Use costs in a two-R&D-sector model

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Abstract

In this paper we assess the properties of scale-free endogenous growth models in presence of use costs for the final users. As benchmark we use Segerstrom(2000) two R&D sector model. When use costs apply to both types of innovation we find counterintuitive results with respect to the standard Endogenous Growth literature: use costs can increase growth. This is due to the presence of both increasing returns in the research functions and the population growth condition. When costs apply to vertical innovations only we can establish more intuitive results: under mild conditions use costs decrease the rate of vertical innovation and of overall economic growth.

Key words: Endogenous Growth, Scale effect, Adoption costs

JEL Classification: O32,O41.

1 Introduction

In order to cope with the so called "scale effect puzzle" which characterizes standard R&D based endogenous growth models, the recent literature (Howitt(1999), Segerstrom(2000) Cozzi and Spinesi(2002)), has introduced models displaying at the same time horizontal and vertical innovation.

Jones (1995) underlines the fact that "scale effect" is one of the most striking features of endogenous growth models: an increase in the level of population leads to an increase in the growth rate of the economy[1]. If a

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[1]For a short discussion of the different definitions of "Scale Effect", as they enter in the literature the reader is referred to Jones(1999)

constant growth rate of population is allowed, the model looses any balanced growth path.

All empirical studies reject the hypothesis of scales effects as simply counterfactual: for example, despite the fact that the number of engineers devoted to R&D have continuously risen after the second world war, there is no evidence of a systematic rise in the economies growth rate in that period[2].

In the new stream of literature two related ingredients are essential in order to cope with population growth. The first is the so called "population growth condition" which relates the innovation growth rates with population growth; the second is the specification of the returns of the research production functions.

Howitt(1999) assumes a set-up in which the growth rate of horizontal innovation is equal to population growth and he assumes that vertical innovations have constant returns to scale i.e. linearity in the input term applies. Segerstrom(2000) generalizes the framework to the case of any return in the two innovations. By doing this he introduces increasing returns in the research functions. His "population growth condition" implies that population growth is related to a linear combination of both types of innovation.

Both models capture the essential feature of Segerstrom(1998): vertical innovations become more and more difficult as far as better intermediate varieties are introduced. This requires that along the balanced growth path more and more resources should be devoted to R&D in order to have a constant growth rate. With respect to one R&D sector models, two sectors of research can prevent the arousal semi-endogenous growth problem, that is growth rates are uniquely determined by population growth.

Segerstrom (2000) shows that the hypothesis of two different technologies in the R&D processes can be used to understand how subsidies to R&D can imply different policy results, depending on some parameters, including the fact that subsidies can produce a *reduction* in the growth rate of the economies. With this respect the model of Segerstrom, in his full generality, does not provide any conclusion about desirability of the two types of innovation, leaving the question to be eventually solved on empirical grounds.

In this paper we study some extensions of scale-effect free models by introducing use costs for the final users in the two R&D sector model proposed by Segerstrom (2000). The gist of employing use costs is that final good firms cannot appropriate the full productivity of the new quality goods but must drop some fraction of final output.

The concept of use costs is closely related with the one of adoption costs and it is well known that adoption costs are a very significant part of expen-

[2]Reported in Jones 1995

diture in modern economies: Jovanovic(1997) reports the figure of a 10% of GDP expenditure in adoption costs.

The economic intuition behind use cost is that high quality intermediate goods need resources to be run. The case of Personal Computer and Networks is striking, in the sense that a battery of technicians is needed in order to aid people use the computers and run a network: moreover this kind of cost do not dissipate once an adoption period has elapsed.

For the sake of simplicity we do not directly model use costs as an allocation problem of labor but we will directly assume that use costs permanently reduce the productivity of intermediate varieties by a fraction z. This assumption keeps the model as simple as possible and it is in line with the assumption that research is conducted by the use of final goods.

As a matter of comparison we implement two different kind of costs: simple and quasi-fixed costs. Simple costs reduce the productivity of intermediate goods and are paid on all varieties, whereas quasi-fixed costs are paid in terms of the same fraction of lost productivity, but only with regards on those varieties that have an increase in productivity attached to them. The latter hypothesis implies that new varieties, ceteris paribus, will be demanded more by final users, giving then more incentives to undertake horizontal rather than vertical research.

In standard endogenous growth models, see Romer(1990) or Aghion-Howitt(1992), the result of an increase in simple use costs would be straightforward: less productive intermediate goods imply a lower demand by the final good producers and therefore less profits for the innovative monopolists. This in turn drives down the innovation effort of the research sector and therefore reduces the growth rate of the economy.

Quite surprisingly we find that in Scale Effect free models we might not observe this behaviour. When we implement simple costs in Segerstrom's (2000) two R&D sector model, we observe that the effort in both kind of innovation will be reduced but if the returns of horizontal innovation decrease at a slower pace than the returns in vertical innovation, then simple costs *increase* the growth rate of vertical research. This may or may not lead to an increase in economic growth, per capita income, according to the value of some parameters.

A more intuitive result applies in the reverse case. When returns to the vertical research decrease more slowly than returns in the horizontal innovation, the fraction of GDP devoted to both kind of research are decreased, but the growth rate in varieties is fostered and the growth rate in quality is reduced. Despite this the overall growth rate of the economy, measured by the growth rate of wages, could still be improved.

After providing an explanation for this behaviour, we analyze the quasi-fixed cost case: we find that the growth rate of vertical innovation is always decreased by quasi-fixed costs. Given the higher incentive in undertaking horizontal research, the rise in the horizontal growth rate of innovation is associated with an increased fraction of resources devoted to horizontal research at the expenses of vertical research. Whatever the returns of innovations, quasi fixed costs decrease the vertical innovation rate by harming the growth rate of productivity and reduce the overall growth rate of the economy.

The purpose of this paper is twofold. First we want to address the question of use/adoption costs in two sector R&D models and discuss the issue of counterintuitive growth enhancing use costs in Segerstrom(2000): we propose quasi-fixed costs as solution to cope with this problem. Second, by proposing a quasi-fixed cost argument, we also take a stand on the relation between horizontal or vertical innovation in theoretical models. That is, using a quasi fixed cost assumption, we obtain the result that horizontal innovation is a "cheaper" but less effective innovation with respect to quality innovation.

Section 2 contains a (brief) review of Segerstrom (2000); in 3 we introduce a simple cost assumption in the model and in 4 we discuss the quasi fixed case in intuitive terms by leaving the proofs to the Appendix. Some conclusions follow.

2 The benchmark model

We use Segerstrom (2000) as a benchmark model. We have a three sector economy: final good producers, intermediate goods producers and the research sector. Final goods can be consumed or used in research, there is no capital. The representative final good firm uses a production function which exhibits constant returns to scale in labor L_{yt} and intermediate varieties x_{it}.

$$Y_t = L_{yt}^{1-\alpha} \int_0^{N_t} A_{it} x_{it}^{\alpha} di \tag{1}$$

Embedded in the production function there are two (three) possibilities of growth: a vertical innovation can increase the productivity parameter A_{it} and horizontal innovation will expand the measure of industries up to N_t. It is assumed that total labor force grows at a constant exogenous rate g_L.

Research is conducted by means of final goods only: there is no labor. This leads to the same market clearing condition in the goods market for both models: final goods can be used for consumption C_t, vertical V_t and horizontal H_t.

$$Y_t = C_t + V_t + H_t \tag{2}$$

In looking at the steady state of our economy we will be interested in the *constant* level of ratios H_t/Y_t and V_t/Y_t[3].

The final firmwill maximize her profits by choosing labor and purchasing intermediate products:

$$\max_{L_{yt},x_{it}} \pi_t = L_{yt}^{1-\alpha} \int_0^{\mathrm{N_t}} A_{it} x_{it}^{\alpha} di - \int_0^{\mathrm{N_t}} p_{it} x_{it} di - w_t L_{yt}$$

The demand of intermediate goods will be such that their price is equal to their marginal productivity. Labor will also be chosen as according to his marginal productivity. The first order condition for intermediate products is:

$$p_{it} = A_{it}\alpha (L_{yt}/x_{it})^{1-\alpha} \tag{3}$$

And for labor:

$$w_t = (1-\alpha) \int_0^{N_t} A_{it} \left(\frac{x_{it}}{L_{yt}}\right)^{\alpha} di \tag{4}$$

Intermediate varieties are treated as a bundle of commodities produced by firms under Dixit-Stiglitz monopolistic competition with the use of labor only according to the technology:

$$x_{it} = L_{it}$$

Skipping the details we can conceive the model as composed of two blocks: a first block dealing with final and intermediate good firms, whose structure we have just described, and the second block dealing with research and development. According to Li(2000), in endogenous two sector R&D models the two blocks are interdependent whereas in semi-endogenous growth the growth rates of the economy can be derived from the research block only. In the latter case, subsidies will be irrelevant for economic growth.

For the first block, we can derive the expression of the steady state income:

$$y_t \equiv \frac{Y_t}{A^d N} = \frac{L_t}{A_t^{d-1} N_t^{\alpha}} \frac{\alpha^{2\alpha}}{\Gamma^{1-\alpha}(1-\alpha)^{\alpha}} \frac{1}{1+\frac{\alpha^2}{1-\alpha}}$$

By log-differentiating we have the so called *population growth condition*:

$$g_L = (d-1)g_A + \alpha g_N \tag{5}$$

[3]This is in Segerstrom, but for example Howitt defines different stationary variables: as a result he gets that the sum $(H+V)/Y$ has to be constant in steady state.

In the same way, solving for the wage and log-differentiating we get that the wage rate grows according to iso-growth condition:

$$g_w = (1-\alpha)g_N + g_A \tag{6}$$

For the second block (research)we have the following structure. We are in a tournament model where the winner of R&D race takes the industry as a monopolist: there is free entry in research. The probability ϕ_t of having a vertical innovation in each industry is described as the instantaneous probability in a Poisson process:

$$\phi_t = \lambda_v \left(\frac{V_t}{Y_t}\right)^{\delta} \frac{Y_t}{A^d N_t} \tag{7}$$

Firms will choose their expenditure V as according to the following first order condition:

$$\frac{\delta \lambda_v \Pi_{vt}}{A_t^d} v_t^{\delta-1} = 1(-s_v)$$

This formulation generalizes Howitt(1999) by allowing for decreasing returns in the fraction of GDP v for the vertical innovation process. Using the law of large numbers, at the economy level, the growth process of leading edge productivity A_t can be described as:

$$g_A = \lambda_v \sigma v^{\delta} y \tag{8}$$

In order to get a constant growth rate level of vertical innovation, ϕ must be constant in steady state. From the resource constraint we know that both $H/Y \equiv h$ and $V/Y \equiv v$ are constant over time. We can then deduce that $y \equiv Y/A^d N$ will be constant in the balanced growth path. Function 8 expresses the fact that with spillovers (y) a decrease in the fraction of GDP allocated to vertical research (v) can be (more than) compensated by an increase in the steady state value of stationarized income. This feature, although not underlined in Segerstrom(2000) is one of the key properties of the model, and it is due to the fact that although the research process has decreasing returns in v only, it shows *increasing* returns in v, y.

The rate of growth for horizontal differentiation is given by the same kind of function as for vertical innovations:

$$\frac{\dot{N}_t}{N_t} = \lambda_h \left(\frac{H_t}{Y_t}\right)^{\gamma} \frac{Y_t}{A^d N_t} \tag{9}$$

And the long run rate of horizontal innovation:

$$g_N = \lambda_h h^{\gamma} y \tag{10}$$

In the solution of the model it is useful to distinguish between two cases: whether research for the vertical sector decrease at a faster pace than in the horizontal case, i.e. $\gamma > \delta$ or if the contrary holds. The solution technique does not change much.

For both kinds of differentiation, the innovator will enjoy some monopoly power, the only difference in the reward being given by the hypothesis that new industries will produce intermediate goods of productivity A_{it} that is randomly assigned to them among the existing ones. Therefore we will have the following relation between the two rewards, Π_h and Π_v:

$$\Pi_h = E((A_{it}/A^{max})^{1/1-\alpha})\Pi_v$$

As in all schumpeterian models the reward of discoveries, Π_h and Π_v, are computed by the perpetual monopolistic rent in each industry, taken into account the well known replacement effect. In multi-industry models we take into account also the so called "crowding out effect", the perpetual rise of wage costs due to technological progress, see Cabellero and Jaffe(1993).

3 First extension: the simple cost model

We introduce a permanent cost that changes the demand for intermediate goods and therefore affects the incentives to undertake R&D. The gist of employing use costs is that final good firms cannot appropriate the full productivity of the new quality goods but must drop some fraction of final output.

Use costs will be an increasing function of the number of workers using the new technology, of the demanded intermediate goods and of their productivity. For the variety i we can therefore write the cost as:

$$C(i) = zL_y^{\beta} A_i x_i^{\delta}$$

By taking constant returns to scale we can simplify our story by assuming that the cost function has the same exponents as the production function. Since this happens for all varieties, the program of the final good firms becomes

$$\max_{L_{yt},x_{it}} \pi_t = (1-z)L_{yt}^{1-\alpha}\int_0^{\mathrm{N_t}} A_{it}x_{it}^{\alpha}di - \int_0^{\mathrm{N_t}} p_{it}x_{it}di - w_tL_{yt} \qquad (11)$$

As a result we get that use costs are a fraction of *final output.*

When we introduce use costs it is easy to see that the equilibrium wage is affected as follows:

$$w^{costs} = (1-z)w_t$$

with costs the wage is reduced by a fixed proportion z: the iso-growth line will not be affected.

The marginal productivity of intermediate goods will be given by:

$$p_{it} = \left(\frac{x_{it}}{L_{yt}}\right)^{\alpha-1} A_{it}\alpha(1-z) \tag{12}$$

And from the maximization of the intermediate firm we get:

$$\left[\frac{A\alpha^2(1-z)}{w}\right]^{1/1-\alpha} = \frac{x_{it}}{L_{yt}} \tag{13}$$

Equations 12 and 13 together imply that the equilibrium price will be given, as before, by: $p = w/\alpha$. In general we obtain that:

Proposition 1 *With simple costs the wage-general equilibrium effect on intermediate goods leaves their production unchanged.*

This result is easily checked in equation 13: once the general equilibrium effect on the wage is taken into account, the optimal quantity produced by the intermediate firms is unchanged by the introduction of the use costs. The cut in wage compensates the reduction of final users demand.

This implies that the market clearing equation of the labor market is unchanged and the usual relation holds:$L_y = \frac{L_t}{1+\frac{\alpha^2}{1-\alpha}}$.

Also the output of the final firms will drop also according to the ratio $1-z$.

$$Y^{costs} = (1-z)Y$$

Therefore we will have the following result:

Proposition 2 *In the simple cost case both the iso-growth line and the population growth condition are unaffected.*

The profit of the intermediate firm which innovates in the vertical dimension will be given by[4]

$$\pi_t^{costs} = (p_{it}^* - w_t^*)x_{it}^* = \left(\frac{w_t^*}{\alpha} - w_t^*\right) x_{it}^* = (1-z)\pi$$

Despite the reduction in profits, simple costs do not give any incentive to undertake more horizontal research. The incentive effect for research is in fact contained in one key equation:

$$\Pi_{ht} = E[(A_{it}/A_t)^{1/1-\alpha}]\Pi_{vt} \tag{14}$$

[4]With a star we indicate the optimal values

When we introduce simple costs, *every firm*, including the new industries that enter the market, will make the final good firm pay the use costs. What happens is that both Π_h and Π_v will be reduced by a factor $1 - z$. The two reductions cancel out in the equation above.

Therefore simple use costs do not alter the behaviour of research toward devoting more resources to horizontal innovation, rather than to vertical: they are neutral from an incentive viewpoint.

3.1 The case $\gamma > \delta$

For the sake of shortness, we omit all the (straightforward) computations and we directly comment on the final result. We start from the case where $\gamma > \delta$, i.e. the returns in vertical innovation decrease with a faster pace than the returns in the horizontal.

$$g_L = g_A(d - 1 + \alpha c_1 v^{\epsilon}) \tag{15}$$

$$\rho - g_L = g_A \left(\frac{\delta \Gamma \alpha (1-\alpha)[(1-z)]}{\sigma(1-s)v} - \frac{1}{\sigma} - \frac{\alpha}{1-\alpha} - \alpha c_1 v^{\epsilon} \right) \tag{16}$$

The second equation represents an arbitrage condition between vertical and horizontal innovation: it is derived from equation 14 by plugging in the expression of profits and making use of the research functions. In square brackets we highlight the wedge in the research condition, $(1-z)$, introduced by the simple costs.

The following figure shows what happens under the hypothesis that the vertical innovation is more growth enhancing than the horizontal one. The contour lines represent the iso-growth lines: we are in the case $d < 1/1 - \alpha$. For higher levels of the iso-growth line there will be more growth in the economy.

When we have costs, the R&D condition, see equation 16, will move upwards, in the opposite direction to the case of a positive subsidy studied by Segerstrom[5].

From the population growth condition

$$g_L = (d-1)g_A + \alpha g_N$$

we know that it is impossible for growth rates of both innovations to grow, or to be reduced, at the same time, population growth being constant. The

[5]The figures are drawn for a specific choice of parameters, the qualitative results hold independently of the specific choice

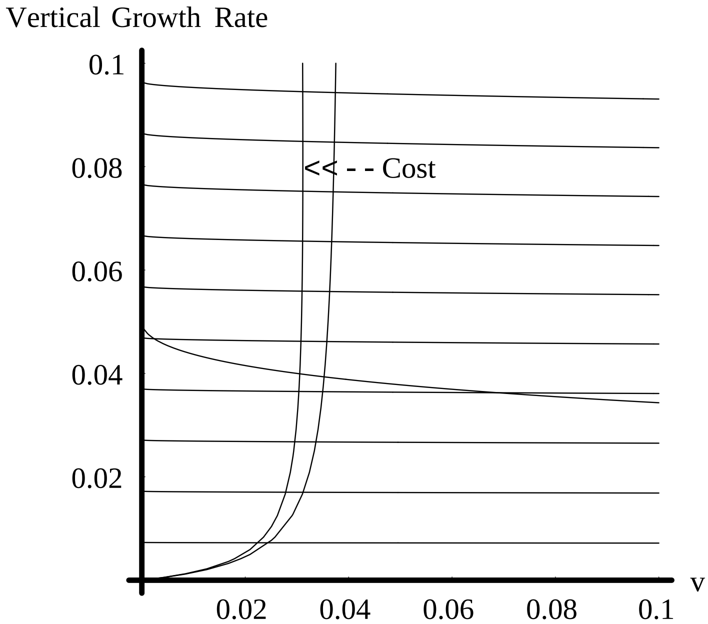

Figure 1: Solution of the system for simple costs

existence of a steady state implies that, when $\gamma > \delta$, g_N declines and g_A rises: this is only possible if the steady state value of y has increased.

Taking a dynamical perspective, at the beginning both rates of innovation will decrease, thus violating the population growth condition. Then since both productivity and industry measure growth rates under-perform the population growth condition, the steady state income will increase since, we remind, $y \propto \frac{L_t}{A^{1-d}N^{\alpha}}$. The increase of the steady state income increases in turn the spillover to research, up to the point in which the vertical innovation is higher than before and the horizontal innovation is reduced[6].

Even if the fraction of GDP devoted to vertical research decreases and there is a boost in vertical innovation, this does not mean that the amount of resources to vertical research decreases, but simply that it increases but this is more than offset by the increase in GDP: i.e. $v = V/Y$ and the numerator V grows less than the denominator[7].

[6]This interpretation, although not explicitly expressed by the author, holds also for the case of uniform subsidies in the original paper of Segerstrom(2000)

[7]An intuition for this claim runs as follows. Suppose that in order to match the population growth condition an increase in g_A of 1% implies a reduction of v of 1%. Then we know that $g_A = v^{\delta} y \sigma \lambda$: Imagine $\delta = 0.4$. The increase in y should be of $1 + 0.4 = 1.4$ in order for g_A to have a 1% growth. Now let us consider the fraction $v = V/Y$, for sure Y

Finally the growth rates of innovation that are compatible with population growth must be matched with the iso-growth line $g_w = (1-\alpha)g_N + g_A$ to determine whether the overall economy growth rate is increased or reduced: since g_N is reduced and g_A is increased, a rise in use costs can either be growth enhancing or growth reducing. The same rule as in Segerstrom(2000) applies: when $d < 1/1-\alpha$, i.e. the population growth line is steeper than the iso-growth line, costs are growth enhancing.

Our result is at odds with economic intuition: in the presence of costs, our fictitious economy can have an higher steady state income and therefore higher growth rates. This feature is not shared by neo-schumpeterian models, where the decline in productivity associated with use costs would simply reduce the growth rate of both productivity and income. The result is due to two features in our model that are not shared by the others: the downward sloping population growth condition, which is likely to be an important component of all scale-free endogenous growth model, and the increasing returns of the R&D functions with respect to the share of GDP and the steady state income.

As we discuss in the section 3.3, the issue does not arise if we stick to the case when the returns of vertical innovation decrease at a lower pace than the returns in the vertical, i.e. $\delta > \gamma$, which is the case in the paper of Howitt: nevertheless if we do not stick exactly to the framework of Howitt(1999), where the horizontal innovation is constrained to be equal to the population growth, we might still have the case of overall growth enhancing use costs.

The other possibility, if we do not want to restrict ourselves to the case $\delta > \gamma$, is to change the cost function by implementing quasi-fixed costs: this issue is discussed in section 4.

3.2 Extension: z is function of the vertical innovation growth rate

Just for the sake of simplicity we have assumed so far that z is a pure number: but nothing would change if we assumed this fraction to be an appropriate function of the vertical growth rate.

Consider now $z(g_A)$ as a positive function of the growth rate of innovation: to describe it we implement the function described in Jovanovic and Greenwood (1998)

$$z(g_A) = \omega(g_A)^\nu$$

cannot grow less than his steady state value y, in order for v to decrease only of the 1% there must be some increase also in V.

The faster the technical progress the higher will be the ratio of production busted by costs: also when there is no innovation there is no cost, a logical consequence. By choosing a cost function which is related to the "amount to be learned" in the literature on learning, we can introduce a reasonable use cost[8].

In the literature of "learning curves" it is assumed that new technologies are not fully productive as soon as they are adopted. Productivity A evolves over time according to learning by doing as for example $A_\tau^{effective} = A_\tau(1 - z^* e^{-\lambda\tau})^{1-\beta}$. At time zero the *amount to be learned* is defined as $1-(1-z^*)^{1-\beta}$: given the absence of learning by doing in our model, if we assume $\beta = 0$ we fall back to our case. Productivity is then permanently cut by the *amount to be learned* z^* that, as described in Greenwood and Jovanovic(1998) is a function of the growth rate of vertical innovation $z^* = \omega(g_A)^\nu$.

A rise in use costs can therefore be represented as a rise in the exogenous component ω of the cost function. In the appendix we show that the same kind of result applies indifferently for the case where z is a number or the above specified function.

By introducing a simple use cost which accelerates with the vertical growth rate we introduce *an externality effect* on the final sector. Final producers perceive a negative pecuniary externality from the research sector since their costs of adoption are increased as far as innovation accelerates. Again the result will be that *the profits of both kind of innovations will be reduced and the fraction of GDP employed in vertical and horizontal differentiation will be reduced too.* Despite this simple costs are neutral from an incentive viewpoint.

3.3 The case $\gamma < \delta$

The analysis of this case can be carried out in the same way as before but with less counterintuitive results. The solution will be now in the space (g_n,h), according to the following system of equations:

$$g_L = g_A(d - 1 + \alpha c_2 h^\mu) \tag{17}$$

$$\rho - g_L = g_N \left(\frac{\gamma\alpha(1-\alpha)[(1-z)]}{(1-s)h} - \left(\frac{1}{\sigma} + \frac{\alpha}{1-\alpha}\right) c_2 h^\epsilon - \alpha \right) \tag{18}$$

Where the expression of the coefficients can be found in the appendix of Segerstrom(2000): here it is sufficient to say that $\mu > 0$ and $c_2 > 0$. As

[8]For $g_A = 3\%$ and $z(g_A) = 0.3(g_A)^{0.5}$ we get a cost of around 5% of GDP, which can be a reasonable figure: the figures in this paper are all drawn by using this specification. The reader is referred to Jovanovic(1997): costs in adoption could amount to 10% of GDP.

shown in the following figure the growth rate of horizontal innovation will be increased by an increase in use costs despite the fact that the ratio of GDP devoted to research is decreased. This happens for the same reason as explained above: the decrease in the expenditure is more than compensated by the movement in the spillover term y. In this case simple costs reduce

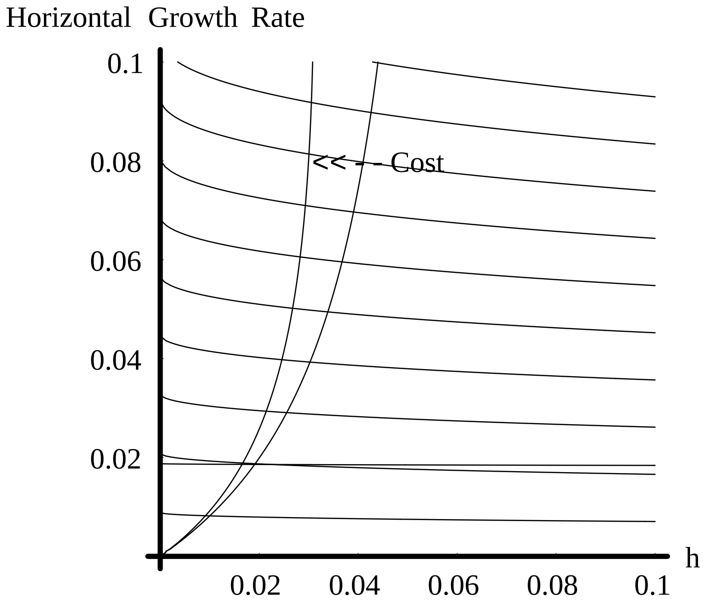

Figure 2: Solution of the system for simple costs

the productivity growth rate and increase the rate of variety growth: this implies that the overall growth rate of the economy (the wage growth rate) is decreased if and only if $d < 1 - \alpha$, which is the case shown in picture 2. Therefore even in the case $\gamma < \delta$ we still might get the result that a rise in use costs is beneficial for economic growth.

4 Second extension: The Quasi Fixed Cost Model

To reconciliate maths and economic intuition for any value of the returns in R&D, we introduce quasi-fixed costs. We will only solve the model in the more counterintuitive case, $\gamma > \delta$: the same conclusions easily apply to the less ill-behaved situation.

As we said above, there are two effects to be taken into account when we introduce use costs: an externality effect and an incentive effect. Simple costs are neutral from the incentive viewpoint, i.e. they do not distort the relation between the two rewards of innovation. Now we address the question of whether the existence of a different kind of use costs may bias the process of innovation towards the horizontal one.

Consider that at time t the final good firm has paid all the costs of adoption on different qualities, up to the maximum level of quality A_t. Once a new industry is created the monopolist in the industry produces an intermediate good that has a level of quality randomly chosen *among all the existing ones.*

Since the final good firm has already paid all the use costs on the existing qualities, there is no reason why it should pay a use cost on the new good. Use costs are in this sense *"quasi-fixed"*[9] in the sense that they are paid proportionally to the quantity bought once the new-quality good is purchased for the first time, but they are not paid anymore for different goods of the same quality that are produced by the new industries.

Since every industry vertical innovations moves the leading edge quality, then an horizontal innovator, ceteris paribus, will profit of an higher demand, as far as it is not replaced by a vertical innovator, since final good firms do not pay an adoption cost. With this hypothesis we address an incentive problem.

With quasi-fixed costs in fact there will be an heterogeneous set of intermediate firms. Until it is replaced, an horizontal innovator will not make final good firm pay use costs: In the steady state time t equilibrium we will have Q_t industries that have never innovated in the vertical direction and therefore produce cost-free intermediate goods. Every new industry enters automatically in the set of Q and it escapes it as soon as an innovation arrives with probability ϕ_t.

To formalize the argument above we assume that in steady state Q is a constant fraction of the N firms. Since all new industries start without use costs attached to them, Q increases as far as N increases; also at any time t there will be a number $\phi_t Q_t$ that will innovate and leave the group of Q. In differential terms we can write:

$$\dot{Q} = \dot{N} - \phi Q$$

If we divide both terms by Q and use the hypothesis that $\frac{Q}{N} = k$ (or in

[9]Another application of quasi-fixed costs can be found in the article of Cozzi and Spinesi(2002). Anyway they consider quasi-fixed costs in research, so there is no economic connection between their model and this one

growth terms $g_Q = g_N$) we get:

$$\frac{\dot{Q}}{Q} = \frac{\dot{N}}{N}\frac{1}{k} - \phi$$

That substituting our assumption $g_Q = g_N$ we can compute the fraction k as:

$$\frac{1}{k} = \frac{\phi}{g_N} + 1 \tag{19}$$

Proposition 3 *The fraction of intermediate firms use-cost-free will be a positive function of the rate of horizontal innovation and an inverse function of the rate of vertical innovation.*

We can rewrite the final good producers profits(π^F) as follows:

$$\pi_t^F = (1-z(g_A))L_{yt}^{1-\alpha}\int_0^{(1-\mathrm{k})\mathrm{N_t}} A_{it}x_{it}^{\alpha}di + L_{yt}^{1-\alpha}\int_{(1-\mathrm{k})\mathrm{N_t}}^{\mathrm{N_t}} A_{it}x_{it}^{\alpha} - \int_0^{\mathrm{N_t}} p_{it}x_{it}di - w_tL_{yt} \tag{20}$$

Where k is determined endogenously and $z(g_A)$ is meant to be the same function as before.

The solution of this case follows the same steps as for the case of general costs but there two main differences in the results. Since firms are heterogeneous, the general equilibrium effect on the wage does not compensate anymore; moreover there is an incentive effect as we have already explained. In what follows we briefly outline these results.

This time the productivity of intermediate goods is given by:

$$p_{it} = A_{it}\alpha\left(\frac{x_{it}}{L_{yt}}\right)^{\alpha-1}$$

for the cost-free intermediate goods and

$$p_{it} = (1-z)A_{it}\alpha\left(\frac{x_{it}}{L_{yt}}\right)^{\alpha-1}$$

For the others. For both cases the reasoning above applies and therefore the price will be fixed by following the standard rule $p = w/\alpha$. The rule for intermediate goods supply is:

$$\left[\frac{A\alpha^2(1-z)}{w_t}\right]^{1/1-\alpha} = \frac{x_{it}}{L_{yt}} \; or \; \left(\frac{A\alpha^2}{w_t}\right)^{1/1-\alpha} = \frac{x_{it}}{L_{yt}}$$

The drop in wage is computed as follows from the marginal productivity of labor for final good firms:

$$w_t = (1-\alpha)(1-z)\int_0^{(1-k)N} A_{it}\left(\frac{(1-z)\alpha^2 A_{it}}{w_t}\right)^{\frac{\alpha}{1-\alpha}} di + (1-\alpha)\int_{(1-k)N}^{N} A_{it}\left(\frac{\alpha^2 A_{it}}{w_t}\right)^{\frac{\alpha}{1-\alpha}} di = \frac{(1-\alpha)^{1-\alpha} A_t N_t^{1-\alpha}\alpha^{2\alpha}[(1-z)(1-k)+k]^{1-\alpha}}{\Gamma^{1-\alpha}} \quad (21)$$

Now the drop in wage corresponds to the term in square brackets. It approximates $1-z$, provided that k is small enough:

$$\zeta = (1-z)(1-k) + k = 1 - z + kz$$

As before the rate of growth of wage is unaffected by the presence of costs. We can conclude that:

Proposition 4 *In the case of quasi fixed costs the arousal of heterogeneity of firms implies that costs are not neutral for the optimal supply of intermediate goods. The drop in wage in fact will under-compensate the firms with costs and over-compensate the firms that are cost-free.*

The same kind of reasoning applies for the income that will be given by:

$$Y_t = \zeta^{1-\alpha}\frac{L_{yt}A_t N_t^{1-\alpha}(1-\alpha)^{\alpha}}{\Gamma^{1-\alpha}}$$

From the equations of the wage and of the income we can conclude that:

Proposition 5 *In the case of quasi-fixed costs, both the population growth condition, and the iso-growth line unaffected.*

Since the production of intermediate goods is affected by the presence of the cost, the market clearing condition will also change and we will get:

$$L_{yt}\left(\zeta\frac{\alpha^2}{1-\alpha}+1\right) = L_t$$

We can compare the profits for a vertical (π_v) and an horizontal(π_h) innovator at time t:

$$\pi_v = (1-z)L_{yt}\alpha(1-\alpha)A_t^{max}\left(\frac{A_t^{max}(1-z)\alpha^2}{\omega_t}\right)^{\alpha/(1-\alpha)} \quad (22)$$

$$\pi_h = L_{yt}\alpha(1-\alpha)A_{it}\left(\frac{A_{it}\alpha^2}{\omega_t}\right)^{\alpha/(1-\alpha)} \quad (23)$$

The reward for vertical innovation can be computed by solving the usual integral:

$$\Pi_{vt} = \int_o^\infty e^{-\int_t^\tau r+\phi_s ds} \pi_{t\tau} d\tau \tag{24}$$

Which takes into account population growth, the probability of being replaced by an innovator and the crowding out effect. This boils down to:

$$\Pi_v^{qcost} = \Pi_v(1-z)\left(\frac{1-z}{\zeta}\right)^{\alpha/1-\alpha}$$

The main difference from the previous case is in the general equilibrium effect on the salary: we do not have a complete compensation in this case.

From an incentive viewpoint we can model the difference between vertical and horizontal innovators by taking into account the profits in 22.

$$\Pi_{ht} = \frac{E[(A_{it}/A_t)^{1/1-\alpha}]\Pi_{vt}}{(1-z(g_A))^{1/1-\alpha}}$$

In the case of quasi-fixed costs the wedge between the horizontal and the vertical differentiation reward is modified with respect to the no-cost case. We show the system of equations that leads us the solution of the problem of allocation of vertical research:

$$g_L = g_A(d - 1 + \frac{(\alpha c_1 v^\epsilon)}{(1-z)^{1/1-\alpha}} \tag{25}$$

$$\rho - g_L = g_A\left(\frac{\delta\Gamma\alpha(1-\alpha)(1-z)\left[\frac{1-z}{\zeta}\right]^{\alpha/(1-\alpha)}}{\sigma(1-s)v} - \frac{1}{\sigma} - \frac{\alpha}{1-\alpha} - \frac{(\alpha c_1 v^\epsilon)}{(1-z)^{1/1-\alpha}}\right) \tag{26}$$

In the case of quasi-fixed costs the *R&D* rotates upward more than in the case of normal cost. This is by no means surprising since in the quasi-cost case both heterogeneity of firms and the incentive effect contribute to modify the research equation.

In the quasi fixed costs case, we find that the combination of the *externality* and the *incentive* effect produce, with respect to the benchmark case, a lower vertical growth, an higher horizontal growth and a reduced overall growth rate. In other words, the incentive effect of the quasi fixed cost case, offsets the counterintuitive result we got in the simple cost case.

In general we find that:

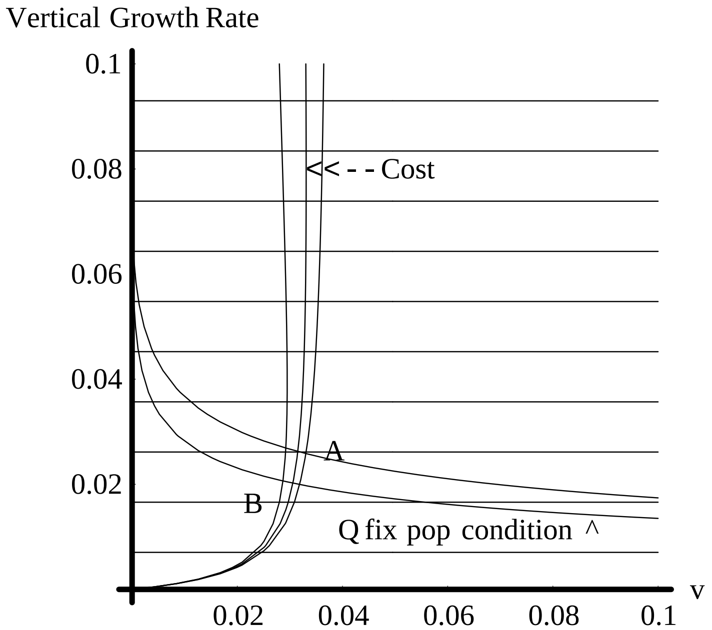

Figure 3: Solution of the system for simple and quasi fixed costs

Proposition 6 *With the exception of the pathological case of no vertical innovation: i) Quasi fixed costs reduce the growth rate of vertical innovation and increase the growth rate of horizontal innovation. ii) In a neighborhood of $z = 0$ (the no cost case), for realistic values of the parameters α and ρ, the resources h allocated to horizontal research will increase when z increases*[10] *iii) Quasi fixed costs reduce the overall growth rate of the economy.*

The reader is referred to the appendix for the proof of proposition 6, point i) and ii). For point iii) a simple graphical argument will suffice and it is presented below.

In the quasi fixed cost case the horizontal innovation is fostered, whereas the vertical innovation is harmed: this is the essence of point i) and ii) in the statement above. Intuitively, in graph 3 the economy passes from point A to point B: point B is below point A in the space v, g_A by part i).

[10] An increase in the fraction h when z rises is a sufficient condition for the the vertical growth rate to decrease: we can prove the argument by contradiction. Suppose that by raising costs h increases and also g_A increases. Then, since v decreases for sure when we raise costs, it must be the case that the spillover term y in $g_A = \lambda_v v^\delta y$ is increased. But since $g_N = \lambda_h h^\gamma y$ holds then g_N should rise too, hence the contradiction.For the necessary part, see the Appendix.

As before the contribution of the two innovations to the overall growth can be assessed by looking at the intercept of the iso-growth lines[11], but, differently from previous cases, no matter what the slope of the iso-growth line is, clearly the intercept of the iso-growth line passing through B is lower than the iso-growth line passing through A: this implies that situation B has a lower overall growth than A. This implies that in the quasi-fixed cost case a reduction of vertical innovation is associated to a decrease in the overall growth rate of the economy.

5 Some conclusive remarks

In this paper we assessed the problem of use costs in scale-effect free endogenous growth model. We have shown that in a fairly general scale effect free set-up, a simple cost argument can convey counterintuitive results. In the paper of Segerstrom this is due to two ingredients: increasing returns in the research production function and a downward sloping condition relating the two innovations, i.e. the population growth condition that enable us to pick up steady states despite population growth.

While the specification of the research functions adopted in Segerstrom (2000) might be regarded as specific to the model, population growth conditions should be seen as general conditions which characterize scale effect free models. Despite this remark it is not entirely clear which of the two hypothesis should be modified and how in order to convey the standard result that use costs reduce the overall growth of the economy.

Therefore, instead of allowing a modification in the set-up of the model, we imagine that the problem might be in the specification of our cost functions. With this respect we consider a different hypothesis: quasi fixed costs. In this case use costs are attached only to the new produced qualities and not to the new varieties that have an already existing quality.

By doing this we introduce heterogeneity among firms and we affect the incentives of undertaking horizontal against vertical research giving an advantage of horizontal innovation with respect the vertical one: under very mild conditions we are able to prove that quasi fixed use costs reduce the growth rate of the economy.

From an empirical viewpoint we might have no direct insight on whether simple costs or quasi-fixed costs apply in reality. Despite this, from a theoretical perspective, the quasi-fixed cost argument could be seen as a rationale

[11] Also the iso-growth formula modifies with quasi-fixed costs: in the picture, as a matter of comparison, we use the iso-growth condition without quasi-fixed costs, since they are steeper. From a numerical viewpoint the difference is very small.

for the common view that, in a creative destruction framework, what really matters for growth are quality-enhancing activities; horizontal innovation can be seen as “cheaper” but less effective innovation.

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7 Appendix: proof of proposition 6

7.1 Proof of part i)

Define $z(g_A)$ as a specific function of the vertical growth rate g_A:

$$z(g_A) = \omega g_A^{\nu} \tag{27}$$

With $\omega > 0$ and $\nu > 0$.

We prove that a rise in the autonomous component ω never increases the vertical growth rate: that is $dg_A/d\omega < 0$.

We proceed in two steps.

First compute $dg_A/d\omega$ by taking the derivative of the function defined in 27. This yields:

$$\frac{dg_A}{d\omega} = \frac{dg_A}{dz}\frac{dz(g_A)}{d\omega} = \frac{dg_A}{dz}\left(g_A^v + \nu\omega g_A^{v-1}\frac{dg_A}{d\omega}\right)$$

By rearranging the expression we can establish:

$$\frac{dg_A}{d\omega}\left[1 - \nu\omega g_A^{v-1}\frac{dg_A}{dz}\right] = \frac{dg_A}{dz}g_A^v \tag{28}$$

Equation 28 implies that when $\frac{dg_A}{dz} < 0$, then also $\frac{dg_A}{d\omega} < 0$ follows.

In the second step we consider the system:

$$g_L = g_A(d - 1 + \frac{(\alpha c_1 v^{\epsilon})}{(1-z)^{1/1-\alpha}} \tag{29}$$

$$\rho - g_L = g_A\left(\frac{\delta\Gamma\alpha(1-\alpha)(1-z)\left[\frac{1-z}{\zeta}\right]^{\alpha/(1-\alpha)}}{\sigma v} - \frac{1}{\sigma} - \frac{\alpha}{1-\alpha} - \frac{(\alpha c_1 v^{\epsilon})}{(1-z)^{1/1-\alpha}}\right) \tag{30}$$

Now we check that $\frac{dg_A}{dz} < 0$ holds $\forall z$ at the points at which condition 29 and 30 are verified. The Implicit Function Theorem yields in general:

$$\frac{dg_A}{dz} = -\frac{\mid J_{z,v} \mid}{\mid J_{g_A,v} \mid} \tag{31}$$

Where with $\mid J_{x,y} \mid$ we define the determinant of the Jacobian matrix computed with respect to the variables x, y.

Recall that since $\zeta = 1 - z + kz$, the endogenous term k, which is a function of the vertical and horizontal growth rate, is part of our problem. Since k is a fraction and we have monotonicity, this issue is overcome by proving that $\frac{dg_A}{dz} < 0$ at the boundaries $k = 0$ and $k = 1$.

Setting $k = 0$ the system(29,30) becomes:

$$g_L = g_A(d - 1 + \frac{(\alpha c_1 v^{\epsilon})}{(1-z)^{1/1-\alpha}} \tag{32}$$

$$\rho - g_L = g_A \left(\frac{\delta\Gamma\alpha(1-\alpha)(1-z)}{\sigma v} - \frac{1}{\sigma} - \frac{\alpha}{1-\alpha} - \frac{(\alpha c_1 v^{\epsilon})}{(1-z)^{1/1-\alpha}} \right) \tag{33}$$

We make use of equations 32 and 33:

$$|J_{g_A,v}| = \begin{vmatrix} -\frac{g_l}{g_A} & -\frac{\alpha c\varepsilon v^{\varepsilon-1}}{(1-z)^{1/1-\alpha}} g_A \\ -\frac{\rho-g_l}{g_A} & g_A \frac{\delta\Gamma\alpha(1-\alpha)(1-z)}{\sigma v^2} + g_A \frac{\alpha c\varepsilon v^{\varepsilon-1}}{(1-z)^{1/1-\alpha}} \end{vmatrix} \tag{34}$$

This determinant is always negative. For the numerator we will have:

$$| J_{z,v} |= \begin{vmatrix} -\frac{1}{1-\alpha} \frac{\alpha c v^{\varepsilon} g_A}{(1-z)^{\frac{2-\alpha}{1-\alpha}}} & -\frac{\alpha c\varepsilon v^{\varepsilon-1}}{(1-z)^{1/1-\alpha}} g_A \\ g_A \left(\frac{\delta\Gamma\alpha(1-\alpha)}{\sigma v} + \frac{1}{1-\alpha} \frac{\alpha c v^{\varepsilon}}{(1-z)^{\frac{2-\alpha}{1-\alpha}}} \right) & g_A \left(\frac{\delta\Gamma\alpha(1-\alpha)(1-z)}{\sigma v^2} + \frac{\alpha c\varepsilon v^{\varepsilon-1}}{(1-z)} \right) \end{vmatrix}$$

Which can be reduced to an always negative expression:

$$| J_{z,v} |= -\frac{1}{1-\alpha} \frac{\alpha c v^{\varepsilon} g_A}{(1-z)^{\frac{2-\alpha}{1-\alpha}}} g_A \frac{\delta\Gamma\alpha(1-\alpha)(1-z)}{\sigma v^2} + \frac{\alpha c\varepsilon v^{\varepsilon-1}}{(1-z)^{1/1-\alpha}} g_A g_A \frac{\delta\Gamma\alpha(1-\alpha)}{\sigma v} < 0$$

This establishes that $\frac{dg_A}{dz} < 0$ for $k = 0$.
The same argument holds for $k = 1$, leading to:

$$g_L = g_A(d - 1 + \frac{(\alpha c_1 v^{\epsilon})}{(1-z)^{1/1-\alpha}} \tag{35}$$

$$\rho - g_L = g_A \left(\frac{\delta\Gamma\alpha(1-\alpha)(1-z)^{1/(1-\alpha)}}{\sigma v} - \frac{1}{\sigma} - \frac{\alpha}{1-\alpha} - \frac{(\alpha c_1 v^{\epsilon})}{(1-z)^{1/1-\alpha}} \right) \tag{36}$$

For $k = 1$ the sign of the Jacobian at the denominator is still negative. The Jacobian at the numerator is:

$$| J_{z,v} |= \begin{vmatrix} -\frac{1}{1-\alpha} \frac{\alpha c v^{\varepsilon} g_A}{(1-z)^{\frac{2-\alpha}{1-\alpha}}} & -\frac{\alpha c\varepsilon v^{\varepsilon-1}}{(1-z)^{1/1-\alpha}} g_A \\ g_A \left(\frac{\delta\Gamma\alpha(1-\alpha)}{\sigma v} + \frac{1}{1-\alpha} \frac{\alpha c v^{\varepsilon}}{(1-z)^{\frac{2-\alpha}{1-\alpha}}} \right) & g_A \left(\frac{\delta\Gamma\alpha(1-\alpha)(1-z)}{\sigma v^2} + \frac{\alpha c\varepsilon v^{\varepsilon-1}}{(1-z)} \right) \end{vmatrix}$$

Which is equal to zero. The growth rate of vertical innovation is reduced by quasi fixed costs, except when all industries in the economy do not make final users pay costs. Nevertheless the case $k = 1$ is purely pathological, implying that there is no vertical innovation at all, and can be disregarded.

Since the population growth condition applies, the decrease in vertical innovation growth rate implies an increase in the horizontal growth rate. This remark completes the proof of part i) of statement 6.

7.2 Part ii)

We make use of the arbitrage condition for research:

$$g_N = c_1 \frac{v^\epsilon}{(1-z)^{1/1-\alpha}} g_A$$

Plugging in the research functions $g_N = \lambda_h h^\gamma y$ and $g_A = \lambda_v v^\delta y$, we obtain after simplifications:

$$H \equiv \frac{\lambda_h}{\lambda_v} h^\gamma = c_1 \frac{v^\epsilon}{(1-z)^{1/1-\alpha}} \tag{37}$$

The sign of dh/dz is the same as the sign of dH/dz.

We invert equation 29 in order to express v in term of g_A and of the parameters of the model and we plug the expression in 37. We compute then dH/dz and we get:

$$\frac{dH}{dz} = -\alpha \frac{g_N}{g_A} \left[\frac{dg_A}{dz} \frac{1}{g_A} (1 + \frac{\delta}{\epsilon}) + (1-\alpha) \right] \tag{38}$$

From the proof of part i) we know that $k = 1$ implies $\frac{dg_A}{dz} = 0$. By continuity, for some high values of z for which we are in the proximity of $k = 1$, $\frac{dg_A}{dz}$ can be arbitrarily low. Therefore we restrict ourselves to the neighborhood of $z = 0$.

According to 38 a sufficient condition for $\frac{dH}{dz} > 0$ to hold is:

$$\mid \frac{dg_A}{dz} \mid > 1 - \alpha$$

We apply formula 31 to the case $z = 0$ and after some algebra we rewrite it as:

$$\frac{N}{D} \equiv \frac{\left(\frac{2-\alpha}{1-\alpha}\right)}{\frac{\rho\sigma v}{(1-\alpha)\Gamma\delta} + \frac{g_L}{c\varepsilon v^\varepsilon}} > \frac{(1-\alpha)}{\alpha} \tag{39}$$

For the realistic values of $\alpha > 0.5$ we have that the following inequality holds:

$$\frac{2-\alpha}{1-\alpha} > \frac{1-\alpha}{\alpha}$$

To establish our final result it is then sufficient to show that $D < 1$ holds. Consider the definition:

$$D \equiv \frac{\sigma v \rho}{(1-\alpha)\Gamma\delta} + \frac{g_L}{c\varepsilon v^{\varepsilon}}$$

We now define D_1 as an upper bound of D that is computed by noting that $\rho > g_L$ and $\Gamma > \frac{\sigma}{1-\alpha}$:

$$D_1 = \rho\left[\frac{v}{\delta} + \frac{1}{c\varepsilon v^{\varepsilon}}\right]$$

Since we are considering the economy close to the zero cost case and assuming that $\delta > \rho$ we conclude that the term D_2 is an upper bound for D_1:

$$D_2 = \left[v + \rho\frac{g_A}{g_N\varepsilon}\right]$$

In the data v and g_A are in hundredths of GDP, whereas $\varepsilon = \frac{1-\delta}{1-\gamma}\gamma - \delta$ is hardly smaller than 0.1. The two innovation rates are roughly comparable as dimension[12]. Then the condition $D_2 < 1$ is hardly a restriction at all.

The inequality $1 > D_2 > D_1 > D$ establishes the result.

[12] A few numerical simulations are also available on request