Tax Progression and Human Capital in a Matching Framework*

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Abstract

This paper investigates the effect of tax progression on labour market outcomes in an equilibrium search model with wage bargain and endogenous human capital. We show that this effect depends on whether the firm and the worker can write a binding contract on human capital investments or not. If complete contracts are not possible, either the firm or the worker invests in human capital. We find that the effect of tax progression on human capital depends crucially on which party invests and the tax function that is considered. When the firm invests, we cannot exclude that a higher tax progression increases human capital. Moreover, we find that when a complete contract is possible or when the firm invests, the optimal tax rate in a model with human capital is at least as high as in a model without human capital.

Key words : Tax progression, Human Capital, Search

JEL Classification : H22, J24, J41

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1 Introduction

As high unemployment in Europe is heavily concentrated among low-skilled workers, several policy advices recommend to cut social security contributions of low-wage earners (see for example Algoskoufis et al., 1995), making thereby the social security contribution system more progressive. However, it is then counter-argued that this tax policy has perverse effects. Tax progressivity is accused to reduce incentives to acquire human capital, and therefore to increase the supply of low-skilled workers. The aim of the present paper is to investigate whether this argument is valid and to check the effects of tax progressivity on human capital and unemployment when labour markets are imperfect.

This paper is inspired by three lines of existing research that try to link tax progression, human capital and imperfect labour markets.

The first and most obvious one connects tax progression to human capital, assuming perfect labour markets. In that case, one of the main arguments against a system of progressive income taxes is the discouragement of investments in human capital. As the marginal tax rate increases with earnings, and therefore with human capital, the benefits of human capital (additional earnings) are taxed at a higher rate than the cost (foregone earnings and/or out-of-pocket expenses) are written off. Therefore, a progressive tax system distorts the investments in human capital downwards. Redistribution of income by taxes has a cost in terms of lower human capital. Taking into account the effects of human capital on growth and unemployment, this cost can be substantial. Although this argument is often used in policy debates, there is not much literature about it. Most papers use general equilibrium models where agents can invest in human and physical capital. Sgontz (1982) shows in a simple model that the effects of tax progression on human capital are ambiguous. On the one hand, as mentioned above, tax progression reduces human capital because the additional earning is taxed at a higher rate than the cost of the human capital is written off. But on the other hand, tax progression also affects investments in physical capital. And if at least part of these investments are in the form of foregone earnings, human capital is favoured to physical capital and agents may shift investments from physical to human capital. However, if the tax is a pure wage tax, the second effect does not take place and tax progression unambiguously reduces human capital. Taber (2000) studies the effects of a shift form the current (progressive) US tax system to a flat tax in an empirical overlapping generations general equilibrium model. Again, both effects mentioned above are present and there is no unambiguous analytical result. His simulations show that tax changes have only little effect on the stock of human capital. The present paper deals with the effects of a pure wage tax. Therefore, this first line of literature suggests a negative effect of tax progression on human capital. But compared to that literature, we assume that labour markets are imperfect.

A second direction of research links tax progression to imperfect labour markets, neglecting human capital. If there is wage bargain in the labour market, tax progression has a distributive effect: progressive taxes reduce the wage claims of the workers and moderate wages. In an environment where labour market imperfections lead to involuntary unemployment, wage moderation can reduce unemployment. Tax progression can then improve efficiency. Hersoug (1984), Lockwood and Manning (1993) and Koskela and Vilmunen (1996) study the effects of tax progression in a model without human capital where the presence of a union gives rise to involuntary unemployment. An increase in tax progression at constant average tax rate makes marginal after-tax wage increases more expensive in terms of pre-tax wage increases. This has two effects. First, if the union is concerned about the employment level, it substitutes employment to wages, because employment has become relatively cheaper. Second, the higher price of a marginal wage increase changes the effective bargaining strength of the agents. The firm is more reluctant to give an increase in after-tax wages because the wage cost for this increase has become higher. A higher tax progression then increases the effective bargaining strength of the firm. This second effect is also present in equilibrium search models where search frictions lead to a rent for every worker-employee match. The worker and the firm bargain about the distribution of the rent. Therefore, as in the union models, tax progression increases the relative bargaining strength of the firm. In efficiency wage models, tax progression has a similar effect on the wage level and involuntary unemployment (Hoel, 1990; Pisauro 1991). With high tax progression, a big part of a marginal wage increase goes to the government, and only a small part is given to the worker as an incentive for effort. Therefore, the firm is more reluctant to give a wage increase and the wage level falls. However, in contrast to the bargaining models, an increase in tax progression (at constant average tax rate) does not lead to a higher output (Rasmussen, 1999). Pissarides (1998) and Sørensen (1999) compare the effects in these different models of imperfect labour markets with fixed skills. They conclude that tax progression reduces wages and unemployment. Our paper uses a static equilibrium search model. Markets are imperfect because of search frictions and tax progression changes the division of the surplus between the worker and the firm. At the same time, we introduce human capital in our model.

Finally, a third direction of research connects human capital to imperfect labour markets. If the imperfections of the labour market give rise to a rent that is shared between the firm and the worker, both parties have incentives to invest in human capital. However, if it is impossible to contract on human capital, a hold-up problem arises. The parties only maximise their part of the rent, not the total surplus, neglecting the external effects the investment has on the part of the rent of the other party. An underinvestment in human capital is the logic consequence of these market imperfections. As Becker (1964) mentions, the firm has incentives to invest in firm-specific human capital. Acemoglu and Pischke (1999) show that the firm has also incentives to invest in general human capital if labour markets are imperfect. Our paper therefore assumes that both the worker and the firm can invest in the human capital of the worker. Due to the static nature of our model, we cannot distinguish between firm-specific and general human capital.

Some recent papers connect the first two lines of research mentioned above. They all assume that only the worker can invest in human capital. Bergström (1997) studies the effect of progressive taxation in a model with general human capital. The worker makes the investment and the government cannot correct distortions by education subsidies. The presence of unions leads to involuntary unemployment. Progressive taxation can reduce this unemployment but leads at the same time to lower investments in human capital by the worker, combining the two effects of tax progression we mentioned above. The total effect on welfare is ambiguous. Van Ewijk and Tang (2001) use a similar environment with general human capital, but introduce education subsidies as an additional instrument for the government to achieve an optimal policy. They show that if education is fully observable, the education subsidies can be used to fully correct the distortions caused by tax progression. This is no longer true if education is not fully observable. In that case, the distortions cannot be corrected by education subsidies, and the negative effect of tax progression on education remains. This negative effect is opposed to the positive effect of tax progression on unemployment in imperfect labour markets. Boone and de Mooij (2000) study the optimal tax rates in a static equilibrium search model where the search intensity of the unemployed is endogenous. The government can choose the tax function and education subsidies to correct the market outcome. They consider both cases where a contract on human capital investments is possible or not.

They find that tax progression is less distortionary when commitment on the human capital investment is possible. In Fuest and Huber (1998), the firm and the worker bargain individually about the wage. Involuntary unemployment arises from a fixed hiring/skill formation cost to the employer. Not all workers that are willing to work are hired. Workers with low human capital are unemployed. Since the (firm-specific) human capital is not observable to a third party and only the worker can determine the level of human capital, a hold-up problem arises. Tax progression decreases the effective bargaining strength of the worker and therefore the wage rate per efficiency unit of human capital. This lower return to human capital in turn decreases the worker's investments. They show that with their specification of the model, this lower human capital increases unemployment more than the effect of lower wages decreases it. Contrary to the mentioned papers, our paper includes the possibility that the firm can invest in the human capital of the worker. We show that this can change the effect of tax progression on human capital radically. We also show that this effect depends on the family of tax functions that is considered. We use the main tax functions used in the above papers and show their consequences on the incentives of the firm to invest in human capital.

Section 2 sets up the basic model we use, before section 3 explores the first-best levels of human capital and labour market tightness. In section 4, we suppose that the firm and the worker can write a complete contract on human capital and the wage and show the effect of tax progression on the labour market outcomes. We derive the optimal (second-best) tax rate and compare it to the optimal tax rate in a model with exogenous human capital endowment. In section 5, we assume that complete contracts are no longer possible, and a hold-up problem arises. We investigate the effects of tax progression and the optimal tax rates then under different tax functions. Section 6 finally concludes.

2 The Model

This section develops a static model of search in the labour market that integrates human capital into the standard model (e.g. Pissarides, 2000 or Cahuc and Zylberberg, 2001) with homogenous firms and workers. The labour force is exogenously given, whereas the number of firms is endogenous. We assume that firms are small and consist of only one worker. Transaction costs make it costly for the firm and the worker to find a partner. A successful match yields therefore a rent. The representative firm and the representative worker bargain about the distribution of the rent.

Matching

We assume that the labour force L is exogenous. At the beginning, all workers are unemployed and search for a job. Firms open a number V of vacancies. The number of matches M is then given by the matching technology

$$M = m(L, V)$$

This matching function is assumed to be increasing and concave in both arguments and homogeneous of degree 1. Moreover, the number of matches cannot exceed nor the labour force nor the number of open vacancies. We assume that the matching function is of the Cobb-Douglas type, an assumption which is supported by empirical findings (see Pissarides, 2000). It is useful to define labour market tightness θ as

V/L. Open vacancies are then filled at a rate

$$q(\theta) = \frac{M}{V} = m(\frac{1}{\theta}, 1)$$

and workers find a job with the probability

$$\theta q(\theta) = \frac{M}{V}\frac{V}{L}$$

Human capital

Human capital is measured in productivity units.[1] Output is given by the amount of human capital H of the worker. Human capital has a cost function $c(H)$ which is assumed to be increasing and convex in H. Put another way, it becomes more and more expensive to increase the productivity of the worker. The firm invests an amount I_f in human capital whereas the worker invests an amount I_w. Total investment is given by the sum of the two, where, by definition, $I_f + I_w = c(H)$.[2]

Workers

The workers are assumed to be risk neutral. An employed worker has a gross income w, which is taxed by an income tax $t(w)$.[3] The average and marginal tax rates are defined as[4]

$$t^a(w) = \frac{t(w)}{w}$$

$$t'(w) = \frac{\partial t(w)}{\partial w}$$

We assume that the worker's outside option equals 0. This means that the worker gets no unemployment benefits if he doesn't find a job.

Firms

The return on a filled vacancy for the firm equals $H - w$, where the worker's gross wage w depends on his level of human capital. For every open vacancy, the firm has to pay a hiring cost k. The expected profit from opening a vacancy is therefore

$$-k + q(\theta)[H - w - I_f]$$

Firms open vacancies as long as this expected profit is positive. Free entry ensures that at the equilibrium this expected profit equals 0, which leads to the following vacancy supply curve:

$$H - w - I_f - \frac{k}{q(\theta)} = 0 \tag{1}$$

[1] One is free to choose the measure of human capital. We opt for a linear production function and a convex cost function. But a model with a concave production function and a linear cost function would intuitively lead to the same results.

[2] Another approach (used among others in Boone and de Mooij, 2000) is to distinguish two levels of productivity only. Human capital increases the probability to get the high productivity output. If $p(H)$ is the probability of attaining the high productivity level, and y_0 and y_1 the two productivity levels, if we assume risk neutrality, we get

$$y(H) = p(H)y_1 + (1 - p(H))y_0$$

[3] Alternatively, the tax function $t(w)$ can be understood as social security contributions.

[4] Note that both the average and the marginal tax rates are functions of the wage.

Wage bargain

Once the firm and the worker are matched, they enjoy a rent because they do not have to invest in costly search any more to produce the output. This rent is shared between the worker and the firm. We assume that they negotiate over the distribution of the rent in a way that the outcome of the bargain is the Nash solution to the bargaining problem. The exogenous relative bargaining powers are β and $1-\beta$ for the worker and the firm respectively, where β is restricted to values between 0 and 1. If there is no agreement, production does not take place. The firm then makes a financial loss, as it has already paid the hiring cost k. The worker gets his outside option, which in absence of unemployment benefits, equals 0. The bargaining problem differs whether complete contracts are possible or not. If complete contracts are possible, the bargain is simultaneously about the wage rate and human capital. On the contrary, if complete contracts are not enforceable, the bargain is about the wage rate only.

Government

The government chooses as its only policy instrument[5] a twice differentiable tax function $t(w)$ subject to its budget constraint. We assume that there is no public good to be financed. Moreover, the government does not pay unemployment benefits to the unemployed. As the workers are homogeneous in our model, a binding budget constraint implies then that at the equilibrium wage the tax $t(w) = 0$. This then implies that the government chooses a tax function such that at the equilibrium wage

$$t^a(w) = 0$$

Moreover, we assume that the marginal tax rate cannot exceed 1.

3 The Social Optimum

We measure social welfare by total output minus the cost of training and posting vacancies. Since agents are homogeneous and risk neutral, distributional considerations between the workers and the unemployed are of no importance for a utilitarian social planner. The social planner therefore chooses human capital and labour market tightness to maximise output net of costs

$$\{\theta q(\theta)[H - c(H)] - k\theta\} L$$

This gives the first-order conditions

$$c'(H) = 1$$

$$q(\theta)[H - c(H)] + q'(\theta)\theta[H - c(H)] = k$$

The first equation simply tells that human capital is at its optimal level if its marginal productivity equals its marginal cost. The second one is similar to Hosios' (1990) equation and determines the optimal

[5] Other instruments like unemployment benefits or profit taxation seem not to be appropriate in this simple model. In fact, the first aim of unemployment benefits is to insure risk averse workers. But workers are assumed risk neutral in our model. Similarly, profit taxation in a model where a free-entry condition drives profits down to zero, makes no sense. On the other hand, a model with risk averse workers and positive profits seems to complicated to us to solve analytically.

level of labour market tightness. The intuition is straightforward: A new vacancy results in a match that gives a net output of $H - c(H)$ with probability $\theta q(\theta)$. The new vacancy has a direct cost of k, and at the same time a negative externality on existing vacancies: The probability that the existing vacancies are matched to a worker and lead to an output $H - c(H)$ decreases by $\theta q\'(\theta)$. The social optimum requires that these costs equal the marginal benefit. In its standard way, this condition is rewritten as

$$[H - c(H)](1 - \eta) = \frac{k}{q(\theta)} \tag{2}$$

where η is the absolute value of the elasticity of the matching function and takes a value between 0 and 1. Under the Cobb-Douglas assumption on the matching function, η is a constant parameter.

The following sections will check to what extent the optimal values for θ and H can be decentralized. Complete and incomplete contracts will be considered in turn.

4 Complete Contracts

In this case, the firm and the worker are able to write a binding contract about the levels of investment in human capital and wages. The sequence of decisions is then the following:

- Step 1: The government sets the tax function $t(w)$.
- Step 2: Firms decide to open vacancies and workers decide to search for a job. Firms and workers are matched.
- Step 3: The firm and the worker bargain about wages and investment levels and the distribution of the investment costs.

We solve backwards, assuming that agents are aware of the value of $t(w)$ for any w, but also of first- and second-order derivatives of the function $t(w)$.

4.1 Bargain about Wages and Human Capital

In step 3, the worker and the firm choose the wage level, human capital and the distribution of the investment costs by maximising the Nash bargain

$$[w - t(w) - \lambda c(H)]^{\beta}[H - w - (1 - \lambda)c(H)]^{1-\beta} \tag{3}$$

with respect to w, H and λ. λ is the part of the investment cost paid by the worker and is restricted to values between 0 and 1, such that $I_w = \lambda c(H)$ and $I_f = (1 - \lambda)c(H)$. The first-order conditions lead to the following equations determining H and w (see Appendix 7.1):

$$c'(H) = \frac{1 - t'(w)}{(1 - \lambda)(1 - t'(w)) + \lambda} \tag{4}$$

$$w = \frac{\beta(1 - t'(w))[H - (1 - \lambda)c(H)] + (1 - \beta)\lambda c(H)}{(1 - \beta)(1 - t^a(w)) + \beta(1 - t'(w))} \tag{5}$$

The Kuhn-Tucker first-order conditions on λ imply $\lambda = 0$ if $t'(w) > 0, \lambda = 1$ if $t'(w) < 0$ and $\lambda \epsilon [0, 1]$ if $t'(w) = 0$. These results then give the following lemma:

Lemma 1 *The investment in human capital is at its optimal level if the marginal tax rate is non-negative. If the marginal tax rate is negative, there is over-investment in human capital.*

Proof. It is straightforward to see that if the marginal tax rate equals 0, equation (4) implies that the marginal cost of human capital equals 1, whatever the value of λ. Similarly, if the marginal tax rate is positive, λ equals 0 and equation (4) shows the same result. However, if the marginal tax rate is negative, λ is chosen to equal 1 and equation (4) shows that the marginal cost of human capital is bigger than 1. This, together with the convexity assumption on the cost function $c(H)$ implies that there is over-investment in human capital. ■

The intuition for this result is simple. If the marginal tax rate is zero, a complete contract on human capital takes the incentives of both the worker and the firm into account. As the party that pays the cost is always reimbursed, it does not matter by which party the investment is paid. They always attain the first-best level of human capital in a world without taxes. But taxes can change this result. The cost of the investment is shared. As in our model, there are no taxes on the firm's profit, it is sufficient to reimburse the exact amount of the investment cost to the firm. On the other hand, the only way of reimbursing the worker for his investment cost is to pay him a higher wage. But this wage increase is taxed at the marginal tax rate. The wage increase has to include this tax and is therefore not the same as the investment cost. In fact, if the marginal tax rate is negative, an increase in human capital, paid by the worker, is subsidised by the government. The worker and the firm then agree that they profit from this subsidy by letting the worker pay the cost. Given the fact that without tax, they attain the first-best level of human capital, a subsidy of the marginal increase in human capital leads to over-investment. Inversely, with a positive marginal tax rate the investment does not deviate from its first-best level: An investment paid by the worker is positively taxed, and if the worker was the only one who could invest, this would lead to under-investment. However, they can also agree that the firm pays the investment. As the reimbursement of the investment cost to the firm is not affected by the marginal tax rate, this leads to the same result as without taxes. The only difference is that now it matters that only the firm pays the investment.

4.2 Labour Market Equilibrium

In step 2 then, the firm decides to open vacancies. The vacancy supply (1) can be rewritten as

$$H - w - (1 - \lambda)c(H) = \frac{k}{q(\theta)} \tag{6}$$

Introducing the results for the wage equation (5) and using the government's budget constraint $t^a(w) = 0$, we get the following results:

Lemma 2 *Labour market tightness θ is at its optimal level if the following condition on the marginal tax rate is satisfied at the equilibrium wage (assuming that the budget constraint of the government is satisfied):*

$$t'(w) = 1 - \frac{\eta}{1 - \eta} \frac{1 - \beta}{\beta} \tag{7}$$

This condition is independent of whether the firm or the worker (or both) invests in human capital. A marginal tax rate that is higher than this value leads to a too high labour market tightness. Inversely, if the marginal tax rate is lower than this critical value, labour market tightness is below the optimal level. Moreover, the critical value of the tax rate (7) is positive if $\beta > \eta$, negative if $\beta < \eta$ and equal to 0 *if* $\beta = \eta$.

Proof. See Appendix 7.2. ■

The intuition is the same as in Hosios' (1990) model. If β equals η, the private and social return on an open vacancy are the same in a model without taxes, which ensures optimal labour market tightness in the decentralized equilibrium. Taxes then only deviate labour market tightness from its optimal value. On the contrary, if β differs from η, tax policy can be used to bring the private return back at its optimal level. More specifically, equation (5) shows that an increase in the marginal tax rate decreases the wage rate and increases therefore the profit of the firm. The higher the marginal tax rate, the more vacancies are opened by firms and the higher the labour market tightness. Inversely, a low marginal tax rate increases the wage claims of the workers and decreases therefore the expected profit of a vacancy for a firm. Less vacancies are opened and labour market tightness decreases.

4.3 Optimal Tax Policy

In step 1, the government chooses the tax function. As the average tax is restricted to 0 by the budget constraint of the government, the only free variable that affects outcome is the marginal tax rate. The government chooses this marginal tax rate to maximise social welfare. However, it faces two variables to adjust - labour market tightness and human capital - but has only one instrument to achieve it. This induces that the first best outcome might not be obtained. The government then chooses the marginal tax rate to achieve the second best. With the help of lemma 1 and 2, we get the following proposition that characterizes the choice of the marginal tax rate:

Proposition 3 *The first-best level of labour market tightness and human capital investment is attained if $\beta \geq \eta$. In that case, the marginal tax rate is non-negative and set at the level given by equation (7). If $\beta < \eta$, the marginal tax rate is negative and the first-best outcome cannot be attained. There is over-investment in human capital and labour market tightness is above its optimal level. The tax rate is between the level given by equation (7) and* 0.

Proof. See Appendix 7.3. ■

Given the lemmas 1 and 2, this result is quite straightforward. The firm pays the investment if the marginal tax is positive and investment is at its optimal level, unaffected by the marginal tax rate (lemma 1). Therefore the marginal tax rate can be used to restore the optimal level of labour market tightness, under the condition that this optimal level of the marginal tax rate is non-negative. This is true as long as $\beta \geq \eta$ (lemma 2). On the contrary, if $\beta < \eta$, lemma 2 tells that the tax level needed to restore the optimal level of labour market tightness is negative. But lemma 1 indicates that a negative marginal tax rate leads to over-investment in human capital. There is then a trade-off between approaching the first-best level of labour market tightness (which needs a negative tax rate given in lemma 2) and the first-best level of human capital (which needs a marginal tax rate at least equal to 0). The second-best is then attained by setting the marginal tax rate somewhere in between these two values. The precise value is given by equation (23) in Appendix 7.3.

These results are conditional on the fact that the government has the marginal tax rate as only policy instrument. In fact, if the government was able to observe costlessly the human capital of the worker, it could use education subsidies or education taxes to attain the first-best level of labour market outcomes. Boone and de Mooij (2000) show this in a model where only the worker can pay the investment cost. The tax rate is then used to put labour market tightness at its optimal level, and the education subsidy is chosen as to achieve the optimal level of human capital. Another possibility for the government might be to increase the worker's outside option by introducing an unemployment benefit financed by taxes on labour. Both the positive average tax rate and the unemployment benefit increase the gross wage and decrease therefore the expected profit form a match that goes to the firm. This can help to decrease labour market tightness to its optimal level.[6] On the contrary, it does not matter whether the tax on labour is formally paid by the worker or the firm. The free-entry condition implies that in any case, expected profits equal zero and therefore, the worker bears the entire tax. The same reasoning applies for profit taxation (see Boone and Bovenberg, 2001).

Comparing our results to the results obtained in a equilibrium search model without endogenous human capital (developed in Appendix 7.4) gives the following corollary:

Corollary 4 *If complete contracts are enforceable, the marginal tax rate in an equilibrium search model with endogenous human capital formation is at least as high as in a similar model with exogenous human capital endowment.*

In fact, when human capital is exogenous, the tax function only has to restore the efficiency condition on labour market tightness. As lemma 2 shows, this can be achieved by setting the marginal tax rate at the level given by equation (7). As human capital is exogenous, this does not enter into conflict with any other endogenous variable. The marginal tax rate then sets labour market tightness at its first-best level. On the contrary, the integration of human capital has the consequence that the first-best outcome cannot always be attained, as the goal to set labour market tightness at its first-best value might be incompatible with setting the human capital at its first-best level. Another interesting implication of proposition 3 is that there is never under-investment in human capital.

However, these results are conditional on the assumption that the worker and the firm can write a binding contract on human capital investments. The next section considers the case where this is not possible.

5 Incomplete Contracts

Human capital is often difficult to observe by a third party. A contract on human capital is then not enforceable and therefore valueless. This leads to a hold-up problem. The two parties can invest in

[6] One can counter-argue that this reasoning misses some important aspects: Unemployment benefits first of all insure risk-averse workers against unemployment. The analysis then should take into account risk-averse workers as well as their search behaviour. Especially, the reasoning made above might decrease the search effort of the unemployed. Lehmann and Van der Linden (2002) analyse this issue in a model without human capital.

human capital, but cannot write a binding contract on it. Wages are then negotiated about separately form the investment decision. The sequence of decisions is the following:

- Step 1: The government sets the tax function $t(w)$.
- Step 2: Firms decide to open vacancies and workers decide to search for a job. Firms and workers are matched.
- Step 3: The firm and the worker decide individually about their level of investment in human capital.
- Step 4: The firm and the worker bargain about the wage rate.

Wages could also be negotiated between steps 2 and 3. However, if we assume that this wage can be renegotiated at every moment in time and at no costs, the first wage contract would then not be enforced. We solve backwards.

5.1 Wage Bargain

In stage 4, the costs of the investment are sunk. Therefore they do not enter the bargain between the worker and the firm. The wage rate is then chosen to maximise the Nash product

$$[w - t(w)]^{\beta}[H - w]^{1-\beta} \tag{8}$$

This gives the wage equation

$$w = \frac{\beta(1 - t'(w))H}{(1-\beta)(1 - t^{a}(w)) + \beta(1 - t'(w))} \tag{9}$$

As in every Nash bargain, if one of the parties has all the bargaining power, the other party gets just its outside option. In our case, if β equals 0, the wage is set at 0. Inversely, if β equals 1, the wage is set at the total output H, and the whole surplus of the match belongs to the worker. Comparing equation (9) to equation (5), it is easily seen that the only difference consists in the fact that equation (9) does not take into account the cost of investment. This is because the investment is made in step 3 and is a sunk cost in step 4.

5.2 Human Capital

In step 3, the firm and the worker invest in firm-specific human capital to maximise their own objective, knowing that the wage will be set in step 4 at the level given by equation (9). Since the investment level is not verifiable to a third party, any contract on it cannot be enforced. When they decide about their investment levels, the firm and the worker therefore only maximise their own part of the surplus and not the total surplus. Any positive externality on the other party is not internalised. The investment is therefore suboptimal. This is in fact a typical hold-up problem.

The worker maximises his utility knowing that the wage rate will be set in step 4 at the level given by equation (9). Taking the investment of the firm I_f as given, he chooses $I_w \geq 0$ to maximise

$$w - t(w) - I_w$$

The wage w depends on the investment level as well. His planned investment is then implicitly given by the first-order condition (for the mathematical development see the Appendix 7.5)

$$(1 - t'(w)) \frac{\beta}{1 + \beta \frac{t''(w)}{1-t(w)}(H - w)} = c'(H)(1 - \mu_w) \tag{10}$$

with the complementary slackness conditions

$$\begin{aligned} I_w &\geq 0 \\ \mu_w I_w &= 0 \end{aligned}$$

where μ_w represents the Lagrange multiplier for the non-negativity constraint on human capital investment of the worker. $t''(w)$ is the second derivative of $t(w)$ with respect to w. The left-hand side is the marginal increase in the after tax wage of a marginal increase in human capital. We assume that this value is non-negative. This means that an increase in human capital does not decrease the worker's utility. Mathematically, this amounts to imposing a lower bound on $t''(w)$.

Taking I_w as given, the firm chooses $I_f \geq 0$ to maximise

$$H - w - I_f$$

This leads to the first-order condition (for the mathematical development see the Appendix 7.5)

$$1 - \frac{\beta}{1 + \beta \frac{t''(w)}{1-t(w)}(H - w)} = c'(H)(1 - \mu_f) \tag{11}$$

with the complementary slackness conditions

$$\begin{aligned} I_f &\geq 0 \\ \mu_f I_f &= 0 \end{aligned}$$

where μ_f represents the Lagrange multiplier for the non-negativity constraint on human capital investment of the firm. Once again, the left-hand side is the marginal increase in the firm's profit of a marginal increase in human capital. We assume that this value is non-negative, i.e. an increase in human capital does not decrease the firm's profit. Introspection of equations (10) and (11) and their associated complementary slackness conditions shows that in general, one of the two non-negativity constraints on investments in human capital is binding. This implies that only one party invests in human capital, the other one sets its investment at zero. In a non-cooperative game, the investment is the higher of the two planned investments (see Appendix 7.6). In fact, if $\overline{I_w}$ and $\overline{I_f}$ are the solutions to (10) and (11) by setting the Lagrange multipliers to 0, the level of human capital equals the maximum of the two. If $\overline{I_w} > \overline{I_f}$, the worker bears all the cost of human capital. Inversely, if $\overline{I_w} < \overline{I_f}$, the firm bears the cost. Introspection of equations (10) and (11), using the convexity assumption on $c(H)$, shows that $\overline{I_w} > \overline{I_f}$ if and only if

$$(1 - t'(w)) \frac{\beta}{1 + \beta \frac{t''(w)}{1-t(w)}(H - w)} > 1 - \frac{\beta}{1 + \beta \frac{t''(w)}{1-t(w)}(H - w)} \tag{12}$$

This equation tells that the party that expects the bigger part of the marginal increase in his own part of the surplus, invest.

The basic intuition behind equations (10) and (11) is easy to see. As equation (9) shows, the marginal tax rate influences the distribution of the surplus between the worker and the firm. The higher the marginal tax rate, the bigger is the part of the output that goes to the firm. When the worker and the firm determine their desired investment, they take this effect into account. An increase in human capital increases the wage rate, and might therefore influence the marginal tax rate $t'(w)$. In fact, if the marginal tax rate is constant, this effect vanishes. The distribution of the output does not change, only the size of the output increases with higher human capital. With a constant marginal tax rate, the second derivative of the tax function equals zero, and it is easy to see from equations (10) and (11) that in that case the firm gets a part $1 - \beta$ of the marginal increase in the output, and the worker's pre-tax wage increases by a part β of the marginal increase[7]. This corresponds to the intuition that in Nash bargains, the parties get a share equal to their relative bargaining powers. Moreover, as the labour demand is perfectly elastic, the tax is entirely borne by the worker (see also Boone and Bovenberg, 2001). On the contrary, if the marginal tax rate is not constant, the change in the distribution of the output has to be taken into account. If the marginal tax rate increases in w, $t''(w)$ is positive. An increase in human capital changes then the distribution of the output in favour of the firm. From equations (10) and (11), it is easy to see that this increases the incentives of the firm to invest in human capital, but at the same time, it decreases the incentives of the worker. Moreover, an increase in the marginal tax rate decreases the part of the marginal increase in the output that goes to both parties, as more taxes have to be paid to the government. Mathematically, introspection of equation (9) shows that an increase in human capital influences the wage through three channels. First, in a direct way: An increase in H increases w, as the output increases. But this wage increase adds two indirect effects of the increase in human capital: On the one hand, a wage increase changes the average tax. This effect depends on the marginal tax rate. If the marginal tax rate is higher than the average tax rate, the average tax rate increases with the wage rate. Intuitively, a higher average tax rate then decreases the surplus that can be divided between the worker and the firm. This indirect effect of human capital on the (pre-tax) wage rate is therefore positive as long as $t^a < t'$. Finally, the third effect shows that the marginal tax rate might change as the wage increases. This effect depends on the second derivative of the tax function. If t'' is positive, an increase in the wage increases the marginal tax rate. But the marginal tax rate influences the distribution of the output that can be divided between the firm and the worker. A high marginal tax rate attributes a bigger part of the output to the firm. This indirect effect of human capital on the wage rate is therefore negative as long as the second derivative of the tax function is positive. As this discussion shows, the shape of the marginal tax rate is of crucial importance for the effect of tax progression on labour market outcomes. One possibility is then to let the government freely choose the tax function. In our case, this amounts to choose freely the marginal tax rate and the second derivative of the tax function with respect to the wage[8]. However, the publications in this field have concentrated - sometimes implicitly - on a specific tax

[7] Note that the part of the worker is taxed at the marginal tax rate $t'(w)$, such that the surplus of the match increases by $(1 - \beta t'(w))$ times the marginal increase in the output. This gives then the traditional result that the worker and the firm get respectively a part $\frac{(1-t'(w))\beta}{1-\beta t'(w)}$ and $\frac{1-\beta}{1-\beta t'(w)}$ of the maginal increase in the surplus of the match.

[8] The results might then differ considerably from the tax functions that are observed in the real world. This might be due to the fact that the agents in our model are homogeneous and redistribution issues are neglected. We therefore restrict ourselves to tax functions that are similar to those observed in the real world. However, the results for the case where the government can choose both the marginal tax rate and the second derivative of the tax function with respect to the wage

function where only one tax parameter is freely chosen by the government. We also follow this approach. But we show the effects under different tax functions, the ones that are used in the recent literature on this subject. This allows us to see that qualitatively different results are obtained with different tax functions. This can partly explain the heterogeneous results in the recent literature about this subject.

First case: the tax function is linear

In this case we restrict ourselves to the family of tax functions where the marginal tax rate does not depend on the wage[9]. This specification is used among others in Bergström (1997) and Boone and de Mooij (2000). As the second derivative of the tax function equals 0, equations (10) and (11) simplify: An increase in human capital does not affect the distribution of the output, and therefore, the indirect effect of human capital on the wage via the second derivative of the tax function is nil. The following lemma can be derived:

Lemma 5 *If the tax function is linear and the firm invests, the investment in human capital is sub-optimal as long as the firm has not all the bargaining power ($\beta = 0$). If the worker invests, the investment is optimal if the following condition for the marginal tax rate is satisfied at the equilibrium wage (assuming that the budget constraint of the government is satisfied):*

$$t'(w) = 1 - \frac{1}{\beta} \leq 0 \tag{13}$$

There is over- (under-)investment if the marginal tax rate is lower (higher) than this value.
An increase in the marginal tax rate decreases human capital if the worker invests and does not affect the level of investment in human capital if the firm invests.

Proof. See Appendix 7.7. ■

Intuitively, with linear tax functions, the distribution of the output does not change as human capital increases. The firm gets then a constant part of the marginal increase in the output. This part does not depend on the marginal tax rate and therefore, tax progression cannot influence human capital if the firm invests. Similarly, the (pre-tax) wage of the worker increases by a constant that is independent of the marginal tax rate. However, the worker's wage is taxed, and an increase in the marginal tax rate reduces his incentives to invest in human capital. As the parties do not take into account the externalities they have on the other party, the hold-up problem implies sub-optimality of the investment level. However, if one party has all the bargaining power, this externality does not exist any more. As the firm's profit is not taxed, the firm sets the human capital investment at its optimal level. On the contrary, attributing all the bargaining power to the worker does not solve the hold-up problem. In fact, the worker's wage is still taxed, such that the investment is sub-optimal as long as the marginal tax rate is positive. However, if the worker invests, the investment can be brought back to its optimal level if investment is sufficiently subsidised (the marginal tax rate is sufficiently below zero).

Second case: The tax function has a constant coefficient of residual income progression (CRIP)

are derived in Appendix 7.11.

[9] i.e. the family of the tax functions with the linear form $t(w) = a + bw$, where a and b are the tax parameters that the government can choose.

The coefficient of residual income progression ν is defined as[10]

$$\frac{1-t'(w)}{1-t^a(w)}$$

This form of tax function is used in van Ewijk and Tang (2001) and implicitly in Fuest and Huber (1998). If ν is bigger than one, the tax system is regressive, whereas a ν that is lower than one indicates a progressive tax system. If the tax functions are restricted to the ones with a constant CRIP, an increase in the marginal tax (which is equivalent to an increase in ν at a constant average tax) also changes the second derivative of the tax function with respect to the wage[11]. A change in tax progression then changes the distribution of the output and we get the following result:

Lemma 6 *If the tax function has a constant CRIP and the firm invests, the investment in human capital is sub-optimal as long as the firm has not all the bargaining power ($\beta = 0$). If the worker invests, the investment is optimal if the following condition for the marginal tax rate is satisfied at the equilibrium wage (assuming that the budget constraint of the government is satisfied):*

$$t'(w) = \frac{1}{2} - \frac{\sqrt{-3+\frac{4}{\beta}}}{2} < 0 \tag{14}$$

There is over- (under-)investment if the marginal tax rate is lower (higher) than this value. An increase in the marginal tax rate decreases human capital if the worker invests but increases it if the firm invests.

Proof. See Appendix 7.8. ■

In fact, the wage equation (9) can be rewritten with ν as the only tax parameter. As ν is assumed to be constant when the tax function has a constant CRIP, this shows that only the direct effect of human capital on the wage remains, and this effect is unambiguously positive. The indirect effects trough the average and marginal tax rate cancel out. It can be shown (see Appendix 7.8) that the direct effect decreases as the marginal tax rate increases, i.e. the higher the marginal tax rate, the lower the wage increase when human capital increases. From equations (10) and (11), we get then the result that a marginal increase in human capital has a higher incentive effect for the firm, but a lower one for the worker.

5.3 Labour Market Equilibrium

In step 2, the firm decides to open vacancies. The vacancy supply is given by equation (1). Using the analytical results used for lemmas 5 and 6, we find the following lemma:

[10] The CRIP is also the elasticity of the net wage with respect to the gross wage w. For more details about the CRIP and its properties, see e.g. Musgrave and Musgrave (1976)

[11] The tax functions with a constant CRIP are of the form $t(w) = w - aw^b$, where a and b are the tax parameters that the government can choose.

Lemma 7 *If the investment is paid by the firm, the level of labour market tightness is optimal if the marginal tax rate solves the following condition at the equilibrium wage (assuming that the budget constraint of the government is satisfied):*

$$t'(w) = 1 - \frac{\eta}{1-\eta}\frac{1-\beta}{\beta}\frac{H - c(H)}{H - c(H) + \frac{c(H)}{1-\eta}} \tag{15}$$

An increase in the marginal tax rate leads then to an increase in labour market tightness.
If the investment is paid by the worker, the level of labour market tightness is optimal if the marginal tax rate solves the following condition at the equilibrium wage (assuming that the budget constraint of the government is satisfied):

$$t'(w) = 1 - \frac{\eta}{1-\eta}\frac{1-\beta}{\beta}\frac{H - c(H) + \frac{c(H)}{\eta}}{H - c(H)} \tag{16}$$

The effect of an increase in the marginal tax rate on labour market tightness is then ambiguous.

Proof. See Appendix 7.9. ■

Note that compared to the marginal tax rate obtained in lemma 2, the marginal tax rate that sets labour market tightness at its optimal level is higher if the firm pays the investment, and it is lower, if the worker pays the investment. This is because in a complete contract, the cost can be shared. With incomplete contracts, this is no longer possible. If the firm pays the entire investment, the cost for the firm is higher than if the firm can share the cost. The expected private return on a vacancy is then lower, but this can be corrected by decreasing the wage through a higher marginal tax rate. Therefore, the marginal tax rate that restores the first-best level of labour market tightness is higher than under complete contracts. The inverse is true if the worker invests. The firm then doesn't have to pay its part of the investment, and the expected private return on a vacancy becomes higher, which can be corrected by a lower marginal tax rate.

There is a direct and an indirect effect of the marginal tax rate on labour market tightness. The direct effect is obtained by the fact that a higher marginal tax rate decreases the wage rate per unit of human capital, given by equation (9). The firm therefore gets a higher part of the output and opens more vacancies. This effect is unambiguously positive. However, the marginal tax rate also indirectly affects labour market tightness trough the level of human capital. As lemmas 5 and 6 show, human capital decreases when the marginal tax rate increases and the worker invests. In that case, the output decreases, and it is possible that this effect dominates the direct effect of the tax rate on labour market tightness[12]. On the contrary, if the firm invests, the indirect effect becomes non-negative, and the total effect is positive.

[12] For the exact conditions, see the proof in Appendix 7.9.

5.4 Optimal Tax Policy

In step 1 finally, the government sets its tax function $t(w)$. The budget constraint implies that the average tax rate equals 0. If the tax functions are restricted to the types discussed in subsection 5.2, the government has only one instrument - the marginal tax rate - to adjust two variables - labour market tightness and human capital. This implies the possibility that the first-best outcome might not be achieved, and the government sets the marginal tax rate to get the second-best. Again, we study the optimal marginal tax rate under the different tax systems that we already used in step 3.

First case: The tax function is linear.
Maximising total output, subject to equations (1), (9), (10) and (11), we get the following result:

Proposition 8 *If the firm invests, the optimal marginal tax rate is at the level that solves for optimal labour market tightness. Compared to the first-best, the investment in human capital is sub-optimal, but labour market tightness is optimal. If the worker invests, the result is ambiguous.*

Proof. See Appendix 7.10. ■

In fact, it the firm invests, human capital investment is not affected by the marginal tax rate, and tax policy cannot be used to correct human capital. As lemma 5 tells, human capital investment is sub-optimal. But the marginal tax rate can then be fully used to correct the labour market tightness and to bring it to its first-best level. There is no trade-off between correcting human capital and correcting labour market tightness. On the contrary, if the worker invests, the marginal tax rate affects human capital and labour market tightness. There is then a trade-off between achieving optimal labour market tightness and the optimal level of human capital. Intuitively, the optimal tax rate lies somewhere in between the tax rates that solve for optimal human capital and optimal labour market tightness, given by equations (13) and (16) respectively. As these values depend on the parameters, we cannot say which one is higher. Therefore, any outcome is possible: Human capital as well as labour market tightness can be too low or too high.

Second case: The tax function has a constant coefficient of residual income progression (CRIP)
By maximising the total output, subject to equations (1), (9), (10) and (11), we get the following result:

Proposition 9 *If the firm invests, the optimal marginal tax rate is above the level that implies optimal labour market tightness. Compared to the first-best, the investment in human capital is sub-optimal, and labour market tightness is too high compared to the first-best. If the worker invests, the result is ambiguous.*

Proof. See Appendix 7.10. ■

For the worker's part, the intuition is the same as with linear taxes. And the reasoning for the firm's part is similar. In fact, an increase in the marginal tax rate approaches the human capital investment to its first-best level. There is a trade-off between approaching the first-best human capital level and the first-best labour market tightness as soon as the marginal tax rate is higher than the level given in equation (15). However, contrary to the case where the worker invests, we know that the tax rate that sets human capital at its optimal level equals 1 and is therefore independent of the parameters. The tax rate is then set somewhere in between the level given by equation (15) and 1. The lemmas 6 and 7 let us finally conclude that labour market tightness is too high and human capital sub-optimal.

As in the case when a complete contract is possible, these results are conditional on the fact that the government has only one policy instrument available. The case where the government can freely choose the marginal tax rate and the second derivative of the tax function is developed and discussed in Appendix 7.11. If the government was able to observe the human capital of the worker without cost, it could use education subsidies or education taxes to attain the first-best level of labour market outcomes. Boone and de Mooij (2000) show this for the case where the worker invests. The marginal tax is used to set labour market tightness at its optimal level, and any distortion on human capital can be corrected by the education subsidy. On the contrary, the introduction of a tax on labour that has to be formally paid by the employer does not change our results. In fact, in our model, it does not matter whether the wage tax is formally paid by the worker or by the firm.

Comparing our results to the results obtained in a equilibrium search model without endogenous human capital (developed in Appendix 7.4) gives the following corollary:

Corollary 10 *In the case where complete contracts are not enforceable, the marginal tax rate in an equilibrium search model with endogenous human capital formation is at least as high as in a similar model with exogenous human capital endowment if the firm decides upon the level of human capital.*

In a model with exogenous human capital, the only task of the marginal tax rate is to put labour market tightness at its optimal level. When human capital is endogenous and the firm invests, lemma 6 indicates that a higher marginal tax rate might increase human capital and approach it to its optimal level. There is a trade-off for the government between getting optimal labour market tightness and optimal human capital, which induces that the choice for the marginal tax rate in this second-best environment is higher than when human capital is exogenous. However, if the worker invests, the effect is ambiguous and we are not able to conclude whether the introduction of human capital in the model increases or decreases the optimal marginal tax rate.

6 Conclusion

This paper has investigated the effect of tax progression on labour market outcomes in a static equilibrium search model. Tax progression was shown to have two effects: First, there is the traditional disincentive effect, as the return on human capital is reduced by more progressive taxation. But wage bargain induces a second effect: Tax progression changes the distribution of the surplus between the worker and the firm. Which one of these two effects dominates, depends then on the types of tax functions considered and whether the firm and the worker are able to write a binding contract on human capital. Two important results emerge from our research: First, a higher tax progression might increase human capital. This is the case if complete contracts are not available, the firm pays the investment and if the tax functions considered are those with a constant coefficient of residual income progression. The second result states that the optimal marginal tax rate is in many cases higher in a model with endogenous human capital compared to a model with exogenous human capital. We showed that the introduction of endogenous human capital decreses the optimal degree of tax progression only if complete contracts are not feasible and if the worker invests.

However, this is certainly not the end of the story. There are three aspects that seem important to us and that are not treated in this paper. First, tax policy is not only an efficiency issue. Tax policy has first of all the aim to change the income distribution. Since workers are homogeneous, the efficiency-equity trade-off is not treated in this paper. It would be interesting to see how the integration of equity concerns changes the results of this paper.

Second, we used in this paper the rule that the party that has the higher marginal benefit form an investment in human capital is the one that invests. However, the worker can be credit constraint. He may then not be able to invest even if he wants to. Another feature that may change the investment pattern is imperfect information. We assumed that both parties are perfectly informed about the possibilities of investment. Finally, we assumed that the investments of both parties are perfect substitutes. This is certainly too simplistic. The results of this paper can be applied to the case where only one party can invest. However, the case where both can invest, but where the investments are imperfect substitutes, is much more complicated.

Third, and still concerning the rule of investment, it seems clear to us that the decision whether the firm or the worker invests depends on labour market institutions as well. Especially, one could think that labour market rigidities that tend to increase the duration of a worker-firm relationship increase the incentives of the firm to invest in human capital. Together with the results of this paper, this would then imply a complementarity of labour market rigidities and progressive taxation. This could help to explain differences between the US and European taxation and labour market policies.

7 Appendix

7.1 Derivation of the Nash bargain solution with complete contracts

One has to maximise equation (3) with respect to w, H and λ, where λ is restricted to values between 0 and 1. We use the Kuhn-Tucker formulation. The Lagrangian is then written as

$$\pounds(H, w, \lambda, \mu_1, \mu_2) = [w - t(w) - \lambda c(H)]^{\beta}[H - w - (1-\lambda)c(H)]^{1-\beta} + \mu_1\lambda - \mu_2(\lambda - 1)$$

The first-order conditions are

$$\frac{\partial \pounds}{\partial w} = \frac{\beta(1-t'(w))}{S_w} - \frac{(1-\beta)}{S_f} = 0 \tag{17a}$$

$$\frac{\partial \pounds}{\partial H} = -\frac{\beta\lambda c'(H)}{S_w} - \frac{(1-\beta)[(1-\lambda)c'(H)-1]}{S_f} = 0 \tag{17b}$$

$$\frac{\partial \pounds}{\partial \lambda} = -\frac{\beta c(H)}{S_w} + \frac{(1-\beta)c(H)}{S_f} + \frac{\mu_1 - \mu_2}{S_w^{\beta} S_f^{1-\beta}} = 0 \tag{17c}$$

$$-\mu_1\lambda = 0 \tag{17d}$$

$$\mu_2(\lambda - 1) = 0 \tag{17e}$$

with $\mu_1 \geq 0$, $\mu_2 \geq 0$ and the inequality constraints $0 \leq \lambda \leq 1$. S_w and S_f are the parts of the surplus that go to the worker and the firm respectively and are defined as

$$\begin{aligned} S_w &= w - t(w) - \lambda c(H) \\ S_f &= H - w - (1-\lambda)c(H) \end{aligned}$$

From equation (17a), we immediately get equation (5). Equation (17a) can also be rewritten as

$$S_w = \frac{\beta(1-t'(w))}{1-\beta} S_f \tag{18}$$

Introducing this result into (17b) leads to equation (4). Introducing (18) into (17c) gives

$$\frac{(1-\beta)c(H)[1-\frac{1}{1-t'(w)}]}{1-\beta} + \frac{\mu_1 - \mu_2}{S_w^\beta S_f^{1-\beta}} = 0 \tag{19}$$

If $t'(w)$ is positive, the first term of (19) becomes negative. By the non-negativity of μ_1 and μ_2, this implies that μ_1 is positive. Given equation (17d), this implies that the constraint $\lambda = 0$ is binding. Inversely, if $t'(w)$ is negative, the first term of (19) becomes positive, which implies that the constraint $\lambda = 1$ is binding. Finally, if $t'(w)$ equals 0, the first term of (19) is zero, which implies that none of the constraints is binding. In that case, λ is indeterminate and can take any value between 0 and 1.

7.2 Proof of lemma 2

Introducing (5) into equation (6) and imposing the government budget constraint $t^a(w) = 0$ at the equilibrium wage w, one gets the equation

$$[H - c(H)]\frac{1-\beta}{1-\beta t'(w)} = \frac{k}{q(\theta)} \tag{20}$$

This equation does not depend on the distribution of the investment cost. Comparing this result with the optimality condition (2), optimal labour market tightness is achieved if

$$\frac{1-\beta}{1-\beta t'(w)} = 1 - \eta$$

Rearranging this equation gives equation (7). It is then straightforward that the marginal tax rate is positive if $\beta > \eta$, negative if $\beta < \eta$ and equal to 0 if $\beta = \eta$.

To get the result that labour market tightness is too high when the marginal tax rate is higher than the value given in equation (7), we show that labour market tightness increases with the marginal tax rate. To get the marginal effect of the marginal tax rate on labour market tightness, we use the implicit function theorem on equation (20). By defining the function

$$F(\theta, t'(w)) \equiv [H - c(H)]q(\theta)(1-\beta) - k(1-\beta) - k\beta(1-t'(w)) \equiv 0$$

one has the partial derivatives

$$\begin{aligned} \frac{\partial F}{\partial \theta} &= q'(\theta)[H - c(H)](1-\beta) \\ \frac{\partial F}{\partial t'(w)} &= k\beta + q(\theta)(1-\beta)\frac{\partial [H - c(H)]}{\partial t'(w)} \end{aligned}$$

which gives the derivative

$$\frac{\partial \theta}{\partial t'(w)} = -\frac{k\beta + q(\theta)(1-\beta)[1-c'(H)]\frac{\partial H}{\partial t'(w)}}{q'(\theta)[H-c(H)](1-\beta)} \tag{21}$$

If the marginal tax rate is non-negative, lemma 1 tells us that $c'(H) = 1$. The numerator is then positive. If the marginal tax rate is negative, lemma 1 tells us that $c'(H) > 1$ and $\lambda = 1$. Putting $\lambda = 1$ into equation (4) and using the implicit function theorem, one gets

$$\frac{\partial H}{\partial t'(w)} = -\frac{1}{c''(H)}$$

Given the convexity assumption on $c(H)$, this derivative is negative. This proves that with negative marginal tax rates, the numerator of (21) is positive as well. As the denominator is negative, we get the result that an increase in the marginal tax rate leads to higher labour market tightness.

7.3 Proof of proposition 3

The government maximises the net output

$$\theta q(\theta)[H - c(H)] - k\theta$$

by choosing the marginal tax rate, subject to equations (6), (5), (4) and the government's budget constraint. Putting (5) into (6) and solving for $[H - c(H)]$ we can rewrite the government's objective as

$$\theta k \frac{\beta}{1-\beta}(1 - t'(w)) \tag{22}$$

The first-order condition of the maximisation of (22) with respect to $t'(w)$ can be rewritten as

$$\frac{\beta}{1-\beta} k[(1-t'(w))\frac{\partial \theta}{\partial t'(w)} - \theta] = 0$$

where $\frac{\partial \theta}{\partial t'(w)}$ is given by equation (21). The first-order condition can then be rewritten as

$$t'(w) = 1 - \frac{\eta(1-\beta)}{(1-\eta)\beta + \frac{(1-\beta)\frac{\partial(H-c)}{\partial t'(w)}}{\frac{k}{q(\theta)}}} \tag{23}$$

First we check the case where $\beta \geq \eta$. We then prove by contradiction that the optimal marginal tax rate is set at the level given by equation (7). Assume that this is not the case. There are two possibilities, either the tax rate is higher or lower than this value. First, assume that the tax rate is higher. Lemma 1 then indicates that $\frac{\partial(H-c)}{\partial t'(w)} = 0$. But then, equation (23) shows that the government sets the tax rate at the level given by equation (7), which contradicts the assumption that the tax rate is higher than this value. Assume then that the tax rate is lower than this value. Then lemma 1 shows that $\frac{\partial(H-c)}{\partial t'(w)} \geq 0$. But under this condition, equation (23) gives a tax rate that is at least as high as the value given by equation (7). This contradicts the assumption that the tax rate is lower than this value. Therefore, the marginal tax rate is set at the level in equation (7), which implies by lemma 2 that labour market tightness is at its first-best level. As the tax rate is positive, lemma 1 lets us conclude that human capital investment is at its first-best level.

Consider now the case where $\beta < \eta$. We prove by contradiction that the optimal tax rate is set between 0 and the value in equation (7). Assume that the tax rate is higher than or equal to 0. From lemma 1 we then know that $\frac{\partial(H-c)}{\partial t'(w)} = 0$. But then equation (23) indicates that the tax rate is set at the value given in equation (7). As $\beta < \eta$, this value is negative. This contradicts the assumption that the tax rate is non-negative. Finally, assume that the tax rate is equal to or lower than the value given in equation (7). As this value is negative, lemma 1 tells that $\frac{\partial(H-c)}{\partial t'(w)} > 0$. But equation (23) then shows that the government sets the tax rate at a level that is higher than the level given by equation (7). This contradicts the assumption that the tax rate is lower than this value. Therefore, the marginal tax rate has a value between 0 and the value described in equation (7). As this tax rate is negative, lemma 1 tells that there is over-investment in human capital. Finally, with the help of lemma 2 we can conclude that labour market tightness is above its optimal level.

7.4 Optimal tax policy in a model with exogenous human capital endowment

In this model, the human capital H is exogenously given. For simplicity, assume that the cost of this human capital $c(H)$ equals 0. The Nash bargain (3) is then only about the wage rate w. This gives the wage equation

$$w = \frac{\beta(1-t'(w))H}{(1-\beta)(1-t^a(w)) + \beta(1-t'(w))} \tag{24}$$

The vacancy supply becomes

$$H - w = \frac{k}{q(\theta)} \tag{25}$$

The government then maximises the net output

$$\theta q(\theta)H - k\theta$$

by choosing the marginal tax rate, subject to equations (25), (24) and the government's budget constraint. Putting (24) into (25) and solving for H we can rewrite the government's objective as

$$\theta k \frac{\beta}{1-\beta}(1-t'(w))$$

The first-order condition of the maximisation of (22) with respect to $t'(w)$ can be rewritten as

$$\frac{\beta}{1-\beta}k[(1-t'(w))\frac{\partial\theta}{\partial t'(w)} - \theta] = 0$$

where $\frac{\partial\theta}{\partial t'(w)}$ can be derived from equations (25) and (24).The first-order condition can then be rewritten as

$$t'(w) = 1 - \frac{\eta(1-\beta)}{(1-\eta)\beta}$$

which indicates the optimal marginal tax rate in a search equilibrium model with exogenous human capital endowment.

7.5 Derivation of the planned human capital investments with incomplete contracts

The problem of the worker is to maximise

$$w - t(w) - I_w$$

with respect to $I_w \geq 0$, taking I_f as given. As the cost function $c(\cdot)$ is convex, its inverse function $c^{-1}(\cdot)$ exists. One can then define human capital H as

$$H = c^{-1}(I_f + I_w)$$

Using this specification in the maximisation problem, one gets the first order-condition

$$(1 - t'(w))\frac{\partial w}{\partial H} - c'(H) + \mu_w = 0$$

where μ_w is the Lagrange multiplier on the non-negativity constraint on human capital investment of the worker.

Similarly, the firm's problem is to maximise

$$H - w - I_f$$

with respect to $I_f \geq 0$, taking I_w as given. This leads to the first order-condition

$$(1 - \frac{\partial w}{\partial H}) - c'(H) + \mu_f = 0$$

where μ_f is the Lagrange multiplier on the non-negativity constraint on human capital investment of the firm.

Using the implicit function theorem on equation (9) gives

$$\frac{\partial w}{\partial H} = \frac{\beta}{1 + \beta\frac{t''(w)}{1-t(w)}(H - w)}$$

Putting these equations together, one gets immediately equations (10) and (11).

7.6 Nash solution to the non-cooperative game in human capital investments

From equation (11) one implicitely gets the firm's investment in human capital as a reaction to the worker's investment. As long as the non-negativity constraint on the investment is not binding, every change in the worker's investment is completly compensated by the investment of the firm, i.e. $\frac{\partial I_f}{\partial I_w} = -1$. This means that the firm never lets the human capital fall under a level H_f, which is the value that solves equation (11) without binding non-negativity constraint. If the non-negativity constraint is binding, the investment of the firm is set at 0.

The worker acts similarly. From equation (10) one implicitely gets the worker's investment in human capital as a reaction to the firm's investment. As long as the non-negativity constraint on the investment

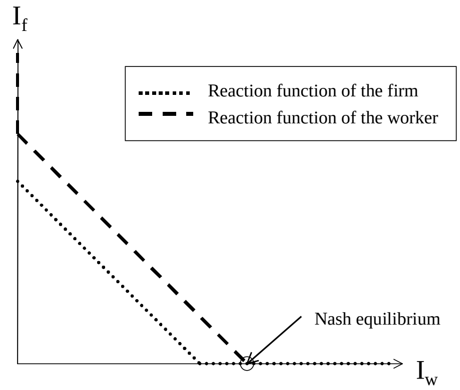

Figure 1: Reaction functions

is not binding, every change in the firm's investment is completly compensated by the investment of the worker, i.e. $\frac{\partial I_w}{\partial I_f} = -1$. This means that the worker never lets the human capital fall under a level H_w, which is the value that solves equation (10) without binding non-negativity constraint. If the non-negativity constraint is binding, the investment of the worker is set at 0.

Figure 1 shows the two reaction functions in the $I_w - I_f$ space for the case where $H_f < H_w$. As it can easily be seen, the only Nash equilibrium is at the point where the investment of the firm equals 0 and the investment paid by the worker equals $c(H_w)$. A similar reasoning for the case where $H_f > H_w$ indicates that in that case the investment of the worker is set at 0, whereas the firm pays an investment of $c(H_f)$. This shows that the party that desires the higher human capital pays it entirely, whereas the other party doesn't paricipate to pay the investment.

Finally, if $H_f = H_w$ (which in general, is not the case[13]), any division of the cost is a Nash equilibrium. Human capital is set at $H_f = H_w$.

[13] This is only the case if the marginal benefit of human capital of the firm and the worker are the same, which is only the case if

$$(1 - t'(w)) \frac{\beta}{1 + \beta \frac{t''(w)}{1-t(w)}(H - w)} = 1 - \frac{\beta}{1 + \beta \frac{t''(w)}{1-t(w)}(H - w)}$$

As this condition is in general not satisfied, we do not treat this case in our analysis.

7.7 Proof of lemma 5

If the tax function is linear, equations (10) and (11) which determine the level of human capital simplify to

$$(1-t')\beta = c'(H) \tag{26}$$

if the worker invests and

$$1-\beta = c'(H) \tag{27}$$

if the firm invests. From (27) it is straightforward that the marginal tax rate does not affect human capital. Hence, if the firm invests, the first-best is only achieved if $\beta = 0$. The implicit derivation of (26) leads to

$$\frac{\partial H}{\partial t'} = -\frac{\beta}{c''(H)} \tag{28}$$

The convexity assumption on $c(H)$ then implies that human capital decreases as the marginal tax rate increases. Assuming that the budget constraint of the government is satisfied, equation (26) tells that human capital is at its optimal level if

$$t' = 1 - \frac{1}{\beta} \leq 0$$

If the tax rate is higher than this value, human capital is sub-optimal, if it is lower, there is over-investment in human capital.

7.8 Proof of lemma 6

If the tax function has a constant coefficient of residual income progression ν, the wage equation (9) can be rewritten as

$$w = \frac{\beta\nu H}{1-\beta+\beta\nu}$$

The derivative of the wage with respect to human capital then becomes

$$\frac{\partial w}{\partial H} = \frac{\beta\nu}{1-\beta+\beta\nu}$$

Equations (10) and (11) which determine the level of human capital simplify to

$$(1-t')\frac{\beta\nu}{1-\beta+\beta\nu} = c'(H)$$

if the worker invests and

$$1 - \frac{\beta\nu}{1-\beta+\beta\nu} = c'(H)$$

if the firm invests. It is easy to see that if the firm invests, the first best level of human capital is only achieved if the firm has all the bargaining power. Otherwise, there is under-investment in human capital. If the worker invests and assuming that the budget constraint of the governement is satisfied, one gets the first-best level of human capital if

$$t' = \frac{1}{2} - \frac{\sqrt{-3 + \frac{4}{\beta}}}{2} < 0$$

If the tax rate is higher than this level, the human capital is sub-optimal. If the tax rate is lower, there is over-investment. The implicit derivation of these two equations then gives the effect of an increase in the marginal tax rate on human capital (under the government's budget constraint $t^a = 0$), which equals

$$\frac{\partial H}{\partial t'(w)} = -\frac{\beta(1 - t'(w)^2)}{c''(H)[1 - \beta t'(w)]} \tag{29}$$

if the worker invests. The convexity assumption on $c(H)$ implies that this term is negative. If the firm invests, the derivative becomes

$$\frac{\partial H}{\partial t'(w)} = \frac{\beta c'(H)}{c''(H)[1 - \beta t'(w)]} \tag{30}$$

which is positive.

7.9 Proof of lemma 7

If the worker invests, combining the vacancy supply curve (1), the wage equation (9) and the government's budget constraint $t^a(w) = 0$, leads to the following equation:

$$H \frac{1 - \beta}{1 - \beta t'(w)} = \frac{k}{q(\theta)} \tag{31}$$

The first-best level of labour market tightness is attained if equation (2) is satisfied. Comparing these two equations, it is easy to check that they are both satisfied if and only if equation (16) is satisfied. Similarly, if the firm invests, combining the vacancy supply curve (1), the wage equation (9) and the government's budget constraint $t^a(w) = 0$ leads to the following equation:

$$H \frac{1 - \beta}{1 - \beta t'(w)} - c(H) = \frac{k}{q(\theta)} \tag{32}$$

Comparing this equation to equation (2), one easily gets the result that labour market tightness is at its optimal level if and only if equation (15) is satisfied.

The implicit derivation of equation (31) gives

$$\frac{\partial \theta}{\partial t'(w)} = -\frac{\beta}{1 - \beta} \frac{q(\theta)}{q'(\theta)} \frac{\frac{k}{q(\theta)} + \frac{\partial H}{\partial t'(w)} \frac{1-\beta}{\beta}}{\frac{k}{q(\theta)}} \tag{33}$$

From lemmas 5 and 6, we know that $\frac{\partial H}{\partial t'(w)}$ is negative if the worker invests. The sign of equation (33) is positive if

$$\frac{k}{q(\theta)} > -\frac{\partial H}{\partial t'(w)}\frac{1-\beta}{\beta}$$

and negative if the inequality is inverted.

The implicit derivation of equation (32) gives

$$\frac{\partial\theta}{\partial t'(w)} = -\frac{\beta}{1-\beta}\frac{q(\theta)}{q'(\theta)}\frac{c(H) + \frac{k}{q(\theta)} + \frac{\partial H}{\partial t'(w)}\frac{1-\beta}{\beta}[1-c'(H)]}{\frac{k}{q(\theta)}} \tag{34}$$

From lemmas 5 and 6, we know that $\frac{\partial H}{\partial t'(w)}$ is non-negative if the firm invests. Equation (34) is then always positive.

7.10 Proof of propositions 8 and 9

The government maximises net output

$$\theta q(\theta)[H - c(H)] - k\theta$$

by choosing the marginal tax rate, subject to equations (1), (9), (10), (11) and the government's budget constraint. The first-order condition is then

$$\frac{\partial\theta}{\partial t'(w)}\frac{1}{\theta}[(1-\eta)(H-c(H)) - \frac{k}{q(\theta)}] + [1-c'(H)]\frac{\partial H}{\partial t'(w)} = 0 \tag{35}$$

Next, let us define

$$x = \frac{\theta(1-c'(H))\frac{\partial H}{\partial t'(w)}}{\frac{\partial\theta}{\partial t'(w)}} \tag{36}$$

If the firm invests, equation (35) can be rewritten by using equations (1) and (9) as

$$t'(w) = 1 - \frac{\eta}{1-\eta}\frac{1-\beta}{\beta}\frac{H - c(H) - \frac{x}{\eta}}{H - c(H) + \frac{c(H)}{1-\eta} + \frac{x}{1-\eta}} \tag{37}$$

As it can easily be seen, this value is bigger than the value given by equation (15) if and only if $x > 0$. Lemma 5 indicates that in the case of a linear tax, $x = 0$. Therefore, equation (37) simplifies to equation (15) which indicates that the optimal tax rate is the one that solves for optimal labour market tightness.

If the tax function has a constant CRIP, lemmas 6 and 7 imply that $x > 0$. Then, the optimal tax rate is higher than the one that solves for optimal labour market tightness.

When the worker invests, we do not get a clear result. In fact, using equations (1) and (9), equation (35) can be rewritten as

$$t'(w) = 1 - \frac{\eta}{1-\eta}\frac{1-\beta}{\beta}\frac{H - c(H) + \frac{c(H)}{\eta} - \frac{x}{\eta}}{H - c(H) + \frac{x}{1-\eta}} \tag{38}$$

But lemma 7 indicates that we cannot determine the sign of x. Therefore, we cannot conclude whether the optimal marginal tax rate is higher or lower than the value that solves for optimal labour market tightness. This holds true for a linear tax as well as for a tax function with a constant CRIP.

7.11 Optimisation with incomplete contracts if the government can freely choose all tax parameters

If the only constraint the government faces when it chooses its tax function is the budget constraint, it has two instruments that it can use and that influence the outcomes: the marginal tax rate, and the second derivative of the tax function with respect to the wage. As there are only two variables that caracterise the optimal labour market outcome (labour market tightness and the investment in human capital), it is not excluded that the government can achieve the first-best outcomes by choosing the optimal tax function. Since the second derivative of the tax function only intervenes in the equations that determine the level of human capital, the government uses this second derivative to set human capital at its optimal level. The marginal tax rate is then chosen to set the labour market tightness at its optimal level.

If the worker invests, we find from equation (10) that human capital is at its optimal level if the second derivative satisfies the following constraint:

$$t''(w) = (1 - t'(w))\frac{1 - \beta t'(w)}{\beta(1-\beta)}[(1 - t'(w))\beta - 1]\frac{1}{H}$$

where $H = c'^{-1}(1)$. The marginal tax rate is then the tax rate that solves equation (16) and sets labour market tightness at its optimal level.

If the firm invests, equation (11) shows that human capital gets to its optimal level as the second derivative of the tax function goes to $+\infty$. The marginal tax rate is then the tax rate that solves equation (15) and sets labour market tightness at its optimal level.

This shows the surprising - and at first sight counterintuitive - result that when the government can freely choose all tax parameters, it is better to have incomplete contracts than complete contracts. This is due to the fact that with incomplete contracts, the government gets a second policy instrument: the second derivative of the tax function. When complete contracts are available, this second derivative has no impact on the outcome. On the contrary, when complete contracts are not enforceable, the double maximisation implies that this second derivative can play a role. The government then has two instruments to adjust two endogenous variables - labour market tightness and human capital. This implies that the first-best outcome can always be achieved when complete contracts are not available. On the contrary, if complete contracts exist, the government has an insufficient number of instruments to adjust the outcome to its first-best level. The availability of complete contracts therefore worsens the outcome.

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