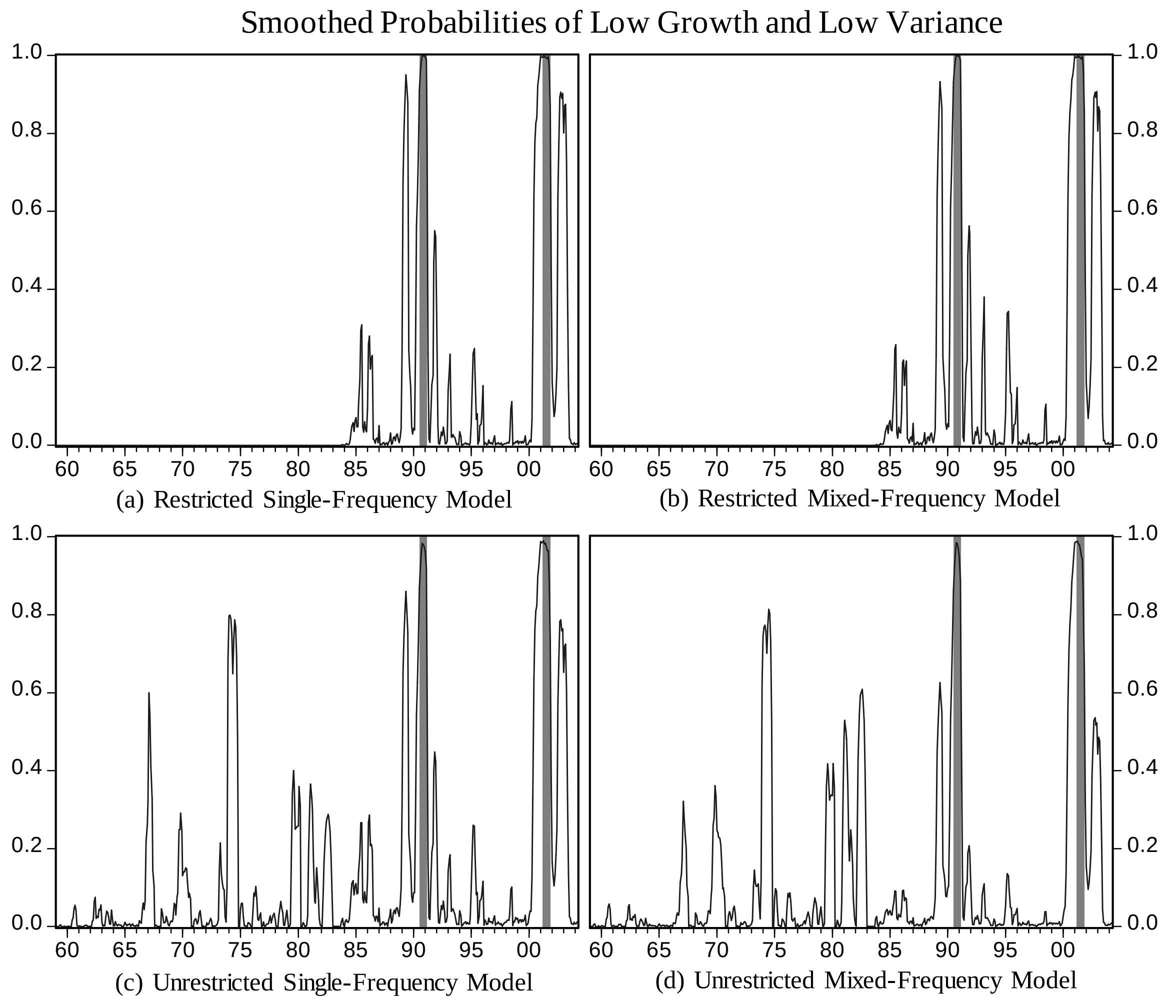

Smoothed Probabilities of Low Variance: Four-Regime Switching Models
1.0
0.8
0.6
0.4
0.2
0.0
60 65 70 75 80 85 90 95 00
(a) Restricted Single-Frequency Model
(b) Restricted Mixed Frequency-Model
(c) Unrestricted Single-Frequency Model
(d) Unrestricted Mixed-Frequency Model

Figure 6.


Smoothed Probabilities of Low Growth and Low Variance
1.0
0.8
0.6
0.4
0.2
0.0
60 65 70 75 80 85 90 95 00
(a) Restricted Single-Frequency Model
(b) Restricted Mixed-Frequency Model
(c) Unrestricted Single-Frequency Model
(d) Unrestricted Mixed-Frequency Model