Two Alternative Approaches to Modelling the Nonlinear Dynamics of the Composite Economic Indicator

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Abstract

The analysis and prediction of the short-run economic dynamics, or the evolution of the business cycle, often require a construction of the composite economic indicator (CEI). This indicator may be endowed with nonlinear dynamics to take care of the possible asymmetries between different phases of the business cycle. This paper suggests using the smooth transition autoregression to model the CEI. The performance of this model is compared to the already classical CEI with regime switching. Both models turn out to produce statistically equally good results in terms of forecasting the business cycle turning points.

Keywords: composite economic indicator, Markov switching, smooth transition autoregression, turning points, NBER dating, forecasting

JEL codes: C4, C5, E3

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1 Introduction

A great deal of the economic and political decision making depends on the forecasts of the state of affairs in the economy. One of the proxies capturing the current business conditions is used to be the so-called composite economic indicator (CEI) estimated using dynamic factor analysis.

The CEI can be constructed assuming that it follows either linear or nonlinear dynamics. Applying the nonlinear models we are able, firstly, to incorporate the business cycle asymmetries, if any, and, secondly, to come up with the endogenous chronologies of the business cycle turns. Taking advantage of these techniques we can predict the turning points, which is impossible when one uses the so-called *ad hoc* methods like the extremely popular Bry-Boschan method[1].

Up to date there was only one nonlinear common factor model considered — the CEI with Markov switching (CF-MS). In this paper we suggest the use of another nonlinear model — CEI with smooth transition autoregressive dynamics (CF-STAR). It might be useful in the cases where CF-MS does not work properly or it might serve as an alternative to the Markovian model when both STAR and regime-switching dynamics are equally probable. Moreover, in some cases the mix of two nonlinear models can possibly allow improved forecasts compared to the predictions of each model made separately.

In the next section we briefly discuss the setup of the two nonlinear models. In section three the two alternative models are estimated and their forecasting performance is evaluated using the Post World War II US macroeconomic series. Concluding remarks section summarizes the main findings of the paper. All the tables are contained in the Appendix.

2 Models

The idea behind the two models examined in this section is that the evolution of the business conditions can be classified into a limited set of the alternating regimes. In the simplest case one distinguishes between two regimes, or states, namely: expansion and recession. One may assume that the switches in the regimes are due to some unobserved state variable.

The economy behaves differently under the different regimes. This is translated into the state-dependent differences between some of the model's parameters. Thus, for example, the growth rates and the volatility may be different under the expansions and recessions.

The models considered below, although stemming from the same premise of the existence of a latent state variable, use different mecha-

[1] For details see Bry and Boschan (1971).

nisms of representing it.

2.1 Dynamic common factor with regime switching

The model of a single dynamic common factor with Markov switching (CF-MS) thanks to the works of C.J.Kim (see, for example, Kim and Nelson (1999)) has almost become classical. Formally in its general form (all common factor's parameters are state-dependent) CF-MS is defined as follows:

$$\Delta y_t = \gamma_i(L)\Delta C_t + u_t \tag{1}$$

$$\Delta C_t = \mu_1^{MS} s_t + \mu_2^{MS}(1-s_t) + \sum_{i=1}^{p} \left[\phi_{1i}^{MS} s_t + \phi_{2i}^{MS}(1-s_t)\right] \Delta C_{t-i} + \varepsilon_t \tag{2}$$

$$\Psi(L)u_t = \eta_t \tag{3}$$

where y_t is the $n \times 1$ vector of the observable time series; C_t is the dynamic common factor in levels; u_t is the $n \times 1$ vector of the idiosyncratic components; s_t is the regime variable taking m values, where m is the number of the regimes; μ_j^{MS} and ϕ_{ji}^{MS} ($j = 1, 2$ and $i = 1, 2, ..., p$) are the common factor's state-dependent intercepts and autoregressive coefficients, respectively. Thus, for $m = 2$, $s_t = 1, 0$. Given that $\mu_1^{MS} > \mu_2^{MS}$, regimes 1 and 2 may be interpreted as an ascending trend and a descending trend states, respectively. In this model the intercept term, μ_i^{MS}, and the residual variance of the common factor, $\sigma_\varepsilon^2(s_t)$, are made state-dependent, that is, they are different for the different regimes, or cyclical phases.

The shocks to the common and specific factors are assumed to be serially and mutually uncorrelated and to be normally distributed:

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim NIID\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{1\varepsilon}^2 s_t + \sigma_{2\varepsilon}^2(1-s_t) & O \\ O & \Sigma_\eta \end{pmatrix}\right)$$

where $\sigma_{j\varepsilon}^2$ ($j = 1, 2$) is the common factor's state-dependent residual variance.

The lag polynomial matrices of the specific factors, Ψ_j ($j = 1, ..., q$), are supposed to be diagonal.

The transition probabilities, $p_{ij} = \Pr(s_t = j | s_{t-1} = i)$, sum up to one when added across all the possible state for the given regime in the previous period: $\sum_{j=1}^{m} p_{ij} = 1 \; \forall i$ for m states.

2.2 Dynamic common factor with smooth transition autoregression

The novelty of this paper is the application of STAR to the unobserved common factor model. The technique itself as applied to the observed univariate time series was developed by Chan and Tong (1986) as well as by Teräsvirta and his coauthors (e.g. Granger and Teräsvirta (1993)).

The common factor model with smooth transition autoregression (CF-STAR) is apparently very similar to its counterpart with regime switching. However, there is a crucial difference between the two approaches: while in CF-MS the state variable determining shifts from one regime to another is unobserved, in CF-STAR the switches between regimes are conditioned upon the past values of the composite indicator itself or upon those of some *observed* regressor. In the present case the situation is complicated by the fact that we do not observe the CEI itself. Hence we should condition the changes in regimes on its past *estimated* values.

The only difference between the two models is the equation describing evolution of the common dynamic factor:

$$\Delta C_t = \mu_1^{STAR} F_t + \mu_2^{STAR}(1 - F_t) + \sum_{i=1}^{p} \left[\phi_{1i}^{STAR} F_t + \phi_{2i}^{STAR}(1 - F_t) \right] \Delta C_{t-i} + \varepsilon_t \tag{4}$$

where $\mu_1^{STAR} > \mu_2^{STAR}$ are the state-dependent intercepts; ϕ_{ji}^{STAR} ($j = 1, 2$ and $i = 1, 2, ..., p$) are the state-dependent autoregressive coefficients; $F_t \equiv F_t(\Delta C_{t-d}; \lambda, r)$ is some smooth transition function. In the present study we are using two specifications of the transition function. Firstly, it is a logistic specification which allows capturing the asymmetries between the business cycle phases:

$$F_t(\Delta C_{t-d}; \lambda, r) = \frac{1}{1 + \exp(-\lambda(\Delta C_{t-d} - r))} \tag{5}$$

where $\lambda > 0$ is the parameter determining the abruptness of transition (the greater is its value the sharper are the switches between the regimes); ΔC_{t-d} is playing the role of the so-called transition variable; $d > 0$ is called the transition delay; r is the transition threshold. Basically, the shifts between the two different regimes (say, high growth and low growth, as in the CF-MS) depend on deviation between the past CEI's growth rate and some threshold, r. If, for instance, the past common factor's growth rate exceeded the threshold, the high growth regime becomes more probable.

Secondly, the exponential specification of the transition function is utilized:

$$F_t(\Delta C_{t-d}; \lambda, r) = 1 - \exp(-\lambda(\Delta C_{t-d} - r)^2) \tag{6}$$

Thus, the CF-STAR model where the common factor dynamics is governed by the equations (4) and (5) will be denoted as CF-LSTAR, while the model where these dynamics are based on the equations (4) and (6) will be denoted as CF-ESTAR.

Again as in the CF-MS case, the residual variance of the common factor can be state-dependent too:

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim NIID\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{\varepsilon 1} F_t + \sigma^2_{\varepsilon 2}(1 - F_t) & O \\ O & \Sigma_\eta \end{pmatrix} \right)$$

where $\sigma^2_{\varepsilon j}$ $(j = 1, 2)$ is the state-dependent residual variance of the common factor.

3 Estimation and evaluation

3.1 Estimation

The composite economic indicators were estimated using four US monthly macroeconomic time series covering the period of 1959:1-1998:12: employees on nonagricultural payrolls (EMP); personal income less transfer payments (INC); index of industrial production (IIP); and manufacturing and trade series (SLS).

As a benchmark the linear CF model was used. We started with determining the optimal lag structure of this benchmark model. By this we mean the order of the autoregressive polynomials of the common and specific factors. The Akaike (AIC) and Schwartz (SBIC) information criteria were applied. The log-likelihood values of the linear CF with different orders of autoregressive polynomials of the common and specific factors together with the corresponding Akaike and Schwartz quantities are presented in Table 1. The AIC and SBIC come up with optimal combinations (1,3) and (1,2), respectively. We chose the combination (1,2) as more parsimonious. It corresponds to the common factor following AR(1) and the specific factors following AR(2).

Next, we have tested the common factor dynamics for linearity. The alternative was the logistic STAR dynamics. The LM-type tests based on the first- and third-order Taylor expansion of the logistic STAR transition function around $\lambda = 0$ were conducted as in van Dijk et al. (2000). For these test the estimated values of the common factor, obtained from the linear CF(1,2) model, were used.

The first-order Taylor expansion of the logistic transition function results in:

$$\Delta\hat{C}_t = \mu_1 + \sum_{i=1}^{p} \phi_{1i}\Delta\hat{C}_{t-i} + \sum_{i=1}^{p} \phi_{2i}\Delta\hat{C}_{t-i}\Delta\hat{C}_{t-d} \tag{7}$$

where $\Delta\hat{C}_t$ is the linear estimate of the growth rate of the common factor.

The null hypothesis (linear CF) is $\phi_{21} = \phi_{22} = ... = \phi_{2p} = 0$. This hypothesis can be tested with F-statistic denoted here as LM_1. In the case when only the intercept is different across different regimes[2] this statistic will not have power and therefore the third-order Taylor approximation is utilized:

$$\Delta\hat{C}_t = \mu_1 + \sum_{i=1}^{p} \phi_{1i}\Delta\hat{C}_{t-i} + \sum_{i=1}^{p} \phi_{2i}\Delta\hat{C}_{t-i}\Delta\hat{C}_{t-d} + \sum_{i=1}^{p} \phi_{3i}\Delta\hat{C}_{t-i}\Delta\hat{C}^2_{t-d} + \sum_{i=1}^{p} \phi_{4i}\Delta\hat{C}_{t-i}\Delta\hat{C}^3_{t-d} \tag{8}$$

Under this condition the null hypothesis is as follows: $\phi_{i1} = \phi_{i2} = ... = \phi_{ip} = 0$ $(i = 2, 3, 4)$. It is denoted as LM_3.

The results of the tests are reported in Table 2. The null of linearity is rejected at 5% significance level for the delays $d = 1$ and $d = 2$. In other words, the STAR nonlinearity can be accepted when the transition variable is ΔC_{t-1} and/or ΔC_{t-2}. This circumstance was used to specify the CF-STAR model. However, since the regime probabilities derived from the CF-STAR model with $d = 2$ were too bad predictors of the NBER dates, we do not present the estimates of this model here.

The CF-MS model is specified as (1,1), because the regime probabilities obtained under CF-MS(1,2) replicate the NBER business cycle chronology much bad. Only common factor's intercept is taken to be state-dependent. Both CF-LSTAR and CF-ESTAR are specified as (1,2) following the optimal lag-structure test conducted for the linear CF model above. The common factor's intercept, autoregressive coefficient, and residual variance are assumed to be state-dependent.

In the case of both CF-STAR models the exponent of the transition function, $F_t(\Delta C_{t-d}; \lambda, r)$, was standardized by division by the common factor's residual variance, σ^2_ε, to make the abruptness parameter λ scale-free and easier to interpret, as suggested by Skalin and Teräsvirta (1999).

Standardized logistic transition function:

$$F_t(\Delta C_{t-d}; \lambda, r) = \frac{1}{1 + \exp(-\lambda(\Delta C_{t-d} - r)/\sigma_\varepsilon)} \tag{9}$$

Standardized exponential transition function:

[2] See van Dijk et al. (2000).

$$F_t(\Delta C_{t-d}; \lambda, r) = 1 - \exp(-\lambda(\Delta C_{t-d} - r)^2/\sigma_\varepsilon^2) \tag{10}$$

Both models were estimated using the method of maximum likelihood. For more details on the maximum likelihood estimation of the CF-MS model see Kim and Nelson (1999). The procedure is easily extended to the case of CF-STAR model. The parameter estimates of the linear CF, both CF-STAR models, and CF-MS (together with their standard errors, t-statistics, and p-values) for these nonlinear models are presented in Tables 3 through 6, correspondingly.

Figure 1 compares the two nonlinear models, on the one hand, with the linear CF model, on the other hand, in terms of the behavior of the common factor in levels. It is constructed as a partial sum of the common factor's growth rates, ΔC_t, being one of the outputs of the CF-model estimation. The profiles of the composite economic indicators constructed using the CF-MS and CF-STAR are pretty similar to that of the CEI estimated using the linear model. Apart from the differences in the levels which are easily explained by the nonstationary nature of C_t, given the way it is constructed, the upward and downward movements of the linear indicator are readily replicated by those of the nonlinear models.

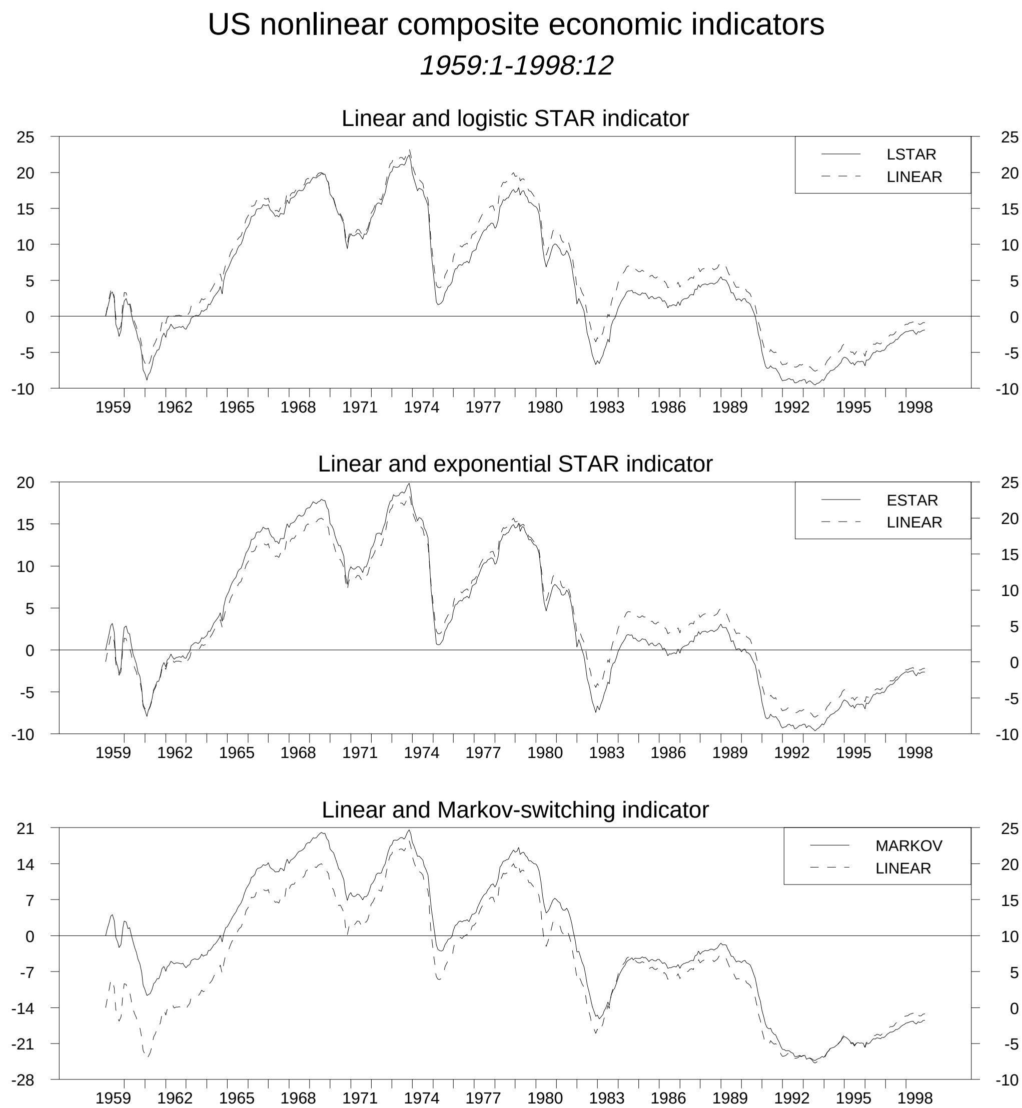

Figure 1. Linear and nonlinear estimates of the common factor

3.2 Evaluation

The forecasting ability of each of the models in question cannot be examined directly, since CEI is unobserved and hence we cannot test which of the models replicates it better. Therefore the performance of the two nonlinear models is evaluated from the viewpoint of capturing and forecasting the turning points of the business cycle. These turns are thought to be captured by the conditional low growth regime probabilities derived from each model. In the case of CF-STAR these probabilities are computed as $1 - F_t(\Delta C_{t-d}; \lambda, r)$, while in the case of CF-MS these are the conditional filtered and smoothed probabilities.

The informal judgement about the "goodness-of-fit" of these models can be made from the visual inspection of Figures 2a-2b displaying the growing trend regime probabilities derived from the CF-STAR and CF-MS, on the one hand, and the US business cycle dating provided by the NBER, on the other hand. The shaded areas correspond to the NBER's recessions, that is, intervals between a peak and a trough. In the case of CF-MS model we dispose of the filtered and smoothed regime probabilities. The CF-STAR regime probabilities and the CF-MS filtered regime probabilities are the most volatile. Anyway, all the regime probabilities seem to sufficiently accurately recognize the NBER dates.

Figure 2a displays the negative growth regime probabilities derived from the CF-LSTAR and CF-ESTAR.

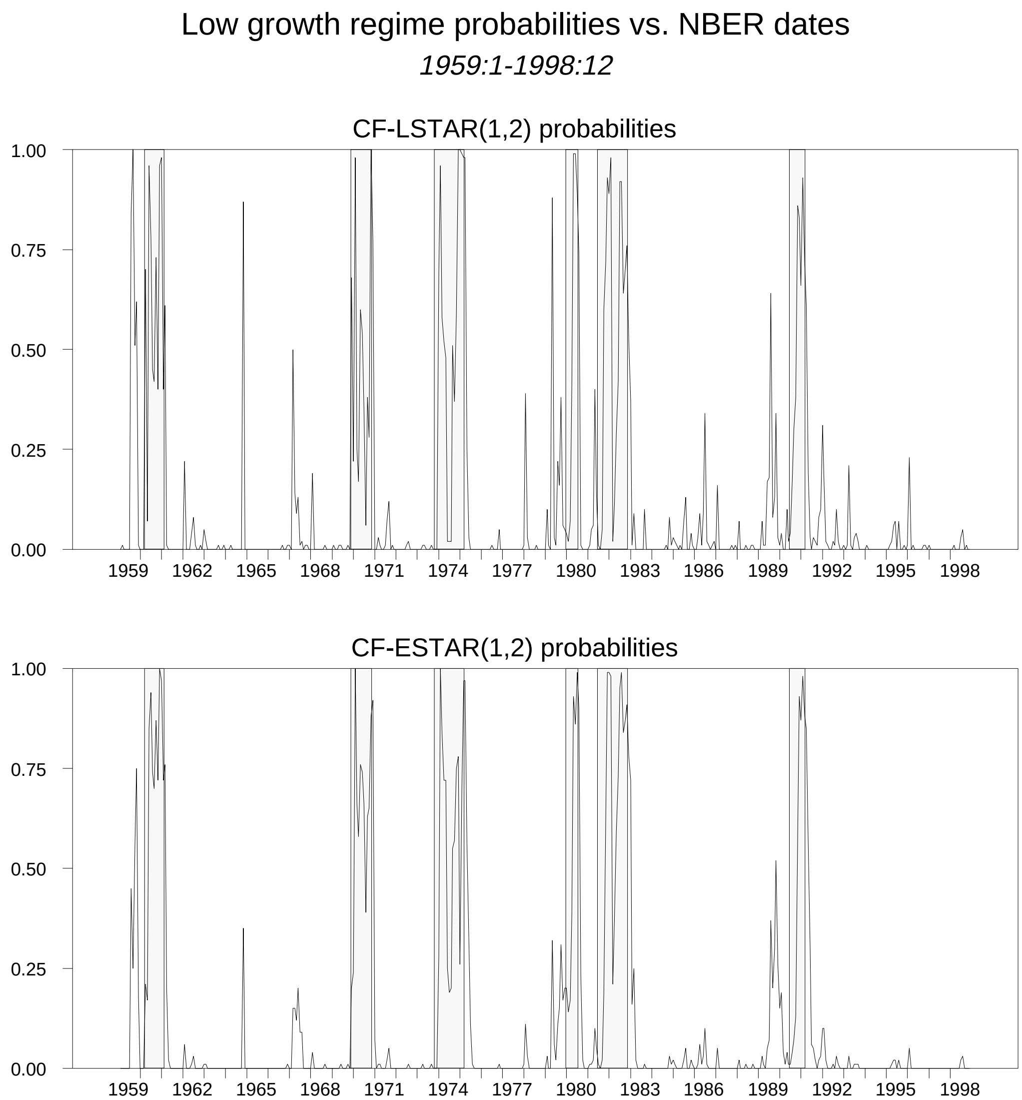

Figure 2a. Estimated low growth regime probabilities of the CF-STAR models

The CF-ESTAR model appears to produce less false alarms than CF-LSTAR. Overall, the CF-ESTAR derived low regime probabilities are much less volatile than those of the logistic model. CF-LSTAR correctly detects six true recessions and signals four false recessions, while CF-ESTAR comes up with six true and two false contractions.

The low growth regime (filtered and smoothed) probabilities corresponding to the CF-MS are graphed on Figure 2b:

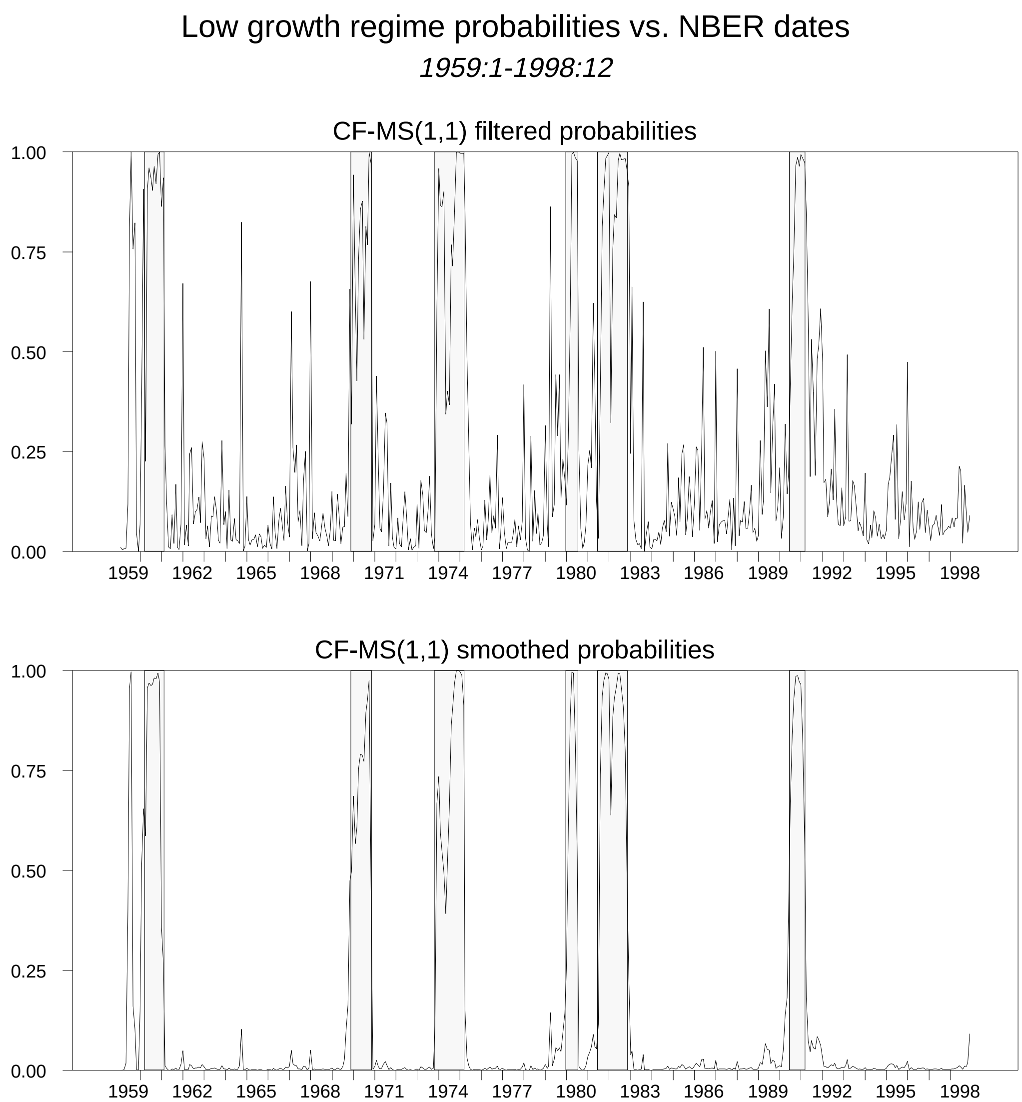

Figure 2b. Estimated low growth probabilities of the CF-MS model

The formal analysis of both in-sample and out-of-sample performance of CF-STAR and CF-MS was undertaken using the quadratic probability score (QPS) suggested by Diebold and Rudebusch (1989). This method compares the recession probabilities derived from some model to a generally accepted business cycle dating. In the case of the US economy one normally takes advantage of the NBER's dates as such "official dating".

The QPS is defined as (see Layton and Katsuura (2001, p.408)):

$$QPS = \frac{1}{T}\sum_{t=1}^{T}(P_t - D_t)^2 \tag{11}$$

where T is the number of observations; P_t is the model-derived probability for the t-th observation; D_t is the binary variable taking value of 1 during the NBER recessions and 0 during the NBER expansions. QPS is limited within the interval [0,1]. The smaller is QPS the better is the correspondence between the model-derived probabilities and "official" business cycle chronology.

To test whether the differences in the QPS of different models are statistically significant we use the Diebold-Mariano statistic (with lag window 5) proposed by Diebold and Mariano (1994).

For the in-sample evaluation we used the conditional recession probabilities — filtered probabilities Pr(low growth regime in period $t|I_t$) and smoothed probabilities Pr(low growth regime in period $t|I_T$) in CF-MS[3], or Pr(low growth regime in period $t|\Delta C_{t-1}$) in CF-STAR — estimated using the whole sample.

The results of the comparison of in-sample forecasting performance of both nonlinear models are presented in Table 6. The second column of the table displays the QPS statistic, while the third and fourth columns report the Diebold-Mariano (DM) statistic and its p-value. The DM-statistic is computed by comparing the filtered and smoothed regime probabilities of CF-MS to the regime probabilities of CF-STAR.

The results of the comparison of in-sample forecasting performance of the three nonlinear models are presented in Table 7. The second column displays the QPS statistic, while the third and fourth columns report the Diebold-Mariano (DM) statistic and its p-value. The DM-statistic is computed by comparing the loss differentials (with respect to the binary coded NBER dating) of the regime probabilities of CF-ESTAR as well as of the filtered and smoothed regime probabilities of CF-MS to the loss differentials of the regime probabilities of CF-LSTAR.

The ranking of different forecasting models according to their point estimated of QPS would be as follows: the smoothed conditional probabilities of CF-MS are characterized by the smallest QPS, then filtered probabilities of CF-MS and CF-ESTAR follow, and finally in the end of the list we find CF-LSTAR. However, when the confidence intervals

[3] $I_t = \{\Delta C_t, \Delta C_{t-1}, ..., \Delta C_1\}$ is the information set consisting of the whole history of the CEI up to the period t.

are taken into account, it turns out that the in-sample performance of the filtered low growth regime probabilities derived from CF-MS is statistically as good as that of the regime probabilities derived from CF-LSTAR. CF-ESTAR in-sample prediction results to be better than that of CF-LSTAR at 10% significance level. This is not the case when we compare the CF-LSTAR conditional probabilities and the filtered CF-MS conditional probabilities. This apparently paradoxical outcome may be due to the high volatility of the latter. The smoothed CF-MS probabilities greatly outperform both the CF-MS filtered probabilities and the CF-LSTAR and CF-ESTAR derived probabilities and this difference is significant at 1% level.

To compare the out-of-sample forecasting accuracy of the three models examined in this paper, the predictions with forecasting horizons ranging from 1 month to 6 months were made. The forecasting period was chosen to be 1980:1-1984:12 since it is characterized by the highest cyclical activity — there are two recessions over this relatively short period. First, each model was estimated for the subsample 1959:1-1979:7 and the 1-, 2-, ..., 6-month ahead forecasts were made. Next, the estimation subsample was augmented by one month and the whole forecasting procedure was repeated until 1984:11 was reached.

The regime probabilities of the CF-MS model were predicted using the forecasting formula from Hamilton (1994, p. 694). The CF-STAR regime probabilities were computed using the following two-step procedure:

$$\hat{F}_{T+1} \equiv F_{T+1}(\Delta \hat{C}_T; \hat{\lambda}, \hat{r}) = \frac{1}{1+\exp(-\hat{\lambda}(\Delta \hat{C}_T - \hat{r}))} \tag{12}$$

$$\Delta \hat{C}_{T+1} = \hat{\mu}_1^{STAR} \hat{F}_{T+1} + \hat{\mu}_2^{STAR}(1-\hat{F}_{T+1}) + \sum_{i=1}^{p} \left[\hat{\phi}_{1i}^{STAR} \hat{F}_t + \hat{\phi}_{2i}^{STAR}(1-\hat{F}_t) \right] \Delta \hat{C}_T \tag{13}$$

where the parameters and variables with hats are those estimated for the period from *1* to *T*. Based on these data the forecasts are made for the period covering h following months, that is, *T+h,* where *h* is the forecasting horizon.

In addition to the "standard" DM-test of the differences in forecasting accuracy, the modified DM-test suggested by Harvey, Leybourne, and Newbold (1997) was applied. This test is especially designed to compare the out-of-sample prediction records. As its authors claim, it is less over-sized than the standard DM-test which tends to over-reject the null hypothesis of no difference in forecasting accuracy of two models being compared. The modified DM-test (DM*) is related to the standard one (DM) in a following way:

$$DM^* = DM\left(\frac{n+1-2h+n^{-1}h(h-1)}{n}\right)^{1/2} \quad (14)$$

where n is the sample size; h is the forecasting horizon. Harvey et al. (1997) report that the best results are obtained when the critical values of the Student's t rather than standard normal distribution are employed. Here we follow their recommendation when computing the p-values of modified DM-test.

The results of testing the out-of-sample forecasting accuracy are reported in Table 8. The second column contains the point estimates of the QPS. In the columns 3 to 4 the DM-statistic and its p-value are presented, while the modified DM-statistic with its p-value can be found in columns 5 to 6. As a benchmark we use CF-MS forecast probabilities to which the other two models are compared. Arithmetically CF-ESTAR dominates both CF-MS and CF-LSTAR over all forecasting horizons. However, this dominance is only significant up to 3-month ahead forecast. Among CF-MS and CF-LSTAR there seems to be no statistically significant difference at any forecasting horizon.

4 Concluding remarks

In this paper we have considered three alternative nonlinear single-factor models of the composite economic indicator: a model with Markov switching and its two counterparts with smooth transition autoregression: CF-LSTAR and CF-ESTAR. For the first time in the literature the composite economic indicator with STAR dynamics is introduced.

The empirical analysis of these three models was conducted based on the Post World War II US monthly macroeconomic series. Both in-sample and out-of-sample turning points forecasting abilities of the models were compared using the quadratic probability score test: the model-derived datings were contrasted to the NBER's business cycle chronology. In the in-sample forecasting it is the CF-MS smoothed regime probabilities which replicate best the NBER recessions. When the out-of-sample forecasting accuracy is concerned, it is the CF-ESTAR who performs the best at 1-, 2-, and 3-month ahead forecast. At higher forecasting horizons all the models produce statistically equivalent results.

Moreover, both CF-ESTAR and CF-LSTAR for the moment appear to be computationally less expensive than the common factor model with regime switching. Hence it can be concluded that CF with smooth transition autoregressive dynamics, especially CF-ESTAR, is a reasonable alternative to the CF-MS model.

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5 Appendix

Table 1. Optimal lag structure of the linear common factor model

Comb	LogLik	AIC	SBIC
(0,0)	-2409.41	-4818.82	-4818.82
(0,1)	-2376.59	-4761.18	-4777.87
(0,2)	-2331.01	-4678.02	-4711.39
(0,3)	-2320.08	-4664.16	-4714.22
(1,0)	-2335.86	-4673.72	-4677.89
(1,1)	-2312.91	-4635.82	-4656.68
(1,2)	-2275.04	-4568.08	**-4605.63**
(1,3)	-2264.05	**-4554.1**	-4608.33
(2,0)	-2331.09	-4666.18	-4674.52
(2,1)	-2309.91	-4631.82	-4656.85
(2,2)	-2274.08	-4568.16	-4609.88
(2,3)	-2263.54	-4555.08	-4613.48
(3,0)	-2330.61	-4667.22	-4679.74
(3,1)	-2309.46	-4632.92	-4662.12
(3,2)	-2273.52	-4569.04	-4614.93
(3,3)	-2263.19	-4556.38	-4618.96

Comb = lag combination; LogLik = value of the log-likelihood function; AIC = Akaike information criterion; SBIC = Schwartz Bayesian information criterion.

Bold entries stand for the minima of the corresponding information criterion: (1,2) is the optimal lag combination according to SBIC, while (1,3) is the optimal lag combination according to AIC.

Table 2. Testing linearity against logistic STAR dynamics

	LM_1		LM_3	
Delay	F-stat	p-value	F-stat	p-value
1	4.720	0.030	2.03	0.003
2	3.780	0.023	3.98	0.001
3	1.620	0.199	3.31	0.140
4	0.558	0.573	5.25	0.764
5	0.429	0.651	2.68	0.860
6	0.936	0.393	1.62	0.468

Linearity tests: 1st (LM1) and 3rd order (LM3) Taylor approximation

Table 3. Estimated parameters of linear CF model
(US macroeconomic monthly data, 1959:1-1998:12)
Log-likelihood: -2275.04

Parameter	Estimate	St. error	t-stat	p-value
γ_{INC}	0.927	0.074	12.5	0.0
γ_{IIP}	1.170	0.081	14.5	0.0
γ_{SLS}	0.785	0.060	13.2	0.0
ϕ	0.572	0.048	12.0	0.0
$\psi_{EMP.1}$	0.100	0.046	2.19	0.015
$\psi_{EMP.2}$	0.450	0.052	8.69	0.0
$\psi_{INC.1}$	-0.016	0.133	-0.119	0.453
$\psi_{INC.2}$	0.039	0.050	0.772	0.220
$\psi_{IIP.1}$	-0.079	0.087	-0.907	0.182
$\psi_{IIP.2}$	-0.089	0.070	-1.28	0.101
$\psi_{SLS.1}$	-0.424	0.052	-8.21	0.0
$\psi_{SLS.2}$	-0.211	0.050	-4.22	0.0
σ^2_{ε}	0.335	0.041	8.17	0.0
σ^2_{EMP}	0.316	0.031	10.2	0.0
σ^2_{INC}	0.567	0.044	12.8	0.0
σ^2_{IIP}	0.315	0.037	8.52	0.0
σ^2_{SLS}	0.554	0.042	13.4	0.0

Table 4. Estimated parameters of CF-LSTAR model with delay d=1
(US macroeconomic monthly data, 1959:1-1998:12)
Log-likelihood: -2235.17

Parameter	Estimate	St. error	t-stat	p-value
λ	3.244	1.516	2.14	0.016
r	-0.737	0.199	-3.70	0.0
$\mu(F_t = 1)$	0.066	0.036	1.85	0.033
$\mu(F_t = 0)$	-0.603	0.564	-1.07	0.143
γ_{INC}	0.898	0.069	12.99	0.0
γ_{IIP}	1.137	0.089	12.78	0.0
γ_{SLS}	0.765	0.061	12.58	0.0
$\phi(F_t = 1)$	0.402	0.075	5.39	0.0
$\phi(F_t = 0)$	0.320	0.308	1.04	0.150
$\psi_{EMP.1}$	0.099	0.047	2.12	0.017
$\psi_{EMP.2}$	0.463	0.052	8.85	0.0
$\psi_{INC.1}$	-0.022	0.053	-0.41	0.342
$\psi_{INC.2}$	0.039	0.053	0.73	0.234
$\psi_{IIP.1}$	-0.071	0.072	-0.99	0.161
$\psi_{IIP.2}$	-0.116	0.068	-1.71	0.044
$\psi_{SLS.1}$	-0.414	0.051	-8.17	0.0
$\psi_{SLS.2}$	-0.201	0.049	-4.10	0.0
$\sigma^2_{\varepsilon}(F_t = 1)$	0.209	0.033	6.35	0.0
$\sigma^2_{\varepsilon}(F_t = 1)$	1.521	0.528	2.88	0.002
σ^2_{EMP}	0.289	0.039	7.34	0.0
σ^2_{INC}	0.578	0.044	13.16	0.0
σ^2_{IIP}	0.320	0.043	7.52	0.0
σ^2_{SLS}	0.568	0.042	13.65	0.0

Table 5. Estimated parameters of CF-ESTAR model with delay d=1
(US macroeconomic monthly data, 1959:1-1998:12)
Log-likelihood: -2231.94

Parameter	Estimate	St. error	t-stat	p-value
λ	0.397	0.141	2.81	0.003
r	-1.645	0.215	-7.65	0.0
$\mu(F_t = 1)$	0.063	0.028	2.21	0.014
$\mu(F_t = 0)$	-0.293	0.360	-0.81	0.209
γ_{INC}	0.899	0.073	12.32	0.0
γ_{IIP}	1.257	0.081	15.43	0.0
γ_{SLS}	0.811	0.060	13.41	0.0
$\phi(F_t = 1)$	0.383	0.067	5.74	0.0
$\phi(F_t = 0)$	0.563	0.292	1.93	0.027
$\psi_{EMP.1}$	0.105	0.045	2.31	0.011
$\psi_{EMP.2}$	0.462	0.049	9.45	0.0
$\psi_{INC.1}$	0.007	0.028	0.25	0.401
$\psi_{INC.2}$	0.066	0.051	1.29	0.099
$\psi_{IIP.1}$	-0.132	0.084	-1.57	0.058
$\psi_{IIP.2}$	-0.163	0.076	-2.14	0.017
$\psi_{SLS.1}$	-0.400	0.050	-8.00	0.0
$\psi_{SLS.2}$	-0.197	0.048	-4.08	0.0
$\sigma^2_{\varepsilon}(F_t = 1)$	0.174	0.024	7.11	0.0
$\sigma^2_{\varepsilon}(F_t = 1)$	1.661	0.337	4.92	0.0
σ^2_{EMP}	0.326	0.031	10.67	0.0
σ^2_{INC}	0.613	0.045	13.67	0.0
σ^2_{IIP}	0.248	0.032	7.72	0.0
σ^2_{SLS}	0.570	0.041	13.83	0.0

Table 6. Estimated parameters of CF-MS model
(US macroeconomic monthly data, 1959:1-1998:12)
Log-likelihood: -2296.78

Parameter	Estimate	St.error	t-stat	p-value
p_{11}	0.976	0.010	101	0.0
$1 - p_{22}$	0.156	0.079	1.95	0.026
μ_1	0.143	0.039	3.62	0.080
μ_2	-0.904	0.161	-5.59	0.0
γ_{INC}	0.823	0.055	15.1	0.0
γ_{IIP}	0.950	0.057	16.5	0.0
γ_{SLS}	0.638	0.049	13.1	0.0
ϕ	0.407	0.066	6.18	0.0
ψ_{EMP}	-0.010	0.037	-0.20	0.421
ψ_{INC}	-0.049	0.054	-0.84	0.2
ψ_{IIP}	0.037	0.056	0.53	0.298
ψ_{SLS}	-0.311	0.047	-6.58	0.0
σ_ε^2	0.312	0.039	7.98	0.0
σ_{EMP}^2	0.320	0.035	9.09	0.0
σ_{INC}^2	0.539	0.041	13.0	0.0
σ_{IIP}^2	0.386	0.036	10.7	0.0
σ_{SLS}^2	0.631	0.046	13.8	0.0

Table 7. In-sample and out-of-sample performance of CF-MS and CF-STAR models

In-sample			
Model	QPS	DM	p-value
CF-LSTAR	0.0723	–	–
CF-ESTAR	0.0617	1.351	0.088
CF-MS:			
filtered	0.0611	0.942	0.173
smoothed	0.0228	3.951	0.0

QPS = quadratic probability score; DM = Diebold-Mariano statistic

Table 8. Out-of-sample forecasting performance of CF-MS and CF-STAR models
Forecasting sample 1980:1-1984:12

Model	QPS	DM	p-value	DM*	p-value
Forecasting horizon: 1 month					
CF-MS	0.182	–	–	–	–
CF-LSTAR	0.191	-0.217	0.414	-0.215	0.415
CF-ESTAR	0.125	2.86	0.002	2.84	0.003
Forecasting horizon: 2 months					
CF-MS	0.247	–	–	–	–
CF-LSTAR	0.243	0.085	0.466	0.083	0.467
CF-ESTAR	0.165	2.83	0.002	2.76	0.004
Forecasting horizon: 3 months					
CF-MS	0.280	–	–	–	–
CF-LSTAR	0.288	-0.162	0.436	-0.155	0.439
CF-ESTAR	0.204	1.40	0.081	1.34	0.093
Forecasting horizon: 4 months					
CF-MS	0.316	–	–	–	–
CF-LSTAR	0.324	-0.159	0.437	-0.150	0.441
CF-ESTAR	0.254	0.986	0.162	0.928	0.179
Forecasting horizon: 5 months					
CF-MS	0.341	–	–	–	–
CF-LSTAR	0.352	-0.251	0.401	-0.232	0.409
CF-ESTAR	0.282	1.10	0.136	1.02	0.157
Forecasting horizon: 6 months					
CF-MS	0.346	–	–	–	–
CF-LSTAR	0.370	-0.556	0.289	-0.505	0.308
CF-ESTAR	0.315	0.617	0.269	0.560	0.289

QPS = quadratic probability score; DM = Diebold-Mariano statistic; DM* = modified Diebold-Mariano statistic